TESTING FOR STATIC RESOURCE AND PRODUCT EQUILIBRIUM UNDER OUTPUT PRICE RISK: WESTERN CANADIAN AGRICULTURE, 1961-84

by

GABRIEL TOICHOA BUAHA

A Thesis
Submitted to the Faculty of Graduate Studies
in Partial Fulfilment of the Requirements
for the Degree of

MASTER OF SCIENCE

Department of Agricultural Economics and Farm Management
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ABSTRACT

This thesis provides econometric tests of the hypothesis of static competitive farm resource and output equilibrium in Western Canadian agriculture over the period 1961-84. These tests are conducted with and without risk aversion and price uncertainty. The theoretical models assumed for this sector included a cost function approach, an indirect utility framework, and a stochastic profit function model. Two different tests are conducted for the proposed hypotheses: (1) Wald chi-square tests of the symmetry restrictions implied by cost minimization and/or utility maximization and/or profit maximization and (2) Wald chi-square, Hauseman specification, and likelihood ratio tests of the first order conditions for static equilibrium for quasi-fixed inputs and outputs.

The symmetry restrictions implied by cost minimization are not rejected given a short run Translog cost function for Western Canadian agriculture. Given these restrictions, static competitive equilibrium is not rejected for dairy, poultry, farm produced capital, and farm land. For farm machinery the outcome of this hypothesis depends on the econometric test conducted. For crop and livestock outputs, similar hypothesis tests are inconclusive due to the significant impact of output price risk and uncertainty on the results of these tests. Overall, static farm resource and product equilibrium is rejected for the whole agricultural sector of Western Canada over the period 1961-84.

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Chapter 1. Introduction

1.1 Statement of problem

A question that has seldom been asked is that given the structure of Canadian agriculture, are resources employed and outputs produced in this sector in static competitive equilibrium? Due to the cyclical nature of agriculture, it is argued that farm resources are seldom in equilibrium since producers are unable to anticipate product prices. The high costs of adjusting quasi-fixed farm inputs also contribute to prolonged periods of farm resource disequilibrium. In the Canadian context, a primary objective in the price pooling system of the Canadian Wheat Board (CWB) is to reduce the impact in price fluctuations on producers' optimizing decisions. A second empirical question that has barely been investigated in Canadian agriculture is whether risk aversion and price uncertainty affect resource and product equilibrium. This study provides tests for the questions raised above.

1.2 Objectives and hypotheses of the study

The objective of this research is to test the following hypotheses given the characteristics of the Canadian agricultural sector over the period 1961-84:

(1.a) were Western Canadian farmers cost minimizers and/or profit maximizers?;

(1.b) were farm resources and outputs in static competitive equilibrium in Western Canadian agriculture? And did farmers' response to price risk and uncertainty affect these conditions?

To evaluate the above hypotheses three different tests are conducted: (1) Wald chi-square tests of the symmetry restrictions implied by static cost minimization and/or profit maximization; (2) Wald chi-square, Hauseman specification, and Likelihood ratio tests of first order conditions for static equilibrium for quasi-fixed inputs and/or outputs under risk neutrality; (3) Wald chi-square test of first order conditions under a simple model of risk aversion and output price uncertainty. These hypothesis tests are conducted under a cost function approach, an indirect utility framework, and a deterministic profit function approach.

1.3 Importance of this research

One of the intended goals of most agricultural policies is to increase the efficient allocation of resources among different enterprises. Analyzing the impacts of risk aversion and output price uncertainty on farm resource and output equilibrium is of upmost importance for the Canadian agricultural sector due to their potential impact on resource allocation. This thesis provides tests of the hypotheses of farm resource and product equilibrium in Western Canadian agriculture.

This study departs from previous research in that here, the hypotheses of static resource and output equilibrium are conducted under a technology that allows for risk aversion and output price uncertainty and incorporates products where quantities are exogenously determined (supply managed products).

1.4 Literature review

The dual profit function has been used to estimate the characteristics (e.g., substitution and expansion effects) of the underlying production technology of an industry. Earlier applications of the profit function approach to agriculture, Lau (1972); Lau and Yotopoulos (1972); and Yotopoulos, Lau, and Lin (1976) have assumed a Cobb-Douglas functional form which is more restrictive than other available forms.

Recent studies have significantly improved on the choice of flexible functional forms. However, a large number of these implicitly assume the equality between shadow and market prices of quasi-fixed factors, Lopez (1980, 1984); Antle (1984); and Dupont (1991). This assumption has led to the estimation of the derived demand, supply, and shadow price equations jointly without any attempt to individually test for the long-run static equilibrium conditions for each quasi-fixed factor. Other studies have assumed long-run static equilibrium that is, all inputs are variable, and proceeded to estimate a multi-product profit function, McKay, Lawrence, and Vlastuin (1983). The down side of this model is the high probability of misspecifying the actual behaviour of producers, i.e., agriculture is unlikely to exhibit long-run equilibrium.

Dynamic equilibrium models have also been employed to model the behaviour of producers, Lopez (1985). In this case, the costs of adjustment are explicitly incorporated in the model and the firm is assumed to be in dynamic rather than static equilibrium. An intertemporal profit maximization or cost minimization problem is solved for equations of motion for the fixed factors. These dynamic equilibrium models have several short-comings. They are empirically difficult to apply and have demanding data requirements, Squires (1987). Furthermore, these models also run the risk of misspecifying the equations of motion for the fixed factors. The restricted profit function framework is an alternative which sidesteps these difficulties but does not provide any explanation on the way fixed factors adjust from one equilibrium point to another.

Few studies have rigorously tested for the validity of static competitive profit-maximising models in the agricultural sector. Junankar (1980a, 1980b) tested for these restrictions but used a nonflexible flexible functional form. Kulatilaka (1985) and, Schankerman and Nadiri (1986) provide a rigorous framework for testing the validity of static equilibrium models and rates of return to fixed factors under a model of short-run cost minimization.[1] They do not specifically test for the restrictions implied by this behavioral assumption. A broader set of tests for static equilibrium and profit maximization in agriculture are conducted by Coyle (1991). This thesis expands upon these tests by allowing for risk aversion and price uncertainty and incorporating restricted outputs.

1.5 Organization of the Study

This thesis consists of five chapters. Chapter 1 has discussed the problem of static resource equilibrium and profit maximization given supply managed policies and price uncertainty in Western Canadian agriculture. This chapter also has described the objectives of the study, the hypotheses to be tested, and the importance of this research. Chapter 2 will provide the theoretical background of the study. This chapter will include three different theoretical models: cost function, indirect utility function, and nonstochastic profit function. Chapter 3 will provide a brief discussion of the data set employed in the study. Chapter 4 will apply the theoretical models described in Chapter II to the

[1] In this model, producers are postulated to minimize costs by choosing their levels of variable factors conditional not only on the level of output, but also on the level of fixed inputs (i.e., factors not easily adjusted from one period to the next.)

econometric study of production decisions in Western Canadian agriculture. Chapter 5 will provide a summary and overall conclusions and limitations of the study. The final data set used in this thesis is provided in the appendix.

Chapter 2. Theoretical Background

The advantages of the duality approach to modelling production decisions are well known (e.g. Fuss and MacFadden, Chambers 1988). Duality allows for the direct specification of the cost or profit functions and by simple differentiation of these with respect to input and/or output prices, factor demand and product supply equations can be derived. Testing for the conditions of static profit maximization is important particularly when a dual approach is chosen. If these conditions are not satisfied but imposed, econometric estimates of the dual function (e.g., profit, cost) or derived equations (e.g., supply, demand) can substantially misspecify the actual behaviour of the agents and should not be used for policy recommendations or other purposes.

To evaluate the hypotheses proposed in Chapter 1, a production process where some of the output prices are assumed stochastic is described next. A dual cost function is considered first, followed by an indirect utility approach. This Chapter concludes with a nonstochastic profit function approach.

2.1 A Short-Run Cost Function Approach

The agricultural industry is characterised by multioutput firms. Suppose that the outputs of such a firm can be partitioned in two groups: restricted and nonrestricted. Now, consider the production decision of this firm which produces M nonrestricted outputs by combining N variable inputs given R quasi-fixed factors and L output restricted products.

Assuming producers are price takers in the factor markets, the variable cost function for a multiproduct firm can be represented as

$$c(w,k,y,q) = \min_{x} \sum_{i=1}^{N} w^i x^i : x \in T(k,y,q) \tag{2.1}$$

where y is an M-dimensional vector of nonrestricted outputs; x is an N-dimensional vector of variable inputs; k is an R-dimensional vector of quasi-fixed factors; q is an L-dimensional quantity vector of supply managed commodities; w is an N-dimensional vector of strictly positive input prices; $T(k,y,q)$ is the set of feasible x that combined with k can produce y,q. To ensure correspondence with a production possibility set or transformation function it is sufficient for the cost function $c(w,k,y,q)$ to be:

(2.1a) non-negative real valued for positive (w);

(2.1b) homogeneous of degree one in (w);

(2.1c) concave and continuous in (w);

(2.1d) non-decreasing in w;

(2.1e) differentiable with respect to all its arguments.

Assuming a constant returns to scale (CRTS) production function for the industry, then the cost function in (2.1) is linearly homogeneous in (k,y,q), i.e., $\lambda c(w,k,y,q) = c(w,\lambda k,\lambda y,\lambda q)$. The variable cost function embodies sufficient information to completely describe the production technology and thus the production possibility set if the restrictions defined in equations (2.1a)-(2.1e) hold. Differentiating the variable cost function (2.1) with respect to variable input prices gives

$$\partial c(w,k,y,q)/\partial w^i = x^i(w,k,y,q) \qquad i=1,\ldots,N \tag{2.2}$$

(Shephard's Lemma), where $x^i(w,k,y,q)$ are the optimum levels of input demand. From properties (2.1a)-(2.1e) of the cost function (2.1), it follows that the derived conditional demand equations (2.2) are homogeneous of degree zero in (w) and the Hessian matrix of second derivatives $c_{w^i w^j}$ of the cost function (2.1) is symmetric negative semidefinite (nsd).[2] It can be shown that these conditions (i.e., homogeneity of degree zero and nsd matrix) represent all the local properties that are imposed on the factor demand equations (2.2) by the hypothesis of competitive cost minimization. If the behavioral model (2.1) describes the actual decision process of the multiproduct firm, then the following symmetry or reciprocity conditions

$$\partial x^i(w,k,y,q)/\partial w^j = \partial x^j(w,k,y,q)/\partial w^i \qquad i,j=1,\ldots,N \tag{2.3}$$

hold. Restrictions (2.3) imply a joint test of the first order conditions for cost minimization and the existence of a parent cost function from which the factor demand equations (2.2) are derived.

To analyze the static competitive equilibrium levels of quasi-fixed factors, nonrestricted outputs, and supply managed products, define the long run profit maximization problem

[2] Homogeneity of degree zero of these functions follows from Euler's Theorem. The symmetric negative semidefiniteness of the Hessian matrix is due to the concavity in (w) of the cost function (2.1).

$$\max_{k,y,q} \ \Sigma_{j=1}^{M} p^j y^j + \Sigma_{i=1}^{L} p^{qi} q^i - c(w,k,y,q) - \Sigma_{i=1}^{R} w^{ki} k^i - (\alpha/2) \Sigma_{i=1}^{M} \Sigma_{j=1}^{M} y^i y^j Vp^{ij} \quad (2.4)$$

where p^j, p^{qi}, and w^{ki} are the prices for nonrestricted outputs, supply managed outputs, and quasi-fixed inputs respectively, $\alpha > 0$ is the coefficient of risk aversion assuming price uncertainty for nonrestricted outputs, and Vp represents the price variance and covariance matrix of these outputs. Model (2.4) assumes a linear mean-variance utility function for the producer defined as

$$u = E\pi - (\alpha/2) V\pi \quad (2.5)$$

where $E\pi$ is expected profits, $V\pi$ is the variance of profits. This model assumes that outputs y, q, inputs x, restricted output prices p^q, and input prices w, w^k are all nonstochastic. The first order conditions for an interior solution to (2.4) are

$$\partial c(w,k^*,y^*,q^*)/\partial k^i + w^{ki} = 0 \qquad i = 1,\ldots,R \quad (2.6)$$

$$\partial c(w,k^*,y^*,q^*)/\partial y^j - p^j + \alpha \Sigma_{i=1}^{M} y^j Vp^{ij} = 0 \qquad j = 1,\ldots,M \quad (2.7)$$

$$\partial c(w,k^*,y^*,q^*)/\partial q^i - p^{qi} = 0 \qquad i = 1,\ldots,L \quad (2.8)$$

where asterisks denote the optimum levels of the choice variables. Note that if prices (w^{ki},p^j,p^{qi}) are identical across firms and if $\alpha = 0$ (risk neutrality), then the cost functions c(w,k,y,q) for individual firms satisfy the conditions for consistent linear aggregation over static competitive profit maximizing combinations of k,y,q, Coyle (1991, p.6). Conditions (2.6)-(2.8) represent the shadow price equals market price principle and if the industry is in full static equilibrium with profit maximization for all inputs and outputs, conditions

(2.6)-(2.8) are satisfied jointly with (2.3). Particularly, (2.8) provides a direct way of testing whether the ex post or fixed prices of supply managed products are equal to the ex ante or cost of producing a unit of these outputs. The above derivatives of the cost function can be interpreted as follows: $\partial c(w,k^*,y^*,q^*)/\partial k^f$ is the shadow price of quasi-fixed input k^f, that is, the impact of a marginal increase in k^f on variable cost; $\partial c(w,k^*,y^*,q^*)/\partial y^j$ is the marginal variable cost of producing output y^j; and $\partial c(w,k^*,y^*,q^*)/\partial q^e$ is the marginal variable cost of producing one unit of supply managed output q^e. Model (2.1) is useful in testing the hypotheses of static profit maximization over outputs and of static resource equilibrium for quasi-fixed factors.

For the empirical analysis of the proposed hypotheses of static resource and product equilibrium, a Translog functional form is postulated for the industry short-run cost function

$$\ln c = a_0 + \Sigma_e a_e D^e + 1/2 \Sigma_e \Sigma_s a_{es} D^e D^s \qquad (2.9)$$

where: D=(ln w^1,...,ln w^N, ln k^1,...,ln k^R, ln y^1,...,ln y^M, ln q^1,...,ln q^L, t); c is total variable cost; and t denotes a time trend intended as a proxy for technological change. Assuming a homothetic production function, Hicks neutral technical change coincides with share neutrality, Chambers (1988). Thus, the share neutrality condition $\partial \log s^i(w,k,y,q)/\partial t = 0$, where s^i is the share of input i in short run variable cost, is equivalent to Hicks neutrality. Furthermore, share-using and share-saving inputs can be defined depending on whether $\partial \log s^i / \partial t$

is positive or negative respectively. Short run cost minimization implies, by Shephard's lemma (2.2), the following conditional cost share equations for variable inputs

$$sx^i = a_i + \textstyle\sum_s a_{is} D^s \qquad i = 1,...,N \tag{2.10}$$

where $sx^i = w^i x^i/c$. For quasi-fixed input k^f, (2.6) and static competitive equilibrium imply the cost share equations

$$\begin{aligned} -sk^f &= \partial \ln c(w,k,y,q,t)/\partial \ln k^f \\ &= a_{N+f} + \textstyle\sum_s a_{N+f,s} D^s \qquad f = 1,...,R \end{aligned} \tag{2.11}$$

where $sk^f = w^{kf} k^f/c$. Also, competitive equilibrium for nonrestricted outputs y^j and supply managed products q^e imply, using (2.7) and (2.8), the following cost share equations

$$\begin{aligned} sy^j &= \partial \ln c(w,k,y,q,t)/\partial \ln y^j \\ &= a_{N+R+j} + \textstyle\sum_s a_{N+R+j,s} D^s + \alpha \sum_{i=1}^M y^i y^j V p^{ij}/c \qquad j = 1,...,M \end{aligned} \tag{2.12}$$

$$\begin{aligned} sq^e &= \partial \ln c(w,k,y,q,t)/\partial \ln q^e \\ &= a_{N+R+M+e} + \textstyle\sum_s a_{N+R+M+e,s} D^s \qquad e = 1,...,L \end{aligned} \tag{2.13}$$

where $sy^j = p^j y^j/c$, $sq^e = p^{qe} q^e/c$ respectively.

Parametric restrictions can be defined for equations (2.10)-(2.13) depending on the behavioral assumption specified. The existence of a translog cost function (2.9) and short run cost minimization imply the following symmetry restrictions

$$a_{ij} = a_{ji} \qquad i,j = 1,\ldots,N \tag{2.14}$$

on coefficients for variable cost share equations (2.10). In addition, if the quasi-fixed inputs k are at their static equilibrium levels, then the additional reciprocity restrictions

$$a_{ij} = a_{ji} \qquad i,j = 1,\ldots,N+R \tag{2.15}$$

are satisfied for equations (2.10)-(2.11). In general, if the industry is at static competitive equilibrium for quasi-fixed inputs and profit maximization over all outputs (nonrestricted and supply managed) then the full set of symmetry restrictions

$$a_{ij} = a_{ji} \qquad i,j = 1,\ldots,N+R+M+L \tag{2.16}$$

holds for equations (2.10)-(2.13). Such restrictions where constrained outputs are explicitly incorporated in the static equilibrium conditions of the industry are not recognized in either Moschini or Lopez.

2.1.1 Hypothesis Testing

Two tests are conducted: (1) Wald chi-square tests of symmetry restrictions across share equations that are implied by static resource and product equilibrium for quasi-fixed inputs and outputs; (2) Hausman specification tests of augmenting the share equations (2.10) for variable inputs by appropriate first order conditions (2.11)-(2.13) for static equilibrium for quasi-fixed inputs and outputs. The parametric restrictions (2.14)-(2.16) are useful in testing hypotheses of static resource and output equilibrium. The reciprocity restrictions (2.14) implied by the existence of a short-run translog cost function (2.9) are tested as a necessary condition for further analysis. Then, the additional symmetry restrictions (2.15)-(2.16) are parametrically tested for each quasi-fixed input, nonrestricted output, and supply managed product. The null hypothesis in this case is that the share equations for variable inputs (2.10) and particular first order conditions for quasi-fixed factors (2.11)/nonrestricted outputs (2.12)/supply managed outputs (2.13) are consistent with a short run Translog cost function and static competitive equilibrium for the quasi-fixed inputs/nonrestricted outputs/supply managed products. The alternative proposition is that equations (2.10) are consistent with a short run Translog cost function, but coefficients of equations (2.11)-(2.13) for quasi-fixed inputs, nonrestricted outputs, and supply managed products are independent of coefficients for all other equations in the system.

This general framework provides a direct way of testing hypotheses (1.a)-(1.c) described in Chapter 1. To further evaluate these propositions, we next derive Hausman specification tests similar to those introduced by Hausman, (1978); Schankerman and Nadiri, (1986);[3]. To begin, suppose that $\hat{\beta}_0, \hat{\beta}_1$ are two estimators of the coefficients (a_i, a_{is}) in equations (2.10) for variable inputs, and that $\hat{\beta}_0$ is asymptotically efficient and $\hat{\beta}_1$ is consistent under an hypothesis H_0 of cost minimization for certain quasi-fixed inputs and/or profit maximization for some outputs; otherwise, only $\hat{\beta}_1$ is consistent under both H_0 and H_1. This in turn implies that $[var(\hat{\beta}_1) - var(\hat{\beta}_0)]$ is positive semidefinite under H_0. The asymptotic covariance of $\hat{\beta}_0$ and $\hat{q} \equiv \hat{\beta}_1 - \hat{\beta}_0$ is equal to zero under H_0 and

[3] These tests have mostly been applied to manufacturing related sectors. The only application to agriculture is provided by Coyle. These procedures have not been applied to Western Canadian agriculture.

$var(\hat{q}) = var(\hat{\beta}_1) - var(\hat{\beta}_0)$,(Hausman). This lead to a direct test of H_0 against a wide set of alternative hypotheses using the test statistic

$$M = \hat{q}^T var(\hat{q})^{-1} \hat{q} \tag{2.17}$$

which is asymptotically distributed as chi-square under H_0. An efficient estimator $\hat{\beta}_0$ of the coefficients of (2.10) under H_0 can be obtained by estimating equations (2.10) jointly with the appropriate static competitive conditions (2.11) for quasi-fixed inputs and/or (2.12)-(2.13) for all outputs. An estimator $\hat{\beta}_1$ that is consistent under both hypotheses can be obtained by estimating equations (2.10) alone.

2.2 An Indirect Short-Run Utility Function with Stochastic Output Prices Approach

The hypotheses put forward in Chapter 1 can also be evaluated under a linear mean-variance utility function framework with stochastic output prices. The properties of this model are well known (e.g., Dhrymes; Robinson and Barry; Chavas and Pope). The incorporation of stochastic output prices into duality models was first introduced by Coyle. A version of this model incorporating restricted outputs is used to test the propositions of static resource and output equilibrium for quasi-fixed inputs and outputs given stochastic prices for nonrestricted products.

Let us start by assuming that all multiproduct firms that produce M nonrestricted outputs by employing N variable inputs given R quasi-fixed inputs and L restricted outputs face stochastic prices for their variable outputs. All firms are price takers in both output and input markets. Then a particular firm's short-run utility maximization problem can be defined as

$$u(p,w,vp,k,q) = \max_{(x,y \in T)} \Sigma_{j=1}^{M} p^j y^j - \Sigma_{i=1}^{N} w^i x^i - (\alpha/2) \Sigma_{i=1}^{M} \Sigma_{j=1}^{M} y^i y^j VP^{ij} \qquad (2.18)$$

where u(p,w,vp,k,q) represents the dual utility function; that is, the maximum feasible utility given the exogenous variables p,w,vp,k,q. Utility is define here as in (2.5). It can be shown that the function (2.18) satisfies the following standard properties (see Coyle for proofs)

2.2a u(.) is increasing in p, decreasing in w and in elements of vp;

2.2b u(.) is linear homogeneous in (p,w,vp);

2.2c u(.) is convex in (p,w,vp); and

2.2d assuming u(.) is differentiable, then the first order conditions for an interior solution to (2.18) imply

$$\partial u(p,w,vp,k,q) / \partial p^j = y^{j*}(p,w,vp,k,q) \qquad j = 1,\ldots,M \qquad (2.19)$$

$$\partial u(p,w,vp,k,q) / \partial w^i = -x^{i*}(p,w,vp,k,q) \qquad i = 1,\ldots,N \qquad (2.20)$$

(Hotelling's lemma) where $y^{j*}(.)$ and $x^{i*}(.)$ are the optimum levels of nonrestricted outputs and variable inputs given k,q.[4] The properties of the Hessian matrix of u(.) and linear homogeneity in (p,w,vp) imply the following symmetry restrictions for utility maximization

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[4] The full set of first order conditions of (2.18) include the partial derivatives of u(.) with respect to the elements of vp. These are omitted for simplicity of presentation.

$$\begin{aligned} &\partial y^{j*}(.)/\partial w^{i} = -\partial x^{i*}(.)/\partial p^{j} \quad j=1,...,M \\ &\qquad\qquad\qquad\qquad\qquad\qquad\quad i=1,...,N \\ &\partial y^{j*}(.)/\partial p^{i} = \partial y^{i*}(.)/\partial p^{j} \quad i,j=1,...,M \\ &\partial x^{i*}(.)/\partial w^{j} = \partial x^{j*}(.)/\partial w^{i} \quad i,j=1,...,N \end{aligned} \tag{2.21}$$

for a multiproduct firm. This set of conditions should hold simultaneously as well as homogeneity of degree zero in (p,w,vp) of the derived supply and variable demand equations (2.19)-(2.20). To evaluate the static competitive equilibrium levels of quasi-fixed inputs k and supply managed outputs q, define the long run optimization problem

$$\begin{aligned} u^{*}(p,w,vp,w^{k},p^{q}) = \max_{k,q} u(p,w,vp,k,q) &+ \textstyle\sum_{i=1}^{L} p^{qi} q^{i} - \sum_{i=1}^{R} w^{ki} k^{i} \\ &- (\alpha/2)\textstyle\sum_{i=1}^{M}\sum_{j=1}^{M} y^{i} y^{j} VP^{ij} \end{aligned} \tag{2.22}$$

where $u^{*}(p,w,vp,w^{k},p^{q})$ denotes the long-run optimum utility of a multiproduct firm given the exogenous variables p,w,vp,w^k,p^q. The first order conditions for an interior solution to (2.22) are

$$\partial u(p,w,vp,k^{*},q^{*})/\partial k^{i} - w^{ki} = 0 \qquad i=1,...,R \tag{2.23}$$

$$\partial u(p,w,vp,k^{*},q^{*})/\partial q^{j} + p^{qj} = 0 \qquad j=1,...,L \tag{2.24}$$

where asterisks denote the optimum levels of the choice variables. The first order conditions (2.23)-(2.24) can be defined as follows: $\partial u(p,w,vp,k^{*},q^{*})/\partial k$ denotes the shadow value of quasi-fixed input k and indicates a one-period increase in utility attainable if, holding restricted output quantities, prices, and variance-covariance of prices constant, the quantity of quasi-fixed factor k is increased by one unit;

$\partial u(p,w,vp,k^*,q^*)/\partial q$ denotes the shadow price of supply managed commodity q and indicates a one-period decrease in utility if, holding quasi-fixed inputs, prices, and variance-covariance of prices constant, the quantity of restricted output is increased by one unit. If the industry is in static equilibrium with utility maximization over all inputs and outputs as in (2.2), conditions (2.23)-(2.24) are satisfied jointly with (2.21). In particular, (2.24) provides a direct way of testing whether the ex post long-run prices of supply managed products are equal to their ex ante or market prices. A Normalized Quadratic functional form is adopted for the industry short-run dual utility function to evaluate the hypothesis of static resource and product equilibrium for quasi-fixed inputs and supply managed outputs. This function is defined as

$$u = a_0 + \Sigma_e a_e D^e + 1/2 \Sigma_e \Sigma_s a_{es} D^e D^s \qquad (2.25)$$

where $D = (p^1/w^4,...,p^M/w^4;\ w^1/w^4,...,w^{N-1}/w^4;\ var(p^1)/w^4,...,var(p^M)/w^4;$ $cov(p^i,p^j)/w^4,\ i,j = 1,...,M;\ k^1,...,k^R;\ q^1,...,q^L;\ t)$. Short-run utility maximization and Hotelling's lemma (2.19)-(2.20) imply the following conditional supply equations for nonrestricted output and variable input demand equations

$$\begin{aligned} y^j &= a_j + \Sigma_s a_{js} D^s \qquad j = 1,...,M \\ -x^i &= a_{M+i} + \Sigma_s a_{M+i,s} D^s \qquad i = 1,...,N \end{aligned} \qquad (2.26)$$

The first order condition (2.23) and static competitive equilibrium for quasi-fixed inputs k yield the inverse demand equations

$$-w^{kf} = \partial u(p,w,vp,k,q)/\partial k^f \\ = a_{M+N+f} + \Sigma_s a_{M+N+f,s} D^s \qquad f=1,\dots,R \tag{2.27}$$

For supply managed outputs, (2.24) and static utility maximization imply the inverse supply equations

$$p^{qe} = \partial u(p,w,vp,k,q)/\partial q^e \\ = a_{M+N+R+e} + \Sigma_s a_{M+N+R+e,s} D^s \qquad e=1,\dots,L \tag{2.28}$$

Given certain behavioral assumptions, parametric restrictions can be defined for equations (2.26)-(2.28). For instance, the existence of a Normalized Quadratic function (2.25) and short run utility maximization imply the following reciprocity restrictions

$$a_{ij} = a_{ji} \qquad i,j = 1,\dots,M+N \tag{2.29}$$

on coefficients for variable supply and input demand equations (2.26). Furthermore, if the quasi-fixed inputs k are at their static equilibrium levels, then the additional symmetry restrictions

$$a_{ij} = a_{ji} \qquad i,j = 1,\dots,M+N+R \tag{2.30}$$

apply to coefficients of (2.26)-(2.27). Finally, if the industry is at static competitive equilibrium for quasi-fixed inputs and utility maximization over supply managed outputs, then the full set of symmetry restrictions

$$a_{ij} = a_{ji} \qquad i,j = 1,...,M+N+R+L \qquad (2.31)$$

are satisfied for equations (2.26)-(2.28).

2.2.1 Hypothesis Testing

Two different tests are conducted: (1) Wald chi-square tests of the additional symmetry restrictions (2.30)-(2.31) implied by static resource equilibrium for quasi-fixed inputs and utility maximization over supply managed outputs; (2) Likelihood ratio tests of the same propositions. The above parametric restrictions (2.30)-(2.31) provide direct procedures for testing the propositions of static resource and output equilibrium . First, the symmetry restrictions (2.29) implied by the existence of a short-run Normalized Quadratic utility function (2.25) are initially tested as a necessary condition for further hypothesis testing. Second, the additional symmetry restrictions (2.30)-(2.31) are parametrically tested for each quasi-fixed input and supply managed product. Under this framework, the null hypothesis is that the variable output supply and input demand equations (2.26) and particular first order conditions for quasi-fixed factors (2.27) and restricted outputs (2.28) are consistent with a short-run Normalized Quadratic utility function and static competitive equilibrium. The alternative proposition is that equations (2.26) are consistent with a short-run Normalized Quadratic utility function, however coefficients of (2.27) for quasi-fixed inputs and (2.28) for restricted outputs are independent of all other coefficients in the system.

To further evaluate the above hypotheses we next derive Likelihood Ratio tests. We start by noting that under H_0, the symmetry restrictions (2.30)-(2.31) are satisfied, and that under H_1 (2.30)-(2.31) do not apply. Now, suppose that L_1 is the log of the likelihood function obtained by maximum likelihood (ML) estimation of equations (2.26) for variable output supplies and input demands jointly with particular first order conditions from (2.27)-(2.28), and that the symmetry restrictions (2.30)-(2.31) are not imposed. L_0 is the log of the likelihood function derived by ML estimation of the same system but imposing the symmetry restrictions (2.30)-(2.31). Then, the likelihood ratio statistic

$$LR = 2 * [\ln(L_1) - \ln(L_0)] \tag{2.32}$$

is asymptotically distributed as chi-square under H_0 with degrees of freedom equal to the difference in free parameters between the two models. L_1 and L_0 can be obtained by Iterative Zellner estimation which yields parameter estimates that are numerically equivalent to those of the maximum likelihood estimator, Berndt (1991, p.463).

The hypotheses of static resource and output equilibrium given the utility function (2.25) can also be tested by Hausman specification tests similar to those derived in section 2.1.1.

2.3 A Nonstochastic Profit Function Approach

The restricted dual profit function for a competitive multiproduct firm can be specified as

$$\pi(p,w,;k,q) \equiv \max_{x,y \in T} \textstyle\sum_{j=1}^{M} p^j y^j - \sum_{i=1}^{N} w^i x^i : F(y,x;k,q) = 0 \tag{2.33}$$