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Symmetrical and Asymmetrical Heat Transfer in Single-Phase Turbulent Flow in Rectangular Channels and Parallel Plates

by

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SYMMETRICAL AND ASYMMETRICAL HEAT TRANSFER IN SINGLE-PHASE TURBULENT FLOW IN RECTANGULAR CHANNELS AND PARALLEL PLATES

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ABSTRACT

The object of this work was to propose methods which are both simple and accurate for the prediction of symmetrical and asymmetrical heat transfer in single-phase turbulent fluid flow in rectangular channels and parallel plates. Existing heat-transfer correlations for circular tubes, symmetrically and asymmetrically heated rectangular channels and parallel plates were reviewed. New correlations, based on the Spalding-Jayatilleke P-function concept, were developed by the present author for symmetrically and asymmetrically heated parallel plates. The approach for the selection of the best methods for heat-transfer prediction was by comparison with all the available published experimental data. In the case of symmetrical heat transfer in rectangular channels and/or parallel plates, both Barrow's correlation and the present author's extension of the Spalding-Jayatilleke P-function method, with root mean square deviations of approximately 23% and 24% respectively, provided better results than the circular-tube correlations. Furthermore, for asymmetrical heat transfer in rectangular channels, the James-Martin-Martin correlation was found to be superior to all others with a root mean square deviation of approximately 13%. In the case of asymmetrical heat transfer in parallel plates, the present author's extension of the Spalding-Jayatilleke P-function method yielded better prediction than any existing correlation with a root mean square deviation of 17%.

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NOMENCLATURE

Symbol	Meaning	Equation of first appearance
A	aspect ratio	(5.2)
b	a number	(6.6)
c	a constant	(3.22)
c_f	friction factor	(2.1)
c_p	specific heat at constant pressure	(3.1)
D	pipe diameter	(3.1)
D_e	hydraulic equivalent diameter	(2.4)
G	mass velocity	(3.1)
h	heat-transfer coefficient	(3.1)
k	(laminar) thermal conductivity	(3.1)
k_{tot}	total thermal conductivity	(3.12)
L	half of width of channel normal to heating surface	(4.4)
m	a number	(6.4)
N	number of data	(8.1)
n	a number	(6.1)
Nu	Nusselt number	(3.3)
P	quantity defined in equations (3.29), (4.16) and (5.10)	
PD_{rms}	percentage root mean square deviation	(8.1)
Pr	(laminar) Prandtl number	(3.3)
Pr_{tot}	total Prandtl number	(3.13)
Pr_{turb}	turbulent Prandtl number	(3.27)

$\dot{q}''$	heat flux	(3.11)
Re	Reynolds number	(2.2)
Re^*	laminar equivalent Reynolds number	(2.6)
s	channel spacing	(2.6)
St	Stanton number	(3.6)
T	absolute temperature	(6.4)
u	local time-mean velocity	(3.8)
$\bar{u}$	bulk velocity	(2.1)
u^+	local time-mean velocity, made dimensionless	(3.7)
$\bar{u}^+$	bulk velocity, made dimensionless	(3.21)
u_R^+	time-mean velocity at axis of pipe, made dimensionless	(3.23)
w	channel width	(2.6)
y	distance from and normal to wall	(3.9)
y^+	distance y, made dimensionless	(3.7)
Γ_{tot}	total thermal exchange coefficient	(3.11)
γ	a number	(4.1)
ε_{tot}	ratio of total viscosity to (laminar) dynamic viscosity	(3.16)
κ	a constant	(3.22)
μ	(laminar) dynamic viscosity	(3.1)
μ_R	viscosity ratio	(5.2)
μ_{tot}	total viscosity	(3.10)
ρ	density of fluid	(2.1)
τ_w	shear stress at wall	(2.1)
ϕ	specific enthalpy	(3.11)
$\bar{\phi}$	bulk value of ϕ over cross-section of flow	(3.25)

ϕ^+	specific enthalpy, made dimensionless	(3.18)
$\bar{\phi}^+$	bulk value of ϕ, made dimensionless	(3.26)
ϕ^*	geometry function	(2.7)

Subscripts

b	bulk condition
cp	constant-property condition
exp	experimental value
L	centre-line condition of rectangular channels or parallel plates
pred	predicted value
w	wall condition

CHAPTER 1

INTRODUCTION

The heat-transfer and flow friction characteristics for single-phase turbulent fluid flow in circular tubes have been the subject of a great deal of theoretical analysis and experimentation in the last seventy years. The circular passage has been analysed extensively because of its importance in technical application and because its simplicity makes it amenable to analysis. In recent years, rectangular channels and parallel plates have been used extensively in engineering systems, e.g. nuclear power reactors, solar energy collectors, compact heat exchangers, ventilating and air conditioning systems, etc. These applications may be under symmetrical or asymmetrical heating conditions. However, they have been investigated to a much lesser extent than the circular tubes.

The object of this present work is to obtain simple and directly-usable correlations for the prediction of symmetrical and asymmetrical heat transfer of single-phase turbulent flow in rectangular channels and parallel plates. Existing heat-transfer correlations for circular tubes, symmetrically and asymmetrically heated rectangular channels and parallel plates were reviewed. New correlations, based on the Spalding-Jayatilleke[22,43]*

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* Superscript numbers refer to literature references listed in the thesis section entitled "References".

P-function concept, were developed by the present author for the symmetrically and asymmetrically heated parallel plates. The advantages of the Spalding-Jayatilleke method are that:

1. it is a delicate blend of theory and empiricism;
2. there is some distinct physical significance to the various terms appearing in the equation(s);
3. it is simple to use.

The approach for the selection of the best correlations for heat-transfer prediction was by comparison with all the available published experimental data.

The body of the work is divided into four major areas. In Chapter 2, the friction factor for turbulent flow in a rectangular geometry and parallel plates is discussed. In Chapter 3, 4 and 5, the heat-transfer correlations are outlined. The effect of fluid property variations on heat transfer is investigated in Chapter 6. Chapter 7, 8 and 9 consist of the test results, conclusions and recommendations.

CHAPTER 2

FRICTION FACTOR

2.1 General

In the solution of practically all heat-transfer convection problems, the corresponding fluid-dynamic problem must first be solved, if it is not indeed completely coupled to the heat-transfer problem. Although this work is not on viscous fluid dynamics, the friction factor in fully-developed turbulent non-circular duct flow is investigated in detail. The effect of the friction factor on heat transfer is demonstrated in the later chapters of this work.

2.2 Circular Tubes

2.2.1 Definition

Consider the steady flow of an imcompressible fluid in a long circular tube. Let the bulk velocity be $\bar{u}$. The friction factor, c_f, is defined by

$$c_f \equiv \frac{\tau_w}{\frac{1}{2}\rho\bar{u}^2} \qquad (2.1)$$

where τ_w is the shear stress at the tube wall and ρ is the density of the fluid.

2.2.2 Developed Turbulent Flow

For turbulent flow in smooth tubes, Blasius's empirical formula[31] gives

$$c_f = 0.079\ Re^{-\frac{1}{4}} \qquad (2.2)$$

where Re is the Reynolds number. It agrees closely with experimental results for Reynolds numbers between 3000 and 10^5. It is both explicit and simple to apply. With some theoretical foundation, the Karman-Nikuradse[24] (also known as Karman-Prandtl and Prandtl-Nikuradse) formula gives

$$\frac{1}{\sqrt{c_f}} = 4.0\ \log_{10}(\ Re\sqrt{c_f})-0.4 \qquad (2.3)$$

Equation (2.3) is inconvenient to use because one cannot solve it directly for c_f. The two formulas (2.2) and (2.3) are plotted in Figure 2.1. They agree very well with one and other for Reynolds numbers between 4000 and 10^5.

2.3. Rectangular Channels and Parallel Plates

It is quite customary to apply the concept of hydraulic equivalent diameter in the calculation of friction factor for developed turbulent flow in channels of

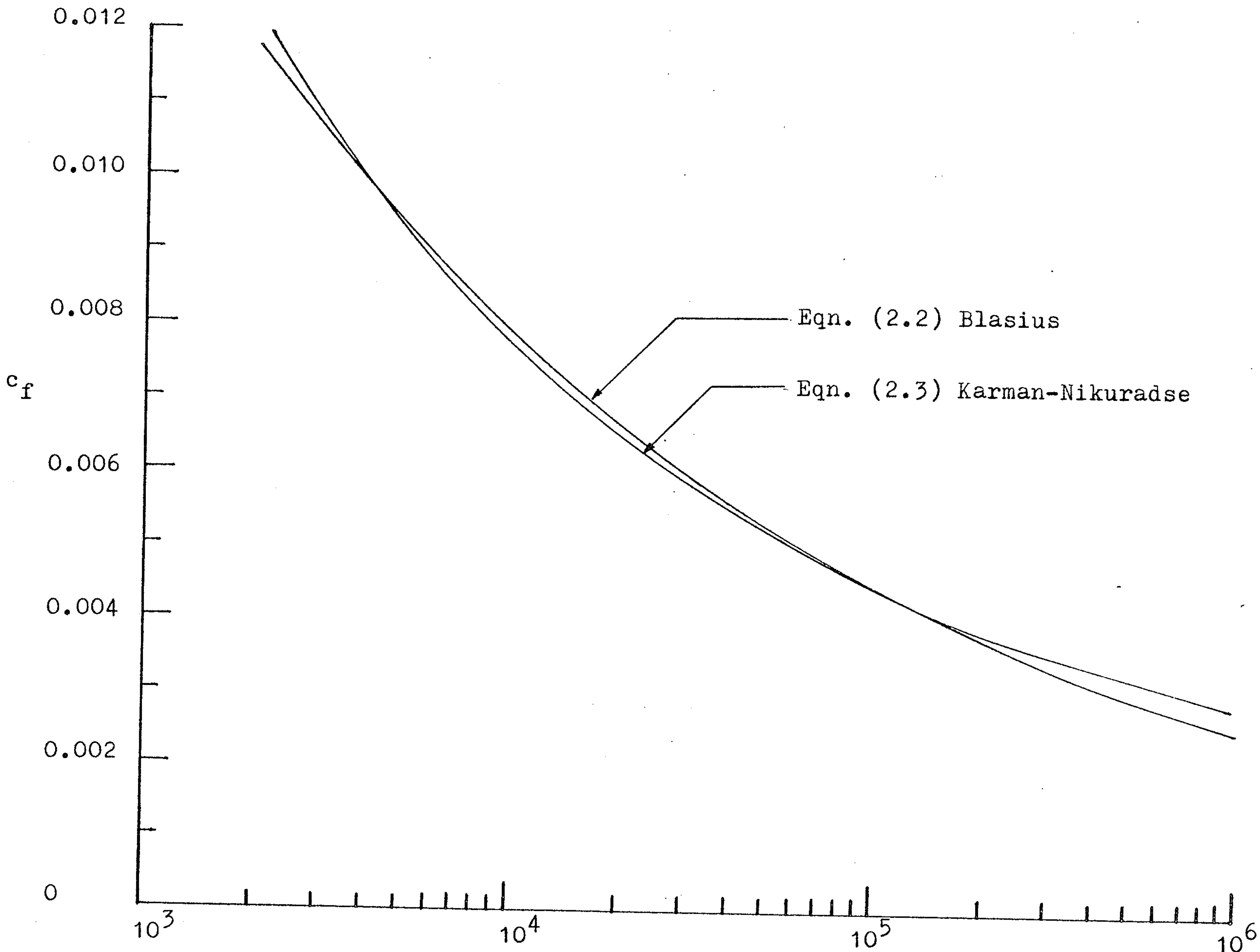

Fig. 2.1 Friction Factor

rectangular or other non-circular cross-section[33]. The hydraulic equivalent diameter, D_e, is defined as

$$D_e \equiv 4 \times \frac{\text{Flow Area}}{\text{Perimeter}} \tag{2.4}$$

However, there have also been theoretical analyses[12] and experiments[16,29] which indicate significant errors may result . Because of these contradictory reports, a review of the literature was done by the present author, as summarized in Table 2.1. Some of the more recent and important papers are discussed in the rest of this chapter.

In 1962, Hartnett et al.[17] made a detailed survey of the literature. In addition, friction factors for ducts of aspect ratio 1:1, 5:1 and 10:1 were predicted by the method of Deissler[10] and Deissler and Taylor[11], and experiments were also performed on these ducts. The calculated and measured results were found to be in agreement for ducts having large aspect ratios. As for aspect ratios less than 5:1, the predicted values of friction factor were lower than the experimental data, with a maximum difference of only 12% evident for the square duct. A plot was made by Hartnett et al. to compare the Blasius correlation with all available experimental data. Those data included

Table 2.1 Investigations on Friction Factor

1. Rectangular Channels

Investigator(s)	Year	Ref.	Test Fluid	Dimensions inches	D_e inches	Re	Channel Length / D_e
Schiller	1923	(41)	Water	0.709x 0.709	0.709	8.6×10^2 to 3.8×10^4	56
				1.098x 0.312	0.486	6.4×10^2 to 5.8×10^4	162
Cornish	1928	(7)	Water	0.464x 0.159	0.237	2.7×10^2 to 6.37×10^4	200
Lea & Tadros	1931	(28)	Water	0.257x 0.257	0.257	3.8×10^2 to 1.6×10^4	-
				0.371x 0.158	0.222	3.5×10^2 to 1.3×10^4	
Allen & Grundberg	1937	(1)	Water	1.004x 0.256	0.410	5.52×10^2 to 3.116×10^3	440
Washington & Marks	1937	(46)	Air	0.563x 5.0	1.010	3.57×10^3 to 3.88×10^4	48
Nikuradse	1939	(35)	Water	1.102x 0.315	0.489	8.8×10^2 to 10^5	-

Table 2.1 (cont'd)

Investigator(s)	Year	Ref.	Test Fluid	Dimensions inches	D_e inches	Re	Channel Length / D_e
Huebscher	1947	(20)	Air	7.88x 7.88	7.88	$5x10^4$ to $5x10^5$	84
				4.46x 36	8.0	$1.8x10^4$ to $4.9x10^5$	84
Eckert & Irvine	1957	(15)	Air	1.5x0.5	0.75	$1.4x10^2$ to $1.4x10^4$	184
Hartnett et al.	1962	(17)	Air	0.5635x 0.5635	0.5635	10^3 to $1.2x10^5$	256
				0.3115x 1.551	0.518	"	278
				0.3115x 3.112	0.566	"	254
Leu-theusser	1963	(29)	Air	3.0x3.0	3.0	10^4 to 10^5	288
				3.0x9.0	4.5	"	192
Maurer & LeTourneau	1964	(32)	Water	0.087x 1.0	1.6	$4x10^3$ to $5x10^5$	169
Brundrett & Burroughs	1967	(5)	Air	3.767x 3.767	3.767	$3x10^4$ to $8x10^4$	96
Jones	1975	(23)	Water	26:1	-	$7x10^3$ to 10^5	260
				31:1	-		500
				12.8:1	-		-

Table 2.1 (cont'd)

2. Parallel Plates

Investigator(s)	Year	Ref.	Test Fluid	Dimensions inches	D_e inches	Re	Channel Length / D_e
Davies & White	1928	(8)	Water	37:1 to 169:1	0.522 to 0.0117	$1.25\times10^2 - 9.2\times10^3$	38 to 170
Washington & Marks	1937	(46)	Air	0.25x5.0	0.476	$1.04\times10^3 - 2.8\times10^4$	101
				0.125x 5.0	0.244	$4.12\times10^2 - 1.7\times10^4$	197
Whan & Rothfus	1959	(47)	Water	0.7x 14.0	1.333	$8\times10^2 - 4\times10^4$	180
Barrow	1962	(3)	Air	9.3x 0.32	0.62	$10^4 - 2.5\times10^4$	156
Patel & Head	1969	(37)	Air	0.25x 12.0	0.49	$2\times10^2 - 8\times10^3$	147

Year: 1923 - 1962

Aspect Ratio: 1:1 - 169:1

Reynolds Number: $1.25 \times 10^2 - 5 \times 10^5$

It was concluded by Hartnett et al. that, with the hydraulic equivalent diameter, the Blasius circular tube correlation accurately predicts the friction factor for flow through rectangular channels at any aspect ratio for Reynolds numbers between 6×10^3 and 5×10^5. Maurer and LeTourneau[32] did experiments on narrow rectangular channels at Reynolds numbers from 4×10^3 to 5×10^5. Their results were in agreement with the works of Hartnett et al., i.e the concept of hydraulic equivalent diameter can be applied. Barrow[3] suggested the using of hydraulic equivalent diameter with the Blasius correlation for parallel plates.

However, the presence of secondary currents in turbulent non-circular conduit flow[4,19] made it doubtful that a single unique resistance law, such as the Blasius equation, could describe friction phenomena in rectangular channels by simply using the hydraulic equivalent diameter as the characteristic length parameter. In Leutheusser's work[29], it was found that, by using hydraulic equivalent diameter, the function relating friction factor with Reynolds number was not unique. He considered the conventional use of hydraulic equivalent diameter not only implied absence of shape effects on the resistance function, but also presumed the existence of wall and Reynolds similarity in the general case of channel flow. His experimental results showed that there was a trend toward a more rapid decrease of friction factor with increasing Reynolds number as compared to the Karman-Nikuradse equation. For Reynolds numbers greater than approximately 5×10^4, all the data lay consistently below the Karman-Nikuradse line by about 20%. Brundrett and Burroughs[5] also found that, with the explanation of stagnation in the duct corners, the friction factor was slightly lower for square ducts as compared with circular tubes for Reynolds numbers between 3×10^4 and 8×10^4. Although these authors have disproved the use of hydraulic equivalent diameter with circular tube correlation, they did not suggest new equations. As for turbulent flow between parallel plates, Patel and Head[37] pointed out the inaccuracy in measurements of the early researchers and proposed a new correlation

$$c_f = 0.0376 \; Re^{1/6} \qquad (2.5)$$

for Re greater than 2.8×10^3 and hydraulic equivalent diameter as the length parameter. However, the Reynolds numbers of their experiments were never greater than 10^4.

Jones[23] obtained friction factor data for channels having aspect ratios between unity and 39:1 in the literature and, in conjunction with his new experimental data, examined for deviations from the smooth circular tube line. It was found that at constant Reynolds number (based on hydraulic equivalent diameter) the friction factor increases monotonically with increasing aspect ratio. It was thus concluded by Jones that the hydraulic equivalent diameter is not the proper length dimension to use in Reynolds number to insure similarity between the circular and rectangular channels. Instead, it was determined that if a modified Reynolds number, Re*, was obtained so that geometric similarity was provided in laminar flow by the relation $c_f = 16/Re^*$ for all geometries, that this Reynolds number also provided good similarity in fully developed turbulent flow within an approximately 5% scatter band about the smooth tube line. By using this "laminar equivalent Reynolds number", Re*, Jones stated that circular tube methods may be readily applied to rectangular ducts thereby eliminating large errors in estimation of friction factor.

Mathematically,

$$Re^* = \phi^*(\frac{w}{s})\ Re \qquad (2.6)$$

where w is the channel width, s is the channel spacing, Re is as before and the geometry function $\phi^*(\frac{w}{s})$ is given by

$$\phi^*(\frac{w}{s}) = \frac{2}{3}\left(1 + \frac{s}{w}\right)^2 \left\{1 - \frac{192}{\pi^5}\frac{s}{w} \sum_{n=0}^{\infty} \frac{1}{(2n+1)^5} \tanh \frac{(2n+1)\pi w}{2s} \right\} \quad (2.7)$$

The function $\phi^*(\frac{w}{s})$ is shown graphically in Figure 2.2. An approximate relationship which will give $\phi^*(\frac{w}{s})$ within about 2% is

$$\phi^*(\frac{w}{s}) \approx \frac{2}{3} + \frac{11}{24}\frac{s}{w}(2 - \frac{s}{w}) \quad (2.8)$$

In this present work, the hydraulic equivalent diameter concept was adopted because it is widely used and easy to apply. Furthermore, correction of friction factor for rectangular channels and parallel plates as proposed by the Jones method[23] was also considered.

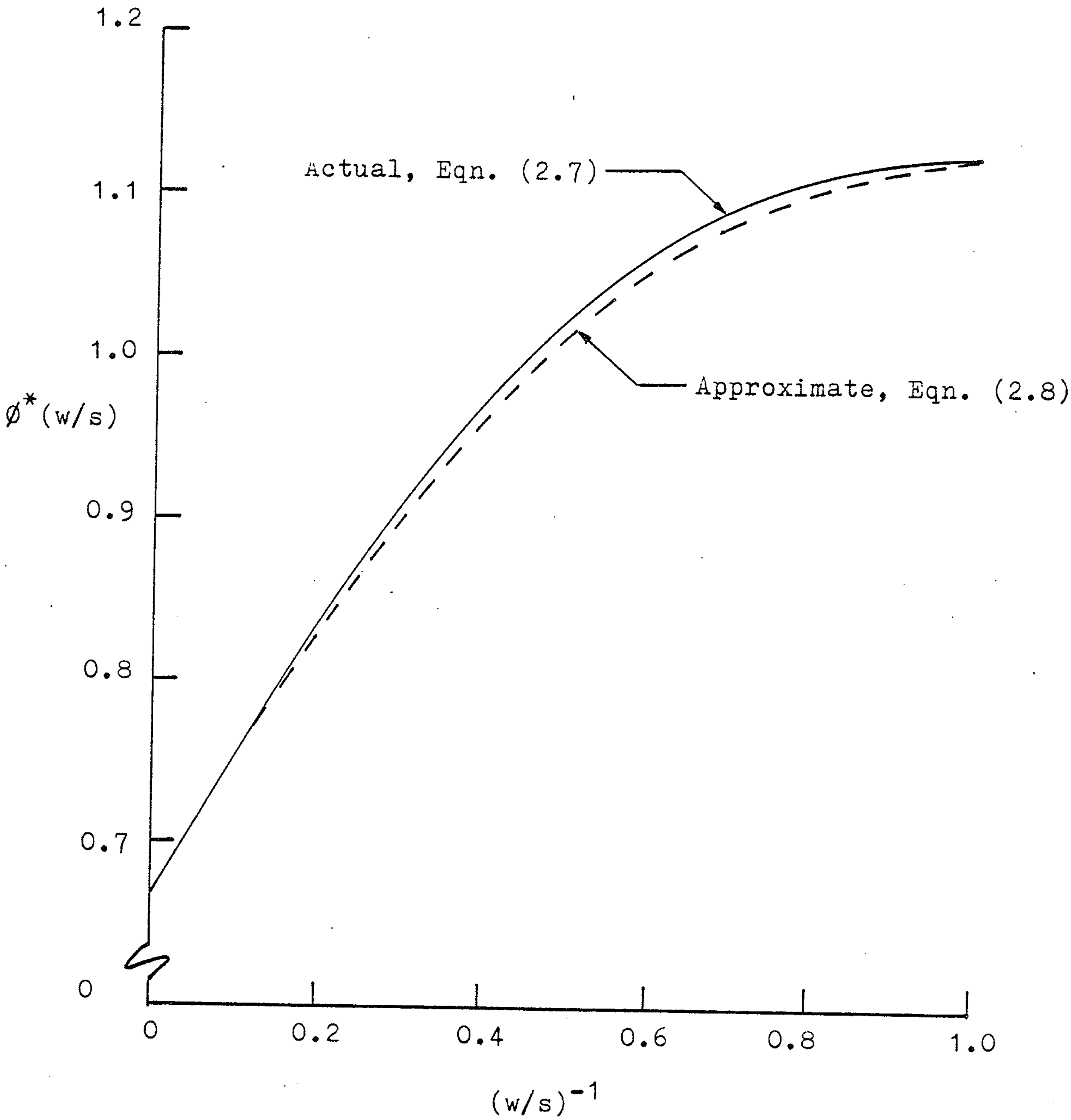

Fig. 2.2 Geometry Function for Calculation of Laminar Equivalent Reynolds Number

CHAPTER 3

CIRCULAR TUBE HEAT-TRANSFER THEORY

3.1 General

There is a voluminous literature on the subject of heating or cooling of fluids inside circular tubes, and many design equations of varying degrees of generality are available. In the calculation of heattransfer in turbulent non-circular duct flow, the practice until recently has been to use circular-tube correlations on the assumption that they can be applied by using the concept of hydraulic equivalent diameter. This practice has been adopted for both symmetrical and asymmetrical heat-transfer conditions. In this chapter, circular-tube heat-transfer correlations are investigated.

3.2 Correlations Obtained from Dimensional Analysis

For turbulent flow in tubes, dimensional analysis gives

$$\frac{hD}{k} \doteq f\left(\frac{DG}{\mu}, \frac{c_p\mu}{k}\right) \qquad (3.1)$$

where the function is to be determined experimentally. For moderate temperature differences between fluid and tube surface, the Dittus-Boelter[13] and Sieder-Tate[42] equations are commonly used. The Dittus-Boelter equation evaluates all physical properties at the bulk temperature of the fluid

$$\frac{hD}{k_b} = 0.024 \left(\frac{DG}{\mu_b}\right)^{0.8} \left(\frac{c_p\mu}{k}\right)_b^{0.4} \tag{3.2}$$

or

$$Nu = 0.024Re^{0.8}Pr^{0.4} \tag{3.3}$$

The Sieder-Tate equation evaluates all physical properties at the bulk temperature, except μ_w in a viscosity-ratio term

$$\frac{hD}{k_b} = 0.027 \left(\frac{DG}{\mu_b}\right)^{0.8} \left(\frac{c_p\mu}{k}\right)_b^{0.33} \left(\frac{\mu_b}{\mu_w}\right)^{0.14} \tag{3.4}$$

or

$$Nu = 0.027Re^{0.8}Pr^{0.33}\left(\frac{\mu_b}{\mu_w}\right)^{0.14} \tag{3.5}$$

3.3 The Petukhov and Popov Equation

Petukhov and Popov[39] used the integral formulation developed by Lyon[27] and numerically solved for the Stanton number using velocity distribution and eddy diffusivity equations proposed by Reichardt[40]. They then developed a Stanton number equation which reasonably approximated the values yielded by their numerical solution. Their recommended equation is

$$St = \frac{\frac{c_f}{2}}{1.07 + 12.7\sqrt{\frac{c_f}{2}}(Pr^{2/3} - 1)} \qquad (3.6)$$

where the friction factor, c_f, may be calculated from the Karman-Nikuradse equation

$$\frac{1}{\sqrt{c_f}} = 4.0 \log_{10} (Re\sqrt{c_f} - 0.4) \qquad (2.3)$$

and St is the Stanton number defined as h/Gc_p.

3.4 The Spalding-Jayatilleke P-Function

Spalding and Jayatilleke[22,43] referred to the extra resistance to heat transfer in the viscous sublayer (see Figure 3.1) of a turbulent flow as $Pr_{turb}P$. A relationship was developed between P and essentially the laminar Prandtl number of the fluid. Then, from the P-function, the Stanton number of the fluid can be calculated. It is beneficial to review their work in detail here because in the next two chapters, the present author extends the P-function to symmetrical and asymmetrical heating conditions in parallel plates.

Based on the observation that pipe velocity profiles resemble that of a Couette flow remarkably, Spalding and Jayatilleke[22,43] applied the Couette flow analysis to pipe flows.

From dimensional analysis, the velocity profile is

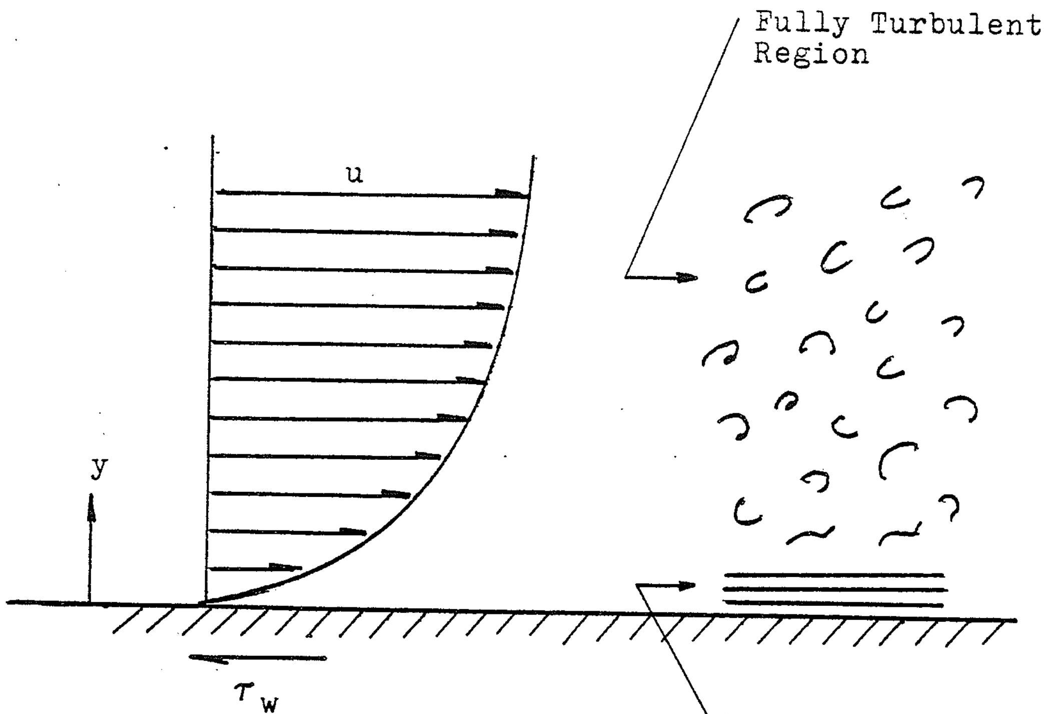

Fig. 3.1 Velocity Profile

expressible by a unique relationship of the form

$$u^+ = u^+(y^+) \tag{3.7}$$

where
$$u^+ \equiv \frac{u}{\sqrt{\frac{\tau_w}{\rho}}} \tag{3.8}$$

and
$$y^+ \equiv \frac{y\sqrt{\tau_w \rho}}{\mu} \tag{3.9}$$

A total viscosity, μ_{tot}, is defined by

$$\mu_{tot} \equiv \frac{\tau}{\frac{du}{dy}} \tag{3.10}$$

and a total thermal exchange coefficient, Γ_{tot}, of specific enthalpy, ϕ, by

$$\Gamma_{tot} \equiv \frac{\dot{q}''_w}{\frac{d\phi}{dy}} \tag{3.11}$$

where $\dot{q}''_w$ is the heat flux from wall into the fluid stream.

So,
$$\Gamma_{tot} = \frac{k_{tot}}{c_p} \tag{3.12}$$

where k_{tot} is the total thermal conductivity and c_p the specific heat at constant pressure of the fluid.

The total Prandtl number, Pr_{tot}, is defined by

$$Pr_{tot} \equiv \frac{\mu_{tot}}{\Gamma_{tot}} \tag{3.13}$$

or $$Pr_{tot} = \frac{\mu_{tot} c_p}{k_{tot}} \tag{3.14}$$

Dimensional analysis leads to the result

$$Pr_{tot} = Pr_{tot}(y^+, Pr) \tag{3.15}$$

where Pr is the laminar Prandtl number of the fluid. The dimensionless total viscosity, $\frac{\mu_{tot}}{\mu}$, is however a function of y^+ alone i.e.

$$\frac{\mu_{tot}}{\mu} = \varepsilon_{tot}(y^+) \tag{3.16}$$

By virtue of the constancy of shear stress and heat flux in Couette flow, it may be shown that

$$\frac{\mu_{tot}}{\mu} = \varepsilon_{tot} = \frac{dy^+}{du^+} \tag{3.17}$$

and $$\frac{\varepsilon_{tot}}{Pr_{tot}} = \frac{dy^+}{d\phi^+} \tag{3.18}$$

where $$\phi^+ \equiv \frac{(\phi - \phi_w)\sqrt{\tau_w \rho}}{\dot{q}''_w} \tag{3.19}$$

By elimination of ε_{tot} from equations (3.17) and (3.18), there is obtained the important result

$$\phi^+ = \int_0^{u^+} Pr_{tot} du^+ \tag{3.20}$$