THE UNIVERSITY OF MANITOBA

A MODEL FOR PREDICTING FUEL CONSUMPTION OF GASOLINE FUELED ROAD VEHICLES.

by

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ABSTRACT

In this study, a model is developed for predicting fuel consumption of gasoline fueled road vehicles. This model, which is developed using the regression analysis, is of a parabolic function, with speed as the independent variable for a particular vehicle on a level tangent paved road. The regression coefficients are then related to the vehicle characteristics that account for fuel consumption and also to the road surface type. The effect of the rise and fall of the road is considered separately and other geometrics of the road are only mentioned because there is not enough information to include them.

The resulting model elicits the vehicle characteristics that account for fuel consumption and the effects of road type on excess fuel consumption. Once the simple vehicular parameters supplied by the manufacturers are known, this model can be used for evaluating fuel savings resulting from road surface improvements. Secondly, the optimum speed at which a vehicle should operate for minimum fuel consumption can be estimated from the model.

Unfortunately, the data used in developing this model is insufficient and not quite reliable, therefore a high precision should not be expected from predictions made using the model. However, this does not make the model inadequate for highway planning purposes.

In general, it is observed that a barrier exists between planning and technical information and this has made it hard to obtain sufficient and reliable information for developing models such as this one. In this study an attempt is made to break through this barrier by setting up a technique that invites an integration of technical systems into highway planning. As a result, problems are encountered in the process and these have helped in outlining the way to overcome the barrier.

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CHAPTER I

INTRODUCTION

Although road improvements involve a large expenditure of money, this is offset in part, at least, by the benefits derived from them. The benefits resulting from road improvements range from saving in fuel costs through to saving lives and, therefore, include some benefits that are easily measured in economic terms and others which are not. Thus, in some cases road improvements may be made on social or political grounds without much reference to purely economic considerations. In most cases, however, road improvements are made for economic advantage and economic arguments dominate such cases. Economic benefits are quantifiable in a manner which is not possible for social and cultural benefits and as a result these latter are either ignored or brought in as secondary arguments in favour of the improvements. But in any discussion of costs and benefits there is a likelihood that the secondary question "Cost to whom and benefit to whom?" is not asked.

In the present study a model is proposed for predicting one of the economic benefits of road improvements to vehicle operators. The limitations of the model are clearly recognized and in particular the fact that it does not include social and environment costs is admitted. With such a model it may be easier to develop more sensitivity to these costs at some later stage.

One of the benefits considered in an economic evaluation of road improvements is the reduction in cost to the road users. The road users' costs are quantitatively represented by the vehicle operating costs, which are measured in terms of fuel consumption, oil consumption, tire wear, depreciation, maintenance and travel time. Fuel consumption constitutes about 30% to 45% of all these costs (ref. 1), and depends upon vehicle type, road type and how the vehicle is operated. Thus, the cost of fuel is a sufficiently large proportion of this total cost of operating a vehicle that fuel economy is worthy of serious attention.

Serious difficulties in supplying fuel have hit the Western world, and have already caused rising fuel prices in Canada and in some other countries have led to shortages and rationing of fuel. This fuel problem has in fact become world wide. It seems likely that fuel costs in the future might well account for something like 60% of the total vehicle operating costs. This calls for a far more comprehensive and critical study of how fuel is being consumed by a vehicle on the roadway and technically accounting for each gallon of fuel that is burnt in moving a vehicle.

For the more detailed discussions of fuel consumption that are to follow it is convenient to classify consumption under two headings, "Normal" and "Excess". "Normal" fuel consumption is defined as the fuel consumed by a vehicle running at constant speed on a very good straight tangent, paved, class "A" road. This can be described as the unavoidable fuel consumption. The "excess" fuel consumption is defined as the fuel consumed over and above the "normal" consumption of a vehicle. Such excess consumption is due to such factors as surface roughness, excessive curvature, rise and fall of the road; traffic congestion, traffic lights and environmental factors such as wind, and variations in temperature and altitude. The "excess" fuel consumption is of such a nature that it can be accounted for easily and its reduction can also be easily effected. The "normal" fuel consumption has been measured experimentally for a number of particular vehicles, but this information is not of direct value in assessing the benefits which might accrue from road improvements. Thus an attempt is made here to fit this experimental evidence into a model in which simple vehicular parameters may be used to predict likely fuel consumption under various road conditions, and hence predict excess consumption.

Fuel consumption is affected by three main factors;

(a) the roadway and the environment

(b) driver behaviour and

(c) the vehicle

Increased resistance to travel due to the roadway and its environment result in an increase in fuel consumed by a vehicle. Similarly the way the driver handles his vehicle plays an important role in fuel economy. It is clear that vehicle design also affects fuel consumption though this design aspect seems to be largely ignored in highway planning.

There are numerous parameters that vary from vehicle to vehicle and they can generally be grouped into two groups; the vehicle's external features and the engine in the vehicle. The external features of the vehicle can be considered as the vehicle without the engine. Each of these two plays an important role in the fuel consumption of a vehicle.

Curves of fuel consumption versus speed can, of course, be obtained for any vehicle under various road and environmental conditions by means of simple but relatively costly experiments. If these curves were available for all vehicles the task of the highway planner would be simplified because the assessment of fuel savings due to road improvements would be straightforward. Such curves are available only for a limited number of vehicles and consequently, the effect of road improvements have to be estimated from a general model of the fuel consumption curves. Unfortunately, highway planners do not seem to have understood the technical parameters involved in this kind of model. Their lack of recognition of the part which engine parameters play is a particularly typical example of their general lack of understanding of the role of technical data in planning decisions. In reading the literature one is left with the impression that the planners and the vehicle designers talk different languages and that there are few, if any, interpreters in the field. In terms of fuel economy studies this linguistic barrier is very apparent and the work reported here is an effort to reduce this barrier.

Highway planners most frequently base their fuel consumption models on one vehicle parameter such as weight, horsepower or tire size. Fuel consumption however does not vary exclusively according to any one of these parameters. But simply because one does not know which parameters account for it does not justify assigning one parameter of the external features, as being most appropriate, just because that parameter has the highest correlation coefficient in the statistical analysis.

A group of data can be analysed in this form to give rise to a good model, but such a model will be good only for those particular data. That model may well fail to apply to other data not included in the analysis, and as a result the model must be checked and, likely, modified to account for new data. This process is probably the best that can be applied to systems that have no technical background, such as the population growth ratio of the French and English speaking people in Canada.

The predictive value of such a model is recognized as being limited because it is based on past events for which no precise laws can be enunciated. When dealing with physical systems, however, the predictive value of a model can, in general, be much greater. Because in these cases the relational laws are usually known or discoverable.

The role of statistical analysis, then, tends to be very different when dealing with physical systems as compared with that when dealing with non-physical systems. A simple example will help to illustrate this difference. The relationship between voltage, current and resistance is known to be linear under certain conditions. Given a resistor of unknown value a simple experiment in which current is recorded at different applied voltages will permit an estimate of the resistor to be made. There will be random errors in reading the voltmeter and ammeter but the fact that, from physical laws, a straight-line relationship is expected permits the use of the linear regression technique to obtain the best estimate of resistance. Once made, this estimate is of predictive value in that the voltage/current relationship for that resistor is known and can be applied at any time in the future. Notice, however, that the predictive value stems not from the statistical technique but from confidence in the repeatable nature of physical events of this kind.

When it comes to building a model of fuel consumption, then, its predictive value will depend on the extent to which the coefficients in the model are understood in terms of the physical phenomena involved.

In summary, for a reasonably effective analysis of a technical system, a good understanding of the basic operation of the system is essential. From such a qualitative knowledge, a foundation is set up on which to base a quantitative analysis. To be able to quantify the fuel savings resulting from road improvement with good accuracy it is necessary to understand the fuel consumption of a vehicle on a roadway and the mechnisms that explain its qualitative and quantitative pattern.

CHAPTER II

EXISTING KNOWLEDGE AND TECHNIQUES

As previously indicated, a good deal of study has been done in this field by both Transportation planners and Automotive Engineers, but relevant information is still quite insufficient. Both Transportation planners and Automotive Engineers have individual approaches to the problem, neither of which gives satisfactory results that are useful for predictions.

Basically, it is known from transportation research that for a specific vehicle, fuel consumption, in gallons per mile, decreases from high values at very low speeds to low values at higher optimum speeds and then starts to increase with speed. A fuel consumption curve for a Plymouth car on a straight, level paved surface is represented in Fig. 1, and is typical of those of other vehicles. This and similar curves are obtained by carrying out test runs on specific roads during which the fuel consumption is measured at a constant speed (ref. 3 & 14). Such tests were run on straight level paved roads and gravel roads to obtain the relative difference in fuel consumption as a result of different road surface types. To evaluate the excess fuel consumption due to other road characteristics for a specific vehicle, the vehicle was simply tested on a road having such characteristics and the excess fuel was measured. A lot of effort has been put into this

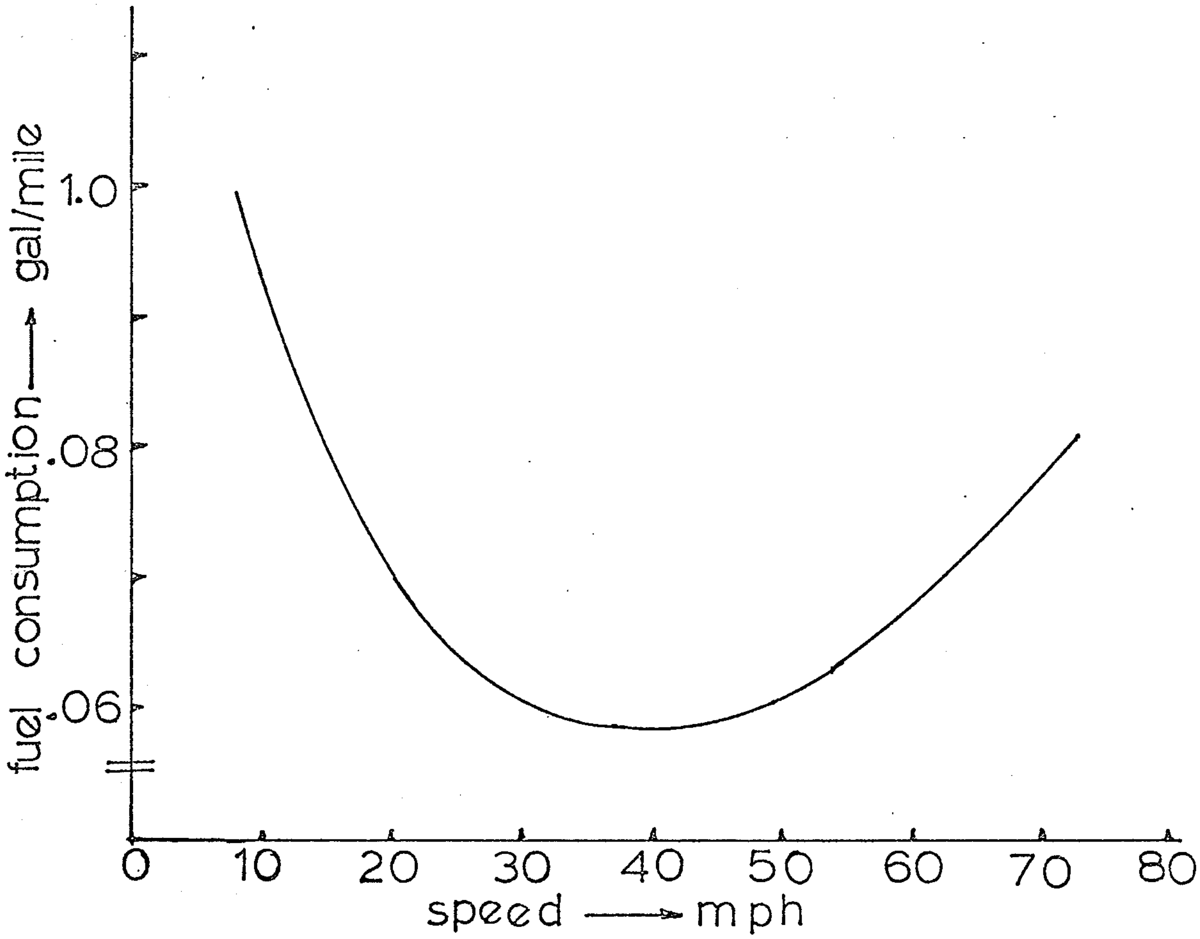

Fig. 1 typical curve of fuel consumption vs speed (for a Plymouth)
(4-door Sedan, 1968 model)

(after Paul J. Claffey, ref. 16)

sort of study, but the approach of each researcher has been very similar and limited. The approach is similar in that test runs were done in the same way with the same limited range of results being yielded. These results restrict prediction of fuel consumption to only those vehicles that had been tested; such results cannot be applied to the untested vehicles because their fuel consumption characteristics are not known, (ref. 1,2,3,14 & 16).

Just observing the traffic on any road for a short time one sees many types of vehicles. If fuel savings are to be reasonably predicted, for that road's improvement, a knowledge of the fuel consumption characteristics of all of these vehicles will be necessary. The fact that these characteristics will vary for each vehicle type is undoubted. It is necessary, then, to develop a format or model by which such curves can be predicted, to a certain degree of accuracy, for any vehicle that has not been tested in the manner described. Since it is not economically justified to test all available vehicles, the few that have been tested have to be taken as a sufficient sample number to be used initially for developing such a model. This model should be such that, knowing certain characteristics of any vehicle, its fuel consumption can be predicted. Carl Saal (ref. 14) in his work did attempt to convey such analysis by developing a graph for predicting fuel consumption for any vehicle, knowing the vehicle's weight. Unfortunately he fell short of the mark, because the vehicle's weight is not the only factor governing fuel consumption. He also used the weight-power ratio which suffered the same shortcoming.

In the work of Roy B. Sawhill and Joseph C. Firey (ref. 3), though they arrived at the same results as others, an attempt was made to bring out the parametric effect of the vehicle on fuel consumption. Predicting fuel consumption for vehicles other than those tested was not their objective, nevertheless, they succeeded in conveying the fact that there is not one specific characteristic of the vehicle that can be used for assessing fuel consumption. From their work, it was reconfirmed that fuel consumption of a vehicle depends on the energy required to overcome friction in the engine and the resistance to the vehicle's motion. However they overlooked one aspect; that these are altogether dependent on the work capacity and efficiency of the engine.

Rolling resistance and wind resistance are the main resistance to the vehicle's motion and they can be evaluated easily. The friction in the engine and engine efficiency which relate to fuel consumption has not received much attention, in the field of Transportation. This is partly because little information is available to the transportation planner and, as well, the little that is available is not well understood. A good understanding of the internal combustion engines would be necessary. This ultimately necessitates a close study combining the efforts of the transportation and automotive engineers.

The approach of the automotive engineers in fuel economy study, regarding application in highway planning, leaves much to be desired. The type of tests they have carried out are of the "test bed" variety and do not take into account the road performance (in gallons per mile) of a specific engine in a specific vehicle. They report results; fuel in lb. per bhp-hr required to drive a specific engine at different engine speeds against a break load on a test-bed. Again, this does not help very much in an analysis in highway planning, since the actual fuel needed to propel the vehicle is not directly obtainable from such results. The resistance to the vehicle's motion is not considered, it is only mentioned. W. S. James (ref. 17a) did attempt to relate engine performance to the vehicle on the road, but he did not arrive at the sort of curves described in Fig. 1. This is perhaps because he used a hypothetical vehicle. Of the other cited literature on Internal Combustion Engines, (ref. 17a,b,c, & d, 18,19, & 20), W.S. James' study is the only one that had a close relation to highway planning. At this point, once more, a mention should be made of the lack of convergence between planning and technical information. From the cited references, it seems as if these two aspects of the system have been kept far apart. To optimize the full use of technology for a better highway system, analysis and planning it is quite necessary to bring these two aspects together.

An attempt is made in this study to interpret the developed fuel consumption curves technically with respect to the vehicles' characteristics, so that a fuel consumption curve can be predicted for any vehicle. This is done by developing similar mathematical models for each of these curves and interpreting the coefficients of these models with respect to the vehicle and the roadway. These models are developed using the available technical information and experimental data. Unfortunately the problems encountered in using the experimental data are two in number. Specifically these are:

(a) the question of the reliability of the data and

(b) the lack of agreement between results from different sources.

This disagreement in results is probably due to differences in environmental conditions at the various researchers test points and periods. The first discrepancy is not unusual in experimental results, so it has to be accepted. The second discrpeancy is of major importance in that it will constitute the primary source of large errors in generalising the fuel consumption pattern of vehicles. The three main sources of data (ref.1,3 & 16), have that problem in that; the paved roads in each case don't seem to be of the same age or standard; altitude, wind and weather conditions were not the same; and the vehicles tested were of different age and mileage accumulation. These might seem trivial at first glance, but they actually are of considerable importance when many vehicles are considered. The effect of each one of these are shown in Figs. 2,3,4 & 5. These initial

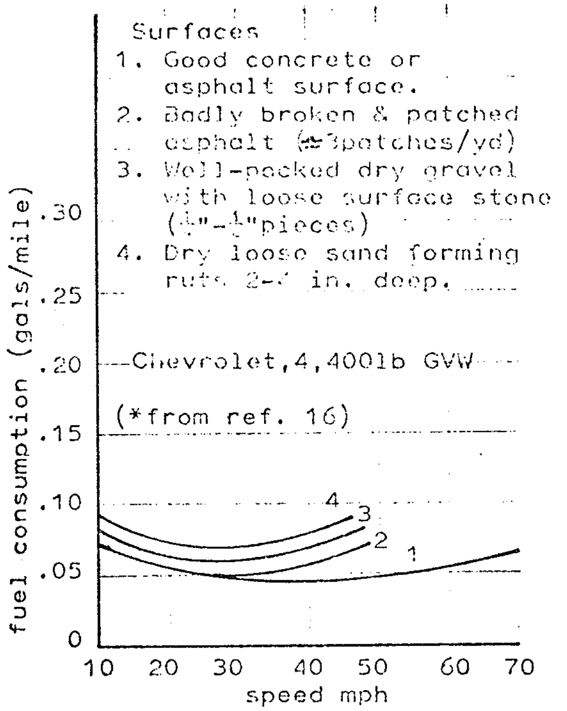

Fig.2 *Effect of surface on fuel consumption-Chevrolet,

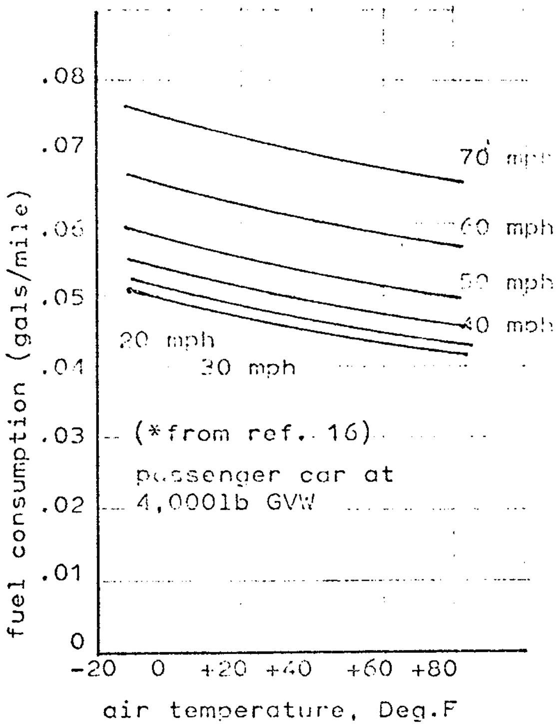

Fig.3*Effect of air temp. on fuel consumption-pass. car.

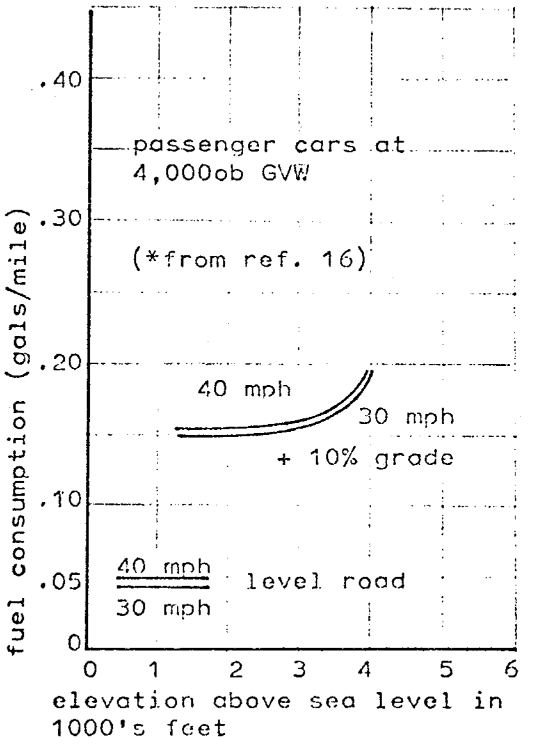

Fig.4*Effect of altitude on fuel consumption-possenger cars

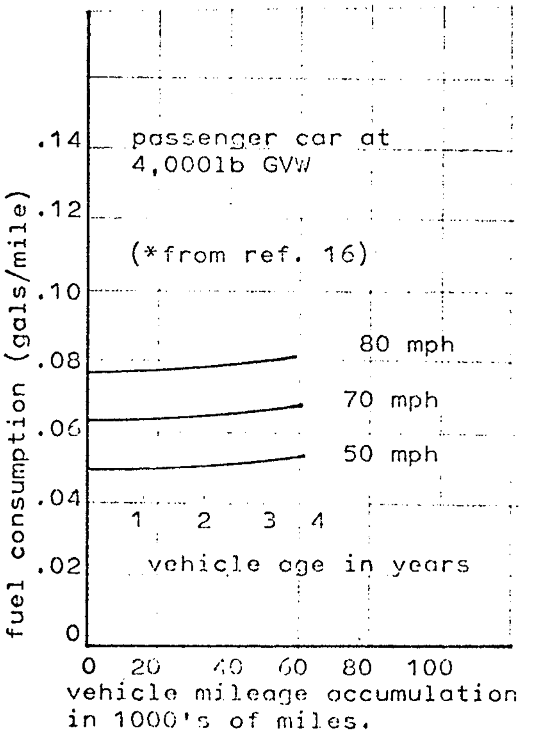

Fig.5*Effect of mileage accum. & age on fuel consumption-passenger car.

sources of primary errors should be born in mind. Despite the fact that an attempt is made to develop an overall general model, its precision may not be expected to be high. Nevertheless a method of developing such a model is introduced and the results may be useful in evaluating relative savings in fuel.

The experimental data extracted from the literature for this analysis are presented in Tables Ia & b, IIa & b, and IIIa & b. In Jan de Weille's work (ref.1), the author mainly compiled information from other publications and put them together. The data from his work were converted to the same units as others: that is, speed was converted from kilometers per hour to miles per hour, and fuel consumption was converted from liters per 1,000 kilometers to gallons per mile. From Roy B. Sawhill and Joseph C. Firey's study, (ref.3), data for trucks was extracted from the curves plotted on pages 53-55. These were plots obtained from actual test runs. Most of the data for the cars were extracted from the curves plotted in Paul J. Claffey's work. (ref.16) pages 48-49.

Table Ia

Data from Jan de Weille's paper (ref. 1)

Technical Description of the Vehicles

Characteristics	European car (volkswagen)	Truck II (van)	Truck III (2-S2 unit)
denoted by	C_1	T_1	T_2
curb weight, lb.	1,630	5,745	24,000
max. payload, lb.	----	7,750	33,200
gross weight, lb.	----	14,000	57,000
piston displacement cu. in.	72.7	477.7*	637.0*
gross horsepower	40	165	210
no. of cylinders	4	6	6
compression ratio	7.0	----	----
wheel base, meters	2.35	----	----
overall length meters	3.96	6.6	13.7
tire size	15x5.6	8x22.5	11x22.5
no. of wheels	4	6	14
no. of axles	1	2	4
no. of doors	2	2	2
type of fuel	gasoline	gasoline	gasoline

* obtained from the Whyte Trucks Co. Ltd. Winnipeg.

Table Ib

Fuel consumption and speed on level tangent paved and gravel roads for the vehicles described in Table Ia, as extracted and converted from Jan de Weille's paper (ref. 1)

Fuel Consumption (gallons per mile)

	paved road			gravel road		
speed (mph)	C_1	T_1	T_2	C_1	T_1	T_2
15	0.0298	0.1070	0.2820	0.0360	0.1260	0.3440
20	0.0269	0.0984	0.2200	0.0320	0.1180	0.2770
25	0.0246	0.0944	0.1855	0.0290	0.1150	0.2400
30	0.0237	0.0927	0.1663	0.0290	0.1150	0.2190
35	0.0240	0.0932	0.1570	0.0290	0.1160	0.2100
40	0.0250	0.0950	0.1553	0.0320	0.1210	0.2110
45	0.0264	0.0984	0.1610	0.0320	0.1260	0.2240
50	0.0282	0.1038	0.1750	0.0340	0.1360	0.2470
55	0.0305	0.1110	0.2060	0.0370	0.1470	0.3010
60	0.0337	0.1215	0.2462	0.0410	------	------
65	0.0386	0.1350	------	------	------	------
70	0.0452	------	------	------	------	------

Table IIa

Data from Roy B. Sawhill & Joseph C. Firey's paper HRB #276 (ref. 3)

Test Vehicle Descriptive Data

test unit no.	1-A	2-B	2-C-D	6	7-C	10
axle class of comb.	3-S2	2-S2	2S1-2	2-bus	2-S1	2-2
denoted by	T_5	T_6	T_3	B_1	T_4	T_7
power vehicle unit	tractor	tractor	tractor	bus	tractor	truck
year of manufacture	1955	1950	1950	1947	1958	1958
fuel type	gasoline	gasoline	gasoline	gasoline	gasoline	gasoline
displacement cu.in.	501	503	503	404	331	332
no. of cylinders	6	6	6	6	6	8
net HP at RPM	184 2,600	185 2,600	185 2,600	180 2,800	122 2,800	187 3,600
rear axle gear ratio	6.69	7.05	7.05	$6^{1}/_{6}$	U8.28 O5.99	U9.77 O7.17
tire size	10x20	10x20	10x20	F10x22 R 9x20	9x20	10x20
comb.overall length ft.	53.3	45.4	62.95	32.8	36.4	51.5
gross wt. empty lb.	26,180	24,430	26,990	15,590	21,580	22,350
gross wt. full lb.	64,650	57,246	72,500	21,350	41,490	58,120

Table IIb

Fuel consumption and speed on level tangent paved & gravel roads for the vehicles described in Table IIa, as extracted from the charts in Roy B. Sawhill & Joseph C. Firey's paper HRB #276 (ref. 3).

Fuel Consumption (gallons per mile)

paved road

speed (mph)	T_3	T_4	T_5	T_6	T_7	B_1
15	0.220	0.240	0.240	0.240	0.260	0.150
20	0.185	0.190	0.205	0.200	0.190	0.100
25	0.165	0.165	0.185	0.165	0.165	0.095
30	0.155	0.145	0.165	0.145	0.150	0.100
35	0.150	0.130	0.160	0.135	0.140	0.110
40	0.145	0.125	0.155	0.135	0.140	0.120
45	0.155	0.125	0.160	0.145	0.145	0.125
50	0.170	0.135	0.165	0.160	0.160	0.140
55	----	0.155	----	0.175	0.175	0.155
			gravel road			
15	0.300	0.260	0.275	0.255	0.290	0.175
20	0.255	0.215	0.245	0.227	0.287	0.130
25	0.235	0.185	0.227	0.215	0.210	0.125
30	0.225	0.170	0.225	0.207	0.195	0.125
35	0.220	0.160	0.227	0.205	0.190	0.130
40	----	0.155	0.235	0.210	0.200	----
45	----	0.155	0.255	0.220	----	----
50	----	----	----	0.237	----	----

Table IIIa

Data from Paul J. Claffey's paper,

National Cooperative Highway Research Program Report #111 (ref.16)

Technical Description of the Vehicles

Characteristics	Volkswagen	Falcon	Chevrolet	Plymouth	Chrysler
denoted by	C_2	C_3	C_4	C_5	C_6
model	2-door	2-door	4-door sedan	4-door sedan	4-door sedan
curb wt. (lb)	2,100	3,000	4,000	4,400	4,400
model year	1965	1965	1964	1968	1966
length (ft.)	14.5	18.0*	18.5**	18.5***	18.5***
frontal area (sq.ft.)	24	26	26	26	26
tire size (in.)	5.60x15	6.50x13	7.50x14	8.55x14	8.25x14
number of cylinders	4	6	8	8	8
compression ratio	7.0:1	9.2:1	9.2:1	9.2:1	10.0:1
displacement (cu.in.)	72	200	283	383	440
net horsepower	42	120	195	290	350
engine speed	3,900	4,400	4,800	4,400	4,400
gasoline	reg.	reg.	reg.	reg.	prem.

* obtained from the Midway Chrysler Plymouth Ltd., Winnipeg.

** obtained from the Gateway Chevrolet Oldsmobile Ltd., Winnipeg.

***obtained from the Ford rep. in Winnipeg.

Table IIIb

Fuel consumption and speed on a level tangent paved road for the vehicles described in Table IIIa, as extracted from the charts in Paul J. Claffey's paper, National Cooperative Highway Research Program Report #111 (ref.16).

Fuel Consumption (gallons per mile)

speed (mph)	C_2	C_3	C_4	C_5	C_6
10	0.045	0.060	0.092	0.092	0.088
20	0.030	0.045	0.050	0.070	0.063
30	0.025	0.035	0.045	0.060	0.055
40	0.025	0.035	0.040	0.059	0.055
50	0.040	0.040	0.048	0.060	0.060
60	0.045	0.050	0.055	0.069	0.072
70	----	0.060	0.060	0.077	0.088

CHAPTER III

APPROACH TO PROBLEM

A model is an effective tool for predictions in system design. Many types of models can be used to represent a system, but they are not equally effective. Graphical models, in particular, have proved very unsatisfactory in a system that has a number of factors contributing to its development, except where some of the factors are neglected. On one plane, graphical models can handle not more than two independent variables effectively. Previous researchers have restricted their model building, for fuel consumption, to the graphical only and that explains why they have found it very hard to incorporate many other necessary parameters that could account for fuel consumption.

In a situation where a multiplane is required, to be able to carry out a very effective graphical representation, mathematical models might be easier to work with. With a mathematical model, as many variables as necessary can be handled. In analysing fuel consumption of a vehicle on a roadway, where so many known and unknown factors are involved, a mathematical model would be the most appropriate form of model to be utilised.

The sort of mathematical model that might be good for representing fuel consumption will be in the general form shown below:

fuel consumption = f(vehicle, road type, operating conditions)

Initially, it will be easier to develop a simple model for an ideal road and simple operating conditions for each vehicle. Then the change in fuel consumption with change in vehicle types, road types and other conditions can be represented on charts, as done by previous researchers. This basic general form of model is built using the multiple regression analysis computer program. The weakness attributed to this program is that the functions of the model are predetermined. The program finds the values of the coefficients of regression and their correlation coefficients. This program obtained from the University of Manitoba Computer Science Dept. (ref. 9), is described in Appendix A.

The fact that this model should have a reasonably technical and conceptual meaning makes it necessary to use a simple enough function that can be explained. From a general survey of the general type of curves already obtained for fuel consumption versus speed (ref. 3), it is reasonable to assume that a shifted parabola would be an adequate function for describing fuel consumption. So a parabolic function was assumed and fuel consumption was described by additive functions of the form below:

$$Q = K_o + K_1 f_1(\text{vehicle}) + K_2 f_2(\text{road}) + K_3 f_3(\text{speed}) \quad \ldots I$$

where, Q is fuel consumption (gals/mile)

K_i are constant coefficients.

f_3(speed) is assumed to be parabolic, and f_1(vehicle) and

f_2(road) are also assumed to be responsible for the shifting of the parabola. This model is further broken down to:

$$Q = K_0 + K_{11}w + K_{12}hp + K_{13}d + K_2r + K_{31}v + K_{32}v^2 \ldots II$$

where:

w = vehicle weight

hp = vehicle horsepower

d = vehicle engine displacement

r = road surface type

v = speed level

K_{ij} = constants

Many models of this form were built, and some of them produced good ststistical results (these models are listed in Appendix B), but they are not technically reasonable. Two weaknesses attributed to such a model are that it assumed:

(i) the effects of each parameter are additive

(ii) that the optimum speed at which fuel consumption is minimum is the same for all vehicles.

$$\frac{\partial Q}{\partial v} = K_{31} + 2K_{32}v = 0, \quad v_{opt} = K_{31}/K_{32}$$

Of course, these two statements are not true. They in fact depict the closeness required between planning and available technical information.

As a result, it became apparent that a more appropriate form of model would be a multiple of functions, i.e. such that:

$$Q = f_1(\text{road } f_2(\text{vehicle } f_3(\text{speed}))) \ldots\ldots\ldots\ldots\ldots \text{III}$$

This is further broken down to:

$$Q_{ijm} = a_{ijm_o} + b_{ij}v_m + c_{ij}v_m^2 \ldots\ldots\ldots\ldots\ldots\ldots \text{IV}$$

where a_{ijm_o}, b_{ij}, and c_{ij} are coefficients that vary with the vehicle type and road type.

i - denotes vehicle type

j - denotes road type

m - denotes speed level, (m_0 = minimum speed $\doteq 0$ but $\neq 0$)

With this sort of model (Eq. IV) items (i) and (ii) above are eliminated. This sort of model is technically reasonable. The other requirement is the statistical fit which is dependent on the experimental accuracy of the experimental data. This model was built in three stages, as follows:

Stage 1: An equation of the form of Eq. IV was developed for each vehicle at different speed levels on a straight tangent paved road. From this, the values of a_{ip}, b_{ip} and c_{ip} were obtained for each vehicle. (p - denotes paved road.)

Stage 2: The values of a_{ip}, b_{ip} and c_{ip} were related to the vehicle types, to allow for their prediction.

Stage 3: Effect of road types and conditions were related to fuel consumption on a chart, for all vehicles generally.

STAGE 1

A basic model (as in Eq. IV) was developed for each vehicle and they were statistically tested. The output and results of the statistical tests are summarised in Appendix C. The models that resulted are shown in Eqs. V to XVIII, below.

CARS:

C_1: $Q_{ipm} = 0.042 - 0.00109v_m + 0.000016v_m^2$......V

C_2: $Q_{ipm} = 0.059 - 0.00190v_m + 0.000029v_m^2$......VI

C_3: $Q_{ipm} = 0.078 - 0.0219v_m + 0.000028v_m^2$.......VII

C_4: $Q_{ipm} = 0.079 - 0.00178v_m + 0.000022v_m^2$......VIII

C_5: $Q_{ipm} = 0.112 - 0.00254v_m + 0.000030v_m^2$......IX

C_6: $Q_{ipm} = 0.110 - 0.0029v_m + 0.000038v_m^2$.......X

BUS:

B_1 : $Q_{ipm} = 0.212 - 0.0069v_m + 0.000109v_m^2$........XI

TRUCKS: (next page)

TRUCKS:

$$T_1: Q_{ipm} = 0.140 - 0.0029v_m + 0.000044v_m^2 \ldots\ldots XII$$

$$T_2: Q_{ipm} = 0.478 - 0.0168v_m + 0.000217v_m^2 \ldots\ldots XIII$$

$$T_3: Q_{ipm} = 0.347 - 0.0109v_m + 0.000147v_m^2 \ldots\ldots XIV$$

$$T_4: Q_{ipm} = 0.403 - 0.0136v_m + 0.000166v_m^2 \ldots\ldots XV$$

$$T_5: Q_{ipm} = 0.367 - 0.0106v_m + 0.000132v_m^2 \ldots\ldots XVI$$

$$T_6: Q_{ipm} = 0.409 - 0.0142v_m + 0.000181v_m^2 \ldots\ldots XVII$$

$$T_7: Q_{ipm} = 0.428 - 0.0150v_m + 0.000101v_m^2 \ldots\ldots XVIII$$

Generally, the values of a_{ij} and c_{ij} turned out to be positive while b_{ij} was negative. This is expected due to the shape of curves being described. These coefficients are given in tabular form in Table IV. As seen in Appendix C, the statistical tests gave fairly positive results.

So far all that has been done is to translate the curves obtained by previous researchers into a mathematical model. For the model to be of predictive value a method must be found for determining the coefficients a_{ip}, b_{ip} and c_{ip} in terms of readily available technical information about the vehicles and their respective engines. If this is achieved, and if the simple parabolic model is adequate, prediction of the fuel consumption of a vehicle for which

TABLE IV

Characteristics and Coefficients a, b, and c.

Coefficients on Paved Road

Vehicle	$a \times 10^{-2}$	$-b \times 10^{-4}$	$c \times 10^{-6}$
C_1	4.204	10.86	16.00
C_2	5.9	19.00	28.6
C_3	7.786	21.85	27.98
C_4	7.871	17.79	22.14
C_5	11.19	25.39	29.64
C_6	11.04	29.18	37.50
B_1	21.15	69.03	109.10
T_1	14.037	29.23	44.0
T_2	47.764	168.40	216.7
T_3	34.73	108.6	147.0
T_4	40.25	136.1	166.0
T_5	36.70	106.0	132.0
T_6	40.93	141.6	181.0
T_7	42.79	150.0	191.30

no test results are available becomes possible. For such a process to be meaningful the technical significance of these coefficients with respect to the vehicles should be well understood, at least in a qualitative form. In other words, a knowledge of the technical factors which cause the fuel consumption versus speed curve to take the form that it has is a necessary precurser to the selection of technical data from which a_{ip}, b_{ip} and c_{ip} are to be calculated. This might be a hard thing to do because of insufficient technical information.

CHAPTER IV

DETERMINATION OF THE REGRESSION COEFFICIENTS 'a', 'b', AND 'c', AND VEHICLE CHARACTERISTICS THAT ACCOUNT FOR THEM.

The values of the coefficients 'a', 'b', and 'c', presented in Table IV may be said to be purely dependent on the vehicles and nothing else in that they represent the "normal" fuel consumption in each case. Therefore these coefficients are directly related to the vehicle characteristics that account for fuel consumption on a straight tangent paved road. Under these conditions it becomes possible to look for relationships between each coefficient and certain vehicle parameters and to assess the goodness of these realtionships. The first step in developing these relationships is, of course, that of understanding the vehicle parameters that affect fuel consumption.

Basically, it is known that a moving vehicle must do work. This work is being done to overcome three resistances; rolling resistance, air resistance and internal friction. To enable a vehicle to overcome such resistances, energy has to be supplied in the form of fuel. Therefore fuel consumed by a vehicle depends on two things; the amount of work done against these resistances and the overall efficiency of the engine in converting fuel to useful mechanical energy. The rolling resistance depends on the vehicle weight and speed, and air resistance depends on the vehicle's frontal area, shape factor and square of the speed. Both of these can be calculated using Equations XIX and XX.

*Rolling Resistance = $w(0.0165 + 0.000165(v30))$....XIX

*Air Resistance = $C_w Av^2/400$ XX

where: w = vehicle weight

A = vehicle frontal area

C_w = shape factor

v = speed

Though these two formulae tend to overestimate the values of the resistances, they at least depict the pattern and the vehicle characteristics involved. Internal friction depends on many factors such as engine size, intake silencer, fan, water pump, generator, muffler and exhaust system. Though it is known to depend on these factors, its value is hard to determine. Rolling resistance, air resistance and internal friction per unit distance account for the amount of foot pounds of work done by a vehicle per unit distance. The amount of foot pounds of work derived from the gasoline is used up by a moving vehicle and depends on total work done by a vehicle, the engine capacity and efficiency.

fuel consumed = f(work done, engine capacity and engine efficiency).............XXI

Engine work capacity is measured as the maximum amount

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* from Sports Car Design #30, (ref.21).

of horsepower that can be developed by an engine at a specific number of revolutions per minute. The engine efficiency, which is related to its thermal efficiency, is the ratio of power output to rate of supply of fuel heating value and depends very much on the engine design.

Generally, the most important vehicle characteristics that account for fuel consumption can be broken up into; weight, frontal area, shape factor, speed, maximum horsepower that can be developed at a particular engine speed, and thermal efficiency and the factors that account for the internal friction. There are other factors which contribute to fuel consumption, such as tire size, valve timing, etc., but they are small and, to simplify the model, are considered to be negligible in this study.

In fuel economy studies for highway planning, fuel savings are determined by thousands of vehicles. In that case it would not be economically justified to carry out very intensive analysis to determine the values of the vehicle parameters that account for fuel consumption, for each vehicle. Usually it will be easier to make use of vehicle characteristics already supplied by the manufacturers; in other words, vehicle characteristics that will not require any further laboratory analysis to determine. As a result, an attempt is made to relate these coefficients to such vehicle parameters with the best possible accuracy. A method of determining fuel consumption with very high accuracy will most likely require an intensive laboratory investigation and such a method will not serve very useful purposes for highway planning.

STAGE 2

Coefficient 'a'

The coefficient 'a', is a constant for each vehicle and might be regarded as that part of the model which accounts for fuel consumption as speed tends to zero. A high proportion of 'a' is due to the vehicle characteristics that account for fuel consumption but which do not vary with speed; the major characteristic in this group is the internal friction.

Usually when the engine of a vehicle is switched on, fuel is consumed whether the vehicle is in motion or not. Though this model is for moving vehicles only, it is still true that immediately a vehicle starts to move at a speed greater than zero, some fuel is consumed. Therefore 'a' can be broken up into two parts 'a_1' and 'a_2', where 'a_1' depends on the internal friction of the vehicle and 'a_2' is the value of fuel that must be consumed by a vehicle immediately it starts to move at the lowest possible speed.

$$a = a_1 + a_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{XXII}$$

$$a_1 = f(\text{internal friction}) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{XXIII}$$

$$a_2 = f(\text{consumption at minimum speed}) \quad \ldots\ldots\ldots\ldots\ldots \text{XXIV}$$

The internal friction of a vehicle depends on many factors such as the size of the engine, the intake and exhaust system, the size of auxiliary services driven by the engine, all of which may be expected to be strongly correlated with vehicle size. But though vehicle size is subjectively well understood a single objective, numerical measure is hard to derive. Consequently, the further correlation between curb weight and vehicle size is employed. The coefficient 'a_2', which is assumed to be dependent on the initial torque required to propel a vehicle, also becomes a function of the vehicle weight. Therefore 'a_1' and 'a_2' should vary with vehicle weight.

$$a = a_1 + a_2 = f(\text{vehicle curb weight}) \ldots\ldots\ldots\ldots\ldots \text{XXV}$$

It is considered that a large improvement in the prediction of coefficient 'a' is not likely to result from using more complicated vehicle parameters. This relationship is graphically represented in Fig. 6. From this chart the value of 'a' can be predicted, to a certain degree, for any vehicle. The charts for cars and trucks have been kept separate because their engines are designed for different purposes; cars for few passengers and high speeds, whereas trucks are designed for freighting goods and not for very high speeds. Thus they are expected to behave differently.

The values of 'a' for cars, increase linearly with curb weight. This is expected as it means that heavier cars have bigger engines and greater intake systems and they require a higher starting torque. Except for one car, all vehicles

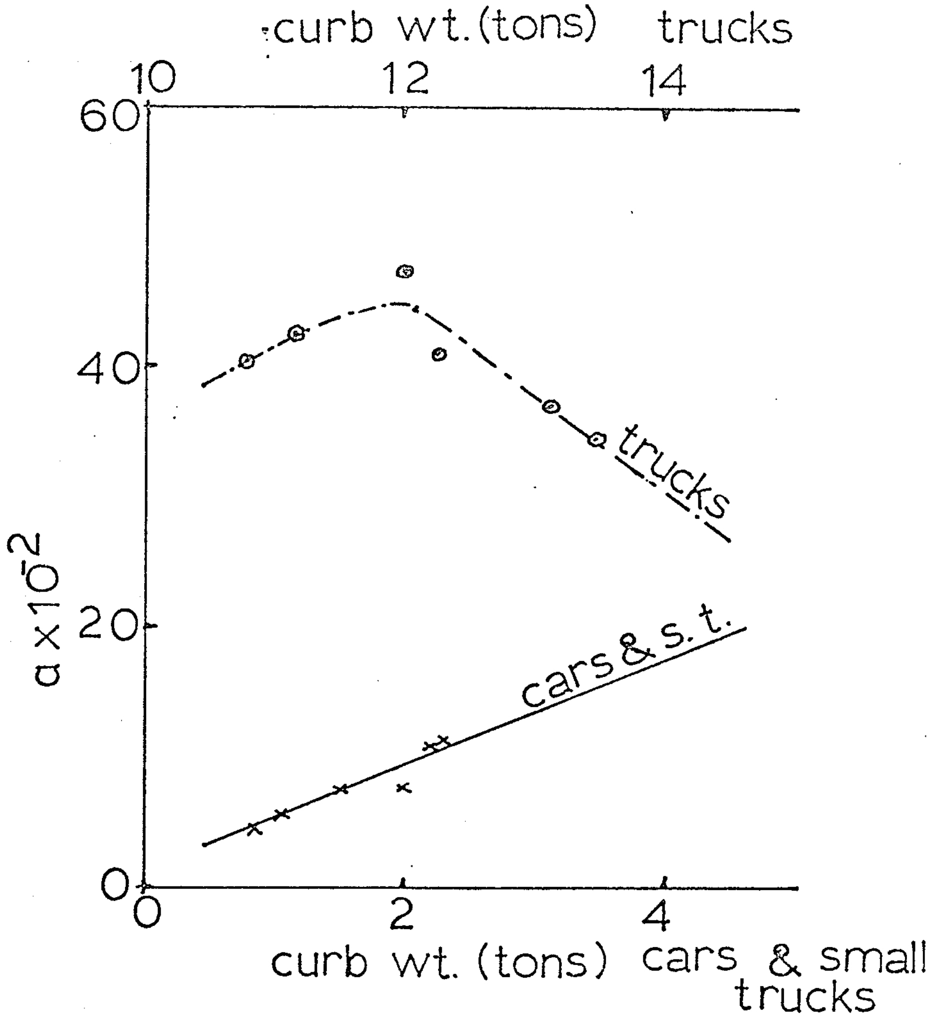

Fig. 6 chart for obtaining coeff. a on paved roads seem to show a fairly good fit. This car, a 1964 Chevrolet, has only two transmission ratios - a unique feature when compared to the other cars. With this aspect taken into consideration, it is not too surprising to have its value of 'a' deviating slightly compared to other cars. As for the trucks, the value of 'a' increases with curb weight untill it reaches a maximum at about 45.0×10^{-2} for a curb weight of 12 tons and then slowly decreases with curb weight. The first part of this curve is as expected but the reduction of 'a' with higher curb weight, beyond 12 tons, can possibly be explained by the fact that because fuel consumption was going to be excessively large for very heavy trucks, a small part of vehicle performance was given up, by the manufacturers, for a reduction in excessive fuel consumption. This reduction is depicted in the values of coefficient 'a', and as a result the value of 'a' reduces for heavier trucks.

Coefficient 'b'

Coefficient 'b', which is a speed term, though constant for a specific vehicle on a paved road, is a very sensitive term. At higher speeds, 'b' contributes a good proportion to fuel consumption. Being a linear term of speed, 'b' will be expected to depend on the rolling resistance, which is assumed to vary linearly with speed. Rolling resistance according to Eq.XIX, varies with the vehicle weight, otherwise it is the same for two vehicles of the same weight at all speeds.

Two vehicle parameters affect rolling resistance; specifically weight and tires. The effects of tires are assumed to be the same for all vehicles and are accounted for by the "0.000165" coefficient in Eq. XIX. The effect of vehicle weight is the one that varies with different vehicles. Therefore, the vehicle's weight is a very good measure for the rolling resistance at any given constant speed.

The contribution of 'b' to fuel consumption is negative as opposed to the contribution expected from rolling resistance which should be positive. The aspect of the vehicle that is responsible for decreasing fuel consumption with speed, is the engine effieiency which increases with speed at least over the range; low to medium engine speeds. Another factor is the engine capacity. These cause a decrease in fuel consumption with speed i.e. engines with high capacity have relatively better performance at higher speeds than at lower speeds. So, 'b', depends also on the engine characteristics that account for the decrease in fuel consumption. 'b', therefore, is a function of two subtractive vehicle parameters; namely the rolling resistance and engine characteristics.

b = f(rolling resistance, engine characteristics)...XXVI

$b = b_1 - b_2$..XXVII

b_1 = f(vehicle weight).............................XXVIII

b_2 = f(engine displacement and no. of cylinders)....XXIX

A reduction in fuel consumption (gallons per mile) is caused by the reduction of the product of engine displacement and indicated mean effective pressure in compression (ref. 17a). Indicated mean effective pressure depends on the number of cylinders and compression ratio and an attempt to measure it for this analysis will be contrary to one of the objectives of this study. Therefore, in the absence of other technical information, the product of the engine displacement (cu. in.) and the number of cylinders is taken as the best available measure of engine characteristics, relevant to reduction in fuel consumption with speed. Thus 'b' is a function of the vehicle weight and the product of engine displacement and number of cylinders according to Eq. XXX.

$$b = f(K_1 w - K_2 D \times N.C.) \qquad \text{XXX}$$

where:

D = engine displacement

N.C. = number of cylinders

For the vehicles used in this study, the values obtained for K_1 and K_2 are:

$K_1 = 140$

$K_2 = 1/6$

This relationship is graphically represented in Fig. 7, where again the charts for the cars and trucks are kept separate. Bearing in mind the extreme simplicity of the parameters used in determining the coefficient 'b', there is good agreement between the experimental points and the predicted values.

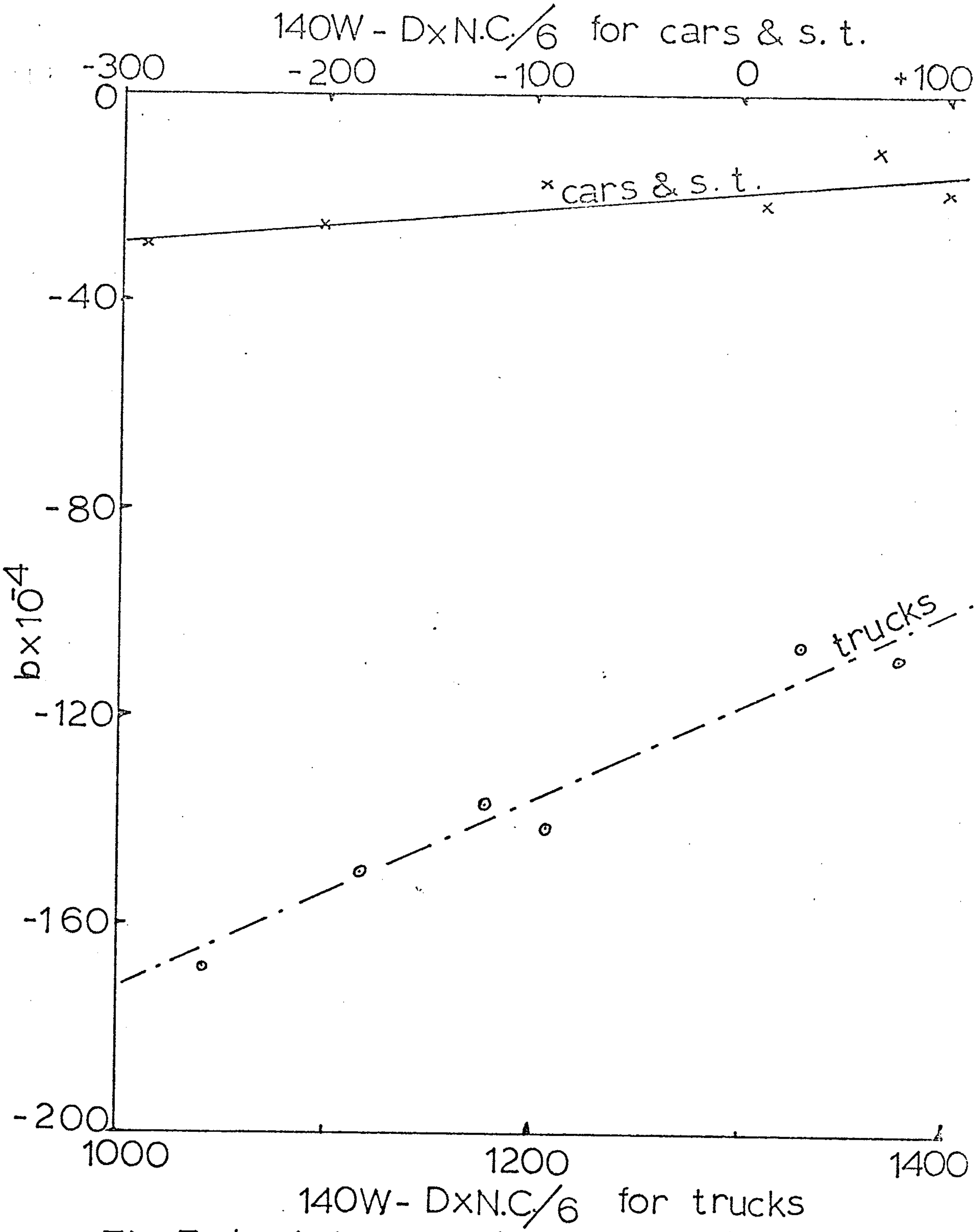

Fig. 7 chart for obtaining coeff. b on paved roads

The values of 'b' increase with an increase in '$K_1w - K_2D \times N.C.$' for both cars and trucks, as was expected. It shows that vehicles with very high ratios of weight to displacement multiplied by the number of cylinders, will consume a lot of fuel at higher speeds.

Coefficient 'c'.

Coefficient 'c', the third coefficient, is a "speed squared" term, and thus it can be postulated that 'c' is related to air resistance, which is known to vary as $(\text{speed})^n$ where $n \doteq 2$. Values of 'c' are generally very small; usually in the order of 10^{-3}. Though it contributes very much more to fuel consumption at very high speeds than at low speeds, over most of the speed range its contribution is very low.

According to Eq. XX, air resistance at any given speed depends on two vehicle parameters; frontal area and shape factor. To some extent, then, one would expect 'c' to vary with the vehicle's frontal area and shape factor.

$$c = f(\text{air resistance factor}) \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{XXXI}$$

Shape factor is governed by the body contours of the vehicle and varies from a highest value of 1.25 for a flat plate (normal to the flow) to a value of about 0.12 for an ultimate "perfect" shape (ref. 21). It is a difficult factor to determine and no reliable values could be found in the literature for the vehicles used in this study. Similarly, the values of the frontal area of the trucks were not available.

This makes it hard to choose any substitutes for the vehicle parameters that can account for 'c'.

Since the shape factor of a vehicle cannot be guessed, and is not normally availabe from manufacturers data, it is ignored in this study. Frontal area is assumed to dominate in determining the coefficient 'c' though, of course, a less reliable determination must be expected. Fortunately, the range of shape factor for American cars is fairly low, from 0.48 to 0.50 (ref.21). Thus, it is fair enough to assume that for cars, ignoring shape factor, will not bring about too much error. Trucks all approximate to rectangular boxes and therefore, as with cars, shape factor will not vary much from truck to truck. For this reason, ignoring variation in the shape factor for trucks will not cause too much error in the determination of coefficient 'c'.

It is regretted that a simple parameter like frontal area is frequently not available from manufacturers published data concerning a given vehicle. Consequently, it has been necessary to derive a measure of this parameter from others which are normally published and which are thought to relate to frontal area.

In the case of trucks, a simplifying assumption is made, namely, that a truck may be represented by a rectangular box and that frontal area is directly related to the cross-sectional area of this box, i.e. the area one would see as the box travels towards one. It is clear that the volume of this box will be related to the maximum payload of the vehicle and that, in turn, the cross-sectional area will be inversely related to length. Thus, for trucks it is assumed that:

frontal area $\propto$ payload/length

and that with available data, this is the best measure upon which to base the calculation of the coefficient 'c' for trucks.

Cars don't have a large load carrying capacity, therefore the ratio of curb weight to length will be a good enough measure related to their frontal areas. Thus, for cars it is assumed that:

frontal area $\propto$ curb weight/length

In summary, then, the coefficient 'c' will be related to the frontal area in terms of payload to length ratio for trucks and curb weight to length ratio for cars or,

c $\propto$ air resistance factor,

c $\propto$ frontal area,

therefore; $$c = K_3 . \frac{\text{payload}}{\text{length}} \qquad \text{(for trucks),}$$

$$c = K_4 . \frac{\text{curb weight}}{\text{length}} \qquad \text{(for cars).}$$

where K_3 and K_4 are constants.

These relationships are graphically represented in Fig. 8. For trucks the fit does not look terribly bad, except for one truck (T_4) which is very far out. The reason for this is not quite obvious but it could be due to the fact that the shape factor has not been incorporated in considering the wind resistance. Nevertheless, in view of the available technical information, the results are not terribly bad, neglecting T_4. Again, it must be remembered that 'c' does not contribute much to fuel consumption, except at very high speeds.

Individual errors in predicting 'a', 'b' or 'c' are as important as the resultant errors occuring in the estimation of fuel consumption. Some of these individual errors could be additive or compensating in some cases. It will be necessary to test this model, by obtaining the values of the coefficients 'a', 'b' and 'c' from the charts on Figs. 6, 7, and 8, and using them to estimate fuel consumption at differrent constant speeds. The estimated values will then be compared to the experimental values.

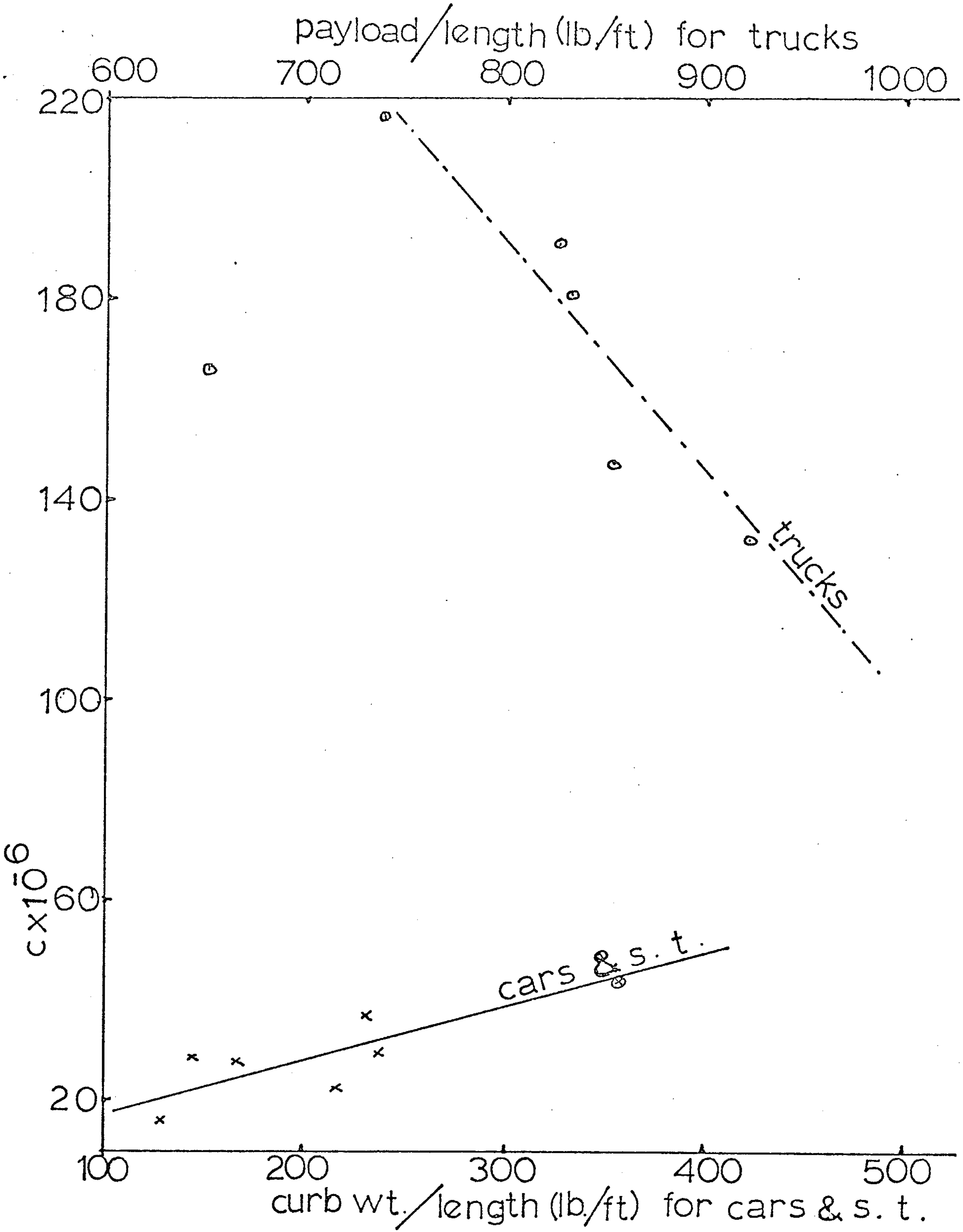

Fig. 8 chart for obtaining coeff. c on paved roads

Table V

Vehicle Characteristics Used in Determining 'a', 'b' and 'c'.

Characteristics	C_1	C_2	C_3	C_4	C_5	C_6	B_1
curb wt. (tons)	0.815	1.050	1.50	2.00	2.20	2.20	10.67
horsepower	40.0	42.0	120.0	195.0	290.0	350.0	180.0
eng.displ.'D' ($in.^3$)	72.7	72.0	200.0	283.0	383.0	440.0	404.0
no. of cylinders	4	4	6	8	8	8	6
length (ft.)	12.7	14.5	18.0	18.5	18.5	19.0	32.8
payload (lb.)	-	-	-	-	-	-	5,760
payload/length	-	-	-	-	-	-	176.0
curb wt./length	128.3	145.0	166.5	216.0	238.0	232.0	-
140w ÷ DxN.C./6	66.5	99.0	10.0	96.0	202.0	287.0	1086.0

Table V (cont.)

Vehicle Characteristics Used in Determining 'a', 'b' and 'c'.

Characteristics	T_1	T_2	T_3	T_4	T_5	T_6	T_7
curb wt.	2.8775	12.00	13.445	10.799	13.090	12.215	11.175
horsepower	165.0	210.0	185.0	122.0	184.0	185.0	187.0
eng.disp.'D' (in.^{3})	477.7	637.0	503.0	331.0	501.0	503.0	332.0
no.of cylinders	6	6	6	6	6	6	8
length (ft.)	21.65	45.0	53.4	30.6	41.7	39.6	43.2
payload (lb.)	7,750	33,200	45,510	19,910	38,470	32,816	35,770
payload/length	358.0	738.0	854.0	651.0	922.0	832.0	826.0
140w - DxN.C/6	75.7	1043.0	1377.0	1179.0	1329.0	1207.0	1118.0

Testing The Model

With the results obtained so far, the fuel consumption curve of any car or truck can be obtained at all speeds on any level tangent paved road. The degree of accuracy with which such a curve can be predicted depends entirely on the availability of the relevant technical information and estimation of the coefficients 'a', 'b', and 'c'. Some of the vehicles used for developing this model will be used for testing the model. Their fuel consumption curve will be predicted and the error in this estimate will reflect the degree to which such a curve can be predicted for other vehicles.

Eight vehicles were used in testing this model; four cars: C_2(Volkswagen), C_3(Falcon), C_4(Chevrolet), and C_5(Plymouth) and four trucks: T_2, T_3, T_5 and T_7. Truck T_4 was deliberately omitted because of its deviation in the value of coefficient 'c'. The fuel consumption was predicted for each of these vehicles at different speeds; the results are shown in Appendix D. They are also shown graphically in Figs. 9 to 16. The percent error in the estimate of fuel consumption was calculated at every speed and an average of the absolute values was taken to be the average deviation. This average error runs from a very low of 4.5% to a high of 13%, except in the case of truck T_3 with an average of 27%. The high of 27% in T_3 is due to the very poor estimate of coefficient 'c'

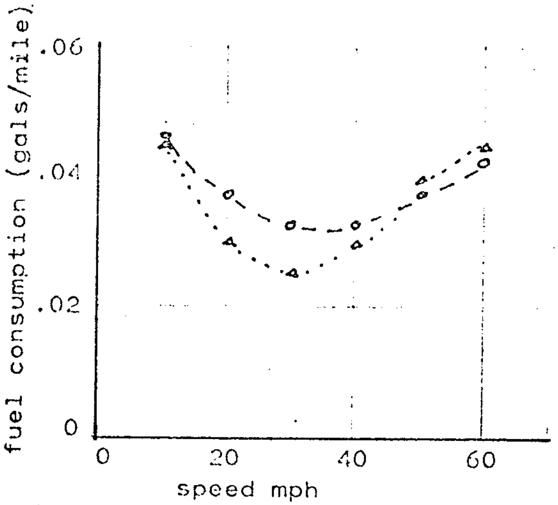

Fig.9 Computed & experimental fuel consumption for Car C_2.

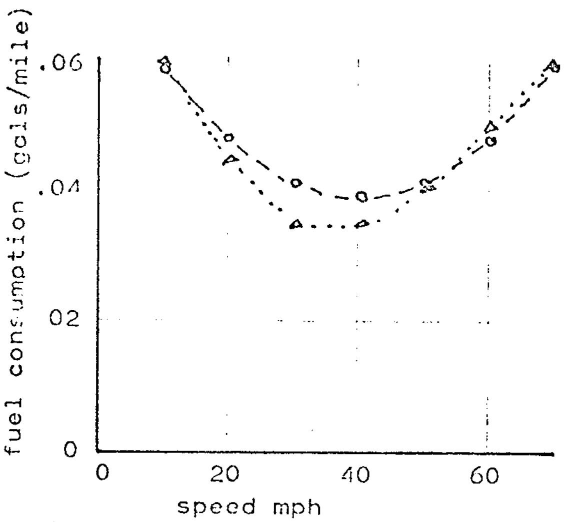

Fig.10 Computed & experimental fuel consumption for Car C_3.

○ – ○ – ○ – ○ Computed

△ · · △ · · △ · · △ Experimental

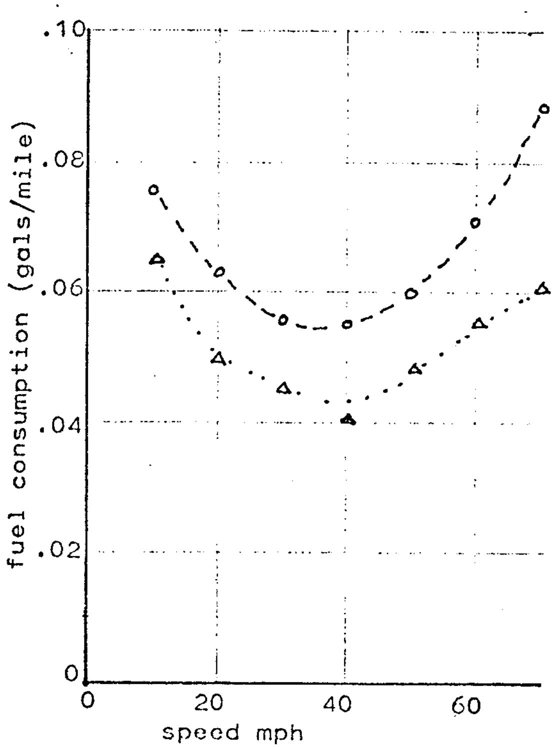

Fig.11 Computed & experimental fuel consumption for Car C_4.

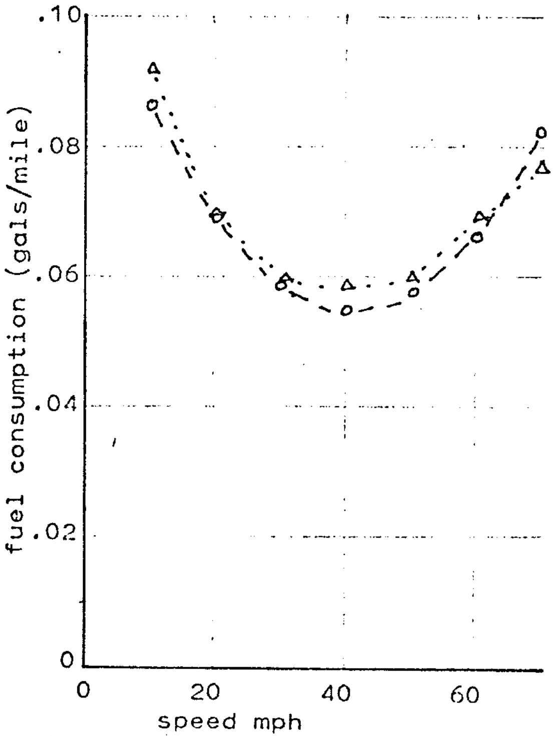

Fig.12 Computed & experimental fuel consumption for Car C_5.

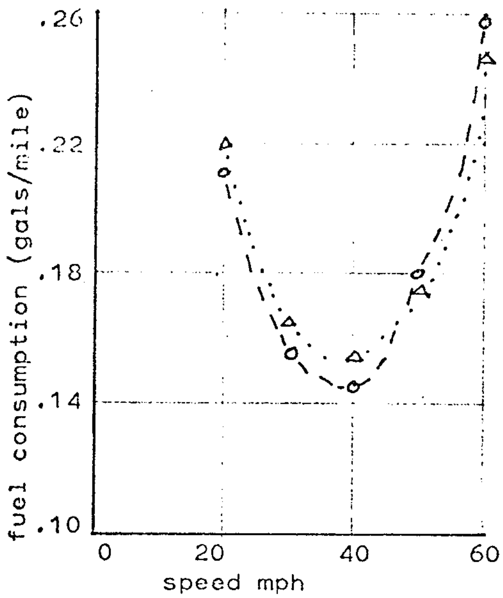

Fig.13 Computed & experimental fuel consumption for Truck T_2.

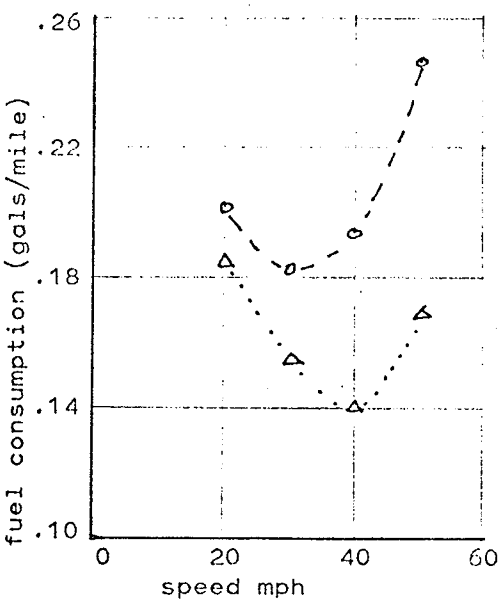

Fig.14 Computed and experimental fuel consumption for Truck T_3.

○ – – ○ – – ○ – – ○ Computed

△ · · · △ · · △ · · △ Experimental

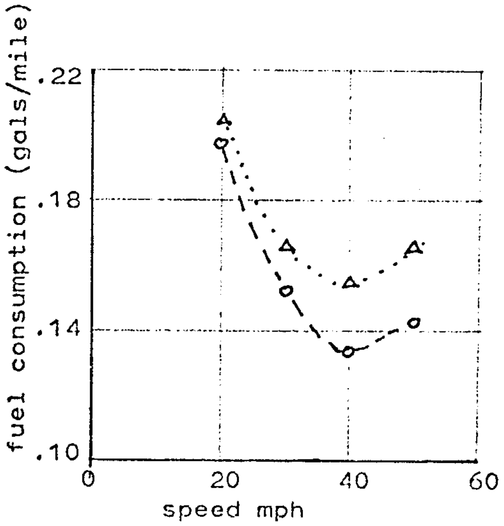

Fig.15 Computed & experimental fuel consumption for Truck T_5.

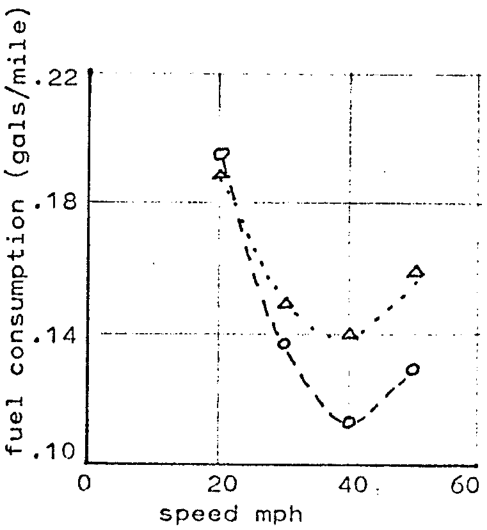

Fig.16 Computed & experimental fuel consumption for Truck T_7.

as shown in Table VI. From these results, it can be re-emphasized that such a poor prediction of coefficient 'c' is not unexpected due to the missing shape factor parameter in its determination. Therefore, the cause of such a large deviation will be attributed to lack of relevant information.

Table VI

Computed Coefficients from Stages 1 and 2

	computed regression coefficients from stage 1			obtained coefficients from charts in stage 2		
vehicle	$ax10^{-2}$	$-bx10^{-4}$	$cx10^{-6}$	$ax10^{-2}$	$-bx10^{-4}$	$cx10^{-6}$
C_2	5.9	19.0	28.6	6.0	16.0	23.0
C_3	7.786	21.85	27.98	7.5	18.0	22.5
C_4	7.871	17.79	22.14	9.5	22.0	30.0
C_5	11.19	25.39	29.64	11.0	27.0	33.0
T_2	47.764	168.40	216.7	45.0	168.0	230.0
T_3	34.73	108.6	147.0	34.5	104.0	165.0
T_5	36.70	106.0	132.0	37.0	113.0	135.0
T_7	42.79	150.0	191.3	42.5	153.0	180.0

Initially, the experimental fuel consumption at different speeds were approximated by a parabola. Then the coefficients 'a', 'b', and 'c', that describe this parabola, were estimated from known vehicle parameters. This estimate of fuel consumption, therefore, has two cumulative errors already. So much relevant technical information is quite lacking for this analysis that even the reliability of the experimental results can be reasonably questioned. In view of these problems, combined with the discrepancies in the experimental values mentioned earlier, one becomes convinced that better results than obtained are very unlikely. Except for the determination of coefficient 'c', in the absence of other useful information this model can be used tentatively with about 10% - 20% error expected.

At this stage it is quite appropriate to accept that a fuel consumption versus speed curve, on a straight tangent paved road, can be modelled for any vehicle, with some reservations. When road conditions change this model can no longer be used. Therefore effects of road conditions such as surface type and rise and fall should be incorporated in the model.

CHAPTER V

EFFECT OF ROAD TYPE AND CONDITIONS ON FUEL CONSUMPTION

A straight tangent paved road is too perfect to be seen often in many parts of the world. In the underdeveloped countries, there are more gravel roads than paved roads and these roads are very hilly. Here on the Prairies, most of the main roads are paved, but surprisingly rise and fall cannot be ignored. In addition, some of the pavings are poor and they are little better than gravel roads. With roads of these types, the model will be consistently underestimating the fuel consumption. Therefore an adjustment will be required in the model, to take care of the changes in the road surface type and geometrics. This takes us to the third stage.

STAGE 3

Effect of Surface Type

On a gravel road, more surface roughness is encountered by a vehicle. This increases the drag force and so results in a higher rolling resistance. Gravel roads usually contain loose gravel, therefore at fairly high speeds, one revolution of the vehicle's tires, makes a shorter horizontal distance than on a paved road, due to shoving action required to propel vehicle tires through the loose gravel. Most of the loose gravel roads and many badly broken or old paved roads have a series of bumps, which bring about excess energy loss in the tires. As a result of all these conditions more fuel is required to supplement these losses.

This excess fuel consumption due to imperfections in the road can be observed in the changes in the coefficients 'a', 'b', and 'c'.

Though no clear explanation for these increases has been attempted in the present work the following qualitative arguments might form the basis for a more detailed study.

Considering the coefficient 'a', it seems likely that it will be increased due to the friction generated between individual stones in the gravel as each wheel of the vehicle generates its own rut. This, of course, will happen at very low speeds.

In the case of the coefficient 'b',it is likely that the negative part of this coefficient will be increased at least over the lower part of the speed range. Such an increase could well be due to a kind of "planing" action of the tires on the gravel surface. In other words, it is likely that as speed increases ruts get less deep.

The increase in coefficient 'c', is quite contrary to what is expected. This could be due to an increase in the propulsion force causing slippage at driving wheels and resultant acceleration of stones.

Generally the ratios: $\frac{a_g}{a_p}$, $\frac{b_g}{b_p}$, & $\frac{c_g}{c_p}$ will be greater than unity.

where; g - denotes gravel and p - denotes paved.

But these ratios are not necessarily equal. For the fact that there will be a general reduction in speed, the optimum speed at which minimum consumption occurs will be expected to reduce. Therefore:

$$\frac{c_g}{c_p} > \frac{b_g}{b_p}$$

For cars, the three ratios are the same, since there is not much difference in the effect of surface type on different cars. In the absence of any other information on the behaviour of cars on gravel roads, it is quite reasonable to state an increase in consumption for all cars on gravel roads as a percentage of the paved road, (ref. 1). This increase is (shown in Table VII) plotted in Fig. 17.

Table VII*

Increase in Fuel Consumption of Gravel Roads as
% Increase of Consumption on Paved Roads for
Cars and Small Trucks.

		% Increase	
Speed	Cars	1.8 ton Truck	2.8 ton Truck
15	17	14	18
20	19	16	20
25	20	16	22
30	21	17	24
35	21	17	27
40	21	17	27
45	22	17	28
50	22	17	29
55	22	18	30
60	23	17	--

* from the World Bank Occasional Paper (ref. 1).

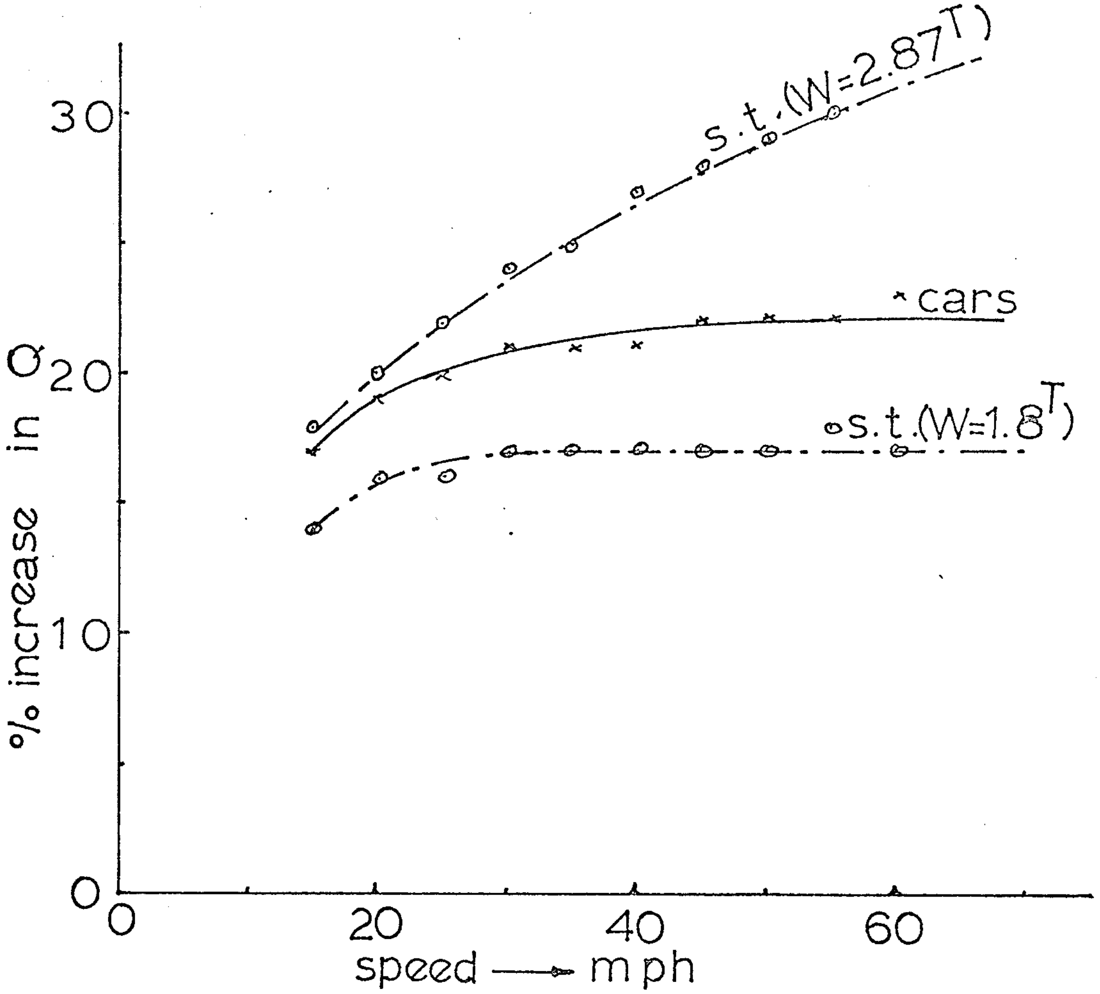

Fig.17 % increase in fuel consumption on gravel roads (over paved roads) for cars & small trucks

This in turn makes the analysis simple for cars, because the coefficients 'a', 'b', and 'c', will just be multiplied by a constant at a specific speed for any car.

Small pickup trucks of less than 3 tons tend to behave in a manner similar to big cars. For any specific pickup truck, the coefficients are multiplied by a single constant, but the constant varies with the size of the truck and the speed. Such a relationship is also shown in Fig. 17. This results in an assumption that small cars and pickup trucks have unchanged optimum speeds whether they are running on gravel or paved roads. Though that is hard to believe, there are not enough experimental results available to disprove it.

From experimental results, the heavier trucks don't show this same sort of behaviour as cars and pickups. The experimental results tend to show a more expected behaviour of the trucks than for cars and pickups. The ratios a_g/a_p, b_g/b_p, and c_g/c_p are not the same and the ratio c_g/c_p is larger than the ratio b_g/b_p. This shows a reduction in the optimum speed on gravel roads. Knowing a truck's optimum speed on a paved road, there seems to exist a relationship between this value and the three ratios of the coefficients (shown in Table VIII). Each of these ratios plotted against b_p/c_p resulted in straight lines with negative slopes. This indicates that the higher the optimum speed of a vehicle the less will be the effect of road surface on its fuel consumption.

Table VIII

Coefficients on Gravel Roads for Trucks

	T_2	T_3	T_4	T_5	T_6	T_7
$a_g \times 10^{-2}$	58.513	48.985	42.14	40.62	35.82	51.98
$-b_g \times 10^{-4}$	206.64	166.56	135.71	116.57	90.29	181.89
$c_g \times 10^{-6}$	281.0	257.0	171.4	184.76	132.39	252.16
a_g/a_p	1.225	1.410	1.045	1.110	0.876	1.213
b_g/b_p	1.225	1.535	0.999	1.100	0.638	1.212
c_g/c_p	1.300	1.750	1.032	1.400	0.730	1.320
b_p/c_p	77.7	73.8	82.1	80.4	78.3	78.4

This seems quite reasonable in that vehicles which attain best fuel economy at very low speeds on good surface roads, stand a chance of consuming a very excessive amount on very poor surface roads. The plots on Figs. 18, 19 and 20, also show that slope of b_g/b_p dips more than that of a_g/a_p, similarily the slope of c_g/c_p dips more than either of them. These only confirm the fact that vehicles that attain optimum speed on paved roads at low speeds, lose a lot more speed on gravel roads, than vehicles with higher optimum speeds on paved roads.

Knowing a truck's behaviour on a paved road, its behaviour on a gravel road can be predicted using the supplied charts on Figs. 18, 19 and 20, except for truck T_6 which

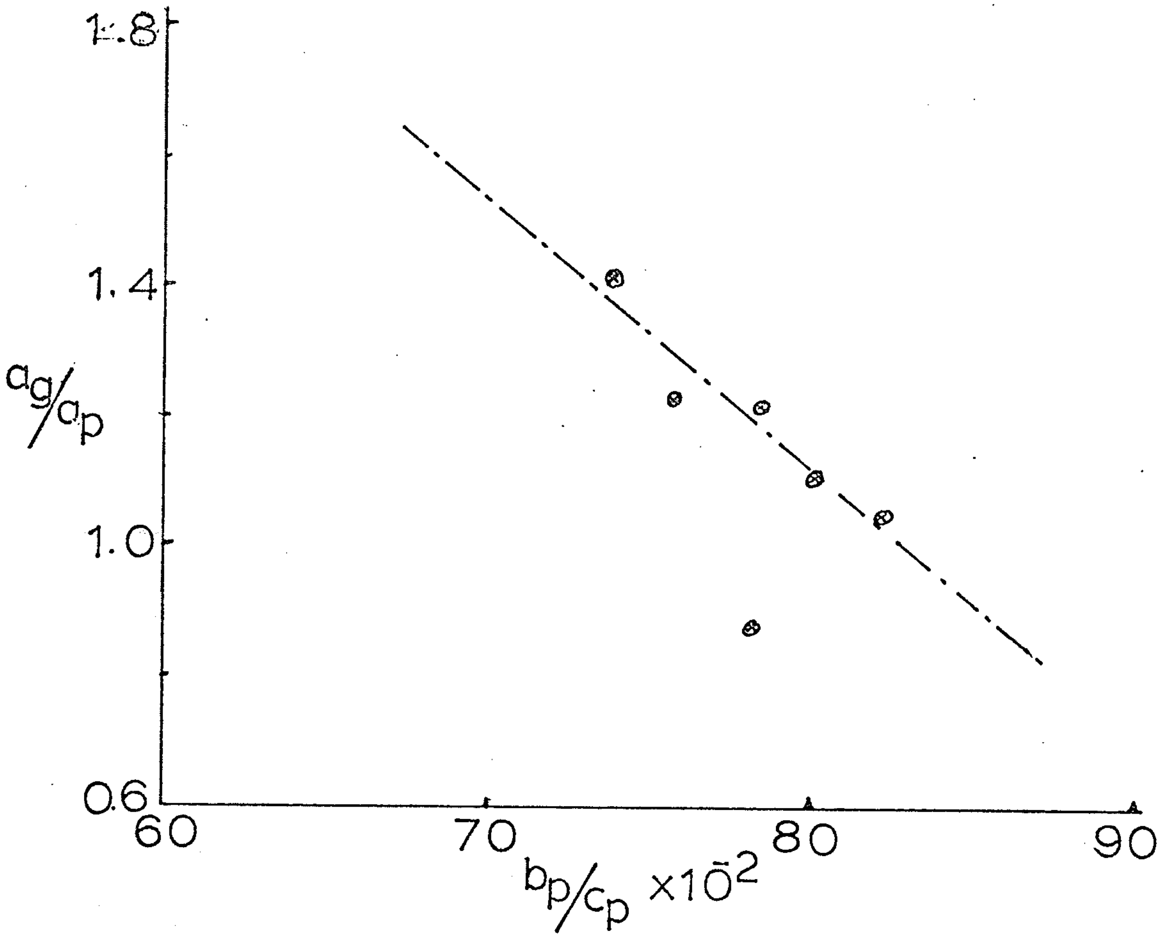

Fig. 18 chart for obtaining coeff. a
on gravel roads (for trucks)

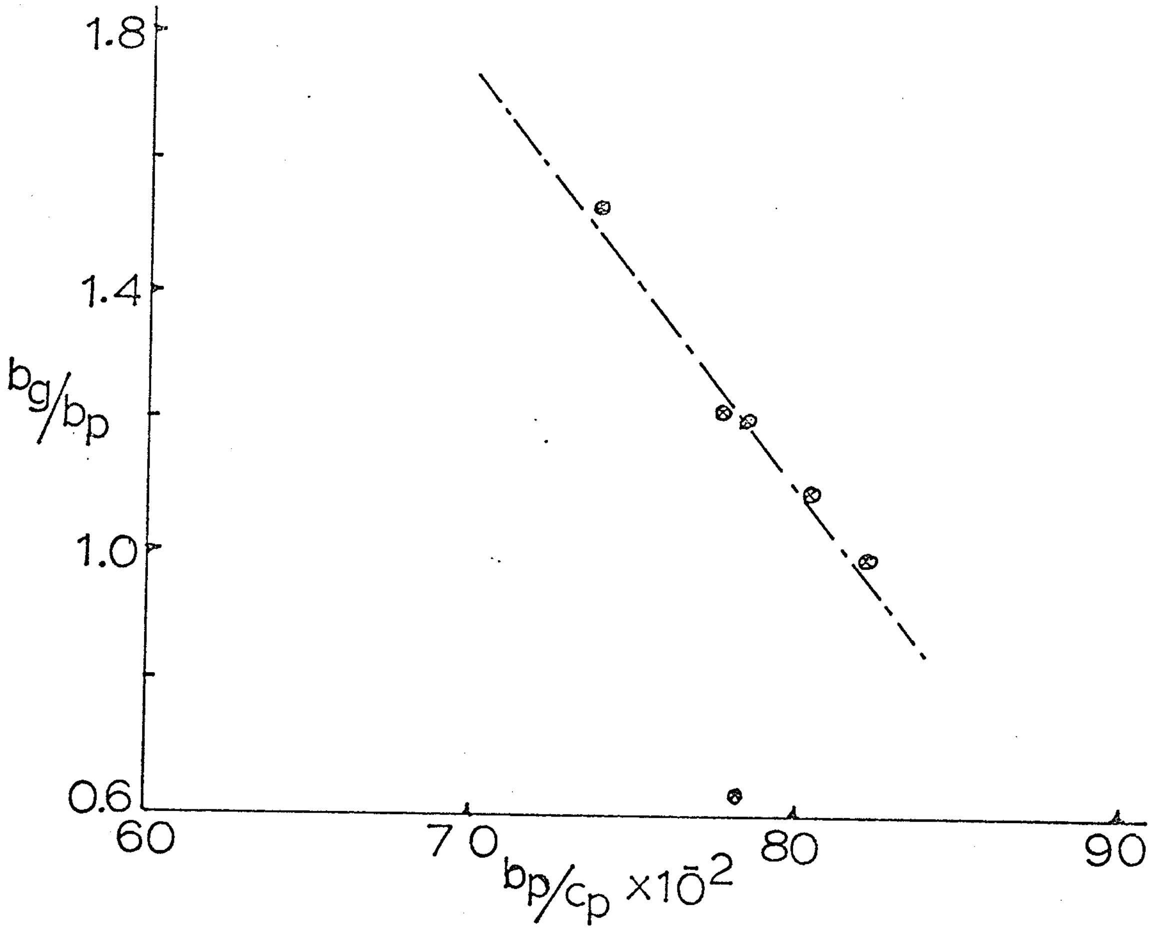

Fig. 19 chart for obtaining coeff. b
on gravel roads (for trucks)

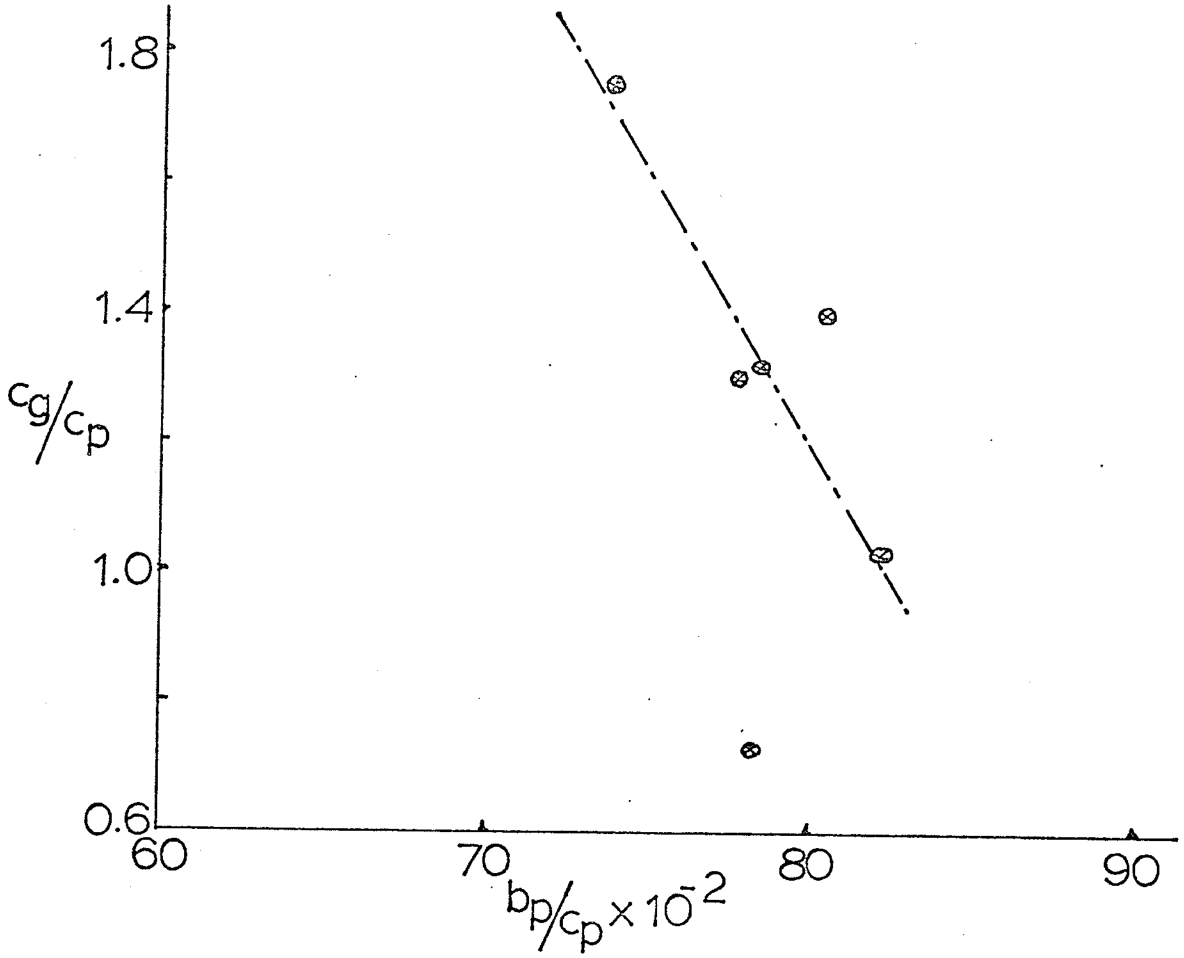

Fig. 20 chart for obtaining coeff. c on gravel roads (for trucks) shows a reduction in the values of the coefficients. There isn't anything specifically abnormal about truck T_6 but there does seem to be an anomaly in the experimentally determined fuel consumption figures. Over parts of the speed range the fuel consumption is shown to be less when the truck is on a gravel road than when on a smooth, paved road. This is quite hard to believe and is another instance of the unreliability of the experimental results.

Effect of Rise and Fall

A vehicle going uphill does some work against gravity depending on the elevation of the vehicle against gravity; and work done per unit time depends on slope and speed of vehicle. When the vehicle goes downhill it regains some, but not all, of the energy lost in going up. Some of the energy is lost in heat, due to factors such as rolling resistance and air resistance. In general the heavier the vehicle, the greater will be the magnitude of this resistance. Thus, the excess fuel consumed while going uphill depends on the slope of the hill and the vehicle's weight. It is found that the amount saved going downhill is often very small and little error results from assuming that there is no fuel saved whilst the vehicle is going downhill.

For a vehicle travelling between two points A and B, uphills in the direction A to B will be downhill in the direction B to A, and vice versa. The average of the sum of the upgrades from A to B and B to A is taken as the rise and fall of the road. Excess fuel consumed due to such uphills is determined on this basis.

Again, cars have a fairly narrow range of weights and they can still accelerate even if going uphill, unlike trucks that have a wide range of weights and are unable to accelerate while going uphill. A good deal of information was available for the behaviour of cars on uphill grades from ref.1, (these are shown in Table IX).

Table IX*

Fuel Consumption and Rate of Rise and Fall
for Passenger Cars

% Increase from Consumption
on Level Road

	Rate of Rise and Fall(ft./100ft.)			
speed (mph)	2	4	6	8
15	-	10	30	54
20	-	10	32	59
25	3	9	32	61
30	3	8	30	60
35	3	8	27	56
40	3	8	22	51
45	3	6	18	48
50	-	5	13	40
55	-	2	11	34
60	-	-	9	28
65	-	-	6	--

* from the World Bank Occasional paper (ref.1).

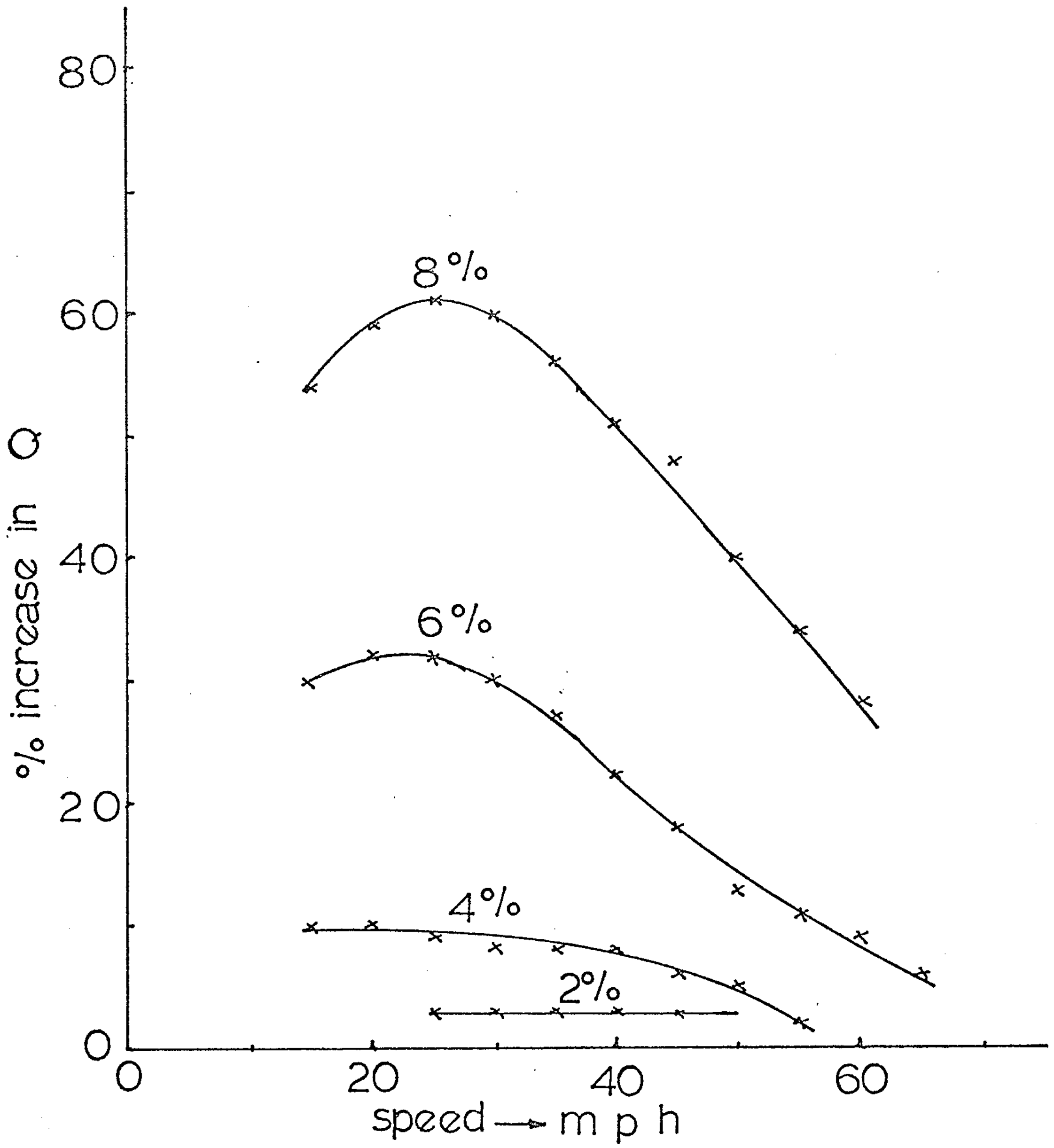

Fig.21 % increase in fuel consumption of cars & small trucks on any rise & fall over that consumed on level roads

This relationship is shown graphically on Fig. 21.

For trucks, very scanty information is available on their behaviour while going on uphill grades. Going uphill, their speeds are reduced and excess consumption goes as high as about 90% above normal for 12 ton trucks on a 6% grade*. Trucks are known to move at fairly constant but low speeds while going uphill. For this reason each truck is assumed to maintain one particular low speed. Excess fuel consumed at this speed was obtained for each truck weight. The values used were extracted from ref.1, for three trucks. In view of the fact that three trucks are not good enough to use for such analysis, the reliability of the results is doutful. Nevertheless, the results give an idea of the pattern of the excess consumption on uphill grades for trucks.

Table X*

Fuel Consumption and Rate of Rise and Fall for Trucks as % Increase from Consumption on Level Road.

Truck wt. (tons)	Rate of Rise and Fall (ft./100ft.)				
	1	2	4	6	8
2.8	--	3	11	25	53
12.0	4	20	53	96	--
15.0	15	44	108	154	--

This is graphically represented in Fig. 22.

As can be seen in Fig. 22, grades of more than 4% affect heavy trucks seriously, whereas passenger cars are not very

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* from the World Bank Occasional paper (ref.1).

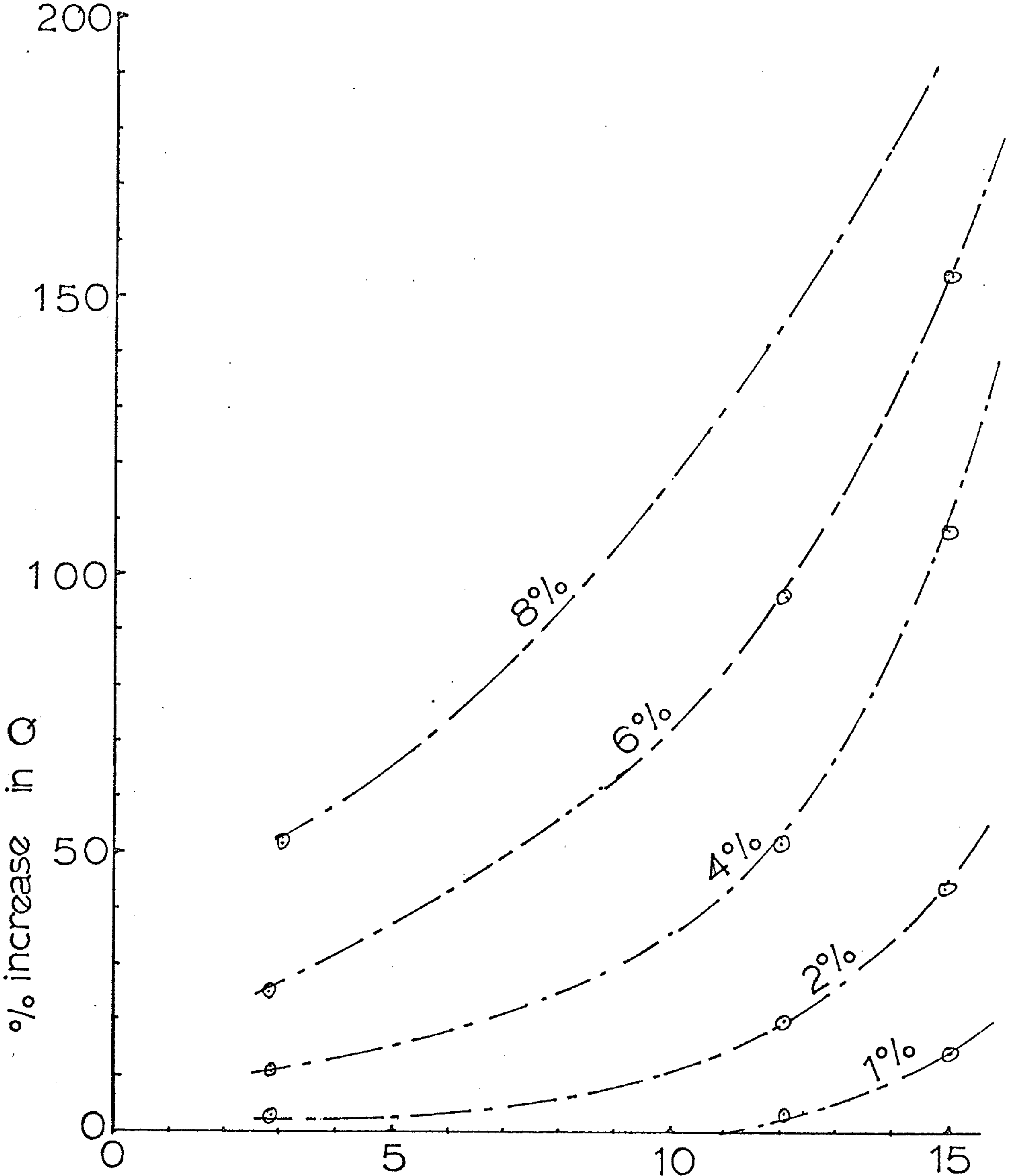

Fig.22 % increase in fuel consumption with rise & fall of the road vs vehicle weight for trucks much affected by grades less than 6%. This in fact explains why high grades are not allowed on roadways used very much by trucks.

Table XI*

Maximum Permissible Grades for Highways

Max. Permissible Grades, %

	Interstate		Highways Other than Freeways		
Design Speed (mph)	Flat & Rolling Country	Mountainous	Flat	Rolling	Mountainous
30	-	-	6	7	9
40	-	-	5	6	8
50	5	7	4	5	7
60	4	6	3	4	6
70	3	5	3	4	-

From Table XI, it can be seen that the maximum permissible grades on an interstate highway (such as Trans Canada Highway) is 3%. Even the 3% grades exert a considerable effect on the fuel consumption of heavy trucks.

Using these measures for excess fuel consumption due to uphills, the model becomes more applicable on any sort of highway.

The road curvature is another road geometric that is of importance in determining excess fuel consumption. Unfortunately, no reliable information on excess fuel

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* from Oglesby & Hewes, "Highway Engineering" (ref. 22)

consumption due to this factor could be found. No attempt has been made, therefore, to include this factor in the present model.

CHAPTER VI

APPLICATION AND LIMITATION OF MODEL. CONCLUSION AND RECOMMENDATIONS.

Application and Limitation of Model

So far this model has been able to incorporate the vehicle characteristics that come into fuel consumption. From a knowledge of readily available and very basic vehicle parameters, the "fuel consumption versus speed" curve can be deduced for all cars and trucks. The technique used in this analysis has a far greater value than the analysis itself. This is simply because the analysis has suffered from data deficiency. As a result, in no way is perfection claimed for this model, but the technique tends to far outreach others, in its comprehensiveness.

In using the charts in this paper, the user should expect errors ranging from 5% - 25%. Higher errors could be expected in special cases where the prediction of the coefficient 'c' is very poor.

The application of this model is limited to gasoline cars and trucks only. Buses have been excluded because information was available for only one bus. This particular bus always has its characteristics and coefficients falling between cars and trucks, which suggests that buses have to be treated as a separate group. An attempt to use this model for buses, without more information, would be inappropriate. At this point the difference between a small truck and other trucks should be clarified. Small pickups are those trucks that behave like cars, but have a bit higher load carrying capacity than cars. Usually their curb weight is less than three tons. This definition was adopted by the author for this study.

Apart from the fact that this model is only applicable to gasoline cars and trucks, it also has some other limitations, which are listed below:

(i) Due to the way the tests were carried out, the experimental results are all for moving vehicles and at top gears. Therefore this model shall be used only for moving vehicles at top gear ratios.

(ii) The speed ranges within which this model is valid are:

cars and small trucks ----10mph to 70 mph
trucks--------------------15mph to 55mph

Outside these speed ranges, the reliability of this model would be doubtful.

(iii) The speeds at which fuel consumptions are measured are assumed to be constant, at each speed level. i.e. excess consumption due to acceleration of these vehicles is not included.

(iv) The model is only applicable in very mild air or still air conditions. This is because the test runs were carried out in still air, or where still air was not present, the fuel consumed was converted to the amount consumed in still air.

The driver behaviour has been brought out only on the speed at which the vehicle is travelling. As it will be hard to bring out the effect of the individual driver's behaviour, it was felt most appropriate to set up the model on the basis of an ideal driver. i.e. one who does not consume excess fuel by unecessary acceleration or braking. Any driver handling his vehicle other than ideally will obviously cause fuel consumption to be greater than that predicted by the model.

Conclusion and Recommendations

One of the basic problems, in transportation planning, that of evaluating road users' savings, can be relatively well handled using this model. Having an idea of the volume and composition of the vehicles that will be using an improved road, the fuel savings can be evaluated. These savings can then be compared to the cost of improving the road.

The results obtained from the model certainly show the right trend. Quantitatively they are as good as can be expected, considering the available technical information. The precision of results, however, is considered to be less important than the lessons to be learned from the approach to the problem. This approach reveals more technical aspects of the fuel consumption pattern than the previous approaches to the problem. The vehicle parameters that come into fuel consumption and how they come in, were fairly well understood. It should be reemphasized that better results could have been obtained if test runs were carried out and relevant information about the vehicles was recorded. Most patently, the development of this model has shown the previous lack of coordination between transportation planning and technical information concerning the fuel consumption of vehicles.

It is recommended that for future research in this area better results are most likely to be achieved if:

(i) A more thorough investigation is made into the vehicle parameters that account for fuel consumption.

(ii) Test runs are carried out with better precision and all vehicle parameters recorded during the test runs.

(iii) The vehicle parameters that are responsible for the coefficients 'a', 'b', and 'c', are known and recorded.

(iv) Test runs are carried out under the same environmental conditions and all the environmental conditions are recorded during these tests.

(v) Fuel consumption figures obtained during the different environmental conditions are converted to the same conditions.

(vi) Test runs on paved roads are carried out on the same type of paved roads and likewise for poor paved roads and gravel roads; (A standardized specification for road surfaces is required so that comparison of different test results can be more meaningful.).

(vii) Vehicles being tested are of the same age and mileage accumulation.

(viii) Speeds are maintained at a constant level for each speed level.

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APPENDICES

LIST OF TABLES IN APPENDIX

APPENDIX A

MULTILINEAR REGRESSION ANALYSIS PROGRAM (18).

The program used in this project for building the models is known as the stats 18 by F. Chebib.

This program is designed to perform a complete multiple regression analysis on a set of one dependent variable and up to 98 independent variables with options to select any combination of variables for the analysis.

The output consists of:

i) A list of input data.

ii) An M x M matrix containing all possible simple correlation coefficients between the variables.

iii) The mean and standard deviation for each variable.

iv) The partial and standardized regression coefficients of each of the selected independent variables on the dependent variables together with its standard error and the t-value for a test of its significance.

v) The value of the intercept and the multiple correlation coefficient.

vi) Analysis of variance of the multiple regression and an F-test of significance.

vii) A list of observed, expected and adjusted values of the dependent variable for each of the actual combinations of the independent variables, (optional).

APPENDIX B

ATTEMPTED FUNCTIONS

1. $Q = a_o + a_1 w + a_2 P + a_3 v$

2. $Q = a_o + a_1 w + a_2 P + a_3 \ln v$

3. $\ln Q = a_o + a_1 w + a_2 P + a_3 \ln v$

4. $Q = a_o + a_1 w + a_2 P + a_3 v^2$

5. $Q = a_o + a_1 w + a_2 P + a_3 v + a_4 v^2$

6. $Q = a_o + a_1 w + a_2 P + a_3 t + a_4 v + a_5 v^2$

7. $Q = a_o + a_1 w + a_2 w/P + a_3 t + a_4 v + a_5 v^2$

8. $Q = a_o + a_1 w/P + a_2 t + a_3 v + a_4 v^2$

9. $Q = a_o + a_1 w/P + a_2 v + a_3 v^2$

10. $Q = a_o + a_1 w/P + a_2 t + a_3 d + a_4 v + a_5 v^2$

11. $Q = a_o + a_1 w + a_2 P + a_3 t + a_4 d + a_5 v + a_6 v^2$

12. $Q = a_o + a_1 w + a_2 P^2 + a_3 t + a_4 d^2 + a_5 v + a_6 v^2$

13. $Q = a_o + a_1 w + a_2 P^2 + a_3 t^2 + a_4 d^2 + a_5 v + a_6 v^2$

a_i = constant coefficients

w = vehicle weight

P = vehicle horsepower

t = vehicle tire size

d = vehicle engine displacement

v = vehicle speed

APPENDIX C

SUMMARY OF COMPUTER OUTPUTS AND STATISTICAL TESTS

Outputs

PAVED ROAD

Table C1

Cars & Bus

Run no.	1	2	3	4	5	6	7
Vehicle	C_1	C_2	C_3	C_4	C_5	C_6	B_1
Multiple Correlation Coefficient	.99315	.93221	.98513	.95172	.97826	.98402	.87454
Standard Error of Estimate	.00089	.04024	.00225	.00327	.00303	.00313	.01260
F'-value	324.895	9.951	65.752	19.225	44.502	61.094	9.757
t'-values	-12.762 +16.419	-4.035 +4.338	-10.873 +11.398	-6.084 +6.200	-9.384 +8.967	-10.425 +10.967	-3.390 +3.799
$ax10^{-2}$	4.20	5.90	7.79	7.87	11.19	11.04	21.15
$-bx10^{-4}$	10.86	19.00	21.85	17.79	25.39	29.18	69.03
$cx10^{-6}$	16.00	28.57	27.98	22.14	29.64	37.50	109.10

PAVED ROAD (contd.)

	Trucks						
Run no.	8	9	10	11	12	13	14
Vehicle	T_1	T_2	T_3	T_4	T_5	T_6	T_7
Multiple Correlation Coefficient	.99413	.99349	.99459	.99687	.99688	.99449	.97419
Standard Error of Estimate	.00162	.00552	.00309	.00346	.00276	.00418	.00987
F'-value	337.629	266.248	229.259	476.212	398.389	270.208	55.873
t'-values	−15.363 +18.569	−23.051 +22.552	−17.788 +15.769	−24.359 +21.019	−18.904 +15.497	−20.958 +19.081	−9.399 +8.505
$ax10^{-2}$	14.04	47.76	34.73	40.25	36.70	40.93	42.79
$-bx10^{-4}$	29.23	168.40	108.60	136.10	106.00	141.60	150.00
$cx10^{-6}$	44.00	216.70	147.00	166.00	132.00	181.00	191.30

GRAVEL ROAD

Table C2

	Car & Bus	
Run no.	15	16
Vehicle	C_1	B_1
Multiple Correlation Coefficient	.97638	.96098
Standard Error of Estimate	.000997	.00837
F'-value	71.474	12.068
t'-values	-8.408 +9.724	-3.752 +3.135
$ax10^{-2}$	4.74	35.70
$-bx10^{-4}$	10.84	169.00
$cx10^{-6}$	16.00	299.99

GRAVEL ROAD (contd.)

	Trucks						
Run no.	17	18	19	20	21	22	23
Vehicle	T_1	T_2	T_3	T_4	T_5	T_6	T_7
Multiple Correlation Coefficient	.99804	.99411	.99508	.99610	.99248	.99480	.93812
Standard Error of Estimate	.00079	.00575	.00456	.00423	.00277	.00206	.02094
F'-value	761.913	252.475	100.862	254.784	131.499	238.634	11.006
t'-values	-23.588 +28.254	-22.226 +21.411	-6.791 +5.279	-12.138 +9.130	-15.895 +15.273	-21.577 +20.806	-2.392 +1.840
$ax10^{-2}$	15.92	58.51	48.99	42.14	40.62	35.83	51.98
$-bx10^{-4}$	30.17	206.64	166.56	135.71	116.57	90.20	181.89
$cx10^{-6}$	51.00	281.00	257.13	171.42	184.76	132.39	252.16

Statistical Tests

F-test (one-tailed test)

This is a variance ratio test. As in other tests of significance, a null hypothesis is adopted which in this case is that the variances of the explained and unexplained variance belong to the same population.

The F-value is calculated as:

$$F' = \frac{\text{Explained (Regression) variance}}{\text{Unexplained (Residual) variance}}$$

Selecting a small probability (usually 0.05 or 0.01), the values of F (v_1, v_2) can be determined from statistical tables.

v_1 = degrees of freedom due to regression.

v_2 = degrees of freedom due to residuals .

Then 'h' is regarded as consistent with the data (or accepted) if the observed value of $F'(v_1, v_2)$ falls below $F\alpha(v_1, v_2)$; otherwise, 'h' is regarded as inconsistent with the data (or rejected).

Choosing α = 5%, results obtained are as follows(on the next page.).

It turned out that the null hypothesis was rejected in all cases except for Run no.16. Run no.16 is for the urban bus, which did not play any important role in this analysis.

Table C3

Run no.	no. of samples	v_1	v_2	F'-value computed	F-value* (table)	Accept or Reject Hypothesis
1	12	2	9	324.895	4.26	reject
2	6	2	3	9.951	9.55	"
3	7	2	4	65.752	6.94	"
4	7	2	4	19.225	6.94	"
5	7	2	4	44.502	6.94	"
6	7	2	4	61.094	6.94	"
7	9	2	6	9.757	5.14	"
8	11	2	8	337.629	4.46	"
9	10	2	7	266.248	4.74	"
10	8	2	5	229.259	5.79	"
11	9	2	6	476.212	5.14	"
12	8	2	5	398.389	5.76	"
13	9	2	6	270.208	5.14	"
14	9	2	6	55.873	5.14	"
15	10	2	7	71.474	4.74	"
16	5	2	2	12.068	19.00	accept
17	9	2	6	761.913	5.14	reject
18	9	2	6	151.475	5.14	"
19	5	2	2	100.862	19.00	"
20	7	2	4	254.784	6.94	"
21	7	2	4	131.499	6.94	"
22	8	2	5	238.634	5.79	"
23	6	2	3	11.006	9.55	"

* obtained from Table A-10, ref.8.

t-test (two-tailed)

This is another test of significance. It is a two tailed test, unlike the F-test which is one tailed. The t-test deals mainly with the regression coefficients. It is used to establish whether a regression coefficient b_j differs significantly from a value (eg. a theoretical value) b_j.

$$t' = \frac{b_j - b_j^o}{S_b}$$

Where S_b is the variance of the regression coefficient. The null hypothesis is rejected at the stipulated level of significance α (usually 5%) if t' values (computed) exceed the critical value given in the table.

t_1' and t_2' are the computed t-values

t_1 and t_2 are the t-values from the table.

$t_1, t_2(n, \alpha) = t_1, t_2(n, 0.05)$.

n = degrees of freedom

At 5% level of significance, the t-test results are as follows:

t-test

Table C4

Run no.	no. of Samples	n	Computed t-values t'_1	t'_2	t_1&t_2 from Table* ±	Accept or Reject Hypothesis
1	12	9	-12.76421	+16.41920	2.262	reject
2	6	3	-4.43454	+4.33835	3.182	"
3	7	4	-10.87333	+11.39808	2.776	"
4	7	4	-6.08377	+6.19973	2.776	"
5	7	4	-9.88440	+8.96710	2.776	"
6	7	4	-10.42474	+10.96653	2.776	"
7	9	6	-3.38996	+3.79889	2.447	"
8	11	8	-15.36338	+18.56941	2.306	"
9	10	7	-23.05092	+22.55147	2.365	"
10	8	5	-17.78824	+15.76883	2.571	"
11	9	6	-24.35870	+21.01851	2.447	"
12	8	5	-18.90372	+15.49647	2.571	"
13	9	6	-20.95750	+19.08125	2.447	"
14	9	6	-9.39865	+8.050496	2.447	"
15	10	7	-8.40766	+9.72400	2.365	"
16	5	2	-3.75200	+3.35357	4.303	accept
17	9	6	-23.58762	+28.25274	2.447	reject
18	9	6	-22.22572	+21.41119	2.447	"
19	5	2	-6.79134	+5.27856	4.303	"
20	7	4	-12.13819	+9.29473	2.776	"
21	7	4	-15.89506	+15.27254	2.776	"
22	8	5	-21.57713	+20.80624	2.571	"
23	6	3	-2.39184	+1.83977	3.182	accept

* obtained from TableA-8, ref.8.

APPENDIX D

COMPUTED AND EXPERIMENTAL FUEL CONSUMPTION FOR THE VEHICLES TESTED WITH THE MODEL.

Car C_2

weight = 1.05 tons

horsepower = 120

displacement = 200 cu. in.

weight/length = 1451b/ft

140w-DxN.C./6 = +99

From charts on:

Fig.6; $a = 6.0\times10^{-2}$

Fig.7; $b = -16.0\times10^{-4}$

Fig.8; $c = 23.0\times10^{-6}$

Q(estimated) $= 0.060 - 0.0016v + 0.000023v^2$

Table D1:Computed and Experimental Q for Car C_2

Speed v(mph)	Fuel Consumption in gals/mile Q comp.	Q exp.	% diff.
10	0.0463	0.045	6
20	0.0372	0.030	24
30	0.0327	0.025	31
40	0.0328	0.030	9
50	0.0375	0.040	6
60	0.0422	0.045	6

total = 82

av. = 12%

Car C_3

weight = 1.5 tons

horsepower = 120.0

displacement = 200.0 cu. in.

weight/length = 166.67lb/ft

140w-DxN.C./6 = +10

From charts on:

Fig.6; $a = 7.5 \times 10^{-2}$

Fig.7; $b = -18.0 \times 10^{-4}$

Fig.8; $c = 22.5 \times 10^{-6}$

Q(estimated) $= 0.075 - 0.0018v + 0.0000225v^2$

Table D2:Computed and Experimental Q for Car C_3

Speed v(mph)	Fuel Consumption in gals/mile Q comp.	Q exp.	% diff.
10	0.05925	0.060	1
20	0.04800	0.045	7
30	0.04125	0.035	18
40	0.03900	0.035	11
50	0.04125	0.040	3
60	0.04800	0.050	4
70	0.05925	0.060	1

total = 45

av. = 6%

Car C_4

weight = 2.00 tons

horsepower = 195

displacement = 283.0 cu. in.

weight/length = 216.0 lb/ft

140w-DxN.C./6 = -96

From charts on:

Fig.6; $a = 9.5 \times 10^{-2}$

Fig.7; $b = -22.0 \times 10^{-4}$

Fig.8; $c = 30.0 \times 10^{-6}$

Q(estimated) $= 0.095 - 0.0022v + 0.00003v^2$

Table D3:Computed and Experimental Q for Car C_4

Speed v(mph)	Fuel Consumption in gals/mile Q comp.	 Q exp.	% diff.
10	0.076	0.065	12
20	0.063	0.050	13
30	0.056	0.045	12
40	0.055	0.040	14
50	0.060	0.048	12
60	0.071	0.055	13
70	0.088	0.060	15
		total =	91
		av. =	13%

Car C_5

weight = 2.20 tons

horsepower = 290

displacement = 383 cu. in.

weight/length = 238.0lb/ft

140w-DxN.C./6 = -287.0

From charts on:

Fig.6; $a = 11.0\times10^{-2}$

Fig.7; $b = -27.0\times10^{-4}$

Fig.8; $c = 33\times10^{-6}$

Q(estimated) $= 0.11 - 0.00270v + 0.000033v^2$

Table D4:Computed and Experimental Q for Car C_5

Speed v(mph)	Fuel Consumption in gals/mile Q comp.	Q exp.	% diff.
10	0.0863	0.092	6
20	0.0692	0.070	1
30	0.0587	0.060	2
40	0.0548	0.059	7
50	0.0575	0.060	4
60	0.0668	0.069	3
70	0.0827	0.077	8

total = 31

av. = 4.5%

Truck T_2

weight = 12.00 tons

horsepower = 210

displacement = 637 cu. in.

payload/length = 738.0lb/ft

140w-DxN.C./6 = +1043

From charts on:

Fig.6; $a = 45.0 \times 10^{-2}$

Fig.7; $b = -164.0 \times 10^{-4}$

Fig.8; $c = 220.0 \times 10^{-6}$

$$Q(\text{estimated}) = 0.45 - 0.0164v + 0.00022v^2$$

Table D5: Computed and Experimental Q for Truck T_2

Speed v(mph)	Fuel Consumption in gals/mile Q comp.	Q exp.	% diff.
10	0.308	-----	--
20	0.210	0.220	6
30	0.156	0.1663	1
40	0.146	0.1553	4
50	0.180	0.1750	17
60	0.258	0.2462	0

total = 28

av. = 6%

Truck T_3

weight = 13.445 tons

horsepower = 185

displacement = 503 cu. in.

payload/length = 854lb/ft

140w-DxN.C./6 = +1377

From charts on:

Fig.6; $a = 34.5\times10^{-2}$

Fig.7; $b = -104.0\times10^{-4}$

Fig.8; $c = 165.0\times10^{-6}$

Q(estimated) $= 0.345 - 0.0104v + 0.000165v^2$

Table D6:Computed and Experimental Q for Truck T_3

Speed v(mph)	Fuel Consumption in gals/mile		% diff.
	Q comp.	Q exp.	
10	0.2575	------	---
20	0.2030	0.185	10
30	0.1815	0.155	17
40	0.1930	0.140	38
50	0.2475	0.170	44
60	0.3150	------	--

total = 109

av. = 27%

Truck T_5

weight = 13.090 tons

horsepower = 184.0

displacement = 501.0 cu. in.

payload/length = 922lb/ft

140w-DxN.C./6 = +1329

From charts on:

Fig.6; $a = 37.0 \times 10^{-2}$

Fig.7; $b = -113.0 \times 10^{-4}$

Fig.8; $c = 135.0 \times 10^{-6}$

Q(estimated) = $0.37 - 0.113v + 0.000135v^2$

Table D7:Computed and Experimental Q for Truck T_5

Speed v(mph)	Fuel Consumption in gals/mile Q comp.	Q exp.	% diff.
10	0.2706	-----	--
20	0.1980	0.205	2
30	0.1515	0.165	8
40	0.1340	0.155	13
50	0.1423	0.165	14
60	0.1780	-----	--

total = 37

av. = 9%

Truck T_7

weight = 11.175 tons

horsepower = 187.0

displacement = 332.0 cu. in.

payload/length = 826.0lb/ft

140w-DxN.C./6 = +1118

From charts on:

Fig.6; $a = 42.5\text{x}10^{-2}$

Fig.7; $b = -150.0\text{x}10^{-4}$

Fig.8; $c = 180.0\text{x}10^{-6}$

Q(estimated) $= 0.425 - 0.0150v + 0.00018v^2$

Table D8:Computed and Experimental Q for Truck T_7

Speed v(mph)	Fuel Consumption in gals/mile Q comp.	Q exp.	% diff.
10	0.293	-----	--
20	0.196	0.190	3
30	0.137	0.150	9
40	0.113	0.140	19
50	0.130	0.160	19
60	0.173	-----	--

total = 50

av. = 13%