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A POTENTIAL FLOW REPRESENTATION OF THE LAMINAR BOUNDARY LAYER ON A LIFTING ELLIPTIC CYLINDER

by

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ABSTRACT

A technique for computing the lift of a two-dimensional elliptic airfoil in incompressible flow is presented. The potential flow, including circulation, was computed around an elliptic airfoil - with a fineness ratio of 6:1 - by using a higher order surface source singularity method. The surface velocity distribution due to the potential flow was then used to calculate the growth of a laminar boundary layer using Thwaites' method at a Reynolds number of 800. The circulation was adjusted to give equal velocities at the separation points and the boundary layer was recalculated. The procedure was iterated until the predicted separation points possessed equal velocities. A maximum lift coefficient of 0.518 was predicted at an angle of attack of 7.1°. The predicted lift is slightly greater than that calculated by Howarth who used a modified Pohlhausen method for the boundary layer.

The effects of boundary layer displacement thickness and separation wake were modelled by using a surface source distribution with boundary conditions such that, in the attached region, the normal velocity was related to the displacement thickness and, in the separated flow region, the pressure was constant. The latter is a non-linear boundary condition and the source distribution was solved iteratively using the Newton-Raphson technique. The subsequent iterations on the boundary layer failed to converge and further work is required.

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NOMENCLATURE

A_{tk}	coefficient matrix in equation 2.14.
c	element curvature.
C_p	pressure coefficient, $(p - p_\infty) \mid \frac{1}{2} \rho W_\infty^2$.
d	the elliptic airfoil chord length which is equal to the major axis length.
g	constant in equation 3.3.
H	shape factor.
$\vec{K}(t,q)$	velocity at point t on the airfoil surface due to a unit source at point q.
ℓ	non-dimensional parameter, $\ell = \frac{\theta}{U_{tt}} (\frac{\partial u}{\partial n})_{wall}$
L(m)	function of m (equation 2.20).
m	non-dimensional parameter, $m = \frac{\theta^2}{U_{tt}} (\frac{\partial^2 u}{\partial n^2})_{wall}$ (wall represents the airfoil surface S).
N	the number of the surface elements.
n	distance in the direction normal to the airfoil surface.
p	pressure.
P_∞	Onset flow pressure (at infinity).
R	element radius.
R_1, R_2	radius of the circles shown in Figure 8.
R_3, Δ, h	distances shown in Figure 9.
r, r_f	distances given by equations 3.5 and 3.6 respectively.
Re	Reynolds number, $Re = W_\infty d/\nu$.
s	distance along the airfoil surface.
S	surface of the airfoil.

u	flow velocity in the boundary layer parallel to the airfoil surface.
U_{tt}	total tangential velocity at the airfoil surface.
U_{t_1}	tangential velocity at the airfoil surface due to U_∞.
U_{t_2}	tangential velocity at the airfoil surface due to V_∞.
U_{t_3}	tangential velocity at the airfoil surface due to the circulation.
U_{t_4}	tangential velocity at the airfoil surface due to the "boundary layer" source distribution.
U_∞	component of W_∞ in the direction of zero angle of attack, $U_\infty = W_\infty \cos\beta$.
V	velocity in the flow field, $\vec{V} = \vec{W}_\infty + \vec{w}$.
V_∞	component of W_∞ in the direction of 90° angle of attack, $V_\infty = W_\infty \sin\beta$.
V_{on}	onset flow which represents either the uniform or circulatory flow.
v	velocity at a point due to the source distribution on one element.
$v^{(0)}, v^{(1)}, v^{(2)}, v^{(c)}$	parts of the velocity v given by euqation 3.10.
$v_\xi^{(0)}, v_\eta^{(0)}$	components of $v^{(0)}$ in the direction of the ξ and η axes of the element.
$v_\xi^{(1)}, v_\eta^{(1)}$	components of $v^{(1)}$ in the direction of the ξ and η axes of the element.
$v_\xi^{(c)}, v_\eta^{(c)}$	components of $v^{(c)}$ in the direction of the ξ and η axes of the element.

$v_\xi^{(2)}, v_\eta^{(2)}$	components of $v^{(2)}$ in the direction of the ξ and η axes of the element.
v_X, v_Y	component of v in the direction of the body axes.
v_n, v_t	normal and tangential components of the velocity v.
w	disturbance velocity due to the airfoil boundary.
W_∞	uniform onset flow with an angle of attack β.
X,Y	body coordinate axes along the major and minor axes of the body.
x,y	coordinates of a general point with respect to the coordinate axes ξ and η.
$\bar{x},\bar{y}$	coordinates of a general point with respect to the coordinate axes $\bar{\xi}$ and $\bar{\eta}$.
α_j	slope angle of the element j.
β	the angle of attack.
γ	uniform vorticity over the airfoil surface.
Γ	resultant circulation in the flow field.
Γ_o	circulation due to a uniform vorticity distribution over the airfoil surface $\gamma = 4\pi$.
δ^*	displacement thickness.
ξ,η	element coordinate axes shown in Figure 10.
$\bar{\xi},\bar{\eta}$	element coordinate axes shown in Figure 9.
θ	momentum thickness.
ν	kinematic viscosity.
ρ	density.
$\sigma^{(0)}, \sigma^{(1)}, \sigma^{(2)}$	constants of the source distribution given by equation 3.2.

σ_k strength of the source distribution of the control point k.

$\sigma(t)$ strength of the source distribution at point t on the airfoil surface.

ϕ velocity potential, $grad(\phi) = -\vec{w}$.

Φ velocity potential, $grad(\Phi) = -\vec{V}$.

1. INTRODUCTION

A knowledge of maximum section lift coefficient (C_{Lmax}) is important, if not crucial to airfoil design. Since C_{Lmax} can not be predicted using existing computer tools, heavy emphasis is placed on expensive wind tunnel tests, even though the tests can not be performed at the high Reynolds number usually used in flight. This has traditionally led to some doubt in airfoil selection and to considerable conservatism in design to reduce the risk of premature stalling.

Full computational analysis of the real flow about airfoil sections has been limited by the inability to compute the effects of separation on section forces and moments. At low angles of attack where the boundary layer is thin and there is little if any separation, potential flow analysis (ignoring the viscosity effects) is a fair approximation to the experimental results but, as the angle is increased, the lift predicted is too high. Obviously, the boundary layer thickness and separation effects must be accounted for to predict maximum lift. In fact, most of the lift curve shows the effects of separation so that accurate results can be obtained in most cases only when both of these effects are accounted for.

Taking the calculations for the GAW-1 reported in Ref. 1 as an example, Figure 1 compares wind tunnel data with a potential flow analysis of the airfoil with no attempt to account for the effects of either boundary layer thickness or separation. Figure 2 displays the effect of modelling the boundary layer thickness (but not separation) and, as shown, the agreement with the wind tunnel data is good but, when the angle of attack is increased and separation begins to occur, the predicted and measured lift diverge. In Figure 3, the boundary layer thickness and separation region have been modelled using Henderson's technique (Ref. 1) and the agreement with the wind tunnel data is good for all values of the angle of attack. Henderson's results are quite satisfactory but the technique is cumbersome especially for multiple element airfoils, therefore a simpler alternative is the motive of the present work.

The current objective is to develop a computational technique to account for the effects of both the boundary layer thickness and separation on the lift coefficient. Although the technique which is presented here is applied to an elliptic airfoil with a laminar boundary layer in incompressible flow, the work can be extended to a general airfoil with a laminar or turbulent boundary layer provided that a suitable technique is used to compute the boundary displacement thickness distribution and to predict the points of separation accurately. The ellipse has been chosen to establish the technique since an exact analytical solution for the potential flow around it can be obtained easily. Also, Howarth (Ref. 2) has made a first approximation of the effects of laminar boundary layer separation on the lift of an elliptic cylinder and his results are available for comparison.

The technique to be presented here is based on the premise that the effects of the boundary layer thickness and the separated wake on the airfoil pressure distribution can be modelled by a source distribution on the surface of the airfoil which will force the dividing streamline away from the body by a distance equal to the displacement thickness, and cause the pressure to be constant in the separated wake region - a feature which experiments show to be nearly true.

Extension of the technique to multi-component airfoils should be possible.

2. LITERATURE REVIEW OF THE PROBLEM OF LOW-VISCOSITY FLOW AROUND AIRFOILS

The solution of the problem of low-viscosity flow around airfoils begins with computing the potential flow around that airfoil. The potential flow pressure distribution is then used to compute the characteristics of the boundary layer such as the displacement thickness and the position of the points of separation. The next step is to use an additional potential flow pattern to model the boundary layer and separation effects. Several iterations of these steps are required because the boundary layer modifies the pressure field. The sequence in this review will follow these steps for the solution of the laminar flow around an elliptic airfoil.

2.1 The Potential Flow Computation

The steady flow of an incompressible inviscid fluid about an arbitrary body is a classical fluid mechanics problem. A good discription of the mathematical formulation is given by Hess and Smith in Ref. 3, and the following discussion is based on that work.

For a steady flow of an incompressible inviscid fluid, the general Navier-Stokes equation reduces to the well known Eulerian equation of motion

$$(\vec{V} \cdot \text{grad})\ \vec{V} = -\frac{1}{\rho}\ \text{grad}\ p \ , \qquad 2.1$$

and the equation of continuity becomes

$$\mathrm{div}(\vec{V}) = 0 \ . \qquad 2.2$$

Equations 2.1 and 2.2 hold in the field of flow which, in the problem under consideration, will be the region exterior to the boundary surfaces. In addition to these equations, the flow field must satisfy certain boundary conditions. Attention will be restricted here to the so-called direct problem of fluid dynamics, where the locations of all boundary surfaces are assumed known and the normal component of fluid velocity is prescribed on these boundaries. For the present problem of a stationary airfoil in an infinite stream the normal component is set equal to zero so that the rigid boundary surface S will be a streamline, and the boundary condition will be written as

$$\vec{V}\cdot\vec{n}\Big|_S = 0 \ . \qquad 2.3$$

For the exterior problem, a regularity condition at infinity must be also imposed: in this case, $\vec{V} \to \vec{W}_\infty$ far from the airfoil.

For the case of potential flow, the velocity field will be irrotational and therefore it can be expressed as the negative gradient of a scalar potential function

$$\vec{V} = -\ \mathrm{grad}\ \Phi \ . \qquad 2.4$$

The velocity field $\vec{V}$ can be expressed as the sum of two velocities

$$\vec{V} = \vec{W}_\infty + \vec{w} \ , \qquad 2.5$$

where the onset flow, $\vec{W}_\infty$, in the airfoil problem will be a uniform flow and the disturbance velocity field, $\vec{w}$, due to the boundaries will be an irrotational flow expressed as the negative gradient of a potential function ϕ; that is

$$\vec{w} = - \text{grad } \phi \; . \qquad 2.6$$

Since $\vec{W}_\infty$ is the velocity of a uniform flow, it satisfies equation 2.2 and therefore it follows that

$$\text{div}(\vec{w}) = 0 \quad . \qquad 2.7$$

Substituting from equation 2.6 into equation 2.7, gives the expected result that the potential ϕ satisfies Laplace's equation

$$\nabla^2 \phi = 0 \quad . \qquad 2.8$$

The boundary conditions on ϕ arise from equations 2.3 and 2.5 in the form

$$\text{grad } \phi \cdot \vec{n} = \vec{W}_\infty \cdot \vec{n} \quad , \qquad 2.9$$

and in the usual exterior problem, the regularity conditions is

$$\text{grad } \phi \rightarrow 0 \text{ at infinity} \quad . \qquad 2.10$$

The essential simplicity of the potential flow derives from the fact that the velocity field is determined by the equations of continuity 2.2 and 2.7 and the condition of irrotationality (implied in equations 2.4 and 2.6): thus the equation of motion 2.1 is not used, and the velocity may be determined independently of the pressure. Once the velocity field is known, the pressure is calculated from equation 2.1 which, when integrated to the well known Bernoulli equation, can be written in terms of the pressure coefficient

$$C_P = \frac{P - P_\infty}{\frac{1}{2}\rho\left|\vec{W}_\infty\right|^2} = 1 - \frac{\left|\vec{V}\right|^2}{\left|\vec{W}_\infty\right|^2} \quad . \qquad 2.11$$

The problem defined by equations 2.8, 2.9 and 2.10 is a well known classic Neumann problem which has certain special features which greatly influenced the development of the methods of solution. In particular, the domain of the unknown ϕ is infinite in extent, but often the solution is of interest only on the boundaries. Despite the fact that Laplace's equation is one of the simplest and best known of all partial differential equations, the number of useful exact analytic solutions is quite small, the difficulty of course lies in satisfying the boundary conditions.

The problem can only be solved analytically by the technique of separation of variables which requires the boundary to be a coordinate surface of one of the special orthogonal coordinate systems for which Laplace's equation can be separated into ordinary differential equations. In the two-dimensional case, Laplace's equation is simply separable in all orthogonal coordinate systems, but this technique is not commonly used. Instead of solving the Laplace's equation, the problem can be replaced by finding a suitable conformal transformation of the boundary into a circle, and the flow around the airfoil will be calculated using the known solution of the flow around this circle.

2.1.1 The Analytic Methods

The Joukowski transformation when applied to an offset circle results in a cambered Joukowski airfoil. The flow around this airfoil can be obtained from the transformed flow around the circular cylinder. The circulation around the airfoil is obtained by applying the Kutta condition which requires the velocity at the trailing edge to be finite, and so the point on the circular cylinder which corresponds to the trailing edge must be a stagnation point. Then the coefficient of lift will be otained from the theorem of Kutta and Joukowski. This gives the ideal lift-curve slope to be equal to 2π/radian for thin airfoils at small incidences.

All Joukowski airfoils have cusps at the trailing edge; the application of von Karman and Trefftz transformation will result in airfoils with finite angle trailing edges.

Different exact and approximate analytical methods have been developed to calculate the steady flow of an incompressible inviscid fluid about two-dimensional arbitrary airfoils. A good survey of these methods has been given by Giesing in Ref. 4.

An exact method to calculate the pressure distribution around an airfoil of given arbitrary shape has been put forward by Theodorsen who maps a single airfoil into a circle. Initially, the airfoil is mapped into a pseudo-circle by an inverse Joukowski transformation and then into an exact circle by a second transformation. The procedures can be generalized and improved by replacing the single Joukowski transformation by one, or more than one, inverse Karman-Trefftz transformation. With such a transformation an airfoil with any number of surface slope discontinuities, in addition to the trailing edge discontinuity, can be mapped into a smooth pseudo-circle.

The application of exact analytic solution to practical problems is generally beyond the capability of hand computation; therefore approximate solutions have in the past received most of the attention of investigators in the field of potential flow. Goldstein developed a series of systematic approximations to the exact Theodorsen method which led to a sufficiently accurate result with little labour in computation. The thin airfoil theory put forward by Munk and refined by Glauert assumes that the general thin airfoil can be replaced by its camber line which is assumed to be only a slight distortion of a straight line and that the perturbation velocity components due to the presence of the thin airfoil are small with respect to the uniform onset flow. The thin airfoil theory, which led to the linearization of the problem, has two different types of analysis. The first one uses an inverse Joukowski transformation of the thin airfoil to a slightly distorted circle which is then transformed to an exact circle and the other one utilizes a distribution of singularities placed along the chord and the strengths of the singularities are determined by satisfying the boundary conditions. The thin airfoil theory does not usually give accurate pressure distributions, especially near the nose where it gives an infinite value, but it does give usable values of the lift coefficient. Riegels established a correction for the pressure distribution in regions of high surface slope, and a similar modification was developed by Weber to improve the pressure distribution, especially near the nose.

The approximate solutions are generally unsatisfactory since they are inapplicable in many cases, their validity in other cases is not predictable, and the accuracy of the computed solutions is usually unknown.

All the exact and approximate analytical solutions mentioned above are unable to handle the flow about multiple bodies and this fact led to the consideration of the exact numerical methods of solution.

2.1.2 Exact Numerical Methods

A distinction must be made between approximate analytic solutions and exact numerical methods. In the latter the analytical formulation, including all equations, is exact and numerical approximations are only introduced for purposes of calculation. Such approximations are numbers having a finite number of decimal places and integrals that are evaluated by quadrature formulas. Exact numerical methods have the property that the errors in the calculated solution can be made as small as desired by sufficiently refining the numerical calculations. In contrast, approximate solutions introduce analytical approximations into the formulation itself and thus place a limit on the accuracy that can be obtained in a given case regardless of the numerical procedures used.

There appear to be two classes of exact numerical solutions that have been applied to the general fluid dynamics problem. The first one is the network method based on finite-difference approximations of the derivatives of the potential; the second is based on the solution of an integral equation over the boundary surface which involves determination of a singularity distribution over the boundary surface. The network method is usually unsuitable for the external fluid dynamics problem since the solution gives the whole field while it is usually only the boundary values which are of interest. The flow field about an airfoil is infinite in extent, which necessitates the distribution of the network control points throughout many body lengths in each direction resulting in a large number of control points and a consequently large number of equations, while the integral methods require only control points distributed on the boundary of the airfoil. This difference is illustrated in Figure 4.