THE UNIVERSITY OF MANITOBA

MEASUREMENT OF WALL SHEAR STRESS WITH HOT WIRE ANEMOMETER

by

Sheikh Burhanuddin

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ABSTRACT

Shear stress measurement at the wall in a boundary layer and in a diffuser flow with a hot wire anemometer is described. The effect of the proximity of the wall on hot wire measurements is discussed and a simple and quite satisfactory method of correction of wall proximity effect is suggested. The correction was originally developed in boundary layer flow (developed on the walls of a pipe) and compared with published data from similar flows. It was found satisfactory and in turn was extended to diffuser flow.

The Preston tube was used to measure shear stress at the wall in a diffuser flow using Patel's calibration. Furthermore, these measurements were corrected by using Frei & Thomann's correction which was developed by using a sealed floating element technique in an adverse pressure gradient. These results agree very well with the measurements made with a hot wire anemometer.

Some quantities, like boundary layer parameters, turbulence intensity, and skewness and flatness of u and $\partial u/\partial t$, were measured to specify the flow in which the effect of the wall proximity on hot wire readings was studied and corrected. The relationship between skewness and flatness of $\partial u/\partial t$ established by Van Atta & Antonia is also valid

in the flows investigated here.

The sub-layer next to the wall exists in all flows (i.e., with different types of pressure gradients) and the velocity in this region is linear. This layer extends to a value of Y^+ approximately equal to 5. Even in turbulent flow, $\tau_w = \mu|\partial\bar{U}/\partial Y|_w$ is valid. Now the characteristics of the sub-layer are,

(a) Intensity of turbulence is maximum at the edge of the sub-layer (i.e., at $Y^+ = 5$).

(b) The skewness of $\partial u/\partial t$ decreases in the sub-layer towards the wall.

(c) The flatness of $\partial u/\partial t$ goes very high towards the wall.

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NOMENCLATURE

$B(u)$	Probability density of u
C_f	Coefficient of friction, $2\left[\frac{U^*}{U_\infty}\right]^2$
d	Outer diameter of Preston Tube
D	Diameter of pipe
E	Voltage
$E(R, 0, \infty)$	The voltage with resistance R, velocity zero away from the wall
$E(R', 0, Y)$	The voltage with resistance R', velocity zero at distance Y
E_p	Energy peak
F	Flatness factor
H	Shape factor, δ_1/δ_2
L	Turbulent eddy length scale
m	Slope of a straight line
P_{st}	Static pressure measured with respect to atmosphere
R	Radius of pipe
r	Variable radius
Re	Reynolds Number, $Re_{av.} = \frac{U_{av}D}{\nu}$ $Re_\infty = \frac{U_\infty D}{\nu}$, $Re_\delta = \frac{U_\infty \delta}{\nu}$ $Re_{\delta_1} = \frac{U_\infty \delta_1}{\nu}$, $Re_{\delta_2} = \frac{U_\infty \delta_2}{\nu}$
S	Skewness factor
T	Time scale, $T = \int_0^T d(t)$
t	Time

U_{av}	Average velocity
U_{cl}	Centre line velocity
U_{∞}	Free stream velocity
$\bar{U}$	Time averaged local mean velocity
U^+	Non-dimensional velocity, $\frac{\bar{U}}{U^*}$
U^*	Friction velocity, $\left\|\frac{\tau_w}{\rho}\right\|^{\frac{1}{2}}$
U^+_{unc}	Non-dimensional uncorrected velocity, $\frac{\bar{U}_{unc}}{U^*}$
ΔU^+_{unc}	Deviation of U^+_{unc} from $U^+ = Y^+$ curve
$\tilde{u}(t)$	Instantaneous velocity
u	Longitudinal or tangential component of turbulent velocity
u'	Root mean square of u, $[\bar{u}^2]^{\frac{1}{2}}$
v	Normal component of turbulent velocity
Y	Distance from the wall
Y_{unc}	Uncorrected distance measured from the wall
Y^+	Non-dimensional distance, $\frac{YU^*}{\nu}$
Y^+_{unc}	Non-dimensional uncorrected distance, $\frac{Y_{unc}U^*}{\nu}$
ρ	Density of air
τ_w	Shear stress at the wall
ν	Kinematic viscosity, μ/ρ
μ	Dynamic visocisty
δ	Boundary layer thickness at 99% of U_{∞}
δ_o	Sub-layer thickness

δ_1	Displacement thickness
δ_2	Momentum thickness
Δ	Difference
ε	Energy loss occurring in the flow
$\partial/\partial t$	Derivative with respect to time

Subscripts

av	Average
cc	Contraction cone
∞	Away from the wall, free stream
cr	Critical
tur	Turbulent or eddy
w	Wall

Superscript

-	Time averaged

1.0 INTRODUCTION

In an adverse pressure gradient, no conclusive method has yet been outlined to measure the shear stress at the wall. One example of an adverse pressure gradient is diffuser flow. For presenting results of turbulent flow close to the wall in a diffuser, the wall velocity scale (i.e., friction velocity), is required. The friction velocity, U^*, can be obtained from the shear stress at the wall. By definition,

$$\tau_W = \mu|\partial\bar{U}/\partial Y|_W$$

since $\mu = \rho\nu$

$$\tau_W/\rho = \nu \; |\partial\bar{U}/\partial Y|_W$$

now $\tau_W/\rho = (U^*)^2$,

therefore

$$(U^*)^2 = \nu|\partial\bar{U}/\partial Y|_W \quad .$$

The shear stress at the wall is proportional to the velocity gradient at the wall. The velocity gradient at the wall is obtained by measuring the mean velocity at several positions close to the wall. The mean velocity can, of course, be measured with a Pitot tube but the sub-layer adjacent to the wall is so thin as compared to the dimensions of the Pitot tube that it is impossible to measure mean velocity with a Pitot tube at several positions within the sub-layer. Another difficulty in measuring mean velocity with a Pitot tube is the very small magnitude of dynamic pressure in the sub-layer which is hard to measure even with a precise micro-manometer. The aerodynamic effect is also a drawback. Therefore a hot wire anemometer was to be used to measure mean velocity close to the wall in the diffuser flow. The boundary layer type of hot wire probe can get very close to the wall. When the tips of the prongs of this probe are just at the wall (but not touching it), the distance of the hot wire from the wall is 0.06 mm (approximately). Therefore the mean velocity can be measured with the hot wire anemometer at several positions in the sub-layer.

Hot wire readings obtained very close to the wall must be corrected because the proximity of a solid boundary, having much higher thermal conductivity than the fluid, affects the heat loss from the hot wire. The boundary is effectively at ambient air temperature so additional heat is extracted from the fluid that is heated by the wire. Therefore, a higher than actual velocity is indicated by the anemometer. The effect increases rapidly as the wire approaches the wall. This effect is typically limited to within 0.5 mm from the wall. The wall conduction effect has been observed by many workers and some have attempted to correct hot wire readings in the proximity of the wall.

In 1924, Van Der Hegge Zijnen obtained a correction by measuring the total heat loss from the wire at a large distance from the wall, and at various closer distances from the wall, all the measurements being taken in still air. The extra heat loss so obtained was then used to correct experimental velocity measurements. The same method was used by Dryden (1936) and Weissberg (1956). Reighardt (1940) calibrated hot wires close to the wall in a laminar flow channel and used this calibration in a channel with a turbulent flow having the same value of wall shear stress. Wills (1962) and Ueda & Hinze (1975) used the same method. This method is limited to the measurement of turbulent flows whose friction velocities are within the range of the laminar flow friction velocities used for calibration.

Oka and Kostic (1972) and Zemkaya et al. (1979) formulated a method to find the friction velocity at the wall. This method assumes the existence of the linear law of velocity, i.e., $U^+ = Y^+$. According to this method, an uncorrected U^+ vs. Y^+ graph is plotted using U^* from some other approximate method and the Y^+_{unc} value corresponding to the minimum position in this graph is noted. Then the uncorrected $\bar{U}$ vs. Y graph is drawn and the Y_{unc} value corresponding to the minimum position in this graph is noted. Then U^* is calculated as follows:

$$Y^+_{unc} = \frac{Y_{unc} U^*}{\nu} ,$$

or

$$U^* = \frac{Y^+_{unc} \nu}{Y_{unc}} .$$

This method requires a friction velocity from some other method such as the Log Law. But once the shear stress is known and, since the velocity profile is linear and passes through the origin ($\bar{U} = 0$, $Y = 0$), it does not require any other method to obtain the true mean velocity near the wall.

Chauve (1977) used two methods of correction of hot wire measurements very close to the wall. The first method was similar to that used by Van Der Hegge Zijnen (1924) described earlier. According to the second method, in the absence of the flow, the resistance of the anemometer is adjusted for each distance Y from the wall such that the voltage remains equal to that far away from the wall. If $E(R, \bar{U}, Y)$ is the voltage E at resistance R, velocity $\bar{U}$ at distance Y then,

$$E(R, 0, \infty) = E(R', 0, Y)$$

In the presence of flow, the voltage measured corresponding to the adjusted resistance R' at a distance Y from the wall is equal to the voltage measured corresponding to resistance R for the same velocity at an infinite distance from the wall. This voltage is converted into velocity using the calibration curve. This method can introduce considerable errors in measurements because a probe calibration at all resistance settings is required.

The present method of correction is similar to that used by Van Der Hegge Zijnen (1924) except that the hot wire output is linearized. It consists of traversing a boundary layer type of probe through the region of wall proximity effect (about 1 mm thick) with and without flow. The voltage at each location from the wall for each condition is noted. The true velocity is obtained by subtracting no flow voltage from the flow voltage.

The correction should be developed in a flow in which the friction velocity is already known so that the corrected friction velocity could be compared with it before applying the correction to the diffuser flow. A boundary layer flow or a developing flow in a pipe (in which the wall region has the characteristics of a boundary layer) can be used for this purpose because the Log. Law is applicable in this flow, and the cross-plot method can be applied to the Log. Law for getting the friction velocity. The corrected friction velocity measurements obtained by Ueda and Hinze (1975) in a boundary layer flow can also be used for comparison in the same velocity (free stream) range.

Considering the available facilities, the correction would be developed in a developing pipe flow and after comparing the results with the available data, it would be applied to the diffuser flow.

In a diffuser flow the shear stress at the wall can also be measured with a Preston tube using Patel's calibration (1965). Recently Frei and Thomann developed a method for correcting Patel's calibration in an adverse pressure gradient. They used a sealed floating element technique for this purpose. The friction velocity obtained from corrected Preston tube measurements in a diffuser flow were to be used for comparing the corrected hot wire measurements in the same flow.

2.0 THEORETICAL CONSIDERATIONS

The central idea of the following discussion of boundary layer structure has been taken from Levich (1962).

The steady advance of a fluid in spearate layers is known as laminar flow. The unsteady chaotic motion in which the flow velocity fluctuates about some average value is known as turbulent flow. At a certain value of the Reynolds number (Re_{cr}), steady laminar flow gives way to distinctly unsteady, chaotic motion in which only on a time average is there net flow in a particular direction. The gross over all motion of a fluid is subject to infinitesimal disturbances. At $R_e < Re_{cr}$, disturbances that occur in the fluid are rapidly damped. At $Re > Re_{cr}$, disturbances are not damped, but rather reinforce each other and result in a chaotic regime having random eddies superposed on the basic motion. At $Re \gg Re_{cr}$ eddy velocities of extremely varied magnitude are superposed upon the motion of a fluid having a local mean velocity, say, $\bar{U}$. Turbulence eddies may be characterized by their velocities and by the distances over which these velocities change significantly. These distances are known as the scale of motion. Let $\Delta\bar{U}$ be the change in the average velocity over a distance equal to the scale of an eddy L. Thus for the example of turbulent motion in a tube, the largest scale L of turbulence eddies is equal to the diameter of the tube and the eddy velocities vary within the range of average velocity over that distance; i.e., they are of the order of the maximum value of the velocity at the centre of tube.

Such large scale eddies contain the main part of the kinetic energy of turbulent motion. Together with these large scale eddies, turbulent flow also includes eddies of smaller scales and lesser velocities. With a large quantity of small scale motion, there is a considerable dissipation of energy, which is transformed to heat. Small scale motions serve as a bridge by means of which the kinetic energy of large motions may be converted into thermal energy. Although turbulent motion occurs only at relatively high Reynolds numbers, it is accompanied by considerable dissipation of energy. From this standpoint it is possible to define a certain effective eddy viscosity, μ_{tur}, appropriate to turbulent flow. This eddy viscosity expresses energy losses occurring in the flow per unit time per unit volume by the equation

$$\varepsilon = \mu_{tur} (\Delta\bar{U}/L)^2 \quad . \qquad (1)$$

ε is not a function of the scale of the motion but is a characteristic constant for a given flow. In particular, for the largest scale motions, it equals the energy dissipated in the process of creating smaller scale motions. This process occurs at high Reynolds numbers and cannot be a function of the molecular viscosity μ of the fluid.

Therefore ε must be determined from the quantities characteristic of large scale turbulent motion. These include the velocity $\Delta\bar{U}$, the scale of motion L and density of the fluid ρ . These quantities can be combined into a single quantity with the same dimensions as ε .

$$\varepsilon \approx \rho \frac{(\Delta\bar{U})^3}{L} \quad . \tag{2}$$

There from (1) and (2)

$$\mu_{tur} \approx \rho \, \Delta\bar{U}L \quad . \tag{3}$$

Following the analogy between turbulent motion and random motion of gas molecules, the scale of motion L may be considered as the analog of the length of the mean free path, and the eddy velocity as the analog of the average velocity of the gas molecules. Therefore the approximation for the eddy viscosity can be written as $\Delta\bar{U} \approx L(\partial\bar{U}/\partial L)$. With the aid of μ_{tur}, the shear stress can be defined,

$$\tau \approx \mu_{tur} \, (\partial\bar{U}/\partial L) \approx \rho L^2 \, (\partial\bar{U}/\partial L)^2 = \alpha\rho L^2 \, (\partial\bar{U}/\partial L)^2 \tag{4}$$

where α is a constant.

Now consider turbulent flow past a flat plate of infinite extent downstream in the plane Y = 0. Let the mean flow be in the x-direction and the time average velocity be $\bar{U}$. The average velocity is, in general, a function of the distance of the fluid layer from the surface of the solid body, and thus $\bar{U} = \bar{U}(Y)$. The function $\bar{U}(Y)$ can be obtained by rewriting equation (4) in the form

$$\partial\bar{U}/\partial L = (\tau/\rho\alpha)^{\frac{1}{2}} (1/L) \quad . \tag{5}$$

In order to integrate the above equation it is necessary to determine the scale of the motion as a function of the distance Y separating the fluid layer from the solid surface. The conditions determining the flow field over a flat plate of infinite extent do not include the dimensions of the body which could be used to describe a characteristic scale of large turbulence eddies L. It is logical, therefore, to assume that

$$L \approx Y \quad . \tag{6}$$

Now equation (5) can be written as

$$\partial\bar{U}/\partial Y = 1/(\alpha)^{\frac{1}{2}} (\tau/\rho)^{\frac{1}{2}} (1/Y) \ .$$

Let $(\tau/\rho)^{\frac{1}{2}} = U^*$, the friction velocity, and introduce the dimensionless velocity $U^+ = \bar{U}/U^*$ and dimensionless distance $Y^+ = (YU^*/\nu)$ so that

$$dU^+/dY^+ = 1/(\alpha^{\frac{1}{2}}Y^+) \quad .$$

Integrating the above equation gives

$$U^+ = \ell_n Y^+/\alpha^{\frac{1}{2}} + C \quad . \tag{7}$$

The values of the constants α and C must be determined experimentally. Equation (7) is usually written as

$$U^+ = A\ \ell_n Y^+ + B \quad .$$

Various experimental measurements of velocity distribution show that the logarithmic relationship is valid only for $Y^+ \geq 30$. The upper limit of Y^+ depends on the Reynolds number.

The reduction in the scale of turbulence eddies as the wall is approached is matched by a corresponding reduction in the Reynolds number,

$$Re = (U^* L/\nu) \quad .$$

The friction velocity U^* is the velocity of turbulence eddies that are characteristic of the flow. At a certain $L = \delta_o$, Re is approximately equal to unity. In the region $Y < \delta_o$, known as the viscous sub-layer, the effects of viscosity are such as to give Newtonian flow. The thickness of the viscous sublayer is given by the condition

$(U^*\delta_o/\nu) \approx 1$. Two hypotheses regarding the velocity distribution in the viscous sub-layer have been suggested:

1) The Prandtl (1952) hypothesis, which has been accepted widely, states that in the region $Y < \delta_o$, the fluid motion is entirely laminar. Prandtl named this region the laminar sub-layer. The shear stress τ in the laminar sub-layer evidently may be expressed by the equation

$$\tau = \rho\nu(d\bar{U}/dY) \quad . \tag{8}$$

The velocity distribution can, therefore, be expressed by the linear equation

$$\bar{U} = (\tau/\rho\nu)Y + C \quad .$$

The integration constant must be equal to zero, since the velocity of the fluid at the solid surface is zero. Therefore, for $Y < \delta_o$,

$$\bar{U} = (\tau/\rho\nu)Y \quad .$$

$$\frac{\bar{U}}{(\tau/\rho)^{\frac{1}{2}}} = (\tau/\rho)^{\frac{1}{2}} (Y/\nu)$$

$$U^+ = Y^+ \quad . \tag{9}$$

Therefore, for the case of turbulent flow past a solid plane, the flow can be divided into three regions:

(a) a region of turbulent flow, (b) a buffer layer, and (c) a laminar sub-layer.

2) The other hypothesis presented by Landau and Levich (1959) states that the turbulent motion in the viscous sub-layer does not suddenly disappear but is gradually damped as it approaches the wall. The dependence of L on Y can no longer be applied on the basis of dimensional consideration, as is the case for the region of developed turbulence. All quantities in the viscous sub-layer may be functions of viscosity, and the distance from the wall is no longer the sole quantity with a linear dimension. The distribution of the average velocity in this layer has the same form as in laminar flow, i.e.,

$$U_x \sim Y$$

where U_x is the x-component of fluid velocity U. Although turbulence eddies do not originate in the viscous sub-layer, they enter it from the side $Y > \delta_o$. The eddy velocities have the same magnitude as the average velocities in the sub-layer. Therefore,

$$u \sim Y$$

In view of the continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

The normal component of eddy velocity is

$$v = - \int \frac{\partial u}{\partial x} dY \sim Y^2 \quad .$$

The proportionality coefficient in the expression for v can be evaluated using the condition that, at $Y \approx \delta_o$, the eddy velocity v at the boundary of the viscous sub-layer is of the same order of magnitude as the characteristic velocity of the turbulent flow U^*. Therefore,

$$v = (U^* \; Y^2/\delta_o{}^2) \quad .$$

Thus, in a viscous sub-layer the tangential and normal components of the average velocity and of the eddy velocities vary as a function of distance in the same way as the distribution of velocities in a laminar boundary layer. This, in essence, is the extent of the resemblance between a viscous sub-layer and a laminar boundary layer.

On the basis of the above discussion and experimental measurements, the turbulent boundary layer can be divided into three regions. In $Y^+ \leq 5$, region, the dimensionless velocity (U^+) is equal to the dimensionless distance (Y^+).

This region is called the viscous sub-layer. This layer is very thin and exists adjacent to the wall. In this region the shear stress contribution from turbulent friction may be neglected compared with molecular friction. The velocity gradient is steepest and constant in this layer. In the region $5 \leq Y^+ \leq 30$ the molecular and turbulent friction are of the same order of magnitude. This region is called the mixing or buffer layer. In the $Y^+ \geq 30$ region the molecular contribution is negligible compared with the turbulent friction. This region is called the turbulent core. A logarithmic curve [Equation (7)] fits the data well in this region. The complete universal profile is given by:

$Y^+ \leq 5$	$U^+ = Y^+$
$5 \leq Y^+ \leq 30$	Mixing
$Y^+ \geq 30$	$U^+ = A \ell_n Y^+ + B$.

There is a small amount of scatter in the values of the constants A and B in the Log. Law. Some experimentalists have suggested the following values of the constants A and B:

A	B	Investigator
2.44	4.9	Clauser (1956)
2.44	5.85	Townsend (1956)
2.5	5.1	Coles (1955)
2.7	4.5-6.0	Comte-Bellot (1965)

The suggestion of a possible slight dependence of A & B on Reynolds number has been made [Hinze (1962), Comte-Bellot (1965)] but it is not confirmed yet.

<u>Cross-Plot Method</u>:

This method can be used to calculate the friction velocity, U^*, from the Log. Law, and the measured velocity profile in the turbulent core region of the boundary layer.

According to this method, a value of Y^+ is assumed in the range where the Log. Law is applicable, say, $30 \le Y^+ \le 100$. By putting this value of Y^+ in the Log. Law we can calculate the corresponding value of U^+. Then the product of U^+ and Y^+ is calculated.

$$U^+Y^+ = \frac{\bar{U}}{U^*} \cdot \frac{YU^*}{\nu} = \frac{\bar{U}Y}{\nu} \quad .$$

So $\frac{\bar{U}Y}{\nu}$ = a numerical value. Now from the above equation $\bar{U}$ is plotted against Y on a linear graph. On the same graph the measured velocity profile is plotted and the value of $\bar{U}$ at the inter-section of the two curves is noted. This value of $\bar{U}$ is divided by the value of U^+ obtained in the first step. The resulting value is the friction velocity U^*. Several values of Y^+ in the applicable range of the Log. Law are selected and U^* is calculated in the manner describe above. These values should be very close if the velocity profile measurements are correct and Y^+ lies in the applicable range.

APPENDIX:

The boundary layer thickness, displacement thickness, momentum thickness, skewness factor and flatness factor have been defined in the appendix.