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THE SENSITIVENESS OF THE EYE TO LIGHT AND COLOR

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*The Sensitiveness of the Eye to Light and Color.**

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Presented by PROF. FRANK ALLEN, Ph. D., F.R.S.C.

(Read May 28, 1913)

In an extensive paper on "The Luminous Equivalent of Radiation," Nutting† thus summarizes some of the more important characteristics of the visual response to radiation:

I. "Sensibility to Slight Differences in Wave Length, has two pronounced maxima, one in the yellow and one in the green; and two slight maxima in the extreme blue and red. These maxima vary considerably with the individual and probably also with the intensity of the radiation used."

II. "Sensibility to Radiation of Varying Intensity:

Sensibility falls off steadily with increasing intensity. It is approximately inversely proportional to the intensity over a wide range. The ratio of optical intensity to intensity of radiation increases more rapidly for red than for blue and green (*Purkinje phenomenon*)."

III. "Sensibility to small Differences in Intensity":

The least perceptible increment measured as a fraction of the whole is approximately:

(1) Independent of Intensity (*Fechner's Law*). It is about ·016 for moderate and high intensities and greater for very low and extremely high intensities.

(2) Independent of Wave Length (*König's Law*) at constant luminosity, extremes again excepted.

(3) Independent of the Individual."

With the exception of number I the above characteristics of the visual response to radiation have been verified by several observers. But concerning the sensibility of the retina to slight changes in the wave-length of the light no investigation of a very exhaustive nature appears to have been conducted. Perhaps the best recent data are those due to Dr. Olga Steindler;‡ but even there a spectrum of only one intensity appears to have been considered. With a view then, in the first place of verifying results already obtained by Steindler and others,

*To be printed also in the Physical Review.

†P. G. Nutting: Bulletin of the Bureau of Standards, 1908, Vol. 5, No. 2, page 265.

‡Wien Sitz.: IIa, 115, pp. 1-24, 1906.

and in the second place, of investigating the behaviour of these maxima with changes in the intensity of the radiation used, the following observations were made.

When it was found that all of the chief characteristics might be investigated by the apparatus to be used in the case of number I, and in view of the fact that it appeared to be an original method, it was decided to investigate all three cases. It was thought that the accuracy with which the well known phenomena were established might indicate the reliability of the results obtained in the first case.

Historical *

Mandelstamm † appears to have been the first to investigate the color sensitiveness of the human eye for slight differences in the wave length of light observed. By shifting the plates of an ophthalmometer so as to give a just perceptible color difference, he obtained a means of observing sensibility in different parts and obtained in this way maxima of sensibility in the region of the D and F lines. Dobrowolosky ‡ came to the same conclusion by similar means. Peirce[4] investigated the sensitiveness of the eye to slight differences of color by having two identical spectral bands one immediately above the other, the upper one of which might be shifted. The object of the experiment was to see how small a displacement could be infallibly *detected* and *named in direction* by the observer in different parts of the spectrum. He found maxima situated similarly to those found by other investigators. More accurate results were first obtained by König and Dieterici.[5] Uhthoff[6] also investigated the differences in wave length for just observable color differences. Brodhun,[7] himself color blind, gives measurements after the method of König for mean intensities. Exner[8] also gives a single service of measurements on a widely dispersed spectrum. In this connection also Steindler[9] reports observations upon twelve subjects. A spectrum with a dispersion of about 85 cm. at the point observed was obtained by means of an arc light and a concave grating. The light from the spectrum at this point fell upon two totally reflecting prisms placed vertically one above the other and after reflections from a second

*Historical references taken from Dr. Steindler's paper.

†Grafe's Archiv: Bd. 13, p. 399.

‡Ebenda, Bd. 18, p. 99.

[4] American Journal of Science, Vol. 26 (1883), p. 299.

[5] Annalen der Physik und Chemie, Bd. 22, p. 579.

[6] Grafe's Archives, Bd. 34, 4, p. 1.

[7] Zeitschr. für Psych. und Phys. Bd. 34 (1892), p. 89.

[8] L. c. p. 875.

[9] Wien Sitz.: IIa, 115, pp. 1-24, 1906.

larger prism two adjacent fields appeared in the eye-piece. Since the upper of the two smaller prisms was moveable along a horizontal scale light of the same or slightly different wave length might be viewed in the field. In this way the amount of change necessary to produce a just perceptible but distinct color difference was measured. Values thus obtained were mean values for at least ten observations made by the same person on the same portion of the spectrum.

Description of Apparatus.

One arrangement of apparatus was found sufficient for all investigations and is essentially that used by Allen* to measure the luminosity of the spectrum. The general arrangement is shown in Fig. 1. The light from an acetylene flame A after concentration by lenses B and B[1], passed through the opening C from a light proof chamber M; then through two nicol prisms (E and F) arranged with their principal

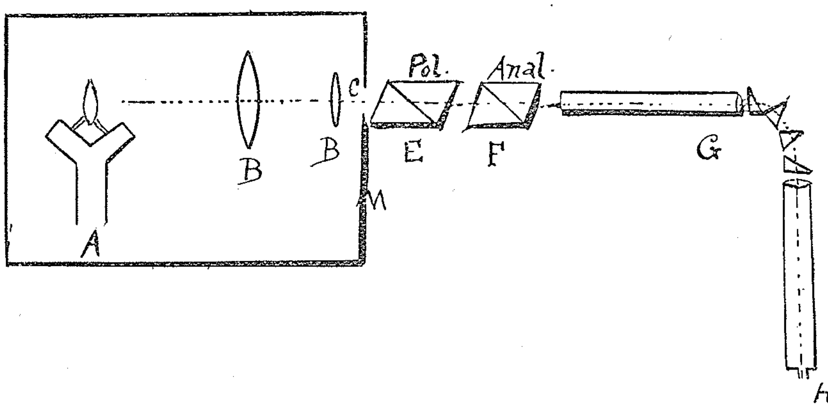

FIG. 1.

sections horizontal; thence through the spectrometer G and was finally viewed in a Hilger eye-piece H in which all the light of the spectrum except a narrow central band was cut off by means of adjustable shutters. The observer was therefore able to subject his eye to a light stimulus of any desired wave-length. A constant gas pressure was taken to indicate a constant luminosity in the source of light. The intensity of the spectrum was controlled by rotating the polariser. The principal plane of the analyser remained horizontal for *all* investigations in order that the displacement of the light waves as they met

*Frank Allen: Phil. Mag. 1911, Vol. 21, No. 125, page 604.

the prism might be perpendicular to its face. In this way it was thought to avoid a variation in intensity of light through reflection from the surfaces of the prisms.

The spectrometer used was of the Hilger Automatic type with four prisms equivalent to three sixty degree prisms and giving a dispersion slightly in excess of twelve degrees.

I.

Each characteristic of the visual response to radiation as outlined in the commencement of this paper, was treated as a separate investigation. Observations were made to show:

1. Whether sensibility to slight difference in wave length has two pronounced maxima, one in the yellow and one in the green, and two light maxima in the extreme blue and red.

2. Whether these maxima vary with the intensity of the radiation used.

1. In the investigation concerning spectra of varying intensity, other conditions remaining constant, spectra of six different intensities were examined. Only observations made with my own eye (the right) which seems quite normal as to color sensations, are considered in this paper. Upon the spectrum of maximum intensity in all parts, which for present purposes, has been represented by unity, and upon a spectrum of intensity ·25, observations were made to determine the minimum *decrease* in intensity by which the initial intensity was changed in effecting a change in sensation that was just perceptible. In the case of spectra at all other intensities considered, the minimum *increase* in the original intensity necessary to produce a just perceptible change in the sensation was determined.

Method of procedure.

The principal sections of the nicols were horizontal and so gave the brightest spectrum obtainable with the chosen arrangement of apparatus. Observations were made upon thirty-two portions of this spectrum; these are given in Table I.

Light of wave length $\cdot 42\mu$ was first observed. This was allowed to act upon the retina for four seconds. Then the polariser was rotated at a uniformly rapid rate; an effort was made to keep the rate at which the polariser was rotated, approximately constant for all observations. As soon as a change in the sensation of light was perceived the rotation was stopped. The angle through which rotation had taken place was noted. This was denoted by α; since unity had been taken to represent the amount of light passing through the prisms before the polariser was rotated, the difference between 1 and $\cos^2 \alpha$ i.e., $\sin^2 \alpha$ will represent the amount by which the original intensity of the light was *diminished* in effecting a change of sensation that was just perceptible. The reciprocal of this value ($\sin^2 \alpha$) taken to represent the sensibility of the eye has been plotted as a function of the corresponding wave length. This is shown in Curve 1, Fig. 2. The other chosen portions of the spectrum were observed in rapid succession in a similar way.

A dimmer spectrum was next considered. This was obtained by setting the polariser before each observation so that its principal plane made an angle of 51° with the horizontal, i.e. with the plane of the analyser. This initial angle was denoted by β. Since $\cos^2 \beta = \cos^2 51°$ is equal to ·395, it was considered that each portion of the spectrum chosen for investigation was 39·5% of the intensity of corresponding portions in the spectrum at maximum intensity. With this initial arrangement light as observed in the eye-piece was, as before, allowed to act upon the retina for four seconds. Then the polariser was rotated so as to *increase* the intensity of the light. As soon as the intensity had just noticeably increased the rotation was stopped. The angle between the planes of the nicols was read and denoted by α. Since $\cos^2 51°$ represents the intensity of the stimulus at the begining and $\cos^2 \alpha$ the intensity of the light affecting the eye at the moment of perceptible change, $\cos^2 \alpha - \cos^2 51°$ will represent the amount by which the original intensity (which for each observation is 39·5% of the maximum intensity of the spectrum at the chosen point) was *increased* in affecting a change of sensation that was just perceptible. As before, the reciprocal of this value was taken to represent the sensibility of the eye and has been plotted as a function of the corresponding wave length. The results are shown graphically in Curve II, Fig. 2. The portions of the spectrum observed and the observations made upon each are given in Table II.

Upon the spectrum of intensity ·25 observations were made to determine the least perceptible *decrease* in the initial stimulus necessary to produce a noticeable change in the sensation of light. This was done by following the method described for the spectrum at maximum intensity. The results are given in Table III and are shown graphically in Curve III, Fig. 2. The spectrum of intensity ·25 was obtained by setting the plane of the polariser at an angle of 60° with the plane of the analyser.

In the case of other spectra investigated observations were made for the least perceptible *increase* in the initial stimulus, following the method used upon the spectrum second in order of brightness, as described above. These spectra were of relative intensities ·060; ·025 and ·0054 respectively,—the spectrum of maximum intensity being

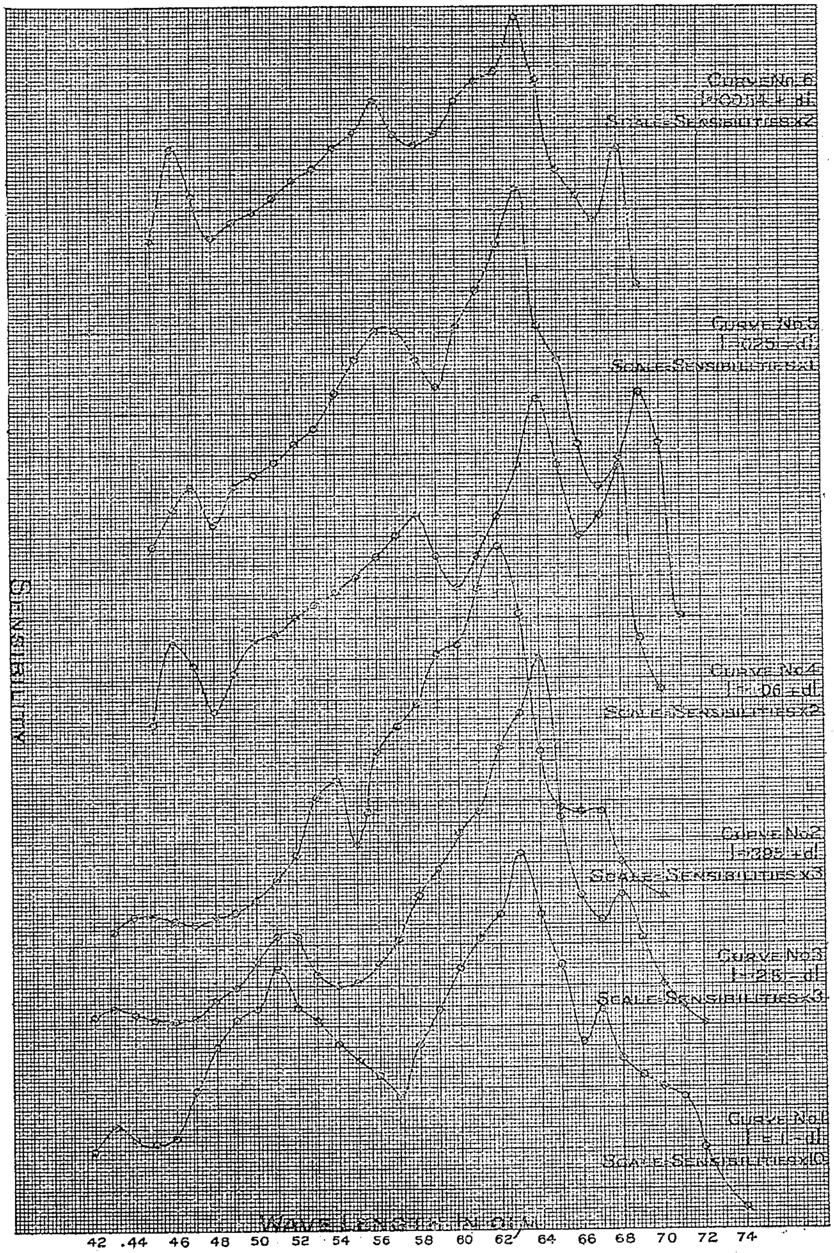
Curve No. 6
Scale-Sensibilities x2
Curve No. 5
Scale-Sensibilities x1
Curve No. 4
Scale-Sensibilities x2
Curve No. 2
Scale-Sensibilities x3
Curve No. 3
Scale-Sensibilities x3
Curve No. 1
Scale-Sensibilities x10
Sensibility
42 .44 46 48 50 52 54 56 58 60 62 64 66 68 70 72 74

Figure 2

considered as equal to unity in all parts. The initial conditions to give spectra of such intensities for each observation were the inclination of the principal planes of the nicols to each other at angles of 75° 45′; 80° 45′ and 85° 45′ respectively. The results are given in Tables IV, V and VI and shown graphically in Curves IV, V and VI, fig. 2.

Tables of data.

In Tables I to VI λ indicates the wave length of light observed. The column headed "Scale Readings" gives the observed reading on the attached scale, after each rotation. With each new intensity the zero of the scale was observed and taken into account to find the angle between the principle sections of the nicols. These values will be found in the column headed (α). Since β denotes the initial angle between the principal sections, in the column under $\text{Cos}^2\beta$ will be found the intensity of the spectrum considered in each case. In the columns under $\text{Sin}^2\alpha$ Table I and under $\text{Cos}^2\beta - \text{Cos}^2\alpha$ Table III will be found the amount by which the initial intensity was *diminished* to effect the minimum noticeable change of sensation. In Tables II and IV to VI, under $\text{Cos}^2\alpha - \text{Cos}^2\beta$ are given the amounts by which the original intensity (as shown under $\text{Cos}^2\beta$) had to be increased to effect a change of sensation just perceptible. Under "Sensibility" are the values of the reciprocals of the amounts of minimum perceptible change in intensity. In order that the various curves might be plotted in one figure these reciprocals, which are the ordinates for the different curves, have been multiplied by arbitrary constants as indicated in the figure and then plotted to the scale shown.

2. *A Spectrum of Equal Luminosity in all parts.*

When measuring the luminosity of the spectrum Allen* took observations upon nineteen portions of it, determining in each case the angle which the principal plane of the polariser should make with the horizontal, i.e., with the plane of the analyser in order that each portion as viewed in the eye-piece should have equal luminosity. Therefore since the apparatus used in the present investigation was also used by Allen in approximately the same arrangement with the same gas jet, gas pressure and slit width, it was considered that, by observing these same portions and setting the plane of the polariser at the corresponding angle for each, equality of luminosity in the parts observed would be obtained. This would it was thought be equivalent to observing a spectrum of equal luminosity in all parts. The next problem considered, then, was that of determining the sensibility of the eye to slight

*Phil. Mag., 1911. Vol. 21., p 604.

differences in intensity when the light stimulus was of equal intensity at each point of observation. For this purpose observations similar in principle to those already described, were made upon this "Equivalent of a Spectrum of Equal Luminosity."

Method of procedure.

The method of procedure was first to adjust the telescope of the spectrometer for light as near the ultra-violet as it was possible to make an observation. This was of wave length $\cdot 460\mu$. The plane of the polariser was adjusted to make an angle of 35° with the plane of the analyser. This gave to the light as viewed in the eye-piece a luminosity equal to the luminosity of the spectrum where the light waves were $\cdot 414\mu$—the principal sections of the nicols then being parallel. This latter luminosity was taken as the standard of brightness since all portions observed were first reduced to this luminosity. After the manner of former observations the light as viewed in the eye-piece was allowed to act upon the retina, the polariser rotated and the angle after rotation noted. This angle was denoted by α. If β denote the initial angle (35°) between the planes of the nicols, then since α was taken to denote the angle between the planes after rotation, $(\text{Cos}^2\alpha - \text{Cos}^2\beta)$ will be proportional to the amount of light by which the initial intensity was *increased* to effect a noticeable change in the sensation. *This amount of light* $(Cos^2\alpha - Cos^2\beta)$, *will be a fraction of the total intensity at the point considered.* But since the luminosity of each part was reduced to that of the standard, it is reasonable to expect that, at those points where the spectrum is brighter and therefore more intense, a smaller portion of the total intensity will be required to produce a noticeable change in the sensation than will be required where the total brightness is much less. That is: the brighter the spectrum at the point of observation the smaller the fraction of total intensity necessary to effect a noticeable change in the sensation. Hence we may say that the luminosity of each part of the spectrum is inversely proportional to the portion of total intensity required to effect a noticeable change in the sensation—the sensation in each case being the result of a light stimulus varying only in wave length. Therefore, plotting the reciprocals of the various values of $(\text{Cos}^2\alpha - \text{Cos}^2\beta)$ as functions of the corresponding wave lengths a luminosity curve for the spectrum should be obtained. This is shown in Fig. 3.

In the above case the reciprocals of the various portions of total light added cannot represent sensibility because in one instance we may take the reciprocal of a large quantity of light at low intensity and

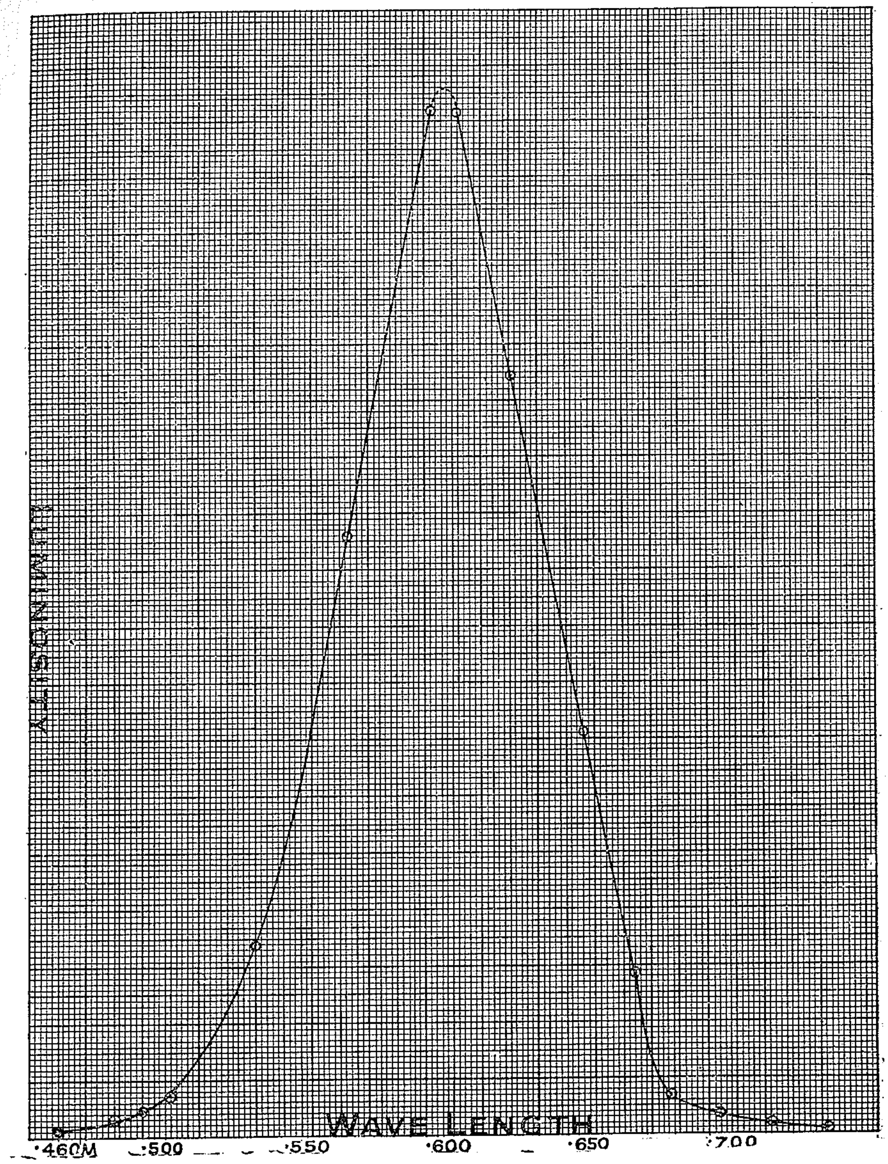

Figure 3

in another the reciprocal of a small quantity at high intensity. And while the reciprocals differ vastly in value the actual illuminating power of the two quantities may be indentical. In the one the increment of light was added at low intensity and therefore more of it was required; in the other it was added at high intensity and less of it was required. Hence before calculating reciprocals for the purpose of showing sensibility, the various increments of light were first reduced to a common standard of intensity. This was the intensity of the standard of brightness, viz., the intensity of light of wave length $\cdot 414\mu$ as observed in the eye-piece while the principal sections of the nicols were parallel.

Method of Reducing to a Common Standard of Intensity.

We may assume that the luminosity of any portion of the spectrum is proportional to its intensity. But the luminosity is inversely proportional to the fraction of total intensity, at the point considered, that will give a luminosity equal to the luminosity of the standard of brightness. But this fraction of total intensity is proportional to the square of the cosine of the angle between the principal sections of the nicols—the light going through giving a luminosity equal to the luminosity of our standard of brightness. That is: the luminosity of any portion of the spectrum is inversely proportional to the square of the cosine of the angle between the principal sections of the nicols when the light going through from that portion gives a luminosity equal to that of the standard. Therefore, taking the luminosity of the standard as unity, the luminosity of the spectrum at any point considered in terms of this standard is given by $1/\cos^2\beta$ where β is the angle between the principal sections of the nicols when just sufficient of the light considered is going through to give a luminosity equal to the luminosity to the standard. And since intensity is assumed proportional to luminosity, the intensity of light at any point in the spectrum in terms of the intensity of the standard is given by $1/\mathrm{Cos}^2\beta$.

Again, α denotes the angle between the principal sections of the nicols after the rotation already explained. Therefore, the fraction of total intensity by which the initial intensity i.e., that represented by $\mathrm{Cos}^2\beta$, was increased to give the least perceptible change in the sensation, may be represented by $(\mathrm{Cos}^2\alpha - \mathrm{Cos}^2\beta)$. The intensity of any given portion of light depends upon its position in the spectrum. Therefore, the fraction of total intensity will have an intensity depending upon the position of the point in the spectrum at which it was added, i.e. depending upon the value of the angle β. But it has been shown that the intensity of light from any portion of the spectrum may be expressed in terms of the standard of brightness by multiplying by $1/\mathrm{Cos.}^2\beta$. Henceit was thought that by multiplying $(\mathrm{Cos}^2\alpha - \mathrm{Cos}^2\beta)$ by $1/\mathrm{Cos}^2\beta$ the various increments of light added to produce a perceptible change in the sensation in each case would be of equal intensity i.e., they would have an intensity equal to the intensity of the light in the standard. The reciprocals of this product it was thought would represent sensibility. The portions of the

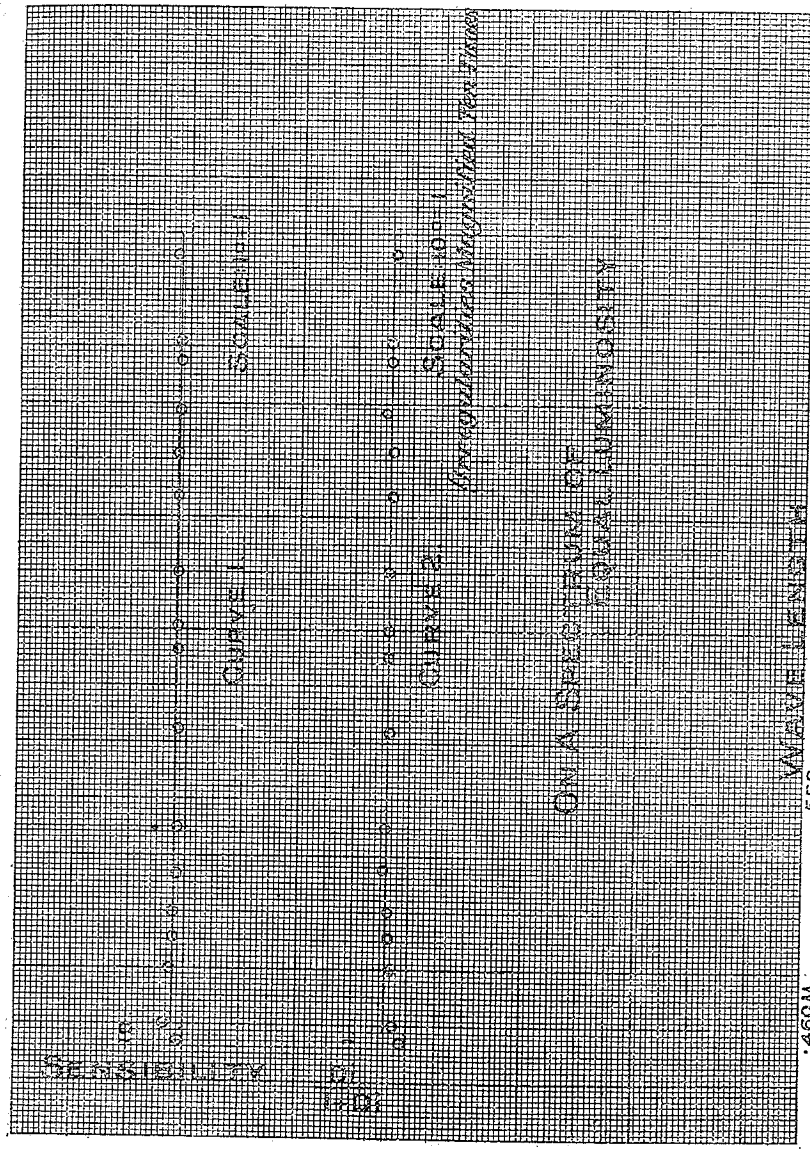

Figure 4

spectrum observed with the corresponding data and calculations are given in Table XV. The sensibility in each case was plotted as a function of the corresponding wave length as shown in Curve 1, Fig. 4, thus showing sensibility to slight differences in intensity using the equivalent of a spectrum of uniform luminosity.

Table of data.

In Table XV, λ indicates the portion of the spectrum upon which observations was made. "β" denotes the inclination of the plane of the polariser to the plane of the analyser. In the column under "α" will be found the inclination of the plane of the polariser to the plane of the analyser *after rotation has taken place.* $\text{Cos}^2\beta$ will represent the amount of light going through the nicols before the rotation of the polariser, and $\text{Cos}^2\alpha$ the amount of light going through when this rotation was completed. In the column under "Least Perceptible Increments" will be found the value of $(\text{Cos}^2\alpha - \text{Cos}^2\beta)$ which will represent the fraction of total intensity which had to be added to the initial intensity ($\text{Cos}^2\beta$) to effect a change of sensation that was just perceptible. The reciprocals of those values plotted as a function of the corresponding wave length gave the luminosity curve found in Fig. 3. As explained above those increments were reduced to light of uniform intensity by dividing by $\text{Cos}^2\beta$ in each case. The reciprocals of these latter results were taken to represent sensibility and will be found under "Sensibility." Under $\text{Di}/(\text{I}+\text{Di})$ will be found the value of the least perceptible increment divided by the total intensity (after rotation) which was used to get curve 2, Fig. 4 to demonstrate König's Law.

II.

Sensibility to Radiation of Varying Intensity.

As a demonstration of the steady falling off of sensibility with increasing intensity and to show that it is approximately inversely proportional to the intensity over a wide range, observations were made as follows:

The same apparatus as used in the preceding was used in this; and the same arrangement. First, the telescope of the spectrometer was adjusted for light of a given wave length, for example $\cdot 420\ \mu$ (see Table VII), and rigidly fixed. The nicols were then crossed so as to exclude all light from the eye-piece. The angle indicated on the attached scale was read and this reading was taken to indicate an inclination of 90° between the principal sections of the nicols. The polariser was then rotated and at the first sensation of light experienced through the eye-piece, rotation was stopped. The angle indicated on the attached scale was again read. Let $\alpha_{,1}$ denote this