

Expectations Theory Of The Term Structure of Interest Rates
And
The Demand For Government of Canada Guaranteed
Marketable Bonds By Five Investor Categories

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in
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By
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THE UNIVERSITY OF MANITOBA
FACULTY OF GRADUATE STUDIES

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EXPECTATIONS THEORY OF THE TERM STRUCTURE OF INTEREST RATES
AND
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ABSTRACT

There have been an enormous number of studies on the subject of The Term Structure of Interest Rates. Most of the studies are in the form of direct estimations, commonly known as term structure approach. As a result, many of the studies are merely trying to marshal evidence to test , particular term structure theories. To change the direction of research and to eliminate any foreseeable biases, this thesis attempts to analyze the theory of term structure indirectly. That is to study an issue in which term structure plays an important role. Evidence from the study is then interpreted in view of term structure theories.

The demand for financial securities is the integral part of the term structure. The demand for financial securities reflects an investor's expectations of the future yields. When one yield is expected to be higher than the other, then demand for that security increases. Conceivably an investor may release some of the latter (if he holds any) so that more funds are available to invest in the former. When an investor(s) begins to readjust the portfolio then the element of arbitrage occurs, which primarily is responsible for expectations theory results coming about.

Evidence about portfolio behavior is difficult to obtain from direct estimation of interest rate structures (the reduced-form approach or the term structure approach). Therefore other approaches should be investigated. The structure model developed by B.M. Friedman seems to provide a relevant model for such analysis. Expectations theory and the structural

model ("the optimal marginal adjustment model for portfolio behavior") are the framework for the analysis. Thus this thesis assumes that investors form their expectations of yields as specified by expectations theory. The structural model enables one to estimate investors' portfolio behavior. From the coefficients of the structural model, one can infer about the expectations theory of the term structure of interest rates. At the same time, one can analyze the portfolio behavior of the different investor categories.

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CHAPTER ONE
INTRODUCTION

In our market oriented economy, price is the engine that drives the economy to equilibrium. All goods (tangible or intangible) and services that are produced in the economy must bear a price. For tangible goods, their prices are easily comprehensible. Intangible goods, on the other hand, have prices which are quite complex. Intangible goods such as financial securities - bonds, stocks, promissory notes and deposits have two sets of prices. They are the face value of the instruments and the carrying costs (interests and dividends). The face values reflect the nominal wealth (or the principal) of the instruments whereas the carrying costs reflect the nominal flow of opportunity costs from the lenders' point of view. These opportunity costs (interest) vary between instruments.

The concern is not about the differences in interest rates between various financial instruments. Instead the concern is more on the differences in interest rates between financial instruments of equal grade that differ only in their term to maturity. For example, in the bonds market, "one of the most intriguing differences among market interest rates concerns the relationship among the yields of high-grade securities that differ only in their term to maturity, that is, in the length of time until the principal amount of the loan becomes due or payable. This relationship is called the "term structure of interest rates", or more popularly, the shape of the yield curve".¹

There are five theories that attempt to explain the term structure of interest rates. There are many empirical studies on the subject, but there is no consensus as to which theory or theories best explains the term structure. Since much of the literature provides inconclusive evidence, perhaps it is time to shift the approach in analyzing the term structure of interest rates. One way is to use an indirect approach to test the expectations theory. As mentioned, the issue of term structure has been well researched by direct estimation. Thus, to further discuss and analyze the issue in that way will only duplicate most of the existing work. Instead one should make full use of the knowledge of the different theories about term structure to study other related issues. In return, evidence from the empirical results from the study can be used to understand the term structure. That is, one can infer from the empirical results the practicality of a particular term structure theory, or for that matter, the validity of the theory.

The indirect approach entails analyzing investors' portfolio behaviour. In this approach, one has to assume that expectation is an important component of term structure. It is also an important factor in the decision making of investors.

The important question is, "How do investors formulate their expectations?" Granted there are many ways by which expectations are formulated. In this thesis, I assume that expectation is both extrapolative and regressive (a concept employed by Modigliani and Sutch [34]). The thesis will first focus on the expectations theory, mostly as a tool representing investors' perceptions about the various yields. Secondly,

it uses the structural model (developed by B.M. Friedman [12]) to analyze the portfolio behaviour of five Canadian investor categories. Details about the different investor categories are discussed in Chapter Four. The objective of this analysis is to observe the portfolio behaviour of these investor categories with respect to expected yields, whether they (the expected yields) be the expected own yield or the expected alternative yields. The implication of the analysis is whether the five investor categories arbitrage, given such information as expected yields (expected own or alternative yields) and increase in wealth.

The specific area the analysis focuses on; (1) the demand for government (federal) guaranteed marketable securities, (2) the sensitivities of demand toward its own yields (elasticity of demand), and (3) the sensitivities of demand toward alternative yields (cross-elasticity of demand). When investors are insensitive to own yield, the short-run elasticity of demand for that security will be low (this is inferred by insignificant coefficient of own yield). By the same token, when investors are sensitive to own yield, the elasticity of demand will be high. For risk-neutral investors (or arbitrageurs), the process of portfolio adjustment is usually characterized by high positive elasticity of demand with respect to expected own yield, or high negative elasticity with respect to expected alternative yields.

The characteristic of high elasticity of demand for risk-neutral investors is that these investors have no preference for particular financial securities. So, whenever a security has a relatively higher expected yield than the alternative securities, they tend to invest in that security regardless of risk. Thus, when expected own yield is

relatively high, the quantity demanded is expected to increase. On the other hand, when investors are risk averse, they tend not to rush their investment into any security which has relatively higher expected yield. They (risk-aversers) in fact treat risk as a cost. This "risk-cost" has a negative impact on the demand of the security. Therefore, the same security may have different expected yield according to risk-neutral and risk-averse investors. Thus their elasticities of demand for the security are different if one expected yield is used. In any event, the own elasticity, the cross elasticity of demand, and O_{ij} and O_{iK} (coefficients of lagged own-stock adjustment and lagged alternative stock adjustment) are analyzed together to consider the source of arbitraging pressure from which the expectations theory results come about.

Any investor category or categories which produce favourable empirical results are considered arbitrageurs. In the empirical analyses followed in Chapter Four, one should not expect every investor category to be the source of arbitraging pressure - for the simple reason that the investment policy and institutional constraints vary between the different investor categories.

To put the discussion in perspective, the theory of the term structure of interest rates traces back to the work of Irving Fisher [11]. His proposition that expectations of future interest rates influence the term structure becomes the foundation of the expectations hypothesis. In the 1930's J.R. Hicks [20] hypothesized the liquidity premium theory of the term structure. He based his theory on the Keynesian notion of normal backwardation in future money markets. He

argued that in order to compensate the investor for assuming the uncertainty of price fluctuations, forward rates would normally exceed the expected interest rates by a risk premium. Since then, there have been extensive studies of the term structure. These studies have resulted in the development and refinement of many variants to interpret and many techniques to analyze the term structure. Most of these techniques and theories are discussed in Chapter Two.

There are five major theories explaining the term structure of interest rates. They are the expectations, liquidity-premium, institutional (hedging-pressure), preferred-habitat and efficient market theories. In most cases, these theories are tested by using a term structure equation (or approach). The term structure approach "assumes that not only the short-term interest rate but also the determinants of long/short spread are exogenous with respect to the actions of participants in the market for long-term debt securities. It further assumes that how participants in the market for long-term securities either individually, or in the aggregate, adjust their actions in that market in response to any or all of the determinants of portfolio behaviour does not matter for the final outcome for the long-term interest rates."² There are several criticisms of the term structure approach. In particular, the second assumption implies that quantities of long-term securities bought or sold in aggregate do not influence the final outcome of the long-term interest rate. This assumption is not compatible with the term structure. Thus when the approach is applied to test the term structure, the full effect of the market is not considered.

Among the researchers who fell into the "reduced-form trap" are Meiselman (1962) Malkiel (1966) and Modigliani and Sutch (1966) - on expectations theory; Hicks (1939) and Kessel (1965) - on liquidity premium theory. Later, a few researchers have suggested relaxing the second assumption by incorporating an exogenous supply of long-term securities as a further determinant of long/short spread. They did so within the framework of the familiar unrestricted reduced-form equation of the term structure approach. These researchers are Feldstein and Eckstein (1970), Fair and Malkiel (1971), and Dobson (1973).

There exists another category of researchers who use structural models to analyze the term structure of interest rates. The structural model drops the second of the two assumptions used in the reduced-form equation. Its (structural model) specification resembles the portfolio behaviour of bond market participants. The structural model imposes a market-clearing constraint, equating the sum of the demands to the sum of supply for long-term debt securities, thus enabling it (the structural model) to determine the equilibrium long-term interest rate (i.e., the own yield). This approach has been used by Duesenberry (1963), Hendershott and Lemmon (1971), B. Friedman (1967) and (1979), and by Christofides, Helliwell and Lester (1976).³

The "term structure" approach to term structure of interest rates is extensively used in both the United States and in Canada. However, it appears to me that a structural model developed by Benjamin M. Friedman [12] has not been adopted to analyze the Canadian case. Although a structural model per se has been used in Canada, let me point out that the structural model (RDX2) used, by Christofides, Helliwell and Lester

[7] is an extremely complicated model. RDX2 may be very capable of illustrating portfolio behaviour, provided one can understand it. In any event, they did not use RDX2 to analyze investors' portfolio behaviour. Instead, they used RDX2 to analyze the impact of Conversion Loan on the term structure of interest rates. For the purpose of this thesis the structural model developed by Friedman, B.M. is sufficient to illustrate investors' portfolio behaviour.

Chapter Two discusses the existing theories and methods related to the study of term structure. Five major theories are discussed; namely the expectations, liquidity-premium, the institutional (or hedging-pressure), preferred-habitat and efficient market theories. Following the discussion of each theory, summary of empirical evidence is presented. To a certain extent, the methods of empirical research are discussed. Last but not least, a general comment is given at the end of the chapter and to reiterate the objective of the thesis.

Chapter Three replicates the structural model as developed by Benjamin M. Friedman. Section 3:1 discusses the rationale for the structural model using different models. The section finishes with the amalgamation of the long-run desired portfolio allocation model with the short-run stock adjustment model for a given investor or a group of investors. The outcome of this is the structural model called the optimal marginal adjustment model of portfolio behaviour. Section 3:2 focuses on how the optimal marginal adjustment model of portfolio behaviour is applied in the Canadian case.

Chapter Four focuses on the estimation procedures, data and the empirical results. Section 4:1 illustrates the estimation procedures -

including the expansion of model (6) and how equation (6) can be made estimable and how the expected yields are estimated. Section 4:2 focuses on the different investment categories and the data used in the empirical analyses. This includes the discussion on the source of data and why certain data are used. Finally, section 4:3 illustrates the empirical results as estimated by the structural model equation (6) with constraints. The estimated results are tested and interpreted.

Chapter Five begins with a general comment pertaining to the analyses and the portfolio management of the institutional investors as a whole. Then the results obtained with Canadian data are compared with those from the United States. Interpretations of these comparisons follow. Additional estimations are illustrated, generated without constraints. Following this are the interpretations of these estimations, and finally a conclusion and summary.

CHAPTER TWO
LITERATURE REVIEW

In the past economists have been studying the term structure of interest rates by explaining the shape of the yield curve from one of a number of theoretical constructs. The yield curve becomes the most widely used graphic device in examining the relationship between yield and term to maturity. The existence of the yield curve stems from the basic differences in the market rates of interest for various types of debt securities (of similar grade) which differ only in their term to maturity. There are a variety of reasons for the differences in the market rates of interest.⁴ The major reasons are (1) the credit risk of the instruments: - the risk of default of the promised interest and principal payments; (2) the differences in the provisions of various sorts of bonds: - whether they are tax-exempt, whether they can be converted into common stocks, whether they can be redeemed at the option of the company, and so forth; (3) perhaps the most celebrated reason, in explaining the differences in market rates among the various types, is the relationship between the yields of various types of debt securities and their term to maturity. That is the length of time before the principal amount of the debt becomes due and payable. This relationship is called the term structure of interest rates.

The term structure is graphically shown by plotting the yields and the terms to maturity for equivalent grade of securities at a point in time. Generally the yield curve approximates one of four shapes as illustrated in Figures I to IV. The typical yield curve is the ascending

curve (Figure I). An ascending yield curve is formed when interest rates are lowest on short-term securities and they rise at a diminishing rate until they level out in the longest maturities. Between 1930 through to the middle 1960's, bond yields in the United States have followed this pattern. In Canada, ascending yield curves were experienced in 1930 to early 1973. This type of yield curve is usually associated with a period when relatively low interest rates prevail for both maturity groups.

When yields are highest on the short-term securities and they decrease at a diminishing rate until they level out in the longest maturities, a descending yield curve is formed (Figure II). The descending yield curve is common during the period when relatively high interest rates prevail for both long and short-term securities and also in periods of restricted credit conditions where short rates tend to rise more than long rates. These types of yield curves have been experienced with Canadian Government securities since April 1979. In the United States, descending yield curves were most common for high-grade corporate bonds between 1906 to 1929 and between late 1960's to early 1970's.

Occasionally, yield curve can be flat (Figure III). A flat curve occurs when yields on short and long-term securities are approximately equal. The average bond yields in Canada from 1966 to early 1970, and late 1975 to 1978 can be classified in this category. Although very few cases had been documented, flat curves appear most frequently when short and long-term rates have been near or somewhat above the middle of the range between the historical highs and lows.

Sometimes securities may have yields that rise in the early maturities, reach a peak, and then decline until they finally level out in

later maturities. When this occurs a humped curve (Figure IV) is formed. Humped curves are most frequent during periods where interest rates are relatively high and stable. During such periods, short-term yields tend to fluctuate greater than long-term yields. Canada has experienced the humped curve briefly in late 1978 to early 1979.

Even though the yield curves are depicted in one of the four shapes, the ascending yield curve is most frequent, as judged by the historical patterns. The others are less frequent. Recently, where short-term interest rates are high, descending curves become more prominent in both the United States and Canada. Figure V exhibits the different types of yield curves plotted with Canadian data. When the yields of securities are plotted according to their term to maturities at any point in time, the curve produced gives investors an additional dimension in their decision-making process.

THEORIES EXPLAINING THE YIELD CURVE (OR THE TERM STRUCTURE)

As mentioned earlier, the study of the term structure of interest rates by economists and econometricians basically tries to explain the shape of the yield curve. The most frequently asked questions are (a) what determines the shape of the yield curve and (b) why are short-term rates of interest sometimes higher and other times lower than long-term yields. There are five competing theories attempting to explain these phenomena. The expectations, liquidity-premium and the institutional or hedging-pressure theories are well researched. Preferred-habitat and efficient market theories have emerged from recent studies.

Yield to
maturity
(per cent)

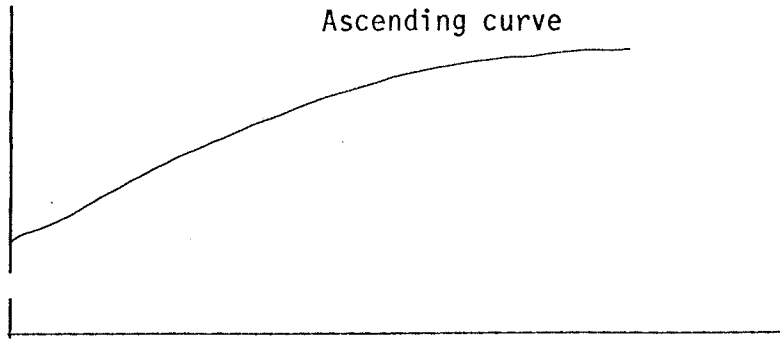


FIGURE I

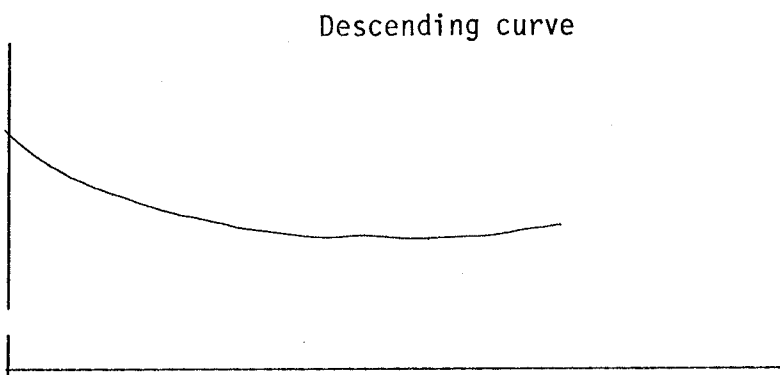


FIGURE II

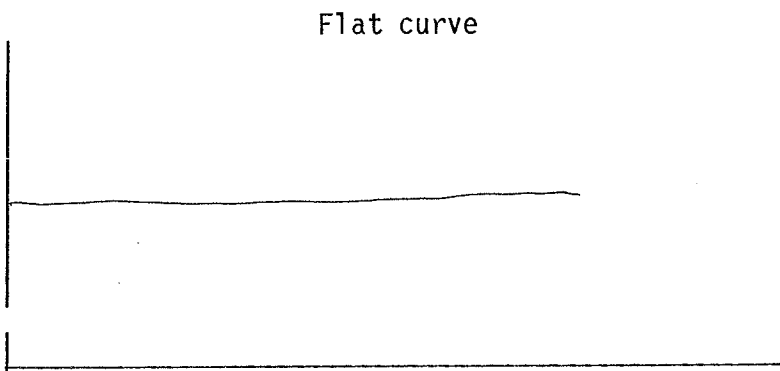


FIGURE III

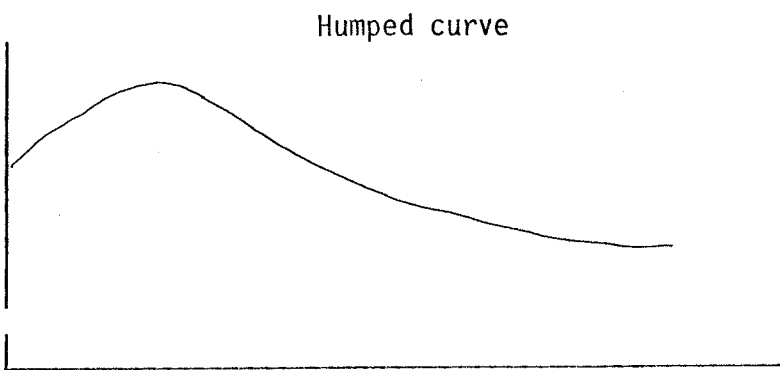
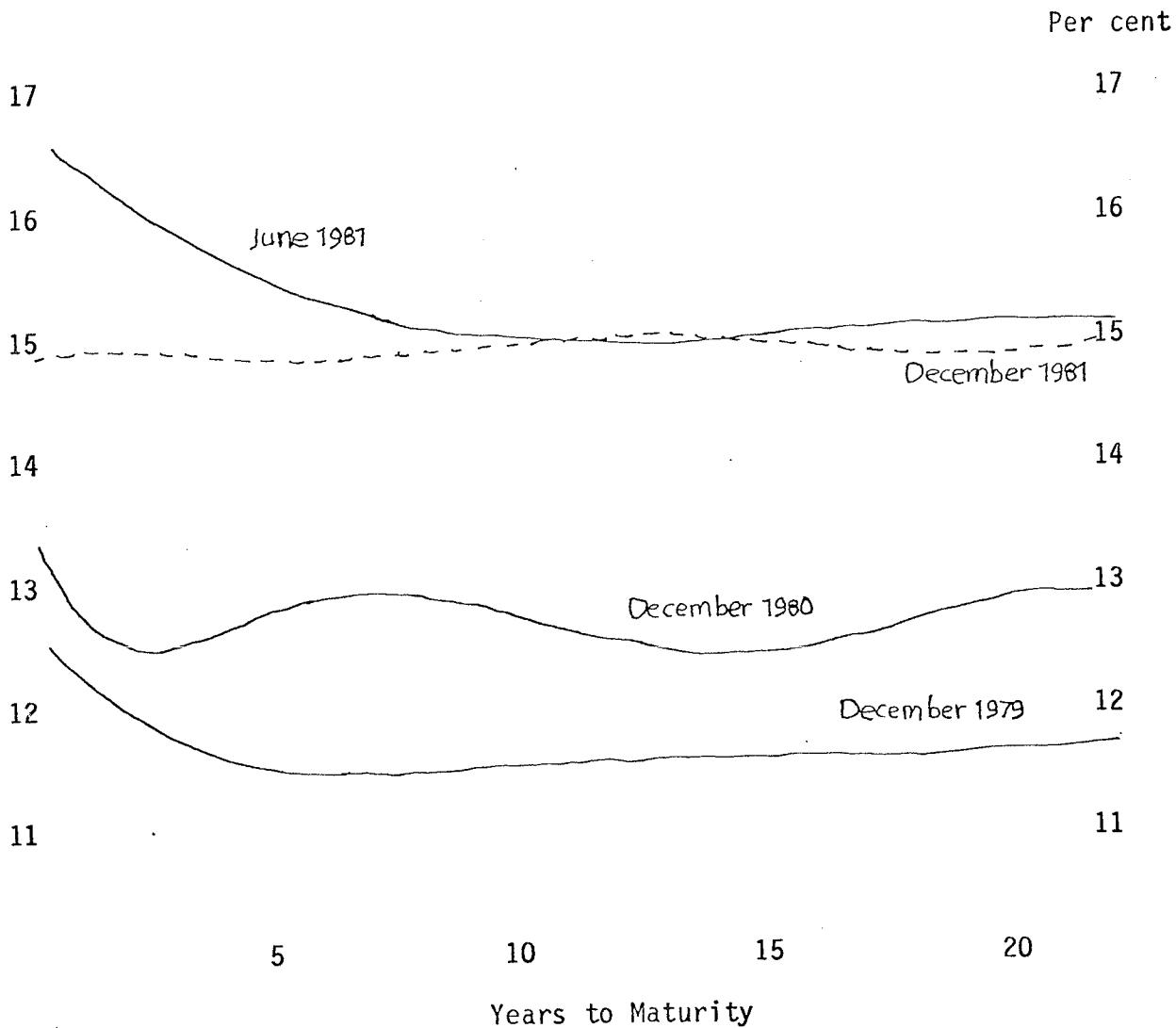


FIGURE IV

Years to maturity

FIGURE V

YIELD CURVES, GOVERNMENT OF CANADA BONDS¹

¹ The yield curves were drawn using the month-end observation for various maturities of Government of Canada bonds.

Source: Economic Review, Department of Finance.

Although the later theories appear to be an extension of the former, they do explain to some degree the shape of yield curves.

THE EXPECTATION THEORY

This theory is sometimes referred to as the pure or unbiased expectations theory. The theory traces back to the work of Irving Fisher [11]. It was developed by F.A. Lutz and polished by J.R. Hicks [20]. The foundation of the theory implies a formal relationship between long and short-term rates of interest. The relationship is such that long rate is an average of current and expected forward short-term rates. In mathematical form, at equilibrium, the theory is explained using the expected forward short-term rates of interest as expressed by equation(1).

$$(1+{}_tR_N)^N = (1+{}_tR_1) (1+{}_{t+1}R_1) \dots (1+{}_{t+N-1}R_1) \quad (1)$$

Where: ${}_tR_N$ = actual rate of interest at time t, on an N period bond;

${}_tR_1$ = actual rate of interest on a 1 period bond;

${}_{t+1}R_1$

${}_{t+2}R_1$

- = are expected market rate of interest at t+1,
- t+2, t+N-1.

${}_{t+N-1}R_1$

According to equation(1), at equilibrium, investors will have no incentive to move from one security to another. The determination and the importance of an equilibrium rate structure can be made clear from the

following examples. First of all, let us assume that there are only two types of securities (one-year bonds and two-year bonds). The expected returns on these bonds differ. Suppose that the coupon rate for one and two-year bonds are 7 and 8 per cent respectively and the future short-rate is expected to be 8 per cent next year. Under these circumstances, investors would tend to buy two-year bonds and sell their one-year bonds.⁵ This process will continue until any expected returns differential over the two investment periods is eliminated.⁶ That is, the expected return on two-year bonds and the expected return on one-year bonds are equal over the same investment period(s); or the two alternatives must offer the same overall yield. Thus one can infer that the long-rate must be an average of present and future short-term rates of interest.

The determination of the equilibrium rates can be shown as follows:- If an investor invests one dollar in one-year bonds and then reinvests the proceeds $(1+tR_1)$ at maturity in another one-year bond next year, his/her total capital will increase to $(1+tR_1) (1+t+1R_1)$ at the end of the two year period. On the other hand, if he/she invests the dollar in a two-year bond (and leaves all interest to be reinvested until final maturity date), his/her total proceeds at maturity will be $(1+tR_2)^2$. Therefore, at equilibrium rates, the total proceeds between the two investment schemes should be identical, i.e.,

$$(1+tR_2)^2 = (1+tR_1) (1+t+1R_1)$$

From equation(2), two-year rates can be expressed as a geometric average of current (1 year) short-term rate and expected future short-rate (for next year),

$$(1+{}_tR_2) = (1+{}_tR_1) (1+{}_{t+1}R_1)^{1/2} \quad (3)$$

Similarly, the long-term rate for a N-period bond can be expressed as a geometric average of a whole series of expected future short-rates.

$$(1+{}_tR_N) = (1+{}_tR_1) (1+{}_{t+1}R_1) (1+{}_{t+2}R_1) \dots (1+{}_{t+N-1}R_1)^{1/N} \quad (4)$$

Thus advocates of expectations theory submit that based on the expression of equation(4), term structure can be explained effectively. From which [equation(4)], they draw the following scenarios. They insist that a descending yield curve is formed when short-rates are expected to be lower in the future. This means long-rate will be below the current short-rate. Conversely, if future short-rates are expected to be higher, then long-rate will exceed the current short-rate; thus an ascending yield curve is formed. The proponents profess that the theory can even encompass the humped yield curve. They argue that when future short-rate rises first and then fall later to much lower levels, a humped yield curve is produced.

Before going into the testing of expectations theory, one has to bear in mind the assumptions underlying the theory. Although some of the assumptions are unrealistic, they do simplify the theory. The basic assumptions are (1) there are no transaction costs and all investors make identical forecasts of future interest rates; (2) investors are

risk-neutral and profit maximizers. Each investor will choose that security or combination of securities which will maximize his/her return for the period of the investment.

EMPIRICAL TESTS OF THE EXPECTATIONS THEORY

Early empirical analysis of the expectation theory was confined to the observations of the relationship between the future short-term rates implied by the yield curve at a given time and the short-term rates that were actually observed. In short, analysts were looking for evidence of accurate forecasts to support the theory. In most cases future short-term rates were regressed on observed actual short-term rates (as expressed by equation[5]). Such empirical analysis assumes perfect certainty.

$${}_{t+1}F_{1,t} = {}_{t+1}R_1 + u_{t+1}$$

(5)

Where:

${}_{t+1}R_1$ = the independent variable which is the observed actual short-term rates

u_{t+1} = the random disturbance term (normal statistical assumptions)

${}_{t+1}F_{1,t}$ = The dependent variable which is the forward rate calculated from market yields during time t.⁷

Early hypothesis tests using methods similar to equation (6) did not produce evidence to support the expectations theory. For example, B.W. Hickman [19], tested the hypothesis by comparing the actual short-rates with those implied by the term structure during the period of 1935-42.

He concluded that the random error (u_{t+1}) swamped any supposed correspondence. Similar tests were performed by Macaulay [1938], Walker and Culbertson [1957]. Their findings rejected the expectations theory.

ERROR-LEARNING MODEL

The theory finally found support from D. Meiselman's [33] error-learning model. First he argued that the previous tests could not be used to reject the expectations theory because the anticipated and the actual-yields would never be equal except in a world of perfect certainty. Therefore, the test of expectations theory should not look for evidence of accurate forecasting. Instead, one should test a systematic hypothesis concerning the formation of expectations.

Meiselman argued that investors should revise their forecasts of future interest rates on the basis of new information. Suppose investors expect next year's one-year rate (${}_{t+1}R_1$) and the subsequent year's one-year rate (${}_{t+2}R_1$) to be 8 percent and 9 per cent respectively as of time t . If after one year the actual one-year rate turns out to be 6 percent (2 per cent below the anticipated 8 per cent), then according to the error-learning approach, investors respond to their error by lowering their forecasts of still unrealized forward rates (${}_{t+1}R_1, \dots, {}_{t+N-1}R_1$). In general, the whole series of expected future shortterm rates (${}_{t+2}R_{1,t+1}, \dots, {}_{t+N-1}R_{1,t+1}$) are adjusted downward in response to the error made in forecasting the present one-year rate. Similarly, future expected short-term rates are adjusted upwards if actual rates turn out to be higher than had been anticipated. Thus, the whole structure of rates could rise or decline depending on the direction of forecasting errors.

The fundamental requirement of the error-learning model, when applied to forward short-term rates in the term structure, is that the implicit short-term rates change on the basis of errors made in forecasting the current short-term rate. The hypothesis may be written as follows:

$${}_{t+N}R_{1,t} - {}_{t+N}R_{1,t-1} = F({}_tR_{1,t} - {}_tR_{1,t-1}), N=1, \dots, n, \dots \quad (6)$$

or

$${}_{t+N}R_{1,t} \equiv g(E_t) \quad (7)$$

Where

${}_{t+N}R_{1,t}$ = the difference between the forward rates on successive dates.

E_t = the forecasting error

By assuming linearity in the functional relationship of equation(8), Meiselman estimated the equation:

$${}_{t+N}R_{1,t} = a + bE_t = u_t \quad (8)$$

Using Durand basis corporate bond yield data from 1901 to 1954 in the regression analysis, Meiselman concluded that a significant part of the movements of forward rates were explained by error in the prediction of the current one-year rate. As his findings suggested, the explanatory power of equation(8) tends to decrease with the increase in time-period. He argued that the decrease in explanatory power is because investors are more inclined to adjust specific forecasts for rates in the immediate future. They are less likely to attempt seriously to forecast short-rates in the distant future. In addition, the constant term a was not

significantly different from zero. Thus, he concluded that the liquidity-premium does not exist. The test, in fact, was consistent with the expectations theory. Therefore, expectations theory supplemented by error-learning mechanism provides a complete description of the term structure.

However, Meiselman's findings did not emerge unchallenged, especially in his treatment about the constant term (a). Later studies found that the constant term in the regression was positive and significantly differs from zero.⁸ The insignificant positive constant term in Meiselman's interpretation was pointed out by John H. Wood [46]. Wood argued that because the Durand bond yield data were biased and overstated the actual rates, these had contributed to the insignificant positive constant terms. Researchers using different data were able to demonstrate the significance of the constant term, thus supporting the notion of liquidity-premium theory.

EXTRAPOLATIVE-REGRESSIVE EXPECTATIONS

The expectation theory was further tested by many researchers, in particular by Modigliani and Sutch [34]. The model used by Modigliani and Sutch was originated by F. Deleeuw. The model entertains how expectations might be formed in the market and whether the shapes of actual yield curves over time were consistently related to these artificially generated expectations in the manner suggested by the theory. According to the model, investors formulate their expectations of future interest rates on the basis of historical experiences. Technically, investors' expectations of future interest rates consist of two parts. First,

investors expect future interest rates to lean toward a normal level of rates that can be estimated on the basis of past experience. For example, when current rates are substantially lower than the normal level, investors expect future rates to rise. Conversely, when current rates are above the normal level, future rates will be expected to fall. Secondly, an increase in rates over the immediate past may lead investors to expect a further rise. Using the hypothesized forecasting techniques of investors and assuming the expectation theory held, Modigliani and Sutch were able to explain interest-rate differentials between long and short rates in the market for U.S. Government securities. They found that the shape of the yield curve had been systematically related to those constructed expectations in precisely the manner suggested by the theory. Thus, their work can be interpreted as offering support for the expectations theory.

There are several criticisms directed at the expectations theory. For other theorists of term structure, they agree that expectation is an important aspect of term structure. However it (expectations theory) alone can not explain the term structure efficiently. Criticism is also directed at the naive expansion of the perfect-certainty variant of the theory to a world of uncertainty. In a world of uncertainty short-term securities are more desirable than long-term securities because the former are more liquid and stable. This leads to the development of the liquidity-premium theory. Expectations and liquidity may be considered as factors influencing the term structure, but there are other factors which warrant some attention. There are the institutional and efficient market factors. The logic underlying each factor will be discussed

individually.

THE LIQUIDITY PREFERENCE THEORY

In a world of perfect certainty, forward rates would be the exact forecast of future short-term rates. However, J.R. Hicks argued that the long-term rate would tend to exceed the value implied by the average of expected future rates by a liquidity or a risk premium. This premium would arise because, when future rates are not known with certainty, the actual short-term yield of long-term securities is uncertain; and given risk aversions, the holders of long-term securities would require compensation for bearing the uncertainty. If the market is dominated by risk averters and filled with uncertainty, then short-term securities are more desirable than long-term securities. Short-term securities are preferred because they are more liquid and their prices do not fluctuate as much as long-term securities. When there is an unexpected change in the level of rates, long-term securities can be expected to have greater potential price volatility than short-term securities, "... for a given change in yields, the fluctuations in market price will be greater the longer the term to maturity."⁹ Therefore, long-term investors stand to have greater potential loss than short-term investors. Thus, liquidity-premium theorists argue that in order to attract long-term investors, long-term securities ought to offer a higher return than short-term securities by the amount of risk premium. If such premiums are not offered to potential long-term investors, those investors will

prefer to hold short-term securities to minimize the variability of the money value of their portfolios. A study by R.A. Kessel [24] found that short-term government securities have averaged less than long-term yields by more than the difference in transaction costs plus any differential attributable to expectations. Similarly, the yield curve for U.S. corporate securities since the Civil War has been decidedly positive. These suggest that risk premium for long-term securities does exist.

On the other hand, borrowers prefer to borrow at long-term to assure themselves that funds are available. This creates an imbalance in the market for different securities. Thus, for borrowers to induce speculators to hold long-term securities, a liquidity premium has to be offered.

When the two sides of the market are considered, if interest rates remain unchanged, one would expect to see an ascending yield curve. This has been consistent with historical evidence. In the present century, on the average, long-term yields have exceeded short-term yields. Conversely, a descending yield curve is conceivable when future short-rates are lower than current short rates by the amount exceeding their respective liquidity premium.

Conceptually, the liquidity premium is typically expressed as an amount that is to be added to the expected future rates. At equilibrium, the relationships are expressed in equation(10):

$$(1+tR_2)^2 \cong (1+tR_1) (1+t+1R_1+L_2) \quad (9)$$

where L_2 is the liquidity premium for two-year security. If L_2 is

positive (which it should always be), then the one-year rate in the second year plus the liquidity-premium (L_2) (${}_{t+1}R_1+L_2$) is greater than the one-year rate (${}_tR_1$). By extending equation(9) to N years, the liquidity model can be written as:

$$(1+{}_tR_N) \equiv (1+{}_tR_1) (1+{}_{t+1}R_1+L_2) \dots (1+{}_{t+N-1}R_1+L_n)^{1/N} \quad (10)$$

If $L_N > L_{N-1} > \dots > L_2$, then it implies that the yield curve is always positively sloped, even when no changes in rates are anticipated. In addition, the longer the term to maturity, the greater is the risk premium.

EMPIRICAL TESTS OF THE LIQUIDITY-PREFERENCE THEORY

The following are empirical studies on the liquidity premium and examinations of their variability over time. Two indirect methods of estimating the size of liquidity premiums have been employed. The method used by Kessel [24] is one that estimates the premiums by taking the average differences between the future short-term rates implied by the current yield curve and the subsequent short rates actually observed. Cagan [4] used a different method of inferring the existence of liquidity premiums. He measured the average holding-period returns that could be earned from securities of different terms to maturities over some specific investment periods.

In Kessel's case, he estimated the premium by taking the average differences between the future short-term rates estimated by equation [F.1] (in footnote 7) and the subsequent short-term rates actually observed. If the future rates calculated from the rate structure exceed

subsequent actual short-term rates on average then the former includes liquidity premiums. The results of Kessel's study show implied forecasts have consistently overstated realized rates. Therefore, the evidence is consistent with the liquidity-premium theory. He argued that if there are expectation errors (as advocated by Meiselman) over a long period of time, these expectation errors ought to cancel out.

Findings from Cagan [4] are more intriguing. Using data from U.S. Government securities during 1951-1965, he found that the holding-period returns (including transaction costs) tend to rise with the maturity of the security. He observed that "the yields of U.S. and municipal securities, plotted by maturity have an upward slope which appears not to reflect differences in transaction cost or expectations. The slope represents a lower pecuniary yield on shorter-term securities, apparently due to the greater stability of their market prices."¹⁰ In any event, he argued that the overall upward slope of those curves can be interpreted as evidence of liquidity premium.

Although studies indicate that the liquidity premium increased with the term to maturity, it (liquidity-premium) also varies directly with the level of interest rates. When rates are unusually high, liquidity premiums are small. Conversely, when rates are unusually below average, liquidity premiums are large. The willingness of investors to accept small liquidity-premium when interest rates are high implies that they (investors) expect the interest rates to come down and vice versa. This merely suggests that expectation does play a role in explaining the term structure of interest rates.

THE INSTITUTIONAL OR HEDGING-PRESSURE THEORY

The institutionalist's view on the term structure has been advanced by several economists, particularly by J. Culbertson [8]. The view has been challenged by many market practitioners having considerable credentials. This notwithstanding, institutionalists believe that all investors do not behave in the way perceived by the expectations theorists. Institutional investors such as life insurance companies, pension funds, and retirement plans have different preferences and needs guided by the nature of their business. Their needs and preferences are different from other investors in the financial market. A life insurance company selling annuity contracts prefers long-term securities to hedge against the risk of interest-rate fluctuations. Hedging consists of matching liabilities with assets. This is because an annuity contract, in essence, guarantees to the annuitant a specified earning over a long period. By investing long-term, the insurance company is guaranteed a stream of income payments regardless of what happens to interest rates over the life of the contract. Similarly, pension funds and retirement plans share the same behaviour. They are concerned with guaranteeing themselves certainty of incomes rather than certainty of principal over the long-run. Their concerns lead them to prefer long-term rather than short-term securities. From the borrowers' and bond issuers' point of view, they tend to tailor the maturity of their offering to the type of asset to be financed, or the length of time over which they required the funds.

Therefore, institutionalists argue that different groups of investors/borrowers have different maturity needs which lead them to confine

their security purchases/issues to restricted segments of the maturity spectrum. Expectations and liquidity are important considerations of term structure, one should not ignore the institutional force. Since institutions are the dominant force in the financial market and are not likely to shift from one segment of the market to the other, in response to rate differentials, these undoubtedly have an impact on the term structure. In short, the demand and supply of securities by institutions can affect the term structure. For example, an exogenous change in the relative supply of securities (long or short) will alter the shape of the yield curve provided that short and long-term securities are imperfect substitutes. To be exact, if quantity demanded for long-term securities is greater than the relative supply, long-term yield decreases ... higher demand bids up the price of long-term securities. When large quantities of long-term securities are offered, long-term rates will rise relative to short-term rates and vice versa. Thus, one can deduce from the above discussion that the maturity composition of the debt plays a central role in the term structure. This explains why most empirical works on the institutional theory focus on maturity compositions. Good examples of such works are the study of "the Operation Twist"¹¹ in the United States and "the Conversion Loan"¹² in Canada.

EMPIRICAL TESTS OF INSTITUTIONAL THEORY

Empirical analysis on the effectiveness of these operations has produced substantial evidence favouring the institutionalists' view. Distinguished economists, who analyze the effect of the Conversion Loan, have come up with different conclusions. Some conclude that Conversion

Loan has affected long-term interest rates. Others who have studied on the same matter have produced contradictory results. Thus, evidence produced cast some doubt on the effect of the maturity composition of debt on the term structure.

Other researchers who have tested this theory are Wallace, N. [44] and Modigliani and Sutch [34&35]. They tested the effect of supply changes on the term structure. In both cases, they found that exogenous changes in the relative supply of debt securities had a negligible influence on the spread between long and short rates.

PREFERRED-HABITAT THEORY

This theory embodies some element of the liquidity theory and the institutional theory. It hypothesizes that the actual yield of a n-period security may differ from the value predicted by the simple expectations theory by a n-period risk premium. The n-period risk premium need not be a monotonic function of length to maturity and may be positive or negative. The premium arises from a possible imbalance between the supply and the demand of funds by lenders and borrowers respectively; especially when the market consists of lenders and borrowers with different preferred habitats. If n-period habitat borrowers exceed lenders, borrowers will be required to offer a premium over and above the expected return from a sequence of one period securities. Conversely, borrowers will be required to offer a discount if n-period lenders exceed borrowers. The discount and premium offered are expected to vary from one maturity to the other in the extreme form of preferred-

habitat. That is, nearby maturities may not be expected as close substitutes for both lenders and borrowers. Advocates of the theory argue that a smooth yield curve can be explained by the substitutability of nearby maturities for both lenders and borrowers.

EMPIRICAL TESTS OF PREFERRED-HABITAT THEORY

Empirical analyses were done by Modigliani and Sutch [35] and J.S. McCallum [29] with regard to preferred habitat. In Modigliani and Sutch's model, they treated long-term rate (L_t) as dependent on current and past short-term rates ($S_t, S_{t-1}, S_{t-2}, \dots, S_{t-N-1}$ respectively) and a risk premium (F_t). The risk premium (F_t) was dropped after the first estimation because it was considered statistically insignificant. This could lead one to reject the liquidity premium theory. Later estimation¹³ (without F_t) yielded little support for preferred habitat theory. This has by no means invalidated the importance of the theory. The reason for the inconclusive results was that the approach encountered numerous econometric difficulties.

Later, J.S. McCallum tested the expectations, liquidity-premium, and the preferred habitat theories using Canadian government securities. His study looked at the risk pattern for a three-month holding period of various securities maturing within three months to twenty years. He found that the risk (measured as the standard deviation and the Beta)¹⁴ increases with maturity of the securities held. While the expected return for short-term securities (three months to three years) rises with maturity, expected return for long-term securities (over three years) decreases with maturity. Since the expected return is neither

horizontal nor ascending, expectations and liquidity-premium theory cannot be supported by the results. Instead, evidence points to a market with distinct short and long horizon participants; a market described by the preferred-habitat theory.

EFFICIENT MARKET THEORY

The pivot of the efficient market theory is that current interest rates fully reflect all available information. It also means that in an efficient-market, prices of securities adjust very rapidly to new information. So, when the market is efficient, interest rates and security prices perform a random walk. Under these circumstances, proponents of the theory argue that investors' expectations of interest rates could not contain both regressive and extrapolative elements. Thus suggesting that future interest rates depend on availability of information at any given point in time. It further implies that term structure of interest rates can change quite rapidly. Therefore it does not follow a normal trend.

Empirical analysis on the theory is not an easy task for researchers. First of all, the definition of the term "fully reflect" is rather ambiguous; especially in modelling. The term used in this context is so general that it has no empirically testable implications. Second, there is the question pertaining to the strength (strength referring to the reliability and the time) of available information. For example, when available information is weak, prices and interest rates may not reflect it fully or vice versa. However researchers such as Marshall Blume, Eugene F. Fama, Maurice G. Kendall, M.F.M. Osborne and many others have devised ingenious methods of testing the efficient market model.

Their work can be divided into three categories depending on the nature of the information subset of interest. The categories are called strong-form, semi-strong-form and weak-form-test. Strong-form tests are concerned with whether individual investors or groups have monopolistic access to any information relevant to price formation. In semi-strong-form-tests, the information subset of interest includes all obviously publicly available information. In the weak-form tests the information subset is just the historical prices and returns sequence.

The empirical results of all these different tests support the efficient market theory. The following are a few quotes from E.F. Fama's article [1970] "weak-form-test of the efficient market are the most voluminous and it seems fair to say that the results are strongly in support ... Semi-strong-form-tests, in which prices are assumed to fully reflect all obviously publicly available information, have also supported the efficient market theory ... The strong-form-test, in which prices are assumed to fully reflect all available information, are probably best viewed as a benchmark against which deviations from market efficiency can be judged."¹⁵ In short, evidence supporting the efficient market theory is extensive and contradictory evidence is sparse.

SUMMARY AND COMMENTS

Five theories have been discussed so far in terms of their ability to explain the shape of the yield curve. The expectations theory argues that the term structure of interest rates can be fully explained by the market's anticipations regarding future interest rates. The liquidity-premium theory argues that short-term securities are more liquid and

premium theory argues that short-term securities are more liquid and stable, so in order to attract long-term investors, a yield premium must be offered in longer-term securities. The institutional or hedging-pressure theory suggests that expectations and liquidity are important factors influencing the term structure of interest rates. They (expectations and liquidity) also influence investors' behaviour, particularly for risk-averse investors with fairly short horizons. However, for investors with long-term obligations such as insurance companies, pension fund and retirement plans, their preference would be in long-term securities which guarantee certainty of income payments over a long period. These, institutionalists argue, should have an effect on the term structure of interest rates.

The preferred-habitat and efficient market theories provide a different view. For the preferred-habitat theory, it suggests that there is more than one factor influencing the term structure at any given point in time. The factors that deem important are liquidity-premium and hedging-pressure. It suggests that liquidity-premium need not be a monotonic function of length to maturity. It (premium) can arise from the imbalance between the supply and demand of funds by lenders and borrowers respectively. This premium is expected to vary from one maturity to the other. In the extreme form of preferred habitat, nearby maturities may not be expected as close substitutes. On the other hand, efficient market suggests investors' participation in the market fully reflect all available information which affects the instrument valuation. This implies that market prices of securities adjust very rapidly to new information. The rapid adjustment of prices in turn reflex the yields of

securities. Thus according to the efficient market theory, the term structure of interest rates (or the yield curve) do not follow a normal trend.

Empirical studies have provided considerable evidence pointing to the importance of expectations. The difficulty lies in the determination of expectation by investors. Studies on liquidity-premium theory focus on measuring the size of the premiums over time. These studies suggest that there is a positive relationship between risk-premiums and term to maturity of securities. Evidence supporting the institutional or hedging-pressure theory is sparse. Empirical tests of the theory examine the effect of the maturity composition of the outstanding debt on the term structure of interest rates. Evidence based on the preferred-habitat theory points to a market with distinct short and long horizon participants. Efficient market theory is supported by extensive evidence provided from different simulation tests. The tests, regardless of the strength of available information, provide positive evidence. In any event, the theories so far have provided different ways to explain the term structure of interest rates. Advocates of each theory are able to marshal strong empirical evidence to support their theory. This may lead one to view these theories as competitive to one another, but one should view them as complementary.

Thus far, most of the tests on the term structure of interest rates are done with single term structure equation. Very few analyses have been done with a structural model. A structural approach, in all cases, offers a more complex and complete explanation of the term structure of interest rates. At the same time, it (the structural approach) offers a

better understanding of the theory and of how the theory's results come about. For the purpose of the thesis, the structural approach developed by Benjamin M. Friedman is adopted to analyze the expectations theory.

CHAPTER THREE

STRUCTURAL MODEL: THE OPTIMAL MARGINAL ADJUSTMENT
MODEL OF PORTFOLIO BEHAVIOUR.

The model, as indicated earlier, was initially developed and used by Benjamin M. Friedman with United States' data. The essence of the model focuses on the behaviour of investors, particularly on how investors adjust their portfolio under certain circumstances. Portfolio adjustment would be easy and quite simple if we were living in a world with certainty and where transaction costs are trivial; and where there are limited choices of securities. Unfortunately, we live in a world of uncertainty; transactions costs are nontrivial and the range of securities are virtually unlimited. Under these circumstances, the portfolio adjustment process becomes extremely complex. Investors not only have to adjust their portfolio such that it will be optimal, but they also have to minimize their transaction costs and allocate new wealth. The structural model, developed by Friedman, is designed to represent the complex portfolio adjustment process. The model combines the adjustment process with the selection process (the expanded version is illustrated by equation(6)) in such a way that the model approximates investors' portfolio behaviour.

In Section 3:1, I will replicate some of the basic framework in the portfolio selection model as well as the portfolio adjustment model. The two models are combined to form the optimal marginal adjustment model of portfolio behaviour. Section 3:2 briefly discusses the adoption of the model to the Canadian case.

3:1 PORTFOLIO SELECTION

According to Friedman, the long-run homogeneous assets demand function (or the model for the selection of desired portfolio allocation) for a given investor or group of investors can be written as:

$$\frac{A_{it}^*}{W_t} = \sum_K \beta_{ik} r_{kt} + \sum_h \gamma_{ih} X_{ht} + \pi_i, \quad i=1, \dots, N, \dots \dots \dots (1) \quad 16$$

Where A_{it}^* , $i=1, \dots, N$, = the investor's desired equilibrium holding of the i th asset at time t . ($\sum_i A_{it}^* = W_t$);

W_t = the wealth of the investor (or the total portfolio size) at time period t ;

r_{kt} , $k=1, \dots, N$ = the expected holding-period yield on the k th asset at time period t ;

X_{ht} , $h=1, \dots, M$ = the value at time period, t , of additional variables which influence the portfolio allocation.

π_i , $i=1, \dots, N$ = the stochastic disturbance term.

The model (1) postulates that given wealth constraint, the investor's desired equilibrium holding of the i th asset at time t , depends on the expected holding-period yield on the k th asset at time (t) and on other non-yield variables which also influence the portfolio allocation. The model (1) is extremely restrictive. For example, by imposing the wealth homogeneity constraint, one attributes the shift in any asset share in the desired equilibrium portfolio to the movements either of the relevant yield (r_k) or of other variables (X_h), rather than to the overall growth of total portfolio itself. The constraint also implies that an increase in any single real asset (i) must be financed by a decrease in other financial assets. This in essence

recognizes some of the elementary interrelationships in the allocative process enforced by the balance-sheet identities. This renders the fixed coefficients β_{ik} and γ_{ih} in equation (1) to satisfy the following properties;

that $\sum_i \beta_{ik} = 0$ for all k , $\sum_i \gamma_{ih} = 0$ for all h .

The analytical role of the expected holding period yield variables (r_{kt} , $k=1, \dots, N$) is quite obvious, but the estimation of r_{kt} may be complicated. The estimation of r_{kt} will be discussed in Chapter Four. In this long-run linear homogeneous asset demand function, the general presumption is that investor's demand for any asset responds positively to the own yield of the asset ($\beta_{ik} > 0$, $k=i$) and negatively to the yields on alternative assets ($\beta_{ik} < 0$, $k \neq i$). In other words, $\beta_{ik} > 0$, $k=i$ is the slope of the demand curve with respect to r_k , normalized by wealth level. When $\beta_{ik} > 0$, $k=i$, it implies that as the yield of i (own-yield) increases the demand for asset i increases. Conversely, demand for asset i decreases when either the own-yield decreases more than alternative yields, or alternative yields increases more than own-yield. This occurrence is implied by the slope of demand coefficient $\beta_{ik} < 0$, $k \neq i$. The model (1) presumes that when there is an equal rise in the yields of all assets, this should not affect the demand for the assets.

However, "because of differing portfolio objectives in terms of return and risk, differing effective transactions cost, and differing institutional and regulatory constraints, the group of assets which compete for a place in the collective portfolio of any investor or group of investors need not coincide with the entire menu or assets available in

the economy."¹⁷ Therefore, the portfolio selection equations for different investor categories do not necessarily include identical groups of yield variables. In fact, one has to predetermine the potential substitutabilities of asset (within a portfolio). The assets which are potential substitutes are included in the portfolio selection equations in each case. Thus, in Chapter Four, one could expect to find different yield variables for different investor categories. This is because, all investors do not share the same preference. They have different needs, wants, and preferences as to which asset to invest.

In addition, there are other non-yield variables (X_h , $h=1, \dots, m$) which influence investors' choice of assets for their desired equilibrium portfolio allocation. The variables (non-yield variables) are proxies representing investors' perceptions of return and risk over the relevant holding period, of anticipated volatility of nominal returns, and of anticipated price inflation. These variables, like the yield-variables, exert different influences on different investor categories. Therefore, in the portfolio selection equations for different investors categories, the equations do not necessarily include identical groups of non-yield variables.

The desired equilibrium portfolio selection equation (1) only illustrates the optimal equilibrium portfolio of an investor or groups of investors with the given wealth constraint and yields. What is missing in equation (1) is the adjustment process of an investor or group of investors. The adjustment process is important to portfolio management and to the analysis of arbitraging pressure in the expectations theory; particularly when cash flow is included in the decision-making. The

adjustment process becomes extremely important when the world we live in is full of uncertainty; and where transactions costs are nontrivial. To approach this methodologically, one familiar representation of the resulting portfolio adjustment process is the stock adjustment model,¹⁸ written as:

$$A_{it} = \sum_k^N \theta_{ik} (A_{kt}^* - A_{k,t-1}), i=1, \dots, N \quad (2)19$$

Where A_{it} = the investor's actual holding of the i th asset time t , ($\sum_i A_{it} = W_t$), and the θ_{ik} are fixed coefficients of adjustment such that $0 \leq \theta_{ik} \leq 1$, $k=1$, and $\sum_i \theta_{ik} = 1$ for all k .

A similar model involving explicit adjustment of portfolio proportions can be written as:

$$\Delta \alpha_{it} = \sum_k^N \theta_{ik} (\alpha_{kt}^* - \alpha_{k,t-1}), i=1, \dots, N \quad (3)20$$

Where $\alpha_{it} = \frac{A_{it}}{W_t}$, ($\sum_i \alpha_{it} = 1$), and the constraint

applicable to the θ_{ik} is the weaker $\sum_i \theta_{ik} = \bar{\theta}$ for all k (which does not require $\theta = 1$). A weaker $\sum_i \theta_{ik} = \bar{\theta} < 1$ indicates the investors' failure to adjust their portfolio fully and immediately to whatever equilibrium allocation is consistent with each period's new values of the relevant variables. Some of the rationales for such failure are conceptually distinct. They are the transaction costs, expectations lags and perception lags. "The lags associated with forming expectations and with perceiving current market developments in large part give rise empirically to the use of distributed lags on market yields to represent the expected holding-period yield variable (r_k) in

portfolio selection model(1)."²¹ Transaction costs (mostly brokerage fees) further complicates the portfolio selection process. In most cases, with the transaction costs imposed upon investors, it is easier and cheaper for most investors to allocate new cash flows -- including both net wealth increments and repayments such as dividends, coupon interest, and maturities, than to reallocate current asset holdings. As a result, the simple stock adjustment models (2) and (3) are rendered inadequate to distinguish clearly between new cash flows and previous-period wealth (including capital gains and net of repayment).

To illustrate the overrestrictiveness of portfolio adjustment models (2) and (3), let us assume that an investor whose portfolio consists of two assets only (A_1 and A_2), and with fixed asset prices p_1 and p_2 respectively, and the initial wealth equals W_{t-1} . Based on model(3), one can illustrate the portfolio adjustment of the investor in figure 3:1.1. The straight line from $[0, (W_{t-1}/P_2)]$ to $[W_{t-1}/P_1, 0]$ represents the investor's wealth constraint in time period (t-1), and the parallel line from $[0, (W_t/P_2)]$ to $[(W_t/P_1), 0]$ represents the wealth constraint after an increment $\Delta W_t > 0$.

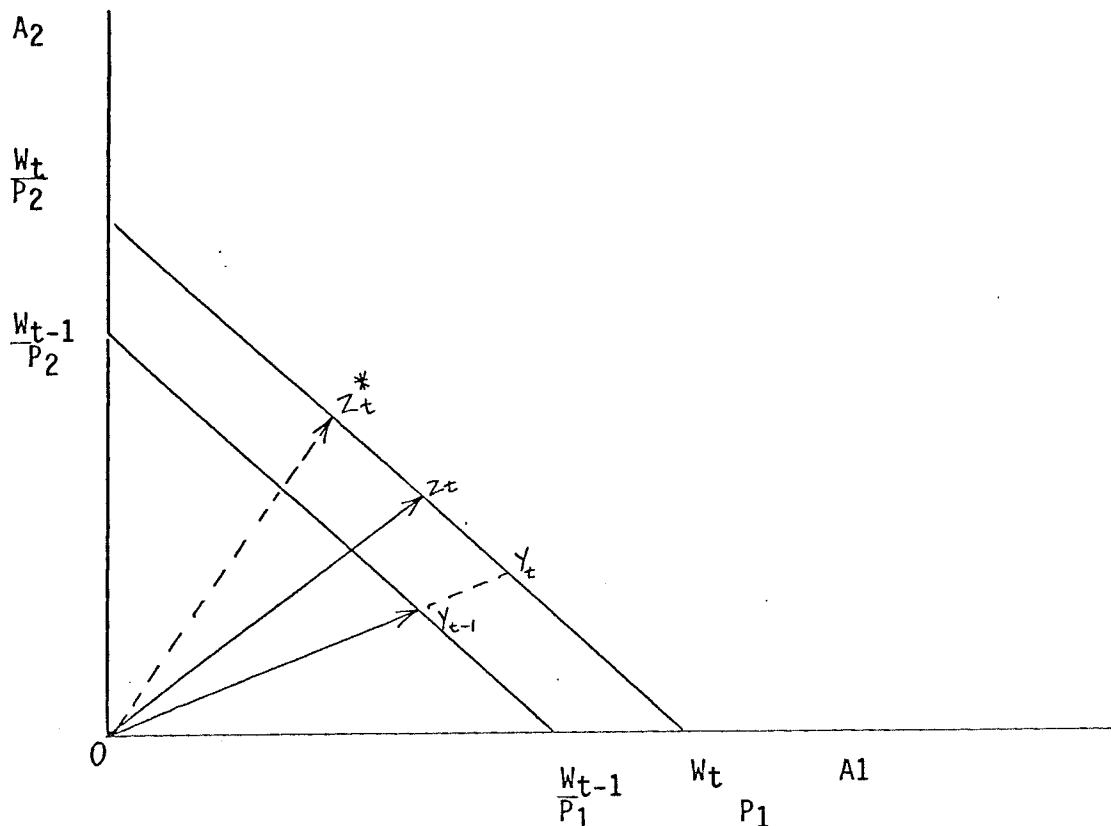
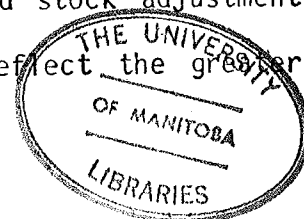


FIGURE 3:1.1 PORTFOLIO ADJUSTMENT MODEL(3), TWO-ASSET CASE

Point Y_{t-1} indicates the investor's previous period asset holding $(A_{1,t-1}, A_{2,t-1})$, and Point Y_t indicates the holding $(A_{1,t}, A_{2,t})$ which would result from allocation of the existing asset holding and allocation of the wealth increment ΔW_t in proportion to the actual holding at time period $(t-1)$. Point Z_t^* indicates the desired equilibrium holdings (A_{1t}^*, A_{2t}^*) . Whereas point Z_t indicates the actual holding (A_{1t}, A_{2t}) which result from portfolio adjustment model(3). The discrepancy between the desired equilibrium holding $(A_{1t}^*, A_{2t}^*$ as represented by Z_t^*) and the actual holding $(A_{1t}, A_{2t}$ indicated by Z_t) is clearly demonstrated by Figure 3:1.1. Therefore, the standard stock adjustment model becomes too restrictive in that it does not reflect the greater



adjustability of new cash flows in comparison with existing asset holdings. To acknowledge the greater adjustability of new cash flows, at the same time without introducing discontinuities, Friedman has postulated that the simple way of incorporating explicit flow effects on portfolio adjustments is to separate the reallocation of the new cash flow by applying fixed coefficients to the flow itself. The postulated equation is written as:

$$\Delta A_{it} = \sum_K^N \theta_{ik} (\alpha_{kt}^* \cdot W_{t-1} - A_{k,t-1}) + \delta_i W_t, \quad i=1, \dots, N, t \quad (4)^{22}$$

Where the δ_i are fixed coefficients such that $0 \leq \delta_i \leq 1$ for all i , and $\sum_i \delta_i = 1$, and the constraint on the θ_{ik} is $\sum_i \theta_{ik} = \bar{\theta}$ for all k , as in model(3). The first term on the right hand side of model(4) represents the reallocation of the existing wealth (W_{t-1}) based on model(3). While the second term represents the reallocation of the flow of new wealth (ΔW_t). Model(5) may represent some improvement over model(2) and (3), in that the adjustments depend explicitly upon ΔW_t . However, because of δ_i (of ΔW_t), it (model (4)) becomes inadequate for most investors. Particularly, when it assumes investors always allocate new cash flows in fixed proportions, regardless of market yields and other variables relevant to portfolio behaviours.

Friedman arrived at a compromise model which allocate the flow of new investable funds according to whatever proportion portfolio selection model(1) indicates are the desired equilibrium proportions for the portfolio as a whole, a model which always forms and uses equilibrium total proportions to allocate at the margin. "The optimal marginal adjustment"

model indicated by this specification is an analogue of model(4). The only difference is that the model's invariant flow allocation coefficient δ_i for each asset is replaced by the corresponding current desired proportion α_{it}^* (from the portfolio selection model(1)). The model is thus called the optimal marginal adjustment model of portfolio behaviour, written as:

$$\Delta A_{it} = \sum_K^N \theta_{ik} (\alpha_{kt}^* \cdot W_{t-1} - A_{k,t-1}) + \alpha_{it}^* \Delta W_t, \quad i=1, \dots, N \quad (5)^{23}$$

The first term on the right hand side is identical to model(4), whereas the second term $(\alpha_{it}^* \cdot \Delta W_t)$ indicates the allocation of new cash flows according to current desired proportions α_{it}^* (from model(1)). Model(5) according to Friedman, is most appropriate for investors participating in private securities markets, in which some level of fairly continuous activity maintains the flow of information about new investment opportunities. In addition, it is also appropriate to institutional investors such as insurance companies and pension funds that participate in both market of publicly offered debt issues and of privately placed corporate debt securities, as well as commercial mortgages and other forms of negotiable instruments. Model(5) is also suitable for institutional investors who are reluctant to undertake sharp negative swings in their new purchasing activities for long periods of time. In short, the model(5) is a gross representation of investors' (institutional or otherwise) adjustment process (in light on transaction costs and lags) and portfolio reallocation (of existing assets) and the allocation of new cash flows among the available assets.

To illustrate the allocative process of the model(5) graphically, Figure 3:1.2 depicts the model in a two-asset case. The first term on the right-hand side of model(5) represents the reallocation of the existing wealth W_{t-1} which is depicted as a move along the W_{t-1} wealth constraint from Y_{t-1} wealth constraint and the ray from the origin to Z_t^* occurs only when there is a full adjustment where $\theta_{ij}=1$ for i , and $\theta_{ik}=0$ for $k \neq 1$. However, the reallocation process leads not to Z_t^* , rather to point Z_t' which lies between Z_t^* and Y_{t-1} on W_{t-1} wealth constraint, for reasons such as transactions costs and lags explained in model(3). The second term on the right hand side of model(5) represents the allocation of wealth increment (ΔW_t) according to the current desired proportions α_{it}^* . The

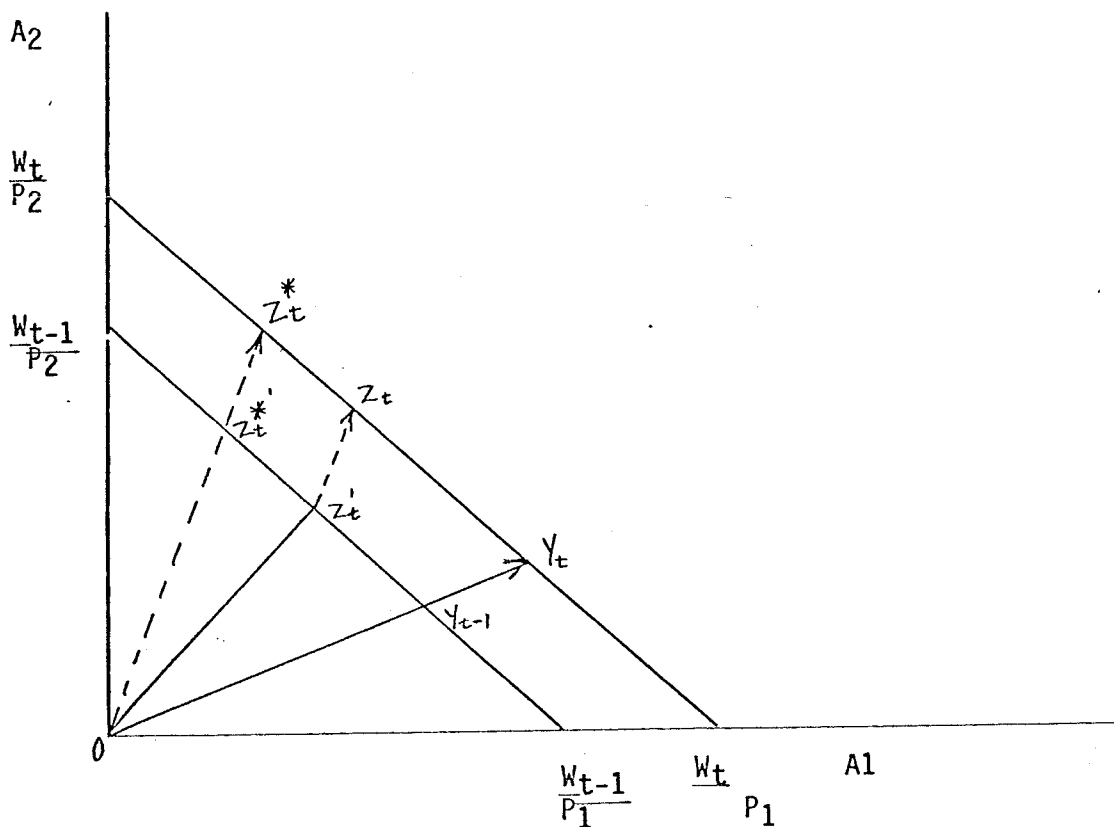


FIGURE 3:1.2 PORTFOLIO ADJUSTMENT MODEL(5), TWO-ASSET CASE

allocation of new wealth is represented by a movement from point Z_t' . Thus point Z_t indicates the new holdings (A_{1t}, A_{2t}) resulted from optimal marginal adjustment model of portfolio behaviour represented by equation(5). In contrast to model(3) illustrated in Figure 3:1.1, Figure 3:1.2 shows that, for a given W_{t-1} , the symmetrical adjustment ratio $(Y_t Z_t) / Y_t Z_T^* = (\alpha_{it} - \alpha_{1,t-1}) / (\alpha_{1t}^* - \alpha_{1,t-1}) = (\alpha_{2t} - \alpha_{2,t-1}) / (\alpha_{2t}^* - \alpha_{2,t-1})$ in model(5) varies positively with the flow W_t . This symmetrical adjustment ratio on W_t in model(5) holds for any number of assets.

The optimal marginal adjustment model of portfolio behaviour developed by Friedman and replicated in Section 3:1 has several advantages over the standard stock adjustment model. First of all, it does not sacrifice either tractability or suitability for empirical application to aggregative data. Second, it relaxes the restrictiveness of the standard stock adjustment model with regard to investors' allocation of their new cash flows. Third, the model reads to an asset demand equation in which the investable cash flow variable matters for short-run allocations. Fourth, it poses a dynamic adjustment process which associates the size of new cash flow with the speed of the portfolio's overall adjustment toward equilibrium allocation.

3.2 THE ADAPTATION OF THE OPTIMAL MARGINAL ADJUSTMENT MODEL OF PORTFOLIO BEHAVIOUR TO THE CANADA CASE

Basically, the optimal marginal adjustment model of portfolio behaviour can be applied to the Canadian case without modification. However, when the model is adopted in this thesis, certain aspects related to investors' portfolio behaviour have to be pointed out so that the

empirical results are interpreted appropriately. First of all, the model is not used to determine long-term interest rates. Instead, it is used to focus on the demand for one asset [Canadian government (federal) marketable securities (3 years and over)], specifically on the demand behaviour of the various investor categories. Thus the yields of Canadian government marketable securities are assumed to be exogenous.

The investor categories are analyzed separately. Analyzing their demand separately permits one to observe how each investor category behaves given the market conditions. The structural model such as the optimal marginal adjustment model of portfolio behaviour will further allow one to observe the way in which each portfolio is being managed, particularly on the portfolio adjustment process which can be interpreted as the source of arbitraging in the bond market. Identifying the source of arbitraging-pressure in the bond market is not an easy task. However, when the demand function is the optimal marginal adjustment model of portfolio behaviour, one can infer and identify the source of arbitraging quite easily.

Thus far the structural model developed by Benjamin M. Friedman has been replicated and quite sufficiently explains the derivation of the optimal marginal adjustment model of portfolio behaviour. The portfolio selection model and the stock adjustment, when applied separately does not amount to very much, as indicated. But when they are combined to form the optimal marginal adjustment model of portfolio behaviour, it closely approximates the investors' portfolio behaviour. Such a model is essential to the analysis of a theory or theories related to the term structure of interest rates, particularly, when the results depend so much on market participant such as the expectations theory.

CHAPTER FOUR

ESTIMATION PROCEDURES, DATA AND EMPIRICAL ANALYSES

This chapter is divided into three sections. Section 4:1 focuses on the estimation procedures. In this section, the extrapolative-regressive method is briefly explained. Section 4:2 discusses the different investor categories to be examined and data which is used in the empirical analyses. This section includes the explanation as to why and how the data are used in the analyses. Section 4:3 illustrates the empirical results of the various investor categories. These results are tested for their significance.

4.1 ESTIMATION PROCEDURES

The optimal marginal adjustment model(5) in Section 3:1, in itself, is not suitable for estimation. The model(5)p.43 has to be elaborated and expanded with the selection model(1)p.36. Appendix I illustrates how model(1) and (5) are expanded together to form model(6). The overall expanded model, the optimal marginal adjustment model of portfolio behaviour can be written as (grouping the variables in the following way):

$$\begin{aligned} \Delta A_{it} = & \pi_j \cdot \Delta W_t + [\sum_k (\pi_k \cdot \theta_{ik})] \cdot W_{t-1} + \beta_{ii} \cdot r_{it}^e \cdot \Delta W_t + \\ & [\sum_k (\beta_{ki} \cdot \theta_{ik})] \cdot r_{it}^e \cdot W_{t-1} + \sum_{k=1} \left\{ \beta_{ik} \cdot r_{kt}^e \cdot \Delta W_t + \right. \\ & \left. [\sum (\beta_{ik} \cdot \theta_{ij}) \cdot r_{kt}^e \cdot W_{t-1}] \right\} + \sum_h \left\{ \gamma_{ih} \cdot X_h \cdot \Delta W_t + \right. \\ & \left. [\sum_k (\gamma_{kh} \cdot \theta_{ik}) \cdot X_h \cdot W_{t-1}] \right\} - \theta_{ii} \cdot A_{i,t-1} - \sum_{k=i} (\theta_{ik} \cdot A_{k,t-1}) \end{aligned} \quad (6)24$$

In the expanded version illustrated in equation(6), the model postulates the change in demand for asset i (A_{it}) is a linear function of a complex overall dynamic adjustment of the existing portfolio and the marginal allocation of new wealth. As pointed out earlier, investors demand for asset i does not entirely depend on its own-yield, but also on the yields of alternative assets. Incidentally, the demand for asset i also indirectly depends on the demand for asset k ($k=1, \dots, N$). Therefore, the model does not only estimate the impact of change in wealth (given own-yield, alternative yields and other variables) on the demand for asset i (as represented by $\pi_i \cdot \Delta W_t$, $\beta_{ii} \cdot r_i e_t \cdot \Delta W_t$, $\sum_{k=i} (\beta_{ik} \cdot r_k e_t \cdot \Delta W_t)$ and $\sum_h (\gamma_{ih} \cdot X_h \cdot \Delta W_t)$). It also illustrates the impact of the adjustment of other existing assets with respect to alternate yields $[\sum_{k \neq i} (\sum_j (\beta_{jk} \cdot \theta_{ij})) \cdot r_k e_t \cdot W_{t-1}]$, with respect to asset i 's yield $[\sum_k (\beta_{ki} \cdot \theta_{ik}) \cdot r_k e_t \cdot W_{t-1}]$, and with respect to other variables $[\sum_h (\sum_k (\gamma_{kh} \cdot \theta_{ik})) \cdot X_h \cdot W_{t-1}]$, on the demand for asset i . The additional variables (X_h) are proxies representing financial securities which do not carry coupon rates, but have to compete with other financial securities (which carry coupon rates) for a place in the portfolio. As equation(6) postulates, these additional variables influence the investor's portfolio allocation. One such important financial security is stocks. The proxy associated with stocks is the Canadian investors' stock market index (standard and poor 500). Vector X_h also represents the relative risks. Since the thesis analyzes the demand for Canada federal government guaranteed marketable bonds (FGMB), there will be no risks of default. However, one should acknowledge the risk involved

in holding these bonds as price changes resulting from the changes in interest rates. This risk increases as term to maturity increases; because prices of long-term bonds are more volatile. Since the analysis focuses on federal government guaranteed marketable bonds as a whole, the risk associating with maturity can be overlooked. Instead, the risk associating with change in interest rates should be included in the analysis. The proxy for such risk is represented by lagged one-period percentage change of FGMB's yields (X_2).

The terms $\theta_{ij} \cdot A_{j,t-1}$ and $\sum_{k \neq i} (\theta_{ik} \cdot A_{k,t-1})$ enter equation(6) linearly; the terms indicate the willingness of investors to adjust their holdings of asset i and the willingness to adjust other assets in the portfolio respectively. This willingness to adjust also has an impact on the demand for asset i .

As to the expected signs of the coefficients contained in equation (6), the coefficients of ΔW_t and lagged wealth stock W_{t-1} are known a priori, where π_i is simply the constant term corresponding to asset A_i and π_k , $k \neq i$ specified in the selection model(1). The coefficients of $(r_i e_t \cdot \Delta W_t)$ term is $\beta_{ii} > 0$ because we expect the slope of demand to be positive with respect to expected own-yield. The coefficient of $(r_i e_t \cdot W_{t-1})$ term is a sum of coefficients, [the cross-elasticity of demand for k asset with respect to the expected yield of asset i (β_{ki}) and the speed and willingness of adjustment (θ_{ik})], which is of an unknown sign a priori.²⁵ The coefficient β_{ik} (for $k \neq i$) is less than zero ($\beta_{jk} < 0$), whereas the sum of coefficients ($\beta_{jk} \cdot \theta_{ij}$) is of unknown sign a priori. The coefficients of the non-yield variables are also of an unknown sign a

priori. The coefficient θ_{ij} and the coefficient θ_{ik} of the lagged stocks of competing asset $A_{k,t-1}, k \neq i$, are of unknown signs. To summarize the expected sign of the coefficients in model(6), Table I illustrates the expected sign.

TABLE I
EXPECTED SIGN OF THE COEFFICIENTS

<u>Coefficients of Variables</u>	<u>Expected Sign</u>
ΔW_t	(+)
W_{t-1}	unknown
$r_{e_{it}} \cdot \Delta W_t$	(+)
$r_{e_{it}} \cdot W_{t-1}$	unknown
$r_{e_{kt}} \cdot \Delta W_t$	(-)
$r_{e_{kt}} \cdot W_{t-1}$	unknown
$X_h \cdot \Delta W_t$	unknown ^a
$X_h \cdot W_{t-1}$	unknown ^a
$A_{i,t-1}$	unknown ^b
$A_{k,t-1}$	unknown ^b

a = The possible signs of these non-yield variables depend on the nature of the variables. If the variables are non-yield assets competing for a place in the portfolio, then the expected sign is expected to be negative. When the variables are complementary to asset i, then the coefficients are expected to be positive.

b = The sign indicates investor's adjustment flexibility. As Significant negative (-) coefficient for $A_{i,t-1}$ and $A_{k,t-1}$ suggests that the investor adjusts the portfolio actively.

Equation(6) is not immediately estimable because of the way in which the expected own yield ($r_{e_{it}}$) and the expected yields on alternative assets ($r_{e_{kt}}, k \neq i$) enter equation(6). The $r_{e_{it}}$ and $r_{e_{kt}}, k \neq i$ are expected yields. There are two ways by which equation(6) becomes estimatable. First, one can estimate $r_{e_{it}}$ and $r_{e_{kt}}$ outside the system, and then enter them into equation(6);

second, one can use the instrumental variable method, that is, to replace r_{it}^e and r_{kt}^e with proxies (such as the actual yields). Since the thesis focuses on the expectations theory and institutions' interest arbitrage, the first method is used. After all expectations theory has to be tested - using the extrapolative - regressive method. Furthermore, because investors' portfolio allocative behaviour reflect on their expected yields of the available assets, it is important to use the generated expected yields. It is based on these expected yields that an investor or a group of investors readjust their portfolio to a more optimal and desirable level.

There are various ways by which the expected yields can be estimated. The method adopted to estimate the expected yields is called the extrapolative-regressive method. The estimate results for r_{it}^e and r_{kt}^e are then entered into equation(6) as variables with known value. This way one can conveniently estimate the single equation using the ordinary - least squares method (OLS).

EXTRAPOLATIVE-REGRESSIVE METHOD

This method combines extrapolative and regressive to the formulation of expectations. The extrapolative hypothesis to expectations was advanced by James Duesenberry [1963]. He argues that a rise in rates may lead to an expectation of a further rise and vice versa. Whereas the regressive hypothesis (associated with Keynes) holds that the market expects the interest rate to regress toward a "normal" level based on past experience. De Leeuw (1965) points out that each of these hypotheses contains an important element of truth. He surmises that expectations contain both

extrapolative and regressive elements. The general model postulating the extrapolative-regressive method (employed by Modigliani and Sutch) can be written as:

$$R_t^e = \psi + \beta_0 r_t + \sum_{i=1}^{\infty} \beta_i r_{t-i} + \epsilon_t, \text{ where } \sum_{i=0}^{\infty} \beta_i = 1 \quad (7)$$

Where R_t^e = expected rate of bond with n periods to maturity;

r_t = the rate on a bond with one period to maturity (short rate);

r_{t-1} = distributed lag on past short-rates;

ϵ_t = the stochastic term.

The distributed lag weights are constrained to follow a third-degree polynomial pattern (with the lead lag weight free of polynomial constraint and the last lag weight constraint to zero).

Equation(7) is used to estimate the various expected yields which are required for the analysis of the demand for government of Canada bonds (in relation with other financial assets). The extrapolative-regressive method is strictly used as a forecasting tool. All proceeding forecasts are based on quarterly data from 1963:I to 1979:IV. The proxy for short-rates is the 90-day treasury bill yields. For expected mortgage rates, the proxy is the average bank loan rates. When these proxies are applied to equation(7) to generate the various expected yields, it is found that the 90-day treasury bill yields are not a good proxy for estimating the expected yields of provincial, municipal and corporate bonds. In other words, the goodness-of-fit is very poor for those estimations. In general, the federal government bond yields, are often considered the pivot of other bonds' yields. Therefore the average

federal government marketable bond yields replace the 90-day treasury bill yields in the estimations of the expected yields of provincial, municipal and corporate bonds. As a result, the goodness-of-fit for those estimations increase notably. For example, the R-square increases from seventy percent to over ninety percent. For further comparison see Appendix II. To illustrate the accuracy of the estimations, the generated expected yields are plotted along with the actual yields. See Figure VI to X). This is, however, not a test of term structure. It is merely getting the best estimate of yields, using the extrapolative-regressive method. The real test is: whether expectation is both extrapolative and regressive. In order to give the method a fair judgement, the most accurate results must be obtained.

The estimations also produce the following polynomial distributed lag coefficients for the various yields generated with the estimation of (7):

- (1) The polynomial distributed lag coefficients for r_{CB}^e (expected Government of Canada bond yields) generated with the estimation of (7) is:

$$r_{CB,t} = a_0 + A_0 r_t + \sum_{i=0}^{16} A_i r_{t-i} + \mu_t$$

<u>Coefficients</u>	<u>t-ratio</u>	<u>Coefficients</u>	<u>t-ratio</u>
$A_0 = 0.6385$	(5.58)	$A_9 = 0.0162$	(1.96)
$A_1 = -0.0473$	(-0.78)	$A_{10} = 0.0194$	(3.08)
$A_2 = -0.0366$	(-0.94)	$A_{11} = 0.0215$	(4.04)
$A_3 = -0.0265$	(-1.17)	$A_{12} = 0.0219$	(3.54)
$A_4 = -0.0172$	(-1.44)	$A_{13} = 0.0210$	(2.66)
$A_5 = -0.0085$	(-1.07)	$A_{14} = 0.0184$	(2.07)
$A_6 = -0.0008$	(-0.09)	$A_{15} = 0.0140$	(1.68)
$A_7 = 0.006$	(0.59)	$A_{16} = 0.0079$	(1.42)
$A_8 = 0.0116$	(1.18)		

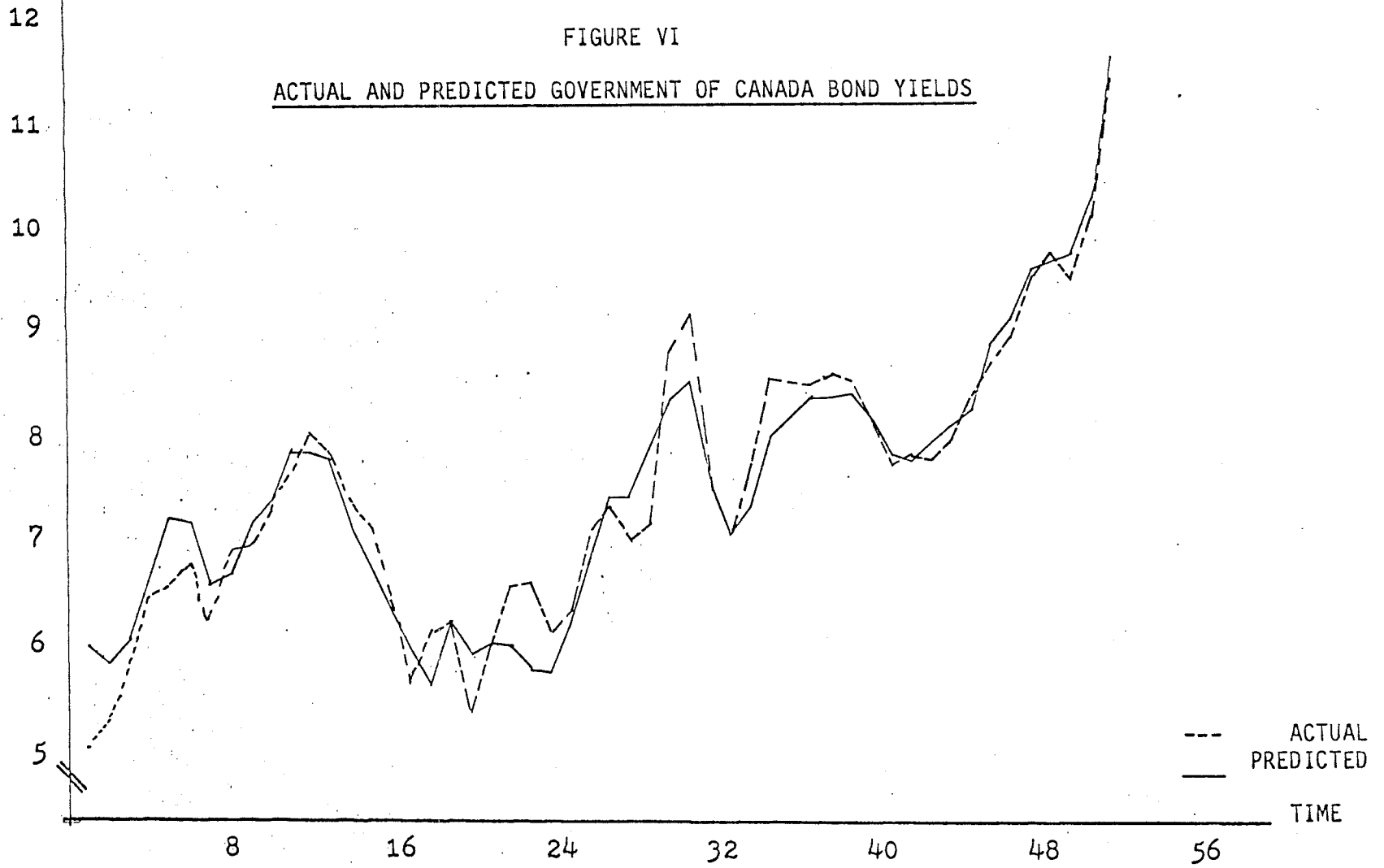
$$\bar{R}^2 = 0.92$$

$$F = 33.75 \quad D-W = 0.66$$

FIGURE VI

ACTUAL AND PREDICTED GOVERNMENT OF CANADA BOND YIELDS

AVERAGE
BOND
YIELD
(%)

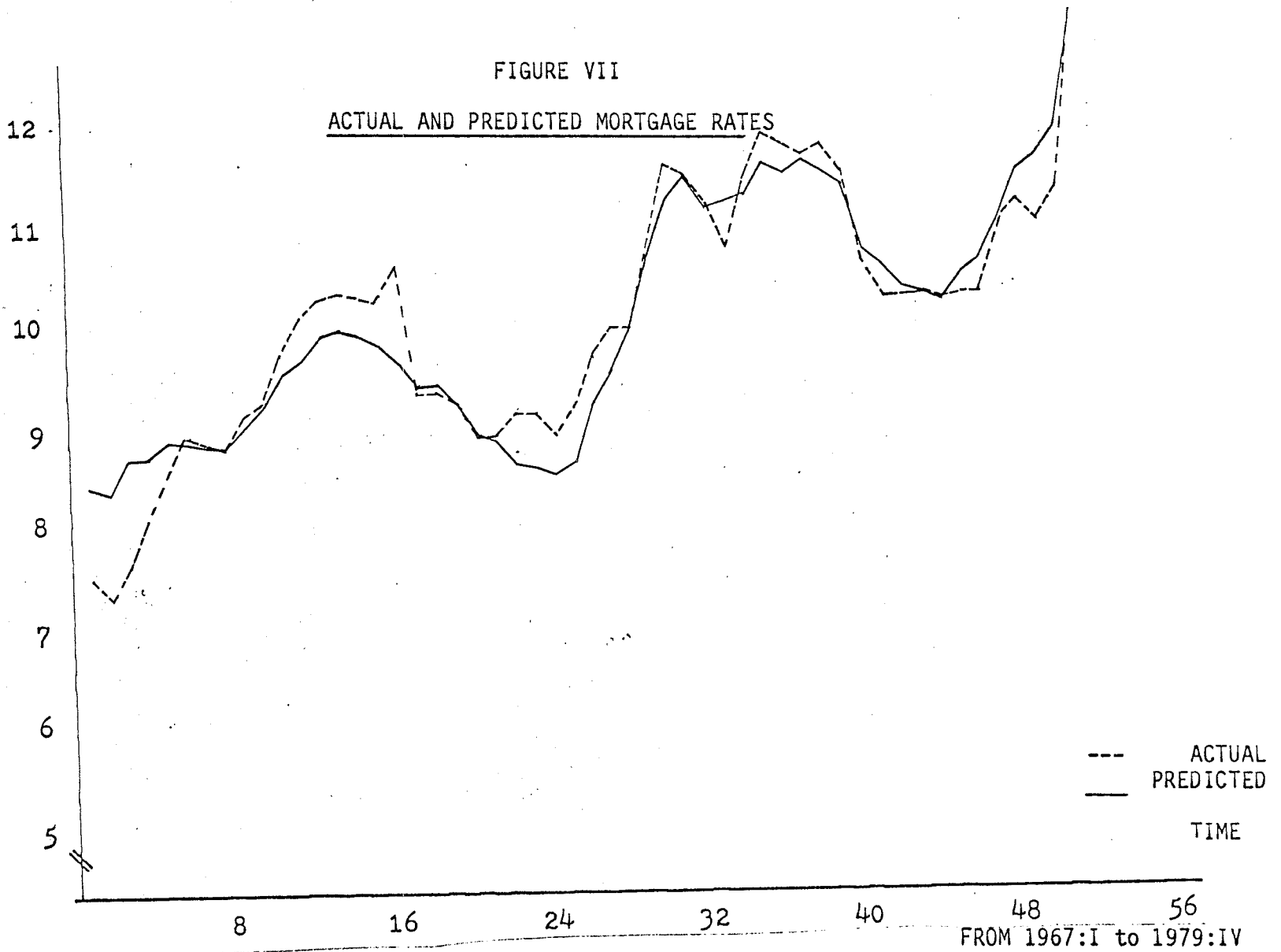


FROM 1967:I to 1979:IV

FIGURE VII

ACTUAL AND PREDICTED MORTGAGE RATES

AVERAGE
MORTGAGE
RATE
(%)

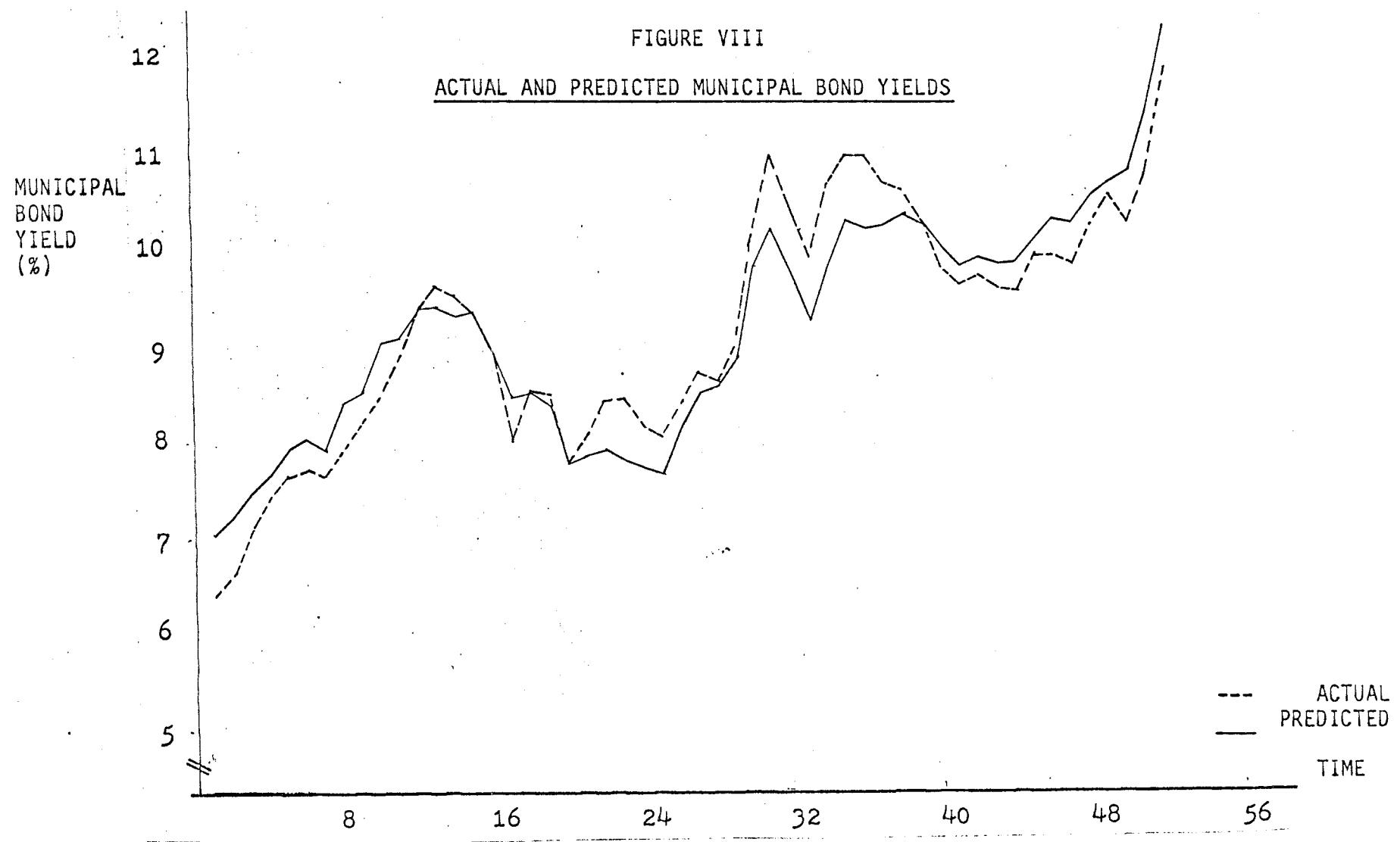


--- ACTUAL
— PREDICTED
TIME

8 16 24 32 40 48 56
FROM 1967:I to 1979:IV

FIGURE VIII

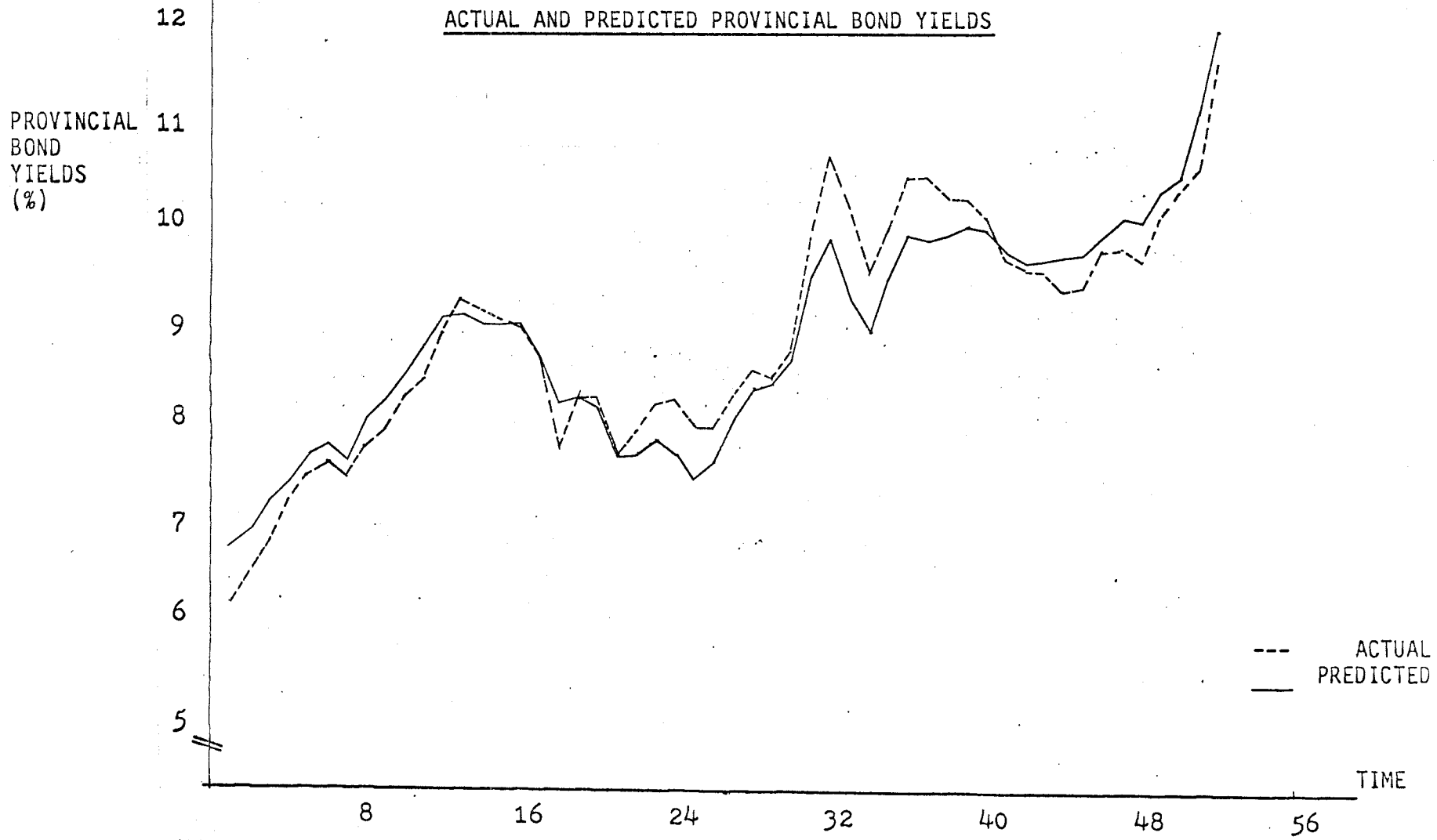
ACTUAL AND PREDICTED MUNICIPAL BOND YIELDS



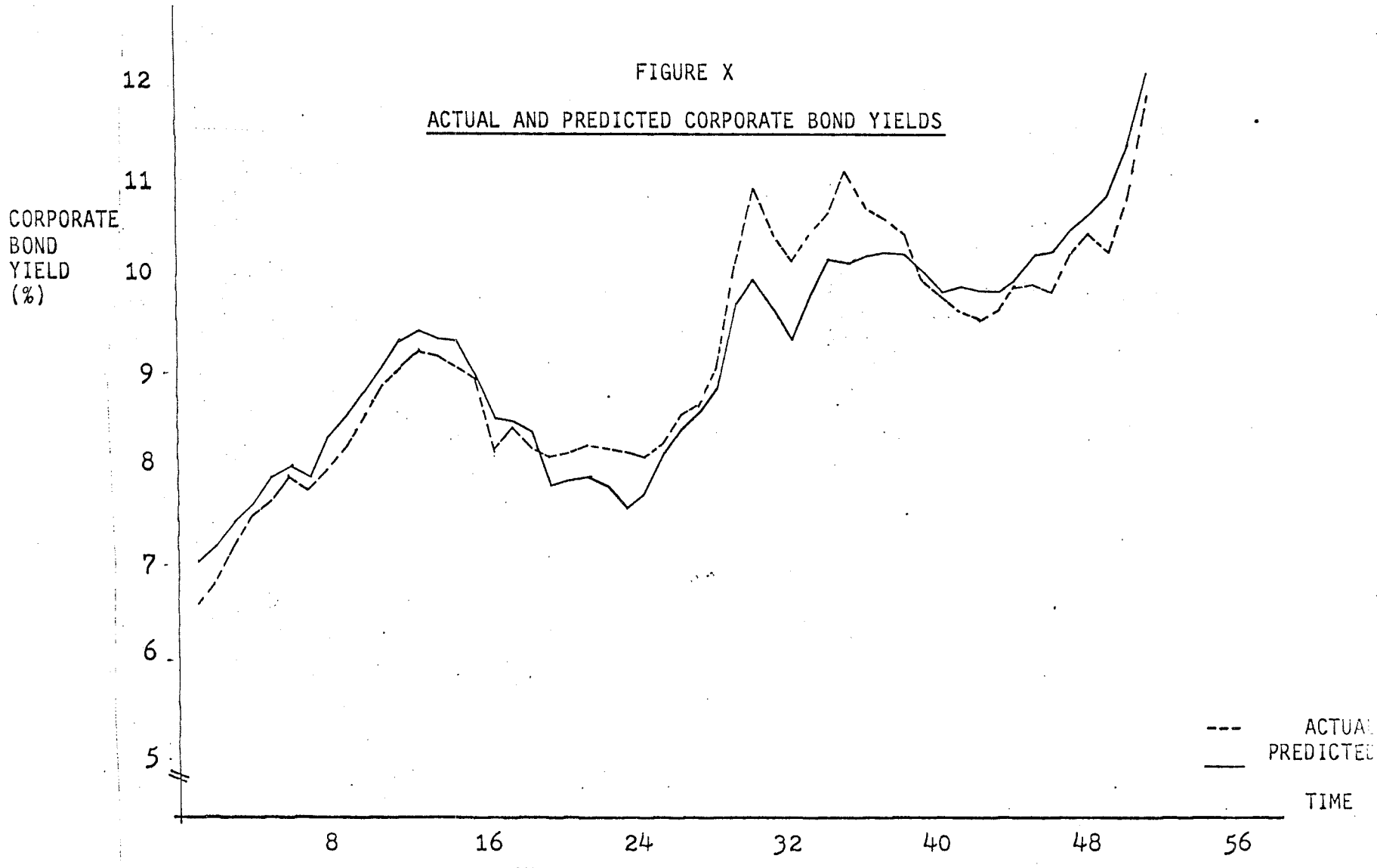
FROM 1967:I to 1979:IV

FIGURE IX

ACTUAL AND PREDICTED PROVINCIAL BOND YIELDS



FROM 1967:I to 1979:IV



FROM 1967:I to 1979:IV

- (2) The polynomial distributed lag coefficients for r_{mt}^e (expected mortgage rates) generated with the estimation of (7) is:

$$r_{mt,t} = a + b_0 r_t^L + \sum_{t=1}^{16} b_i r_{t-i}^L + \mu_t$$

<u>Coefficients</u>	<u>t-ratio</u>	<u>Coefficients</u>	<u>t-ratio</u>
b ₀ = 0.3327	(2.42)	b ₉ = 0.0223	(2.21)
b ₁ = 0.0061	(0.08)	b ₁₀ = 0.0081	(0.79)
b ₂ = 0.0333	(0.73)	b ₁₁ = -0.0054	(-0.45)
b ₃ = 0.0505	(1.95)	b ₁₂ = -0.0166	(-1.22)
b ₄ = 0.0593	(3.94)	b ₁₃ = -0.0244	(-1.76)
b ₅ = 0.0609	(4.78)	b ₁₄ = -0.0273	(-2.28)
b ₆ = 0.0567	(4.22)	b ₁₅ = -0.0240	(-2.34)
b ₇ = 0.0479	(3.65)	b ₁₆ = -0.0132	(-0.76)
b ₈ = 0.0360	(3.10)		

$$\bar{R}^2 = 0.87$$

$$F = 19.61 \quad D-W = 0.43$$

- (3) The polynomial distributed lag coefficients for r_{MB}^e (expected yields for provincial bonds) generated with estimation of (7) is:

$$r_{MB,t} = a + x_0 r_t^{CB} + \sum_{j=1}^{16} x_j^{ce} r_{t-j} + \mu_t$$

<u>Coefficients</u>	<u>t-ratio</u>	<u>Coefficients</u>	<u>t-ratio</u>
x ₀ = 0.5508	(2.85)	x ₉ = 0.0383	(2.35)
x ₁ = -0.0553	(-0.47)	x ₁₀ = 0.0058	(0.34)
x ₂ = 0.0259	(0.37)	x ₁₁ = -0.0241	(-1.13)
x ₃ = 0.0797	(2.08)	x ₁₂ = -0.0476	(-1.88)
x ₄ = 0.1099	(4.49)	x ₁₃ = -0.0609	(-2.35)
x ₅ = 0.1203	(4.91)	x ₁₄ = -0.0603	(-2.77)
x ₆ = 0.1146	(4.41)	x ₁₅ = -0.0419	(-2.60)
x ₇ = 0.0965	(3.98)	x ₁₆ = -0.0021	(-0.75)
x ₈ = 0.0698	(3.48)		

$$\bar{R}^2 = 0.89$$

$$F = 23.77 \quad D-W = 0.18$$

- (4) The polynomial distributed lag coefficients for r_{OB}^e (expected yields or corporate bonds) generated with estimation of (7) is:

$$r_{OB,t} = a + z_0 r_t^{CB} + \sum_{l=1}^{16} z_l r_{t-l}^{CB} + \mu_t$$

<u>Coefficients</u>	<u>t-ratio</u>	<u>Coefficients</u>	<u>t-ratio</u>
$z_0 = 0.4189$	(2.23)	$z_9 = 0.0331$	(2.20)
$z_1 = 0.0107$	(0.10)	$z_{10} = 0.0050$	(0.32)
$z_2 = 0.0646$	(0.99)	$z_{11} = -0.0203$	(-1.04)
$z_3 = 0.0981$	(2.78)	$z_{12} = -0.0399$	(-1.72)
$z_4 = 0.1142$	(5.07)	$z_{13} = -0.0509$	(-2.14)
$z_5 = 0.1156$	(5.12)	$z_{14} = -0.0504$	(-2.52)
$z_6 = 0.1053$	(4.40)	$z_{15} = -0.0419$	(-2.60)
$z_7 = 0.0965$	(3.98)	$z_{16} = -0.0032$	(-0.12)
$z_8 = 0.0611$	(3.31)		

$$\bar{R}^2 = 0.89$$

$$F = 23.77 \quad D-W = 0.21$$

- (5) The polynomial distributed lag coefficients for r_{PB}^e (expected yields for provincial bonds) generated with estimation of (7) is:

$$r_{PB,t} = a + \theta_0 r_t^{CB} + \sum_{j=1}^{16} \theta_j r_{t-j}^{CB} + \mu_t$$

<u>Coefficients</u>	<u>t-ratio</u>	<u>Coefficients</u>	<u>t-ratio</u>
$\theta_0 = 0.466$	(2.78)	$\theta_9 = 0.040$	(2.86)
$\theta_1 = 0.032$	(0.35)	$\theta_{10} = 0.022$	(2.13)
$\theta_2 = 0.061$	(1.05)	$\theta_{11} = 0.006$	(0.63)
$\theta_3 = 0.079$	(2.35)	$\theta_{12} = -0.009$	(-0.87)
$\theta_4 = 0.088$	(4.67)	$\theta_{13} = -0.020$	(-1.52)
$\theta_5 = 0.088$	(5.88)	$\theta_{14} = -0.026$	(-1.76)
$\theta_6 = 0.082$	(4.85)	$\theta_{15} = -0.026$	(-1.84)
$\theta_7 = 0.071$	(2.95)	$\theta_{16} = -0.017$	(-1.88)
$\theta_8 = 0.056$	(3.36)		

$$\bar{R}^2 = 0.90$$

$$F = 29.41 \quad D-W = 0.21$$

EMPIRICAL TEST OF EXTRAPOLATIVE-REGRESSIVE METHOD

The polynomial distributed coefficients, of the distributed lags generated with estimation of (7), provide some interesting results. In all cases, the R^2 seem to be reasonably high which means high percentages of variation is in the dependent variable are explained by the independent variables. To test the significance of these estimations, the F-Test (see Appendix III for procedures and details) is used. The F-Test applied on the estimations are conducted at ninety-nine percent confidence level. As it turns out, the F-Test demonstrates that the estimations are significant. Table II summarizes the F-Test for the estimations.

TABLE II

Summary of F-tests	Degree of Freedom		Critical Value (99%)	F-Value	Hypothesis	
	N	D			$H_0: R^2=0$	$H_1: R^2 \neq 0$
Expected Government Bond Yields	17	52	2.44	33.75	reject	accept
Expected Mortgage Rates	17	52	2.44	19.61	reject	accept
Expected Provincial Bond Yields	17	52	2.44	29.4	reject	accept
Expected Municipal Bond Yields	17	52	2.44	23.77	reject	accept
Expected Corporate Bond Yields	17	52	2.44	23.77	reject	accept

For the individual coefficient and to test its significance, the t-ratio test is employed. The t-ratio test is to test if the individual coefficient equals zero. At a ninety percent confidence interval, the critical value for t-ratio is ± 2.11 for two tails test. Any coefficient with t-ratio that lies within this range (± 2.11) is considered insignificant. Judging from the t-ratios provided by the estimations, the significance of the distributed lag variables differ. In the Government of Canada bond yields, the distributed lag variables are insignificant from one to eight lags. For other estimations, distributed lag variables have significant impact on the expected yields from the third to the ninth lags. This lack of total significant impact from the distributed lag variables on expected yields seems to suggest that expectation is partially regressive.

As for the extrapolative component, all coefficients (A_0 , b_0 , θ_0 , X_0 and Z_0) are significant. This suggests that expectation contains an extrapolative element. Another factor that may affect the estimations is the problem of serial correlation. That is, the error terms are not randomly distributed through time. Instead, a strong continuity of error terms exists; this is reflected in the low Durbin-Watson statistics.

4.2 DATA

The primary data source for the stock and flow quantities used in model(6) is from The Statistics Canada's Financial Flow Accounts (1966 to 1981) and Statistics Canada's Financial Institutions' Financial Statistics, Quarterly (1967 to 1979) and from The Bank of Canada Review

(1967 to 1979) and The Report of Superintendent on Insurance Companies. The sample consists of 52 quarterly observations beginning in 1967:I and ending in 1979:IV.

Since the analysis focuses on the demand for government (federal) marketable bonds, it is advisable to examine the distribution of these bonds, that is, the holders of these bonds. By the year end of 1979, the percentages of government marketable bonds held by the types of holder are as follows (excluding Treasury bills):

TABLE III

DISTRIBUTION OF GOVERNMENT MARKETABLE BONDS BY TYPES OF HOLDER

<u>Types of Holder</u>	<u>Amount (millions)</u>	<u>Percentages</u>
General Public	\$21,493	61.61
Bank of Canada	9,311	26.69
Chartered Banks	3,471	9.96
Government of Canada	610	1.74
	<u>\$34,888</u>	<u>100.00%</u>

Source: Table III is compiled with data from The Bank of Canada Review, Bank of Canada.

Among these holders, the Bank of Canada, Chartered Banks and the Government of Canada are not considered to be the source of arbitraging-pressure in the bond market. In the case of the Bank of Canada and the Government of Canada, they hold government marketable bonds - not as an investment, but as an instrument to monitor money supply and the bond market. The chartered banks, on the other hand, treat their government marketable bonds holding as an investment, and, to a certain extent, they directly expand the money supply because when chartered banks buy

government marketable bonds, say x million dollars, they, in essence, create an equal x million dollars of deposits. Therefore, the holder that is relevant to the analysis of this thesis is the general public. After all the general public held 61.61 percent of the outstanding government marketable bonds as of year-end 1979. If one has to find the source of arbitraging-pressure in the bond market, there is nowhere better than the investors contained in the general public. However, not all investors are active market participants in the bond market. There are some investors contained in the general public, that hold government marketable bonds as a captive fund. These holdings are not marketable. Thus, in the empirical analyses, certain categories of investors are selected. The selection is based on the amount of government guaranteed marketable bonds and the types of financial assets they hold.

The distribution of government guaranteed marketable bonds, held by categories of investor in the general public as of year-end 1979, is listed on the following page.

Among the categories of investor listed in Table IV, Persons and Unincorporated Business, Trusteed Pensions Funds, Life Insurance Companies, Fire and Casualty Insurance Companies and Trust and Mortgages Companies are analyzed. By the year-end 1979, the above five categories of investor held 37.26 (13,001 ÷ 34,888) percent of the total outstanding government guaranteed marketable bonds. And, they represent 60.50 (13,001 ÷ 21,493) percent of the outstanding government guaranteed marketable bonds held by the general public.

The majority of the investors examined here hold at least some amount of the large number of different types of financial assets. In

TABLE IV
DISTRIBUTION OF GOVERNMENT MARKETABLE BONDS HOLDING YEAR-END 1979

<u>Bonds Held By</u>	<u>Amount (million)</u>	<u>Percentages</u>
Persons and Unincorporated Businesses	4820	22.43
Trusted Pension Funds	3280	15.26
Life Insurance Companies	1964	9.14
Fire and Casualty Insurance Companies	1875	8.72
Provincial and Municipal Governments	1134	5.28
Trust and Mortgages Companies	1062	4.94
Local & Central Credit Union and Caises Populaires	512	2.28
Social Security Funds	107	0.50
Rest of the World	<u>5220</u>	<u>24.29</u>
	<u>\$19,974</u>	<u>92.84%</u>

Source: Table IV is compiled from The Bank of Canada Review, Bank of Canada and Financial Flow Account, Statistic Canada.

the analysis, the procedure used in estimation is selecting the few assets which, along with the Government of Canada marketable bonds, comprise the major elements in the investors' aggregate portfolios. Table V lists the likely asset groups for each of the five investor categories and shows the percentage of the sectors' total financial portfolio for which each asset was accounted for, as of the year-end 1979.

Before discussing the actual data, perhaps it is beneficial to briefly define each investor category and the financial assets in the

TABLE V
PRINCIPAL ASSETS FOR SIX INVESTOR CATEGORIES

<u>Category and Assets</u>	<u>% of Total Financial Assets</u>
Persons and Unincorporated Business:	
Currency and Deposits	40.64
Stocks	19.46
Mortgages	5.01
Canada Savings Bonds	4.58
Government of Canada Bonds	1.17
Subtotal	<u>70.86</u>
Trust and Mortgages Companies:	
Mortgages	75.75
Currency and Deposits	4.71
Stocks	4.12
Other Bonds	2.45
Government of Canada Bonds	2.34
Provincial Bonds	0.88
Municipal Bonds	0.45
Subtotal	<u>90.70</u>
Trusted Pension Plans:	
Provincial Bonds	27.78
Stocks	22.78
Mortgages	15.74
Other Bonds	15.54
Municipal Bonds	3.87
Currency and Deposits	3.75
Government of Canada Bonds	2.50
Subtotal	<u>91.96</u>
Fire and Casualty Insurance Companies:	
Government of Canada's Bonds	18.50
Other Bonds	15.76
Provincial Bonds	14.75
Stocks	13.46
Currency and Deposits	6.58
Municipal Bonds	4.82
Mortgages	3.56
Subtotal	<u>77.43</u>
Life Insurance Companies:	
Mortgages	43.10
Other Bonds	21.72
Provincial Bonds	7.89
Government of Canada Bonds	6.20
Stocks	6.08
Municipal Bonds	2.44
Subtotal	<u>87.43</u>

Source: Table V is compiled with the data from The Bank of Canada Review, Bank of Canada and Financial Flow Account, Statistics Canada.

aggregate portfolio. The Persons and Unincorporated Business, Trust and Mortgages and Life Insurance Companies and Fire and Casualty Insurance Companies categories of investor are self explanatory. The Trusteed Pension Plans category consists of both private and other industrial Pension plans.

The make-up of the financial assets in the portfolio listed on Table V are hereby explained. Financial assets such as Canada Savings Bonds is self explanatory. Currency and deposits include short-term deposits, demand deposits and cash balances. Government of Canada bonds, Provincial bonds, and Municipal bonds are the total outstanding marketable bonds (excluding Treasury bills), regardless of term to maturity. Other bonds consist mostly of corporate bonds and therefore are treated as such. For Mortgages, they consist of residential, commercial and National Housing Association (NHA) mortgages. The items contained in the stocks are the common shares and preferred-shares.

The yields for currency and deposits are yields based on the chartered banks' 90-day deposit rates. The yields of Canada Savings Bonds are the actual coupon rates. The average expected yields for government marketable bonds are generated with treasury bills' yields as proxy. The average Government bond yields are established with the average yields of all outstanding government marketable bonds of different maturity (excluding treasury bills). For provincial, municipal and other bonds, their expected yields are estimated with past average government marketable bonds' yields as proxy. Of course, the dependent variables for these estimations are their own yields. The mortgage rates are the average rates of residential, commercial and National Housing Association (NHA)

mortgages. The expected mortgage rates are estimated with average bank loan rates as proxy. Other non-yield variables which influence investors' choice are represented by X_1 and X_2 ; the investors' stock market index (Canada); and lagged one-period percentage change of r_{CB}^e respectively. X_2 can be considered the risk factor when investing in federal government marketable bonds.

The use of average rates may prompt some criticism. For example, using the expected average rates for mortgage may diminish the impact of demand for other financial assets. Because for some investors, their mortgages consist mostly of commercial mortgages. For others, the mortgages are mostly residential. By the same token, for the various bonds, it is equally correct to argue that there are some investors who prefer to invest in long-term bonds rather than short-term, or vice versa.²⁶ To overlook this point may result in a gross over-estimate or under-estimate of the results. However, one can argue that because the mortgage rates for residential and commercial, short-term bonds and long-term bonds are so closely (positively) correlated, using the expected average rate and expected average yield would probably not alter the accuracy of the estimations (unless the theoretical foundation of expectations theory is questionable). If all expected yields are based on averages, then there should not be any skepticism of the estimations.

As for the data, the bond demand variable -- Δi_t in equation(6) -- is the net purchases of government of Canada marketable bonds during the quarter for all five investor categories. The wealth flow variable -- ΔW_t in equation(6) -- is the net acquisitions of financial assets for four investor categories: Persons and Unincorporated

Businesses, Trusteed Pension funds, Fire and Casualty Insurance Companies, Trust and Mortgages Companies. For Life Insurance Companies the wealth flow variable, ΔW_t , is the net acquisitions of financial assets less net policy loans. This is because policy loans are exogenous to the portfolio behaviour of life insurance companies. In other words, the portion of cash flow available for investment is the portion of wealth flow variable remaining after policy loans. The asset stock variables A_{kt} and ΔW_t , for all five investor categories, are constructed by decrementing backward from the end-of-year stocks and wealth for 1979 using quarterly flows data.²⁰ The lag wealth (W_{t-1}) variable is represented by the total financial assets of the previous period.

EMPIRICAL RESULTS

The equations listed below are the results of estimating equation(6) for each of the five major categories of investors' demand for government marketable bonds. The dependent variable in each case is denominated in millions of dollars. The numbers in parenthesis are ratios of the estimated values to standard errors for each coefficient. In other words, they are the asymptotic t-ratios (or t-statistics). The R^2 (adjusted) is the coefficient of determination adjusted for degrees of freedom. SE is the standard error of estimation, and D-W is the Durbin-Watson statistic. For clarity and convenience, the variable symbols listed below are consistent for all five equations, with letter superscripts indicating distinctions among corresponding variables for different investor categories.

Because of the number of financial assets available in the financial

market, and because of the diversity of financial assets within each investor's portfolio, it is extremely difficult to pin-point the securities that have significant impact on the demand for government marketable bonds. Therefore, the results of the estimations presented below are the results by the trial-and-error method. The estimations that produce the best significant goodness-of-fit and significant coefficients are listed. The asterisk indicates that the variable is generated by the extrapolative-regressive method.

- CB = holding of Government of Canada Marketable bonds,
- PB = holding of Provincial bonds,
- MB = holding of Municipal bonds,
- OB = holding of other bonds (mostly corporate bonds),
- MT = holding of mortgages (both commercial and residential),
- CSD = holding of cash and deposits,
- CSB = holding of Canada Savings Bonds,
- r^e_{CB} = expected government of Canada marketable bonds yields,*
- r^e_{PB} = expected Provincial bond yields,*
- r^e_{MB} = expected Municipal bond yields,*
- r^e_{OB} = expected Corporate bond yields,*
- r^e_{MT} = expected mortgage rates,*
- r_{SR} = short-term bank rates,
- x_1 = Investors' stock market index (Canada),
- x_2 = lagged one-period percentage change of r^e_{CB} ,

The following are the estimated results. These estimations are based on fifty-two quarterly observations from 1967:1 to 1979:IV. The following estimations have the following constraints. (1) The estimations must

include the combined term of expected own-yield and change in wealth, $(r_{CB}^e, t \cdot \Delta W_t)$; (2) The sum of all own-yield and alternative yields coefficients are restricted to zero. The first constraint is required because demand for Government of Canada marketable bonds should be sensitive to own-yield. The second constraint is important in maintaining the balance sheet identity. If there is no change in yields, or if all yields change equally, then no portfolio share should be affected. The values in parenthesis are the t-ratios.

(1) Pension Plan:

$$\begin{aligned}
 CB_t = & 47.13 + 0.199 \Delta W_t^P + 0.130 (r_{CB}^e, t \cdot \Delta W_t^P) \\
 & (0.57) \quad (2.96) \quad (3.82) \\
 & - 0.346 (r_{PB}^e, t \cdot \Delta W_t^P) + 0.118 (r_{MB}^e, t \cdot \Delta W_t^P) \\
 & (-1.39) \quad (0.78) \\
 & + 0.098 (r_{OB}^e, t \cdot \Delta W_t^P) + 0.085 CB_{t-1}^P \\
 & (0.39) \quad (2.59) \\
 & + 0.011 MTP_{t-1} - 0.013 X_1 \\
 & (0.86) \quad (-1.80) \\
 \bar{R}^2 = & 87 \quad F = 36.25 \quad SE = 54.81 \quad D-W = 1.60
 \end{aligned}$$

(2) Life Insurance Companies:

$$\begin{aligned}
 CB_t = & 153.62 + 0.278 \Delta W_t^L + 0.006 W_{t-1}^L \\
 & (2.01) \quad (3.23) \quad (0.24) \\
 & + 0.024 (r_{CB}^e, t \cdot \Delta W_t^L) + 0.001 (r_{CB,zt}^e \cdot W_{t-1}^L) \\
 & (0.33) \quad (1.18) \\
 & + 0.730 (r_{PB;t}^e \cdot \Delta W_t^L) - 0.756 (r_{OB;t}^e \cdot \Delta W_t^L) \\
 & (2.47) \quad (-2.88)
 \end{aligned}$$

$$+ 0.0140 (BL_{t-1} - 0.198 PB_{t-1} - 0.014 X_1)$$

$$(0.21) \quad (-0.97) \quad (-2.37)$$

$$\bar{R}^2 = 63 \quad F = 7.95 \quad SE = 40.88 \quad D-W = 2.19$$

(3) Fire and Casualty Insurance Companies:

$$CB_T = -45.10 - 0.026W_{t-1}^F + 0.085(r_{CB,t}^e \cdot \Delta W_t^F)$$

$$(-2.81) \quad (-1.87) \quad (3.90)$$

$$+ 0.003(r_{CB,t}^e \cdot W_{t-1}^F) - 0.082(r_{SR,t} \cdot \Delta W_t^F)$$

$$(2.60) \quad (-3.18)$$

$$+ 0.122CSD_{t-1}$$

$$(1.81)$$

$$\bar{R}^2 = .73 \quad F = 24.01 \quad SE = 27.27 \quad D-W = 1.95$$

(4) Trust and Mortgage Companies:

$$CB_t = 141.20 + 0.062 \Delta W_t^T + 0.026 (r_{CB,t}^e \cdot \Delta W_t^T)$$

$$(2.43) \quad (1.60) \quad (1.65)$$

$$- 0.003 (r_{MT,t}^e \cdot \Delta W_t^T) + 0.009 (r_{SR,t}^e \cdot \Delta W_t^T)$$

$$(-0.16) \quad (1.19)$$

$$- 0.033 (r_{OB,t}^e \cdot \Delta W_t^T) + 0.022 CB_{t-1}^T - 0.164 OB_{t-1}^T$$

$$(-1.25) \quad (0.35) \quad (-1.53)$$

$$- 0.011 X_1$$

$$(-1.92)$$

$$\bar{R}^2 = 36 \quad F = 3.02 \quad SE = 31.51 \quad D-W = 2.12$$

(5) Persons and unincorporated Businesses:

$$CB_t = -10807 - 0.034 \Delta W_t^H + 0.083 W_{t-1}^H$$

$$(-2.77) \quad (-1.05) \quad (2.82)$$

$$= 0.013 (r_{CB,t}^e \cdot \Delta W_t^H) + 0.0003 (r_{CB,t}^e \cdot W_{t-1}^H)$$

$$(-1.35) \quad (1.09)$$

$$\begin{aligned}
 &+ 0.013 (r_{SR,t}^e \cdot \Delta W_t^H) - 0.445 CB_{t-1}^H \\
 &\quad (1.29) \qquad \qquad \qquad -3.88) \\
 &- 0.112 CSD_{t-1}^H - 0.179 CSB_{t-1}^H) \\
 &\quad (-2.70) \qquad \qquad \qquad (-2.86)
 \end{aligned}$$

$$\bar{R}^2 = 39 \quad F = 3.44 \quad SE = 305.86 \quad D-W = 2.58$$

EMPIRICAL TESTS ON THE SIGNIFICANCE OF THE COEFFICIENTS

To test the significance of individual coefficients, the t-statistic test is again applied. In each case, the t-ratio is tested at ninety percent confidence interval (a two tail test). The critical values for t-ratios depend on the degree of freedom numerator and denominator. If the estimated t-ratio lies within the critical values, the coefficient (associated with that t-ratio) is considered insignificant. That is, statistically, ninety percent of the time the estimated coefficient is zero. Thus we accept the null hypothesis that $\beta_i = 0$. When the coefficient is significant, then we accept the alternative hypothesis that $\beta_i \neq 0$.

The tests are summarized in Table VI. The asterisk beside the t-ratio indicates that the coefficient is insignificant.

For the five estimations, the results indicate the significance of the expected yields and the flow variable. For Life Insurance Companies, Pension Plan and Trust and Mortgage Companies, the flow variable is important and has significant positive impact on the demand for federal government marketable bonds (CB). On the other hand, the flow variable is insignificant in Fire and Casualty Insurance Companies. It has a significant negative impact in Person and Unincorporated Businesses.

TABLE VI
T-TESTS

Expected signs for the Co-efficients	Variables	Life Insurance companies	Pen-sion Plan	Fire & Casualty Ins. Cos.	Trust & Mort-gage Co.	Person & un-corporated Bus.
	Intercept	2.01	0.57*	-2.81	2.43	-2.77
+	ΔW_t	3.23	2.96		1.60*	-1.05*
unknown	W_{t-1}	0.24*		-1.87		2.82
+	$re_{CB,t} \cdot \Delta W_t$	0.33*	3.82	3.90	1.65*	-1.35*
unknown	$re_{CB,t} \cdot W_{t-1}$	1.18*		2.60		1.09*
-	$re_{PB,t} \cdot \Delta W_t$	2.47	-1.39*			
-	$re_{DB,t} \cdot \Delta W_t$	-2.88	0.39*		-1.25*	
-	$re_{MB,t} \cdot \Delta W_t$		0.78*			
-	$re_{MT,t} \cdot \Delta W_t$				-0.16*	
-	$re_{SR,t} \cdot \Delta W_t$			-3.18	1.19*	1.29*
unknown	CB_{t-1}	0.21*	2.59		0.35*	-3.88
unknown	PB_{t-1}	-0.97*				
unknown	MB_{t-1}					
unknown	OB_{t-1}				-1.53*	
unknown	CSD_{t-1}			1.81		-2.70
unknown	CSB_{t-1}					-2.86
unknown	MT_{t-1}		0.86*			
unknown	X_1	-2.37			-1.92	
unknown	X_2		-1.80			
	Critical value	<u>+1.68</u>	<u>+1.68</u>	<u>+1.67</u>	<u>+1.68</u>	<u>+1.68</u>
	\bar{R}^2	63	87	73	36	39

The lagged wealth (W_{t-1}) has significant (negative and positive) impact in Fire and Casualty Insurance Companies and Person and Unincorporated Businesses respectively. W_{t-1} is insignificant for other investors.

The coefficient of the combined expected own yield and the flow term ($r_{CB,t}^e \cdot \Delta W_t$) is significant in Pension Plan, Fire and Casualty Insurance Companies, and Trust and Mortgage Companies. The coefficient bears a positive sign (+) as postulated by Model(6). This suggests that these investors' demand for CB is sensitive to the movement in the expected own yield ($r_{CB,t}^e$) and on the positive current - quarter flow variable (ΔW_t). As indicated by the coefficient (of $r_{CB,t}^e \cdot \Delta W_t$), the parameter of demand for CB is quite high. The parameters of demand in Pension Plan, Fire and Casualty Insurance companies, and Trust and Mortgage companies are +0.130, +0.085 and +0.026. The estimation of Fire and Casualty Insurance Companies also indicates that combined expected own yield and lagged wealth variable (W_{t-1}) is significant. This suggests the demand for CB depends on the expected own yield ($r_{CB,t}^e$), flow variable (ΔW_t) and past wealth (W_{t-1}).

For the cross-parameter of demand, that is the impact of the combined terms from the yields of alternative financial securities e.g., ($r_{MB,t}^e \cdot \Delta W_t$), ($r_{PB,t}^e \cdot \Delta W_t$), ($r_{OB,t}^e \cdot \Delta W_t$), ($r_{MT,t}^e \cdot \Delta W_t$) and ($r_{SR,t} \cdot \Delta W_t$), the model postulates that their coefficients are negative, suggesting the inverse relationship to the demand for government of Canada marketable bonds. However, some of the empirical results show otherwise. The institution which has positive

significant coefficients of $(r_{PB,t}^e \cdot \Delta W_t)$ is Life Insurance Companies. The inconsistency may be attributable to the positive correlation of yield among the various financial securities. For Life Insurance Companies, the combined term $(r_{OB,t}^e \cdot \Delta W_t)$ shows a significant negative impact on the demand for CB. The cross-parameter of demand is -0.756. Similarly, $r_{SR,t}^e \cdot \Delta W_t$ has a significant negative impact on the demand for CB in Fire and Casualty Insurance Companies. The cross-parameter of demand is -0.082.

When each estimation is interpreted as a whole, these investors seem to exhibit different techniques in portfolio management. For Life Insurance Companies and Pension Plan, Portfolio Managers seem to focus on the allocation of new wealth. This allocation of new wealth depends on the expected yields of various financial securities. For Fire and Casualty Insurance Companies, the investment decision depends on the expected yields, change in wealth, past wealth and on the liquidity requirement. The liquidity requirement factor is indicated by the significant coefficient of the combined term $(r_{SR,t}^e \cdot \Delta W_t)$. In Person and Unincorporated Businesses, investment decisions basically depend on the past wealth (W_{t-1}). The estimation also indicates that investors in this category are very active in their stock adjustment. Thus suggesting that this category of investors provide a strong source of arbitraging pressure.

COMMENTS AND SUMMARY

For all five institutional investors, their wealth flows (ΔW_t) are important in their bond purchases. This does not suggest that these

investors are totally unresponsive to the expected own yield if the wealth flow is zero. As postulated, investors may be willing to fund their purchases through stock adjustments. From the estimations there is little evidence suggesting such stock adjustments occur. The only significant lagged own-stock and lagged alternative stocks adjustments are from Persons and Unincorporated Business. Limited stock adjustments also occur in Pension Plan, Fire and Casualty Insurance Companies and Trust and Mortgage Companies. This evidence suggests that these institutional investors manage their portfolios in a more conservative and inactive manner. In essence, one can conclude that these institutions are more inclined to allocate new wealth among the available financial assets rather than readjust the existing portfolio in conjunction with new wealth to achieve a new desirable portfolio equilibrium (based on equation(6)). In addition, four out of five institutions have produced insignificant coefficients for W_{t-1} and $(r_{CB,t}^e \cdot W_{t-1})$. This, together with the lack of stock adjustments, provide further evidence to support the unwillingness of these investors to release funds from existing financial assets to support new purchases. The adjusted R-squares for Trust and Mortgage Companies, and for Persons and Unincorporated Business do not suggest an acceptable goodness-of-fit. This leads to questionable adoption of equation(6) in the study of Canadian Institutional investors. Comments on this are reserved for Chapter Five.

However, when the overall significance of the estimated regressions are tested (see Appendix II), the results suggest that the coefficients are jointly or simultaneously NOT equal to zero at the ninety-nine

percent confidence level. As illustrated in Appendix II, the F-Test rejected the hypothesis ($H_0: R^2 = 0$) and accepted the alternative hypothesis ($H_1: R^2 \neq 0$).

The significant positive β_{ii} , negative β_{ik} , $k \neq i$ and some degree of stock adjustments can be drawn as evidence substantiating the existence of arbitraging. The significant positive β_{ij} and negative β_{ik} suggest that when there is a significant increase in the expected yield of government bonds, relative to alternative securities, investors will shift to purchase government bonds. By the same token, investors will also readjust their existing stocks. That is, to release funds for the purchase of additional government bonds. This sort of arbitraging will put downward pressure on the yield of government bonds and put upward pressure on alternative yields. Until the yields for all securities become relatively equal, arbitraging continues. Before then, β_{ii} and β_{ik} will continue to be significantly positive and negative respectively. Empirical results also suggest that the degree of arbitraging varies among the five institutional investors. This suggests the maturity preferences of different investors are so strong that they never purchase securities outside their preferred maturity range to take advantage of yield differentials (an institutional or hedging-pressure theory argument).

The empirical analyses on the demand for the Federal Government marketable bonds for five institutional investors provides several conclusions. First, it is reasonable to assume (based on the evidence) that the five institutionals are quite inactive in their portfolio readjustment. However, there exists a marginal difference in degree of

inactiveness among these institutions. For example, Life Insurance Companies and Trust and Mortgage Companies are content with the allocation of new wealth among the wide spectrum of financial assets whereas other institutions (although to a limited degree) are engaged in some form of stock adjustment. Second, the importance of investor's expectations on the various yields and the wealth flow are acknowledged.

Though the empirical study does not provide any convincing evidence to support portfolio readjustment among the institutions, it provides some evidence to support the importance of expectations and flow variables in the analysis. Above all, why are these institutions different from those in the United States? (comparing this study with the study done by B. Friedman 1977). Comments about the differences and the ways in which Canadian institutions manage their portfolio are reserved for Chapter Five.

CHAPTER FIVE

COMMENTS, SUMMARY, AND CONCLUSION5:1 Comments

Several comments can be deduced from the empirical study. First of all, the study points out some important aspects of Canadian institutional investors and the way they manage their investment portfolio. Second, the study induces one to re-examine the Canadian financial market and the role of the government regarding institutional investment.

In the former case, most of it has been discussed in the latter part of Chapter Four. However, with regard to the way in which Canadian institutional investors manage their portfolio, empirical results estimated with equation(6) do not suggest active portfolio readjustment. Instead, the results point toward reallocation of new wealth (indicated by the significance of ΔW_t and $r_{CB,t}^e \cdot \Delta W_t$).

The inconsistency of significant negative impact by the combined alternative yields and flow variable can be attributable to the wide spectrum of financial assets that are available to the investors. Another contributing factor is the relatively close correlation of their yields (particularly the various bonds). That is, when the yields of the available bonds are so closely and positively correlated, they pose the problem of multicollinearity. This is actually what happens when all expected yields are included in the analysis; the results are high R^2 and insignificant coefficients. Yet another problem which may be attributable to the less-than-acceptable equation(6) is the size of government of Canada marketable bonds in a portfolio. This suggests a

preference for other financial securities rather than government of Canada marketable bonds. Unless the expected yield for government bonds becomes more attractive or the expected yield becomes relatively higher than normal compared to other expected yields, institutional investors prefer other financial securities. When investors have strong preferences, it is difficult to estimate their behaviour based on equation(6). In addition to these arrays of complexity, investment management of these institutions often try to match their long-term liabilities with their long term assets. In some cases, government regulatory constraints play an important role.

COMPARISON OF U.S. AND CANADA'S EMPIRICAL RESULTS

When the empirical results presented in this thesis are compared with the results provided by B. Friedman on United States financial institutions, one can draw several different conclusions. In particular, the ways in which the institutions manage their portfolio; and the degree of market pressure exists in the bond market. The following estimations illustrate the results obtained in United States and in Canada. Table VII illustrates the estimated results of the institutions. Values in parenthesis are t-statistics.

Because of the varieties of financial assets available in Canada and the diversity of the portfolio (of Canada institutions) it is difficult to compare the portfolio behaviour of U.S. institutions and Canada institutions. If one has to estimate Canadian institutions with specification similar to the U.S., the estimations may not illustrate the full impact of expected yields and change in wealth. Therefore, in most cases,

Canadian estimations contain more independent variables.

Table VII compares only four investor categories. Trust and Mortgage Companies are excluded for the simple reason that U.S. estimation on this investor category is not available. When comparing the U.S. estimations with the Canadian estimations, one has to bear in mind that U.S. estimations are not constrained. That is, the sum of all own-yield and alternative yields coefficients for each investor category are not restricted to zero whereas Canadian estimates are restricted.

The estimations illustrated in Table VII imply a diverging portfolio managing technique between the United States' and Canada's portfolio managers. In the U.S. the demand for financial securities is always sensitive to own-yield and change in wealth. In all cases, the combine term $(r_{CB,t} \cdot \Delta W_t)$ always produces a significant positive coefficient. By the same token, alternative yields always have a negative impact. Whereas in Canada, some of the institutions display their insensitivities toward own-yield. In all cases the coefficient is less significant than the U.S. In some cases, their sensitivities toward alternative yields are quite opposite to the U.S. institutions (take for example the Life Insurance Companies and Pension Plan).

In terms of the willingness to adjust existing portfolios, U.S. institutions are far more robust than their counterparts. From the empirical results, their portfolio readjustments are very significant (indicated by high t-ratio), and are an important process of portfolio management. In all cases, except for households, the coefficients of

TABLE VII
 COMPARISON OF EMPIRICAL RESULTS ESTIMATED WITH EQUATION(6)
 USING DATA FROM UNITED STATES AND CANADA

Institutions Variables	Life Insurance Companies		Other Insurance Companies		Pension Plan		Households	
	U.S.	Canada	U.S.	Canada*	U.S.	Canada**	U.S.	Canada
Intercept	-3354 (-3.9)	153.62 (2.01)		-45.10 (-2.81)		47.13 (0.57)		-1080T (-2.77)
ΔW_t	-1.177 (-1.7)	0.278 (3.23)				0.199		-0.034 (-1.05)
W_{t-1}	0.1465 (3.5)	0.006 (0.24)	1696 (12.0)	-0.026 (1.87)		0.2407 (3.3)		0.084 (2.82)
<u>Wt times</u> own-yield	0.455 (3.6)	0.024 (0.33)	0.1401 (2.5)	0.085 (3.90)	0.1546 (6.5)	0.130 (3.82)	0.0314 (4.4)	-0.013 (-1.35)
alternative yields	-0.2624 (-3.4)	0.730 (2.47)	-0.131 (-1.8)	-0.082 (-3.18)	-0.8157 (-4.6)	-0.346 (-1.39)	-0.022 (-1.6)	+0.013 (1.29)
	-0.0084 (-2.5)	-0.756 (-2.88)	-0.0225 (-7.2)			0.118 (0.78)	-0.0163 (-3.5)	
<u>Lagged</u> <u>Wealth</u>						0.098 (0.39)		
<u>(W_{t-1})times</u>								
own-yield	-0.003 (-2.5)	0.001 (1.18)					0.003 (3.5)	0.0003 (1.09)
alternative yields				0.003 (2.60)				
<u>Lagged</u> <u>Stocks</u>								
own-stock	-0.1491 (-1.9)	0.014 (0.21)	-0.1390 (-5.3)		-0.373 (-3.5)	0.085 (2.59)	-0.1167 (-7.1)	-0.445 (-3.88)
alternative stocks	-0.1638 (3.7)	-0.198 (-0.97)	-0.064 (-4.9)	0.122 (1.81)	0.2146 (3.6)	0.011 (0.86)	0.008 (2.2)	-0.112 (-2.70)
			-0.1590 (-15.8)		-0.2493 (-3.4)		-0.006 (-3.9)	-0.179 (-2.86)
<u>Other factors</u>								
X ₁		-0.014						
X ₂		(-2.37)				-0.013 (-1.80)		
\bar{R}^2	.80	.63	.92	.73	.67	.87	.79	.39

* Fire and Casualty Insurance companies

** Both Private and Public Pension Plan whereas only private pension plan in U.S.

lagged stocks are larger and more significant for U.S. institutions than for Canadian institutions. The coefficients of lagged own stock and lagged alternative stocks for Life Insurance Companies are -0.1491 and -0.1638 respectively; whereas in Canada, they are 0.014 and -0.198. Other comparisons can be observed in Table VII. In Canada, there is some degree of portfolio readjustment. The most significant readjustment is experienced by households. In Life, Fire and Casualty Insurance Companies, empirical results indicate that portfolio readjustment is relatively insignificant.

From this comparison, one can infer that portfolio managers in U.S. are far more aggressive. The financial market is more competitive, thus more prone to arbitrageur's actions. In Canada, the financial market (particularly the bond market) is less prone to arbitrageur's actions because institutional investors are more willing to allocate new wealth than to readjust existing wealth, thus suggesting the arbitrageurs' actions alone cannot be solely responsible for how the expectations theory's results coming about. One will find that the tentative conclusion drawn is consistent with the conclusion drawn at the end of the chapter.

OTHER ESTIMATIONS

Furthermore, when no restrictions are applied to equation(6), the following estimations are obtained. These estimations provide the best estimation of the demand for government marketable bonds (in terms of significant coefficients and the overall significance of the estimations). Estimations for Fire and Casualty Insurance Companies remains the same (with or without restrictions).

(1) Life Insurance Companies:

$$\begin{aligned}
 CB_t = & 189.46 + 0.052 (r^{e}_{CB,t} \cdot W^L_t) + 1.18 (r^{e}_{PB,t} \cdot W^L_t) \\
 & (3.34) \qquad (1.74) \qquad (3.57) \\
 & -1.32 (r^{e}_{OB,t} \cdot W^L_t) + 0.119 (r^{e}_{MT,t} \cdot W^L_t) \\
 & (-3.76) \qquad (2.56) \\
 & -1.78 X_1 \\
 & (-3.53)
 \end{aligned}$$

$$R^2 = .69 \quad F = 23.00 \quad SE = 27.27 \quad D-W = 1.95$$

(2) Pension Plan:

$$\begin{aligned}
 CB_t = & 169.85 - 0.764 W^P_t + 0.980 (r^{e}_{PB,t} \cdot W^P_t) \\
 & (4.5) \quad (-5.1) \quad (4.7) \\
 & -0.335 (r^{e}_{MB,t} \cdot W^P_t) - 0.771 (r^{e}_{OB,t} \cdot W^P_t) \\
 & (-2.1) \qquad (-3.0) \\
 & +0.208 (r^{e}_{MT,t} \cdot W^P_t) + 1.701 X_1 \\
 & (4.5) \qquad (1.9) \\
 & -0.123 PB_{t-1} + 0.327 MT_{t-1} - 0.147 OB_{t-1} \\
 & (-2.7) \qquad (4.4) \qquad (-3.5)
 \end{aligned}$$

$$R^2 = .93 \quad F = 23.00 \quad SE = 40.85 \quad D-W = 1.68$$

(3) Trust and Mortgage Companies:

$$\begin{aligned}
 CB_t = & 138.87 + 0.027 (r^{e}_{CB,t} \cdot W^T_t) - 0.020 (r^{e}_{MT,t} \cdot W^T_t) \\
 & (3.70) \quad (3.84) \qquad (-3.24) \\
 & -1.32 X_1 \\
 & (-3.75)
 \end{aligned}$$

$$R^2 = .40 \quad F = 12.42 \quad SE = 30.54 \quad D-W = 2.15$$

(4) Households:

$$CB_t = -8586.3 - 0.312 W_t^H + 0.071 W_{t-1}^H + 0.032 (r_{CB,t}^e \cdot W_t^H)$$

$$(-2.31) \quad (-2.79) \quad (2.61) \quad (2.66)$$

$$-0.519 CB_{t-1}^H - 0.176 CSB_{t-1}^H - 0.089 CSD_{t-1}^H$$

$$(-4.34) \quad (-2.90) \quad (-2.27)$$

$$R^2 = .33 \quad F = 5.178 \quad SE = 296.96 \quad D-W = 2.52$$

The unrestricted estimations show that the demand for CB is sensitive to the expected own yield and the change in wealth, except for Pension Plan. For Pension Plan and Life Insurance Companies, the coefficients of the combined term $(r_{PB,t}^e \cdot \Delta W_t)$ is positive (as indicated by the parameters +0.980 and +1.18 respectively). These results may be attributable to the fact that provincial bond yields are correlated to the yields of CB. The results may also be attributable to the investors preference for provincial bonds. In each case, provincial bonds account for a significant percentage of total portfolio; for example, Pension Plan has 27.78 percent of the total portfolio invested in provincial bonds while Life Insurance Companies have 7.89 percent. Positive signs are also obtained from the combined term $(r_{MT,t}^e \cdot \Delta W_t)$. This is due to the fact that these investors tend to match long-term liabilities with long-term assets.

To explain these results in another way, suppose there is no change in wealth (ΔW_t) then the change in CB (ΔCB) can occur only when there is an equal change in other assets. That is, $CB_t = \theta_{ki} (A^*_{Kt} - A_{K,t-1})$ where $K =$ other assets. When investors are insensitive to the expected yields and do not adjust their stocks, θ_{ki}

equals zero. Theoretically, θ_{ki} can be negative or positive. The following illustrates when θ_{ki} is positive or negative:

- (1) θ_{ki} is positive when ΔCB_t is negative and $A^*_{kt} - A_{k,t-1}$ is negative;
- (2) θ_{ki} is negative when ΔCB_t is negative and $A^*_{kt} - A_{k,t-1}$ is positive.

If θ_{ki} is negative and β_{ki} is negative, the combined coefficient turns positive. In the Life Insurance Companies and Pension Plan Cases, such occurrences exist as investors decide to put less emphasis on CB. In the Pension Plan portfolio, CB accounted for over six percent of the total portfolio in 1967. By the end of 1979, the percentage has reduced to 2.5 percent. For Life Insurance Companies, the percentage declined from 4% at the end of 1967 to 2.5% at the end of 1976. Since then, the percentage has been increasing steadily.

Despite the above inconsistency, results from the rest of the estimations do agree with the postulations of model(6).

The unrestricted estimations, once again, reiterate the conclusions arrived with restricted estimations. That is, some investors seem to focus on the reallocation of new wealth while others are quite sensitive to expected yields, change in wealth and past wealth. Above all, there is little evidence that suggests stock adjustment.

SUMMARY AND CONCLUSIONS

To summarize the thesis, Chapter One discusses the problems related to the study of term structure of interest rates. Because of the tremendous amount of work done on the subject, perhaps attention should

be focused on other subjects, making use of the knowledge available on term structure of interest rates. Such subject is the portfolio behaviour of investors. Thus the object of the thesis is to observe the portfolio behaviour of five investor categories with respect to expected own-yield and expected alternative yields. The model used for the analysis is one developed by B. Friedman, called the Optimal Marginal Adjustment Model of Portfolio Behaviour. In essence, the model consists of portfolio selection and stock adjustment. The combination of the two closely approximate investors portfolio behaviour. Based on the Optimal Marginal Adjustment Model of Portfolio Behavior, the demand for government of Canada marketable bonds by five institutional investors are analyzed.

In spite of the many theories advanced to explain the term structure of interest rates, the notion of expectations seems to prevail. In fact, other theorists do not deny the importance of expectations in explaining the term structure. They (opponents of expectations theory) argue that besides expectations, there are additional factors that ought to be considered, such as liquidity and hedging-pressure. In any case, the advocates of the respective theory are able to marshal strong empirical evidence in support of their theory.

In the context of investment and managing an investment portfolio, expectations on the market and yields have always been the priority. Of course, the needs and transactions costs of the investor are important too. The latter are reflected within the system of estimation -- for example, if transaction costs are high, it is cheaper (and easier) for most investors to allocate new cash flows rather than to readjust existing stocks. Therefore, the analysis not only deals with the

portfolio behaviour of the five investor categories, it also deals with the expectations theory and the element of arbitraging from which the expectations results come about.

In the structural approach to investors' portfolio behaviour -- particularly their demand for Federal Government marketable bonds, expectations of the various yields (for the various available financial assets in the market) are very crucial. In this case, the expected yields for the various instruments are generated by the extrapolative- regressive method, a method which has been widely used to test the expectations theory. The generated expected yields and the flow variables are used in the estimation of equation(6). The institutions in the analysis are the Pension Plan, Life Insurance Companies, Fire and Casualty Insurance Companies, Trust and Mortgage Companies, and Persons and Unincorporated Businesses. These institutions together hold over fifty percent of outstanding government guarantee marketable securities. In addition, they all hold a wide variety of financial securities with various maturity structures. In short, their portfolios are very diversified.

The empirical study though, does not produce any decisive evidence. Nevertheless, it produces sufficient evidence to address the objective of the thesis. First, the results provided by the extrapolative-regressive method does produce significant evidence to support that the expectation is extrapolative. Evidence also suggests that expectations is partially regressive. Second, estimation with equation(6) provides significant evidence to support the importance of flow variables and expected own yield as well as alternative yields. There is also evidence suggesting that portfolio management of the institutions analyzed is quite

conservative. They are more inclined to allocate new wealth rather than to reallocate existing wealth together with new wealth to achieve at a more desirable portfolio equilibrium. This leads one to postulate that portfolio management among these institutions is mostly in the form of portfolio selection rather than an optimal marginal adjustment of portfolio behaviour as postulated by equation(6).

As implied by the evidence, there exists some degree of arbitraging among the institutions. Though the degree varies from one institution to another, the act of arbitraging by these institutions enables the expectations results to come about. The difference in the degree of arbitraging further suggests and supports the hedging-pressure theory of term structure. That is, different institutions have different maturity preferences; and some institutions have such strong preferences that they never purchase securities outside their preferred maturity range regardless of yield differential.

In the final analysis, the implications provided from the empirical results and empirical analysis reiterate the complexity involving the explanation of the term structure. It acknowledges the importance of expectations theory in the term structure as well as the role it plays in the demand for financial assets. It also acknowledges the importance of arbitraging which enables expectations results come about. However, not all institutions are arbitraguers as suggested, thus acknowledging the notion that different institutions have different maturity preferences. Therefore, I submit, and it is generally accepted by many others, that the term structure of interest rates is better explained by two or more theories together. That is, the theories should be used to compliment

each other rather than to oppose each other.

Evidence related to stock adjustments are important. It represents the source of arbitraging pressure in the bond market. In Canada, the source of arbitraging pressure is provided mostly by households and Trusted Pension Plan. They, together, hold over thirty-seven percent of the total outstanding Federal Government marketable bonds held by the general public (refer to table IV). The other institutions do not seem to exert arbitraging pressure in the bond market. If thirty-seven percent of outstanding Federal Government marketable bonds held by the general public are actively traded, this can have quite a substantial effect on term structure. Therefore any term structure theory that includes expectations-based arbitrage, the theory can quite adequately explain the term structure of interest rates in Canada.

APPENDIX I

MODEL (6)

The optimal marginal adjustment model of portfolio behaviour is expanded from stock adjustment and the portfolio selection models. In the original form of the model, as postulated B.M. Friedman, is as follows:

$$(1) \Delta A_{it} = \sum_k^N \theta_{ik} (\alpha_{kt}^* \cdot W_{t-1} - A_{k,t-1}) + \alpha_{it}^* W_t, \quad i=1, \dots, N. \quad (A-1)$$

substitute α_{kt}^* and α_{it}^* in equation (1) where

$$\alpha_{it}^* = \sum_k^N \beta_{ik} r_{kt} + \sum_h^M \gamma_{ih} X_{ht} + \bar{\pi}_i, \quad i=1, \dots, N, \quad (A-2)$$

and

$$\alpha_{kt}^* = \sum_j^N \beta_{kj} r_{jt} + \sum_h^M \gamma_{kh} X_{ht} + \bar{\pi}_k, \quad k=1, \dots, N \quad (A-3)$$

Equations (A-2) and (A-3) are identical; the only difference is that of the yield-variables, r_{kt} in (A-2) and r_{jt} in (A-3). In both cases, the selection process is not only sensitive to alternative yield, but also to own yield. This can be reflected by adding the appropriate yield in each case. Substituting α_{kt}^* and α_{it}^* into equation (A-1) gives the following:

$$(2) \Delta A_{it} = \sum_k^N \theta_{ik} \left[\left(\sum_k^N \beta_{ki} \cdot r_{it} + \sum_k^N \gamma_{kh} X_{ht} + \bar{\pi}_k \right) \cdot W_{t-1} - A_{k,t-1} \right] + \left(\sum_k^N \beta_{ik} r_{kt} + \sum_h^M \gamma_{ih} X_{ht} + \bar{\pi}_i \right) \cdot \Delta W_t$$

$$(3) A_{it} = \sum_k^N \theta_{ik} \left[\sum_k^N \beta_{ki} \cdot r_{it} \cdot W_{t-1} + \sum_k^N \gamma_{kh} X_{ht} W_{t-1} + \bar{\pi}_k \cdot W_{t-1} - A_{k,t-1} \right] + \sum_k^N \beta_{ik} r_{kt} \cdot \Delta W_t + \sum_h^M \gamma_{ih} X_{ht} \cdot \Delta W_t + \bar{\pi}_i \cdot \Delta W_t \quad (A-4)$$

Knowing that the selection process is sensitive to own-yield and alternative yields, equation (A-4) is expanded in the following form:

$$(4) \Delta A_{it} = \sum_k^N (\theta_{ik} \sum_k \beta_{ki}) r_{it} \cdot W_{t-1} + (\sum_j \theta_{ij} \beta_{ik}) r_{kt} \cdot$$

$$W_{t-1}$$

$$+ (\sum_k \theta_{ik} Y_{kh}) X_{ht} \cdot W_{t-1} + (\sum_k \theta_{ik} \pi_k) W_{t-1}$$

$$+ (\sum_k \beta_{ik} \cdot r_{kt}) \cdot \Delta W_t + (\beta_{ii} \cdot r_{it}) \Delta W_t + (\sum_h Y_{ih} \cdot X_h) \Delta W_t$$

$$+ \pi_i \cdot \Delta W_t$$

(A-5)

By grouping the variables, equation (A-5) is written as follows:

$$(5) \Delta A_{it} = \pi_i \cdot \Delta W_t + (\sum_k \theta_{ik} \pi_k) \cdot W_{t-1}$$

$$+ (\beta_{ii} \cdot r_{it}) \cdot \Delta W_t + \left[\sum_k (\beta_{ki} \cdot \theta_{ik}) \right] \cdot r_{it} \cdot W_{t-1}$$

$$+ \sum_{k \neq i} \left\{ (\beta_{ik} \cdot r_{kt}) \cdot \Delta W_t + (\sum_j \beta_{jk} \cdot \theta_{ij}) \cdot r_{kt} \cdot W_{t-1} \right\}$$

$$+ \sum_h \left\{ Y_{ih} \cdot X_h \cdot \Delta W_t + \sum_k (Y_{kh} \cdot \theta_{ik}) \cdot X_h \cdot W_{t-1} \right\}$$

$$- \theta_{ii} \cdot A_{i,t-1} - \sum_{k \neq i} (\theta_{ik} \cdot A_{k,t-1})$$

(A-6)

APPENDIX II

The following estimations are the results of the extrapolative-regressive method. The expected yields for provincial, municipal, and other bonds are regressed with treasury bills yields. Numbers in the parentheses are the t-ratios.

For Provincial bonds the polynomial distribution lag proxy r^{ePB} (expected provincial bond's yields) generated with estimation of (7) is:

$$r_{PB,t} = \theta + a_0 r_{TB,t} + \sum_{i=0}^{16} a_i r_{TB,t-1} + \mu_t$$

<u>Coefficients</u>	<u>t-ratio</u>	<u>Coefficients</u>	<u>t-ratio</u>
$a_0 = 0.228$	1.21	$a_9 = 0.014$	1.05
$a_1 = 0.052$	0.53	$a_{10} = 0.012$	1.05
$a_2 = 0.045$	0.71	$a_{11} = 0.009$	1.08
$a_3 = 0.039$	1.03	$a_{12} = 0.007$	0.87
$a_4 = 0.034$	1.64	$a_{13} = 0.006$	0.53
$a_5 = 0.029$	2.33	$a_{14} = 0.004$	0.31
$a_6 = 0.025$	1.90	$a_{15} = 0.003$	0.19
$a_7 = 0.022$	1.39	$a_{16} = 0.001$	0.13
$a_8 = 0.018$	1.14		
$\bar{R}^2 = 0.68$		$F = 6.50$	

The polynomial distribution lag proxy for r^{eOB} (expected other bond's yields) generated with estimation of (7) is:

$$r_{OB,t} = a + b_0 r_{TB,t} + \sum_{i=0}^{16} b_i r_{TB,t-1}^{+} \mu_t$$

<u>Coefficients</u>	<u>t-ratio</u>	<u>Coefficients</u>	<u>t-ratio</u>
$b_0 = 0.23$	(1.27)	$b_9 = 0.014$	(1.08)
$b_1 = 0.043$	(0.46)	$b_{10} = 0.009$	(0.86)
$b_2 = 0.044$	(0.71)	$b_{11} = 0.004$	(0.54)
$b_3 = 0.042$	(1.13)	$b_{12} = 0.003$	(-0.33)
$b_4 = 0.039$	(1.94)	$b_{13} = 0.006$	(-0.46)
$b_5 = 0.036$	(2.89)	$b_{14} = 0.007$	(-0.50)
$b_6 = 0.031$	(2.41)	$b_{15} = 0.006$	(-0.51)
$b_7 = 0.026$	(1.73)	$b_{16} = 0.004$	(-0.51)
$b_8 = 0.021$	(1.34)		
$R^2 = 0.69$		$F = 6.81$	

The polynomial distribution lag proxy for r_{MB}^e (expected marginal bond's yields) generated with estimation of (7) is:

$$r_{MB,t} = z + \theta_0 r_{TB,t} + \sum_{i=0}^{16} \theta_i r_{TB,t-i} + \mu_t$$

<u>Coefficients</u>	<u>t-ratio</u>	<u>Coefficients</u>	<u>t-ratio</u>
$\theta_0 = 0.23$	(1.14)	$\theta_9 = -0.015$	(0.96)
$\theta_1 = 0.048$	(0.47)	$\theta_{10} = -0.009$	(0.78)
$\theta_2 = 0.046$	(0.68)	$\theta_{11} = -0.005$	(0.49)
$\theta_3 = 0.043$	(1.06)	$\theta_{12} = -0.002$	(-0.21)
$\theta_4 = 0.040$	(1.78)	$\theta_{13} = -0.005$	(-0.33)
$\theta_5 = 0.036$	(2.60)	$\theta_{14} = -0.006$	(-0.37)
$\theta_6 = 0.031$	(2.14)	$\theta_{15} = -0.005$	(-0.38)
$\theta_7 = 0.025$	(1.53)	$\theta_{16} = -0.003$	(-0.38)
$\theta_8 = 0.019$	(1.16)		
$\bar{R}^2 = 0.69$		F = 6.81	

The above estimated results show that the prediction of expected provincial, municipal, and other bond's yields are clearly less accurate than the estimated results provided by regressing government of Canada bond's yields. First of all, the former produces a low adjusted R-square. Second, the t-ratios show that treasury bills yields have insignificant influence after seven period (as indicated by the t-ratio when tested at the 95 percent confidence level).

APPENDIX III

F-Test

The F-Test, in essence, tests the significance of the estimated partial regression coefficients. It can also be used as a joint hypothesis test. The F-Test postulates two hypotheses (null hypothesis and alternative hypothesis). The null hypothesis is a joint hypothesis that all coefficients in a regression are jointly or simultaneously equal to zero (i.e., $H_0: R^2 = 0$); whereas the alternative hypothesis suggests that all coefficients in a regression are not jointly or simultaneously equal to zero (i.e., $H_1: R^2 \neq 0$). For the regression results, the F-Test requires a certain degree of freedom (df) numerator and denominator. In each case the df differs. The table below summarizes the F-test for the five estimations (including accepting or rejecting the null hypothesis).

Institutions	Degree of Freedom		Critical Value (99%)	F-Value	Hypothesis	
	N	D			$H_0: R^2=0$	$H_1: R^2 \neq 0$
Pension Plan	9	41	2.89	35.25	reject	accept
Life Insurance Co.	10	40	2.80	7.95	reject	accept
Fire & Casualty Co.	5	45	3.48	24.01	reject	accept
Trust & Mortgage	9	41	2.89	3.02	reject	accept
Persons & Unincorp. Business	9	41	2.89	3.44	reject	accept

FOOTNOTES

Chapter One

1. The term structure of interest rates; Theory, Empirical evidence and application by Malkiel B.G. in Boorman J.T. and Havrilesky T.M. [1] pp. 395.
2. Financial flow variables and the short-run determination of long-term rates, J.P.E. 1977, pp. 672.
3. The structural model used by Christofides, Helliwell and Lester (1976) is part of a more complex model called RDX 2 (developed by the Bank of Canada).

Chapter Two

4. The algebra of bond yields and bond prices is calculated as follows:

$$P = \frac{C}{(1+R)} + \frac{C}{(1+R)^2} + \dots + \frac{C}{(1+R)^N} + \frac{C}{(1+R)^N} \quad (1)$$

where P is the market price of bonds, which is simply the sum of the present values of all the coupons (C) to be received as interest and the principal amount to be received as interest and the principal amount to be paid at maturity;

C is the coupon or interest paid periodically to the bondholder;

F is face value of the bond, that is the principal amount to be paid at maturity;

R is the effective rate per period, which is referred to as the bond's yield to maturity; and

N is the number of years to maturity.

Therefore, according to equation (1), the market value of a bond is determined by four factors.

For any given bond issued, the asking or bidding price (P) of the bond is observed. The coupon (C), the face value (F) and the length of time to maturity (N) are given. By solving equation (1), the annual yield (R) of the bond is derived (with the aid of Bond Tables).

A simplified rule of thumb for the calculation of bond yield to maturity is approximately equal to the average annual interest payment and capital gain return expressed as a percentage of the investor's average investment. Suppose investor A purchased a twenty-year #100 bond at \$85.50 and will receive \$6 annual interest. The average annual capital gain is equal to \$0.725. \$14.50 (100 - 85.50) divided by twenty. Thus the average annual over-all return is \$6.725 per bond. The average investment over the life of the bond is approximated by averaging the current value \$85.00 and the maturity value \$100 (\$92.75). So, yield to average yearly returns = $(6.725/92.75) \times 100 = 7.25\%$. This 7.25% is not too far from the correct yield of 7.40 percent obtained from bond tables.

5. In this example, if investors have funds to invest into these securities, they would be most likely to invest in two-year securities. If the investor invests in a one-year security of 7% and reinvests the proceeds in 8% issues next year, the average yield on the investments is only 7.5%. Whereas, by buying the two-year securities, the investors would earn 8%, regardless of the holding period. Conversely, only 7% would be earned by holding one-year securities to maturity.
6. When investors begin to invest in one specific issue, (say two-year bonds), the price of the bonds would be bid up. The increase in price cause the capital gain (loss) to decrease/increase. The net result is, the return over the investement period declines as calculated by the rule of thumb formula:

$$\text{Return over investment period} = \frac{\text{Coupon payment} + \text{Capital gain (loss)}}{\text{Purchase price}}$$

If investors have to sell off short-term securities to purchase longer-term securities, then the prices of short-term securities fall (yields rise).

7. The forward one-year rate of interest (${}_{t+1}F_1$) for next year is calculated by this formula:

$${}_{t+1}F_1 \equiv \frac{(1+{}_tR_2)^2}{(1+{}_1R_t)} - 1, \text{ as derived from equation (2).}$$

In general, the forward rate of one year securities for any future period can be estimated from this equation:

$${}_{t-N-1}F_1 \equiv \frac{(1+{}_tR_N)^N}{(1+{}_tR_{N-1})^{N-1}} - 1 \quad (\text{F.1})$$

8. See articles by Buse, A., Van Horne, J., and Wallace, N., listed in the Bibliography.
9. Investment Fundamentals by W.C. Freund, Washington (1970), p. 51.
10. A study of Liquidity Premiums on Federal and Municipal Government Securities by Cagan, P., p. 138.
11. Operation Twist: The basic objective of the operation is to twist the structure of interest rates in order to lower long-term rates..., at the same time raising short-term rates. In this operation, long-term government securities are converted into short-term government securities. Thus it shortens the maturity composition of the debt.

12. **Conversion Loan:** It is the reverse of Operation Twist. Conversion Loan converts all short-term government securities into four categories of long-term government securities into four categories of long-term securities. However, one has to bear in mind that monetary authority did not set out to raise long-term rates. The operation was set out to lengthen the average term to maturity of the federal debt. Incidentally, it serves extremely well for researchers studying the term structure of interest rates.

Economists who have studied the effect of the Conversion Loan are Barber, C.L., Christofides, L.N., Dobson, S.W., Lang, W.R., Rasche, H.R., and Pesando, J.E.

13. The equation used in this estimation is:

$$L_t = \alpha + \beta_0 S_t + \sum_{i=1}^{16} 1 S_{T-1} + \eta_t$$

This equation is also used by Phillips and Pippenger.

14. Beta is a measure of uncertainty based on:

$$\beta_n = \frac{\rho_{nm} \sigma_n \sigma_m}{\sigma_m^2}, \text{ where } \sigma_{nm} \text{ is the correlation between}$$

one-period bond returns and return on the market portfolio.

σ_n is the standard deviation of one-period returns on a one-period bond.

σ_m is the standard deviation of one-period returns on the market portfolio.

15. **Efficient capital market:** a review of recent empirical work by E.F. Fama, *J. of Finance*, 1970, pp. 412.

Chapter Three

16. This model is quoted from B.M. Friedman [12], 1977, pp. 663.
17. Friedman, M.B., *Financial Flow Variables and the Short-Run Determination of Long-term Interest Rates*, *Journal of Political Economy*, 1977, Vol. 85, No. 4, pp. 664.
18. Anderson (1965), deLeeuw (1965), and Goldfield (1966) are a few examples of the application of the stock adjustment model to portfolio behaviour.
19. B.M. Friedman [12], pp. 665.
20. *Ibid.* pp. 665.

21. Ibid. pp. 665.
22. Ibid. pp. 668.
23. Ibid. pp. 669.
24. Ibid. pp. 676.

Chapter Four

25. The coefficient ($\beta_{ki} \cdot \theta_{ik}$) is of unknown sign a priori, because θ_{ik} , $k \neq i$ is of unknown sign and since $\beta_{ii} > 0$, presumably $\beta_{ik} < 0$, $k \neq i$, similarly, $\beta_{ki} < 0$, $k \neq i$. If θ_{ik} is assumed negative, then the sum of coefficient ($\beta_{ki} \cdot \theta_{ik}$) is rendered unambiguously positive.
26. For example, by the year-end of 1979, the average-term to maturity of government of Canada marketable bonds held by the general public was 10 years and one month (10:1). Whereas the average-term to maturity of total government outstanding marketable bonds was seven years and two months (7:2). Throughout the history of government marketable bonds, the average-term to maturity of these bonds held by the general public has always been longer than the average-term to maturity of total government outstanding marketable bonds.
27. The object of this procedure is to generate a time-series of end-of-quarter stocks and wealth. The Financial Flow Accounts only produce year-end financial assets and liabilities and quarterly flows.

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