Optimization of semi-flexible transit operation for low demand scenarios

By

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Abstract

Many transit agencies in North America suggest semi-flexible transit (SFT) as a viable solution to the growing demand for highly personalized and expensive paratransit services with an increasingly aging population, high operating costs associated with low demand bus transit routes, and lacking adaptability of fixed-route bus transit to serve increasingly diverse spatiotemporal travel needs. This thesis proposes an effective methodology for the optimization of SFT for operation along an under-performing low demand bus transit route in Regina, Canada.

In this thesis, three research questions are addressed: (1) What levels of demand are optimal for SFT operation, given the two service delivery models, in-house transit, IHT, and contracted-out taxi, COT? (2) How to optimally design service headway (h) and slack time per trip for route-deviation (Δt) in an integrated SFT that serves both fixed-route and paratransit demand? and (3) What is the optimal vehicle size and vehicle technology for SFT operation when comparing two technologies: battery-electric vehicles (BEV) and diesel-based vehicles (ICEV)? Analytical and metaheuristic optimization techniques are employed to determine the optimal value for decision variables. The findings suggest that SFT with COT delivery model is most economical in terms of operator cost when demand is unexpectedly low, SFT with IHT delivery model is more economical when demand is low to medium, and conventional bus transit operating in-house is more cost-effective when transit demand is high. Operator cost favors solutions with low service frequency (i.e., high h), user cost favors lower ranges of h and Δt, and higher service benefit is derived from high Δt; thus, medium ranges of h and Δt appear to provide the most reasonable tradeoff for service. It is also observed that for low demand (5-15 pass/hr), in terms of the total cost (i.e., operator, user, and environmental) a minivan ICEV outperforms all other scenarios as the potential savings in energy cost in favor of BEB were offset by the present high cost of installing fast chargers and when demand increases, minivan BEV outperforms. Besides contributing to the state-of-the-art research on SFT optimization, the study models are used as part of a decision support tool to establish contracting, transit network planning, vehicle technology, operation, and fare policies.
Acknowledgment

This thesis is the culmination of a long and rewarding journey at the University of Manitoba with the support and guidance of remarkable individuals whom I wish to acknowledge.

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I would like to thank the members of my thesis committee, Dr. Garreth Rempel and Dr. Olanrewaju Ojo, for their encouragement, insightful comments, and challenging questions that helped me to strengthen my research from a practical standpoint. I would like to thank Dr. Jonathan Regehr and Dr. Babak Mehran for providing excellent graduate-level courses on urban mobility and transportation, which contributed greatly to my thesis development and other projects. I would also like to thank Dr. Prasanta Sahu for his valuable feedback and comments on the preparation of several research articles.

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Contribution of Authors

The thesis is organized as a "sandwich" or grouped-manuscript thesis with chapters 3, 4, and 5 containing material from journal articles that have been published and are awaiting publication. Following are the titles of the prepared research articles, the manuscript review status at the time of submission of this thesis, the authors' contributions, and relevant chapters of the thesis:

1. Chapter 3 includes the content published in the Transport Policy journal manuscript titled “Assessment of delivery models for semi-flexible transit operation in low-demand conditions”. DOI: [https://doi.org/10.1016/j.tranpol.2020.09.004](https://doi.org/10.1016/j.tranpol.2020.09.004) (Submitted: 26 March 2020, Accepted 4 September 2020)
   Author contributions:
   Sushreeta Mishra (Thesis author): Conceptualization, Visualization, Methodology, Software, Writing original draft
   Babak Mehran: Conceptualization, Methodology, Supervision, Writing - review & editing
   Prasanta Sahu: Conceptualization, Writing - review & editing

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   Author contributions:
   Sushreeta Mishra: Conceptualization, Visualization, Methodology, Software, Writing original draft
   Babak Mehran: Conceptualization, Methodology, Supervision, Writing - review & editing

3. Chapter 5 includes the content of the manuscript under review in Transportation Research Part D journal and titled “Cost analysis of different vehicle technologies for semi-flexible transit operations”. Manuscript ID: TRD-D-23-00097 (Submitted: 17 January 2023)
   Author contributions:
   Sushreeta Mishra: Conceptualization, Visualization, Methodology, Software, Writing original draft
   Babak Mehran: Conceptualization, Methodology, Supervision, Writing - review & editing
# Key Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>FRBT</td>
<td>Fixed Route Bus Transit</td>
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<tr>
<td>DRT</td>
<td>Demand Responsive Transit</td>
</tr>
<tr>
<td>SFT</td>
<td>Semi-Flexible Transit</td>
</tr>
<tr>
<td>COT</td>
<td>Contracted-Out Taxi</td>
</tr>
<tr>
<td>IHT</td>
<td>In-House Transit</td>
</tr>
<tr>
<td>PBD</td>
<td>Poisson Binomial Distribution</td>
</tr>
<tr>
<td>MC</td>
<td>Marginal Cost</td>
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<tr>
<td>AC</td>
<td>Average Cost</td>
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<tr>
<td>MOEA</td>
<td>Multi-Objective Evolutionary Algorithm</td>
</tr>
<tr>
<td>NSGA-II</td>
<td>Non-Dominated Sorting Genetic Algorithm-II</td>
</tr>
<tr>
<td>SMPSO</td>
<td>Speed-Constrained Multi-Objective Particle Swarm Optimization</td>
</tr>
<tr>
<td>HV</td>
<td>Hypervolume</td>
</tr>
<tr>
<td>GD</td>
<td>Generation Distance</td>
</tr>
<tr>
<td>IGD</td>
<td>Inverted Generational Distance</td>
</tr>
<tr>
<td>CT</td>
<td>Computation Time Per Run</td>
</tr>
<tr>
<td>TOPSIS</td>
<td>Technique for Order Performance by Similarity to Ideal Solution</td>
</tr>
<tr>
<td>CUTA</td>
<td>Canadian Urban Transit Association</td>
</tr>
<tr>
<td>OC</td>
<td>Operating Cost</td>
</tr>
<tr>
<td>UC</td>
<td>User Cost</td>
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<tr>
<td>SB</td>
<td>Service Benefit</td>
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<tr>
<td>TC</td>
<td>Total Cost</td>
</tr>
<tr>
<td>ICEV</td>
<td>internal combustion engine vehicle</td>
</tr>
<tr>
<td>BEV</td>
<td>Battery electric vehicle</td>
</tr>
<tr>
<td>SoC</td>
<td>state-of-charge</td>
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Chapter 1. Introduction

1.1. Research motivation

The current trends indicate transit ridership plateaued or declined in many North American cities, especially for bus transit (Boisjoly et al., 2018). Several studies have associated the decline in bus transit ridership with its service pattern, which lacks the adaptability to serve increasingly diverse spatiotemporal travel needs (Taylor et al., 2009). The service pattern of a conventional fixed-route bus transit (FRBT) includes operation along pre-defined routes, stops, and schedules, typically ensuring greater cost efficiency and resource sharing along a high demand corridor. Hence, low travel demand conditions, including demand in suburban/rural areas and during off-peak hours and weekends in urban areas, are postulated to possess a challenge for FRBT, subject to fiscal constraints (Fittante & Lubin, 2015). Yet, arguably, access to FRBT or viable public transit alternatives is essential for citizens to maintain an acceptable quality of living. The first initiative in this direction constitutes demand-responsive transit (DRT), an informal transport service primarily designed to serve passengers with limited mobility and later augmented to serve the general public in low demand conditions (Cervero & Golub, 2007). However, the DRT service pattern offering door-to-door service is deprecated because of its relatively high cost of operation and inefficiency in managing high passenger requests (Davison et al., 2014). Semi-flexible transit (SFT) which combines the rigidity of FRBT, and the flexibility of DRT is the most discussed alternative in the past two decades to serve low-demand travel needs (Errico et al., 2013; Mehran et al., 2020; Mishra et al., 2020). Due to its composite nature, SFT operates with a fixed/flexible route, stop, and schedule to accommodate a few route deviation requests.

1.2. Introduction

SFT service pattern is typically adopted in tech-enabled app-based shared ride services commonly delivered by transportation network companies (TNCs), taxi companies, and microtransits, to provide an on-demand, convenient, flexible, and cost-effective ride (Rayle et al., 2016). Consequently, several transit agencies in North America are recently interested in exploring opportunities to minimize operating costs in low travel demand conditions by delivering SFT service in-house by public transit agencies or contracting-out to private operators. (Westervelt et
The economies of scale in operating cost attained from adopting a delivery model are attributed to its structure (i.e., fully in-house, fully contracted, or hybrid) influenced by the interaction of supply and demand components (Palmer et al., 2008). For instance, small-sized transit agencies offering low passenger-revenue hours provide most of their services in-house to avoid additional costs involved in the bidding process and contractor oversight. However, these agencies often lack technological advancement and contract-out their scheduling, routing, and dispatching services to provide a cost-efficient service (Edrington et al., 2013). Curtis et al. (2019) reviewed 36 partnerships between transit agencies and TNCs in the US and revealed that most agencies use TNCs through fully contracted agreements to supplement FRBT by first mile/last mile service or as an alternative to FRBT for late night or early morning, and on-demand service. Livermore Transit, CA experienced a 38.6% reduction in cost per trip on replacing an unproductive FRBT service along suburban routes with SFT in contract with both Uber and Lyft. Similarly, Pinellas County Transit saved $40,000 per year in operating costs while collaborating with Uber and United Taxi to provide first-/last-mile service. Thus, for a given service pattern/delivery model, it is essential to identify the factors affecting the operating cost and, subsequently, develop a quantitative model to assist policymakers in assessing the potential for cost reduction.

Several transit agencies in North America suggest SFT as an interesting solution to respond to the growing demand for expensive paratransit services and the high operating cost of FRBT in low-demand conditions (Koffman, 2004). Regina Transit in Canada facing a similar situation has recently piloted a flexible transit service along a least frequently used route hoping to increase ridership and the vehicle used is accessible with spots reserved for wheelchairs/mobility devices (Klumpenhouwer et al., 2020). While paratransit demand continues to grow in many North American communities, some transit operators, including Regina Transit, are providing training and incentives to help shift trips from paratransit/DRT to less costly modes like FRBT and SFT (Weiner, 2008). Regina paratransit, like most others, is currently operating at its capacity (City of Regina, 2021). Consequently, SFT with route deviations, when provided along an under-performing low-demand bus route, is capable of substituting both FRBT and DRT in the service zone and is beneficial to both shifted regular transit and paratransit users as well as the operator (Koffman, 2004; Weiner, 2008). Paratransit users will benefit from improved response times and
flexible mobility, thereby improving their social inclusion. There will be more resources at the disposal of transit agencies for paratransit service in the same or other zones, in addition to cost savings. General transit users could expect a discounted fare as the total operating cost reduces.

In addition to the development of flexible mobility services like DRT or SFT, urban transit systems are experiencing and undertaking the rapid evolution of electrification because of its benefits, including operational cost savings, reduced energy consumption and emissions, and ease of deployment and maintenance (Islam & Lownes, 2019). As far as electric bus operation is concerned, Canada has several demonstration projects underway (Li, 2016). For instance, three 5.3 meters compact electric buses manufactured by Tecnobus, have been operating in Quebec since 2005. A 12.2m lithium-ion-powered bus developed by Mitsubishi Heavy Industries and the New Flyer is operating on Winnipeg Transit route 20 Academy-Watt. Despite its considerable potential, implementation of SFT is limited due to uncertain economic stability due to smaller vehicle sizes and circuitous routes, and only in low demand density scenarios are SFT services more competitive than FRBT services (Davison et al., 2014). With SFT electrification, the expected reduction in operating costs related to energy consumption and environmental costs could open new scenarios for SFT to be applied more widely, and transit agencies may benefit more greatly. Additionally, the flexibility and adaptability of SFT can help transit agencies offset the high capital costs and infrastructure costs (charging stations) associated with transit electrification through contracting and shifting paratransit trips to comparatively cheaper modes like SFT as discussed above.

1.3. Need for this research

Several studies in recent decades have attempted to develop rigorous operating and user cost models to quantify the economic feasibility of SFT (Errico et al., 2013). However, the studies assessing SFT delivery models mostly focused on qualitative aspects including policy issues and service quality through case studies (Brake et al., 2007; Curtis et al., 2019; Haglund et al., 2019; Jokinen et al., 2019; Mulley et al., 2012; Schwieterman, 2019; Sharmeen & Meurs, 2019). These studies focused on deriving policy lessons to overcome a range of barriers to implementing SFT. The barriers are (i) economic (financing and user cost), (ii) operational (fleet utilization and management), (iii) institutional (legislation, market regulation, and sectoral policies), (iv) cultural, and (v) information and education. Among these, the economic barrier is the most attended by past
research efforts because of its importance in making budgetary decisions by transit agencies. However, the previous research focusing on economic issues in implementing SFT delivery models mostly used rudimentary information provided by transit agencies to quantify operating costs which include aggregated costs of service components like average annual maintenance, fuel, and administrative costs (Curtis et al., 2019; Fittante & Lubin, 2015). The aggregate cost possesses a uniform cost for SFT operation per unit time; however, the operating cost changes according to spatial and temporal variations in demand and service. Thus, there is a need for an analytical disaggregate model that accounts for marginal variations in operating cost due to the change in demand and service characteristics. Such a model can assist policymakers in assessing the economic feasibility of SFT delivery models.

The modeling and optimization of SFT to provide an efficient service is a challenging and complex task since it requires retaining both FRBT and DRT properties (Errico et al., 2013). Due to its composite nature, SFT operates with a fixed/flexible route, stop, and schedule to accommodate a few curb-to-curb stop requests. Key design parameters for SFT include zone size/service area, headway, slack time, and demand patterns (Errico et al., 2013; Koffman, 2004). While several design parameters govern SFT, the determination of the amount of slack time allocated in the schedule to accommodate route deviations for serving paratransit users is the most critical (Fu, 2002). However, it is noted that studies optimizing the decision variables that are elements of strategic planning or tactical planning for SFT like headway and slack time are very limited and are essential to define a timetable for SFT operation (Vansteenwegen et al., 2022).

Battery electric vehicles (BEVs), powered by electricity stored exclusively in a battery package, can achieve zero emissions, but their design entails additional components, primarily battery capacity and charging strategy, which makes them a more complex powertrain typology than diesel-powered internal combustion engines (ICEV) (Li, 2016). BEVs are typically charged in four ways: overnight charging, fast/opportunity charging, battery swapping, and wireless charging. The cost of battery swapping and wireless charging infrastructures, however, is significantly higher than the cost of the fast-charging system, and overnight charging will require a greater fleet size of small-capacity electric vehicles characteristic of SFT since the maximum allowed battery capacity and vehicle size are limited. Therefore, the BEVs must be off duty until they have been
fully charged at the overnight charging stations equipped at the depot (Ke et al., 2016). Thus, the opportunity charging strategy is more prevailing and allows BEVs to be recharged without significantly reducing the useful operation time or availability of the vehicle. Additionally, studies implementing opportunity charging focus on two strategies: (a) charging within a single trip assuming consecutive trips are independent and irrelative (Badia & Jenelius, 2021), and (b) charging between multiple trips because the state-of-charge (SOC) of the electric bus after the last trip influences the route range for the following trip (Bektaş et al., 2016). Based on existing knowledge, SFT operation with small-sized battery-operated vehicles and opportunity charging strategies can be a competitive solution in the future mobility industry while studies in this direction are limited.

In summary, three research questions are addressed in this thesis: (1) At what levels of demand is SFT economically feasible, given the various service delivery models?, (2) How to optimally design the schedule parameters of an integrated SFT that can meet both fixed-route and paratransit demand through route-deviation?, and (3) Can SFT be economically feasible with battery-powered vehicles as opposed to diesel-powered vehicles?

1.4. Thesis objectives

Following the background and the identified research gaps, the purpose of this thesis is to develop an effective methodology for economic assessment, design, and electrification of SFT for operation along an under-performing low demand transit route. This purpose is accomplished through three research objectives, framed below as a set of questions:

1) What levels of demand justify switching between competing service patterns (i.e., FRBT and SFT) and service delivery models (i.e., in-house and contract-out)?

To address this objective, analytical operating cost models are used to identify the critical passenger demand when service patterns (SFT and FRBT) with one/both delivery models (in-house transit, IHT, and contracted-out taxi, COT) could substitute one another to minimize the total cost of operation considering daily temporal demand distribution.

2) How to optimally design an integrated SFT that can meet both fixed-route and paratransit demand through route-deviation?
This objective was addressed by implementing meta-heuristic algorithms to optimize service headway \((h)\) and slack time per trip \((\Delta t)\) (i.e., additional time allocated in the schedule to accommodate route deviation requests) that minimize operator and user costs, while maximizing service benefits defined as the cost incurred to serve paratransit passengers using a dedicated DRT service if not served by SFT.

(3) Can SFT be economically feasible with battery-powered vehicles as opposed to diesel-powered vehicles?

The objective is addressed by using analytical models for estimation of the total cost incurred by the transit agency, including operator, user, and environmental costs, providing a comparative analysis of battery electric vehicle technology (BEV) with opportunity charging and internal combustion engine (ICEV) vehicles, considering three vehicle sizes: minivans, standard vans, and minibuses.

1.5. Thesis scope

As described in Section 1.2, the economic assessment and design space for SFT services is vast, and much of it is beyond the scope of this thesis. To establish the scope of this thesis and the connections between the chapters, a taxonomy for SFT service economic assessment and design consisting of the following dimensions is described in detail below: (i) the type of competing transit systems, (ii) the type of SFT operating policy, (iii) SFT application area, (iv) SFT service delivery model types, (v) passenger types, (vi) spatial and temporal patterns of demand, (vii) vehicle stopping policy, (viii) vehicle capacity and technology and (ix) optimization model properties.

(i) Competing transit systems

The three major road public transit service patterns adopted by transit agencies in North America include fixed route bus transit (FRBT), semi-flexible transit (SFT), and demand responsive transit (DRT) with varied levels of flexibility/rigidity offered in route, pick-up and drop-off locations, and schedule. A comparison of the economic feasibility of FRBT and SFT systems is presented in Chapter 3, while Chapter 4 and Chapter 5 are devoted to the design and electrification of SFT systems only.
(ii) Flexible transit service pattern

SFT operating policy is further classified into six sub-categories with decreasing order of flexibility: demand-responsible connector, zone route, point deviation, route deviation, flexible route segments, and request stops (Koffman, 2004). SFT service pattern adopted in Chapter 3 includes most characteristics of a “request stops” type, where vehicles operate in a fixed route, flexible schedules, and serve a limited number of requested stops within a zone. In Chapters 4 and 5, the “route-deviation” operating policy is adopted where vehicles follow a fixed route and deviate to serve curb-to-curb requests, with a maximum allowable deviation on both sides. This operating policy accepts two types of stop requests: flag requests and curb-to-curb requests. Flag requests involve vehicles stopping at any location along the route, which may or may not correspond to a marked stop. Curb-to-curb requests involve vehicles deviating from their fixed route to serve pick-up and drop-off locations requested by passengers in advance (usually 1 hour).

(iii) SFT application area

In addition to providing service to limited areas that are otherwise difficult to reach due to demographics and road network layouts, SFT can also be used to replace a costly transit alternative by DRT/FRBT for paratransit passengers and general passengers in limited routes/zones or at certain low-demand time periods during off-peak hours and weekends, as well as as a primary transit mode in small cities, low-density suburban areas, and rural areas (Koffman, 2004). This research focuses on replacing an existing DRT/FRBT along an underperforming low-demand bus route.; however, the models developed can also be used to assess the replacement of transit service during low-demand time periods.

(iv) SFT service model delivery types

Commonly adopted delivery model structures include fully in-house, fully contracted, and hybrid (Palmer et al., 2008). In Chapter 3, in-house delivery of FRBT is assumed, while for SFT, two delivery models, Contract-Out Taxi (COT) and In-House Transit (IHT), are compared, while for SFT, in-house delivery is assumed in Chapters 4 and 5.

(v) Passenger type
SFT serves as a substituting or complementing transit service for two existing transit modes FRBT and DRT, SFT could be used as a service for two types of passengers: (1) Type G- existing FRBT demand commonly designed for the general public and (2) Type S-existing DRT demand referring to users that are eligible for paratransit service in the study area. Chapter 3 develops an analytical model focusing on the Type G passengers while analytical models in Chapters 4 and 6 are developed considering both Type G and Type S passengers.

(vi) Spatial and temporal patterns of demand

The study area in this research is modeled as a rectangle with dimensions $W$ (km) and $L$ (km) along an existing fixed bus route defined by two terminal stations. It is assumed in Chapter 3 that passenger arrivals are Poisson distributed, and the non-uniformity in the spatial distribution of passenger demand is captured by adopting a Poisson binomial distribution (PBD). Chapters 4 and 6 assume that demand per trip is uniformly and independently distributed within the service area for ease of analysis; however, the derived models are based on the fact that low-demand routes typically have a higher headway (i.e., >10 minutes) and published schedules; therefore, passengers may or may not exhibit random arrival at stop locations and may adjust their arrival time at departure stops to minimize waiting times (Ansari Esfeh et al., 2021).

(vii) Vehicle stopping policy

Three typical stopping regimes are usually found in urban transit operations: (1) demand stopping, (2) on-call stopping, and (3) fixed stopping (Kikuchi & Vuchic, 1982). Demand stopping is the most flexible and allows passengers to board and alight anywhere within the service area. On-call stopping is slightly more restrictive, as passengers must board at fixed stops, but vehicles stop only when there is demand. Fixed stopping is the most regulated, as vehicles will stop at all stops or stations, regardless of demand. In Chapter 3, it is assumed that FRBT and SFT vehicles adhere to on-call stopping when operated in-house and demand stopping when contracted out. In Chapters 4 and 6, SFT vehicles follow on-call stopping for serving Type G passengers and demand stopping for serving Type S passengers.

(viii) Vehicle capacity and technology
This thesis compares the economic feasibility of different vehicle capacities in each competing transit alternative based on passenger demand levels, such as a bus in FRBT and a taxi, minibus, or van in SFT. Vehicle capacity is an important design parameter for flexible transit (M. Kim et al., 2019) handled differently in each chapter as constraint, analytical modeling parameter, or sensitivity analysis. In Chapter 5, which examines the cost-effectiveness of alternative vehicle technologies for SFT, two types of propulsion systems are considered: diesel-powered internal combustion engines (ICEVs) and battery electric vehicles (BEVs), whereas the initial chapters assume ICEV technology.

(ix) Optimization model properties

This section focuses on defining the key elements of flexible transit optimization models studied in this thesis including decision variables, objective(s), constraints, and solution method. In Chapter 3, critical passenger demand is considered as a decision variable for performance evaluation of service patterns (i.e., FRBT and SFT) and service delivery models (i.e., IHT and COT). Numerical analysis is implemented to derive optimal values of decision variables that minimize the operating cost as the objective function. In Chapter 4, using heuristic methods, the optimal values of slack time and headway are derived that minimize operator and user costs while maximizing the benefit defined as the cost of serving paratransit demand in an expensive DRT mode if not serviced by SFT, when vehicle capacity is constrained. Chapter 5 identifies the optimal vehicle technology and vehicle size that minimizes the total system cost including operator, user, and environmental cost using numerical analysis.

1.6. Thesis organization

This thesis is structured as a grouped manuscript or sandwich-style thesis and is organized into seven chapters. In Chapter 1, the topic is introduced, the research objectives are specified, the value of the research is argued, and the scope of the papers included is outlined. In Chapter 2, studies focusing on several subfields of literature, including SFT service delivery design, SFT system design elements including headway and slack time, electrification, and automation of flexible transit, are reviewed and summarized to reveal existing gaps in the literature. Each of Chapters 3, 4, and 5 pertains to a research module and focuses on the overarching research
objectives discussed in section 1.3. Each chapter contains a methodology section, results and discussion section, and conclusion section. Lastly, Chapter 6 presents the conclusion, outlining the findings and contributions of Chapters 3, 4, and 5 in relation to the formulated research objectives, potential applications, limitations of the methodology, and future research prospects.
Chapter 2. Literature review

A large body of literature concurs that it is cost-efficient to operate SFT in low-demand conditions and FRBT when the demand for transit is high (Errico et al., 2013; Koffman, 2004). The concept has been emphasized in several studies, including this one, which expands on previous research by addressing what service delivery models minimize operator costs, how to best operate an integrated SFT that can meet both fixed-route and paratransit demand, and if battery-powered vehicles are economically feasible for SFT. The following sections provide a detailed discussion of the studies related to each of these topics, which are summarized in Tables 2.2, 2.3, and 2.4.

2.1. Economic assessment of flexible transit

2.1.1. Service pattern classification

Characteristics of three major public transit service patterns - FRBT, SFT, and DRT adopted by transit agencies in North America are illustrated here. Table 2.1 compares the service patterns based on the level of flexibility offered in route, pick-up and drop-off locations, and schedule along the base route and/or the surrounding zone. SFT service pattern is further classified into six sub-categories with decreasing order of flexibility: demand-responsible connector, zone route, point deviation, route deviation, flexible route segments, and request stops (Koffman, 2004). SFT service pattern adopted in this study, as presented in Table 2.1 includes most characteristics of a “request stops” type, where vehicles operate in a fixed route, flexible schedules, and serve a limited number of requested stops within a zone.

2.1.2. SFT evaluation method

Table 2.2 lists the most relevant studies on on-demand services, labeled by model characteristics and solution methods. Evidently, SFT services designed by transit agencies often serve the general public along mainline routes or are coordinated with major bus transit routes as a feeder service. Several studies considered critical passenger demand as a decision variable for the performance evaluation of a service pattern based on user cost and/or operator cost. Based on operating cost, the critical demand is determined at which it is desirable to shift from FRBT to SFT and in-house to contracted SFT delivery model along a low-demand bus route.
2.1.3. SFT service delivery design

Westervelt et al. (2017) and studies in Table 2.2 revealed that the taxi companies, TNCs, and microtransit providers hold the majority of fully or partially contracted projects with transit agencies to provide cost-efficient SFT service in low demand conditions. Fittante and Lubin (2015) evaluated the performance of SFT, offering a feeder service to FRBT and rail transit in suburban counties in Middlesex County, United States. The operating cost per hour for shuttle service was $50.00 compared to $75.00 for the counterpart, FRBT, given both are delivered in-house. Table 2.2 suggests that contrary to the promising economic potential of SFT in terms of travel time and operating cost irrespective of the delivery model, SFT is expensive in terms of user fare. However, other relevant studies suggest that on-demand service users have a higher willingness to pay for ‘in-vehicle time, perceived comfort, and safety’ than traditional public transport (Alonso-González et al., 2020; Weckström et al., 2018). Focussing on operating cost reduction through contracting, Turmo et al. (2018) reported potential savings in providing DRT by contracting taxi companies or TNCs to serve a proportion of ADA-eligible customers; alongside existing in-house paratransit service. However, as discussed most studies have employed basic aggregate cost models to evaluate the efficiency of adopting a delivery model or used disaggregate level analysis while focussing on DRT. Mehran et al. (2020) are the first to evaluate the operational feasibility of in-house and contracted SFT along a low demand bus route using disaggregate cost models. Nevertheless, the cost model adopted for in-house SFT and FRBT, assumes operating cost to be only a linear function of vehicle-hours traveled, and regression analysis was performed to determine the cost coefficient that incorporates the influence of other cost components. Thus, the application of the cost models is limited to the availability of past data and prone to overestimation; hence, it cannot be applied for the evaluation of routes with no pre-existing transit service like routes connecting newly developed regions. As an extension of our previous research, this study attempts to formulate mathematically the operating cost models, including all contributing components to perform a more fine-grained analysis for better decision-making.
### Table 2.1 Elements of service design for FRBT, SFT, and DRT

<table>
<thead>
<tr>
<th>Service pattern</th>
<th>Delivery model</th>
<th>Route features</th>
<th>Advance booking</th>
<th>Target user</th>
<th>Mode</th>
<th>Vehicle capacity</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>FRBT</td>
<td>IH</td>
<td>FX</td>
<td>FX</td>
<td>N</td>
<td>Type 1</td>
<td>B</td>
<td>≥ 30</td>
</tr>
<tr>
<td>Proposed</td>
<td>COT</td>
<td>CT</td>
<td>FX</td>
<td>Y (same day)</td>
<td>Type 1</td>
<td>T</td>
<td>1-4</td>
</tr>
<tr>
<td>(Chapter 3)</td>
<td>IHT</td>
<td>IH</td>
<td>FX</td>
<td>Y (same day)</td>
<td>Type 1</td>
<td>T/M</td>
<td>1-15</td>
</tr>
<tr>
<td>Request stops</td>
<td>IH/CT</td>
<td>FX/FL</td>
<td>FX</td>
<td>Y</td>
<td>Type 1</td>
<td>T/M</td>
<td>1-15</td>
</tr>
<tr>
<td>DRT</td>
<td>IH/CT</td>
<td>FL</td>
<td>FL</td>
<td>Y</td>
<td>Type 2</td>
<td>T/M</td>
<td>1-15</td>
</tr>
</tbody>
</table>

Delivery model: **IH**- In-house, **CT**- Contracted; Route features: **FX**- Fixed, **FL**- Flexible; Advance booking: **Y**- Yes, **N**- No; Target user: **Type 1**- General public, elderly, and disabled, **Type 2**- Mostly elderly and disabled; Mode: **B**- Bus; **T**- Taxi; **M**- Micro transit

### Table 2.2 SFT service designs

<table>
<thead>
<tr>
<th>Reference</th>
<th>Route type</th>
<th>Service Pattern</th>
<th>Delivery model</th>
<th>Operator</th>
<th>Performance measure</th>
<th>Evaluation technique</th>
<th>Decision variable</th>
<th>Cost model</th>
<th>Target user</th>
<th>Cost (Pr/ Ex)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turmo et al. (2018)</td>
<td>ML</td>
<td>DRT DRT</td>
<td>CO</td>
<td>T/ TNCs</td>
<td>UC (TT)</td>
<td>AN (MD)</td>
<td>CD (Yearly)</td>
<td>DA</td>
<td>Type 2</td>
<td>0.9-0.67</td>
</tr>
<tr>
<td>Quadrifoglio and Li (2009)</td>
<td>FD</td>
<td>FRBT SFT</td>
<td>IH</td>
<td>PTA</td>
<td>UC (TT)</td>
<td>AN (MD)</td>
<td>CD (Areal density)</td>
<td>DA</td>
<td>Type 1</td>
<td>&lt; 1</td>
</tr>
<tr>
<td>Zheng et al. (2018a)</td>
<td>ML</td>
<td>SFT</td>
<td>IH</td>
<td>PTA</td>
<td>UC (TT)</td>
<td>AN (MD)</td>
<td>CD (Hourly)</td>
<td>DA</td>
<td>Type 1</td>
<td>-</td>
</tr>
<tr>
<td>Rahimi et al. (2018)</td>
<td>ML</td>
<td>DRT DRT</td>
<td>CO</td>
<td>T/ TNCs</td>
<td>UC</td>
<td>AN (MD)</td>
<td>CD (Yearly)</td>
<td>DA</td>
<td>Type 2</td>
<td>&lt; 1</td>
</tr>
<tr>
<td>Qiu et al. (2015)</td>
<td>ML</td>
<td>FRBT SFT</td>
<td>IH</td>
<td>PTA</td>
<td>UC</td>
<td>AN (MD)</td>
<td>CD (Hourly)</td>
<td>DA</td>
<td>Type 1</td>
<td>&lt; 1</td>
</tr>
<tr>
<td>Haglund et al. (2019)</td>
<td>ML</td>
<td>DRT DRT</td>
<td>IH</td>
<td>PTA</td>
<td>UC</td>
<td>AN (MD)</td>
<td>D (Hourly)</td>
<td>A</td>
<td>Type 1</td>
<td>4.33</td>
</tr>
<tr>
<td>Schwieterman (2019)</td>
<td>ML</td>
<td>FRBT DRT &amp; SFT</td>
<td>CO</td>
<td>TNCs/M</td>
<td>UC</td>
<td>EMP (CS)</td>
<td>D (Hourly), TY</td>
<td>A</td>
<td>Type 1</td>
<td>2.81-7.22</td>
</tr>
<tr>
<td>Fittante and Lubin (2015)</td>
<td>FD</td>
<td>FRBT SFT</td>
<td>CO</td>
<td>PTA</td>
<td>UC</td>
<td>EMP (CS)</td>
<td>SF</td>
<td>A</td>
<td>Type 1</td>
<td>0.83</td>
</tr>
</tbody>
</table>

Route type: **FD**- Feeder, **ML**- Mainline; Service pattern: **Ex**- Existing, **Pr**- Proposed, **FRBT**- Fixed route bus transit, **DRT**- Demand responsive transit, **SFT**- Semi-Flexible transit; Delivery model: **IH**- In-house, **CT**- Contracted; Operator: **T**- Taxi, **TNCs**- Transit network companies, **M**- Microtransit companies, **PTA**- Public transit agency; Performance measure: **OC**- Operating cost, **UC**- User cost, **TT**- Travel time, **F**- Fare; Evaluation technique: **AN**- Analytical, **EMP**- Empirical; Study type: **MD**- Methodological, **CS**- Case study; Decision variable: **CD**- Critical demand, **D**- Demand, **TY**- Trip type classified by neighbourhood, **SF**- Service frequency; Cost Model: **DA**- Disaggregate, **A**- Aggregate; Target user: **Type 1**- General public, elderly, and disabled, **Type 2**- Mostly elderly and disabled
2.2. Flexible transit system design

2.2.1. Key elements of flexible transit service design

This section focuses on defining the key elements of flexible transit optimization models studied in literature including decision variables, objective(s), constraints, and solution method as illustrated in Table 2.3. The decision variables or optimized variables for SFT commonly include routing and scheduling (Pei et al., 2019), service zone size (M. Kim et al., 2019; Smith et al., 2003; L. Wang et al., 2018), passenger request (Zheng et al., 2019), headway (Estrada et al., 2021; M. Kim et al., 2019; M. (Edward) Kim & Schonfeld, 2014), velocity (Quadrifoglio et al., 2006), and slack time (Alshalalfah, 2009; Alshalalfah & Shalaby, 2012; Fu, 2002; Smith et al., 2003). Most objectives can be classified into two categories: operator-related and user-related cost and service benefit. Operator costs include the minimization of fleet acquisition and operation costs (Estrada et al., 2021). User-related objectives include the minimization of travel time components such as access time, waiting time, and in-vehicle time (Alshalalfah, 2009; Fu, 2002). To attain system-wide savings, few studies optimize total cost considering both operator and user-related objectives (M. (Edward) Kim & Schonfeld, 2014). The benefit associated is specific to the operators’ intent like increasing revenue/fare income (Pei et al., 2019), reducing parking infrastructure investment (Alshalalfah, 2009), increasing mobility, reducing vehicle miles and emissions, or replacing a costly transit alternative DRT/FRGBT for paratransit passengers and passengers in suburban or rural areas (Fu, 2002). The design of SFT includes constraints characteristic of regular bus transit design including capacity (M. Kim et al., 2019), vehicle arrival and departure schedule (Pei et al., 2019), travel time (Fu, 2002), and fleet size (Zhao et al., 2021) in addition to including constraints characteristic of DRT like zoning (Smith et al., 2003) and passenger pick-up and drop-off schedule (Pei et al., 2019). Finally, for a given set of objective functions and constraints, the optimal value of decision variables can be derived using analytical models (M. Kim et al., 2019), numerical approximation (Nourbakhsh & Ouyang, 2012), simulation (Fu, 2002), and heuristics (Lai et al., 2022). In this study, using heuristic methods, the optimal values of slack time and headway are derived that minimize operator and user costs while maximizing the benefit defined as the cost of serving paratransit demand in an expensive DRT mode if not serviced by SFT, when vehicle capacity is constrained.
2.2.2. *Studies focusing on slack time optimization for flexible transit*

Fu (2002) proposed the first analytical model to determine the optimal slack time for a flex service to accommodate door-to-door paratransit requests while serving mandatory stops along the route. This model minimizes the total net cost to all stakeholders, including the operator, and regular and paratransit passengers. The fundamental relationships between system performance and design parameters revealed using an analytical model are further validated using simulation. Despite capturing some general trends, the models developed failed to capture the details of system behavior. Smith et al. (2003) implemented a heuristic method to optimize two key design variables in flex-route service planning: service area and slack time distribution. The optimization problem included two objectives: maximization of feasible deviations (i.e., from the operator’s perspective) and minimization of dwell time/unused slack time (i.e., from the user’s perspective). Two existing fixed routes with a maximum of five major fixed stops were chosen to serve as flex routes. Assuming a deterministic scenario, this study uses the gradient method to derive Pareto-optimal solutions for transit planners to assess design trade-offs. Alshalalfah (2009) implemented analytical modeling and constraint programming for static and dynamic flex-route service optimization. The system is designed to cover mandatory stops with a predetermined schedule while accommodating on-demand route deviation requests constrained by slack time in the schedule. The study derived optimal values for service area and slack time that minimize the operator and user costs and maximize the savings in parking costs when encouraging people to switch from using their cars to using transit when accessing a regional rail network. Zheng et al. (Zheng et al., 2018b) proposed a slack arrival strategy to improve the acceptance rate of the flex-route service at both expected and unexpected demand levels. Analytical and simulation models are developed to investigate the optimal slack time window based on system cost, including the operator and user costs. Studies cited above have focused on designing SFT and most studies, except these, analyze slack time differently. Alshalalfah and Shalaby (2012) conducted sensitivity analyses with various slack time values (0 to 12 minutes) to study its effect on the number of accepted demand-responsive requests. Quadrifoglio et al. (2006) suggested that the maximum slack time between checkpoints for MAST vehicles could be set by the minimum threshold longitudinal velocity value while minimizing the total distance traveled. Lai et al. (2022) considered slack ratio in designing flexible transit system elements such as path, pick-up and drop-
off location, and schedule that maximizes vehicle sharing and the number of accepted requests while minimizing the walking time. These studies do not, however, focus on identifying the optimal slack time value.

2.2.3. Studies focusing on headway optimization for flexible transit

Kim and Schonfeld (2014) implemented a probabilistic optimization model to determine optimal vehicle capacity, headway, and fleet size that minimizes the passenger transfer cost in integrated conventional and flexible feeder systems with coordinated transfers. The feeder system offers door-to-door service and follows a predetermined schedule to make timed transfers. Wang et al. (2018) derived an analytical model to identify zone size and headway that minimizes both operator and user costs when designing a many-to-one DRT between a residential area and a terminal. Nourbakhsh et al. (2012) proposed a DRT bus service along a service area designed as a hybrid of hub-and-spoke and grid networks. The study optimizes the network layout, service area, and headway based on operator and user cost using numerical approximation. Likewise, for a many-to-one flexible door-to-door feeder system, Kim et al. (2019) suggested that joint optimization of service headway and zone size is essential for minimizing the total system cost. This paper implemented Newton’s method to solve for the optimal values while constraining the vehicle capacity. Estrada et al. (2021) determined the optimal vehicle technology (i.e., diesel, electric, and autonomous), service pattern (i.e., SFT, FRBT, and DRT), and vehicle size (i.e., mini-bus, van, bus, and car) for varying demand density. Enumeration procedure implemented to identify headway, stop spacing, and waiting time for the above scenarios that minimizes the total cost to the operator and user constrained by capacity.

As a final consideration, it is noted that studies optimizing the decision variables that are elements of strategic planning or tactical planning for SFT like headway and slack time are very limited and are essential to define a timetable for SFT operation (Vansteenwegen et al., 2022). Although Alshalalalfah (2009) and Fu (2002) suggested some interaction between slack time and headway, they focused on the optimization of slack time, assuming a fixed value of headway for system design. Joint optimization of slack time and headway for an integrated service has not yet been addressed in the literature, which mostly concentrates on optimizing them separately. The motivation for joint optimization is derived from Kim et al. (2019) who compared two optimization
scenarios for flexible-bus service: 1) One decision variable, zone size considering the maximum allowable headway policy, and 2) Joint optimization of headway and zone size. According to the study, scenario 1 has a 26% greater average cost per passenger trip and an 83% larger optimal zone size than scenario 2. When compared to scenario 2, scenario 1 proposes solutions that reduce operator costs but increase in-vehicle and waiting costs.
## Table 2.3 Summary of studies related to flexible transit system design

<table>
<thead>
<tr>
<th>Year</th>
<th>Reference</th>
<th>System</th>
<th>Method</th>
<th>Objective</th>
<th>Constraint</th>
<th>Decision variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>2002</td>
<td>Fu (2002)</td>
<td>SFT</td>
<td>AN, SM</td>
<td>Max. OC, Min. UC, &amp; Max. SB</td>
<td>TT, SC</td>
<td>Slack time</td>
</tr>
<tr>
<td>2003</td>
<td>Smith et al. (2003)</td>
<td>SFT</td>
<td>HT</td>
<td>Min. OC &amp; Min UC</td>
<td>SC</td>
<td>Service area and slack time distribution</td>
</tr>
<tr>
<td>2006</td>
<td>Quadrifoglio et al. (2006)</td>
<td>DRT</td>
<td>AN, SM</td>
<td>Min. OC</td>
<td>SC</td>
<td>Longitudinal velocity</td>
</tr>
<tr>
<td>2009</td>
<td>Alshalalfah (2009)</td>
<td>SFT</td>
<td>AN, SM</td>
<td>Max. OC, Min. UC, &amp; Max. SB</td>
<td>SC, CP</td>
<td>Service area and slack time distribution</td>
</tr>
<tr>
<td>2012</td>
<td>Alshalalfah and Shalaby (2012)</td>
<td>SFT</td>
<td>AN, SM</td>
<td>Max. OC</td>
<td>SC, CP</td>
<td>Slack time</td>
</tr>
<tr>
<td>2012</td>
<td>Nourbakhsh et al. (2012)</td>
<td>DRT</td>
<td>NM</td>
<td>Min. OC &amp; Min UC</td>
<td>SC, CP</td>
<td>Network layout, service area, and headway</td>
</tr>
<tr>
<td>2014</td>
<td>Kim and Schonfeld (2014)</td>
<td>SFT</td>
<td>AN, HT</td>
<td>Min. OC &amp; Min UC</td>
<td>SC</td>
<td>Vehicle capacity, headway, and fleet size</td>
</tr>
<tr>
<td>2018</td>
<td>Wang et al. (2018)</td>
<td>DRT</td>
<td>AN</td>
<td>Min. OC &amp; Min UC</td>
<td>CP</td>
<td>Service area</td>
</tr>
<tr>
<td>2018</td>
<td>Zheng et al. (2018b)</td>
<td>SFT</td>
<td>AN, SM</td>
<td>Min. OC &amp; Min UC</td>
<td>SC</td>
<td>Slack arrival strategy</td>
</tr>
<tr>
<td>2019</td>
<td>Kim et al. (2019)</td>
<td>DRT</td>
<td>AN</td>
<td>Min. OC &amp; Min UC</td>
<td>CP</td>
<td>Service area and headway</td>
</tr>
<tr>
<td>2019</td>
<td>Pei et al. (2019)</td>
<td>SFT</td>
<td>HT</td>
<td>Min.[SB-(OC+UC)]</td>
<td>SC</td>
<td>Stop locations and routes</td>
</tr>
<tr>
<td>2021</td>
<td>Estrada et al. (2021)</td>
<td>DRT &amp; FRB T</td>
<td>AN</td>
<td>Min. OC &amp; Min UC</td>
<td>CP</td>
<td>Stop distances, headways, or waiting time</td>
</tr>
<tr>
<td>2022</td>
<td>Lai et al. (2022)</td>
<td>SFT</td>
<td>HT</td>
<td>Min. OC &amp; Min UC</td>
<td>CP</td>
<td>Path, pick-up and drop-off location, vehicle schedule</td>
</tr>
<tr>
<td>Proposed</td>
<td></td>
<td>SFT</td>
<td>HT</td>
<td>Max. OC, Min. UC, &amp; Max. SB</td>
<td>CP</td>
<td>Service headway and slack time</td>
</tr>
</tbody>
</table>

*Method*: NM - Numerical, HT- Heuristic, SM- Simulated, AN- Analytical; *Objective*: OC- Operating cost, UC- User cost, TC- Total cost, SB- Service benefit; *Constraints*: CP- Capacity, TT- travel time for vehicle/passenger, SC- vehicle/passenger schedule constraints, ZN- zoning/service area constraints, F- Fleet size
2.3. Electrification of flexible transit

Estrada et al. (2021) proposed the first analytical model for determining the optimal vehicle technology for semi-flexible transit and taxi in the domain of intermediate demand (refer to Table 2.4). This model identifies an optimal headway range for semi-flexible transit that minimizes the total cost to the operator and user. Despite capturing some general trends, the models developed failed to capture the details of system behavior as they do not include essential components like environmental cost, capital costs associated with charging infrastructure, and estimation of additional fleet size requirements due to charging time. Badia and Jenelius (2021) studied the impact of automation and electrification on feeder transit systems serviced by fixed routes and door-to-door services in suburban areas. Analytical models based on continuous approximations are used to identify the configuration of decision variables that minimize total costs per passenger. Due to electrification, the authors modified the cost structure by considering charging time in estimating fleet size and the number of charging stations. Although extensive, this study estimates fleet size by adding a fixed amount of charging time per cycle, with the conservative assumption that this time cannot overlap with layover time, and it does not consider the impact of battery size variation on operating costs. Barraza and Estrada (2021) developed robust analytical models to design an efficient transit network operated by battery electric and diesel buses in cities with a grid-shaped road network, based on continuous approximations. The study identifies the optimal vehicle size, vehicle technology, and charging strategy that minimizes operator cost, emissions cost, and the travel time of transit users, while optimizing the network design (spatial and temporal coverage) parameters. Estrada et al. (2022) derived analytical operating cost models to estimate the number of resources to be deployed for different vehicle powertrain technologies and charging schemes. Costs incurred by the transit agency include vehicle depreciation and labour, battery cost, distance cost, and charging infrastructure cost. Several charging schemes are evaluated for battery electric vehicles, including (a) day charging at bus garages with and without advancing the charging period to reduce fleet sizes during peak hours, and (b) opportunity charging at electric stations located at both or one end or terminal stop with or without skipping charging operations during peak hours. Although the above-cited studies summarized in Table 2.4 evaluated the cost effectiveness of vehicle technology and charging scheme using analytical most that modified the
cost structure due to electrification, many studies concerning demand responsive transit focus on autonomous electric vehicles (Militão & Tirachini, 2021) and estimate the change in unit cost parameters with the level of automation or use aggregated annual costs or simulation for estimation (Bösch et al., 2018; Schlüter et al., 2021).

There is a substantial body of literature that performed feasibility analysis confirming that SFT is cost-effective under low-demand conditions and regular bus transit is cost-effective under high-demand conditions (Mehran et al., 2020; Mishra et al., 2020). In terms of the design of SFT, a large body of literature focus on operational aspects such as route optimization and transit coordination but studies optimizing variables related to tactical or strategic planning such as headway and slack time for SFT are limited. While Alshalalfah (2009) and Fu (2002) suggested that slack time and headway have some interaction, they focused on the optimization of slack time and assumed a fixed value for headway when designing the system. Mishra & Mehran (2023b) optimized slack-time and headway to accommodate both paratransit and general transit users. However, these studies do not focus on electric vehicles or automation technology. This study expands on previous research by investigating whether electrifying SFT is economically feasible. The study will address the optimization of slack time and headway for a battery electric vehicle-based SFT that has not been addressed in the literature.
<table>
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*System:* FRBT- Fixed route bus transit, DRT- demand responsive transit (door-to-door service), SFT- semi-flexible transit; *Method:* NM-Numerical, HT- Heuristic, SM- Simulated, AN- Analytical; *Objective:* OC- Operating cost, UC- User cost, EC- Environmental cost, SB- Service benefit; *Technologies:* EV- Electric vehicle, ICEV- diesel-powered internal combustion engines, HEV- Hybrid electric vehicle, AV- Autonomous vehicle
2.4. Identified research gaps and standing of current work

(1) Earlier studies typically used aggregate operator cost models to evaluate the effectiveness of in-house or outsourced delivery models or performed disaggregate level analyses focusing on DRT. Study by Mehran et al. (2020), co-authored by the thesis author, developed disaggregate cost models for evaluating SFT delivery models assuming linearity between operator cost and vehicle-hours travelled and implemented regression analysis for unit cost estimation. As a result, the developed cost models are limited by the availability of historical data and are subject to overestimation, making them inapplicable to routes without pre-existing transit services, such as routes connecting newly established regions. As an extension of our previous research, Chapter 3 of this thesis proposes analytical disaggregate models for both delivery models that incorporate marginal variation in the operating costs of SFT resulting from changes in demand and service characteristics. The operator cost models include all contributing components to provide a more thorough analysis to facilitate better decision making.

(2) From section 2.2, it is noted that studies optimizing the decision variables that are elements of strategic planning or tactical planning for SFT like headway and slack time for route-deviation are very limited (Vansteenwegen et al., 2022). While Alshalalfah (2009) and Fu (2002) suggested some interaction between slack time and headway, they emphasized optimization of slack time assuming a fixed value of headway, ignoring the interdependency of the two variables. The literature has not yet addressed the joint optimization of slack time and headway, which is generally confined to optimizing them separately. In Chapter 4 this gap is addressed by performing joint optimization of slack time and headway which is essential for designing a schedule for the operation of an integrated SFT that can meet both fixed-route and paratransit demand through route-deviation. The individual effects of slack time and headway as well as their interdependencies on operator costs, user costs, and service benefits are examined.

(3) Most studies that used analytical methods to evaluate the cost effectiveness of vehicle technology and charging schemes with modified cost structures due to electrification focused on FRBT, while few evaluated alternative vehicle technologies for flexible transit systems like DRT and SFT. The models developed in studies relating to flexible transportation systems do not adequately capture the details of the system behavior, as they fail to include one or more
essential components such as environmental costs, capital costs associated with charging infrastructure, estimation of additional fleet sizes due to charging time, battery costs or make the conservative assumption that charging time cannot overlap with layover time or use over-simplified aggregated cost models. In chapter 5, the impact of the new requirements for battery electric vehicles (BEV) on the operation of SFT services is examined while accounting for most of the essential components outlined above. This chapter focuses on developing rigorous analytical models for detailed estimation of the total cost incurred by the transit agency, including operator, user, and environmental costs, allowing a comparison with conventional vehicle technology: internal combustion engines (ICEV), and comparing three vehicle sizes: minivans (7-seaters), standard vans (15-seaters), and minibuses (25-seaters).
Chapter 3. Assessment of delivery models for semi-flexible transit operation in low-demand conditions

3.1. Abstract

Semi-flexible transit (SFT) service pattern coupled with real-time dynamic scheduling of passenger requests and dynamic route assignment of vehicles is considered as a cost-effective alternative to serving public transportation users in low demand conditions. Transit agencies tend to provide SFT by delivering it in-house by public transit agencies or contracting-out to private operators like transportation network-, taxi- or microtransit companies. Past studies focusing on economic issues in implementing SFT delivery models mostly used rudimentary information provided by transit agencies to quantify the operating cost, including budgetary inputs such as average annual maintenance, fuel, and administrative costs. These aggregate costs imply a uniform cost for SFT operation per unit time; however, the cost is subject to change through spatial and temporal variations in demand and service. This study developed analytical disaggregate operating cost models that could account for marginal variations in cost due to the change in demand and service characteristics. Such a model can assist policymakers while evaluating the economic feasibility of SFT delivery models. Rigorous and approximate operating cost models are developed for two competing service patterns: commonly adopted fixed-route bus transit (FRBT) and proposed SFT. Assuming in-house delivery of FRBT, SFT operating cost equations are derived for two service delivery models, namely Contract-Out Taxi, and In-House Transit. Subsequently, these cost models are used to identify the critical passenger demand, where service patterns or delivery models could replace one another to minimize the total cost of operation. Study models can be used as part of a decision support tool to establish contracting, planning, and operating policies to optimize SFT operation subjected to required service quality.

Keywords: Transit operating policy; Semi-flexible transit; Transit contracting; In-house transit; Critical passenger demand; Low demand
3.2. Methodology

3.2.1. Model description

3.2.1.1. Transit service pattern and delivery model description

FRBT offers continuous two-way service from fixed terminal stations along a fixed route of length, \( L \); stops, \( n \); and a pre-defined schedule, as illustrated in Figure 3.1(a). COT refers to the app-based taxi services administered and operated by TNCs or taxi companies. As illustrated in Figure 3.1(b), COT follows the SFT service pattern, and the taxis are expected to be flexible to adopt any shortest route to the requested stop location(s), which does not necessarily follow the existing fixed route. IHT refers to a taxi, van, or mini-bus service (similar to microtransit), and it follows the SFT service pattern offering a shared ride terminal-to-stop (and stop-to-terminal) service. Vehicles of different capacities are considered in each competing transit alternative to compare economic feasibility based on passenger demand levels, such as a bus in FRBT and a taxi or mini-bus, or van in SFT. It is assumed that app-based service requests in SFT are managed by an online dispatch system ensuring maximum vehicle utilization in each trip while satisfying vehicle capacity and time window constraints.

3.2.1.2. Economic performance measure

The performance of a transit service pattern and delivery model can be measured by operator cost and user cost. However, the effect of the delivery model on the earlier is more significant if the fare is insensitive to the delivery model. Thus, this study evaluates SFT delivery model efficiency only based on operating costs in $/passenger. Here, the demand is measured in passengers per hour to enable agencies to evaluate the service patterns/delivery models based on demand at any hour of a day.

The list of symbols, definitions, and units used in deriving analytical cost models are discussed in Appendix D. The frameworks illustrated in Appendix E serve as a comprehensive guide to detailing inputs, outputs, and analytical procedures for implementing the study models developed in this chapter for any other low demand route.
3.2.2. Analytical modelling

3.2.2.1. Probability of stopping along a low demand bus route

Along a low demand route, on-call stop policy is practiced (Kikuchi & Vuchic, 1982), and the general expression for the expected number of stops, \( E(s) \) along a route is given in Eq. (3.1). Transit stopping is assumed to follow a binomial distribution, and passenger arrival is Poisson distributed at a stop. The binomial distribution assumes equal passenger demand for boarding/alighting (\( \lambda \)) at any stop location along a route.

\[
E(s) = n \left[ 1 - e^{(-\lambda)} \right] = n \left[ 1 - e^{\left(\frac{2ph}{n}\right)} \right] \quad \text{Eq. 3.1}
\]

\[
E(s) = \begin{cases} 
  n, & \text{if } ph \to \infty \\
  2ph, & \text{if } ph < n 
\end{cases} \quad \text{Eq. 3.2}
\]
where, $p$ is hourly passenger boarding demand (pass/hr), $n$ is the number of stops; $h$ is service headway (hr/TU). So, the probability of transit stopping at a stop is given as $(1 - e^{-\lambda})$ where $\lambda = 2ph/n$. In Eq. (3.2), the upper limit indicates that transit stops at all stop locations ($n$) in a high demand case, and the lower limit indicates that for a very low passenger demand condition, the passengers tend to board and alight at distinct stop locations.

Tirachini (2014) reported that the assumption of Poisson arrival pattern and uniform passenger distribution along a low demand route leads to an overestimation of $E(s)$. Although in this chapter the Poisson arrival pattern is assumed, the non-uniformity in passenger demand is captured by adopting a Poisson binomial distribution (PBD) for transit stopping. PBD ensures the probability of stopping at a stop is independent and non-identical for each stop along the route (Y. H. Wang, 1993). The probability mass function and properties of the transit stopping process following a PBD is defined in Appendix A. Eq. (3.3) determines the $E(s)$ following a PBD.

$$E(s) = \sum_{i=1}^{n} \left[ 1 - e^{-\lambda_i} \right] = \sum_{i=1}^{n} \left[ 1 - e^{(-\frac{2p_i h}{n})} \right] \quad \text{Eq. 3.3}$$

where, $\lambda_i$ is the boarding/alighting demand observed at each stop $i$ and $p_i$ is the average hourly passenger demand for boarding and alighting at stop $i$ ($1 \leq i \leq n$). $E(s)$ estimate is subsequently used to calculate the time spent in acceleration and deceleration of FRBT and SFT vehicles.

3.2.2.2. Rigorous analytical operating cost formulation for service patterns

I. Fixed Route Bus Transit (FRBT)

FRBT operation cost per passenger ($C_{FRBT}$) determinants as expressed as in Eq. (3.4) are Fleet Size ($N$), Vehicle-miles travelled ($VMT$), and Vehicle-hours travelled ($VHT$).

$$C_{FRBT} = c_0 + \frac{c_1 \times N}{p} + \frac{c_2 \times VMT}{p} + \frac{c_3 \times VHT}{p} \quad \text{Eq. 3.4}$$

where, the cost coefficient $c_0$ is the sum of fixed costs associated with the three determinants ($$/passenger), $c_1$ is the cost of acquisition per vehicle ($$/veh), $c_2$ is the cost of operation per vehicle-mile ($$/veh-km), and $c_3$ is the cost of operation per vehicle-hour ($$/veh-hr). The variable
component of operating cost is expressed as a function of total annual ridership ($P$) along the route, which is the product of $p$ and the total annual service hours of all vehicles in the system ($H_s$).

Eq. (3.5) expresses $N$ as a ratio of total cycle time, $T_c$ (hr), and service headway, $h$ (hr/TU). $T_c$ consists of operating time, $T_o$ (hr), and terminal time, $T_t$ (hr) while $\gamma = T_t / T_o$.

$$N = \left[ \frac{T_c}{h} \right]^+ = \left[ 2\left( \frac{T_o + T_t}{h} \right) \right]^+ = \left[ \frac{2T_o(1 + \gamma)}{h} \right]^+$$  \hspace{1cm} \text{Eq. 3.5}

$T_o$ consists of three components: total riding time ($T_r$), total time spent in acceleration and deceleration of vehicle ($T_{loss}$), and time spent in boarding and alighting of passengers ($T_p$) as given in Eq. (3.6).

$$T_o = T_r + T_{loss} + T_p = \frac{L}{V_R} + t_{loss} \times E(s) + 2pht$$  \hspace{1cm} \text{Eq. 3.6}

$$t_{loss} = \frac{V_R}{\bar{a}} + \frac{V_R}{\bar{b}}$$  \hspace{1cm} \text{Eq. 3.7}

where, $L$ is the length of the route (km) and $V_R$ is the average riding speed (km/hr). $t_{loss} = \text{time spent in acceleration (}\bar{a})$ and deceleration (\bar{b}) of the vehicle at each stop, and $t = \text{time spent in boarding and alighting per passenger (hr)}$ as expressed in Eq. (3.7).

The cost components corresponding to VMT include fuel cost, maintenance cost, and vehicle depreciation cost, and VHT includes hourly wage and benefits of staff. The VMT and VHT models follow linear form as expressed in Eq. (3.8) and (3.9), respectively.

$$VMT = N \times D$$  \hspace{1cm} \text{Eq. 3.8}

$$VHT = N \times H$$  \hspace{1cm} \text{Eq. 3.9}

where, $D$ (in km) and $H$ (in hour) represent the annual distance travelled and annual service hours for each vehicle. Eq. (3.10) indicates a distinct relationship between VMT and VHT, where VHT is a function of VMT and total delay (Chen & Petty, 2001). Total delay per trip consists of two components: demand delay and congestion delay. Demand delay in Eq. (3.11) is the delay caused in serving the passenger demand with an average cycle speed ($V_c$). Congestion delay, $d$ (hr/km) in
Eq. (3.12) is a function of the speed of the transit vehicle \((V_R)\) and the speed of the adjacent traffic \((V)\).

\[
VHT = \frac{VMT}{V_c} + VMT \times d \quad \text{Eq. 3.10}
\]

\[
\frac{1}{V_c} = \frac{T_c}{L} \quad \text{Eq. 3.11}
\]

\[
d = \left(\frac{1}{V} - \frac{1}{V_R}\right)^+ \quad \text{Eq. 3.12}
\]

II. Semi-flexible transit

A. Contract-out taxi (COT) delivery model

Typically, in contracting, the private taxi operators periodically produce the invoice generated during operation to the transit agencies, to claim generated revenues. Operating cost per passenger in COT \((C_{COT})\) is thus assumed to follow a regulated fare structure similar to a taxi (Turmo et al., 2018). \(C_{COT}\) is a ratio of average fare and average passenger occupancy of taxi \((m)\) as shown in Eq. (3.13). Average fare includes a base fare and variable fare in terms of trip length, \(l^t\) (km), and delay due to congestion, \(d^t\) (hr). \(l^t\) in Eq. (3.14) is the average of the shortest paths \((d_{ij}^k; i,j \in [1, n])\) from stop \(i\) to \(j\) along the network of ‘\(k\)’ routes weighted over passenger demand at origin stop location \(i\) \((P_i)\). \(d^t\) is a function of the average riding speed of the taxi \((V_R^k)\) and speed of the adjacent traffic \((V)\) as expressed in Eq. (3.15).

\[
C_{COT} = \frac{[\beta_0 + (\beta_1 \times l^t) + (\beta_2 \times d^t \times l^t)]}{m} \quad \text{Eq. 3.13}
\]

\[
l^t = \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} P_i \times d_{ij}^k}{\sum_{i=1}^{n} P_i} \quad \text{Eq. 3.14}
\]

\[
d^t = \left(\frac{1}{V} - \frac{1}{V_R^k}\right)^+ \quad \text{Eq. 3.15}
\]

here, \(\beta_0\) represent fixed cost to hire a taxi (\($\)), \(\beta_1\) operating cost per distance travelled (\($/km\)), \(\beta_2\) represent operating cost per delay time experienced (\($/hr\)).
B. In-house transit (IHT) delivery model

C_{IHT} - the operation cost model of IHT is similar to FRBT, as it is completely operated and administered by the public transit agency. The operating cost coefficients $c_4$, $c_5$, $c_6$, and $c_7$ in Eq. (3.16) represent the same parameters as in Eq. (3.4). Cycle time ($T_c^h$) in Eq. (3.17) for IHT fleet size ($N^h$) consists of dead-heading time ($T_d$) in addition to transit operation time ($T_0^h$) and terminal time ($T_t^h$) while $\gamma^h = T_t^h / T_0^h$. Dead-heading time refers to scheduled time spent traveling from the terminal station to the origin and from the final destination to the terminal station with no passenger boarded. $T_d$ is defined as a percentage ($f$) of the maximum possible dead-heading time, $(T_d)_{max}$. The maximum dead-heading time for a trip of length $l^h$ occurs when the origin or the destination point is the last stop location along the route, as expressed in Eq. (3.18). IHT service headway, $h_0$ (hr) as given in Eq. (3.19) is a function of minimum ($W_{min}$) and maximum waiting time ($W_{max}$) representing the available service time window for pick-up operation. $l^h$ in Eq. (3.20), is the weighted linear average of shortest paths from a stop $i$ to $j$ weighted over passenger demand. The definitions of components of $T_0^h$, VMT, and VHT in Eq. (3.21), Eq. (3.22), and Eq. (3.23-3.24) respectively follow the same definitions as provided for FRBT.

$$C_{IHT} = c_4 + \frac{c_5 \times N^h}{P} + \frac{c_6 \times VMT^h}{P} + \frac{c_7 \times VHT^h}{P} \tag{3.16}$$

$$N^h = \left[ \frac{T_c^h}{h_0} \right]^+ = \left[ \frac{2T_0^h + T_t^h + T_d}{h_0} \right]^+ = \left[ \frac{2T_0^h + \gamma^h T_0^h + T_d}{h_0} \right]^+$$

$$= \left[ T_0^h (2 + \gamma^h) + (f \times (T_d)_{max}) \right]^+ \tag{3.17}$$

$$(T_d)_{max} = \frac{2L - l^h}{V_R^h} \tag{3.18}$$

$$h_0 = W_{min} + \left( \frac{W_{max} - W_{min}}{2} \right) \tag{3.19}$$

$$l^h = \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} P_i \times d_{ij}}{\sum_{i=1}^{n} P_i} \tag{3.20}$$

$$T_0 = \frac{l^h}{V_R^h} + t_{loss}^h \times E(s) + 2p h_0 t^h \tag{3.21}$$
\[ VMT = N^h \times D \quad \text{Eq. 3.22} \]
\[ VHT = N^h \times D \times \left[ \frac{1}{V_C^h} + d^h \right] \quad \text{Eq. 3.23} \]
\[ d^h = \left[ \frac{1}{V} - \frac{1}{V_R^h} \right]^+ \quad \text{Eq. 3.24} \]

where, \( V_R^h \) represents average riding speed (km/hr), \( t_{\text{loss}}^h \) represents time spent in acceleration and deceleration of transit in each stop (hr), and \( t^h \) represent time spent in boarding and alighting of a passenger (hr), \( V_C^h \) is average cycle speed, and \( V_R^h \) is average riding speed.

### 3.2.2.3. Analytical operating cost approximation for SFT delivery models

The approximate model assumes that FRBT is a cost-inefficient alternative for low demand conditions and hence defines approximate operating cost formulations only for evaluating SFT delivery models (i.e. COT and IHT). The two space-time approximations employed over the models to derive solutions are discussed below.

**Approximation 1 (A1):** Low traffic volumes and uncongested roads characterize the low demand bus routes. So, congestion delay in cost models can be approximated to 0, see Eq. (3.25).

\[ d(\text{or } d^t \text{ or } d^h) \equiv 0 \quad \text{Eq. 3.25} \]

**Approximation 2 (A2):** For a low demand condition, the lower limit of \( E(s) \) as given in Eq. (3.26) is adopted. Here, \( E(s) \) is twice the number of passengers boarded in a vehicle.

\[ E(s) = 2ph, \quad \text{for } \sum_{i=1}^{n} \lambda_i = ph < n \quad \text{Eq. 3.26} \]

Hence, applying A1 and A2, the operating cost per passenger for COT and IHT delivery models are expressed in Eq. (3.27) and (3.28).

\[ cco_c = \frac{[\beta_0 + (\beta_1 \times l^t)]}{m} \quad \text{Eq. 3.27} \]
\[ C_{\text{IHT}} = c_5 + \left( \frac{c_6 + c_7 \times D + c_8 \times \frac{D}{V_c}}{P} \right) \left( f \times \left( T_d \right)_{\text{max}} \frac{h_0}{h_0} \right) \]
\[ + \frac{2 + \gamma^h}{h_0} \left( \frac{t^h}{V_R^h} + 2p h_0 t^h + 2p h_0 t^h \right) \]

Eq. 3.28

3.2.3. Performance evaluation

3.2.3.1. Critical passenger demand \((P_c)\)

Rigorous and approximate cost models defined in the previous section are used for estimating critical passenger demand \((P_c)\), which is a decision variable to evaluate the performance of service patterns and delivery models. \(P_c\) is the demand at which two service patterns/delivery models are equally competent in terms of operating cost. In essence, at \(P_c\), it is desirable to shift to a service pattern/delivery model offering reduced cost of service. \(P_c\) estimated using rigorous analytical models corresponds to the intersection points of operating cost curves of FRBT, COT, and IHT. However, \(P_c\) for the delivery model estimated using approximate formulation is defined by simply equating the per-passenger operating cost equation of COT and IHT as defined in Eq. (3.27) and (3.28), respectively. Eq. (3.29) is the resulting expression for the approximate estimation of \(P_c\) for SFT delivery model.

\[
P_c = \frac{G \bar{m} \left[ (f \div (T_d)_{\text{max}} \times V_R^h) + (2 + \gamma^h) t^h \right]}{h_0 \times V_R^h \left[ (\beta_0 + (\beta_1 \times t^l)) H - c_5 \bar{m} H - 2G \bar{m} (2 + \gamma^h)(t^h \text{loss} - t^h) \right]}
\]

Eq. 3.29

where, \(G = \left( c_6 + c_7 \times D + c_8 \times \frac{D}{V_c} \right) \)

3.2.3.2. Compare system configurations

As discussed in the previous section, the economic feasibility of the operation of a service pattern/delivery model is dependent on hourly passenger demand. Thus, to account for real-time variation in hourly demand along the route, it is suggested to compare the economic feasibility of service patterns/delivery models when operated independently or integrated within (i.e., a combination of service patterns or delivery models). The average operating cost per passenger \((\bar{c})\) for an independent/integrated system is used as a metric for comparison and can be estimated using
Eq. (3.30). The system configuration offering the least $\bar{c}$ is further analyzed to define a plan for operations.

$$\bar{c} = \begin{cases} 
\sum_{p=1}^{(P_c)_1} (C_1)_p \times f_p + \sum_{p=(P_c)_1}^{(P_c)_2} (C_2)_p \times f_p + \sum_{p=(P_c)_2}^{p_{max}} (C_3)_p \times f_p, & \text{for three integrated system} \\
\sum_{p=1}^{(P_c)_1} (C_1)_p \times f_p + \sum_{p=(P_c)_1}^{p_{max}} (C_2)_p \times f_p, & \text{for two integrated system} \\
\sum_{p=1}^{p_{max}} (C_1)_p \times f_p, & \text{for independent system}
\end{cases}$$

where,

$p =$ hourly passenger demand value (pass/hr)

$p_{max} =$ maximum observed hourly passenger demand value (pass/hr)

$C_1, C_2, and C_3 =$ Cost of operation per passenger ($/pass) for transit alternatives 1, 2, and 3 respectively

$f_p =$ Proportion of total annual trips corresponding to ‘$p$’ hourly passenger demand value

$(P_c)_1$ and $(P_c)_2 =$ Critical passenger demand to shift from 1 to 2 and 2 to 3 respectively

3.2.4. Operations planning

3.2.4.1. Scheduling

Scheduling is used to allocate daily service hours (i.e., peak and off-peak hours) along the route to each service pattern/delivery model for the proposed system configuration in section 3.2.3.2. Hence, the observed average hourly passenger demand values and $P_c$ are plotted against the hour of operation in a day to define the schedule for shifting between service pattern/delivery models. The hour of the day corresponding to the intersection of the hourly demand profile and $P_c$ line represents the switching time.
3.2.4.2. Vehicle utilization

From section 3.2.4.1, it is apparent that if the proposed system is an integrated system, then vehicles utilized in each service pattern/delivery model remain unutilized for a proportion of service hours distinctly peak or off-peak hours. Low demand routes are characterized by fewer peak hours and more off-peak hours. So, it could be hypothesized that diverting a percentage of trips during peak hours to vehicles used in off-peak hours could possibly reduce the fleet size acquired for peak hour service and maximize the utilization of all operating vehicles during the day. For instance, using microtransits to serve a proportion of peak demand along with bus transit. The stated hypothesis is reviewed by comparing the marginal cost (MC) and the average cost (AC) of providing a service pattern/delivery model. Eq. (3.31) represents the marginal cost is defined as the cost of serving an additional trip by a transit unit used in a service pattern/delivery model. Thus, it is suggested to shift the proportion of trips corresponding to values of passenger demand when the MC of a service pattern/delivery model used in an off-peak hour is less than the one used in peak hours.

\[ MC = AC + P \times \left( \frac{d(AC)}{dP} \right) \]  

Eq. 3.31

3.3. Results

3.3.1. Study area

The proposed models are employed for a low demand bus route in the city of Regina with a population density of 1,195.2/km², a medium-sized city in Western Canada. FRBT is the primary public transit mode, provided by Regina transit, a public transportation agency operated by the City of Regina. As per the data obtained from Regina transit for 2015, the FRBT serves an annual transit ridership of 47,971,184 passengers in 118 buses (37-seated) operating along 20 routes on weekdays, 12 routes on Saturdays, and 8 routes on Sundays. CUTA (2015) report found that Regina transit serves an average ridership of 25 pass/veh-hr with an average revenue-to-cost ratio (R/C) of 26.27%; thus, the remaining 73.73% of the cost is subsidized by the non-users essentially the city and taxpayers. This R/C is attained while maintaining a subsidized fare of $0 for a child (i.e., < 4 years of age), $2.75 for youth (5-14 years of age), and $3.25 for others representing 1.6%,
9.8%, and 88.6% of the total annual ridership, respectively. Analysis of Regina transit data demarcates routes 14, 15, 16, and 6 with low annual ridership as underperforming routes when compared to the average R/C. Bus route 6 selected for analysis is characterized by: low ridership of 9 pass/veh-hr, R/C of 15.3%, high operating cost of $10.84/pass, and declined ridership rate of 1.04% from the previous year. The passenger boarding data used in the analysis are also obtained from Regina transit. The values of cost parameters discussed in previous sections are assumed or estimated by analyzing the data included in Appendix B.

3.3.2. Effect of passenger demand on the expected number of stoppings, $E(s)$

Figure 3.2 compares $E(s)$ for FRBT and SFT vehicle stopping processes following a binomial distribution (Eq. 3.1) and Poisson binomial distribution, PBD (Eq. 3.3). Results indicate that the binomial distribution overestimates $E(s)$ for both FRBT and SFT since, in low-demand routes, a large proportion of passenger demand for boarding and alighting is concentrated in only a few stops. For a given demand, $E(s)$ is higher for FRBT with higher service headway than SFT. As the demand increases, $E(s)$ for both distributions converge for FRBT, while a clear gap in the prediction of $E(s)$ is observed for SFT. This is because PBD considers the demand at each stop location to define its corresponding probability of stopping; hence, a lesser service headway available in SFT warrants lesser demand at each stop location.

![Figure 3.2 An illustration of $E(s)$ for binomial and Poisson-binomial stopping pattern](image)
3.3.3. Performance evaluation results

3.3.3.1. Critical passenger demand

Figure 3.3 demonstrates the variation of operating cost per passenger with hourly passenger demand \((p)\) along Route 6 using rigorous cost equations. Evidently, SFT service pattern following COT delivery model offers the lowest operating cost till 3 pass/hr and 3 pass/hr is the critical passenger demand \((P_c)\) at which it is desirable to shift from COT to IHT delivery model. Further, 27 pass/hr is the \(P_c\) to shift from a SFT following IHT delivery model to FRBT. In essence, 27 pass/hr is the demand at which both SFT and FRBT are equally competent and for \(p > 27\) pass/hr FRBT is more cost efficient. Further, \(P_c\) for SFT delivery models calculated using Eq. (3.29) is 2.89 pass/hr, indicating high accuracy of the approximate formulation to represent the scenario in a low demand condition. This indicates the applicability of the approximate formulation as a decision support tool to evaluate SFT delivery models in a low demand condition.

3.3.3.2. Compare system configurations

The average operating cost per passenger \((\bar{c})\) for different system configurations compared in Table 3.1 suggests that the cost for implementing systems including independent service patterns or delivery models are expensive than integrated systems. The existing FRBT system for route 6 is the most expensive service when operated independently with \(\bar{c}\) as $8.83/passenger, which is slightly less than the observed value of $10.83/passenger as per Regina transit data. SFT service pattern with COT delivery model offers 15\% less operating cost than existing FRBT and is followed by SFT with IHT delivery model, the least expensive independent system offering nearly 42\% less operating cost than FRBT.

Besides, an integrated FRBT & SFT system with SFT following both delivery models is the most cost-efficient configuration offering an average cost of $4/passenger. This system includes assigning all trips to SFT when \(p < 27\) pass/hr, which accounts for 82.9\% of total trips, with 27.5\% served under COT and 55.4\% under IHT. However, providing three types of services increases the complexity of management and reduces the fleet utilization secured in each transit service. On the contrary, an integrated SFT system with a unique service pattern and combined COT and IHT delivery models has only a 2.5\% higher annual operating cost than the previous system and is
comparatively a less complex structure and hence, recommended to improve the operational efficiency of the transit system along route 6.

**Table 3.1 Average operating cost per passenger for different service configurations**

<table>
<thead>
<tr>
<th>Service pattern</th>
<th>Delivery model</th>
<th>$(P_c)_1$</th>
<th>$(P_c)_2$</th>
<th>$f_p$ when $p &lt; (P_c)_1$</th>
<th>$f_p$ when $p \geq (P_c)_1$ &amp; $p \leq (P_c)_2$</th>
<th>$f_p$ when $p &gt; (P_c)_2$</th>
<th>$\bar{c}$</th>
<th>Annual operating cost ($\times 10^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Independent systems</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FRBT</td>
<td>IHT</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>8.83</td>
<td>479</td>
</tr>
<tr>
<td>SFT</td>
<td>IHT</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>5.15</td>
<td>279</td>
</tr>
<tr>
<td>SFT</td>
<td>COT</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>7.5</td>
<td>407</td>
</tr>
<tr>
<td>Integrated systems</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SFT</td>
<td>IHT &amp; COT</td>
<td>3</td>
<td>-</td>
<td>0.275</td>
<td>0.725</td>
<td>-</td>
<td>4.1</td>
<td>222</td>
</tr>
</tbody>
</table>

**Figure 3.3 Comparison of operating cost per passenger for the competing service patterns**
<table>
<thead>
<tr>
<th>FRBT &amp; SFT</th>
<th>COT</th>
<th>3</th>
<th>0.275</th>
<th>0.725</th>
<th>5.1</th>
<th>277</th>
</tr>
</thead>
<tbody>
<tr>
<td>FRBT &amp; SFT</td>
<td>IHT</td>
<td>27</td>
<td>0.829</td>
<td>0.171</td>
<td>4.9</td>
<td>265</td>
</tr>
<tr>
<td>FRBT &amp; SFT</td>
<td>IHT &amp; COT</td>
<td>3</td>
<td>27</td>
<td>0.275</td>
<td>0.554</td>
<td>0.171</td>
</tr>
</tbody>
</table>

3.3.3.3. *Error in the approximate operating cost model*

Figure 3.4 represents the errors in approximate solutions obtained by implementing A1 and A2 to rigorous cost models. Approximating congestion delay to 0 (i.e., A1) results in a slight underestimation of operating cost for both delivery models since low demand routes are often characterized by very light traffic volume. For $p < 20$ pass/hr, A2 underestimates the cost of IHT, and the error rapidly approaches zero when demand increases. On the contrary, for $p > 20$ pass/hr, the operating cost of IHT delivery model is overestimated, and the error continuously increases with demand; thus, suggesting low accuracy of approximate expression in the estimation of cost in high demand conditions. The overestimation of operating cost is a result of the assumption that each passenger riding in a vehicle boards and alights at a distinct stop location, as in high demand conditions, there can be multiple boarding and alighting at a stop location. These results further underpin the use of the approximate formulation as a decision-support tool for evaluating SFT delivery models in low demand.
3.3.4. Operation planning results

3.3.4.1. Scheduling

Route 6 is presently served by FRBT operating for 12 hours (6 a.m. to 6 p.m.) only on weekdays. As illustrated in Figure 3.5(a), the temporal distribution of passenger demand along the route follows a bimodal distribution with two distinct peaks: morning and evening peaks from 6 a.m. to 8:30 a.m. and 2:30 p.m. to 5:30 p.m. respectively. Results from section 3.3.3.1 suggest the implementation of IHT during peak hours when the demand is higher than $P_c$ (i.e., 3 pass/hr) and COT during off-peak hours from 8:30 a.m. to 2:30 p.m. and 5:30 p.m. to 6 p.m. The daily and annual average operating costs of SFT following a combined IHT and COT delivery model along Route 6 are $5.14/passenger and $4.1/passenger, respectively. Figure 3.5(b) illustrates the shift in demand from COT delivery model to IHT based on $P_c$. Evidently, if $P_c$ increases due to variation in service characteristics, the proportion of demand served by COT increases, and demand served under IHT reduces. Given that along low demand route, the frequency of observing low values of $p$ is high; the above observation favors the implementation of an independent SFT service with COT delivery model during all service hours in a day for a high value of $P_c$. 

![Graph showing observed variation of error in operating cost with approximation](image)
Figure 3.5 Variation of passenger demand by the hour of the day for Route 6

3.3.4.2. Vehicle utilization: Use of taxi or existing paratransit vehicle to serve peak demand

Vans or mini-buses are typically used to deliver in-house DRT-based paratransit service for ADA-eligible users, and the elderly population remains underutilized for most service hours (Davison et al., 2014). So, when providing SFT with IHT delivery model in peak hours and COT in off-peak hours, diverting a percentage of peak demand to existing paratransit vehicles or taxi in COT could maximize the utilization of all operating vehicles during the day. Assuming, the existing paratransit operates in a fashion similar to IHT delivery model, its operation cost is calculated using Eq. (3.16) while neglecting the fleet size (N) component. Figure 3.6 is a pictorial representation of the variation of average cost (AC) and marginal cost (MC) of COT, IHT, and paratransit service with passenger demand. Clearly, MC and AC of the COT delivery model are equal owing to no change in AC per unit increase in passenger demand. Although MC of paratransit and IHT are equal, AC of paratransit is always less than IHT for all values of passenger demand since the cost of acquiring fleet in paratransit is excluded. However, specifically at higher levels of demand, the MC of paratransit is lower than AC of paratransit and AC and MC of the taxi. This indicates that the cost of serving an additional trip by an existing paratransit fleet is more economical than assigning that trip to a taxi or IHT vehicle. Thus, along Route 6, instead of combined operation, distinct operation of taxi and IHT/paratransit vehicles during off-peak and peak periods respectively, is suggested.
Put differently, the maximum passenger demand that the COT delivery model can serve efficiently is 3 pass/hr (i.e., $P_c$), and assigning any further trips to a taxi could result in an increased average operating cost of the integrated system. Finally, $\bar{c}$ for combined operation of COT and in-house paratransit delivery model as estimated using Eq. (3.30) is 3.5$/passenger; offering 14.6% reduced cost than combined COT and IHT delivery model. Interestingly, Turmo et al. (2018) examined that providing a DRT service pattern with IHT delivery model for high demand values results in a substantial increase in AC and MC. Hence, unlike this study in peak hours economy is achieved in shifting demand to taxi in COT as MC of a COT is less than IHT.

![Graph showing the average and marginal cost of COT and IHT delivery models](image)

**Figure 3.6 An illustration of the average and marginal cost of COT and IHT delivery models**

**3.3.5. Sensitivity analysis results**

As discussed in section 3.3.4.1, $P_c$ is subjected to change due to variations in service and demand characteristics. Hence, a critical evaluation of $P_c$ and subsequently $\bar{c}$ for implementing SFT with COT or IHT delivery model would assist transit planners in making a policy decision. Critical parameters affecting $P_c$ in SFT delivery models are average taxi occupancy ($m$), dead-heading time ($T_d$), and service headway ($h_0$); as the other parameters in Eq. (3.29) including average speed values, average trip lengths, cost coefficients, and other fixed-route characteristics can be assumed as reasonably constant. For a given fixed taxi fare structure, $m$ is an essential parameter in COT,
and $T_d$ and $h_0$ mainly govern the operating cost of IHT. Thus, sensitivity analysis of $P_c$ w.r.t. $m$, $T_d$, and $h_0$ is performed.

3.3.5.1. Effect of $m$, $T_d$, and $h_0$ on $P_c$

The base values adopted so far for analysis correspond to $m = 2$ for COT and $f=0.3\%$ and $h_0 = 22.5$ minutes for IHT (Refer to Appendix B). Figure 3.7 illustrates the sensitivity of $P_c$ with $m$, $T_d$, and $h_0$ calculated using approximate formulation in Eq. (3.29). Results suggest that $m$ and $T_d$ are positively related to $P_c$, while $h_0$ is negatively related to $P_c$. An increase in $P_c$ is a consequence of the rightward shifting of IHT cost curve associated with increased $T_d$ and reduced $h_0$ and downward shifting of COT cost curve with increased $m$ (Refer Figure 3.3). Further, high occurrence of low values of $P_c$ is associated with the expensive fare structure of taxis adopted in Regina.

3.3.5.2. Effect of base fare on $P_c$

The operating cost for COT delivery model follows a taxi fare structure consisting of base fare and variable fare. Assuming variable fares cannot be reduced significantly, a reduction in the base fare of taxis could be achieved through the partnership design between transit agencies and private operators. A sensitivity of benefit in $\bar{c}$ corresponding to base fare ($\beta_0$) is illustrated in Figure 3.8(a) illustrating the variation of $\bar{c}$ and corresponding cost-to-revenue ratio (C/R) w.r.t. $\beta_0$, $m$, $T_d$, and $h_0$ characterize the best- and worst-case scenarios for the implementation of SFT under integrated IHT and COT delivery model. Revenue (R) for the integrated system is calculated assuming that the fare charged for both FRBT and SFT service pattern is the same and is equivalent to $3/passenger. As the figure suggests, the scenarios contributing to $\bar{c}$ for which C/R $\leq 1$ are classified as best-case and C/R $\geq 1$ as worst-case. Apparently, low values of $\bar{c}$ or C/R mostly correspond to high values of $m$, $h_0$, and $P_c$ and low values of $T_d$ and $\beta_0$. Further, the variation of $\bar{c}$ and C/R with $P_c$ follows an interesting pattern. Initially as $P_c$ increases, $\bar{c}$ increases to attain maximum value and then follows a declining curve while the rate of attaining a maximum $\bar{c}$ and maximum $\bar{c}$ value per se reduces as $m$ increases. For a given $m$, maximum $\bar{c}$ value is attributed to maximum $\beta_0$ contributing to the high operating cost of COT and rightward shifting of IHT cost curve due to increased cost with high $T_d$ and low $h_0$ (Refer Figure 3.3). In addition, the declining...
curve for \( \bar{c} \) is concurrent to the observation in Figures 3.5(b) \& 3.3 with a larger proportion of ridership being diverted to COT with increasing \( P_c \) and reduced slope of the operating cost curve of IHT with the increase in demand. The authors are keen to derive the functional forms for the discussed scenario in future studies. Finally, the set of points corresponding to the best-case scenario is further analyzed and the results are shown in Figure 3.9. The results suggest that, for a given \( R \), low values of \( T_d \) and \( \beta_0 \) and high values of \( m \) and \( h_0 \) increase the probability to obtain the best-case scenario. Further, since \( R \) is constant, low values of \( m \) and \( h_0 \) or high values \( T_d \) has to be compensated with high values of the \( \beta_0 \) to obtain the best-case scenario. Given, user fare is assumed to be insensitive to changes in \( \beta_0 \), the revenue (R) here is constant.
Figure 3.7 An illustration of variation of $P_c$ with $m$, $T_d$, and $h_0$ for base scenario
Figure 3.8 An illustration of variation of $\bar{c}$ with (a) $P_c$ and $\beta_0$ and (b) $m$, $T_d$, and $h_0$
3.4. Discussion

3.4.1. Policy implications and recommendations

The study findings provide the basis for certain policy implications and associated general recommendations that could play a key role in improving the implementation of SFT:

1. Contracting: If the unit cost of distance and time component is held constant, negotiation of a suitable taxi base fare ($\beta_0$) by transit agencies with TNCs or taxi-companies can further minimize the operating cost of SFTs with COT delivery model operated independently or integrated with IHT delivery model.

2. Tactical and strategic planning: maximization of taxi occupancy ($\bar{m}$) and minimization of dead-heading time ($T_d$) and service headway ($h_0$) is essential in cost reduction and obtaining a cost-to-revenue ratio $\leq 1$. Thus, policymakers should attempt to identify strategic
potential locations for depot stations that minimize the dead-head kilometers, when implementing IHT delivery model for SFT. Further, SFT service headway and vehicle occupancy are highly subjective to the technology adopted, including algorithms for passenger request management, vehicle assignment, and routing; hence, suggesting high potential for technology enhancement as a tactical tool for cost reduction.

3. Operations: combining different service patterns/delivery models to form an integrated system to serve seasonal and daily varying demand patterns is also key to reducing operating costs.

3.4.2. Suggested model applications

In addition to focusing on replacing an underperforming bus route, the study methodology can also be used to evaluate the potential of SFT delivery models and the service pattern per se to fortify an existing FRBT service network through network-wide planning and partnership design.

1. Transit network planning: Study methodology application can be extended to evaluate partnerships with private operators (i.e., TNCs or taxi or microtransit companies) for providing first mile/last mile service to FRBT stations, mobility in suburban areas, and service along bus routes in early mornings or late nights. Subsequently, for a given low demand profile, the service parameters like service headway and vehicle size can be changed dynamically that simultaneously optimizing the operating cost and vehicle utilization along with the entire FRBT network.

2. Partnership design: Proposed methodology can further be exploited to design the level of subsidization in partnership between transit agencies and private operators. Revenue generated on implementing SFT is estimated by assuming user fare for SFT is the same as FRBT. However, for any trip in a contracted SFT, fare subsidy or revenue charged can take any of the three forms: the passenger pays an initial fare, and the remaining is paid by the transit agency or vice-versa, the passenger pays a flat fare, and the remaining is paid by the agency, and passengers pay the full fare. Thus, different revenue models can be plugged into cost models for evaluation.
3.5. Conclusion

The present study develops rigorous cost models for the economic evaluation of semi-flexible transit as compared to traditional fixed bus route transit. In particular, proposed models enable estimating the critical passenger demand that justifies switching between competing service patterns i.e. fixed-route bus transit (FRBT) and semi-flexible transit (SFT), and service delivery models i.e. in-house transit (IHT) and contract-out taxi (COT). Approximate formulations with reduced data needs and complexity were also developed and evaluated for practical applications. To demonstrate the performance and applicability of the developed models as potential decision support tools, a detailed case study was conducted to evaluate SFT service delivery models for a low demand bus route in Regina, Canada. In the case study, for instance, the models suggested that it is cost-efficient to provide SFT following a COT delivery model for demand ≤ 3 pass/hr, SFT with IHT delivery model for demand between 3 to 27 pass/hr, and FRBT for demand > 27 pass/hr. Additionally, SFT under IHT delivery model is the least expensive independent system offering nearly 42% less operating cost than the existing FRBT followed by SFT with COT delivery model offering 15% less operating cost than FRBT. An integrated SFT with a unique service pattern and combined COT and IHT delivery models is the least expensive and less complex configuration offering 53.6% and 20.4% less operating cost than independently operating FRBT and SFT under IHT delivery model, respectively. For the scenarios evaluated in this study, the findings suggest that it is cost-efficient to operate SFT service pattern in low demand conditions than FRBT. When implementing an independent system, if the fare structure is expensive, exclusive operation of SFT with IHT delivery model is more economical than SFT with COT delivery model. When implementing an integrated system (i.e. SFT with combined COT and IHT delivery model), it is recommended to operate under COT and IHT during off-peak hours and peak hours, respectively. Further, in an integrated system to ensure maximum vehicle utilization during peak periods, it is suggested to shift a proportion of trips from IHT delivery model to COT or existing paratransit vehicles for values of passenger demand when MC of a COT or paratransit is less than the MC of IHT. The policy recommendations based on study analysis suggest that increased involvement of policymakers could minimize the cost of SFT by negotiating a reduced base fare, adopting strategic and tactical planning approaches to improve operating standards, and
tackling varying demand patterns. In essence, the study encourages decision-makers to adopt a methodological approach to evaluate SFT delivery models to fortify an existing FRBT service network through network-wide planning and partnership design.

We lay out three future scopes of investigations to fully explore the economies of scale achieved through the implementation of SFT delivery models and address some of the limitations of this study. Transit service planning process involves the inclusion of conflicting criteria held by two major stakeholders i.e., operator and user. SFT is a promising alternative in terms of travel time and operating cost but is expensive in terms of user fare. Thus, the first scope is to study the effects of the inclusion of user cost on the total system cost of SFT delivery models. In addition, this study assumes two extreme levels of contracting SFT: either fully contracted or fully in-house (i.e., no contracting). Thus, the second scope includes investigating the economies of scale achieved through partially contracting out a portion of services while effectively reducing the size of in-house delivery. Finally, the developed models are purely deterministic and include simplifying assumptions such as Poisson passenger arrival pattern and uniformity in service headway. Thus, the third scope highlights the inclusion of stochastic vehicle arrivals and passenger arrivals in the cost models for a more extensive analysis.
Chapter 4. Optimal design of integrated semi-flexible transit services in low-demand conditions

4.1. Abstract

Semi-flexible transit (SFT) is commonly discussed as a cost-effective alternative to serving public transportation users in low-demand conditions. It is hypothesized that joint optimization of service headway and slack time per trip for route deviation is essential for designing a schedule for the operation of an integrated SFT that can meet both fixed-route and paratransit demand. An integrated SFT has the potential to lower the cost of transportation for regular transit users (both operators and riders) while redirecting potential paratransit riders to less expensive transit modes; thus, reducing demand for overwhelmed paratransit services operating with limited resources. The optimization problem has three competing objectives: minimizing operator costs, minimizing user costs, and maximizing service benefits. Two state-of-the-art multi-objective evolutionary algorithms NSGA-II and SMPSO are compared to obtain the most representative deterministic Pareto optimal solution set. This study has three major contributions. First, quantile regression is used to suggest multiple slack time values for a given headway that transit planners can consider when generating a static schedule for SFT operation. Second, relationships derived to analyze cost trade-offs suggest that headway governs operator cost and is negatively correlated, user cost is positively and equally influenced by both variables, and slack time governs service benefit and is positively correlated. Third, sensitivity analysis for an integrated SFT operation reveals that low-capacity minivans and standard vans offer higher vehicle occupancy and cost efficiency, mostly economical for low to medium demand (5-20 pass/hr), low permissible deviation from the fixed route is desirable during peak hours to avoid delays for passengers on-board, and extreme weather conditions dramatically and negatively influence costs. Policy recommendations for integrated SFT implementation include a recommendation for fare structure design addressing service equity through surcharges/discounts, vehicle technology and service booking technology advancements for cost reduction, and fleet mix design through estimation of passenger loading profile. The application of the study methodology is demonstrated for a low-demand bus route in Regina, Canada.
Keywords: Paratransit; Semi-flexible transit; Service headway; Slack time

4.2. Problem description

4.2.1. Service area and demand

An existing underperforming fixed bus route is used as the study area, which is defined by two terminal stations and modelled as a rectangle with dimensions $W$ (km) and $L$ (km) (see Figure 4.1). SFT in this study is designed to serve two types of passenger demand: (a) $Q_G$- existing FRBT demand (Type G), and (b) $Q_S$- existing DRT demand referring to users that are eligible for paratransit service in the study area (Type S). It is assumed that the demand per trip is uniformly and independently distributed within the service area.

![Figure 4.1 A schematic representation of route-deviation operating policy](image)

4.2.2. Operating policy

For SFT, the route-deviation policy defined by Koffman (2004) is adopted where vehicles follow a fixed route and deviate to serve curb-to-curb requests, with a maximum allowable deviation of $W/2$ on both sides (see Figure 4.1). This operating policy accepts two types of stop requests: flag requests and curb-to-curb requests. Flag requests involve vehicles stopping at any location along the route, which may or may not correspond to a marked stop. Curb-to-curb requests involve vehicles deviating from their fixed route to serve pick-up and drop-off locations requested by passengers in advance (usually 1 hour). Flag-stop in an integrated service can be requested by both Type G and Type S passengers; however, curb-to-curb stop requests in this study are restricted to Type S passengers. An online dispatch system is assumed to handle curb-to-curb service requests and routing, and a predetermined timetable, including slack time, is published for all or a few stops along the fixed route to assist Type G and Type S passengers in planning their arrival at the route.
Within the service area, Type S passengers can request four possible types of requests with proportions of $\eta_{R1}$, $\eta_{R2}$, $\eta_{R3}$, and $\eta_{R4}$ ($\eta_{R1} + \eta_{R2} + \eta_{R3} + \eta_{R4} = 1$) as follows:

R1: Both pick-up and drop-off locations are along the fixed route (Both flag stops)

R2: Pick-up location deviates from the fixed route and the drop-off location is along the fixed route (flag stop and curb-to-curb stop)

R3: Drop-off location deviates from the fixed route and the pick-up location is along the fixed route (flag stop and curb-to-curb stop)

R4: Both pick-up and drop-off locations deviate from the fixed route (Both curb-to-curb stops)

4.2.3. Problem definition

In SFT, adding slack time increases the one-way running time of vehicles, which can be compensated for by increasing the service headway to reduce fleet size requirements, this, however, will increase the waiting time and reduce the available vehicle capacity for serving Type S passengers with more Type G passengers queued up for the service. Thus, joint optimization of slack time and headway will ensure maximum vehicle utilization for a desired level of service.

The list of symbols, definitions, and units used in the deriving analytical cost models are discussed in Appendix D. The frameworks illustrated in Appendix E serve as a comprehensive guide to detailing inputs, outputs, and analytical procedures for implementing the study models developed in this chapter for any other low demand route.

4.3. Problem formulation

4.3.1. Decision variable

The study presents a multi-objective optimization model for service headway, $h$ (hr), and slack time required to serve Type S passengers, $\Delta t$ (hr) within the service area. Here, $h$ represents the time difference between the arrival of two vehicles at any stop, and $\Delta t$ represents the one-way trip time difference when the potential Type S passengers are accommodated in the trip against serving only Type G passengers.
4.3.2. Objective function

The three conflicting objective functions considered are $f_1$: minimization of operator cost ($C_{OC}$), $f_2$: minimization of user cost ($C_{UC}$), and $f_3$: maximization of service benefit ($C_{SB}$), as shown in Eq. (4.1 – 4.3).

$$\text{Min } f_1 \rightarrow C_{OP}(h, \Delta t) \quad \text{Eq. 4.1}$$

$$\text{Min } f_2 \rightarrow C_{UC}(h, \Delta t) \quad \text{Eq. 4.2}$$

$$\text{Max } f_3 \rightarrow C_{SB}(h, \Delta t) \quad \text{Eq. 4.3}$$

4.3.3. Constraints

**Constraint 1:** Eq. (4.4) depicts that $h$ is constrained by a minimum and maximum service headway. Literature consensus that for a low-demand route characterized by low-frequency service, the minimum headway can be set to 10 minutes (Ansari Esfeh et al., 2021). The minimum desired level of service is set through policy headway, $h_p$, and the vehicle capacity, $C$ governs the maximum value.

$$0.167 \leq h \leq \min\left\{\frac{C}{Q_G + Q_S}, h_p\right\} \quad \text{Eq. 4.4}$$

where $C$ is capacity in pass/veh, $Q_G$, and $Q_S$ represent hourly Type G and Type S demand in pass/hr, and $h_p$ is the policy headway in hr.

**Constraint 2:** As shown in Eq. (4.5), slack time, $\Delta t$, should vary between 0 and a maximum value based on the SFT vehicle capacity ($C$) and the average time to serve one passenger/paratransit user ($\delta$). A minimum value corresponds to no Type S passengers served (i.e., Type G passengers only), and a maximum value corresponds to all passengers on board are Type S (i.e., No Type G passengers).

$$0 \leq \Delta t \leq C \delta \quad \text{Eq. 4.5}$$

**Constraint 3:** Constraint 3 in Eq. (4.6) limits the number of Type S passengers served per one-way trip to the available vehicle capacity after serving Type G passengers. This imposes Type G
flow and holds priority in the assignment of the vehicle capacity since an alternate mode DRT is available to Type S passengers if not accommodated in SFT.

\[ Q_R h + \frac{\Delta t}{\delta} \leq C \]  

**Eq. 4.6**

**C4:** Constraint 4 in Eq. (4.7) ensures that the number of Type S passengers served cannot exceed the demand received during the service interval (i.e., \( h \)).

\[ \frac{\Delta t}{\delta} \leq Q_S h \]  

**Eq. 4.7**

**Estimation of \( \delta \)**

Based on Table 4.1, the time required to serve a given request type, R1 to R4 is composed of (a) riding time, (b) acceleration and deceleration time, and (c) dwell time. For instance, the riding time required to serve request type R4 includes the time required to deviate an average of \( W/4 \) from the fixed route to serve a curb-to-curb stop and the same \( W/4 \) distance back to the fixed route for both pickup and drop-off. Eq. (4.8) based on conditional probability theory, is used to estimate the expected time to serve one Type S passenger (\( \delta \)).

\[
\delta = \eta_{R1}\delta_{R1} + \eta_{R2}\delta_{R2} + \eta_{R3}\delta_{R3} + \eta_{R4}\delta_{R4} \\
= \frac{W}{2V_R} \left( \eta_{R2} + \eta_{R3} \right) + 2t_{ad} + 2t_d
\]  

**Eq. 4.8**

where \( V_R \) is average riding speed (km/hr), \( t_{ad} \) is acceleration and deceleration per stopping (hr), and \( t_d \) is dwell time per stopping for boarding or alighting (hr).

**4.3.4. Analytical cost models for deterministic analysis**

**4.3.4.1. Operator cost**

SFT operating cost (\( C_{OC} \)) estimated in $/hr is defined as a function of Fleet Size, \( M \) as given in Eq. (4.9).

\[ C_{OC} = c_1[M] \]  

**Eq. 4.9**
where $c_1$ ($$/veh-hr$) is the unit cost of operating a transit unit including fleet acquisition cost, and distance and time-based cost (C. F. Daganzo, 1978).

Eq. (4.10) expresses $M$ as a ratio of the total round-trip time, $T_R$ (hr), and headway, $h$ (hr). $T_R$ consists of three components: (a) time required for serving Type G passengers, $T_v$ (hr), (b) layover time at the terminal station, $T_l$ (hr), and (c) slack time to serve Type S passengers, $\Delta t$ (hr). $T_v$ includes total riding time, time for vehicle acceleration and deceleration, and dwell time as given in Eq. (4.11). For the estimation of dwell time, the number of stops is assumed to be twice the number of passengers boarded in a vehicle, which holds in low-demand situations (Kikuchi & Vuchic, 1982).

$$M = \left[\frac{T_R}{h}\right]^+ = \left[\frac{2(T_v + T_l + \Delta t)}{h}\right]^+ = \left[\frac{2(T_v(1 + \mu) + \Delta t)}{h}\right]^+ \quad \text{Eq. 4.10}$$

$$T_v = \frac{L}{V_R} + (2t_{ad} + 2t_d)(Q_R h) \quad \text{Eq. 4.11}$$

where $\mu = T_l / T_v$.

4.3.4.2. User cost

The user cost ($C_{UC}$) is the sum of the costs of the three equivalent time components, access/egress time, $C_A$, waiting time, $C_W$, and in-vehicle time, $C_I$, estimated in $$/hr given in (12). The product of passenger value of time, $c_2$ ($$/pass-hr$), and equivalent time components $T_a$ (hr), $T_w$ (hr), and $T_v$ (hr) as given in Eq. (4.12) are used to estimate $C_{UC}$. Estimation of $T_a$ and $T_v$ are based on the microeconomic models for vehicle resource consumption derived by Mohring (Mohring, 1972) and the model for $T_w$ estimation is based on vehicle and passenger arrival patterns derived by Ansari Esfeh et al. (Ansari Esfeh et al., 2021).

$$C_{UC} = C_A + C_W + C_I = c_4(T_a + T_w + T_v) \quad \text{Eq. 4.12}$$

When requesting a flag stop, Type G and Type S passengers must walk/wheel an average vertical distance of W/4 from their origin/destination to the fixed route, and curb-to-curb pick-ups/drop-offs for Type S passengers involve no walking (see Table 4.1). Hence, the expected walking time cost can be estimated using Eq. (4.13).
\[ C_A = c_2 \left[ \frac{W}{4V_a} (2\eta_{R1} + \eta_{R2} + \eta_{R3}) \times \frac{\Delta t}{\delta h} + \frac{W}{2V_a} \times Q_R \right] \]  

**Eq. 4.13**

where, \( V_a \) (km/hr) is the passenger walking speed and \( \Delta t/\delta h \) is the accepted Type S demand passenger (pass/hr) which is always less than or equal to the received demand, \( Q_S \).

Most studies assume that passengers arrive at bus stops at random; therefore, the mean waiting time equals half of the service headway. Low-demand routes usually have a higher headway (i.e., \( h>10 \) minutes) and published timetable; thus, passengers may or may not exhibit random arrival, and instead may adjust their arrival time at the departure stop to minimize the waiting time; thus, the mean waiting time is less than half the headway (Ansari Esfeh et al., 2021). The mean waiting time for passengers requesting a flag stop pick-up can be calculated using Eq. (4.14), proposed by Ansari Esfeh et al. (2021) for low-demand routes. Curb-to-curb pickup does not incur any additional waiting time for passengers. The expected value of waiting time can therefore be estimated using Eq. (4.15) derived from conditional probability theory.

\[ E(W) = \left[ \frac{1}{2} - \frac{\alpha(1 - \beta)}{2} \right] h \]  

**Eq. 4.14**

\[ C_W = c_2 \left[ \left[ \frac{1}{2} - \frac{\alpha(1 - \beta)}{2} \right] h(\eta_{R1} + \eta_{R2}) \times \frac{\Delta t}{\delta h} + \left[ \frac{1}{2} - \frac{\alpha(1 - \beta)}{2} \right] h \times Q_R \right] \]  

**Eq. 4.15**

where \( \alpha \) and \( \beta \) are the proportion of planning passengers and the proportion of planning passengers with fixed arrival times, respectively.

Similarly, passengers can be dropped off/picked up uniformly anytime in the trip between two terminals. Thus, the average in-vehicle time for Type G and Type S passengers is half of the total travel time between the two terminals with the expected value given in Eq. (4.16).

\[ C_I = c_2 \left[ \frac{T_v + \Delta t}{2} (\eta_{R1} + \eta_{R2} + \eta_{R3} + \eta_{R4}) \times \frac{\Delta t}{\delta h} + \frac{T_v + \Delta t}{2} \times Q_R \right] \]

\[ = c_2 \left[ \frac{T_v + \Delta t}{2} \left( \frac{\Delta t}{\delta h} + Q_R \right) \right] \]  

**Eq. 4.16**
4.3.4.3. Service benefit

SFT service benefits are specific to the operators' intent (Fu, 2002). This analysis defines the service benefit in Eq. (4.17) as the cost incurred to serve paratransit passengers (Type S) using a dedicated DRT service if not served by SFT.

\[ C_{SB} = c_3 \times \frac{\Delta t}{\delta h} \]  \hspace{1cm} \text{Eq. 4.17}

where \( c_3 \) ($/pass) is the average operating cost of providing paratransit service (assumed).

### Table 4.1 Values of \( \delta \) and user time components are classified by demand and request type.

<table>
<thead>
<tr>
<th>Demand type</th>
<th>Request type</th>
<th>Demand proportion</th>
<th>( \delta )</th>
<th>Access time</th>
<th>Waiting time</th>
<th>In-vehicle time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type G</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>( \frac{W}{2V_a} )</td>
<td>( \frac{1}{2} - \frac{\alpha(1 - \beta)}{2} )</td>
<td>( \frac{T_v + \Delta t}{2} )</td>
</tr>
<tr>
<td>Type S</td>
<td>R1</td>
<td>( \eta_{R1} )</td>
<td>2( t_l + 2t_d )</td>
<td>( \frac{W}{2V_a} )</td>
<td>( \frac{1}{2} - \frac{\alpha(1 - \beta)}{2} )</td>
<td>( \frac{T_v + \Delta t}{2} )</td>
</tr>
<tr>
<td></td>
<td>R2</td>
<td>( \eta_{R2} )</td>
<td>( \frac{W}{2V_R} + 2t_l )</td>
<td>( \frac{W}{4V_a} )</td>
<td>( \frac{1}{2} - \frac{\alpha(1 - \beta)}{2} )</td>
<td>( \frac{T_v + \Delta t}{2} )</td>
</tr>
<tr>
<td></td>
<td>R3</td>
<td>( \eta_{R3} )</td>
<td>( \frac{W}{2V_R} + 2t_l )</td>
<td>( \frac{W}{4V_a} )</td>
<td>0</td>
<td>( \frac{T_v + \Delta t}{2} )</td>
</tr>
<tr>
<td></td>
<td>R4</td>
<td>( \eta_{R4} )</td>
<td>( \frac{W}{V_R} + 2t_l + 2t_d )</td>
<td>0</td>
<td>0</td>
<td>( \frac{T_v + \Delta t}{2} )</td>
</tr>
</tbody>
</table>

4.4. Solution method

Based on the general formulation of the optimization problem presented in Eq. (4.1) - (4.3), the objective functions are defined as functions of the decision variables by expanding them using the cost component equations in (4.8) - (4.17) before reducing them to a simpler form as given in equations (4.18) – (4.20) after initialization. Essentially, the constants \( \theta_0 \) to \( \theta_k \) are estimated based on initialized values of cost coefficients (i.e., \( c_1, c_2, \) and \( c_3 \)); and other parameters for the case study (i.e., \( L, W, t_{ad}, t_d, \) etc.). Eq. (4.18) suggests that \( f_1 \) is inversely related to \( h \) and includes an interaction term (\( \Delta t/h \)) which indicates that the effect of one decision variable (\( h \) or \( \Delta t \)) on \( f_1 \) is
based on the level or magnitude of another decision variable. $f_2$ in Eq. (4.19) is linearly related to $h$ and $\Delta t$ and includes interaction terms with linear and quadratic relationships and $f_3$ in Eq. (4.20) is only a function of interaction term $\Delta t/h$. Essentially, the optimization problem is non-linear since the effect of both decision variables on $f_1, f_2$, and $f_3$ takes both linear and non-linear effects.

$$f_1 = \theta_0 + \theta_1 \left( \frac{1}{h} \right) + \theta_2 \left( \frac{\Delta t}{h} \right) \quad \text{Eq. 4.18}$$

$$f_2 = \theta_3 + \theta_4 (h) + \theta_5 (\Delta t) + \theta_6 \left( \frac{\Delta t}{h} \right) + \theta_7 \left( \frac{\Delta t^2}{h} \right) \quad \text{Eq. 4.19}$$

$$f_3 = \theta_8 \left( \frac{\Delta t}{h} \right) \quad \text{Eq. 4.20}$$

Multi-objective evolutionary algorithms (MOEAs) are generally considered mainstream methods for solving these problems. Our study compares the quality of Pareto solutions obtained using two state-of-the-art MOEAs: Non-dominated Sorting Genetic Algorithm-II (NSGA-II) proposed by Deb et al. (2002) and Speed-constrained Multi-objective PSO Algorithm (SMPSO) proposed by Nebro et al. (2009). Pseudocodes to implement both algorithms are provided in Appendix B.

To obtain the most representative deterministic Pareto optimal solution set, the procedure described below is followed.

Step 1: Obtaining Pareto optimal solution sets for each MOEA based on different parameter settings.

- NSGA-II and SMPSO implementation requires the initialization of the following parameters: chromosome population size ($N_P$) or swarm size ($N_S$), number of generations ($G$), crossover probability ($p_c$), mutation probability ($p_m$), crossover distribution index ($\tau_c$), and mutation distribution index ($\tau_m$).
- For ‘$X$’ combinations of $N_S$ or $N_P$ and $G$, ‘$Y$’ runs of each combination obtaining $XY$ Pareto set for each algorithm is performed.

Step 2: Performance evaluation of Pareto optimal sets.
• For performance evaluation, the five indicators used are hypervolume (HV), generation distance (GD), inverted generational distance (IGD), epsilon (\( \varepsilon \)), and computation time per run (CT).
• HV, GD, and IGD measure the diversity and/or convergence of solutions compared to a reference Pareto front RF or reference point RP which can be obtained by selecting non-dominated solutions from all Pareto solutions obtained in Step 1 (discussion in detail in Ishibuchi et al. (2014)).
• Finally, the average indicator values across ‘Y’ runs are reported for ‘X’ combinations in each algorithm.

Step 3: Ranking of Pareto optimal sets.

• TOPSIS ranking method proposed by Tzeng and Huang (1981) is implemented.
• In TOPSIS, Pareto optimal sets are ranked by minimizing GD, IGD, \( \varepsilon \), and CT, and maximizing HV while assigning equal weightage to indicators. Select a solution set at random from all ‘Y’ runs corresponding to the top-ranked parameter setting of the best-performing algorithm (i.e., the lowest sum of the ranks).

4.5. Results and discussions

4.5.1. Study area description

As a case study, the optimization problem is applied to a low-demand bus route served by Regina Transit, Canada. Analysis conducted by CUTA (2015) indicates that a total annual ridership of 6,434,022 is served by Regina Transit with revenue to cost ratio of 26.27%; thus, the non-passengers (i.e., the city and taxpayers) subsidize the remaining 73.73%. The analysis of ridership and operating data obtained from Regina Transit for 2015 shows that routes 6, 14, 15, and 16 exhibit underperformances based on average R/C (Mehran et al., 2020). Route 6 selected for the
case study highlighted in Figure 4.2 is attributed to low ridership (Q_R) of 9 passengers/hour, R/C of 15.3\%, the high operating cost of $10.84/pass, and declining ridership of 1.04\% from 2014.

![Route 6: Westhill - Ross Industrial](image)

**Figure 4.2 Route 6: Westhill to Ross industrial**

### 4.5.2. Multi-objective optimization results

#### 4.5.2.1. Identification of the best set of Pareto optimal solutions

Initialization of parameters in analytical cost models and parameter settings for NSGA-II and SMPSO are outlined in Table B1 in Appendix. Constraint 1 in the optimization problem limits h from 10 minutes to 1 hour, Constraint 2 limits Δt from 0 to 24 minutes, and the average time required to serve one Type S passenger/paratransit user (δ) is estimated as 1.6 minutes. NSGA-II and SMPSO algorithms are implemented in Python to obtain Pareto solution sets for 100 runs of 18 different parameter settings in each algorithm, subsequently used to estimate the reference front and reference point. According to TOPSIS analysis, NSGA-II yields a lower sum of ranks than SMPSO; therefore, it outperforms SMPSO. Table 3 shows the top-ranked parameter settings for both algorithms. SMPSO requires twice the number of function evaluations and hence, much higher computation time than NSGA-II to obtain Pareto fronts that do not differ greatly in terms of convergence and diversity. From 100 runs of the NSGA-II algorithm with the parameter settings described in Table 4.2, a Pareto set is selected at random for further analysis.
Table 4.2 TOPSIS results

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>NSGA-II</th>
<th>SMPSO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sum of ranks</td>
<td>306</td>
<td>360</td>
</tr>
<tr>
<td>Rank</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>$N_P$ or $N_S$</td>
<td>500</td>
<td>1000</td>
</tr>
<tr>
<td>$G$</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>Runs</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>Values averaged across runs</td>
<td>CT (sec)</td>
<td>49.82</td>
</tr>
<tr>
<td></td>
<td>GD</td>
<td>0.0007</td>
</tr>
<tr>
<td></td>
<td>IGD</td>
<td>0.0198</td>
</tr>
<tr>
<td></td>
<td>$\epsilon$</td>
<td>0.1113</td>
</tr>
<tr>
<td></td>
<td>HV</td>
<td>0.782</td>
</tr>
</tbody>
</table>

4.5.2.2. Descriptive analysis of Pareto solutions

The hypervolume indicator value for NSGA-II-based Pareto solutions shown in Figure 4.3(a) is 0.79, which indicates a good convergence of solutions since 79% of the volume represents the dominated space. For NSGA-II, the knee-point solution with the highest hypervolume contribution marked in Figure 4.3(a) is usually a preferred trade-off solution. The knee-point solution has the following properties: $h = 41$ minutes; $\Delta t = 6.4$ minutes; operator cost, $f_1 = \$122/hr$; user cost, $f_2 = \$393/hr$; and service benefit, $f_3 = \$253/hr$. With 10 passengers per trip, 6 Type G and 4 Type S, and 60% capacity utilization, the knee-point solution is feasible from an operator cost and service benefit perspective, but it isn't from a user cost perspective. According to Figure 4.3(b), operator cost ($f_1$) and user cost ($f_2$) are inversely related; for a given value of $f_1$, an increase in $f_2$ increases $f_3$; for a given value of $f_2$, an increase in $f_1$ increases $f_3$; and the lowest values of $f_1$ and $f_2$ provide almost no service benefit ($f_3$). Thus, the transit operator must carefully compromise between $f_1$ and $f_2$ to maximize $f_3$. Figure 4.3(c) indicates that adopting solutions with $h$ and $\Delta t$ above 16.3 minutes and 2.5 minutes, respectively, would result in lower costs for serving the same number of Type S passengers via integrated SFT service compared to the existing DRT service. Thus, operators will gain monetary benefits from an integrated SFT service when high values of $h$ and $\Delta t$ are used. Additionally, $h$ has a U-shaped distribution with a mean of 31 minutes and $\Delta t$ has a right-skewed
distribution with a mean of 2.6 minutes. Mean values of \( f_1, f_2, \) and \( f_3 \) are $187.1/hr, $281.1/hr, and $129.1/hr respectively.

4.5.2.3. Statistical analysis of Pareto solutions

I. Relationship between \( h \) and \( \Delta t \)

The relationship between \( h \) and \( \Delta t \) cannot be modeled using linear regression approaches, as indicated by the violation of homogeneity in Figure 4.4(a); thus, quantile regression is applied in this case. For quantile levels ranging from 0.1 to 0.9, Figure 4.4(a) shows the fitted quantile regression line, including the median regression line (i.e., the 50th quantile regression). For a significance level of 95%, the coefficients for all quantile levels are significant. The quantile regression models the conditional quantiles of \( \Delta t \) given the input variable, \( h \). For example, when
the quantile level of regression is 0.9, an intercept value of -0.23 and a coefficient of 0.15 is obtained; therefore, for \( h = 30 \) minutes, the 90\(^{th}\) percentile of \( \Delta t \) is expected to be 4.37 minutes. Table 4.3 illustrates that as the quantile level increases, the slope coefficient increases, but the rate of increase in slope decreases. The lower and upper quartiles differ significantly from the least squares estimate. Thus, with an increase in \( h \), it is more likely to obtain higher \( \Delta t \) values, but it is less likely to obtain Pareto solutions in general. The intercepts for most quantile levels are smaller than those for least squares; thus, quantile regression is more predictive than linear regression. Based on the quantile regression model parameters shown in Table 4.3, the value of \( \Delta t \) is estimated for values of \( h \) from 10 to 60 minutes and illustrated in Figure 4.4(b). Considering that it takes approximately 1.6 minutes to serve one Type S passenger (\( \delta \)), Figure 4.4(b) suggests that the probability \( P \) (Number of Type S passengers served per trip \( \geq 1 \)) increases with \( h \), while \( P \) (Number of Type S passengers served per trip = 0) decreases with \( h \). Also, \( \Delta t \) increases by 0.2 minutes for each 0.1 increment in quantile level for \( h \) between 10 and 20 minutes, and by 0.4, 0.6, 0.7, and 0.9 minutes for \( h \) between 20 to 30, 30 to 40, 40 to 50, and 50 to 60 minutes. If an agency wishes to operate its fleet at \( h = 25 \) minutes, the optimal slack time, \( \Delta t \), is between 0.4 and 3.6 minutes which means the optimal solutions are 10\(^{th}\) likely to be \( \leq 0.4 \) minutes and 90\(^{th}\) likely to be \( \leq 3.6 \) minutes. The average value of \( \Delta t \) as a percentage of one-way operating time without route deviation (\( T_v \)) is 8.9\%. Furthermore, 50\(^{th}\) of the ratio \( \Delta t / T_v \) falls between 2.86\% and 13.48\%, with a median of 6.49\% and a maximum and minimum of 28.5\% and 0\%, respectively. In contrast to our study, most studies recommend a fixed slack time for a given set of conditions. Fu (2002) suggested that \( \Delta t \) of 6 minutes is optimal to accommodate two deviated stops requested per analysis period of \( T_v + \Delta t \) with a maximum allowable deviation ratio of \( \Delta t / T_v + \Delta t \) of 40\% and vehicle capacity of \( C = 9 \) seats. To accommodate route deviation requests, Potomac and Rappahannock Transportation Commission (PRTC) has included approximately 20\% slack time in their basic schedules for medium-duty, 28-passenger buses (Koffman, 2004). Careful consideration should be given to the amount of slack time to be built into each route (in each direction, i.e., inbound/outbound). \( \Delta t \) should be sufficient to process the desired number of Type S requests without causing excessive idle time downstream if no Type S requests are received.
Figure 4.4 (a) Quantile regression of $h$ and $\Delta t$ for quantile levels and (b) Variation of $\Delta t$ with $h$ and quantile level.

Table 4.3 Quantile regression coefficient estimates

<table>
<thead>
<tr>
<th>Quantile level</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>Least square method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>-0.07</td>
<td>-0.29</td>
<td>-0.49</td>
<td>-0.66*</td>
<td>-0.76*</td>
<td>-0.69*</td>
<td>-0.50*</td>
<td>-0.39*</td>
<td>-0.23*</td>
<td>-0.39</td>
</tr>
<tr>
<td>Slope</td>
<td>0.02*</td>
<td>0.05*</td>
<td>0.07*</td>
<td>0.10*</td>
<td>0.12*</td>
<td>0.13*</td>
<td>0.13*</td>
<td>0.14*</td>
<td>0.15*</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Note: Significance levels: *$p < .05$.

II. Relationship between $h$ and $\Delta t$ and $f_1$, $f_2$, and $f_3$

Figure 4.5(a) and 4.5(b) confirms the relationship in Eq. (4.18) and suggest that $f_i$ and $M$, the operator cost and fleet size, are negatively correlated with $h$ and positively correlated with $\Delta t$. By increasing $h$, the fleet size requirements will be reduced, thereby reducing $f_i$. An increase in $\Delta t$ will result in more Type S users being served per trip, increasing round-trip time; hence, larger fleet size is needed to maintain the same service frequency, which ultimately increases $f_i$. As shown in Figure 4.5(a), the range of $\Delta t$ values observed is highest for $h$ between 50-60 minutes with the standard deviation in $f_i$ values observed within this range being 6.3. When $\Delta t$ is between 0-2 minutes, $h$ values range from 10-60 minutes with the standard deviation in $f_i$ values observed within this $\Delta t$ range being 81.8. This standard deviation in $f_i$ values decreases with an increase in $h$ and $\Delta t$ ranges. Thus, $f_i$ is primarily influenced by $h$ with the influence of decision variables reducing for high $h$ and $\Delta t$ values. Alshalalfah (2009) suggested that fleet size would increase by
a fraction equal to the ratio of slack time to original headway, similar to (4.10). Figure 4.5(c) suggests that user cost, $f_2$ is positively correlated to $h$ and $\Delta t$. The standard deviation in $f_2$ values for $h$ between 50-60 minutes is 59.4 while the standard deviation is 40.9 for $\Delta t$ between 0-2 minutes. The standard deviation increases with $h$ and reduces with $\Delta t$ with low values occurring for $h$ and $\Delta t$ between 40-50 and 2-4 minutes, respectively. Thus, $f_2$ is influenced by both variables with $\Delta t$ having slightly greater influence than $h$ and the variation in $f_2$ values is lowest for the medium range of values of decision variables followed by high range values and low range values.

$C_A$ in Figure 4.5(d) is primarily a function of $\Delta t$ since the number of Type S passengers increases with $\Delta t$ but decreases with $h$ since the number of Type G passengers increases with $h$ (as per Constraint 3). As shown in Figure 4.5(e), $C_W$ is a function of both $h$ and $\Delta t$ as the wait time per passenger varies directly with $h$, and the accepted Type S passenger demand, which increases with $\Delta t$ but decreases with $h$, with $h$ having a greater impact on the former than the latter. $C_I$ in Figure 4.5(f) increases with $\Delta t$ as it increases the one-way travel time (i.e., $C_I$ exhibits positive quadratic growth as it is proportional to the square of $\Delta t$). $h$ increases Type G passengers in the system, thus increasing $T_{TV}$, but also limits Type S passengers, with a greater impact on the latter. From Figure 4.5(g) it is evident that $f_3$ is directly proportional to $\Delta t$ and inversely proportional to $h$. Additionally, the standard deviation in $f_3$ values does not vary significantly with $h$ ranges and is averaged at 68.4, while the standard deviation values decrease from 67.1 to 11.3 with the increase in $\Delta t$ value ranges.

Thus, $\Delta t$ primarily impacts $f_3$ irrespective of $h$, and as $\Delta t$ increases variation in $f_3$ reduces. Figure 4.5(h) graphically confirms that total cost, TC defined as the sum of $f_1$ and $f_2$ minus $f_3$, is a convex function with the minimum value of $TC = $242.2/hr. The Pareto solution corresponding lowest TC is $h = 21$ minutes and $\Delta t = 3$ minutes as shown in Figure 4.5(i). In comparison to the "knee-point solution", this solution with $f_1 = $197.8/hr, $f_2 = $285.3/hr, and $f_3 = $240.9/hr would drive $f_2$ down by 27.4% and $f_3$ down by 4.8%, and $f_1$ up by 62%. Additionally, Figure 4.5(i) suggests that extreme values of $h$ and $\Delta t$ yield higher TC values and that opting for mid-range values would minimize TC. For pruning the Pareto optimal set, the analysis described here can be useful.
Figure 4.5 For Pareto optimal solutions $\mathcal{F}^*$, heatmap illustrating the variation of (a) $f_1$, (b) fleet size, (c) $f_2$, (d) access time cost, (e) wait time cost, (f) in-vehicle time cost, (g) $f_3$, and total cost (h) 3D-plot and (i) 2D-plot.
4.5.3. Sensitivity analysis

4.5.3.1. Vehicle capacity

Vehicle capacity ($C$) is an important design parameter for flexible transit (Estrada et al., 2021; M. Kim et al., 2019). In this study, constraints limit the range of decision variables and ensure that no proposed solution exceeds $C$. Accordingly, vehicle occupancy, passenger composition, and system costs also vary with $C$. In this chapter, three common vehicle types in SFT are considered with varying capacities: Mini-van (7-passenger vehicle excluding the driver), standard van (15-passenger vehicle excluding the driver), and mini-bus (25-passenger vehicle excluding the driver). As $C$ increases, vehicle occupancy decreases due to low passenger demand along the route, as shown in Figures 4.6(a), 4.6(b), and 4.6(c). Average occupancy drops from 52.5% to 42.4% when $C$ increases from 7 to 15 seats/vehicle, and even further to 25.1% when $C$ increases from 15 to 25 seats/vehicle. From Figure 4.6(d) and 4.6(e), as $C$ increases from 7 to 15 seats/vehicle, the average $h$ across the Pareto set ($\overline{h}$) increases from 18 to 31 minutes, while $\overline{\Delta t}$ increases from 1.6 to 2.6 minutes. When $C$ increases from 15 to 25 seats, little or no difference is observed in $\overline{h}$ and $\overline{\Delta t}$. As expected, the average number of Type G and Type S passengers served per trip increases from 3 to 5 and 1 to 2, respectively, when $C$ increases from 7 to 15 seats/vehicle. A similar finding was reported by Kim et al. (2019) where optimal zone size and optimal headway for operating flexible buses increased rapidly when $C$ increased from 5 to 15 seats; however, when vehicles had sufficient capacity (i.e., greater than 15 seats/bus), the increase was less rapid. Owing to an increase in $\overline{h}$ and $\overline{\Delta t}$ when $C$ increases from 7 to 15 seats/vehicle, $f_1$ decrease significantly, while user costs $f_2$ increase. When $C > 15$ seats/vehicle, the difference becomes less evident as little, or no difference is observed in $\overline{h}$ and $\overline{\Delta t}$. While $f_2$ does not improve much across vehicle sizes. Precisely, with an increase of $C$ from 5 to 15 and 15 to 25 seats/vehicle, $\overline{f_1}$ decreases by 21.5% and 2%, respectively, while $\overline{f_2}$ increases by 22.7% and 1%. Accordingly, a larger fleet with smaller capacity vehicles results in shorter passenger travel times, but from the operator’s perspective, a few larger capacity vehicles are more cost-effective. It is concluded that minivans are more appropriate for SFT services in terms of vehicle occupancy and user costs, and that standard vans are the most cost-effective from the standpoint of operator costs, while minibuses offer no benefit from an operator, user, or service standpoint. If more paratransit users are accommodated, the standard van
and minibus occupancy may increase, but the service may become less competitive as the overall travel time increases. Estrada et al. (2021) also reported that flexible services with cars ($C = 4$ pass/veh) are most economical in terms of operator cost than minibuses ($C = 22$ pass/veh), and standard buses ($C = 70$ pass/veh).

Figure 4.6 Sensitivity analysis results for vehicle capacity

4.5.3.2. Hourly demand ($Q_G$ and $Q_S$)

Figure 4.7 illustrates the sensitivity of Pareto solutions to average hourly Type G ($Q_G$) and Type S ($Q_S$) passenger demand, with each varying from 5 to 30 passengers/hour. According to Figure 4.7(a), when the demand for the service increases, a high-frequency service is required to achieve system optimality. With $Q_S$ at 5 pass/hr, the average decrease in $\bar{h}$ with $Q_G$ is $10.89\%$, reaching $12.29\%$ at $Q_S = 10$ pass/hr, and continuously declining to $6.61\%$ at $Q_S = 30$ pass/hr. Figure 4.7(b) demonstrates an expected phenomenon, $\Delta t$ increases with $Q_S$ since more Type S passengers are available, and decreases with $Q_G$ as the capacity available to serve Type S passengers reduces with
Q_G. The decrease in $\Delta t$ with Q_G reducing as Q_S increases, varying from 12.39% to 7.34%. Figure 4.7(c) shows that optimization problem constraints always ensure that the number of Type S passengers served per SFT trip is always less or equal to the total observed Type S demand. An increase in Q_G suggests high service frequency (i.e., $\bar{h}$ reduces) and reduced available capacity for Type S passengers (i.e., $\Delta t$ reduces) while the increase in Q_S suggests higher slack time to accommodate Type S passengers (i.e., $\Delta t$ increases) and increased service frequency as demand increases (i.e., $\bar{h}$ reduces). Thus, as demand increases, average operating cost ($\bar{f}_1$) and average user cost ($\bar{f}_2$) increases but the rate of increase with Q_G reduces as Q_S increases, ranging from 6.3% to 3.5% for $\bar{f}_1$ and 30.5% to 15.5% for $\bar{f}_2$. $\bar{f}_3$ increases with Q_S while decreasing with Q_G, with the rate being significantly higher in the former case. Simply put, operator cost and user cost are directly proportional to Q_G and Q_S whereas service benefit is directly proportional to Q_S but inversely proportional to Q_G. Hence, a reasonable trade-off in cost and benefit is possible when demand is low to medium (5-20 passes/hr), whereas high demand dramatically increases costs. In their initial feasibility analysis along this route, Mishra et al. (2020) recommended regular bus transit (FRBT) over SFT when Type G demand exceeds 27 passengers/hour. A transit planner can utilize this analysis to develop an integrated service schedule based on the observed temporal distribution of passenger demand along the route, which is typically bimodal with two distinct peak periods for Type G, and peaks during noon off-peak periods for Type S. For example, it may be recommended to adopt relatively lower $\Delta t$ and $\bar{h}$ values during peak hours than in off-peak hours.
When transit operators agree to serve curb-to-curb requests outside of the designated service area shown in Figure 4.1, sensitivity analysis with permitted deviations ($D_P$) will help us understand its impact on Pareto solutions. $D_P$ is 0.5 km in the base case, which is equivalent to half the width of the service area, 1 km (see Appendix Table B1). Typically, in real-world situations, $D_P$ range between 0.5 km and 2.5 km from the fixed route (Koffman, 2004). Figure 4.8 depicts the results of the sensitivity analysis using Eq. (4.21) in this situation to estimate $\delta$.

$$\delta = \frac{D_P \left( \frac{\eta_{R2} + \eta_{R3}}{2} + \eta_{R4} \right)}{V_R} + 2\tau_{ad} + 2\tau_d$$

*Eq. 4.21*

where, $D_P$ represents the permitted deviation from a fixed route, in km.

Based on Figure 4.8, $D_P$ has a minimal effect on $h$, but increases $\delta$, thereby affecting $\Delta t$ in the Pareto set significantly. When $D_P$ increases by 0.25 km, $\bar{h}$ and $\Delta\bar{t}$ increase by 0.15% and 10%, respectively; therefore, $\bar{f}_1$, $\bar{f}_2$, and $\bar{f}_3$ increase by 0.95%, 0.83%, and 0.5%, respectively; and the rate of increase decreases with increasing $D_P$. Percentage change in $\bar{f}_3$ with $D_P$ is the least, since the increase in $\Delta t$ does not necessarily help serve more type S passengers as the time required to serve a paratransit request $\delta$ also increases with $D_P$. Alshalalfah (2009) also demonstrated that as
the width of the service area increases, the percentage of feasible deviations decreases systematically. The benefits derived from adopting high $\Delta t$ values suggested for systems with higher $D_P$ are less than the increased travel time and the delay imposed on regular passengers by deviation services. By providing higher $D_P$ at off-peak hours, when fewer passengers are on board, the user cost can be reduced while operator costs can be reduced by using larger headways.

![Figure 4.8 Sensitivity analysis results for permitted deviation](image)

4.5.3.4. Weather conditions

In extreme weather conditions, such as snow, ice, or sleet, commonly observed in parts of Canada during Winter, using a wheelchair or walking to the bus stop often causes significant discomfort or poses a safety risk to transit users. This scenario is replicated by reducing walking/wheeling speed, vehicle riding speed, and increasing the proportion of Type S passengers requesting curb-to-curb service for both pickup and drop-off, as shown in Table 4.4. For a route-deviated pickup and drop-off in extreme weather conditions (Case 1), it takes 3.16 minutes ($\delta$), almost twice as long as for normal weather conditions (Case 2). According to Figure 4.9(a), the probability of observing lower values of $h$ is relatively higher in Case 1 than in Case 2, suggesting more frequent service is required to achieve system optimality during extreme weather conditions; the $\bar{h}$ for Case 1 is 26.9 minutes, which is 5% less than Case 2. As $\delta$ increases in Case 2, the upper limit of $\Delta t$ in Constraint 2 also increases; therefore, $\bar{\Delta}t$ for Case 1 is 1.5 times higher than Case 2, as shown in Figure 4.9(b). Case 1 and Case 2 serve the same number of Type S passengers per trip, but Case 1 serves two passengers on average, while Case 2 serves one passenger. $\bar{f}_1$ and $\bar{f}_2$ are higher for Case
1 compared to Case 2 by 2.2 and 2.7 times, respectively, whereas the increase in $\bar{f}_3$ for Case 1 compared to Case 2 is negligible by 6.1%. This is expected since in Case 1 walking time increases dramatically, and $\Delta t$ values in Case 1 are higher, meaning that one-way travel times are longer, resulting in a need to increase fleet size requirements. Thus, normal weather conditions are more conducive to the operation of an integrated SFT than extreme weather conditions, and in extreme weather conditions, slack time should be approximately twice as long as normal weather conditions, and headway should be approximately the same as normal weather conditions or slightly higher. Under adverse weather conditions, Nourbakhsh and Ouyang (2012) compared two systems, FRBT and SFT, by lowering walking speed to 0.1 km/hr and found that FRBT bears a significant increase in total costs (i.e., operator and user costs) as compared to SFT, which can handle a broader range of demand densities.

Table 4.4 System characteristics based on weather conditions

<table>
<thead>
<tr>
<th>Weather condition</th>
<th>$Q_G$</th>
<th>$Q_S$</th>
<th>$C$</th>
<th>$V_R$</th>
<th>$V_a$</th>
<th>$\eta_{R1}$</th>
<th>$\eta_{R2}$</th>
<th>$\eta_{R3}$</th>
<th>$\eta_{R4}$</th>
<th>$\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>pass/hr</td>
<td>pass/hr</td>
<td>seats</td>
<td>km/hr</td>
<td>km/hr</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>hr/pass</td>
</tr>
<tr>
<td>Case 1: Adverse</td>
<td>9</td>
<td>6</td>
<td>15</td>
<td>15</td>
<td>0.5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0.053</td>
</tr>
<tr>
<td>Case 2: Normal</td>
<td>9</td>
<td>6</td>
<td>15</td>
<td>35</td>
<td>4</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.026</td>
</tr>
</tbody>
</table>

Figure 4.9 Sensitivity analysis results for weather conditions
4.6. Policy recommendations

4.6.1. Fare policy

Several fare policies can be adopted during the pilot phase. First, existing bus transit users (Type G) and paratransit users (Type S) may pay the same fare, and deviations outside the service area (\(D_P > W\)) are not subject to a surcharge. Secondly, transit operators may impose a deviation surcharge on Type S users requesting deviations outside the designated service area or permitted deviation set by them. Transit agencies may offer Type G passengers a discounted fare for their degraded service quality since their travel time increases with deviations (Shen et al., 2019). To encourage Type S passengers to switch from an overburdened paratransit service to the SFT, a reduced fare may be offered. When optimizing the transit system under different fare policies, the fare surcharge/discount can be incorporated into the service benefit objective.

4.6.2. Technology

Developments in automotive technology and technology for deviated service booking are fundamental decisions regarding technology in integrated SFT. When requesting curb-to-curb service, Type S passengers may reserve a ride via phone or mobile app about an hour in advance, providing trip details only for the deviated portion of the trip. Type S passengers may then be provided with real-time information, such as available pick-up or drop-off times based on the request type R2, R3, or R4, and slack time available. Unit operator costs accounting for fleet purchases, fuel purchases, and driver wages (nearly 40-80%) can be reduced through technology advancement. Through electrification, a reduction in energy costs, and automation, which eliminates the need for drivers, operating costs can be reduced.

4.6.3. Operations

To estimate the passenger loading profile along the route, optimal headway, and slack-time values may be adopted for a given hourly demand profile. Using this graph, it is possible to determine the required number of seats for each trip, which may facilitate the design of a vehicle mix that maximizes vehicle occupancy.
4.7. Conclusion

We performed joint optimization of service headway \((h)\) and slack time per trip \((\Delta t)\) utilizing operator cost \((f_1)\), user cost \((f_2)\), and service benefit \((f_3)\) to design a semi-flexible transit (SFT) system along an existing low demand bus route that serves both general and special-need passengers. A detailed case study is conducted to demonstrate the methodology application for a low-demand bus route, Route 6 in Regina, Canada. These are the main contributions.

1. A relationship between optimal \(h\) and \(\Delta t\) is modeled using quantile regression, where conditional quantiles of \(\Delta t\) can be suggested for a given value or a range of \(h\). This analysis helps transit planners evaluate different levels of flexibility that can be introduced into the timetable through slack time for a given service frequency to generate a static schedule for SFT operation that maximize cost efficiency.

2. We established a relationship between decision variables and objective functions to analyze the trade-offs in costs between alternative Pareto solutions essential for decision-makers in pruning the Pareto optimal sets. \(f_1\) is negatively correlated and primarily impacted by \(h\), \(f_2\) is positively correlated to both variables, but \(\Delta t\) has a greater impact than \(h\), and \(\Delta t\) is positively correlated and primarily influences \(f_3\). \(f_1\) and \(f_3\) exhibit relatively low sensitivity to decision variables for high and medium value ranges, respectively. Essentially, operator cost favors solutions with low service frequency (i.e., high \(h\)), user cost favors lower ranges of \(h\) and \(\Delta t\), and higher service benefit is derived from high \(\Delta t\); thus, medium ranges of \(h\) and \(\Delta t\) appear to provide the most reasonable tradeoff for service.

3. Sensitivity analysis reveals that low-capacity vehicles are more cost competitive with 7-seater minivans offering higher vehicle occupancy and lower user costs and 15-seater standard vans offering lower operator costs. A reasonable trade-off can be achieved between cost and benefit under low to medium demand (5-20 passes/hr), but high demand significantly increases costs. To minimize the possibility of causing passenger delays on board and to reduce user costs, a low permissible deviation from the fixed route is desirable during peak hours. When extreme weather conditions prevail, vehicle and passenger walking speeds are reduced and door-to-door services are demanded more frequently, resulting in higher operator and user costs. The purpose of sensitivity analysis for transit operators is to gain a greater understanding of the
cost-effectiveness of the system under varying environmental conditions as well as determine if changes should be made to the schedule design based on variation in optimal slack time and headway.

Policy recommendations for integrated SFT implementation include a recommendation for fare structure design addressing service equity through surcharges/discounts, vehicle technology and service booking technology advancements for cost reduction, and fleet mix design through estimation of passenger loading profile. Certain aspects limiting the implementation of this study will be investigated in future extensions. The simplified environment in terms of the service area and demand for defining analytical cost models could be enhanced to reflect a more realistic environment, including accounting for stochasticity in vehicle arrival and demand.
Chapter 5. Cost analysis of different vehicle technologies for semi-flexible transit operations

5.1. Abstract

In low-demand areas, semi-flexible transit systems (SFTs) operated by battery electric vehicles (BEVs) can reduce operational costs and achieve zero emissions, allowing SFT to be used more widely, and transit agencies to benefit more significantly. This paper is aimed at analyzing the effect of the new requirements of BEVs on the operation of SFT services along a low demand bus route, considering different headways and slack time to accommodate route-deviation. Analytical models are used for detailed estimation of the total cost incurred by the transit agency, including operator, user, and environmental costs, allowing a comparison with internal combustion engine (ICEV) vehicle technology, and three vehicle sizes: minivans, standard vans, and minibuses. This information is crucial for transit agencies and city councils to determine which electric vehicle technology and size is most cost-competitive, and to allocate budgets to replace existing ICEV-based standard buses along an underperforming low demand bus route with BEV-based SFT. The application of the study methodology is demonstrated for a low-demand bus route in Regina, Canada.

5.2. Basic considerations

The defined transit system is based on the following considerations.

A: Service area- The service area is modeled as a rectangle with dimensions $W$ (km) and $L$ (km) (see Figure 5.1).

B: Operating policy- The route-deviation policy defined by Koffman (2004) is adopted where vehicles follow a fixed route and deviate to serve curb-to-curb requests, with a maximum allowable deviation of $W/2$ on both sides (see Figure 5.1(a)).

C: Demand distribution- The passenger demand is uniformly and independently distributed along the route.
D: Request type- Within the service area, passenger requests are of two types: flag requests and curb-to-curb requests.

E: Vehicle capacity- Three common vehicle types in SFT with varying capacities are considered: (a) Mini-van (7-passenger vehicle excluding the driver), (b) standard van (15-passenger vehicle excluding the driver), and (c) mini-bus (25-passenger vehicle excluding the driver).

F: Charging scenarios- In this study, the opportunity charging scheme is considered with the following strategy: vehicles are charged after the state-of-the-charge (SOC) of the battery drops to a minimum value. In these cases, fast charging stations may be deployed at the last stop/terminal or located at $L_D$ units of distance away from the terminal station (see Figure 5.1(a)). Thus, for recharging if $L_D > 0$, each vehicle travels an $L_D$ kilometer from and to the terminal station before serving the next trip.

G: Vehicle technology- This chapter focuses on comparing the costs of systems composed of (a) internal combustion engine vehicles (ICEV) and (b) battery electric vehicles (BEV). ICEVs are powered by a conventional internal combustion engine, typically using fossil fuels such as gasoline or diesel as shown in Figure 5.1(b). BEVs are powered by chemical energy that is stored in an onboard lithium-ion-based battery package, which is charged by connecting a plug to an external power source as shown in Figure 5.1(c). In addition, the configuration for vehicles irrespective of capacity and technology includes auxiliary devices, an electric motor, a transmission system, and a final drive. The engine configuration of this technology does not include any mechanical parts.

The list of symbols, definitions, and units used in the deriving analytical cost models are discussed in Appendix D. The frameworks illustrated in Appendix E serve as a comprehensive guide to detailing inputs, outputs, and analytical procedures for implementing the study models developed in this chapter for any other low demand route.
5.3. Cost calculation

5.3.1. Operator cost

$C_{OC}$ in Eq. (5.1) is calculated as the sum of (a) time-related costs including amortized vehicle cost, labor, and staff costs, (b) distance-related costs including energy and maintenance costs, (c) battery purchase cost, and (d) charging or refueling infrastructure cost.

$$C_{OC} = C_1 \times F + C_2 \times L_F + C_3 \times E_b \times F + C_4 \times N_{st}$$

Eq. 5.1

Here, $C_1$ is unit temporal cost ($/veh-hr$), $C_2$ is unit distance cost ($/veh-km$), $C_3$ is unit temporal battery cost ($/kWh-hr$), $C_4$ is unit charging facility cost or refueling station cost ($/charger-hr$), $F$
is fleet size, $L_F$ is vehicle-kilometres travelled by fleet per unit time (veh-km/hr), $E_b$ is battery capacity per vehicle (kWh/veh), and $N_{st}$ is the number of charging or refuelling stations.

### 5.3.1.1. Estimation of fleet size, $F$

Eq. (5.2) expresses $F$ as the sum of the minimum fleet size required for SFT operation ($F_{min}$) and the additional fleet required because of the inclusion of charging/refueling time in the schedule to maintain the same service frequency ($F_{add}$). In the case of ICEV, range limitations are not generally a concern since the size of the fuel tank typically allows a vehicle to complete all scheduled trips without having to refuel; thus, $F$ is equal to $F_{min}$ as $F_{add}$ is negligible. $F_{min}$ is defined as the ratio of cycle time, $T_C$ (hr), and scheduled headway, $h$ (hr/veh) and $F_{add}$ is substantial for BEV and is defined as the ratio of required charging time ($T_{Chg}$) and the available charging time ($T_A$).

$$F = F_{min} + F_{add} = \left[ \frac{T_C}{h} \right]^+ = \left[ \frac{2(T_O + T_S)}{h} + \frac{T_{Chg}}{T_A} \right]^+ \tag{Eq. 5.2}$$

$T_C$ consists of three components: (a) time required to serve on-route requests, $T_O$ (hr), and (b) slack time to serve route deviation requests, $T_S$ (hr). $T_O$ includes total riding time, vehicle acceleration and deceleration time, and dwell time for passenger boarding and alighting as given in Eq. (5.3). $T_S$ represents the slack time allocated to the timetable to accommodate route-deviation requests, also represented as one-way trip time difference when the route deviations are accommodated in the trip against serving only on-route requests. Thus, $N_{RD}$ in Eq. (5.4) represents the number of route deviation requests that can be accommodated for a fixed amount of slack time built into the timetable.

$$T_O = \frac{L}{V_R} + \left( \frac{V_R}{a} + t_d \right) 2Ph \tag{Eq. 5.3}$$

$$N_{RD} = \frac{T_S}{\varphi} \tag{Eq. 5.4}$$

Here, $L$ is the length of the route (km), $V_R$ is the riding speed (km/hr), $V_R/a$ is time spent in acceleration and deceleration per stopping with an acceleration/deceleration rate $a$, $t_d$ is per passenger boarding/alighting time, $2Ph$ is the number of stoppings that is assumed to be twice the
number of passengers boarded in a vehicle, which holds true in low-demand situations (Kikuchi & Vuchic, 1982).

φ in Eq. (5.5) is the average time required to serve one route deviation request which includes the time required to deviate an average of W/4 from and to the fixed route to serve a curb-to-curb stop request for a pickup/drop-off (see Figure 5.1).

\[ \phi = \frac{\left(\frac{W}{4} + \frac{W}{4}\right)}{V_R} = \frac{W}{2V_R} \]  

Eq. 5.5

T\text{Chg} for opportunity charging is the time required to charge a BEV after serving one-way trip including deadheading estimated using Eq. (5.6) and is a function of TL and TA. The layover time at terminal stations, TL (hr), is a fixed time interval inserted in the schedule for drivers’ rest or boarding/alighting process and is proportional to operating time. TA is equivalent to service interval (h). In Eq. (5.6), if TL greater than or equal to T\text{Chg}, vehicle charging operation is made within a predefined operation time slot TL; thus, the charging process, in this case, would not impact the fleet size and introducing additional charging time will result in more idle time in the schedule. Otherwise, fleet size increases as vehicles need to spend additional time at header stations to perform charging operations.

\[ T_{\text{Chg}} = \max \left[ T_L, \frac{S \{L + N_{RD} \left(\frac{W}{2}\right)} + 2L_D \} E_C}{B_{\text{Chg}}} \right] \]  

Eq. 5.6

Here, EC is the energy consumption factor (kWh/veh-km), B\text{Chg} is the charging speed (kWh/h), and LD is the deadheading distance. S in Eq. (5.7) represents the number of trips served before recharging operation and ensures that this number although limited by the battery capacity does not exceed the number of trips served by the ICEV counterpart during the entire day’s operation with H service hours.

\[ S = \min \left[ \frac{E_b(1 - SOC_{min})}{\left[ \left\{ L + N_{RD} \left(\frac{W}{2}\right) \right\} + 2L_D \} E_C \right]} \right] \frac{H}{T_O + T_S + T_L} \]  

Eq. 5.7
Here, $E_b$ is the battery capacity of the vehicle and $SOC_{\text{min}}$ is the minimum state of charge of the batteries maintained to avoid complete depletion of battery reducing the damage to the energy storage system and consequently, shorter lifetime. $SOC_{\text{min}}$ is defined as the ratio of available to nominal capacity that varies between 0 to 1. Essentially, $E_b \times SOC_{\text{min}}$ is the minimal energy that buses must present at any moment of the service in case of emergency.

5.3.1.2. Estimation of energy consumption factor, $E_C$

We have adopted the energy demand model derived by Gallet et al. (2018), which is based on a common longitudinal dynamics model for electric vehicles and incorporates real-world bus route characteristics. $E_C$ between two consecutive vehicle stoppings defined in Eq. (5.8) consists of two main energy systems: (1) propulsion system ($E_{\text{Prop}}$) and (2) auxiliaries ($E_{\text{Aux}}$) (see Appendix C for details). $E_{\text{Prop}}$ is the energy demand due to tractive and $E_{\text{Aux}}$ is the energy consumed providing auxiliary power for air conditioning and various auxiliary services (such as operating doors, powered steering, lighting, in-vehicle displays, etc.). $l$ is the inter-stop distance and as shown in Eq. (5.9) is defined as the ratio of total one-way distance travelled and the number of identical stopping phases. Now, as shown in Eq. (5.8) $E_{\text{Prop}}$ for each phase (i.e., inter-stop) has two components (1) energy consumed during start and end consisting of constant acceleration/deceleration with rate ‘$a$’ over distance $l_0$ estimated as $V_R^2/2a$, (2) energy consumed during riding with constant speed $V_R$ over distance $l-2l_0$. The interaction of traction force components like aerodynamic drag force, rolling friction, grade force, and inertia force in $E_{\text{Prop}}$ calculation is derived in Appendix C. $P_{\text{aux}}$ in kW is the constant auxiliary power required within the vehicle over the driving duration between two stoppings.

\[
E_C = \frac{E_{\text{Prop}} + E_{\text{Aux}}}{l} \quad \text{Eq. 5.8}
\]

\[
= \frac{2l_0(Mgf_t) + (l - 2l_0)(Mgf_t + 0.5\rho C_d A_f V_R^2) + P_{\text{aux}}\left(\frac{T_b + T_S}{2Ph + 1}\right)}{l} \quad \text{Eq. 5.8}
\]

\[
l = \left\{\frac{L + N_{RD}(\frac{W}{2})}{2Ph + 1}\right\} \quad \text{Eq. 5.9}
\]

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Here, total mass of vehicle \( M \) is the sum of the curb weight \( M_{\text{curb}} \) and the total mass of the passengers on board with the average weight of passenger as \( m_{\text{pass}} \) (i.e., \( M = M_{\text{curb}} + Ph_{\text{pass}} \)), \( g \) is acceleration due to gravity (m/s\(^2\)), \( f \) is the coefficient of rolling, \( C_d \) is the drag co-efficient, \( \rho \) is the air density (kg/m\(^3\)), and \( A_f \) is the vehicle frontal area (m\(^2\)).

5.3.1.3. Estimation of distance travelled by the fleet per unit time, \( L_F \)

As shown in Eq. (5.10), \( L_F \) is estimated as the ratio of the length of the roundtrip and service headway as there will be a vehicle that has completed an entire roundtrip with charging at each time headway \( h \).

\[
L_F = \frac{2\left\{L + N_{RD}\left(\frac{W}{2}\right)\right\} + 2L_D}{h} \quad \quad \quad \text{Eq. 5.10}
\]

5.3.1.4. Estimation of battery capacity on-board, \( E_b \)

Opportunity charging require a battery package of capacity \( E_b \) defined in Eq. (5.11) as the product of the number of batteries and the nominal capacity of the battery \( (E_{b,\text{nominal}}) \). Due to the installation of charging stations at the end of the route, the number of batteries is estimated as the ratio of the energy required to complete a single one-way trip including the deadheading and the available battery capacity based on the minimum state of charge.

\[
E_b = \left[\frac{\left\{L + N_{RD}\left(\frac{W}{2}\right)\right\} + 2L_D}{E_{b,\text{nominal}}(1 - \text{SOC}_{\text{min}})}\right]^+ \times E_{b,\text{nominal}} \quad \quad \text{Eq. 5.11}
\]

5.3.1.5. Estimation of the number of chargers, \( N_{Ch} \)

The number of chargers to be deployed at both ends of the route for opportunity charging is given in Eq. (5.12). In case of opportunity charging, \( T_{\text{Chg}} > h \) does not allow dispatching vehicles from the initial stop at time headway; thus, transit agencies are required to deploy multiple charging stations at the terminals to charge vehicles simultaneously.

\[
N_{Ch} = 2\left[\frac{T_{\text{Chg}}}{h}\right]^+ \quad \quad \quad \text{Eq. 5.12}
\]
5.3.2. User cost

The user cost \( (C_{UC}) \) is the sum of the costs of the three equivalent time components, access/egress time, \( C_A \), waiting time, \( C_W \), and in-vehicle time, \( C_I \), estimated in $/hr given in Eq. (5.13).

\[
C_{UC} = C_A + C_W + C_I \tag{5.13}
\]

5.3.2.1. Access time cost

When requesting a flag stop, passengers must walk/wheel an average vertical distance of \( W/4 \) from their origin/destination to the fixed route, and curb-to-curb pick-ups/drops involve no walking (see Table 5.1). Hence, the expected walking time cost can be estimated using Eq. (5.14).

\[
C_A = C_5P \left[ \frac{W}{4V_a} \left( \frac{1}{4} + \frac{1}{4} \right) + \frac{W}{2V_a} \times \left( \frac{1}{4} \right) \right] = C_5P \left[ \frac{W}{4V_a} \right] \tag{5.14}
\]

where, \( V_a \) (km/hr) is the passenger walking speed, \( C_5 \) ($/pass-hr) is the passenger value of time.

5.3.2.2. Waiting time cost

The mean waiting time for passengers requesting a flag stop pick-up can be calculated using the value in Table 5.1 proposed by Ansari Esfeh et al. (2021) for low-demand routes characterized by low-frequency service and published timetable. Curb-to-curb pickup does not incur any additional waiting time for passengers. The expected value of waiting time can therefore be estimated using Eq. (5.15).

\[
C_W = C_5P \left[ h \left( \frac{1}{2} - \frac{\alpha(1 - \beta)}{2} \right) \left( \frac{1}{4} + \frac{1}{4} \right) \right] = C_5P \left[ \frac{h}{2} \left( \frac{1}{2} - \frac{\alpha(1 - \beta)}{2} \right) \right] \tag{5.15}
\]

where \( \alpha \) and \( \beta \) are the proportion of planning passengers and the proportion of planning passengers with fixed arrival times, respectively.

5.3.2.3. In-vehicle time cost

Similarly, passengers can be dropped off/picked up uniformly anytime in the trip between two terminals. Thus, the average in-vehicle time for passengers is half of the total travel time between the two terminals with the expected value given in Eq. (5.16). In addition, charging time does not
affect the in-vehicle time since buses will be charged during the lay-over time with no passengers on board.

\[ C_t = C_5 P \left[ \frac{T_O + T_S}{2} \right] \]  
Eq. 5.16

<table>
<thead>
<tr>
<th>Flexibility increases</th>
<th>Request type</th>
<th>Pick-up location</th>
<th>Drop-off location</th>
<th>Demand proportion</th>
<th>Access time</th>
<th>Waiting time</th>
<th>In-vehicle time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed</td>
<td>Fixed route</td>
<td>Fixed route</td>
<td>1/4</td>
<td>(\frac{W}{2V_a})</td>
<td>(\left[ 1 - \frac{\alpha(1 - \beta)}{2} \right] h)</td>
<td>(\frac{T_O + T_S}{2})</td>
<td></td>
</tr>
<tr>
<td>Demand responsive</td>
<td>Fixed route</td>
<td>Curb-to-curb</td>
<td>1/4</td>
<td>(\frac{W}{4V_a})</td>
<td>(\left[ 1 - \frac{\alpha(1 - \beta)}{2} \right] h)</td>
<td>(\frac{T_O + T_S}{2})</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Curb-to-curb</td>
<td>Fixed route</td>
<td>1/4</td>
<td>(\frac{W}{4V_a})</td>
<td>0</td>
<td>(\frac{T_O + T_S}{2})</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Curb-to-curb</td>
<td>Curb-to-curb</td>
<td>1/4</td>
<td>0</td>
<td>0</td>
<td>(\frac{T_O + T_S}{2})</td>
<td></td>
</tr>
</tbody>
</table>

5.3.3. Environment cost

The emission monetization associated with the circulation of vehicles per unit time can be estimated by Eq. (5.17), considering the total distance run by the whole fleet per unit time. It is considered that BEBs have zero greenhouse gas tailpipe emissions.

\[ C_{ENV} = L_F \times \sum_{x \in X} Z_x \varepsilon_x \]  
Eq. 5.17

Here, \(\varepsilon_x\) a proxy parameter to monetize the effect of producing one unit of greenhouse gas \(X \in (\text{in } \$/gm \text{ of greenhouse gas } X)\) and \(Z_x\) is the mass of greenhouse gas \(x (x \in X)\) produced per kilometer run (gm/veh-km).
5.4. Problem optimization

The optimal equilibrium between the transit operator, transit user, and environment comes from the problem minimization of the total cost ($C_{TC}$) incurred by the transit agency with two continuous decision variables: (a) service headway ($h$) and slack time ($T_s$). Eq. (5.18) states the mathematical formulation of the problem, $C_{TC}$ expressed in units of $$/hour is the sum of the operator cost ($C_{OC}$), user cost ($C_{UC}$), and environment cost ($C_{ENV}$). The problem constraints ensure that decision variables must take positive values with thresholds defined by transit agencies and the vehicle occupancy must be equal to or lower than the vehicle capacity ($C$). The objective function is evaluated for multiple combinations of the decision variable values for each vehicle technology and vehicle type discussed in section 5.2 using numerical analysis.

$$\min_{h,y} C_{TC} = C_{OC} + C_{UC} + C_{ENV} \quad \text{Eq. 5.18}$$

subject to

$$h_{min} \leq h \leq \min \left( \frac{C}{p}, h_p \right) \quad \text{Eq. 5.19}$$

$$0 \leq T_s \leq 2Php \quad \text{Eq. 5.20}$$

Eq. (5.19) depicts that $h$ is constrained by a minimum and maximum service headway. Literature consensus that for a low-demand route characterized by low-frequency service, the minimum headway can be set to 10 minutes (Ansari Esfeh et al., 2021). The minimum desired level of service is set through policy headway, $h_p$, and the vehicle capacity governs the maximum value. As shown in Eq. (5.20), $T_s$ should vary between 0 and a maximum value based on observed demand ensuring that slack time cannot exceed the time required to provide route deviations for both pick-up and drop-off for all passengers in the system.

5.5. Results and discussion

5.5.1. Case study

An analysis of ridership and operating data obtained from Regina Transit, Canada for 2015 reveals that route 6, is an underperforming bus route with low ridership ($P$) of 9 passengers/hour, a low revenue-to-cost ratio of 15.3%, a high operating cost of $10.84/pass, and decline in ridership of
1.04% from 2014; hence used as a case study. Our first step in solving the optimization problem involves defining a base case study based on the input parameters of the model described in Table C1 in the Appendix. The results of optimization are presented in the following sections. In Eq. (5.19), the constraint limits $h$ between [10, 46] minutes for the minivan and [10, 60] minutes for the standard van and minibus, while in Eq. (5.20), the constraint limits $T_s$ between [0, 11] minutes for the minivan and [0, 15] minutes for the standard van and minibus.
5.5.2. Comparison of costs for different vehicle technologies

Figure 5.2 Average of total cost and cost components incurred in the provision of ICEV, and BEV-based SFT service

Figure 5.2 summarizes the average of the operator, user, and total cost components considering different SFT vehicle technologies in Route 6. Despite the charging strategy, the average operating cost with BEV is higher than the corresponding cost to ICEV counterparts and this cost increases with vehicle size with a rate higher in the case of ICEV than BEV. In the BEV, the average operating cost increases by 2.8%, 7.6%, and 13.1% from ICEV counterparts for minivan, standard
van, and minibus, respectively. It is observed that for both ICEVs and BEVs, the standard van has the lowest average fleet cost. ICEV fleet costs are lower for minibuses than minivans, while BEV fleet costs are lower for minivans than minibuses. Service headway, $h$ range suggests greater average fleet size for minivan and the same for a standard van and minibus. However, this difference in fleet costs is a result of the increase in unit temporal costs with vehicle size, which is more significant for BEVs than for ICEVs. Additionally, the average energy consumption factor ($E_C$) increases with vehicle size with values 0.07kWh/km, 0.17kWh/km, and 0.26kWh/km for BEV-based minivan, van, and minibus, respectively, contributed by increasing gross vehicle weight and the number of stoppings. Consequently, given that each vehicle is equipped with a 25kWh battery pack, and the minimum required state of charge is constrained to 0.2, the average number of one-way trips that can be served with 80% charging decreases with vehicle size with 17, 7, and 4 trips, respectively, for minivans, vans, and minibuses. For a given layover time ($T_L$) of 5 minutes, $T_L$ exceeds the required charging time ($T_{Chg}$) for all instances varying from 2-2.6 minutes; therefore, charging of a BEV can be accomplished within a predefined operation time slot without changing the existing schedule if the transit agency wishes to transition from ICEV to BEV. The average energy costs increase with vehicle size and for each vehicle size, the cost drops approximately 70% from ICEV to BEV. It is estimated that the average range of a minivan BEV is 277km, while that of a minibus BEV is 68km, with battery costs remaining the same regardless of vehicle size. Finally, the infrastructure costs are only a function of vehicle technology and are significantly higher for BEV (i.e., $8/\text{hr}$) than ICEV (i.e., $0.14/\text{hr}$). Interestingly, if infrastructure cost is excluded similar to results reported by Estrada et al. (2021), BEV always outperforms ICEV for all vehicle sizes; thus, the potential savings in energy cost in favor of BEV was neutralized by the current expensive cost of installing fast chargers. Similarly, Tirachini and Antoniou (2020) suggested that the operator cost of BEV comprising of vehicle capital costs, driver costs, and running costs, (e.g., energy consumption and maintenance) increases with vehicle size. The total operator cost for the electric car, van, and minibus are €18.9/veh-h, €20/veh-h, and €28.7/veh-h. While the average access costs are insensitive to vehicle size and technology, vehicle capacity determines the maximum headway, which in turn affects average waiting and in-vehicle costs with the lowest values observed for minivans for a maximum $h$ of 47 minutes. In addition, there is no environmental associated with BEV and this cost increases with vehicle size for an ICEV. Finally,
in terms of the total cost, a minivan ICEV outperforms all other scenarios and for standard van and minibus, BEV technology offers lower total cost than ICEV with the total cost increasing with vehicle size irrespective of technology.

5.5.3. Variation of cost components with \( h \) and \( T_S \)

In this section, general relationships between \( C_{TC} \), \( C_{OC} \), and \( C_{UC} \) with decision variables are established using descriptive analysis plots for all scenarios in Figures 5.3 and 5.4. Figures 5.3 and 5.4 demonstrate that the \( C_{TC} \) is non-linearly related to \( h \) and \( T_S \). The shape of the curve between \( C_{TC} \) and \( h \) is a U-shaped curve suggesting that mid-range headway values appear to be optimal, whereas the curve between \( C_{TC} \) and \( T_S \) shows an approximate linearly increasing trend suggesting that low \( T_S \) values minimize \( C_{TC} \). Additionally, the range of \( C_{TC} \) values decreases with \( T_S \) and increases with \( h \), since a higher service interval/headway indicates more passengers in the system, resulting in more slack time required to accommodate route deviations with higher \( T_S \) values are only associated with high \( h \) values. According to Figures 5.3 and 5.4, the rate of increase in \( C_{TC} \) with \( h \) and \( T_S \) is higher for ICEVs than for BEVs at values of \( h \) less than the optimal value, whereas the opposite holds for values of \( h \) greater than the optimal value. In Figure 5.4, a linear relationship between \( C_{TC} \) and \( T_S \) is a reasonable approximation for BEVs; however, for ICEVs, the relationship departs from a linear trend, indicating the presence of a nonlinear relationship caused by high \( C_{TC} \) values observed for low \( h \) values. Consequently, from Figure 5.5 graphically, it can be concluded that the optimal range of \( h \) is between 20-30 minutes and \( T_S \) is between 0-5 minutes. Alternatively, for a given \( h \), as \( T_S \) increases \( C_{TC} \) increases, and for a given \( T_S \), as \( h \) increases, \( C_{TC} \) increases for low and high value ranges. In essence, it is imperative that careful consideration is given to the choice of headways and slack times in SFT operations, as although mid-range values of \( h \) and low \( T_S \) values minimize \( C_{TC} \); however, when non-optimal values are used, the difference in \( C_{TC} \) from minimum value is more significant for BEVs than for ICEVs. Using Figure 5.3 (g), 5.3 (h), 5.4 (g), and 5.4 (h), the relationship of the operator and user cost with \( h \) and \( T_S \) is explored for minivan BEV, however, the relationship derived holds for all other scenarios as well. As the figure suggests, operator cost, \( C_{OC} \), is inversely related to \( h \) and \( T_S \), while user cost, \( C_{UC} \), is directly related to \( h \) and \( T_S \), while linear relationship approximations are better established with \( h \) than with \( T_S \). With respect to \( C_{OC} \), an increase in \( h \) results in a reduction in the required fleet size, thereby reducing the battery

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cost and the distance that the fleet travels per unit of time, contributing to lower energy costs. Furthermore, as $T_S$ increases, round trip distance increases, which increases energy costs and battery costs; however, high $T_S$ values are only associated with high $h$ values, which reduces fleet size, explaining the inverse relationship between $C_{OC}$ with $T_S$. Infrastructure costs, however, are not affected by $h$ since the required charging time is always less than the headway. In addition, for $C_{UC}$, access/egress time, $C_A$, is not determined by $h$ and $T_S$; however, waiting time, $C_W$, is only positively related to $h$, while in-vehicle time, $C_I$, is positively related as higher $h$ and $T_S$ increases the one-way travel time. Estrada et al. (2021) also demonstrated that for flexible and fixed-route service with diesel and battery-operated vehicles, the operator and user costs usually present a strictly decreasing and increasing behavior, respectively with regard to $h$. For diesel-based flexible-route systems, Alshalalfah (2009) estimated that fleet size would increase as a function of the ratio of slack time to original headway, similar to Eq. (5.2). Yoon et al. (2022) suggested that compliance of timetables with slack time could result in longer travel times since there will be longer waits and in-vehicle time. $T_S$, however, affects wait time only when the fleet size is fixed, since $T_S$ increases the round-trip time, which implies higher headway values; therefore, long waits.
Figure 5.3 Variation of total cost, operator cost, and user cost with $h$ color coded by $T_s$
Figure 5.4 Variation of the total cost, operator cost, and user cost with $T_s$ color-coded by $h$
Figure 5.5 Heatmap illustrating the variation of the total cost with $h$ and $T_s$ for all scenarios

5.5.4. Sensitivity analysis

5.5.4.1. Charging speed, $B_{Chg}$

This section analyses the impact of variation in charging rate at the end stop location on the fleet cost and charging infrastructure cost; thus, total cost, where the charging rate takes values between [0.5, 7.5] kWh/min as shown in Figure 5.6. In contrast to the base case scenario where the charging rate is 7.5 kWh/min, when charging rates are varied, BEVs are most affected, resulting in a difference between 0.3 to 23.3% in total costs. In the base case scenario, the average $T_{Chg}$ for a minivan BEV is 2.3 minutes; however, at the lowest charging rate of 0.5 kWh/min, it is 39 minutes; because the fixed layover time of 5 minutes is less than the average $T_{Chg}$, the fleet size requirements are between 3-13 vehicles instead of 2-6 vehicles, and the number of chargers varies between 2-8 compared to 2 chargers in the base case scenario; thus, the average total costs increase by 23.3% from the base case. Additionally, as the vehicle size increases, the increase in total cost relative to
the base case scenario decreases, and this difference is further reduced as charging speed increases, with no difference observed for charging speeds $\geq 4.5$ kWh/minute.

**Figure 5.6 Sensitivity analysis of the total cost based on the charging rate**

5.5.4.2. Hourly demand, $P$

In their initial feasibility analysis along this route, Mishra et al. (2020) recommended regular bus transit over SFT when demand exceeds 27 pass/hr; thus, in this analysis, the demand is varied over a range of 5, 15, and 25 pass/hr. Figure 5.7 illustrates that when the demand for service increases, the average total cost, $C_{TC}$, increases regardless of vehicle size and technology. For each technology, when demand increases from 5 to 15 pass/hr, standard vans experience the greatest increase in $C_{TC}$, followed by minibuses and minivans; however, when demand increases from 15 to 25 pass/hr, minibuses experience the greatest increase in $C_{TC}$ followed by minivans and standard vans. In addition, from 5-15 pass/hr, the change in $C_{TC}$ is 136% while for 15-25 pass/hr, the change is 55%. With respect to technology, the change in total costs with demand is comparatively more significant for BEV than ICEV. Interestingly, minivan ICEV outperforms all other scenarios when demand levels are 5 pass/hr and 15 pass/hr, whereas when demand increases to 25 pass/hr, minivan BEV outperforms all other scenarios. Further, at all levels of demand and technology, minivans are more economical than standard vans and minibuses. The results are consistent with those reported by Estrada et al. (2021) suggesting that for low to medium demand densities (i.e., 3 to
92pass/km²-h), flexible service operated by electric cars with a capacity of 4pass/veh minimizes the total cost (user and operator cost), followed by diesel-based cars.

Figure 5.7 Sensitivity analysis of the total cost based on hourly demand

5.5.4.3. Battery size and state of the charge

Figure 5.8 illustrates that as battery size increases the total cost increases by 0.64% to 1.73%. For a given battery size, if the minimum required state of charge (SOC) increases from 20% to 50%, the total cost may decrease varying between 0% and -1.55%. For a standard van with a 100kWh battery, as the minimum SOC increases from 20% to 50%, the amount of charging required after completing the maximum number of trips decreases; thus, average $T_{Chg}$ decreases from 8 to 6.5 minutes, so fleet size decreases, resulting in a 1.55% decrease in total cost due to fleet and battery costs. Additionally, for any given SOC, as the battery size increases, the number of one-way trips will increase before recharging; this will result in a greater charging time; thus, a larger fleet size and higher total costs. Moreover, it is important to note that battery size has a negligible impact on the total cost compared to charging speed and demand, so modifying this parameter would have little impact on the total cost.
5.6. Conclusion

There is a limited implementation of semi-flexible transit (SFT) services as they are generally costly to operate due to smaller vehicle sizes and circuitous routes, and only in low demand density scenarios are SFT services more competitive than fixed-route transit services. With SFT electrification, the expected reduction in operating costs related to energy consumption and environmental costs could open new scenarios for SFT to be applied more widely, and transit agencies may benefit more greatly. Nonetheless, it is necessary to assess the cost effectiveness of electrification by considering the additional resources required for electric bus operations. For transit agencies and city councils, this information is essential for determining which electric charging technology is most cost-competitive and allocating budgets to replace existing standard bus fleets with electric SFT along low demand underperforming bus routes. This paper is aimed at analyzing the effect of the new requirements of battery electric vehicles (BEVs) on the operation of SFT services along a low demand bus route, considering different headways \( h \) and slack time \( T_s \). The analytical models developed are used to provide a detailed estimation of the total cost incurred by the transit agency, including operator, user, and environmental costs, allowing a comparison with conventional vehicle technology: internal combustion engines (ICEV), and
comparing three vehicle sizes: minivans (7-seaters), standard vans (15-seaters), and minibuses (25-seaters). The application of the study methodology is demonstrated for a low-demand bus route in Regina, Canada. These are the main contributions. Firstly, in terms of the total cost, a minivan ICEV outperforms all other scenarios as the potential savings in energy cost in favor of BEB were offset by the present high cost of installing fast chargers, and these observations remain valid when demand levels are 5 and 15 passes/hr. In contrast, when demand increases to 25 passes/hr, the minivan BEV outperforms because the savings in energy costs offset the infrastructure costs. Second, at all levels of demand, charging speed, battery size, and powertrain technology, minivans are more economical than standard vans and minibuses. Third, it is imperative that a careful consideration is given to the choice of \( h \) and \( T_S \) in SFT operations, as although mid-range values of \( h \) and low \( T_S \) values minimize \( C_{TC} \); however, when non-optimal values are used, the difference in \( C_{TC} \) from minimum value is more significant for BEVs than for ICEVs.
Chapter 6. Conclusion

The purpose of this thesis is to develop a methodology for the economic assessment, design, and electrification of semi-flexible transit (SFT) along low demand transit routes. To demonstrate the performance and applicability of the developed methodology, a detailed case study was conducted along an existing underperforming low demand bus route, Route 6 in Regina, Canada. For transit agencies and city councils, this methodology will be useful for determining if SFT can be a competitive solution to address high operating costs associated with bus transit operated in-house on routes with low demand, growing demand for highly personalized and expensive paratransit services with an increasing elderly population in North America, and increasing vehicle emissions. A summary of the key findings from each research module, contributions to research including policy implications and applications, and recommendations for future research are presented in this concluding chapter of the thesis.

6.1. Key findings

The first research module case study findings indicate that under low demand conditions, SFT, whether operated as in-house transit (IHT) or as contracted out taxi (IHT) offers lower operator cost and FRBT operating in-house is only cost-efficient when transit demand is high. When implementing an independent system, if the fare structure is expensive, exclusive operation of SFT with IHT delivery model is more economical than SFT with COT delivery model. When implementing an integrated system (i.e., SFT with combined COT and IHT delivery model), it is recommended to operate under COT and IHT during off-peak hours and peak hours, respectively. Further, in an integrated system to ensure maximum vehicle utilization during peak periods, it is suggested to shift a proportion of trips from IHT delivery model to COT or existing paratransit vehicles for values of passenger demand when the marginal cost (MC) of a COT or paratransit is less than the MC of IHT. This analysis encourages decision-makers to adopt a methodological approach to evaluate SFT delivery models to fortify an existing public transit service network by cost-effectively integrating low demand.

The findings of the second research module suggest that joint optimization of slack time and headway is essential to design a static schedule for integrated SFT operations that accommodates
both existing fixed route transit demand and shifted paratransit demand. Analysis of optimal solutions suggests that it is possible to derive multiple optimal slack time values indicating the level of flexibility offered by route deviation with varying probabilities of occurrence in the Pareto set for a given optimal value/range of headway indicative of transit’s level of service. Relationships derived to analyse cost trade-offs suggest that headway primarily governs operator cost and is negatively correlated, user cost is positively and equally influenced by both variables, and slack time primarily governs service benefit and is positively correlated. Sensitivity analysis reveals that low-capacity minivans and standard vans offer higher vehicle occupancy and cost efficiency, the service is most economical for low to medium demand (5-20 pass/hr), low permissible deviation from the fixed route is more desirable during peak hours to avoid delays for passengers on-board, and extreme weather conditions dramatically and negatively influence costs.

The third research module indicates that the cost-effectiveness of vehicle technology in SFT varies with demand as for low values of demand, ICEV outperforms because the potential savings in energy cost in favor of battery electric vehicles (BEV) are offset by the current high cost of installing fast chargers, and when demand increases, BEV outperforms because the savings in energy costs offset the infrastructure costs. Regardless of the level of demand, the charging speed, the battery size, and the powertrain technology, low-capacity vehicles such as minivans offer the lowest total cost of service, including the operator cost, the user cost, and the environmental cost. A further observation is that the total cost of service in BEVs is more sensitive to variations in headway and slack time than in ICEVs. This information is useful to decision makers in determining which vehicle technology and vehicle size is the most cost competitive for SFT.

6.2. Contributions

The first research module of this thesis contributes methodologically to the literature focusing on the planning of SFT by developing robust operator cost models for the economic evaluation of delivery models and by developing approximate formulations with reduced data needs and complexity for practical applications. In terms of practical applications, the study methodology can be used to fortify an existing FRBT service network by evaluating partnerships with private operators for providing SFT substituting or complementing existing public transit on low demand routes or time periods. This methodology can be applied to the design of a subsidy level for a
partnership between transit agencies and private operators by integrating different revenue models into derived cost models for contracted-out taxis (COT). Policy recommendations based on the study findings include: (1) by negotiating a reduced taxi base fare, the operator cost of SFT with COT delivery model can be further minimized and its applicability widened, (2) strategic planning of depot locations and investment in technology for demand management are key to cost reduction when implementing IHT delivery model for SFT, (3) combining different service patterns and delivery models to meet varying daily and seasonal demand patterns may enable greater cost savings.

The second research module of this thesis contributes methodologically to the design of SFTs by jointly optimizing the two schedule design variables: slack time to accommodate route deviations and service headway required to maintain the desired level of service. The literature has not yet addressed the joint optimization of slack time and headway, which is generally confined to optimizing them separately. It is possible to apply the study methodology to different scenarios by expressing the service benefit derived from SFT differently depending on the operator's intent, such as in this study where the benefit is expressed in terms of cost savings when accommodating a few paratransit trips using a less costly mode of transportation. Service benefits can also be expressed as an increase in revenue by attracting more passengers to the zone through curb-to-curb service, a reduction in parking infrastructure investment through encouraging people to use transit instead of their cars when accessing the public transit network, or an increase in mobility. When designed for paratransit passengers, this system provides the riders with an alternate mode of transportation that is more accessible with greater service frequency since paratransit service usually requires reservations up to 24 hours in advance. Policy recommendations for integrated SFT implementation include: (1) the use of surcharges and discounts in fares to address service equity between general transit and paratransit users can help define service benefits more accurately, (2) vehicle technology and service booking technology advancements can further reduce costs, and (3) the optimal headway and slack-time values can be used to estimate passenger loading profiles, which may facilitate the development of vehicle mixes that maximize vehicle occupancy.
The third research module is the first to develop robust analytical models for comprehensive evaluation of the total cost of electrification in SFT systems, which includes operator, user, and environmental costs. The total cost of SFT system with battery-operated electric vehicles is analyzed over a range of headway and slack time for three commonly used vehicle sizes in SFT, varying demand, charging speed, battery size, and state-of-charge and comparing the results with operating conventional diesel-powered SFT vehicles. This study methodology allows transit agencies and city councils to determine which technologies and vehicle sizes are most cost-competitive under the various operating conditions, as well as allocate budgets to replace existing ICEV-powered bus fleets with BEV-based SFT, based on the additional resources required for electric vehicle operations. When SFT is electrified, the expected reduction in operating costs related to energy consumption and environmental impacts may open up new scenarios for the application of SFT more widely, and transit agencies may benefit more from it since the implementation of SFT is currently limited due to its higher operator costs resulting from smaller vehicles and circuitous routes.

The primary contribution of this thesis to the literature lies in its methodological innovation, deriving handy but powerful analytical cost models for decision makers using simplified service environment and fewer data needs, aiding the planning of SFT operations in low-demand scenarios. This thesis expands the knowledge base on SFT focussing on studying the effect of decision variables that are elements of strategic and tactical planning on cost when earlier studies only focussed on operational aspects. Additionally, it conducts comprehensive sensitivity analysis through controlled experiments to explore how changes in individual parameters affect optimal decision variables and, consequently, overall costs.

6.3. Justifying thesis assumptions and recommendations for future research

We point out two directions for future research based on the assumptions of this thesis.

6.3.1. Direction 1: Improving cost models estimation accuracy

6.3.1.1. Considering varying temporal and spatial patterns of demand

In Chapters 4 and 5, passenger demand is assumed to be uniformly and independently distributed along the route and the boarding and alighting pattern at a stop is assumed to be Poisson distributed.
These assumptions are examined in the studies discussed in detail below, and the results indicate that these assumptions are valid for this study focusing on a low demand route. Section 3.3.2, in Chapter 3 investigated the effect of non-uniformity in demand distribution along a low demand route using Poisson binomial distribution to ensure that the probability of stopping at each stop is independent and non-identical. Results suggest that, for low demand, less than 20 pass/hr, there is no noticeable gap in the expected number of vehicle stoppings (i.e., $E(s)$) observed for SFT; therefore, in-vehicle time remains the same. In practical situations, there can be numerous instances of service areas with limited demand, where the demand is focused in proximity to areas of significant interest. Our future research efforts will focus on addressing spatial demand variations along the main route, incorporating multiple service zones with similar or diverse service patterns and transfer points.

The study assumed Poisson passenger arrival pattern at stops and suggested that for low demand route adopting the lower limit of $E(s)$ (refer Appendix A) estimated at twice the number of passengers boarded in the vehicle is a slight underestimate of the operator's cost (i.e., the error is -10% for 1 pass/hr, approaches rapidly to 0% at 20 pass/hr, and then gradually increases for demand >20 pass/hr). Mishra et al. (2023) examined whether a negative-binomial distribution with overdispersion would be more indicative of a route with low demand and a Poisson distribution would be characteristic of a route with high demand. For demand less than 20 pass/hr, both distributions predict the same $E(s)$; however, for medium range demand, the gap widens and the Poisson distribution overestimates $E(s)$, while both distributions converge for higher demand ranges. Analogously, we are interested in further investigating the zero-inflated models to account for overdispersion in demand distribution that is commonly observed on low demand routes.

In light of the above observations, the following considerations should be taken into account when implementing the study models for high-demand scenarios:

1. Implement equation 3.1 or 3.3 for estimation of $E(s)$ since in high demand there can be multiple boardings and alightings at a stop and assumption A2 in section 3.2.2.3 can result in overestimation.
2. In high demand scenarios served with high frequency service, passengers arrive at random to stops and hence waiting time in Chapter 4 and 5 can be approximated to half of the service headway.

6.3.1.2. Considering irregularity of vehicle arrivals on cost models

This analytical cost models developed in this thesis does not consider the stochasticity in vehicle arrivals or delays en route.

This assumption is generally true for low demand routes which are characterized by uncongested roads and low occupancy throughout the route which limits the impact of schedule changes including cancellations and delays in the arrival of passengers at pick-up locations on vehicle delays. The advantage of SFT over fixed-route bus transit is that the vehicle might use the slack time assigned to a timetable for route deviation can be used to recover small delays en-route, which means that the stochasticity element can be minimized at each stop (Alshalalfah, 2009; Mishra & Mehran, 2023a). However, stochastic vehicle arrival may be significant when changes to itinerary concerning existing users are significant and during adverse road-weather conditions (Ho et al., 2018) since the extreme temperatures during winters are often encountered throughout the year in parts of Canada. Hence, consideration coefficient of variation in scheduled headways are critical to account for service irregularities.

6.3.1.3. Considering the effect of variation in the value of time

Research suggests that passengers perceive out-of-vehicle time, including walking/access and waiting time, differently than in-vehicle time based on service period (peak/off-peak), adverse weather conditions, stop facilities, and surrounding environment (Wardman, 2004). In fact, the value of access time is usually twice the value of in-vehicle time, and the value of waiting time is higher than access time and three times the value of in-vehicle time. Hence, \( c_9 \) or \( c_{15} \) representing the passenger value of time ($/pass-hr) in Chapter 4 and Chapter 5, respectively must be assigned differently for user travel time components.
6.3.1.4. Considering the additional parameters for estimating operator and environmental cost of BEV-based

The simplified velocity profile implemented in this thesis to estimate the energy consumption factor ($E_C$) only considers acceleration and deceleration phases at stops as discussed in Appendix C assuming low demand routes serving majorly residential areas have uncontrolled intersections. Future research efforts will focus on imitating of urban driving conditions more accurately, where vehicles may frequently be required to stop at intersections or traffic lights and go through a series of acceleration and deceleration phases. Environmental cost in Chapter 5 considers only the monetization of tail-pipe emissions resulting from vehicle operation but ignores the contribution of vehicle manufacturing and infrastructure construction to emissions for battery-electric vehicles and diesel-operated vehicles that can be considered in the future.

6.3.1.5. Considering advancements over implemented optimization method

In Chapter 4, the analysis accounts for the multi-objective nature of objectives and implements Pareto optimization techniques, whereas in Chapter 3, the analysis is based on a single objective, while in Chapter 5, multiple objectives are transformed into a single objective optimization. Even though transit planning is a fairly complex problem involving varying levels of importance given to objectives by different stakeholders in the transit system, there is a need for implementing advanced optimization techniques that take variable weights into account.

6.3.2. Direction 2: Widen the application area of this research

6.3.2.1. Considering the effect of the shape of the service area

In this thesis, the service area is modeled as a rectangle with dimensions $W$ (km) and $L$ (km). The rectangular service area is an approximation of similar shapes to get a closed-form solution, and this approximation is used in similar works (Barraza & Estrada, 2021; C. Daganzo, 2005; Fu, 2002). While the rectangular area should realistically represent many practical situations of low demand route comprising majorly residential areas, further research might be needed for other shape types.
6.3.2.2. Considering the effect of network-level parameters

In regions with established fixed-route transit network, SFT terminal stations within the service area act as links to larger fixed-route transit systems and may or may not function as transfer points to other routes. Moreover, transfer stations connecting high-frequency routes may not substantially impact the total waiting times of passengers, given that headways along low-demand routes are generally much longer. However, in regions with limited or no fixed-route transit network, SFT replaces the entire fixed-route network during off-peak or daily service hours, necessitating that analytical cost models for SFT incorporate considerations of fleet management, routing, interlining, and coordination or consolidation with paratransit service which will be explored in future studies.

6.3.2.3. Considering the advancement in vehicle technologies

The third research module of this thesis exclusively examines the cost efficiency of widely adopted opportunity charging based battery-electric vehicle technology. However, we are also keen on assessing the cost efficiency of operating hybrid and fuel-cell electric vehicles, considering potential infrastructure upgrades or modifications. Further, this thesis can further be extended to study the effect of automation on the total cost of the system since unit operator costs are expected to decrease significantly as driver wages account for 40-80% of operating costs in most cases and automation eliminates the need for drivers.

6.3.2.4. Considering the partial contracting delivery model

In the first research module, two extreme levels of contracting are assumed: fully contracted or fully in-house (i.e., no contracting), allowing us to examine the economies of scale that can be achieved by partially outsourcing a portion of services and effectively reducing the size of in-house delivery.

6.3.2.5. Considering the different service delivery patterns

This thesis examined only two types of flexible transit operating policies, namely "request stops" in the first research module and "route deviation" in the second and third research modules. There is potential for future research to examine the application of the study model to all six key
categories of SFT operating policies: demand-responsible connectors, zone routes, point deviations, route deviations, flexible route segments, and request stops.
References


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Appendix

Appendix A. Properties of vehicle stopping process following a Poisson binomial distribution

Let \( Y_i, i = 1, 2, 3, \ldots, n \) be the random indicator representing the vehicle stopping event, where ‘\( n \)’ is the number of stops along the route. Each stopping event is independent and follows

\[ Y_i \sim \text{Bernoulli} \left( S_i \right), i = 1, 2, 3, \ldots, n \]  

Eq. A 1

Where \( S_i = \text{Pr}(Y_i = 1) \) represents the success probability of \( Y_i \) and all \( S_i \)’s are different. The Poisson binomial random variable \( X \) is defined as the sum of independent and non-identical random indicators (i.e. \( X = \sum_{i=1}^{n} Y_i; X = \{0, 1, 2, 3, \ldots, n\} \)) represents the number of stoppings. Let \( f_k = \text{Pr}(X = k), k = 0, 1, 2, 3, \ldots, n \) be the probability mass function (pmf) for the Poisson binomial random variable \( X \); representing the probability of \( k \) stoppings from \( n \) number of stops. The pmf \( f_k \) can be expressed as:

\[ f_k = \sum_{A \in \mathcal{F}_k} \left( \prod_{i \in A} p_i \right) \left( \prod_{i \in A^C} (1 - p_i) \right) \]  

Eq. A 2

where \( \mathcal{F}_k \) is the set of all subsets of \( k \) integers that can be selected from \( \{1, 2, 3, \ldots, n\} \), and \( A^C \) represent the complement of set A (i.e., \( A^C = \{1, 2, 3, \ldots, n\} \setminus A \)). If \( p_i = p \ \forall \ i \), Eq. (A2) reduces to pmf of binomial distribution. The average number of stoppings for the described Poisson binomial process is:

\[ E(k) = \sum_{i=1}^{n} p_i \]  

Eq. A 3

Table A 1 Model parameter values for Chapter 4

<table>
<thead>
<tr>
<th>Symbols</th>
<th>Values</th>
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<td></td>
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<tr>
<td>( L )</td>
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</tr>
<tr>
<td>( N )</td>
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<td>-</td>
</tr>
<tr>
<td>( H )</td>
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<td>Hours/vehicle</td>
</tr>
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<td>Value</td>
<td>Description</td>
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<td>-------</td>
<td>-------------</td>
</tr>
<tr>
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<td>21970[^1]</td>
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<tr>
<td>$H_S$</td>
<td>5710[^1]</td>
<td>Hours</td>
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### FRBT, IHT and COT operating cost parameters

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<td>$</td>
</tr>
<tr>
<td>$c_2$</td>
<td>0.2[^2]</td>
<td>$/vehicle-mile</td>
</tr>
<tr>
<td>$c_3$</td>
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<td>$/vehicle-hour</td>
</tr>
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</tr>
<tr>
<td>$c_5$</td>
<td>25000[^2]</td>
<td>$</td>
</tr>
<tr>
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<td>0.2[^2]</td>
<td>$/vehicle-mile</td>
</tr>
<tr>
<td>$c_7$</td>
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<td>$/passenger</td>
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<td>$/kilometre</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>36[^3]</td>
<td>$/hour</td>
</tr>
<tr>
<td>$\bar{m}$</td>
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<td>passengers per vehicle</td>
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<td>minutes</td>
</tr>
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<td>$h_0$</td>
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<td>minutes</td>
</tr>
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<td>$l$</td>
<td>6[^5]</td>
<td>kilometres</td>
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<td>$F$</td>
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<td>$W_{max}$</td>
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<td>minutes</td>
</tr>
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</tr>
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<td>km/hr</td>
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<tr>
<td>$V_R^h$</td>
<td>35[^2]</td>
<td>km/hr</td>
</tr>
<tr>
<td>$t_{loss}$</td>
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<td>seconds</td>
</tr>
<tr>
<td>$t_{loss}^h$</td>
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<td>seconds</td>
</tr>
<tr>
<td>$T$</td>
<td>5[^2]</td>
<td>seconds</td>
</tr>
<tr>
<td>$t^h$</td>
<td>5[^2]</td>
<td>seconds</td>
</tr>
<tr>
<td>$\gamma$</td>
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<td>-</td>
</tr>
<tr>
<td>$\gamma^h$</td>
<td>0.2[^2]</td>
<td>-</td>
</tr>
</tbody>
</table>

### Sources:

[^1]: City of Regina transit ridership data
[^2]: Assumed based on study reports published by government and non-government organizations
Appendix B. Pseudocode for NSGA-II and SMPSO

Algorithm 1
Input: \( N_p, G, p_c, p_m \)
1: Initialize the population \( P \) with \( N_p \) random individuals
2: Evaluate fitness of \( P \)
3: for \( g = 1 \) to \( G \)
4: perform binary tournament selection
5: for \( i = 1 \) to \( N_p/2 \)
6: two parents are selected at random from the population \( P \)
7: if \( r < p_c \)
8: generate two offspring using SBX-crossover
9: bound the offspring
10: else
11: duplicate the selected parents as offspring
12: end
13: end
14: for \( i = 1 \) to \( N_p \)
15: if \( r < p_m \)
16: perform polynomial mutation of \( p_m \) offspring
17: bound the mutated offspring
18: else
19: duplicate the selected offspring
20: end
21: end
22: calculate the objective values of offspring
23: evaluate the fitness of combined set of population and offspring
24: select \( N_p \) best individuals as \( P \) for next generation
25: end
Output: Pareto-optimal set \( F^* \) and feasible solution set \( F \)

Algorithm 2
Input: \( N_s, G, p_m \)
1: Initialize the swarm population \( S \) with \( N_s \) random swarms
2: Evaluate fitness of \( S \)
3: Initialize the external archive \( A \) with the non-dominated solutions of the swarm
4: for \( g = 1 \) to \( G \)
5: for \( s = 1 \) to \( N_s \)
6: Use constrained binary tournament to select a leader solution from the external archive \( A \) based on crowding distance
7: Compute the velocity of \( s \)
8: Constrain the velocity of \( s \)
9: Update the position of \( s \) according to the velocity
10: Apply the polynomial mutation
11: Evaluate the fitness of the new particle
12: end
13: Update the particle \( s \)'s memory and the external archive \( A \)
14: if size of the external archive \( A \) exceeds the limit
15: remove particle from \( A \) with lowest crowding distance
16: end
17: end
Output: Return the set of feasible non-dominated solutions in external archive \( A \)
### Table B.1 Case study parameter setting for Chapter 5

<table>
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<td>km</td>
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<tr>
<td>$W$</td>
<td>km</td>
<td>$1^{[2]}$</td>
</tr>
<tr>
<td>$Q_R$</td>
<td>pass/hr</td>
<td>$9^{[1]}$</td>
</tr>
<tr>
<td>$Q_S$</td>
<td>pass/hr</td>
<td>$6^{[2]}$</td>
</tr>
<tr>
<td>$C$</td>
<td>pass/veh</td>
<td>$15^{[2]}$</td>
</tr>
<tr>
<td>$V_R$</td>
<td>km/hr</td>
<td>$35^{[2]}$</td>
</tr>
<tr>
<td>$V_a$</td>
<td>km/hr</td>
<td>$4^{[2]}$</td>
</tr>
<tr>
<td>$t_{ad}$</td>
<td>hr</td>
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<td>$t$</td>
<td>hr</td>
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</tr>
<tr>
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</tr>
<tr>
<td>$h_p$</td>
<td>hr</td>
<td>1.5$^{[2]}$</td>
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<td>$\mu$</td>
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</tr>
<tr>
<td>$\alpha$</td>
<td>-</td>
<td>0.5</td>
</tr>
<tr>
<td>$\beta$</td>
<td>-</td>
<td>0.5</td>
</tr>
<tr>
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<td>0.25</td>
</tr>
<tr>
<td>$\eta_{R2}$</td>
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<td>0.25</td>
</tr>
<tr>
<td>$\eta_{R3}$</td>
<td>-</td>
<td>0.25</td>
</tr>
<tr>
<td>$\eta_{R4}$</td>
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<tr>
<td><strong>NSGA-II and SMPSO Parameter setting</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N_P$</td>
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<td>100, 500, 1000</td>
</tr>
<tr>
<td>$N_S$</td>
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<td>$G$</td>
<td>-</td>
<td>10, 50, 100, 500, 1000, 1500</td>
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<tr>
<td>$\tau_m$</td>
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<td>20$^{[3]}$</td>
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</table>

**Sources:**

1. City of Regina transit ridership data
2. Assumed based on study reports published by government and non-government organizations or literature
Appendix C. Estimation of $E_{\text{Prop}}$ using simplified velocity profile

Using the simplified velocity profile in Figure C1, $E_{\text{Prop}}$ can be estimated using known parameters and the trip variables from the real-world data set and consists of three phases: acceleration, coasting, and deceleration as given in Eq. (C1) - (C3) adopted from Gallet et al. (2018).

$$E_{\text{Prop}}^{\text{acceleration}} = l_0 \left( M g f_r \cos(\theta) + M g \sin(\theta) + 0.5 \rho C_d A_f a l_0 + \delta M a \right)$$ \hspace{1cm} \text{Eq. C 1}

$$E_{\text{Prop}}^{\text{deceleration}} = l_0 \left( M g f_r \cos(\theta) + M g \sin(\theta) - 0.5 \rho C_d A_f a l_0 + \delta M (-a) \right)$$ \hspace{1cm} \text{Eq. C 2}

$$E_{\text{Prop}}^{\text{coasting}} = l_0 \left( M g f_r \cos(\theta) + M g \sin(\theta) - 0.5 \rho C_d A_f V_R^2 \right)$$ \hspace{1cm} \text{Eq. C 3}

$E_{\text{Prop}}$ between two consecutive stops is estimated based on the following four assumptions:

(i) the grading is insignificant and can be neglected ($\theta = 0$)
(ii) the flow velocity of air around the vehicle is equal to the vehicle speed; thus, wind speed is neglected,
(iii) for a low demand route, the distance between two stoppings is long enough for the coasting speed $V_R$ to be reached
(iv) the rate of acceleration denoted by ‘a’ is equal to the rate of deceleration but with a negative sign.

On applying assumptions on Eq. (C2) - (C4), the final expression is derived for $E_{\text{Prop}}$ in Eq. (C4) given as the sum of energy consumption during acceleration, coasting, and deceleration.

$$E_{\text{Prop}} = 2l_0 (M g f_r) + (l - 2l_0) \left( M g f_r + 0.5 \rho C_d A_f V_R^2 \right)$$ \hspace{1cm} \text{Eq. C 4}
Figure C.1 Simplified speed profile
### Table C 1 Parameters used in this case study depending on propulsion type, charging scheme, and vehicle type.

<table>
<thead>
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<th>BEV</th>
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<td>$C$</td>
<td>pass</td>
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<tr>
<td>$\sum_{x \in X} Z_x e_x$</td>
<td>$$/veh-km$$</td>
<td>0.105$[1]$</td>
<td>0.225$[1]$</td>
</tr>
<tr>
<td>$E_{P_{nominal}}$</td>
<td>kWh</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

[1] Barraza and Estrada (2021) - Unit costs are estimated in CAD assuming linear relationship between values and vehicle capacity
[2] Estrada et al. (2022)
[3] Assumed based on multiple studies
[4] Average parameter values adopted from Abdelaty and Mohamed (2021) and values for vehicle type estimated using linear relationship

### Table C 2 Constant parameters used in the case study.

<table>
<thead>
<tr>
<th>L</th>
<th>W</th>
<th>$V_R$</th>
<th>$V_a$</th>
<th>$P$</th>
<th>$a$</th>
<th>$t_d$</th>
<th>$\mu$</th>
<th>$a$, $\beta$</th>
<th>$L_D$</th>
<th>$g$</th>
<th>$m_{pass}$</th>
<th>$f_t$</th>
<th>$C_d$</th>
<th>$\rho$</th>
<th>$SOC_{mi}$</th>
<th>$C_5$</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>km</td>
<td>km</td>
<td>km/hr</td>
<td>km/hr</td>
<td>pass/hr</td>
<td>m/s²</td>
<td>sec</td>
<td>-</td>
<td>-</td>
<td>km</td>
<td>m/s²</td>
<td>kg</td>
<td>-</td>
<td>-</td>
<td>kg/m³</td>
<td>-</td>
<td>-</td>
<td>hr</td>
</tr>
</tbody>
</table>

[1] City of Regina transit ridership data
[2] Assumed based on research articles and study reports published by government and non-government organizations
Appendix D. List of parameters and symbols

Table D 1 List of symbols used in analytical models

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{\text{FRBT}}$</td>
<td>Fixed-route bus transit operator cost</td>
<td>$/\text{pass}$</td>
</tr>
<tr>
<td>$C_{\text{COT}}$</td>
<td>Contracted-Out Taxi based SFT operator cost</td>
<td>$/\text{pass}$</td>
</tr>
<tr>
<td>$C_{\text{IHT}}$</td>
<td>In-House Transit based SFT operator cost</td>
<td>$/\text{pass}$</td>
</tr>
<tr>
<td>$C_{\text{OC}}$</td>
<td>SFT operator cost</td>
<td>$$/\text{hr}$</td>
</tr>
<tr>
<td>$C_{\text{UC}}$</td>
<td>SFT user cost</td>
<td>$$/\text{hr}$</td>
</tr>
<tr>
<td>$C_{A}$</td>
<td>passenger cost of access/egress time to/from stops</td>
<td>$$/\text{hr}$</td>
</tr>
<tr>
<td>$C_{W}$</td>
<td>passenger cost of waiting time at stops</td>
<td>$$/\text{hr}$</td>
</tr>
<tr>
<td>$C_{I}$</td>
<td>passenger cost of in-vehicle time</td>
<td>$$/\text{hr}$</td>
</tr>
<tr>
<td>$C_{\text{SB}}$</td>
<td>SFT service benefit</td>
<td>$$/\text{hr}$</td>
</tr>
<tr>
<td>$C_{\text{ENV}}$</td>
<td>SFT environment cost</td>
<td>$$/\text{hr}$</td>
</tr>
<tr>
<td>$C_{\text{TC}}$</td>
<td>SFT total cost including operator, user, and environment cost</td>
<td>$$/\text{hr}$</td>
</tr>
<tr>
<td>$c_{0} \text{ or } c_{4}$</td>
<td>sum of fixed costs associated with fleet size, VMT, and VHT for FRBT and IHT, respectively in Chapter 3</td>
<td>$$/\text{pass}$</td>
</tr>
<tr>
<td>$c_{1} \text{ or } c_{5}$</td>
<td>unit cost of vehicle acquisition for FRBT and IHT, respectively in Chapter 3</td>
<td>$$/\text{veh}$</td>
</tr>
<tr>
<td>$c_{2} \text{ or } c_{6}$</td>
<td>unit cost of operation per vehicle-mile for FRBT and IHT, respectively in Chapter 3</td>
<td>$$/\text{veh-km}$</td>
</tr>
<tr>
<td>$c_{3} \text{ or } c_{7}$</td>
<td>unit cost of operation per vehicle-hour for FRBT and IHT, respectively in Chapter 3</td>
<td>$$/\text{veh-hr}$</td>
</tr>
<tr>
<td>$c_{8}$</td>
<td>unit cost of operating a integrated SFT transit unit including fleet acquisition cost, and distance and time-based cost in Chapter 4</td>
<td>$$/\text{veh-hr}$</td>
</tr>
<tr>
<td>$c_{9} \text{ or } c_{15}$</td>
<td>passenger value of time in Chapter 4 and Chapter 5, respectively</td>
<td>$$/\text{pass-hr}$</td>
</tr>
<tr>
<td>$c_{10}$</td>
<td>average operating cost of providing paratransit service in Chapter 4</td>
<td>$$/\text{pass}$</td>
</tr>
<tr>
<td>$c_{11}$</td>
<td>unit temporal cost in Chapter 5</td>
<td>$$/\text{veh-hr}$</td>
</tr>
<tr>
<td>$c_{12}$</td>
<td>unit distance cost in Chapter 5</td>
<td>$$/\text{veh-km}$</td>
</tr>
<tr>
<td>$c_{13}$</td>
<td>unit temporal battery cost in Chapter 5</td>
<td>$$/\text{kWh-hr}$</td>
</tr>
<tr>
<td>$c_{14}$</td>
<td>unit charging facility cost or refueling station cost in Chapter 5</td>
<td>$$/\text{charger-hr}$</td>
</tr>
<tr>
<td>$\beta_{0}$</td>
<td>fixed cost to hire a taxi in Chapter 5</td>
<td>$\text{charger-hr}$</td>
</tr>
<tr>
<td>$\beta_{1}$</td>
<td>operating cost per distance travelled in Chapter 5</td>
<td>$$/\text{km}$</td>
</tr>
<tr>
<td>$\beta_{2}$</td>
<td>operating cost per delay time experienced in Chapter 5</td>
<td>$$/\text{hr}$</td>
</tr>
<tr>
<td>$L$</td>
<td>length of the study route</td>
<td>km</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Unit</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
<td>--------------</td>
</tr>
<tr>
<td>$W$</td>
<td>width of the study route</td>
<td>km</td>
</tr>
<tr>
<td>$n$</td>
<td>number of stops</td>
<td>stops</td>
</tr>
<tr>
<td>$h$</td>
<td>vehicle headway</td>
<td>hr</td>
</tr>
<tr>
<td>$H$</td>
<td>service hours per vehicle along study route</td>
<td>hr/veh</td>
</tr>
<tr>
<td>$D$</td>
<td>annual distance travelled by vehicles servicing the study route</td>
<td>km</td>
</tr>
<tr>
<td>$P$</td>
<td>total annual ridership along the route</td>
<td>pass</td>
</tr>
<tr>
<td>$H_S$</td>
<td>total annual service hours of all vehicles along study route</td>
<td>hr</td>
</tr>
<tr>
<td>$p$</td>
<td>average hourly passenger demand</td>
<td>pass/hr</td>
</tr>
<tr>
<td>$E(s)$</td>
<td>expected number of stoppings along the route</td>
<td>stoppings/veh</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>demand arrival rate at any stop location along the route</td>
<td>pass/stop</td>
</tr>
<tr>
<td>$N$ or $M$</td>
<td>fleet size</td>
<td>veh</td>
</tr>
<tr>
<td>$T_c$ or $T_R$</td>
<td>total cycle time to complete one roundtrip</td>
<td>hr</td>
</tr>
<tr>
<td>$T_o$ or $T_v$</td>
<td>operating time for one-way travel</td>
<td>hr</td>
</tr>
<tr>
<td>$T_t$ or $T_l$</td>
<td>terminal time for layover at end stations</td>
<td>hr</td>
</tr>
<tr>
<td>$T_d$</td>
<td>dead-heading time</td>
<td>hr</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>ratio of terminal time tp operating time</td>
<td>-</td>
</tr>
<tr>
<td>$V_R$</td>
<td>average riding speed of transit vehicle</td>
<td>km/hr</td>
</tr>
<tr>
<td>$t_{loss}$ or $t_{ad}$</td>
<td>time spent in acceleration and deceleration of the vehicle at each stop</td>
<td>hr</td>
</tr>
<tr>
<td>$a$</td>
<td>vehicle acceleration rate</td>
<td>km/hr$^2$</td>
</tr>
<tr>
<td>$b$</td>
<td>vehicle deceleration rate</td>
<td>km/hr$^2$</td>
</tr>
<tr>
<td>$t$ or $t_d$</td>
<td>time spent in boarding and alighting per passenger (hr)</td>
<td>hr</td>
</tr>
<tr>
<td>$V_c$</td>
<td>average cycle speed in completing a round trip</td>
<td>km/hr</td>
</tr>
<tr>
<td>$V_a$</td>
<td>passenger walking speed</td>
<td>km/hr</td>
</tr>
<tr>
<td>$d$</td>
<td>congestion delay due to interaction with adjacent traffic</td>
<td>hr/km</td>
</tr>
<tr>
<td>$V$</td>
<td>speed of the adjacent traffic</td>
<td>km/hr</td>
</tr>
<tr>
<td>$l$</td>
<td>trip length per passenger</td>
<td>km</td>
</tr>
<tr>
<td>$W_{min}$</td>
<td>Minimum waiting time for requested ride</td>
<td>hr</td>
</tr>
<tr>
<td>$W_{max}$</td>
<td>Maximum waiting time for requested ride</td>
<td>hr</td>
</tr>
<tr>
<td>$P_c$</td>
<td>critical passenger demand to switch between competing systems</td>
<td>pass/hr</td>
</tr>
<tr>
<td>$m$</td>
<td>passenger occupancy of taxi</td>
<td>pass/veh</td>
</tr>
<tr>
<td>$\bar{c}$</td>
<td>average operating cost per passenger</td>
<td>$$/pass</td>
</tr>
<tr>
<td>$f_p$</td>
<td>proportion of total annual trips corresponding to ‘p’ hourly passenger demand value</td>
<td>-</td>
</tr>
<tr>
<td><strong>MC</strong></td>
<td>marginal cost of providing a service pattern/delivery model</td>
<td>$/pass</td>
</tr>
<tr>
<td><strong>Q_G</strong></td>
<td>existing FRBT demand (Type G)</td>
<td>pass/hr</td>
</tr>
<tr>
<td><strong>Q_S</strong></td>
<td>existing paratransit demand (Type S)</td>
<td>pass/hr</td>
</tr>
<tr>
<td><strong>( \eta )</strong></td>
<td>proportion of possible types of requests placed by Type G and S passengers</td>
<td>-</td>
</tr>
<tr>
<td><strong>C</strong></td>
<td>Capacity of vehicle</td>
<td>pass/veh</td>
</tr>
<tr>
<td><strong>h_p</strong></td>
<td>policy headway set by transit agency to provide minimum desired level of service</td>
<td>hr</td>
</tr>
<tr>
<td>**( \Delta t ) or <strong>( T_S )</strong></td>
<td>slack time required to serve Type S passengers</td>
<td>hr</td>
</tr>
<tr>
<td>**( \delta ) or <strong>( \varphi )</strong></td>
<td>average time to serve one paratransit passenger in SFT</td>
<td>hr</td>
</tr>
<tr>
<td><strong>( \alpha, \beta )</strong></td>
<td>proportion of planning passengers and the proportion of planning passengers with fixed arrival times, respectively</td>
<td>-</td>
</tr>
<tr>
<td><strong>D_P</strong></td>
<td>permitted deviation from a fixed route</td>
<td>km</td>
</tr>
<tr>
<td><strong>L_D</strong></td>
<td>distance between fast charging stations and last stop/terminal</td>
<td>km</td>
</tr>
<tr>
<td><strong>L_F</strong></td>
<td>vehicle kilometres travelled by fleet per unit time</td>
<td>veh-km/hr</td>
</tr>
<tr>
<td><strong>E_b</strong></td>
<td>battery capacity per vehicle</td>
<td>kWh/veh</td>
</tr>
<tr>
<td><strong>N_st</strong></td>
<td>number of charging or refuelling stations</td>
<td>-</td>
</tr>
<tr>
<td><strong>T_{Chg}</strong></td>
<td>required charging time</td>
<td>hr</td>
</tr>
<tr>
<td><strong>T_A</strong></td>
<td>available charging time</td>
<td>hr</td>
</tr>
<tr>
<td><strong>N_{RD}</strong></td>
<td>number of route deviation requests</td>
<td>-</td>
</tr>
<tr>
<td><strong>E_C</strong></td>
<td>energy consumption factor</td>
<td>kWh/veh-km</td>
</tr>
<tr>
<td><strong>B_{Chg}</strong></td>
<td>charging speed</td>
<td>kWh/h</td>
</tr>
<tr>
<td><strong>S</strong></td>
<td>number of trips served before recharging operation</td>
<td>-</td>
</tr>
<tr>
<td><strong>E_b</strong></td>
<td>battery capacity of the vehicle</td>
<td>kWh</td>
</tr>
<tr>
<td><strong>SOC_{min}</strong></td>
<td>the minimum state of charge of the batteries maintained</td>
<td>-</td>
</tr>
<tr>
<td><strong>P_{aux}</strong></td>
<td>constant auxiliary power required within the vehicle over the driving duration between two stoppings</td>
<td>kW</td>
</tr>
<tr>
<td><strong>E_{b, nominal}</strong></td>
<td>nominal capacity of the battery</td>
<td>kWh</td>
</tr>
<tr>
<td><strong>Z_x</strong></td>
<td>mass of greenhouse gas ( x ) produced per kilometer run</td>
<td>gm/veh-km</td>
</tr>
<tr>
<td><strong>( \varepsilon_x )</strong></td>
<td>proxy parameter to monetize the effect of producing one unit of greenhouse gas</td>
<td>$/gm</td>
</tr>
</tbody>
</table>
Appendix E. Framework for application

Figure E1 Framework for model application derived in Chapter 3
Figure E2 Framework for model application derived in Chapter 4
Figure E3 Framework for model application derived in Chapter 5