Three Essays on Stock Market Risk Estimation and Aggregation

by

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ABSTRACT

This dissertation consists of three essays. In the first essay, I estimate a high dimensional covariance matrix of returns for 88 individual stocks from the S&P 100 index, using daily return data for 1995-2005. This study applies the two-step estimator of the dynamic conditional correlation multivariate GARCH model, proposed by Engle (2002b) and Engle and Sheppard (2001) and applies variations of this model. This is the first study estimating variances and covariances of returns using a large number of individual stocks (e.g., Engle and Sheppard (2001) use data on various aggregate sub-indexes of stocks). This avoids errors in estimation of GARCH models with contemporaneous aggregation of stocks (e.g. Nijman and Sentana 1996; Komunjer 2001). Second, this is the first multivariate GARCH adopting a systematic general-to-specific approach to specification of lagged returns in the mean equation. Various alternatives to simple GARCH are considered in step one univariate estimation, and econometric results favour an asymmetric EGARCH extension of Engle and Sheppard's model.

In essay two, I aggregate a variance-covariance matrix of return risk (estimated using DCC-MVGARCH in essay one) to an aggregate index of return risk. This measure of risk is compared with the standard approach to measuring risk from a simple univariate GARCH model of aggregate returns. In principle the standard approach implies errors in estimation due to contemporaneous aggregation of stocks. The two measures are compared in terms of correlation and economic values: measures are not perfectly correlated, and the economic value for the improved estimate of risk as calculated here is substantial.

Essay three has three parts. The major part is an empirical study of the aggregate risk-return tradeoff for U.S. stocks using daily data. Recent research indicates that past risk-return studies suffer from inadequate sample size, and this suggests using daily rather than monthly data. Modeling dynamics/lags is critical in daily models, and apparently this is the first such study to model lags correctly using a general-to-specific approach. This is also the first risk-return study to apply Wu tests for possible problems of endogeneity/measurement error for the risk variable. Results indicate a statistically significant positive relation between expected returns and risk, as is predicted by capital asset pricing models.

Development of the Wu test leads naturally into a model relating aggregate risk of returns to economic variables from the risk-return study. This is the first such model to include lags in variables based on a general-to-specific methodology and to include covariances of such variables. I also derive coefficient links between such models and risk-return models, so in theory these models are more closely related than has been realized in past literature. Empirical results for the daily model are consistent with theory and indicate that the economic and financial variables explain a substantial part of variation in daily risk of returns.

The first section of this essay also investigates at a theoretical and empirical level several alternative index number approaches for aggregating multivariate risk over stocks. The empirical results indicate that these indexes are highly correlated for this data set, so only the simplest indexes are used in the remainder of the essay.

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DEDICATION

To my father

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CHAPTER ONE: GENERAL INTRODUCTION

This thesis presents three essays on the measurement of multivariate risk and applications with aggregate risk. The approach taken here is to measure multivariate risk of stock market returns directly at the level of individual stocks; using multivariate GARCH methods (essay one). These estimates of multivariate risk are to be converted to an aggregate measure of risk, and for this purpose two types of aggregation problems are addressed. The standard approach to measuring aggregate risk is to apply univariate GARCH to data on aggregate returns, but such contemporaneous aggregation implies errors in econometric model specification and estimation that have not been evaluated empirically. This thesis evaluates these errors for a particular data set (essay two). A second problem is how to aggregate estimates of multivariate risk (from essay one) into an aggregate measure of stock market risk. This thesis considers several alternative index approaches (beginning of essay three).

Given an aggregate measure of risk for stock market returns, this thesis then addresses two major areas of econometric application in finance: risk-return models and models explaining risk (essay three). Moreover it is shown that these two types of models are closely connected in theory.

The first essay estimates a variance-covariance matrix of returns for a large number of individual stocks. Central inquiries in finance such as portfolio diversification, risk management, and asset pricing require estimation of the covariance between asset returns as well as variances. There is substantial academic research on variance models of stock market returns. The univariate ARCH and GARCH models initiated by Engle and Bollerslev have shown success in modeling stock return volatilities. However, extension of GARCH models to multivariate estimation is more problematic; in particular one step maximum likelihood estimation procedures are not tractable for a large number of stocks.

Recently, Engle and Sheppard proposed a two-step procedure that substantially simplifies MGARCH estimation (Engle 2002b, Engle and Sheppard 2001). In this model, step one involves estimating the parameters of conditional variances using univariate GARCH models. Given the results in the first step, the next step estimates the correlation coefficients. Their MGARCH model assumes that correlation coefficients can vary over time, providing a dynamic conditional correlation MGARCH model (DCC-MGARCH).

This essay first estimates DCC-MGARCH models using daily holding period returns for 88 individual stocks (1995 - 2005). The 88 stocks are components of the S&P 100 index. In contrast, all other MGARCH studies use indexes of stock returns. A serious weakness of such studies is the errors in aggregation over stocks of GARCH models, as is discussed in the second essay. Another unique contribution of this study is the systematic specification of lags in returns in the mean equation. Other GARCH studies either assume no lags in returns or specify lags in an ad-hoc manner. This study adopts a general-to-specific approach to specifying the lag structure (e.g. Sargan 1980; Hendry and Richard 1982), leading to significant lags in returns for many models.

This essay also considers alternative specifications for step-one univariate models. There is strong support for AR lags in return and for an asymmetric relationship between stock return and volatility. Regarding specifications of correlation in step two, test results reject the constant correlation hypothesis under all specifications of step one, suggesting that a dynamic conditional correlation structure is very important in modeling covariances.

The second essay addresses the empirical importance of aggregation error in contemporaneous aggregation of GARCH models. The standard approach to measuring aggregate risk is to estimate univariate GARCH models of aggregate returns, but it is well known that in principle this leads to errors in model specification and estimation. These errors can be avoided by multivariate GARCH estimation of return data for individual stocks. This alternative approach to measuring aggregate risk is based on MGARCH estimates from essay one and simple procedures for aggregating the multivariate risk over stocks (analogous to value-weighted and Laspeyres). This is the first study to compare the two approaches empirically. The two measures of aggregate risk are not perfectly correlated (correlations approximating +0.8) and calculations suggest that investors would be willing to pay a substantial premium for the improved estimates of aggregate risk (approximately 4% of portfolio return for our data set between 1995 - 2005).

Essay three addresses two major areas of econometric research in finance: risk-return tradeoffs and relating stock market risk to economic fundamentals. This essay shows that daily risk-return models can address problems of inadequate sample size and that these two areas of research are closely connected.

Capital asset pricing models are central to theory in finance and many related empirical studies have attempted to estimate risk-return tradeoff, primarily at the aggregate level for a stock market. However empirical results have generally been poor, with relatively few studies estimating a significant positive tradeoff as implied by standard theory.

Recent literature has suggested that poor results may largely be due to inadequate sample size in previous studies (e.g. Lundblad 2007). One plausible response to this problem is to use daily data, rather than monthly or quarterly data as in most studies. However daily models require a systematic specification of dynamics/lag structures and this is missing in the literature.

This essay develops and estimates an aggregate risk-return model using daily return data on S&P 100 stocks over 1995 - 2005.This is the first risk-return tradeoff study to use a general to specific approach to specify lags, leading to a simple autoregressive distributed lag model ADL(2,1). The estimated tradeoff is positive and statistically significant, as predicted by the intertemporal CAPM model of Merton (1973, 1980). Results are insignificant using monthly data for the same time period. This is also the first risk-return study to conduct a specification test for endogeneity of risk. Within the framework of a Wu test, the null hypothesis of zero covariance between risk and disturbance is not rejected in better specified models.

Development of the Wu test leads naturally to a model relating aggregate risk of returns to economic variables from the risk-return model. This model is novel in several respects. As in the risk-return model, this is the first model explaining risk to include lags in variables based on a general-to-specific approach. In addition, this study includes covariances of economic variables and clarifies coefficient links with risk-return models. This is one of few models at the daily level rather than monthly or quarterly level. Empirical results are consistent with theory and indicate that the economic and financial variables explain a substantial part of variation in daily risk of returns.

Essay three first considers procedures for constructing aggregate measures of risk of returns from multivariate risk over stocks. At a theoretical level, a Fisher-type index of aggregate risk is slightly better than a Laspeyres or Paasche-type index (or a value-weighted index) of aggregate risk, but all are very highly correlated in our data set. The remainder of the essay simply uses a Laspeyres or value-weighted type index of aggregate risk of returns.

CHAPTER TWO: AN EMPIRICAL APPLICATION OF DYNAMIC MULTIVARIATE GARCH

Abstract

Estimates of variance and covariance of stock returns are essential to risk measurement and portfolio construction. This study estimates a high dimensional covariance matrix of returns for 88 individual stocks from the S&P 100 index, using daily return data for 1995 - 2005. This study applies the two-step estimator of the dynamic conditional correlation multivariate GARCH model, proposed by Engle (2002b) and Engle and Sheppard (2001) and applies variations of this model.

This study makes three contributions to empirical literature. To the best of our knowledge, this is the first study estimating variances and covariances of returns using a large number of individual stocks (e.g., Engle and Sheppard (2001) use data on various aggregate sub-indexes of stocks). This avoids loss of information in the process of risk estimation due to any aggregation of stocks (e.g. Nijman and Sentana 1996; Komunjer 2001). One recent study (Engle, Shephard and Sheppard, 2009) comes close to this by estimating multivariate GARCH models with large numbers of individual stocks, but this study also includes an aggregate index of stocks in the estimation which introduces the aggregation bias. Second, this is a first study adopting a systematic general-to-specific approach to specification of lagged returns in the mean equation (e.g. Sargan 1980; Hendry and Richard 1982). In contrast, other GARCH studies typically assume that expected returns are constant over time (e.g. Engle and Sheppard) or specify lags in returns in a non-systematic, ad-hoc manner. A third contribution to empirical multivariate

GARCH literature is that various alternatives to simple GARCH are considered in step one univariate estimation (Sheppard's thesis considered more specifications but with fewer stocks). Econometric results favour an asymmetric EGARCH extension of Engle and Sheppard's model (their model assumes simple GARCH (1, 1) in step one. Estimates of multivariate risk from this chapter are essential to the following chapters. Chapter three measures the empirical and economic significance of the aggregation problem related to multivariate risk and chapter four considers alternative index measures for aggregating multivariate risk.

Keywords: Variance-covariance Matrices of Large Dimension, Multivariate GARCH, Two-Step Estimation, Maximum Likelihood

2.1 Introduction

The main goal of this paper is to estimate a variance-covariance matrix of returns for a large number of individual stocks. Central inquiries in finance, (such as portfolio diversification, risk management, and asset pricing) require estimation of the covariance between asset returns as well as variances. However, one of the key challenges of empirical financial econometrics is the development of an easy to use, tractable, and theoretically valid method of estimating these covariances. The most popular methods for estimating large variance-covariance matrices used in the financial industry are relatively simple. Some examples include the historical rolling-window method and the RiskMetrics method. However, there are serious theoretical drawbacks to these methods (e.g. Engle 2002 b, 2004, Andersen, Bollerslev, Christoffersen and Diebold 2007).

There is substantial academic research on variance models of stock market returns. Univariate ARCH and GARCH models initiated by Engle and Bollerslev have shown success in modeling stock return volatilities. However, the extension of GARCH models to multivariate covariance estimation is still limited. Earlier research in the 1980s and 1990s extended univariate GARCH methods to multivariate covariance models, such as VECH (e.g. Bollerslev et al 1998), BEKK (e.g. Engle and Kroner, 1995), Factor-GARCH (e.g. Engle, Ng and Rothschild 1990b), and Orthogonal GARCH (e.g. Alexander and Chibumba, 1997). However, these methods use a one-step maximum likelihood estimation procedure which is not tractable for a large number of stocks. A one step procedure estimates variances and covariances jointly rather than in two steps in which covariances are estimated after variances. Estimating variances and covariances in a single step (guaranteeing positive semi-definiteness) requires nonlinear estimation with many parameters, and the number of parameters increases exponentially with the number of stocks. Thus, application of these MGARCH models has been limited to no more than five stocks.

Recently, Engle and Sheppard proposed a two-step procedure that substantially simplifies MGARCH estimation (Engle 2002b, Engle and Sheppard 2001). In this model, step one involves estimating the parameters of conditional variance using univariate GARCH models. Given the results in the first step, the next step estimates the correlation coefficients. Their MGARCH model assumes that correlation coefficients can vary over time, providing a dynamic conditional correlation MGARCH model (DCC-MGARCH). Engle and Sheppard (2001) have applied this method to estimate covariance of returns for 100 stock sector indices in the New York Stock Exchange and the components of Dow

Jones index. The constant correlation MGARCH model from Bollerslev (1990) is a special case of DCC-MGARCH. Indeed, a two-step approach makes CCC-MGARCH a very easy and tractable method for variance-covariance matrix estimation for a very large number of assets.

This chapter estimates DCC-MGARCH and CCC-MGARCH models using daily holding period returns for 88 individual stocks (1995 - 2005). The 88 stocks are components of the S&P-100 index. In contrast, all other MGARCH studies use indexes of stock returns. Almost all univariate GARCH studies use indexes of returns (exceptions include Kim and Kon 1994, Lamoureaux and Lastrapes 1990 a, b). A serious weakness of such studies is errors in aggregation over stocks of GARCH models, as is discussed in Chapter three.

Another unique contribution of this study is the systematic specification of autocorrelation (AR) lags in returns. Other GARCH studies either assume no lags in returns (constant expected returns, AR(0)) or specify lags in an ad-hoc manner. This study adopts a general-to-specific approach to specifying lag structure (e.g. Sargan 1980; Hendry and Richard 1982), leading to significant lags in returns for many models.

This study also considers alternative specifications for step-one univariate models. Engle and Sheppard (2001) assumed standard GARCH(1,1) for simplicity in step one. Sheppard in his thesis considered a broad variety of alternatives to GARCH(1,1). This study considers fewer alternatives applied to more stocks, including asymmetric volatility and GARCH-in-mean models. Our test results suggest that variance of returns can be excluded from the mean relationship, i.e. ARCH-in-mean models are rejected for individual stocks in most cases. There is strong support for AR lags in returns and for an asymmetric relationship between stock return and volatility. Test results indicate that an ARMA(2,1)-EGARCH(1,1) model is the preferred model among alternatives for univariate stage one models. Regarding specification of correlations in step two, test results reject the constant conditional correlation hypothesis under all specifications of step one. This test result suggests that a dynamic conditional correlation structure is very important in modeling co-variances.

The chapter proceeds as follows: Section 2 briefly reviews literature on high dimensional variance-covariance estimation. Section 3 reports summary statistics for the data set. Section 4 discusses return modeling, especially the general to specific approach for studying 88 individual stocks. Section 5 discusses estimation results using the two-step estimator for MGARCH models. The paper concludes with section six.

2.2 Literature Review

2.2. A. Introduction for Conditional Variance and Covariance Specification

The composition of the return of an asset includes both the expected return and the error term:

$$(1)\ r_{it} = E_{t-1}(r_{it}) + \varepsilon_{\mathrm{t}}$$

Where r_{it} denotes the return of asset i at time t, and ε_t denotes the error of the expectation. Estimation of the expected return is one of the primary and most difficult tasks in finance (e.g. Black 1993, Elton 1999). Expected return models include the Capital Asset Pricing Model (CAPM) and the Arbitrage Pricing Theory (APT). There is no consensus about the preferred model. Financial time series studies deviate from examining return using fundamentals, preferring to examine the statistical models of the data. An ARMA (p, q) process is a general time series model describing the data generating process of a stock return:

$$(2)\ r_{i,t} = c + \sum_{m=1}^{p} \alpha_m r_{i,t-m} + \sum_{n=1}^{q} \beta_n \varepsilon_{i,t-n} + \varepsilon_{i,t}$$

Expected return and risk are the two most important aspects of financial investment. Variances of returns are generally used as measures of risk in investment. GARCH models have been established as one of the best classes of econometric models to estimate variances. The empirical distributions of financial time series have three features invalidating the treatment of returns as independent identical normal random variables: fat tails, skewness and volatility clustering. Research has established that GARCH models successfully capture these stylized facts. GARCH(1, 1) is a milestone in this model family since it can capture long lags of error terms with a very parsimonious specification. A simple GARCH (p, q) process is as follows:

$$(3.1)\ h_{it} = E_{t-1}(\varepsilon_{it}^2)\text{, or alternatively, } \varepsilon_{it} = \sqrt{h_{it}}\, u_{it}$$

$$(3.2)\ h_{i,t} = c_h + \sum_{p=1}^{p} \alpha_{h,m}\varepsilon_{i,t-m}^2 + \sum_{n=1}^{q} \beta_{h,n} h_{i,t-n}$$

Where u_{it} is the standardized variance, distributed with mean $E(u_{it}) = 0$, and variance $Var(u_{it}) = 1$.

A major criticism of simple GARCH models is that they do not incorporate asymmetric volatility effects in stock market returns. A common observation is that stock market volatility goes up when the market price is going down and vice versa (e.g. Black

1976). Hence, many popular models have been developed to capture the asymmetric volatility effect, which includes EGARCH (exponential GARCH) developed by Nelson (1991), and GJR developed by Glosten, Jagannathan and Runkle (1993). Such models that capture asymmetric volatility effects are superior to standard GARCH models (e.g. Kim and Kon, 1994). Also, see Bollerslev, Chou, and Kroner (1992) for a detailed literature review on univariate GARCH models.

Since investors typically hold more than one asset, and returns between assets co-vary, it is important to measure covariances and correlations in returns. The conditional correlation coefficient between two assets is:

$$(4) \quad \rho_{ij,t} = \frac{E_{t-1}(\varepsilon_i \varepsilon_j)}{\sqrt{E_{t-1}(\varepsilon_i^2) E_{t-1}(\varepsilon_j^2)}}$$

Substituting for ε_{it} in equation (4) from equation (3.2) gives:

$$(4.1) \quad \rho_{ij,t} = \frac{E_{t-1}(\sqrt{h_{it}}\, u_{it} \sqrt{h_{jt}}\, u_{jt})}{\sqrt{E_{t-1}(h_{it} u_{it}^2) E_{t-1}(h_{jt} u_{jt}^2)}}$$

Since the h_{it}'s are predetermined, given information at t-1, the correlation reduces to

$$(4.2) \quad \rho_{ij,t} = \frac{E_{t-1}(u_{it} u_{jt})}{\sqrt{E_{t-1}(u_{it}^2) E_{t-1}(u_{jt}^2)}}$$

Then by definition, the conditional covariance is:

$$(5) \quad h_{ij,t} = \rho_{ij,t} \times \sqrt{h_{it}} \times \sqrt{h_{jt}}$$, where $\rho_{ij,t} \in [-1, +1]$, or in matrix form:

$$(5.1) \quad H_t = E_{t-1}(\Sigma_t{}'\Sigma_t)$$, where $\Sigma_t = H_t^{1/2} U_t$.

Corresponding to ε_{it} in the two-asset case, Σ_t is an n by one vector stochastic process.

Corresponding to u_{it} in the two-asset case, U_t is distributed i.i.d. ($E(U_t) = 0, \quad Var(U_t) = I$). The distribution of U_t is not necessarily normal, and H_t is a conditional variance-covariance matrix.

The traditional methods of estimating the covariance of returns, widely applied in industry, are rolling historical correlation and RiskMetrics. The rolling historical method gives equal weights to past observations. Engle (2002b) noted that there is no clear theoretical justification whether, or under what assumptions this method will provide consistent estimation of conditional correlation.

RiskMetrics was developed by J.P. Morgan researchers in 1994 (Jia, Miao, and Christian L. Dunis, 2005). It applies an exponential smoother to reduce the weight of observations from further back in time, but it imposes an extremely restrictive specification of weights. The RiskMetrics approach cannot capture the mean-reverting nature of volatility or covariance. Nevertheless, these have been the most popular approaches to estimation of covariance in practice due to the complexities of MGARCH.

2.2. B. Development of MGARCH Models

Bauwens, Laurent and Rombouts (2006) offer a comprehensive survey of MGARCH models. Silvennoinen and Timo (2008) offer another review on MGARCH models. Here we only discuss literature directly related to the model used in this empirical research, i.e. the DCC-MGARCH model.

The key difference between MGARCH models is the structure imposed on the variance-covariance dynamics. Kraft and Engle (1983) initiated the study of multivariate linear ARCH(q) immediately after the univariate ARCH model of Engle (1982). Analogously to the univariate model, they specify the conditional variance-covariance matrix as a linear function of contemporaneous product of past error terms. Just as Bollerslev (1986) generalized the univariate ARCH model into the GARCH model, Bollerslev, Engle and Wooldridge (1988) generalized multivariate linear ARCH(q) into an MGARCH (p, q) model.[1] It is often referred to as the VECH model in literature. The VECH model is the most general functional form among the existing models, since it allows the conditional variance-covariance matrix to be a function of past residuals, the cross products of past residuals, and its past variance-covariance matrix. That is to say, it allows the variance transmission or substitution between different assets. However, the number of parameters to be estimated is very large. For example, we need to estimate 6,075 parameters for a MGARCH(1,1) model of ten assets. It is also very difficult to ensure that the conditional variance-covariance matrix is positive semi-definite.

Bollerslev, Engle and Wooldridge (1988) imposed the restriction on the VECH model that the coefficient matrices are diagonal to reduce the number of parameters to be estimated. The imposition of a diagonal matrix form implies that there is no variance transmission between different assets. This is very restrictive, while the number of parameters to estimate when n is large is still daunting since it is a function of order n^2.

To overcome some of the VECH model's failures, and to ensure an estimated positive semi-definite conditional variance-covariance matrix, Engle and Kroner (1995) adopted a quadratic conditional variance-covariance matrix specification, now called the BEKK

[1] In MGARCH (p, q), q represents the number of lags of the matrix of error terms, and p the number of lags of the variance-covariance matrix.

model. This representation assures that the conditional variance-covariance matrix is positive semi-definite for any ε_t. The number of parameters in a BEKK (1, 1) model is $n\times((5n+1)/2)$ for n assets. The complexity of MGARCH models and the computational burden remains when the number of dimensions increases. Another drawback is that the estimated unconditional covariance matrices may not be positive semi-definite during the estimation. In addition, the interpretation of the coefficients is more difficult. Diagonal BEKK is another way to simplify the number of the parameters to be estimated.

The models mentioned above model the covariances directly. These models encounter two problems. First, the number of parameters increases dramatically when the number of assets increases. This is also difficult for one-step maximum likelihood estimation. Since the covariance matrix appears in the likelihood, it has to be inverted for every period for each iteration of the optimization process. When the number of assets and number of parameters increases, the calculation is very time consuming. Second, ensuring a positive semi-definite covariance matrix often leads to more parameters to estimate and difficulty in interpreting these parameters. Researchers start searching for models that are flexible to capture the variance and covariance dynamics, and are parsimonious and easy to interpret. Among many different models proposed (see Silvennoinen and Timo, 2008), one line of research models variance and covariance (correlation) separately instead of modeling covariance directly.

Bollerslev (1990) proposed an MGARCH model, which assumed constant conditional correlation among securities over time, henceforth CCC-MGARCH. This model is conceptually different, in the sense that it models conditional correlation instead of conditional covariance. Bollerslev (1990) defined the conditional variance-covariance matrix as: $H_t = D_t \Omega D_t$, where D_t is an n by n diagonal matrix with the only nonzero elements being the conditional standard deviations. i.e., D_t = diag $\sqrt{h_{it}}$, where $\sqrt{h_{it}}$ denotes the standard deviation of asset i. Ω denotes the constant conditional correlation coefficient matrix. Therefore, the conditional covariance is proportional to variances over time.

Although it is still a nonlinear estimator, the CCC-MGARCH (1, 1) contains n (n+5)/2 parameters, which is much less than previous models. Under MLE, the estimated correlation coefficient is the same as the sample correlation matrix of normalized residuals. The CCC-MGARCH model is popular among empirical researchers because of its computational simplicity. This model also assures a positive semi-definite conditional variance-covariance matrix. However, the CCC assumption can be very restrictive.

In line with the principle of finding a parsimonious model,[2] and inspired by the idea of modeling correlation instead of covariance, Engle (2002b), and Engle and Sheppard (2001) generalized CCC-MGARCH to DCC-MGARCH. In their model, the correlation matrix is not constant but follows a dynamic similar to the GARCH(1, 1) process. Assuming dynamic conditional correlation requires the correlation matrix to be inverted for each t during every iteration. Therefore, the computation burden is still a problem with one-step estimation. The most significant contribution of Engle (2002b), Engle and Sheppard (2001) is that they proposed a two-step estimation procedure for the conditional variance-covariance matrix: first variances are estimated by standard GARCH models,

[2] See Diebold, Francis X., 2004, The Nobel Memorial Prize for Robert F. Engle, Scand. J. of Economics 106, 165-185.

second correlations are estimated assuming common dynamic parameters across stocks that permit non-constant correlations. They show that their two-step estimators are consistent and asymptotically normal under standard assumptions. They demonstrate that the DCC-MGARCH model performs better than other correlation estimation methods by several test standards. By adding suitable restrictions on the conditional correlation matrix equation, DCC-MGARCH leads to a positive semi-definite variance-covariance matrix. They note that two step methods also further simplify estimation of CCC models.

Researchers have developed other MGARCH procedures to address the high dimensionality problem. Tse and Tsui (2002) independently develop a DCC-MGARCH model. Their model differs from Engle (2002b) in defining the conditional correlation matrix to follow an autoregressive moving average (ARMA) process. However, they did not suggest a two-step estimation method, so the methodology is more complex than Engle and Sheppard's method. Palandri (2005) extends the DCC model by breaking the conditional correlation matrix into a product of a sequence of matrices ensuring semi positive definiteness. This SCC (Sequential Conditional Correlation) model allows multiple steps in estimation of the variance-covariance matrices through a series of simple estimations. This allows more complex functional forms for a single asset. However, the number of estimated parameters in the SCC model can be very large and statistical properties of final estimated parameters are too complex to permit statistical inference.

Engle, Shephard and Sheppard (2009) have recently suggested a different modification of the DCC model for a very large number of assets. Rather than estimating a simple stage two quasi-likelihood functions defined directly over all correlations, they suggest defining a likelihood function for each pair of stocks, summing these likelihood functions over all pairs, and then maximizing this sum of likelihood functions for the common dynamic parameters. By summing likelihoods over pairs, there is no need to invert large dimensional covariance matrices, and biases in estimation are also reduced. These issues seem particularly important for DCC models with very large dimensions of stocks (e.g. 250). The study also estimates MGARCH models of returns for 95 components of S&P 100 and 480 components of S&P 500 (using daily return data for 1997 through 2006). However, the index itself is also included in MGARCH estimation, which introduces to some extent errors in GARCH estimation due to aggregation of stocks (e.g. Nijman and Sentana 1996; Komunjer 2001).

An obvious serious restriction of models related to DCC is the assumption of common dynamic parameters for correlations across all pairs of stocks. However, in the absence of restrictions on these parameters across stocks, there are too many parameters for estimating large dimensional models. Moreover, even if models can be computed, estimated covariance matrices are unlikely to be positive semi-definite.

Ledoit, Santa-Clara and Wolf (2003) suggest one approach to modeling differences in correlation dynamics. First, simple MGARCH models are estimated for each pair of stocks. Then estimated coefficient matrices are transformed to obtain a positive semi-definite covariance matrix, where the transformation is chosen to be least disruptive by some metric (here minimizing the Frobenius norm between the estimated and transformed/selected matrix). However, any choice of metric is somewhat arbitrary. This was the first feasible approach to estimating unrestricted correlation dynamics for more than five stocks.

Other papers have relaxed to some extent the DCC assumption of common correlation dynamics across all stocks. The most obvious approach is to assume a block–diagonal DCC model, where correlation dynamics are identical for stocks within a block but vary across multiple blocks of stocks (Billio, Caporin and Gobbo 2006). However, this approach is only appropriate for a small number of blocks. Another approach, supported by several empirical studies including this chapter, is to restrict one dynamic parameter (β) but not both to be identical across pairs of stocks. However, estimation with many stocks also requires additional restrictions on variation of the other dynamic correlation parameter (α) across stocks (Hafner and Fransces 2003). Engle and Kelly (2007) assume correlations change over time and are constant across the cross-section of stocks, but this equi-correlation model is too restrictive and cannot model diversity of correlations across stocks.

Later models, for example STCC-GARCH models (Silvennoinen and Terasvirta 2005), DSTCC-GARCH models (Silvennoinen and Terasvirta 2007), RDSC-GARCH models (Pellitier 2006) and Berbal and Jansen (2005) further generalize the correlation dynamics of the simple DCC models proposed by Engle. Pelletier (2006) proposed a regime-switching model. In his model, correlations are constant within each regime. Regimes are determined by an unobserved discrete state variable that follows a first-order Markov Chain. The changes between states are governed by transition probabilities. The correlation, however, changes abruptly between states. The STCC-GARCH (Smooth Transitional Conditional Correlation-GARCH) model extends Pelletier's model by allowing an observable state variable and the transition is smooth. DSTCC-GARCH allows two observable state variables and nests the Berbal and Jansen (2005) model.

Applications of these models are limited to a small number of assets so far. In addition, other studies encompass more features of the stock market into the correlation structure by extending the DCC-MGARCH model. For example, Cappiello, Engle and Sheppard (2004) incorporate asymmetric responses to innovations. Kasch-Haroutounian (2005) proposes a volatility-threshold DCC-MGARCH model. Kawakatsu (2006) extended the EGARCH model and proposed a matrix exponential MGARCH model to ensure positive semi-definitiveness of the variance-covariance matrix. Again, we can see that it is difficult to balance between flexibility of correlation structure and easy application to a large number of stocks.

2.3 Data

The ultimate goal of this thesis is to examine the risk structure of the overall stock market. Hence, we would like to choose stocks that are representative of the overall stock market. The S&P 100 index is composed of the largest 100 companies listed in the U.S stock market according to market capitalization. The index is often used as an approximation for overall market portfolio in financial research. This paper uses daily holding period returns of the components of the S&P 100 index. Components of the S&P 100 index are changing over time and are listed on the Standard and Poor's website. We considered stocks in S&P 100 index at the time of downloading data (August 6, 2006). The stock return data are extracted from the Centre for Research in Security Prices (CRSP) database, through the platform provided by CHASS at the University of Toronto. The data covers the period January 3, 1995 to December 30, 2005. Stocks that are not listed for the entire period are removed from the analysis leaving 88 stocks over 1995 -

2005 (eleven years of daily return data). See Appendix 2.C, Table 2.11 for the list of the 88 companies included in this study.

Holding period returns include both capital gains and dividend payouts. In contrast, to the best of our knowledge, most studies consider capital gains only since they mostly work with index data. However, investors may care about both dividend and capital returns. For example, GE stock has long been viewed as a dividend stock. For completeness, it is appropriate to include the dividend return risk in examining return variance covariance among stocks. Shiller (1981) showed that volatility of dividends is small relative to stock index return volatility. So this may justify ignoring dividends when the purpose is to estimate variance of stock returns. However, recent financial turmoil also shows that dividends can vary significantly at certain times (see Hauser 2011).

Table 2.1 reports the descriptive statistics of daily returns. The number of observations for each security is 2,771. Where the raw data downloaded miss observation in between data, a simple linear interpolation was used to replace the missing data. The statistics testing the null hypotheses of an independent and identically distributed normal distribution are reported in the table as well. The descriptive statistics are the mean, standard deviation, skewness, kurtosis, and Jaque-Bera test.

From the descriptive statistics, we can see some of the characteristics documented in previous research. Eighteen out of 88 stocks' returns are negatively skewed. The kurtosis statistics range from 4.4141 to 114.472. Kurtosis values are all greater than 3, which imply that the stock returns have a much lower concentration around the center of the distribution than normal. Since the companies we choose are among the liquid and largest companies, we expect that the kurtosis should not be too high. There are a few stocks with very high kurtosis. The stock with highest kurtosis is Williams COS (WMB), an energy company. The next highest kurtosis at 66.95 is RAYTHEON CO. (RTN), a defence system, defence and electronics company. Procter and Gamble (PG) has kurtosis 46.72, a business in consumer goods. The next highest at 31.54 is American Electric Power Co Inc. (AEP). Most others have kurtosis below 20. These numbers are not surprising. Engle (2004) examined daily return for the Standard and Poor 500 Composite index from 1963 to 2003. He found the kurtosis for the full sample period is "*a dramatic 41*". Kim and Kon's (1994) study on daily return data of 30 individual stocks in the Dow Jones Industrial Average found that excess kurtosis ranges in value from 3.32 to 73.7, for the period of 1962 to 1990. High kurtosis implies these companies' returns are riskier than in a normal distribution. Our results are also consistent with previous findings that fat tails are more prominent than skewness. The Jaque-bera test rejects the null hypothesis of a normal distribution for all stocks. The daily standard deviation is between 1.3689 to 4.0221 percent. The lowest daily return is -61.05%, and the highest daily return is 87.74%. These statistics imply substantial risk for non-diversified investors. The mean stock return is generally higher than the median, which is another way of describing the asymmetric nature of the stock returns. In sum, the summary statistics capture the well-documented character of stock returns: skewness and fat tails.

An important issue is the degree of autocorrelation of returns, i.e. the correlation of current return with lagged returns for a stock. Autocorrelation in aggregate indexes of daily returns has been well documented in research. Various studies relate autocorrelation in index returns with nonsynchronous trading of individual stocks (e.g. Fischer 1966; Scholes and Williams 1977; and Lo and MacKinlay 1990). However, previous studies conclude that only a small part of this autocorrelation is due to non-synchronous trading (e.g. Lo and MacKinlay 1990, Atchison et al 1987) and the remainder presumably reflects autocorrelation at the level of individual stocks. So it is important to examine the autocorrelation of individual stock returns as well.

We test the autocorrelation of stock returns on the 88 time series up to 36 orders. Table 2.2 reports the results of the test for the first order, second order, 20th order and 36th order autocorrelation. Except for five stocks, there are certain orders of autocorrelation detected in the sample return data using the Ljung-box test, with a significance level of up to 5%. There are 22 stocks with first order autocorrelation at the 5% significance level. One noticeable feature is that there are 51 stocks with second order autocorrelation at the 5% significance level. 77 of the stocks exhibit a negative second order autocorrelation. 58 stocks show 20th order autocorrelation, while 66 shows 36^{th} order autocorrelation. These test results show that it is important to model the autocorrelation of stock market returns in this study. The correlations among stocks range from a low of -0.01014 to a high of 0.737694.

2.4 Methodology

2.4. A. Mean and Variance Specification

As discussed before, a two-step estimation procedure is very flexible in modeling variances and covariances. However, a correct specification of mean and variance in the first step is essential to correct specification of covariances in the second step. Thus, this study first considers different univariate specifications for mean and variance in the first step.

The first decision is whether we should model stock returns or excess returns. Nelson (1991) modeled excess returns from risk-less rate series (proxy by Treasury bill returns) and value-weighted CRSP daily market returns, and the estimated parameters of the fitted variances are virtually identical. Therefore, this study models stock returns directly.

As mentioned in section 2.3, there are autocorrelations detected in the stock return data. In fact, most previous researchers have modeled the mean process using ad-hoc autocorrelation structure. Nelson (1991) modeled the index data with an AR(1) model. Kim and Kon (1994) modeled the mean process with an ARMA(2, 1) model for the individual stocks in Dow Jones Index. Apparently these lag specifications are quite arbitrary. Lo and Mackinlay (1988) noted that such simple models do not adequately explain the short-term autocorrelation behaviour of the market indices, and no fully satisfactory model exists yet.

Whereas previous studies have either ignored lagged returns in the mean equation or apparently specified these in an ad-hoc manner, this study adopts a general-to-specific approach to specification of lags. Given that lags in dynamic models can be long and are generally unknown a priori, this general-to-specific approach has become the standard approach to lag specification in time series econometrics (e.g. Hendry and Richard 1990; Hendry 1995). Here we follow a simplified version of this methodology that is common in spirit with studies mentioned above.[3] Since a long autoregressive (AR) process can in

[3] Use of various general-to-specific specification searches is common in dynamic econometrics. Hendry and others have further developed this into a specific "GETS" methodology involving more particular evaluation criteria (Hendry and Krolzig 2005; Krolzig and Hendry 2001). This methodology is less common in empirical research and is not adopted here. For a simple brief standard introduction to the rationale for general-to-specific specification searches, see e.g Greene (2010 pp 133-37, 676-77).

effect be shortened by augmentation with a moving average (MA) process (e.g. Greene 2008), we begin by assuming an ARMA(20,1) for all mean equations. This accommodates substantial lags in returns and presumably encompasses true lags. Then we conduct standard tests (e.g. F test or Wald test) to reduce AR lags to a more compact model (e.g. Sargan 1980; Davidson and Mackinnon 1993). For simplicity (and to avoid the possibility of over-fitting), we assume identical lag patterns across stocks. We start with an ARMA (20, 1) - EGARCH (1, 1)-M model defined as follows,

$$(6)\ r_{it} = c_i + \alpha_{i1} r_{i,t-1} + \alpha_{i2} r_{i,t-2} + \alpha_{i3} r_{i,t-3} + ... + \alpha_{i20} r_{i,t-20} + \beta_i \varepsilon_{i,t-1} + \gamma_i * h_{it} + \varepsilon_{\text{it}}$$

$$(6.1)\ \varepsilon_{it} = \sqrt{\text{h}_{\text{it}}}\ \text{u}_{\text{it}}$$

$$(6.2)\ \ln(h_{it}) = c_{ih} + \beta_{\text{ih}} \ln(h_{i,t-1}) + \alpha_{ih} |u_{i,t-1}| + \gamma_{ih} u_{i,t-1}$$

This model allows up to twenty lags in returns, which is longer than previous regression models. An EGARCH model captures volatility clustering and asymmetric volatility. An EGARCH model is less stringent on parameter constraints than a GARCH model and guarantees the variance term is positive (e.g. Jondeau et al 2008). I also include a variance term in the mean equation, allowing for a GARCH-in-mean model. There are several hypotheses that I will test and see whether the model can be reduced to a more parsimonious model.

Hypothesis 1: $\gamma_i = 0$

Although a risk-return tradeoff relation is often assumed for index return data, empirical test results are not clear (see Engle 2004). Based on the CAPM model, investors should only care about the systematic risk part of a stock and not the total risk of the stock. However, if investors are not holding the market portfolio, this may not be true. Investors might price the total risk of the individual stock; hence we will observe mean variance tradeoff for individual stock returns. We will test the null hypothesis with 88 stocks. Hypothesis 1 will not be rejected at the 0.05 significance level for most stocks (hence for simplicity we will assume $\gamma_i = 0$ for all stocks). These results are consistent with the CAPM model's prediction. Given hypothesis 1, the model reduces to an ARMA (20, 1) - EGARCH (1, 1):

(7) $r_{it} = c_i + \alpha_{i1} r_{i,t-1} + \alpha_{i2} r_{i,t-2} + \alpha_{i3} R_{i,t-3} + ... + \alpha_{i20} r_{i,t-20} + \beta_i \varepsilon_{i,t-1} + \varepsilon_{\text{it}}$

(7.1) $\varepsilon_{it} = \sqrt{\text{h}_{\text{it}}}\, u_{\text{it}}$

(7.2) $\ln(h_{it}) = c_{ih} + \beta_{\text{ih}} \ln(h_{i,t-1}) + \alpha_{ih} \left| u_{i,t-1} \right| + \gamma_{ih} u_{\text{i,t-1}}$

In the general-to-specific dynamic literature, typically models with long lags can be reduced to parsimonious models such as ADL(1,1). Conditional on hypothesis one, we test for an appropriate lag length in returns. We are particularly interested if ARMA(20,1) models can be reduced to parsimonious models such as ARMA(2,1), which is our second hypothesis. Similar test results held for intermediate lags.

Hypothesis 2: $\alpha_{i3} = ... = \alpha_{i20} = 0$

This hypothesis is not rejected at the 0.05 significance level for most stocks.

This leads to an ARMA (2, 1) - EGARCH (1, 1) model.

(8) $r_{i,t} = c_i + \alpha_{i1} r_{i,t-1} + \alpha_{i2} r_{it-2} + \beta_i \varepsilon_{i,t-1} + \varepsilon_{\text{it}}$

(8.1) $\varepsilon_{it} = \sqrt{\text{h}_{\text{it}}}\, \text{u}_{\text{it}}$

(8.2) $\ln(h_{it}) = c_{ih} + \beta_{\text{ih}} \ln(h_{i,t-1}) + \alpha_{ih} \left| u_{i,t-1} \right| + \gamma_{ih} u_{i,t-1}$

Then we test whether an ARMA(2,1) reduces to an ARMA(0,1) or ARMA(0,0).

Hypothesis 3: (a) $\alpha_{i1} = \alpha_{i2} = 0$ **and/or (b)** $\beta_i = 0$

An ARMA(0,0) would reduce the model to a simple EGARCH model,

(9) $r_{it} = c_i + \varepsilon_{it}$

(9.1) $\varepsilon_{it} = \sqrt{h_{it}}\, u_{it}$

(9.2) $\ln(h_{it}) = c_{ih} + \beta_{ih} \ln(h_{i,t-1}) + \alpha_{ih}|u_{i,t-1}| + \gamma_{ih} u_{i,t-1}$

However, hypothesis 3 is rejected at the 0.05 level for all stocks.

Ideally we should also take a general-to-specific approach in modeling volatility as well as lags in returns in the mean equation. However, it is difficult to find a model that nests a broad variety of GARCH models or more generally to find a broad encompassing model for volatility.[4] Here we will simply test for asymmetric volatility within an EGARCH framework.

Given an ARMA(2,1)-EGARCH(1,1) model, we test if the asymmetric effect drops out of the model:

Hypothesis 4: $\gamma_{ih} = 0$

This is a test for a symmetric rather than an asymmetric volatility effect. The asymmetric volatility hypothesis states a negative correlation between stock return volatility and stock returns (Black 1976). It implies that bad news decreases stock returns and increases volatilities. It would be interesting to examine asymmetric volatility at the firm level. If returns above expected returns (positive error ε) leads to a decrease in volatility, there is asymmetric volatility with $\gamma_{ih} < 0$. This phenomenon has long been

[4] Formal extensions of the general-to-specific approach to the modeling of volatility are just beginning (Bauwens and Sucarrat 2010).

observed and is explained in terms of leverage and volatility feedback effects (e.g. Wu 2001). The coefficient γ_{ih} is significantly negative at the 0.05 level for almost all stocks.

Nevertheless for comparison, we will also estimate GARCH (1, 1) models, which ignore asymmetric volatility. Due to the popularity of univariate GARCH(1,1) models, the extension of GARCH(1,1) to MGARCH has been of great interest (e.g. Andersen, Bollerslev, and Lange 1999 , Lee and Sltoglu 2001 , and Ledoit, Santa-Clara and Wolf 2003). A simple GARCH(1,1) model is

(10) $r_{it} = c_i + \varepsilon_{\text{it}}$

(10.1) $\varepsilon_{it} = \sqrt{\text{h}_{\text{it}}}\ \text{u}_{\text{it}}$

(10.2) $h_{it} = c_{ih} + \alpha_{i\text{h}}\varepsilon^2_{i,t-1} + \beta_{ih}h_{i,t-1}$

2.4. B. Conditional Correlations

The specification for the conditional covariance matrix follows the Engle (2002b) and Sheppard (2001) model. The conditional variance-covariance is related to correlation as

(11) $H_t = E_{t-1}(\Sigma'_t\, \Sigma_t)$

(11.1) $U_t = H_t^{\,-1/2}\ \Sigma_t$

(11.2) $H_t = D_t \Omega_t D_t$.

Where $D_t = \begin{bmatrix} h_{1,t}^{1/2} & 0 & \ldots & 0 \\ 0 & h_{2,t}^{1/2} & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & h_{n,t}^{1/2} \end{bmatrix}$.

The diagonal components are estimates from the first step. Ω_t denotes the dynamic conditional correlation matrix, i.e.

$$\Omega_t = \begin{bmatrix} 1 & \rho_{12,t} & \cdots & \rho_{1n,t} \\ \rho_{21,t} & 1 & \cdots & \rho_{2n,t} \\ \cdots & \cdots & \cdots & \cdots \\ \rho_{n1,t} & \rho_{n2,t} & \cdots & 1 \end{bmatrix}.$$

Here the elements of Ω_t are guaranteed to be in the interval [-1,+1]. Thus H_t is:

$$(11.3)\quad H_t = D_t \Omega_t D_t = \begin{bmatrix} h_{1,t} & \cdots & \cdots & \rho_{1n} \\ \rho_{21}\sqrt{h_{1t}h_{2t}} & h_{2t} & \cdots & \rho_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \rho_{n1}\sqrt{h_{1t}h_{nt}} & \rho_{n2}\sqrt{h_{nt}h_{2t}} & \cdots & h_{nt} \end{bmatrix}$$

The simplest dynamic structure of Engle's DCC estimator is DCC (1, 1), where the correlations obey the following process:

$$(11.4)\quad \Omega_t = (1-\alpha-\beta)\,\overline{\Omega} + \alpha(\mathrm{U}_{t-1}\mathrm{U}'_{t-1}) + \beta\Omega_{t-1},$$

where $\alpha \geq 0$, $\beta \geq 0$, $\alpha + \beta \leq 1$. Here β is a common parameter associated with the autocorrelations Ω_t and α is a common parameter for correlation innovations $U_{t-1}U'_{t-1}$ in $\alpha + \beta < 1$ implies a mean reverting model.

This specification ensures that the diagonals of the correlation matrix will be ones. This study considers a DCC (1, 1) process. $\overline{\Omega}$ is the unconditional covariance matrix estimated from the sample. This specification is analogous to GARCH (1, 1) for a univariate variance process. This is a very restrictive model for correlations since it restricts all the correlation processes to the same dynamics and also ignores asymmetry in correlations (i.e. correlation may go up when stock prices go down). However, this greatly reduces the number of parameters to estimate. IDCC and CCC models are nested in the simple DCC(1,1) model, so we test the following hypothesis

Hypothesis 5: $\alpha = 0$ and $\beta = 1$ or zero

This hypothesis implies a constant conditional correlation model.

Hypothesis 6: $\alpha + \beta = 1$

This implies an integrated model (IDCC).

2.5 Estimation and Hypothesis Test Results

2.5. A. Introduction

The two-step estimation procedure proposed by Engle and Sheppard is as follows. First, estimate univariate GARCH models for each individual stock using the method of maximum likelihood. The standardized residuals U_t from these regressions are the inputs into the second step calculating correlations. The standardized residuals are not orthogonalized. Details of the estimation procedure are in the appendix. For the CCC-MGARCH model, the estimated correlations are simply the correlations of the standardized residuals.

From equation 11.4, it is evident that only two parameters estimate, α and β, are required in the second step of this method. This two-step estimation retains the flexibility of estimating the variance of each variable and the interpretation is very straightforward. Two-step estimation makes it relatively easy to estimate a high dimensional variance covariance matrix. Sheppard's PhD thesis shows that estimators of the MGARCH (1, 1)-DCC (1, 1) model are consistent and asymptotically normal, with known variances under certain assumptions. However, it is not an efficient estimator similar to most multistep estimators.

As for each univariate estimate, the Maximum Likelihood (ML) estimator is consistent and asymptotically normal under sufficient regularity conditions, even when the assumption of a conditionally normal distribution is violated. However, Nelson (1991) and Weiss (1986) both mentioned it is very difficult to verify that these conditions will hold for ARCH-M, GARCH-M and EGARCH models. Nelson simply assumed that the ML estimator is consistent and asymptotically normal. For a model with such large number of parameters, this assumption may not be appropriate (Palandri 2005). Nevertheless, we maintain this assumption in statistical inference as in other studies.

2.5. B. Testing Hypotheses on the Mean Equation

Properties of GARCH(1,1)-DCC(1,1) model will generally hold for other univariate GARCH model specifications according to Sheppard, such as our model. The mean equation is initially specified as an ARMA(20,1)-EGARCH(1,1)-M and tests are done over all 88 stocks to simplify this model. Hypothesis 1 is that variance of returns can be excluded from the main equation, i.e. GARCH-in-mean can be excluded from model. For 71 of 88 stocks, this hypothesis is not rejected at the 5% level. Coefficients γ_i are generally positive but insignificant. This is similar to results from Kim and Kon (1994) on the thirty individual stocks in the Dow Jones index. Thus there is no strong support for including a variance term in the ARMA model of the mean (for most stocks).

Next we considered the number of lagged returns in the mean model. For the ARMA(20,1)-EGARCH(1,1) model, we did a Wald test of hypothesis 2 (i.e. whether ARMA(20,1) reduces to ARMA(2,1)). F statistics and chi square test results reject the null hypothesis for only 15 stocks at the 5% significance level, i.e. an ARMA(20,1) can be reduced to an ARMA(2,1) for 73 of 88 stocks. Similar test results held for intermediate lags. For most stocks, one or two lags in returns are sufficient for an ARMA process. To the best of my knowledge, this is the first study to adopt a systematic general-to-specific specification of lags in mean equation for GARCH models. Other models assume zero lags (constant mean) or apparently specify lags in an ad-hoc manner. For example Nelson (1991) and Akgiray (1989) use an AR (1) model. Schwert and Seguin (1990) use an AR(2), Kim and Kon (1994) use an ARMA(2,1) and French et al (1987) use an MA(1). All specifications appear to be ad-hoc.

Since different lag specifications influence residuals, they also influence estimated variances. To illustrate how different lag specifications can affect estimates of variances, Table 2.3 presents estimation results of the coefficients for EGARCH under the following lag specifications for 10 stocks: ARMA(20,1)-M, ARMA(20,1), ARMA(2,1), ARMA(1,1), ARMA(0,1) and ARMA(0,0). For example the estimated coefficients (α, β, γ) for Unix are (0.3373, 0.8466, -0.0965) respectively under ARMA(2,1), but they are (0.1444, 0.9660, -0.0946) under ARMA(0,1). For another stock, GD, the estimated coefficients (α, β, γ) are (0.2039, 0.9703, -0.0614) under ARMA (2,1) model, but they are (0.0642, 0.9967, -0.0261) under ARMA(0,1) model (differences are statistically different at 0.05 level). Here estimated coefficients α and γ vary substantially with lag length. We also calculate impacts of different lag lengths on estimated variance. We calculated the absolute value of differences in estimated variances for different lag lengths, divided by variances for the ARMA(2,1) model. For example, the mean ratio is 23% for stock GD. Therefore, different lag lengths can have a major impact on both estimated coefficients and estimated variances. Influences of lag lengths on estimated correlations are summarized in a later table (Table 2.5).

Next we estimate ARMA(2,1)-EGARCH models for individual stocks. The estimated coefficients for first lag in returns are significant at 0.05 level for 76 of the 88 stocks, and the coefficients for MA(1) are significant for 75 stocks. Hence in most cases we reject H3. The coefficients for second lag on returns show mixed results (only 18 of the coefficients are significant at 0.05 level). Then asymmetric volatility is tested for ARMA(2,1)-EGARCH(1,1) models. Coefficients of the asymmetric terms γ_{ih} in EGARCH models are negative and significant at 0.05 level for 75 of 88 stocks. Hence, it is important to model asymmetric volatility effects. There is a large literature to explain the causes and effects of asymmetric volatility see Bekaert and Wu (2000) for a comprehensive empirical study. Therefore, we reject null hypothesis 4 and maintain EGARCH. For completeness, an EGARCH(1,1) model without lags in returns (ARMA(0,0)) was also estimated.

2.5. C. Testing the Hypothesis of Constant Conditional Correlation without Estimating DCC Model

Before carrying out complex DCC-MGARCH estimation, we test whether correlations are constant. Engle and Sheppard (2002) proposed a procedure to test for correlation dynamics without estimating a full MGARCH model, and we will apply this test. Let Ω_t denote the conditional correlation matrix, the null hypothesis is H$_0$: $\Omega_t = \overline{\Omega}$, where $vech(\overline{\Omega})$ denote unconditional correlations. Assuming that we have a consistent estimate of the unconditional correlation, the alternative hypothesis is a vector auto regression, i.e.:

$$H_1: vech(\Omega_t) = C + \beta_1 vech(\Omega_{t-1}) + \beta_2 vech(\Omega_{t-2}) + \ldots + \beta_p vech(\Omega_{t-p})$$

First, we estimate a univariate GARCH process, then standardize the residuals, and estimate the correlation from the standardized residuals, $vech(\Omega_t) = vech(\hat{\varepsilon}_t \hat{\varepsilon}_t^{'})$. As seen in the above equation, the hypothesis test is a test of the autocorrelation structure in the correlations. The advantage of this test is that it can be applied to high dimensional data as in this study. Table 2.4 gives the CCC test results when the mean equation is specified as ARMA(2,1)-EGARCH(1,1), ARMA(0,0)-EGARCH(1,1) and ARMA(0,0)-GARCH(1,1) model.

Following Engle and Sheppard (2001), the tests on a different number of assets are conducted in an expanding manner. That is: the three assets tested include the first two assets; the four assets tested include the first three assets, and so on. None of the test results for the above models favors constant correlation. We do the test with all different models proposed in the paper, and the conclusions are the same.

Table 2.5 shows correlation estimates based on different lag specifications for five stocks. For example, consider the correlation between GM and ETR. Correlations are 0.0210 under ARMA(20,1), 0.0189 under ARMA(2,1) , and 0.0192 under ARMA(0,1). For the other four stocks, correlations are larger but differences are smaller between lag lengths.

2.5. D. Statistical Results for DCC Model with Simple Mean and Variance Model

We first discuss DCC estimation results for the simplest model: a GARCH(1,1) process and no ARMA process, i.e a GARCH(1,1)-DCC(1,1) model. This excludes lags in returns for the mean and asymmetric volatility. We apply the GARCH(1,1)-DCC(1,1) model to a different number of assets in an expanding manner, as in tests for constant correlation.

Table 2.6 summarizes the estimation results for α and β . The estimated coefficients α and β are reported for mean reverting DCC (1, 1). The range of α is 0.0016 to 0.018, and the range of β is 0.9657 to 0.99. Broadly similar results are reported by Engle and Sheppard (2001) and Engle, Shephard and Sheppard (2009). High β implies high persistency (slow decay) of correlation among stocks, similar to the high persistency of variances. High persistency implies most recent information forecast future correlations and long memory of the information (Kim and Kong 1994). Low α implies new information changes the correlation slightly (e.g. Engle 2004). α tends to decrease as the number of assets under estimation increases, while the value of β is quite stable. T-ratios for both α and β are very high, increasing with the number of assets included in the test. For this reason, I only report t-ratios for small number of stocks. Since the model imposes the same dynamic for all correlations, an increase in number of stocks in effect leads to an increase in imposed restrictions on DCC. Imposing restrictions (even if false) generally leads to a decrease in standard errors of coefficient estimates.[5]

It is of interest to investigate how α and β in DCC may vary across stocks, even though this procedure complicates estimation. Results can also indicate if the significance

[5] Regarding effects of imposed restrictions on standard errors of coefficients for standard one step estimators, this is most frequently shown in textbooks in the case of specification analysis (omitting a relevant variable in effect imposes a restriction that its coefficient is zero, leading to a reduction in standard error of the estimate). For e.g. see Greene 2008, pp. 133-5. Also see Davidson and Mackinnon (1993, pp. 94-96) and Aitchison and Silvey (1958).

of α and β in the standard DCC model is an artifact of imposing restrictions of identical coefficients α and β across pairs. We estimate separate pair-wise DCC correlations of six stocks using GARCH(1,1)_DC(1,1) model. The six stocks are Microsoft, General Electric, Exxon Mobile Corporation, Pfizer Inc. Wal-mart and Intel. Estimated standard errors of coefficients are averaged across stocks. Results are presented in Table 2.7. Results show that the estimated coefficients are statistically significant at 0.01 level even without imposing identical coefficients across stocks.

Estimated α varies from 0.0092 to 0.042, and estimated β varies from 0.92 to 0.99 across these pairs of stocks. In percentage terms, estimated α varies between different pairs of stocks more than does β. Another study (Hafner and Franses 2003) also concludes that parameters α appear to vary more over stocks than parameters β, using data on German and UK stocks. Somewhat similar results are in Engle and Sheppard (2007, Figure 1). This suggests that a model allowing for α (but not β) to vary across stocks might be of interest (See Hafner and Franses 2003).

Since the DCC assumption of common dynamic parameters across correlations appears to be wrong, an alternative MGARCH model was considered early in this study. Perhaps the most well-known MGARCH model allowing for dynamic parameters of correlation to vary across pairs of stocks is the Flex-GARCH approach of Ledoit, Santa-Clara and Wolf (2003). Here simple MGARCH models are estimated for each pair of stocks and then a Frobenius transformation is used to calculate a positive semi-definite covariance matrix. Using a computer algorithm provided by the authors, MGARCH models could be estimated for 25 stocks in the data set for this chapter. However, for 50 stocks the estimation did not converge. So the Flex-GARCH model is not used in this study.

Table 2.6 also reports likelihood ratio test (LR) results for CCC and IDCC models within the framework of estimating DCC models. Here CCC-MGARCH and Integrated MGARCH (i.e. IDCC are restricted models within a DCC-MGARCH framework. Standard LR tests can be applied.[6] Likelihood values for the IDCC and CCC models are compared with the likelihood of DCC. The CCC hypothesis 5 ($\alpha = 0$ and β =1 or zero) was rejected at the 0.01 level for all groups of stocks considered. The IDCC hypothesis 6($\alpha + \beta$ =1) was rejected at the 0.01 level for all groups of stocks considered.

2.5. E. Statistical Results for DCC Models with More General Mean and Variance Models

We also estimate DCC(1,1) models for all 88 stocks using more general mean and variance models. These DCC(1,1) models still impose identical α and β coefficient across all pairs of stocks. Correct specification of univariate models is important for precision of second step estimation of covariances. Stage two estimation results for coefficients are reported in Table 2.8 for the following EGARCH(1,1) models: ARMA(20,1)-EGARCH-M, ARMA(20,1)-EGARCH, ARMA(2,1)-EGARCH models and also EGARCH (ARMA(0,0), i.e. no lags in returns and disturbance). All coefficient estimates are statistically significant at the .01 level.[7]

[6] When maximizing a log-likelihood function, dropping variables leads to a smaller log-likelihood. The log-likelihood test statistic is twice the difference in the log-likelihoods. LR= 2(Lur - Lr), where Lur is the unrestricted log-likelihood, and Lr is the restricted log-likelihood. LR has an asymptotic chi-square distribution with q exclusion restrictions.

[7] Engle et al (2007) mentioned that allowing asymmetric volatility in modeling variances will not change the statistical properties of MGARCH models.

Except for the general model with GARCH-in-mean, estimates of β are similar. Reducing lagged returns from 20 to 2 has little effect on estimates of β but has a larger proportional effect on estimates of α. Thus, different lag lengths for returns in the mean equation have a significant impact on estimates of the coefficient α in models for conditional correlations.

2.5. F. Specification Tests on MGARCH Models

As a rule of thumb, if the residual terms are i.i.d., the model is probably a good fit (e.g. Engle and Sheppard 2002). Here we extend the above rule to test multivariate variance standardized residuals. We know that $U_t = H_t^{-1/2}\,\Sigma_t$. If H_t is the true variance-covariance matrix, then U_t should have i.i.d as in the univariate case. The Ljung-Box portmanteau test (1978) is a standard test for serial correlation of single time series. The multivariate form of the test was proposed by Hosking (1980). The test statistic is: H(p) = $T^2 \sum_{i=1}^{p} \frac{1}{T-i} Tr(\hat{C}_i \hat{C}_0^{-1} \hat{C}_i \hat{C}_0^{-1})$,where $\hat{C}_i = \frac{1}{T}\sum_{t=i+1}^{T}(U_t U_{t-i}^T)$ is the sample auto-covariance matrix of order i of U_t. Under the null hypothesis of no autocorrelation of the time series under investigation, the statistic H(p) is distributed as $\chi^2(n^2 p)$. P is the order of autocorrelation, and n is 88 in this study. Rejecting the null hypothesis implies at least one of the time series is not white noise.

Using the estimated variance covariance matrix for all 88 stocks under different models specified in the methodology section, we conducted the tests for all models up to 4 lags. Test results imply that the null hypothesis is rejected for all models. These results suggest that all models considered here are inadequate in this respect. Table 2.9 shows test results for different estimation methods.

2.6 Conclusions

Knowledge of risk is critical in making investment decisions. Variance and covariance among returns are often used as proxies for measuring portfolio risk in risk management. Estimation of return covariance among a large number of assets is very difficult. This chapter first reviews MGARCH models that estimate variances and covariances. Then the DCC-MGARCH model proposed by Engle and Sheppard (2001) is discussed. We apply this model to estimate variance and covariance of returns among 88 individual stocks which are components of the S&P 100 stocks, using daily return data for 1995 - 2005.

This study makes three contributions to the empirical literature. This is the first study estimating variances and covariances using data for a large number of individual stocks. As will be explained in the next chapter, this avoids any loss of information in risk estimation due to aggregation of stocks. One recent study (Engle, Shephard and Sheppard 2009) comes close to this by estimating models with large numbers of individual stocks, but this study also includes an aggregate index of stocks in the estimation. Second, this is the first study adopting a systematic general-to-specific approach to specification of lagged returns in mean equations for returns. In contrast, other GARCH studies assume that expected returns are constant over time or specify lags in a nonsystematic manner. Results in this study suggest that lags in returns are important in estimation of risk for most stocks. A third contribution to empirical multivariate GARCH literature is that various alternatives to simple GARCH are considered in step one univariate estimation (Sheppard's thesis considered more specifications but with fewer stocks). Results favour an asymmetric EGARCH extension of Engle and Sheppard's model.

This econometric study begins by assuming a general mean equation with long lags in returns. More specially, the step one regression model is specified as ARMA(20,1)-M-EGARCH(1,1). First, GARCH-in-mean is rejected. Then we test for a simpler lag structure in returns and conclude that a 2 period lag in returns seems adequate for most stocks, leading to an ARMA(2,1)-EGARCH(1,1) model. This is the final model for step one (GARCH (1, 1) is rejected, i.e. asymmetric volatility is important). In step two, correlations are dynamic conditional correlations modeled as DCC (1,1), and the constant conditional correlations (CCC) hypothesis is rejected. However, specification test results suggested problems with all models. One problem is that the DCC model imposes identical coefficients α and β across all stocks, but results here suggest that one coefficient α may vary across stocks. Estimates of the variance-covariance matrix of returns for all 88 stocks (daily over 1995 - 2005) are obtained for this model and for other models considered here.

The MGARCH estimates of return risk for 88 individual stocks obtained in this chapter are essential to the following chapters of this thesis. Chapter three combines the MGARCH estimates for 88 stocks into a measure of aggregate risk and compares this to a common aggregate measure of aggregate risk based on a univariate GARCH model of aggregate returns. This aggregate approach is inferior in theory, and empirical results in chapter three indicate that it is also inferior in practice. Thus, MGARCH estimates of return risk for individual stocks are essential to proper measures of aggregate risk.

Chapter four considers several alternative (index number) approaches to combining these MGARCH estimates into an aggregate measure of stock market risk, and it uses these measures in econometric models of the aggregate risk-return tradeoffs. In contrast, previous regressions studies of aggregate risk-return tradeoffs have employed aggregate measures of aggregate risk.

Table 2.1. Descriptive Statistics

Table 2.1 reports descriptive statistics of daily holding period returns for the 88 stocks used in this study. Each stock is coded with its orginal column number from the full S&P 100 stocks as listed in CRSP (The column numbers here are not continuous since 12 stocks are deleted from the data set). The 88 stocks are the components of S&P 100 index as of Aug. 6, 2006. The total numbers of observations is 2,771. The data spans the period from Jan. 1995 to Dec. 30, 2005. See Appendix for a list of companies' names.

STOCKS	Mean	Median	Maximum	Minimum	Std. Dev.	Skewness	Kurtosis	Jarque-Bera	p_value
COL1	0.0013	0.0000	0.3109	-0.2915	0.0354	0.3189	10.2452	6107.73	0.00
COL2	0.0010	0.0000	0.1957	-0.1560	0.0228	0.1342	7.9573	2845.63	0.00
COL3	0.0007	0.0000	0.2822	-0.1778	0.0228	0.1916	17.1549	23150.44	0.00
COL4	0.0013	0.0000	0.2486	-0.2807	0.0366	0.0385	7.6399	2486.35	0.00
COL5	0.0005	0.0000	0.3240	-0.3757	0.0340	-0.4489	20.1840	34186.59	0.00
COL6	0.0019	0.0010	0.2076	-0.1894	0.0318	0.1212	6.1105	1123.90	0.00
COL7	0.0004	0.0000	0.0965	-0.1048	0.0165	0.0084	6.8368	1699.67	0.00
COL8	0.0005	0.0000	0.0987	-0.1104	0.0187	0.1395	5.7537	884.48	0.00
COL9	0.0001	0.0000	0.1271	-0.2479	0.0205	-0.9759	18.0033	26429.45	0.00
COL10	0.0007	0.0008	0.1105	-0.0846	0.0153	0.1338	5.8294	932.54	0.00
COL11	0.0009	0.0000	0.0912	-0.1238	0.0171	-0.1116	7.2195	2061.43	0.00
COL12	0.0008	0.0000	0.1246	-0.1067	0.0181	0.1839	6.6523	1555.74	0.00
COL13	0.0002	-0.0002	0.1811	-0.1397	0.0209	0.2147	7.6399	2507.00	0.00
COL14	0.0008	0.0003	0.1316	-0.1554	0.0211	0.2648	9.1992	4469.43	0.00
COL15	0.0007	0.0000	0.1614	-0.1116	0.0176	0.5993	10.6064	6845.99	0.00
COL16	0.0009	0.0006	0.1627	-0.1386	0.0205	-0.0075	10.2414	6054.38	0.00
COL17	0.0013	0.0000	0.1510	-0.1341	0.0256	0.3455	6.2183	1250.94	0.00
COL18	0.0010	0.0000	0.1034	-0.1219	0.0224	0.1952	4.8829	426.92	0.00
COL19	0.0006	0.0001	0.0946	-0.0659	0.0151	0.1395	4.4141	239.85	0.00
COL21	0.0007	0.0000	0.1410	-0.2649	0.0275	-0.5174	10.6617	6901.21	0.00
COL22	0.0014	0.0000	0.2407	-0.1822	0.0338	0.4138	5.3168	698.81	0.00
COL23	0.0010	0.0005	0.0983	-0.2825	0.0188	-1.2149	23.3500	48495.67	0.00
COL24	0.0007	0.0004	0.0952	-0.3138	0.0171	-2.3283	46.7204	223199.00	0.00
COL25	0.0007	0.0000	0.0918	-0.0847	0.0137	0.1717	6.5187	1443.11	0.00
COL26	0.0009	0.0000	0.1084	-0.1216	0.0208	0.1137	5.4019	672.05	0.00
COL27	0.0007	0.0000	0.2032	-0.1546	0.0176	0.3036	14.3611	14945.40	0.00
COL28	0.0006	0.0000	0.1464	-0.2241	0.0200	-0.5231	14.5652	15569.43	0.00
COL29	0.0007	0.0000	0.1163	-0.1763	0.0209	-0.3397	9.0574	4289.65	0.00
COL30	0.0008	0.0000	0.1377	-0.1595	0.0209	0.4041	8.1350	3119.88	0.00
COL31	0.0006	0.0000	0.1247	-0.1614	0.0179	-0.0990	8.0414	2938.93	0.00
COL32	0.0006	0.0000	0.1139	-0.1058	0.0191	0.2361	6.6888	1596.82	0.00
COL33	0.0003	0.0000	0.1189	-0.1046	0.0199	0.3572	5.7071	905.03	0.00
COL34	0.0008	0.0000	0.0971	-0.1115	0.0197	-0.0520	5.2264	573.54	0.00
COL35	0.0008	0.0000	0.0821	-0.1585	0.0156	-0.2276	9.0446	4242.49	0.00
COL36	0.0007	0.0000	0.1107	-0.0959	0.0160	0.3445	6.1304	1186.24	0.00

Table 2.1. Descriptive Statistics (Continued)									
STOCKS	Mean	Median	Maximum	Minimum	Std. Dev.	Skewness	Kurtosis	Jarque-Bera	p_value
COL37	0.0005	0.0004	0.1303	-0.2678	0.0188	-1.0068	20.8219	37139.99	0.00
COL38	0.0004	0.0000	0.1410	-0.0983	0.0163	0.3793	8.5362	3605.13	0.00
COL39	0.0004	0.0000	0.1087	-0.0825	0.0146	0.3232	6.5575	1509.43	0.00
COL40	0.0011	0.0000	0.2435	-0.4245	0.0296	-0.4875	22.6607	44739.41	0.00
COL41	0.0008	0.0000	0.1060	-0.1810	0.0155	-0.5335	14.8312	16292.96	0.00
COL42	0.0011	0.0000	0.1468	-0.1650	0.0252	0.0830	6.6875	1573.18	0.00
COL43	0.0005	0.0000	0.1984	-0.2278	0.0164	-0.0205	31.5437	94068.89	0.00
COL44	0.0007	0.0000	0.1406	-0.1101	0.0223	0.3691	5.7982	966.94	0.00
COL45	0.0006	0.0000	0.2676	-0.4360	0.0227	-2.2960	66.9543	474677.10	0.00
COL47	0.0004	0.0000	0.1429	-0.1316	0.0170	0.3683	9.6775	5210.84	0.00
COL49	0.0005	0.0000	0.1526	-0.1836	0.0214	0.0552	9.8068	5350.90	0.00
COL50	0.0009	0.0000	0.2091	-0.1870	0.0280	0.2102	8.0921	3014.15	0.00
COL51	0.0007	0.0003	0.1091	-0.2628	0.0197	-1.3958	21.8241	41811.97	0.00
COL52	0.0007	0.0000	0.3906	-0.2575	0.0320	0.3493	20.8579	36876.57	0.00
COL53	0.0014	0.0000	0.8774	-0.6105	0.0402	2.3124	114.4720	1437155.00	0.00
COL54	0.0009	0.0000	0.1000	-0.0878	0.0169	0.2423	5.9307	1018.76	0.00
COL55	0.0007	0.0000	0.2071	-0.2167	0.0247	-0.0601	10.4488	6407.83	0.00
COL56	0.0006	0.0000	0.1175	-0.1194	0.0196	0.2856	5.4897	753.35	0.00
COL57	0.0006	0.0000	0.1809	-0.3956	0.0256	-1.2329	27.6501	70857.38	0.00
COL58	0.0009	0.0000	0.1919	-0.2767	0.0215	-0.2616	19.4805	31390.95	0.00
COL60	0.0004	0.0000	0.1559	-0.1152	0.0227	0.5118	6.7249	1722.94	0.00
COL61	0.0009	0.0000	0.1964	-0.1647	0.0285	0.5685	8.1391	3198.57	0.00
COL62	0.0006	0.0000	0.1086	-0.1282	0.0182	0.1149	7.0345	1885.41	0.00
COL63	0.0011	0.0000	0.4582	-0.3082	0.0294	0.6690	44.7943	201884.90	0.00
COL64	0.0009	0.0000	0.1604	-0.1811	0.0228	0.3594	8.4082	3436.61	0.00
COL65	0.0011	0.0000	0.1497	-0.1636	0.0232	0.2170	7.4296	2287.20	0.00
COL67	0.0008	0.0000	0.1143	-0.1256	0.0179	0.1282	6.8511	1719.97	0.00
COL68	0.0012	0.0000	0.2094	-0.3435	0.0389	0.0636	7.1755	2014.90	0.00
COL69	0.0012	0.0000	0.1508	-0.1157	0.0251	0.2996	5.5789	809.30	0.00
COL70	0.0008	0.0000	0.0943	-0.0975	0.0199	0.2534	5.5050	754.15	0.00
COL71	0.0010	0.0000	0.1277	-0.1360	0.0213	0.1084	6.0425	1074.20	0.00
COL72	0.0012	0.0003	0.2012	-0.2203	0.0291	-0.1063	7.5903	2438.04	0.00
COL73	0.0009	0.0006	0.0880	-0.1089	0.0192	-0.0729	5.9760	1025.01	0.00
COL74	0.0011	0.0006	0.1190	-0.1169	0.0202	0.1981	6.4092	1360.04	0.00
COL75	0.0009	0.0010	0.1893	-0.3807	0.0207	-2.1705	50.7837	265800.00	0.00
COL76	0.0008	0.0000	0.1422	-0.1306	0.0231	0.1940	6.4607	1400.14	0.00
COL77	0.0007	0.0000	0.1541	-0.1049	0.0208	0.3411	6.7965	1717.89	0.00
COL78	0.0004	0.0000	0.1226	-0.1185	0.0186	0.2563	6.7347	1640.75	0.00
COL79	0.0005	0.0000	0.0924	-0.1266	0.0195	0.0676	5.8273	925.02	0.00
COL80	0.0008	0.0005	0.1532	-0.2776	0.0205	-0.7576	20.8778	37167.55	0.00
COL81	0.0008	0.0000	0.1289	-0.2874	0.0228	-0.6148	14.8237	16315.75	0.00
COL82	0.0008	0.0000	0.1103	-0.1043	0.0186	0.2254	6.1305	1154.96	0.00
COL83	0.0012	0.0000	0.1834	-0.1573	0.0219	0.2579	8.1677	3114.08	0.00

Table 2.1. Descriptive Statistics (Continued)									
STOCKS	Mean	Median	Maximum	Minimum	Std. Dev.	Skewness	Kurtosis	Jarque-Bera	p_value
COL84	0.0009	0.0000	0.1862	-0.1444	0.0263	0.3170	6.2461	1263.00	0.00
COL85	0.0014	0.0012	0.2439	-0.1353	0.0314	0.4168	7.9471	2905.89	0.00
COL86	0.0008	0.0000	0.1596	-0.1718	0.0240	0.1427	7.4541	2299.99	0.00
COL87	0.0006	0.0000	0.1382	-0.1988	0.0215	0.0094	9.9395	5560.14	0.00
COL88	0.0024	0.0000	0.2707	-0.2654	0.0382	0.4780	8.2015	3229.29	0.00
COL89	0.0014	0.0000	0.4103	-0.4948	0.0400	0.0683	28.2574	73657.43	0.00
COL90	0.0018	0.0000	0.1795	-0.1714	0.0353	0.2966	5.8378	970.41	0.00
COL91	0.0006	0.0000	0.3302	-0.3565	0.0311	-0.5522	29.9513	84006.41	0.00
COL92	0.0008	0.0000	0.1043	-0.1262	0.0191	-0.1655	7.4678	2317.32	0.00
COL93	0.0016	0.0000	0.1929	-0.1857	0.0280	0.5105	8.3539	3429.92	0.00
		Min	0.0821	-0.6105	0.0137	-2.3283	4.4141		
		Max	0.8774	-0.0659	0.0402	2.3124	114.4720		

Table 2.2. Autocorrelation Test Results												
Table 2.2 presents autocorrelation test results for each individual stock return. The order of autocorrelation tested is up order 36. Only statistical results for AR(1), AR(2), AR(20) and AR(36) are reported here due to space constraints. AR is autoregression. Q-stat is Ljung-Box(1973) statistic with a Chi-square distribution with q degrees of freedom. Q is number of lags. The null hypothesis is the time series is white noise.												
	COL1			COL2			COL3			COL4		
lags	AC	Q-Stat	Prob	AC	Q-Stat	Prob	AC	Q-Stat	Prob	AC	Q-Stat	Prob
1	-0.048	6.430	0.011	-0.014	0.545	0.460	0.020	1.064	0.302	0.012	0.380	0.537
2	-0.061	16.601	0.000	-0.015	1.176	0.555	-0.018	1.981	0.371	-0.062	11.219	0.004
20	-0.003	35.185	0.019	0.011	22.890	0.294	0.019	12.371	0.903	-0.012	61.503	0.000
36	0.023	54.587	0.024	0.018	59.127	0.009	-0.051	37.192	0.414	-0.030	79.978	0.000
	COL5			COL6			COL7			COL8		
lags	AC	Q-Stat	Prob	AC	Q-Stat	Prob	AC	Q-Stat	Prob	AC	Q-Stat	Prob
1	-0.040	4.547	0.033	0.008	0.177	0.674	0.032	2.899	0.089	0.003	0.023	0.879
2	-0.018	5.415	0.067	-0.061	10.397	0.006	-0.038	6.936	0.031	-0.061	10.378	0.006
20	-0.020	32.090	0.042	-0.025	34.895	0.021	-0.012	32.886	0.035	0.013	21.516	0.367
36	0.001	47.140	0.101	-0.001	49.402	0.068	0.032	47.575	0.094	-0.001	35.317	0.501
	COL9			COL10			COL11			COL12		
lags	AC	Q-Stat	Prob	AC	Q-Stat	Prob	AC	Q-Stat	Prob	AC	Q-Stat	Prob
1	0.041	4.727	0.030	-0.056	8.554	0.003	-0.016	0.699	0.403	0.000	0.000	0.984
2	0.023	6.244	0.044	-0.062	19.094	0.000	-0.030	3.175	0.204	-0.053	7.848	0.020
20	-0.022	22.871	0.295	-0.006	51.841	0.000	0.020	27.511	0.121	-0.023	27.747	0.116
36	-0.004	38.344	0.364	0.008	79.668	0.000	-0.018	43.353	0.186	-0.030	55.655	0.019
	COL13			COL14			COL15			COL16		
lags	AC	Q-Stat	Prob	AC	Q-Stat	Prob	AC	Q-Stat	Prob	AC	Q-Stat	Prob
1	-0.044	5.338	0.021	-0.030	2.553	0.110	-0.065	11.757	0.001	-0.024	1.542	0.214
2	-0.022	6.724	0.035	-0.013	2.997	0.223	-0.037	15.629	0.000	-0.028	3.721	0.156
20	-0.008	24.042	0.241	0.011	29.770	0.074	-0.013	51.516	0.000	-0.018	37.279	0.011
36	-0.013	44.512	0.156	-0.023	60.930	0.006	-0.011	73.707	0.000	-0.017	61.570	0.005
	COL17			COL18			COL19			COL21		
lags	AC	Q-Stat	Prob	AC	Q-Stat	Prob	AC	Q-Stat	Prob	AC	Q-Stat	Prob
1	-0.033	3.102	0.078	0.012	0.370	0.543	-0.048	6.439	0.011	0.040	4.331	0.037
2	-0.069	16.449	0.000	-0.082	19.240	0.000	-0.065	18.119	0.000	0.002	4.341	0.114
20	0.014	62.982	0.000	0.001	59.585	0.000	0.014	32.788	0.036	-0.006	28.068	0.108
36	0.008	77.088	0.000	-0.028	80.533	0.000	-0.007	57.243	0.014	0.000	54.641	0.024

Table 2.2. Autocorrelation Test Results (Continued)												
	COL22			COL23			COL24			COL25		
lags	AC	Q-Stat	Prob	AC	Q-Stat	Prob	AC	Q-Stat	Prob	AC	Q-Stat	Prob
1	0.024	1.558	0.212	-0.027	2.015	0.156	-0.036	3.537	0.060	-0.021	1.218	0.270
2	-0.044	6.855	0.032	-0.037	5.874	0.053	-0.042	8.325	0.016	-0.019	2.222	0.329
20	-0.018	57.976	0.000	-0.010	38.293	0.008	-0.050	45.027	0.001	-0.012	23.028	0.287
36	0.013	75.134	0.000	-0.027	56.635	0.016	-0.001	72.342	0.000	-0.002	40.168	0.291
	COL26			COL27			COL28			COL29		
lags	AC	Q-Stat	Prob	AC	Q-Stat	Prob	AC	Q-Stat	Prob	AC	Q-Stat	Prob
1	-0.010	0.291	0.590	0.020	1.120	0.290	-0.003	0.022	0.882	-0.005	0.077	0.782
2	-0.033	3.367	0.186	-0.047	7.324	0.026	-0.055	8.503	0.014	-0.012	0.459	0.795
20	-0.024	22.224	0.328	0.004	56.337	0.000	-0.034	61.635	0.000	-0.015	22.059	0.337
36	0.031	41.007	0.260	-0.013	88.435	0.000	-0.040	94.796	0.000	0.011	77.737	0.000
	COL30			COL31			COL32			COL33		
lags	AC	Q-Stat	Prob	AC	Q-Stat	Prob	AC	Q-Stat	Prob	AC	Q-Stat	Prob
1	0.011	0.340	0.560	0.000	0.000	0.996	-0.013	0.471	0.492	-0.031	2.612	0.106
2	-0.047	6.446	0.040	-0.035	3.492	0.174	-0.027	2.437	0.296	-0.007	2.749	0.253
20	0.032	44.904	0.001	0.011	53.834	0.000	0.032	25.049	0.200	0.032	28.637	0.095
36	-0.002	71.447	0.000	0.003	68.902	0.001	-0.010	50.725	0.053	0.021	51.726	0.043
	COL34			COL35			COL36			COL37		
lags	AC	Q-Stat	Prob	AC	Q-Stat	Prob	AC	Q-Stat	Prob	AC	Q-Stat	Prob
1	0.030	2.422	0.120	0.029	2.306	0.129	-0.029	2.393	0.122	0.031	2.623	0.105
2	-0.095	27.362	0.000	-0.100	30.113	0.000	-0.056	11.119	0.004	-0.035	5.988	0.050
20	-0.008	47.358	0.001	-0.010	79.235	0.000	-0.036	36.766	0.012	0.017	39.751	0.005
36	0.015	68.485	0.001	0.006	89.756	0.000	0.045	61.745	0.005	-0.034	57.588	0.013
	COL38			COL39			COL40			COL41		
lags	AC	Q-Stat	Prob	AC	Q-Stat	Prob	AC	Q-Stat	Prob	AC	Q-Stat	Prob
1	-0.007	0.141	0.707	-0.068	12.657	0.000	0.029	2.287	0.130	-0.041	4.761	0.029
2	-0.059	9.913	0.007	-0.021	13.844	0.001	-0.063	13.476	0.001	-0.010	5.045	0.080
20	-0.023	32.940	0.034	-0.010	46.346	0.001	0.023	59.969	0.000	0.017	24.273	0.231
36	0.015	62.935	0.004	-0.015	56.427	0.016	-0.010	86.874	0.000	-0.035	51.272	0.047
	COL42			COL43			COL44			COL45		
lags	AC	Q-Stat	Prob	AC	Q-Stat	Prob	AC	Q-Stat	Prob	AC	Q-Stat	Prob
1	0.031	2.604	0.107	0.056	8.809	0.003	0.001	0.002	0.967	0.069	13.334	0.000
2	-0.054	10.640	0.005	-0.008	9.002	0.011	-0.043	5.231	0.073	-0.010	13.618	0.001
20	0.031	31.985	0.043	-0.026	71.622	0.000	-0.011	31.178	0.053	-0.013	59.690	0.000
36	-0.029	93.252	0.000	0.023	113.470	0.000	-0.008	55.742	0.019	-0.033	103.560	0.000
	COL47			COL49			COL50			COL51		
lags	AC	Q-Stat	Prob	AC	Q-Stat	Prob	AC	Q-Stat	Prob	AC	Q-Stat	Prob
1	-0.090	22.688	0.000	-0.008	0.161	0.688	-0.021	1.200	0.273	0.015	0.644	0.422
2	-0.012	23.071	0.000	-0.027	2.197	0.333	-0.040	5.712	0.058	-0.039	4.772	0.092
20	0.014	52.962	0.000	-0.021	26.984	0.136	-0.032	38.331	0.008	-0.028	43.131	0.002
36	-0.004	82.262	0.000	-0.003	38.359	0.363	-0.023	67.642	0.001	0.014	71.456	0.000

Table 2.2. Autocorrelation Test Results (Continued)												
	COL52			COL53			COL54			COL55		
lags	AC	Q-Stat	Prob	AC	Q-Stat	Prob	AC	Q-Stat	Prob	AC	Q-Stat	Prob
1	-0.020	1.126	0.289	0.170	80.244	0.000	-0.015	0.648	0.421	0.009	0.213	0.645
2	0.005	1.191	0.551	0.028	82.354	0.000	-0.014	1.217	0.544	0.004	0.259	0.879
20	-0.027	29.960	0.071	-0.004	400.170	0.000	0.009	40.164	0.005	-0.004	29.009	0.088
36	-0.028	46.699	0.109	-0.029	475.910	0.000	0.003	58.135	0.011	-0.037	69.919	0.001
	COL56			COL57			COL58			COL60		
lags	AC	Q-Stat	Prob	AC	Q-Stat	Prob	AC	Q-Stat	Prob	AC	Q-Stat	Prob
1	-0.062	10.512	0.001	-0.009	0.249	0.618	0.029	2.409	0.121	-0.063	11.057	0.001
2	-0.011	10.821	0.004	-0.022	1.622	0.445	-0.045	8.084	0.018	0.004	11.103	0.004
20	0.034	42.554	0.002	-0.021	26.087	0.163	-0.009	30.275	0.066	0.030	56.382	0.000
36	-0.005	70.445	0.001	-0.045	52.306	0.039	0.036	56.711	0.015	0.004	78.774	0.000
	COL61			COL62			COL63			COL64		
lags	AC	Q-Stat	Prob	AC	Q-Stat	Prob	AC	Q-Stat	Prob	AC	Q-Stat	Prob
1	-0.035	3.317	0.069	0.002	0.013	0.911	-0.021	1.187	0.276	-0.009	0.206	0.650
2	0.025	4.996	0.082	-0.029	2.292	0.318	-0.044	6.471	0.039	-0.016	0.933	0.627
20	-0.017	37.992	0.009	-0.013	13.244	0.867	-0.008	52.817	0.000	0.002	23.836	0.250
36	0.006	47.655	0.093	-0.002	27.464	0.846	-0.016	181.750	0.000	0.001	55.650	0.019
	COL65			COL67			COL68			COL69		
lags	AC	Q-Stat	Prob	AC	Q-Stat	Prob	AC	Q-Stat	Prob	AC	Q-Stat	Prob
1	-0.039	4.171	0.041	-0.045	5.605	0.018	0.031	2.678	0.102	0.035	3.464	0.063
2	-0.052	11.803	0.003	-0.037	9.325	0.009	-0.055	11.160	0.004	-0.042	8.346	0.015
20	0.001	30.563	0.061	0.014	46.890	0.001	-0.014	47.514	0.000	-0.042	31.951	0.044
36	0.028	57.868	0.012	0.025	71.633	0.000	0.003	85.361	0.000	-0.012	50.759	0.052
	COL70			COL71			COL72			COL73		
lags	AC	Q-Stat	Prob	AC	Q-Stat	Prob	AC	Q-Stat	Prob	AC	Q-Stat	Prob
1	-0.012	0.433	0.511	-0.005	0.075	0.784	-0.026	1.863	0.172	0.010	0.281	0.596
2	-0.053	8.154	0.017	-0.046	5.980	0.050	-0.026	3.791	0.150	-0.015	0.931	0.628
20	0.009	54.493	0.000	-0.018	30.618	0.060	-0.008	50.486	0.000	-0.031	44.429	0.001
36	-0.004	83.354	0.000	0.032	69.929	0.001	0.005	70.285	0.001	0.022	62.071	0.004
	COL74			COL75			COL76			COL77		
lags	AC	Q-Stat	Prob	AC	Q-Stat	Prob	AC	Q-Stat	Prob	AC	Q-Stat	Prob
1	-0.014	0.524	0.469	0.029	2.382	0.123	-0.012	0.391	0.532	-0.030	2.486	0.115
2	-0.027	2.489	0.288	0.031	5.090	0.078	0.017	1.200	0.549	-0.009	2.702	0.259
20	-0.002	29.074	0.086	-0.021	35.318	0.018	0.030	25.083	0.198	-0.037	30.940	0.056
36	0.043	46.955	0.105	-0.004	42.719	0.205	-0.023	49.948	0.061	0.014	53.557	0.030

Table 2.2. Autocorrelation Test Results (Continued)												
	COL78			COL79			COL80			COL81		
lags	AC	Q-Stat	Prob	AC	Q-Stat	Prob	AC	Q-Stat	Prob	AC	Q-Stat	Prob
1	-0.079	17.401	0.000	-0.026	1.901	0.168	0.000	0.000	0.990	0.024	1.641	0.200
2	0.003	17.428	0.000	0.001	1.903	0.386	-0.021	1.171	0.557	-0.049	8.331	0.016
20	0.024	53.158	0.000	-0.011	31.869	0.045	0.006	47.260	0.001	0.016	39.974	0.005
36	-0.015	78.161	0.000	-0.041	62.352	0.004	0.003	73.841	0.000	0.028	56.216	0.017
	COL82			COL83			COL84			COL85		
lags	AC	Q-Stat	Prob	AC	Q-Stat	Prob	AC	Q-Stat	Prob	AC	Q-Stat	Prob
1	0.042	4.997	0.025	0.016	0.681	0.409	-0.013	0.454	0.500	-0.046	5.806	0.016
2	-0.031	7.749	0.021	-0.025	2.435	0.296	-0.060	10.453	0.005	-0.033	8.799	0.012
20	-0.032	44.099	0.001	0.014	28.566	0.097	0.026	56.344	0.000	-0.006	45.977	0.001
36	-0.005	73.379	0.000	0.030	63.824	0.003	0.010	95.460	0.000	-0.009	73.030	0.000
	COL86			COL87			COL88			COL89		
lags	AC	Q-Stat	Prob	AC	Q-Stat	Prob	AC	Q-Stat	Prob	AC	Q-Stat	Prob
1	-0.014	0.581	0.446	0.021	1.209	0.271	-0.043	5.038	0.025	0.087	20.872	0.000
2	-0.022	1.951	0.377	-0.058	10.578	0.005	-0.058	14.477	0.001	-0.015	21.495	0.000
20	-0.028	19.151	0.512	-0.016	48.726	0.000	-0.030	32.634	0.037	-0.079	72.635	0.000
36	0.004	45.851	0.126	-0.007	59.958	0.007	-0.002	47.425	0.096	-0.036	123.560	0.000
	COL90			COL91			COL92			COL93		
lags	AC	Q-Stat	Prob	AC	Q-Stat	Prob	AC	Q-Stat	Prob	AC	Q-Stat	Prob
1	0.054	8.139	0.004	0.090	22.569	0.000	-0.008	0.179	0.672	-0.010	0.294	0.588
2	-0.032	11.067	0.004	-0.034	25.748	0.000	-0.046	6.096	0.047	-0.045	5.844	0.054
20	-0.049	44.653	0.001	-0.057	71.619	0.000	-0.018	21.491	0.369	-0.010	31.481	0.049
36	0.020	92.349	0.000	-0.037	142.810	0.000	0.025	49.563	0.066	-0.031	55.379	0.020

Table 2.3. EGARCH Models with Different Return Lags

The following table presents estimation results for ten stocks of the EGARCH coefficients and the corresponding T-stat under different specifications of lags in returns. The EGARCH model is specified as:

$$\ln(h_{it}) = c_{ik} + \beta_{ik}\ln(h_{i,t-1}) + \alpha_{ik}\left|u_{i,t-1}\right| + \gamma_{ik}u_{i,t-1}$$

	ARMA(20,1)-M		ARMA(20,1)		ARMA(2,1)		ARMA(1,1)		ARMA(0,1)		ARMA(0,0)	
	Coef.	T-stat	Coef.	T-stat	Coef.	T-stat	Coef.	T-stat	Coef.	T-stat	Coef.	T-stat
Oracle												
C_h	-0.1469	-8.0639	-0.1865	-9.1642	-0.1202	-9.2698	-0.1220	-9.5092	-0.1196	-9.3136	-0.1245	-9.6435
α_h	0.1084	10.4317	0.1276	9.9931	0.0986	11.1062	0.0988	11.2691	0.0980	11.1980	0.1007	11.4350
γ_h	-0.0512	-7.2857	-0.0566	-7.6597	-0.0464	-8.0437	-0.0540	-9.1797	-0.0497	-8.5717	-0.0547	-9.4480
β_h	0.9905	484.3506	0.9869	511.5227	0.9932	823.4643	0.9929	825.2953	0.9932	832.5766	0.9928	826.4537
Microsoft												
C_h	-0.1948	-7.7509	-0.1905	-8.1248	-0.1591	-8.0799	-0.1696	-8.2480	-0.1604	-8.0856	-0.1612	-8.0377
α_h	0.1538	11.5812	0.1530	11.5174	0.1352	11.5136	0.1415	11.8019	0.1357	11.4931	0.1362	11.4696
γ_h	-0.0302	-3.9490	-0.0304	-4.0444	-0.0302	-5.4036	-0.0306	-5.3030	-0.0303	-5.4395	-0.0313	-5.5669
β_h	0.9900	380.6061	0.9905	417.7622	0.9928	504.9119	0.9920	480.3656	0.9927	502.0530	0.9926	493.9481
Unisys												
C_h	-0.3517	-9.4459	-0.3596	-11.2552	-1.3031	-22.1364	-0.3609	-14.7641	-0.3343	-14.9760	-0.3272	-15.0740
α_h	0.1497	11.7739	0.1512	11.9906	0.3373	17.8453	0.1506	14.0478	0.1444	14.0150	0.1422	14.1510
γ_h	-0.1055	-14.3871	-0.1093	-13.1136	-0.0965	-11.5155	-0.0983	-16.5958	-0.0946	-18.7057	-0.0972	-19.6879
β_h	0.9641	202.8953	0.9633	261.2728	0.8467	112.2037	0.9628	350.0799	0.9660	395.0597	0.9668	407.0007
GD												
C_h	-0.3659	-9.1346	-0.0380	-6.7524	-0.3973	-11.0708	-0.0587	-8.7685	-0.0746	-9.3720	-0.0738	-9.3841
α_h	0.2030	14.1202	0.0406	9.5377	0.2039	14.6713	0.0543	11.8949	0.0642	13.0012	0.0635	12.9253
γ_h	-0.0544	-7.5808	-0.0224	-5.7099	-0.0614	-9.1646	-0.0233	-6.3478	-0.0261	-6.7265	-0.0261	-6.6235
β_h	0.9741	230.2623	0.9991	1616.4533	0.9703	256.3465	0.9978	1294.5929	0.9967	1080.1067	0.9968	1094.3550
GM												
C_h	-0.2027	-6.8099	-0.2145	-8.9364	-0.2063	-9.9248	-0.1919	-8.9954	-0.2069	-10.0419	-0.2079	-10.1639
α_h	0.0933	8.3222	0.0925	8.4430	0.0878	8.4072	0.0766	7.5216	0.0885	8.4389	0.0894	8.4966
γ_h	-0.0527	-8.4275	-0.0538	-8.9261	-0.0534	-11.5333	-0.0579	-11.9809	-0.0527	-11.4392	-0.0534	-11.5251
β_h	0.9831	297.9426	0.9815	404.7070	0.9820	497.0642	0.9832	461.8341	0.9821	501.9039	0.9820	506.9968

Table 2.3. EGARCH Models with Different Return Lags (Continued)												
ETR												
	ARMA(20,1)-M		ARMA(20,1)		ARMA(2,1)		ARMA(1,1)		ARMA(0,1)		ARMA(0,0)	
	Coef.	T-stat	Coef.	T-stat	Coef.	T-stat	Coef.	T-stat	Coef.	T-stat	Coef.	T-stat
C_h	-0.2153	-8.1160	-0.3276	-9.8797	-0.1689	-9.3855	-0.1982	-10.3459	-0.1828	-9.7808	-0.1790	-9.9109
α_h	0.1262	12.4418	0.1672	13.5488	0.1213	14.0709	0.1238	14.3984	0.1206	13.4997	0.1182	13.7322
γ_h	-0.0624	-8.4599	-0.0609	-7.2613	-0.0578	-9.8601	-0.0631	-9.6270	-0.0524	-9.7256	-0.0504	-9.6512
β_h	0.9861	357.4382	0.9769	294.2873	0.9909	555.7159	0.9878	501.8279	0.9894	529.0491	0.9896	545.7185
Disney												
C_h	-0.2125	-8.1998	-0.2041	-8.5759	-0.2765	-11.1116	-0.2626	-10.8167	-0.2539	-10.7706	-0.2542	-10.8088
α_h	0.1545	11.9010	0.1521	11.8012	0.1858	15.8593	0.1784	15.3603	0.1754	15.3501	0.1756	15.5798
γ_h	-0.0374	-5.5149	-0.0400	-5.8289	-0.0410	-6.2761	-0.0409	-6.6121	-0.0403	-6.5513	-0.0402	-6.6181
β_h	0.9878	356.9249	0.9886	399.8963	0.9826	349.0796	0.9836	365.8459	0.9845	375.3252	0.9844	374.8926
Walmart												
C_h	-0.1265	-5.6227	-0.1151	-6.1288	-0.1158	-6.2162	-0.1248	-6.3912	-0.1261	-6.3727	-0.1254	-6.3279
α_h	0.0921	7.6791	0.0908	7.6720	0.0916	7.9151	0.0982	8.1206	0.0986	8.0946	0.0982	8.0144
γ_h	-0.0436	-6.0314	-0.0435	-6.1047	-0.0439	-6.4122	-0.0491	-7.0176	-0.0482	-6.9045	-0.0492	-6.9977
β_h	0.9932	459.0839	0.9944	611.1973	0.9944	622.4907	0.9939	585.5113	0.9938	580.2634	0.9939	582.2306
Cisco												
C_h	-0.2336	-7.7635	-0.2030	-7.8292	-0.2116	-8.5489	-0.2177	-8.7578	-0.2049	-8.6018	-0.2042	-8.5993
α_h	0.1352	9.4851	0.1341	9.5776	0.1381	10.0723	0.1400	10.0894	0.1355	10.0605	0.1354	10.0238
γ_h	-0.0751	-8.9351	-0.0730	-8.9546	-0.0700	-9.3804	-0.0755	-10.2644	-0.0720	-10.1639	-0.0735	-10.4138
β_h	0.9823	287.5814	0.9863	364.6573	0.9855	385.5069	0.9849	381.6384	0.9862	401.6141	0.9862	403.6791
Leh												
C_h	-0.0996	-6.9441	-0.1001	-8.4666	-0.1001	-8.9909	-0.0995	-9.0752	-0.0989	-9.0174	-0.0988	-9.0474
α_h	0.0944	9.9741	0.0951	9.9769	0.0958	10.4502	0.0938	10.3490	0.0935	10.3124	0.0930	10.2997
γ_h	-0.0403	-6.6665	-0.0403	-6.7047	-0.0388	-7.0898	-0.0429	-7.6603	-0.0430	-7.5672	-0.0428	-7.6900
β_h	0.9964	651.9285	0.9964	871.2594	0.9964	913.0745	0.9963	925.2299	0.9964	926.9900	0.9963	931.6379

Table 2. 4. Test Results of Constant Correlation

Table 4 presents results for testing constant correlation among stocks based on different univariate GARCH models.The null hypothesis is constant correlation. The test statistic Chi has asymptotic χ2 distribution, with 3 degree of freedom. Chi statistics and the corresponding P-value (Pval) is reported for three models.

	ARMA(20,1)-EGARCH-M		EGARCH		GARCH	
Number of stocks	Chi	Pval	Chi	Pval	Chi	Pval
2	446.0839	0.0000	440.8242	0.0000	17.6590	0.0071
3	756.4787	0.0000	759.3550	0.0000	22.7594	0.0009
4	1659.1000	0.0000	1674.5000	0.0000	27.2013	0.0001
5	2181.6000	0.0000	2222.7300	0.0000	31.2076	0.0000
10	6675.8000	0.0000	6915.6000	0.0000	66.6526	0.0000
20	24566.0000	0.0000	25847.0000	0.0000	65.0398	0.0000
50	13223.0000	0.0000	14423.0000	0.0000	196.6750	0.0000
88	40220.0000	0.0000	47083.0000	0.0000	268.3760	0.0000

Table 2.5. Correlation Estimates With Different Return Lags in Mean Specification

The following table shows how correlation estimates (under assumption of constant correlation) vary under different univariate specifications of lag lengths, in the case of 5 stocks.

ARMA(20,1)-EGARCH-M

STOCK	CSCO	DIS	ETR	GD	GM
CSCO	1	0.312968	0.021547	0.172839	0.296531
DIS		1	0.097665	0.133189	0.308701
ETR			1	0.161038	0.140175
GD				1	0.224812
GM					1

ARMA(20,1)-EGARCH

STOCK	CSCO	DIS	ETR	GD	GM
CSCO	1	0.313233	0.021032	0.176503	0.296029
DIS		1	0.099242	0.135424	0.308316
ETR			1	0.163294	0.139599
GD				1	0.225498
GM					1

ARMA(2,1)-EGARCH

STOCK	CSCO	DIS	ETR	GD	GM
CSCO	1	0.315604	0.018928	0.173943	0.296087
DIS		1	0.102264	0.136891	0.307736
ETR			1	0.159829	0.142218
GD				1	0.221796
GM					1

ARMA(1,1)-EGARCH

STOCK	CSCO	DIS	ETR	GD	GM
CSCO	1	0.314177	0.017296	0.174993	0.295457
DIS		1	0.101979	0.138004	0.308142
ETR			1	0.162157	0.144126
GD				1	0.220575
GM					1

ARMA(0,1)-EGARCH

STOCK	CSCO	DIS	ETR	GD	GM
CSCO	1	0.315415	0.019211	0.175059	0.296596
DIS		1	0.103705	0.137860	0.309515
ETR			1	0.161665	0.144055
GD				1	0.222324
GM					1

ARMA(0,0)-EGARCH

STOCK	CSCO	DIS	ETR	GD	GM
CSCO	1	0.313661	0.019833	0.176169	0.295891
DIS		1	0.102732	0.138465	0.308522
ETR			1	0.162077	0.145297
GD				1	0.221774
GM					1

Table 2.6. Estimation Results for DCC(1,1) Model

Table 6 presents the parameters estimated for DCC (1,1) model. Estimation results are shown for different numbers of stocks. T-ratios presented in parentheses. T-ratios increase as the number of stock increases(see the text for explanations). All the estimation results for these parameters are statistically significant above 1%. This table also presents the likelihood ratio test results for integrated DCC model and constant correlation model. The right side x2 value is for the null of constant correlation, while the left side x2 is for the null of integrated dynamic correlation model. The degrees of freedom equal to the number of restrictions imposed.

				H0: α +β=1				H0: α =0 and β=1 or 0		
No.of stocks		α	β	DCC-log-likelihood	IDCC-log-likelihood	x^2	P-value	CC log-likelihood	x^2	P-value
2	Parameter	0.01811	0.96784	12858.16822	12853.75590	8.82465	0.00160	12836.78670	42.76305	0.00000
	T-ratio	(104.33179)	(717.38900)							
3	Parameter	0.01624	0.96571	19888.95645	19884.18580	9.54130	0.00110	19857.72470	62.46350	0.00000
	T-ratio	(53.68)	(299.51)							
4	Parameter	0.00840	0.98562	25832.49039	25818.96950	27.04178	0.00000	25755.46740	154.04598	0.00000
	T-ratio	(482.41)	(9632.87)							
5	Parameter	0.00758	0.98707	31632.56392	31610.29600	44.53583	0.00000	31556.22920	152.66943	0.00000
	T-ratio	(1928.59)	(47564.53)							
6		0.00698	0.98791	38229.76380	38199.81500	59.89761	0.00000	38132.52940	194.46881	0.00000
7		0.00694	0.98782	46097.40417	46052.78090	89.24654	0.00000	45094.40170	2006.00494	0.00000
8		0.00646	0.98895	53774.48937	53724.26830	100.44213	0.00000	53614.08080	320.81713	0.00000
9		0.00568	0.98944	60878.62083	60818.60840	120.02487	0.00000	60727.22460	302.79247	0.00000
10		0.00529	0.98983	69024.35066	68862.75470	323.19192	0.00000	68861.92760	324.84612	0.00000
20		0.00307	0.99022	144733.45443	144559.46000	347.98885	0.00000	144556.05000	354.80885	0.00000
25		0.00276	0.98896	183131.49803	182937.94700	387.10207	0.00000	182933.20000	396.59607	0.00000
50		0.00209	0.98243	375587.63751	375345.60000	484.07503	0.00000	375328.38000	518.51503	0.00000
88		0.00163	0.96611	658145.68535	657909.60000	472.17070	0.00000	657838.72000	613.93070	0.00000

Table 2.7. Results for Pair-Wise Parameter Estimation and GARCH(1,1)-DCC(1,1) Model

Table 2.7 presents pair-wise correlation parameter estimation results and statistics for GARCH(1,1)-DCC(1,1) model. The six stocks are Microsoft, GE, Exxon Mobil, Pfizer, Walmart and Intel. They are coded as 1, 2, 3, 4, 5 and 6 in the table. The second last column presents the average of α, β and their statistics. The last column present the correlation parameter estimation of the same model but for six stocks altogether.

	1,2	1,3	1,4	1,5	1,6	2,3	2,4	2,5
α	0.0265	0.0092	0.0185	0.0181	0.0413	0.0142	0.0226	0.0215
β	0.9621	0.9870	0.9738	0.9777	0.9248	0.9827	0.9663	0.9720
SE(α)	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
t-ratios	512.2805	1,495.2982	689.0575	1,002.5041	1,074.7001	1,089.4824	598.5533	875.5364
SE(β)	0.0002	0.0000	0.0001	0.0000	0.0016	0.0000	0.0001	0.0001
t-ratios	6,212.1481	75,926.9291	12,940.8624	25,415.1080	562.4150	40,384.2711	8,248.8977	18,853.8641

	2,6	3,4	3,5	3,6	4,5	4,6	5,6	Average
α	0.0213	0.0096	0.0146	0.0112	0.0173	0.0123	0.0157	0.0183
β	0.9672	0.9889	0.9807	0.9843	0.9710	0.9808	0.9746	0.9729
SE(α)	0.0001	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000
t-ratios	406.9945	1,264.6645	682.7525	725.8920	199.6559	512.3123	327.5534	763.8158
SE(β)	0.0001	0.0000	0.0001	0.0000	0.0004	0.0001	0.0002	0.0002
t-ratios	6,862.8762	73,739.9033	19,424.2547	28,266.7751	2,367.6277	14,994.5797	5,829.2701	22,668.6521

DCC estimation for six stocks	
α	0.0125
β	0.9778
SE(α)	0.0000
t-ratios	3,483.8120
SE(β)	0.0000
t-ratios	57,171.8809

Table 2.8 DCC(1,1) estimation results for different specifications for EGARCH(1,1) Model

	ARMA(20,1)-M	ARMA(20,1)	ARMA(2,1)	ARMA(0,0)
α	0.001357	0.001681	0.001407	0.001718
β	0.806863	0.96211	0.971573	0.964214

Table 2.9 Ljung-Box Portmanteau Specification Test 9

Table 2.9. Ljung-Box Portmanteu Specification Test										
Table 2.9 presents multivariate Ljung-Box Portmanteu test results on the standardized residuals Ut from different DCC model examined in this study. Under the null hypothesis of constant correlation, the statistics is distributed as $\chi^2(n^2p)$. p is number of lags, n is number of time series in the test (n=88 in our study).										
P	GARCH(1,1)-DCC(1,1)		EGARCH-AR(20)-MA(1)-M-DCC(1,1)		EGARCH-AR(20)-MA(1)-DCC(1,1)		EGARCH-AR(2)-MA(1)-DCC(1,1)		EGARCH-DCC(1,1)	
	Q-Stat	P-Value	Q-Stat	P-Value	Q-Stat	P-Value	Q-Stat	P-Value	Q-Stat	P-Value
1	9364.1043	0.0000	11915.4602	0.0000	9084.3352	0.0000	8963.8005	0.0000	9221.9925	0.0000
2	18024.9322	0.0000	22820.7706	0.0000	17186.4239	0.0000	17128.2041	0.0000	17692.3197	0.0000
3	26420.9373	0.0000	33804.6170	0.0000	25384.6250	0.0000	25261.1448	0.0000	26032.6504	0.0000
4	34522.1086	0.0000	44502.0833	0.0000	33326.7188	0.0000	33122.9314	0.0000	34028.4519	0.0000

Appendix 2.A Chronological Development of Related MGARCH Models

Table 2.10 lists the major papers in the development of MGARCH.

Table 2.10 Chronological Development of MGARCH

Functional Form	Paper	Estimation Method
ARCH (q)	Engle (1982)	MLE
GARCH (p, q)	Bollerslev (1986)	MLE
Multivariate ARCH (q)	Kraft and Engle (1983)	MLE
VECH model	Bollerslev, Engle and Wooldridge (1988)	MLE
CCC- MGARCH	Bollerslev (1990)	One-step Quasi-MLE
BEKK MGARCH	Engle and Kroner (1993)	MLE
Constant correlations test in MGARCH model	Bollerslev (1990), Tse (2000), Engle and Sheppard (2001)	
DCC-MGARCH	Engle (2002b), Engle and Sheppard (2001)	Two-step MLE
DCC-MGARCH	Tse and Tsui (2001)	One-step MLE
Asymmetric DCC-MGARCH	Cappiello, Engle and Sheppard (2004)	Two-step MLE
Bi-variate MGARCH estimation to high dimension MGARCH	Ledoit, Santa-Clara and Wolf (2003)	Two-step MLE
DCC-MGARCH	Palandri (2005, August)	Multi-Step MLE
MLE = Maximum Likelihood Estimation		

Example 1: Extension of ARCH model to multivariate Models

For two assets, the model MGARCH (q) will be:

$$h_1 = \alpha_1 + \beta_1 \varepsilon_{1,t-1}^2 + \ldots + \beta_q \varepsilon_{1,t-q}^2 \quad ,$$

$$h_{12} = \alpha_1 + \beta_1 \varepsilon_{1,t-1} \varepsilon_{2,t-1} + \ldots + \beta_q \varepsilon_{1,t-q} \varepsilon_{2,t-q} \quad ,$$

$$h_2 = \alpha_1 + \beta_1 \varepsilon_{2,t-1}^2 + \ldots + \beta_q \varepsilon_{2,t-q}^2$$

Just as Bollerslev (1986) generalized the univariate ARCH model into the GARCH model, Bollerslev, Engle and Wooldridge (1988) generalized the multivariate linear ARCH(q) into an MGARCH (p, q) model.[8] In this model, they specified the conditional variance-covariance matrix to have the following process:

$$vech(H_t) = W + \sum_{i=1}^{q} A_i \times Vech(\Sigma_{t-i}\Sigma'_{t-i}) \sum_{i=1}^{P} B_i \times Vech(H_{t-i})$$

VECH is a vector constructed by stacking the columns of the conditional variance-covariance matrix. Henceforth this is the VECH model. The dimension of Vech (H_t) is $\frac{n(n+1)}{2}$, where W is an $\frac{n(n+1)}{2}$ dimension vector. A_i and B_i are $\frac{n(n+1)}{2} \times \frac{n(n+1)}{2}$ matrices. The total number of parameters to estimate in the model is: $\frac{n(n+1)}{2}$ + (p+q)$\times \frac{n^2(n+1)^2}{4}$.

Example 2: Two assets MGARCH (1,1) in VECH form will be:

$$\begin{bmatrix} h_1 \\ h_{1,2} \\ h_2 \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} + \begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} \\ \beta_{21} & \beta_{22} & \beta_{23} \\ \beta_{31} & \beta_{32} & \beta_{33} \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t-1}^2 \\ \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ \varepsilon_{2,t-1}^2 \end{bmatrix} + \begin{bmatrix} \lambda_{11} & \lambda_{12} & \lambda_{13} \\ \lambda_{21} & \lambda_{22} & \lambda_{23} \\ \lambda_{31} & \lambda_{32} & \lambda_{33} \end{bmatrix} \begin{bmatrix} h_{1,t-1} \\ \sqrt{h_{1,t-1}h_{2,t-1}} \\ h_{2,t-1} \end{bmatrix}$$

Example 3: BEKK Model for two asset case

$$\begin{bmatrix} h_1 & h_{1,2} \\ h_{2,1} & h_2 \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} + \begin{bmatrix} a_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t-1}^2 & \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ \varepsilon_{1,t-1}\varepsilon_{2,t-1} & \varepsilon_{2,t-1}^2 \end{bmatrix} \begin{bmatrix} a_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix}^T +$$

$$\begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} h_{1,t-1} & \sqrt{h_{1,t-1}h_{2,t-1}} \\ \sqrt{h_{1,t-1}h_{2,t-1}} & h_{2,t-1}^2 \end{bmatrix} \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}^T$$

To overcome some of the VECH model's failures, and to ensure an estimated positive semi-definite conditional variance-covariance matrix, Engle and Kroner (1995) adopted a quadratic conditional variance-covariance matrix specification, now called the BEKK model. The simple version of their model defines the conditional variance-covariance

[8] In MGARCH (p, q), q represents the number of lags of the matrix of error terms, and p the number of lags of the variance-covariance matrix.

matrix as:

$$H_t = C_0^{'}C_0 + \sum_{i=1}^{q} A_i^{'}\Sigma_{t-i}\Sigma'_{t-i}A_i + \sum_{i=1}^{p} G_i^{'}H_{t-i}G_i$$

The elements of H_t depend on their lags as well as the square and cross product of the residuals. C_0, A_i , and G_i are $n \times n$ parameter matrices. This representation assures that the conditional variance-covariance matrix is positive semi-definite for any ε_t . The number of parameters in a BEKK (1, 1) model is $n \times ((5n+1)/2)$. The complexity of MGARCH models and the computational burden remains when the number of dimensions increases. Another drawback is that the estimated unconditional covariance matrices may not be positive semi-definite during the estimation. In addition, the interpretation of the coefficients is more difficult. Diagonal BEKK is another way to simplify the number of the parameters to be estimated.

Appendix 2.B Maximum Likelihood Estimation of DCC Model

The conditional covariance matrix in the Engle (2002b) and Sheppard model (2001) is specified as:

$$H_t = D_t \Omega_t D_t,$$

where $D_t = \begin{bmatrix} h_{1,t}^{1/2} & 0 & \ldots & 0 \\ 0 & h_{2,t}^{1/2} & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & h_{n,t}^{1/2} \end{bmatrix}$, Ω_t denotes the dynamic correlation matrix,

$$\Omega_t = \begin{bmatrix} 1 & \rho_{12} & \cdots & \rho_{1n} \\ \rho_{21} & 1 & \cdots & \rho_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \rho_{n1} & \rho_{n2} & \cdots & 1 \end{bmatrix},$$

Here the elements of Ω_t are guaranteed to be in the interval [-1, +1]. Thus H_t is:

$$H_t = D_t\Omega_t D_t = \begin{bmatrix} h_{1,t} & \cdots & \cdots & \rho_{1n} \\ \rho_{21}\sqrt{h_{1t}h_{2t}} & h_{2t} & \cdots & \rho_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \rho_{n1}\sqrt{h_{1t}h_{nt}} & \rho_{n2}\sqrt{h_{nt}h_{2t}} & \cdots & h_{nt} \end{bmatrix}$$

For the univariate GARCH model, the estimation method is full information ML. It is assumed that the residuals of the mean equations follow a conditional multivariate joint normal distribution. When the assumption of a conditionally normal distribution is violated, the Quasi-Maximum Likelihood (QML) approach is still consistent and asymptotically normal under certain regularity conditions. The contribution of observation t toward the likelihood function can be written as:

$$\ell_t = -\frac{1}{2}\log 2\pi - \frac{1}{2}\ln\left|\sigma_t^2\right| - \frac{1}{2}\ln(\varepsilon_t^2 / \sigma_t^2).$$

Analogously, in the case of the MLE for the multivariate case, with n assets, the contribution of observation t toward the likelihood function is:

$$\ell_t = \frac{1}{2}n\log 2\pi - \frac{1}{2}\ln\left|H_t\right| - \frac{1}{2}\ln(\Sigma_t H_t^{-1}\Sigma_t')$$

The likelihood function for the overall observations will be:

$$L_t = -\frac{1}{2}\sum_t (n\log 2\pi + \ln\left|D_t\Omega_t D_t\right| + \ln \Sigma_t D_t^{-1}\Omega_t^{-1}D_t^{-1}\Sigma_t')$$

$$= -\frac{1}{2}\sum_t \; (n\log 2\pi + 2\ln|D_t| + \ln|\Omega_t| + \ln \Sigma_t D_t^{-1}\Omega_t^{-1}D_t^{-1}\Sigma_t^{'})$$

$$= -\frac{1}{2}\sum_t \; (n\log 2\pi + 2\ln|D_t| + \ln|\Omega_t| + \ln U_t^{'}\Omega_t^{-1}U_t^{'-1})$$ [9]

From the equation above we can see that if the parameters in D_t, the diagonal matrix with variances of each series on the diagonal, are known, we only need to estimate the parameters in Ω_t, the correlation matrix. U_t is the IID matrix, so there is no need to estimate Ut. Based on Newey and McFadden (1994), D_t can be estimated as a univariate GARCH model. Then the estimated parameters can be considered as given and we can use a second step maximization to get the parameters in Ω_t. This two-step procedure provides a consistent and asymptotic normal estimator of the parameters. The estimators are easy to interpret and maintain the intuition of univariate GARCH. A consistent estimator of standard errors is also achieved. (Sheppard 2000).

The two-step estimation method can be outlined as follows:

Step I: replace the matrix Ω_t with the identity matrix I_t, which implies that the correlation among assets is zero. The likelihood function becomes:

$$L_t = -\frac{1}{2}\sum_t \; (N\log 2\pi + 2\ln|D_t| + \Sigma_t^{-1}D_t^{-1}D_t^{-1}\Sigma_t^{-1})$$

$$= -\frac{1}{2}\sum_{t=1}^{T} \; (N\log 2\pi + 2\sum_{i=1}^{N} \; (\ln|h_{it}| + \frac{\varepsilon_{it}^2}{h_{it}})),$$

Changing the sequence of summation over t and n, we get

$$= -\frac{1}{2}\sum_{t=1}^{N} \; (T\log 2\pi + 2\sum_{t=1}^{T} \; (\ln|h_{it}| + \frac{\varepsilon_{it}^2}{h_{it}}))$$

---

[9] $U_t = H_t^{-1/2}\,\Sigma_t = D_t^{-1}\Sigma_t$

This is the sum of the likelihoods of the individual GARCH models for each asset. The summation can be maximized by jointly maximizing the components of each asset, i.e., we can estimate the individual univariate GARCH equations using QML method.

The conditional variance in the multivariate model can be estimated as GARCH (p, q):

$$h_{it} = \omega_i + \sum_{p=1}^{p} \alpha_{i,p}(\varepsilon_{i,t-p}^2) + \sum_{q=1}^{q} \beta_{iq} h_{it-q},$$

There are three components of the GARCH process. The first term is the long run average volatility, the second term is the volatility due to new information, and the third term is the forecast made in the previous period. Here, the conditional variance is a function of its own lagged values and the past error terms, and is not affected by the covariance and the cross product between past errors terms. Hence, this specification is more restrictive than the VECH model.

Step 2 is to maximize the likelihood function conditional on the parameters of the variances estimated in step one. From equation (13),

$$L_t = -\frac{1}{2}\sum_t \left(N \log 2\pi + 2\ln|D_t| + \ln|\Omega_t| + \ln U_t' \Omega_t^{-1} U_t'^{-1}\right)$$

Since we know the parameters in D_t, the only part of the likelihood that will be affected will be parameters determining correlation function, so we only need to maximize

$$L_t = -\frac{1}{2}\sum_t \left(\log|\Omega_t| + \ln U_t' \Omega_t^{-1} U_t'^{-1}\right)$$

It can be shown that the conditional correlation matrix is identical to the expected value of the standardized residuals.

$E_{t-1}(U_t U_t^{'})$

$= E_{t-1}(D_t^{-1}\Sigma_t (D_t^{-1}\Sigma_t)^{'})$

$= E_{t-1}(D_t^{-1}\Sigma_t \Sigma_t^{'} D_t^{-1^{'}})$

$= D_t^{-1} E_{t-1}(\Sigma_t \Sigma_t^{'}) D_t^{-1^{'}}$

$= D_t^{-1} H_t D_t^{-1^{'}} = \Omega_t$

This is analogous to univariate case. Therefore, to estimate correlation matrix Ω_t , we could use the following dynamics similar to GARCH (m, n) process,

$$\Omega_t = (1 - \sum_{m=1}^{M} a_m - \sum_{n=1}^{N} b_n)\,\Omega + \sum_{m=1}^{M} a_m (U_{t-m} U'_{t-m}) + \sum_{n=1}^{N} b_n \Omega_{t-n}$$

However, the estimated Ω_t is guaranteed to be positive semi-definite but is not constrained to be a correlation matrix (i.e. elements are not necessarily bounded by +1 and -1). To circumvent the problem, Engle and Sheppard introduced another matrix, Q_t , which follows the following process.

$$Q_t = (1 - \sum_{m=1}^{M} a_m - \sum_{n=1}^{N} b_n)\,\overline{Q} + \sum_{m=1}^{M} a_m (U_{t-m} U'_{t-m}) + \sum_{n=1}^{N} b_n Q_{t-n}$$ [10]

where $Q_t = \{q_{ij}\}_t$ is the conditional variance and covariance. $\overline{Q} = \sum_{t=1}^{T} U_t U'_t / T$ is the estimation for unconditional variance and covariance. Again, Q_t is guaranteed to be positive semi-definite but is not constrained to be a correlation matrix (i.e. elements are not necessarily bounded by +1 and -1). Q_t has to be transformed to get Ω_t. Let Q^*_t be

[10] This equation is analogous to GARCH (p, q) specification.

the square root of the diagonal element from Q_t.

$$Q^*_t = \begin{bmatrix} q_{1,t}^{1/2} & 0 & ... & 0 \\ 0 & q_{2,t}^{1/2} & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & q_{n,t}^{1/2} \end{bmatrix},$$

Let $\rho_{ij,t} = q_{ij,t} / \sqrt{q_{iit} q_{jjt}}$, then the correlation matrix will be estimated as:

$$\Omega_t = \mathrm{Q}^{*-1}_{\mathrm{t}} \mathrm{Q}_{\mathrm{t}} \mathrm{Q}^{*-1'}_{\mathrm{t}}$$

In the DCC-MGARCH model, let $Q_t = (1-\alpha-\beta)\overline{Q} + \alpha(\mathrm{U}_{\mathrm{t-1}}\mathrm{U'}_{\mathrm{t-1}}) + \beta Q_{t-1}$ [11] and, assume $\alpha = 0, \beta = 0$, so $Q_t = \overline{Q}$. The model drops simple constant correlation model. This CCC-MGARCH estimator is a special case of Engle and Shepard (2001) DCC-MGARCH. An extension or simplification is to assume constant conditional correlation, and (unlike Bollerslev (1990)) use a two-step procedure to estimate variance and covariance. That is, first use univariate GARCH to estimate variances, and then average the standardized residuals to estimate covariances. Therefore, the theoretical justification for two-step DCC-MGARCH parameter estimation should also apply to CCC-MGARCH.

Bollerslev (1990) simplified one-step estimation of MGARCH models by assuming constant conditional correlations. However, he used one-step ML, so the computation remains quite difficult. Nevertheless, one-step CCC-MGARCH is popular due to its relative computational simplicity. Among others, Bollerslev (1990), Kroner and Claessens (1991), Kroner and Sultan (1993), Park and Switzer (1995) and Lien and Tse (1998) used the CCC-MGARCH model of Bollerslev. Tse (2000) developed a test for

[11] This is a special case of equation (17).

constant conditional correlations in an MGARCH model. He found that the CCC hypothesis cannot be rejected for spot-futures markets and foreign exchange markets, but is rejected for cross-country stock markets..

Bollerslev (1990) pointed out that the MLE of the correlation matrix is equal to the sample correlation matrix under CCC. The second step of the estimation is simply $\hat{\Omega} = \frac{1}{T}\sum_{i=1}^{T}\hat{U}_t\hat{U}'_t$. This guarantees that the CCC matrix is positive semi-definite. This corollary to Engle and Sheppard has been noted by Andersen, Bollerslev, Christoffersen and Diebold (2005)(ABCD).

The major advantage of the two-step CCC-MGARCH is its simplicity and the ease with which it can be applied to a large number of assets. This MGARCH model is much more tractable for many assets than is either the one-step CCC-MGARCH model of Bollerslev or the two-step DCC-MGARCH model of Engle and Sheppard. Indeed, this MGARCH model can be implemented simply with standard computer programs for univariate GARCH. Therefore, this two-step CCC-MGARCH model holds considerable promise for estimating covariances of returns where there are a large number of stocks.

Proceeding from the two-step DCC-MGARCH of Engle and Sheppard (independently of ABCD), Coyle (2007) proposed an equivalent two-step CCC-MGARCH approach to estimate the variance and covariance matrix for agriculture commodity prices. He first estimated the variance for each price with a GARCH (1, 1) model with conditional disturbances $\varepsilon_{it} = \sqrt{h_{it}}\, u_{it}$. Then he ran pair-wise regressions on the estimated standardized residuals from the first step to estimate the correlation matrix. The pair-wise regressions are

$$\hat{u}_{it} = a_{ij,0} + \beta_{ij}\hat{u}_{jt} + v_{ij,t}\text{, then } \beta_{ij} = \frac{Cov(u_{it}, u_{jt})}{Var(u_{it})} = \rho_{ij}.$$[12]

The estimate for the time varying covariance is: $Cov(\varepsilon_{it}, \varepsilon_{jt}) = \beta_{ij}\sqrt{h_{it}h_{jt}}$. By construction, the CCC matrix is positive semi-definite.

[12] The results depend on the standard assumption that u has mean zero and variance one.

Appendix 2.C. 88 Companies from S&P 100 Index Components

Table 2.11 88 Companies from S&P 100 Index Components

Appendix C: 88 Companies from S&P 100 index Components					
No.	**siccode**	**variable**	**Name**	**CUSIP**	**Ticker**
1	7370	COL1	ORACLE CORP	68389X10	ORCL
2	7370	COL2	MICROSOFT CORP	59491810	MSFT
3	3724	COL3	HONEYWELL INTERNATIONAL INC	43851610	HON
4	3572	COL4	E M C CORP MA	26864810	EMC
5	3571	COL5	UNISYS CORP	90921410	UIS
6	3570	COL6	DELL INC	24702R10	DELL
7	2086	COL7	COCA COLA CO	19121610	KO
8	2892	COL8	DU PONT E I DE NEMOURS & CO	26353410	DD
9	3861	COL9	EASTMAN KODAK CO	27746110	EK
10	2911	COL10	EXXON MOBIL CORP	30231G10	XOM
11	3731	COL11	GENERAL DYNAMICS CORP	36955010	GD
12	3641	COL12	GENERAL ELECTRIC CO	36960410	GE
13	3711	COL13	GENERAL MOTORS CORP	37044210	GM
14	3571	COL14	INTERNATIONAL BUSINESS MACHS COR	45920010	IBM
15	2086	COL15	PEPSICO INC	71344810	PEP
16	2111	COL16	ALTRIA GROUP INC	02209S10	MO
17	2830	COL17	AMGEN INC	3116210	AMGN
18	1381	COL18	SCHLUMBERGER LTD	80685710	SLB
19	2911	COL19	CHEVRON CORP NEW	16676410	CVX
20	5732	COL21	RADIOSHACK CORP	75043810	RSH
21	3674	COL22	TEXAS INSTRUMENTS INC	88250810	TXN
22	3724	COL23	UNITED TECHNOLOGIES CORP	91301710	UTX
23	2841	COL24	PROCTER & GAMBLE CO	74271810	PG
24	4911	COL25	SOUTHERN CO	84258710	SO
25	3531	COL26	CATERPILLAR INC	14912310	CAT
26	2844	COL27	COLGATE PALMOLIVE CO	19416210	CL
27	2834	COL28	BRISTOL MYERS SQUIBB CO	11012210	BMY
28	3721	COL29	BOEING CO	9702310	BA
29	3546	COL30	BLACK & DECKER CORP	9179710	BDK
30	2834	COL31	ABBOTT LABORATORIES	282410	ABT
31	2821	COL32	DOW CHEMICAL CO	26054310	DOW
32	2621	COL33	INTERNATIONAL PAPER CO	46014610	IP
33	2834	COL34	PFIZER INC	71708110	PFE
34	2834	COL35	JOHNSON & JOHNSON	47816010	JNJ
35	2891	COL36	3M CO	88579Y10	MMM

Appendix C: 88 Companies from S&P 100 index Components (continued)					
No.	siccode	variable	Name	CUSIP	Ticker
36	2834	COL37	MERCK & CO INC	58933110	MRK
37	2013	COL38	SARA LEE CORP	80311110	SLE
38	2033	COL39	HEINZ H J CO	42307410	HNZ
39	1389	COL40	HALLIBURTON COMPANY	40621610	HAL
40	4911	COL41	ENTERGY CORP NEW	29364G10	ETR
41	7311	COL42	CLEAR CHANNEL COMMUNICATIONS INC	18450210	CCU
42	4911	COL43	AMERICAN ELECTRIC POWER CO INC	2553710	AEP
43	3334	COL44	ALCOA INC	1381710	AA
44	3812	COL45	RAYTHEON CO	75511150	RTN
45	2032	COL47	CAMPBELL SOUP CO	13442910	CPB
46	7996	COL49	DISNEY WALT CO	25468710	DIS
47	3571	COL50	HEWLETT PACKARD CO	42823610	HPQ
48	3841	COL51	BAXTER INTERNATIONAL INC	7181310	BAX
49	3577	COL52	XEROX CORP	98412110	XRX
50	2879	COL53	WILLIAMS COS	96945710	WMB
51	6021	COL54	WELLS FARGO & CO NEW	94974610	WFC
52	3577	COL55	SPRINT NEXTEL CORP	85206110	S
53	2421	COL56	WEYERHAEUSER CO	96216610	WY
54	7373	COL57	COMPUTER SCIENCES CORP	20536310	CSC
55	2844	COL58	AVON PRODUCTS INC	5430310	AVP
56	5111	COL60	OFFICEMAX INC	67622P10	OMX
57	3312	COL61	ALLEGHENY TECHNOLOGIES	01741R10	ATI
58	5812	COL62	MCDONALDS CORP	58013510	MCD
59	3678	COL63	TYCO INTERNATIONAL LTD NEW	90212410	TYC
60	6021	COL64	JPMORGAN CHASE & CO	46625H10	JPM
61	5331	COL65	TARGET CORP	876120000000000	TGT
62	4011	COL67	BURLINGTON NORTHERN SANTA FE CP	12189T10	BNI
63	3674	COL68	NATIONAL SEMICONDUCTOR CORP	63764010	NSM
64	6211	COL69	MERRILL LYNCH & CO INC	59018810	MER
65	5331	COL70	WAL MART STORES INC	93114210	WMT
66	6141	COL71	AMERICAN EXPRESS CO	2581610	AXP
67	3679	COL72	INTEL CORP	45814010	INTC
68	6021	COL73	BANK OF AMERICA CORP	6050510	BAC
69	3845	COL74	MEDTRONIC INC	58505510	MDT
70	6324	COL75	C I G N A CORP	12550910	CI

Appendix C: 88 Companies from S&P 100 index Components (continued)					
No.	siccode	variable	Name	CUSIP	Ticker
71	5621	COL76	LIMITED BRANDS INC	53271610	LTD
72	4011	COL77	NORFOLK SOUTHERN CORP	65584410	NSC
73	4813	COL78	VERIZON COMMUNICATIONS	92343V10	VZ
74	4813	COL79	A T & T INC	00206R10	T
75	6021	COL80	U S BANCORP DEL	90297330	USB
76	5211	COL81	HOME DEPOT INC	43707610	HD
77	6331	COL82	AMERICAN INTERNATIONAL GROUP INC	2687410	AIG
78	6021	COL83	CITIGROUP INC	17296710	C
79	3533	COL84	BAKER HUGHES INC	5722410	BHI
80	3674	COL85	CISCO SYSTEMS INC	17275R10	CSCO
81	7993	COL86	HARRAHS ENTERTAINMENT INC	41361910	HET
82	8060	COL87	H C A INC	40411910	HCA
83	2830	COL88	MEDIMMUNE INC	58469910	MEDI
84	4911	COL89	A E S CORP	00130H10	AES
85	7812	COL90	TIME WARNER INC NEW	88731710	TWX
86	4922	COL91	EL PASO CORP	28336L10	EP
87	6331	COL92	ALLSTATE CORP	2000210	ALL
88	6211	COL93	LEHMAN BROTHERS HOLDINGS INC	52490810	LEH

Footnotes for Appendix 2.C: Variable name for each stock were kept as the result from the time downloading data from CRSP, i.e the original column number. For example No. 88 stock is COL93. This is done in case there is need to track back to original data set.

CHAPTER THREE: AN EMPIRICAL COMPARISION OF STANDARD MEASURES OF AGGREGATE RISK OF STOCK MARKET RETURNS VERSUS MEASURES BASED ON DISAGGREGATE RETURN DATA

Abstract

Due to loss of information and misspecification, simple aggregate modeling of risk at a portfolio level using GARCH models in theory leads to errors in estimation of risk (Nijman and Sentana 1996; Komunjer 2001; Jondeau 2008). We aggregate a variance-covariance matrix of returns (estimated in chapter two using DCC-MGARCH) to an aggregate risk index. We also examine the empirical differences between the two measures of risk. The correlations between them range from +0.80 to 0.83 for our data set depending on methods of aggregating return and GARCH modeling. T-ratio tests reject the null hypothesis that the correlation is one.

In addition, we quantify the value to stockholders of switching from aggregate to disaggregate estimation of risk. The economic value (performance fee) for the improved estimates of risk is approximated in two ways: a comparison of portfolios based directly on aggregate versus disaggregate estimates of risk, and of more interest a comparison of these portfolios using realized returns. Results for the two approaches are broadly similar. On average, investors are willing to pay 4% of their total return to switch to a disaggregate measure of risk using in-sample data. This suggests that shareholders can benefit from using estimates of risk calculated by the disaggregate approach.

Keywords: Aggregation, MGARCH, Loss of Information, Aggregate Risk, Correlation, Performance Fee

3.1 Introduction

Financial econometrics centers on how to get good estimates and predictions of risk, which is of particular interest for risk management and portfolio construction. Andersen, Bollerslev, Christoffersen and Diebold (2007) pointed out that one important issue in measuring risk is the level of aggregation. There are two practices in risk modeling: aggregate modeling of risk at the portfolio level and modeling at the individual asset level. Aggregate modeling of risk at the portfolio level typically is to apply a GARCH model on a stock return index, for example, the S&P 100 index, and get the estimated risk of the stock return. However, economic and econometric theory suggests that there may be significant errors in estimation of aggregate return models.

Stock market price and return indexes are designed for different purposes. For example, Gourieraux and Jasiak (2001) classify these as measures of asset price evolutions, benchmarks for portfolio management, support of derivatives, and economic indicators. These can involve various weighting schemes for prices or returns, e.g. equal weights, fixed weights, particular weights corresponding to a hypothetical portfolio different from the market, and "value weights" corresponding to capitalizations in the market as a whole. In principle, weights may also be chosen in accordance with index number theory, although this is unusual in finance.

In many cases price or return indexes are intended to measure average experience in the historical stock market. For example, Cowles wrote: "*The purpose of the Cowles Commission common-stock indexes is to portray the average experience of those investing in this class of security ... The indexes of stock prices are intended to represent ... what would have happened to an investor's funds if he had bought at the* *beginning ... all stocks quoted on the New York Stock Exchange, allocating his purchases among the individual issues in proportion to their total monetary value, and each month ... had by the same criterion redistributed his holdings among all quoted stocks*" (Cowles 1939, p. 2). S&P price indexes and other value-weighted indexes continue this tradition.

There has also been considerable interest in measuring average experience with risk in the historical stock market. The standard approach in finance has been to proxy aggregate risk of returns as an estimate of a variance of a return index (e.g. value-weighted) in levels. Many studies estimate univariate GARCH models using such return indexes (e.g. see Bollerslev, Chou and Kroner, 1992 for references). We will refer to this standard approach as an "aggregate" approach to measuring aggregate risk. This is in contrast to approaches (developed here) based on estimation using disaggregate returns for individual stocks.

This chapter investigates the value of a more theoretically justified approach to measuring aggregate risk directly from disaggregate data, in comparison to the standard aggregate approach. This alternative approach is based on MGARCH estimates of variances and covariances of individual returns (from the previous chapter). In finance, it is important to estimate stockholders' willingness to pay for improved information about aggregate risk from the complex disaggregate approach relative to the simpler aggregate approach. This has not been done in previous studies. We quantify the value to stockholders of switching from aggregate to disaggregate estimation of risk.

The chapter is outlined as follows. Section 3.2 addresses the issue of univariate estimation of aggregate returns versus multivariate estimation of disaggregated returns (individual stock returns) as steps in measuring aggregate risk of returns. Univariate GARCH estimation is certainly much simpler than MGARCH estimation, but in theory, there are substantial errors (discussed below) in aggregation/specification in an aggregate returns econometric model. Therefore, in this respect multivariate estimation of individual stock returns should provide a more accurate measure of aggregate risk than does the simpler aggregate_approach. This section shows (for the data set in chapter two) that there is a significant empirical difference between indexes as measured by these univariate and multivariate approaches. These results together with theory suggest that MGARCH estimation of individual stock returns is more appropriate than univariate GARCH estimation of an aggregate return index as in the standard aggregate approach, unless there is inadequate time to estimate MGARCH models.[13]

Section III quantifies the value to stock holders of this disaggregate approach to measuring aggregate risk of returns. Portfolios are calculated for both the simpler aggregate and improved measures of aggregate risk. Then the economic value (performance fee) for the improved estimates of risk are approximated in two ways: a comparison of portfolios based directly on aggregate versus disaggregate estimates of risk, and of more interest a comparison of these portfolios using realized returns. Results are broadly similar for in sample data, suggesting a substantial economic benefit to stockholders using estimates of risk calculated by disaggregate rather than aggregate methods.

[13] Bauwens, Laurent and Rombouts (2006) mentioned that: "*A GARCH model can be fit to the portfolio returns for given weights. If the weight vector changes, the model has to be estimated again.*"

3.2 Univariate versus Multivariate Approaches to Measuring Aggregate Risk for Stock Market Returns

In finance an aggregate return index is commonly constructed as a ratio of the value of a stock market in adjacent periods, e.g. as a ratio of the S&P 500 in adjacent periods. These are “value-weighted" indexes. Denote the value of the stock market at time t as p_t y_t , where p is a row vector of stock prices and y is a column vector of quantities of stocks. Then the aggregate return is

(1) $R_t = p_t\, y_t \,/\, p_{t-1}\, y_{t-1}$

$= \Sigma_i\, (p_{it}/p_{it-1})\, (p_{it-1}\, y_{it} \,/\, p_{t-1}\, y_{t-1})$

$= \Sigma_i\, r_{it}\, w_{it-1}$

where $r_{it} = p_{it}/p_{it-1}$ (gross returns for stock i) and $w_{it-1} = p_{it-1}\, y_{it} \,/\, p_{t-1}\, y_{t-1}$. Here R_t is an aggregate return index in levels (not ratios). If $y_t = y_{t-1}$, then w_{it-1} is the share of stock i in total value of the stock market at t-1. Then R_t weights the vector of returns r_t by a vector of shares w_{t-1}, and the weights w change over time. Since w_t and w_{t-1} are likely to be very similar (especially for daily data), a closely related index would weight r_t by shares w_t. These can be interpreted as value-weighted indexes of aggregate returns. Of course in the long run, stock market quantities y cannot be constant, as stocks enter and exit the market.

Alternatively an aggregate return index could be defined with fixed weights as r_t w_0 where w_0 is a vector of fixed weights (constant for all periods in the index formula). This can be interpreted loosely as a fixed base Laspeyres index in levels.

Based on the above discussion, this section will focus on two simple indexes for aggregate return. A value-weighted aggregate return index is defined as $R^{VW}{}_t = \Sigma_i\, r_{it}\, w_{it}$ where w_t is a vector of capitalization shares for period t ($w_{it} = p_{it}\, y_{it} / \Sigma_j\, p_{jt}\, y_{jt}$). A fixed base Laspeyres return index is defined as $R^L{}_t = \Sigma_i\, r_{it}\, w_{i0}$ where w_0 are weights (capitalization shares) that are constant over time in the index formula. These indexes are defined in levels (rather than in ratios).

In finance, aggregate risk for returns is typically constructed as a measure of variance for an aggregate return index in levels, such as above. For example in the case of a fixed base Laspeyres return index $R^L{}_t = r_t\, w_0$, a variance of index R^L can be expressed equivalently as $var(r_t\, w_0) = w_0{}^T Vr_t\, w_0$ (treating w_0 as non-stochastic) where Vr is a variance- covariance matrix of returns.[14] Here the variance of the index R corresponds to an index aggregating Vr for individual stocks. The variance is estimated simply from univariate GARCH models of the aggregate return index or by more traditional ad-hoc methods (e.g. historical method).[15] We will refer to these univariate approaches to measuring aggregate risk from aggregate returns as "aggregate" methods of estimating aggregate risk for returns.

However economic and econometric theory suggests two difficulties with this aggregate approach: there may be significant errors in estimation of aggregate return models, and simple indexes such as (1) or Laspeyres of aggregate risk in prices or returns may not be appropriate (based on basic index number theory, a simple Laspeyres return

[14] The covariance matrix of returns Vr corresponds in notation to Ht in chapter one. This notation Vr is more convenient to this and the next chapter.

[15] Market risk management practitioners in financial institutions typically measure aggregate risk for a portfolio using historical simulation of risk for aggregate returns for the portfolio. However apparently these models are outperformed by a simple univariate GARCH model for aggregate returns (Berkowitz and O'Brien 2002; Andersen, Bollerslev, Christoffersen and Diebold 2005).

index reflects the contribution of return to investor welfare, i.e. utility, only under very restrictive assumptions on the utility function, as discussed in chapter 3). This section addresses the first problem, and the second problem is discussed in the next chapter.

It is well known in economics that aggregation of information (e.g. regarding commodities or agents) leads to a loss of information, which usually implies incorrect specification of theoretical or econometric models.[16] In the case of linear models, aggregate models are correctly specified if (key) coefficients of the micro models are identical, or if information for all commodities or agents covaries perfectly over the period of a study (or, less restrictively, if changes in the relative distribution of micro variables are independent of changes in the mean of the distribution of the variables, Lewbel 1992). Neither of these conditions is likely. As a result, misspecification in aggregate econometric models leads to poor estimators and tests, and these problems may often be quite serious.

There is a very large literature on this subject in the context of optimization models (e.g. Deaton and Muellbauer 1980; Blundell and Stoker 2005), and there is also a substantial literature in the general context of econometric estimation (e.g. Kelejian 1980; Pesaran, Pierse and Kumar 1989). Tests have been developed for choosing between alternative disaggregate and aggregate specifications to predict aggregate variables (Grunfeld and Griliches, 1960; Pesaran, Pierce and Lee, 1994).

[16] To give a simple example, consider two regression equations $y_1=a_1+b_1*x_1+e_1$ and $y_2=a_2+b_2*x_2+e_2$, and define two aggregates $y= y_1+y_2$, $x=x1+x2$. Then aggregate y cannot generally be specified correctly in terms of aggregate x, i.e. as $y=a+bx+e$, unless $b_1=b_2$ (Note that by definition $y= y_1+y_2= a1+a2+ b_1*x_1+ b_2*x_2+ e_1+e_2 \neq a1+a2+ b(x_1+ x_2)+ e_1+e_2$ unless $b_1=b_2$, i.e. y depends not just on x but also on the distribution of x between x_1 and x_2.

Issues in aggregation of nonlinear models are much more complex, particularly when the underlying models are dynamic (e.g. Kelejian, 1980; van Garderen, Lee and Pesaran, 2000, VLP henceforth). Nevertheless selection techniques have been developed for choosing between simple nonlinear aggregate and disaggregate models (VLP,2000).

Since GARCH models are nonlinear and dynamic, we would expect (from the general literature) substantial errors in aggregation of GARCH models of returns for individual stocks. Moreover, given the complexity of GARCH models, development of formal tests for choosing between aggregate and disaggregate models of risk may be quite difficult. Based on the related GARCH literature, these conjectures seem to be correct.

There is a small literature on contemporaneous aggregation of GARCH models (in contrast to a larger literature on temporal aggregation). Nijman and Sentana (1996) considered a simple case of the sum of two independent univariate GARCH(1,1) processes. Aggregation leads to a substantially more complex parametric structure (a "weak" GARCH(2,2) rather than a "strong" GARCH(1,1)), and standard techniques (e.g. quasi-maximum likelihood) lead to inconsistent estimators of parameters of the aggregate process. Inconsistency is in part due to misspecification of conditional variance for the aggregate (Komunjer 2001). There are no tests for choosing between alternative disaggregate and aggregate specifications to predict aggregate risk within a GARCH framework.

Jondeau (2008) studies GARCH(1,1) for individual asset returns and a diagonal vec multivariate model of conditional covariances (Bollerslev, Engle and Wooldridge 1988). The aggregate return model is a weak GARCH process with an infinite number of lags reflecting moments of cross section distributions of parameters. Jondeau proposes a truncated estimation procedure adopting a flexible parametric approximation to a cross section distribution. This appears to improve upon standard procedures (and other proposed procedures, e.g. Komunjer). However, this estimator depends critically upon an assumption of fixed weights for individual returns in aggregation. Variable weights would substantially complicate cross section distributions and estimation of the aggregate model, as is recognized by Jondeau. Unfortunately, fixed weights are contrary to the popular value-weighted indexes of returns and are also restrictive in terms of index number theory (index number theory is discussed in the next chapter).

Jondeau also presents simulations to evaluate the importance of the aggregation bias to potential investors in portfolios. The standard aggregate GARCH(1,1) approach is compared to his proposed approach to measuring aggregate risk for portfolios. Treating his approach as correct, the aggregation bias has substantial implications for investors (in these simulations, investors would be willing to pay approximately one fifth of expected return in exchange for switching from aggregate to proposed measures of aggregate risk).

Alternatively we can avoid these errors in aggregation by estimating risk of returns directly at the level of individual stocks and then aggregating variance-covariance matrices of return risk up to an aggregate index of return risk. Multivariate GARCH models can be estimated directly from data on returns for individual stocks (or ad-hoc methods such as RiskMetrics can be applied to this data), and estimates of variance-covariance matrices of returns can be combined into an aggregate index of return risk.

Here we compare the aggregate univariate GARCH approach and the MGARCH approach to measuring aggregate risk of returns using data and MGARCH estimates from chapter two. There is no formal test in the literature for such a comparison. Instead, we are limited to comparing measures of aggregate risk for the two approaches, assuming that the alternative to the aggregate approach is more correct (Jondeau offers one approach to such a comparison). Here we shall simply calculate that the correlation between the two measures of aggregate risk is positive but far from perfect, so that there is a substantial difference between the two measures of aggregate risk in empirical practice. Given the strong theoretical arguments against the aggregate approach, these differences in correlation suggest the use of the MGARCH-based approach rather than the aggregate approach in empirical practice.

Laspeyres-type indexes of aggregate risk for returns can be constructed from estimated variance-covariance matrices of returns as follows. Let Vr_t be the nxn variance-covariance matrix of returns for n stocks at time t (relative to t-1). Assuming a fixed base 0, a Laspeyres-type return risk index in levels can be defined as

(2) $VR_t = w_0{}^T Vr_t\, w_{0,}$

where $w_{i0} = p_{i0}\ y_{i0} / p_0\ y_0$ (share of stock i in total capitalization in period 0). Apparently such approaches to an index of aggregate risk have not been applied in finance.[17] This is only value-weighted at t=0 unless you rebalance continuously.

In this section we study how sensitive aggregate risk is to alternative MGARCH specifications. Univariate and multivariate approaches to Laspeyres-type indexes of aggregate risk in returns are compared using data on daily returns for 88 individual stocks on the S&P 100 index (January 1995 - December 2005) and multivariate GARCH

[17] Assuming a moving base t-1, the Laspeyres-type return risk index in levels is $VR_t = w_{t-1}{}^T Vr_t\ w_{t-1}$ where $w_{it-1} = p_{it-1}\ y_{it-1} / p_{t-1}\ y_{t-1}$ (share of stock i in total capitalization in period t-1). For simplicity, such moving base models will not be considered in this chapter.

estimates from chapter two. First, a Laspeyres-type index (in levels) of aggregate returns is constructed as

(3) $R_t = \Sigma_i\ r_{it}\ w_i$

where $r_{it} = (p_{it}\ /\ p_{it-1})$ (gross return of stock i in period t) and fixed weights w are calculated using average capitalizations over the data set (w_i = $avgcap_i\ /\ \Sigma_j\ avgcap_j$). Then a variance measure of aggregate risk of returns is calculated by applying the following four univariate methods to this index of aggregate returns: a simple univariate GARCH(1,1), a GARCH(1,1)-ARMA(2,1), an EGARCH(1,1) and an ARMA(2,1)-EGARCH(1,1). These simple univariate GARCH models exclude GARCH-in-mean (to be discussed later in section).

Second, multivariate estimates of variance-covariance matrices (88x88) Vr_t of risk in individual returns are used to construct Laspeyres-type indexes of aggregate returns. Using estimates of Vr_t, a Laspeyres-type index (in levels) of aggregate risk for returns is constructed as

(4) $VR_t = w^T Vr_t\ w$

where fixed weights w are calculated using average capitalizations over the data set (w_i = $avgcap_i$).

MGARCH models include both simple constant conditional correlations (CCC) and dynamic conditional correlations (DCC) of Engle and Sheppard. Variance-covariance matrices Vr_t of risk in returns for individual stocks are estimated by the following six multivariate methods: GARCH(1,1)-CCC, ARMA(2,1)-EGARCH(1,1)-CCC, GARCH(1,1)-DCC, ARMA(2,1)-EGARCH(1,1)-DCC, historic and the RiskMetrics

Method. These estimates of Vr_t by the six methods are applied to indexes (4) of aggregate risk for returns.

Correlations between all these indexes of aggregate risk of returns for alternative univariate and multivariate GARCH approaches are presented in Tables 3.1 - 3.3. Table 3.1 presents correlations within four univariate (aggregate) measures with fixed bases. For Laspeyres-type indexes, correlations exceed +0.99 within GARCH models and within EGARCH models (+0.92 across GARCH and EGARCH models). Similar results hold for value-weighted indexes. A univariate GARCH model can be estimated for a value-weighted aggregate return index (e.g.) $R_t = w_t \; r_t$, where w_t are shares in capitalizations on day t.

Table 3.2 presents correlations within three multivariate measures, with both fixed bases and value-weighted bases. A value-weighted aggregate risk index can be constructed from MGARCH estimates of risk as (e.g.) $VR_t = w_t^T Vr_t \, w_t$. For multivariate measures all correlations exceed +0.98.

Table 3.3 presents correlations between (across) analogous univariate and multivariate measures, and this table is of the most interest. For example, consider the most general univariate GARCH model, EGARCH(1,1)-ARMA(2,1) and the corresponding DCC-MGARCH model. The correlation between the associated simple aggregate risk index and the aggregate risk index based on MGARCH (with time trend) is +0.839 and +0.813 for Laspeyres indexes and value-weighted indexes, respectively. The average (over all days) ratio of MGARCH to aggregate approach is 1.1382. Similar correlations hold for other models. Correlations in Tables 3.3 are calculated by running regression between standardized measures of risk. Simple T-ratios are calculated to test the null hypothesis that the correlation is one. To correct for heteroskedasticity, white-corrected ratio is also calculated. Both suggest a rejection of the null hypothesis. A time trend is also added to the regression. The coefficient for the time trend is insignificant.

An alternative aggregate approach to measuring aggregate risk is to estimate a GARCH-in-mean model of aggregate returns rather than a simple univariate GARCH model of aggregate returns.[18] Hence, a univariate GARCH-in-mean approach to measuring aggregate risk is briefly considered here. The EGARCH-in-mean model is $R_t = \alpha_0 + \alpha_1 R_{t-1} + \alpha_2 h_t + \varepsilon_t$, $\varepsilon = h^{1/2} u$, $\ln(h_t) = \beta_0 + \beta_1 \ln(h_{t-1}) + \beta_2 u_{t-1} + \beta_3 |u_{t-1}|$. The correlation between aggregate risk for this aggregate (univaraite) approach and the previous aggregate (univariate) approach is +0.99 for both Laspeyres and value-weighted indexes. The correlation between aggregate risk for this aggregate (univariate) approach and the MGARCH (disaggregate) approach is +0.82 for Laspeyres and +0.81 for value-weighted indexes, which is similar to results for the first aggregate approach.

In sum, in the cases of both Laspeyres and value-weighted index approaches, correlations between aggregate (univariate) and MGARCH-based indexes of aggregate risk (albeit high at approximately +0.8) are substantially lower than correlations between alternative aggregate indexes and between alternative MGARCH-based indexes (primarily +0.98). In principle, both Laspeyres-type, or value-weighted aggregate and MGARCH-based risk indexes in levels are equivalent overlooking econometric issues, but in theory errors in aggregation in estimation of aggregate models are serious.

[18] However this alternative approach assumes an aggregate return-aggregate risk tradeoff and the empirical literature on such a tradeoff is quite mixed (Lettau and Ludvigson 2010). Therefore, the simple univariate GARCH aggregate approach may be more robust.

Empirical results on correlations between the two measures provide preliminary evidence that the errors in aggregation may not be trivial.

The substantial empirical differences between these two methods suggest (but of course do not prove) that the more complex MGARCH approach to a Laspeyres-type or value-weighted index of aggregate risk in returns is preferable, at least for this data set. However the critical issue for investors is whether empirical differences between aggregate and disaggregate methods have economic value, i.e. would investors be willing to pay substantial performance fees for disaggregate estimates of risk relative to aggregate estimates. The next section addresses this important question.

3.3 Measuring the Value to Stock Holders of the Disaggregate Approach to Estimating Aggregate Risk of Returns

In this section I measure the value to stock holders of the more theoretically justified disaggregate approach to measuring aggregate risk of returns, relative to the standard aggregate approach. First, in part A, alternative portfolios are calculated based on these two measures of aggregate risk and mean-variance risk preferences, and preliminary measures of the economic value of the disaggregate approach are presented. Then, in part B, the economic value of the disaggregate approach is calculated in an alternative and more interesting manner related to realized returns. Results suggest substantial economic benefit to stockholders using estimates of risk calculated by disaggregate rather than aggregate methods.

3.3. A Comparisons of Portfolios based on Aggregate and Disaggregate Approaches to Aggregate Risk, and Preliminary Estimates of Economic Value of Disaggregate Approach

This section of the paper calculates alternative portfolios based on measures of aggregate risk using a disaggregate approach and a aggregate approach. Preliminary estimates of the economic value of the disaggregate approach are also calculated. These calculations assume a standard portfolio optimization problem in finance. Most investors make asset allocation decisions among different sets of stocks rather than between individual stocks. The 88 stocks from the S&P 100 are placed into 5 groups based on the Fama-French 5 industry classifications, resulting in aggregate returns for these 5 groups. The individual asset weights in each group are exogenous and value-weighted (rather than Laspeyres). Table 3.4 present descriptive statistics of return for the five groups over the sample period from 1995 to 2005. The goal is to construct a covariance matrix of risk across these 5 groups that can be used by stockholders in investing across these 5 groups. In this context, a "aggregate" approach is to estimate an MGARCH model directly from data on aggregate returns for these 5 groups, resulting in a 5x5 covariance matrix VR^{sc} of risk across the 5 groups. In contrast, a disaggregate approach is to estimate an MGARCH model over return data for the individual 88 stocks (rather than the 5 groups) and then aggregate estimates of multivariate risk across the 88 stocks into a 5x5 covariance matrix VR^{mv} across the 5 groups. In detail, the aggregate group variances and covariances were calculated as $Vr_m = q_m^T Vr_m q_m$, $Vr_{mn} = q_m^T Vr_{mn} q_n$, where q_m is the value weight for stocks in group m, Vr_m is the covariance matrix of returns for stocks in group m, and Vr_{mn} is the covariance matrix of returns for stocks across the two groups m and n (m=1,2..5; n=1,2…5). In the end, we get a corresponding daily aggregate 5 by 5 variance covariance matrix.

Suppose a representative stockholder chooses an optimal portfolio among the 5 groups and has mean-variance risk preferences. Assuming returns are joint normal or quadratic utility function over wealth, a mean variance utility function is sufficient to capture the investors' portfolio allocation decision. To develop our methodology, assume a representative investor has the following mean variance utility function,

(5) $\text{Max}_{w}\, U = \bar{r}w - \frac{\lambda}{2} w'VRw$

where $\bar{r}$ is a column vector of (expected) returns for the 5 groups and risk free return r_0; w is a column vector of portfolio shares for the 5 groups and a risk free asset (sum of shares equals 1); VR is a 5x5 covariance matrix of returns across the 5 groups, and λ is the coefficient of absolute risk aversion (e.g. Ingersoll 1980). Investors will allocate their wealth between the risk free asset and the risky assets based on the estimated variance covariance matrix. Expected returns simply calculated as the average daily returns over our sample period, i.e. expected return vector $\bar{r}$ is held constant over time. Since the purpose is to see how different measures of risk affect investors' portfolio decisions, holding expected returns constant for individual groups seem proper. Each day the investor rebalances his portfolio according to the estimation of variance covariance of returns. Let (w^{*sc}, w_0^{*sc}) be the utility maximizing portfolio given VR^{SC}, and let (w^{*mv}, w_0^{*mv}) be the utility maximizing portfolio given VR^{MV}. Here, w* denotes the allocation of investment to the five group stocks, and w_0* denotes the allocation of investment to the risk free asset. The six weights add up to one.

We can easily calculate a preliminary crude estimate of the economic value of information from the disaggregate approach relative to the aggregate approach as follows.

Assuming that VR^{MV} (rather than VR^{SC}) is the true measure of aggregate risk VR (5x5), then the willingness to pay (performance fee) for information VR^{MV} relative to VR^{SC} is defined as

(6) $v = U(w^{*mv}, w_0{}^{*mv}, R, r_0, VR^{MV}) - U(w^{*sc}, w_0{}^{*sc}, R, r_0, VR^{MV})$

I.e., as a difference in certainty equivalents evaluated at the "true" VR^{MV}. R denotes the expected returns for the 5 groups. In this study, we assume that no short sales on risky assets are allowed, but allow for risk free borrowing and lending.

Note that v>0 by construction, since both portfolios are assessed at the presumed true measure VR^{MV} of aggregate risk. Therefore, these measures of economic value may be of less interest than measures to be presented in the next section, where economic value is not positive by construction. This provides a preliminary measure of economic loss from using aggregate approach assuming that the MGARCH disaggregate model is the true model. In this respect, a comparison of the alternative measures of economic value is of some interest.

Optimal portfolios and performance fees were calculated on a daily basis both in sample and out of sample. In sample calculations used the daily data for 1995 - 2005 and MGARCH constant conditional correlation and dynamic conditional correlation models. Theory implies that investors will choose the same risky portfolio composed by the five group returns independent of risk preferences, but of course, this will vary with the variance covariance matrix. A comparison of this optimal risky portfolio for different measures of risk is presented in Table 3.5. For each of the five groups of stocks, differences in shares for the two approaches are calculated daily, $(w^{mv} - w^{sc})_{it}$, and the absolute values are averaged over 1995 - 2005. w^{mv} and w^{sc} denote the weights of five group stocks in the optimal risky portfolio. By this criterion there are substantial differences in optimal risky portfolios: (e.g. for CCC models) 11% for group 2 and 20% for group 5 (the smallest difference is 2% for group 1). Since the final decision of asset allocation is simply to allocate assets between risk free asset and risky portfolio, alternative measures of risk VR^{mv} versus VR^{sc} have a substantial impact on final optimal portfolio choice.

In sample estimates of performance fees (willingness to pay) for information VR^{mv} relative to VR^{sc} are presented in Table 3.6 for several common values for λ in finance literature. [19] These are averages of daily performance fees over 1995 - 2005 (by construction performance fees are positive for all days). The performance fee is positive and large for all λ . For example for CCC models and λ = 2, the average daily performance fee is 3 basis points. The mean daily-expected return for the optimal portfolio is calculated as 46 basis points for an investor using an aggregate approach model. Therefore, the performance fee here is, on average, 7% of the optimal return. Performance fees range from 3.12 to 0.62 basis points, i.e. from 6% to 7.3% of optimal return (and are decreasing in λ , because investors substitute away from risky assets as λ increases and the importance of correct risk measurement declines). When expected returns are high as in this data set (1995 - 2005), investors may well be willing to pay

[19] Here the portfolio vector w is defined as shares, so $\Sigma_i w_i = 1$. This means that, in this particular mean-variance maximization problem, the total initial wealth allocated between stocks is 1. Then in this case the coefficient of absolute risk aversion λ is identical to the coefficient of relative risk aversion ρ, i.e. $\lambda = \rho$ if wealth W = 1 (see next section).

higher fees for more accurate information about risk than in normal times. In any case, these are substantial fees.[20]

The performance fee measure derived above is calculated as a difference in utility, which depends on differences in expected returns as well as differences in risk. Merton (1980) argues that variances and covariances of returns can typically be estimated with greater precision than expected returns. In order to relate performance fees more directly to risk, we also consider the following risk minimization problem given a target portfolio return R*:

$$(7)\ \min_{q}\ q^{T}\, VR\, q$$
$$\text{s.t.}\quad q^{T} R = R^{*}$$

where q is a column vector of portfolio shares for the 5 risky groups (sum of shares equals 1), VR is a 5x5 covariance matrix of returns across the 5 groups, and R is a column vector of (expected) returns for the 5 groups. Here we assume no risk free asset. Let q^{*sc} be the minimum variance portfolio given VR^{SC}, and let q^{*mv} be the minimum variance portfolio given VR^{MV}. Hence, we also examine the risk difference for a minimum variance portfolio with given target return between our two approaches of measuring risk. The risk difference, $q^{*sc,T} VR^{mv} q^{*sc} - q^{*mv,T} VR^{mv} q^{*mv}$, is a measure of the risk reduction from adopting an individual asset based MGARCH method rather than a aggregate method. The comparison is based on the assumption that MGARCH variance

[20] Sharpe ratios for optimal risky portfolios are reported in Table 3.6 as well. The Sharpe ratio increases when we allocate assets using the disaggregate model of risk. It should be noted here that the Sharpe ratio will grow at the rate of the square root of time assuming returns are iid. If we want to compare the Sharp ratio calculated from a series of daily holding period rates of return with those from series of weekly holding period returns, we should multiply the daily sharp ratio by the square root of 7 (see Bodie, Kane, Marcus, Perrakis, Ryan, 2000). Therefore, the measure is very comparable to Jondeau's results based on weekly data.

covariance matrix VR^{mv} is true measure of risk. Note that the economic value of aggregate risk information for the disaggregate approach is again positive by construction.

We applied the above method for risk minimization (7) assuming R*=0.001.[21] The daily minimum variance portfolio with expected return equal to 0.001 is calculated based on the two approaches of measure of risk. The mean of the risk difference is one basis point, which is 10% of the target return R*=0.001. To convert this to certainty equivalent terms, you would multiply this risk difference by λ /2. From Table 3.6, the average expected return for the utility maximization model was approximately 0.004 (including the risk free asset) for λ = 2. The risk free asset makes it difficult to compare differences in risk for the risk minimization model with previous performance fees. Nevertheless, the risk differences are consistent with the substantial performance fees in the previous model. Figure 3.2 shows the ratio of the portfolio risk derived from individual based MGARCH method and aggregate method. The mean of the ratio is 1.0202 with a standard deviation of 0.0225. For each of the five groups of stocks, the optimal weights difference is calculated similarly as before: we take the average value of share difference. By this criterion, there are substantial differences: (e.g., for CCC models) 20.01% for group 5 stock choice, 10.77% group 2 stocks, 8.54% group 3, 6.83% for group 4, and 2.16% for group one.

For out of sample calculations, an additional year (2006) of daily data on returns for the 88 stocks was collected. Similarly to the in sample case, these stocks are placed in

[21] The range of expected return over five group stocks is quite small. This leads to the difficulty to choose a target expected return that will not lead to a corner solution for risk minimization with five assets. Using a trial and error method, a target of 0.001 is the one that finally provides an optimal portfolio over ten years of data. This is the only one that can give me continuous comparison over a long period.

Fama French five industry groups. The five groups of stock returns are constructed similarly to the in sample case. Table 3.7 describes summary statistics for the out of sample industry returns. The goal is to forecast the variances and covariances of the five groups of stocks. An individual based MGARCH forecast implies that we need to forecast the variance covariance matrix of 88 stocks.

Given estimates Vr^{mv} and Vr^{sc} from 1995 - 2005 as before, daily forecasts[22] for group variances and covariances (2006) are constructed in a standard manner as explained below. As in the case of estimation, forecasts are done separately in two steps. First, we forecast the variance of each individual stock, then one step ahead forecasts of correlations are constructed assuming CCC or DCC, leading to daily forecasts Vr^{mvf} and Vr^{scf} for 2006.

A standard one step ahead GARCH (1, 1) forecast (Jondeau, Poon & Rockinger, 2007, page 88) for σ_{t+1}^2 is $\sigma_{t+1}^2 = \hat{\omega} + \hat{\alpha}_1 \hat{\varepsilon}_t^2 + \hat{\beta}_1 \hat{\sigma}_t^2$. Our forecast is a sequence of one-step ahead forecasts, using the actual, rather than the forecasted values for lagged dependent variables. However, the parameters for the forecasting are only estimated once as in Engle, Shephard, Sheppard (2008).

The second step is to forecast correlation. For constant correlation MGARCH model, the correlation forecast is simply the sample correlation. One step ahead forecast for correlation in the DCC model is: $\Omega_t = (1-\alpha-\beta)\,\overline{\Omega} + \alpha(U_{t-1}U'_{t-1}) + \beta\Omega_{t-1}$. For a large

[22] One day forecast horizons are appropriate for trading stocks, and there is agreement that volatility can be forecast with some accuracy one day ahead. Much longer forecasts are more appropriate for many other applications of financial risk management such as long term solvency of firms. However it has been argued that volatility may not be forecast-able for stocks over a horizon beyond ten days (Christofferson and Diebold 2000). This paper also argues that, by focusing on variances/covariance rather than extreme values, forecasting models in finance have missed the most important issues in risk management.

matrix it is very difficult to come up with an unbiased forecast. However, according to Sheppard's thesis, the empirical bias is very small. During the procedure of estimation, we cannot guarantee that Ω_t will have well defined variance covariance matrix with ones on the diagonal. To get a well-defined correlation matrix, we did the following transformation $R_t = \Omega_t{}^* \Omega_t \Omega_t{}^*$, where $\Omega_t{}^*$ is a diagonal matrix composed of the inverse of the square root of the diagonal element of Ω_t. Given the forecasted correlation and variance, we will be able to calculate the forecasted variance covariance matrix. Based on the forecasted variance and covariance for the 88 stocks, a 5*5 covariance matrix VR^{mvf} was constructed similarly to the in sample case. A aggregate approach to forecast the variance covariance matrix for the five groups of stock returns would simply apply the MGARCH forecast approach to the five groups of stocks and get $VR^{scf.}$.

In order to obtain an estimate of the "true" VR in 2006, MGARCH is applied to return data for the 88 stocks over 1995-2006, which leads to $VR^{mv,true}$ over 2006. Daily optimal portfolios for 2006 are calculated given VR^{mvf} and VR^{scf} as above. Performance fees for 2006 are estimated as (8) $v = U(w^{*mvf}, w_0{}^{*mvf}, R, r_0, VR^{mvtrue}) - U(w^{*scf}, w_0{}^{*scf}, R, r_0, VR^{mvtrue})$.

Out of sample portfolios and average performance fees are summarized in Tables 3.8 and 3.9. Results are fairly similar to in sample. Average absolute values of differences in optimal risky allocations by stock group vary between 2.6 to 13.8% (CCC) and between 0.1 to 13.6% (DCC), which are substantial. Average performance fees for 2006 are substantial, e.g. 4.4 basis points for CCC and λ =2. Average performance fee varies from 4.4 to 0.9 basis points.

Similarly, we apply the model of minimizing variance with target return for out of sample data. With the same target return, we can calculate the corresponding $q^{*sc,f}$ and $q^{*mv,f}$ based on two different forecast of risk. Corresponding to $VR^{mv,\,true}$,we get the true value of optimal weight $q{*^{true}}_f$. Then we can compare the ex post difference in variance of the two optimal portfolios by comparing: $q^{*true,f^T} VR^{true} q{*}^{true,f}$ - $q^{*mv,f^T} VR^{mv} q^{*mv,f}$ and $q^{*true,f^T} VR^{true} q{*}^{true,f}$ - $q^{*sc,f^T} VR^{mv} q^{*sc,f}$.

Setting the target return as 0.0011, the mean risk difference is 0.394 basis points under assumption of constant correlation. The risk difference is very small indeed. However, the share differences are substantial: 0.0388 for group one, 0.0794 for group two, 0.0910 for group three, 0.0020 for group four, and 0.1548 for group five. Setting the target return as 0.001169, the mean risk difference is 0.492 basis, mean risk ratio is 1.0061, the standard deviation of the risk ratio is 0.0904, the weight difference for group one to five are (0.0171, 0.0350, 0.0004, 0.0020, 0.0526) respectively.

In addition, we briefly consider how forecasted covariance matrices compare with a covariance matrix VR* estimated by the disaggregate method over all data including out of sample data. Perhaps VR* is a somewhat more accurate proxy for VR in the future period than are the forecasted ${VR^{SC}}_f$ and ${VR^{mv}}_f$. We compare $w^T VR * w$, $w^T VR^{sc,f} w$, $w^T VR^{mv,f} w$, where weights w are the same. We use the value weight of the five groups. Figure 3.3 shows the results of these measures of aggregate risk based on the constant correlation assumption. From the figure, we can see that $w^T VR^* w$ moves much more closely with $w^T VR^{mv,f} w$ than with $w^T VR^{sc,f} w$. Then we see the percentage difference between the estimated VR and VR*. The percentage difference between aggregate measure and the VR* matrix is calculated as ($w^T VR^{mv,f} w$ - $w^T VR^* w$)/ $w^T VR^* w$. We calculate the difference between the disaggregate forecast and the VR* measure similarly. Then we take the absolute value of the difference. The average absolute percentage differences for shortcut methods is 14.23%, and for disaggregate method it is 10.43 %.

3.3. B. Economic Value based on Realized Aggregate Returns

An alternative utility based approach for assessing volatility estimation and forecasts was developed by West, Edison and Cho (1993) and adapted to finance by Fleming, Kirby and Ostdiek (2001). This approach has various applications in finance (e.g. Fleming et al 2003; Chou and Liu, 2010; Han 2005; Thorp and Milunovich, 2007) and we shall adapt it here.

Unlike the approach in the previous section where performance fee for the disaggregate method of measuring aggregate risk is by construction positive, here performance fees can be negative as well as positive. Therefore, in this respect the approach in this section provides more interesting results for performance fees. Nevertheless, results for the two approaches are similar using in sample data.

The basic approach of Fleming et al is very simple. Assume a utility function (typically quadratic) for an event representing aggregate stock market return R_i: $U(R_i)$.

Denote sample returns i=1,..., n periods as R_1,.., R_n. Then expected utility EU for this time period is approximated as

$$(9)\ EU = \frac{\sum_{i=1,,n} U(R_i)}{n}$$

Such approximations to EU can be used to calculate the economic value of volatility forecasts.However, there is a critical assumption in this approximation: the sampled returns $R_1,\ldots,R_n$ closely approximate the prior probability distribution for returns R over the period. This assumption is questionable especially for short periods. For example, consider returns sampled daily for one year. Suppose that there is a prior non-negligible probability of an extreme event such as a stock market crash (e.g. Johansen and Sornette 1999), but the crash does not occur during the year. Then the sampled returns are not adequate proxy for the prior distribution. Moreover, aggregate returns generally reflect a complex joint distribution of returns for many stocks that may not be stationary over the period. Therefore, it appears that such an approach as (9) may well provide a very noisy approximation to expected utility. Moreover, in the literature there is a mistaken explanation of how this approach can be related to risk preferences. The following correct explanation is standard (e.g. Varian 1992).

Assume a quadratic utility function $u(W) = W - b/2\ W^2$, where W is wealth known with certainty and b is a constant independent of W. Then expected utility is

(10) $EU = EW - b/2\ EW^2$,

where E is the expectation operator. The Arrow-Pratt measure of relative risk aversion is defined as $\rho = - u''(W)\ W / u'(W)$, where u' and u'' are first and second derivatives of u with respect to W. The coefficient of absolute risk aversion is defined as $\lambda = - u''(W) /$

u'(W), so $\rho = \lambda W$ (Varian pp 189, 178). Then for the quadratic utility function $\lambda = b/(1-bW)$ and $\rho = bW/(1-bW)$. In turn $\rho * (1-b W) = bW$ implies $\rho = b(W+W \rho)$, so

(11) $b = \rho/\{W(1+\rho)\} = \lambda/(1+\lambda W)$ by $\rho = \lambda W$.

Using an average value for W and a guess for ρ or λ, we can approximate the constant coefficient b.

The analogous discussion in the literature is confused. West, Edison and Cho (1993) initially assume that b (in their notation γ) is constant, but then they "consider fixing the coefficient of relative risk aversion" ρ (p. 30), which would imply b is not constant (unless W is constant), i.e. b=b(W) (or as they say, "by fixing relative risk aversion rather than γ, we are implicitly interpreting quadratic utility as an approximation to a nonquadratic utility function, with the approximating choice of γ dependent on wealth", p. 30). However, in this case the simple relation $\rho = bW/(1-bW)$ derived above (for quadratic utility with b constant) no longer holds, although the authors invoke it. For example, suppose $u(W) = W - b(W)/2\ W^2$. Then $\rho = - u''(W)\ W / u'(W)$ does not reduce to $\rho = bW/(1-bW)$ unless b is constant, since $u' = 1 - bW - 1/2\ b'\ W^2$ and $u'' = - b - 2b'W - 1/2\ b''W^2$.

Extensions in the finance literature of the West et al. approach sometimes compound the confusion. The initial paper by Fleming, Kirby, and Ostdiek (2001) refer to the coefficient b in the utility function as absolute risk aversion (the correct interpretation is as above). Jondeau (May 2008) confuses absolute and relative risk aversion in his formula for performance fee (in (21) he mistakenly substitutes absolute risk aversion for relative risk aversion in (8) of Fleming et al (2001)).

Extensions in the finance literature can be explained more clearly and more correctly as follows. Given the above discussion, the economic value of a portfolio plan with realized wealth W_t over T periods can be approximated as

$$(12)\ EU \approx \frac{1}{T}\sum_t W_t - \frac{b}{2}\frac{1}{T}\sum_t W_t^2 ,$$

where the constant coefficient b is evaluated from a guess for ρ and average W over the period (11). Given two portfolios A and B over time with realized wealth W_t^A and W_t^B, the performance fee v relating EU for W_t^A and W_t^B is

$$(13)\ \sum_t (W_t^A - v) - b/2 \sum_t (W_t^A - v)^2 = \sum_t W_t^B - b/2 \sum_t W_t^{B\,2}$$

In order to relate this discussion directly to realized returns, suppose that in each period of one day, there is a static portfolio problem allocating shares among stocks (max EU s.t. $\sum s = 1$). For this problem initial wealth W_0 is 1 ($\sum s = 1$) and wealth at the end of the day is $W_t = W_0 R_t = R_t$. Then the EU approximation (12) can be expressed as

$$(14)\ EU \approx \frac{1}{T}\sum_t R_t - \frac{b}{2}\frac{1}{T}\sum_t R_t^2$$

If we approximate the constant b assuming W=1, then b= ρ/(1+ρ)(11) can be substituted into the above expression. This leads to the performance fee measure v in Fleming et al (2001)

$$(15)\ \sum_t (R_t^A - v) - \{\rho/2(1+\rho)\} \sum_t (R_t^A - v)^2 = \sum_t R_t^B - \{\rho/2(1+\rho)\} \sum_t R_t^{B\,2}$$

More specifically, the performance fee, v, in our case can be defined as in Fleming et al,

$$(16)\quad \left(\sum_{t=0}^{T-1}[(R^{mv}_{p,t+1}-\mathrm{v})-\frac{\rho}{2(1+\rho)}(R^{mv}_{p,t+1}-\mathrm{v})^2]\right)=\left(\sum_{t=0}^{T-1}[R^{sc}_{p,t+1}-\frac{\rho}{2(1+\rho)}(R^{sc}_{p,t+1})^2]\right)$$

$R^{mv}_{p,t+1}$ and $R^{sc}_{p,t+1}$ are the realized optimal portfolio return at each point of time obtained based on two different risk estimates or forecasts in our study. For example, $\mathrm{R}^{\mathrm{mv}}_{\mathrm{p,t+1}}=\mathrm{r_t w_t^{*mv}}$ and $\mathrm{R}^{\mathrm{sc}}_{\mathrm{p,t+1}}=\mathrm{r_t w_t^{*sc}}$, where $\mathrm{w}^{*\mathrm{mv}}$ and $\mathrm{w}^{*\mathrm{sc}}$ are optimal portfolio weights for utility maximization (5) given VR^{mv} and VR^{sc}, respectively, and r_t is a vector of realized rates of return for the five groups at time t.

Estimates for this portfolio performance fee are presented in Table 3.10. Alternative assumptions for the relative risk aversion coefficient ρ are 2, 5, and 10. The performance fees are reported for both in and out of sample. The range of the performance fee is broadly similar to the results based directly on different estimates of risk reported in the previous section. All results favor the disaggregate over the aggregate approach. The in sample fee ranges from 0.9 to 3.3 basis points daily for CCC (versus 0.6 to 3.1 basis points for the previous section) and from 0.5 to 2.3 basis points for DCC (versus 0.7 to 3.6 basis points for the previous section). The out of sample results in Table 3.11 range from 2.5 to 10.2 basis points for CCC (versus 0.9 to 4.4 basis points for the previous section), and from 2.6 to 11.0 basis points for DCC (versus 0.9 to 4.4 basis points for the previous section). On average the performance fee is around 4% of the average optimal portfolio return for in sample data, and 10% for out of sample data. Estimates of performance fees in this section and previous section are similar for in sample calculations, but estimates are quite different for out of sample calculations. The reason for the larger difference in out of sample calculations may be that the out of sample period (one year) is much smaller than the in sample period (ten years). As we mentioned at the beginning of this section, the critical assumption in this expected utility approach is that sample returns closely approximate the prior probability distribution for returns over the period. This assumption is more reasonable for sample returns over a ten year period than over a one year period.

3.3. C. Forecasting Volatility using Squared Returns/Residuals, and Utility as a Loss Function

The obvious difficulty in evaluating forecasts of volatility is that volatility is never directly measured in either the estimation or forecast periods. The standard approach has been to use the square of observed returns or residuals as a proxy for volatility in the forecast period, and to regress this against forecasts of volatility, e.g. as in Mincer and Zarnowitz (1969). By such criteria forecast methods such as GARCH were judged to be poor.

However Anderson and Bollerslev (1998) argued that these poor regression results may largely reflect the fact that squared returns or residuals are a very noisy proxy for volatility. They suggested that cumulative intra-daily (e.g. every 30 minutes) squared returns (or residuals) provides a much more accurate measure of daily volatility in returns, and by this measure forecasts from univariate GARCH models were judged to be much more accurate. This use of "realized volatility" has become the standard approach for evaluating univariate GARCH forecasts of volatility (along with intra-daily range).

Ideally, we would like to forecast multivariate stock market volatility using intra-daily data, by estimating a realized covariance matrix of intra-daily returns. However this complex task has only recently been addressed (Bauer and Vorkink 2011), this is left for future research.

Evaluating volatility forecasts is further complicated here where we are comparing two such different aggregation procedures. Risk is better proxied by squared residuals than by squared returns (since expected return is not zero). Residuals from regressions for 88 stocks could be aggregated to a 5x5 covariance matrix for groups by aggregation procedures similar to above. Given aggregation problems in grouping, this should provide more accurate proxies for risk covariance for the five groups than would regressions for the five groups. Using this proxy for risk covariance for the groups and risk forecasts for our two methods (aggregate and disaggregate), a loss function (e.g. Patton 2006) can be evaluated for each variance and covariance and for each method. Then loss functions for the two methods can in principle be compared. However, due to the complexity of this process, it does not seem entirely clear that standard theoretical results for testing the difference between two loss functions (Diebold and Mariano 1995; West 1996) are feasible here.

It should be noted that West et al. present their contribution as a utility approach to a loss function for evaluating forecasts of conditional variances (Fleming et al reinterpret this). To the extent that volatility studies are intended to provide information to risk averse investors, this seems more appropriate than standard loss functions (e.g. mean squared error) and may often lead to different inferences. Indeed, other papers have supported this argument although deferring to analyze this more complex loss function (e.g. Anderson and Bollerslev 1998).

Thus performance fee measures in the previous section can be viewed as a comparison of loss functions for our two approaches in terms of their forecast ability for volatility. This may be more appropriate than the other comparisons of forecast ability in this section, although there is also considerable noise as discussed in the previous section. Consequently these other comparisons of forecast ability are not applied here.

3.4 Conclusion

Finance literature shows considerable interest in measuring an aggregate risk of returns that reflects average historical experience. A standard aggregate approach is to estimate a variance for an aggregate index of returns (usually a value-weighted return index in levels). Typically, aggregate risk is estimated (by academics) from a univariate GARCH model of aggregate returns.

This chapter considers an alternative approach to measuring aggregate risk of returns based on disaggregate data on returns for individual stocks. First, we consider whether it is appropriate to estimate risk from data on aggregate returns (as in the aggregate method) or from data on disaggregate returns. This is discussed in section II. Theoretically, GARCH models of risk are not aggregation invariant. I find that univariate GARCH models of aggregate returns and MGARCH models of individual stock returns lead to significantly different measures of aggregate risk: correlation between the two measures are approximately +0.80, and the hypothesis that correlation equals one is rejected. This result gives a motivation to study further whether the aggregate method is appropriate in practice.

Section 3 of this chapter provides quantitative results on the value to stock holders of measures of aggregate risk based on disaggregate data relative to the aggregate approach. The approximate economic value (performance fee) for the improved estimates of risk is substantial. On average, the fee is approximately 4% of the total optimal portfolio return using in sample data.

The results of this chapter quantify the value of developing indexes of aggregate risk of returns using multivariate measures of risk (e.g., MGARCH) for individual stocks, as an alternative to standard aggregate approaches. However, the discussion of the merits of multivariate measures in this chapter relies only on the value-weighted and Laspeyres indexes to aggregate multivariate risk over all stocks. The next chapter considers alternative index approaches to aggregate multivariate risk in detail.

Table 3.1Aggregate Measure ofAggregate Risk of Returns

A: Using Laspeyres-Type Index of Aggregate Return

Table 3.1.A presents the summary statistics and correlations of different univariate (aggregate) measures of aggregate risk. These risks are estimated for a Laspeyres-Type index of aggregate return, $R^L{}_t = r_t\, w_0$. w_0 denotes constant weights for individual stock returns , which are the average share of each stock in capitalization over the period from Jan 1995-to Dec. 2005.

	Mean	Max	Min	Standard Deviation
GARCH(1,1)	0.00014	0.000897	1.81E-05	0.000122
GARCH(1,1)_ARMA(2,1)	0.000139	0.000899	1.23E-05	0.000121
EGARCH(1,1)	0.000132	0.00084	1.66E-05	0.000106
EGARCH(1,1)_ARMA(2,1)	0.000132	0.00085	1.23E-05	0.000107

Correlations

	GARCH(1,1)	GARCH(1,1)_ARMA(2,1)	EGARCH(1,1)
GARCH(1,1)	1		
GARCH(1,1)_ARMA(2,1)	0.9999	1	
EGARCH(1,1)	0.919889	0.910928	1
EGARCH(1,1)_ARMA(2,1)	0.910338	0.920517	0.99534

B: Using Value-Weighted Index of Aggregate Return

Table 3.1.B presents the summary statistics and correlations of different aggregate measures of aggregate risk. These risks are estimated for a value-weighted Index of aggregate return, $R^{VW}{}_t = \Sigma_i\, r_{it}\, w_{it}$. w_{it} denotes weights for individual stock returns, which are share of each stock in capitalization at each day.

	Mean	Max	Min	Standard Deviation
GARCH(1,1)	0.000138	0.000817	9.24E-06	0.000118
GARCH(1,1)-ARMA(2,1)	0.000138	0.000818	9.82E-06	0.000118
EGARCH(1,1)	0.000131	0.000803	7.49E-06	1.03E-04
EGARCH(1,1)-ARMA(2,1)	0.000131	0.000813	1.16E-05	0.000105

Correlations

	GARCH(1,1)	GARCH(1,1)-ARMA(2,1)	EGARCH(1,1)
GARCH(1,1)	1		
GARCH(1,1)-ARMA(2,1)	0.999877	1	
EGARCH(1,1)	0.914970	0.914286	1
EGARCH(1,1)-ARMA(2,1)	0.906837	0.906152	0.995764

Table 3.2 Laspeyres and Value-Weighted Indexes of Aggregate Risk Based on MGARCH Estimation of Variances and Correlation

A: Using Laspeyres-Type Index to Aggregate Variances and Covariances

Table 3.2.A presents summary statistics and correlations among different measures of Laspeyres-Type aggregate risk index. Different GARCH models are applied to each individual stocks and then risk are aggregated as $VR_t = w_0{}^T Vr_t w_0$, using constant weights as defined in Table 3.12.A. Vr_t is the estimated daily variance-covariance matrix .

	Mean	Max	Min	Standard Deviation
GARCH(1,1)-CC	0.000133	4.97E-04	1.02E-05	7.94E-05
GARCH(1,1)-DCC	0.000135	5.19E-04	1.03E-05	8.08E-05
EGARCH(1,1)-ARMA(2,1)-DCC	0.000127	0.000426	3.53E-05	7.20E-05

Correlations

	GARCH(1,1)-CC	GARCH(1,1)-DCC
GARCH(1,1)-CC	1	
GARCH(1,1)-DCC	0.998854	1
EGARCH(1,1)-ARMA(2,1)-DCC	0.981337	0.983337

B: Using Value-weighted Index to Aggregate Variances and Covariances

Table 3.2.B presents summary statistics and correlations among different measures of value-weighted-type aggregate risk index. Different GARCH models are applied to each individual stocks and then risk are aggregated as $VR_t = w_t{}^T Vr_t w_t$, using value weights as defined in Table 3.12 B. Vr_t is the estimated daily variance covariance matrix .

	Mean	Max	Min	Standard Deviation
GARCH(1,1)-CC	0.000131	0.000504	8.42E-06	8.79E-05
GARCH(1,1)-DCC	0.000132	0.0005	8.53E-06	8.86E-05
EGARCH(1,1)-ARMA(2,1)-DCC	0.000124	0.000448	2.75E-05	7.84E-05

Correlations

	GARCH(1,1)-CC	GARCH(1,1)-DCC
GARCH(1,1)-CC	1	
GARCH(1,1)-DCC	0.999071	1
EGARCH(1,1)-ARMA(2,1)-DCC	0.983483	0.985292

Table 3.3 Correlations between Analogous Aggregate and Multivariate Measures of Aggregate Risk

Table 3.3 presents the correlations between analogous aggregate and multivariate measures of aggregate risk. These correlations are calculated by running regressions between standardized measures of risk. Corresponding T-ratios against the null hypothesis that the correlation is one are also reported.

A: Laspeyres Type Indexes		
1. GARCH(1,1) Aggregate and Multivariate Measure		
	No Time Trend	Time Trend
Correlations	0.818941	0.827666
OLS standard error	0.010904	0.010798
T-ratio*	-16.604823	-15.959807
White corrected standard error	0.021853	0.021803
White corrected T-ratio*	-8.2853025	-7.9041416
Time		0.00012
OLS standard error		1.35E-05
2. EGARCH(1,1)_ARMA(2,1) Aggregate and Multivariate Measure		
Correlations	0.833875	0.839022
OLS standard error	0.010495	0.010447
T-ratio*	-15.828966	-15.409016
White corrected standard error	0.019607	0.019536
White corrected T-ratio*	-8.4727393	-8.2400696
Time		8.57E-05
OLS standard error		1.31E-05
B: Value-weighted Indexes		
1. GARCH(1,1) Aggregate and Multivariate Measure		
	No Time Trend	Time Trend
white standard error		8.37E-06
Correlations	0.805226	0.80684
OLS standard error	0.011271	0.011212
T-ratio*	-17.28098	-17.227970
White corrected standard error	0.020915	0.02085
White corrected T-ratio*	-9.3126464	-9.3126464
Time		7.91E-05
standard error		1.40E-05
2. EGARCH(1,1)_ARMA(2,1) Aggregate and Multivariate Measure		
correlations	0.811958	0.812548
OLS standard error	0.011101	0.011078
T-ratio*	-16.939194	-16.921104
White corrected standard error	0.019153	0.019089
White corrected T-ratio*	-9.820964	-9.8198962
Time		4.91E-05
standard error		0.011078
T-ratio=(correlation -1)/standard error		

Table 3.4 Descriptive Statistics for In-Sample Period

Table 3.4 presents descriptive statistics of stock returns over the five group stocks for in sample period (Jan 1995 to Dec. 2005). The groups of stocks are based on Fama-French 5 industry classifications based on CRSP SIC codes for the 88 stocks. The first group is consumer durables, nondurables, wholesale, retail, and some services, including 15 companies. The second group includes manufacturing, energy, and utilities pair shops, including 25 companies. The third group includes high-tech business equipment, telephone and television transmission (18 companies). The fourth group includes healthcare, medical equipment, and drugs (10 companies). The fifth group includes others -- mines, construction, building materials, trans, hotels, bus services, entertainment, finance (20 companies).

	INDUSTRY 1	INDUSTRY 2	INDUSTRY 3	INDUSTRY 4	INDUSTRY 5
Mean	0.000817	0.000818	0.001168	0.000825	0.000968
Median	0.000756	0.000910	0.001303	0.000486	0.000892
Maximum	0.068785	0.072753	0.133537	0.084902	0.080086
Minimum	-0.077851	-0.073349	-0.083349	-0.083757	-0.071175
Std. Dev.	0.012496	0.010501	0.017820	0.013452	0.014133
Skewness	0.134831	0.007545	0.370834	0.036273	0.119279
Kurtosis	6.206040	5.995558	6.240651	5.907114	6.013451
Jarque-Bera	1195.156	1036.076	1276.033	976.3818	1055.035
Probability	0.000000	0.000000	0.000000	0.000000	0.000000

Table 3.5 Summary Statistics for Optimal Risky Portfolio

Table 3.5 presents descriptive statistics for optimal risky portfolios based on the aggregate approach or the disaggregate approach to estimating aggregate risk. Expected return over five groups and expected risk free rate are the historical average of the sample period 1995 - 2005. Investors follow a volatility timing strategy. Each day an investor constructs an optimal risky portfolio based on expected return and the estimated variance and covariance matrix for the day. Mean is the average value of the optimal risky portfolio expected return. Standard deviation is the average standard deviation of the daily optimal risky portfolio. We present results based on both constant correlation and dynamic conditional correlation. The table also reports difference in shares for the optimal portfolios constructed based on two approaches in the last row. Share differences are calculated daily, and the absolute values are averaged over the sample period.

	Univariate-GARCH (1,1)-CCC	MGARCH(1,1) - CCC	Univariate-GARCH (1,1)-DCC model	MGARCH(1,1)-DCC(1,1)
Mean	0.000930	0.000938	0.000944	0.000939
Standard Deviation	0.0009568	0.0009965	0.0009651	0.0010011
Minimum	0.000818	0.000825	0.000819	0.000827
Maximum	0.001146	0.001086	0.001168	0.001091
Skewness	0.682695	0.343217	0.724099	0.377028
Kurtosis	3.079469	2.811173	2.948456	2.757126
Weight difference (average)			Weight difference (average)	
0.0547, 0.1156, 0.0889, 0.0761 , 0.1515			0.0429, 0.1191, 0.0807, 0.0814 , 0.0851	

Table 3.6 Summary Statistics on Portfolio Returns and Performance Fees Measure (in-sample)

Table 3.6 presents summary statistics of optimal portfolio returns with absolute risk aversion coefficient equal to 2, 5 and 10. The performance fee is calculated as $v_t = (\mu^{*,mv}_{p,t} - \mu^{*,sc}{}_{p,t}) - \frac{\lambda}{2}(\sigma^{*,mv^2}_{p,t} - \hat{\sigma}^{*,sc^2}{}_{p,t})$, where $\mu^{*,mv}_{p,t}$ and $\sigma^{*,mv2}_{p,t}$ denotes the expected return and variance for the optimal strategy, $\mu^{*,sc}_{p,t}$ and $\sigma^{*,sc,2}_{p,t}$ denotes the corresponding values for suboptimal strategy. Average sharp ratios, calculated as the excess expected return of the optimal portfolio divided by its standard deviation, are presented in the table as well.

	Univariate-GARCH (1,1)-CCC	MGARCH(1,1)- CCC	Univariate-GARCH(1,1)-DCC(1,1)	MGARCH(1,1) -DCC(1,1)
Risk Aversion =2				
Mean	0.004601	0.004046	0.004745	0.004021
Standard Deviation	0.00049354	0.00042510	0.00050968	0.00042371
Minimum	0.000901	0.000666	0.000617	0.000863
Maximum	0.037070	0.003724	0.035602	0.036639
Skewness	2.298901	2.737453	2.177370	2.704333
Kurtosis	15.58162	26. 90670	14.25688	26.33375
Sharp ratio	0.0828	0.0850	0.0832	0.0847
Performance fee (basis points)		3.117		3.6486
Risk Aversion =5				
Mean	0.001707	0.001929	0.001986	0.001697
Standard Deviation	0.019742	0.017004	0.0204	0.0169
Minimum	0.000355	0.000449	0.000335	0.000433
Maximum	0.014171	0.014916	0.014329	0.014744
Skewness	2.298901	2.737453	2.177370	2.704333
Kurtosis	15.58162	26.90670	14.25688	26.33375
Performance fee (basis points)		1.2471		1.4594
Risk Aversion =10				
Mean	0.001038	0.000927	0.0011	0.00092186
Standard Deviation	0.0009871	0.0008502	0.00102	0.00085
Minimum	0.000298	0.000251	0.00024104	0.000292
Maximum	0.002730	0.003883	0.0072	0.0074
Skewness	2.298901	2.737453	2.177370	2.704333
Kurtosis	15.58162	26.90670	14.25688	26.33375
Performance fee (basis points)		0.623		0.729

Table 3.7 Summary Statistics for Five Industry Returns (out-of-sample)

Table 3.7 presents summary statistics for 5 industry returns during out-of-sample period, Jan. 2006 to Nov. 2006. These stocks are placed into five groups according to the Fama-French five industry classifications.

	INDUSTRY1	INDUSTRY2	INDUSTRY3	INDUSTRY4	INDUSTRY5
Mean	0.000630	0.001117	0.000989	0.000717	0.000878
Median	0.000535	0.001417	0.000731	0.000408	0.000850
Maximum	0.013776	0.023756	0.022118	0.024205	0.023877
Minimum	-0.021596	-0.023677	-0.028460	-0.023830	-0.021368
Std. Dev.	0.005838	0.007775	0.008395	0.006800	0.006880
Skewness	-0.155627	-0.086412	-0.025559	0.073941	-0.016846
Kurtosis	3.420959	3.786009	3.451569	4.175623	4.477183
Jarque-Bera	2.546703	6.018016	1.918988	13.04512	20.28562
Probability	0.279892	0.049341	0.383087	0.001470	0.000039
Sum	0.140502	0.249080	0.220623	0.159923	0.195820
Sum Sq. Dev.	0.007565	0.013420	0.015645	0.010266	0.010508
Observations	223	223	223	223	223

Table 3.8 Summary Statistics for Out-of-Sample Optimal Risky Portfolios 19

Table 3.8 presents descriptive statistics for optimal risky portfolios with variances and covariances estimated using aggregate and individual based MGARCH models for out of sample data. The expected returns for each group are also estimated using GARCH model forecast. Each day, optimal risky portfolio is constructed based on mean variance utility function. The risk free rate is the historical average of the sample period. Mean is the average optimal expected return over the out of sample period. Standard deviation is the average standard deviation for the optimal portfolio. We present results based on both constant correlation and dynamic conditional correlation. The table also reports differences in shares for the optimal portfolios constructed based on the two approaches. Share differences are calculated daily, and absolute values are averaged over year 2006.

	Univariate-GARCH(1,1)-CCC	MGARCH(1,1)-CCC	Univariate-GARCH (1,1)- DCC(1,1)	MGARCH (1,1)-DCC
Mean	0.001072	0.00 1066	0.001082	0.001098
Standard Deviation	0.00004	0.00003	0.00006	0.00002
Minimum	0.000974	0.001013	0.0009313	0.001021
Maximum	0.001133	0.001112 b	0.001154	0.001133
Skewness	-1.0085	-0.2344	-1.0192	-1.0304
Kurtosis	2.7452	1.7232	2.9681	3.8494
Weight difference (average) [0.0858, 0.08367, 0.100, 0.02645, 0.1377]		Weight difference (average) [0.0797, 0.0012, 0.0663, 0.0406, 0.1364]		

Table 3.9 Summary Statistics for Out-of-Sample Performance Fees Measure

Table 3.9 presents the summary statistics of optimal portfolio returns with absolute risk aversion coefficient equal to 2, 5 and 10 for out of sample data. The performance fee is calculated as $v_t = (\mu^{*,mv}_{p,t} - \mu^{*,sc}{}_{p,t}) - \frac{\lambda}{2}(\sigma^{*,mv2}_{p,t} - \hat{\sigma}^{*,sc2}{}_{p,t})$,

Where $\mu^{*,mv}_{p,t}$ and $\sigma^{*,mv2}_{p,t}$ denotes the expected return and variance for the optimal strategy, $\mu^{*,sc}_{p,t}$ and $\sigma^{*,sc,2}_{p,t}$ denotes the corresponding values for suboptimal strategy. Sharpe ratios are presented in the table as well.

	Univariate -GARCH (1,1)-CCC	MGARCH(1,1)-CCC	Univariate-GARCH(1,1)-DCC(1,1)	MGARCH(1,1)-DCC(1,1)
Risk Aversion =2				
Mean	0.01091	0.009411	0.01106	0.009729
Standard Deviation	0.002959	0.001495	0.003158	0.001473
Minimum	0.0058	0.0066	0.005786	0.007024
Maximum	0.0170	0.0123	0.01765	0.01266
Sharp ratio	0.1334	0.1358	0.1356	0.138 0
Performance fee (basis points)		4.4165		4.3678
Risk Aversion =5				
Mean	0.004452	0.0039	0.0045	0.0040
Standard Deviation	0.001184	0.000598	0.0013	0.0006
Minimum	0.002425	0.0027	0.0024	0.0029
Maximum	0.006897	0.0050	0.0071	0.0052
Performance fee (basis points)		1.7667		1.7471
Risk Aversion =10				
Mean	0.002299	0.002003	0.0023	0.0021
Standard Deviation	0.000592	0.000299	6.1371-004	1.9993-004
Minimum	0.001286	0.001447b	0.0013	0.0015
Maximum	0.003521	0.002586b	0.0036	0.0026
Performance fee (basis points)		0.8833		0.8735

Table 3.10 Economic Value based on Realized Aggregate Returns (In-sample)

Table 3.10 presents the economic value of switching from aggregate measure of aggregate risk to individual based aggregate risk using realized aggregate return. The performance fee v satisfies the following equation

$$\left(\sum_{t=0}^{T-1}[(R^{mv}_{p,t+1}-\mathrm{v})-\frac{\rho}{2(1+\rho)}(R^{mv}_{p,t+1}-\mathrm{v})^2]\right)=\left(\sum_{t=0}^{T-1}[\mathrm{R}^{sc}_{p,t+1}-\frac{\rho}{2(1+\rho)}(\mathrm{R}^{sc}_{p,t+1})^2]\right)$$

	Univariate-GARCH(1,1)-CCC	MGARCH(1,1)-CCC	Univariate-GARCH(1,1)-_DCC(1,1)	MGARCH(1,1)-DCC(1,1)
Risk Aversion =2				
Mean	0.0039	0.0034	0.00378	0.003366
Standard Deviation	0.0482	0.0427	0.04917	0.0429
Minimum	-0.268	-0.248	-0.241	-0.2419
Maximum	0.2254	0.1503	0.2722	0.1473
Performance fee (basis points)		3.3443		2.1378
Risk Aversion =5				
Mean	0.001651	0.001451	0.001604	0.001434
Standard Deviation	0.01929	0.0171	0.01967	0.01691
Minimum	-0.1071	-0.0991	-0.09633	-0.09668
Maximum	0.09203	0.06021	0.109	0.05899
Performance fee (basis points)		1.6709		1.2763
Risk Aversion =10				
Mean	0.0009	0.0008	0.0009	0.0008
Standard Deviation	0.0097	0.0086	0.0098	0.0085
Minimum	-0.05348	-0.04958	-0.04809	-0.04827
Maximum	0.04519	0.03018	0.05456	0.02957
Performance fee (basis points)		0.9111		0.7337

Table 3.11 Economic Value Based on Realized Aggregate Returns (Out-of-Sample)

Table 3.11 presents the economic value of switching from aggregate measure of aggregate risk to individual based aggregate risk using realized aggregate return. The performance fee v satisfies the following equation $\left(\sum_{t=0}^{T-1}[(R_{p,t+1}^{mv}-\mathrm{v})-\frac{\rho}{2(1+\rho)}(R_{p,t+1}^{mv}-\mathrm{v})^2]\right)=\left(\sum_{t=0}^{T-1}[\mathrm{R}_{p,t+1}^{sc}-\frac{\rho}{2(1+\rho)}(\mathrm{R}_{p,t+1}^{sc})^2]\right)$

	Univariate-GARCH(1,1)-CCC	MGARCH(1,1)-CCC	Univariate-GARCH (1,1)-DCC(1,1)	MGARCH(1,1)-DCC(1,1)
Risk Aversion =2				
Mean	0.008966	0.007603	0.0094	0.0080
Standard Deviation	0.0681	0.06017	0.06762	0.0611
Minimum	-0.2445	-0.2171	-0.221	-0.1879
Maximum	0.2702	0.2124	0.2666	0.2145
Performance fee (basis points)		10.227		11.0247
Risk Aversion =5				
Mean	0.003675	0.003129	0.0038	0.0033
Standard Deviation	0.02724	0.02407	0.0271	0.02445
Minimum	-0.09772	-0.08674	-0.0883	-0.0751
Maximum	0.1082	0.08504	0.1067	0.0859
Performance fee (basis points)		4.7749		4.9709
Risk Aversion =10				
Mean	0.001911	0.001638	0.0020	0.0017
Standard Deviation	0.01362	0.01203	0.01352	0.0122
Minimum	-0.04879	-0.04329	-0.04411	-0.03748
Maximum	0.05415	0.04259	0.05341	0.04299
Performance fee (basis points)		2.5416		2.6124

Figure 3.1 Average Returns of 88 Stocks

The following figure shows the variation of daily average return for each stock from S&P 100 index (88 after eliminating stocks disappearing within the period) over Jan. 1995 to Dec. 2005. The minimum of the average return is 0.000117, the maximum 0.002384, and the mean is 0.000845.

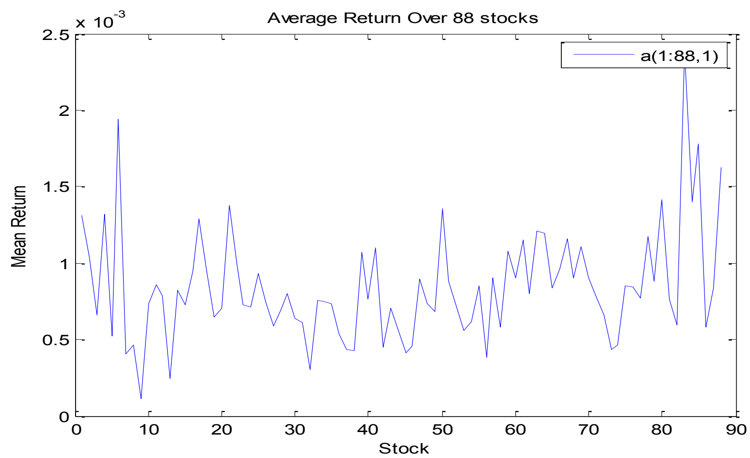


Figure 3.1 Average Return of 88 Stocks

Figure 3.2 Ratio of Optimal Portfolio Risk between MGARCH and Aggregate Method

Figure 3.2 shows the ratio of the optimal portfolio risk derived from MGARCH method and aggregate method. The mean of the ratio is 1.0202 with a standard deviation of 0.0225. The mean of the risk difference is one basis point. The comparison was based on the assumption that MGARCH variance covariance matrix is true. The portfolio weight differences between aggregate and MGARCH approach is 2.16%, 10.77%, 8.54%, 6.83%, 20.01%

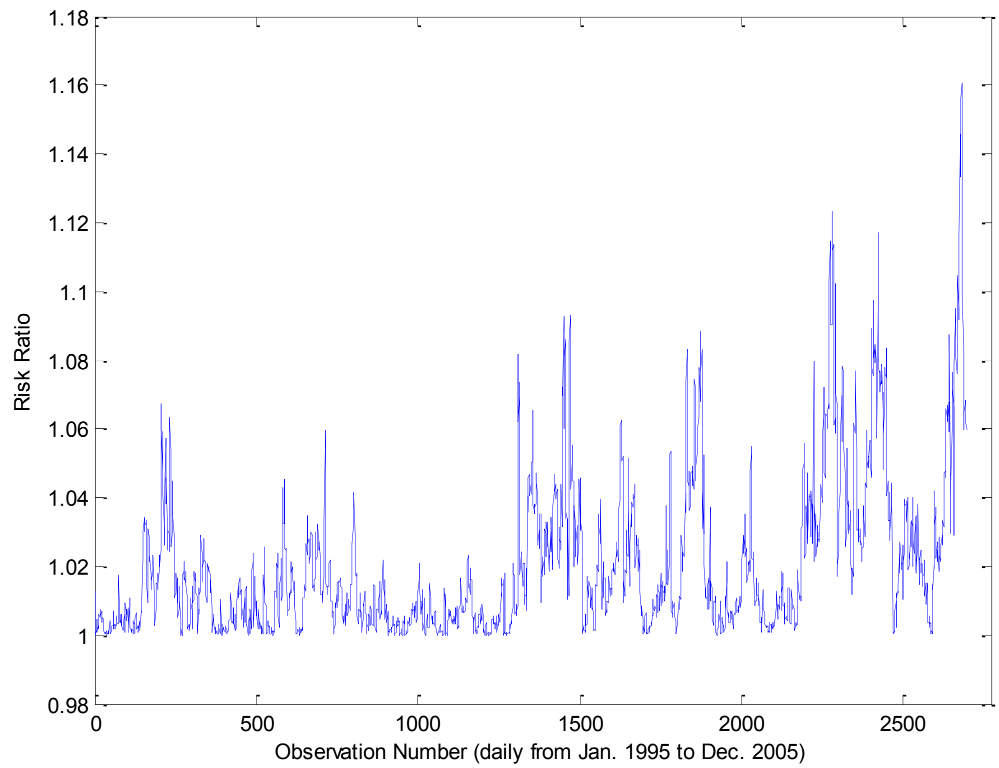
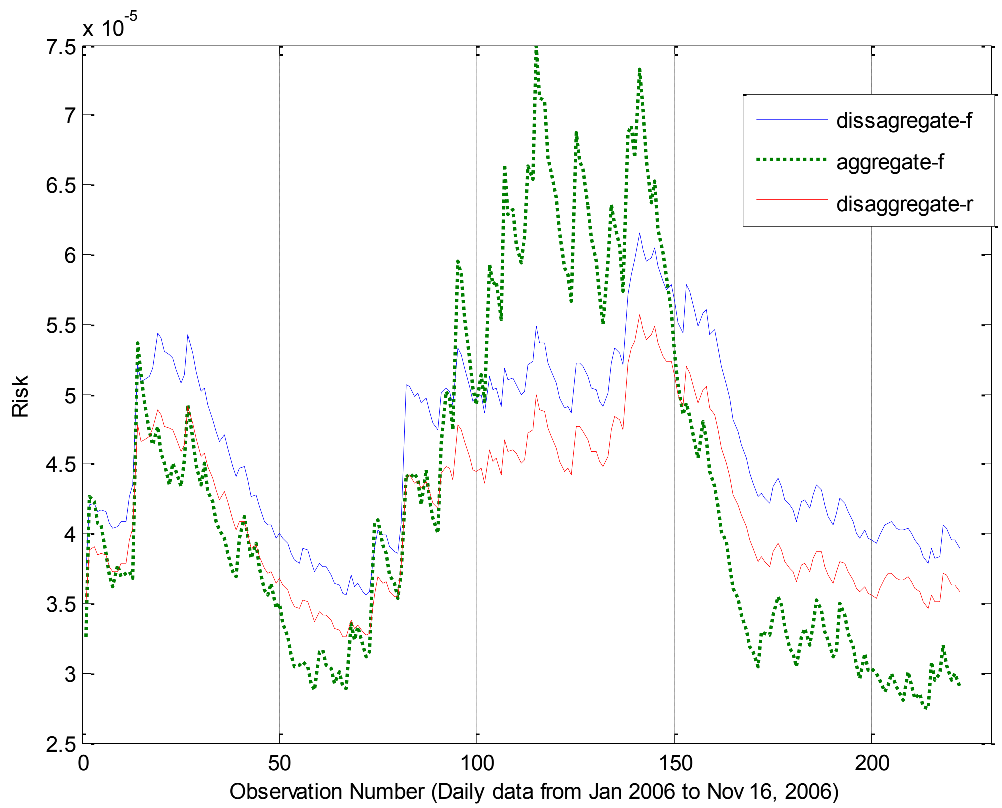


Figure 3.3 Comparison of Variance Forecast Based on MGARCH or Univariate Aggregate Method

Figure 3.3 shows the difference between the forecasted variance and the true measure of risk. With the forecasted $VR^{SC}{}_f$ and $VR^{mv}{}_f$, we compare them with VR^{mv} by comparing $wVR^{mv}w'$ (disaggregate-r), $wVR^{sc,f}w$ (aggregate-f) and $wVR^{mv,f}w$ (disaggregate-f), where the weights for each matrix are the same. We use value weights for each group.



CHAPTER FOUR: AN ECONOMETRIC STUDY OF STOCK MARKET AGGREGATE RISK-RETURN TRADEOFF, EXPLAINING AGGREGATE RISK, AND INDEX MEASURES OF AGGREGATE RISK

Abstract

This chapter has three parts. The major part is an empirical study of the aggregate risk-return tradeoff for U.S. stocks using daily data. Modeling dynamics/lags is critical in daily models, and apparently this is the first such study to model lags correctly using a general-to-specific approach. This is also the first risk-return study to apply Wu tests for possible problems of endogeneity/measurement error for the risk variable. Results indicate a statistically significant positive relation between expected returns and risk, as is predicted by capital asset pricing models. Few empirical studies have analyzed risk-return relations at a daily level, but this is one obvious approach to addressing a problem that is now recognized as fundamental (past risk-return studies suffer from insufficient observations). For this reason, this study should be a significant contribution to the literature.

Development of the Wu test leads naturally into a model relating aggregate risk of returns to economic variables from the risk-return study. This is the first such model to include lags in variables based on a general-to-specific methodology and to include covariances of such variables. The study also derives coefficient links between such models and risk-return models. Empirical results for the daily model are consistent with theory and indicate that the economic and financial variables explain a substantial part of variation in daily risk of returns.

The chapter first considers procedures for constructing aggregate measures of risk of

returns. This section investigates at a theoretical and empirical level several alternative index number approaches for aggregating multivariate risk over stocks. The empirical study concludes that these indexes are highly correlated for this data set. For this reason, this study uses only simple Laspeyres or value-weighted type indexes of aggregate risk of returns.

Keywords: Risk-Return Tradeoff, Wu Test, Index Number Theory, Aggregate Risk

4.1 Introduction

The primary purpose of this chapter is to develop and estimate an empirical model of aggregate stock market risk-return tradeoff at a daily level. This also leads to development and estimation of an empirical model explaining aggregate risk of returns in terms of variables explaining returns. However, it is first important to address issues in constructing an aggregate index for risk of stock market returns.

Section three of this chapter presents an empirical model of the aggregate stock market risk-return tradeoff using daily data. This research is novel in two major respects. First, modeling dynamics/lags is critical in daily models, and this is the first such study to model lags using a general-to-specific approach. Results indicate a statistically significant positive relation between stock market expected returns and risk, as is predicted by capital asset pricing models. Second, the model is viewed as a structural equation with possible problems of endogeneity/measurement error for the risk variable, leading to a Wu test for these problems. This is the first risk-return study to apply such a test.

A recent study by Lundblad (2007) has provided a plausible explanation for past poor results in empirical studies of risk-return tradeoff: the nature of the data implies that many more observations are required than have been used in the past. An obvious approach to dealing with this problem is to use daily data (most studies have estimated tradeoffs at monthly or quarterly levels). By showing how to formulate a model more appropriately at a daily level, this study makes an important contribution to the general risk-return tradeoff literature.

Development of the Wu test leads naturally to a model relating aggregate risk of returns to economic variables from the risk-return model (section four). This model is novel in several respects. As in the risk-return model, this is the first model explaining risk to include lags in variables based on a general-to-specific approach. In addition, this study includes covariances of economic variables and clarifies coefficient links with risk-return models. This is one of few models at the daily level rather than monthly or quarterly level. Empirical results are consistent with theory and indicate that the economic and financial variables explain a substantial part of variation in daily risk of returns.

This chapter connects econometric models of risk-return tradeoffs and models explaining risk much more closely than in past literature. In part this is done by Wu tests. In addition we prove that in theory the two models share common economic variables and there are simple (non-linear) restrictions on coefficients across models. Thus, in theory, these two major econometric models in finance are more closely related than has been realized in the past.

The chapter first considers procedures for constructing aggregate measures of risk of returns from multivariate risk over stocks. At a theoretical level, a Fisher-type index of aggregate risk is slightly better than a Laspeyres or Paasche-type index (or a value-weighted index) of aggregate risk, but all are very highly correlated in our data set. In the remainder of the chapter we simply use a Laspeyres or value-weighted type index of aggregate risk of returns.

The chapter proceeds as follows. Section two outlines the theoretical analysis related to the types of risk-return indexes under consideration, and presents empirical comparisons. Formal arguments and proofs are presented in Appendix 4.III. The risk-return tradeoff study is discussed in section three, and the model explaining aggregate risk of returns is presented in section four. Section five concludes the chapter.

4.2 Alternative Index Number Formulations for Aggregating Risk of Returns over Stocks

The previous chapter indicates that it is important in finance to develop indexes of aggregate risk of returns using multivariate measures of risk (e.g. MGARCH) from disaggregate data for individual stocks, rather than adopting the standard aggregate approach (essentially applying univariate GARCH to an index of aggregate returns) which involves substantial errors in aggregation of risk. Although there are many studies of MGARCH models of stock returns, none of these studies considers issues in aggregation and apparently none even constructs an index of aggregate risk. This is the first study to consider such indexes.[23] As the first study, there is no precedent indicating how to construct such indexes. In the previous chapter we used ad-hoc analogs to value-weighted and Laspeyres return indexes. In this study, we investigate in a systematic manner, at both theoretical and empirical levels, alternative index number approaches to

[23] Coyle 2007 emphasized indexes for aggregate risk based on aggregate quantity indexes, but these seem less appropriate for finance.

aggregating multivariate risk over all stocks.

In the previous chapter a Laspeyres-type or value-weighted index of aggregate risk was defined in terms of levels, as is common in finance. However, in order to consider alternative indexes analogous to a Paasche or Fisher, it is more appropriate to define indexes in terms of ratios, as in general economics.

4.2. A. Behavioral Model

The following behavioral model underlies this section. Assume that a representative agent in a stock market decides at the beginning of time period t how to allocate total money Q_t among n stocks, i.e. how to choose a portfolio $q_t = (q_1,\ldots,q_n)_t$ of investment \$ in n stocks, satisfying the constraint $\Sigma_{i=1,\,n}\ q_{it} = Q_t$. Portfolio q_t is chosen by the agent and hence known by the agent at time t, so q is non-stochastic rather than a random variable at t. In summary, for simplicity we can view $q_t = (q_1,\ldots,q_n)_t$ as non-stochastic portfolio decisions made at the beginning of period t, and returns $r_t = (r_1,\ldots,r_n)_t$ at the end of the period t are stochastic.

Denote the vector of expected returns and variance-covariance matrix of returns as $Er_t = (Er_1,..,Er_n)_t$ and $Vr_{t\ nxn}$, respectively. Total dollar returns on the portfolio q_t at the end of period are $R_t = \Sigma_{i=1,..,n}\ r_{it}\ q_{it}$. Since r_t is stochastic and q_t is non-stochastic during t, the expectation and variance during t of total dollar returns R_t are $ER_t = \Sigma_{i=1,..,n}\ Er_{it}\ q_{it} = Er_t\ q_t$ and $VR_t = \Sigma_{i=1,..,n}\ \Sigma_{j=1,..,n}\ Vr_{ij}\ q_{it}\ q_{jt} = q_t{}^T Vr_t\ q_t$, respectively.

Suppose that the agent's risk preferences can be represented by a mean-variance utility function $U = U(ER_t, V_{Rt}) = ER_t - \alpha(ER_t, V_{Rt})/2 * VR_t$,where α is the coefficient of absolute risk aversion. α is generally a function of Er and Vr, i.e. α = α(Er, Vr). Assume that the agent chooses portfolio q_t to maximize his mean-variance utility function as follows (deleting subscripts t):

$$(1) \quad \max_q \quad U = U(Er\ q, q^T\ Vr\ q)$$

$$\text{s.t.} \qquad \Sigma_i\ q_i = Q$$

and denote the optimal portfolio decision as q*. Denote the relation between maximum utility and exogenous parameters Er, Vr and Q as the dual utility function U* = U*(Er, Vr, Q).

4.2. B. Alternative Index Numbers for Aggregate Risk of Returns

Within the framework of a decision-maker as in the above portfolio choice problem, the "economic" significance of expected returns Er for the n stocks is that they contribute (jointly with capitalizations q) to total market expected return ER = Er q, which in turn influences utility U. Thus, in order to preserve the economic significance of vector Er_t at time t (the contribution of Er to U) as we aggregate over stocks, Er_t should be weighted by some portfolio measure q_w (Er_t q_w). An appropriate measure q_w should preserve the contribution of Er_t to utility, and the index number problem is how to choose q_w. This is the spirit of the economic approach to index numbers in the case of aggregating (expected) returns over stocks.

Similarly, within the framework of a decision-maker as in the above portfolio choice problem, the "economic" significance of risk variances and covariances Vr (nxn) for individual stocks is that they contribute (jointly with capitalizations q) to total market risk VR = q^T Vr q, which in turn influences utility. Thus, in order to preserve the economic significance of risk variances and covariances Vr_t at time t (the contribution of Vr to U) as we aggregate over stocks, and following basic statistics, Vr_t should be weighted by some portfolio measure q_w as follows: $q_w{}^T Vr_t\, q_w$. An appropriate measure q_w should preserve the contribution of Vr_t to utility, and the index number problem is how to choose q_w. This is the spirit of the economic approach to index numbers in the case of aggregating risk of returns over stocks.

Ideally an index number aggregate of return risks Vr_{nxn} should accurately reflect their contribution to maximum utility U*. For a particular return risk index number formula, the economic approach to analysis of index numbers attempts to identify restrictions on the dual U*(Er, Vr, Q),when this will be the case, i.e. when the formula accurately reflects the contribution of Vr_{nxn} to the dual U*. The index is described as "exact" in this case. If these restrictions are extremely unrealistic, then the particular return risk index number formula provides a poor approximation to the economic contribution of Vr_{nxn} and is judged to be a poor index number formula in theory.

Consider the following alternative index numbers for aggregate risk of returns. These fixed base indexes are analogous to Laspeyres, Paasche and Fisher indexes, respectively:

(2) a) $(VR_t/VR_0)^L = q_0{}^T Vr_t\, q_0 / q_0{}^T Vr_0\, q_0$

b) $(VR_t/VR_0)^P = q_t{}^T Vr_t\, q_t / q_t{}^T Vr_0\, q_t$

c) $(VR_t/VR_0)^F = \{(VR_t/VR_0)^L\ (VR_t/VR_0)^P\}^{1/2}$.

Here Vr_t is the variance-covariance matrix (nxn) of return risk at time t, and q^T is the transpose (1xn) of q. Vr_0 and q_0 are Vr and q for the fixed base period 0. In empirical applications, rather than choosing an arbitrary day as the fixed base, q_0 will be defined as the average of capitalizations of each stock over all days, and Vr_0 will be defined as the average of each estimated variance and covariance of returns over all days (for the particular MGARCH model).

The above economic criterion is applied to these alternative indexes in Appendix III. The Laspeyres-type return risk index (2a) accurately reflects the contribution of Vr_{nxn} to U* only if ratios of portfolio decisions q_i / q_k are independent of all Er, Vr, and Q (see discussion of Proposition 1). Since this condition is restrictive and unrealistic, the Laspeyres return risk index presumably provides a poor approximation to the economic contribution of Vr_{nxn} and is judged to be a poor index number formula in theory.

This conclusion is not surprising. Casual inspection of the index (2a) suggests that it is an adequate index only if the weightings q are constant or equiproportional over time (otherwise weightings misrepresent contributions of Vr to U* over time). A similar conclusion applies to a Paasche-type index (2b).

The above economic criterion can also be applied to the Fisher-type return risk index (2c). This index accurately reflects the contribution of Vr_{nxn} to U* under conditions that are somewhat less restrictive than the conditions justifying the Laspeyres making the Fisher return risk index superior to the Laspeyres return risk index.

To be more specific, the Fisher-type return risk index (2c) accurately reflects the contribution of Vr_{nxn} to U* only if ratios of portfolio decisions q_i / q_k are independent of all Er, Q, but ratios can vary quite generally with variances and covariances of risk Vr (see discussion of Proposition 2 in Appendix III). In sum, by the economic criterion, a Fisher return risk index is superior in principle to a Laspeyres index, but the Fisher index also is quite restrictive. This contrasts with Fisher return indexes under risk neutrality, which satisfy economic criteria under quite general conditions (ratios of portfolio decisions vary generally with Er, Q).

The economic approach to index numbers for aggregate risk of returns is extended further in a separate paper (Chen and Coyle, 2011). This paper considers index numbers constructed in terms of differences as well as ratios, among other matters. By adopting a differences approach, we can construct index numbers that meet the economic criterion for models such as (1) with risk aversion under much more general conditions than a Laspeyres or Fisher (see discussion of Proposition 3). For further details, we refer the reader to Chen and Coyle 2011.

4.2. C. Empirical Applications of Alternative Index Numbers for Aggregate Risk of Returns

The three alternative index measures of aggregate risk for return defined above (2a-c) are compared using alternative MGARCH models and daily data on 88 stocks on the S&P 100 index over 1995-2005. I choose a common GARCH (1,1) specification for the mean and the ARMA(2,1)-EGARCH(1,1) model (from chapter one results). I also consider both constant correlation and dynamic correlation models. The resulting MGARCH models are: GARCH(1,1)-CCC(1,1), ARMA(2,1)-EGARCH(1,1)-CCC, GARCH(1,1)-DCC, and ARMA(2,1)-EGARCH(1,1)-DCC.

Correlations between the three index measures are presented in Table 4.1. For example, consider indexes using estimates of Vr from the EGARCH(1,1)-ARMA(2,1)-DCC(1) model of MGARCH (this is the preferred estimation approach). The correlation between the Laspeyres and Paasche is very high (+0.996), and these are also very highly correlated with the Fisher, the geometric mean of the two indexes (+0.999). Similar results are obtained for other methods of estimating risk at the disaggregate level (GARCH(1,1)- CCC, GARCH(1,1)-CCC, historical estimation, RiskMetrics).

The levels of these three indexes are graphed over all days (1995 - 2005) in Figure 4.1 (based on EGARCH(1,1)-ARMA(2,1)-DCC(1) estimates). By definition, the indexes are identical on the first calculated day. Thereafter the indexes are almost identical. Aggregate risk as measured by these indexes shows substantial variation over time, and it tends to be highest in the middle period from days 900 to 2000 (between year 2000 and year 2003).

Consider two value-weighted indexes of aggregate risk

(3) a) $(VR_t/VR_0)^A = q_t{}^T Vr_t\, q_t \,/\, q_0{}^T Vr_0\, q_0$

b) $(VR_t/VR_0)^B = s_t{}^T Vr_t\, s_t \,/\, s_0{}^T Vr_0\, s_0$

where q is capitalization levels for stocks and s is capitalization shares (portfolio weights). In finance, value-weighted indexes of returns typically weight by shares rather than levels of capitalization, so the second value-weighted risk index (3b) is more common in finance. Risk index (3a) does not attempt to control at all for changes in capitalizations q, in contrast to the previous three indexes. Here Vr_t is simply weighted by current capitalizations q_t, and Vr_0 is simply weighted by q_0, in contrast to the previous indexes. The Laspeyres, Paasche and Fisher-type indexes have lower correlations with the index (3a) (+0.761 to +0.766). This index is also graphed in Figure 4.1. It has the same general pattern as the other indexes, i.e. measured aggregate risk is highest in the middle period. However, the measure of aggregate risk tends to be much higher over this period than for the other indexes. Moreover the variation in aggregate risk, from low levels in early days to high levels, is substantially larger for this index than for the other three indexes. All four indexes use the same MGARCH estimates of Vr over t, so this greater variation in measured risk is because this index uses current capitalizations rather than trying to control for changes in capitalizations over time. In measuring aggregate risk, these changes in capitalizations tend to magnify effects of changes in commodity-level risk Vr (nxn) in the index.

It can be shown that this difference between indexes (3a) and (2) is primarily due to variations in total market capitalization over time rather than to variation in individual stock shares in total capitalization. Consider risk index (3b) where capitalization levels q are replaced by capitalization shares, controlling for variations in total market capitalization over time by re-specifying q as shares. Correlation with indexes (2) increases substantially to +0.99. This index (3b) is analogous to a variance of a value-weighted risk index (using MGARCH rather than univariate GARCH estimation).

An interesting empirical result (from the viewpoint of standard index number theory) in this section is that the Laspeyres-type and Paasche-type measures of aggregate risk are very similar in this study. In standard consumer price indexes (CPI), a Laspeyres and Paasche CPI bound a true cost of living index under general conditions (see Appendix I). However it is not at all clear that an analogous result holds for indexes of aggregate risk (Chen and Coyle 2011). Moreover the theoretical arguments (following an economic approach) for a Fisher risk index are much weaker than in cases such as a CPI (see Appendix III). Therefore alternative index numbers for aggregate risk should be considered, beyond Laspeyres, Paasche and Fisher.

4.3 An Econometric Study of Aggregate Risk-Return Tradeoff with Daily Data

Lundblad (2007) indicates that estimation of risk-return tradeoff requires many more observations than has been used in the past. One possible approach to addressing this problem is to estimate risk-return models using daily data rather than monthly or quarterly data as has been common in the past. This section develops and estimates an econometric model of the aggregate stock market risk-return tradeoff using daily data. Results indicate a statistically significant positive relation between expected returns and risk for daily models (in contrast to simple monthly models).

4.3. A. Literature Review

The intertemporal CAPM model of Merton (1973, 1980) implies the following equilibrium relation for any risky asset i at time t:

(4) $ER_{it} = \theta \operatorname{cov}(R_{it}, R_{mt}) + \lambda \operatorname{cov}(R_{it}, X_t)$

where R_i and R_m are returns on asset i and the market portfolio m net of risk-free return, X is a vector of state variables influencing future investment opportunities and returns, and expectations and covariances are conditional on information available at time t (Bali and Engle 2010a,b). Here, $\operatorname{cov}(R_i, R_m)$ and $\operatorname{cov}(R_i, X)$ are conditional covariances of R_i with R_m and X, respectively. In the standard model, assuming identical risk preferences and constant relative risk aversion, then θ is constant over cross sections (different assets/portfolios), and θ can be interpreted as a coefficient of relative risk aversion (λ is price of risk for innovations in X).[24] (4) implies the following risk-return relation at the level of the market portfolio m:

[24] More generally, assuming variances and covariances of assets change over time, Campbell (1993) derives a relatively simple risk-return formula (44, page 502) somewhat similar to (5)-(6) here, but the coefficient of risk cannot generally be interpreted as relative risk aversion. Also, in his simplest discrete time analogue (25, page 496) to Merton's (1973) continuous model, the coefficient analogous to θ in (6) is relative risk aversion and the coefficient analogous to λ is θ-1.

(5) $ER_{mt} = \theta \text{ var}(R_{mt}) + \lambda \text{ cov}(R_{mt}, X_t)$.

Few studies directly estimate cov(R,X) (e.g., Bali and Engle 2010b). Instead most studies including state variables do not model cov(R,X) and instead include X as follows;

(6) $ER_{mt} = \theta \text{ var}(R_{mt}) + \lambda X_t$

(λ is now the market's aggregate reaction to changes in X) (e.g., Bali and Engle 2010a). Many early studies of risk-return tradeoff ignore state variables and estimate $ER_{mt} = \theta \text{ var}(R_{mt})$.

There have been many econometric studies of risk-return tradeoff. However, results have been mixed, with many studies reporting a negative or statistically insignificant relationship (e.g. see literature reviews in Lettau and Ludvigson 2010; Muller, Durand and Maller 2011; Bali and Engle 2010a; and Lundblad 2007). Estimates of the tradeoff are "mostly insignificant or even negative" (Bali and Engle 2010b, p. 1). Poor results are often attributed to difficulties in modeling unobserved expected returns and risk. For example, in their recent review, Lettau and Ludvigson 2010 (hereafter L&L) suspect problems are largely due to the paucity of predetermined conditioning variables in most studies. Others attribute poor results to inappropriate models of risk (common approaches include realized volatility, i.e. sample standard deviations constructed from high frequency return data, and various parametric conditional volatility models such as GARCH, GARCH-in-mean, EGARCH, stochastic volatility). Moreover in response to poor results, several authors have developed models where a negative relation is consistent with equilibrium (Abel 1988; Backus and Gregory 1993; Gennotte and Marsh 1993).

Ghysels, Santa-Clara and Valkanov (2005) obtain significant estimates of risk-return tradeoff by mixing monthly and daily data using a "MIDAS" estimator of monthly conditional variance. The tradeoff is modeled at the monthly level, but daily data are used in a more flexible manner in modeling monthly risk than in other studies. In the literature, risk at the monthly level is often defined using a simple sum of squared daily returns over the month. Instead, Ghysels et al. define monthly risk as a weighted sum of squared daily returns, where the weights are a two-parameter polynomial function with weights declining over lag time (one year). Aggregate returns are defined at the monthly level as usual. All parameters are estimated jointly in the risk-return equation. Using a MIDAS conditional variance in a simple risk-return model and CRSP value-weighted portfolio 1928-2000, significant positive estimates of a tradeoff were obtained, in contrast to many other studies.

Yu and Yuan (2011) have recently argued that investor sentiment may help explain poor empirical results for risk-return models. Investor sentiment is a frequently considered departure from rational asset pricing models (naïve or inexperienced investors behave irrationally or have poor information about risk and expected return) and there is some empirical support that it influences stock prices and expected returns. Using an investor sentiment index, the authors define two regimes (high and low sentiment) over 1963-2004 NYSE and estimate standard risk-return models with monthly data. Coefficient estimates for the risk-return tradeoff are highly significant positive during low sentiment but are insignificant during high sentiment and ignoring sentiment.

Recent literature (Lundblad 2007) has emphasized a fundamental problem that may explain poor empirical results: studies may require many more observations than have been used in the past. Models explaining returns in terms of risk have extremely low R^2's, reflecting high variation in returns relative to persistent measures of conditional risk. Lundblad (2007) argues that this is the fundamental problem in risk-return studies rather than volatility specification. In a Monte Carlo study within a GARCH-in-mean framework, he shows that a very large number of observations are required to estimate successfully a tradeoff irrespective of volatility model. From the simulations, he speculates that successful estimation requires that observations be increased by adopting a greater time span more so than a greater frequency of data. Lundblad also estimated risk-return models using monthly data on U.S. equities from 1836-2003, a much longer time period than in other studies. He estimated a statistically significant positive relation for all volatility models considered (four GARCH models), but the relation was insignificant for the more common sub-period 1950-2003 and for other 50 year sub-periods. An obvious weakness of this approach, recognized by Lundblad, is the assumption that coefficients in regression models are constant over almost 200 years. He does not estimate a model using daily data in order to check the implication of Monte Carlo simulations that data frequency is of second order importance. As in many studies, other risk factors serving as state variables influencing future investment opportunity set were excluded from the GARCH-in-mean models.

An earlier study by French, Schwert and Stambaugh (1987) directly addressed the issue of gains in precision using daily rather than monthly data. They estimated simple risk-return models at both monthly and daily levels for 1928-84 and two subperiods 1928-52 and 1953-84 (using Standard and Poor's composite portfolio). Regressions of monthly excess returns on realized volatility (based on daily returns) led to insignificant estimates of the tradeoff. Of more interest, simple GARCH-in-mean models were estimated with both daily and monthly data, and the tradeoff was more significant with daily data for 1953-84 (results were insignificant for both daily and monthly data 1928-52). For 1953-84, the coefficient of return variance and its standard error were 7.22 and 2.8 for daily data, and 7.81 and 4.2 for monthly data. Thus standard error of the coefficient for return risk was 50% larger for monthly data than for daily data, indicating a substantial increase in precision with daily data rather than monthly data for this time period.[25]

Duffee (1995) also estimates risk-return models at daily and monthly levels, over 1977-91. Similarly to French et al, he finds a more statistically significant (positive) tradeoff at the daily level than at the monthly level. However, the study uses different measures of risk in the two cases (monthly and daily risks are approximated as sample standard deviation of daily returns within a month and absolute value of the day's return, respectively), so it is unclear whether the increase in precision is due to daily data or differences in methodology. Models are estimated separately for individual stocks, i.e. without imposing common coefficients across stocks (in contrast to Bali and Engle, discussed below).

In contrast to other risk-return studies at the daily level, Schwert (1990) includes lags in return and risk. Daily stock returns are specified as a function of daily dummy variables and 22 period lags (approximating a month) on returns and a daily measure of

[25] French et al (1987) speculate in a brief footnote that there may be a generated regressor problem in their GARCH-in-mean risk-return models, citing Pagan (1984). However assuming a correct model specification, GARCH (and multivariate GARCH) models have a well-established asymptotic theory, implying that GARCH provides consistent estimators of "true" conditional risk. In this sense, the generated regressors problem does not apply assuming sufficiently large sample size. Note that this concern is not raised in risk-return studies using GARCH by Bali and Engle or other authors. Of course, in practice there may be substantial measurement errors using GARCH, and we will address this issue later in this study by proposing a specification test such as a Wu test.

volatility (essentially absolute residuals from a regression of returns in terms of dummies and lagged returns). Macroeconomic variables are excluded from the model. The model is estimated over 1885-1987 using Cowles and CRSP value-weighted return indexes for NYSE stocks. The sum of lag coefficients for risk is insignificant (positive) and the coefficient for first period lag is significant (positive); whereas the sum of lag coefficients for returns is significant (positive). Schwert concludes that there is only weak support for a positive risk-return tradeoff, and he does not investigate if this support can be strengthened by testing for shorter lag lengths.

Bali and Engle (2010a,b) and Bali (2008) have adopted an alternative approach to obtaining adequate observations in risk-return studies, i.e. without either increasing time span or data frequency. They note that, assuming the inter-temporal CAPM model of Merton (1973), expected return for any portfolio should be positively related to the covariance of the stock or portfolio with the market portfolio. Assuming identical risk preferences across investors, the coefficient for risk in risk- return regressions should be identical across stocks and portfolios, and this is the common relative risk aversion. Consequently Bali and Engle specify regression equations for different portfolios and pool them for estimation, assuming a common coefficient for risk across all equations. This greatly increases the number of observations, since almost all other econometric studies of risk-return tradeoff are limited to data on returns and risk for a total market portfolio, i.e. a single portfolio. The model is estimated by DCC-MGARCH and the common coefficient is statistically significant positive (Bali and Engle 2010a use both multiple portfolios and daily data to increase observations).

This is a very interesting, important and largely successful approach to addressing the problem of inadequate sample size noted by Lundblad. However, this approach depends critically upon the assumption of common coefficients for risk across different portfolios; but different portfolios may well attract investors with different risk preferences. In a more general model, Campbell (1993) argues that the coefficient of risk cannot usually be interpreted as relative risk aversion (see earlier footnote). Bali and Engle (2010b) test the hypothesis of common coefficients and do not reject it. On the other hand, Ng (1991) rejected such a hypothesis, and Duffee (1995) concluded that estimates of coefficients of risk in risk-return equations for different stocks were correlated with size of stock.

In sum, there appear to be two interesting approaches to increasing observations in risk-return studies (beyond increasing time span). Risk-return models for various portfolios (not simply the total market portfolio) can be pooled and estimated assuming common coefficients for risk in regression equations for different portfolios, as in Bali and Engle. Alternatively, time frequency can be increased by estimating tradeoff relations at a daily level rather than monthly or quarterly level. Both approaches have their advantages and drawbacks (and the two approaches can be combined, as in Bali and Engle 2010a). Modeling lags is particularly important in daily models and apparently has not yet been done appropriately in the risk-return literature. This study focuses on the second approach using daily data and shows that it can effectively address the problem of inadequate sample size.

Although Lundblad speculated that frequency of data is of second order importance in addressing the problem of adequate observations, several studies addressing the problem since then have used daily data (Bali and Engle 2010a; Muller, Durand and

Maller 2011). However these studies ignored lags in specifying risk-return models with daily data.

The importance of including lags is emphasized by Lettau and Ludvigson (2010) (L&L) even for risk-return models with monthly or quarterly data. Standard risk-return tradeoff models exclude lags in risk and return, and L&L refer to these as unconditional models. In contrast L&L specify a model adding one period lags in both return and risk, and they refer to this as a conditional model. Using quarterly data for aggregate U.S. stock market (CRSP data) over 1953-2000, the coefficient of risk is negative in the unconditional model but often significant positive in the conditional model (p. 669). Other studies also favor conditional models over unconditional models, although results can differ from LL (Whitelaw 1994; Brandt and Kang 2004; Ludvigson and Ng 2007).

We conclude the literature review with a brief summary of the rationale for lags. Lags have been justified in empirical risk-return models on the basis of empirical results rather than theory. The most recent authoritative review of the literature (Lettau and Ludvigson 2010) argues that "the empirical risk- return relation is characterized by important lead-lag interactions" and this "is crucial for understanding the empirical risk-return relation" (p. 621). The argument is based on empirical results in this and other studies rather than on theory (also see pp. 662-3, 670, 682). An earlier study (Schwert 1990) estimated a daily risk-return model with 22 period lags in both return and risk. The lag in returns is justified in terms of estimating "short-term movements in conditional expected returns" (p. 82) and the lag in risk reflects "persistence of volatility" (p. 86). Brandt and Kang (2004) conclude that the motivation for lags is "mostly empirical" and that "lead-lag interactions are a systematic but until-now overlooked feature of the relationship between expected returns and risk" (p. 224). Whitelaw (1994) makes similar statements and concludes that "volatility appears to lead expected returns over the course of the business cycle" (p. 517) and that "empirical results bring into doubt the value and validity of focusing on the contemporaneous relation between expected returns and volatility at the market level" (p. 540). Ludvigson and Ng (2007) state that lags are "crucial for understanding the empirical risk-return situation" (p. 174). In the absence of a theoretical justification for lags in the empirical risk-return literature, we can refer to the extensive theoretical arguments by economists for generally including lags in time series econometric models that underlie the general-to- specific approach to dynamic econometric models. Lags should be more important in a daily model than in a monthly model (for example, a two period lag in a daily model may be represented more closely as a zero period lag in a monthly model than as a one period lag).

4.3. B. Methodology

Given the historically poor results for empirical risk-return studies, the primary concern in the empirical literature has been to obtain statistically significant (and positive) estimates of a tradeoff. Recent literature suggests that poor results may be largely due to inadequate sample size.

This study estimates a risk-return tradeoff at a daily level, whereas most other studies estimate a tradeoff at a monthly or quarterly level. Our primary interest in a daily level is that this may help address the problem of insufficient sample size, leading to more precise estimators of the tradeoff.

The effect on estimator precision of increasing data frequency from monthly to daily is an empirical issue not clearly established by theory. Higher data frequency generally improves volatility estimation using both realized volatility and GARCH; but this may increase noise in estimating expected returns, especially in relation to various microstructure effects. Moreover modeling dynamics/lags may be particularly important for daily models. So the net effect of daily data on precision is unclear a priori.

Literature on market microstructure highlights effects of nonsynchronous trading and bid-ask spreads on distribution of returns at daily and intradaily levels. These lead to negative lag-1 serial correlations and covariances in stock returns and negative lag-1 serial correlation in returns for a portfolio (e.g. Tsay 2010). Market conditions or frictions in the pricing process at daily level may complicate the relation between current asset price and future expected cash flows, perhaps increasing noise in the risk-return relation.

However, we accommodate possible lag-1 serial correlations in aggregate returns by including lagged returns as an explanatory variable in our empirical model (this is an additional rationale for lagged returns in the case of daily models). Regarding increased noise at the daily level, we will not find that R^2's for empirical models are lower at the daily level than at the monthly level. This suggests that microstructure noise in daily data may not add much noise to already serious noise in monthly models, or at least that other factors (e.g. perhaps more precise estimates of risk) compensate for this.

The empirical study by French et al (1987) suggests that daily data can increase precision, at least in a time span of thirty years (1953-84). Merton (1980) argues that a long time span is required to model expected returns, but more recent studies conclude that expected returns change over time and that risk aversion may change over time as institutions change. Adopting both a substantial time span and daily data may help address these problems.

The data used here is from essay one, covering 1995-2005. This time period of eleven years is much shorter than in any risk-return study using monthly or quarterly data and is also shorter than for other studies with daily data (Duffee 1995 used 15 years). A short time period is chosen in order to highlight the role of daily data in contributing to significant estimates of risk coefficients, and this period has the advantage of including substantial variation in stock market conditions (two periods of high returns with a recession in the middle). Estimates of the tradeoff are statistically significant and positive (and of course estimates are insignificant using monthly data over this short horizon). Thus using daily data 1995-2005 is sufficient for significant estimates of a tradeoff. This suggests that, using daily data with a more substantial time span, we may be able to accommodate and test for changes in coefficients of risk over at least ten year intervals.

The method presented here is unique in two respects. First and most important, lags are specified in the model using a general-to-specific approach, as is common in dynamic econometric models outside the risk-return literature. Second, the model is viewed as a structural equation with possible endogeneity/measurement error problems, as is also common in econometric models outside the risk-return literature.

The risk-return relation is specified at the level of the market portfolio, as in most studies. Aggregate risk for the portfolio is calculated from multivariate GARCH estimates of individual returns for 88 stocks, as discussed in chapter two. Results in chapter two favored an EGARCH(1,1)-ARMA(2,1) model for estimating univariate risk together with a DCC(1) model for covariances, and here we use estimates of this

MGARCH model. MGARCH results are aggregated into an index of portfolio returns using a value-weighted index and Laspeyres index as discussed in chapter three.

The choice of state variables for an ICAPM model is ambiguous. According to Cochrane (2005), variables that can forecast asset returns can be the candidate based on permanent income logic. The following state variables influencing returns were included in the model: a relative bill rate defined as the three month Treasury bond yield less its four quarter moving average, a term spread defined as difference between the 10 year Treasury bond yield and the 3 month Treasury bond yield, and a default spread between Baa corporate bonds and Aaa corporate bond rates. These three macroeconomic variables are denoted as RREL, TEF and DEF, respectively. Daily data on these variables is obtained from the H.15 data base of the Federal Reserve Board. Similar variables have been incorporated into other risk-return studies (e.g., Lettau and Ludvigson 2010; Bali and Engle 2010a, b; Pollet and Wilson 2008; Ang and Bekaert 2007; Campbell 1987, 1991; Fama and French 1988; Fama and Schwert 1977). RREL and TRM represent innovations in short and long term interest rate (Campbell 1991). Quarterly data on ratio of consumption to aggregate wealth (CAY) is also included in the model; since Lettau and Ludvigson (2010) concludes that this is particularly important (data are downloaded from Lettau's website). This quarterly variable is included in the daily model simply by assuming CAY is constant for each day in a quarter. Obviously, this is a poor proxy for daily CAY data, but we have no basis for transforming this to a more smooth proxy (we estimate models with and without CAY). Summary statistics for dependent and independent variables are presented in Table 4.2, and correlations are presented in Table 4.3.

The risk-return regression equation is written similarly to (6) except with lags. This study adopts a general-to-specific approach to specifying lags similar to chapter two. The tradeoff is estimated with time series data, so the relation is essentially dynamic. Since the exact structure of dynamic relations is seldom known a priori, it is common practice in economics to estimate dynamic models as autoregressive distributed lag (ADL) models (e.g. Davidson and MacKinnon 1993; Hendry, Pagan and Sargan 1984). ADL models include lags in all variables (including both the dependent and independent variables) as explanatory variables. ADL models provide reduced form approximations to true dynamic models, and relatively short lag lengths are often sufficient.

Given that lags in dynamic models can be long and are generally unknown a priori, a general-to-specific approach has become the standard approach to lag specification in time series econometrics (e.g. Hendry and Richard 1990; Hendry 1995; Sargan 1980). Long lags are specified for ADL models that nest a correct approximation to the dynamic model. Then nested tests are used to reduce lag lengths.

In chapter two we begin by assuming a general ARMA(20,1) autoregressive model for returns of individual stocks with daily data, i.e., the mean equation for return is initially specified with a 20 period (20 day) lag in returns and an MA(1) disturbance. Wald test results indicate that an ARMA(20,1) can be reduced to an ARMA(2,1) for 73 of 88 stocks at the 5% level.

Similarly, here we begin by specifying a general ADL(20,20)- MA(1) model for the risk-return model, i.e., the risk-return equation is initially specified with 20 period (20 day) lags in all variables (dependent and independent). The hypothesis ADL(2,1)-MA(1) is not rejected at the .05 level, i.e., the general model can be reduced to a 2 period lag in return and a 1 period lag in independent variables. We also confirmed that ADL models can be reduced by interval of (5, 5) from an ADL(20,20) to an ADL (5,5), and then to an ADL (2,1). This result of a very simple dynamic structure is not surprising: it is well known that dynamic models with long lags in independent variables (no lag in dependent variable) can often be approximated as simple ADL models such as ADL(1,1). [26]

Our ADL(2,1)-MA(1) model for expected returns-risk of returns tradeoff at the aggregate portfolio level is

(7) $R_{mt} = \alpha_0 + \Sigma_{i=1,2}\alpha_1 \; R_{i,t-i} + \theta_1 \; var(R_{mt}) + \theta_2 \; var(R_{m,t-1}) + \Sigma_{i=1,.,4} \; \lambda_{1i} \; X_{it} + \Sigma_{i=1,.,4} \; \lambda_{2i} \; X_{i,t-1}$

$+\beta \; e_{t-1} + e_t$

($\alpha_0 = 0$ under standard CAPM). R_m is the aggregate return on market portfolio net of risk free return, $var(R_m)$ is conditional variance of R_m, and X are four other state variables. Initially, we also include daily dummy variables, but they are statistically insignificant separately and jointly.

Thus, we arrive at a simple dynamic structure for the risk-return model with daily data in an appropriate manner. In contrast, other risk-return studies with daily data generally ignore the issue of lags. A notable exception is Schwert (1990), who specifies in effect an ADL(22,22) model. However, he does not consider how to reduce this to a simpler ADL structure, so he can only conclude that there is weak support for a tradeoff in his study.

Empirical studies of risk-return tradeoff adopt many estimation methods. These

[26] This is analogous to the observation in heteroskedastic models that an ARCH model with long lags can often be approximated as a simple GARCH model such as GARCH(1,1).

include: (a) after estimation of risk, estimate risk-return regression equations by ordinary least squares (OLS) or simple variants; (b) estimate GARCH-in-mean models by maximum likelihood; (c) project returns and risk onto state variables and then analyze the correlation between the fitted mean and volatility from these projections (Lettau and Ludvigson 2010).

This study adopts a slightly different approach that is standard outside the risk-return literature, i.e., in general econometrics literature. The risk-return model is viewed as a structural equation where the major issue is endogeneity of risk. Risk var(R) and the disturbance e may co-vary for one of two reasons: expected return and risk of returns may be determined jointly in the economy, or there may be measurement errors in risk. The consequences, tests and corrections for these problems are standard (so long as variables measured with error are linear in the model). The standard test for covariance of var(R) and e is a Wu test, or more generally a Durbin-Wu-Hausman test. Risk-return studies emphasize difficulties in measuring risk and possible sensitivity of results to alternative methods of modeling risk, so one might expect Wu tests to be common in the literature. Nevertheless, we were unable to find Wu tests in any risk-return study.[27] This may be explained in part because measures of risk are typically generated variables rather than data, but Wu tests can still be appropriate (see footnote 28).

A Wu test can be illustrated by the following simple example. Given a simple model $y= \beta x+e$, where variable x may co-vary with disturbance e, regress x versus instrument z

[27] There is a large literature estimating CAPM models using instrumental variables within the framework of GMM (e.g. see Ferson and Jagannathan 1996 for references). Testing is often limited to J-tests of over identifying moment conditions (orthogonality of instruments with disturbance) without testing specifically whether return risk is endogenous. In principle Durbin-Wu-Hausman tests for endogeneity of regressors can be closely related to appropriate GMM tests of orthogonality conditions (e.g. Baum, Schaffer and Stillman, 2002), but such tests have not been conducted for return risk.

(cov(z,e)=0) using OLS to obtain predicted $\hat{x}$, and then specify an augmented model y= $\beta x+\gamma \hat{x}+e^*$. Then the null hypothesis H_0: cov(x,e)=0 can be tested as a simple t (or F) test of H_0: $\gamma=0$ for OLS estimation of the augmented model. In our case we use an analogus procedure to test if risk varR covaries with the disturbance in the risk-return relation.

A Wu test requires identification of the risk-return equation in the case where risk is endogenous, i.e., it is necessary to find sufficient instruments for risk beyond X to imply identification. Often it is difficult to find such instruments. If R is related to variables X as X β (abstracting from a risk-return tradeoff), then the simple variance of R is related to X as β^T CovX β, where CovX is a covariance matrix for X (see any elementary text in statistics). Then the simple variance of R depends on variances of individual variables in X and also depends on covariances of variables in X. We assume that X directly influences R or ER but that CovX does not directly influence R or ER (CovX only influences R or ER indirectly, through its influence on varR). This suggests that conditional variances and covariances may be appropriate additional instruments for risk. Relations between coefficients in risk-return models and models explaining risk will be discussed in more detail in section 4.4 B.

Then a Wu test for endogeneity of risk can be outlined as follows. VarR is estimated as a linear function of Z and CovZ (and 2 period lag in R assuming this does not co-vary with e), and the predicted VarR from this regression (and its one period lag) is added to equation (7). Then the joint hypothesis that predicted varR and its lag can be dropped from the augmented equation is tested as an F test in OLS regression of the augmented equation, assuming large samples and some qualifications.[28]

If the hypothesis is rejected, then the endogeneity problem is likely and consistent estimation of (8) requires instrumental variable (IV) methods including CovX as additional instruments for varR. If the hypothesis is not rejected, then we can use simpler methods than IV and are more likely to uncover precise estimates of the risk-return tradeoff (IV increases standard errors of coefficient estimates relative to OLS). In our study, in most cases we do not reject this joint hypothesis, so varR does not appear to co-vary with the disturbance and IV estimation seems unnecessary.

4.3. C. Econometric Results

The main OLS results for the final ADL(2,1) model (8) are summarized in Tables 4.4 and 4.5 (model one) for value-weighted and Laspeyres aggregations, respectively. As expected, R^2 is low at .046 and .026. R^2 drops to .006 and .004 when omitting all X variables, and to .003 and .002 when omitting lags in return and risk as well. Test results suggest heteroskedasticity but no autocorrelation in e. Both standard and HAC (heteroskedasticity and autocorrelation corrected) Newey-West standard errors were calculated (Newey and West 1987), and corresponding probabilities are reported. Although residuals do not have a normal distribution, the asymptotic distribution of the

[28] If under the null hypothesis explanatory variables in (7) are exogenous (not merely predetermined) and the disturbance is normal, then the F statistic for the joint hypothesis does have an F distribution (e.g. Davidson and MacKinnon 2004). Lagged returns are predetermined, but a more serious problem is that the proxy for risk is a generated variable that is clearly stochastic. However GARCH coefficient estimators are consistent under standard assumptions, and similar results hold for MGARCH-DCC (Engle 2002b; Engle and Sheppard 2001). So, assuming our MGARCH model of risk is correct, we have consistent estimates of true varR, which perhaps can be exogenous (under this version of the null hypothesis, the F statistic can have an asymptotic F distribution).

OLS estimator for coefficients is still normal under standard assumptions,[29] which justifies hypothesis testing based on large samples.

Sum of current and lagged coefficients for varR and X are reported. Sum of coefficients for lagged returns R is +0.68 and +0.66 for value-weighted and Laspeyres models. The constant is statistically insignificant, as is suggested by standard CAPM theory.

In the final ADL(2,1) models with all X (model one), the sum of coefficients for return risk varR are +12.62 and +13.41 for value-weighted and Laspeyres models, respectively, and these are significant at the .01 level using both OLS and HAC standard errors. The sum of coefficients is interpreted as the total impact of risk on returns R_t. Thus there is a statistically significant positive relation between risk and return.[30] An ADL(0,0) model (no lags, model three) leads to similar results. Dropping the insignificant quarterly variable CAY (ratio of consumption to wealth) from the ADL(2,1) (model two), the sum of coefficients for risk varR are +8.73 and +9.71 (and significance levels drop slightly, ranging from .01 to .03) (model two). The sum of coefficients is insignificant for separate state variables X, but the joint hypothesis excluding all state variables is rejected at the .05 level based on a Wald test. Nevertheless, the model is estimated omitting all state variables (ADL(1,0) model four, which also omits the lag in

[29] Normality of disturbance e is rejected (probability 0.00 for Jarque-Bera test, but in a histogram residuals at least show an approximate bell shape). Nevertheless, OLS estimates of coefficients are asymptotically normal assuming no autocorrelation in the disturbance e and non-stochastic explanatory variables, by the Lindberg-Feller central limit theorem (e.g. Greene 2008). The major difficulty is that aggregate risk varR may be stochastic, but Wu test results suggest this is not a problem.

[30] Individual coefficients for current and lagged risk varR have opposite signs and no ready interpretation, as is common in ADL models. So it is the sum of coefficients that is of interest. The standard error of the sum of coefficients is most easily calculated by transforming the regression equation as discussed in (e.g.) Davidson and MacKinnon (1993, pages 673-4). This leads to standard errors ignoring heteroscedasticity and to HAC standard errors for the sum, and to associated probabilities.

risk and the second period lag in returns). Then the coefficient for risk is +5.99 and +5.76 (significant at .01 to .06 levels). Finally, an ADL(0,0) model omitting all lags and all X is estimated (model five). The estimated coefficient for VarR is 7.73 and 7.29 for value-weighted and Laspeyres (significance levels range from .01 to .05). [31, 32]

Thus the estimated impact/tradeoff of risk varR is positive and statistically significant in all the above specifications, with ADL (2,1) to ADL(0,0) lag structures, and irrespective of state variables X. Although the time span is considerably shorter than in most studies (11 years), the use of daily data implies more observations than in most studies (2,771 here). [33]These estimated tradeoffs between risk and expected return are higher than in most studies, and the tradeoff is usually interpreted as a measure of relative risk aversion (see caveats in footnote 24). The specification most comparable to previous studies is value-weighted model five, excluding state variables and lag in risk, and here the coefficient of risk is 7.73 (with probability .01 based on HAC standard errors). [34] This is very similar to French et al's estimate of 7.22 for daily data for 1953-84. A recent study by Yu and Yuan (2011) also estimates high coefficients for risk during periods of

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[31] Similar results were obtained using aggregate methods, but these results are not reported since in principle this is an inferior approach due to errors in contemporaneous aggregation in GARCH models (as discussed in chapter three).

[32] If a model is correctly specified leading to consistent estimators, then substantial increases in sample size generally leads to statistically significant estimates close to the truth (which would be 0 if there is no risk-return tradeoff). Since estimated coefficients of risk in this study are larger than in most studies (as well as statistically significant), there is no indication that daily data is leading to statistical significance for an irrelevant risk variable.

[33] Two five-year sub periods were also briefly considered (1995-99, 2001-2005, omitting recession year 2000). Coefficient estimates of risk were statistically significant positive for these sub periods.

[34] In dynamic risk-return models with lags in risk, it is not clear that the sum of coefficients for risk should be closely related to relative risk aversion. An ADL model can be interpreted as a reduced form for quite general dynamic optimization models (e.g., regarding the simplest case of an error correction model or equivalently simple ADL(1,1), see Hendry and von Ungern-Sternberg 1981; Salmon 1982; Nickell 1985; Davidson and MacKinnon 1993), and models such as Merton's inter-temporal CAPM is only one of many possibilities. In contrast, models specified without lags (as in most risk-return studies) cannot readily accommodate most possibilities, and in that sense they are biased in favor of simple models such as CAPM.

low sentiment, when investment decisions presumably are most likely to follow rational asset pricing models. For monthly GARCH models during low sentiment, coefficient estimates range from 7.6 to 15.7 and are generally significant; whereas estimates are insignificant during high sentiment (our study includes periods of both low and high sentiment).

However, the range of estimates for relative risk aversion (RRA) in finance studies is quite broad. For example, Brown and Gibbons (1985) estimated RRA using a utility based model of asset pricing and reported a wide range of estimates for different time periods in 1926-81, with estimates of relative risk aversion between 0.1 and 7.3 (7.0 to 7.3 with standard error 2.35 using value-weighted return index monthly data for 1953-67, and 0.1 insignificant for 1967-81). Litzenberger and Ronn (1986) estimated RRA at 4.2 (standard error 0.25) using an utility-based model and aggregate stock price, dividend and consumption data over 1926-82, and they note that this is similar to the guess (4.0) by Grossman and Shiller (1981). Using a somewhat similar methodology, Karson, Cheng and Lee (1995) present estimates of RRA from 1.0 to 2.1 (t-ratios approximately 2.0) based on 1926-83 stock return index data. Another related paper by Lee and Lee (2004) estimate RRA distribution for 1991-2001 centered at 5.0 (length of 95% confidence interval is 15) and for 1967-80 centered at 1.2 (with similar size confidence interval).[35]

For comparison, data in the daily models were aggregated to a monthly level and several specifications of the risk-return model were re-estimated by OLS. As expected using monthly data over a short time span, the estimated tradeoff was generally less

[35] Following Rabin (2000), Neilson and Winter (2002) argue that RRA compatible with portfolio choice data (moderate risk) are larger than RRA compatible with wage-fatality risk premium data (large risk situations).

statistically significant than with daily data. Results are reported in table 4.6. Since lags in daily and monthly models are not equivalent, the most instructive comparison between monthly and daily models is in the case of no lags. For example in the daily Laspeyres model with no lags, the coefficient of aggregate risk is significant at the .05 level (probability .047) using HAC standard errors (model five Table 4.5). In contrast for the monthly Laspeyres model with no lags, the coefficient of aggregate risk is not significant (probability .88) using HAC standard errors. Since the econometric method was identical for these daily and monthly models, these results suggest that increasing time frequency from monthly to daily can substantially increase precision of estimates of risk-return tradeoff, in contrast to speculation by Lundblad (2007). Other studies also obtained more precise estimates of tradeoffs at daily than monthly level (French, Schwert and Stambaugh 1987; Duffee 1995).

A comparison of R^2's in daily and monthly models also may be of some interest. In the monthly Laspeyres model without lags (model five), R^2 is 0.0001 versus 0.0020 for the analogous daily model. In the monthly value weighted model without lags, R^2 is 0.0015 versus 0.0027 for the analogous daily model. By this measure, it is not apparent that aggregating from daily to monthly data reduced noise (e.g. related to microstructure) in the risk-return model.

This is the first study applying a specification test for covariance between risk and disturbance in a risk-return equation. A covariance between risk and disturbance reflects either joint determination of return and risk in the economy or measurement error in risk. Pollet and Wilson (2006) conjectured that risk and return may be determined simultaneously. In this case, consistent estimation of the model requires instrumental variable (IV) methods.

Additional instruments for the Wu test are conditional variances and covariances of daily state variables X. Only monthly data are available for the ratio of consumption to wealth CAY, so it is omitted from these calculations.[36] Since there are only three daily state variables X here, covX can be estimated by the diagonal VECH model, and this allows a more general dynamic correlation model than does the two-step DCC model of Engle (2001). Maximum likelihood estimates of the diagonal VECH model are shown in Table 4.7, and estimated variances and covariances are shown in Figure 4.3.

Wu tests are conducted as discussed above.[37] Risk varR is regressed against all instruments including covX to obtain a predicted varR. The predicted varR (RISKHAT) and its lag are added to model (8) and an F test is conducted for their joint significance.

Wu test results for the ADL (2, 1) model (8) with value-weighted and Laspeyres indexes are reported in Tables 4.8 and 4.9. Step one is OLS estimation of risk varR against all instruments. For the value-weighted model, R^2 is .464 and F-statistic for significance of all instruments is 144.1, and for the Laspeyres model $R^2 = .479$ and F-statistic is 153.6.[38] Step two is addition of predicted risk from step one to model (8) and OLS. Predicted risk (RISKHAT) and its lag are insignificant separately and jointly. The

[36] Omitting variance and covariances with CAY does not alter properties of the Wu test, since the other variances and covariances are more than sufficient to achieve identification of the regression equation (7) if risk is endogenous.

[37] A standard assumption for the Wu test is that the disturbance e has a normal distribution, but this is rejected by tests. Nevertheless, under the null hypothesis that aggregate risk varR is non-stochastic, the OLS estimator is asymptotically normal (see earlier footnote), and a similar conclusion applies to an IV estimator. This implies that a Hausman-Wu test is appropriate even if e does not have a normal distribution.

[38] Wu-Hausman tests are invalid in the case of weak instruments (Hahn, Ham and Moon 2011). The standard ad-hoc diagnosis for weak instruments is a low R^2 or F-statistic below 10 over all instruments for stage one regression (Hahn and Hausman (2002, 2003) propose a more sophisticated approach). However here the F- statistic is 140-150 and R^2 is moderate (especially in comparison to R^2 for risk-return regressions). This suggests we do not have a problem of weak instruments.

F statistic for the joint hypothesis that RISKHAT can be dropped has probability 0.23 and 0.35 for value-weighted and a Laspeyres model, respectively, i.e., the hypothesis is not rejected at the .05 level. We conclude from these test results that risk does not co-vary with the disturbance in risk-return model (7). For this reason, IV methods are not required for consistent estimation of the model. [39]

4.4. Relating Aggregate Risk to Economic and Financial Variables

A unique and important aspect of the above risk-return study is Wu tests of covariance between risk and disturbance. This test requires regression of risk versus instruments including variances and covariances of state variables. Essentially, this is a reduced form model explaining aggregate stock market risk in terms of state variables at the daily level.

This section considers in more detail models relating aggregate risk to state variables. In addition, links are established to risk return models.

There is an important caveat: as in the related literature, we are essentially estimating correlations between risk of returns and economic and financial variables rather than analyzing causality. As noted by (e.g.) Engle, Ghysels and Sohn (2009), all such models are reduced form models delinked from structural models of the macro economy.

4.4. A. Literature Review

[39] Wu tests were also conducted for simpler (more poorly specified) risk-return models, and in some cases the hypothesis of zero covariance between risk and disturbance was rejected. Perhaps the frequent acceptance of the null hypothesis is not surprising, since risk-return models have low R^2's and generally difficulty in precise estimation.

Although there has been substantial progress in modeling time variation of volatility of stock market returns, progress in explaining this volatility empirically in terms of economic fundamentals has been limited. The classic study is Schwert (1989). Monthly volatility in returns is proxied as a simple variance of daily returns (1885-1987, S&P and Dow Jones composite portfolios). Volatility for monthly economic variables is measured from absolute value of residuals from a 12th-order auto regression of macroeconomic data, including inflation, money growth, industrial production, interest rate. However, none of these volatilities are significant in explaining return volatility, although it is observed that return volatility is higher during recessions. In multiple regressions (quarterly data 1900-87) for return volatility in terms of macroeconomic volatility (industrial production, inflation, and money base), recession and leverage, only recession is generally statistically significant.

Recently progress has been made in explaining return volatility in terms of economic volatility. Two studies have pooled data across stock markets for many countries, assuming identical coefficients across countries. Diebold and Yilmaz (2008) pool annual data 1983-2002 on stock markets of approximately 40 countries (and pool quarterly data for fewer countries). Conditional volatilities for stock returns, real GDP and real personal consumption are calculated similarly to Schwert, as residuals from an AR(3) model of returns, GDP and consumption. Stock market volatility is significant positive in volatility of GDP and consumption separately (results are not reported for a regression model with both variables).

Engle and Rangel (2008) specify high frequency (daily) return volatility in terms of a standard GARCH model and a slow-moving trend modeled as an exponential quadratic spline. These are short term and long term components of volatility (the slow-moving trend does not revert to a constant level and allows for changing unconditional volatility, unlike most studies). Daily returns are used to estimate a spline-GARCH volatility model for each country, leading to estimates of low frequency and high frequency volatility. Volatilities for macroeconomic variables (real GDP, inflation, short-term interest rate, exchange rate) are calculated by estimating AR(1) models with quarterly data and then summing absolute values of residuals over quarters. Annual models of low-frequency return volatility (unconditional volatility) versus macroeconomic volatility and other variables are estimated over 48 countries 1997-2003 by SUR (seemingly unrelated regressions). Volatility of real GDP and inflation has large significant positive effects, volatility of interest has small significant positive effect, and volatility of exchange rate has an insignificant effect.

Engle, Ghysels and Sohn (2009) advocate a different approach to relate macroeconomic variables to return volatility. High frequency return volatility is again decomposed into a standard GARCH model and a slow-moving trend, but now the trend is specified as a MIDAS model that directly incorporates macroeconomic variables of any data frequency, rather than as a spline.[40] The study uses daily returns for U.S. stock market over 1885-2004. Levels and volatilities (calculated as above) for two quarterly macro variables are included in the MIDAS model of trend, with endogenous weights for

[40] Colacito, Engle and Ghysels (2011) have recently extended the GARCH-MIDAS model to a DCC-MIDAS model by replacing the univariate GARCH process (ignoring correlations between, e.g., returns of different stocks) with a DCC-GARCH multivariate model of correlations. This short run component of volatility is still combined with a long run component specified as a MIDAS model. This approach could be useful in explaining return risk at a disaggregate level rather than (as in most studies) at an aggregate level (i.e. explaining aggregate risk, as here).

16 lags (4 years). The macro variables are inflation and industrial production (earlier versions of the paper also included monetary base, term spread and GDP). Daily data on returns are used to estimate a MIDAS-GARCH volatility model, providing direct estimators of impacts of macro variables on long term component of return risk. Models are estimated with just one macro variable at a time. When both level and volatility of the single macro variable are included, impact of volatility is significant positive but impact of level is statistically insignificant.

Lettau and Ludvigson (2010) also estimated models of return volatility. Using daily data on CRSP value-weighted return index (1952-2000), monthly volatility is measured as a realized volatility from squared daily returns over the quarter (rather than as a MIDAS process). Explanatory variables are ratio of consumption to wealth CAY, dividend yield, and default spread (Baa corporate bond rate minus Aaa corporate bond rate), difference between yields on six-month commercial paper and three-month Treasury bond, and one-year treasury yield. Return risk for various forecast horizons (1 to 24 quarters) are regressed by OLS against these variables in levels (not volatilities) and also current and one period lag in return risk. This is somewhat similar to an ADL(2,0) model. All variables except default spread were statistically significant.

Empirical studies of stock market return volatility have examined many other possible explanations in addition to macroeconomic volatility. Schwert (1989) addressed the major considerations in the literature. Although emphasizing the role of macroeconomic volatility, he also estimated the relation of return volatility to returns, recessions, corporate profitability (dividends and earnings yields, spreads between yields on Baa and Aaa-rated corporate bonds, financial leverage (debt-equity ratio), and volume of stock market trading. Levels but not volatilities of noneconomic variables were included in regressions for return volatility. In separate regressions, return volatility was positively related to spread between yields on Baa versus Aa-rated corporate bonds (a measure of default risk), unrelated to dividend or earnings yields, unrelated to leverage, and positively related to stock market trading volume. Schwert concluded that all variables explained little of return volatility.

4.4. B. Methodology

This section develops a model relating aggregate risk $varR_m$ of stock market returns to state variables X in the previous risk-return model (7). The model is novel in several respects, in comparison to models explaining aggregate risk in terms of economic variables. This is the first model to include lags in variables based on a general-to-specific methodology and to include covariances of variables X (other studies only included variances). In addition, this is one of few models at the daily level rather than monthly or quarterly level (Engle, Ghysels and Sohn 2009 estimate a model using daily data on returns).

In our earlier discussion of the Wu test and additional instruments for risk, we noted the following: ignoring a risk- return tradeoff, if returns R are related to state variables X simply as $R = X\beta$, then variance of R is related to variances and covariances of X simply as $var(R) = \beta^T CovX \beta$, where CovX is a variance-covariance matrix of variables X. However this well-known result (see any introductory text in statistics) is not directly relevant since it ignores a risk-return tradeoff.

Appendix IV extends the above analysis to risk-return models. Now consider a risk-return model where returns R are related to variables X and return risk varR. Let

(9) $R_t = X_t \beta + \gamma \, varR_t + e_t$ $Ee=0$ $cov(X,e)=0$ $cov(varR,e)=0$.

The conditional variance for R_t is

(10) $varR_t = \beta^T cov(X_t) \beta + var(e_t) + 2\gamma \, cov(X_t\beta, varR_t) + E\,(varR_t - E\,varR_t)^2$

where the last term is the conditional variance of the random variable varR (proof is in Appendix IV). If risk and return are jointly determined (so $cov(varR,e)\neq 0$), then a more complex result holds: $2\gamma \, cov(varR_t,e_t)$ is added to the right hand side of (10) (see proof). On the other hand, in the special case where varR is not a random variable (is non-stochastic), then (10) reduces to $varR_t = \beta^T cov(X_t) \beta + var(e_t)$. This result (10) presumably is not known, at least within the context of risk-return models. This links coefficients β of X in risk-return models to coefficients of covX in models explaining risk, as $\beta^T cov(X)\beta$. The connection between coefficients of the risk-return model and of the model explaining risk is more complex than in the absence of risk-return tradeoffs.

The above theoretical result has broad implications for econometric models of risk-return and models explaining risk. In their review of risk-return models, Lettau and Ludvigson (2010) note that there is a general perception these models do not share any explanatory variables. In contrast, LL conclude from their empirical results that these models share at least one variable, consumption to wealth ratio CAY (p.667); but LL do not refer to a theoretical argument supporting this conclusion. In contrast, we have proved that in principle, these two models share variables (and there are nonlinear restrictions between coefficients). Thus our results help to unify in principle these two major models in empirical finance.

This result (10) implies that the variance of R is positive in variances of individual variables in X and also depends on covariances of variables in X. This suggests that models explaining conditional variance of returns should include conditional covariances as well as variances of state variables X. [41] In contrast, apparently all other studies ignore covariances.

Results (9) - (10) linking coefficients β of risk-return models and models explaining risk suggest that these two models share common state variables X and these two models may be estimated jointly. Of course one complication in joint estimation is that the equation for varR is nonlinear in β.

It is important to include lags in our model, especially since it is specified with daily data. For simplicity we began by assuming a similar ADL(2,1) structure as in the risk-return model (several simple tests with longer lags did not reject this assumption). This reduced to an ADL(1,1) model (the second period lag on $varR_m$ was insignificant due to high multicollinearity). The resulting ADL(1,1) model is [42]

$$(11)\quad var(R_{mt}) = \beta_0 + \beta_1\, var(R_{m,t-1}) + \Sigma_{i=1,..,4}\, \lambda_{1i}\, X_{it} + \Sigma_{i=1,..,4}\, \lambda_{2i}\, X_{i,t-1} + \Sigma_{i=1,..,3}\, \gamma_{1i}\, var(X_{it}) + \Sigma_{ij=1,..,3;i\neq j}\, \varphi_{2ij}\, cov(X_i X_{j,t-1}) + e_t$$

Test results indicate that the lag structure can be simplified further by eliminating lags in variances and covariances (since these are highly auto correlated), resulting in a hybrid ADL(1,1)/ADL(1,0) model.

The four state variables X are the conditioning variables in the risk-return model: a

[41] Moreover some studies include levels of X rather than variances of X. As noted above, Engle, Ghysels and Sohn (2009) include both levels and variances, and find that variances but not levels are significant when both are included in the same regression. We will include levels of X as well as variances and covariances of X.

[42] In the Wu test for the risk-return model (7), it is necessary to include all possible instruments from (7) as well as additional instruments in the regression equation for risk on the assumption that expected returns and risk are determined jointly, so lagged returns was included in the equation. Since the Wu test results suggest that expected returns and risk are not determined jointly, now we exclude lagged returns from the regression model (8) for risk.

relative bill rate (three month Treasury bond yield less its four quarter moving average), a term spread (difference between 10 year Treasury bond yield and 3 month Treasury bond yield), a default spread (between Baa corporate bonds and Aaa corporate bond rates), and ratio of consumption to aggregate wealth. These are denoted as RREL, TEF, DEF and CAY, respectively. These variables have been included in various other studies explaining aggregate risk.

The model includes levels of X as well as variances and covariances of X. Level of CAY (consumption/wealth ratio), but not variance or covariances with CAY, are included in the model since we only have quarterly data for this variable.

There is high correlation in our measure of aggregate risk $var(R_m)$ (unlike aggregate returns), so the R^2 for this ADL(1,1) model is very high. In order to obtain a simple measure of the contribution of X to $var(R_m)$, we also estimate the following ADL(0,1) model deleting lags in $var(R_m)$:

$$(12)\quad var(R_{mt}) = \beta_0 + \Sigma_{i=1,.,4}\, \lambda_{1i}\, X_{it} + \Sigma_{i=1,.,4}\, \lambda_{2i}\, X_{i,t-1} + \Sigma_{i=1,.,3}\, \gamma_{1i}\, var(X_{it}) + \Sigma_{ij=1,.,3;i\neq j}\, \varphi_{2ij}\, cov(X_i X_{j,t-1}) + e_t$$

4.4. C. Econometric Results

The model explaining aggregate risk uses the same data (daily data 1995 - 2005) and the same measures of risk as in the risk-return model. The model is specified at the level of the market portfolio, as in most studies. Aggregate risk for the portfolio is calculated from multivariate GARCH estimates of individual returns for 88 stocks, as discussed in chapter two. Estimates from an EGARCH(1,1)-ARMA(2,1)-DCC(1) model are aggregated into an index of portfolio returns using a value weighted index and Laspeyres index as discussed in chapter three. Conditional variances and covariances for the three daily state variables are estimated by the diagonal VECH model, and this allows a more general dynamic correlation model than does the two-step DCC model of Engle (2001).

Measures of the aggregate risk of returns are graphed versus time (1995 - 2005) in Figure 4.3. Series 1 is Laspeyres risk, series 2 is value-weighted risk, and series 3 is the difference between the two. The two series move similarly and are highly correlated (+0.99). Volatilities are relatively high during the middle of the time period. This is related to bursting of the high tech bubble around year 2000.

OLS estimates of Laspeyres and value-weighted models are reported in Tables 4.10 - 4.11. Both standard and HAC Newey-West standard errors are calculated, and corresponding probabilities are reported.

Results for the general ADL(1,1) model are shown in model one of both tables. This model includes a lagged risk of returns, current and lagged levels of all X, and current and lagged variances and covariances of all X excluding CAY. Results are similar for both Laspeyres and value-weighted models. Test results indicate autocorrelation and heteroskedasticity and that the disturbance does not have a normal distribution (Jarque-Bera probability 0.00, and a histogram of residuals suggests skewness). Levels of all four X's are statistically significant at .05 level for either the current or lag specification. Sum of coefficients for variances of TRM and RREL are positive and statistically significant at the .01 level, as suggested by theory. The third variance (DEF default risk) is insignificant. One covariance (TRM, RREL) is positive and significant at the .01 level, and the other two covariances are insignificant. Thus variances and covariances involving TRM, RREL (but not DEF) are related to aggregate risk of returns in a positive and significant manner. Lagged variances and covariances are insignificant due to high autocorrelations, so they are dropped in other models reported here without changing results (compare models one and two). Due to high correlation in the measure of return risk, including lagged return risk leads to a very high R^2 (0.989).

The coefficient of lagged CAY is negative and significant at the .05 level. A similar result is reported by Lettau and Ludvigson (2010) and explained as follows: high CAY predicts high excess returns and an improving economy, and as the economy improves over time volatility of returns is likely to fall. Consumption and wealth are often viewed as cointegrated, i.e., they tend to move together. So an increase in CAY implies a ratio higher than normal and the economy is expected to perform better than normal, which suggests lower volatility of returns.

Other results for levels of X are as predicted. The coefficient for (lagged) relative bill rate is significantly negative: a high relative bill rate implies an increasing interest rate which is an indicator of an improving economy with less volatility. The coefficient for (lagged) term spread is negative and significant: a positive term spread suggests higher short term interest rates in future reflecting an improving economy with presumably lower volatility. The coefficient for default spread is positive and significant: as the economy becomes worse and more volatile, the default spread should increase (as the economy worsens, more risky or lower credit companies suffer higher risk of default, so the yield that investors demand from lower grade stocks will be higher than normal, i.e. default spread increases).

Although levels of X are significant and levels are commonly used in models explaining aggregate risk, theory suggests the importance of covX. To this end, in model three we drop levels of X while retaining variances and covariances. The only level not dropped is CAY, since there are no variances or covariances of CAY in the model. Results for variances and covariances of state variables show little change (the decrease in R^2 is negligible, presumably because lagged risk of return explains most of current risk).

In order to obtain a simple measure of the relation between aggregate risk of returns and state variables X, lagged risk of returns is excluded in model four, leading to an ADL(0,0) model with no lags. R^2 falls substantially but is still 0.479 (Laspeyres) and 0.464 (value-weighted). This indicates that a substantial part of the variation in our measures of aggregate risk of returns can be attributed to state variables.

Model five excludes levels of X (except CAY), as well as lagged risk of returns. Then, relative to model four, R^2 falls substantially to 0.246 (Laspeyres) and 0.221 (value-weighted). This further illustrates that levels of X, as well as covX, are important in explaining risk of returns.

Dropping lagged risk of returns from the regression model (10) has the following interpretation in terms of dynamic models. According to econometric theory of dynamic models and empirical results, the ADL(1,1) model (10) or an ADL(1,0) model is a reasonable reduced form approximation to a true structural dynamic model. Then dropping the highly significant lagged risk of returns seriously mis-specifies the dynamic model (not surprisingly this leads to autocorrelation of residuals, as indicated by extremely small Durbin-Watson d-statistics). In turn, we would expect estimates of the mis-specified model to be less in agreement with theory.

It is interesting to note this is what happens with estimates of models four and five, which exclude lagged risk of returns. In model four Laspeyres, one variance of X is positive and significant, another is insignificant, and a third is negative and significant at the .01 level (somewhat similar results hold for value-weighted). However, theory suggests that coefficients of variances should not be negative. In contrast, results for correctly specified dynamic models (one and two) are more in accord with theory (coefficients are positive and significant for two variances and insignificant for one variance).

The analysis in Appendix IV, as summarized above in (9) - (10), suggests links between risk-return models and models explaining return risk. In principle, coefficients in the two models are linked: coefficients β of Z in risk-return models are linked to coefficients of covZ in models explaining risk, as β^T cov(Z) β.

This possibility of a link in coefficients was tested simply as follows: +/- the square root of estimate of γ_{1i} from model two (Tables 10-11) explaining risk was compared to a 95/99% confidence interval for the estimate of λ_i in risk-return model one (using HAC standard errors), and this was done for both Laspeyres and value-weighted models. However results did not support a link in coefficients (square root of estimates of γ_{1i} were outside confidence intervals for the estimate of λ_i). A more rigorous test would involve joint estimation of the two models (risk- return, explaining risk) as in the next section.

4.5 Joint Estimation of Aggregate Risk-Return Models and Models Explaining Aggregate Risk

Results (9)-(10) show in theory links between aggregate risk-return models and models explaining aggregate risk, and these links have not been recognized in the finance literature. However these links imply that the model explaining risk is nonlinear in coefficients as β^T cov(X) β, where β are coefficients of state variables X in the risk-return model, and there are substantial problems in nonlinear estimation.

This section briefly considers estimation of a simple joint model (ADL(2,0) risk-return, ADL(0,0) explaining risk)

(13a) $R_{mt} = \alpha_0 + \alpha_1 R_{m,t-1} + \alpha_2 R_{m,t-2} + \theta \text{ var}(R_{mt}) + \Sigma_{i=1,..,3} \beta_i X_{it} + e_t$

(13b) $\text{var}(R_{mt}) = \beta_0 + \Sigma_{i=1,..,3} \beta_i^{\,2} \text{var}(X_{it}) + 2 \beta_1 \beta_2 \text{ cov}(X_{1t}X_{2t}) + 2 \beta_1 \beta_3 \text{ cov}(X_{1t}X_{3t})$

$+ 2 \beta_2 \beta_3 \text{ cov}(X_{2t}X_{3t}) + e_t$

where state variables X are DEF, RREL and TRM respectively. This model omits the state variable CAY with quarterly data, but we also estimate models with levels CAY added to both equations as before. For simplicity equation (13b) also excludes other terms implied by theory (9)-(10): $\text{cov}(X_t\beta, \text{var}R_t)$, $E(\text{var}R_t - E \text{ var}R_t)^2$ and perhaps $\text{cov}(\text{var}R_t, e_t)$.

In order to test the nonlinear restrictions on β, we first specify an unrestricted model as (13a) plus

(13b') $\text{var}(R_{mt}) = \beta_0 + \Sigma_{i=1,..,3} \gamma_i \text{ var}(X_{it}) + 2 \gamma_4 \text{ cov}(X_{1t}X_{2t}) + 2 \gamma_5 \text{ cov}(X_{1t}X_{3t}) + 2 \gamma_6$ $\text{cov}(X_{2t}X_{3t})$

$+ e_t$

where the six nonlinear restrictions are $\gamma_i = \beta_i^{\,2}$(i=1,2,3), $\gamma_4 = \beta_1 \beta_2$, $\gamma_5 = \beta_1 \beta_3$, $\gamma_6 = \beta_2 \beta_3$. Then the nonlinear restrictions can be tested without a nonlinear regression: the unrestricted model can be estimated by linear methods and the nonlinear restrictions can be tested from model estimates as a Wald test (e.g. Greene 2008). Test results are reported in Table 12 (for linear seemingly unrelated regressions SUR using Shazam).

Wald chi-square statistics are quite high, so the null is rejected at any level of significance[43] (Shazam also reports a similar F statistic). Thus we conclude, from linear estimates of the simple unrestricted model, that the nonlinear restrictions implied by theory are rejected in this case.

Next we estimated model (13a-b) imposing the nonlinear restrictions, using nonlinear SUR algorithms in Shazam. Results for Laspeyres and value weighted models (without and with CAY) are presented in Tables 13 and 14, based on starting values derived from coefficient estimates of linear models in the previous section. Coefficients β_1,β_2,β_3 of state variables are sometimes significant at .05 level, especially in models with corrections for first order autocorrelation in disturbances. As expected, imposing restrictions reduced standard errors of coefficients for state variables in risk-return equations.

However results varied substantially with starting values for coefficients, so no conclusions can be drawn.[44] Much more exploration is required with alternative algorithms/packages, and perhaps tailoring procedures to the specific nonlinear structure here.

4.6 Conclusion

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[43] Greene (2008, p. 502) notes that Wald test statistics are often quite (perhaps unduly) large in applications. In any case, since the distribution-free Chebychev inequality on the upper bound of the probability of the hypothesis is calculated as .01, it seems apparent that the null hypothesis should be rejected at the .01 level and presumably the .05 level.

[44] As in many nonlinear models, the likelihood function was relatively flat, so it was difficult to choose between apparent local solutions. Moreover search procedures may not even converge on a local solution. Shazam nonlinear SUR uses a Davidon- Fletcher-Powell algorithm as default (this was used for results reported here), and similar results were obtained for alternative algorithms (primarily Broyden-Fletcher-Goldfarb-Shanno).

This chapter is an empirical study of the aggregate risk-return tradeoff for U.S. stocks using daily data. This is the first such study to model lags using a general-to-specific approach and to apply Wu tests for possible problems of endogeneity/measurement error for the risk variable. Results for aggregate returns and volatility 1995 - 2005 indicate a statistically significant positive relation between expected returns and risk, as in standard theory. Few empirical studies have analyzed risk-return relations at a daily level, but this is one obvious approach to solving a problem that is now recognized as fundamental (past risk-return studies suffer from insufficient observations).

A major advantage of estimating risk-return models at the daily level is that this does not require us to assume model parameters are constant over extremely long time periods. In the future we plan to apply this method to daily models with long time periods and test for changes in model structure over time.

We also develop an empirical model relating aggregate risk of returns to levels and volatility of economic and financial variables from the risk-return model. This is a natural extension of the Wu test. This is the first such model to include lags in variables based on a general-to-specific methodology and to include covariances of such variables. Empirical results are consistent with theory and indicate that the economic and financial variables explain a substantial part of variation in daily risk of returns.

We show that econometric risk-return models and models explaining risk are closely related. In part this is achieved by Wu tests for risk-return models. In addition, we prove that in principle the two models share variables and there are nonlinear restrictions on coefficients across models. This suggests that, in future, researchers should consider joint estimation of these two models. A very brief exercise in nonlinear joint estimation is included here.

One possible extension to our model is to include fundamental macroeconomic variables that have not been considered in risk-return studies but have been included in several studies explaining risk. Data for several variables are available on a daily basis (short term interest rates, exchange rates). Incorporating variables available only at a monthly or quarterly level is much more problematic and might require a MIDAS approach; although we were able to incorporate quarterly consumption/wealth ratio to some extent using levels (various studies use only levels, not volatilities). In addition, several recent studies explaining aggregate risk of returns have decomposed risk into short term and long term components, and it may be of interest to extend the current study in this manner.

The initial section of this chapter has investigated at a theoretical and empirical level several alternative index number approaches for aggregating multivariate risk over stocks. Such an investigation is essential in choosing an appropriate aggregator of multivariate risk for use in the rest of the chapter. The empirical study concludes that these indexes are highly correlated for this data set, so only simple Laspeyres and value-weighted type indexes of aggregate risk of returns are used in other sections of the chapter.

Further theoretical research on index numbers for aggregating volatility over stocks is included in a preliminary working paper separate from this thesis. Alternative (more complex) index number approaches suggested there will be investigated empirically in the future.

Another interesting topic for the future is theoretical and empirical work on alternative index numbers appropriate for aggregating returns (as distinct from volatility) over stocks in finance. This is motivated by empirical results in Appendix I on standard index number formulas applied to stock returns (indicating significant differences between Laspeyres and Paasche return indexes).

Table 4.1 Simple Indexes of Aggregate Risk of Returns Based on MGARCH Estimation of Returns for Individual Stocks

The following tables A, B, C, D, E present the summary statistics for three types of aggregate risk of return constructed in ratio forms. The formulas are

(2)a) $(VR_t/VR_0)^L = q_0{}^T Vr_t\, q_0 \,/\, q_0{}^T Vr_0\, q_0$ for Laspeyres index

b) $(VR_t/VR_0)^P = q_t{}^T Vr_t\, q_t \,/\, q_t{}^T Vr_0\, q_t$ for Paasche index

c) $(VR_t/VR_0)^F = \{(VR_t/VR_0)^L\,(VR_t/VR_0)^P\}^{1/2}$ for Fisher index

* Value weighted A, B indexes correspond to 3 a, b respectively

(3) a) $(VR_t/VR_0)^A = q_t{}^T Vr_t\, q_t \,/\, q_0{}^T Vr_0\, q_0$

b) $(VR_t/VR_0)^B = s_t{}^T Vr_t\, s_t \,/\, s_0{}^T Vr_0\, s_0$

A. Using GARCH(1,1)_CC estimation of variance covariance matrix

	Mean	Max	Min	Standard Deviation
Laspeyres	1	3.724	0.07651	0.5959
Paasche	0.9699	3.645	0.07948	0.5753
Fisher	0.9848	3.685	0.07798	0.5848
Value Weighted A*	1.27	7.83	0.005688	1.383
Value Weighted B*	0.9794	3.781	0.06316	0.6595

Correlations

	Laspeyres	Paasche	Fisher	Value Weighted A*
Laspeyras	1			
Paasche	0.9961	1		
Fisher	0.999	0.9991	1	
Value Weighted A*	0.7941	0.7885	0.7919	1
Value Weighted B*	0.9767	0.9756	0.9772	0.8984

* Value weighted A, B indexes correspond to 3 a, b respectively

B. Using GARCH(1,1)_ DCC estimation of variance covariance matrix

	Mean	Max	Min	Standard Deviation
Laspeyres	1	3.856	0.07667	0.6004
Paasche	0.9718	3.778	0.0797	0.5808
Fisher	0.9856	3.817	0.5897	0.07817
Value Weighted A*	1.264	7.723	0.005706	1.359
Value Weighted B*	0.98	3.715	0.06336	0.6586

Correlations

	Laspeyres	Paasche	Fisher	Value Weighted A*
Laspeyras	1			
Paasche	0.9961	1		
Fisher	0.999	0.999	1	
Value Weighted A*	0.7817	0.7758	0.7793	1
Value Weighted B*	0.9762	0.9755	0.9768	0.8903

Table 4.1 Continued

C. Using EGARCH(1,1)_ARMA(2,1)_ DCC estimation of variance covariance matrix

	Mean	Max	Min	Standard Deviation
Laspeyres	1	3.343	0.2774	0.5657
Paasche	0.9681	3.281	0.2689	0.5468
Fisher	0.9837	3.312	0.2755	0.5555
Value Weighted A*	1.25	7.358	0.01966	1.264
Value Weighted B*	0.975	3.515	0.2156	0.6153
Correlations				
	Laspeyres	Paasche	Fisher	Value Weighted A*
Laspeyres	1			
Paasche	0.9963	1		
Fisher	0.999	0.9991	1	
Value Weighted A*	0.7663	0.7609	0.7642	1
Value Weighted B*	0.9766	0.9758	0.9771	0.8789

D. Using Historical estimation of variance covariance matrix

	Mean	Max	Min	Standard Deviation
Laspeyres	1	3.881	0.2008	0.7246
Paasche	0.9978	3.832	0.208	0.7166
Fisher	0.9985	3.855	0.2072	0.72
Value Weighted A*	1.237	4.617	0.02146	1.088
Value Weighted B*	0.9977	3.651	0.165	0.7193
Correlations				
	Laspeyres	Paasche	Fisher	Value Weighted A*
Laspeyres	1			
Paasche	0.9967	1		
Fisher	0.9992	0.9992	1	
Value Weighted A*	0.7288	0.7175	0.7235	1
Value Weighted B*	0.9849	0.9838	0.9851	0.8266

E. Using Riskmetrics Estimation of variance covariance matrix

	Mean	Max	Min	Standard Deviation
Laspeyres	1	4.939	0.1623	0.8193
Paasche	1.0001	4.973	0.1676	0.8113
Fisher	0.9998	4.956	0.1704	0.8141
Value Weighted A*	1.198	6.449	0.0182	1.161
Value Weighted B*	0.9936	4.725	0.1343	0.8046
Correlations				
	Laspeyres	Paasche	Fisher	Value Weighted A*
Laspeyres	1			
Paasche	0.995	1		
Fisher	0.9987	0.9988	1	
Value Weighted A*	0.7319	0.7256	0.7291	1
Value Weighted B*	0.9836	0.9856	0.9857	0.8276

Table 4.2 Summary Statistics for Variables in Risk-Return Tradeoff

Table 4.2 presents summary statistics for the dependent variable and independent variables in risk return tradeoff models. Where RREL is relative bill rate, TEF is term spread, DEF is default spread, CAY is ratio of consumption to wealth. LASP RETURN is Laspeyres return index, VWRETURN is value weighted return index. LASP_Var(Rm) is Laspeyres risk and VW_Var(Rm)is value weighted risk.

	LASP RETURN	CAY	DEF	VWRETURN	RREL	TRM	LASP _Var(Rm)	VW_Var(Rm)
Mean	0.000765	0.000555	0.820415	0.000970	-0.068698	1.559924	0.000127	0.000124
Median	0.000846	0.001447	0.760000	0.001032	-0.050923	1.380000	0.000110	9.76E-05
Maximum	0.060732	0.029647	1.480000	0.060774	1.456586	3.870000	0.000426	0.000448
Minimum	-0.073563	-0.024372	0.500000	-0.072535	-5.488945	-0.77	3.53E-05	2.75E-05
Std. Dev.	0.011671	0.016989	0.224120	0.011621	0.754551	1.065221	7.20E-05	7.84E-05
Skewness	0.043611	0.166970	1.042486	0.071664	-1.132788	0.353968	1.091274	1.111748
Kurtosis	6.195767	1.801812	3.223498	6.131273	6.399835	2.174822	3.833319	3.573750
Jarque-Bera	1179.194	178.5049	507.3107	1133.607	1925.806	136.3839	629.7096	608.3868
Probability	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
Sum	2.118103	1.536165	2271.730	2.684562	-190.2258	4319.430	0.352609	0.343804
Sum Sq. Dev.	0.377033	0.798918	139.0360	0.373822	1575.951	3140.836	1.44E-05	1.70E-05
Observations	2769	2769	2769	2769	2769	2769	2769	2769

Table 4.3.A Correlations for Variables in Laspeyres Risk-Return Tradeoff Models

Table 4.3.A presents the correlations between dependent variables and independent variables in risk-return tradeoff model. This table includes Laspeyres aggregate return and risk (LASP _Var(Rm))

	LASP RETURN	LASP _Var(Rm)	CAY	DEF	RREL	TRM
LASP RETURN	1.000000	0.044395	0.020465	-0.021278	0.012713	-0.019265
LASP _Var(Rm)		1.000000	0.097359	0.273925	-0.318758	-0.249548
CAY			1.000000	-0.311814	-0.256643	-0.229855
DEF				1.000000	-0.360989	0.542001
RREL					1.000000	-0.267014

Table 4.3.B Correlations for Variables in Value-Weighted Risk-Return Tradeoff Models

Table 4.3.B presents the correlations between dependent variables and independent variables in risk-return trade off model. This table includes value-weighted aggregate return and risk (VW_Var(Rm)).

	VWRETUTN	VW_Var(Rm)	CAY	DEF	RREL	TRM
VWRETUTN	1.000000	0.051432	0.013698	-0.01886	0.011695	-0.022332
VW_Var(Rm)		1.000000	-0.009179	0.251940	-0.246626	-0.284225
CAY			1.000000	-0.311814	-0.256643	-0.229855
DEF				1.000000	-0.360989	0.542001
RREL					1.000000	-0.267014
TRM						1.000000

Table 4.4 Estimates of Value-Weighted Risk-Return Tradeoff Model

This table reports OLS estimates for the value weighted risk return tradeoff model (daily data 1995 -2005). The dependent variable is the value weighted return. The regressors are as follows: Var(Rm) is aggregate volatilitility, CAY is ratio of consumption to wealth. RREL is relative bill rate, TRM is term spread, and DEF is default spread. P-values of coefficients are based on both OLS standard errors and HAC Newey-West standard errors (correcting for heteroskedaticity and autocorrelations). Only sum of coefficients (and probability of the sum) is reported for variables with lags.

	Model One (ADL (2,1))			Model Two (ADL(2,1))			Model Three (ADL(0,0))			Model Four(ADL(1,0))			Model Five (ADL(0,0))		
Variables	Coef	OLS Prob.	HAC Prob.	Coef	OLS Prob.	HAC Prob.	Coef	OLS Prob.	HAC Prob.	Coef	OLS Prob.	HAC Prob.	Coef	OLS Prob.	HAC Prob.
C	0.0003	0.7852	0.8334	0.0015	0.0929	0.1516	0.0007	0.4815	0.5730	0.0002	0.5391	0.5161	0.0000	0.9815	0.9777
Var(Rm)	12.6160	0.0008	0.0063	8.7351	0.0142	0.0319	11.9859	0.0008	0.0018	5.9964	0.0170	0.0582	7.7313	0.0061	0.0126
CAY	0.0234	0.1674	0.1544				0.0132	0.3890	0.3448						
DEF	-0.0015	0.2684	0.3165	-0.0022	0.0983	0.1083	-0.0022	0.1254	0.1427						
RREL	0.0006	0.1458	0.2156	0.0003	0.3818	0.4444	0.0005	0.1831	0.2381						
TRM	0.0001	0.6608	0.6281	0.0001	0.6306	0.5950	0.0004	0.2197	0.1698						
AR(1)	0.7627	0.0003	0.0029	0.7481	0.0020	0.0133				0.7103	0.0000	0.0003			
AR(2)	-0.0799	0.0001	0.0030	-0.0795	0.0002	0.0009									
MA(1)	-0.6973	0.0009	0.0065	-0.6761	0.0053	0.0256				-0.7517	0.0000	0.0000			
R-squared			0.04651			0.040918			0.004945			0.006			0.002717
Breusch-Godfrey Serial Correlation LM Test P-Value															
			0.0624			0.1063			0.0691			0.5321			0.0752
Heteroskedasticity Test: White P-value															
			0			0			0			0			0

Table 4.5 Estimates of Laspeyres Risk-Return Tradeoff Model

This table reports OLS estimates for the Laspeyres risk return tradeoff model (daily data 1995 - 2005). The dependent variable is the Laspeyres return. The regressors are as follows: Var(Rm) is aggregate volatilitility, CAY is ratio of consumption to wealth. RREL is relative bill rate, TRM is term spread, and DEF is default spread. P-values of coefficients are based on both OLS standard errors and HAC Newey-West standard errors (correcting for heteroskedaticity and autocorrelation). Only sum of coefficients (and probability of the sum) is reported for variables with lags.

	Model One (ADL (2,1))			Model Two (ADL(2,1))			Model Three (ADL(0,0))			Model Four (ADL(1,0))			Model Five (ADL(0,0))		
Variables	Coef	OLS Prob.	HAC Prob.	Coef	OLS Prob.	HAC Prob.	Coef	OLS Prob.	HAC Prob.	Coef	OLS Prob.	HAC Prob.	Coef	OLS Prob.	HAC Prob.
C	0.0002	0.8233	0.8563	0.0014	0.0901	0.1614	0.0007	0.5089	0.6105	0.0001	0.7559	0.7410	-0.0001	0.8890	0.8793
Var(Rm)	13.4109	0.0005	0.0055	9.7083	0.0088	0.0287	12.1912	0.0020	0.0074	5.7642	0.0431	0.0867	7.2912	0.0179	0.0472
CAY	0.0230	0.1366	0.1416				0.0120	0.4337	0.3987						
DEF	-0.0017	0.1753	0.2195	-0.0025	0.0425	0.0436	-0.0024	0.1054	0.1252						
RREL	0.0008	0.0496	0.1165	0.0003	0.3027	0.2948	0.0005	0.1697	0.2346						
TRM	0.0001	0.5948	0.5687	0.0002	0.5084	0.4625	0.0004	0.2067	0.1608						
AR(1)	0.7094	0.0000	0.0007	0.6795	0.0008	0.0053				0.7390	0.0000	0.0001			
AR(2)	-0.0449	0.0460	0.0683	-0.0444	0.0392	0.0536									
MA(1)	-0.7073	0.0000	0.0007	-0.6722	0.0009	0.0060				-0.7709	0.0000	0.0000			
R-squared			0.0258			0.0193			0.0044			0.0042			0.0020
Breusch-Godfrey Serial Correlation LM Test P-value															
			0.0612			0.0945			0.131			0.3988			0.1437
Heteroskedasticity White Test: P-Value															
			0			0			0			0			0

Table 4.6 Simple Risk-Return Tradeoff Model with Monthly Data

This table presents regression results for monthly risk-return tradeoff. The monthly return is calculated as the summation of daily return within the month, and the monthly volatility is the summation of daily volatility of returns within the month. P-values of coefficients are based on both OLS standard errors and HAC Newey-West standard errors (correcting for heteroskedaticity and autocorrelation).

Dependent Variable: Laspeyres Return									
Variable	Coeff.	OLS Prob.	HAC Prob.	Coeff.	OLS Prob.	HAC Prob.	Coeff.	OLS Prob.	HAC Prob.
C	-0.0038	0.6400	0.5080	0.0130	0.1195	0.0975	0.0133	0.0976	0.0617
LASP_VAR_MONTH	2.3591	0.3816	0.2863	0.4220	0.8829	0.8836	0.3630	0.8947	0.8847
AR(1)	0.9665	0.0000	0.0000	-0.0114	0.9021	0.9106			
MA(1)	-0.9899	0.0000	0.0000						
R-squared	0.0846				0.0004			0.0001	

Dependent Variable: Value weight Return									
Variable	Coeff.	OLS Prob.	HAC Prob.	Coeff.	OLS Prob.	HAC Prob.	Coeff.	OLS Prob.	HAC Prob.
C	-0.0033	0.6700	0.4931	0.0137	0.0596	0.0274	0.0136	0.0619	0.0217
VW_VAR_MONTH	3.3753	0.1998	0.0806	0.9818	0.6923	0.6453	1.0870	0.6603	0.5813
AR(1)	0.9668	0.0000	0.0000	-0.0363	0.6889	0.7040			
MA(1)	-0.9902	0.0000	0.0000						
R-squared	0.0736				0.0031			0.0015	

Table 4.7 Diagonal VECH Estimation of Variance-Covariance for Economic Variables in Risk-Return Model

This table presents the variance and covariance estimation results of term spread (TRM), relative interest rate (RREL) and default spread (DEF) based on diagnal vech model. The mean equation is AR(1) Model as follows:

TRM=C(1)+C(2)*TRM(-1)

RREL=C(3)+C(4)*RREL(-1)

DEF=C(5)+C(6)*DEF(-1)

Covariance specification is Diagonal VECH

GARCH = M + A1*RESID(-1)*RESID(-1)' + B1*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	-0.003872	0.001821	-2.126322	0.0335
C(2)	1.000053	0.000946	1,057.547000	0
C(3)	0.002088	0.000571	3.658932	0.0003
C(4)	0.993775	0.000682	1,457.811000	0
C(5)	0.002799	0.000908	3.083667	0.0020
C(6)	0.996609	0.001121	888.964000	0
Variable	Coefficient	Std. Error	z-Statistic	Prob.
M(1,1)	0.000284	0.000044	6.485030	0
M(1,2)	-0.000035	0.000005	-7.243574	0
M(1,3)	0.000000	0.000000	-2.110884	0.0348
M(2,2)	0.000049	0.000006	8.391740	0
M(2,3)	-0.000001	0.000001	-1.139188	0.2546
M(3,3)	0.000003	0.000000	6.593152	0
A1(1,1)	0.075014	0.006979	10.748210	0
A1(1,2)	0.089618	0.007920	11.315440	0
A1(1,3)	0.008522	0.002258	3.774854	0.0002
A1(2,3)	0.039202	0.015213	2.576922	0.0100
A1(3,3)	0.129793	0.004535	28.618680	0
B1(1,1)	0.853725	0.016208	52.674640	0
B1(1,2)	0.861269	0.010134	84.984830	0
B1(1,3)	0.987725	0.003275	301.625900	0
B1(2,2)	0.820405	0.009337	87.865090	0
B1(2,3)	0.916899	0.031951	28.697120	0
B1(3,3)	0.889398	0.004607	193.034300	0

Table 4.8 WU Test Results for Value-weighted Aggregate Risk

This table reports results of the Wu test for the value weighted risk return tradeoff model. Step one is to estimate risk of returns as a function of instrumental variables. The second step is to include predicted risk (risk hat) from step one into original risk return model and then test the joint significance of the risk hat and its lag. Var(Rm) is the aggregate risk estimated using EGARCH-AR(2)-MA(1)-DCC(1,1) model and Rm is aggregate return (only sum of coefficients for all variables excluding risk hat is reported).

Step One				Step Two				Step Three			
Dependent Variable: var(Rm)				Dependent Variable: VW-RETURN				Wu Test (Wald Test)			
Variable	Coefficient	t-Statistic	Prob.	Variable	Coefficient	t-Statistic	Prob.		Value	df	Probability
C	0.00005	6.64466	0.00000	C	-0.00118	-0.80539	0.42070	F-statistic	1.46534	(2, 2662)	0.23120
CAY	-0.00081	-8.31668	0.00000	VAR(Rm)	10.94833	2.82250	0.00480	Chi-square	2.93068	2	0.23100
DEF	0.00013	14.44090	0.00000	RISKHAT	13.48504	0.41258	0.67990				
RREL	-0.00003	-15.02698	0.00000	RISKHAT(-1)	6.27862	0.19615	0.84450				
TRM	-0.00004	-31.56033	0.00000	CAY	0.03964	1.95719	0.05040				
TRM_VAR	0.01064	10.11225	0.00000	DEF	-0.00402	-1.77820	0.07550				
RREL_VAR	0.00018	0.18866	0.85040	RREL	0.00137	2.08736	0.03700				
DEF_VAR	-0.00793	-5.08772	0.00000	TRM	0.00093	1.44152	0.14960				
TRM_RREL_COV	0.00418	1.89004	0.05890	AR(1)	0.77847	4.08819	0.00000				
TRM_DEF_COV	0.02544	2.14156	0.03230	AR(2)	-0.08237	-3.98897	0.00010				
RREL_DEF_COV	-0.06566	-3.39479	0.00070	MA(1)	-0.71297	-3.73250	0.00020				
Rm(-1)	0.00001	0.09569	0.92380								
Rm(-2)	-0.00003	-0.36456	0.71550								
R-squared			0.46401	R-squared			0.04911				

Table 4.9 WU Test Results for Laspeyres Aggregate Risk

This table reports results of Wu test for Laspeyres risk return model.Step one is estimate risk of returns as a function of instrument variables. The second step is to include predicted risk (risk hat) from step one into orginal risk return model and then test the joint significance of the risk hat and its lag. Var(Rm) is the aggregate risk estimated using EGARCH-AR(2)-MA(1)-DCC(1,1) model and Rm is aggregate return (only sum of coefficients for all variables excluding risk hat is reported).

Step One				Step Two				Step Three			
Dependent Variable: var(Rm)				Dependent Variable: Rm				Wu Test (Wald Test)			
Variable	Coefficient	t-Statistic	Prob.	Variable	Coefficient	t-Statistic	Prob.		Value	df	Prob .
C	0.00003	4.73417	0.00000	C	-0.00093	-0.70982	0.47790	F-statistic	1.048683	(2, 2662)	0.3505
CAY	-0.00008	-0.85556	0.39230	VAR(Rm)	11.80615	2.93263	0.00340	Chi-square	2.097367	2	0.3504
DEF	0.00015	17.78410	0.00000	RISKHAT	32.39264	0.99161	0.32150				
RREL	-0.00003	-15.06423	0.00000	RISKHAT(-1)	-15.12598	-0.46999	0.63840				
TRM	-0.00004	-29.24666	0.00000	CAY	0.02648	1.69744	0.08970				
TRM_VAR	0.00958	10.03837	0.00000	DEF	-0.00392	-1.82818	0.06760				
RREL_VAR	0.00131	1.52961	0.12620	RREL	0.00133	2.23706	0.02540				
DEF_VAR	-0.00880	-6.23117	0.00000	TRM	0.00073	1.36016	0.17390				
TRM_RREL_COV	0.00528	2.63145	0.00860	AR(1)	0.72144	4.48486	0.00000				
TRM_DEF_COV	0.00013	0.01182	0.99060	AR(2)	-0.04566	-2.02618	0.04280				
RREL_DEF_COV	-0.07662	-4.36867	0.00000	MA(1)	-0.71901	-4.46367	0.00000				
Rm(-1)	-0.00005	-0.60080	0.54800								
Rm(-2)	-0.00009	-1.00788	0.31360								
R-squared			0.47996	R-squared			0.028592				

Table 4.10 Relating Laspeyres Aggregate Risk to Economic Variables

This table reports OLS results relating Laspeyres aggregate risk to level and volatility of economic variables. CAY is ratio of consumption to wealth. RREL is relative bill rate, TRM is term spread, and DEF is default spread. VAR denotes variances and COV denotes covariances of these variables. One period lags in variables are denoted as -1 (and in the case of quarterly variable CAY) -90. P-values of coefficients are based on both OLS standard errors and HAC Newey-West standard errors (correcting for heteroskedaticity and autocorrelation).

	Model One (ADL(1,1))			Model Two (ADL(1,1))			Model Three (ADL(1,0))			Model Four (ADL(0,0))			Model Five (ADL(0,0))		
Variable	Coef	Prob	HAC Prob.	Coef	Prob	HAC Prob.	Coef	Prob	HAC Prob.	Coef	Prob	HAC prob	Coef	Prob	HAC Prob
C	0.00013	0.0000	0.0000	0.00013	0.0000	0.0000	0.00012	0.0000	0.0001	0.00003	0.0000	0.0879	0.00011	0.0000	0.000
CAY	-0.00007	0.6462	0.6332	-0.00007	0.6679	0.6555	-0.00012	0.4501	0.4623	0.00197	0.0000	0.0005	0.00249	0.0000	0.000
CAY(-90)	-0.00034	0.0245	0.0432	-0.00034	0.0249	0.0413	-0.00035	0.0237	0.0374	-0.00205	0.0000	0.0000	-0.00219	0.0000	0.000
DEF	0.00002	0.0466	0.0482	0.00002	0.0390	0.0380				0.00007	0.2784	0.4106			
DEF(-1)	-0.00001	0.4187	0.5216	-0.00001	0.4447	0.5473				0.00008	0.1984	0.3477			
RREL	-0.00001	0.0361	0.1792	-0.00001	0.0216	0.1427				-0.00001	0.6155	0.6206			
RREL(-1)	-0.00002	0.0000	0.0001	-0.00002	0.0000	0.0001				-0.00002	0.5087	0.5070			
TRM	0.00000	0.4249	0.4469	0.00000	0.4274	0.4492				-0.00002	0.3605	0.3914			
TRM(-1)	-0.00001	0.0000	0.0000	-0.00001	0.0000	0.0001				-0.00002	0.2466	0.2840			
VARTRM	0.00264	0.0000	0.0000	0.00264	0.0000	0.0000	0.00224	0.0000	0.0000	0.00954	0.0000	0.0003	0.00621	0.0000	0.026
VARRREL	0.00165	0.0000	0.0000	0.00162	0.0000	0.0000	0.00176	0.0000	0.0000	0.00137	0.1081	0.4776	0.00035	0.7276	0.902
VARDEF	-0.00118	0.0687	0.1911	-0.00118	0.0644	0.1768	-0.00125	0.0502	0.1780	-0.00881	0.0000	0.0038	-0.00707	0.0000	0.071
COV(TRMRREL)	0.00521	0.0000	0.0000	0.00515	0.0000	0.0000	0.00495	0.0000	0.0000	0.00538	0.0072	0.3021	-0.00321	0.1609	0.612
COV(TRMDEF)	0.02689	0.1321	0.3344	0.02644	0.1336	0.3307	0.02425	0.1715	0.3926	0.00054	0.9603	0.9847	0.09930	0.0000	0.000
COV(RRELDEF)	-0.01258	0.0635	0.2016	-0.01279	0.0581	0.1901	-0.01179	0.0825	0.2433	-0.07704	0.0000	0.0623	-0.07560	0.0003	0.149
VARTRM(-1)	0.00008	0.7733	0.7568												
VARRREL(-1)	-0.00018	0.3978	0.6365												
VARDEF(-1)	0.00014	0.8262	0.8100												
COV(TRMRREL)(-1)	-0.00053	0.3696	0.6429												
COV(TRMDEF)(-1)	-0.01777	0.3176	0.3566												
COV(RRELDEF)(-1)	-0.01435	0.0327	0.1119												
AR(1)	0.99347	0.0000	0.0000	0.99339	0.0000	0.0000	0.99477	0.0000	0.0000						
R-squared	0.989583			0.989557			0.989312			0.479698			0.246156		
Breusch-Godfrey Serial Correlation LM Test:															
F-statistic	25.34365	Prob.	0	26.11086	Prob.	0	29.69016	Prob.	0	39069.85	Prob.	0	44121.34	Prob.	0
Heteroskedasticity Test: white															
F-statistic	8.659198	Prob.	0.0033	8.594898	Prob. F(1,2677)	0.0034	2.311601	Prob. F(1,2677)	0	39.08737	Prob.	0	26.95732	Prob.	0

Table 4.11 Relating Value-weighted Aggregate Risk to Economic Variables

This table reports OLS results relating value weighted aggregate risk to level and volatility of economic variables. CAY is ratio of consumption to wealth. RREL is relative bill rate, TRM is term spread, and DEF is default spread. VAR denotes variances and COV denotes covariances of these variables. One period lags in variables are denoted as -1(and in the case of quarterly variable CAY) -90. P-values of coefficients are based on both OLS standard errors and HAC Newey-West standard errors (correcting for heteroskedaticity and autocorrelation) .

Variable	Model One (ADL(1,1))			Model Two (ADL(1,1))			Model Three (ADL(1,0))			Model Four (ADL(0,0))			Model Five (ADL(0,0))		
	Coef	Prob	HAC Prob.	Coef	Prob	HAC Prob.	Coef	Prob	HAC Prob.	Coef	Prob	HAC prob	Coef	Prob	HAC Prob
C	0.00011	0.0001	0.0005	0.00011	0.0001	0.0004	0.00012	0.0001	0.0001	0.00005	0.0000	0.0134	0.00011	0.00000	0.00000
CAY	-0.00002	0.9062	0.9101	-0.00002	0.9090	0.9123	-0.00008	0.6336	0.6632	0.00153	0.0000	0.0115	0.00245	0.00000	0.00000
CAY(-90)	-0.00051	0.0020	0.0388	-0.00050	0.0023	0.0408	-0.00050	0.0024	0.0400	-0.00234	0.0000	0.0000	-0.00264	0.00000	0.00000
DEF	0.00002	0.0145	0.0240	0.00002	0.0117	0.0191				0.00005	0.4967	0.5569			
DEF(-1)	-0.00001	0.4013	0.4849	-0.00001	0.4306	0.5153				0.00008	0.2122	0.3043			
RREL	-0.00001	0.0826	0.3057	-0.00001	0.0629	0.2868				-0.00001	0.6924	0.6887			
RREL(-1)	-0.00003	0.0000	0.0006	-0.00002	0.0000	0.0003				-0.00002	0.4451	0.4314			
TRM	0.00001	0.0022	0.0334	0.00001	0.0023	0.0355				-0.00001	0.5107	0.5252			
TRM(-1)	-0.00002	0.0000	0.0000	-0.00002	0.0000	0.0000				-0.00003	0.1138	0.1363			
VARTRM	0.00281	0.0000	0.0000	0.00283	0.0000	0.0000	0.00240	0.0000	0.0001	0.01063	0.0000	0.0005	0.00546	0.00000	0.05430
VARRREL	0.00142	0.0000	0.0001	0.00145	0.0000	0.0000	0.00159	0.0000	0.0001	0.00018	0.8473	0.9204	-0.00172	0.12040	0.51840
VARDEF	-0.00159	0.0225	0.1489	-0.00155	0.0237	0.1337	-0.00167	0.0153	0.1391	-0.00793	0.0000	0.0178	-0.00621	0.00090	0.14740
COV(TRMRREL)	0.00500	0.0000	0.0000	0.00507	0.0000	0.0000	0.00480	0.0000	0.0001	0.00419	0.0580	0.4156	-0.00818	0.00120	0.17570
COV(TRMDEF)	0.03658	0.0566	0.2524	0.03540	0.0615	0.2435	0.03337	0.0804	0.2993	0.02550	0.0317	0.4013	0.09468	0.00000	0.00000
COV(RRELDEF)	-0.01398	0.0548	0.1552	-0.01396	0.0540	0.1517	-0.01285	0.0786	0.2166	-0.06576	0.0007	0.1543	-0.07918	0.00060	0.16680
VARTRM(-1)	-0.00015	0.6264	0.5982												
VARRREL(-1)	-0.00010	0.6720	0.8251												
VARDEF(-1)	0.00050	0.4654	0.5275												
COV(TRMRREL)(-1)	-0.00022	0.7281	0.8643												
COV(TRMDEF)(-1)	-0.02279	0.2328	0.2863												
COV(RRELDEF)(-1)	-0.01078	0.1349	0.2917												
AR(1)	0.99406	0.0000	0.0000	0.99398	0.0000	0.0000	0.99471	0.0000	0.0000						
R-squared	0.9898			0.9898			0.9895			0.4640			0.2210		
Breusch-Godfrey Serial Correlation LM Test:															
F-statistic	19.904	Prob	0	19.055	Prob	0	20.583	Prob	0	43,885.220	Prob	0	49,377.46	Prob	0
Heteroskedasticity Test: white															
F-statistic	2.128	Prob	0	3.327	Prob	0	4.482	Prob	0	40.823	Prob	0	16.804	Prob	0

Table 4.12 Test Results for Nonlinear Restrictions on Coefficients Across Aggregate Risk-Return Models and Models Explaining Aggregate Risk

This table presents test results for the coefficient links between risk-return model and model explaining risk (see 13a, 13b') for both Laspeyres and value weighted models.

A. Laspeyres Model

		Df	Prob
Wald Chi-Square Statistic	505.43	6	0
F Statistic	82.24	6 5224	0
Chebychev Inequality Upper Bound on Probability			0.012

B. Value Weighted Model

		Df	Prob
Wald Chi-Square Statistic	405.08	6	0
F Statistic	67.51	6 5224	0
Chebychev Inequality Upper Bound on Probability			0.015

Table 4.13 Nonlinear SUR Estimates of Model (13a-b): Laspeyres

This table presents joint estimation results for risk-return model and model explaining risk (see 13a, 13b) with and without CAY. It also reports estimation results allowing autocorrelation (using coefficient estimates from linear models as starting values).

					Autocorrelation			
	Coef.	T-ratio	Coef	T-ratio	Coef.	T-ratio	Coef	T-ratio
α0	0.00124	1.28	0.00105	0.97	-0.01130	5.77	-0.02209	3.41
α1	0.00000	0.26	0.00000	0.27	0.83060	4,370.00	0.88341	1,327.00
α2	0.00000	0.19	0.00000	0.15	-0.00240	87.59	0.00357	33.53
θ	10.61500	10.58	9.32200	8.44	10.50400	10.60	6.05420	5.88
β0	0.00013	92.68	0.00013	91.99	0.00020	25.55	0.00372	14.58
β1	-0.00275	1.98	-0.00236	1.58	0.01413	5.58	0.03431	7.01
β2	0.00041	1.10	0.00053	1.37	-0.00139	1.30	0.00845	3.17
β3	0.00035	1.16	0.00038	1.25	-0.00102	1.73	-0.00109	0.89
CAY(13a)*			0.01318	0.86			0.00038	0.01
CAY(13b)*			0.00041	5.06			0.00024	1.50

* Coefficient of CAY in risk-return and explaining risk equations, respectively.

Table 4.14 Nonlinear SUR Estimates of Model (13a-b): Value Weighted

This table presents joint estimation results for risk-return model and model explaining risk(see 13a, 13b) with and without CAY. It also reports estimation results allowing autocorrelation (using coefficient estimates from linear models as starting values).

					Autocorrelation			
	Coef.	T-ratio	Coef	T-ratio	Coef.	T-ratio	Coef	T-ratio
α0	0.00149	1.52	0.00081	0.77	0.00819	3.95	-0.00045	0.31
α1	0.00000	0.24	0.00000	0.25	0.70206	2,441.00	0.79546	5,723.00
α2	0.00000	0.09	0.00000	0.05	-0.00030	179.00	-0.00040	1.67
θ	8.78640	7.62	11.31700	11.32	13.56700	13.52	7.52120	2.35
β0	0.00012	81.83	-0.00040	0.46	0.00232	285.00	-0.00039	16.34
β1	-0.00257	1.87	-0.00220	1.52	-0.02703	10.79	-0.00094	0.54
β2	0.00037	1.03	0.00052	1.26	-0.00369	2.94	0.00009	0.20
β3	0.00033	1.11	0.00038	1.23	0.00482	5.81	0.00029	0.50
CAY(13a)*			0.00012	82.19			-0.00126	0.06
CAY(13b)*			-0.00040	0.46			-0.00005	0.15

* Coefficient of CAY in risk-return and explaining risk equations, respectively.

Figure 4.1 Risk Index in Ratio Form Using Market Capitalization as Weight

This figures shows the Laspeyres, Paasche and Fisher risk indexes constructed using the following formulas (2) a), b) c) and formula (3) (a) for value-weighted index of aggregate risk.

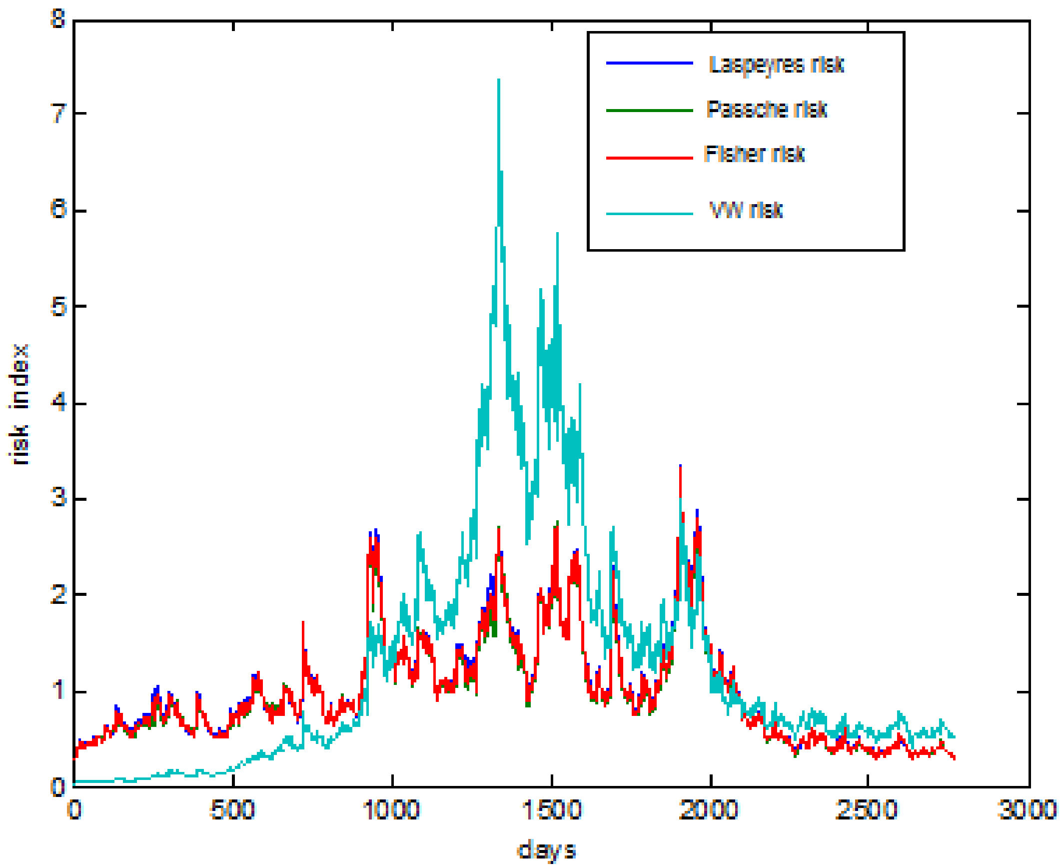

Figure 4.2 shows the variances of term spread (TRM), relative interest rate spread (RREL) and default spread (DF), and the covariances among these variables over time (1995-2005).

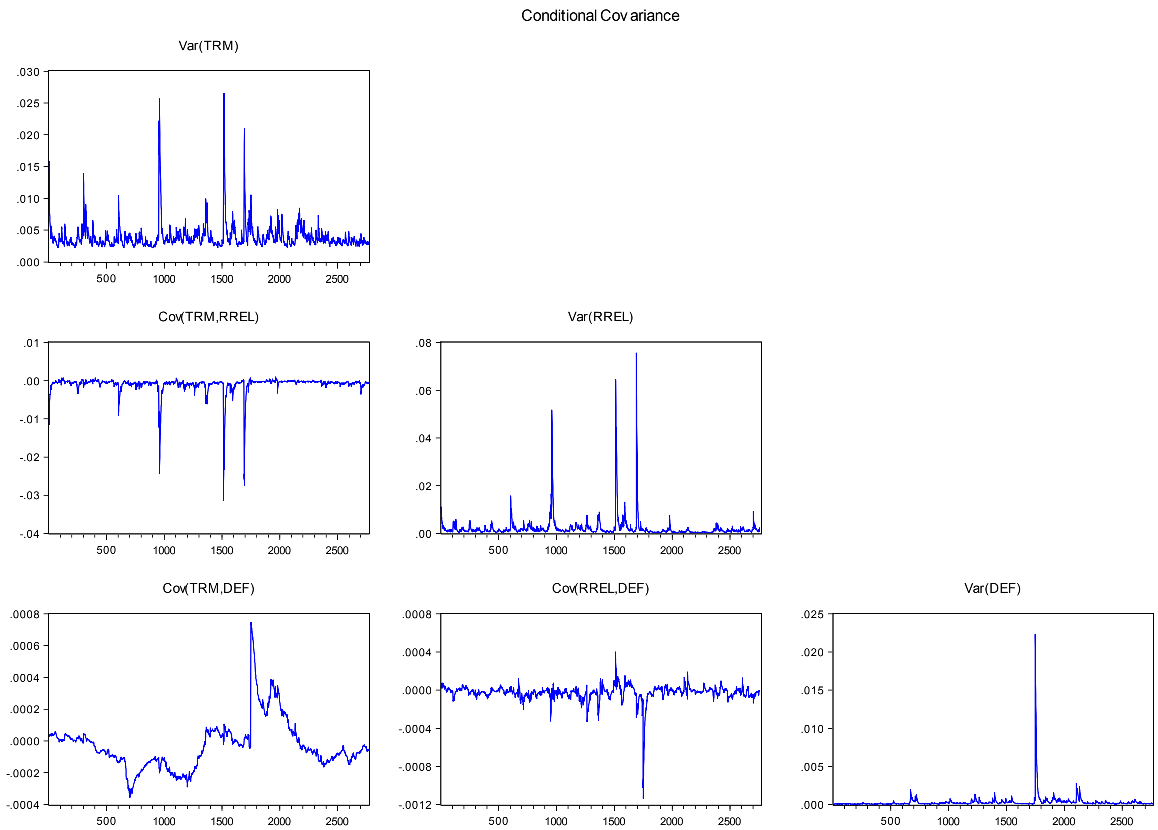

Figure 4.2 Variance-Covariance among Macroeconomic Variables

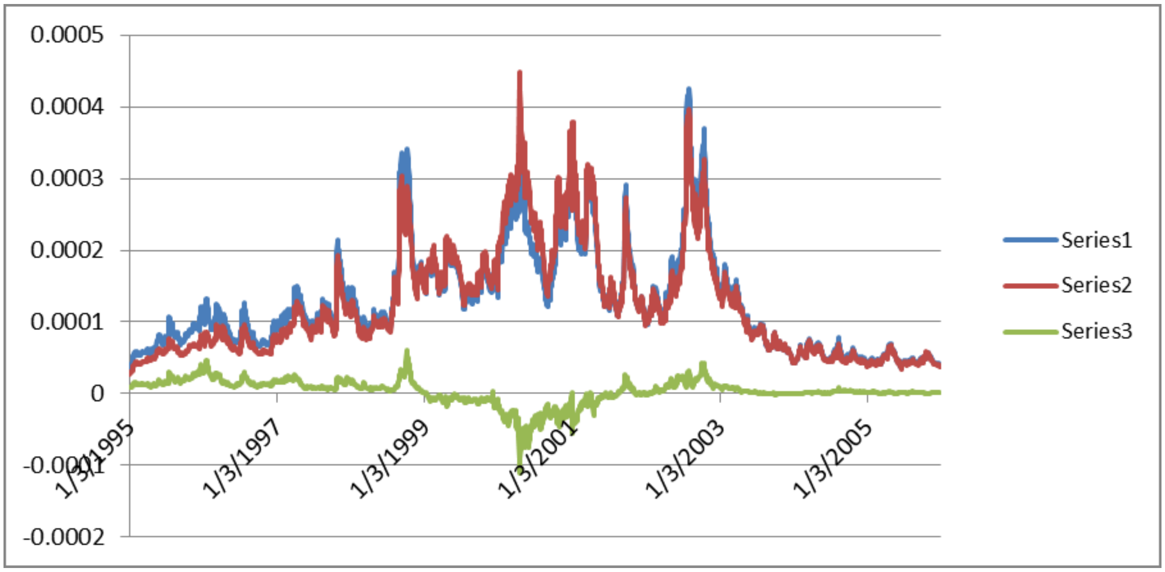

Figure 4.3 Comparison of Laspeyres and Value Weighted Aggregate Risk

Figure 4.3 compares the aggregate risk (1995-2005) constructed using different methods. Series one is Laspeyres aggregate risk. Series 2 is value weighted aggregate risk. Series 3 is the difference between series one and two.

APPENDIX 4.I A Brief Summary of Index Number Theory, its Applications in Economics, and a Cursory Extension to Aggregate Returns for Stock Markets

4.I.A. Introduction

This Appendix to Chapter 4 first briefly discusses standard index number theory and its applications. This discussion is in terms of consumer price indexes, since this is the major application in economics. The standard fixed base Laspeyres index is inappropriate in theory and apparently in practice, i.e. the underlying theoretical arguments appear to be important in empirical applications.

Then this Appendix briefly considers index numbers for aggregate returns in stock markets. Although the finance literature largely ignores such matters, some early and current researchers consider these. The standard economic approach to index number theory assumes risk neutrality, but this is inappropriate for finance. Extensions to risk aversion for the economic theory approach to aggregation of prices or returns are very limited (Barnett, Lui and Jensen 1997; Chen and Coyle 2010). This Appendix simply compares alternative indexes of aggregate returns using daily data on 88 stocks on the S&P 100 over 1995 - 2005. There are substantial differences between Laspeyres and other indexes and between fixed base and moving base (chained) indexes. This suggests that, when more fully developed, an economic approach to index numbers for aggregate returns in stock markets may well be of empirical importance.

4.I.B. Economic Index Number Theory for Consumer Price Indexes

Here we provide a brief summary of several aspects of the "economic approach" to index number theory, and we also briefly mention the test approach (see Diewert 2004 for a recent discussion of index number theory). The most common application of index numbers is to Consumer Price Indexes (CPI's). A CPI is intended to measure changes in representative cost of living due to changes in consumer prices. So here we present index number theory in the context of a CPI.

Consider a consumer purchasing m commodities at prices $p = (p_1,.,p_m)$, and corresponding quantities consumed are $x = (x_1,.,x_m)$. Consumer preferences are represented by a "utility function" u = u(x). The consumer chooses x at level x* so as to maximize u(x) given a budget constraint p x = B. Equivalently the consumer chooses x to solve the following cost minimization problem:

(1) $\min_x \; p\,x$

s.t. $u(x) = u^*$

where u* denotes the equilibrium indifference curve (standard of living) for the maximization problem. Denote the relation between minimum expenditure/cost p x* and parameters (p, u) as the dual cost function E = E(p, u) (E = p x*). E(p, u) represents the minimum "cost of living" given prices p and indifference curve (standard of living) u.

Standard index number theory poses the following question: how to aggregate prices p for all commodities so that the aggregate represents the cost of a representative (constant) level of living given these prices, and changes in the aggregate represent changes in cost of living (for a representative constant standard of living u) due to changes in prices p over time. If the consumer's consumption levels were constant over time at level x_0, then it would be very simple to aggregate prices p_t at time t so as to represent the cost of living: $P_t = p_t\, x_0$. However this a biased (upwards) measure of cost of living given that decisions x change with prices p. Total cost $p_t\, x_t$ over time t cannot be interpreted as an index of cost for a constant level of living since the level of living u generally changes with x. In constructing a CPI, we need to assign weights to prices p_t that reflect importance of these prices in consumption costs at time t yet also reflect a representative (constant) standard of living.

In principle it is extremely difficult to construct a CPI that accurately measures changes in cost of living. Index number theory considers various possible candidates for a CPI and in effect attempts to analyze by various criteria which candidates come "closest" to answering the above question.

Index number theory is almost entirely based on indexes formulated as ratios of costs. By expressing the aggregates of prices in periods 0,1 in a ratio form rather than simply in separate levels for the two periods, the comparison between the aggregates is explicit rather than visual. This greatly simplifies an analytical approach to index numbers. Index number theory would lose most of its richness and simplicity if indexes were expressed directly in terms of levels of costs rather than directly in terms of ratios (an alternative approach based on differences rather than ratios is summarized in Diewert 2005).

Until recently the most common price indexes in empirical studies were the Laspeyres and Paasche. These are expressed in terms of two periods as $(P_1/P_0)^L = p_1\, x_0 / p_0\, x_0$ and $(P_1/P_0)^P = p_1\, x_1 / p_0\, x_1$, respectively. The first approach calculates cost by weighting the two price vectors p_1,p_0 by the quantities x_0 for period 0, and the second approach weights the price vectors by the quantities x_1 for period 1. A Laspeyres index has been more common than a Paasche in empirical work, but a Paasche index is just as appropriate since the choice between x_0 and x_1 as weights is essentially arbitrary.

An important result in theory is that the Laspeyres and Paasche generally place bounds on a true CPI. By definition $p_0\, x_0 = E(p_0,u_0)$, and $p_1\, x_0 > E(p_1,u_0)$ since x_0 is feasible for but is not the solution for the following problem: $\min_x\ p_1\, x$ s.t. $u(x) = u_0$ (as (relative) prices change from p_0 to p_1, the cost minimizing solution changes from x_0). Similarly $p_1\, x_1 = E(p_1,u_1)$ and $p_0\, x_1 > E(p_0,u_1)$. So $(P_1/P_0)^L > E(p_1,u_0) / E(p_0,u_0)$ and $(P_1/P_0)^P < E(p_1,u_1) / E(p_0,u_1)$. Under standard conditions this implies $(P_1/P_0)^L > (P_1/P_0)^P$ and a true CPI is bounded by the Laspeyres and Paasche.[45] Thus, in general, a Laspeyres overestimates a true CPI and a Paasche underestimates a true CPI.

For multiple time periods t=0,1,.,N, fixed base Laspeyres and Paasche CPI's are defined as

(2) $(P_t/P_0)^L = p_t\, x_0 / p_0\, x_0$

$(P_t/P_0)^P = p_t\, x_t / p_0\, x_t$

where (e.g.) period t=0 is chosen as the fixed base period. Alternatively indexes can be defined using moving bases. Moving base Laspeyres and Paasche CPI's are

(3) $(P_t/P_{t-1})^L = p_t\, x_{t-1} / p_{t-1}\, x_{t-1}$

$(P_t/P_{t-1})^P = p_t\, x_t / p_{t-1}\, x_t$.

In order to be comparable to fixed base indexes P_t/P_0 as in (2), these moving base indexes can be multiplied (chained) as $P_t/P_0 = P_1/P_0\ \ P_2/P_1\ ....\ P_{t-1}/P_{t-2}\ \ P_t/P_{t-1}$. Historically statistical agencies have usually used fixed bases, but economists have typically argued that moving bases are more appropriate.

[45] Assuming that indifference curves have the same shapes (more precisely u(x) is homothetic), then the cost function E(p, u) can be decomposed as u E(p,1) □ u e(p). Then E(p1,u0) / E(p0,u0) = e(p1)/e(p0) and E(p1,u1) / E(p0,u1) = e(p1)/e(p0). Then (P1/P0)L > e(p1)/e(p0) > (P1/P0)P where e(p1)/e(p0) is a true CPI.

Moving bases have been recommended primarily for two reasons. First, unlike the fixed base Laspeyres with a fixed consumption bundle x_0, the reference consumption bundle is updated over time as goods enter and leave the market, so the reference bundle remains more current. Second, chaining usually reduces the spread between Laspeyres and Paasche indexes and hence tightens the bound about the true CPI, at least for annual data. This reduction in spread occurs if price and quantity patterns are more similar for adjacent periods than for more distant periods. This is generally true for annual data but not for monthly data, which may have substantial seasonality.

Economists have argued that Laspeyres and Paasche approaches to a CPI are less appropriate than a Fisher or Tornqvist/Divisia index. The "economic approach" assesses these indexes in terms of the following question: under what restrictions does an index represent a true cost of living index? Conditions are much more restrictive for Laspeyres and Paasche, so the Fisher and Tornqvist are preferred. A Laspeyres CPI is a true cost of living index essentially only if actual consumption levels x are constant over time (and hence independent of prices). This is intuitively obvious and is extremely restrictive. Obviously the same criticism applies to a Paasche. Assessment of Fisher and Tornqvist indexes are not so obvious.

A Fisher index is simply the geometric mean of the Laspeyres and Paasche:

(4) $(P_t/P_0)^F = [(P_t/P_0)^L (P_t/P_0)^P]^{1/2}$.

Nevertheless this is much less restrictive than the Laspeyres or Paasche. A Tornqvist/Divisia index P^{Div} can be represented in logarithmic form as

(5) $\log (P_t/P_0)^{Div} = 0.5\ \Sigma_i\ (s_{it} + s_{i0}) \log(p_{it}/p_{i0})$

$\log (P_t/P_{t-1})^{Div} = 0.5\ \Sigma_i\ (s_{it} + s_{it-1}) \log(p_{it}/p_{it-1})$

where s_i is the share of commodity i in total expenditure for the time period. These are fixed and moving bases, respectively. It can be shown that Fisher and Tornqvist/Divisia indexes can each be interpreted as a true cost of living index under relatively general conditions (homotheticity and quadratic/flexible functional forms for the dual cost function) that imply a relatively general model of how consumption decisions x are influenced by prices p. Thus these indexes are relatively general and should provide closer approximations to a true cost of living index than do a Laspeyres or Paasche CPI.

In index number theory, there is a second approach (in addition to the above economic approach) used in assessing alternative index numbers. This is called the axiomatic or test approach. This approach also favors Fisher over other common indexes. Index number theorists have proposed various properties or tests that a price index should satisfy. Alternative indexes are assessed in terms of these properties rather than in terms of a behavioral model of cost of living as cost minimization. For example, Diewert (2004, Ch. 3) lists 20 such properties. A Fisher price index satisfies all 20 properties. Laspeyres and Paasche price indexes fail 3 tests, including an essential time reversal test (if the price and quantity data for two periods 0,1 are interchanged, then the resulting price index should equal the reciprocal of the original price index). Failure of this test is viewed as a serious logical weakness of Laspeyres and Paasche. The Tornqvist/Divisia price index fails 9 tests, but it does pass the critical time reversal test. In sum, the axiomatic approach favors a Fisher price index over the other three indexes considered here.

4.I.C. Empirical Implications of Index Number Theory for U.S. Consumer Price Indexes: the Boskin Commission

There have been many theoretical and empirical studies by economists of biases in common CPI's, particularly for the U.S. One survey of this literature by leading academics has been quite influential, so it is summarized here.

In 1996 the Boskin Commission (BC) assessed these studies of U.S. CPI's and provided suggestions for improvements (Boskin et al, 1996, 1997, 1998). This commissioned study has been referred to as "probably ... the most important measurement paper of the century in terms of its impact" (Diewert 1998, p. 56). The study has led many statistical agencies in the world to reevaluate their price measurement techniques.

The BC concluded that the standard CPI for the Bureau of Labor Statistics (BLS) overestimated cost of living by approximately 1.1% per year (plausible range of 0.8-1.6% per year) circa 1996. A fixed base Laspeyres index introduced a bias of 0.5% per year, with errors in modeling new products and quality change accounting for the remaining bias. The BC emphasized that these estimates of bias probably were conservative. The BC concluded that these biases were substantial and had important implications for measuring economic progress and economic policy, and also for finance.

Boskin (2005) emphasizes implications of biases in the CPI for measuring real returns to stocks and bonds, which have been calculated as nominal returns minus inflation in CPI. Over 1946 - 2001, real returns to stocks and bonds are calculated as 7.1% and 1.3%, with inflation at 4.1% (Seigel 2002) (long-run real returns to stocks are relatively stable at approximately 7% over all major sub-periods since 1800). However an upward bias of 1.1% in CPI implies that real returns should be revised to 8.2% and 2.4% for stocks and bonds, respectively. Boskin notes that these differences in real returns have important implications in finance (Boskin 2005, p. 9).

Moreover index number theory may well imply further biases in calculations of aggregate real returns for stocks. In principle index number theory applies to aggregation of nominal returns for stocks. Just as there is an upward bias in a standard CPI, there may be biases in standard estimates of aggregate returns to stocks. If aggregate nominal returns are underestimated, this would imply further upward revisions in real aggregate rates of return.

4.I.D. Index Number Theory in the Context of Aggregate Returns for Stocks

4.I.D.i. History

The earliest stock market index is the Dow Jones (beginning 1885), which is a simple sum of stock prices divided by the number of stocks. Obviously this is a poor measure of overall market performance, since it provides equal weights to all stocks in the index rather than weighting by relative capitalization. In 1923 the Standard Statistics Company began to calculate a weekly stock price index using capitalization weights and the Fisher ideal index formula (Wilson and Jones 2002, p. 507; Standard Statistics Company 1928).

Thus apparently a Fisher index was adopted in important early research on aggregate stock prices. Also, rather than adopting a fixed base, indexes were chained.

In contrast, most current researchers and users of indexes for stock market aggregate prices or returns seem unaware of index number theory and its relevance. However there are exceptions. For example, the Princeton Series in Finance commissions books "written by top experts" in finance and financial economics (jacket of GJ). Gourieroux and Jasiak (GJ) include a chapter on Market Indexes in their book "Financial Econometrics" (2001) (Gourieroux is Director of the Laboratory for Finance and Insurance at CREST, Paris).

They state that "the use of stock market indexes follows from the tradition of computing standard consumer price indexes" and reference Laspeyres, Paasche and Fisher indexes (p. 409). Although the discussion of index number theory is quite limited, they do alert the reader to some critical issues in application. They show that a Laspeyres CPI generally exceeds a Paasche index (so a more appropriate index is likely to be in the middle, e.g. a Fisher index), and they briefly discuss the alternatives of fixed base and moving base (chained) indexes. They then note that index number theory is relevant in some but not all uses of market indexes in finance: stock indexes intended to be representative of the market or to be a benchmark for portfolio management should use more general indexes than a fixed base Laspeyres.

4.I.D.ii. An Introduction to Index Number Theory for Aggregate Returns in Stock Markets

The standard economic approach to index number theory assumes risk neutrality, whereas in finance it is generally assumed that agents making portfolio decisions in stock markets are risk averse. This is a fundamental difference in economic models.

Apparently index number theory under risk aversion is limited to Barnett, Liu and Jensen (1997). BLJ derive a generalized Divisia (Tornqvist) index for price or monetary aggregation assuming risk aversion, where cost share weights include risk-adjusted user costs. Emphasis is on aggregation of monetary assets rather than commodities or stocks. So far applications are limited (Barnett and Liu, 2000).

A behavioral model underlying index number theory for aggregation of stocks must assume risk aversion and risk in returns. Assume that a representative agent in a stock market makes portfolio decisions so as to maximize a mean-variance utility function given the agent's perceived expected returns and variance-covariance matrix of returns for stocks.

Given this behavioral model, Chen and Coyle (2011) analyze several index numbers for aggregate risk of returns, but there is also a brief analysis of several index numbers for aggregate returns. The paper concludes that Fisher (and Tornqvist/Divisia) indexes of aggregate returns are superior to Laspeyres and Paasche. However the Fisher (and Tornqvist/Divisia) indexes of aggregate returns are much less satisfactory under risk aversion than under risk neutrality. Alternative indexes are also considered briefly.

Finally, the axiomatic/test approach to analyzing index numbers extends in a simple manner to aggregation of returns for stocks under risk aversion. Results are the same as under risk neutrality. A Fisher index of aggregate returns for a stock market passes all 20 tests. Laspeyres and Paasche fail the important time reversibility test, and a Divisia return index fails 9 other tests. So a Fisher index of aggregate returns is highly superior to these other indexes in terms of axiomatic theory.

4.I.D.iii. An Introductory Application of Alternative Index Numbers for Aggregate Returns in a Stock Market

Thus, in principle, economic and axiomatic theory of index numbers is relevant to various studies of aggregate indexes in finance. As discussed above, the practical importance of the theory has been established for empirical research on CPI's. However apparently no studies have assessed the practical importance of the theory for finance, and this issue cannot be answered from empirical research on CPI's.

Here we make a very brief first attempt to assess the practical importance of index number theory to empirical studies of aggregate stock market returns. Various index numbers are constructed and compared using daily data on 88 stocks on the S&P 100 over 1995 - 2005. We briefly consider two major issues: Laspeyres versus other indexes, and fixed base versus moving base (chained) indexes. Here returns are defined in gross form (p_{it}/p_{it-1}) rather than in net form (p_{it}/p_{it-1} - 1). This is so returns will always be defined as positive (negative returns complicate calculation and interpretation of chains). Correlations are presented in Table 4A.1 for alternative aggregate indexes of returns.

Correlations for fixed base indexes of aggregate returns are summarized as follows: +0.7501 for Laspeyres and Paasche, +0.9344 for Laspeyres and Fisher, and +0.9363 for Paasche and Fisher. The interesting result is that there is a substantial difference between the Laspeyres and Paasche indexes of aggregate returns. This suggests that an appropriate index of aggregate returns may be quite different empirically from a Laspeyres or a Paasche. (In standard indexes such as a consumer price index, a Laspeyres and Paasche generally bound a true index, but this is not so clear for indexes of stock market returns). The Laspeyres and Paasche is more highly correlated with the Fisher (a geometric average of the two indexes), but this difference is also significant.

These indexes were also compared with a value-weighted return index, which is popular in finance. The value-weighted index is $R_t = w_t\ r_t$, where r is gross returns and w is shares in total capitalization (equivalently in ratios $R_t/R_0 = w_t\ r_t\ /\ w_0\ \ r_0$). The value-weighted return index has a correlation of +0.7562 with the Laspeyres index and a correlation of +0.9955 with the Paasche index. So there is a substantial difference between the value-weighted and Laspeyres indexes of aggregate returns. This suggests that an appropriate index of aggregate returns may be quite different empirically from a value-weighted Laspeyres or a Paasche. However this requires much further study that is beyond the paper.

Correlations are also presented regarding chained indexes. A moving base R_t/R_{t-1} is calculated using Laspeyres, Paasche and Fisher formulas, then these moving bases are chained/multiplied together to produce an index analogous to a fixed base: $R_t/R_0 = R_1/R_0 \;\; R_2/R_1 \; ... \; R_{t-1}/R_{t-2} \;\; R_t/R_{t-1}$ (t=0 is the first day of the data set). Then these are compared with analogous fixed base indexes. The correlation between the Laspeyres fixed base and chained indexes is +0.7123. This substantial difference suggests that the issue of fixed base versus chained indexes deserves further consideration in finance, although the issue is quite complicated especially with daily data.

Table 4 A.1 Aggregate Indexes of Returns

A. Fixed Base				
	Mean	Max	Min	Standard Deviation
Laspeyres	0.9978	1.45	0.9322	0.01359
Paasche	0.996	1.055	0.9221	0.01189
Fisher	0.9969	1.209	0.9271	0.01179
Value Weighted A*	0.9975	1.057	0.923	0.01181

Correlations				
	Laspeyres	Paasche	Fisher	Value Weighted A*
Laspeyres	1			
Paasche	0.7501	1		
Fisher	0.9344	0.9363	1	
Value Weighted A*	0.7562	0.9955	0.9375	1

B: Moving Base				
	Mean	Max	Min	Standard deviation

Laspeyres	1	1.132	0.9244	0.0171
Paasche	0.996	1.055	0.9221	0.01189
Fisher	1.001	1.672	0.5787	0.03753
Value Weighted A*	1	1.721	0.5782	0.02332

Correlations

	Laspeyres	Paasche	Fisher	Value Weighted A*
Laspeyres	1			
Paasche	0.7653	1		
Fisher	0.9216	0.9549	1	
Value Weighted A*	0.7424	0.9992	0.9439	1

C. Comparison of Fixed and Moving Bases

	Laspeyres	Paasche	Fisher	Value Weighted A*
Laspeyres	0.7123			
Paasche		0.6632		
Fisher			0.9618	
Value Weighted A*				0.6707

APPENDIX 4.II Alternative Index Number Formulas for Aggregate Returns and Aggregate Risk of Returns, and Alternative Aggregate Methods for Aggregate Risk of Returns

4.II.A. Common Indexes in Levels

In finance an aggregate return index is commonly constructed as a ratio of the value of a stock market in adjacent periods, e.g. as a ratio of the S&P 500 in adjacent periods. These are "value-weighted" indexes. Denote the value of the stock market at time t as $p_t\ y_t$, where p is a vector of stock prices and y is a vector of quantities of stocks. Then the aggregate return is

(1) $R_t = p_t\, y_t \,/\, p_{t-1}\, y_{t-1}$

$= \Sigma_i\, (p_{it}/p_{it-1})\, (p_{it-1}\, y_{it} \,/\, p_{t-1}\, y_{t-1})$

$= \Sigma_i\, r_{it}\, w_{it-1}$

where $r_{it} = p_{it}/p_{it-1}$ (gross returns) and $w_{it-1} = p_{it-1}\, y_{it} \,/\, p_{t-1}\, y_{t-1}$. Here R_t is an aggregate return index in levels (not ratios). If $y_t = y_{t-1}$, then w_{it-1} is the share of stock i in total value of the stock market at t-1. Then R_t weights the vector of returns r_t by a vector of shares w_{t-1}, and the weights w change over time. Of course in the long run stock market quantities y cannot be constant, as stocks enter and exit the market.

Alternatively an aggregate return index could be defined with fixed weights as $r_t\ w_0$ where w_0 is a vector of fixed weights (constant for all periods). This can be interpreted loosely as a fixed base Laspeyres index in levels.

In finance, aggregate risk for returns is typically constructed as a measure of variance for an aggregate return index in levels, such as above. For example in the case of a fixed base return index $R_t = r_t\ w_{t-1}$, a variance of index R can be expressed equivalently as

$var(r_t \; w_{t-1}) = w_{t-1}{}^T Vr_t \; w_{t-1}$ where Vr is a variance-covariance matrix of returns, i.e. variance of the index R corresponds to an index aggregating Vr for individual stocks. The variance is estimated simply from univariate GARCH models of the aggregate return index or by more traditional ad-hoc methods. We will refer to these univariate approaches to measuring aggregate risk from aggregate returns as "aggregate" methods of estimating aggregate risk for returns.

4.II.B. Return Indexes in Ratio Form: Fixed and Moving Bases

In economics indexes are usually expressed in ratio form. This permits specification of more general indexes than a level index such as (1). The most common indexes are Laspeyres, Paasche, Fisher and Tornqvist/Divisia. These are defined in terms of fixed bases (at t=0) as follows (e.g. Aizcorbe and Jackman, 1993):

(2) $(R_t/R_0)^L = r_t \, q_0 \, / \, r_0 \, q_0$

$(R_t/R_0)^P = r_t \, q_t \, / \, r_0 \, q_t$

$(R_t/R_0)^F = \{(R_t/R_0)^L \, (R_t/R_0)^P\}^{1/2}$

$\log \{(R_t/R_0)^{Div}\} = \Sigma_i \, 0.5 \, (w_{it} + w_{i0}) \log(r_{it}/r_{i0})$

where q_t is the vector of capitalizations (values $p_i \; y_i$) for period t, and $w_{it} = q_{it} \, / \, \Sigma_j \, q_{jt}$ (share of commodity i in total capitalization at time t). The analogous moving base indexes are (Aizcorbe and Jackman):

(3) $(R_t/R_{t-1})^L = r_t \, q_{t-1} \, / \, r_{t-1} \, q_{t-1}$

$(R_t/R_{t-1})^P = r_t \, q_t \, / \, r_{t-1} \, q_t$

$(R_t/R_{t-1})^F = \{(R_t/R_{t-1})^L \, (R_t/R_{t-1})^P\}^{1/2}$

$\log \{(R_t/R_{t-1})^{Div}\} = \Sigma_i \, 0.5 \, (w_{it} + w_{it-1}) \log(r_{it}/r_{it-1})$.

These moving bases can be multiplied (chained) as follows to form indexes R_t/R_0 analogous to fixed base indexes: $R_t/R_0 = R_1/R_0 \;\; R_2/R_1 \ldots R_{t-1}/R_{t-2} \;\; R_t/R_{t-1}$.

4.II.C. Indexes for Risk of Returns in Ratio Form: Fixed and Moving Bases

Here we propose indexes of aggregate risk for returns analogous to (2) - (3). These indexes are based on estimates of the variance covariance matrix of return risks, Vr_t. Fixed base indexes analogous to Laspeyres, Paasche and Fisher are:

(3) $(VR_t/VR_0)^L = q_0{}^T Vr_t \, q_0 \,/\, q_0{}^T Vr_0 \, q_0$

$(VR_t/VR_0)^P = q_t{}^T Vr_t \, q_t \,/\, q_t{}^T Vr_0 \, q_t$

$(VR_t/VR_0)^F = \{(VR_t/VR_0)^L \; (VR_t/VR_0)^P\}^{1/2}$.

Tornqvist-type risk indexes are not considered since these do not readily accommodate negative covariances of returns. The analogous moving base risk indexes are

(4) $(VR_t/VR_{t-1})^L = q_{t-1}{}^T Vr_t \, q_{t-1} \,/\, q_{t-1}{}^T Vr_{t-1} \, q_{t-1}$

$(VR_t/VR_{t-1})^P = q_t{}^T Vr_t \, q_t \,/\, q_t{}^T Vr_{t-1} \, q_t$

$(VR_t/VR_{t-1})^F = \{(VR_t/VR_{t-1})^L \; (VR_t/VR_{t-1})^P\}^{1/2}$.

These moving bases can be multiplied (chained) as follows to form indexes VR_t/VR_0 analogous to fixed base indexes: $VR_t/VR_0 = VR_1/VR_0 \;\; VR_2/VR_1 \ldots VR_{t-1}/VR_{t-2} \;\; VR_t/VR_{t-1}$.

4.II.D. Aggregate Methods for Indexes of Aggregate Risk for Returns

In principle price or return risk indexes discussed above can be constructed by estimating univariate GARCH rather than multivariate MGARCH models of prices. This is overlooking problems in estimation of mis-specified aggregate econometric models.

Consider fixed base and moving base Laspeyres return risk indexes:

$$(VR_t/VR_0)^L = q_0{}^T Vr_t\, q_0 \,/\, q_0{}^T Vr_0\, q_0$$

$$= var(r_t\ q_0) / var(r_0\ q_0)$$

$$\neq var\ (r_t\ q_0 / r_0\ q_0)$$

$$(VR_t/VR_{t-1})^L = q_{t-1}{}^T Vr_t\, q_{t-1} \,/\, q_{t-1}{}^T Vr_{t-1}\, q_{t-1}$$

$$= var(r_t\ q_{t-1}) / var(r_{t-1}\ q_{t-1})$$

$$\neq var(r_t\ q_{t-1} / r_{t-1}\ q_{t-1})\ .$$

The inequalities follow from the fact that (by construction) rt and r_{t-1} are not independent (moreover p_t and p_{t-1} are seldom independent in practice). In the economics literature, there are many cases where a variance of a price index in ratios has been mistakenly used as a proxy for a price risk index.

Thus we can use return data r and capitalization data q for all stocks to construct three time series on total value r q defined as $r_t\ q_t$, $r_t\ q_0$ and $r_t\ q_{t-1}$. Then univariate Garch models can be estimated for these three series. The resulting estimates of $var(r_t\ q_t)$, var $(r_t\ q_0)$ and $var(r_t\ q_{t-1})$ can be plugged into the above formulas to calculate $(VR_t/VR_0)^L$ and $(VR_t/VR_{t-1})^L$, overlooking problems in estimating mis-specified aggregate econometric models.

Similarly consider fixed base and moving base Paasche return risk indexes:

$$(VR_t/VR_0)^P = q_t{}^T Vr_t\, q_t \,/\, q_t{}^T Vr_0\, q_t$$

$$= var(r_t\ q_t) / var(r_0\ q_t)$$

$$\neq var(r_t\ q_t / r_0\ q_t)$$

$(VR_t/VR_{t-1})^P = q_t^T Vr_t q_t / q_t^T Vr_{t-1} q_t$

$= var(r_t q_t) / var(r_{t-1} q_t)$

$\neq var(r_t q_t / r_{t-1} q_t)$

Then we construct time series $r_t q_t$, $r_0 q_t$ and $r_{t-1} q_t$, estimate corresponding univariate GARCH models, and plug variance estimates $var(r_t q_t)$, $var(r_0 q_t)$ and $var(r_{t-1} q_t)$ into the above equation to calculate and $(VR_t/VR_0)^P$ and $(VR_t/VR_{t-1})^P$. Then Fisher price risk indexes can be calculated from estimated Laspeyres and Paasche indexes as $VR^F = (VR^L VR^P)^{1/2}$.

Although univariate GARCH estimation is certainly much simpler than MGARCH for many stocks, univariate estimation of an aggregate return or price index model is inappropriate in theory. Regression models that are highly aggregated over commodities or stocks are generally highly misspecified by omitting relevant information (see section 2 of chapter 2).

APPENDIX 4.III A Brief Economic Index Number Analysis of Simple Alternative Return Risk Indexes in Finance: Laspeyres and Fisher

This Appendix extends the economic analysis of index numbers to aggregation of return risk in finance. We assume that investors choose a portfolio of stocks with uncertain returns so as to maximize a mean-variance utility function. According to the economic approach to index number analysis, aggregation of variance-covariances of return risk should preserve their collective contribution to the objective function. If one index number formula does this under less restrictive assumptions than does an alternative formula, then the first formula is judged to be the superior index according to the economic criterion.

The main results of this Appendix can be summarized as follows. First, a Laspeyres-type index of return risk is considered (variances-covariances of return risk are weighted by dollar portfolios). This index only meets the economic criterion under extremely restrictive conditions similar to standard theory. Second, a related Fisher-type index of return risk is considered. This index meets the economic criterion under more general conditions than the Laspeyres, but these conditions are still quite restrictive - a separability restriction between impacts of return risk and expected returns on portfolio decisions. See Chen and Coyle (2011) for more general discussions of these indexes and other approaches to indexes of aggregate risk of returns for stocks.

Assume that a representative agent in a stock market decides in time period t how to allocate total money Q_t among n stocks, i.e. how to choose a portfolio/expenditures $q_t = (q_1,..,q_n)_t$ satisfying the constraint $\Sigma_{i=1,..,n}\ q_{it} = Q_t$. Expenditures/portfolio q_t are chosen/determined by the agent, so they are non-stochastic rather than a random variable in t. Q_{it} is equal to the product of stock price p_i at the time of purchase and a quantity of shares implicit in the purchase, where this price is known (non-stochastic) at the time of purchase. The return r_{it} on decision q_{it} in period t is essentially the ratio of the final price p_i in period t to the initial price p_i at the time of the decision q_{it}. Since the final price is unknown at the time of the decision, the final price and the return are random variables, i.e. stochastic, in the period. For simplicity, we can view $q_t = (q_1,..,q_n)_t$ as non-stochastic portfolio decisions made at the beginning of period t, leading to stochastic returns $r_t = (r_1,..,r_n)_t$ at the end of the period t.

Denote the vector of expected returns and variance- covariance matrix of returns as $Er_t = (Er_1,..,Er_n)_t$ and $Vr_{t\ nxn}$, respectively. Total returns on the portfolio q_t at the end of period are $R_t = \Sigma_{i=1,.,n}\ r_{it}\ q_{it}$. Since r_t is stochastic and q_t is non-stochastic during t, the expectation and variance during t of total returns R_t are $ER_t = \Sigma_{i=1,.,n}\ Er_{it}\ q_{it} = Er_t\ q_t$ and $VR_t = \Sigma_{i=1,.,n}\ \Sigma_{j=1,.,n}\ Vr_{ij}\ q_{it}\ q_{jt} = q_t^{\ T} Vr_t\ q_t$, respectively.

Suppose that the agent's risk preferences can be represented by a mean-variance utility function $U = U(ER_t, VR_t) = ER_t - \alpha(ER_t, VR_t)/2\ \ VR_t$ where α is the coefficient of absolute risk aversion. α is generally a function of Er and Vr, i.e. $\alpha = \alpha(Er, Vr)$. Assume that the agent chooses portfolio q_t to maximize his mean-variance utility function as follows (deleting subscripts t):

(A1) $\max_q \ U = U(Er\ q, q^T\ Vr\ q)$

s.t. $\Sigma_i\ q_i = Q$

and denote the optimal portfolio decision as q*. Denote the relation between maximum utility and exogenous parameters Er, Vr and Q as the dual utility function U* = U*(Er, Vr, Q). The envelope theorem implies

(A2) $\partial U^*(.)/\partial Er_i = \partial U(ER,VR)/\partial ER \quad q_i$

$\partial U^*(.)/\partial Vr_{ij} = \partial U(ER,VR)/\partial VR \quad q_i\, q_j$ for all i,j .[46]

A Laspeyres-type index of return risk is:

(1) $(VR_1/VR_0)^L = (q_0{}^T Vr_1\, q_0) / (q_0{}^T Vr_0\, q_0)$

where return covariance matrices Vr for periods 1 and 0 are weighted by portfolio expenditures q for period 0. Similarly a Paasche-type index for return risk uses q for period 1 as weights:

(2) $(VR_1/VR_0)^P = (q_1{}^T Vr_1\, q_1)/ (q_1{}^T Vr_0\, q_1)$.

A Fisher-type index for return risk is the geometric mean of the Laspeyres and Paasche:

(3) $(VR_1/VR_0)^F = [(VR_1/VR_0)^L \ (VR_1/VR_0)^P]^{1/2}$.

The economic approach to index number aggregation in the context of the portfolio choice problem (A1) can be stated as follows. The joint contribution of exogenous parameters $Er_{nx1} = (Er_1,.,Er_n)$ and Vr_{nxn} and Q to maximum utility U* is summarized by the dual U*(Er, Vr, Q). This is the economic importance of Er, Vr, Q within the context of the choice problem: they contribute to maximum utility U* as

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[46] Envelope relations (A2) can be proved as follows (standard proof). First order conditions for (A1) in Lagrange form $\max_{q,\gamma}$ $L = U(Er\ q, q^T\ Vr\ q) - \gamma\ (\Sigma_i\, q_i - Q)$ are (a) $\partial L/\partial q_i = U_{ER}\, Er_i + U_{VR}\ 2\ \Sigma_j\, Vr_{ij}\, q_j - \gamma = 0$ and (b) $\Sigma_i\, q_i = Q$. Total differentiating $U^*(Er, Vr, ER, Q) = L^* = U(Er\ q^*, q^{*T}\ Vr\ q^*) - \gamma^*\ (\Sigma_i\, q_i^* - Q)$ with respect to Er_i, $\partial U^*(.)/\partial Er_i = U_{ER}\, q_i + U_{ER}\, \Sigma_j\, Er_j\ \partial q_j^*/\partial Er_i + U_{VR}\ 2\ \Sigma_j\, \Sigma_k\, Vr_{jk}\ q_k\, \partial q_j^*/\partial Er_i - \gamma^*\ \Sigma_j\ \partial q_j^*/\partial Er_i - \partial \gamma^*/\partial Er_i\ (\Sigma_j\, q_j^* - Q) = U_{ER}\, q_i + \Sigma_j\, (U_{ER}\, Er_j + U_{VR}\ 2\ \Sigma_k\, Vr_{jk}\ q_k - \gamma^*)\ \partial q_j^*/\partial Er_i - (\Sigma_j\, q_j^* - Q)\ \partial \gamma^*/\partial Er_i = U_{ER}\, q_i$ by first order conditions (a)-(b). Total differentiating $U^*(Er, Vr, ER, Q) = L^* = U(Er\ q^*, q^{*T}\ Vr\ q^*)$ with respect to Vr_{ij}, $\partial U^*(.)/\partial Vr_{ij} = U_{ER}\, \Sigma_j\, Er_j\ \partial q_j^*/\partial Vr_{ij} + U_{VR}\ q_i\, q_j + U_{VR}\ 2\ \Sigma_j\, \Sigma_k\, Vr_{jk}\ q_k\ \partial q_j^*/\partial Vr_{ij} - \gamma^*\ \Sigma_j\, \partial q_j^*/\partial Vr_{ij} - \partial \gamma^*/\partial Vr_{ij}\ (\Sigma_k\, q_k^* - Q) = U_{VR}\, q_i\, q_j + \Sigma_j\, (U_{ER}\, Er_j + U_{VR}\ 2\ \Sigma_k\, Vr_{jk}\ q_k - \gamma^*)\ \partial q_j^*/\partial Vr_{ij} - (\Sigma_k\, q_k^* - Q)\ \partial \gamma^*/\partial Vr_{ij} = U_{VR}\, q_i\, q_j$ by first order conditions (a)-(b).

summarized by the dual U*(Er, Vr, Q). So ideally an index number aggregate of return risks Vr_{nxn} will accurately reflect their contribution to U*. For a particular return risk index number formula, the economic approach to analysis of index numbers attempts to identify restrictions on the dual when this will be the case, i.e. when the formula accurately reflects the contribution of Vr_{nxn} to the dual U*. The index is described as "exact" in this case. If these restrictions are extremely unrealistic, then the particular return risk index number formula presumably provides a poor approximation to the economic contribution of Vr_{nxn} and is judged to be a poor index number formula in theory.

We now apply the above economic criterion to the Laspeyres- type return risk index (1). We show that this index accurately reflects the contribution of Vr_{nxn} to U* only under extremely restrictive conditions. So this is a very poor index in theory.

First, the following proposition shows that this index accurately reflects the contribution of Vr_{nxn} to U* if the dual has a linear functional form. This is similar to standard analyses of Laspeyres price indexes, and it is known that this index is not exact for nonlinear functional forms (Diewert 1981, pp. 182-183).

Proposition 1. Assume the maximization problem (A1) and U*(Er, Vr, Q) = h(Er, Vr, Q) = a z where z is elements of Er, Vr, Q and a is a vector of constants. Then $(VR_1/VR_0)^L$ = $h(0,Vr_1,0)$ / $h(0,Vr_0,0)$.

Proof. By definition

(A3) $(VR_1/VR_0)^L = \{[q_0{}^T Vr_1 q_0) / (q_0{}^T Vr_0 q_0)]$

$$= \Sigma_i\Sigma_j Vr_{ij1} q_{i0} q_{j0} / \Sigma_i\Sigma_j Vr_{ij0} q_{i0} q_{j0}$$

By envelope theorem results (A2),

(A4) $q_i\, q_j\, \partial U(ER,VR)/\partial VR = \partial U^*(Er,\ Vr)/\partial Vr_{ij}$

$= a_{ij}$

assuming $U^* = h(Er,\ Vr, Q) = a\ z$. Substituting into (A3),

(A5) $(VR_1/VR_0)^L = \Sigma_i\Sigma_j\ Vr_{ij1}\ a_{ij}\ /\ \partial U(ER_0,VR_0)/\partial VR\ \ /\ \Sigma_i\Sigma_j\ Vr_{ij0}\ a_{ij}\ /\ \partial U(ER_0,VR_0)/\partial VR$

$= \Sigma_i\Sigma_j\ Vr_{ij1}\ a_{ij}\ /\ \Sigma_i\Sigma_j\ Vr_{ij0}\ a_{ij}$

$= h(0,Vr_1,0)\ /\ h(0,Vr_0,0)$. Q.E.D.

However the assumptions justifying the Laspeyres return risk index are extremely restrictive. Placing (A4) in ratio form, q_i / qk = a_{ij} / a_{jk}, i.e. the ratio is independent of all Er, Vr, Q. Since this condition is so restrictive and unrealistic, the Laspeyres return risk index presumably provides a poor approximation to the economic contribution of Vr_{nxn} and is judged to be a poor index number formula in theory.

This conclusion, that a Laspeyres-type return risk index is a good index by an economic criterion only if ratios q_i / q_j are constant, is not surprising. Casual inspection of the index (1) suggests that it is an adequate index only if the weightings qs are constant or equiproportional over time. Analyses of other indexes such as a Fisher are more interesting.

We now apply the above economic criterion to the Fisher-type return risk index (3). We show that this index accurately reflects the contribution of Vr_{nxn} to U* under conditions that are somewhat less restrictive than the above linearity conditions justifying the Laspeyres. So by the economic criterion the Fisher return risk index is in principle superior to the Laspeyres return risk index.

The following proposition shows that the Fisher return risk index accurately reflects the contribution of Vr_{nxn} to U* if the dual is separable in the following form

(A6) $U^*(Er, Vr, Q) = U^{\sim}(Er, Q, h(Vr))$

and if h is quadratic in Vr as $h = z^T A z$, where A is a matrix of constants (h does not include any linear terms in Vr).

Proposition 2. Assume the maximization problem (A1), separability condition (A6) and h $= \Sigma_i \Sigma_j a_{ij} z_i z_j$ where z is elements of Vr. Then $(VR_1/VR_0)^F = \{h(Vr_1)/h(Vr_0)\}^{1/2}$. If instead $h = (\Sigma_i \Sigma_j a_{ij} z_i z_j)^{1/2}$, then $(VR_1/VR_0)^F = h(Vr_1)/h(Vr_0)$.

Proof.

By definition

(A7) $(VR_1/VR_0)^F = \{[(q_0{}^T Vr_1 q_0) / (q_0{}^T Vr_0 q_0)] \; [(q_1{}^T Vr_1 q_1)/ (q_1{}^T Vr_0 q_1)]\}^{1/2}$

$= \{[\Sigma_i\Sigma_j Vr_{ij1} q_{i0} q_{j0} / \Sigma_i\Sigma_j Vr_{ij0} q_{i0} q_{j0}] \; / [\Sigma_i\Sigma_j Vr_{ij0} q_{i1} q_{j1} / \Sigma_i\Sigma_j Vr_{ij1} q_{i1} q_{j1}]\}^{1/2}$.

By (A2) and separability (A6),

(A8) $q_i q_j = \partial U^{\sim}(Er, Q, h)/\partial h \; \partial h(Vr)/\partial Vr_{ij}$.

By assumption $h(Vr) = Vr^T A Vr$ where Vr is the elements of Vr_{nxn} expressed as a column vector and A is a symmetric matrix of constants, so $\partial h/\partial Vr = 2 A Vr$. Then by (A8),

(A9) $\Sigma_i\Sigma_j Vr_{ijs} q_{it} q_{jt} = Vr_s{}^T \partial U^{\sim}(.)_t/\partial h \, \partial h_t/\partial Vr$

$= Vr_s{}^T \partial U^{\sim}(.)_t/\partial h \; 2 A Vr_t$.

Substituting (A9) into (A7),

(A10) $(VR_1/VR_0)^F = \{[Vr_1{}^T \partial U^{\sim}(.)_0/\partial h \; 2 A Vr_0 / Vr_0{}^T \partial U^{\sim}(.)_0/\partial h \; 2 A Vr_0]$

$/ [Vr_0{}^T \partial U^{\sim}(.)_1/\partial h \; 2 A Vr_1 / Vr_1{}^T \partial U^{\sim}(.)_1/\partial h \; 2 A Vr_1]\}^{1/2}$.

By rules of transposition $Vr_s{}^T A Vr_t = Vr_t{}^T A Vr_s$, so (A10) reduces to

(A11) $(VR_1/VR_0)^F = \{1 / Vr_0{}^T A\, Vr_0] / [1 / Vr_1{}^T A\, Vr_1]\}^{1/2}$

$$= \{h(Vr_1) / h(Vr_0)\}^{1/2} .$$

Now assume instead $h(Vr) = (Vr^T\ A\ Vr)^{1/2}$, so $\partial h/\partial Vr = 2\ A\ Vr /\ h^{1/2}$. Proceeding as above leads to the first line of (A11), so $(VR_1/VR_0)^F = h(Vr_1) / h(Vr_0)$. Q.E.D.

The assumptions justifying a Fisher return risk index are somewhat less restrictive than in the case of a Laspeyres. (A8) implies $q_i / q_k = \partial h(Vr)/\partial Vr_{ij} / \partial h(Vr)/\partial Vr_{jk}$; so the separability condition (A6) used in Proposition 2 implies that ratios q_i / q_j are independent of Er, Q, which is similar to a Laspeyres. On the other hand, since h(Vr) is assumed to be quadratic, ratios q_i / q_j do vary with Vr, in contrast to the Laspeyres case. In sum, by the economic criterion, a Fisher return risk index is superior in principle to a Laspeyres index, but the Fisher index also is quite restrictive.

APPENDIX 4.IV: Coefficient Links Between Risk-Return Models and Models Explaining Risk

As an introduction, suppose for simplicity that returns R is related to variables Z but not to return risk varR. Let

(A1) $R_t = Z_t\, \beta + e_t \quad Ee = 0 \quad cov(Z, e) = 0$

where R_t is 1x1 and Z_t is 1xk . A1 is conditional on all prior information Ω_{t-1}. Then the conditional expectation for R_t is $ER_t = EZ_t\, \beta$. The conditional variance for R_t is

(A2) $varR_t = \beta^T\, cov(Z_t)\, \beta + var(e_t)$

where cov(Z) is the conditional covariance matrix (kxk) for Z . This result is well known (see any elementary text in statistics), but a proof is attached.

Proof.

$varR_t = E\,(R_t - ER_t)^2$

$= E\,(Z_t\, \beta + e_t - EZ_t\, \beta)^2$

$= E\,((Z_t - EZ_t)\, \beta + e_t)^2$

$= E\,((Z_t - EZ_t)\, \beta + e_t)^T\, ((Z_t - EZ_t)\, \beta + e_t)$

since the transpose of a scalar $((Z_t - EZ_t)\, \beta + e_t)_{1x1}$ is itself

$= E\,(\beta^T\, (Z_t - EZ_t)^T + e_t)\, ((Z_t - EZ_t)\, \beta + e_t)$

$= E\,(\beta^T\, (Z_t - EZ_t)^T\, (Z_t - EZ_t)\, \beta) + E(e_t^{\ 2}) + 2\, E\,((Z_t\text{-}EZ_t)\beta e_t)$

$= \beta^T\, E\,(Z_t - EZ_t)^T (Z_t - EZ_t)\, \beta + var(e_t)$

$= \beta^T\, cov(Z_t)\, \beta + var(e_t)$.

Now consider a risk-return model where returns R are related to variables Z and return risk varR. Let

(A3) $R_t = Z_t\, \beta + \gamma\, varR_t + e_t \quad Ee=0 \quad cov(Z, e)=0 \quad cov(varR, e)=0$

and in general $cov(varR, Z) \neq 0$. The conditional variance for R_t is

(A4) $varR_t = \beta^T cov(Z_t)\, \beta + var(e_t) + 2\gamma\, cov(Z_t\beta, varR_t) + E\,(varR_t - E\, varR_t)^2$

where the last term is the conditional variance of the random variable varR. This result presumably is not known, at least within the context of risk-return models. This links coefficients β of Z in risk-return models to coefficients of covZ in models explaining risk, as $\beta^T\; cov(Z)\, \beta$. However the connection between coefficients of the risk-return model and of the model explaining risk is more complex than in (A2). This result can be proved almost as simply as (A2).

Proof.

$varR_t = E\,(R_t - ER_t)^2$

$= E\,(Z_t\, \beta + \gamma\, varR_t + e_t - \gamma\, E\, varR_t - EZ_t\, \beta)^2$

$= E\,((Z_t - EZ_t)\, \beta + e_t + \gamma\, (varR_t - E\, varR_t))^2$

$= E\,((Z_t - EZ_t)\, \beta + e_t + \gamma\, (varR_t - E\, varR_t))^T \quad ((Z_t - EZ_t)\, \beta + e_t + \gamma\, (varR_t - E\, varR_t))$

since the transpose of a scalar is itself

$= E\,(\beta^T\, (Z_t - EZ_t)^T + e_t + \gamma\, (varR_t - E\, varR_t)) \;\; ((Z_t - EZ_t)\, \beta + e_t + \gamma\, (varR_t - E\, varR_t))$

$= E\,(\beta^T\, (Z_t - EZ_t)^T\, (Z_t - EZ_t)\, \beta) + E(e_t\,^2)$

$+ \gamma^2\, E\,(varR_t - E\, varR_t)^2 + 2\, E\,((Z_t\text{-}EZ_t)\, \beta\;\; \gamma\, (varR_t - E\, varR_t))$

$+ 2\, E\,((Z_t - EZ_t)\, \beta\, e_t) + 2\, \gamma\, E\,((varR_t - E\, varR_t)\, e_t)$

$= \beta^T\, cov(Z_t)\, \beta + var(e_t) + \gamma^2\, E\,(varR_t - E\, varR_t)^2 + 2\gamma\, cov(Z_t\beta, varR_t)$.

CHAPTER FIVE: GENERAL CONCLUSION

This thesis has studied measurement of multivariate risk and applications in finance. This thesis has estimated a high dimensional multivariate GARCH model of stock market returns, empirically evaluated errors in contemporaneous aggregation of GARCH models, investigated alternative index measures of aggregate risk, and estimated econometric risk-return tradeoff models and models relating stock market risk to economic fundamentals. The major contributions of this thesis to the finance literature can be stated briefly as follows: errors in contemporaneous aggregation of GARCH models are substantial and of economic value, risk-return models at a daily level can effectively address the problems of inadequate sample size, and risk-return models are closely connected to models explaining aggregate risk of stock returns.

The contributions of each essay have been summarized in their respective conclusions. Here we provide a briefer summary. Essay one makes three contributions to empirical literature. This is the first study to estimate variances and covariances using data for a large number of individual stocks, which avoids any loss of information in risk estimation due to aggregation of stocks. Second, this is the first study adopting a systematic general-to-specific approach to specification of lagged returns in mean equation for returns. Third, various alternatives to simple GARCH are considered in step one univariate estimation. Results favor an asymmetric EGARCH extension of Engle and Sheppard's model.

Essay two compares measures of aggregate risk from standard univariate GARCH models of aggregate return with measures based on multivariate GARCH estimates for individual stocks (from essay one). Results suggest that errors in contemporaneous aggregation of GARCH models are important empirically: correlations between the two approaches are +0.8 (not 1.0) and the economic value (performance fee) for the alternative measure of aggregate risk is calculated as approximately 4% of portfolio return for the data period. This is the first study to evaluate empirically the significance of errors in contemporaneous aggregation of GARCH models of stock market returns.

Essay three shows that specifying risk-return models at a daily level rather than monthly or quarterly level can effectively address the serious problem of insufficient sample size in previous studies. Apparently this problem can be addressed most effectively using daily data as in this study or by estimating risk-return models across portfolios with common coefficients as in Bali and Engle, and these two approaches can be combined. Results for aggregate returns and volatility 1995-2005 indicate a statistically significant positive relation between expected returns and risk, as in standard theory.

This is the first risk-return study to incorporate systematic specification of lags (a critical matter for daily models) and specification tests for endogeneity of risk of returns. Specification test link risk-return models and models explaining risk, and we also link in theory coefficients of the two models. Empirical results for models relating return risk to state variables are consistent with theory and indicate that the economic and financial variables explain a substantial part of variation in daily risk of returns.

Essay three also includes the first theoretical and empirical study of alternative indexes of aggregate risk for stock market returns. In theory, Fisher-type indexes are less restrictive than Laspeyres or value weighted-type indexes, but differences are small (in contrast to return indexes under risk neutrality). In the empirical study, all indexes are very highly correlated.

Important extensions of the research in this thesis are mentioned in the conclusion to essay three. The risk-return model was estimated over a short time period (1995-2005) with daily data. Next this study will be extended to a longer period: first at least 20 years and then perhaps back to 1885 (Schwert uses daily data back this far). The study will also incorporate additional economic variables available on a daily basis (short term interest rates, exchange rates), since these variables have been used in modeling aggregate risk. Then we can test for structural changes in coefficients of risk over time. Models relating aggregate stock market risk to economic fundamentals will also be estimated, following the links to risk-return models noted in this thesis.

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