

Stochastic optimization of a closed-loop fashion supply chain.

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Abstract

The fashion industry, characterized by rapid changes in consumer preferences, high demand volatility, and intense global competition, faces challenges in maintaining profitability while addressing sustainability concerns. The fast-changing nature of fashion trends leads to a high turnover of products, contributing to waste and environmental degradation. These issues underscore the need for fashion companies to adopt sustainable practices and enhance their supply chain management to remain competitive and ecologically responsible. This research proposes a novel mathematical modeling approach aimed at minimizing the cost of fashion companies within a closed-loop supply chain framework, incorporating sustainability considerations, and addressing uncertainties through stochastic optimization.

In the fast fashion industry, companies grapple with fluctuating consumer demands and the imperative for environmental stewardship. The perishability of fast fashion exacerbates waste, as products quickly fall out of favor, leading to environmental impacts. This thesis emphasizes the necessity of transitioning towards sustainable practices within the fashion industry, advocating for implementing closed-loop supply chains to mitigate environmental degradation while optimizing efficiency and profitability.

We introduce deterministic and stochastic models tailored for the fashion industry. Our approach uses two-stage stochastic optimization to manage uncertainties in demand within the closed-loop supply chain. This model incorporates sustainability into supply chain practices, reducing environmental harm and enhancing resilience and adaptability in the face of market volatility. Furthermore, a comparative analysis of the deterministic and stochastic model results underscores the importance of accounting for demand uncertainty in optimizing fashion supply chain operations.

Central to our methodology is the Sample Average Approximation (SAA) method, employed to solve the two-stage stochastic problem presented by the supply chain's dynamics. The SAA method is chosen for its robustness in handling uncertainty and its capacity to provide actionable insights based on empirical data. By applying this method, I demonstrate the feasibility of our model in real-world scenarios, guiding fashion companies toward sustainable practices without compromising profitability. This study adds to the knowledge of sustainable fashion by providing a practical solution to sector problems. Integrating sustainability and mathematical modeling opens

the door to a more successful fashion business where economic goals and environmental concerns are both addressed.

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Chapter 1: Introduction

In the past, people sewed their clothes, and fashionable clothes were just for the Bourgeoisie class. After Britain's Industrial Revolution, thrift stores became more common, increasing the demand for ready-made garments. By the middle of the 1800s, Britain had developed and used sewing machines to create clothing in large quantities. The improvement of factories and the emergence of sewing machines helped the fashion industry's growth. The clothing business has historically relied heavily on labor and has low capital. Because of the affordable labor, tax benefits, and tax legislation in developing nations, the fashion industry outsourced its production there. This transition, which decreased production costs, accelerated the emergence of fast fashion (Linden et al., 2016).

The fast fashion industry produces new low-quality collections and short product cycles at lower prices every two to eight weeks. Fast fashion has increased the variety of clothes. Companies that implement fast fashion are supported by consumers who want affordable clothes in a wide variety to keep up with new trends. Fast fashion decreases inventory costs in factories because the lead time is shorter (Linden et al., 2016).

Although the fast-fashion industry has some advantages for customers and companies, it has a detrimental environmental impact. This sector produces 4-5 billion tons of CO₂ annually, or 8-10% of the world's total emissions. Significant water consumption is related to the clothing sector. Textile treatment and dyeing have caused 20% of industrial water pollution. The dramatic increase in apparel consumption and subsequent growth in textile production is responsible for the growing environmental impact. For example, global textile production per person increased from 5.9 kg to 13 kg annually (Niinimäki et al., 2020).

A study regarding plastic pollution in Lake Winnipeg showed that Microfibers like synthetic textiles were the most prevalent kind of microplastic detected in the lake (Anderson et al., 2017). The fast-fashion industry is growing daily, and the demand for it is also rising, which can harm the environment. So, it is essential to find ways to reduce its harmful effects on the environment. One option is to reuse or recycle the clothes that have been returned or used. By minimizing the costs of production, inventory, and recycling, I can provide a good incentive for companies to reproduce clothes and gain profit while minimizing environmental harm.

A closed-loop supply chain represents a transformative approach to mitigate these environmental impacts. This model fundamentally differs from traditional linear supply chains, which typically follow a 'take-make-dispose' trajectory, by embracing a regenerative cycle. In a closed-loop supply chain, the focus is on maintaining the value of products, components, and materials for as long as possible, reducing waste to a minimum. It encompasses strategies for reusing, recycling, and refurbishing goods and materials, effectively looping them back into the production and supply chain after their initial use phase ends (Abbey & Guide, n.d.).

In the fashion industry, implementing a closed-loop supply chain has significant benefits for the environment. On the environmental front, it drastically diminishes the need for virgin raw materials, cuts down on emissions linked to production, and significantly lowers waste generation. It also addresses the industry's intense water usage by promoting the recycling of textiles, which reduces the demand for water in fabric dyeing and treatment processes, directly combating water pollution (Hu et al., 2014).

Embracing a closed-loop supply chain drives innovation in sustainable materials, processes, and technologies, encouraging the industry to rethink how garments are designed, produced, and disposed of. It requires a collaborative endeavor across the supply chain, involving suppliers, manufacturers, and retailers, to redesign products and processes with sustainability at the core.

From an economic standpoint, closed-loop systems can bolster the efficiency and sustainability of fashion businesses. By maximizing resource utilization and incorporating recycling into their operations, businesses can uncover efficiencies and cost reductions. This approach is also in line with the rising consumer demand for sustainable products, offering brands a competitive edge in the market that increasingly values environmental consciousness (Bozarth et al, 2022). However, reverse logistics, such as collecting returned products or recycling process can increase the costs. That is why it is essential to research the most economically optimized methods and strategies for establishing a closed-loop supply chain to make it profitable.

In summary, transitioning to a closed-loop supply chain is not merely an environmental imperative but a holistic strategy that reimagines the fashion industry's operational framework. It provides a viable route for the industry to balance economic growth with environmental sustainability, ensuring that fashion can thrive without compromising its ecological responsibilities. This research underscores the critical role of closed-loop supply chains in transforming fashion practices towards

enhanced sustainability, profitability, and ethical accountability, marking a crucial step towards resolving the fast fashion dilemma.

Chapter 2: Literature review

Currently, the environmental crisis exacerbated by excessive exploitation and CO₂ emissions underscores the urgency for sustainable industrial practices. The oil and fuel sectors are widely recognized as major contributors to pollution, yet the significant environmental impact of the fashion industry, ranked as the third largest polluting industry, is often underestimated (Climate Trade, 2023). Consequently, prioritizing sustainability within the fashion sector is imperative for mitigating its considerable ecological footprint.

Most fashion companies have forward supply chains, which means they have knitting, cutting, embroidery, cloth making packaging, distribution centers, and stores to deliver fashion products to the customers but products will be disposed by customers when they come to the end of their life cycle (Felfel et al., 2018). In the forward fashion supply chain literature, articles developed mathematical models to minimize cost or maximize their profit and give a better insight to managers in order to implement better strategies to manage logistics, inventory, labor, production, and equipping centers (Jun-jun Gao et al, 2010) (Imen Safra, et al, 2019) (Gabriel R. Bitran et al , 1986) (Huang et al., 2014) (Ait-Alla et al, 2014). Other articles concentrating on forward fashion supply chains optimize over-quota and under-quota costs for production planning (Yue Wu, 2011), minimize carbon emissions and late deliveries, and maximize social value (Ghasemy Yaghin, R. and Sarlak, P., 2022).

In the dynamic realm of fashion, an understated yet profound shift is taking place, targeting the fashion industry's considerable environmental footprint. Central to this shift is the adoption of closed-loop supply chains (CLSCs), a pivotal movement for an industry responsible for large amounts of CO₂ emissions every year. This drive is not merely about environmental stewardship but about redefining the lifecycle of fashion items to integrate sustainability with the industry's fast-paced nature. Research by Ji a et al. (2020) explain the principles of integrating circular economy within the textile and apparel sectors. It outlines the driving forces behind this movement, including government support, management commitment, consumer demand for sustainability, and societal pressures. A deeper exploration into the unique challenges faced by the fashion industry, especially when compared to the electronics sector, offers fascinating insights. The

difficulties associated with material reusability and the complexities of recycling underscore the necessity for industry-specific solutions (Denizel & Schumm, 2023).

In the forward fashion supply chain, especially fast fashion industry, customers will dispose products frequently and demand new trends which will both increase CO₂ emissions related to disposing fashion products and environmental pollution caused by producing higher quantity of new cloth in smaller amount of time. Designing closed-loop supply chains in the fashion industry can be an effective approach to decrease environmental pollution. Closed-loop supply chains integrate forward and reverse logistics, which means sold products can be returned and collected again and be reused. Utilizing closed-loop supply chains will increase life cycle of items and decrease resource exploitation for producing new products.

Further enriching the dialogue on CLSCs within the fashion industry are studies focusing on the operational challenges and strategic implementations of these systems, particularly highlighting the experiences of developing countries and smaller enterprises. Muhwati & Salisbury (2022), and Ya-Jun Cai et al (2022), contribute valuable perspectives, stressing the need for customized approaches and collective efforts such as involving collaboration between fashion brands, remanufacturers, and charity organizations to optimize used apparel collections and achieve both profitability and social responsibility goals. These discussions not only deepen the understanding of the existing obstacles but also illuminate the path toward innovation and advancement in the industry's pursuit of sustainability and resilience. The evolving narrative from one of challenges to one of optimism showcases the fashion industry's journey toward embracing a sustainable and circular future.

Considering the positive impact of employing closed-loop supply chain network in the fashion industry, it is unfortunate that not many articles focus on developing mathematical modeling techniques to optimize profit for this type of supply chains. Providing tools to maximize profit or minimize costs in this context can be a great incentive for fashion companies to change the structure of their supply chains. Shafiee Roudbari et al., (2023) develop a two-stage stochastic mixed-integer linear programming model for enhancing closed-loop supply chain efficiency, focusing on economic gains, reducing carbon emissions, and creating jobs through the strategic recovery and processing of returned fashion products. The research meticulously examines a comprehensive closed-loop supply chain, encompassing inspection, dismantling, disposal,

recycling, spare part markets, repair, refurbishing, suppliers, manufacturing, warehousing, and distribution centers. The solution method used in their research is Augmented epsilon constraint method to find the optimal result for their three objectives including Maximizing profit, minimizing CO2 emissions, and maximizing job opportunities. Another article which studied profit maximization in closed-loop fashion supply chain contained manufacturer, dispatching center, retail outlets, and customers. Authors crafted an optimization model to refine dynamic pricing and inventory strategies in the apparel sector, leveraging Bayesian methods for enhanced demand forecasting. Their findings reveal that demand learning substantially improves supply chain profitability and aids in optimizing supply chain coordination (Jun-jun Gao et al, 2010).

In the fast-paced world of the fashion industry, accurately forecasting demand and being responsive to its uncertainties, especially demand uncertainty, stand out as critical challenges. Fashion items are mostly seasonal, and the trends change so fast each season, so fashion companies face difficulties when aiming to predict demand accurately based on the last year's demand in that season. Moreover, responsiveness to customer's demand in the fashion industry is crucial due to the sector's fast-paced nature, characterized by rapidly changing trends, seasonality, and high consumer expectations for novelty and customization. This agility allows brands to minimize overproduction and markdowns, thereby increasing profitability and reducing waste. Furthermore, being responsive enables companies to better meet consumer preferences, enhancing customer satisfaction and loyalty in a highly competitive market where brand differentiation is key.

Deterministic optimization cannot address uncertainties in fashion industry. Despite the shortcomings of deterministic optimization, some work in the fashion supply chain literature assumes deterministic demand when modelling issues like assigning inventory between retail stores (Naderi, Siamak et al., 2020) and managing logistics like vehicle routing within fashion supply chains using cross-docking (Zhi-Hua Hu et al., 2013). Many articles have used stochastic optimization to address uncertainties in different industries including fashion industry. Usually demand in forward and closed-loop supply chains and quantity of return in closed-loop supply chains are considered stochastic parameters in the articles aiming to optimize profit or cost using stochastic optimization modeling technique (Lee et al 2010), (Feng et al., 2022), (Li et al, 2017), (Badri et al, 2017), (Ait-Alla et al, 2014). Other uncertainties considered in developing models are return quality (Kadir Biçe & Sakine Batun, 2021), disposal fraction, variable processing cost,

penalty cost (Mir Saman Pishvae et al, 2009), and workload-dependent lead times (Aouam & Uzsoy, 2015).

Ayvaz et al, (2015) developed a Reverse Logistics Network Design model to optimize the management of returned electrical and electronic equipment in Turkey, considering uncertainties like return quantity and quality. Their approach, a two-stage stochastic programming model aiming to maximize profit for recycling firms. Litvinchev et al., (2014) develop two mathematical models for optimizing closed-loop supply chains, focusing on strategic decisions like the placement of facilities and setting acquisition prices under uncertain demand. Their innovative use of the Golden Section Search method for pricing used products enhances profitability and operational efficiency in competitive and uncertain market conditions. Li, Congbo, et al, (2009) propose a stochastic dynamic programming model to optimize lot sizes in remanufacturing systems' production planning, aiming to minimize costs related to manufacturing, holding, and backlogs. This model addresses remanufacturing risks by considering the quality and timing of returns, alongside uncertain demand and return quantities, employing a policy iteration approach for optimal production planning.

Articles used different methods To solve the stochastic optimization problem. Hajiaghaei-Keshteli et al., (2018) and Soleimani & Kannan (2015) both explore stochastic optimization in closed-loop supply chain networks and leverage metaheuristic algorithms to navigate the complexities of network design and planning under uncertainty. They utilize Particle Swarm Optimization (PSO) and Genetic Algorithm (GA), enhancing their approaches with innovative methods such as the Response Surface Method (RSM) and hybrid algorithms. These studies highlight the effectiveness of combining PSO and GA in addressing the stochastic nature of supply chain challenges, demonstrating significant advancements in environmental sustainability and operational efficiency.

Karabuk (2008) and Wai Man Kong (2008) both address the uncertainties in the textile and apparel industries by applying stochastic programming models using scenario-based method, integrating discrete demand scenarios for yarn and apparel production planning, respectively. They focus on enhancing computational efficiency and flexibility in planning through methodological innovations, such as a two-phase preprocessing technique and multi-stage planning, demonstrating

the benefits of stochastic models over deterministic ones in managing demand and supply uncertainties.

Y. -x. Yang et al. (2014) and Agra et al. (2018) underscore the importance of the Sample Average Approximation (SAA) method in solving stochastic optimization problems within supply chain management. This method, crucial for both studies, enhances the practicality of addressing uncertainties in demand and return rates by efficiently estimating the expected values of random variables, thus enabling the identification of robust solutions that account for economic, environmental, and logistical complexities. The SAA method's applicability in generating realistic scenarios and optimizing supply chain configurations underlines its significance in operational research. Albritton M. et al. (2000) introduce simulation-based optimization (SBO) techniques for production planning under stochastic demand, distinguishing their work with two SBO methods: one for known continuous demand distributions and another, the Discrete Distribution Approach (DIS SBO) for uncertain demand scenarios. This innovative approach was enhanced by the Sample Average Approximation (SAA) method.

The literature review emphasizes the critical need for sustainable practices in the fashion industry, particularly through the adoption of closed-loop supply chains (CLSCs), to mitigate its substantial environmental footprint. It explores various research efforts on mathematical modeling to optimize profits within these sustainable frameworks, addressing the challenges of demand uncertainty and material reusability.

Research gap

In the existing literature, very few articles have researched stochastic optimization for a closed-loop supply chain. The articles typically focus on limited types of supply chain costs without integrating multiple costs to have a more holistic approach to managing supply chain costs. Specifically, a gap in addressing the simultaneous minimization of costs including minimizing facility equipping costs, purchasing material costs, transportation costs, manufacturing costs, distribution costs, inspection costs, disposing of products cost, holding costs in stores, holding costs in thrift stores, recycling costs, inventory costs, unutilized capacity cost, and cost of buying back returned products altogether .

Moreover, sustainable fashion supply chain articles concentrated on recycling and refurbishing clothes, neglecting the potential of reusing them. Also, these studies did not address the price

elasticity on returns. which is important in understanding how the price paid for returned products impacts the volume of returns.

In terms of transportation costs, most articles associated variable transportation costs with the number of products transported from one center to another; However, associating variable transportation costs with the number of trucks that should travel between centers is more realistic.

In order to fill the research gap, I conducted an optimization model that minimized all the costs mentioned together, implemented a price elasticity factor in the model which denotes how the price paid to customers for returning products can affect the number of returns, considered thrift stores in the close-loop supply chain to enable repairing products .

Our presented model also redefines the variable transportation cost by associating it with the number of trucks traveling between centers, which makes the model more realistic compared to previously proposed optimization models .

I used the Sample Average Approximation (SAA) method to solve the model, a novel application in the context of fashion closed-loop supply chains. This comprehensive approach increases the realism and practicality of the model, providing a more effective solution to the challenges faced by closed-loop supply chains in the fashion industry.

Table1 - Literature review

Authors	Fashion Industry	Network type			Facilities	Multi period	Multi product
		Forward	Reverse	Closed-loop			
Shafiee Roudbari, E., et al. (2023)	*			*	Inspection, dismantling, disposal, recycling, spare part markets, repair, refurbishing, suppliers, manufacturing, warehousing, and distribution centers	*	*
Naderi. Siamak, Kilic. Kemal, Dasci. Abdullah (2020)	*	*			Stores		*
Yue Wu (2011)	*	*			Manufacturing plants and cutomers	*	*
Y. -x. Yang, Y. -s. Guo and Z. -q. Huang, (2014)				*	Manufacturing/remanufacturing plants, distribution/recovery centers, cutomers		
Jun-jun Gao, Yun Zheng, & Xia Mao.(2010)	*			*	Manufacturer, dispatching center, retail outlet, and customers		*
Agra, A., Requejo, C., & Rodrigues, F., (2018)		*			Manufacturer, customer	*	
Soleimani, H., Seyyed-Esfahani, M. & Shirazi, M.A. (2016)				*	supplier, manufacturer, warehouses, distribution center, retailers/customer, collection and disassembly center, redistributors, second customers and disposal center	*	*
Inen Safra, et al. (2019)	*	*			Manufacturers, warehouses, and retailers	*	*
Narita, Hirohisa et al. (2005)	*	*			Manufacturers, warehouses, and retailers		
Felfel, H., Yahia, W.B., Ayadi, O. et al. (2018)	*	*			knitting and dyeing, cutting, embroidery, cloth making, packaging centers, and customers	*	*
Karabuk, S. (2008).	*	*			Manufacturer, Customer	*	
Wai Man KONG (2008)	*	*			Manufacturer, Customer	*	*
Gabriel R. Bitran, Elizabeth A. Haas, Hirofumi Matsuo, (1986)	*	*			Manufacturer, Customer	*	*
Albritton M, Shapiro A, Spearman M. (2000).		*			Manufacturer, Customer	*	*
Ghasemy Yaghin, R. and Sarlak, P. (2022)	*	*			Supplier, distribution centers, and customers.	*	*
Ayvaz, B., Bolat, B., & Aydin, N. (2015)			*		Regions of waste, collection centers, sorting centers, recycling centers, refinery centers, raw material markets and disposal centers	*	*
Kadir Biçe, Sakine Batun, (2021)				*	supplier, plant, disassembly center, distribution centers, recycling center, disposal center, spare part market and customers	*	
Litvinchev, I, Rios, Y. A., Özdemir, D., & Hernández-Landa, L. G. (2014)				*	Factories, Distribution or inspection centers, customers	*	
Soleimani, H., & Kannan, G. (2015)				*	suppliers, manufacturers, warehouses, distributors, retailers, collection and disassembly centers, disposal centers, redistributors, and customers	*	*
Jeihoonian, M., Zanjani, M. K., & Gendreau, M. (2017)				*	Part suppliers, raw material supplier, module supplier, manufacturing centers, distribution centers, end users, collection centers, disassembly centers, remanufacturing centers, modules markets, bulk recycling centers, disposal centers, material recycling centers, recycled material markets		*
Chouinard, M., D'Amours, S., & Kadi, D. A. (2008)				*	Supplier, warehouses, repair center, disassembling and refurbishing center, and disposal center, service centers, customer		*
Lee, D. H., Dong, M., & Bian, W. (2010)				*	Manufacturers, depots, customers		*
Hajiaghahi-Keshтели et al. (2018)				*	Supplier, manufacturer, recycle center, retailer, customer		*
Zhi-Hua Hu, Yingxue Zhao, and Tsan-Ming Choi (2013)	*	*			supplier, cross docks, customer		
Huang, Y.-S., Hsu, C.-S., & Ho, J.-W. (2014).	*	*			warehouse and retailer		*
Li, Congbo & Liu, Fei & Cao, Huajun & Wang, Qiulian.(2009)			*		Stores, warehouse for recovery, remanufacturer, serviceable inventory warehouse	*	
Mir Saman Pishvae, Jafar Razmi, and Fariborz Jolai (2009)				*	Manufacturer, Distribution centers, and customers		
Feng, Y. et al. (2022)				*	Supplier, manufacturer, remanufacturing centers, recycling centers, preprocessing centers, processing or disposal centers.	*	
Aouam, T., & Uzsoy, R. (2015)		*			warehouse and customers	*	
Li, S.-X., et al (2017)		*			Supplier, terminals and customers.		
Badri, H., Fatemi Ghomi, S. M. T., & Hejazi, T.-H. (2017)				*	Remanufacturing plants, collection zones, customers	*	*
Ait-Alla, A., et al (2014)	*	*			Manufacturer, customers	*	*

Authors	Criteria	Uncertainty	parameters	Solution method	Mathematical modeling technique
Shafiee Roudbari, E. et al, (2023)	Maximizing profit, minimizing CO2 emissions, maximizing job opportunities	Demand, quality and quantity of returns, percentage of products sent to each center	Transportation, processing, locating, inventory costs, and product price	Augmented epsilon constraint method	Two-stage stochastic mixed-integer linear programming
Naderi, Siamak, Kilic, Kemal, Dasci, Abdullah (2020)	Maximizing profit	-	Transportation and Inventory costs, price	annealing-based metaheuristic method	Mixed integer linear programming model
Yue Wu (2011)	Minimizing cost	Demand, under/over quota cost, shortage and surplus cost	Labor, quota, and production costs	Scenario-Based Method	Two-stage stochastic programming model
Y. -x. Yang, Y. -s. Guo and Z. -q. Huang. (2014)	Minimizing cost	Demand, return rates	processing, production, locating, transportation, and carbon emission costs	SAA method and GA	Two-stage stochastic programming model
Jun-jun Gao, Yun Zheng, & Xia Mao.(2010)	Maximizing profit	Demand	Transportation, inventory, production costs, price	Bayesian method	Stochastic programming with demand learning
Agra, A., Requejo, C., & Rodrigues, F., (2018)	Minimizing cost	Demand	Transportation, production, inventory and backlog cost	SAA method	Stochastic Production Inventory Routing Problem
Soleimani, H., Seyyed-Esfahani, M. & Shirazi, M.A. (2016)	Maximizing profit	demand, quantity of returns, number of centers, and price	Transportation, production, processing, inventory, over and under production costs, price	Scenario-Based Method	Stochastic Mixed Integer Linear Programming
Imen Safra, et al. (2019)	Minimizing cost	-	Transportation, production	-	Integer linear programming
Narita, Hirohisa et al. (2005)	Maximizing profit	Demand	leasing and renting capacity	Not mentioned	Stochastic programming-the newsboy problem
Y. -x. Yang, Y. -s. Guo and Z. -q. Huang. (2014)	Maximizing profit, risk minimization	Demand, quantity of returns, price	production, inventory, backorder, and transportation costs.	Scenario-Based Method	Two-stage stochastic programming
Karabuk, S. (2008).	Minimizing cost	Demand	Transportation, Inventory costs	Scenario-Based Method	Robust stochastic programming
Wai Man KONG (2008)	Minimizing cost	Demand	Outsourcing, production, and inventory cost	Scenario-Based Method	Stochastic linear programming
Gabriel R. Bitran, Elizabeth A. Haas, Hirofumi Matsuo, (1986)	Maximizing profit	Demand	Inventory, under and over production costs, price	Langrangian multiplier method and Density function	Stochastic programming
Albritton M, Shapiro A, Spearman M. (2000).	Maximizing profit	Demand	Inventory cost, price	Discrete Distribution Approach (DIS SBO), SAA method	Stochastic programming
Ghasemy Yaghin, R. and Sarlak, P. (2022)	Maximizing profit, Maximizing total social value, Minimizing late deliveries, Minimizing carbon emissions	Demand	Labor, inventory, transportation, and production costs	fuzzy multi-objective mathematical optimization	Fuzzy multi-objective programming
Ayvaz, B., Bolat, B., & Aydin, N. (2015)	Maximizing profit	Return quantity, quality, percentage of products sent to each center, transportation costs	Transportation, locating, and processing cost, price	SAA method	Two-stage stochastic programming
Kadir Biçe, Sakine Batun, (2021)	Maximizing profit	Demand, return quantity, quality	Transportation, locating, and processing cost, price	L-shaped method in iterative, branch-and-cut framework	Two-stage stochastic mixed-integer program (SMIP)
Litvinchev, I. Rios, Y. A. Özdemir, D. & Hernández-Landa, L. G. (2014)	Maximizing profit	Demand, return quantity	Transportation, locating, price	the Golden Section Search method	Stochastic Mixed Integer Linear Programming
Soleimani, H., & Kannan, G. (2015)	Maximizing profit	-	Transportation, inventory, locating, shortage and non-utilization, processing cost, and price	GA and PSO methods	Mixed integer linear programming (MILP)
Jeihoonian, M., Zanjani, M. K., & Gendreau, M. (2017)	Maximizing profit	Quality of returns	Locating costs, processing cost, transportation cost, price	L-shaped method	Two-stage stochastic mixed-integer programming model
Chouinard, M., D'Amours, S., & Kadi, D. A. (2008)	Minimizing cost	Demand, quantity of items in each center	Transportation costs, processing costs	SAA method	Stochastic programming model
Lee, D. H., Dong, M., & Bian, W. (2010)	Minimizing cost	Demand, quantity of returns	Transportation, locating, processing costs.	SAA method	Two-stage stochastic programming model
Hajjaghaei-Keshтели et al. (2018)	Minimizing cost	Demand	Transportation, manufacturing, locating, processing costs.	GA, PSO, and RSM methods	Two-stage stochastic programming model
Zhi-Hua Hu, Yingxue Zhao, and Tsan-Ming Choi (2013)	minimizing travel time, distance, waiting time at the CDC	-	Transportation	-	Mixed integer linear programming model
Huang, Y.-S., Hsu, C.-S., & Ho, J.-W. (2014).	Maximizing profit	Demand dependent on price	Pricing, inventory costs, over and under production cost	-	Discrete dynamic pricing
Li, Congbo & Liu, Fei & Cao, Huajun & Wang, Qiulian.(2009)	Minimizing costs	Demand and quantity of returns	Manufacturing, inventory and backlog costs	Scenario-Based Method	Stochastic dynamic programming-based model
Mir Saman Pishvae, Jafar Razmi, and Fariborz Jolai (2009)	Minimizing costs	Demand, quantity of returns, disposal fraction, variable processing cost and penalty cost	Transportation cost, locating cost, processing cost, penalty cost.	Scenario-Based Method	Stochastic Mixed Integer Linear Programming
Feng, Y. et al. (2022)	Minimizing costs	Demand and quantity of returns	Transportation, locating, replenishment, and inventory costs.	nonlinear grey Bernoulli-Markov model	Dynamic multi-objective model
Aouam, T., & Uzsoy, R. (2015)	Minimizing costs	Demand, workload-dependent lead times	Production, backordering and inventory costs.	-	chance-constrained models, two-stage stochastic programming, robust optimization
Li, S.-X., et al (2017)	Minimizing costs	Demand	Terminal locating, transportation and routing costs	Scenario-Based Method	Two-stage stochastic model
Badri, H., Fatemi Ghomi, S. M. T., & Hejazi, T.-H. (2017)	Maximizing economic value added (EVA)	Demand, quantity of returns	Transportation, production, processing, and inventory costs, market value, cost of capital, interest rate of debts.	Scenario-Based Method	Stochastic Mixed-Integer Linear Programming (MILP)
Ait-Alla, A., et al (2014)	Maximizing profit	Demand	Transportation, production, under and over production, and inventory costs, conditional value at risk	Scenario-Based Method	Stochastic programming -CVaR-Optimization

Chapter 3: Methodology

This research offers a stochastic programming technique to construct an integrated forward-reverse logistics network under uncertain conditions. I first build a deterministic mixed integer linear programming (MILP) model to ensure efficiency. Then, using a two-stage stochastic optimization technique, I create a stochastic variant of the MILP model.

To effectively manage uncertainties in supply chain modeling different approaches can be taken. These approaches include the two-Stage Stochastic Linear Programs with Fixed Recourse, chance constraint method, stochastic Mixed integer programming, two-stage stochastic nonlinear programs with recourse, multistage stochastic programming, scenario-based method, stochastic dynamic programming, and robust optimization.

Modeling Approaches

The two-stage stochastic linear programs with fixed recourse addresses decision-making under uncertainty by planning in two phases. Initially, decisions are made before the uncertainty is revealed; subsequently, corrective actions (recourse) are taken. It's suitable for financial planning and supply chain management where future conditions are uncertain.

Meanwhile, the constraints in the probabilistic or chance constraints method ensure that certain conditions are met with a predefined probability, allowing for flexibility in constraint satisfaction. This is pivotal in risk management and investment decisions where adherence to constraints within probabilistic bounds is critical.

Additionally, stochastic integer programs integrate the complexities of discrete decisions with uncertainty, this approach is vital for logistics and scheduling where decisions are not continuous but have to be made from a set of discrete choices.

Moreover, two-stage stochastic nonlinear programs with recourse expand the linear programming framework to encompass nonlinear relationships, essential in energy planning and resource allocation where the relationships between decision variables are not linear.

Multistage stochastic programs with recourse further extend the decision-making horizon across multiple stages, allowing for a series of decisions in response to unfolding uncertainties. It's crucial in dynamic strategies for investment and strategic planning over long horizons.

Moreover, the scenario-based method involves creating a set of discrete scenarios representing different possible outcomes, each with an associated probability. The decision variables are

optimized across these scenarios, usually with the aim of minimizing expected cost or maximizing expected profit (Birge & Louveaux, 2011).

In addition to previously explained methods, stochastic dynamic programming can be utilized for sequential decision-making problems where uncertainties unfold over time. It emphasizes breaking down complex problems into manageable stages and solving them piece by piece. Finally, although robust optimization is not a traditional stochastic programming approach, it optimizes against the worst-case scenario, ensuring solutions are viable under all possible outcomes (Kall & Mayer, 1995).

Solution methods

There are various methods to solving the mentioned stochastic programming models. The most used methods based on the literature are Sample average approximation (SAA) method using Monte Carlo simulation, Bender's decomposition or L-shaped method, Genetic algorithm and Particle swarm optimization, and Fuzzy optimization.

Sample Average Approximation Method Generates a sample of scenarios from the probability distribution of the uncertain parameters with a known probability distribution using the Monte Carlo simulation method. The objective is to approximate the expected value of the solution by averaging the outcomes across all samples. This method allows for the conversion of a stochastic problem into a deterministic equivalent, which can be solved with standard optimization techniques. SAA is particularly useful in large-scale problems where exact stochastic programming methods may be computationally infeasible (Kleywegt et al, 2002).

The L-shaped method, also referred to as Benders' decomposition, facilitates the efficient solution of large-scale stochastic linear programming problems by decomposing them into a master problem and one or more subproblems.

The Benders Decomposition method involves three main steps. First, solve a relaxed Master Problem to obtain a solution that guides the decision-making process. Second, solving subproblems based on the solution from the master problem to generate Benders cuts, which are constraints that refine the master problem. Third, iteratively updating and solving the master problem with the new Benders cuts until convergence is achieved, meaning no further improvement is possible. This iterative process aims to find an optimal solution by efficiently navigating the solution space. However, its disadvantages involve potential slow convergence rates

and the complexity of generating effective Benders cuts. (amírez-Pico et al., 2022), (Kall & Mayer, 1995).

Fuzzy logic for stochastic optimization refers to a computational paradigm that integrates the concepts of fuzzy logic with stochastic optimization techniques to handle uncertainty and imprecision in optimization problems. Stochastic optimization deals with optimization problems under uncertainty where some parameters of the problem are not known with certainty but can be described by probabilistic distributions. Fuzzy logic, on the other hand, deals with reasoning that is approximate rather than fixed and exact. It mimics the way humans make decisions, allowing for a range of values between "true" and "false" to capture the idea of partial truth. "Fuzzy Sets" by L.A. Zadeh, published in 1965, introduces the concept of fuzzy sets, which are classes of objects with grades of membership ranging between zero and one. This foundational work describes how fuzzy sets are characterized by a membership function assigning each object a grade of membership, thus extending traditional notions like inclusion, union, intersection, and complement to accommodate the gradation of membership. This concept provides a mathematical framework to deal with imprecision and uncertainty.

According to Goldberg and David (1989), genetic Algorithms (GAs) are a class of evolutionary algorithms inspired by natural selection mechanisms to solve complex optimization and search problems. They operate on a population of candidate solutions, represented as chromosomes, which evolve over successive generations. Each chromosome's fitness is evaluated using a problem-specific fitness function, guiding the selection process where fitter individuals are more likely to reproduce. Through genetic operators such as crossover (recombination) and mutation, GAs explore the solution space, generating new offspring that inherit characteristics from their parents. This process allows GAs to efficiently navigate large and complex search spaces by simulating the evolutionary principle of survival of the fittest, thereby evolving solutions towards optimality. The algorithm iterates through these steps selection, crossover, mutation, and replacement until a termination criterion is met, such as reaching a maximum number of generations or achieving a satisfactory level of fitness.

Genetic algorithms are notable for their flexibility, robustness, and ability to find near-optimal solutions in problems where other optimization methods might fail, making them applicable across diverse fields such as engineering, economics, machine learning, and artificial intelligence. Due to their stochastic nature and mechanisms like crossover and mutation, GAs have a strong ability

to escape local optima, making them effective in searching for global solutions in complex landscapes. While GAs are good at escaping local optima, they can still converge prematurely to suboptimal solutions if not properly tuned or if the genetic diversity in the population is not maintained (Goldberg and David, 1989)

Model description

I propose a new closed-loop fashion supply chain that has not been designed previously in the literature. This model comprises supplier, manufacturers/remanufacturers, distribution centers, stores, inspection centers, thrift stores, disposal centers, and recycling centers. The supply chain studied produces demanded wool clothes of different types and then sends them to distribution centers and stores. The demand for each type of product is uncertain as a result of the uncertain nature of the fashion industry. Then, at an uncertain rate, used clothes can get returned. Next, they are sent to inspection centers to evaluate the quality of returned clothes. If they are highly damaged, they will be sent to disposal centers. If products are in a good shape and can be reused, they will be sent to thrift stores, but if they are not wearable right away, they will be sent to recycling centers which clean clothes and turn them into raw material and fabric and recover a proportion of repairable clothes. In addition, the extra clothes in the thrift stores that are left unsold will be sent to the recycling centers. Finally, repaired fabrics are sent to distribution centers and recycled materials are sent to manufacturing centers. Our model considers multiple items in multiple months with demand changes.

The objective function of this model is to minimize the total costs of inventory, logistics, manufacturing, processing in the fashion closed-loop supply chain. The costs of processing in manufacturing centers, distribution centers, disposal centers, and recycling centers, in addition to equipping centers costs, the manufacturing cost of each product in each manufacturing center and the transportation cost of sending clothes between centers to reach stores and back to manufacturing centers to meet customer's demand for each product type in each store, the processing cost in distribution centers, the inventory cost of holding products in each store until its sold or returned, recycling costs, and finally unutilized capacity costs will be minimized.

The decision variables in this model will be the number of products that should be produced in each month, number of products transported from each center to the other, and deciding on which facilities that should be active and which ones should be deactivated.

There are uncertain variables in this model, such as demand for each product in stores, the return rate of products, the disposal fraction, the fraction of products sent to recycling centers directly, and the fraction of clothes sent to recycling centers from thrift stores.

The data for solving the model will be the secondary data available in different articles. Data will be scaled down proper to the size of the problem studied. Specifically, data from articles on fashion supply chains and closed-loop supply chains were used. The data was generated following a normal distribution, using the averages and standard deviations proposed in these articles. To fit the scale of a fashion supply chain with three products over three months, the mean values from these articles were divided by a specific factor to ensure appropriateness.

The Two-stage stochastic optimization model will be solved with sample average approximation method. SAA method is chosen to solve this problem as the Sample Average Approximation (SAA) method is particularly well-suited for optimizing stochastic mathematical models in complex systems such as closed-loop fashion supply chains.

SAA Method directly tackles uncertainty by generating many scenarios based on the probability distributions of the uncertain parameters. By solving the optimization problem for these scenarios, it provides an approximation of the expected value of the objective function, which is inherently suited for stochastic problems. While Fuzzy Optimization also deals with uncertainty, it does so by using fuzzy sets to model uncertainty rather than probabilistic models, which might not be as effective in capturing the variability and randomness in supply chain parameters. The L-shaped method is designed for two-stage stochastic problems with recourse, but it might not capture the full distribution of outcomes as effectively as SAA. Genetic Algorithms, though powerful in exploring a wide solution space, do not inherently model uncertainty but rather search for optimal solutions through evolution-inspired operations.

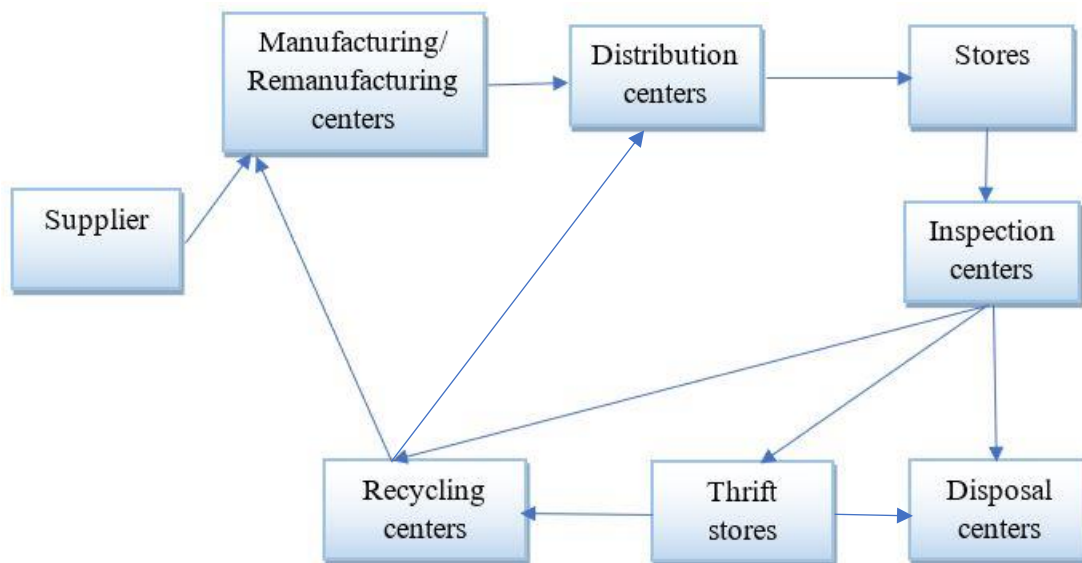
Moreover, SAA Method offers high accuracy in estimating the objective function and constraints by averaging over a large number of samples, leading to more reliable solutions. As the number of scenarios increases, the solution obtained by SAA converges to the true optimal solution of the stochastic problem.

Also, in term of computational efficiency for certain problem structures, especially those with a large number of scenarios that can be easily parallelized, SAA can be computationally more efficient. The method allows for exploiting modern computational architectures to handle large-scale problems effectively.

In conclusion. The SAA method offers a robust and efficient framework for addressing the inherent uncertainty and complexity of closed-loop fashion supply chains.

Finally, a sensitivity analysis will be performed to give insights to the companies about the importance of their decisions related to transportation, locating centers, inventory, and recycling costs in each center.

Figure 1- Fashion Supply Network studied



Data Collection

I conducted a comprehensive data collection from various articles to gather relevant information for my research. Specifically, I examined apparel industry articles to obtain data on wool suppliers and apparel manufacturing. For data related to distribution, transportation, inspection, recycling, and disposal, as well as stores and thrift stores, I consulted articles focused on closed-loop supply chains for durable products. For demand data, I gathered information on fast fashion items.

To expand the dataset, I utilized the average and standard deviation values from the mentioned articles and generated additional data through Monte Carlo simulation, following a normal

distribution. Given that the data from these articles was tailored to their specific contexts, I scaled the numbers down to fit the scope of our problem more appropriately.

To ensure the model's effectiveness, it was crucial that the data be proportional. For instance, if the number of products to be processed at a single center was significantly less than the available capacity, the feasible region would become excessively large, thereby extending the time required for the solver to obtain optimal solution.

To address this issue, I scaled down the problem, ensuring that all parameters were proportionate and consistent. By doing so, I was able to match the parameters more closely, which in turn reduced the solution time.

Variable cost of processing

Table 2-Supplying variable cost per raw material in US dollar

Supplier1	Supplier 2	Supplier 3
4	4.5	5.6

Table 2 shows the variable cost of supplying wool from suppliers 1, 2, and 3. The raw material used for producing all three products is wool. The average price of a skein of wool is \$4 with a standard deviation of \$1.5. (MacAusland's Woolen Mills, 2023). Data was generated following a normal distribution.

Table 3- Production variable cost per product in US dollar

Product	Manufacturing center 1 cost	Manufacturing center 2 cost	Manufacturing center 3 cost	Manufacturing center 4 cost
1	7.08	3.50	6.14	3.89
2	6.94	12.02	6.34	8.40
3	9.45	4.14	6.25	5.81

Table 3 demonstrates the variable production cost of products 1, 2, and 3 by manufacturing centers 1, 2, 3, and 4. Production, pattern making, and finishing of a short-sleeved shirt have been stated to be 7.08 (Jin Gam et al, 2009). Our problem studies three different types of shirts, so the average variable cost can be set to 7.08 with variation based on the model or pattern. Production variable cost data has been generated using a normal distribution with a mean of 7.08 and a standard deviation of 2.

Table 4-Variable distribution cost per product in US dollar

Distribution center	Cost of distributing
1	2
2	1.91
3	3.36

Table 4 displays variable distribution costs per product for each distribution center. Variable distribution costs include packaging, labeling costs, and expenses related to loading and unloading goods. Data has been generated with a mean of 2 and a standard deviation of 1.5 (Huang et al., 2014).

Table 5-Variable holding cost per product in stores in US dollar

Products	Holding cost in Store 1	Holding cost in Store 2	Holding cost in Store 3
1	2.27	3.03	2.91
2	5.03	2.49	3.58
3	3.01	1.24	2.44

Table 5 depicts the variable holding cost per each type of product at each store. It is stated that the average total cost of a retail store is \$650,000 annually and average sales is \$10,000,000 every

year. The total variable expenses for a retail store include organizing clothes, temporary staff, promotional events, and other costs like sales commissions, credit card fees, discounts, seasonal decorations, and miscellaneous variable expenses, amounting to an average \$650,000. The exact total cost will face variations based on the location or staffing (Solink., n.d.). Also, the average price of fast fashion products like Zara clothes is \$34 (McKinsey & Company, 2023). Hence, based on the average sales and average price, fast fashion companies can sell approximately 294,117 items annually. Variable cost per product can be calculated by dividing the average cost of total sales by number of items sold. The average variable cost per product will be \$2.21. Data has been generated with a mean of 2.21 and a standard deviation of 1.

Table 6-Inspecting variable cost per product in US dollar

Inspection center	cost
1	3
2	2.83
3	4.1

Table 6 illustrates inspecting variable cost per product. Inspection variable costs can include testing kits, sample containers, and variable utility expenses associated with running inspection equipment, such as electricity or water. The inspection variable cost's mean is 3 and the standard deviation is 1.5 (Feng et al, 2022).

Table 7-Disposing variable cost per product in US dollar

Disposal center	Cost
1	2
2	3.16
3	2.17

Table 7 shows variable disposal costs per product, like landfill fees. Data has been generated with a mean of 2 and a standard deviation of 1.5 (Jeihoonian et al., 2017).

Table 8-Variable holding cost in thrift stores per product in US dollar

Product	Holding cost in thrift store 1	Holding cost in thrift store 2	Holding cost in thrift store 3
1	2.27	1.06	1.05
2	0.92	3.62	2
3	1.45	2.1	2.42

Table 8 represents the variable holding cost per product at thrift stores. Holding costs in thrift stores have been calculated similarly to the information provided for holding costs at stores.

Table 9-Variable repairing cost per product in US dollar

Product	Repairing center 1	Repairing center 2
1	1	0.8
2	0.95	0.71
3	1.05	0.68

Table 10-Variable recycling cost per product in US dollar

Product	Recycling center 1	Recycling center 2
1	1.5	2
2	1.73	1.53
3	1.57	1.61

Tables 9 and 10 show variable repairing and recycling costs per each type of product respectively. This cost includes the variable cost of the process of making each product usable again. The mean of variable repairing cost is 1, and the standard deviation is 0.2. Variable recycling cost has a mean of 1.5 and a standard deviation of 0.2 (Fashion Takes Action. et al., 2023).

Table 11-Cost of buying back products from customers in US dollar

Product	Cost of buying back
1	3.5
2	4
3	5

Table 11 demonstrates the cost of buying each product back from customers. fashion companies employ various strategies to incentivize customers to return products for recycling purposes. Many companies in the fashion industry offer discounts ranging from 5% to 20% on the next purchase as an incentive. For instance, the clothing retailer J. Jill provides a 10% discount on the next purchase for each gently worn garment returned by customers (Graves, 2023) .

Based on the sustainable strategies implemented in the fashion industry, I have assumed the price of buying back the products to be 10% of the price of a full-price product, which can be offered as cash, a voucher, or a discount. Hence, the cost of buying back has been considered as 10% of the product's price. Also, As mentioned earlier average price of fast fashion clothes like Zara is \$34 (McKinsey & Company,2023). So, the average cost of buying back products will be \$3.4 with a standard deviation of 0.5.

Fixed cost of equipping centers

Table 12-Fixed cost of operating as suppliers in US dollar

Supplier	Cost
1	6,300,000
2	7,321,161
3	5,460,197

The average fixed cost of operating as a supplier is \$6.3 million for wool suppliers in Western Australia. Data available in table 12 has been generated with a mean of 6.3 million and a standard deviation of 1 million (Layman, 1999). Examples of fixed costs associated with wool supply include the expenses related to producing the wool, such as costs for primary factors like land, labor, and capital. Other costs encompass fertilizers, shearing, taxes (such as wholesale sales tax), and margins like brokers' fees., Legal and administrative expenses related to drafting and negotiating supply agreements, and investment in equipment and facilities required to handle and process wool from the supplier.

Table 13-Fixed cost of operating manufacturing centers in US dollar

Manufacturing center	Cost
1	16,105,000
2	15,187,000
3	7,509,000
4	7,673,000

Table 13 shows the fixed cost of operating, property, plant, and equipment of manufacturing centers. The data has been collected from the balance sheet of Inditex fashion company (Inditex, 2024). Since Inditex is a very large company with extensive facilities, we acknowledge that the fixed costs might be high. However, our presented model focuses on only three types of product, whereas Inditex manages a much larger product portfolio. Most fashion supply chains indeed handle numerous products, so our model assumes similar facilities as large fashion companies but applies the strategies to a limited number of products for validation purposes. This approach allows any company to input their specific data into the model to achieve optimal results tailored to their unique situation.

Table 14-Fixed cost of operating distribution centers in US dollar

Distribution center	Cost
1	7,200,000
2	7,000,000
3	8,588,176

Table 14 demonstrates the fixed cost of operating distribution centers. Regular payments for leasing or owning the distribution center facility, wages for employees working at distribution centers, and utility bills for electricity, water, heating, and cooling are all considered as fixed costs of operating distribution centers. The average fixed cost associated with a distribution center is 7.2 million dollars with a standard deviation of 0.5 million (Kadir Biçe & Sakine Batun, 2021).

Table 15- Fixed cost of operating stores in US dollar

Store	Cost
1	2,575,000
2	4,447,043
3	1,343,528

Table 16- Fixed cost of operating thrift stores in US dollar

Thrift stores	cost
1	2,575,000
2	1,388,500
3	1,007,407

Fixed costs related to stores and thrift stores like rent, salaries, utilities, insurance, advertising, depreciation, licensing and permits, security, maintenance, tech services, and amortization expenses have been estimated to be on average \$2,757,000. I generated data with an average of \$2,757,000 and a standard deviation of 1,500,000 (Solink., n.d.).

Table 17- Fixed cost of operating inspection center in US dollar

Inspection center	Cost
1	6,000,000
2	9,000,000
3	7,500,000

The average fixed cost of operating an inspection center, like plant and equipment costs, is 6 million dollars (Feng et al., 2022). The data was generated with a mean of 6 million and a standard deviation of 1.5 million.

Table 18- Fixed cost of operating disposal center in US dollar

Disposal center	Cost
1	6,000,000
2	4,986,444
3	6,636,600

Table 18 displays the fixed cost of operating disposing centers. The average fixed cost has been considered to be 6 million with a 1.5 million standard deviation (Jeihoonian et al, 2017).

Table 19- Fixed cost of operating recycling center in US dollar

Recycling center	Cost
1	10,000,000
2	8,340,916

Table 19 illustrates the fixed costs of operating a recycling center such as lease or mortgage, recycling facilities, salaries, and utilities costs with a mean of 10 million dollars and a standard deviation of 1.5 million. (Fashion Takes Action et al., 2023).

Transportation costs

Each selected and equipped location within the supply chain needs the possession of trucks for product transportation between centers. The cost of purchasing commercial and semi-trucks averages \$150,000, with a standard deviation of \$70,000. This variation in cost depends on factors such as the model and capacity of the trucks (Rechtien International Trucks., n.d.).

The costs that are related to the level of transportation are considered variable transportation costs such as fuel costs and insurance costs. The variable cost of transportation for trucks is on average \$47 per mile with a \$1 standard deviation (Octi, 2021)

The fixed cost of transportation and variable cost of transportation in each period have been generated based on the mentioned mean and standard deviation. The variable cost of transportation increases by %1 each month because of the increase in fuel cost.

I did not differentiate between the variable transportation costs incurred by various actors within the supply chain, such as store to recycling facility and supplier to manufacturer. Instead, all variable transportation costs were simulated using the same distribution.

Table 20- Fixed and variable cost of transportation from each supplier to other centers in US dollar in each period per mile

Supplier	Fixed cost	Variable cost in period 1	Variable cost in period 2	Variable cost in period 3
1	150,000	47.5	47.97	48.45
2	200,000	48.13	48.61	49.09
3	100,000	46.33	46.79	47.26

Table 20 shows the fixed cost of buying trucks for each supplier and the variable cost of transportation from each supplier to other centers at every period.

Table 21-Fixed and variable cost of transportation from each manufacturing center to other centers in US dollar in each period per mile

Manufacturing center	Fixed cost	Variable cost in period 1	Variable cost in period 2	Variable cost in period 3
1	188,275	46.64	47.10	47.57
2	218,106	47.15	47.62	48.09
3	233,856	47.85	48.32	48.81
4	220,913	47.35	47.82	48.30

Table 21 displays the fixed cost and the variable cost of transportation from each manufacturing center in different periods.

Table 22-Fixed and variable cost of transportation from each distribution center to other centers in US dollar per mile

Distribution center	Fixed cost	Variable cost in period 1	Variable cost in period 2	Variable cost in period 3
1	82,028	45.74	46.19	46.65
2	139,663	47.18	47.65	48.12
3	143,494	48.82	49.30	49.80

Table 22 demonstrates the fixed cost of acquisition of trucks for each distribution center and the variable cost of transportation per mile from each distribution center to stores in each period.

Table 23-Fixed and variable cost of transportation from each store to other centers in US dollar per mile

Stores	Fixed cost	Variable cost in period 1	Variable cost in period 2	Variable cost in period 3
1	165,923	47.99	48.47	48.95
2	136,832	46.70	47.17	47.64
3	102,249	46.19	46.65	47.12

Table 23 represent the fixed cost of buying trucks for stores and the variable transportation cost of sending goods from each store to inspection centers in each period.

Table 24-Fixed and variable cost of transportation from each inspection center to other centers in US dollar per mile

Inspection centers	Fixed cost	Variable cost in period 1	Variable cost in period 2	Variable cost in period 3
1	184,944	46.03	46.49	46.96
2	104,398	44.28	44.72	45.17
3	114,468	45.74	46.20	46.66

Table 24 represents fixed cost of possessing trucks for inspection centers, and the variable cost of transportation for sending products from each inspection center to other centers in each period.

Table 25-Fixed and variable cost of transportation from each thrift store to other centers in US dollar per mile

Thrift stores	Fixed cost	Variable cost in period 1	Variable cost in period 2	Variable cost in period 3
1	80,192	45.84	46.30	46.76
2	95,248	46.04	46.50	46.97
3	115,162	49.71	50.21	50.71

Table 25 shows the fixed cost and variable cost of transportation for sending goods from thrift stores to other centers at different periods.

Table 26-Fixed and variable cost of transportation from each recycling center to other centers in US dollar per mile

Recycling center	Fixed cost	Variable cost in period 1	Variable cost in period 2	Variable cost in period 3
1	177,785	47.61	48.09	48.57
2	187,222	48.25	48.73	49.22

Table 26 exhibit the fixed and variable cost of transportation from each recycling center to other centers in each period.

Truck capacities or batch sizes

The average capacity of a heavy truck is 16,000 pounds (Badger Truck & Auto Group., n.d.), and a wool shirt weighs around 0.4 pounds (Gam et al., 2009). Hence, each heavy truck can carry 40,000 wool shirts on average. It is assumed that the weight of the packaging is negligible.

I generated truck capacities for each center with a mean of 40,000 and a standard deviation of 15000. The truck capacity is the same for each period. Moreover, the capacity of trucks is related to the cost of possessing them. So, I set the capacity of trucks of each center based on the fixed cost of buying trucks for that center. For example, the numbers 15,000, 25,000, and 35,000 were generated for supplier truck capacities, and supplier 2 has the highest fixed cost of transportation, hence 35,000 which is the highest capacity was assigned to supplier 2.

Table 27-Truck capacity of each supplier at each time period

Truck capacity	Period 1	Period 2	Period 3
Supplier 1	25,000	25,000	25,000
Supplier 2	35,000	35,000	35,000
Supplier 3	15,000	15,000	15,000

Table 28-Truck capacity of each manufacturing center at each time period

Truck capacity	Period 1	Period 2	Period 3
Manufacturing center 1	22,818	22,818	22,818
Manufacturing center 2	35,384	35,384	35,384
Manufacturing center 3	67,838	67,838	67,838
Manufacturing center 4	44,582	44,582	44,582

Table 29-Truck capacity of each distribution center at each time period

Truck capacity	Period 1	Period 2	Period 3
Distribution center 1	19,629	19,629	19,629
Distribution center 2	49,594	49,594	49,594
Distribution center 3	53,945	53,945	53,945

Table 30-Truck capacity of each store at each time period

Truck capacity	Period 1	Period 2	Period 3
Store 1	41,527	41,527	41,527
Store 2	36,158	36,158	36,158
Store 3	27,365	27,365	27,365

Table 31-Truck capacity of each inspection center at each time period

Truck capacity	Period 1	Period 2	Period 3
Inspection center 1	44,262	44,262	44,262
Inspection center 2	27,353	27,353	27,353
Inspection center 3	35,826	35,826	35,826

Table 32-Truck capacity of each thrift store at each time period

Truck capacity	Period 1	Period 2	Period 3
Thrift store 1	26,624	26,624	26,624
Thrift store 2	27,885	27,885	27,885
Thrift store 3	31,575	31,575	31,575

Table 33-Truck capacity of each recycling center at each time period

Truck capacity	Period 1	Period 2	Period 3
Recycling center 1	50,989	50,989	50,989
Recycling center 2	60,984	60,984	60,984

Tables 27 to 33 show the capacity of trucks of every center in the supply chain in each period.

Distances

To generate data for the distance between centers, I observed the distance between the distribution center and the store of Aritzia, a fashion company, on Google Maps. The distance between the distribution center and the store in Vaughan, Ontario is 15 miles. I used this data to generate data for distances between each pair of centers. The numbers have been generated with a mean of 15 and a standard deviation of 5 miles following a normal distribution.

Table 34- Distance between each supplier and each manufacturing center in miles

Distance	Manufacturing center 1	Manufacturing center 2	Manufacturing center 3	Manufacturing center 4
Supplier 1	7.78	18.6	12.56	22.28
Supplier 2	24.64	15.9	17.47	24.4
Supplier 3	16.57	9.73	14.84	18.36

Table 35-Distance between each manufacturing center and each distribution center in miles

Distance	Distribution center 1	Distribution center 2	Distribution center 3
manufacturing 1	13.33	13.18	18.64
manufacturing 2	15.32	10.38	21.85
manufacturing 3	10.53	9.24	17.63
manufacturing 4	12.06	15.16	17

Table 36-Distance between each distribution center and each store in miles

Distance	Store 1	Store 2	Store 3
Distribution center 1	10.5	9.39	14.61
Distribution center 2	15.79	16.3	23.41
Distribution center 3	15.23	24.27	20.55

Table 37-Distance between each store and each inspection center in miles

Distance	Inspection center 1	Inspection center 2	Inspection center 3
Store 1	6.58	13.05	20.4
Store 2	16.69	14.08	8.6
Store 3	18.3	14.32	13.29

Table 38-Distance between each inspection center and each disposal center in miles

Distance	Disposal center 1	Disposal center 2	Disposal center 3
Inspection center 1	12.42	17.26	16
Inspection center 2	18.16	10.58	20.21
Inspection center 3	10.79	19.69	20.57

Table 39-Distance between each inspection center and each thrift store in miles

Distance	Thrift store 1	Thrift store 2	Thrift store 3
Inspection center 1	18.93	16.6	22.68
Inspection center 2	12.71	18.07	10.18
Inspection center 3	14.62	15.33	10.35

Table 40- Distance between each inspection center and each recycling center in miles

Distance	Recycling center 1	Recycling center 2
Inspection center 1	13.36	11.25
Inspection center 2	16.11	11.84
Inspection center 3	21.29	18.38

Table 41-Distance between each thrift store and each disposal center in miles

Distance	Disposal center 1	Disposal center 2	Disposal center 3
Thrift store 1	23.11	20.14	13
Thrift store 2	11.59	18.84	11.27
Thrift store 3	8.48	16.72	15.32

Table 42-Distance between each thrift store and each recycling center in miles

Distance	Recycling center 1	Recycling center 2
Thrift store 1	14.58	17.67
Thrift store 2	9.73	10
Thrift store 3	11.6	12.09

Table 43-Distance between each recycling center and each manufacturing center in miles

Distance	Manufacturing center 1	Manufacturing center 2	Manufacturing center 3	Manufacturing center 4
Recycling center 1	11.29	16.74	19.91	25.07
Recycling center 2	11.35	12.82	16.04	14.11

Table 44-Distance between each recycling center and each distribution center in miles

Distance	Distribution center 1	Distribution center 2	Distribution center 3
Recycling center 1	12.25	16.08	13.67
Recycling center 2	17.43	10.06	18.43

Tables 34 to 44 demonstrate the distance between each two centers in miles.

Capacities

A supplier for fabric stretchers stated that they can support supplying between 18000 to 42000 raw materials each month (Soleimani & Kannan, 2015). I generated random data with an average of 30,000 and a standard deviation of 15,000 following a normal distribution using Minitab software. Then, as the supply chain studied only has 3 products, I scaled the numbers down to be appropriate for the capacity that a supplier can assign to providing raw materials for only 3 products. Hence, all the numbers were divided by 3 and rounded down to make them integers. The unit of raw material is in skeins of wool.

Table 45- Supplying capacity of each supplier in each period

Capacity	Period 1	Period 2	Period 3
Supplier 1	8,333	8,333	8,333
Supplier 2	14,000	14,000	14,000
Supplier 3	10,000	10,000	10,000

The manufacturing capacity of an apparel manufacturing center is an average of 60,000 products with a standard deviation of 1,500 items every month (Wai Man KONG, 2008). Considering that I only have three products while a manufacturing center produces multiple products every month, I generated data based on the mentioned information and divided it by 3. This model focuses on three specific products that every manufacturing center studied produces.

Table 46-Manufacturing capacity of each manufacturing center in each period

Capacity	Period 1	Period 2	Period 3
Manufacturing center 1	5,209	5,209	5,209
Manufacturing center 2	5,510	5,510	5,510
Manufacturing center 3	6,079	6,079	6,079
Manufacturing center 4	2,667	2,667	2,667

The average capacity of distribution centers, thrift stores, inspection centers, and disposal centers is based on the information provided by Kadir Biçe and Sakine Batun (2021). They presented a two-stage stochastic profit maximization model for any closed-loop supply chain and used generated data to assess the validity of their model. I will be explaining how I generated capacity data for each of these centers based on data utilized by Kadir Biçe and Sakine Batun .(2021)

The average capacity of a distribution center data is generated with an average of 20,000 and a standard deviation of 2000 units for three products (Kadir Biçe and Sakine Batun, 2021).

Table 47- Capacity of each distribution center in each period

Capacity	Period 1	Period 2	Period 3
Distribution center 1	20,000	20,000	20,000
Distribution center 2	16,558	16,558	16,558
Distribution center 3	16,907	16,907	16,907

In Turkey, the average capacity of a clothing store can vary widely, accommodating between 100 to 2,000 units of product per week depending on various scenarios and situations. This translates to a monthly handling capacity ranging from approximately 400 to 8,000 units. To fit the scale of this problem, I generated data with an average store capacity of 4,000 units and a standard deviation of 1,000 units, providing a realistic and dynamic range for analysis. (Naderi Siamak et al., 2020)

Table 48- The handling capacity of each store in each period

Capacity	Period 1	Period 2	Period 3
Store 1	4,167	4,167	4,167
Store 2	3,963	3,963	3,963
Store 3	4,545	4,545	4,545

The average capacity of inspection centers each month is 30,000 units, with a standard deviation of 1,500 units (Kadir Biçe and Sakine Batun, 2021). Given that the supply chain produces three products and only 0.125% of these products are returned on average, the figures needed to be scaled down to reflect this return rate accurately. To achieve this, both the average and the standard deviation were divided by 800. As a result, the adjusted mean capacity for handling three types of products is 37 units, with a corresponding standard deviation of 2 units.

Table 49- Capacity of each inspection center in each period

Capacity	Period 1	Period 2	Period 3
Inspection center 1	37	37	37
Inspection center 2	27	27	27
Inspection center 3	29	29	29

The average capacity of the disposal center is approximately 18,000 units (Kadir Biçe and Sakine Batun, 2021). To reflect the appropriate scale, this data was scaled down by dividing by 900. Consequently, the adjusted average capacity for the disposal center is 20 units, with a standard deviation of 2 units.

Table 50-Capacity of each disposal center in each period

Capacity	Period 1	Period 2	Period 3
Disposal center 1	20	20	20
Disposal center 2	22	22	22
Disposal center 3	22	22	22

The average capacity of the thrift store is the same as stores with an average of 4000 items, with a standard deviation of 1000 units. To appropriately scale this data, the figures were divided by 300 and rounded up. As a result, the adjusted average capacity is 14 units, with a standard deviation of 2 units (Kadir Biçe and Sakine Batun, 2021).

Table 51- Capacity of each thrift store in each period

Capacity	Period 1	Period 2	Period 3
Thrift store 1	14	14	14
Thrift store 2	14	14	14
Thrift store 3	16	16	16

A textile recycling center can recycle 800 tons of textiles annually, which equates to 133,333.2 pounds per month (Fashion Takes Action et al., 2023). Considering each wool shirt weighs 0.4 pounds, a recycling center can process 363,762 wool shirts every month. However, since we are dealing with three products with a low return rate, the full capacity of a recycling center is not necessary for our problem scale. Therefore, to make the figures reasonable, I scaled them down by dividing by 10,000. The scaling was done to match my model and the computational resources I had available. This adjustment results in an average of 36 units, with a standard deviation of 2 units for three products.

Table 52- Capacity of each recycling center in each period

Capacity	Period 1	Period 2	Period 3
Recycling center 1	36	36	36
Recycling center 2	36	36	36

Ratios

To knit a women's shirt, 1,175 yards of wool are required (Jimmy Beans Wool, n.d.). Given that each skein of wool contains an average of 535 yards (Sarah Maker, 2023), approximately 2.23 skeins are needed to produce one shirt.

Table 53- Coefficient of skein of wool needed to produce each product

F_{ap}	Product 1	Product 2	Product 3
Raw material 1	1	1	4

From the raw materials supplied by the supplier, specific proportions will be allocated to produce three different products. For example, if 1000 skeins of wool are supplied, based on Table 54, 160 skeins will be used for producing product 1, 160 skeins for product 2, and 680 skeins for product 3. The percentage of raw material that should be used for each product is shown in Table 54. These percentages have been set based on the stated amount of raw material required to produce each product in table 53.

Table 54- The percentage of available raw material assigned to producing each product

Ratio _p	Product 1	Product 2	Product 3
Wool	0.16	0.16	0.68

The mean and standard deviation of all the data related to the following ratios have been appropriately scaled to align with the scope of the problem.

Table 55-Return rate of each product in each period

α_{pt}	Period 1	Period 2	Period 3
Product 1	0.00125	0.0012	0.001325
Product 2	0.0013	0.00125	0.0015
Product 3	0.00115	0.0013	0.001325

Less than 1% of garments are collected for reusing and recycling into new garment purposes (Hedrich et al, 2022). Table 55 shows the return rate of each product in each period. Appropriate

to the scale of the problem, the data has been generated following a normal distribution with the mean of %0.125 and the standard deviation of %0.0125. The scaling was done to match my model and the computational resources I had available.

Table 56- Ratio of each product sent to thrift stores from inspection centers in each period

β_{Pt}	Period 1	Period 2	Period 3
Product 1	0.22	0.29	0.22
Product 2	0.31	0.29	0.18
Product 3	0.28	0.26	0.21

Approximately 27% of apparel recollected in the fashion industry consists of reusable or second-hand clothing (Statista, 2021). Table 56 demonstrates the percentage of each product sent to thrift stores from inspection centers in each period. To generate the data in this table the mean has been considered to be 0.27 with the standard deviation of 0.05.

Table 57-Ratio of each product sent to recycling centers from inspection centers in each period

μ_{Pt}	Period 1	Period 2	Period 3
Product 1	0.44	0.45	0.44
Product 2	0.46	0.51	0.48
Product 3	0.45	0.37	0.37

Fifty percent of clothing that is given or returned is used to produce lower-quality textiles (Howlett, 2023). Table 57 presents the ratio of each product sent from inspection centers to recycling centers for each period. The mean ratio has been set at 0.5, with a standard deviation of 0.05, to be appropriate for the scale of the analysis.

Table 58- Ratio of each product sent to recycling centers from thrift store in each period

G_{pt}	Period 1	Period 2	Period 3
Product 1	0.42	0.38	0.47
Product 2	0.47	0.49	0.4
Product 3	0.43	0.4	0.49

Table 59- Ratio of each product that can be repaired at recycling centers in each period

η_{pt}	Period 1	Period 2	Period 3
Product 1	0.42	0.46	0.37
Product 2	0.51	0.44	0.45
Product 3	0.52	0.45	0.38

Table 58 represents the ratio of each product sent to recycling centers from thrift stores in each period. Table 59 shows the ratio of each product that can be repaired at recycling centers in each period. For both tables, the mean is set to 0.45 with a standard deviation of 0.05.

Number of centers

In this study, the company intends to engage with a maximum of three suppliers. The operational limits for other facilities are set at a maximum of four manufacturing centers, one distribution center, three stores, two inspection centers, one disposal center, one thrift store, and one recycling center. These limits are based on the company's resource capacity and represent the maximum number of centers manageable within the assumptions of the problem. Consequently, the company must select from these maximum allowable centers among all available options.

Inventory cost

The average value of fast fashion products is 34 US dollars as stated earlier. Typically, a business's inventory holding costs amount to approximately 20% to 30% of its total inventory value

(NetSuite, 2020). So, the inventory holding cost per product should be between 6.8 and 10.2. I generated data with a mean of 8.5 with a standard deviation of 1.

Table 60- Inventory holding cost of each product in US dollar

Product	Inventory holding cost
1	8.75
2	10.5
3	9.4

Unutilized capacity cost

The cost of unused capacity refers to the financial loss faced by a business due to maintaining resources that are not fully utilized. This means that the company is paying for the potential output or service capabilities of its resources, such as machinery, labor, or facilities, but these resources are not being used to their full potential. So, the business incurs costs for idle capacity, which could otherwise be employed productively.

Cost of unutilized capacity can be calculated as the portion of the total capacity that was unutilized divided by the total available capacity multiplied by the total fixed cost associated with maintaining the total capacity (Masschelein, 2023).

- Cost of unused capacity = (Amount of the allocation base at capacity – Actual amount of allocation base) * Predetermined overhead rate
- Predetermined overhead rate= Total fixed cost of facilities to hold inventory / Total capacity

The formula for cost of unutilized capacity for every unit of capacity can be written as below:

- Cost of each unit of unused capacity = $1 * \frac{\text{Total fixed cost of facilities to hold inventory}}{\text{Total capacity}}$

Based on the explained formula, the unutilized cost of capacity at manufacturing centers, distribution centers, and recycling centers has been calculated. Tables 61, 62, and 63 respectively

demonstrate the cost of unused capacity per unit at manufacturing centers, distribution centers, and recycling centers.

Table 61- Unutilized capacity cost per unit at each manufacturing center

Manufacturing centers	unutilized capacity cost
1	3091.764254
2	2756.261343
3	1235.236059
4	2877.015373

Table 62-Unutilized capacity cost per unit at each distribution center

Distribution centers	Unutilized capacity cost
1	360
2	422.7563715
3	507.9656947

Table 63-Unutilized capacity cost per unit at each recycling center

Recycling center	Unutilized capacity cost
1	274.8989746
2	229.7962917

Virgin and raw material proportion

In order to achieve greater strength in the final product, it is important to combine virgin and raw materials. While recycled fibers offer benefits in terms of reduced imperfections and better yarn quality, they fall short in tensile and burst strength compared to virgin fibers. By blending virgin and recycled materials, manufacturers can enhance the overall durability and robustness of the yarns and fabrics (Gribaa et al, 2016).

To enhance the strength of the fabric, it is essential to incorporate a proportion of virgin materials. Although the optimal blend of raw and recycled materials has not been definitively determined, Patagonia uses a blend of 89% recycled wool and 11% virgin wool (Patagonia, n.d.). This consideration is captured by a parameter named W in our model, which indicates the minimum percentage of raw material that must be virgin. In this study, I have set the parameter W to 30% for higher durability.

Price elasticity parameter

Additionally, it is assumed that the number of returns is influenced by the price customers receive for returning products. This relationship is quantified by a price elasticity parameter set to 1. This implies that for every dollar increase in the price paid to customers, the number of returns increases by one unit.

Demand

The normal distribution is often considered an appropriate model to predict demand in the apparel and fashion industry. This is due to the Central Limit Theorem, which states that the sum or average of many independent and identically distributed random variables tends to follow a normal distribution (Hanley, 2007). This theorem applies well to fashion demand, where many small factors like customer preferences, economic conditions, and seasonal trends collectively influence overall demand, resulting in a bell-shaped curve. Additionally, the symmetry and mean reversion properties of the normal distribution align well with observed patterns in apparel sales, where extremely high or low values are less frequent than moderate ones (Christopher et al., 2004). Additionally, the CPLEX solver used in Gams to solve the problem only supports continuous

distributions and the mathematical convenience of the normal distribution facilitates various analytical methods essential for informed decision-making in the fashion industry.

The data of demand for each product at each store in each period has been generated following a normal distribution with a mean of 740 and a standard deviation of 100. The average was selected based on the data presented by Foster (2016). The numbers were rounded down to make them integers.

Table 64- Demand at each store for each product type in each period

Store	Product	Period	Demand
1	1	1	708
1	1	2	772
1	1	3	786
1	2	1	557
1	2	2	666
1	2	3	642
1	3	1	700
1	3	2	833
1	3	3	664
2	1	1	740
2	1	2	649
2	1	3	774
2	2	1	673
2	2	2	691
2	2	3	928

2	3	1	667
2	3	2	610
2	3	3	602
3	1	1	771
3	1	2	858
3	1	3	624
3	2	1	811
3	2	2	841
3	2	3	667
3	3	1	640
3	3	2	740
3	3	3	640

Chapter 4: Deterministic Optimization

Definition of deterministic optimization

Deterministic optimization is a mathematical optimization algorithm where all the parameters are certain. In deterministic optimization, one or multiple objective functions will be proposed with the goal of maximization or minimization. Next, decision variables will be defined, these are the variables that should be controlled or adjusted in order to optimize the objective function.

Deterministic optimization has a faster convergence to a solution than stochastic optimization because it generally needs fewer function evaluations and simulations as there is less evaluation needed to reach the optimality because all the parameters are determined, and it is less complex. Also, the outcomes for deterministic models are easily replicable. However, deterministic optimization algorithms could locate local optimums instead of global optimums as they aim for points of stability in the response variable (Cavazzuti, 2013).

Model description

In this section, I propose a deterministic mixed-integer programming model for a closed-loop fashion supply chain in order to meet the demand at stores with minimum costs. The studied problem is multi-period and multi-product. This fashion supply chain includes a maximum number of 3 suppliers, 4 manufacturers, 3 distribution centers, 6 stores, 3 inspection centers, 3 disposal centers, 3 thrift stores, and 1 recycling center. Transport flows are only between levels of the supply chain, and transportation within a level is not allowed. The presented model aims to minimize total costs, including facility location and equipping cost, material cost, transportation cost, manufacturing cost, distributing cost, inspecting cost, disposal cost, holding cost in stores, holding cost in thrift stores, recycling and repairing cost, inventory cost, unutilized capacity cost, and cost of buying returned products.

Tasks of each supply chain actor

- Suppliers: Provide raw materials for manufacturing centers.
- Manufacturing centers: Produce 3 types of clothes and send them to distribution centers.
- Distribution centers: Clothes are stored in distribution centers and transported to stores to satisfy customer demand and replenish inventory.

- Stores: The goods received from the distribution centers reach the customers in the stores and satisfy the customer's demand for each clothing design.
- Inspection centers: The quality of the goods returned is inspected and divided into three categories, reusable, recyclable or repairable, and very bad quality. Reusable products are sent to thrift stores, recyclable or repairable products are sent to recycling centers, and very bad-quality groups of products are sent to disposal centers.
- Disposal centers: Items sent from inspection centers and thrift stores are disposed of at these centers.
- Thrift stores: Repairable products are sent to thrift stores to be sold again. If items aren't sold at thrift stores at the end of a period, a proportion of them that are recyclable are sent to recycling centers, and the rest will be sent to disposal centers.
- Recycling centers: Recycling centers fix repairable products and send them to distribution centers, while recyclable products are processed into raw materials and sent to manufacturing centers.

Decision variables

1. variables denoting if a facility location should be selected and equipped.
2. Quantity of each material that should be processed and sent to each center from another center to meet the demand in each period.
3. Residual inventory of each product at each distribution center for each period.

Model assumptions

1. The model is multi-echelon, multi-product, and multi-period.
2. Fixed costs of equipping and variable costs of operating each center in each period are determined.
3. Fixed and variable costs of transportation in each period between each two centers are determined.
4. Inventory cost and unutilized capacity cost are determined.
5. Distances between centers are determined.
6. Capacity of each center is determined.
7. Ratios of return, percentage of reusable, repairable, and recyclable products are determined.

8. Capacities of trucks of each center are determined.
9. Demand of each product at each store in each period is determined.

Mathematical model

Indexes

S: Set of suppliers

M: Set of manufacturers

D: Set of distribution centers

C: Set of stores

I: Set of inspection centers

K: Set of disposal centers

H: Set of thrift stores

R: Set of recycling centers

P: Set of products

A: Set of raw material

T: Index of periods

Parameters

- Fixed costs of locating

F_s : Fixed cost of operating supplier S

F_m : Fixed cost of operating manufacturer M

F_d : Fixed cost of operating distribution center D

F_c : Fixed cost of operating store C

F_i : Fixed cost of operating inspection center I

F_k : Fixed cost of operating disposal center k

F_h : Fixed cost of operating thrift store H

F_r : Fixed cost of operating recycling center R

- Variable costs

C_{mp} : Unit manufacturing cost of product p in manufacturing center M

C_d : Cost of processing in distribution center D per product

C_i : Cost of processing in inspection center I per product

C_k : Cost of processing in disposal center K per product

C_{rp} : Cost of repairing in recycling center R for product P

W_{rp} : Cost of processing in recycling center R for product P

C_{as} : Procurement cost of raw material A from supplier S

C_{cp} : Cost of keeping product P in store C

C_{hp} : Cost of keeping product P in thrift store H

C_p : Cost of buying back returned product P from customers

- Transportation cost

V_{st} : Variable cost of shipment from supplier S at time T

V_{mt} : Variable cost of shipment from manufacturer M at time T

V_{dt} : Variable cost of shipment from distribution center D at time T

V_{ct} : Variable cost of shipment from store C at time T

V_{it} : Variable cost of shipment from inspection center I at time T

V_{ht} : Variable cost of shipment from thrift store H at time T

V_{rt} : Variable cost of shipment from recycling center R at time T

x_s : Fixed cost of shipment from supplier S

x_m : Fixed cost of shipment from manufacturer M

x_d : Fixed cost of shipment from distribution center D

x_c : Fixed cost of shipment from store C

x_i : Fixed cost of shipment from inspection center I

x_h : Fixed cost of shipment from thrift store H

x_r : Fixed cost of shipment from recycling center R

- Inventory cost

H_p : Holding cost per unit of product P in distribution centers.

- Unutilized capacity cost

M_m : The cost of unutilized capacity in manufacturing centers.

M_d : The cost of unutilized capacity in distribution centers.

M_r : The cost of unutilized capacity in recycling centers.

- Distances

Dis_{sm} : Distance between supplier S and manufacturing center M

Dis_{md} : Distance between manufacturing M and distribution center D

Dis_{dc} : Distance between distribution center D and store C

Dis_{ci} : Distance between store c and inspection center I

Dis_{ik} : Distance between inspection center I and disposal center K

Dis_{ih} : Distance between inspection center I and thrift store H

Dis_{ir} : Distance between inspection center I and recycling center R

Dis_{hk} : Distance between thrift store H and disposal center K

Dis_{hr} : Distance between thrift store H and recycling center R

Dis_{rd} : Distance between recycling center R and distribution center D

Dis_{rm} : Distance between recycling center r and manufacturing center M

- Capacities

Cap_{st} : Capacity of supplier S in period T

Cap_{mt} : Capacity of manufacturing center M in period T

Cap_{dt} : Capacity of distribution center D in period T

Cap_{ct} : Capacity of store C in period T

Cap_{it} : Capacity of inspection center I in period T

Cap_{kt} : Capacity of disposal center K in period T

Cap_{ht} : Capacity of thrift store H in period T

Cap_{rt} : Capacity of recycling center R in period T

- Ratios and rates

$ratio_p$: Percentage of raw material that should be used for product P

Y: The price elasticity of return quantity

W: Minimum percentage of raw material that should be virgin.

α_{pt} : Return rate of product P in period T

β_{pt} : Ratio of product P that could be sent to thrift stores from inspection centers in period T

μ_{pt} : Ratio of product P that could be sent to recycling centers from inspection centers in period T

σ_{pt} : Ratio of product P that could be sent to recycling centers from thrift stores in period T

η_{pt} : Ratio of product p that can be repaired in recycling centers in period T

f_{ap} : Coefficient of raw material A needed for product P

- Batch sizes

B_{st} : Capacity of trucks going from supplier S to manufacturing centers in period T

B_{mt} : Capacity of trucks going from manufacturing center M to distribution centers in period T

B_{dt} : Capacity of trucks going from distribution center D to stores in period T

B_{ct} : Capacity of trucks going from store C to inspection centers in period T

B_{it} : Capacity of trucks going from inspection centers to other centers in period T

B_{ht} : Capacity of trucks going from thrift store H to recycling centers in period T

B_{rt} : Capacity of trucks going from recycling center R to other centers in period T

L_s : Maximum number of suppliers that can be operated

L_m : Maximum number of manufacturing centers that can be operated

L_d : Maximum number of distribution centers that can be operated

L_c : Maximum number of stores that can be operated

L_i : Maximum number of inspection centers that can be operated

L_k : Maximum number of disposal centers that can be operated

L_h : Maximum number of thrift stores that can be operated

L_r : Maximum number of recycling centers that can be operated

- Demand

D_{pct} : Demand of product P at store C in period T

Decision variables

- Binary variables

O_s : Binary variable denoting if supplier S is selected

O_m : Binary variable denoting if manufacturing center M is selected

O_d : Binary variable denoting if distribution center D is selected

O_c : Binary variable denoting if store C is selected

O_i : Binary variable denoting if inspection center I is selected

O_k : Binary variable denoting if disposal center K is selected

O_h : Binary variable denoting if thrift store H is selected

O_r : Binary variable denoting if recycling center R is selected

- Integer variables

Q_{asmt} : Volume of raw material A that should be sent to manufacturing center M from supplier S in period T

Q_{pmdt} : Number of product P that should be sent to distribution center D from manufacturing center M in period T

Q_{pdct} : Number of product P that should be sent to store C from distribution center D in period T

Q_{pcit} : Number of product P that should be sent to inspection center I store C in period T

Q_{pikt} : Number of product P that should be sent to disposal center K from inspection center I in period T

Q_{piht} : Number of product P that should be sent to thrift store H from inspection center I in period T

Q_{pirt} : Number of product P that should be sent to recycling center R from inspection center I in period T

Q_{phkt} : Number of product P that should be sent to disposal center K from thrift store H in period T

Q_{phrt} : Number of product P that should be sent to recycling center R from thrift store H in period T

Q_{prdt} : Number of product P that should be sent to distribution center D from recycling center R in period T

Q_{armt} : Volume of raw material A that should be sent to manufacturing center M from recycling center R in period T

RE_{pdt} : Residual inventory of product P at distribution center D in period T

Objective function

Min Z = Facility Location and equipping cost+ material cost+ transportation cost + manufacturing cost + distributing cost + inspecting cost + disposing cost + holding cost in stores + holding

cost in thrift stores + recycling and repairing cost+ inventory cost+ unutilized capacity cost+ cost of buying returned products

Location cost (1)

$$\begin{aligned} & \sum_s F_S O_S + \sum_\mu F_\mu O_\mu + \sum_d F_d O_d + \sum_C F_C O_C + \sum_I F_I O_I + \sum_k F_k O_k \\ & + \sum_h F_h O_h + \sum_r F_r O_r \end{aligned}$$

Fixed cost of transportation (2)

$$\begin{aligned} & \sum_s X_S O_S + \sum_\mu X_\mu O_\mu + \sum_d X_d O_d + \sum_C X_C O_C + \sum_I X_I O_I + \sum_k X_k O_k \\ & + \sum_h X_h O_h + \sum_r X_r O_r \end{aligned}$$

Transportation cost (3)

$$\begin{aligned} & \sum_S \sum_m \sum_t \left[\frac{\sum_a Q_{asmt}}{B_{st}} \right] \times Dis_{SM} \times V_{st} + \sum_M \sum_d \sum_t \left[\frac{\sum_p Q_{pmdt}}{B_{mt}} \right] \times Dis_{md} \times V_{mt} \\ & + \sum_D \sum_C \sum_t \left[\frac{\sum_p Q_{pdct}}{B_{dt}} \right] \times Dis_{dc} \times V_{dt} + \sum_C \sum_i \sum_t \left[\frac{\sum_p Q_{pcit}}{B_{ct}} \right] \times Dis_{ci} \times V_{ct} \end{aligned}$$

$$\begin{aligned}
& + \sum_i \sum_t V_{it} \times \left(\sum_k \left[\frac{\sum_p Q_{pikt}}{B_{it}} \right] \times Dis_{ik} + \sum_h \left[\frac{\sum_p Q_{piht}}{B_{it}} \right] \times Dis_{ih} \right. \\
& + \left. \sum_r \left[\frac{\sum_p Q_{pirt}}{B_{it}} \right] \times Dis_{ir} \right) + \sum_h \sum_t V_{ht} \times \left(\sum_k \left[\frac{\sum_p Q_{phkt}}{B_{ht}} \right] \times Dis_{hk} \right. \\
& + \left. \sum_r \left[\frac{\sum_p Q_{phrt}}{B_{ht}} \right] \times Dis_{hr} \right) + \sum_r \sum_t V_{rt} \times \left(\sum_d \left[\frac{\sum_p Q_{prdt}}{B_{rt}} \right] \times Dis_{rd} \right. \\
& + \left. \sum_m \left[\frac{\sum_a Q_{armt}}{B_{rt}} \right] \times Dis_{rm} \right)
\end{aligned}$$

Material cost (4)

$$\sum_a \sum_S \sum_m \sum_t C_{as} \times Q_{asmt}$$

Manufacturing cost (5)

$$\sum_p \sum_m \sum_d \sum_t C_{mp} \times Q_{pmdt}$$

Distributing cost (6)

$$\sum_p \sum_d \sum_c \sum_t C_d \times Q_{pdct}$$

Inspecting cost (7)

$$\sum_p \sum_c \sum_i \sum_t C_i \times Q_{pcit}$$

Disposing cost (8)

$$\sum_k (C_k \times \sum_p \sum_t (\sum_i Q_{pikt} + \sum_h Q_{phkt}))$$

Holding cost in stores (9)

$$\sum_p \sum_d \sum_c \sum_t C_{cp} \times Q_{pdct}$$

Holding cost in Thrift stores (10)

$$\sum_p \sum_i \sum_h \sum_t C_h \times Q_{piht}$$

Recycling and repairing cost (11)

$$\sum_p \sum_r \sum_t (C_{rp} \times \sum_d Q_{prdt} + w_{rp} \times (\sum_h Q_{phrt} + \sum_i Q_{pirt} - \sum_d Q_{prdt}))$$

Inventory cost (12)

$$\sum_p \sum_d \sum_t RE_{pdt} \times H_p$$

Unutilized Capacity cost (13)

$$\begin{aligned} & \sum_t \left(\sum_m (M_m \times (Cap_{mt} \times OM - \sum_p \sum_d Q_{pmdt})) \right. \\ & \left. + \sum_d (M_d \times (Cap_{dt} \times Od - (\sum_p (\sum_c Q_{pdct} + RE_{pdt})))) \right) \\ & \left. + \sum_r (M_r \times (Cap_{rt} \times Or - (\sum_p (\sum_h Q_{phrt} + \sum_i Q_{pirt})))) \right) \end{aligned}$$

Buying returned products cost (14)

$$\sum_p \sum_c \sum_i \sum_t Q_{pcit} \times C_p$$

- Flow balances

Manufacturing center (15)

$$\left(\sum_s Q_{asmt} + \sum_r Q_{arnt} \right) \times \text{ratio}_p \geq F_{ap} \times \sum_d Q_{pmdt}, \quad \forall a, m, t, p$$

Distribution centers (16)

$$\sum_r Q_{prdt} + \sum_m Q_{pmdt} + RE_{pd(t-1)} = \sum_c Q_{pdct} + RE_{pdt}, \forall t, p, d$$

Stores (17)

$$\sum_d Q_{pdct} = D_{pct}, \forall p, c, t$$

Maximum number of each product sent to inspection center (18)

$$\sum_d Q_{pdct} \times \alpha_{pt} + Y \times C_p \geq \sum_i Q_{pcit}, \forall p, c, t$$

Minimum number of each product sent to inspection center (19)

$$\sum_d Q_{pdct} \times \alpha_{pt} \leq \sum_i Q_{pcit}, \forall p, c, t$$

Maximum number of each product sent to inspection center (20)

$$\sum_i Q_{pcit} \leq \sum_d Q_{pdct}, \forall p, c, t$$

Inspection center flow in and out (21)

$$\sum_h Q_{piht} + \sum_r Q_{pirt} + \sum_k Q_{pikt} = \sum_i Q_{pcit}, \forall p, i, t$$

Inspection center to thrift stores (22)

$$[\sum_c Q_{pcit} \times \beta_{pt}] = \sum_h Q_{piht}, \forall p, i, t$$

Inspection center to recycling centers (23)

$$\left[\sum_c Q_{pcit} \times \mu_{pt} \right] = \sum_r Q_{pirt}, \forall p, i, t$$

Thrift stores flow in and out (24)

$$\sum_i Q_{piht} = \sum_k Q_{phkt} + \sum_r Q_{phrt}, \forall p, h, t$$

Thrift store to recycling centers (25)

$$\left[\sum_i Q_{piht} \times G_{pt} \right] = \sum_r Q_{phrt}, \forall p, h, t$$

Repairing at recycling center (26)

$$\left[\left(\sum_h Q_{phrt} + \sum_i Q_{pirt} \right) \times n_{pt} \right] = \sum_d Q_{prdt}, \forall p, r, t$$

Recycling at recycling center (27)

$$\left[\left(\sum_h Q_{phrt} + \sum_i Q_{pirt} - \sum_d Q_{prdt} \right) \times F_{ap} \right] = \sum_m Q_{armt}, \forall p, r, a, t$$

- Capacity constraints

Supplier capacity (28)

$$\sum_a \sum_m Q_{asmt} \leq Cap_{st} \times Os, \forall s, t$$

Manufacturing center capacity (29)

$$\sum_p \sum_d Q_{pmdt} \leq Cap_{mt} \times Om, \forall m, t$$

Distribution center capacity (30)

$$\sum_p (\sum_c Q_{pdct} + RE_{pdt}) \leq Cap_{dt} \times Od, \forall d, t$$

Store capacity (31)

$$\sum_p \sum_d Q_{pdct} \leq Cap_{ct} \times Oc, \forall c, t$$

Inspection center capacity (32)

$$\sum_p \sum_c Q_{pcit} \leq Cap_{it} \times Oi, \forall i, t$$

Disposal center capacity (33)

$$\sum_p (\sum_i Q_{pikt} + \sum_h Q_{phkt}) \leq Cap_{kt} \times Ok, \forall k, t$$

Thrift store capacity (34)

$$\sum_p \sum_i Q_{piht} \leq Cap_{ht} \times Oh, \forall h, t$$

Recycling center capacity (35)

$$\sum_p (\sum_h Q_{phrt} + \sum_i Q_{pirt}) \leq Cap_{rt} \times Or, \forall r, t$$

Virgin raw material (36)

$$w \times \left(\sum_s Q_{asmt} + \sum_r Q_{armt} \right) \leq \sum_s Q_{asmt}, \forall a, m, t$$

- Number of centers

Suppliers (37)

$$\sum_S O_S \leq L_s$$

Manufacturers (38)

$$\sum_m O_m \leq L_m$$

Distribution centers (39)

$$\sum_d O_d \leq L_d$$

Stores (40)

$$\sum_c O_c \leq L_c$$

Inspection centers (41)

$$\sum_i O_i \leq L_i$$

Disposal centers (42)

$$\sum_k O_k \leq L_k$$

Thrift stores (43)

$$\sum_h O_h \leq L_h$$

Recycling centers (44)

$$\sum_r O_r \leq L_r$$

Binary variables (45)

$$O_s, O_m, O_d, O_c, O_i, O_k, O_h, O_r \in \{0, 1\}, \forall s, m, d, c, i, k, h, r$$

Integer variables (46)

$$Q_{asmt}, Q_{pmdt}, Q_{pdct}, Q_{pcit}, Q_{pikt}, Q_{piht}, Q_{pirt}, Q_{phkt}, Q_{phrt}, Q_{prdt}, Q_{arnt}, RE_{pdt} \in \mathbb{Z}$$

Non-negative variables (47)

$$Q_{asmt}, Q_{pmdt}, Q_{pdct}, Q_{pcit}, Q_{pikt}, Q_{piht}, Q_{pirt}, Q_{phkt}, Q_{phrt}, Q_{prdt}, Q_{arnt}, RE_{pdt} \geq 0$$

Model explanation

- Objective function

Formula 1, and 2 show the fixed cost of operating centers respectively, and the fixed cost of transportation from each center, so if the variable "O" that specifies whether a facility should be operated or not is 0 then there will be no costs associated with that center .

The 3rd formula demonstrates the total variable transportation cost. The variable transportation cost from each center is calculated by multiplying the distance between centers by the variable cost of transportation in thin period by the number of trucks that should be sent from that specific center to the next centers in the supply chain network .

To calculate the number of trucks that should travel between centers, I used the ceiling function and divided the number of products that should be sent out from each center by the truck capacity of the trucks available at that specific center. The justification for using the ceiling function in the model is that this approach rounds up to the next whole number, so if the division of products by truck capacity results in a non-integer, like 5.5, I would use 6 trucks to accommodate all products.

Formula 4 shows the cost of buying material from suppliers. Formulas 5 to 8 represent the operating costs of manufacturing, distribution, inspection, and disposal centers. Disposing cost is calculated by multiplying the cost of disposing per product by the number of products sent to disposal centers either from inspection centers or thrift stores .

Formulas 9 and 10 represent the holding costs in stores and thrift stores. Formula 11 shows recycling and repairing costs. Repairing cost is calculated as the cost of repairing per product multiplied by the number of products that will be sent from the recycling center to distribution centers, which are the repairable products .

The recycling cost is computed by multiplying the recycling cost per product by the total number of products that arrived at the recycling center either from thrift stores or inspection centers, except the number of products that are repairable .

Formula 12 represents inventory holding costs at distribution centers. Formula 13 is responsible for unutilized capacity costs at manufacturing centers, distribution centers, and recycling centers. Unutilized capacity cost is calculated by multiplying the unutilized capacity cost per unit of capacity by the number of products processed in that center minus the total capacity available in an equipped center, which is defined by multiplying capacity by the binary variable denoting if a location should be equipped or not .

Finally, formula 14 represents the cost of buying returned products from customers. In order to calculate total costs for this closed-loop supply chain, formula 1 to formula 14 are added together.

- Constraints

Constraints 15 to 27 are flow balance constraints. These constraints guarantee that the number of products flowing out of each center is equal to the number of products entering that center in each period.

Constraint 15 balances the number of raw materials that a manufacturing center receives with the number of products it can produce from those raw materials in a specific period.

Constraint 16 ensures that the total number of products sent to a distribution center, whether from recycling centers or a manufacturing center, plus the number of products held as residual inventory from the previous period, equals the total number of products sent out from the distribution center to stores and the residual inventory at the distribution center for that period.

Constraint 17 guarantees that the demand at each store in each period is met, with no shortages allowed.

Constraints 18 to 20 manage the flow of products between sales and returns sent to inspection centers. Constraint 18 ensures that the number of products sent to inspection centers does not exceed the total number sold, adjusted by the expected percentage of returns and additional returns due to price elasticity. Constraint 19 specifies that at least the minimum percentage of sold products typically returned must be sent to inspection centers. Constraint 20 ensures that the number of returns cannot exceed the number sold, even if the price of buying returned products is increased.

Constraint 21 is the flow balance constraint for inspection centers. Constraint 22 specifies that the number of products sent to thrift stores from an inspection center should be equal to the percentage of reusable products available at the inspection center. The floor function is used to ensure the equation results in an integer, as the number of products transported between centers cannot be non-integer. Constraint 23 ensures that the percentage of repairable and recyclable products in inspection centers is sent to recycling centers.

Constraint 24 guarantees that the number of products entering a thrift store is equal to the number of products sent out of it. Constraint 25 manages the percentage of recyclable products at thrift stores to be sent to recycling centers. Constraint 26 ensures that the percentage of repairable

products sent to a recycling center from thrift stores and inspection centers are repaired and sent to distribution centers. Constraint 27 is set to control the balance between the number of recyclable products at each recycling center and the number of raw materials that can be sent to manufacturing centers from each recycling center.

Constraints 28 to 35 are capacity constraints. They are set to ensure that the number of items processed at each center does not exceed its capacity if the center is equipped.

Constraint 36 ensures that a specific minimum percentage of raw materials sent to a manufacturing center should be virgin raw material. Constraints 37 to 44 specify the maximum number of centers that the company is willing to equip from all available options.

Constraint 45 indicates the binary variables. Constraint 46 shows integer variables, and constraint 47 specifies that all variables should be non-negative.

To reduce the complexity of the model, I will linearize it by removing the non-linear components. The proposed model is non-linear because it includes ceiling and floor functions. Non-linear problems often require complex algorithms, while linear problems can be solved efficiently using well-established algorithms such as the Simplex method or Interior-Point methods. These algorithms handle the linearity and convexity of the problem, providing fast and reliable solutions.

Non-linear constraints can create non-convex feasible regions, resulting in multiple local optima, making it difficult to find the global optimum. In contrast, linear programming problems define a convex polytope, ensuring that a local optimum is also a global optimum. Hence, I present a new linear model based on the previous model to make the solving process faster and easier.

Linearized model formulation

To linearize the model integer and continuous variables were added to the initial deterministic model. These variables have been presented below.

- Integer variables

L_{pht} : Integer number of recyclable product P at thrift store H in period T.

L_{prt} : Integer number of repairable product P at recycling center R in period T.

L_{prat} : Integer number of recycled raw material A at recycling center R in period T.

L_{smt} : Integer number of trucks that should be sent to manufacturing center from supplier in period T.

L_{mdt} : Integer number of trucks that should be sent to distribution center from manufacturing center in period T.

L_{dct} : Integer number of trucks that should be sent to stores from manufacturing center in period T.

L_{cit} : Integer number of trucks that should be sent to inspection center from store in period T.

L_{ikt} : Integer number of trucks that should be sent to disposal center from inspection center in period T.

L_{iht} : Integer number of trucks that should be sent to thrift store from inspection center in period T.

L_{irt} : Integer number of trucks that should be sent to recycling center from inspection center in period T.

L_{rdt} : Integer number of trucks that should be sent to distribution center from recycling center in period T.

L_{hkt} : Integer number of trucks that should be sent to disposal center from thrift store in period T.

L_{hrt} : Integer number of trucks that should be sent to recycling center from thrift store in period T.

L_{rmt} : Integer number of trucks that should be sent to manufacturing center from recycling center in period T.

LH_{pit} : Integer number of reusable product P at inspection center I in period T.

LR_{pit} : Integer number of recyclable or repairable product P at inspection center I in period T.

- Continuous variables

TH_{pit} : Decimal part of the number of reusable product P at inspection center I in period T.

TR_{pit} : Decimal part of the number of recyclable or repairable product P at inspection center I in period T.

T_{pht} : Decimal part of the number of recyclable product P at thrift store H in period T.

T_{prt} : Decimal part of the number of repairable product P at recycling center R in period T.

T_{prat} : Decimal part of the number of recycled raw material A at recycling center R in period T.

T_{smt} : Decimal number deducted from the integer number of trucks that should be sent to manufacturing center from supplier in period T.

T_{mdt} : Decimal number deducted from the integer number of trucks that should be sent to distribution center from manufacturing center in period T.

T_{dct} : Decimal number deducted from the integer number of trucks that should be sent to stores from manufacturing center in period T.

T_{cit} : Decimal number deducted from the integer number of trucks that should be sent to inspection center from store in period T.

T_{ikt} : Decimal number deducted from the integer number of trucks that should be sent to disposal center from inspection center in period T.

T_{iht} : Decimal number deducted from the integer number of trucks that should be sent to thrift store from inspection center in period T.

T_{irt} : Decimal number deducted from the integer number of trucks that should be sent to recycling center from inspection center in period T.

T_{rdt} : Decimal number deducted from the integer number of trucks that should be sent to distribution center from recycling center in period T.

T_{hkt} : Decimal number deducted from the integer number of trucks that should be sent to disposal center from thrift store in period T.

T_{hrt} : Decimal number deducted from the integer number of trucks that should be sent to recycling center from thrift store in period T.

T_{rmt} : Decimal number deducted from the integer number of trucks that should be sent to manufacturing center from recycling center in period T.

Location cost (1)

$$\begin{aligned} & \sum_s F_S O_S + \sum_\mu F_\mu O_\mu + \sum_d F_d O_d + \sum_C F_C O_C + \sum_I F_I O_I + \sum_k F_k O_k \\ & + \sum_h F_h O_h + \sum_r F_r O_r \end{aligned}$$

Fixed cost of transportation (2)

$$\begin{aligned} & \sum_s X_S O_S + \sum_\mu X_\mu O_\mu + \sum_d X_d O_d + \sum_C X_C O_C + \sum_I X_I O_I + \sum_k X_k O_k \\ & + \sum_h X_h O_h + \sum_r X_r O_r \end{aligned}$$

Transportation cost (3)

$$\begin{aligned}
 & \sum_S \sum_m \sum_t T_{smt} \times Dis_{sm} \times V_{st} + \sum_M \sum_d \sum_t T_{mdt} \times Dis_{md} \times V_{mt} + \\
 & \sum_D \sum_C \sum_t T_{dct} \times Dis_{dc} \times V_{dt} + \sum_C \sum_i \sum_t T_{cit} \times Dis_{ci} \times V_{ct} \\
 & + \sum_i \sum_t V_{it} \times (\sum_k T_{ikt} \times Dis_{ik} + \sum_h T_{iht} \times Dis_{ih} + \sum_r T_{irt} \times Dis_{ir}) \\
 & + \sum_h \sum_t V_{ht} \times (\sum_k T_{hkt} \times Dis_{hk} + \sum_r T_{hrt} \times Dis_{hr}) \\
 & + \sum_r \sum_t V_{rt} \times (\sum_d T_{rdt} \times Dis_{rd} + \sum_m T_{rmt} \times Dis_{rm})
 \end{aligned}$$

Material cost (4)

$$\sum_a \sum_S \sum_m \sum_t C_{as} \times Q_{asmt}$$

Manufacturing cost (5)

$$\sum_p \sum_m \sum_d \sum_t C_{mp} \times Q_{pmdt}$$

Distributing cost (6)

$$\sum_p \sum_d \sum_c \sum_t C_d \times Q_{pdct}$$

Inspecting cost (7)

$$\sum_p \sum_c \sum_i \sum_t C_i \times Q_{pcit}$$

Disposing cost (8)

$$\sum_k (C_k \times \sum_p \sum_t (\sum_i Q_{pikt} + \sum_h Q_{phkt}))$$

Holding cost in stores (9)

$$\sum_p \sum_d \sum_c \sum_t C_{cp} \times Q_{pdct}$$

Holding cost in Thrift stores (10)

$$\sum_p \sum_i \sum_h \sum_t C_h \times Q_{piht}$$

Recycling and repairing cost (11)

$$\sum_p \sum_r \sum_t (C_{rp} \times \sum_d Q_{prdt} + w_{rp} \times (\sum_h Q_{phrt} + \sum_i Q_{pirt} - \sum_d Q_{prdt}))$$

Inventory cost (12)

$$\sum_p \sum_d \sum_t RE_{pdt} \times H_p$$

Unutilized Capacity cost (13)

$$\begin{aligned}
 & \sum_t \left(\sum_m (M_m \times (Cap_{mt} \times OM - \sum_p \sum_d Q_{pmdt})) \right. \\
 & \left. + \sum_d (M_d \times (Cap_{dt} \times Od - (\sum_p (\sum_c Q_{pdct} + RE_{pdt})))) \right) \\
 & + \sum_r (M_r \times (Cap_{rt} \times Or - (\sum_p (\sum_h Q_{phrt} + \sum_i Q_{pirt}))))
 \end{aligned}$$

Buying returned products cost (14)

$$\sum_p \sum_c \sum_i \sum_t Q_{pcit} \times C_p$$

- Flow balances

Manufacturing center (15)

$$\left(\sum_s Q_{asmt} + \sum_r Q_{arnt} \right) \times \text{ratio}_p \geq F_{ap} \times \sum_d Q_{pmdt}, \quad \forall a, m, t, p$$

Distribution centers (16)

$$\sum_r Q_{prdt} + \sum_m Q_{pmdt} + RE_{pd(t-1)} = \sum_c Q_{pdct} + RE_{pdt}, \quad \forall t, p, d$$

Stores (17)

$$\sum_d Q_{pdct} = D_{pct}, \forall p, c, t$$

Maximum number of each product sent to inspection center (18)

$$\sum_d Q_{pdct} \times \alpha_{pt} + Y \times C_p \geq \sum_i Q_{pcit}, \forall p, c, t$$

Minimum number of each product sent to inspection center (19)

$$\sum_d Q_{pdct} \times \alpha_{pt} \leq \sum_i Q_{pcit}, \forall p, c, t$$

Maximum number of each product sent to inspection center (20)

$$\sum_i Q_{pcit} \leq \sum_d Q_{pdct}, \forall p, c, t$$

Inspection center flow in and out (21)

$$\sum_h Q_{piht} + \sum_r Q_{pirt} + \sum_k Q_{pikt} = \sum_i Q_{pcit}, \forall p, i, t$$

Inspection center to thrift stores (22)

$$\sum_c Q_{pcit} \times \beta_{pt} = TH_{pit} + LH_{pit}, \quad \forall p, i, t$$

Integer- products sent to thrift stores (23)

$$\sum_h Q_{piht} = TH_{pit}, \quad \forall p, i, t$$

Inspection center to recycling centers (24)

$$\sum_c Q_{pcit} \times \mu_{pt} = TR_{pit} + LR_{pit}, \quad \forall p, i, t$$

Integer- products sent to recycling centers from inspection center (25)

$$\sum_r Q_{pirt} = TR_{pit}, \quad \forall p, i, t$$

Thrift stores flow in and out (26)

$$\sum_i Q_{piht} = \sum_k Q_{phkt} + \sum_r Q_{phrt}, \quad \forall p, h, t$$

Thrift store to recycling centers (27)

$$\sum_i Q_{piht} \times G_{pt} = T_{pht} + L_{pht}, \quad \forall p, h, t$$

Integer- products sent to recycling centers from thrift store (28)

$$\sum_r Q_{phrt} = T_{pht}, \quad \forall p, h, t$$

Repairing at recycling center (29)

$$(\sum_h Q_{phrt} + \sum_i Q_{pirt}) \times n_{pt} = T_{prt} + L_{prt}, \quad \forall p, r, t$$

Repaired products sent to distribution centers (30)

$$\sum_d Q_{prdt} = T_{prt}, \quad \forall p, r, t$$

Recycling at recycling center (31)

$$(\sum_h Q_{phrt} + \sum_i Q_{pirt} - \sum_d Q_{prdt}) \times F_{ap} = T_{prat} + L_{prat}, \quad \forall p, r, a, t$$

Recycled material sent to manufacturing centers (32)

$$\sum_m Q_{armt} = T_{prat}, \quad \forall p, r, a, t$$

The decimal part of the number of reusable products at inspection center (33)

$$LH_{pit} < 1, \quad \forall p, i, t$$

The decimal part of the number of recyclable products at inspection center (34)

$$LR_{pit} < 1, \quad \forall p, i, t$$

The decimal part of the number of recyclable products at thrift store (35)

$$L_{pht} < 1, \quad \forall p, h, t$$

The decimal part of the number of repairable products at recycling center (36)

$$L_{prt} < 1, \quad \forall p, r, t$$

The decimal part of the number of recycled raw material (37)

$$L_{prat} < 1, \quad \forall p, r, a, t$$

- Capacity constraints

Supplier capacity (38)

$$\sum_a \sum_m Q_{asmt} \leq Cap_{st} \times Os, \forall s, t$$

Manufacturing center capacity (39)

$$\sum_p \sum_d Q_{pmdt} \leq Cap_{mt} \times Om, \forall m, t$$

Distribution center capacity (40)

$$\sum_p (\sum_c Q_{pdct} + RE_{pdt}) \leq Cap_{dt} \times Od, \forall d, t$$

Store capacity (41)

$$\sum_p \sum_d Q_{pdct} \leq Cap_{ct} \times Oc, \forall c, t$$

Inspection center capacity (42)

$$\sum_p \sum_c Q_{pcit} \leq Cap_{it} \times Oi, \forall i, t$$

Disposal center capacity (43)

$$\sum_p \left(\sum_i Q_{pikt} + \sum_h Q_{phkt} \right) \leq Cap_{kt} \times Ok, \forall k, t$$

Thrift store capacity (44)

$$\sum_p \sum_i Q_{piht} \leq Cap_{ht} \times Oh, \forall h, t$$

Recycling center capacity (45)

$$\sum_p \left(\sum_h Q_{phrt} + \sum_i Q_{pirt} \right) \leq Cap_{rt} \times Or, \forall r, t$$

Virgin raw material (46)

$$w \times \left(\sum_s Q_{asmt} + \sum_r Q_{arnt} \right) \leq \sum_s Q_{asmt}, \forall a, m, t$$

- Number of trucks sent to centers

Supplier to manufacturing center trucks (47)

$$\sum_a Q_{asmt} / B_{st} = T_{smt} - L_{smt}, \quad \forall s, m, t$$

Manufacturing center to distribution center trucks (48)

$$\sum_p Q_{pmdt} / B_{mt} = T_{mdt} - L_{mdt}, \quad \forall m, d, t$$

Distribution center to store trucks (49)

$$\sum_p Q_{pdct}/B_{dt} = T_{dct} - L_{dct}, \quad \forall d, c, t$$

Store to inspection center trucks (50)

$$\sum_p Q_{pcit}/B_{ct} = T_{cit} - L_{cit}, \quad \forall c, i, t$$

Inspection center to disposal center trucks (51)

$$\sum_p Q_{pikt}/B_{it} = T_{ikt} - L_{ikt}, \quad \forall i, k, t$$

Inspection center to thrift store trucks (52)

$$\sum_p Q_{piht}/B_{it} = T_{iht} - L_{iht}, \quad \forall i, h, t$$

Inspection center to recycling center trucks (53)

$$\sum_p Q_{pirt}/B_{it} = T_{irt} - L_{irt}, \quad \forall i, r, t$$

Thrift store to disposal center trucks (54)

$$\sum_p Q_{phkt}/B_{ht} = T_{hkt} - L_{hkt}, \quad \forall h, k, t$$

Thrift store to recycling center trucks (55)

$$\sum_p Q_{phrt}/B_{ht} = T_{hrt} - L_{hrt}, \quad \forall h, r, t$$

Recycling center to distribution center trucks (56)

$$\sum_p Q_{prdt}/B_{rt} = T_{rdt} - L_{rdt}, \quad \forall r, d, t$$

Recycling center to manufacturing center trucks (57)

$$\sum_a Q_{armt}/B_{rt} = T_{rmt} - L_{rmt}, \quad \forall r, m, t$$

- Decimal part of number of trucks

Decimal part of the number of trucks sent from supplier to manufacturing center (58)

$$L_{smt} < 1, \quad \forall s, m, t$$

Decimal part of the number of trucks sent from Manufacturing center to distribution center (59)

$$L_{mdt} < 1, \quad \forall m, d, t$$

Decimal part of the number of trucks sent from Distribution center to store (60)

$$L_{dct} < 1, \quad \forall d, c, t$$

Decimal part of the number of trucks sent from Store to inspection center (61)

$$L_{cit} < 1, \quad \forall c, i, t$$

Decimal part of the number of trucks sent from Inspection center to disposal center (62)

$$L_{ikt} < 1, \quad \forall i, k, t$$

Decimal part of the number of trucks sent from Inspection center to thrift store (63)

$$L_{iht} < 1, \quad \forall i, h, t$$

Decimal part of the number of trucks sent from Inspection center to recycling center (64)

$$L_{irt} < 1, \quad \forall i, r, t$$

Decimal part of the number of trucks sent from Thrift store to disposal center (65)

$$L_{hkt} < 1, \quad \forall h, k, t$$

Decimal part of the number of trucks sent from Thrift store to recycling center (66)

$$L_{hrt} < 1, \quad \forall h, r, t$$

Decimal part of the number of trucks sent from Recycling center to distribution center (67)

$$L_{rdt} < 1, \quad \forall r, d, t$$

Decimal part of the number of trucks sent from recycling center to manufacturing center (68)

$$L_{rmt} < 1, \quad \forall r, m, t$$

- Number of centers

Suppliers (69)

$$\sum_S O_S \leq L_s$$

Manufacturers (70)

$$\sum_m O_m \leq L_m$$

Distribution centers (71)

$$\sum_d O_d \leq L_d$$

Stores (72)

$$\sum_c O_c \leq L_c$$

Inspection centers (73)

$$\sum_i O_i \leq L_i$$

Disposal centers (74)

$$\sum_k O_k \leq L_k$$

Thrift stores (75)

$$\sum_h O_h \leq L_h$$

Recycling centers (76)

$$\sum_r O_r \leq L_r$$

Binary variables (77)

$$O_S, O_m, O_d, O_c, O_i, O_k, O_h, O_r \in \{0, 1\}, \forall s, m, d, c, i, k, h, r$$

Integer variables (78)

$$T_{pht}, T_{prt}, T_{prat}, T_{smt}, T_{mdt}, T_{dct}, T_{cit}, T_{ikt}, T_{iht}, T_{irt}, T_{rdt}, T_{hkt}, T_{hrt}, T_{rmt}, TH_{pit}, TR_{pit},$$

$$Q_{asmt}, Q_{pmdt}, Q_{pdct}, Q_{pcit}, Q_{pikt}, Q_{piht}, Q_{pirt}, Q_{phkt}, Q_{phrt}, Q_{prdt}, Q_{armt}, RE_{pdt} \in \mathbb{Z}$$

Non-negative variables (79)

$$T_{pht}, T_{prt}, T_{prat}, T_{smt}, T_{mdt}, T_{dct}, T_{cit}, T_{ikt}, T_{iht}, T_{irt}, T_{rdt}, T_{hkt}, T_{hrt}, T_{rmt}, TH_{pit}, TR_{pit},$$

$$LH_{pit}, LR_{pit}, L_{pht}, L_{prt}, L_{prat}, L_{smt}, L_{mdt}, L_{dct}, L_{cit}, L_{ikt}, L_{iht}, L_{irt}, L_{hkt}, L_{rdt}, L_{rmt}, L_{hrt},$$

$$Q_{asmt}, Q_{pmdt}, Q_{pdct}, Q_{pcit}, Q_{pikt}, Q_{piht}, Q_{pirt}, Q_{phkt}, Q_{phrt}, Q_{prdt}, Q_{armt}, RE_{pdt} \geq 0$$

The fundamentals of this model, its objective function, and its constraints are similar to the non-linear model, however, in this model ceiling and floor functions have been redefined as linear equations.

In order to make the ceiling function linear, I set the equation equal to an integer variable and a continuous variable between 0 and 1. To make the equation integer, I replace the equation with the newly defined integer variable. For example, the number of raw materials that should be sent to the manufacturing center from the supplier divided by the capacity of trucks of the supplier results in the number of trucks that should be sent to the manufacturing center; However, this equation might give a non-integer result, while we can only send integer number of trucks.

Constraint 48 equals the number of trucks that should be sent to the manufacturing center with a continuous variable between 0 and 1 named " L_{smt} " deducted from an integer variable defined as " T_{smt} "; Then, in objective function part 3, instead of using the ceiling function of " $\frac{\sum_a Q_{asmt}}{B_{st}}$ ", I replace it with " T_{smt} ", which is the integer part of that equation. Every ceiling function in the first non-linear model has been replaced in the same manner.

Constraints 47 to 57 divide the number of trucks into an integer part and a decimal part. In the objective function 3, the number of trucks has been substituted with the integer variable defined in constraints 47 to 57. Constraints 58 to 68 control that the continuous variables used in constraints 47 to 57 are less than 1.

The equation inside the floor function in the non-linear model was set equal to a continuous variable between 0 and 1 and added to an integer variable to make the floor functions linear for the balance constraints. In all the constraints, the floor function equation was replaced with the mentioned integer variable. Constraints 15 to 32 represent the balance constraints in the linear model.

For example, constraint 22 divides the number of reusable products at the inspection center into an integer number and its decimal part. Constraint 23 guarantees that the integer number of reusable products at the inspection center is sent to thrift stores.

Similar to the logic behind constraint 22, constraint 24 divides the number of recyclable products at every inspection center into an integer number and its decimal part, and constraint 25 ensures that the integer number of recyclable products at the inspection center are sent to recycling centers.

Constraint 27 separates the integer part and decimal part of the recyclable number of products at a thrift store, and constraint 28 ensures that the integer number of recyclable products at the thrift store are sent to recycling centers.

Constraint 29 divides the decimal part and integer part of the number of repairable products at a recycling center, and constraint 30 guarantees that the integer number of repairable products at the recycling center are sent to distribution centers.

Finally, constraint 31 separates the integer and decimal parts of raw materials that can be obtained through recycling, and constraint 32 controls that the integer number of recyclable products are recycled, converted into reusable raw materials, and sent to manufacturing centers.

Constraints 33 to 37 show that the decimal parts of floor function equations are less than 1. All other constraints are the same as the non-linear model.

Deterministic model results

To validate the deterministic model, I solved the problem using the CPLEX solver in GAMS (The general algebraic modeling system) software. The results show the locations that should be equipped, the number of products to be processed at each center during each period, the logistics of transferring products to other centers for further processing, and the number of products that

should be held as inventory in order to meet the demand of each product at each store in each period all while minimizing total costs. The results are shown below.

Figure2 -Quantity of raw material sent to each manufacturing center from each supplier in each period

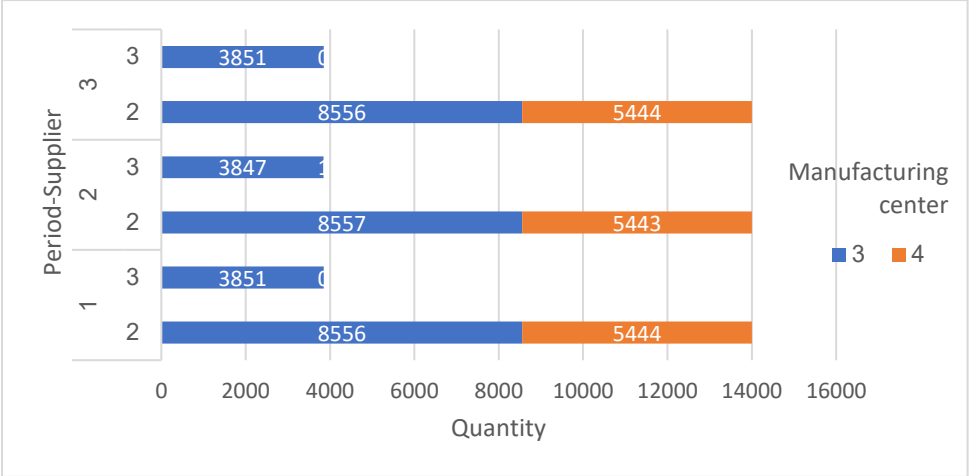


Figure 2 illustrates the selection of suppliers and manufacturing centers, indicating that Supplier 2, Supplier 3, Manufacturing Center 3, and Manufacturing Center 4 should be chosen from the available options.

Additionally, Figure 2 details the allocation of skeins of wool from each supplier to each manufacturing center for each period. For instance, it specifies that Supplier 3 should provide 3,851 skeins of wool to Manufacturing Center 4, while Supplier 2 should deliver 8,556 skeins of wool to Manufacturing Center 3 and 5,444 skeins of wool to Manufacturing Center 4 in the first period.

Figure 3-Quantity of products sent to each distribution center from each manufacturing center in each period

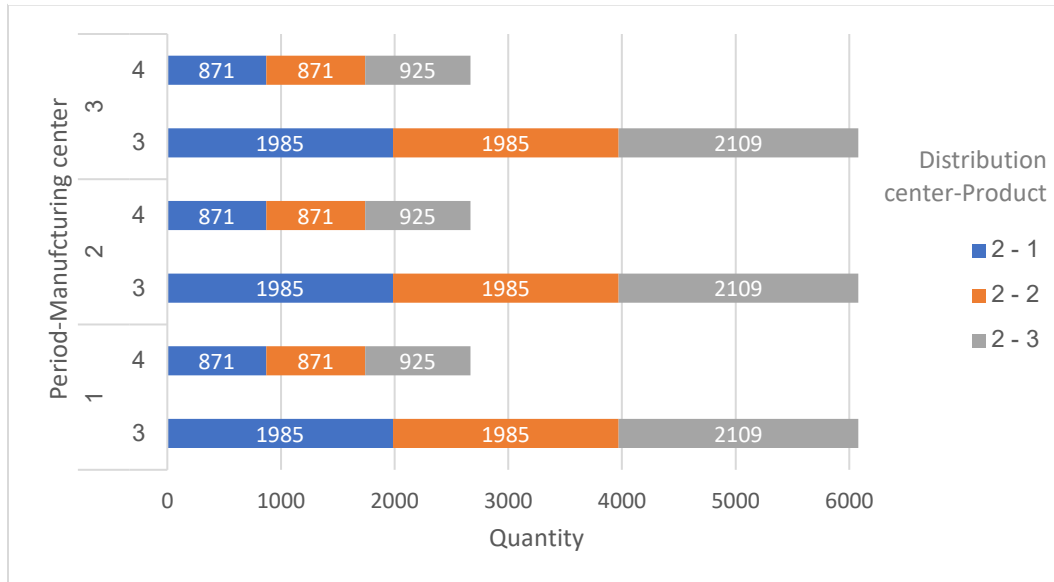


Figure 3 indicates that distribution center 2 is the optimal choice based on its capacity and associated costs. The figure also outlines the production quantities at each manufacturing center, based on the raw materials received, with all outputs sent to distribution center 2. This figure presents that in period 1, manufacturing center 3 should produce 1985 units of product 1, 1985 units of product 2, and 2109 units of product 3, all of which are to be sent to distribution 2.

Figure4 -Quantity of products sent to stores from distribution center in each period

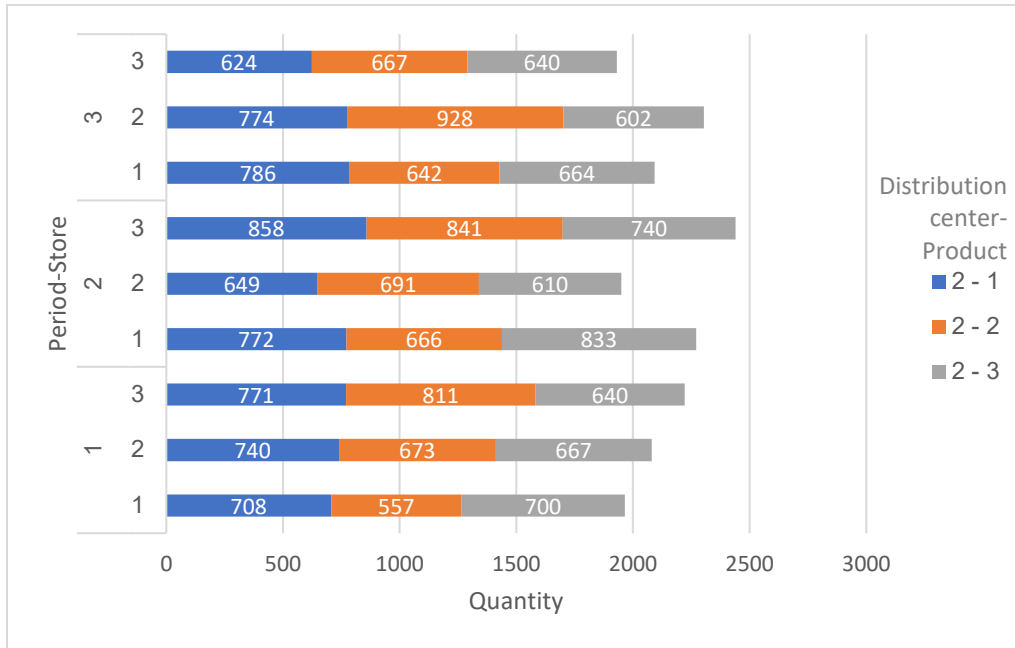


Figure 4 specifies the quantities of each product that should be sent to each store from distribution center 2 in each period. In the first period 771 units of product 1 should be sent to store 3, 740 units to store 2, and 708 units sent to store 1 to satisfy the demand for product 1 at these stores.

Figure 5-Quantity of products held in distribution center 2 in each period

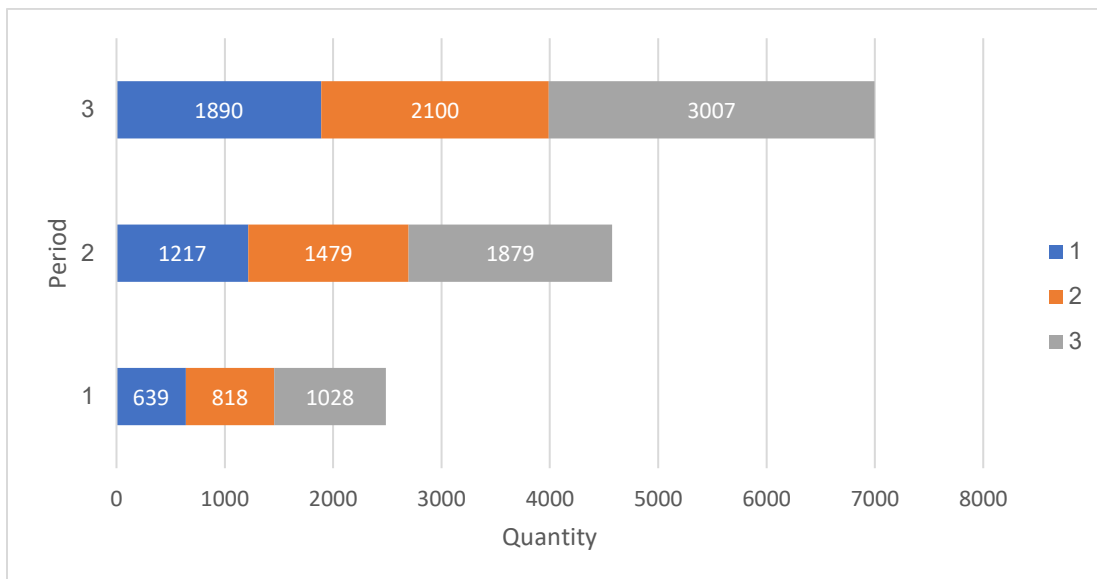


Figure 5 shows the number of products that should be kept in distribution center 2 to be sent in the next periods to satisfy future demand. As illustrated in period 1, 639 units of product 1, 818 units of product 2, and 1028 units of product 3 should be kept at the distribution center.

Figure6 -Quantity of returned products sent to each inspection center from stores in each period

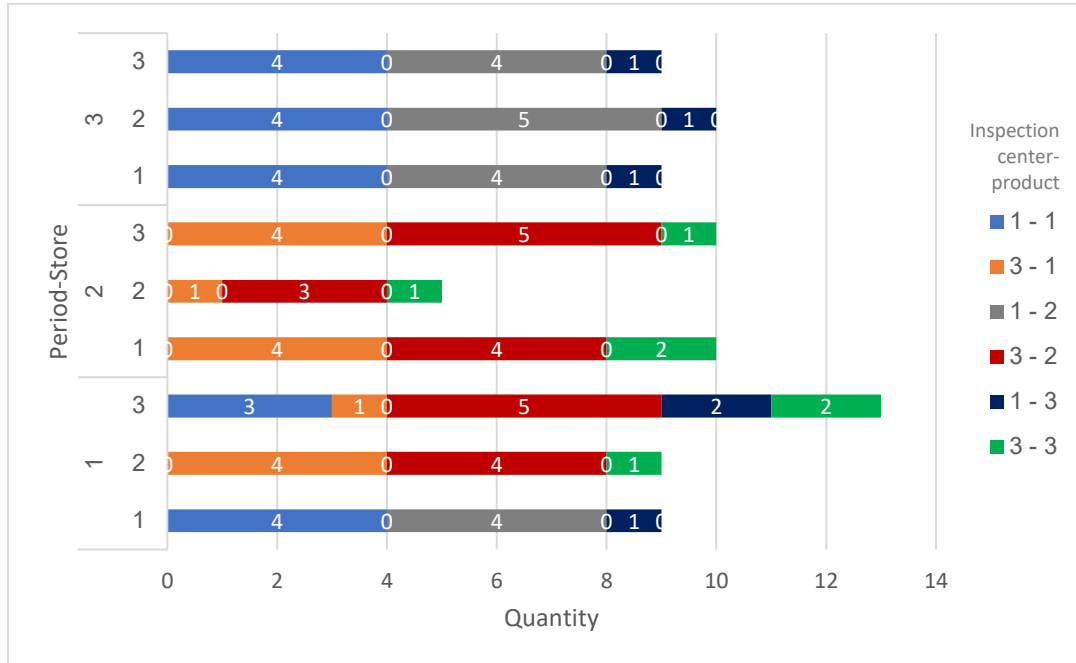


Figure 6 identifies inspection centers 1 and 3 as the most optimal facilities to be utilized. Additionally, the figure details the quantity of each product that should be transferred from each store to the designated inspection centers during each period. For instance, during the first period, Store 1 is expected to receive returns of 4 units of product 1, 4 units of product 2, and 1 unit of product 3 from customers; all these products should be sent to inspection center 1. In the same period, store 2 will receive 4 units of product 1, 4 units of product 2, and 1 unit of product 3, all of which should be sent to inspection center 3. Concurrently, store 3 will receive 4 units of product 1, sending 3 units to inspection center 1 and 1 unit to inspection center 3. Additionally, store 3 will receive 5 units of product 2, which should all be sent to inspection center 3, and 4 units of product 3, with 2 units of it sent to inspection center 3, and 2 units to inspection center 1.

Figure7 -Quantity of products sent to each disposal center from each inspection center in each period

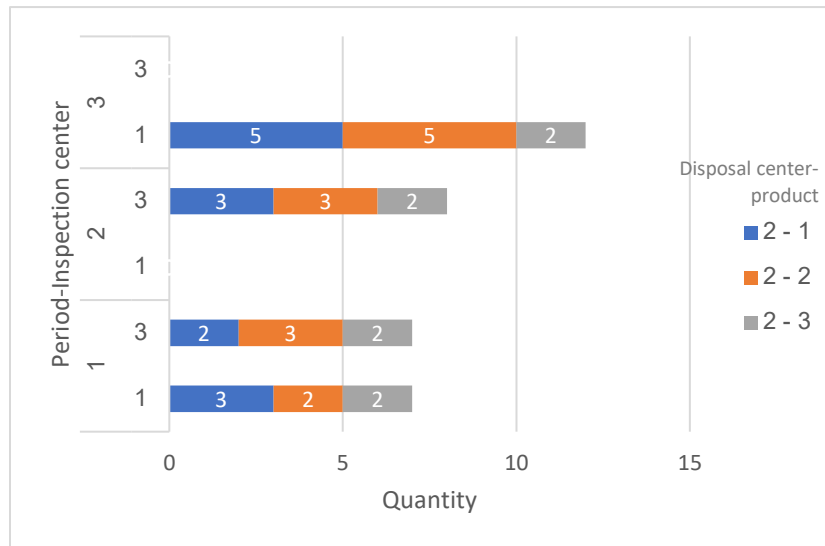


Figure 7 shows that the most optimal disposal center to be equipped is disposal center 2. The figure specifies the number of products that should be sent to disposal center 2 from each inspection center in each period. For instance, the figure shows that in period 1, 3 units of product 1, 2 units of product 2, and 2 units of product 3 should be sent to disposal center 2 from inspection center 1.

Figure8 -Quantity of products sent to each thrift store from each inspection center in each period

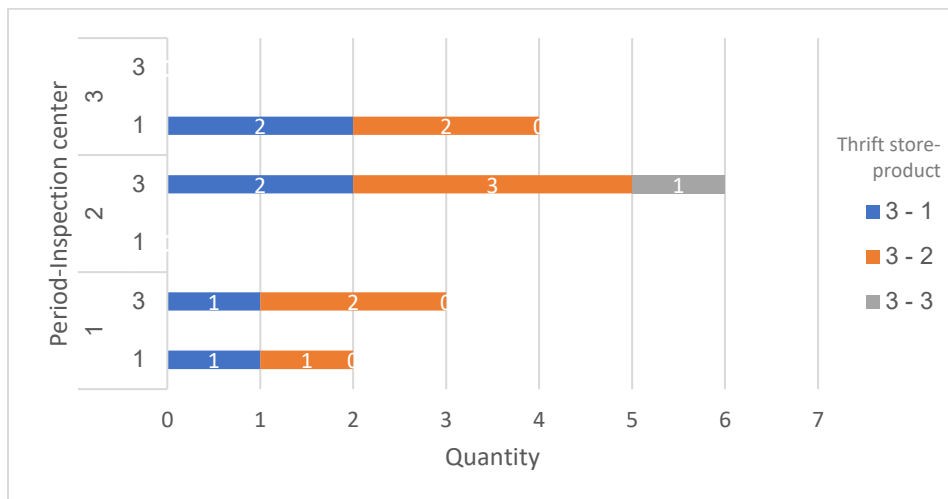


Figure 8 details the allocation of products to each thrift store from the respective inspection centers for each period. The figure indicates that only thrift store 3 should be utilized. During the first

period, 1 unit of product 1, and 1 unit of product 2 should be sent from inspection center 1 to thrift store 3, in addition to 1 unit of product 1 and 2 units of product 2 from inspection center 3.

Figure9 -Quantity of products sent to each recycling center from each inspection centers in each period

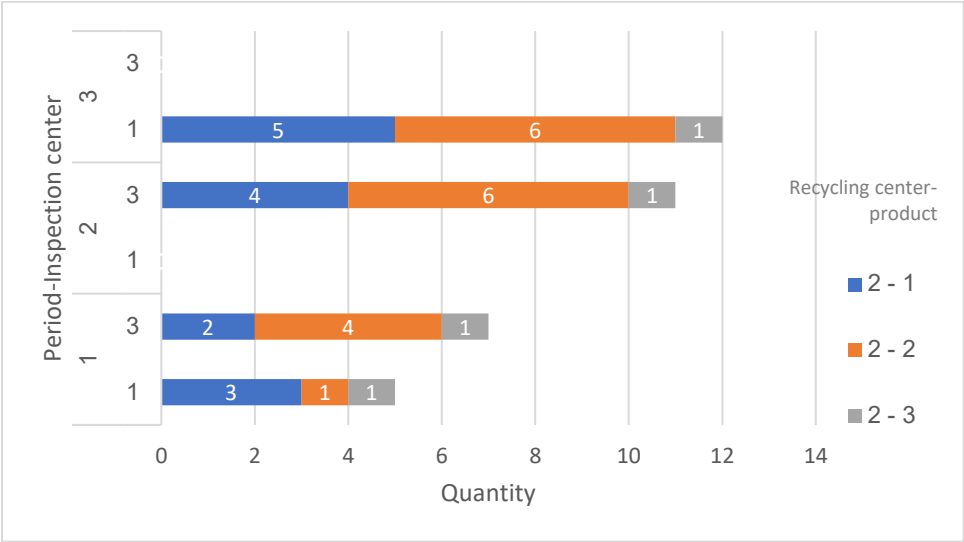


Figure 9 displays the number of products that should be sent to each recycling store from each inspection center in each period based on the return rates and ratio of recyclable products. Recycling center 2 should be chosen between available options. As shown in this figure, in period 1, inspection center 1 should send 3 units of product 1, 1 unit of product 2, and 1 unit of product 3, and inspection center 3 should send 2 units of product 1, 4 units of product 2, and 1 unit of product 3 to recycling center 2.

Figure10 -Quantity of products sent to each recycling center from each thrift store in each period

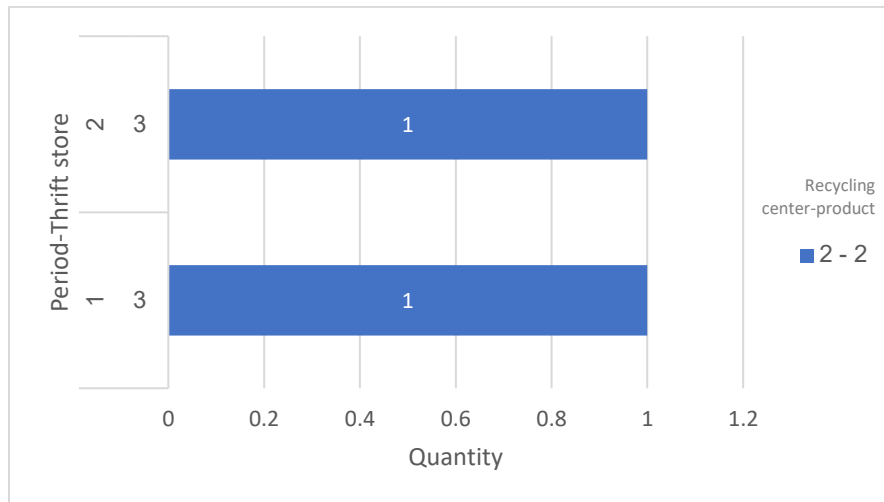


Figure 10 illustrates the number of products that should be sent to recycling center 2 from each thrift store. In period 1 and 2, 1 unit of product 2 should be sent to recycling center 2 from thrift store 3.

Figure11 -Quantity of products sent to each disposal center from each thrift store in each period

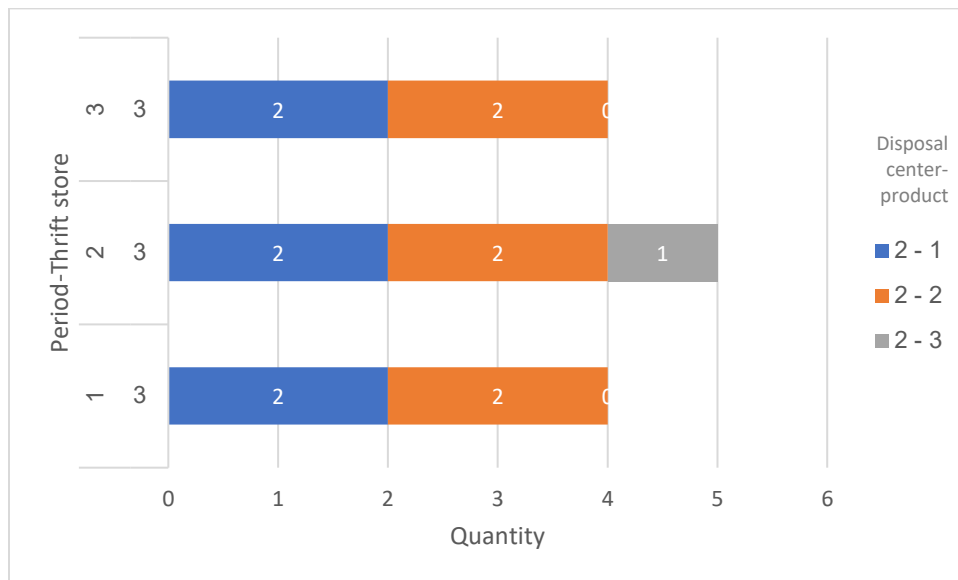


Figure 11 shows the number of products that should be sent to disposal center 2 from thrift store 3 in each period. In the first period, 2 units of Product 1 and 2 units of Product 2 should be sent to Disposal Center 2 from Thrift Store 3. In the second period, 2 units of Product 1, 2 units of Product

2, and 1 unit of Product 3 should be sent. During the third period, 2 units of Product 1 and 2 units of Product 2 should again be transferred from Thrift Store 3 to Disposal Center 2.

Figure12 -Quantity of products sent to each distribution center from each recycling center in each period

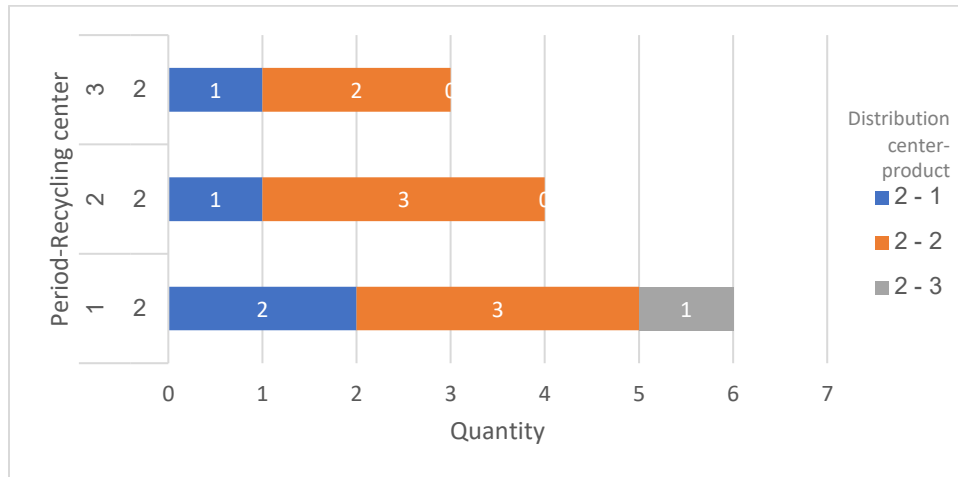


Figure 12 shows the number of products that should be repaired and sent to distribution center 2 from recycling center 2 in each period. In the first period, 2 units of product 1, 3 units of product 2, and 1 unit of product 3 should be repaired at recycling center 2 and sent to distribution center 2.

Figure13 -Quantity of raw material sent from each recycling center to each manufacturing center in each period

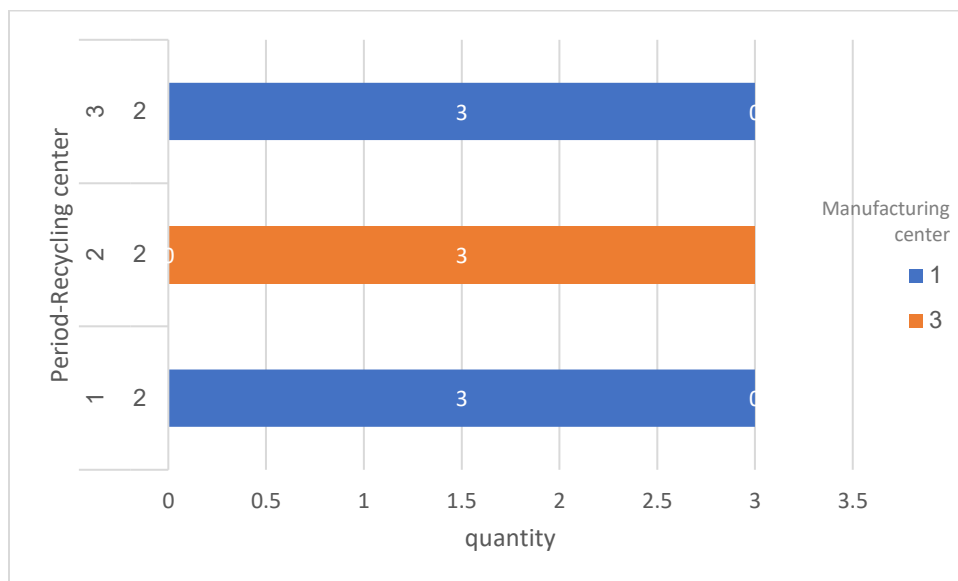


Figure 13 presents the quantity of raw material, wool, that should be sent to manufacturing centers 1 and 3 after getting recycled at recycling center 2 in each period. In the first period, 3 skeins of wool are designated for Manufacturing Center 1. During the second period, 3 skeins are to be sent to Manufacturing Center 3. Finally, in the third period, another 3 skeins of wool are to be allocated to Manufacturing Center 1. These allocations follow the recycling process completed at Recycling Center 2.

In our analysis, we utilized the data to determine the appropriate locations to equip and the quantity of items to dispatch for the three specified products. It is important to note that distribution centers, recycling facilities, thrift stores, and disposal centers will receive products from multiple companies concurrently and will not operate solely to manage the output from this specific supply chain. Therefore, we have accounted for the costs associated with equipping and processing these three products, and the strategies for their production and transportation are detailed within the presented model. Due to computational resource limitations, we focused on three types of products handled at these centers; however, these equipped centers will be capable of handling many types of clothing items in practice.

To enhance our understanding of the data, I will provide a detailed explanation of the optimal product flow of product 1 to distribution center 2 in the first period, illustrating how the results should be interpreted.

Figure14 -Quantity of product 1 sent to distribution center 2 from manufacturing centers at the first period

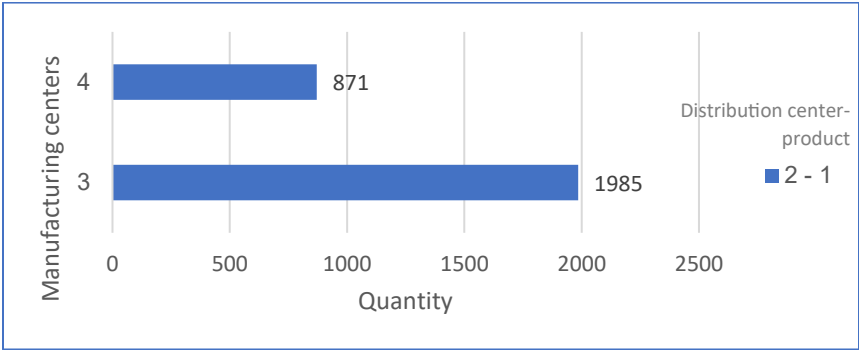
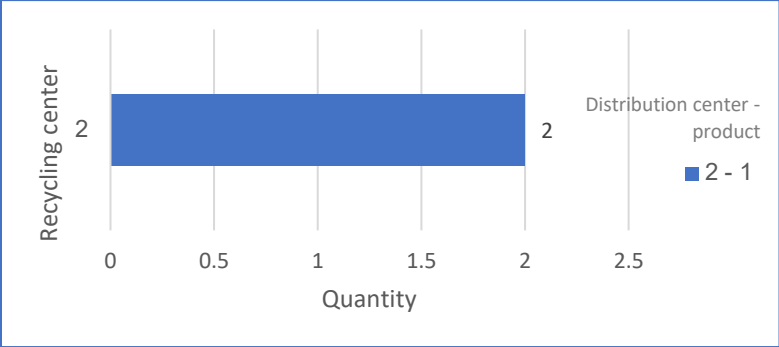


Figure 14 demonstrates the quantity of product 1 that should be sent from manufacturing centers 3 and 4 to distribution center 2 in the first period. The results specify that 1985 units of product 1

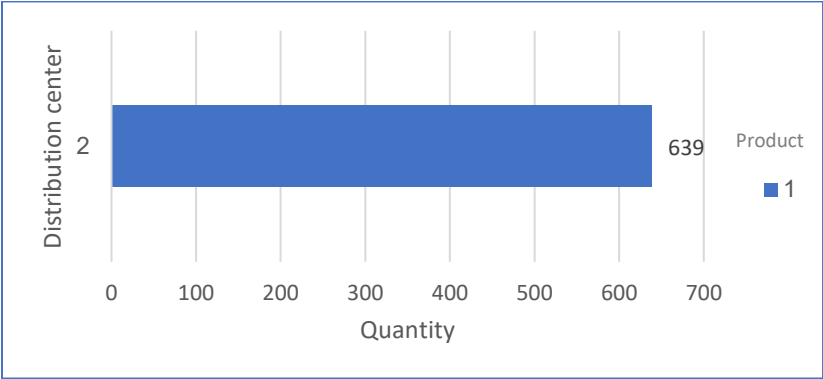
should be manufactured at manufacturing center 3 and 871 units should be manufactured at manufacturing center 4 and then delivered to distribution center 2.

Figure15 -Quantity of product 1 sent to distribution center 2 from recycling center 2 at the first period



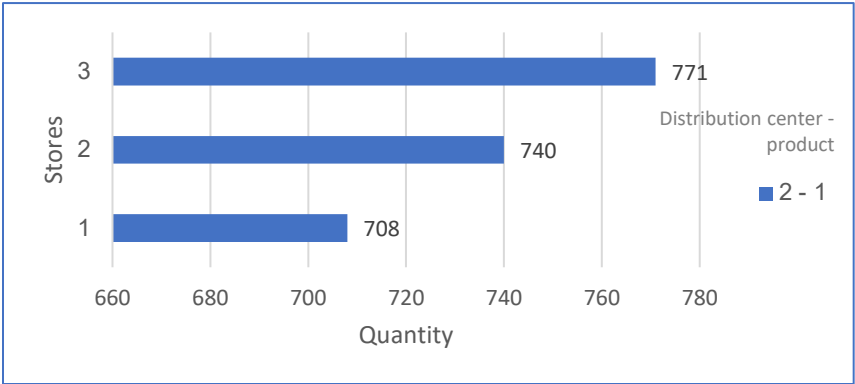
Recycling center 2 can repair 2 units of product 1 and send it to distribution center 2 in period 1. In total, in the first period, distribution center 2 should receive 2858 units of product 1.

Figure16 -Quantity of product 1 that should be held in distribution center at the first period



The results show that 639 units of product 1 should be held at distribution for the next period's demand. The remaining products which contain 2219 units of product 1 should be transferred to stores to satisfy the demand.

Figure17 -Quantity of product 1 that is sent to each store from distribution 2 at the first time period



Based on the demand for product 1 at each store, 708 units should be sent to store 1, 740 units to store 2, and 771 units to store 3.

In conclusion, the model was validated by presenting reasonable results that specify which centers should be facilitated, the quantity of products to be processed at each center, and the transportation schedules. The model was executed using the CPLEX solver within the GAMS software, successfully processing and providing the most optimal solution. The accuracy of the flow balances was confirmed, ensuring that the quantities of products entering and exiting the stores, as well as those processed at each center, are aligned accurately with the demand. This demonstrates the model's capability to effectively manage and optimize the supply chain operations.

Chapter 5: Two-stage Stochastic programming

Stochastic optimization

Deterministic optimization assumes all the parameters are determined and fixed. However, when facing a real-world problem there might be uncertain parameters that should be addressed. Stochastic optimization is a mathematical model technique that is used to solve problems that involve uncertainty or randomness. It considers that objective function and decision variables might depend on parameters that are uncertain but follow a certain probabilistic distribution. Stochastic optimization often involves generating samples to estimate the expected value of objective function in different scenarios, aiming to find the best solution that performs well under various possible future situations. Moreover, stochastic optimization is practical in situations where key parameters are uncertain, such as demand forecasts, future prices, or supply availability. It is also useful in dynamic environments where conditions change over time, and robust decision-making is required.

There are several common approaches to stochastic optimization. Stochastic programming uses scenario-based methods to represent uncertainty. It solves the problem by minimizing the expected cost or maximizing the expected benefit across all the scenarios. On the other hand, Simulation-based optimization combines simulation models with optimization techniques to assess system performance under uncertainty, allowing for complex and dynamic interactions to be considered. Heuristic and metaheuristic methods, such as Genetic Algorithms or Simulated Annealing, consider randomness in their search processes to explore the solution space more effectively and escape local optima.

In summary, stochastic optimization is a powerful tool for addressing problems involving uncertainty. It is widely applicable in various fields where decisions must be robust to unpredictable changes, providing a way to optimize performance in the face of variability and incomplete information.

Two-stage stochastic optimization

Two-stage stochastic optimization is useful when decisions need to be made sequentially. The first-stage decisions should be made before having information regarding uncertainties. These decisions are based on initial information available and are strategic long-term decisions such as

setting up infrastructure, equipping centers, or allocating resources. These decisions should be robust enough to handle various possible future scenarios.

The second-stage decisions are mostly operational or short-term decisions. These short-term decisions modify the first stage decisions. In the second stage, the uncertainty is revealed, and decisions should be made considering the uncertain nature of the problem. For example, when the probability distribution of demand levels is known, the company can decide on how many products to process at each center and plan for its transportation schedule.

Mathematical Formulation of the two-stage stochastic optimization problem

$$\min_x c^T x + \mathbb{E}_\xi [Q(x, \xi)] \quad (\text{Birge \& Louveaux, 2011}) \quad (1)$$

In formula (1) proposed, x represents the first-stage decision variables, C^T is the cost associated with these decisions, and $\mathbb{E}_\xi [Q(x, \xi)]$ is the expected value of the second-stage cost given the decisions x and the uncertainty ξ .

$$Q(x, \xi) = \min_y q^T y \quad \text{s.t.} \quad Tx + Wy = h(\xi), \quad y \geq 0 \quad (\text{Birge \& Louveaux, 2011}) \quad (2)$$

Formula (2) demonstrates the second-stage problem. In this problem, y represents the second-stage decision variables, $q^T y$ is the second-stage cost, and the constraints $T_x + W_y = h(\xi)$ represent the constraints in the second-stage problem.

T_x reflects the impact of the first-stage decisions on the second stage. As an example, if ' x ' represents the initial number of locations that should be equipped, T_x can describe how much of the resources or capacities are allocated and affect the second stage.

W_y shows the direct influence of second-stage decisions on the outcomes. It demonstrates how the second-stage variables y such as adjustments in production, inventory, or operations, meet the requirements or constraints at the second stage.

Finally, $h(\xi)$ represents the realization of uncertain parameters ξ in the second stage, such as the uncertain demand. It indicates the right-hand side of the constraints that depend on the scenario or state of the problem.

In conclusion, two-stage stochastic optimization provides a structured approach to decision-making under uncertainty. It divides the decision process into two distinct stages: first-stage strategic decisions followed by second-stage tactical planning based on realized outcomes. This methodology enables the creation of robust plans that can adapt to uncertain future events, leading to more resilient and effective decision-making for companies (Birge & Louveaux, 2011; Kall & Wallace, 1995)

Model assumptions

1. The model is multi-echelon, multi-product (including three types of wool clothes), and multi-period (consisting of a three-month period)
2. Fixed costs of equipping and variable costs of operating each center in each period are determined.
3. Fixed and variable costs of transportation in each period between each two centers are determined.
4. Inventory cost and unutilized capacity cost are determined.
5. Distances between centers are determined.
6. Capacity of each center is determined.
7. Ratios of return, percentage of reusable, repairable, and recyclable products are determined.
8. Capacities of trucks of each center are determined.
9. Demand of each product at each store in each period is stochastic and follows a normal distribution.

Decision variables

1. Variables denoting if a facility location should be selected and equipped that can cover the resources needed in all possible scenarios.
2. Quantity of each material that should be processed and sent to each center from another center to meet the demand in each period in each scenario.
3. Residual inventory of each product at each distribution center for each period in each scenario.

Stochastic model

Indexes

S: Set of suppliers

M: Set of manufacturers

D: Set of distribution centers

C: Set of stores

I: Set of inspection centers

K: Set of disposal centers

H: Set of thrift stores

R: Set of recycling centers

P: Set of products

A: Set of raw material

T: Index of time periods

E: Index of scenarios

Parameters

- Fixed costs of locating

F_s : Fixed cost of operating supplier S

F_m : Fixed cost of operating manufacturer M

F_d : Fixed cost of operating distribution center D

F_c : Fixed cost of operating store C

F_i : Fixed cost of operating inspection center I

F_k : Fixed cost of operating disposal center k

F_h : Fixed cost of operating thrift store H

F_r : Fixed cost of operating recycling center R

- Variable costs

C_{mp} : unit manufacturing cost of product p in manufacturing center M

C_d : cost of processing in distribution center D per product

C_i : cost of processing in inspection center I per product

C_k : cost of processing in disposal center K per product

C_{rp} : cost of repairing in recycling center R for product p

W_{rp} : cost of processing in recycling center R for product p

C_{as} : procurement cost of raw material A from supplier S

C_{cp} : cost of keeping product P in store C

C_{hp} : cost of keeping product P in thrift store H

C_p : cost of buying back returned product P from customers

- Transportation cost

V_{st} : variable cost of shipment from supplier S at time T

V_{mt} : variable cost of shipment from manufacturer M at time T

V_{dt} : variable cost of shipment from distribution center D at time T

V_{ct} : variable cost of shipment from store C at time T

V_{it} : variable cost of shipment from inspection center I at time T

V_{ht} : variable cost of shipment from thrift store H at time T

V_{rt} : variable cost of shipment from recycling center R at time T

x_s : Fixed cost of shipment from supplier S

x_m : Fixed cost of shipment from manufacturer M

x_d : Fixed cost of shipment from distribution center D

x_c : Fixed cost of shipment from store C

x_i : Fixed cost of shipment from inspection center I

x_h : Fixed cost of shipment from thrift store H

x_r : Fixed cost of shipment from recycling center R

- Inventory cost

H_p : holding cost per unit of product P in distribution centers.

- Unutilized capacity cost

M_m : the cost of unutilized capacity in manufacturing center

M_d : the cost of unutilized capacity in distribution centers

M_r : the cost of unutilized capacity in recycling centers.

- Distances

Dis_{sm} : Distance between supplier S and manufacturing center M

Dis_{md} : Distance between manufacturing M and distribution center D

Dis_{dc} : Distance between distribution center D and store C

Dis_{ci} : Distance between store c and inspection center I

Dis_{ik} : Distance between inspection center I and disposal center K

Dis_{ih} : Distance between inspection center I and thrift store H

Dis_{ir} : Distance between inspection center I and recycling center R

Dis_{hk} : Distance between thrift store H and disposal center K

Dis_{hr} : Distance between thrift store H and recycling center R

Dis_{rd} : Distance between recycling center R and distribution center D

Dis_{rm} : Distance between recycling center r and manufacturing center M

- **Capacities**

Cap_{st} : Capacity of supplier S at time period T

Cap_{mt} : Capacity of manufacturing center M at time period T

Cap_{dt} : Capacity of distribution center D at time period T

Cap_{ct} : Capacity of store C at time period T

Cap_{it} : Capacity of inspection center I at time period T

Cap_{kt} : Capacity of disposal center K at time period T

Cap_{ht} : Capacity of thrift store H at time period T

Cap_{rt} : Capacity of recycling center R at time period T

- Ratios and rates

$ratio_p$: percentage of raw material that should be used for product p

Y: the price elasticity of return quantity

W: minimum percentage of raw material that should be virgin.

α_{pt} : return rate of product P at time period T

β_{Pt} : ratio of product P that could be sent to thrift stores from inspection centers at time period T

μ_{Pt} : ratio of product P that could be sent to recycling centers from inspection centers at time period T

γ_{pt} : ratio of product P that could be sent to recycling centers from thrift stores at time period T

η_{pt} : ratio of product p that can be repaired in recycling centers at time period T

f_{ap} : coefficient of raw material A needed for product p

- Batch sizes

B_{st} : capacity of trucks going from supplier S to manufacturing centers at time period T.

B_{mt} : capacity of trucks going from manufacturing center M to distribution centers at time period T.

B_{dt} : capacity of trucks going from distribution center D to stores at time period T.

B_{ct} : capacity of trucks going from store C to inspection centers at time period T.

B_{it} : capacity of trucks going from inspection centers to other centers at time period T.

B_{ht} : capacity of trucks going from thrift store H to recycling centers at time period T.

B_{rt} : capacity of trucks going from recycling center R to other centers at time period T.

L_s : Maximum number of suppliers that can be operated.

L_m : Maximum number of manufacturing centers that can be operated.

L_d : Maximum number of distribution centers that can be operated.

L_c : Maximum number of stores that can be operated.

L_i : Maximum number of inspection centers that can be operated.

L_k : Maximum number of disposal centers that can be operated.

L_h : Maximum number of thrift stores that can be operated.

L_r : Maximum number of recycling centers that can be operated.

- Demand

D_{pcte} : demand of product P at store C in period T in scenario e

Decision variables

- First stage decision variables

O_s : binary variable denoting if supplier S is selected.

O_m : binary variable denoting if manufacturing center M is selected.

O_d : binary variable denoting if distribution center D is selected

O_c : binary variable denoting if store C is selected

O_i : binary variable denoting if inspection center I is selected

O_k : binary variable denoting if disposal center K is selected

O_h : binary variable denoting if thrift store H is selected

O_r : binary variable denoting if recycling center R is selected

- Second stage variables

Q_{asmt_e} : Volume of raw material A that should be sent to manufacturing center M from supplier S in period T in scenario e

Q_{pmdt_e} : number of product P that should be sent to distribution center D from manufacturing center M in period T in scenario e

Q_{pdct_e} : number of product P that should be sent to store C from distribution center D in period T in scenario e

Q_{pcit_e} : number of product P that should be sent to inspection center I store C in period T in scenario e

Q_{pikt_e} : number of product P that should be sent to disposal center K from inspection center I in period T in scenario e

Q_{piht_e} : number of product P that should be sent to thrift store H from inspection center I in period T in scenario e

Q_{pirt_e} : number of product P that should be sent to recycling center R from inspection center I in period T in scenario e

Q_{phkt_e} : number of product P that should be sent to disposal center K from thrift store H in period T in scenario e

Q_{phrt_e} : number of product P that should be sent to recycling center R from thrift store H in period T in scenario e

Q_{prdte} : number of product P that should be sent to distribution center D from recycling center R in period T in scenario e

Q_{armte} : Volume of raw material A that should be sent to manufacturing center M from recycling center R in period T in scenario e

RE_{pdte} : residual inventory of product P at distribution center D in period T in scenario e

- Integer variables

L_{phte} : integer number of recyclable product P at thrift store H in period T in scenario e.

L_{prte} : integer number of repairable product P at recycling center R in period T in scenario e.

L_{prate} : integer number of recycled raw material A at recycling center R in period T in scenario e.

L_{smte} : integer number of trucks that should be sent to manufacturing center from supplier in period T in scenario e.

L_{mdte} : integer number of trucks that should be sent to distribution center from manufacturing center in period T in scenario e.

L_{dcte} : integer number of trucks that should be sent to stores from manufacturing center in period T in scenario e.

L_{cite} : integer number of trucks that should be sent to inspection center from store in period T in scenario e.

L_{ikte} : integer number of trucks that should be sent to disposal center from inspection center in period T in scenario e.

L_{ihste} : integer number of trucks that should be sent to thrift store from inspection center in period T in scenario e.

L_{irte} : integer number of trucks that should be sent to recycling center from inspection center in period T in scenario e.

L_{rdte} : integer number of trucks that should be sent to distribution center from recycling center in period T in scenario e.

L_{hkte} : integer number of trucks that should be sent to disposal center from thrift store in period T in scenario e.

L_{hrte} : integer number of trucks that should be sent to recycling center from thrift store in period T in scenario e.

L_{rmte} : integer number of trucks that should be sent to manufacturing center from recycling center in period T in scenario e.

$LH_{pите}$: integer number of reusable product P at inspection center I in period T in scenario e.

$LR_{pите}$: integer number of recyclable or repairable product P at inspection center I in period T in scenario e.

- Continuous variables

$TH_{pите}$: Decimal part of the number of reusable product P at inspection center I in period T in scenario e.

$TR_{pите}$: Decimal part of the number of recyclable or repairable product P at inspection center I in period T in scenario e.

$T_{phте}$: Decimal part of the number of recyclable product P at thrift store H in period T in scenario e.

$T_{prте}$: Decimal part of the number of repairable product P at recycling center R in period T in scenario e.

T_{prate} : Decimal part of the number of recycled raw material A at recycling center R in period T in scenario e.

T_{smte} : Decimal number deducted from the integer number of trucks that should be sent to manufacturing center from supplier in period T in scenario e.

T_{mdte} : Decimal number deducted from the integer number of trucks that should be sent to distribution center from manufacturing center in period T in scenario e.

T_{dcte} : Decimal number deducted from the integer number of trucks that should be sent to stores from manufacturing center in period T in scenario e.

T_{cite} : Decimal number deducted from the integer number of trucks that should be sent to inspection center from store in period T in scenario e.

T_{ikte} : Decimal number deducted from the integer number of trucks that should be sent to disposal center from inspection center in period T in scenario e.

T_{ihcte} : Decimal number deducted from the integer number of trucks that should be sent to thrift store from inspection center in period T in scenario e.

T_{ircte} : Decimal number deducted from the integer number of trucks that should be sent to recycling center from inspection center in period T in scenario e.

T_{rdte} : Decimal number deducted from the integer number of trucks that should be sent to distribution center from recycling center in period T in scenario e.

T_{hkcte} : Decimal number deducted from the integer number of trucks that should be sent to disposal center from thrift store in period T in scenario e.

T_{hrcte} : Decimal number deducted from the integer number of trucks that should be sent to recycling center from thrift store in period T in scenario e.

T_{rmcte} : Decimal number deducted from the integer number of trucks that should be sent to manufacturing center from recycling center in period T in scenario e.

Two-stage stochastic model formulation

1. First-stage costs

Location cost (1)

$$\begin{aligned} & \sum_s F_S O_S + \sum_\mu F_\mu O_\mu + \sum_d F_d O_d + \sum_C F_C O_C + \sum_I F_I O_I + \sum_k F_k O_k \\ & + \sum_h F_h O_h + \sum_r F_r O_r \end{aligned}$$

Fixed cost of transportation (2)

$$\begin{aligned} & \sum_s X_S O_S + \sum_\mu X_\mu O_\mu + \sum_d X_d O_d + \sum_C X_C O_C + \sum_I X_I O_I + \sum_k X_k O_k \\ & + \sum_h X_h O_h + \sum_r X_r O_r \end{aligned}$$

2. Second-stage costs

Transportation cost (3)

$$\begin{aligned} & \sum_e \text{Prob}_e \left(\sum_S \sum_m \sum_t T_{smte} \times \text{Dis}_{SM} \times V_{st} + \sum_M \sum_d \sum_t T_{mdte} \times \text{Dis}_{md} \times V_{mt} \right. \\ & + \sum_D \sum_C \sum_t T_{dcte} \times \text{Dis}_{dc} \times V_{dt} + \sum_C \sum_i \sum_t T_{cite} \times \text{Dis}_{ci} \times V_{ct} \\ & + \sum_i \sum_t V_{it} \times \left(\sum_k T_{ikte} \times \text{Dis}_{ik} + \sum_h T_{ihte} \times \text{Dis}_{ih} + \sum_r T_{irte} \times \text{Dis}_{ir} \right) \\ & + \sum_h \sum_t V_{ht} \times \left(\sum_k T_{hkte} \times \text{Dis}_{hk} + \sum_r T_{hrte} \times \text{Dis}_{hr} \right) \end{aligned}$$

$$+ \sum_r \sum_t V_{rt} \times \left(\sum_d T_{rdte} \times \text{Dis}_{rd} + \sum_m T_{rmte} \times \text{Dis}_{rm} \right)$$

Material cost (4)

$$\sum_e \text{Prob}_e \sum_a \sum_S \sum_m \sum_t C_{as} \times Q_{asmte}$$

Manufacturing cost (5)

$$\sum_e \text{Prob}_e \sum_p \sum_m \sum_d \sum_t C_{mp} \times Q_{pmdte}$$

Distributing cost (6)

$$\sum_e \text{Prob}_e \sum_p \sum_d \sum_c \sum_t C_{dp} \times Q_{pdcte}$$

Inspecting cost (7)

$$\sum_e \text{Prob}_e \sum_p \sum_c \sum_i \sum_t C_{ip} \times Q_{pcite}$$

Disposing cost (8)

$$\sum_e \text{Prob}_e \sum_k C_{kp} \times \sum_p \sum_t \left(\sum_i Q_{pikte} + \sum_h Q_{phkte} \right)$$

Holding cost in stores (9)

$$\sum_e \text{Prob}_e \sum_p \sum_d \sum_c \sum_t C_{cp} \times Q_{pdcte}$$

Holding cost in Thrift stores (10)

$$\sum_e \text{Prob}_e \sum_p \sum_i \sum_h \sum_t C_{hp} \times Q_{pihte}$$

Recycling and repairing cost (11)

$$\sum_e \text{Prob}_e \sum_p \sum_r \sum_t \left(C_{rp} \times \sum_d Q_{prdte} + w_{rp} \times \left(\sum_h Q_{phrte} + \sum_i Q_{pirte} - \sum_d Q_{prdte} \right) \right)$$

Inventory cost (12)

$$\sum_e \text{Prob}_e \sum_p \sum_d \sum_t RE_{pdte} \times H_p$$

Unutilized Capacity cost (13)

$$\begin{aligned} & \sum_e \text{Prob}_e \times \left(\sum_t \left(\sum_m \left(M_m \times \left(\text{Cap}_{mt} \times O_M - \sum_p \sum_d Q_{pmdte} \right) \right) \right) \right) \\ & + \sum_d M_d \times \left(\text{Cap}_{dt} \times O_d - \sum_p \left(\sum_c Q_{pdcte} + RE_{pdte} \right) \right) \\ & + \sum_r M_r \times \left(\text{Cap}_{rt} \times O_r - \sum_p \left(\sum_h Q_{phrte} + \sum_i Q_{pirte} \right) \right) \end{aligned}$$

Buying returned products cost (14)

$$\sum_e \text{Prob}_e \sum_p \sum_c \sum_i \sum_t Q_{pcite} \times C_p$$

- Flow balances

Manufacturing center (15)

$$\left(\sum_s Q_{asmte} + \sum_r Q_{armte} \right) \times \text{ratio}_p \geq F_{ap} \times \sum_d Q_{pmdte}, \forall a, m, t, p, e$$

Distribution centers (16)

$$\sum_r Q_{prdte} + \sum_m Q_{pmdte} + RE_{pd(t-1)e} = \sum_c Q_{pdcte} + RE_{pdte}, \forall t, p, d, e$$

Stores (17)

$$\sum_d Q_{pdcte} = D_{pcte}, \forall p, c, t, e$$

Maximum number of each product sent to inspection center (18)

$$\sum_d Q_{pdcte} \times \alpha_{pt} + Y \times C_p \geq \sum_i Q_{pcite}, \forall p, c, t, e$$

Minimum number of each product sent to inspection center (19)

$$\sum_d Q_{pdcte} \times \alpha_{pt} \leq \sum_i Q_{pcite}, \forall p, c, t, e$$

Maximum number of each product sent to inspection center (20)

$$D_{pcte} \geq \sum_i Q_{pcite}, \forall p, c, t, e$$

Inspection center flow in and out (21)

$$\sum_h Q_{pihte} + \sum_r Q_{pirte} + \sum_k Q_{pikte} = \sum_i Q_{pcite}, \forall p, i, t, e$$

Inspection center to thrift stores (22)

$$\sum_c Q_{pcite} \times \beta_{pt} = TH_{pите} + LH_{pите}, \forall p, i, t, e$$

Integer- products sent to thrift stores (23)

$$\sum_h Q_{pihte} = TH_{pите}, \forall p, i, t, e$$

Inspection center to recycling centers (24)

$$\sum_c Q_{pcite} \times \mu_{pt} = TR_{pите} + LR_{pите}, \forall p, i, t, e$$

Integer- products sent to recycling centers from inspection center (25)

$$\sum_r Q_{pirte} = TR_{pите}, \forall p, i, t, e$$

Thrift stores flow in and out (26)

$$\sum_i Q_{pihte} = \sum_k Q_{phkte} + \sum_r Q_{phrte}, \forall p, h, t, e$$

Thrift store to recycling centers (27)

$$\sum_i Q_{pihte} \times G_{pt} = T_{phте} + L_{phте}, \forall p, h, t, e$$

Integer- products sent to recycling centers from thrift store (28)

$$\sum_r Q_{phrte} = T_{phте}, \forall p, h, t, e$$

Repairing at recycling center (29)

$$\left(\sum_h Q_{phrte} + \sum_i Q_{pirte} \right) \times n_{pt} = T_{prte} + L_{prte}, \forall p, r, t, e$$

Repaired products sent to distribution centers (30)

$$\sum_d Q_{prdte} = T_{prte}, \forall p, r, t, e$$

Recycling at recycling center (31)

$$\left(\sum_h Q_{phrte} + \sum_i Q_{pirte} - \sum_d Q_{prdte} \right) \times F_{ap} = T_{prate} + L_{prate}, \forall p, r, a, t, e$$

Recycled material sent to manufacturing centers (32)

$$\sum_m Q_{armte} = T_{prate}, \forall p, r, a, t, e$$

The decimal part of the number of reusable products at inspection center (33)

$$LH_{pite} < 1, \forall p, i, t, e$$

The decimal part of the number of recyclable products at inspection center (34)

$$LR_{pite} < 1, \forall p, i, t, e$$

The decimal part of the number of recyclable products at thrift store (35)

$$L_{phte} < 1, \forall p, h, t, e$$

The decimal part of the number of repairable products at recycling center (36)

$$L_{prte} < 1, \forall p, r, t, e$$

The decimal part of the number of recycled raw material (37)

$$L_{prate} < 1, \forall p, r, a, t, e$$

- Capacity constraints

Supplier capacity (38)

$$\sum_a \sum_m Q_{asmte} \leq Cap_{st} \times Os, \forall s, t, e$$

Manufacturing center capacity (39)

$$\sum_p \sum_d Q_{pmdte} \leq Cap_{mt} \times Om, \forall m, t, e$$

Distribution center capacity (40)

$$\sum_p \left(\sum_c Q_{pdcte} + RE_{pdte} \right) \leq Cap_{dt} \times Od, \forall d, t, e$$

Store capacity (41)

$$\sum_p \sum_d Q_{pdcte} \leq Cap_{ct} \times Oc, \forall c, t, e$$

Inspection center capacity (42)

$$\sum_p \sum_c Q_{pcite} \leq Cap_{it} \times Oi, \forall i, t, e$$

Disposal center capacity (43)

$$\sum_p \left(\sum_i Q_{pikte} + \sum_h Q_{phkte} \right) \leq Cap_{kt} \times Ok, \forall k, t, e$$

Thrift store capacity (44)

$$\sum_p \sum_i Q_{pihte} \leq Cap_{ht} \times Oh, \forall h, t, e$$

Recycling center capacity (45)

$$\sum_p \left(\sum_h Q_{phrte} + \sum_i Q_{pirte} \right) \leq Cap_{rt} \times Or, \forall r, t, e$$

Virgin raw material (46)

$$W \times \left(\sum_s Q_{asmte} + \sum_r Q_{armte} \right) \leq \sum_s Q_{asmte}, \forall a, m, t, e$$

- Number of trucks sent to centers

Supplier to manufacturing center trucks (47)

$$\sum_a \frac{Q_{asmte}}{B_{st}} = T_{smte} - L_{smte}, \forall s, m, t, e$$

Manufacturing center to distribution center trucks (48)

$$\sum_p \frac{Q_{pmdte}}{B_{mt}} = T_{mdte} - L_{mdte}, \forall m, d, t, e$$

Distribution center to store trucks (49)

$$\sum_p \frac{Q_{pdcte}}{B_{dt}} = T_{dcte} - L_{dcte}, \forall d, c, t, e$$

Store to inspection center trucks (50)

$$\sum_p \frac{Q_{pcite}}{B_{ct}} = T_{cite} - L_{cite}, \forall c, i, t, e$$

Inspection center to disposal center trucks (51)

$$\sum_p \frac{Q_{pikte}}{B_{it}} = T_{ikte} - L_{ikte}, \forall i, k, t, e$$

Inspection center to thrift store trucks (52)

$$\sum_p \frac{Q_{pihte}}{B_{it}} = T_{ihite} - L_{ihite}, \forall i, h, t, e$$

Inspection center to recycling center trucks (53)

$$\sum_p \frac{Q_{pirte}}{B_{it}} = T_{irte} - L_{irte}, \forall i, r, t, e$$

Thrift store to disposal center trucks (54)

$$\sum_p \frac{Q_{phkte}}{B_{ht}} = T_{hkte} - L_{hkte}, \forall h, k, t, e$$

Thrift store to recycling center trucks (55)

$$\sum_p \frac{Q_{phrte}}{B_{ht}} = T_{hrte} - L_{hrte}, \forall h, r, t, e$$

Recycling center to distribution center trucks (56)

$$\sum_p \frac{Q_{prdte}}{B_{rt}} = T_{rdte} - L_{rdte}, \forall r, d, t, e$$

Recycling center to manufacturing center trucks (57)

$$\sum_a \frac{Q_{armte}}{B_{rt}} = T_{rmte} - L_{rmte}, \forall r, m, t, e$$

- Decimal part of number of trucks

Decimal part of the number of trucks sent from supplier to manufacturing center (58)

$$L_{smte} < 1, \forall s, m, t, e$$

Decimal part of the number of trucks sent from Manufacturing center to distribution center (59)

$$L_{mdte} < 1, \forall m, d, t, e$$

Decimal part of the number of trucks sent from Distribution center to store (60)

$$L_{dcte} < 1, \forall d, c, t, e$$

Decimal part of the number of trucks sent from Store to inspection center (61)

$$L_{cite} < 1, \forall c, i, t, e$$

Decimal part of the number of trucks sent from Inspection center to disposal center (62)

$$L_{ikte} < 1, \forall i, k, t, e$$

Decimal part of the number of trucks sent from Inspection center to thrift store (63)

$$L_{ihte} < 1, \forall i, h, t, e$$

Decimal part of the number of trucks sent from Inspection center to recycling center (64)

$$L_{irte} < 1, \forall i, r, t, e$$

Decimal part of the number of trucks sent from Thrift store to disposal center (65)

$$L_{hkte} < 1, \forall h, k, t, e$$

Decimal part of the number of trucks sent from Thrift store to recycling center (66)

$$L_{hrte} < 1, \forall h, r, t, e$$

Decimal part of the number of trucks sent from Recycling center to distribution center (67)

$$L_{rdte} < 1, \forall r, d, t, e$$

Decimal part of the number of trucks sent from recycling center to manufacturing center (68)

$$L_{rmte} < 1, \forall r, m, t, e$$

- Number of centers

Suppliers (69)

$$\sum_S O_S \leq L_s$$

Manufacturers (70)

$$\sum_m O_m \leq L_m$$

Distribution centers (71)

$$\sum_d O_d \leq L_d$$

Stores (72)

$$\sum_c O_c \leq L_c$$

Inspection centers (73)

$$\sum_i O_i \leq L_i$$

Disposal centers (74)

$$\sum_k O_k \leq L_k$$

Thrift stores (75)

$$\sum_h O_h \leq L_h$$

Recycling centers (76)

$$\sum_r O_r \leq L_r$$

Binary variables (77)

$$O_S, O_m, O_d, O_c, O_i, O_k, O_h, O_r \in \{0, 1\}, \forall s, m, d, c, i, k, h, r$$

Integer variables (78)

$$Q_{asmte}, Q_{pmdte}, Q_{pdcte}, Q_{pcite}, Q_{pikte}, Q_{pihte}, Q_{pirte}, Q_{phkte}, Q_{phrte}, Q_{prdte}, Q_{armte},$$

$$RE_{pdte}, TH_{pите}, TR_{pите}, T_{phte}, T_{prte}, T_{prate}, T_{smte}, T_{mdte}, T_{dcte}, T_{cite}, T_{ikte}, T_{ihте}, T_{irte},$$

$$T_{hkте}, T_{hrte}, T_{rdte}, T_{rmte} \in \mathbb{Z}$$

Non-negative variables (79)

$$Q_{asmte}, Q_{pmdte}, Q_{pdcte}, Q_{pcite}, Q_{pikte}, Q_{pihte}, Q_{pirte}, Q_{phkte}, Q_{phrte}, Q_{prdte}, Q_{armte},$$

$$RE_{pdte}, LH_{pите}, LR_{pите}, L_{phte}, L_{prte}, L_{prate}, L_{smte}, L_{mdte}, L_{dcte}, L_{cite}, L_{ikte}, L_{ihте}, L_{irte},$$

$$L_{hkте}, L_{hrte}, L_{rdte}, L_{rmte}, TH_{pите}, TR_{pите}, T_{phte}, T_{prte}, T_{prate}, T_{smte}, T_{mdte}, T_{dcte},$$

$$T_{cite}, T_{ikte}, T_{ihте}, T_{irte}, T_{hkте}, T_{hrte}, T_{rdte}, T_{rmte} \geq 0$$

Two-stage stochastic model description

In the objective function, formulas 1 and 2 show first-stage costs. The costs of selecting and equipping centers and the costs of buying trucks and implementing transportation systems for these centers. First-stage variables should be planned before knowing the demand. So, the model chooses which locations to equip in order to support various scenarios of demand, and then through formulas 1 and 2, the costs associated with first-stage variables can be calculated. After fixating first-stage decision variables, which are the decisions on which locations to equip, the model can find second-stage variables, which are variables specifying how many products to process in these centers, how many products to send to another center, and how much inventory to keep at the distribution center for all scenarios of demand.

Formulas 3 to 14 demonstrate second-stage costs. Second-stage costs calculate the expected value of material cost, variable transportation cost, manufacturing cost, distribution cost, inspection cost, disposal cost, holding cost in stores, holding cost in thrift stores, recycling cost, inventory cost, unutilized capacity cost, and the cost of buying returned products under all demand scenarios. The second-stage decision variables, like the quantity of materials that should be processed to meet demand with minimal cost, vary for each scenario. The expected value of second-stage costs is calculated by multiplying the probability of demand scenario 'e' occurring by the second-stage costs for each scenario and then adding them all together.

In the constraint section, constraints 69 to 76 are first-stage constraints. These constraints define the maximum number of locations that the company is willing to equip. Constraints 38 to 45 show the influence of the first-stage decisions on the second-stage decisions. Based on the first-stage decisions regarding which locations to equip, these constraints determine the number of resources available to the company to satisfy its demand.

The remaining constraints are second-stage constraints which control the impact of second-stage decisions on the results based on different demand scenarios. The difference between stochastic problem constraints and deterministic constraints is that in the stochastic problem, all constraints should be true for every demand scenario generated.

Sample average approximation method

Sample Average Approximation (SAA) is a method in stochastic optimization that approximates the expected value of the objective function through sampling, converting a stochastic problem

into more manageable deterministic problems. It is particularly useful for large-scale optimization problems under uncertainty, involving generating numerous samples from the probability distribution of the random variables (Birge & Louveaux, 1997). For each set of samples, a deterministic equivalent problem is formulated. This problem approximates the original stochastic problem by replacing the expectation in the objective function with the sample average. Mathematically, the SAA problem can be expressed as minimizing the average of the objective function over the generated samples. This transforms the original stochastic optimization problem into a deterministic one that can be solved using traditional optimization techniques, such as linear programming or mixed-integer programming. The solution obtained provides an approximation of the optimal solution to the original stochastic problem.

The process involves solving the SAA problem for multiple sets of samples to ensure robustness. The quality of the solution is evaluated based on the sample average objective values and their variance. If necessary, the sample size is increased to improve the accuracy of the approximation (Birge & Louveaux, 1997).

Figure18 - Algorithm of SAA method

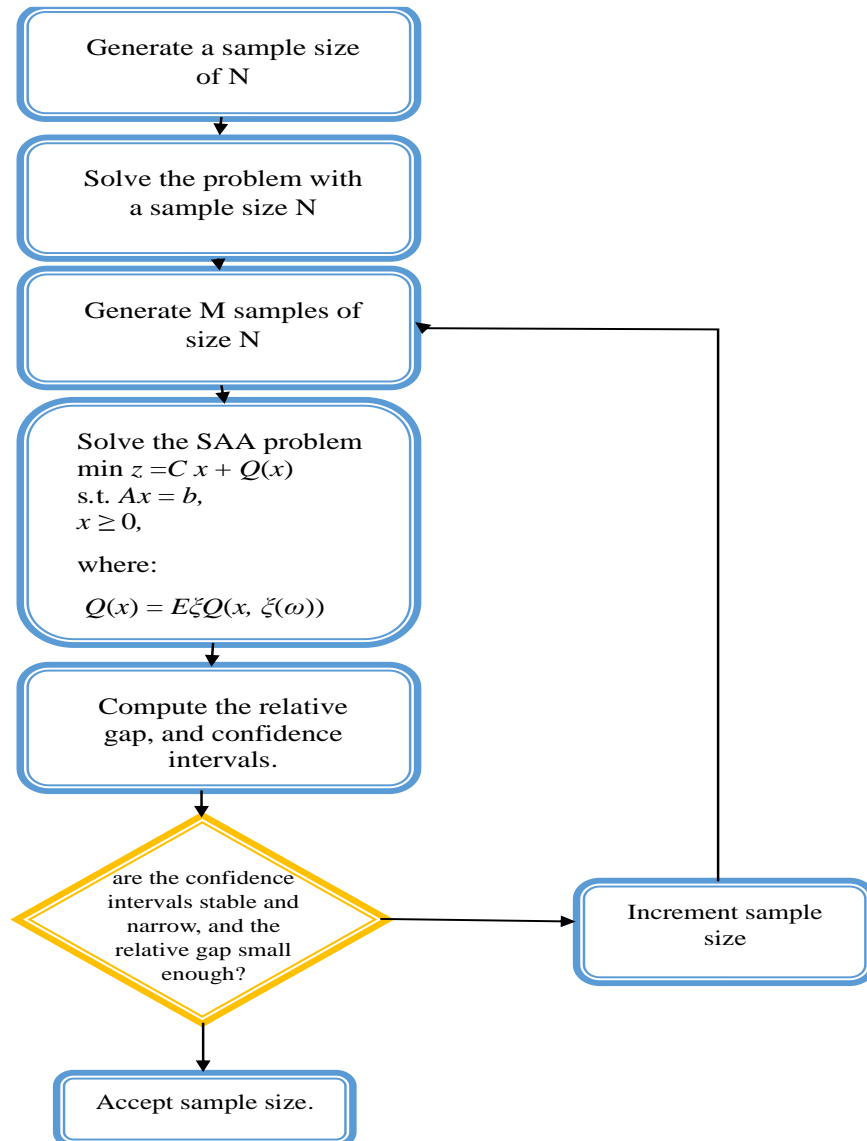


Figure 18 illustrates the algorithm for the Sample Average Approximation (SAA) method. The initial step involves generating a sample size of N using the Monte Carlo Simulation method. This sample represents the stochastic variables, allowing the problem to be treated as a deterministic model .

In this context, the stochastic variable is the demand for each product at each store during each period. By employing Monte Carlo simulation, a sample is generated from a normal distribution with a mean of 740 and a standard deviation of 100 .

Next, this deterministic equivalent of the problem is solved, using the generated sample to approximate the original stochastic model.

Subsequently, I assess if the solution is close to optimality by analyzing the optimality gap, upper bound and lower bound, and confidence intervals. The best possible solution found by the solver, the solution when the constraints are relaxed, and the mixed integer programming model's solution will be recorded to calculate the relative gap between solutions. In other words, the difference in objective values between a candidate solution and an ideal solution is known as the optimality gap. The smaller the optimality gap, the closer the solution is to optimality (Bagaram & Tóth, 2020).

Upper and lower bounds are estimates that enclose the true optimal value of the objective function. The upper bound represents an estimate that the true optimal value will not exceed, while the lower bound indicates an estimate that the true optimal value will not fall below. These bounds are crucial in assessing the quality of approximate solutions. They provide a measure of how far the current solution is from the true optimal solution, which helps in determining whether further optimization is necessary.

Confidence intervals in the context of optimization provide a range within which the true value of the objective function is expected to lie with a certain probability. For example, a 95% confidence interval suggests that there is a 95% probability that the true objective value falls within the specified range. These intervals are important for understanding the reliability and robustness of the obtained solutions. They help in assessing the stability of the solution under uncertainty and provide insights into the variability of the outcomes (Mak al, 1999).

If the confidence intervals are stable, narrow, and show convergence in results, and the relative gap is small enough, it shows that the sample size is acceptable to give an accurate result that considers uncertainty effectively; Otherwise, the number of samples should be increased. When there are multiple samples, the solver will show results on which locations to facilitate in which all the scenarios of demand can be satisfied while minimizing the average cost through all the scenarios.

The Sample Average Approximation (SAA) method can yield insights into the average costs associated with managing the supply chain. Additionally, it provides the company with information

on the optimal first-stage decision variables, such as which locations to equip in order to meet demand, even in the presence of uncertainty.

Stochastic model results

Table 65-Sample average approximation algorithm results with different number of samples

Number of samples	Best possible solution	MIP solution	Absolute gap	Relative gap	CPU time (seconds)
1	95385676.858	105673353.616	10287676.758	0.097	6.06
2	96264045.334	106297178.935	10033133.601	0.094	50.38
3	95382534.500	98555622.400	3173087.900	0.032	223.11
4	96116734.724	98119173.808	2002439.083	0.020	62.52
5	94722124.060	104589686.538	9867562.478	0.094	107.05
6	95432256.248	104606412.448	9174156.201	0.088	23.28
7	100620335.312	105875246.007	5254910.694	0.050	625.64
8	102845491.095	107364048.716	4518557.622	0.042	2326.14
9	101040710.321	107503309.279	6462598.958	0.060	2467.25
10	103536098.668	108789116.852	5253018.184	0.048	688.03
12	102409137.556	113266007.181	10856869.625	0.096	1549.22

Table 65 displays the average cost of the supply chain under different number of demand scenarios. The results in the table have been obtained with the CPLEX solver. The mixed integer programming (MIP) solution shows the budget that the supply chain should be prepared for in order to cover all costs. Additionally, the table includes the best possible solution, representing the costs under relaxed constraints. The absolute gap is calculated as the difference between the MIP

solution and the best possible solution, while the relative gap is the ratio of the absolute gap to the MIP solution.

Table 66- Results of sample average approximation algorithm with 95% confidence intervals

Number of samples	Lower bound	Upper bound	Margin of error
1	85509507.170	125837200.061	20163846.445
2	92391965.196	120202392.674	13905213.739
3	94964935.419	102146309.380	3590686.980
4	96156783.506	100081564.110	1962390.302
5	95940386.673	113238986.403	8649299.865
6	97265558.621	111947266.276	7340853.827
7	101982353.686	109768138.327	3892892.320
8	104232848.036	110495249.397	3131200.681
9	103281077.960	111725540.598	4222231.319
10	105533262.450	112044971.254	3255854.402
12	107123154.912	119408859.450	6142852.269

Table 66 provides the upper bound, lower bound, and margin of error based on 95% confidence intervals. These results demonstrate that there is a 95% probability that the costs, across various scenarios, will remain within the specified lower and upper bounds.

Figure19 -MIP solutions and the 95% confidence intervals

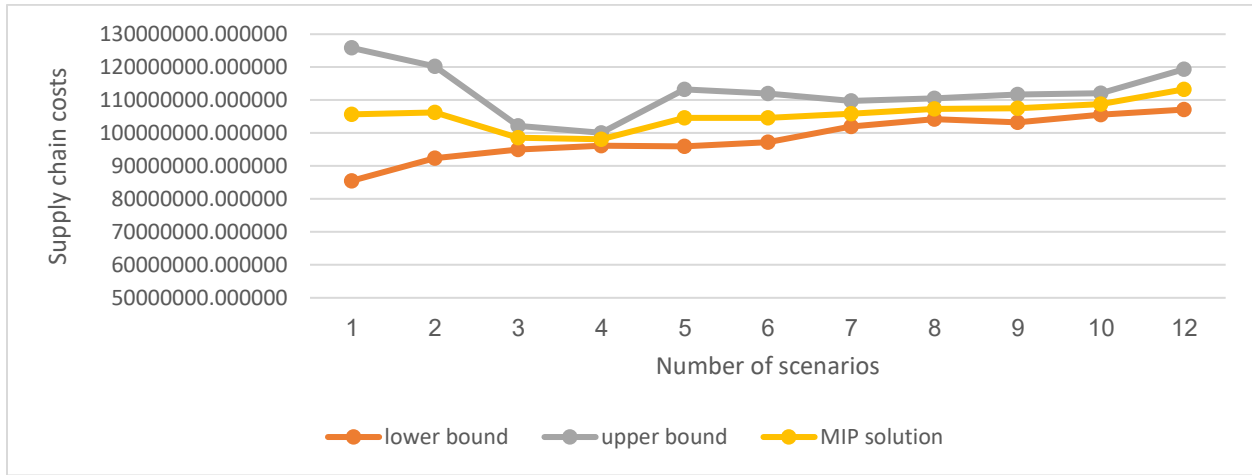


Figure 19 presents the mixed integer programming results across different scenarios and their corresponding confidence intervals. The gray and red lines represent the 95% confidence intervals, indicating the range within which the results are expected to fall. Initially, the intervals are volatile with sample sizes ranging from 1 to 7. However, as the sample size reaches 7 or more, the results show convergence.

To determine the most optimal solution under various scenarios, it is essential to select a sample size that strikes a balance between computational time and solution quality. The literature shows that 1 to 5% relative gap can be acceptable to ensure the results are reliable (Seyfi et al, 2023). Among sample sizes greater than 7, using 8 samples yields the smallest relative optimality gap of 4.2%. The decision to stop at 12 samples was due to increasing computational time, with a maximum CPU time set at 18,000 seconds, as running 13 or more samples would have exceeded this limit and the resources available couldn't handle longer computational time. Therefore, using 8 demand scenarios and applying the Sample Average Approximation method provides an efficient solution with a relative gap below 5%, allowing the company to plan its budget more precisely in advance.

Comparison of stochastic and deterministic model

Table 67-Comparison between stochastic and deterministic problem results

Problem type	MIP solution
Deterministic	105673353.6
Stochastic	107364048.7

Table 67 presents the total cost associated with managing the supply chain for both deterministic and stochastic models. The results indicate that, without accounting for uncertainty, the proposed model suggests a budget of \$105,673,353.60 to meet all demands at minimum cost. However, when demand uncertainty is considered, the required budget increases to \$107,364,048.70. This represents a 1.6% increase over the deterministic model, highlighting the financial risk of not considering demand uncertainty. Consequently, failure to incorporate stochastic parameters could result in unforeseen demand, costs, and reduced profitability. Therefore, it is crucial for companies to account for uncertainties when developing strategies such as pricing.

Table 68- Suppliers and manufacturing centers that should be equipped with different number of demand samples

No.Samples	Suppliers			Manufacturing centers			
	1	2	3	1	2	3	4
1							
2							
3							
4							
5							
6							

7							
8							
9							
10							
12							

Table 69-Distribution centers and stores that should be equipped with different number of demand samples

No.Samples	Distribution centers			Stores		
	1	2	3	1	2	3
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						
12						

Table 70-Inspection centers and disposal centers that should be equipped with different number of demand samples

No.Samples	Inspection centers			Disposal centers		
	1	2	3	1	2	3
1	■				■	
2	■		■	■		
3	■			■		
4	■				■	
5	■				■	
6	■				■	
7	■		■		■	
8	■	■			■	
9	■		■			■
10		■	■		■	
12	■		■		■	

Table 71-Thrift stores and recycling centers that should be equipped with different number of demand samples

No.Samples	Thrift stores			Recycling centers	
	1	2	3	1	2
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
12					

The results of the stochastic model, determined by various demand scenarios, indicate which locations to equip in order to meet the demand at minimum cost. A single sample represents the deterministic model while increasing the number of samples alters the optimal locations to equip. The blue cells in the tables above highlight the recommended locations. The most accurate results are obtained with eight samples, showing that the supply chain should equip suppliers 2 and 3, manufacturing centers 3 and 4, distribution center 2, stores 1, 2, and 3, inspection centers 1 and 2, disposal center 2, thrift store 3, and recycling center 2. The tables also provide a comparative analysis of location recommendations based on different number of demand scenarios.

The difference between the optimal stochastic model with eight samples and the deterministic model lies in the recommendations for inspection centers. The deterministic model suggests equipping only inspection center 1, whereas the stochastic model advises equipping both inspection centers 1 and 2.

This indicates that relying solely on the deterministic model and neglecting demand uncertainty could result in insufficient capacity at the inspection centers. Demand uncertainty refers to the variability in demand, where it can deviate from expected values either positively or negatively. In a deterministic model, this variability is not accounted for, potentially leading to insufficient capacity at inspection centers. Consequently, if actual demand exceeds the expected levels, even within its probability distribution, the inspection centers may not be able to meet this demand, resulting in unmet needs. Consequently, it is crucial for the company to consider stochastic modeling to ensure adequate processing capacity and decrease potential risks associated with demand variability.

Chapter 6: Conclusion

In this thesis, I presented a comprehensive approach to optimizing costs within a closed-loop fashion supply chain. The deterministic model developed addresses various costs, including location equipping, material, transportation, manufacturing, distribution, inspection, disposal, holding costs in stores and thrift stores, recycling, inventory, unutilized capacity, and the cost of buying returned products. This model demonstrates that fashion supply chains can achieve sustainability while remaining profitable.

Using the deterministic model, companies can formulate long-term strategies, such as determining which locations to equip, and make short-term decisions regarding the number of items to process at each center and the quantities to transport between centers, all aimed at minimizing total costs and satisfying demand. This model enables fashion businesses to manage their costs efficiently across multiple products and different sales periods. The accuracy of this model was validated using the CPLEX solver in GAMS software.

I also introduced a two-stage stochastic mixed integer programming model to manage costs and make strategic decisions under demand uncertainty. This model helps closed-loop fashion supply chains determine which locations to equip, the number of products to process at each center in each period, the inventory levels required for future demand, and the quantities to transport between centers. The stochastic model was solved using the sample average approximation (SAA) method, demonstrating its effectiveness in accounting for demand uncertainty.

The results indicate that the financial risk associated with the deterministic model is higher compared to the stochastic model, as the stochastic model proposed a 1.6% higher budget to cover costs. Additionally, the inclusion of demand uncertainty affects the recommended locations to equip.

In conclusion, the two-stage stochastic mixed integer programming model proposed in this paper provides an efficient framework for closed-loop fashion supply chains to implement strategies that minimize total costs and meet demand in a timely manner. This approach not only ensures profitability but also supports the sustainability of fashion supply chains.

Contributions

The innovative aspect of our research is encapsulated in the formulation of a comprehensive closed-loop supply chain network design, specifically tailored for the fashion industry. This model distinguishes itself through its multi-period, multi-product framework.

Contrary to the predominant focus on forward supply chains within existing fashion industry literature as shown in table 1, our study pioneers in constructing and solving a mathematical model dedicated to elucidating such a nuanced network within this sector. Also, no prior research has applied the Sample Average Approximation (SAA) method for solving the model for the fashion industry. This not only marks a departure from traditional supply chain analyses but also introduces a novel approach to addressing the complexities inherent in the fashion industry's supply chain dynamics.

The Sample Average Approximation (SAA) method is particularly well-suited for two-stage stochastic models in the fashion industry due to its ability to effectively handle the high variability and rapid changes in consumer demand and trends. By generating numerous scenarios based on probabilistic models, SAA provides a reliable approximation of the expected costs and decisions, ensuring that the solutions are robust against uncertainty. Additionally, its capacity for parallelization and handling large-scale problems makes it computationally efficient, allowing fashion companies to make timely and informed decisions in a dynamic market environment.

Fashion supply chains are unique compared to other product supply chains due to the rapid changes in trends and the potential 'spoilage' of goods as seasons and consumer preferences shift. Unlike other industries, where products may have a longer shelf life, fashion items quickly become obsolete if not sold within a particular season. Therefore, the ability to meet the demand accurately is critical in this industry to avoid overproduction and markdowns, ensuring that businesses remain competitive and profitable.

This work expands the literature by including a reverse supply chain, highlighting the environmental benefits of closed-loop versus linear supply chains. The closed-loop system significantly reduces waste and resource consumption, promoting sustainability. The importance of reverse logistics in the fashion industry is also underscored, particularly for managing returns, which is a growing concern as consumers become more conscious of sustainable practices.

The contributions of this thesis are valuable to multiple audiences. For researchers, this work proves the feasibility of developing and solving such models, potentially inspiring further studies in this domain. For managers, the models provide practical tools to make informed, sustainable decisions and implement closed-loop supply chains effectively. This dual utility underscores the broader impact of this research on both academic and industry practices.

In conclusion, the two-stage stochastic mixed integer programming model proposed in this thesis provides an efficient framework for closed-loop fashion supply chains to implement strategies that minimize total costs and meet demand in a timely manner. This approach not only ensures profitability but also supports the sustainability of fashion supply chains.

Limitations and future studies

The model proposed in this thesis does not account for shortage costs, future research could integrate shortage costs into the model to provide a more comprehensive analysis of the total costs associated with managing a closed-loop fashion supply chain. Incorporating shortage costs would offer deeper insights into the trade-offs between inventory holding and potential stockouts, thereby enhancing the model's practical applicability.

For future studies, it is recommended to utilize empirical data to develop more cohesive datasets, which can lead to more accurate and practical results. Additionally, higher computational resources would enable larger computational tasks, allowing for more centers and products to be processed, which was limited in this study due to resource constraints.

Additionally, the current model was solved using the sample average approximation (SAA) method. Future studies could explore the application of other stochastic optimization techniques, such as genetic algorithms, robust optimization, and fuzzy logic. Comparing the results obtained from these methods with those from the SAA method could help identify which approach yields more accurate results with less computational time. This comparative analysis would be valuable for practitioners seeking efficient and effective optimization strategies under uncertainty.

Furthermore, future research could expand the scope of the model to incorporate sustainability metrics beyond cost minimization, such as carbon footprint, waste reduction, and employee salaries. By integrating environmental impact assessments, the model could support more holistic decision-making processes, aligning with the growing emphasis on sustainable practices in the

fashion industry. This extension would not only enhance the model's relevance but also contribute to the development of greener and more responsible supply chain strategies.

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