THE PROBABILITY OF DROUGHTS IN THE ASSINIBOINE BASIN

BY

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MASTER OF SCIENCE

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ABSTRACT

Accumulated Basin Storage (ABS) which is an estimate of soil moisture levels in a river basin is used to quantify agricultural droughts. The relevance of ABS to agricultural droughts is investigated by means of its correlation with crop yield within the basin. It is observed that crop yield is more strongly correlated with ABS than with other parameters such as precipitation and runoff and thus the ABS will be the better drought estimating parameter. In the method, agricultural drought is related to soil moisture deficit, when the ABS falls below a reference base level.

The ABS is a random variable and its annual sequence is a random time series with a specific correlation structure. A statistical experimental method of generating a large number of time series samples can be used to estimate probability distributions of characteristic drought variables. The drought variables considered here are the number of droughts occurring in a given time interval, the magnitude of soil moisture deficits, the rate at which soil moisture deficit progresses (or drought intensity), and the duration of drought events. Also, the largest drought deficit, intensity and longest durations are considered. The

theory of the maximum of a random number of random variables is used to interprete the experimental results. This approach is based on the assumption that drought is independent identically distributed random variable whose occurrence follows the Poisson probability law. These assumptions are checked and found to be satisfied. The comparison between the experimental and theoretical results are shown graphically and statistical tests were conducted to show their goodness of fit.

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THE RELEVANCE OF ACCUMULATED BASIN STORAGE TO DROUGHT MEASUREMENT

CHAPTER 1

INTRODUCTION

1.1 INTRODUCTION

The objective of this study is to use a physically derived basin parameter, called Accumulated Basin Storage (abbreviated ABS), to analyse agricultural droughts in the Assiniboine River Basin.

In Assiniboine River Basin, one of the Prairie river basins, droughts as well as floods occur. Both events have serious economic consequences in the region and pose threats to the people who settle in the area. Several hydrologic studies have therefore been conducted aimed at alleviating the adverse effects of floods and droughts through physical control structures or planning and managerial measures.

The Assiniboine valley is of great economic importance to the province, because of its contribution to the economy in the agricultural sector. Agriculture is the main-stay of the economy here as oppose to the manufacturing industries found in the east and oil in the west.

Floods and droughts however are persistent nuisances to farmers in the area. Much of the hydrologic research has been directed towards floods. All that has been done about droughts deals with streamflow droughts. This is because both floods and streamflow droughts concern streamflow magnitudes which are easily quantifiable, measurable, and records on them are available. Agricultural drought on the other hand has no universally accepted measure. It involves not only meteorological factors such as temperature, humidity, precipitation, but also soil drainage properties, evapo-transpiration rate, and other factors such as type of vegetation cover and even agricultural economics.

A single parameter, which combines at least all the hydro-meteorological factors would greatly simplify the problem of agricultural drought analysis.

The ABS, which will be defined in the next chapter, is such a parameter. The mean ABS level within the growing season from May to September was used in this study to define agricultural drought conditions in the basin. In addition, mathematical model of the mean seasonal ABS was developed and used to conduct statistical studies.

1.2 THE ACCUMULATED BASIN STORAGE (ABS)

The Accumulated Basin Storage was developed [Booy & Lye, 1986] as a wetness index in an explanation of the clustering observed in peak flows of the Red River. It is a physically based parameter that measures the average soil moisture conditions in the drainage basin.

The river basin acts as a reservoir, storing precipitation and releasing it in the form of runoff and evapotranspiration. The ABS is obtained as a time series by taking a hydrologic inventory of all inputs and outputs for this reservoir with respect to time.

Since no historical record of soil moisture exists for the basin, the ABS series must be calculated from other meteorological and hydrological time series for which records are available. This process is discussed in the next chapter.

Accumulated Basin Storage is indicative of the soil moisture levels in the upper layers of the soil where the roots of vegetation reach to extract water for transpiration. Since plants also depend on this stored moisture for their existence, very low ABS levels will signify agricultural drought conditions. Therefore ABS, which is a basin wide parameter that averages the soil moisture level over the whole basin, can be expected to be a very good parameter for quantifying agricultural drought.

1.3 ASSINIBOINE RIVER BASIN CHARACTERISTICS

The Assiniboine River is the second major river in the Province of Manitoba, takes its source in Saskatchewan and confluences with the Red River in the heart of the city of Winnipeg.

Its location is between latitudes 49.5° N and 51.5° N and longitudes 97° W and 103° W, as shown in Figure 1. The basin area under this study, upstream of the city of Brandon, covers 86,000 square kilometers. It lies at an altitude of 610 m above sea level at its source to 400 m above sea level at Brandon, and has a length of about 1250 kilometers. Thus it has a very flat gradient of between 0.09 m to 0.19 m per kilometer.

Its main tributaries are the Qu' Appelle and Minnedosa Rivers upstream of Brandon, and the Souris River downstream of Brandon. The average natural flow at Brandon is about 32.0 cumecs with a low flow of 0.15 cumecs and high flow of 435 cumecs. The natural flow has been changed by the construction of Shellmouth dam on the Assiniboine between 1968 and 1972, the Rivers Dam on the Minnedosa River in 1960 and the Qu' Appelle River diversion since 1970. The Shellmouth Reservoir is primarily for flood control but also augments low flows to 0.7 cumecs. The Rivers Dam also adds another 0.15 cumecs to the low flows.

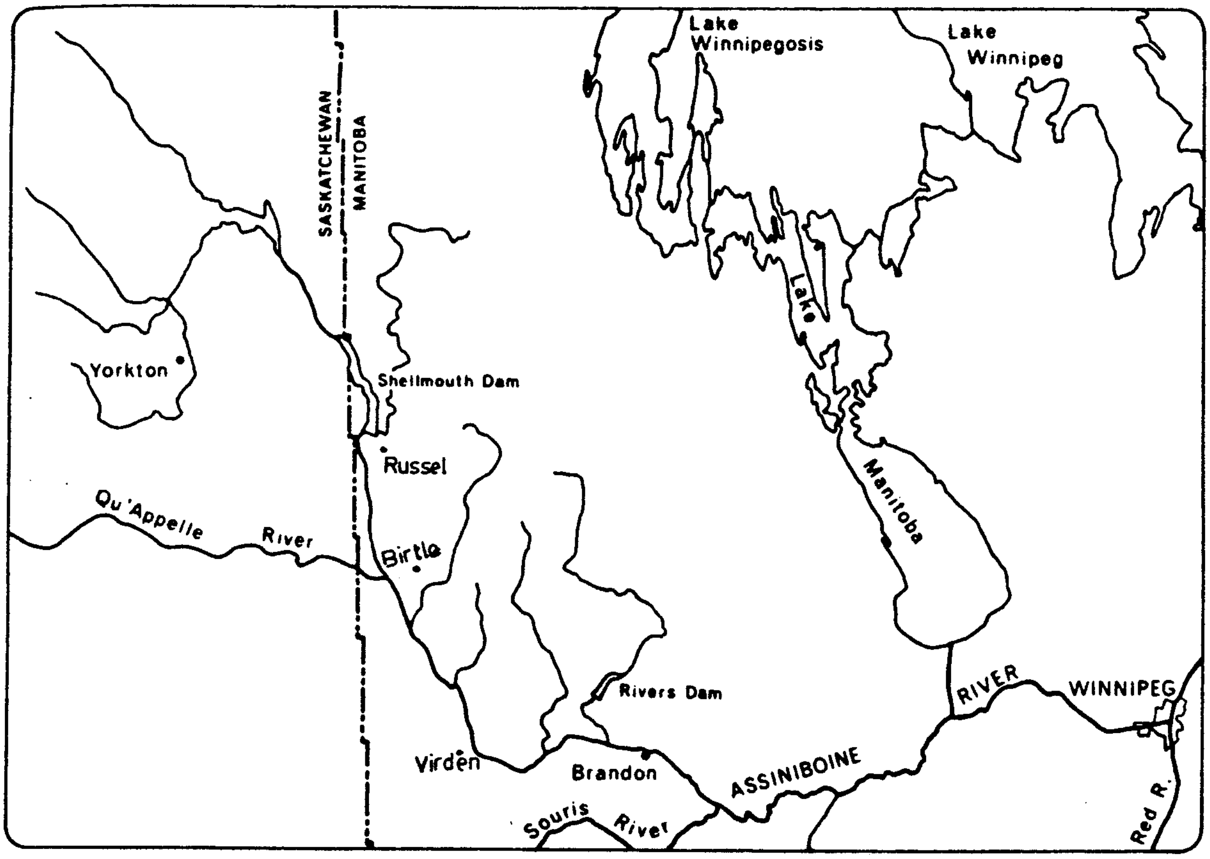

Figure 1 : Assiniboine River

The Assiniboine River valley above Brandon is wide and deep. The banks of the valley are stable and the river has lost its erosive power. The bottom of the valley consists of mainly alluvial deposits.

Studies for groundwater resources for the region show that only a small patch of blanket sand aquifer exists in the Assiniboine valley around the Shellmouth dam and also downstream of Brandon up to the confluence with the Souris River. [Ref: Saskatchewan-Nelson Basin Board, 1972].

CHAPTER 2

ESTIMATING THE HISTORICAL ABS

2.1 DATA REQUIREMENTS

ABS is the soil moisture accumulation obtained after taking a hydrological inventory of the hydrological cycle. Therefore all hydrometeorological variables should be included in the balancing if possible, and any that are not included should not have any significant effect on the magnitude of the estimated ABS.

These hydrometeorological variables include precipitation in the form of snow and rain, basin drainage as indicated by streamflow, evapotranspiration, and groundwater flow. Data will also be needed for natural minimum streamflow.

Records for all the above are available or can be constructed except groundwater flow into and out of the basin. However, in Section 1.3, it was noted that the basin has a very flat gradient. It can be assumed the variations of movement of groundwater in and out of the basin are small. A constant inflow or outflow will automatically be incorporated in the necessary coefficients of the simulation model, hence it will be ignored in the moisture balancing process.

The water year will be taken as the period between April 30th in one calendar year to March 31st the following calendar year. This is to enable a distinction to be made between winter (when precipitation is snow and no infiltration takes place) and summer (when precipitation is rain, and infiltration takes place).

All data used in this study are obtained from four recording stations within the river basin. The data are averaged over the basin. More stations could not be used because the other stations have either very short records or a lot of missing records. Relatively little error is expected by this limitation since there is not much variation in seasonal precipitation from station to station as will be observed in subsequent tables. The length of records obtained was 63 years, covering a period from 1913 to 1975 at all four stations in the basin.

Some of the incoming precipitation in the basin is either intercepted or stored. Interception occurs when vegetation canopy or foliage traps some of the rainfall before it can infiltrate into the soil. Some of this intercepted rainfall will be evaporated directly and the rest will find its way onto the soil. Storage includes snow pack and storage of rain in depressions in the form of ponds and lakes. Evaporation from snow is insignificant and will therefore be neglected. Apart from the artificial lakes of Shellmounth and Rivers Dams, practically no natural ponds or lakes exist in the basin. All these minor evaporations will be lumped together with the basin evapotranspiration.

2.2 PRECIPITATION AND RUNOFF

Precipitation is an input into the hydrologic model and runoff or drainage is its output.

Monthly precipitation data was obtained from four recording stations and averaged over the basin using the Thiessen Polygon method. Precipitation occurs in two physical states as snow and as rain. A distinction is therefore made between winter precipitation and summer precipitation. Winter precipitation is what occurs between November 1st and March 31st and what occurs from April 1st to October 30th is summer precipitation.

From the temperature records shown in Table 1, it can be observed that winter precipitation usually melts sometime in April. Therefore it will be assumed that the winter precipitation will not contribute to the ABS during the winter months until April when it will be lumped to the April precipitation.

Table 2 shows the mean values of precipitation obtained for the stations and the basin averages.

TABLE 1

MEAN DAILY MAXIMUM TEMPERATURES

Station	Mean Daily Maximum Temperature (°C) Jan	Feb	Mar	Apr	May
Brandon	-13.4	-10.1	-3.1	9.7	18.2
Birtle	-14.4	-10.5	-3.1	7.9	16.9
Russel	-15.0	-11.8	-4.3	7.7	16.6
Virden	-14.5	-10.9	-3.5	7.8	15.9

TABLE 2

PRECIPITATION DATA FOR STATIONS IN THE ASSINIBOINE RIVER BASIN

Station	Mean Winter Ppt (mm)	Mean Summer Ppt (mm)	Mean Annual Ppt (mm)
Brandon	134.1	338.0	472.1
Russel	135.0	315.0	450.0
Birtle	128.5	337.0	465.9
Virden	131.6	331.0	462.6
BASIN AVG.	132.6	329.6	462.2

There are two types of streamflow used in the study. First is the mean monthly streamflow which will be used for developing the monthly ABS series. Since the problem involves taking the moisture balance, it is very important that the streamflows used are the natural flows which has not been augmented or controlled by upstream reservoirs. Data of natural streamflows are available for the period of record to be used at the location of Brandon. These natural flows were derived from historical observations which were modified to remove the effects of minor regulations that took place. The second type of streamflow needed is the minimum winter streamflows. This should also be natural, unaugmented flow. From the record of natural streamflows, the minimum winter flows were obtained as the minimum monthly flows in each year, and occurred mostly in February.

Table 3 is a summary of the streamflow record obtained for the Assiniboine River.

The distribution of the annual minimum winter flows is important in the estimation of the historical ABS. Therefore its normal probability plot was made and shown on Figure 2.1 with 95% confidence band. It can be observed that the data departs very much from normality. After taking logarithms of the data however, the normal plot in Figure 2.2 approximates to the normal distribution. Therefore the logarithm of the minimum winter flows will be used for the ABS fitting process.

TABLE 3

MEAN MONTHLY FLOW DATA FOR ASSINIBOINE RIVER AT BRANDON

Month	Mean Flow (m^3/s)
January	4.99
February	4.33
March	7.81
April	84.7
May	114.0
June	60.3
July	44.6
August	20.0
September	13.2
October	13.7
November	11.5
December	6.9

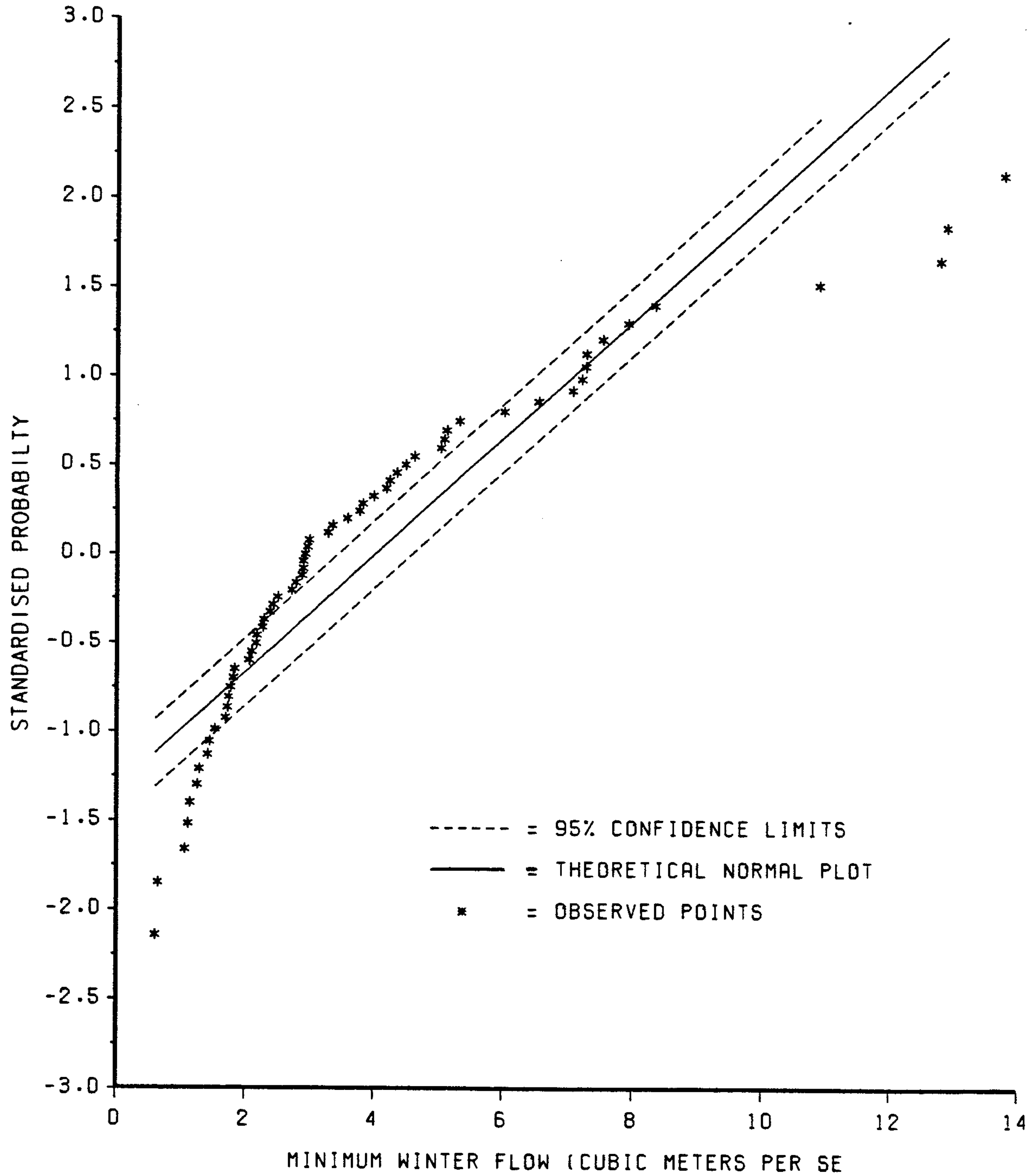

FIG. 2.1 NORMAL PROBABILITY PLOT OF MINIMUM WINTER FLOWS WITH THE 95% CONFIDENCE LIMITS

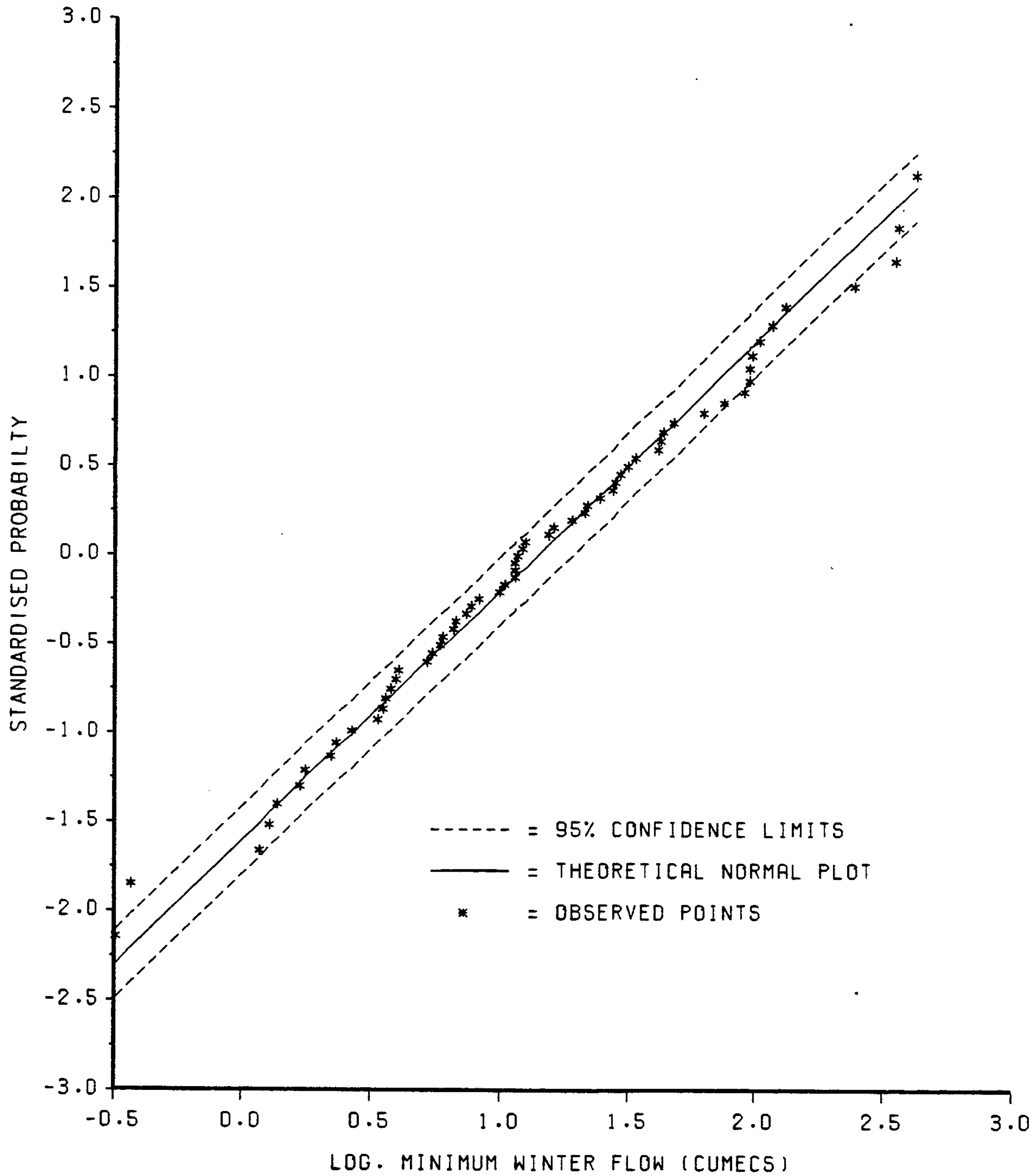

FIG. 2.2 NORMAL PROBABILITY PLOT OF LOG OF MINIMUM WINTER FLOWS WITH THE 95% CONFIDENCE LIMITS

2.3 POTENTIAL AND ACTUAL EVAPOTRANSPIRATION

Evapotranspiration is one of the required variables. All basin moisture is removed through evapotranspiration and runoff. Of the two, evapotranspiration alone removes about 90% of the total precipitation, therefore its accurate estimation is very important in the estimation of the ABS.

Evapotranspiration is the process whereby soil moisture is removed by plants and vaporized into the atmosphere through transpiration. The rate of evapotranspiration depends on the energy available for vaporisation, the amount of moisture available in the soil, and the type of vegetative cover.

For the short term evapotranspiration rate, vegetative cover type and growing stage are very important factors. However, since the vegetative type and growing patterns do not change much from year to year, if basin wide average is considered and from year to year, differences in vegetation cover can be ignored. Therefore, evapotranspiration can be determined on the basis of available energy and soil moisture alone.

The concept of potential evapotranspiration gives the evapotranspiration that is obtained if the available energy alone is considered, without the limiting factor of amount of moisture available for evapotranspiration of the type of vegetation. If this potential evapotranspiration could be estimated, the the actual field evapotranspiration could be estimated by multiplying it by a coefficient to account for the other limiting factors.

Many complicated relationships have been proposed for deriving the potential evapotranspiration, but the one which is suitable for application to the study is that of Lowry and Johnson (1942). Their method is an empirical relationship between temperature and potential evapotranspiration developed for the Mid-Western U.S.A. Their formula is given as:

$$PE = 0.8 + 0.156F \quad \text{... (2.1)}$$

where the potential evapotranspiration PE is in feet of water per year over the growing season (April to October), and F is the sum of the effective heat units per month.

$$F = \sum f \quad \text{... (2.2)}$$

where,

$$f = (t_m - 32°F)\ n / 1000 \quad \text{... (2.3)}$$

t_m = mean maximum monthly temperature in °F

n = the number of days in which the daily maximum temperature exceeds 32°F.

For any month, i, the evapotranspiration will be given as:

$$PE_i = (f_i/F) \times PE \quad \text{...(2.4)}$$

Temperature records within the Assiniboine River Basin show that the potential evapotranspiration for the winter months between November to March is zero because the daily maximum temperatures are all less than 32°F.

Using the Lowry-Johnson method, the average annual potential evapotransiration for the basin was computed and a plot of it shown on Figure 2.3. Having a mean of 572 mm and a standard deviation of only 18 mm, it can be observed that the potential evapotranspiration does not change much from year to year.

The potential evapotranspiration values obtained using the Lowry-Johnson method compares very well with the evapotranspiration rates obtained within the region for some of the major crops, as shown in Table 4 (F. Penkava, 1977). Comparing the Lowry-Johnson method mean potential evapotranspiration rate of 572 mm with actual evapotranspiration of the major crops within the basin shows the method gives a good estimate.

Having obtained the potential evapotranspiration, the actual year to year or month to month evapotranspiration can be obtained by using a coefficient that will account for the other factors.

$$E = C \times PE \qquad \ldots (2.5)$$

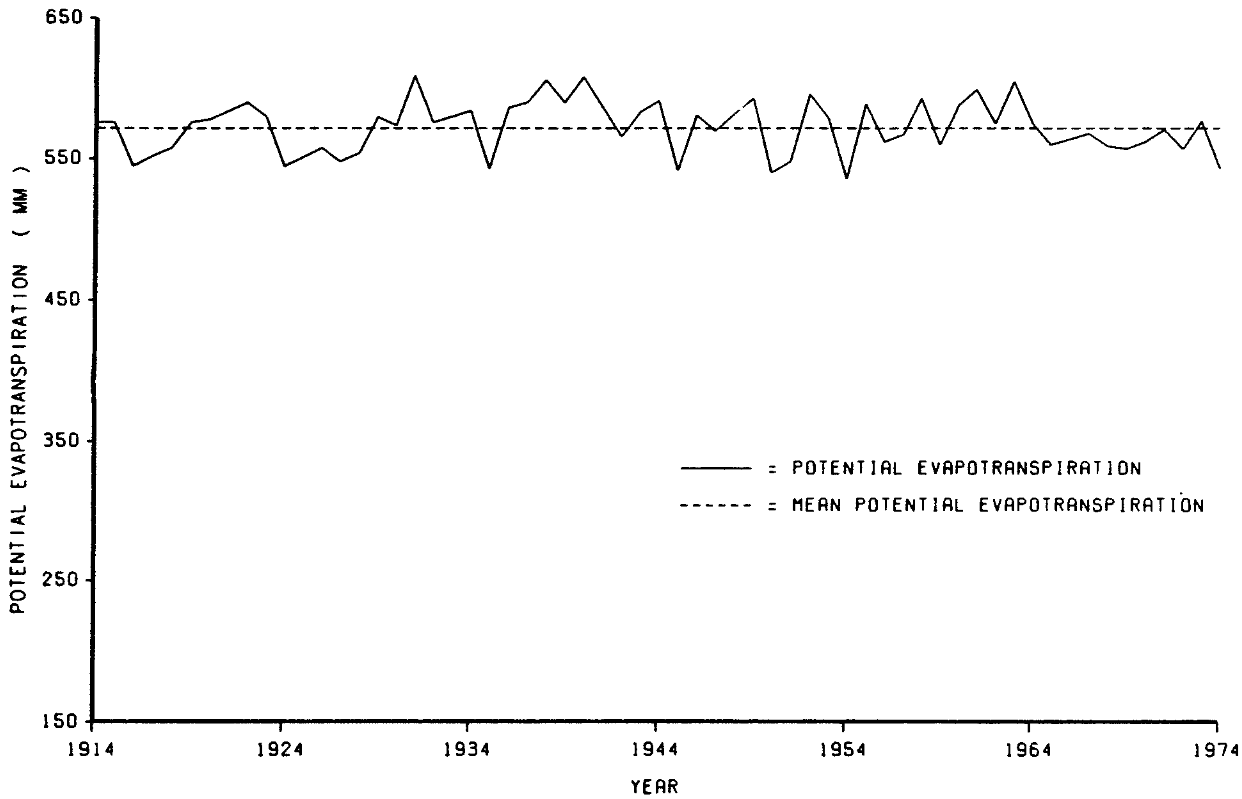

FIG. 2.3 POTENTIAL EVAPOTRANSPIRATION OF THE BASIN

TABLE 4

FIELD EVAPOTRANSPIRATION RATES OF SOME MAJOR CROPS IN SOUTHERN MANITOBA

(Penkava, F. "Principles and Practices of Commercial Farming". Irrigation and Drainage, University of Manitoba, 1977).

Crop	Evapotranspiration (mm) Total for May to Sept
Alfalfa	525
Pastures and Meadows	540
Potatoes	455
Wheat	340 (May to August)
Cabbage and Lettuce	600
Strawberries	525

The coefficient can be considered constant from year to year, or can vary with the amount of soil moisture available.

A constant coefficient of evapotranspiration will be justified if on the average, the basin can be considered to have a vegetation cover with uniform root depth. On the other hand, a variable coefficient will be valid for a mixture of deep and shallow root systems. In this case the coefficient will vary with the amount of soil moisture available, from a minimum value corresponding to very dry conditions, to a maximum value corresponding to very wet conditions. It is difficult to check which of these conditions will apply. Consequently the two conditions will be used to derive the ABS and the one that gives the better results will be taken. This process is explained in more detail in the next section.

2.4 FITTING THE HISTORICAL ABS

Since the study is concerned with the soil moisture storage within the growing season, the ABS will be fitted on monthly basis enabling the months of the growing season to be identified.

In general, at any time t, the basin moisture balance could be defined mathematically by:

$$ABS_t = ABS_{t-1} + PW_{t-1} + PS_t - E_t - R_t \qquad \ldots(2.6)$$

where,

ABS = the accumulated basin moisture storage

PW = winter precipitation

PS = summer precipitation

E_t = actual evapotranspiration

R_t = the stream runoff

E_t = C x PE where C is a coefficient and PE is the potential evapotranspiration.

All the above variables have been obtained from records except C and the initial ABS (at the beginning of the historical period).

In order to estimate the coefficient C and the initial ABS, use can be made of the fact that the base flow of a river is highly correlated to the soil moisture level in the basin. The minimum winter flow is only the base flow because in winter only stored water in the soil contributes to any flow that takes place in the river. Therefore the ABS at the end of the winter season and the minimum winter flow should be highly correlated. The technique to be used is to find, by trial and error, the values of the initial ABS and C, that will give the best fit between the minimum winter flow series and the ABS series.

On monthly basis, we have the moisture balance as:

$$ABS_{j,t} = ABS_{j-1,t} + P_{j,t} - C_j \times PE_{j,t} - R_{j,t} \quad \ldots (2.7)$$

where, j refers to the month in year t, and $P_{j,t}$ refers to the total precipitation available for infiltration in month j. P will therefore be zero for the winter months, and will be the sum of all winter precipitation plus April precipitation for the month of April.

The best way to compare the two time series, the normalised minimum winter flows (more precisely the logarithms of minimum flows) and the ABS, is by comparing their standardised values. This is done by subtracting their respective means from their values and dividing them by their standard deviations. Since the minimum winter flow occurs mostly in February, the February ABS series is fitted to the logarithm of the minimum flows. The best fit is the one for which the SUM OF THE SQUARED DEVIATIONS (SSE) between the two series is minimum.

The fitting was done for the two types of coefficients of evapotranspiration. For the uniform coefficient the coefficient was varied in increments from a small value to a maximum of 1. For each increment the corresponding SSE was calculated. The coefficient that gives the minimum value of the SSE (with a corresponding initial ABS) gives the best fit.

In the case of variable coefficient, it was assumed that a linear relationship exists between the coefficient of evapotranspiration and the ABS within an interval between some minimum and maximum values for the coefficient and the ABS (see diagram).

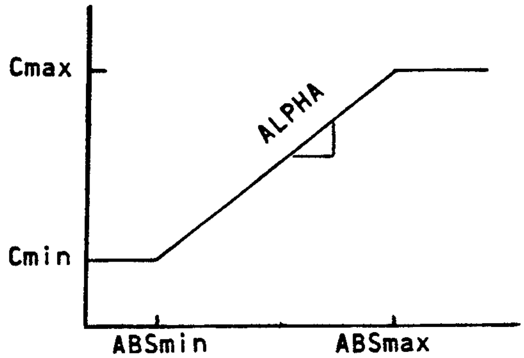

FIG. 2.4 RELATIONSHIP BETWEEN ABS AND COEFFICIENT OF EVAPOTRANSPIRATION

By varying the four key points, C_{max}, C_{min}, ABS_{max} and ABS_{min}, together with the initial ABS, the combination that gave the best fit was obtained. If C_t is the coefficient of evapotranspiration in month t, then

$$C_t = C_{min} \quad \text{for } ABS_{t-1} \leqslant ABS_{min} \qquad \ldots(2.8)$$

$$C_t = C_{min} + ALPHA \times ABS_{t-1} \quad \text{for } ABS_{min} < ABS_{t-1} \leqslant ABS_{max} \qquad \ldots(2.9)$$

$$C_t = C_{max} \quad \text{for } ABS_{t-1} > ABS_{max} \qquad \ldots(2.10)$$

where ALHPA is $(C_{max} - C_{min})/(ABS_{max} - ABS_{min})$.

From the results shown below, the varying coefficient of evapotranspiration gave the better result.

Constant Coefficient:

$$C = 0.76$$

$$\text{Initial ABS} = 350 \text{ mm}$$

$$SSE_{min} = 51.67$$

Variable Coefficient:

$$C_{max} = 1.0$$

$$C_{min} = 0.6$$

$$ABS_{max} = 500 \text{ mm}$$

$$ABS_{min} = 0.0 \text{ mm}$$

$$\text{Initial ABS} = 200 \text{ mm}$$

$$SSE_{min} = 43.4$$

A graphical comparison of the two time series is shown on Figure 2.5 and from it we can observe that both follow the same pattern.

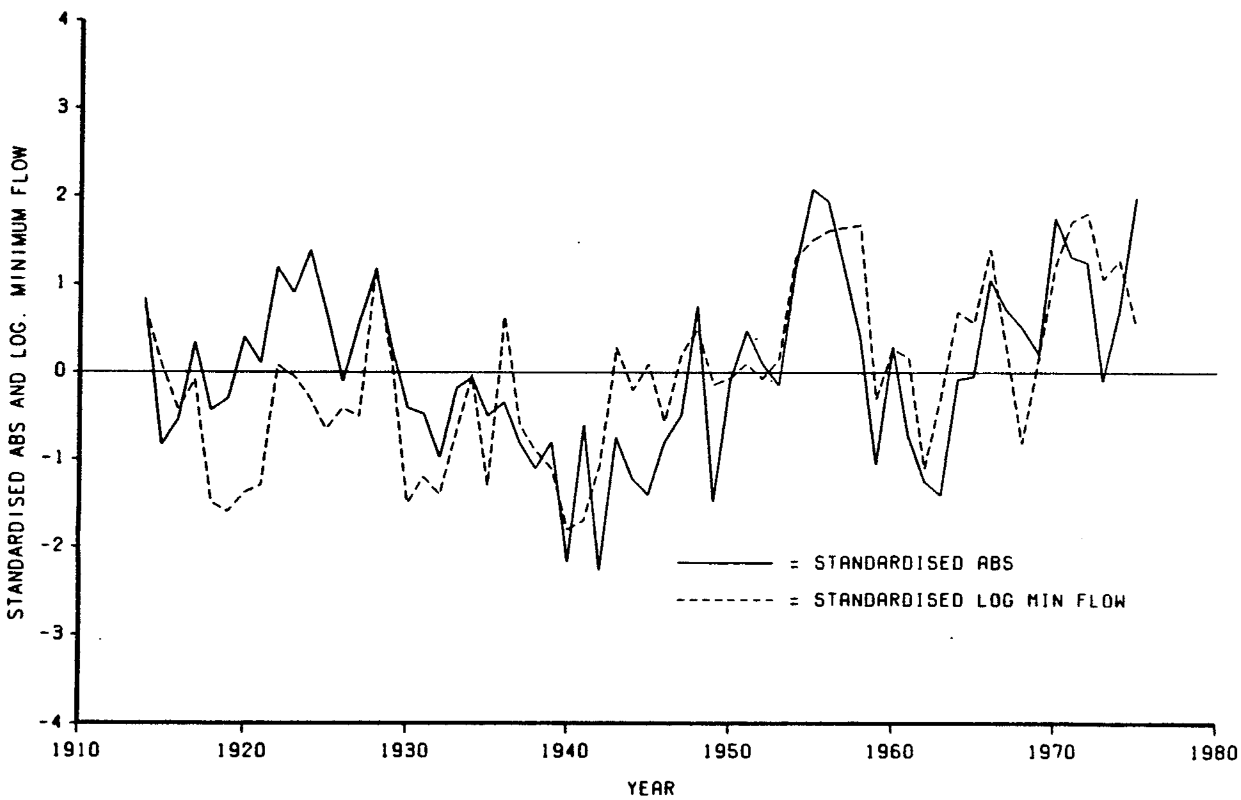

FIG. 2.5 PLOTS OF STANDARDISED ABS AND STANDARDISED LOG OF MINIMUM FLOW

2.5 STATISTICAL PROPERTIES OF THE ABS SERIES

Table 5 is a summary of some of the statistical parameters obtained for the ABS series.

From the table of the mean monthly ABS values, one can observe that on the average, within a complete water year, the largest ABS occurs in April during spring melt. In the months of summer, the high summer precipitation is removed by the high evapotranspiration rate and so the ABS falls off. In October at the beginning of winter, the ABS falls sharply from the summer values and thereafter gradually tapers to a minimum in March. Since there is practically no evapotranspiration and precipitation does not contribute to the ABS in winter, the ABS remains almost constant, with the gradual decrease being due to its depletion by the river.

It can also be observed that all the monthly ABS series exhibit very low skewness which implies that they are close to a normal distribution. They also have high first order autocorrelation coefficient showing that they are serially dependent series.

TABLE 5

STATISTICAL PARAMETERS OF ABS SERIES

Month	Mean ABS (mm)	Standard Deviation	Skewness Coeff	Lag One Ar Coeff	Hurst Stat
April	266.6	93.5	0.21	0.68	0.90
May	246.6	91.7	0.11	0.63	0.92
June	248.4	92.4	0.24	0.63	0.88
July	222.5	92.2	0.24	0.55	0.81
August	192.2	85.5	0.08	0.58	0.87
September	180.3	86.3	0.12	0.62	0.86
October	142.5	79.5	0.18	0.63	0.86
November	141.9	79.2	0.18	0.63	0.86
December	141.5	79.0	0.18	0.63	0.86
January	141.1	78.8	0.18	0.63	0.86
February	140.7	78.6	0.18	0.63	0.86
March	138.0	76.5	0.11	0.65	0.86

The ABS of the months of the farming season from May to September are of interest in this study. Their ABS series are checked for normality by plots of their normal probability shown in Figures 2.6, 2.7, 2.8, 2.9 and 2.10. The mean seasonal ABS series obtained by taking their averages is also checked for normality and shown in Figure 2.11. From the plots, in each case, the number of points outside the confidence band is less that the 5% maximum required for the 5% significance level.

Table 6 shows the correlation between the ABS at the beginning of the growing season (May) with some of the parameters of interest.

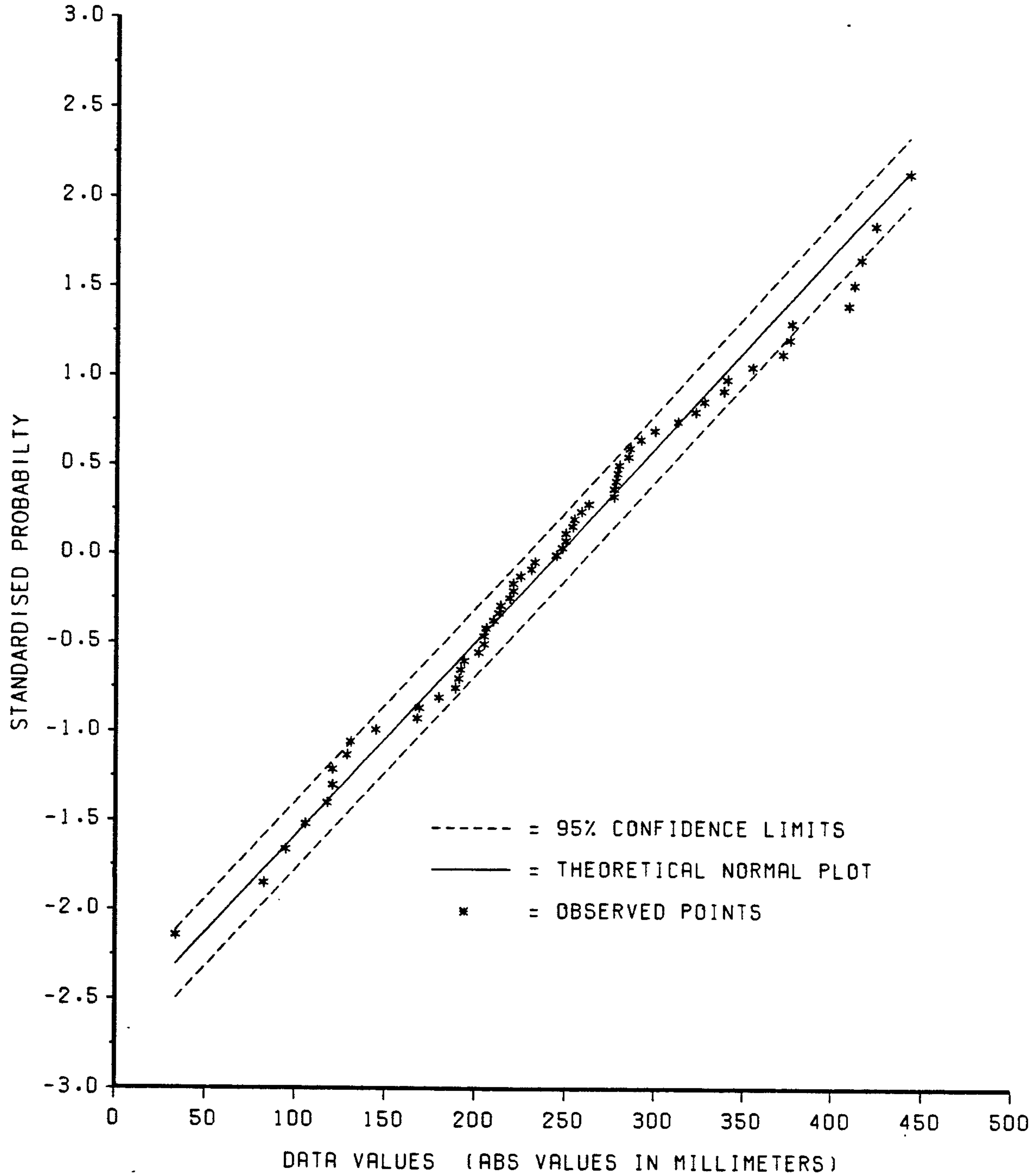

FIG. 2.6 NORMAL PROBABILITY PLOT OF MAY ABS WITH THE 95% CONFIDENCE LIMITS

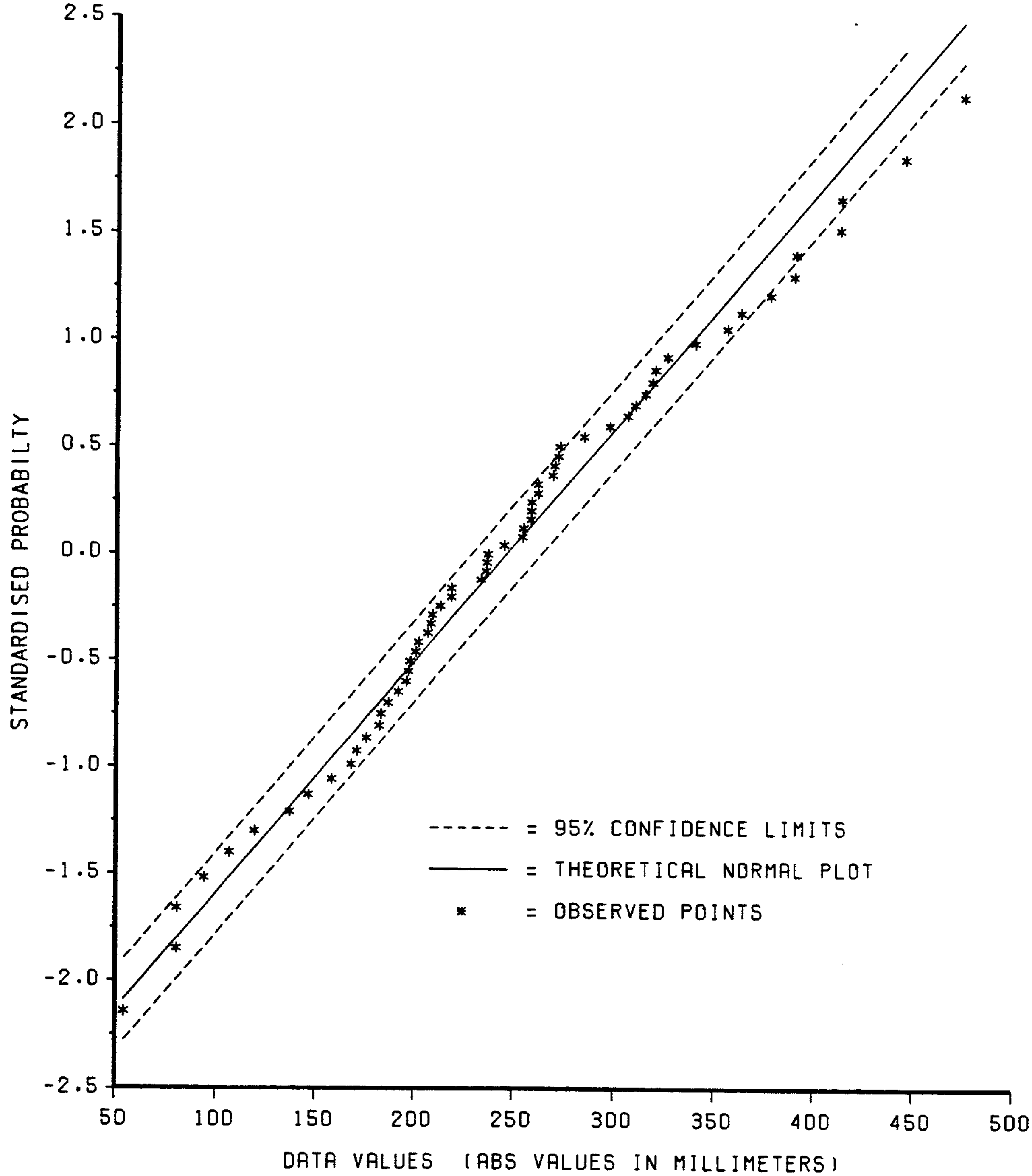

FIG. 2.7 NORMAL PROBABILITY PLOT OF JUNE ABS WITH THE 95% CONFIDENCE LIMITS

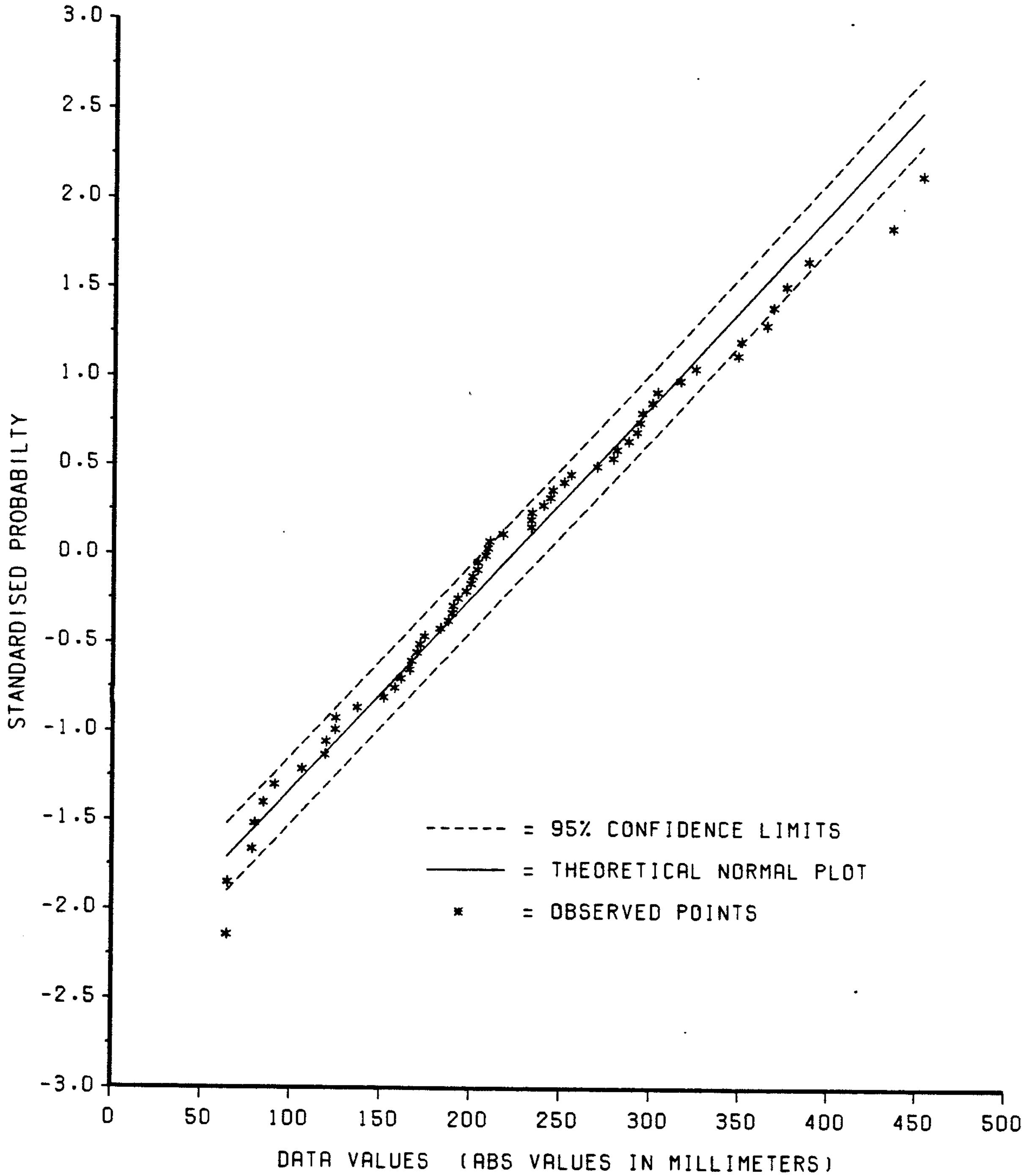

FIG. 2.8 NORMAL PROBABILITY PLOT OF JULY ABS WITH THE 95% CONFIDENCE LIMITS

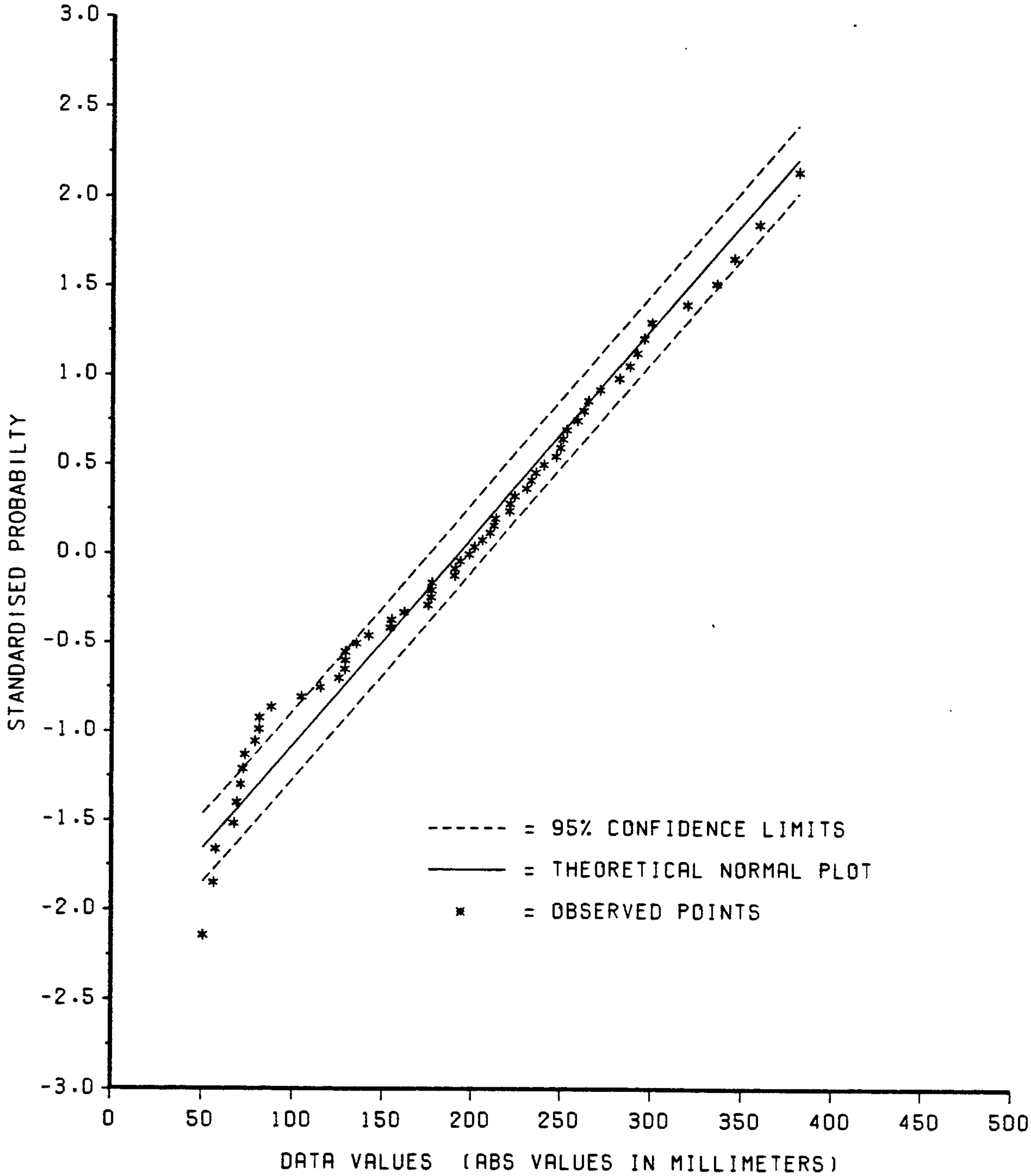

FIG. 2.9 NORMAL PROBABILITY PLOT OF AUGUST ABS WITH THE 95% CONFIDENCE LIMITS

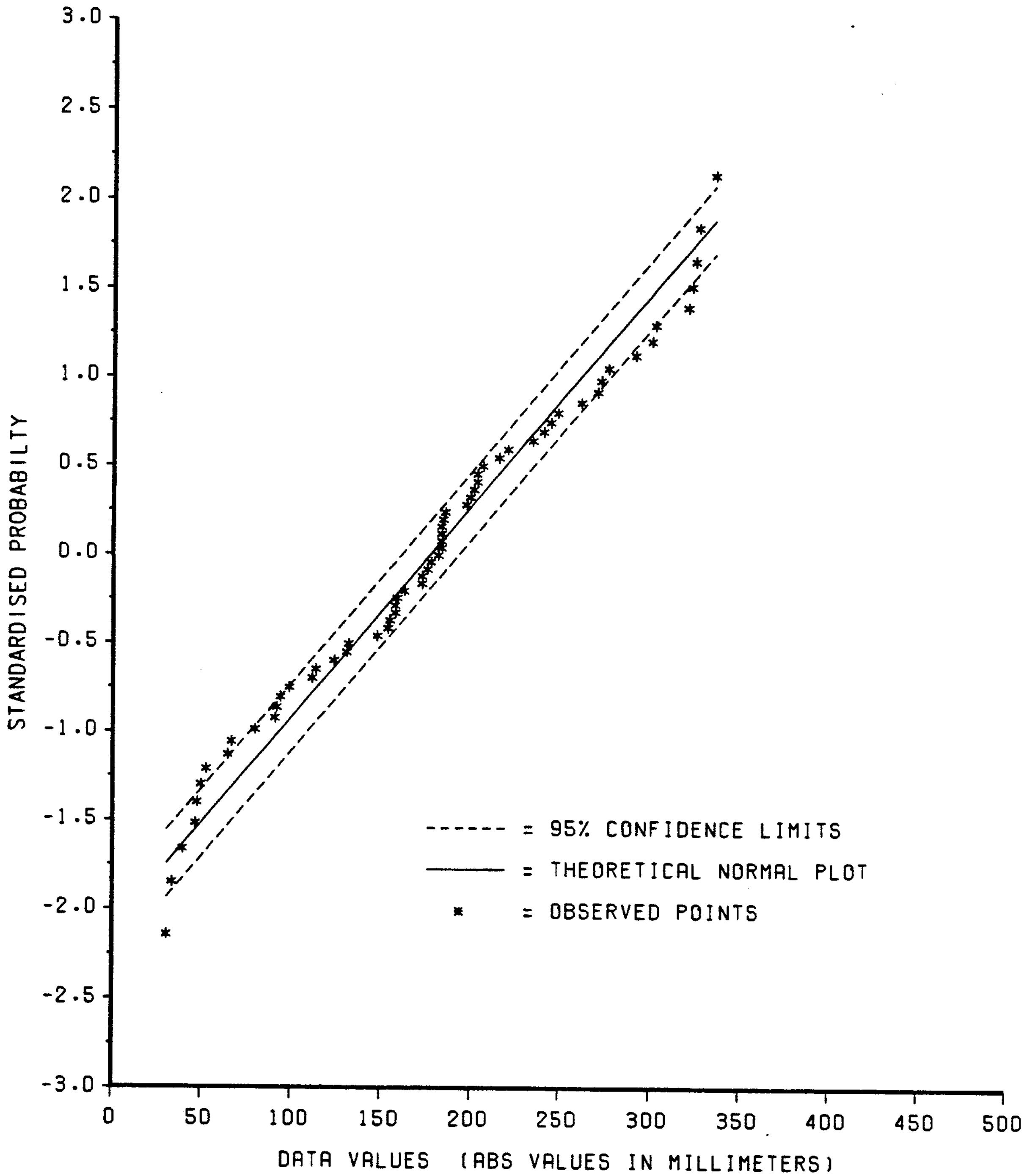

FIG. 2.10 NORMAL PROBABILITY PLOT OF SEPTEMBER ABS WITH THE 95% CONFIDENCE LIMITS

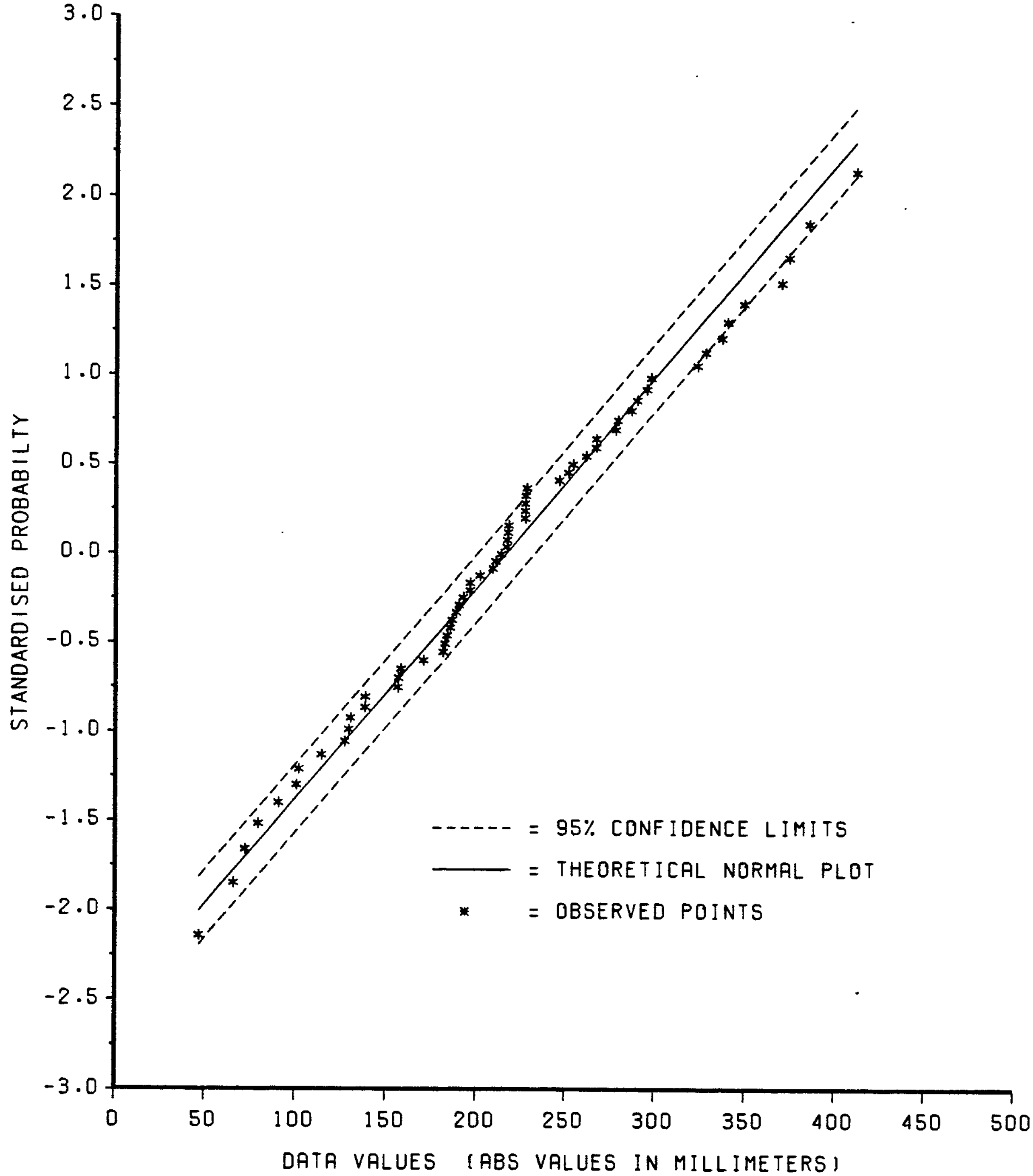

FIG. 2.11 NORMAL PROBABILITY PLOT OF AVERAGE GROWING SEASON ABS WITH THE 95% CONFIDENCE LIMITS

TABLE 6

PARAMETER CORRELATION WITH MAY ABS

Winter Precipitation	0.497*
Summer Precipitation	0.087
Potential Evapotranspiration	-0.240
Annual Streamflow	0.450*
Minimum Winter Flow	0.574*
June ABS	0.927*
July ABS	0.841*
August ABS	0.777*
September ABS	0.744*
Crop Yield Per Acre (Wheat)	0.453*
Average of Season ABS	0.907*

* Significant at 5% level

CHAPTER 3

THE RELEVANCE OF ABS TO AGRICULTURAL DROUGHTS

3.1 WHAT ARE DROUGHTS?

Drought has been called 'the scourge of mankind' since Biblical times. Although the term 'drought' is used loosely to imply the scarcity of anything, (for example; the province is suffering from a drought of investment capital), it is used mainly to describe the shortage of water or moisture within an environment.

The presence of drought is manifested by certain distresses to humans and animals alike in the affected region such as famine as a result of crop and pasture failures, serious bush and forest fires, and severe economic disruptions. Like floods, it brings economic woes to the affected people, but unlike floods, little can be done to protect society from its effects.

Several reasons account for the reason why flood is a well quantified and statistically predictable phenomenon while drought remains a mystery. There is no universally acceptable definition of droughts. Criteria used to identify droughts have been arbitrary because drought is a "non-event" unlike a distinct event like flood. A flood occurs at a distinct time such as immediately after spring thaw or the time of heavy rains. A drought on the other hand has no distinct onset and it is only recognisable after a period of time. Furthermore, it is a rare event so that a statistical analysis very difficult.

3.2 TYPES OF DROUGHTS

A universal drought definition is impossible because the importance of a moisture deficiency is related to the moisture requirement. For example, a 10 mm soil moisture shortage in Kansas (U.S.A.) will not mean the same thing as a 10 mm shortage in Central Iowa (U.S.A), because of different normal climates and different economic activities. Further, the time of occurrence and the duration factors can either classify a moisture shortage into a 'drought' or a 'non-drought' period. A soil moisture shortage in winter will not constitute a drought as far as agriculture is concerned because crop farming does not take place then.

Drought also has different meaning to different people depending on their specific interest. The following are some types of definitions used to identify droughts.

1. "A period with precipitation less than some small amount (say 0.1 inches) in 48 hours" (G. Blumenstock, 1942).

2. "A period of more than some number of days with precipitation less than some amount" (The Meteo Glossary, 1951).

3. "A period of strong winds, low precipitation, high temperatures and unusually low relative humidity" (G. Condra, 1944).

4. "A day on which the available soil moisture was depleted to some small percentage of available capacity" (C.H.M. van Bavel, et al., 1956).

5. "A condition that may be said to prevail whenever precipitation is insufficient to meet the needs of human activities" (J.C. Hoyt, 1938).

One can see from the above definitions, that there is difficulty in placing figures on variables such as 'some small percentage' or 'some number of days'. How small is the 'small percentage of available soil moisture capacity'?

However, one can observe that, despite the arbitrariness of each definition, the concept of moisture shortage is implied in all of them. Lacking in all the definitions, however, is the essence of "prolongness" of the moisture deficit that constitute a drought. A short dry spell may not constitute a drought and is usually necessary to get out of a very wet period.

On the basis of the previous discussion, three types of droughts may be defined. These are agricultural, hydrological and meteorological droughts.

Hydrological drought concerns the falling of water levels in streams, lakes, reservoirs and groundwater table below average so as to affect the supply of water seriously.

Agricultural drought occurs when soil moisture in the root zone of crops become depleted such that wide-spread crop failure occurs.

Both types of droughts may occur simultaneously in an environment of meteorological drought which is the climatic case where the moisture deficiency is within the atmosphere.

A study of hydrologic drought will involve meteorology, hydrology, geology and other geophysical sciences whilst a study of agricultural drought will include in addition, soil physics, plant physiology and agricultural economics.

The causes of droughts is still a subject of controversy, but droughts are known to be associated with anomalous atmospheric circulation patterns. While some researchers believe the causes of the circulation anomaly are extraterrestial, others believe they are self evolving.

3.3 THE RELEVANCE OF ABS TO AGRICULTURAL DROUGHT

Accumulated Basin Storage should be used for agricultural drought conditions because agricultural drought is primarily concerned with soil moisture shortage.

More specifically the ABS has the following advantages over a simple precipitation based index of agricultural drought. The amount of moisture retained in the soil for plant use depends on the soil drainage properties, the amount of moisture accumulated in the soil previously, and present evapotranspiration level. Therefore the lack of precipitation over a period of time does not necessarily mean drought conditions are prevailing since accumulated soil moisture could still be available for plant use. Precipitation alone does not give the overall moisture balance within the basin.

Crop yield per unit area was used to measure how well ABS or any other variable such as precipitation accounts for crop failures within the basin. It should be noted however that crop yield is also dependent on other factors like improvement in seed types, use of chemicals for weed control and fertilization, and cultural practices that go to improve soil conditions, reduce evapotranspiration or retain soil moisture.

Table 7 shows the correlation between crop yield per hectare and the variables of interest. The crop yield data was taken from the annual 'Year Book - Manitoba

Agriculture' which had earlier been known as (between 1914 to 1936) 'Report On Crops And Livestock - Department of Agriculture And Immigration'. Records of two reporting stations within the basin, Russell and Virden, were averaged and used as a basin average. Specifically, the total annual wheat crop is used because wheat is the largest cultivated crop in the Assiniboine River Basin and it is not irrigated. Forage yield is also a good indicator but data for it was not available. The ABS values used here are the mean seasonal ABS for the growing season between May and September.

TABLE 7

RELATION BETWEEN CROP YIELD AND OTHER FACTORS

Variable	Correlation With Crop Yield Per Acre
ABS	0.53 (Significant at 95% level)
Total Annual Ppt	0.25 (Significant at 95% level)
Annual Runoff	0.15 (Not significant at 95% level)
Evapotranspiration	-0.09 (Not significant at 95% level)
Winter Precipitation	0.22 (Not significant at 95% level)
Summer Precipitation	0.17 (Not significant at 95% level)

From the above table, one can conclude that a properly derived ABS is a better parameter for measuring agricultural drought than precipitation. To obtain a visual comparison between the growing season average ABS and crop yield per unit area, a plot was made of their standardised values and shown in Figure 3.1. One can observe that both variables follow the same pattern quite well.

Therefore the ABS should be useful as a drought parameter. It would be useful for the analysis of droughts and also for the prediction of agricultural droughts.

3.4 DEFINITION OF AGRICULTURAL DROUGHT FOR THIS STUDY

In order to develop an objective definition for agricultural drought for this study, the following generalised 'guideline' definition will be used as a basis.

> "A drought period is an interval of time during which a given place experiences a prolonged and abnormal moisture deficiency so as to disrupt the established economy".

Abnormality in this definition means a pronounced deviation from what has been established as the middle point between the extremes of a variable (the mean or median). 'Prolonged' is the duration factor that

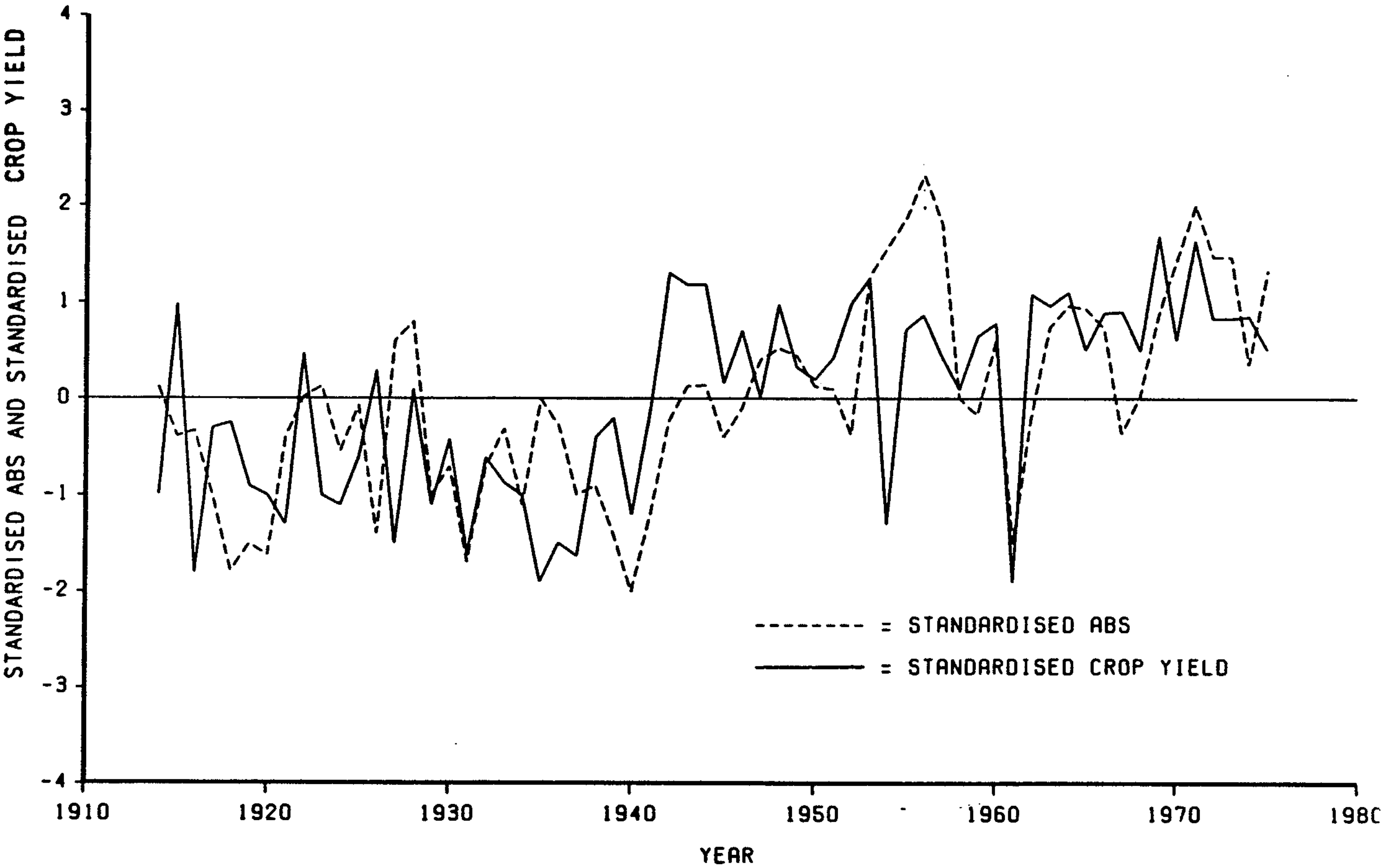

FIG. 3.1 PLOTS OF STANDARDISED GROWING SEASON ABS AND STANDARDISED CROP YIELD PER HECTARE

distinguishes a brief dry spell from a disastrous drought period. 'Established economy' is the human economic activities which are based on available moisture within the study area.

Therefore, based on the basin variable, ABS, an agricultural drought occurs when the ABS value of the river basin falls below a certain value over a period of time. This reference level should be so low as to constitute a condition of abnormality.

As noted earlier the ABS is measured with respect to time, and in this study, it is derived on monthly basis. However, for the purpose of drought definition, short averaging periods such as daily or monthly means may give dry spells which are not droughts. The dryness should be prolonged over a period such as the whole growing season in order to be called a drought. The averaging period to be used is therefore the growing season, May through September. The ABS within the winter season of the water year is not critical to crop farming, and is therefore not considered. The mean ABS of the growing season is also used because there are a variety of crops cultivated within the basin, each with its own critical period with respect to moisture needs. It is therefore not possible to use the ABS values of a particular month for the purpose of drought definition.

To determine the reference level of ABS that separates drought periods from wet periods, it will be assumed that farming in any region is adapted to the prevailing climatic pattern. This implies that due to variations in annual precipitation, a seasonal drought of certain durations and magnitudes are often a normal feature of that particular climate. Thus, as the hydrological inputs of ABS vary from year to year, farming is adjusted to this variability to a considerable extent. Consequently, only ABS values in the extremely low levels will constitute droughts.

The observed historical droughts can be used to give an indication of the reference level. Using average wheat yield per acre in the region, Table 8 states some observed historical droughts. The variables in the table are standardised by subtracting their mean from their values and dividing them by their respective standard deviations.

These obvious historical drought periods can be obtained depending on the reference level used. Using a reference 101 mm which is the value of ABS that is exceeded 90% of the time, it can be seen from Figure 3.2 that only these historical droughts are identified. The reference level can therefore be the 90% greater exceedence value. In the study, the 90% value will be used throughout. A series of drought events will therefore be a partial duration series of the ABS series.

TABLE 8

HISTORICAL DROUGHTS

Year of Drought	Standardised ABS	Standardised Crop Yield
1918	-1.78	-0.24
1919	-1.49	-0.89
1920	-1.62	-0.99
1931	-1.71	-1.61
1939	-1.35	-0.2
1940	-2.00	-1.2
1961	-1.50	-1.9

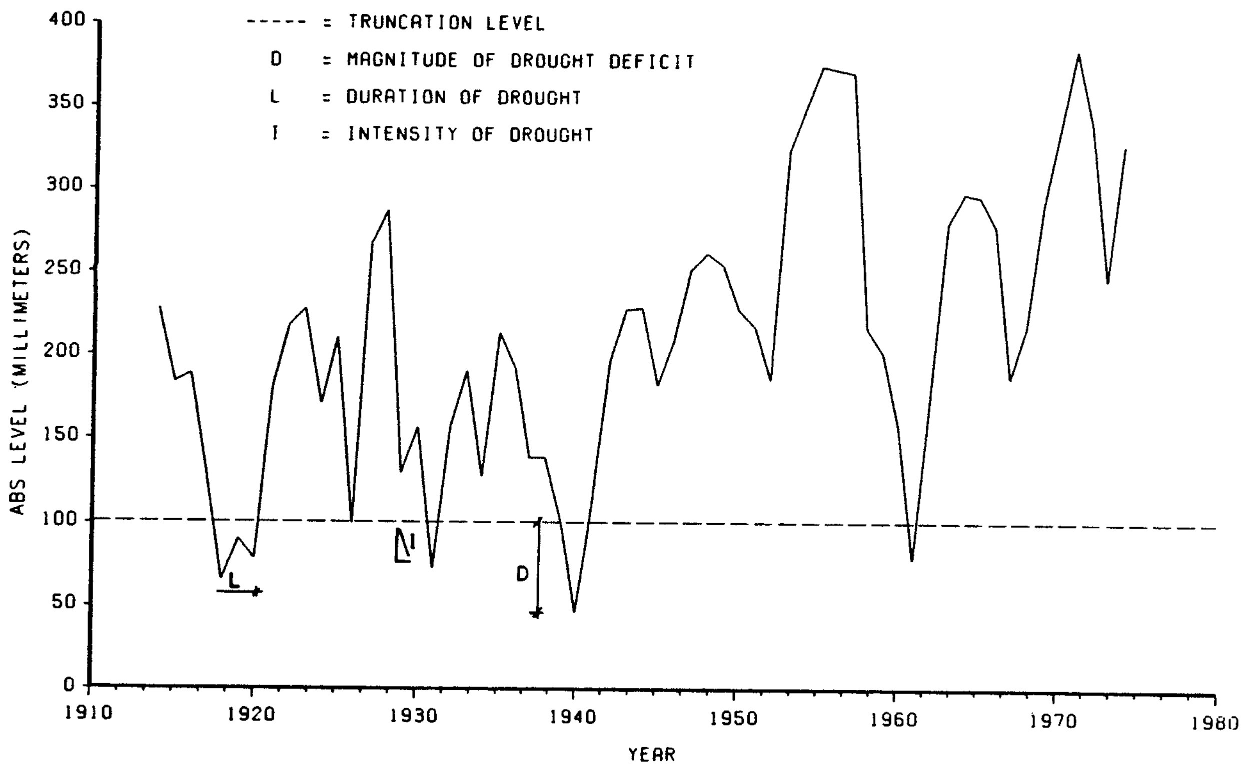

FIG. 3.2 HISTORICAL DROUGHT DEFINITION USING A.B.S

3.5 DROUGHT VARIABLES RELEVANT TO AGRICULTURE

A drought event can be defined by the magnitude of moisture deficit observed and the duration over which the drought extends. If it is the aim of the planner to store water for averting droughts in time of need, he will be interested in knowing the total volume needed and the period over which he will have to supply this volume. In particular, he will be interested in the probabilities of the maximum deficit and the longest durations for design purposes.

Another drought variable that will be of interest is Drought Intensity, which is the rate at which the moisture deficit occurs. This will give a measure of how severe the drought is, since the effect of a drought with a large deficit occurring within a short time will not be the same as one of the same magnitude occuring over a relatively longer period. The intensity is taken as the slope of the ABS curve when the drought begins up to when the maximum deficit is registered. This will give the rate at which water will be needed within a drought period.

One would also be interested in the rate of occurrence of drought, that is the expected number of droughts within a given period, in the basin. For example, consider a multipurpose water reservoir project to generate power, supply water for irrigation on contingency basis, for fishing and other purposes. One would be interested in knowing how many times within the project that droughts will occur so that it would be necessary to supply the farms with water, besides the need to know the total volume to supply and the duration.

Other possible uses of these probabilities are for determining the economic benefit of the projects, the risk of crop failure in the basin for insurance purposes, and for fire protection measures.

Since a drought event is defined by two variables, it will be necessary to consider both simultaneously. This can be achieved by considering the maximum deficit as the primary variable and determine the conditional probability of drought duration given the maximum deficit is equal to a given value.

SECTION II

STATISTICAL PROPERTIES OF DROUGHT VARIABLES

CHAPTER 4

ABS DATA GENERATION MODEL

4.1 THE NEED FOR SIMULATION OF THE ABS

Simulation of a water resources system is the method of obtaining hidden information about its performance by performing a large number of experiments using a model of the system. For example we might want to have an idea about its reliability or the number of failures in a unit time.

For the problem at hand, the system is the river basin, with inputs as precipitation and output as runoff and evapotranspiration. Its performance is measured by the amount of moisture retained in the soil to support plant life and is indicated by the index ABS. If the generating mechanism of the input and output of the system can be adequately represented by a mathematical model, then one can use this model to perform experiments to extract information on the rate at which the system fails (in this case, the occurrence of agricultural droughts).

It is observed that in practice, decisions requiring hydrologic knowledge is usually based on some hydrologic model of some kind. For example, in a flood control project, why is the highest flood magnitude observed historically not used as the design magnitude, but instead a model is developed to give a series of flood magnitudes? This is because it is recognised that the historical record is simply a sample of the parent population hydrological series and there is no guarantee that future realisations will be the same or less severe than the historical one. Furthermore, the historical records are usually short, and also, due to sample fluctuations, generally contain unusually large or unusually small values of the parameter of interest.

In the case of agricultural droughts, the inadequacy of the historical record is even more severe. Unlike streamflow drought which may occur annually, agricultural droughts occurrence is in decades. This makes statistical studies of this 'non-event' near impossible and the only approach is by the experimental method. For the basin in the study for example, out of the 62 years of ABS record, there were only four drought events based on the definition described earlier. One cannot therefore make any probability statements about the historical droughts and hence the need for simulation.

4.2 STOCHASTIC MODEL SELECTION

The first step in the simulation study of any hydrological system is the selection of a generating model for the time series. The appropriate type of model for this study is the STOCHASTIC type whose generation mechanism is probabilistic.

The model parameters must be estimated from the historical data which are assumed to be representative of the population time series. Since 62 years of historical record are available, this assumption sounds reasonable considering a planning horizon of 100 years.

Several types of stochastic models are available to model the ABS time series. These include autoregressive models, fractional gaussian noise, autoregressive moving average (ARMA), broken line model, etc. The type of model that fits best will be the one whose theoretical parameters compare well with the observed ones. The selected model should also pass a diagnostic test, and if possible, be justified by physical considerations of the hydrologic system.

Table 9 is a summary of the statistical parameters of relevance to the generating model.

TABLE 9

STATISTICS OF THE ABS RECORD

Parameter	Value
Mean (mm)	218.8
Standard Deviation	85.3
Skewness	0.17
Serial Correlation Coefficient (1st Order)	0.69
Partial Autocorrelation Coefficient (1st Order)	0.69
P.A.C.F (2nd Order)	-0.003
Hurst Coefficient	0.88
Rescaled Range	20.7

The low skewness value, and the plots and tests of normality in Section 2.5 showed that the data is approximately normal. The plot of the correlogram shown on Figure 4.1 suggest an autoregressive model, since the correlogram dies out rapidly, and plots of the tolerance limits showed the data is not independent. Since the averaging period used is the seasonal one (May to September) which is equivalent to annual series, it is safe to assume there is no periodicity inherent in the

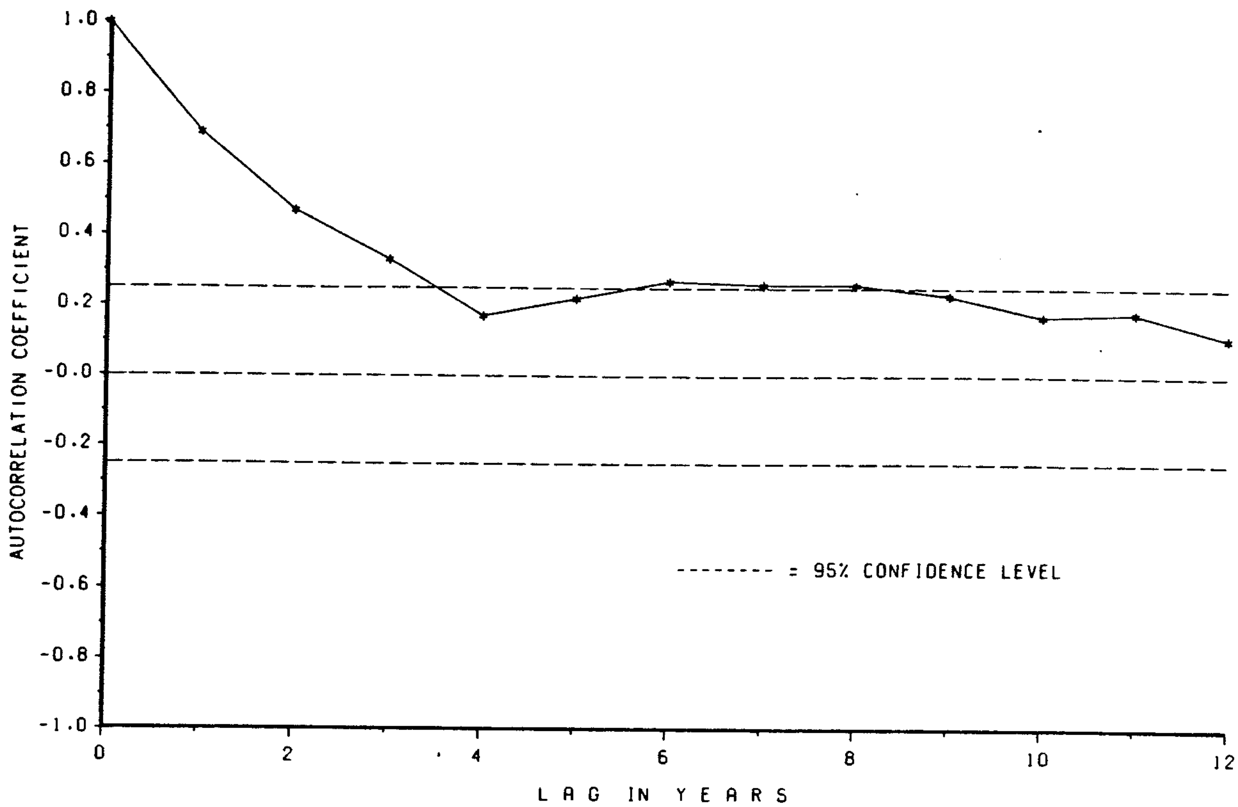

FIG. 4.1 PLOT OF CORRELOGRAM OF AVERAGE ABS

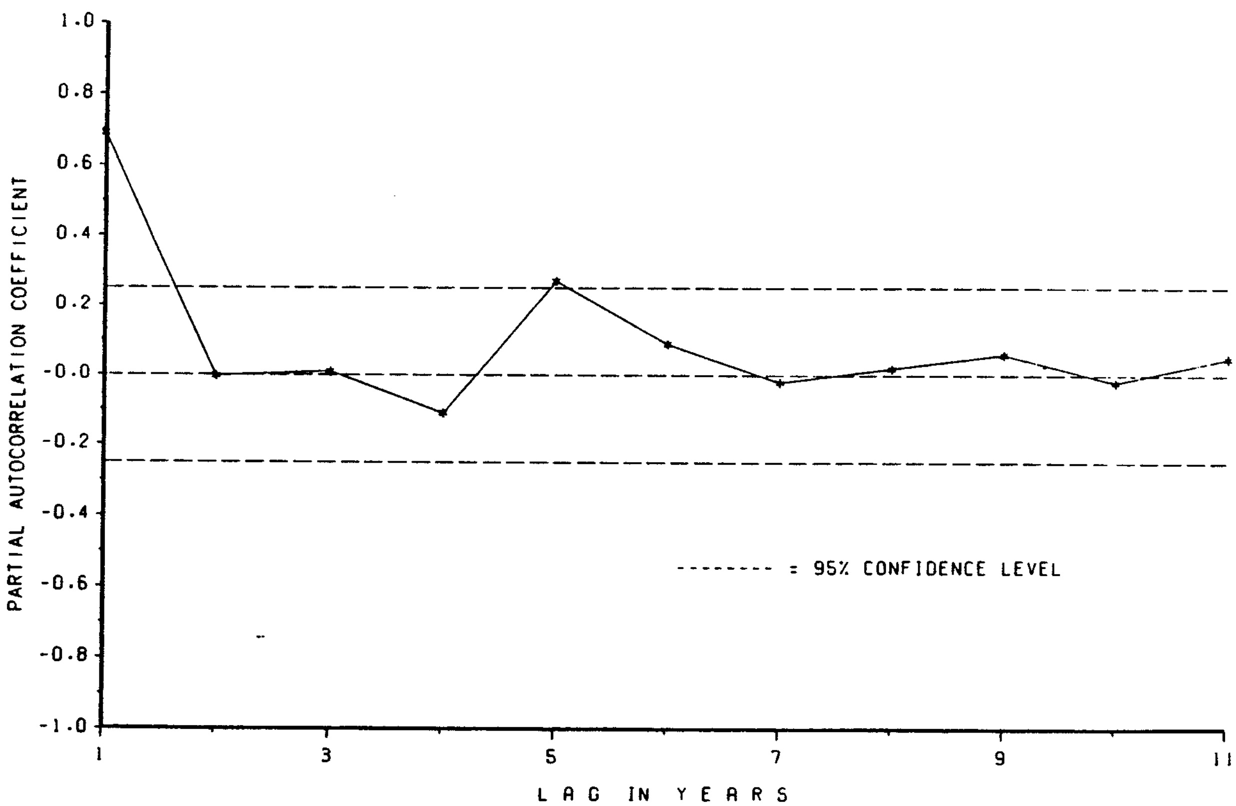

FIG. 4.2 PLOT OF P.A.C.F OF SEASONAL ABS

series. We can therefore consider only linearly dependent autoregressive models and not periodic models.

Two types of linearly dependent models are reviewed here. These are autoregressive (or Markov) model and a variation of it called fractional gaussian noise (FGN). FGN is used to overcome the shortcoming of Markov models when it does not reproduce observed persistence in the time series. This occurs when the serial correlation coefficient is relatively small whilst the measure of persistence, the Hurst coefficient is relatively large. In the problem at hand, we observe a very high serial correlation which accounts for the corresponding high Hurst coefficient. Hence the Markov model should reproduce this persistence. The problem is now to determine the order of the Markov model that will adequately represent the series. It should be noted that most natural hydrologic series follow the lag one model. The lag one autoregressive parameter is usually large compared with the lag two parameter if a multi-lag model is adopted, thus the second order terms tend not to improve the model very much.

To determine the order of the Markov model, a plot of the partial autocorrelation function was made, and is shown in Figure 4.2. It can be observed from the plot that the lag one partial autocorrelation coefficient is significant. The lag 5 coefficient also lies just above the tolerance limit but this may be due to chance only, since those of lags 2, 3, and 4 are not significant. Therefore the lag one Markov model will be selected and justified by diagnostic test. After parameter estimates, the model becomes:

$$ABS_t = \overline{ABS} + \rho(ABS_{t-1} - \overline{ABS}) + \sigma\sqrt{(1-\rho^2)}\ \varepsilon_t \qquad \ldots (4.1)$$

where;

ABS_t = accumulated basin storage in year t

ABS_{t-1} = accumulated basin storage in year t-1

ρ = first order serial correlation coefficient = 0.69

σ = standard deviation of the ABS record

$\overline{ABS}$ = mean of the ABS record

ε_t = a normal random variate.

For a standardized series, which is obtained by subtracting the mean from the variable ABS and dividing it by its standard deviation, the mean becomes zero and its standard deviation one. Hence we have:

$$ABS'_t = ABS'_{t-1} + \sqrt{(1-\rho^2)}\ \varepsilon_t \qquad \ldots (4.2)$$

If the above model is appropriate, then the following test should be passed:

From Equation (4.2) we obtain:

$$ABS'_t - ABS'_{t-1} = (1-\rho^2)^{1/2}\ \varepsilon_t \qquad \ldots (4.3)$$

Since is a constant and ε_t is an independent variable, then the residuals, which is $ABS'_t - ABS'_{t-1}$, should be independent.

Test of independence of the residuals was conducted by plotting its correlogram. From the plots in Figures 4.3 and 4.4 one can conclude that the residuals of the lag one Markov model are independent and normally distributed. This condition holds if and only if the model were lag one model. For example, consider the lag two model:

$$ABS'_t = \phi_1 ABS'_{t-1} + \phi_2 ABS'_{t-2} + \phi_3 \varepsilon_t \qquad \ldots(4.4)$$

then the residuals,

$$ABS'_t - \phi_1 ABS'_{t-1} = \phi_2 ABS'_{t-2} + \phi_3 \varepsilon_t \qquad \ldots(4.5)$$

will not be independent because of the presence of the term $\phi_2 ABS'_{t-2}$ in the right hand side of equation (4.5).

4.3 A PHYSICAL JUSTIFICATION FOR THE SELECTED MODEL

To find a physical basis for the above model, consider the ABS as groundwater storage with a linear outflow [Salas and Smith, 1981].

Let X_t be the total precipitation in year t.

Let aX_t of the total precipitation infiltrate into ground.

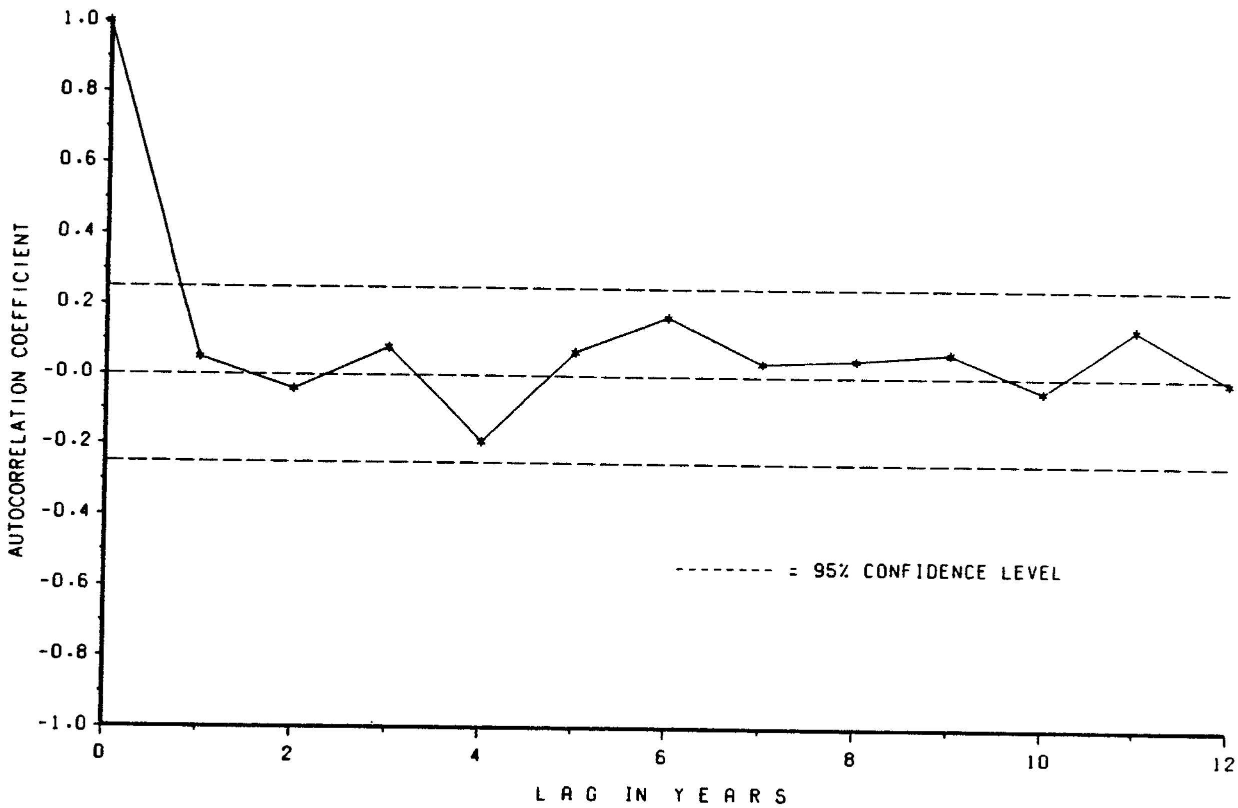

FIG. 4.3 PLOT OF CORRELOGRAM OF RESIDUALS

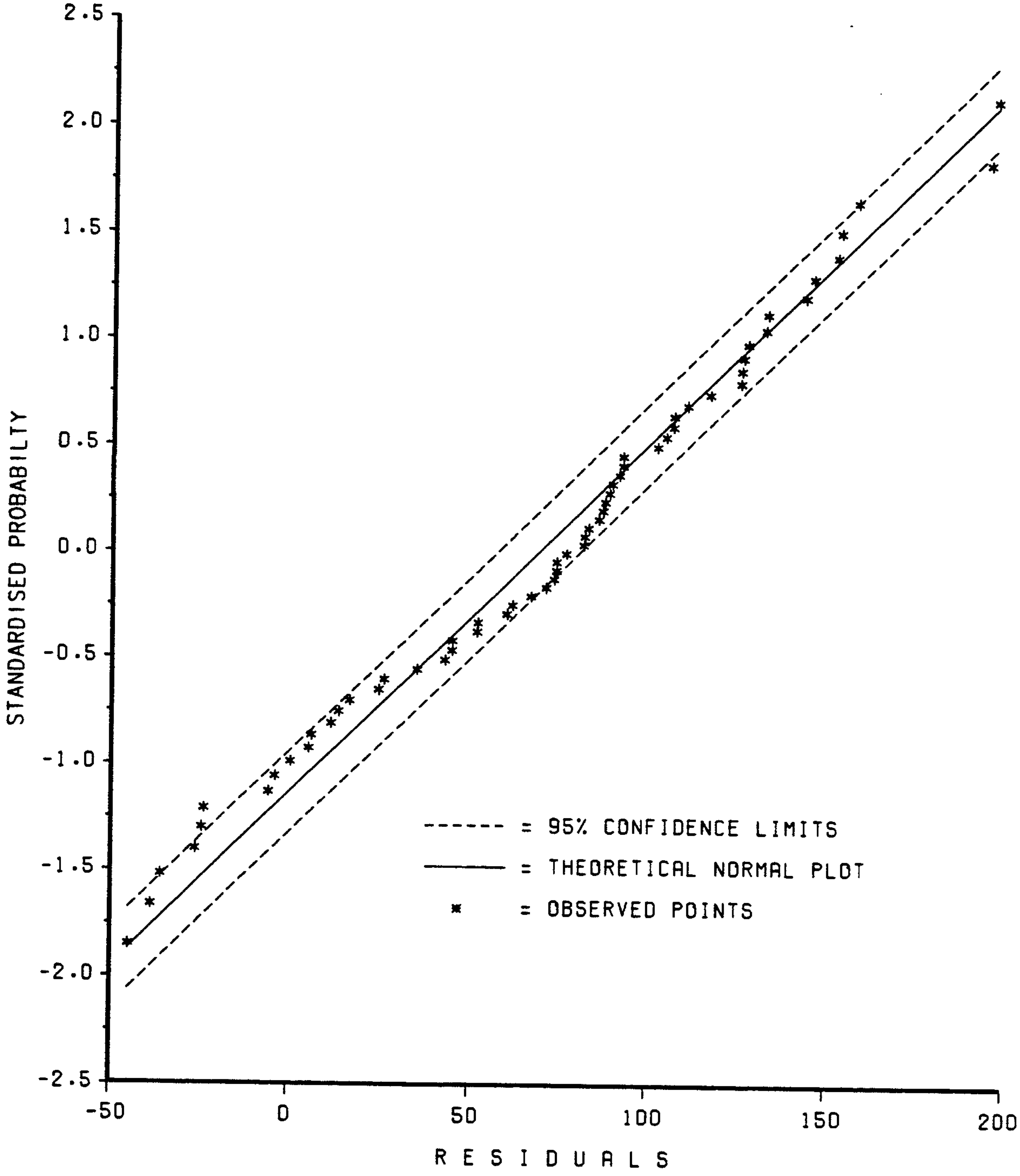

FIG. 4.4 NORMAL PROBABILITY PLOT OF RESIDUALS WITH THE 95% CONFIDENCE LIMITS

Also let S_{t-1} be the groundwater level at the end of the year.

Also let $c(S_{t-1})$ be the contribution of it to the stream. Then in year t, the mass balance for groundwater storage is:

$$S_t = (1 - c)S_{t-1} + aX_t \qquad \ldots (4.6)$$

Whether the above physical model is lag one or multilag type depends on the correlation structure of the precipitation term X_t.

For the case of a precipitation which is an INDEPENDENT PROCESS, then Equation (4.6) can be written as:

$$S_t = \phi S_{t-1} + \varepsilon_t \qquad \ldots (4.7)$$

which is similar to lag one model.

If the precipitation were an AR-1 process, then we will have an AR-2 model for the groundwater storage as proved below.

The AR-1 precipitation can be written as:

$$X_t = \phi X_{t-1} + \varepsilon_t \qquad \ldots (4.8)$$

From Equation (4.6) we obtain $aX_t = S_t - (1 - c)S_{t-1}$ and hence,

$$aX_{t-1} = S_{t-1} - (1 - c)S_{t-2} \qquad \ldots (4.9)$$

Putting Equation (4.8) into Equation (4.6) we obtain:

$$S_t = (1 - c)S_{t-1} + a(\phi X_{t-1} + \hat{\varepsilon}_t) \text{ or}$$

$$S_t = (1 - c)S_{t-1} + \phi(aX_{t-1}) + a\,\hat{\varepsilon}_t \qquad \ldots(4.10)$$

Putting Equation (4.9) into Equation (4.10) we obtain:

$$S_t = (1-c)S_{t-1} + \phi[S_{t-1} - (1-c)S_{t-2}] + a\,\hat{\varepsilon}_t \quad \text{or}$$

$$S_t = (1 - c + \phi)S_{t-1} - (1 - c)\phi S_{t-2} + a\,\hat{\varepsilon}_t \qquad \ldots(4.11)$$

Let $\alpha = 1 - c + \phi$

$\beta = \phi\,(c - 1)$, then we have;

$$S_t = \alpha\,S_{t-1} + \beta\,S_{t-2} + \hat{\varepsilon}_t \qquad \ldots(4.12)$$

which is the form of AR-2 process.

For the ABS system, the possible parameters that can be put in a simulation model can be obtained by considering the correlation between them. The ABS model is given as:

$$ABS_t = ABS_{t-1} + PW_{t-1} + PS_t - c.PE_t - R_t$$

In Table 10 the correlation coefficients are given among the other variables.

From Table 10, we observe that the runoff term is significantly correlated to all the other variables whilst there is no correlation amongst winter precipitation, summer precipitation, evapotranspiration and ABS_{t-1}.

TABLE 10

CORRELATION MATRIX

	ABS_{t-1}	PW_{t-1}	PS_t	PE_t	R_t
ABS_{t-1}	1.00				
PW_{t-1}	0.179	1.00			
PS_t	-0.064	0.008	1.00		
PE_t	-0.212	-0.021	-0.179	1.00	
R_t	0.442*	0.412*	0.28*	-0.33*	1.00

* SIGNIFICANT CORRELATION AT 5% LEVEL

Since the runoff term is correlated to all the other terms it cannot be used directly in the simulation model.

The runoff term will therefore be deleted and expressed as a linear function of the other variables plus a random error term. Therefore, the ABS model can be reduced to the form:

$$ABS_t = aABS_{t-1} + bPW_{t-1} + cPS_t + dPE_t + \mathcal{E}_t \quad (4.13)$$

where a, b, c and d are constants and $\mathcal{E}_t$ is a random term.

If the winter precipitation, summer precipitation and evapotranspiration are all independent processes, then, as shown for the case of groundwater storage, the ABS model will be a lag one A-R process, since we can combine all the independent terms into one as ϵ_t. Therefore,

$$ABS_t = a\ ABS_{t-1} + \epsilon_t \qquad \ldots(4.14)$$

The independence of winter, summer precipitation and evapotranspiration were tested by the plot of their correlograms in Figures 4.5, 4.6, and 4.7. From the plots we conclude that these variables are all independent processes.

After checking for stationarity of the series, and using Yule-Walkers autoregressive parameter estimates, the final model for generating the ABS series is:

$$ABS_t = 218.8 + 0.69(ABS_{t-1} - 218.8) + 61.7\ \mathcal{E}_t \qquad (4.15)$$

This model was used to generate ABS series of the length equal to the historical record of 62 years. The Normal Random Deviate was generated from the generator GGUBFS available in the Mantes computer Library for this purpose. The generation process was replicated 100 times and the average values of the statistical parameters taken, with the results below:

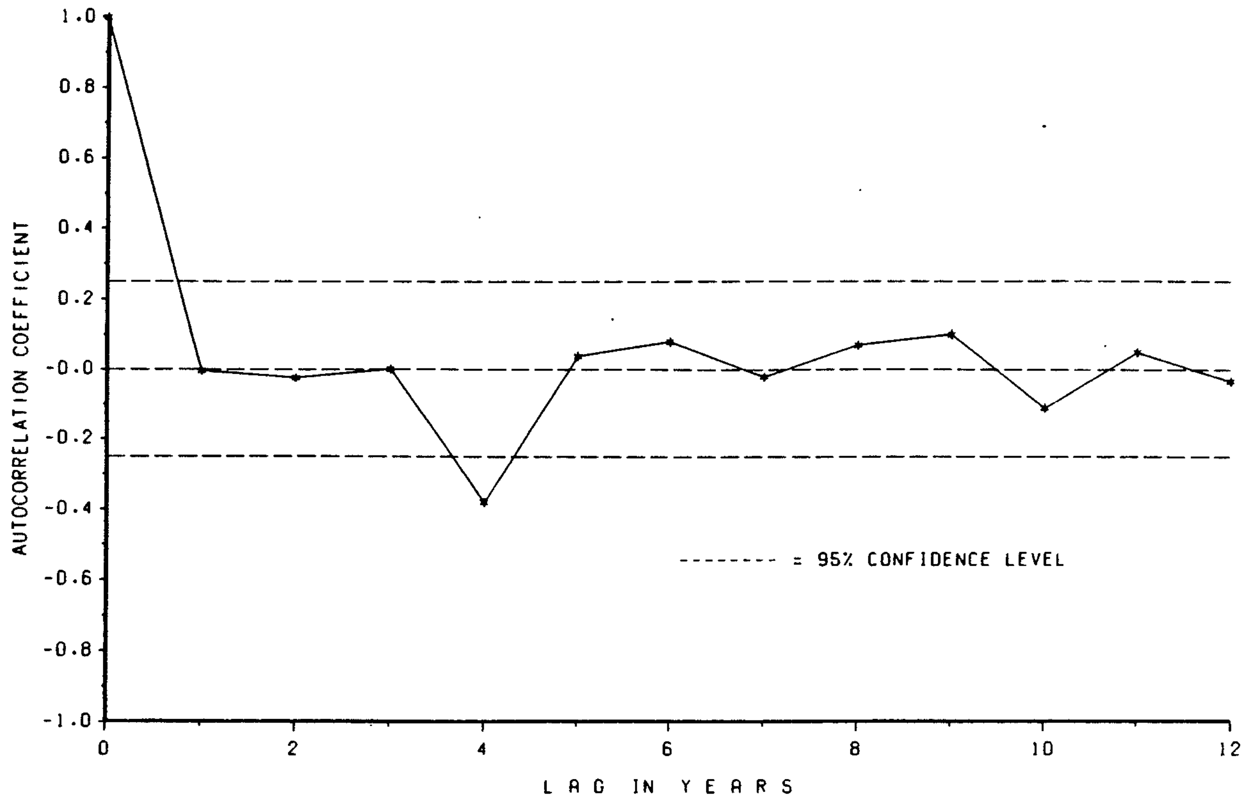

FIG. 4.5 PLOT OF CORRELOGRAM OF SUMMER PRECIPITATION

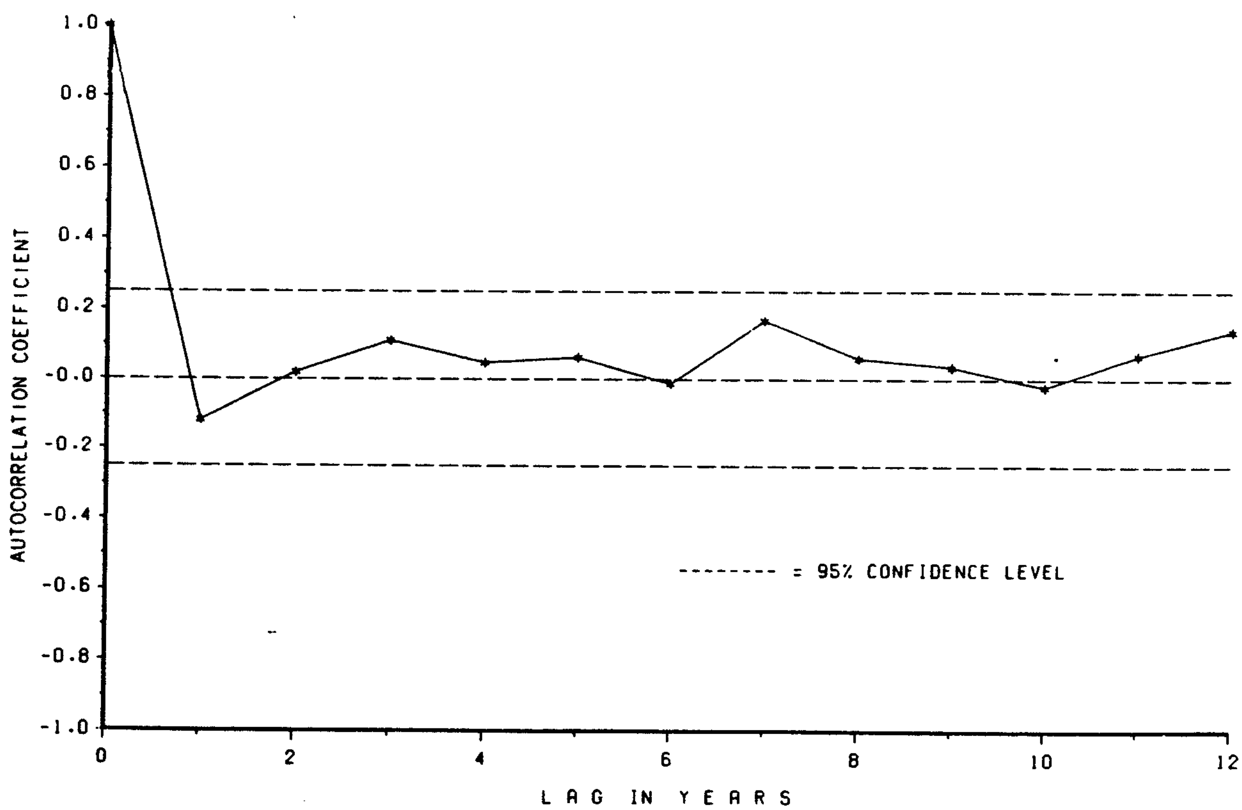

FIG. 4.6 PLOT OF CORRELOGRAM OF WINTER PRECIPITATION

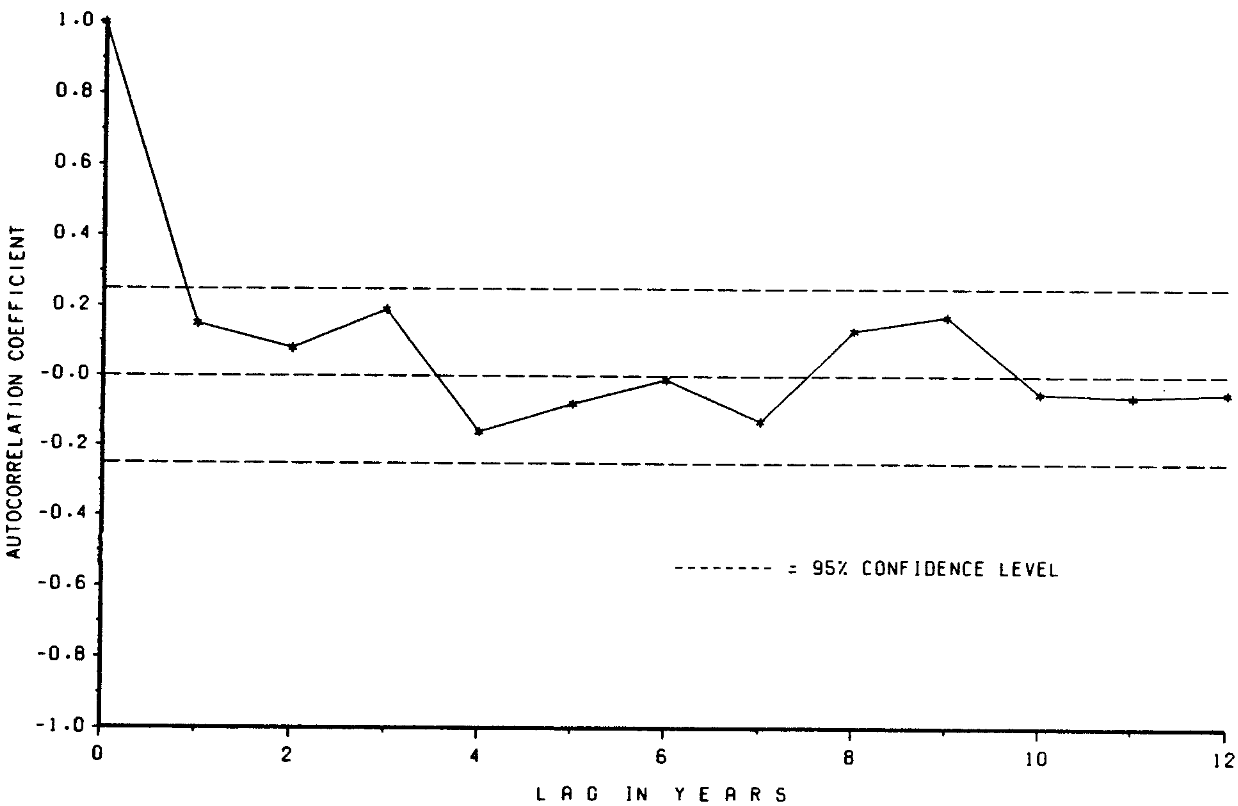

FIG. 4.7 PLOT OF CORRELOGRAM OF EVAPOTRANSPIRATION

TABLE 11

COMPARISON OF THE HISTORICAL PARAMETERS WITH THE GENERATED ONES

	Historical	Synthetic	U95% Limit	L95% Limit
Mean	218.8	219.3	266.5	172.2
Std Deviation	85.3	82.0	88.6	79.4
Serial Correlation Coefficient	0.69	0.63	0.83	0.43
Hurst Coefficient	0.88	0.86	1.0	0.71
Skewness	0.17	0.10	0.8	0.0

The distributions of the Hurst statistics and the first-order autocorrelation coefficients of the generated samples were determined. They were found approximate to the theoretical normal distribution as shown in Figures 4.8 and 4.9. These historical values of the Hurst statistic and first order autocorrelation coefficients can be observed to be well within the confidence limits of the model (given in Table 11).

Since all the historical parameters fall within the confidence limits, we conclude that at 95% level, the model adequately reproduces the historical sample.

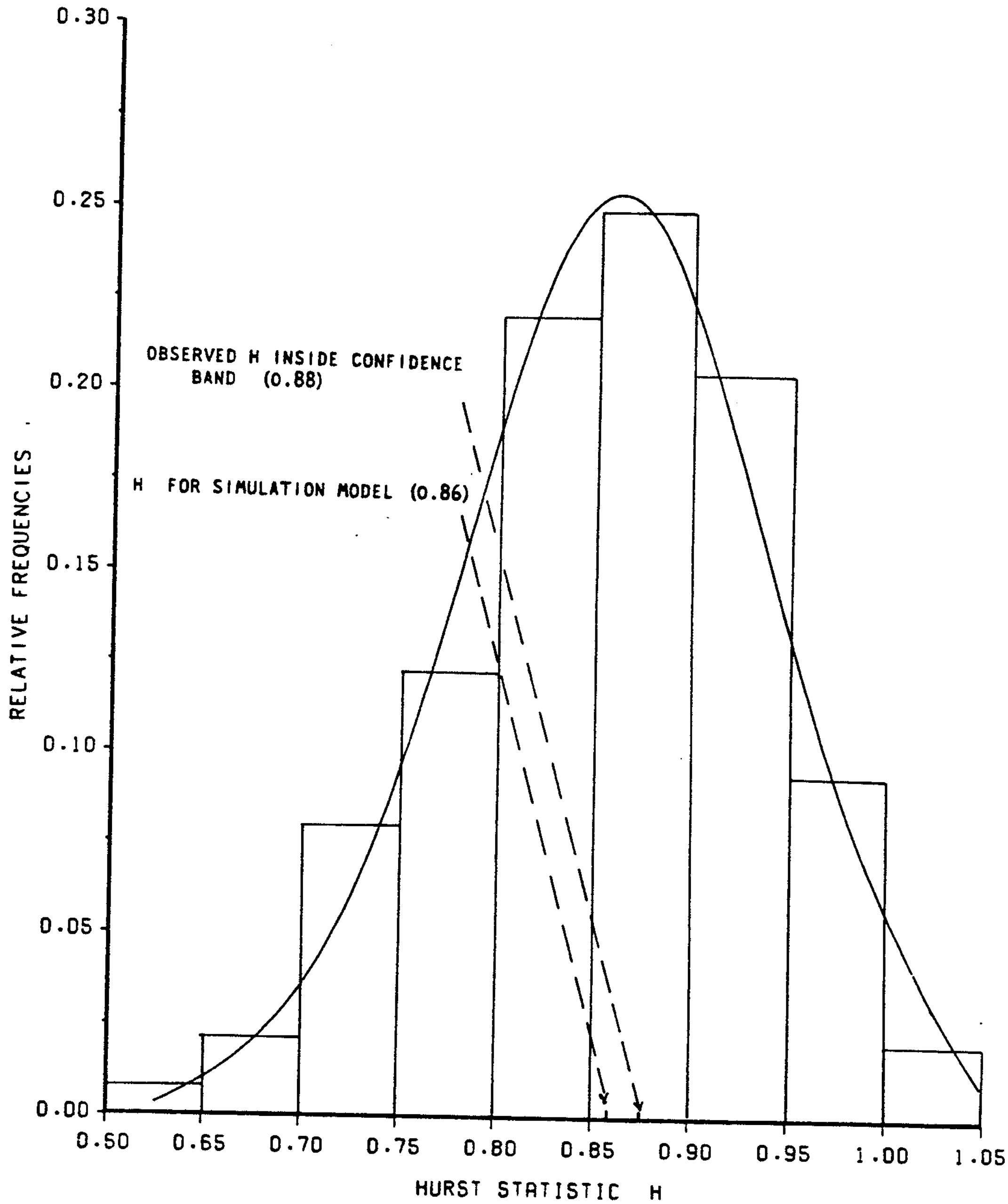

FIG. 4.8 PROBABILITY DISTRIBUTION OF THE HURST STATISTIC FOR THE GENERATED ABS SERIES

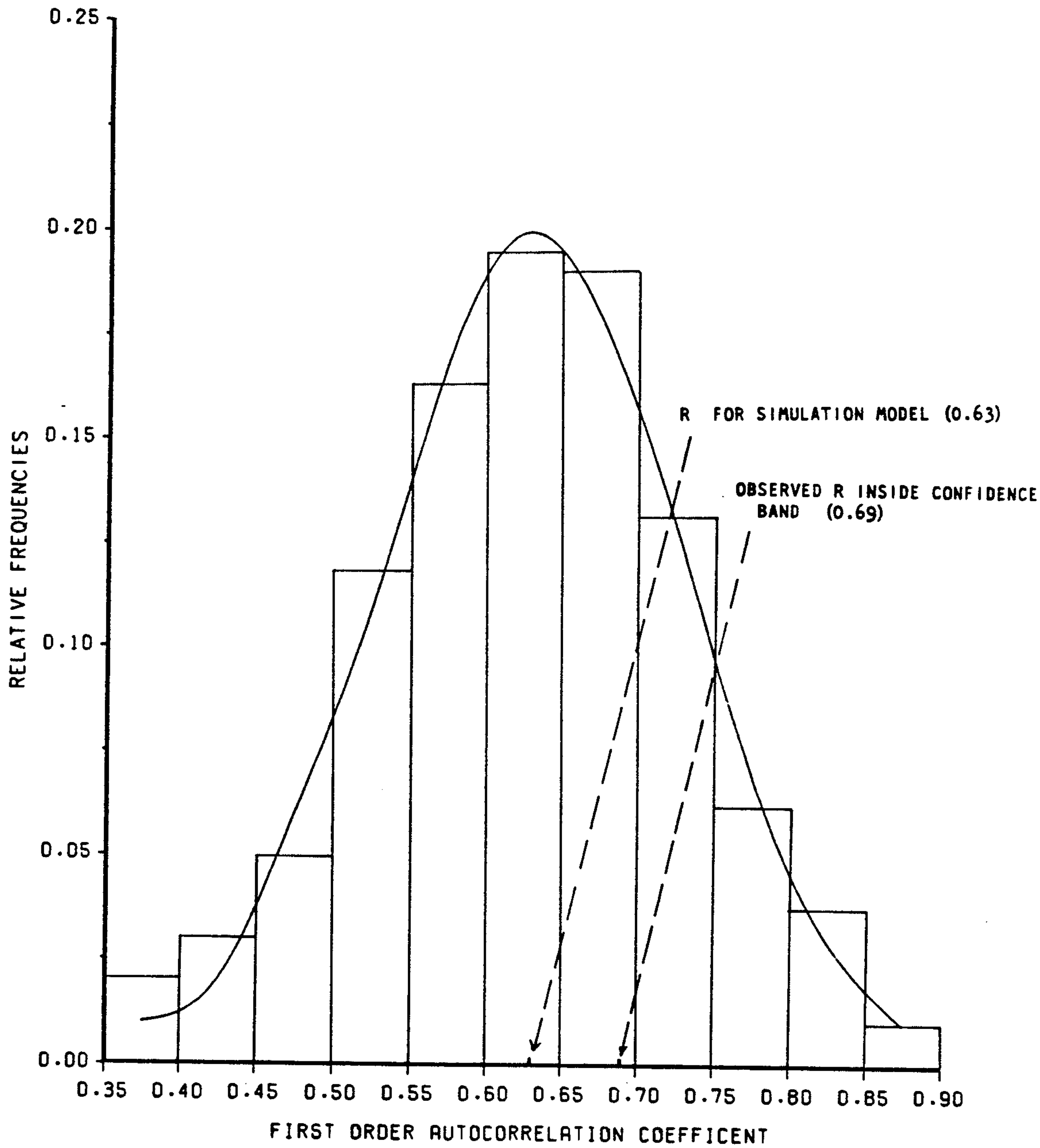

FIG. 4.9 PROBABILITY DISTRIBUTION OF CORRELATION COEFFICIENT FOR THE GENERATED ABS SERIES

CHAPTER 5

PROBABILITY DISTRIBUTION OF GENERATED DROUGHT VARIABLES

5.1 THE DISTRIBUTION OF THE NUMBER OF DROUGHTS

The probability distribution of the number of droughts in a period of time is an important aspect of the stochastic studies of agricultural droughts. All those concerned with agricultural economic activity in the area would be interested in how many droughts to expect in a project life. Furthermore, the distribution of the number of droughts will be used subsequently to derive the distribution for other drought variables.

In fitting a distribution to a random variable, if a parallel can be drawn between the problem at hand and any other phenomenon for which there was a solution, then one can narrow down the range of possible distributions.

Droughts and floods can be considered as extreme values and this is the basis of drought definition outlined in Section 3.4, so a close look will be taken at recent development in the theory of extreme values. Based on the theory of extreme values, Todorovic and Zelenhasic (1970) developed a stochastic model for flood analysis. In their model, floods are treated as a maximum term among a random number of a random variable in an interval of time. Todorovic's approach is becoming a popular method for streamflow drought analysis and many recent researchers like Sen (1980), Guven (1983) and Zelenhasic (1987) have tried to develop analytical models based on it. Zelenhasic in particular (1987) applied the Todorovic flood model directly to streamflow droughts with very good results.

Their results showed that streamflow floods or droughts follow a time dependent Poisson probability law. This is based on the assumption that the events (drought or floods) are independent.

The Todorovic-Zelenhasic flood models can be applied to agricultural droughts defined by the basin soil moisture parameter, ABS. Therefore, one can define agricultural drought as a set of minimum values among a random number of the random variable of ABS. As defined earlier (see Figure 3.1), agricultural drought events are a sequence of ABS whose values fall below a threshold value. Each event is considered as a two dimensional variable, defined by magnitude and duration.

Before one can apply the flood model to droughts, it is necessary to prove that the events, as defined by magnitude and duration are independent. The events are very rare and the number of droughts that occurred in the historical record is too few to be able to apply the usual statistical tests of independence. Practically, since the drought events are separated by relatively long time intervals, they should be independent.

To test the independence of the events, it is necessary to extend the historical record in order to yield sufficient drought events. Since an event is defined by magnitude (deficit sum) and duration, both the durations and the deficit sums were tested for independence. This was done by generating 300 years of data to yield almost 50 drought events. The third drought variable, drought intensity, was also tested for independence.

Serial correlation coefficients for drought deficit durations and intensity and tests of significance at 5% for the correlogram and Partial Autocorrelation Function show that they are independent. Plots of these parameters are shown on Figures 5.1 to 5.6.

With the assumption of independence satisfied, one can now proceed to fit a distribution to the number of droughts in a given time interval. Now, consider the significance of time interval to the distribution of the number of droughts. Since droughts are rare events, the probability that there will be no drought in one year is high. If we increase the time interval to say 10 years, we can expect a low number of droughts, hence the probability of zero number of droughts will decrease. If we keep increasing the time, to about a 100 year interval, for example, the zero or 'no drought' event will become increasingly unlikely.

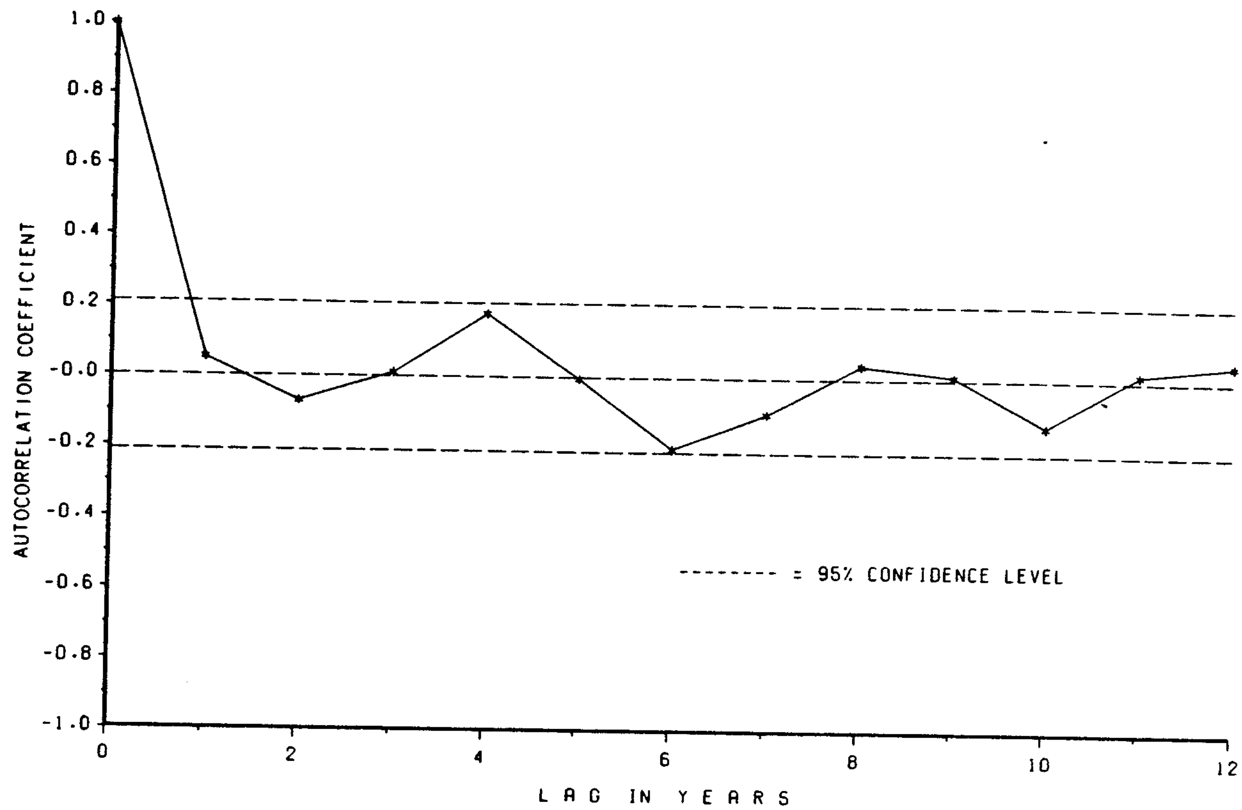

FIG. 5.1 PLOT OF CORRELOGRAM OF DROUGHT DEFICITS

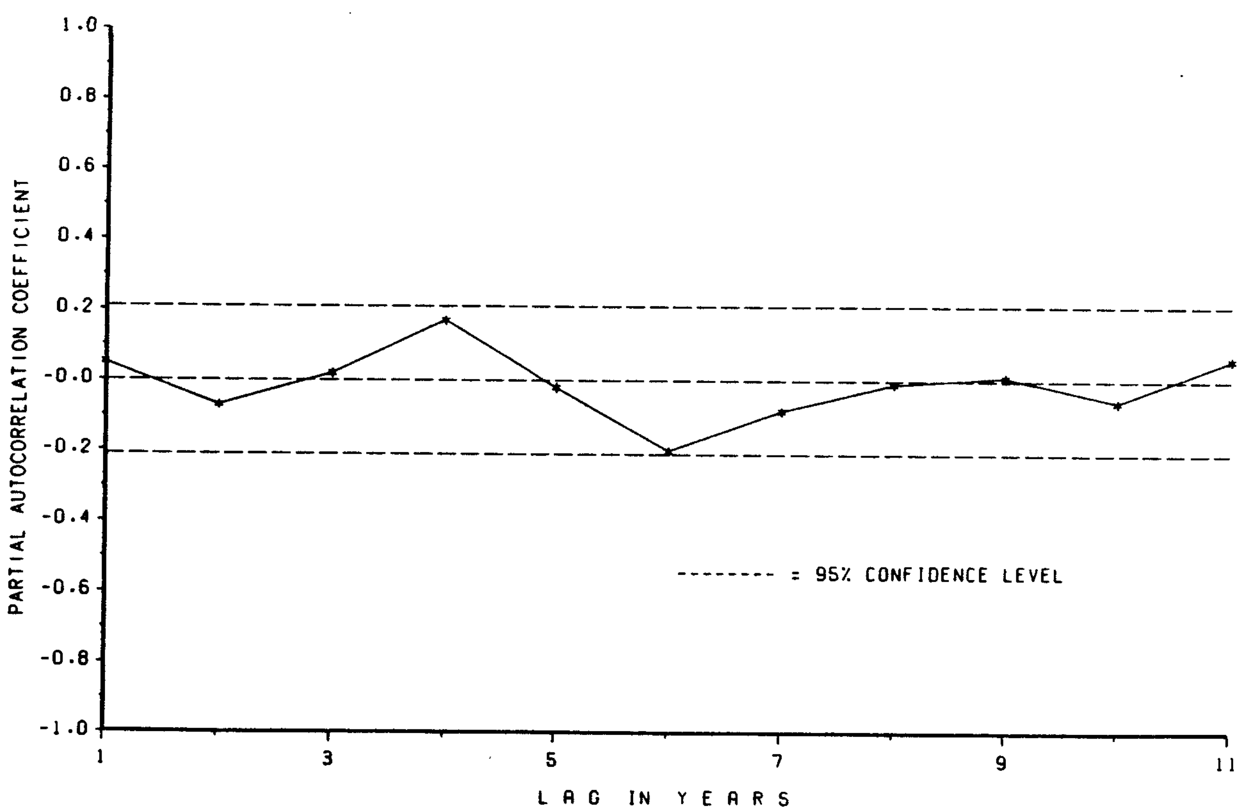

FIG. 5.2 PLOT OF P.A.C.F OF DROUGHT DEFICITS

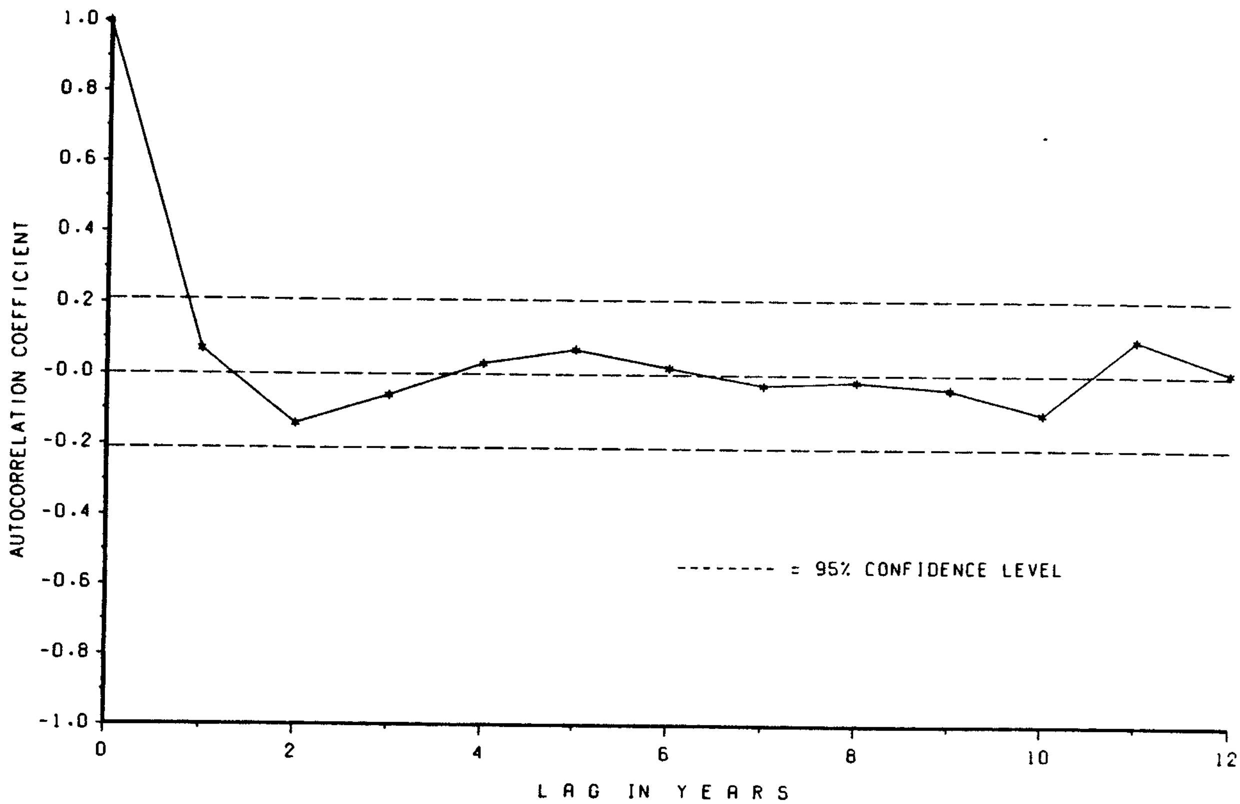

FIG. 5.3 PLOT OF CORRELOGRAM OF DROUGHT DURATION

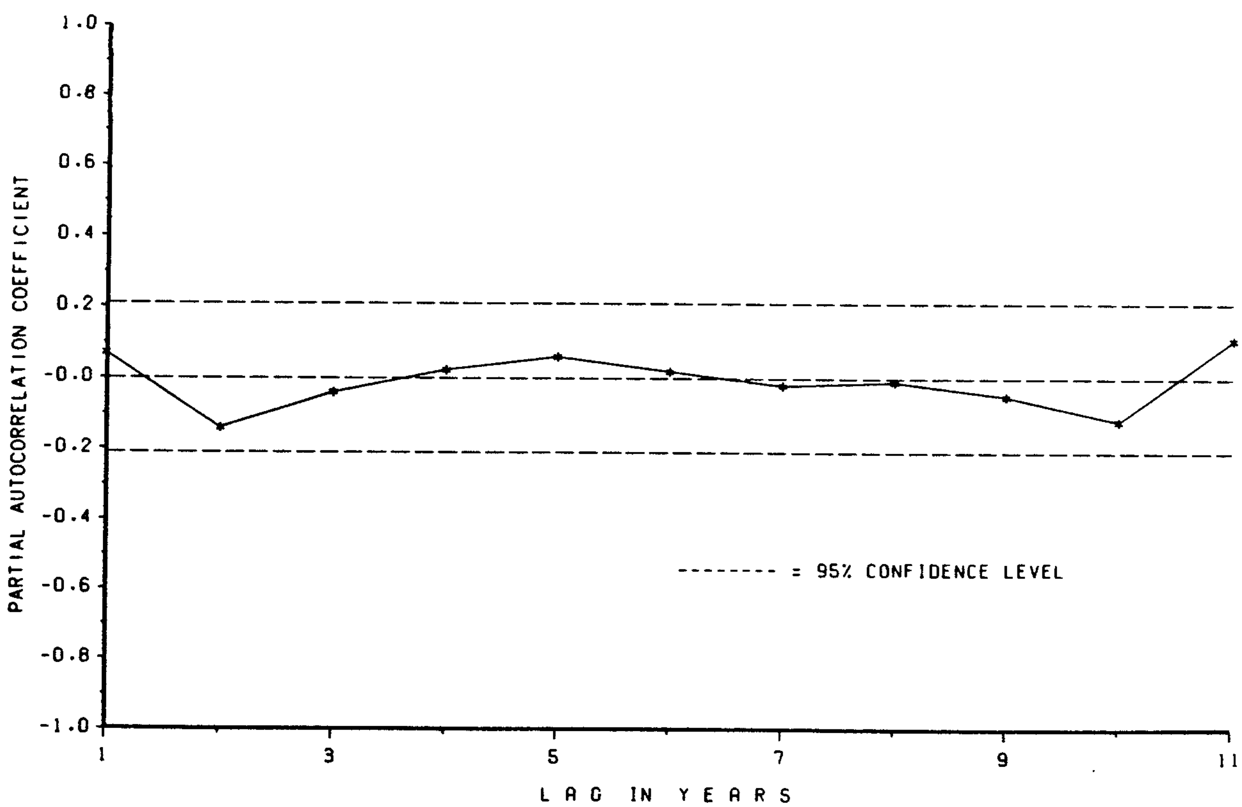

FIG. 5.4 PLOT OF P.A.C.F OF DROUGHT DURATION

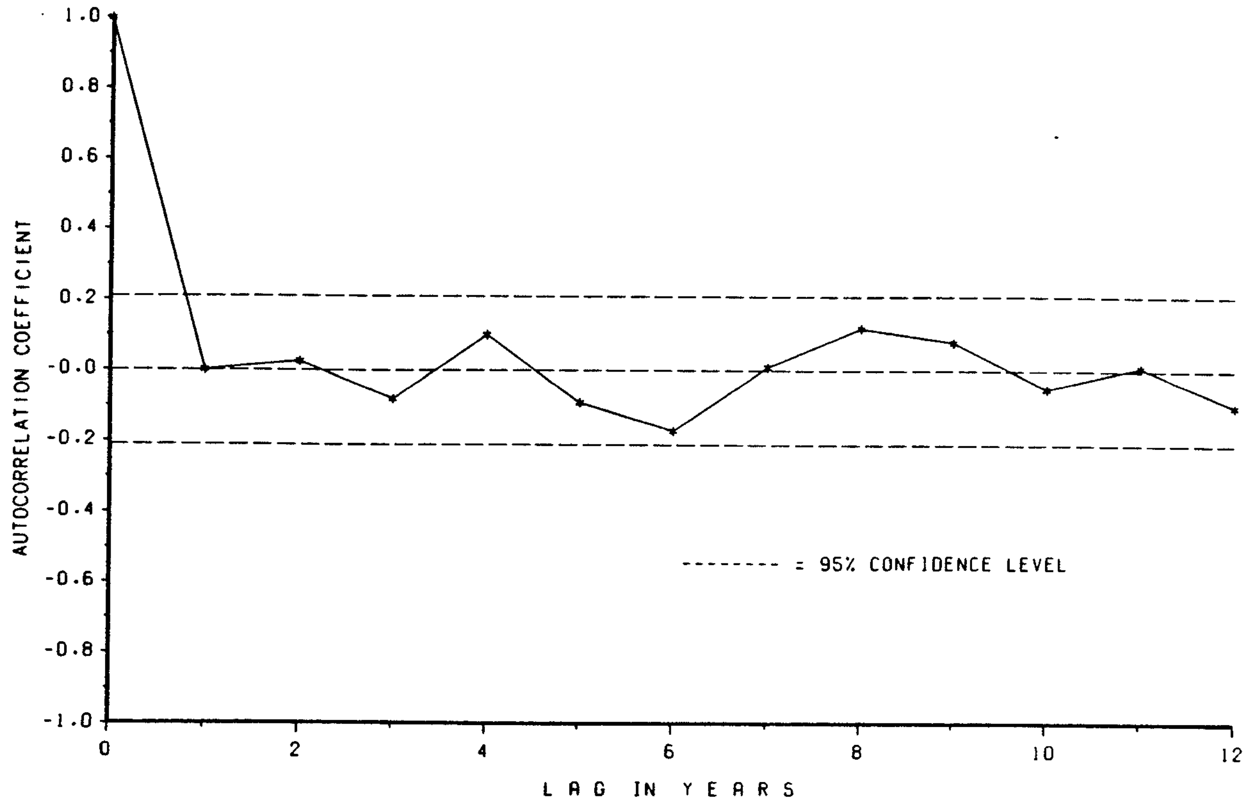

FIG. 5.5 PLOT OF CORRELOGRAM OF DROUGHT INTENSITY

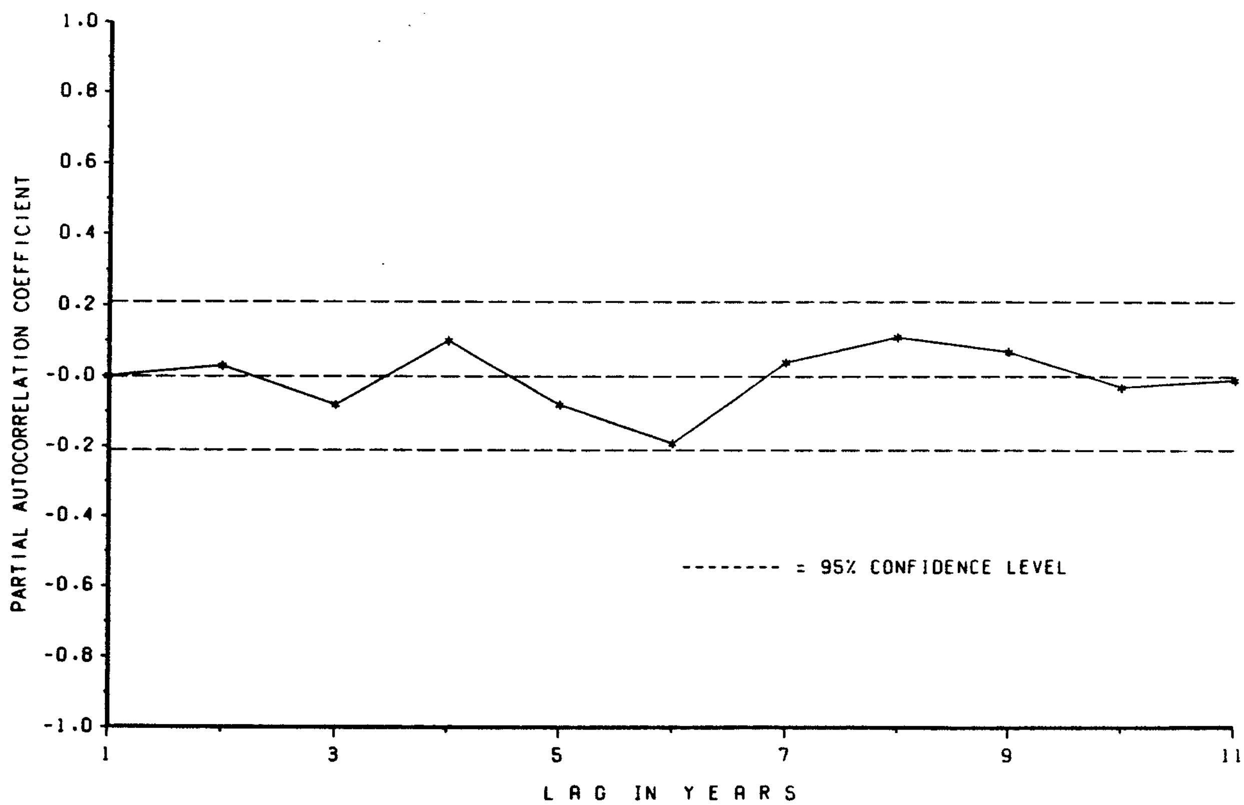

FIG. 5.6 PLOT OF P.A.C.F OF DROUGHT INTENSITY

The same analogy will apply to one event, two events, etc. Hence the shape or the parameter of the distribution will depend on the time interval considered. This will be illustrated by determining the distribution for different time intervals. Time intervals of 30, 40 and 50 years will be considered, with 50 years being of specific interest since the life of an agricultural project can be reasonably taken as 50 years.

As noted earlier, the reconstructed historical ABS record is too short for a statistical analysis of drought events to be done, so there is a need for its extension. 3000 years of ABS data were therefore generated using the previously developed lag one autoregressive model drought events were then extracted according to the proposed definition of droughts. A series of deficits were thus created with wet periods given zero deficits. The generated data was then subdivided into non-overlapping periods of 30, 40 or 50 years as the case may be. Relative frequencies for the number of droughts, 0, 1, 2, 3, etc within each time interval were then computed. The whole process was replicated 50 times and the mean of all the 50 frequencies were taken.

Results for the period of 30 years are shown in Table 12 below.

Average number of droughts in 30 year period:

$$\bar{x} = 1.83$$

TABLE 12

RELATIVE FREQUENCIES OF NUMBER OF DROUGHTS IN 30 YEAR INTERVAL

Number of Droughts in 10 Years	Relative Frequency
0	0.13
1	0.26
2	0.38
3	0.14
4	0.06
5	0.03

Assuming a Poisson distribution, the probability distribution of x number of droughts is given as:

$$f_x(x, \lambda) = \lambda^x e^{-\lambda}/x! \qquad \ldots (5.1)$$

where the parameter of the distribution λ is the average number of droughts.

Here $\lambda = 1.83$

Hence if the distribution is Poissonian, then the theoretical distribution is:

$$f_x(x, \lambda) = (1.83^x \quad e^{-1.83}) / x! \qquad \ldots (5.2)$$

A comparison between the observed (simulated) and the theoretical Poisson distributions are shown below.

TABLE 13

THEORETICAL AND SIMULATED RELATIVE FREQUENCIES
OF THE NUMBER OF DROUGHTS IN 30 YEAR INTERVAL

No. of Droughts	Simulated	Theoretical
0	0.130	0.160
1	0.260	0.290
2	0.380	0.270
3	0.140	0.164
4	0.060	0.075
5	0.030	0.027

TABLE 14

THEORETICAL AND SIMULATED CUMULATIVE FREQUENCIES
OF THE NUMBER OF DROUGHTS IN 30 YEAR INTERVAL

No. of Droughts	Simulated	Theoretical
0	0.130	0.160
1	0.390	0.450
2	0.770	0.720
3	0.910	0.884
4	0.970	0.960
5	1.000	0.990

Plots of the simulated probability distribution and the theoretical Poissonian are shown on Figure 5.7. Also is their cumulative distribution functions are plotted on Figure 5.8. These plots are step functions since the number of droughts are integers. However, in order to show the comparison between the simulated and theoretical functions better, the points will be joined.

Visually, from the plots, one can note that the theoretical distribution fits the simulated one. However, statistical goodness of fit test will be conducted to conclude this.

The Kolmogorov-Smirnov goodness of fit test is conducted as below.

TABLE 15

THEORETICAL AND SIMULATED CUMULATIVE FREQUENCIES FOR THE NUMBER OF DROUGHTS IN 30 YEARS

No. of Droughts	Observed CDF	Expected CDF	Deviation
0	0.130	0.160	0.03
1	0.390	0.450	0.06
2	0.770	0.720	0.05
3	0.910	0.884	0.03
4	0.970	0.960	0.01
5	0.030	0.027	0.01

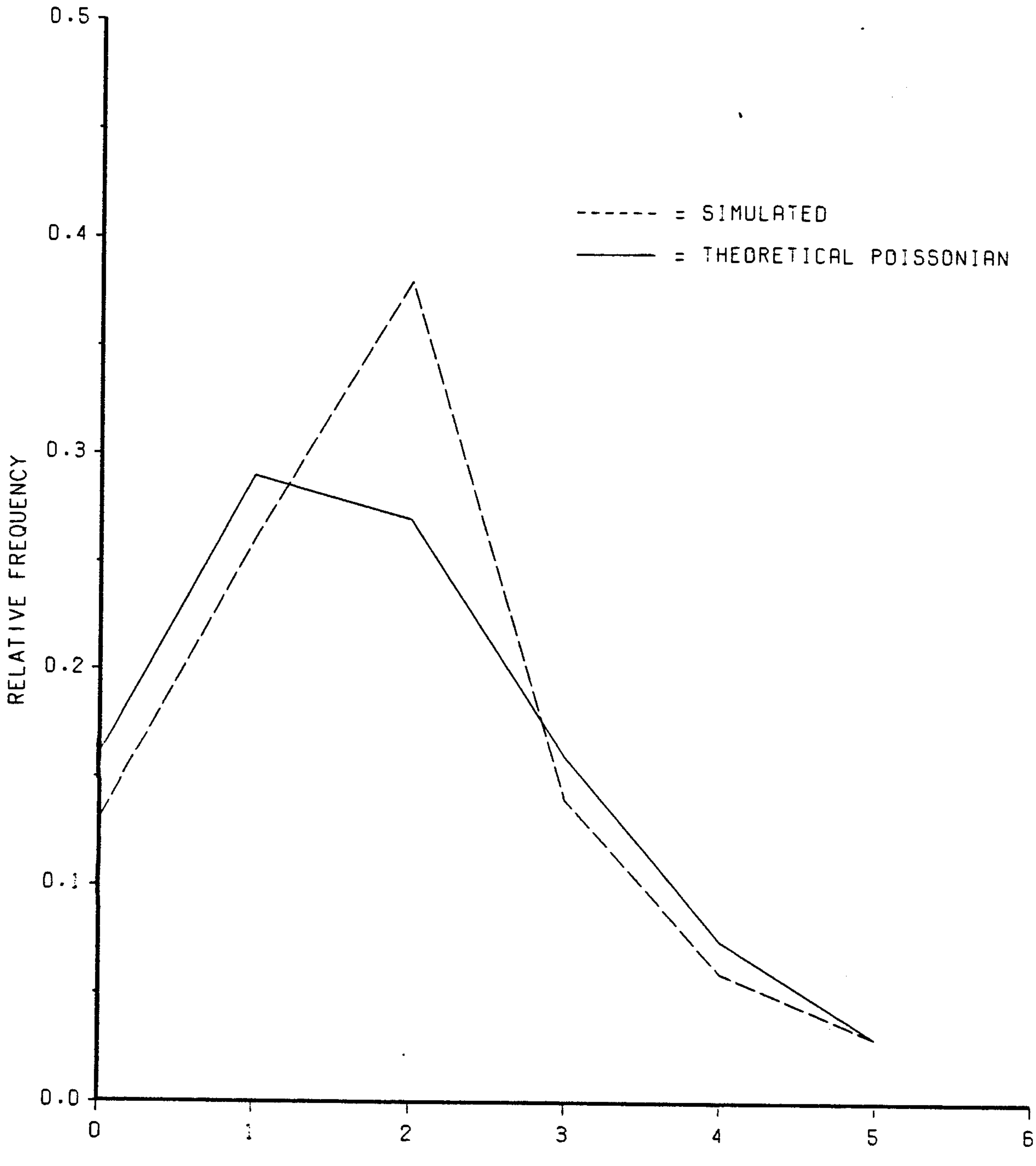

FIG. 5.7 RELATIVE FREQUENCIES OF THE NUMBER OF DROUGHTS IN PERIODS OF 30 YEARS

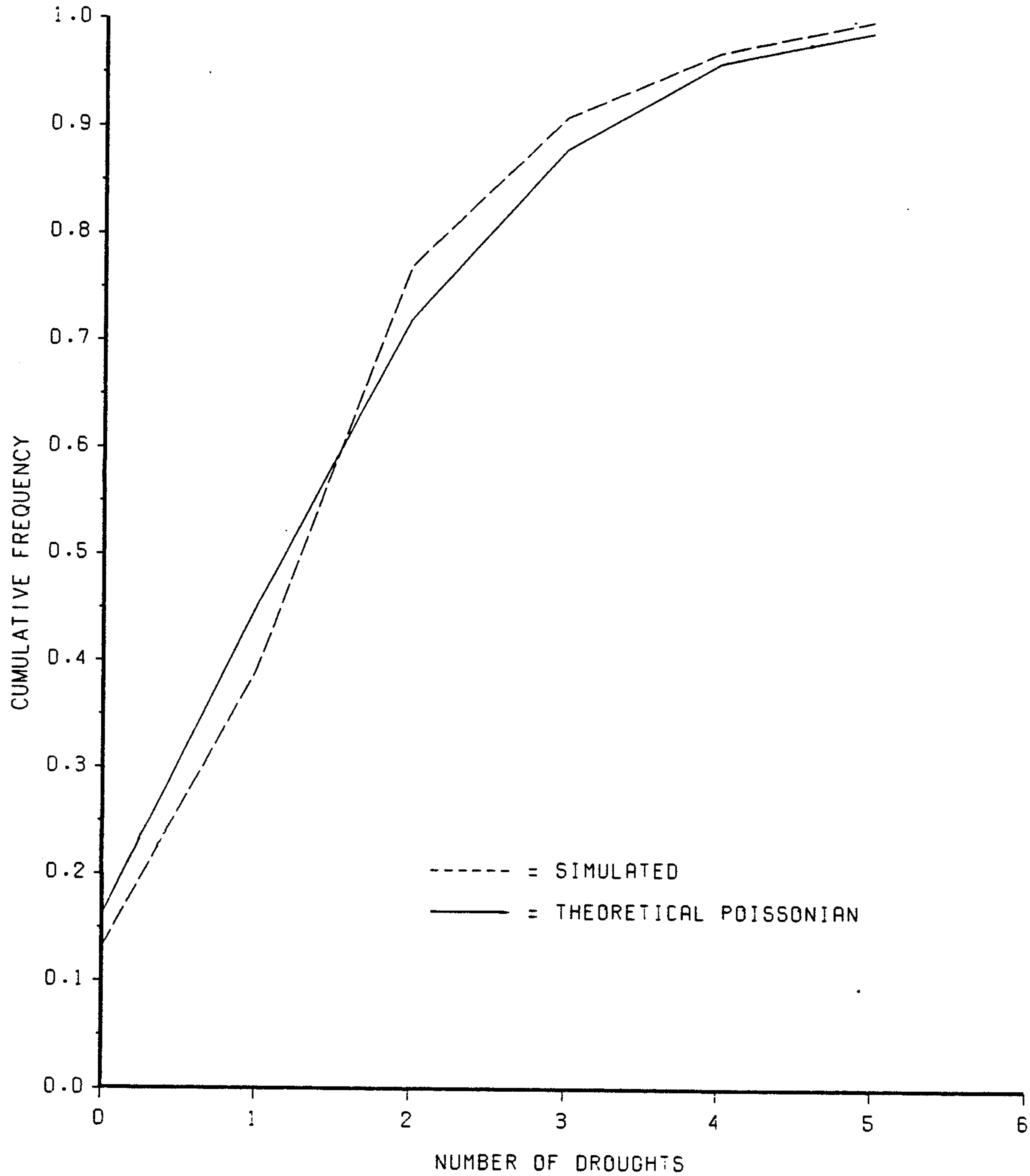

FIG. 5.8 CUMULATIIVE FREQUENCY DISTRIBUTION OF THE NUMBER OF DROUGHTS IN PERIODS OF 30 YEARS

The maximum deviation D = 0.05

Sample size = 100

Therefore, critical Kolmogorov-Smirnov test statistic at 5% level is C = 0.14

Since the parameters were estimated from the historical record, the test statistic should be modified, reduced approximately by two-thirds, thus:

Critical Test Statistic C = 0.093

Since D << C, we conclude that the Poisson distribution fits the number of droughts well.

The procedure was repeated for 40 year time interval. In this case, the average number of droughts $\bar{x}$ = 2.41.

TABLE 16

RELATIVE FREQUENCIES FOR NUMBER OF DROUGHTS IN PERIOD 40 YEARS

No. of Droughts	Simulated Relative Frequency	Theoretical Relative Frequency
0	0.080	0.089
1	0.187	0.216
2	0.253	0.261
3	0.307	0.210
4	0.080	0.126
5	0.080	0.061
6	0.013	0.024

TABLE 17

THEORETICAL AND SIMULATED CUMULATIVE FREQUENCIES AND TEST OF FITNESS

No. of Droughts	Simulated CDF	Theoretical CDF	Deviation
0	0.080	0.089	0.009
1	0.270	0.305	0.035
2	0.523	0.566	0.043
3	0.830	0.780	0.050
4	0.910	0.906	0.004
5	0.990	0.967	0.023
6	1.000	0.991	0.009

Maximum deviation $D = 0.050$

Modified critical test statistic $C = 0.10$

Since $D \ll C$, we conclude the theoretical distribution fits the observed data well.

Plots of the simulated and theoretical PDF abd CDFs are shown in Figures 5.9 and 5.10 respectively.

For the period of 50 years, the results are as below.

Mean number of droughts $\bar{x} = 2.84$

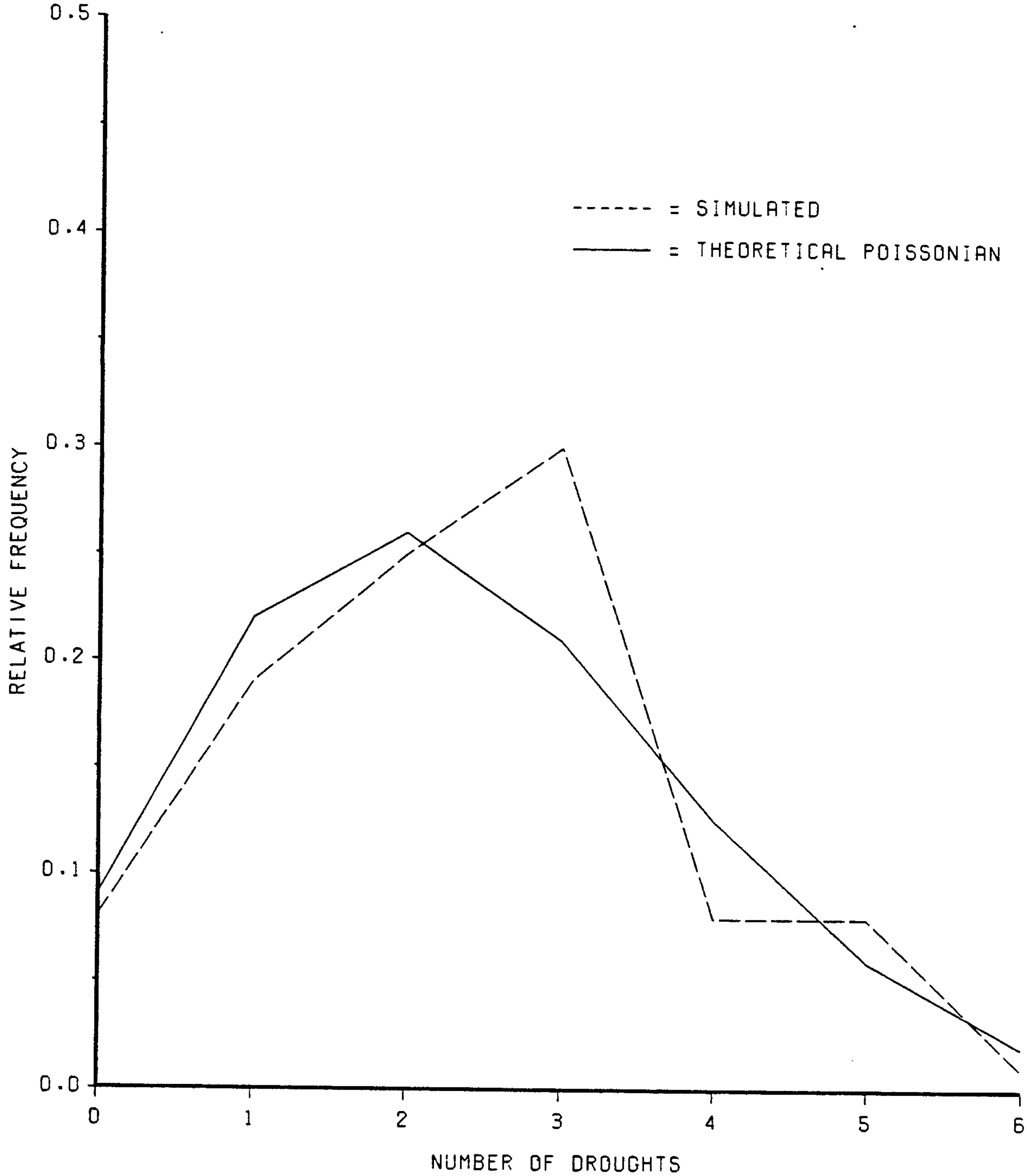

FIG. 5.9 RELATIVE FREQUENCIES OF THE NUMBER OF DROUGHTS IN PERIODS OF 40 YEARS

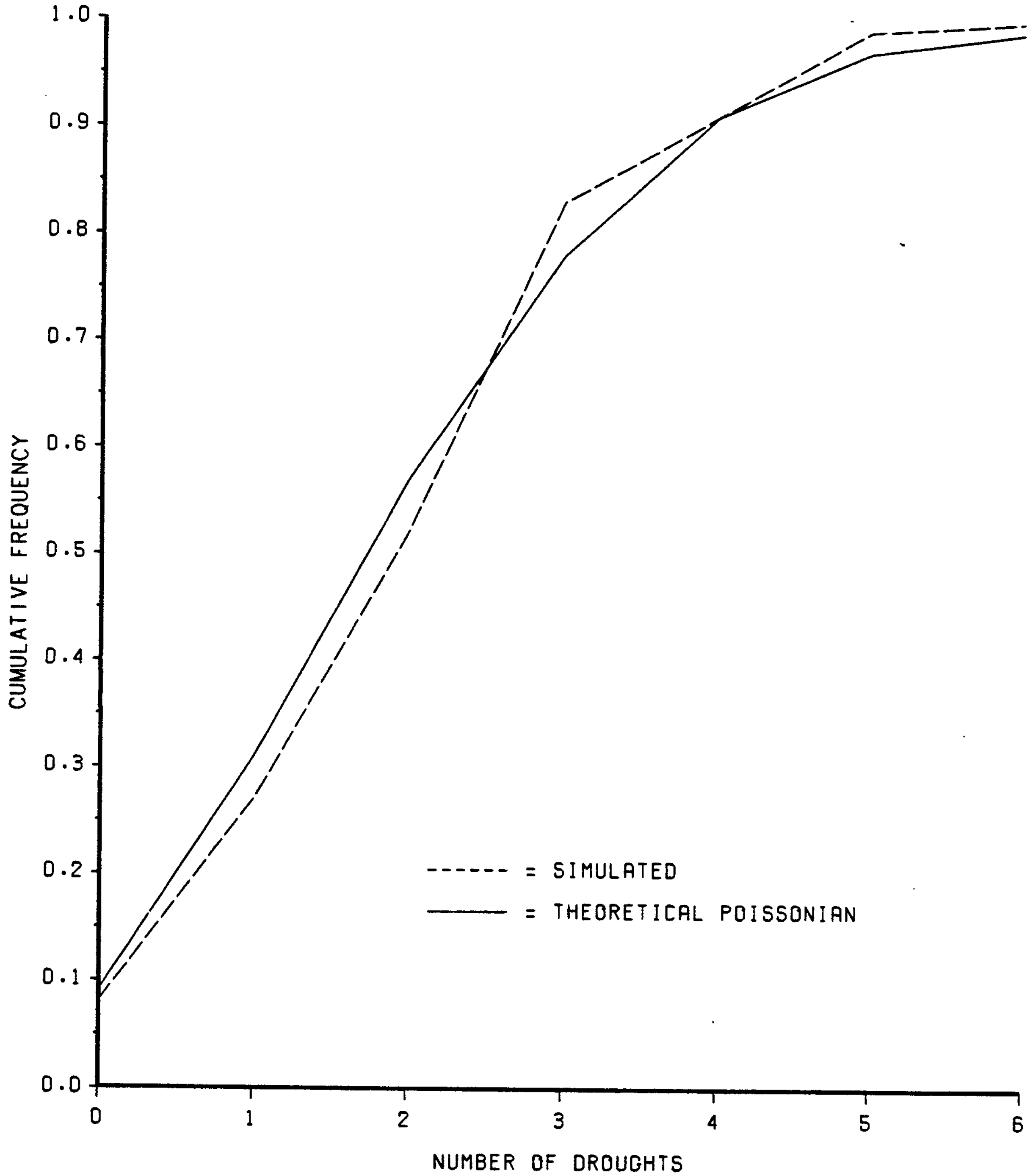

FIG. 5.10 CUMULATIIVE FREQUENCY DISTRIBUTION OF THE NUMBER OF DROUGHTS IN PERIODS OF 40 YEARS

TABLE 18

RELATIVE FREQUENCIES OF THE NUMBER OF DROUGHTS IN 50 YEAR INTERVAL

No. of Droughts	Simulated Rel. Freq	Theoretical Rel. Freq
0	0.04	0.058
1	0.187	0.166
2	0.240	0.236
3	0.227	0.223
4	0.173	0.158
5	0.08	0.090
6	0.013	0.043
7	0.013	0.017
8	0.013	0.006
9	0.013	0.002

TABLE 19

CUMULATIVE FREQUENCIES OF THE NUMBER OF DROUGHTS IN 50 YEAR INTERVALS

No. of Droughts	Simulated CDF	Theoretical CDF	Deviation
0	0.04	0.058	0.018
1	0.227	0.224	0.003
2	0.467	0.460	0.007
3	0.694	0.683	0.011
4	0.867	0.841	0.026
5	0.947	0.931	0.016
6	0.960	0.974	0.014
7	0.973	0.991	0.018
8	0.986	0.997	0.011
9	0.999	0.999	0.001

The maximum deviation D = 0.026

The modified critical test statistic C = 0.10

Hence we conclude that Poisson distribution fits the number of droughts well.

Plots for the 50 year period theoretical and simulated PDF and CDFs are shown on Figures 5.11 and 5.12 respectively.

As predicted earlier, the average number of droughts increases with the time interval and for this study, for the periods 30, 40 and 50 years, the parameters 1.83, 2.41 and 2.84 respectively were obtained.

These results confirm the observation that the distribution of the number of droughts follow the TIME DEPENDENT POISSON DISTRIBUTION.

The theoretical Poisson cumulative distribution functions for 30, 40 and 50 years time intervals for this study are compared on Figure 5.13.

5.2 THE DISTRIBUTION OF DROUGHT DEFICITS

Drought deficit is one of the defining variables of a drought event and it will therefore be necessary to determine its distribution. Drought deficit is defined as the magnitude of the lowest level to which the ABS falls below the reference level within a drought period. This value will give the amount of moisture needed to raise the

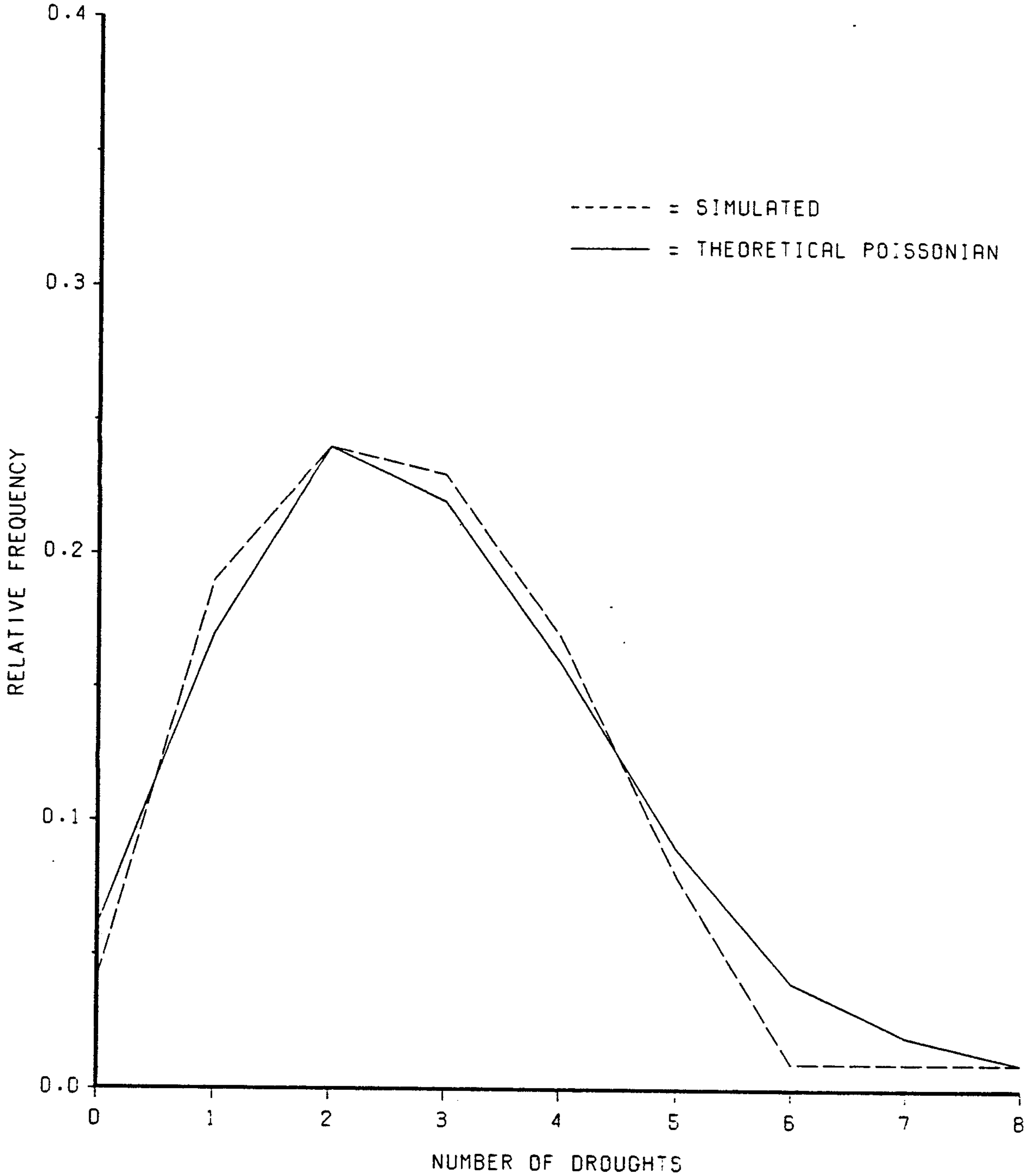

FIG. 5.11 RELATIVE FREQUENCIES OF THE NUMBER OF DROUGHTS IN PERIODS OF 50 YEARS

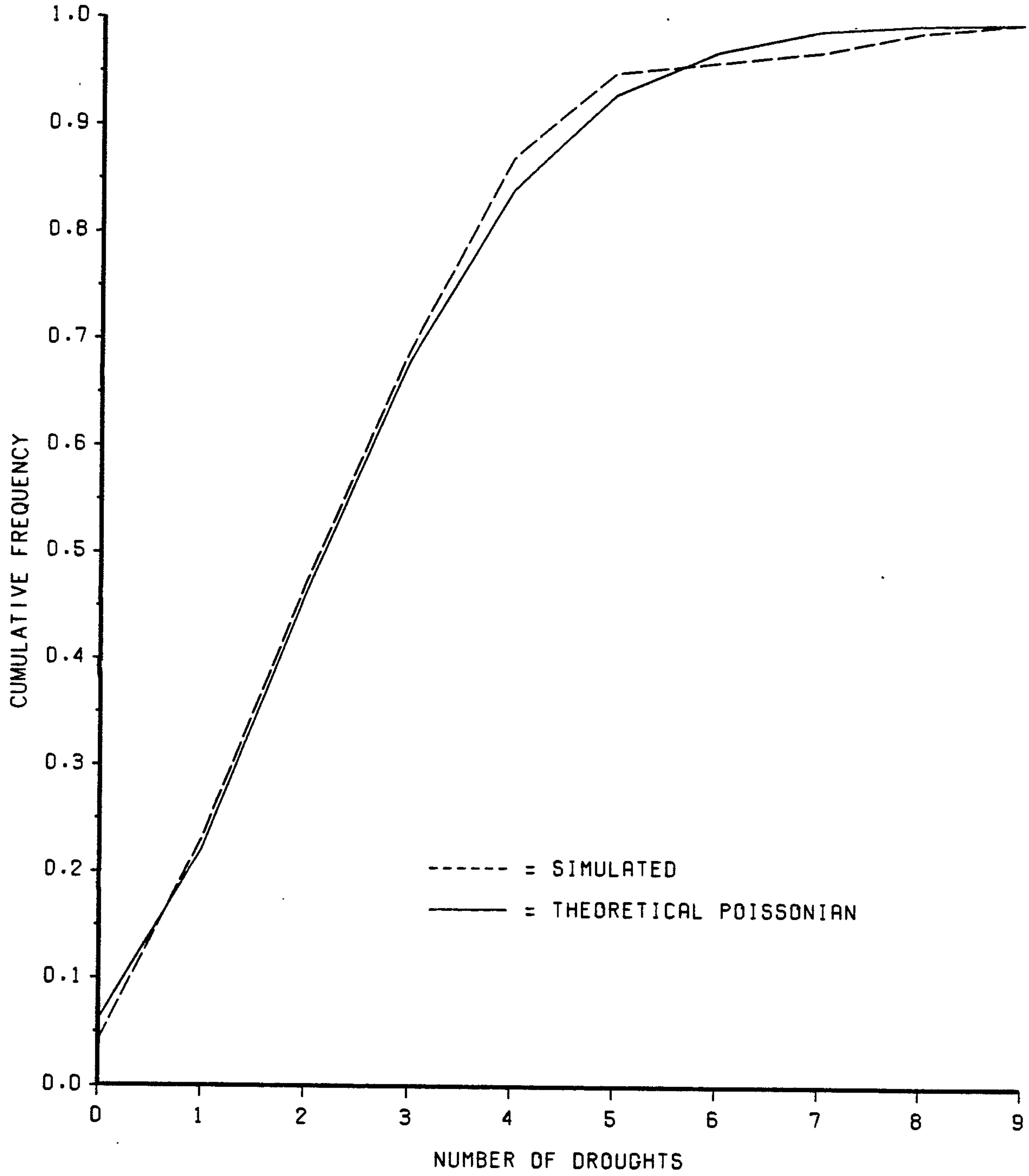

FIG. 5.12 CUMULATIIVE FREQUENCY DISTRIBUTION OF THE NUMBER OF DROUGHTS IN PERIODS OF 50 YEARS

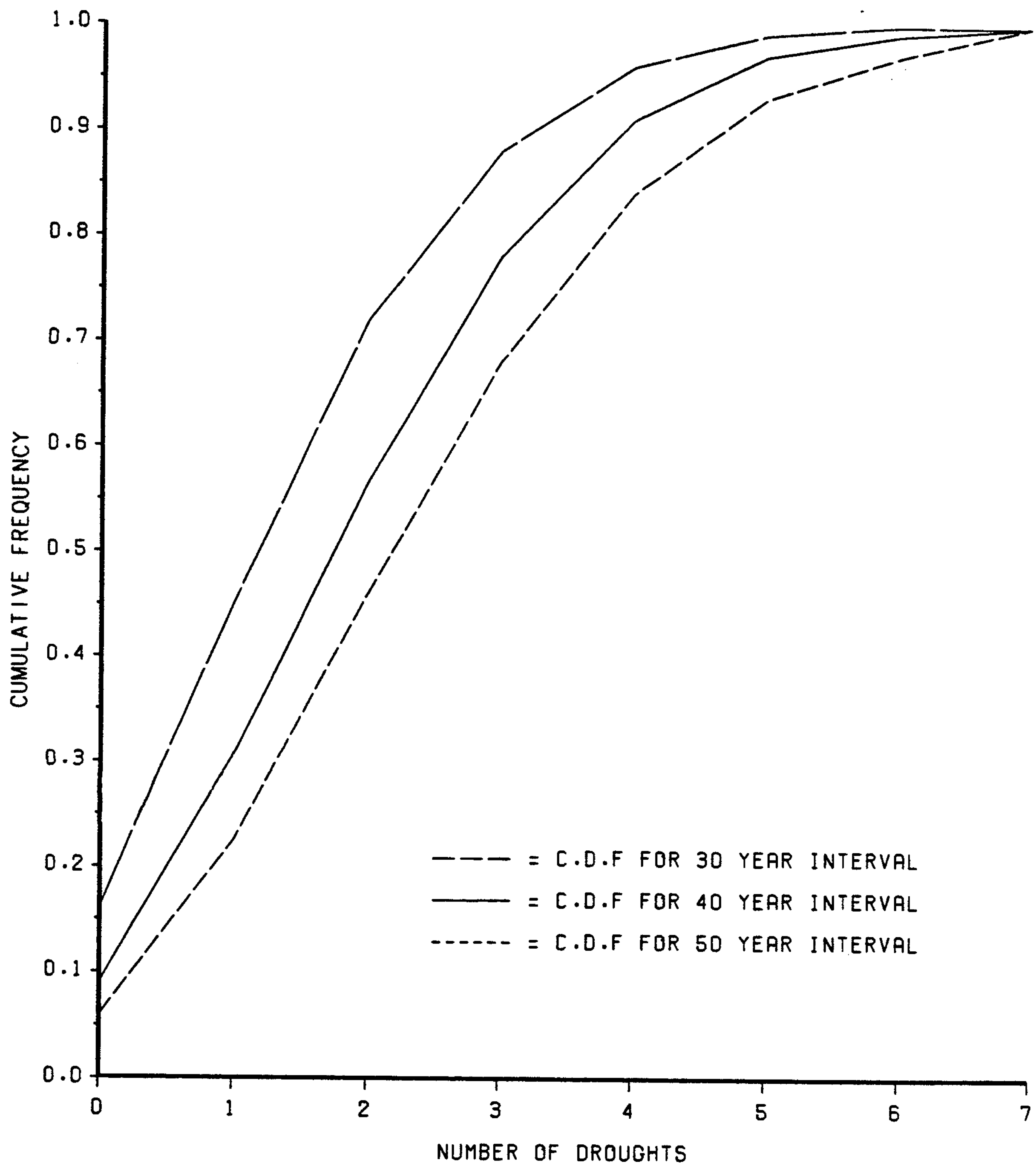

FIG. 5.13 THEORETICAL CUMULATIIVE FREQ. DISTRIBUTION FOR DIFFERENT TIME INTERVALS

ABS to the threshold level. This will be a continuous variable and therefore, class intervals are used to form a frequency analysis for the deficits from the generated data. The table below shows the resulting relative frequencies.

TABLE 20

THEORETICAL AND SIMULATED RELATIVE FREQUENCIES FOR DROUGHT DEFICITS

Class Interval (mm)	Simulated Relative Frequency	Theoretical Relative Frequency
0.5 - 20.5	0.327	0.387
20.5 - 40.5	0.246	0.234
40.5 - 60.5	0.215	0.141
60.5 - 80.5	0.102	0.085
80.5 - 100.5	0.070	0.052
100.5 - 120.5	0.039	0.019

The shape of the resulting histogram shown on Figure 5.14, indicates an exponential distribution function for the drought deficits.

The estimate of the parameter of the distribution was 0.03.

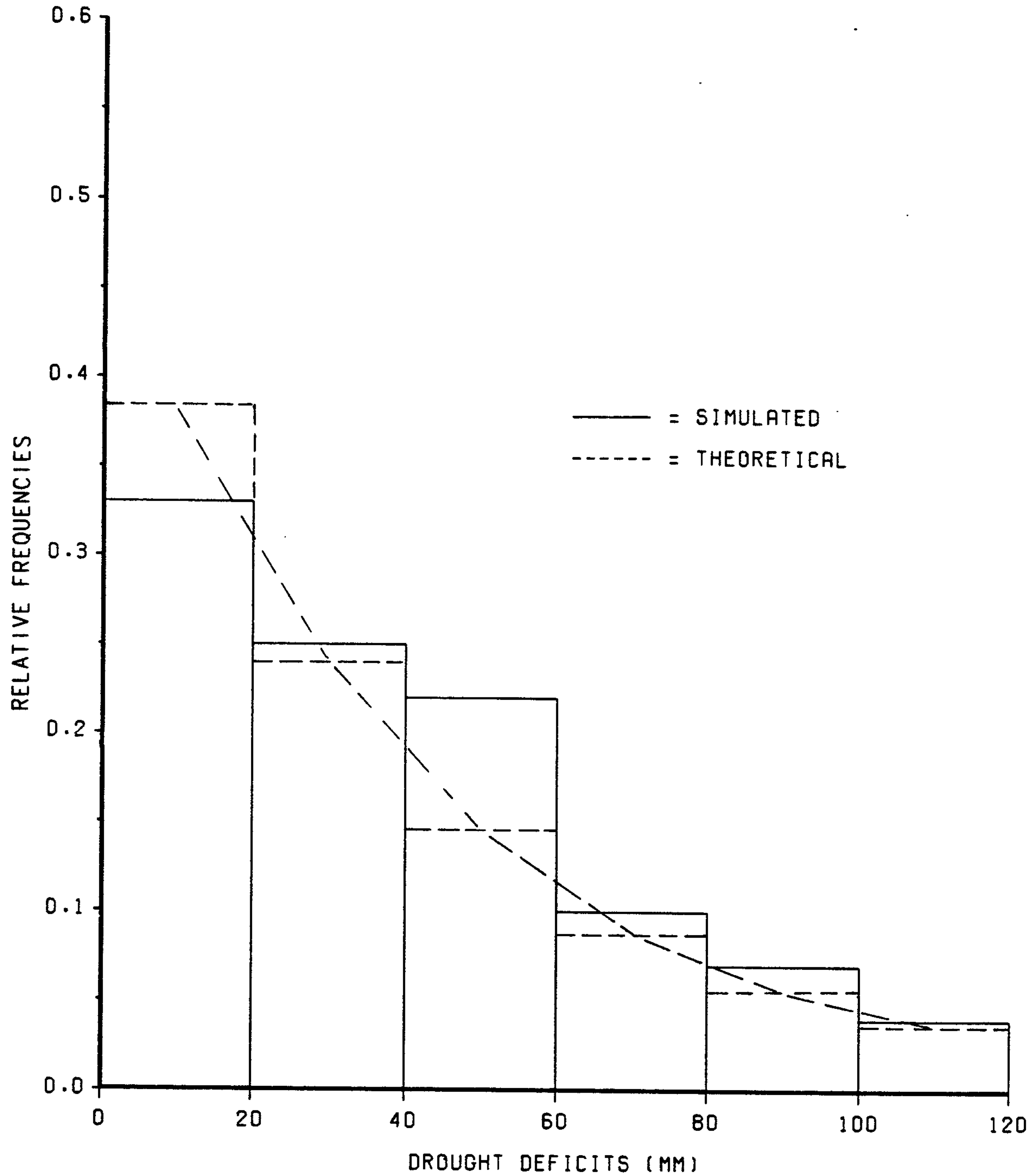

FIG. 5.14 A PLOT OF RELATIIVE FREQUENCY OF DROUGHT DEFICITS

Therefore the probability distribution function for the drought deficit is:

$$PDF = 0.03 \Delta x \exp[-0.03 x] \quad \ldots (5.3)$$

where Δx is the class interval, and x the class midpoints. The cumulative frequency distribution = 1 - exp[-0.03x].

Plot of the simulated relative frequencies and the theoretical ones are shown in Figure 5.14.

The resulting theoretical exponential distribution and the simulated distribution with their corresponding departures are shown on Table 21 below.

TABLE 21

CUMULATIVE FREQUENCY OF DROUGHT DEFICIT AND TEST OF FITNESS

Class Interval	Observed Cumulative Frequency	Theoretical Cumulative Frequency	Deviation
0.5 - 20.5	0.327	0.270	0.057
20.5 - 40.5	0.573	0.600	0.027
40.5 - 60.5	0.788	0.780	0.008
60.5 - 80.5	0.890	0.879	0.011
80.5 - 100.5	0.960	0.934	0.026
100.5 - 120.5	0.999	0.964	0.035
120.5 - 140.5	0.999	0.980	0.020

Applying the Komogorov-Smirnov test of goodness of fit, the maximum observed deviation is 0.057. The critical modified test statistic for average sample size of 99 and 5% level is 0.093. Since 0.057 < 0.093, we conclude that the exponential distribution fits the observed data.

Plots of the theoretical function and the one obtained by simulation are shown in Figure 5.15.

5.3 DISTRIBUTION OF DROUGHT INTENSITY

The concept of drought intensity arises out of the fact that the magnitude of drought deficits will be realised over a period of time. A drought deficit of a certain magnitude may take several years to be realised so that one would be interested in the rate of soil moisture depletion. The drought intensity will be defined in terms of this rate and will be in units of millimeters per year.

The drought intensity is strongly correlated to drought deficit as indicated by a correlation coefficient of 0.75 and will most likely follow its distribution.

As drought intensity is also a continuous variable, class intervals are used to compute relative frequencies for the drought intensity from the generated data. A summary of the results is shown on Tables 22 and 23.

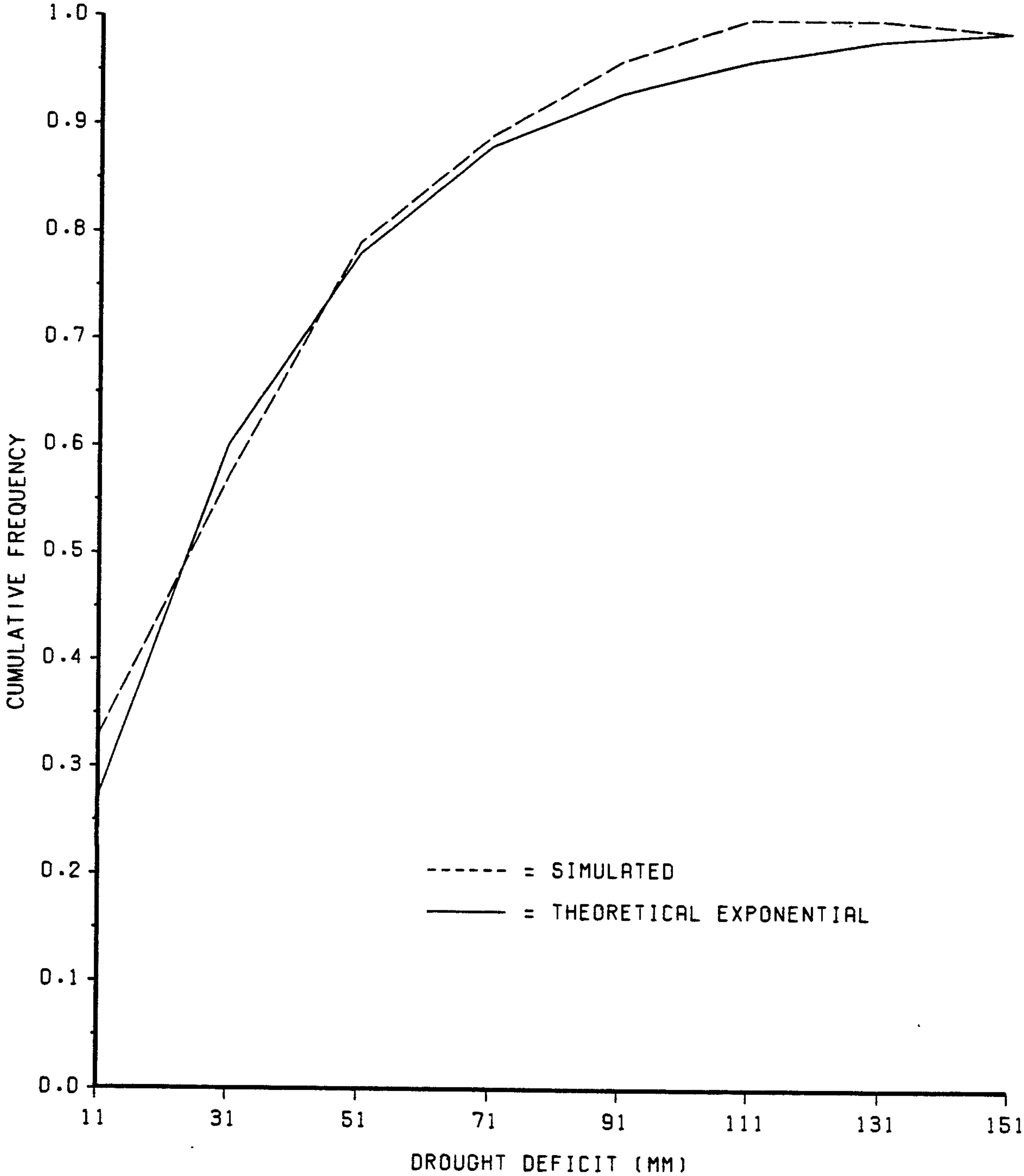

FIG. 5.15 CUMULATIIVE FREQUENCY DISTRIBUTION OF DROUGHT DEFICITS IN PERIODS OF 50 YEARS

TABLE 22

RELATIVE FREQUENCIES OF DROUGHT INTENSITY

Class Interval	Observed Relative Frequency	Theoretical Exponential R/F
0.5 - 20.5	0.505	0.526
20.5 - 40.5	0.212	0.236
40.5 - 60.5	0.182	0.106
60.5 - 80.5	0.061	0.048
80.5 - 100.5	0.040	0.021

Estimated mean intensity = 25.6

For an exponential distribution, the parameter = 1/25.6 = 0.039 and the exponential distribution = 0.039 x exp[-0.039x]. The table below shows the CDFs.

TABLE 23

CUMULATIVE FREQUENCIES OF DROUGHT INTENSITY

Class Interval	Observed CDF	Theoretical CDF	
0.5 - 20.5	0.505	0.526	0.021
20.5 - 40.5	0.717	0.762	0.045
40.5 - 60.5	0.899	0.868	0.031
60.5 - 80.5	0.960	0.916	0.044
80.5 - 100.5	1.000	0.937	0.063

Applying the Kolmogorov-Smirnov test of fit, the maximum observed deviation is 0.063. The critical modified test statistic for sample size of 99 and at 5% level is 0.093. Since 0.063 < 0.093, we conclude that the exponential distribution fits the observed data well.

Plots of the theoretical and observed distributions are shown in Figures 5.16 and 5.17.

5.4 THE DISTRIBUTION OF DROUGHT DURATION

The probability distribution of the duration of droughts is necessary since drought duration is one of the variables that defines a drought event. Furthermore, agricultural project planners would be interested in knowing the return periods for drought durations so as to provide enough water storage to last over the period of drought.

To fit a distribution to the duration of droughts, the same generated data used in the previous section was used. The relative frequencies of the duration of drought events were computed and the generating process replicated fifty times.

The resulting histogram shown on Figure 5.18 indicates an exponential distribution. This can also be

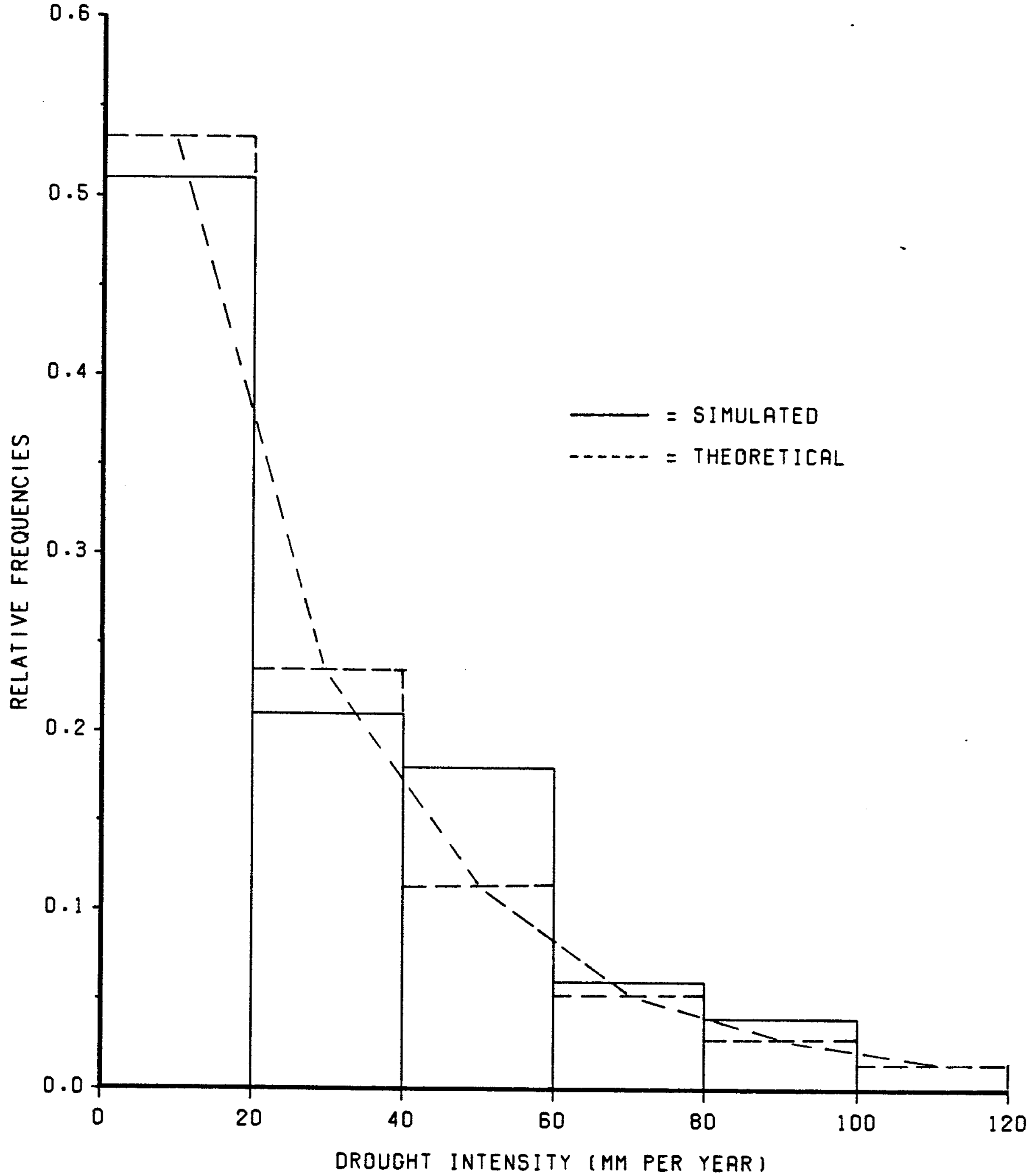

FIG. 5.16 A PLOT OF RELATIIVE FREQUENCY OF DROUGHT INTENSITIES

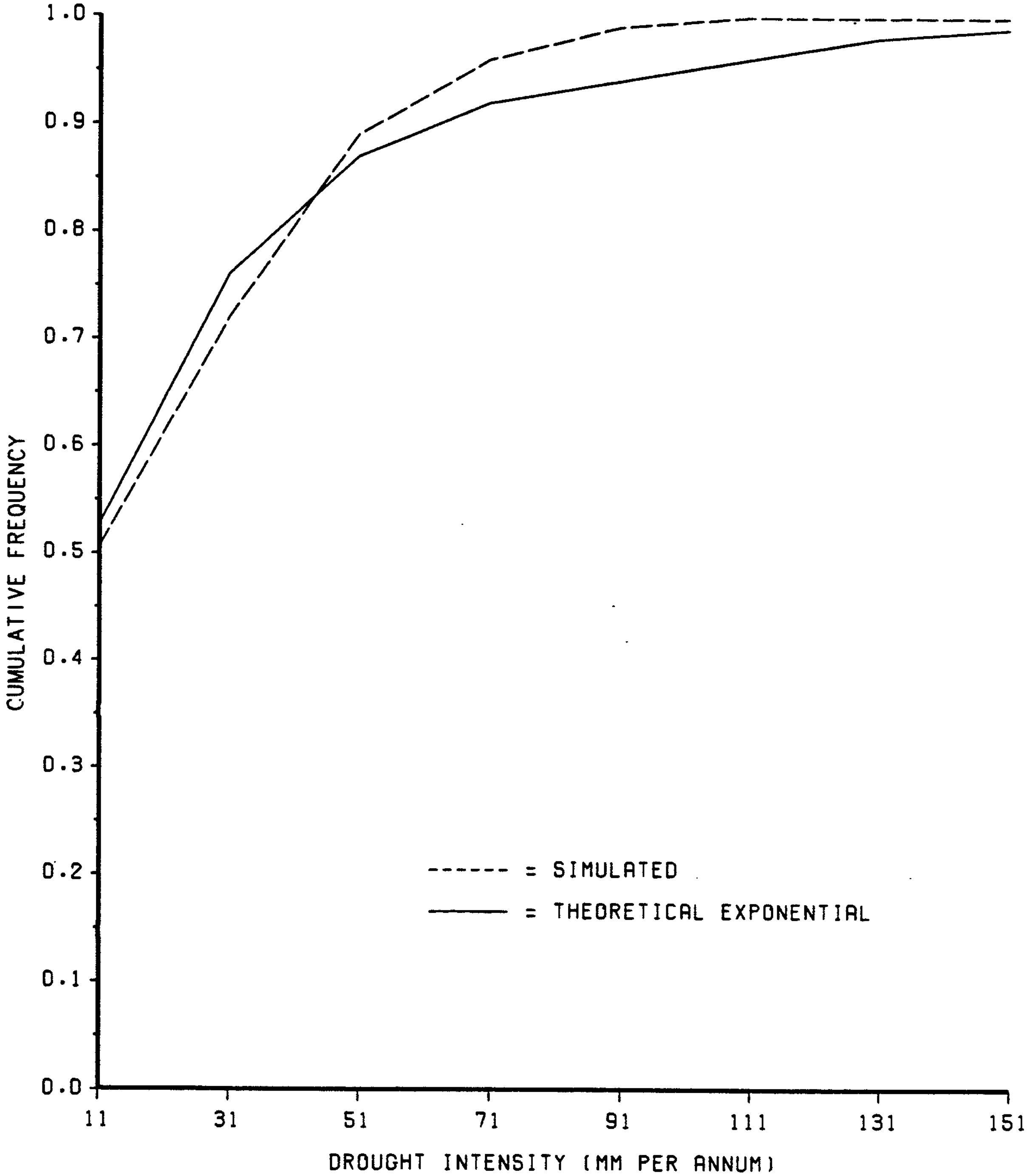

FIG. 5.17 CUMULATIIVE FREQUENCY DISTRIBUTION OF DROUGHT INTENSITY IN PERIODS OF 50 YEARS

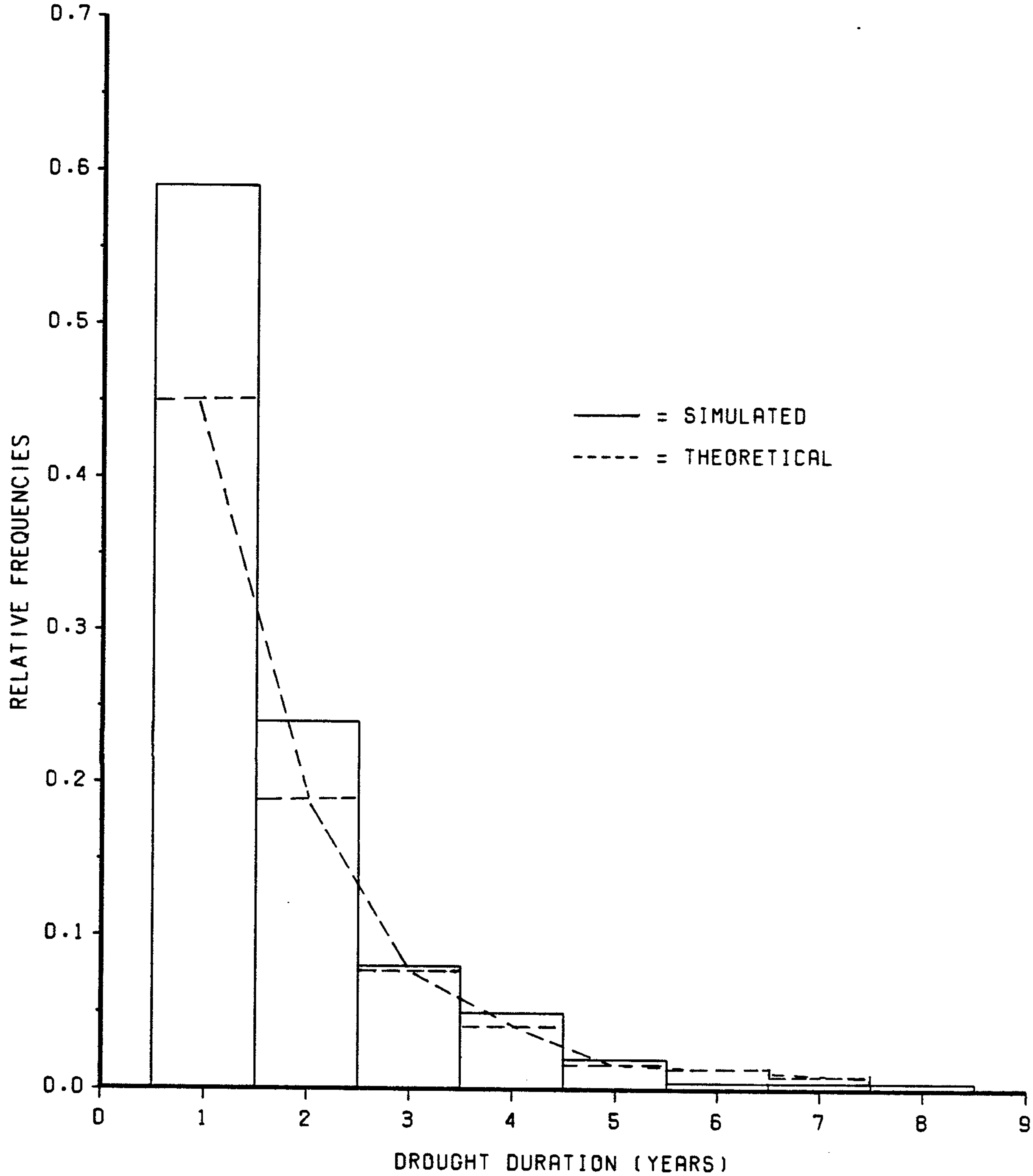

FIG. 5.18 A PLOT OF RELATIIVE FREQUENCY OF DROUGHT DURATIONS

inferred from the fact that the exponential distribution arises as the distribution of the time between occurrences of events of a Poisson process.

However, this theoretical exponential distribution failed to pass the test of goodness of fit at 5% significance level.

A further examintion of available theoretical results for the distribution of run lengths was therefore undertaken. If it is assumed that the streamflows may be represented as a first-order, two-state Markov chain, where the two states are defined below the reference level ($Z_i < Z_o$) and above

TABLE 24

RELATIVE FREQUENCIES OF DROUGHT DURATION

Drought Duration	Simulated Relative Frequency
1	0.586
2	0.240
3	0.084
4	0.050
5	0.0223
6	0.0056
7	0.0056
8	0.0056

the reference level ($Z_i > Z_o$) where Z_o is the constant reference or cut off streamflow level, it has been shown (Cox and Miller, 1968; Heiny, 1968; Bayazit and Sen, 1976) that:

$$P(L > j) = r^j \qquad j = 0, 1, 2 \ldots \qquad \ldots (5.4)$$

where L is the duration of the drought (or negative run length) and r is the conditional probability of a flow to be less than or equal to the truncation level, given that the previous years' was less than or equal to the truncation level, that is:

$$r = P(Z_i < Z_o / Z_{i-1} < Z_o) \qquad \ldots (5.5)$$

The expression (3) has been shown to be approximately valid for first-order autoregressive process (lag one Markov process) by Guerrero-Salazar and Yevjevich, 1975.

Since the ABS series can be generated by the lag one Markov model, the above theory can be applied to it directly.

Sen (1978) suggests that r, which he terms the autorun coefficient, can be estimated by:

$$\hat{r} = [n_{dd}/(n_R - 1)] / (n_{dy}/n_R) \qquad \ldots (5.6)$$

where; n_{dd} is the number of 'dry-dry' transitions, which is the number of pairs of consecutive 'dry' years such that $Z_i < Z_o$ and $Z_{i-1} < Z_o$ in the record. n_{dy} is the number of dry years for which $Z_i < Z_o$, and n_R is the length of record.

From the generated ABS record of 3000 years, after replication 50 times, the following were obtained:

$n_R = 3000$

$n_{dd} = 134$

$n_{dy} = 310$

Hence, $\hat{r} = 0.4325$

Therefore, the theoretical distribution is:

$$P(L > j) = r^j = (0.4325)^j \qquad j = 1, 2, 3 \ldots.$$

Therefore the theoretical cumulative distribution is

$$P(L < j) = 1 - P(L > j) = 1 - (0.4325)^j \qquad j = 1,2..$$

Table 25 below shows the cumulative distribution function of the theoretical function and that actually observed from the simulated data.

TABLE 25

CUMULATED FREQUENCIES OF DROUGHT DURATION

Duration j	Theoretical PDF	Theoretical CDF	Observed CDF	Deviation
1	0.433	0.567	0.586	0.019
2	0.187	0.813	0.826	0.013
3	0.081	0.919	0.910	0.009
4	0.035	0.965	0.960	0.005
5	0.015	0.985	0.982	0.003
6	0.0065	0.994	0.988	0.006
7	0.0028	0.997	0.994	0.003
8	0.0012	0.998	0.999	0.001

For the Kolmogorov-Smirnov goodness of fit test, the maximum deviation was 0.019. The modified critical test at 5% level was 0.093 which is far larger than the registered deviation of 0.019. Hence the theoretical cumulative distribution function fits the data well.

Plot of the theoretical and observed CDFs are shown on Figure 5.19.

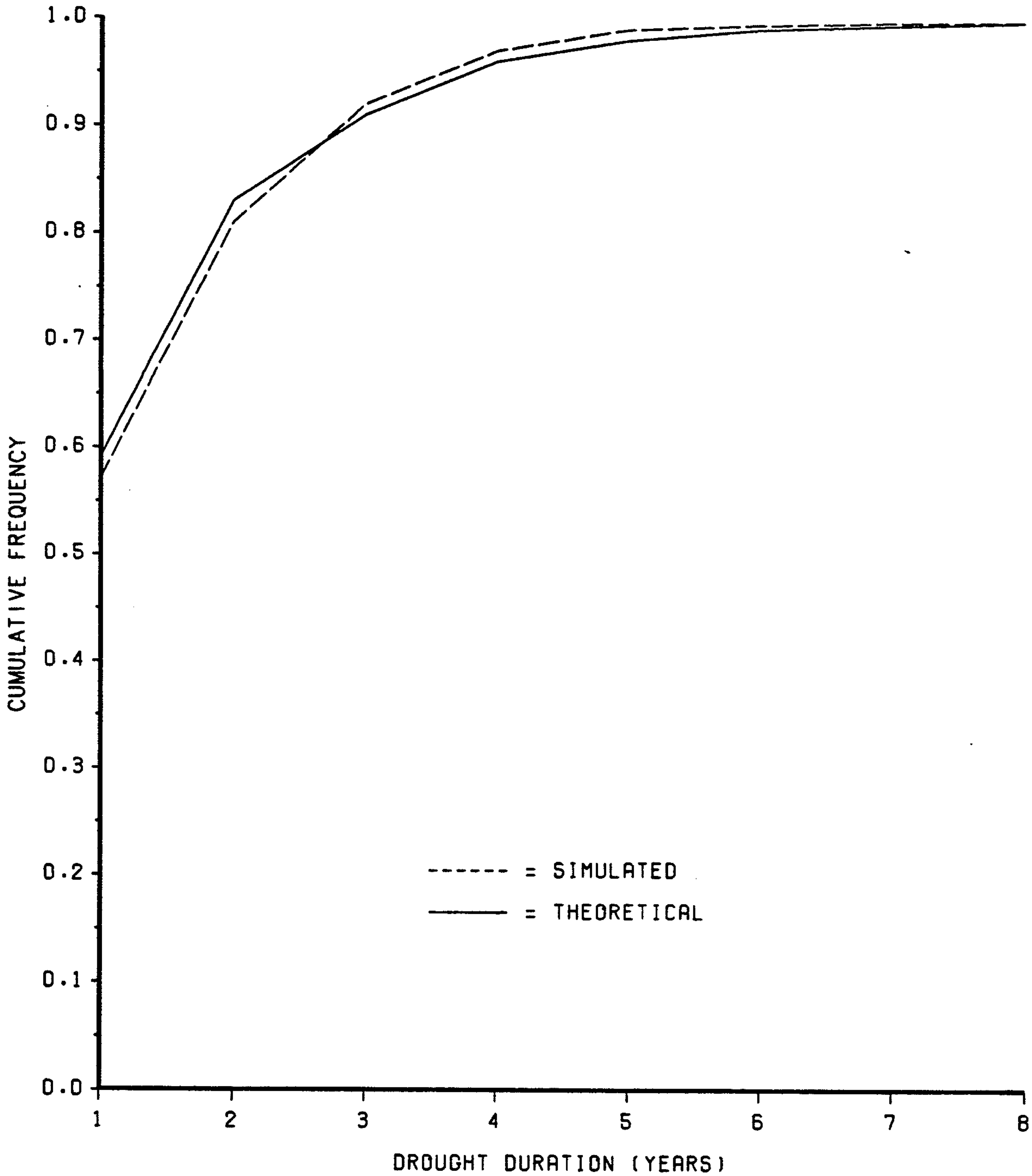

FIG. 5.19 CUMULATIIVE FREQUENCY DISTRIBUTION OF DROUGHT DURATIONS

CHAPTER 6

PROBABILITY DISTRIBUTION OF THE LARGEST DEFICIT, DURATION AND INTENSITY

6.1 THE DISTRIBUTION OF THE LARGEST DROUGHT DEFICIT

The largest drought deficit in the life of a project is one random variable which will be of particular interest and will be the critical drought variable that one would design for.

The distribution of drought deficits was found to be exponential in Section 5.2. Thus, to find the distribution of the largest drought deficit is the same as finding the distribution of the largest value from a population that has exponential distribution. This immediately brings to mind the Extreme Value Type 1 or Gumbel's Extreme Value Distribution. The Extreme Value Distribution will arise when:

1) The deficits are independent.
2) The distribution of the deficits is exponential.
3) The data of deficits is a sufficiently large sample.

The sample of drought deficits satisfy all but the last requirement, therefore this will not exactly fit the Gumbel's distribution which is asymptotic. However, it should bear a resemblance to it. Therefore we should expect the distribution function of the largest drought deficit can be expected to be double exponential.

This double exponential nature of the distribution has been recognised by many researchers in drought analysis. Guven (1983), Zekai Sen (1980) and Kisisel (1979) has shown that the probability distribution of the largest drought duration is approximately double exponential.

Todorovic and Zelenhasic (1970) obtained the distribution of the largest flood as:

$$F_t(x) = \exp(- \wedge(t) [1 - H(x)]) \qquad \ldots (6.1)$$

where $\wedge(t)$ is the average number of floods in the time interval (0 - t) and H(x) is the distribution function of floods.

An elaboration of the derivation of the above function can be found in Appendix A. This approach can be adapted to analyse droughts and this was done by Zelenhasic (1987).

The basic assumptions in arriving at the above function is that the sequence of deficits is independent and identically distributed, and the occurrence of drought events is Poissonian. Since all these assumptions have been satisfied in previous sections, one can apply the Todorovic model to the ABS drought deficits. Since the distribution function of the drought deficits was found to be exponential in Section 5.2, the function of $F_t(x)$ above becomes double exponential.

From Section 5.2, $H(x) = 1 - \exp(-0.03x)$ for time interval of 50 years. Also, from section 5.1, $\Lambda(t = 50) = \lambda = 2.84$.

Hence, $F_t(x) = \exp\{-2.84 \exp(-0.03x)\}$ ...(6.2)

Using simulation, the following results for the cumulative frequency distribution for the largest drought deficit and that from the theoretical distribution are compared below.

TABLE 26

CUMULATIVE FREQUENCIES OF THE LARGEST DROUGHT DEFICIT

Class Interval	Simulation CDF	Theoretical CDF	Deviation
0.5 - 20.5	0.160	0.126	0.034
20.5 - 40.5	0.270	0.321	0.051
40.5 - 60.5	0.540	0.536	0.004
60.5 - 80.5	0.740	0.710	0.030
80.5 - 100.5	0.890	0.829	0.061
100.5 - 120.5	1.000	0.902	0.098

For the Kolmogorov-Smirnov test of goodness of fit, the maximum deviation obtained was:

$$D_{max} = 0.098$$

The modified critical test statistic at 5% level for $N = 60$ is $D_c = 0.113$

Since $D_c > D_{max}$, we conclude that the proposed distribution fits the observed data.

Plots of the CDF obtained by simulation and the theoretical one are shown in Figure 6.1 and one can see the closeness between them.

6.2 THE DISTRIBUTION OF THE LONGEST DROUGHT DURATION

The longest drought duration within a period of time can also be a critical drought variable that will be of interest and therefore its distribution is necessary.

Finding the distribution function of the longest drought duration is equivalent to finding the distribution of the largest term from a set of values. Since the drought durations had been shown to be independent, and on assuming it is identically distributed, then one can apply the Todorovic's Extreme Value theory to obtain the distribution of the largest duration.

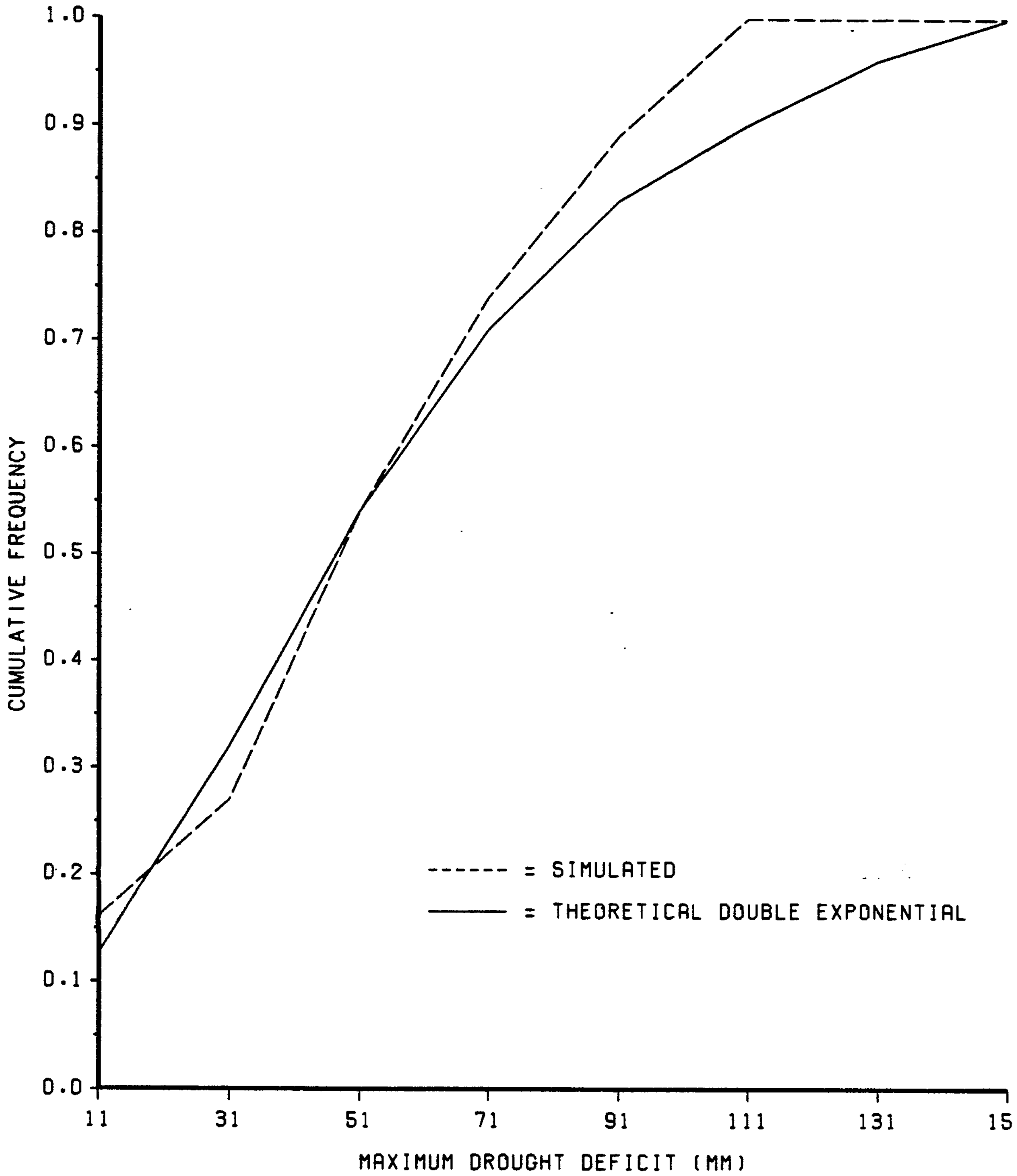

FIG. 6.1 CUMULATIIVE FREQUENCY DISTRIBUTION OF THE MAXIMUM DROUGHT DEFICIT

From the theory we obtain:

$$P(L_m \leqslant j) = P(N_m = 0) + \sum_{n=1}^{\infty} [P(L < j)^n \cdot P(N_m = n)] \quad ..(6.3)$$

where; L_m = the longest drought duration

j = integer ranging from 0, 1, 2 ...

N_m = the number of droughts in time interval m,

and n = a real number (integers)

It has been shown in Section 5.1 that the probability of the number of droughts in a time interval m is Poissonian, that is:

$$P(N_m = n) = ([\wedge(t)^n]/n!) \exp(-\wedge(t))$$

where $\wedge(t)$ is the average number of droughts in time t.

After some algebra, the Expression (6.3) will become, as Expression (6.1) in Section 6.1, as:

$$P(L_m \leqslant j) = \exp[-\wedge(t) \cdot P(L > j)] \quad ... (6.4)$$

$P(L > j)$ had been defined in Section 5.4 as $P(L > j) = r^j$ $(j = 1, 2 ....)$ also $P(L > j) = 1 - P(L \leqslant j) = 1 - H(x)$ in Expression (6.1) of Section 6.1.

From Section 5.4, we obtained:

$$P(L > j) = (0.4325)^j \qquad j = 1, 2 \ldots .$$

and Section 5.1, $\wedge(t = 50) = 2.84$.

Hence the theoretical distribution will be given as

$$P(L_m \leqslant j) = \exp[-2.84\ (0.4325)^j] \qquad j = 1, 2 \ldots$$

From the generated data, the relative frequencies were computed for the longest drought durations in 50 year intervals and the obtained cumulative frequencies with the corresponding theoretical cumulative frequencies are shown in Table 27.

For the Kolmogorov-Smirnov test, the maximum deviation $D_m = 0.054$.

The critical value for sample size 60 at 5% level is $D_c = 0.113$.

Since $D_c > D_m$, the data fits the theoretical model well.

TABLE 27

CUMULATIVE FREQUENCIES OF DROUGHT DURATION

Drought Duration	Simulated Rel. Freq	Simulated Cum. Freq	Theoretical Cum. Freq	Deviation
0	0.0583	0.058	0.058	0.00
1	0.214	0.272	0.293	0.21
2	0.262	0.534	0.588	0.054
3	0.223	0.757	0.795	0.038
4	0.126	0.883	0.905	0.022
5	0.078	0.961	0.958	0.003
6	0.0097	0.971	0.982	0.011
7	0.0194	0.990	0.992	0.002
8	0.0097	0.999	0.996	0.003

Plots of the theoretical and simulated cumulative frequency distributions are shown on Figure 6.2

6.3 THE DISTRIBUTION OF THE LARGEST DROUGHT INTENSITY

The largest drought intensity can also be of interest to the designer. Its distribution will follow that of the largest drought deficit since it also satisfies the same conditions for the Todorovic's extreme value theory.

Therefore, we obtain the distribution of the largest drought intensity as:

$$F_t(x) = \exp(-\Lambda(t)\ [1 - H(x)])$$

with $\Lambda(t)$ as the expected number of droughts in time interval (0 - t), and H(x) the distribution function of the variable.

For a 50 year time span, from Section 5.1, $\Lambda(t = 50) = 2.84$, and H(x) = 1 - exp(-0.004x) in Section 5.3.

Therefore the theoretical distribution of the largest drought intensity in 50 years is:

$$F_t(x) = \exp(-2.84 \exp(-0.04x))$$

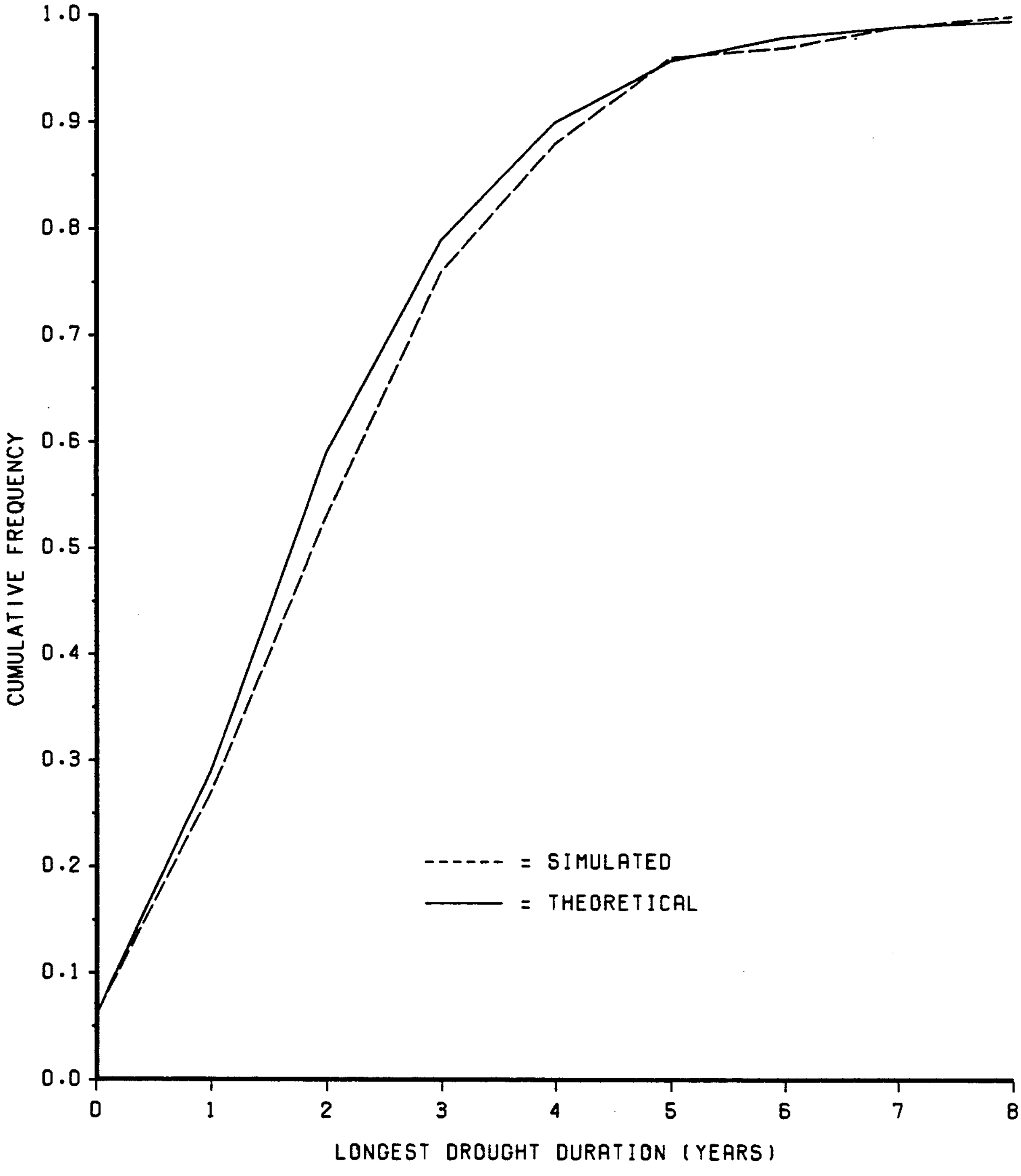

FIG. 6.2 CUMULATIIVE FREQUENCY DISTRIBUTION OF THE LONGEST DROUGHT DURATION

The actual distribution function obtained from the generated data and the above theoretical one are shown in Table 28.

For the Kolmogorov-Smirnov test, the maximum deviation D_{max} = 0.073. The critical value for sample size of 60 at 5% level is D_c = 0.113.

Since $D_c > D_{max}$, it can be concluded that the model fits the data.

Plots of the simulated and theoretical distribution functions are shown in Figure 6.3.

TABLE 28

CUMULATIVE FREQUENCIES OF DROUGHT INTENSITY

Class Interval (mm)	Observed Relative Frequency	Observed Cumulative Frequency	Theoretical Cumulated Frequency	Deviation
0.5 - 20.5	0.216	0.216	0.155	0.061
20.5 - 40.5	0.196	0.412	0.432	0.020
40.5 - 60.5	0.340	0.750	0.686	0.064
60.5 - 80.5	0.155	0.907	0.844	0.063
80.5 - 100.5	0.093	0.99	0.927	0.073
100.5 - 120.5	0.0	1.00	0.966	0.034
120.5 - 140.5	0.0	1.00	0.999	0.001

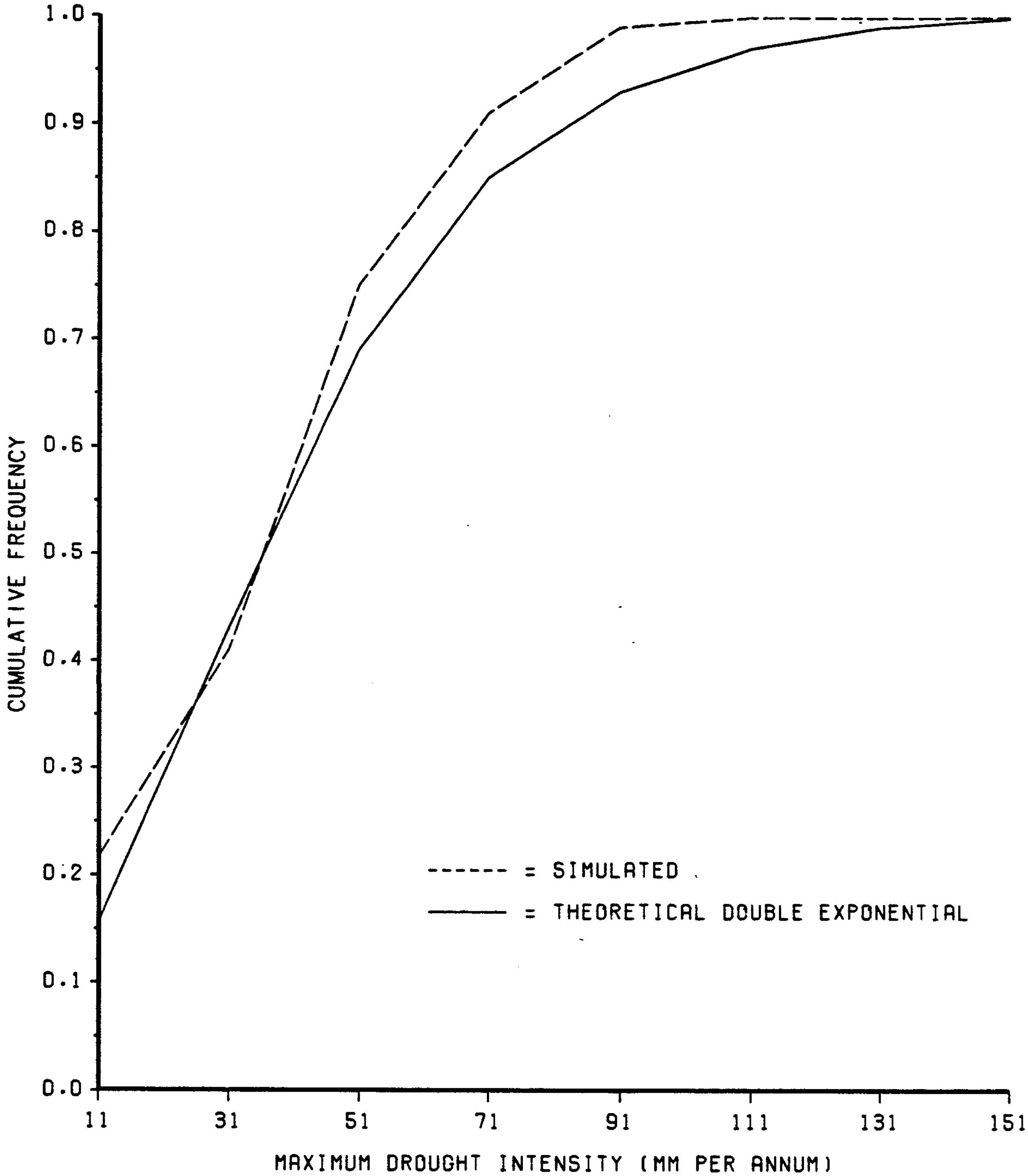

FIG. 6.3 CUMULATIIVE FREQUENCY DISTRIBUTION OF THE MAXIMUM DROUGHT INTENSITY

CHAPTER 7

CONCLUSION AND RECOMMENDATION

7.1 CONCLUSION

From the previous chapters, it has been shown that Accumulated Basin Storage can be used as an index for quantifying agricultural droughts. The relevance of ABS to agricultural droughts is due to its significant correlation to crop yield per unit area. The ABS can therefore be used for:

i) Agricultural drought analysis.

ii) Drought prediction.

For design or planning purposes, the probability of occurrence of the magnitudes of the drought variables are essential and so their distributions were computed. In particular, the probabilities of the largest values of the drought variables will be the critical design parameters and thus their distributions were computed. These probability distributions will be useful for giving better drought prediction.

The distribution of the number of drought events in a given interval of time have been found to be Poissonian. The distribution of drought deficit, intensity and duration have also all been found to be exponentially distributed. In the case of their largest values, the theory of extreme values of a random variable was applied to derive theoretical models for the distributions. In this case, Todorovic's Flood Model was found to be applicable.

Due to limitation of time and data, the study could not be extended to other rivers so that the present results cannot be generalised. Until other basins are also studied, one can only conclude that they are applicable to the Assiniboine River Basin.

7.2 RECOMMENDATION

There is the need for this study to be extended to other river basins before generalised conclusions can be drawn about the distribution of the drought variables using ABS as an index.

The ABS appears to have good potential for use in irrigation design. The drought deficit defined in this study cannot be said to be the exact moisture deficiency for plants. It is the amount of moisture needed to raise the soil moisture level above that which gives drought conditions. To obtain the exact amount of water to provide for in an irrigation scheme, one can use the ABS fitting process to obtain the difference between the 'ideal evapo-transpiration' of the particular plant in question and the actual evapotranspiration that took place as a result of drought. This differences will yield a series of deficits whose statistical properties can be used for irrigation planning. This was not done in this study as no particular plant was under investigation but only the general effect on all vegetation in the whole basin was being considered.

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APPENDIX A

THE DISTRIBUTION OF THE LARGEST DEFICIT AND DURATION

Given the ABS hydrograph (a plot of ABS against time), consider only those values, in some interval of time, that are smaller than a reference level (given drought events).

Let D be the total deficit of ABS within the drought period. Also let the time of occurrence of the drought be Z, its duration T and i be the order number of the event.

The largest drought deficit in time interval 0 - t is:

$$X_{(t)} = \text{Sup } D_i \qquad \text{...(A1)}$$

and the longest drought duration is:

$$X_1(t) = \text{Sup } T_i \qquad \text{...(A2)}$$

where 'Sup' refers to Supremo or the largest value among a term of values.

The distribution function of $X_{(t)}$ is denoted as:

$$F_t(X) = P(X(t) \leqslant x), \quad t \geqslant 0, x \geqslant 0 \qquad \text{...(A3)}$$

Using the result of Theorem 1 in his paper on random number of random variables (Todorovic, 1970A), Todorovic obtained $F_t(X)$ as the mathematical expectation of the conditional probability:

$$P\,(\,\text{Sup}\ D_i \leqslant x \,/\, \wedge(t);\ \ \tilde{Z}(i) \leqslant t\,) \qquad \ldots(A4)$$

where ^(t) is the number of drought events in an interval of time (0 - t). [Todorovic, 1970]. His result, proved in Theorem 1 (Todorovic, 1970A) was:

$$F_t(X) = \sum_{k=0}^{\infty} P\,[\,\bigcap_{i=0}^{K}(D_i \leqslant x) \cap E_k^t\,] \qquad \ldots(A5)$$

which is the probability that all deficits D_i, in an interval of time (0, t) will be less than or equal to a value of x, and E_k^t is the event that exactly k droughts occur in time interval (0 - t).

Expression (A5) can be written as:

$$F_t(X) = P\,[E_o^t] + \sum_{k=1}^{\infty} P\,[\,\bigcap_{i=0}^{K}(D_i \leqslant x) \cap E_k^t\,] \quad ..(A6)$$

To solve for Equation (A6), one has to determine the probabilities in:

$$P\,[\,\bigcap_{i=0}^{K}(D_i \leqslant x) \cap E_k^t\,] \qquad \ldots(A7)$$

Under the following assumptions:

(i) D_i is a sequence of independent identically distributed random variables with $H(x) = P(D_i \leqslant x)$.

(ii) D_i and $\tilde{Z}(i)$ are mutually independent, the

$$F_t(x) = P[E_o^t] + \sum_{K=1}^{\infty} [H(x)^k \ .\ P(E_k^t)] \qquad \ldots(A8)$$

For the first assumption, it had already been shown that the sequence of ABS deficits are independent. Also since only one season (growing season) is being considered, for practical purposes, one can consider the deficits to be identically distributed [Chow, 1964]. Also because of the physical intuition, there is no reason to believe that the deficits are mutually dependent on their times of occurrence [assumption (ii)].

Furthermore, if the distribution of the number of drought events is Poissonian, that is if:

$$P(E_k^t) = [\Lambda(t)]^k \exp[-\Lambda(t)] / k!$$

then Equation (A8) has been shown [Todorovic, 1970] to become:

$$F_t(X) = \exp(-\Lambda(t) [1 - H(x)])$$

in which H(x) is the distribution function of all deficits in a given time interval (0 - t).

The above will also hold for $F_t(X_1)$, the distribution function for the longest drought duration if it satisfies all the preceeding assumptions.