FAULT ISOLATION IN LINEAR ELECTRONIC CIRCUITS BY UTILIZING THE FREQUENCY RESPONSE AND THE MOEBIUS TRANSFORMATION

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ABSTRACT

FAULT ISOLATION IN LINEAR ELECTRONIC CIRCUITS
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THE MOEBIUS TRANSFORMATION

A method of isolating (diagnosing) a single fault in a conventional linear electronic circuit, where only the input and output terminals are available for test measurements, is described.

A faulty circuit element is identified by comparing the measured magnitude and phase response of the faulty circuit, at a selected set of test frequencies, with a precomputed set of curves corresponding to all possible single fault conditions. These curves are shown to be arcs of circles, where the straight line is considered to be the special case of a circle with an infinite radius. Although the state equation approach is used in calculating the n-port parameters required for the precomputed curves, other methods such as topological formulas, ECAP(Electronic Circuit Analysis Program, supplied by IBM), or laboratory measurements could also be utilized.

The success of the Moebius transformation method, in identifying faults, was demonstrated by laboratory measurements and digital computer simulations. The results of fault diagnosis in several test circuits are presented.

This new approach to fault diagnosis provides a visual (graphical) identication of the faulty component and a precise evaluation of the faulty component value.

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INTRODUCTION

Motivation for the development of new fault isolation techniques

Prior to the introduction of a wide variety of complex electronic equipment, a technician could acquire adequate experience with the circuitry of the equipment in order to repair it effectively. However, with todays rapid change in circuitry, it is difficult for technicians to gain this experience and other methods must be introduced as an aid to the technician.

Another difficulty experienced in the fault diagnosis of a circuit is due to high packaging density of modern equipment. The procedure of probing the signal path has become difficult even in the discrete component circuitry. In an integrated circuit, however, once the device has failed, it is impossible to probe the signal path unless test points have been provided. Test points are generally undesirable since they cause a deterioration in the high frequency performance and, consequently, are usually not provided. Although an integrated circuit cannot be repaired, the manufacturer may want to identify the fault in order to modify the manufacturing process.

Statement of the problem and inherent assumptions

The problem that will be considered is the one of identifying a single fault in a linear, lumped, time-invariant network with a known topology and a set of known nominal component values. Identification of a fault requires that the faulty element and the value of the element be determined. If test points are required for unique fault identification, only a minimum number will be permitted; if possible, only the normal input and output terminals of the network will be used for test purposes.

The solution of the problem is based on the following assumptions:

(i) The fault is not intermittant, or the fault persists throughout the test measurement period.

(ii) Only those catastrophic failures, having a non-zero response at some terminal, can be identified uniquely, except in the case where a zero response at all terminals is unique to one fault.

(iii) Mutual inductance and the transformer turns ratio cannot be isolated as faults, since they violate the single element fault assumption. However, circuits, with mutual inductance and/or transformers with turns ratios, can be diagnosed if these parameters are excluded as possible faults.

If the value of the faulty parameter is not required, assumption (i) above can be relaxed and restated as:

(iv) The fault persists long enough to permit one significant test measurement.

A violation of assumption (ii), above, will be observable during the initial analysis of the circuit, and test points could be inserted into the circuit to satisfy the assumption.

Review of previous work

Seshu and Waxman[6] have given a solution to the problem by recognizing that the transfer function magnitude will change at some frequency if a circuit element changes. In terms of the Bode plot, this will be observable by a change in the location of the break frequencies, or the gain constant, or both. Thus by precomputing the gain between the break frequencies for all possible fault conditions, a fault dictionary of gain signatures is obtained. The magnitude response of the faulty circuit is then measured and compared to the list in the fault dictionary, and the faulty element is identified.

Since the number of transfer functions that must be evaluated for a reasonably complete fault dictionary is large, the symbolic transfer function is derived by topological formulas, and then, the fault signatures are calculated simply by substituting the appropriate values. This technique is necessary to shorten the computation time of the digital computer.

The method of Seshu and Waxman was investigated by Maenpaa, Stehman, and Stahl [2] and apparently proved to be quite successful. However, some of the problems encountered were: (i) the computation time and storage space required in the digital computer becomes quite large as the network size increases; (ii) no method is provided for interpolating between fault values, if a measured fault happens to fall between values listed in the dictionary.

Neu [4] has developed a method whereby a quantity $k(x_i)$ remains invariant from one n-port parameter to another if x_i is the circuit element that has changed. If x_i is another circuit element (i.e. one which could not produce the observed changes in the n-port parameters), $k(x_i)$ will be different for different n-port parameters. The quantity $k(x_i)$ is defined as:

$$k(x_i) = \frac{\frac{\partial T(x_i)}{\partial x_i}}{\Delta T}$$

where $k(x_i)$ = the "constant" of Neu's method

x_i = circuit element i

T = the n-port parameter

ΔT = the amount T has changed from its nominal value

To isolate a fault, $\frac{\partial T(x_i)}{\partial x_i}$ must be computed for all circuit elements and for two or more n-port parameters. The change in the corresponding n-port parameters due to the fault is then measured and $k(x_i)$ is computed. That circuit element x_i, for which all the $k(x_i)$ are constant, is then the faulty component.

The amount Δx_i, that the faulty component x_i has changed from its nominal value, can then be calculated from the equation:

$$\Delta x_i = \frac{Q+x_i}{k(Q+x_i)-1}$$

where Q is either the Thevenin immittance or transmittance at x_i . Neu only discusses resistive networks, although the procedure could probably be extended to include capacitors and inductors by using phasor analysis. The evaluation and measurement of several n-port parameters is still required, however, and a technique based on a single n-port parameter would probably be preferable.

Moroz[3] has shown that the coefficient of each power of s, the Laplace transform variable, of a transfer or driving point function can be written as:

$$k = \frac{ax_i + b}{cx_i + d} \qquad (1.1)$$

where: k is any power of s of the transfer function (or driving point function),

x_i is any network parameter other than mutual inductance or a transformer turns ratio, and

a, b, c, d are real constants independent of x_i.

The constants a, b, c, and d must be computed for each power of s, for each circuit component. If the symbolic transfer function is used, the constants can be obtained directly by substitution. If a numerical circuit analysis technique is used, a, b, c, and d can be calculated by solving equation (1.1) for four values of k corresponding to four values of x_i. Three solutions are adequate in obtaining the ratio of three of the constants to the fourth one. The array of all coefficients a, b, c, and d is known as the coefficient signature.

The coefficients of the transfer function of the faulty network must be computed from test measurements and a curve fitting procedure. The measured coefficients are then used in solving equation (1.1) for x_i. This produces as many solutions for each x_i as there are independent coefficients in the transfer function polynominal ratio. If x_i is the faulty component, which caused the nominal transfer function to change to the new value, the solutions for x_i will all be constant and equal to the new value of x_i. If x_i is not the faulty component, the solutions for x_i will vary. As was suggested by Moroz, the solutions of the faulty x_i are not exactly constant, due to errors in the measurement procedure and in the curve fitting method. The x_i, which exhibits the least variance in the solution, is then chosen as the faulty component.

Significance of the new technique

One of the problems associated with the fault dictionary isolation technique is that of interpolating between the listed entries(gain signatures) in the dictionary when a fault falls between those listed. Since each element could fail with any value between zero and infinity, it is impossible to list all possible faults, and interpolation between the discrete steps of listed faults will often be required. The new technique, which is developed in the next section, permits a direct calculation of the value of the faulty component; the gain signature need not be evaluated and, therefore, the interpolation problem associated with the fault signature method is avoided.

It is shown that the use of the Moebius transformation method substantially reduces the number of n-port parameters that must be evaluated. Consequently, it becomes practical to use numerical evaluation of the n-port parameters by the state equation technique, rather than by the topological approach, without an increase in computer computation time. Since the state equation method requires much less storage than the topological formula method, the new technique also eliminates the large storage problem of the fault dictionary diagnosis.

If the network which is being investigated has two or more ports, more than one n-port parameter describing its behaviour exists. A two-port, for example, could be described by:

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

where, in general, the hybrid parameters: h_{11}, h_{12}, h_{21}, h_{22}, all differ. Seshu and Waxman[6] use only one of the h-parameters, in the above case, to determine a faulty element. However, in some cases, the same frequency signature exists for several different faults. Although a fault dictionary for another of the h-parameters might identify the fault uniquely, the computation time, required to compute the h-parameter and compile the additional fault dictionary, would increase significantly. The state equation technique of circuit analysis, which is impractical to use for compiling a fault dictionary, generates all n-port parameters simultaneously. Consequently, the new technique of fault diagnosis, which is suitable for state equation circuit analysis, can be used to investigate all of the h-parameters with only a small amount of additional computation.

An outline of the diagnosis technique

It is shown that the locus of an n-port parameter of a linear electronic network, as an element varies from zero to infinity, is an arc of a circle. This holds for all components except mutual inductances and transformer turns ratios. Since a circle is uniquely determined by any three points on the circle, we must evaluate or measure the n-port parameter for three values of each circuit element at each test frequency. Although any method may be used to evaluate the n-port parameter (e.g. ECAP, topological formula, state equations or laboratory measurements), the state equation approach was chosen as the most suitable , for reasons discussed in part IV. After the n-port parameters have been evaluated for three values of each component, the coefficients of the Moebius transformation are used in calculating the loci of the n-port parameters. The loci of the n-port parameters, at the test frequencies, are then plotted on graphs with one test frequency plot per graph.

The magnitude and phase of the n-port parameter of the faulty circuit are measured at the test frequencies and plotted on the $T(x_i)$ graphs. The plots are examined and the component x_i, which corresponds to the n-port parameter arc on which the measured n-port parameter of the faulty circuit falls, is the faulty component. After the faulty element has thus been detected, the coefficients of the Moebius transformation are used to calculate the value of the faulty component.

APPLICATION OF THE MOEBIUS TRANSFORMATION TO N-PORT PARAMETERS

Some of the properties of the well known Moebius transformation[1,8] are developed in the following section. The development, which is not essential for an understanding of the fault diagnosis technique, is included for completeness and ease of reference. The results of the development are used in the fault isolation computer program which is included in Appendix A.

The Moebius transformation

The Moebius transformation, which is also known by the names, bilinear transformation, fractional linear transformation and linear substitution, is a transformation of the form:

$$w = \frac{Az+B}{Cz+D} \qquad AD-BC \neq 0 \qquad (2.1)$$

where A,B,C,D are complex numbers and z is a complex variable. The condition $AD-BC \neq 0$ is necessary, as otherwise w will reduce to a constant. In general, the transformation depends on three essential constants, namely, the ratio of any three of the constants A, B, C, D to the fourth one. If we choose three w_i corresponding to three z_i, then

$$\frac{(w-w_1)(w_2-w_3)}{(w-w_3)(w_2-w_1)} = \frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)} \qquad (2.2)$$

The R.H.S. fraction is known as the cross ratio of four numbers z, z_1, z_2, z_3 and the L.H.S. is the cross ratio of w, w_1, w_2, w_3. Thus, if we know w and three w_i corresponding to three z_i we can solve for z. To do this we note that there is a one to one correspondence between equation (2.2) and the following equation:

$$Sw = Tz$$

where $S = \begin{bmatrix} w_2-w_3 & -w_1(w_2-w_3) \\ w_2-w_1 & -w_3(w_2-w_1) \end{bmatrix}$

and $T = \begin{bmatrix} z_2-z_3 & -z_1(z_2-z_3) \\ z_2-z_1 & -z_3(z_2-z_1) \end{bmatrix}$

then $z = T^{-1}Sw$

where $T^{-1} = \begin{bmatrix} -z_3(z_2-z_1) & z_1(z_2-z_3) \\ -(z_2-z_1) & (z_2-z_3) \end{bmatrix}$

Then by the one to one correspondence:

$$z = \frac{aw + b}{cw + d} \tag{2.3}$$

where $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = T^{-1}S$

Solving (2.1) for z and equating to (2.3) we get:

$$z = \frac{Dw - B}{-Cw + A} = \frac{aw + b}{cw + d}$$

If z is real, $z = \bar{z}$, where $\bar{z}$ is the complex conjugate of z;

that is $\frac{aw + b}{cw + d} = \overline{\left(\frac{aw + b}{cw + d}\right)}$

Cross multiplying and simplifying we get:

$$(a\bar{c}-c\bar{a})|w|^2 + (a\bar{d}-c\bar{b})w + (b\bar{c}-d\bar{a})\bar{w} + (b\bar{d}-d\bar{b}) = 0 \tag{2.4}$$

Multiplying the numerator and denominator of the last term by $(\bar{a}c-\bar{c}a)$:

$$(a\bar{c}-c\bar{a})|w|^2 + (a\bar{d}-c\bar{b})w + (b\bar{c}-d\bar{a})\bar{w} + \frac{\bar{a}c(b\bar{d}-d\bar{b}) - \bar{c}a(b\bar{d}-d\bar{b})}{(\bar{a}c-\bar{c}a)} = 0$$

Adding and subtracting $b\bar{b}c\bar{c}$ and $\bar{a}a\bar{d}d$ to the numerator of the last term and regrouping terms we get:

$$(a\bar{c}-c\bar{a})|w|^2 + (a\bar{d}-c\bar{b})w + (b\bar{c}-d\bar{a})\bar{w} + \frac{(\bar{a}d-\bar{c}b)(a\bar{d}-c\bar{b})}{(\bar{a}c-\bar{c}a)} = \frac{(ad-bc)\overline{(ad-bc)}}{(\bar{a}c-\bar{c}a)}$$

Dividing by $a\bar{c}-c\bar{a}$, provided $a\bar{c}-c\bar{a} \neq 0$:

$$|w|^2 + \frac{(a\bar{d}-c\bar{b})w}{(a\bar{c}-c\bar{a})} + \frac{(\bar{a}d-\bar{c}b)\bar{w}}{(\bar{a}c-\bar{c}a)} + \frac{(\bar{a}d-\bar{c}b)(a\bar{d}-c\bar{b})}{(\bar{a}c-\bar{c}a)(a\bar{c}-c\bar{a})} = \frac{(ad-bc)\overline{(ad-bc)}}{(\bar{a}c-\bar{c}a)(a\bar{c}-c\bar{a})}$$

Therefore:

$$\left(w + \frac{(\bar{a}d-\bar{c}b)}{(\bar{a}c-\bar{c}a)}\right)\left(\bar{w} + \frac{(a\bar{d}-c\bar{b})}{(a\bar{c}-c\bar{a})}\right) = \frac{(ad-bc)\overline{(ad-bc)}}{(\bar{a}c-\bar{c}a)(a\bar{c}-c\bar{a})}$$

But since $(w+m)(\overline{w+m}) = (w+m)(\overline{w+m}) = |w+m|^2$

we have:

$$\left|w + \frac{(\bar{a}d-\bar{c}b)}{(\bar{a}c-\bar{c}a)}\right|^2 = \left|\frac{ad-bc}{\bar{a}c-\bar{c}a}\right|^2$$

After taking the square root of both sides we note that we have the equation of a circle:

$$\left|w + \frac{(\bar{a}d-\bar{c}b)}{(\bar{a}c-\bar{c}a)}\right| = \left|\frac{ad-bc}{\bar{a}c-\bar{c}a}\right| \qquad (2.5)$$

with centre: $\frac{(\bar{a}d-\bar{c}b)}{(\bar{a}c-\bar{c}a)}$

and radius: $\frac{(ad-bc)}{(\bar{a}c-\bar{c}a)}$

However, if $\bar{a}c-\bar{c}a = 0$, (2.6)

equation (2.4) becomes:

$$(a\bar{d}-c\bar{b})w + (b\bar{c}-d\bar{a})\bar{w} + (b\bar{d}-d\bar{b}) = 0, \qquad (2.7)$$

which is a straight line in w.

The initial assumption that $ad-bc \neq 0$ implies:

(i) $\overline{ad} \neq \overline{bc}$

(ii) a and c are not both zero

(iii) $\bar{a}$ and $\bar{c}$ are not both zero

(iv) d and b are not both zero

(v) $\bar{d}$ and $\bar{b}$ are not both zero

Assume that $a\bar{d}-c\bar{b} = 0$ (2.8)

Then, since $\bar{d}$ and $\bar{b}$ are not both equal to zero we can write:

$$a = \frac{c\bar{b}}{\bar{d}} \tag{2.9}$$

or

$$c = \frac{a\bar{d}}{\bar{b}} \tag{2.10}$$

Because of points (ii) and (iii) above, (2.6) can be rewritten as:

$$a = \frac{\bar{a}c}{\bar{c}} \tag{2.11}$$

or

$$c = \frac{a\bar{c}}{\bar{a}} \tag{2.12}$$

A substitution of (2.11) or (2.12) into (2.9) or (2.10) depending which equations exist, produces equations of the form:

$$\frac{\bar{a}}{\bar{c}} = \frac{\bar{b}}{\bar{d}} \tag{2.13}$$

or

$$\frac{\bar{a}\bar{d}}{\bar{c}\bar{b}} = 1. \tag{2.14}$$

But since the initial assumption that $ad \neq bc$ implies that $\bar{a}\bar{d} \neq \bar{b}\bar{c}$, equations (2.13) and (2.14) are invalid, and therefore the assumption (2.8) that $a\bar{d}-c\bar{b} = 0$ is incorrect.

Equation (2.7) can then be solved for the real and imaginary axis intercepts of w, written as w_R and w_I respectively:

$$w_R = \frac{-\mathrm{Imag}(b\bar{d})}{\mathrm{Imag}(u)} \qquad \text{where } u = a\bar{d}-c\bar{b} \neq 0 \tag{2.15}$$

$$w_I = \frac{-\mathrm{Imag}(b\bar{d})}{\mathrm{Real}(u)} \qquad \text{where } u = a\bar{d}-c\bar{b} \neq 0 \tag{2.16}$$

Application to n-port parameters

Many authors [3,5,7] have used the Moebius representation of the n-port parameters of linear active networks:

$$T = \frac{Ax_i + B}{Cx_i + D} \qquad (2.17)$$

where T is the n-port parameter as a function of the Laplace transformation variable s,

x_i is any circuit element (excluding mutual inductance and the turns ratio of transformers) value or the multiplier of a dependent source, and A, B, C, D. are polynominals in s and any of the network components other than x_i.

After the well known substitution of $s = j\omega$, A,B,C,D and T become complex numbers and x_i is a real number. It is noted that equation (2.17) is equation (2.1) with T and x_i corresponding to w and z respectively.

Since the circuit parameter, x_i, can only have positive values (except, perhaps, in certain, special cases such as negative resistance), only the portion of the locus of T corresponding to values of x_i between zero and infinity, is of interest. Thus, from equation (2.17) we get:

$$T(0) = B/D$$

$$T(\infty) = A/C$$

where $T(0)$ is the limit of $T(x_i)$ as x_i approaches zero and $T(\infty)$ is the limit of $T(x_i)$ as x_i approaches infinity.

Summarizing the procedure used in determining the circles (the straight lines are considered as being circles with an infinite radius and a centre at infinity):

(i) At each test frequency some method is used to obtain three values T_1, T_2, T_3 of an n-port parameter corresponding to the three values x_1, x_2, x_3 of the circuit parameter x_i.

(ii) The coefficients a, b, c, and d for x_i are computed from:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -x_3(x_2-x_1) & x_1(x_2-x_3) \\ -(x_2-x_1) & (x_2-x_3) \end{bmatrix} \begin{bmatrix} T_2-T_3 & -T_1(T_2-T_3) \\ T_2-T_3 & -T_3(T_2-T_3) \end{bmatrix} \quad (2.18)$$

(iii) The arc of the circle of T or the straight line of T is calculated: if $a\bar{c}-c\bar{a} \neq 0$, we have a circle centered at $\frac{\bar{a}d-\bar{c}b}{\bar{a}c-\bar{c}a}$, and radius of $\frac{ad-bc}{\bar{a}c-\bar{c}a}$ with the ends of the arcs of interest at $T(0) = -b/a$, and $T(\infty) = -d/c$; however, if $a\bar{c}-c\bar{a} = 0$, we have a straight line with the real axis intercept $T_R = \frac{-\text{Imag}(b\bar{d})}{\text{Imag}(a\bar{d}-c\bar{b})}$ and the imaginary axis intercept $T_I = \frac{-\text{Imag}(b\bar{d})}{\text{Real}(a\bar{d}-c\bar{b})}$.

(iv) Steps i, ii, iii are repeated for all of the circuit parameters, x_i, which are not mutual inductances or turns ratios.

The digital computer program, used in steps ii, iii and iv, above, is included in Appendix A.

III

FAULT DIAGNOSIS

Test frequency selection

As was pointed out by Seshu and Waxman[6], a change in a circuit parameter causes a shift in one or more of the breakpoints (poles or zeroes) and/or a shift in the gain constant of the n-port parameter. At complex poles or zeroes, a change in a circuit parameter may result in no change of gain or location of the breakpoint but only in the value of the resonant dip or peak. In order to detect the change corresponding to any possible change in the breakpoint, or gain, we can choose test frequencies as follows: one test frequency below the lowest non-zero breakpoint, one frequency above the highest finite breakpoint, and at least one frequency between successive breakpoints. In addition, there must be at least one test frequency at each complex pole or zero. Generally the test frequencies between successive breakpoints are chosen at the logarithmic midpoint between the breakpoints.

Although the above scheme of chosing test frequencies is sufficient in detecting all unique faults (except perhaps if we require a certain amount of redundancy), fewer test frequencies may be sufficient to diagnose the faults. Since, generally, the number of poles of a circuit equals the number of reactive components, the poles should be adequate in identifying the reactive elements. One or two additional test frequencies should be sufficient in detecting the components which are not frequency dependent. The sufficiency condition for the test frequency selection is: the test frequencies must be chosen in such a manner that, at least at one frequency, a change in any circuit parameter produces a significant change in the n-port parameter. A significant change is here defined as one which is readily distinguishable from a change in another component or from no change at all. Examples of non-significant loci are shown in Figures 1 to 3 . For example, in Fig. 1 , the locus of $T(x_1)$, as x_1 varies from zero to infinity, is almost a point coinciding with the nominal value of the n-port parameter, and cannot be distinguished from the nominal value.

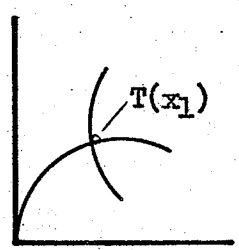


Fig.1. $T(x_1)$
a "point"

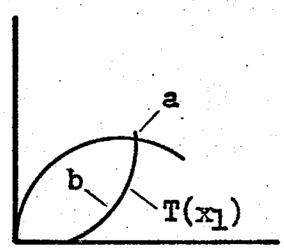


Fig.2. Part of $T(x_1)$
a "point"

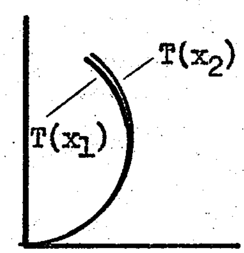


Fig.3. Two $T(x_i)$
almost coinciding

That part of the $T(x_1)$ locus in Fig. 2 denoted by "a" is very short and a value of $T(x_1)$ on that part of the curve could not be distinguished from the nominal $T(x_1)$. Thus the curve could not be used to identify a fault of x_1, that corresponds to a faulty n-port parameter T_F, which is on part "a" of the locus. The loci of $T(x_1)$ and $T(x_2)$ in Fig. 3 nearly coincide and, since a change in x_1 could not be distinguished from a change in x_2, loci of this form are not considered significant. In some cases $T(x_1)$ and $T(x_2)$ coincide at all frequencies and a different n-port parameter must be chosen to distinguish between faults of x_1 and x_2. Sometimes no distinction may be obtainable regardless of the n-port parameter chosen. An example of this is a circuit with parallel resistors. In these cases we must resort to test points in order to isolate all faults.

Since the plots of $T(x)$ indicate whether the test frequencies which have been chosen are adequate for isolating all faults, we need not rely on a method that computes the critical frequencies of a network. We can choose a set of frequencies, plot the loci of $T(x_i)$, and, if some are of no significance, disregard them. An examination of the loci also indicates whether we have chosen the test points so that all faults can be isolated. Although this trial and error method may not be appealing if an analytical solution can be readily used, the method could be used if no means is available to compute the critical frequencies or for large networks where the problem of finding the poles and zeroes becomes difficult.

Test frequencies were selected by the criteria of Seshu and Waxman. The graphs were plotted and examined for significant changes of each element. If some of the test frequencies were not required for fault diagnosis, they were disregarded.

Identification of the faulty circuit parameter

The curves of $T(x_i)$ $i = 1, 2, \cdots n$, (where n is the number of circuit parameters), are plotted for each test frequency. The n-port parameter of the faulty circuit is then measured and plotted for each test frequency. Since it is assumed that only one fault has occured, T_F, the faulty n-port parameter, will only satisfy those $T(x_i)$ where some value of that x_i could produce T_F, i.e. some circuit parameters can be eliminated as possible faults and in some cases there is only one possible faulty parameter.

Calculation of the value of the faulty circuit parameter

After the faulty component has been identified, its value can be calculated using equation (2.3):

$$x_F = \frac{aT_F + b}{cT_F + d} \tag{3.1}$$

where T_F is the measured n-port parameter,

x_F is the value of the faulty component and

a,b,c,d, are given by equation (2.18).

Example # 1

Digital computer simulation was used in fault isolation of the network shown in Fig. 4. The critical frequencies of the nominal Z_{11} are:

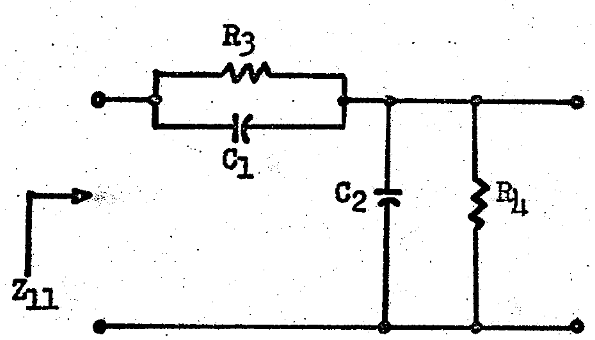
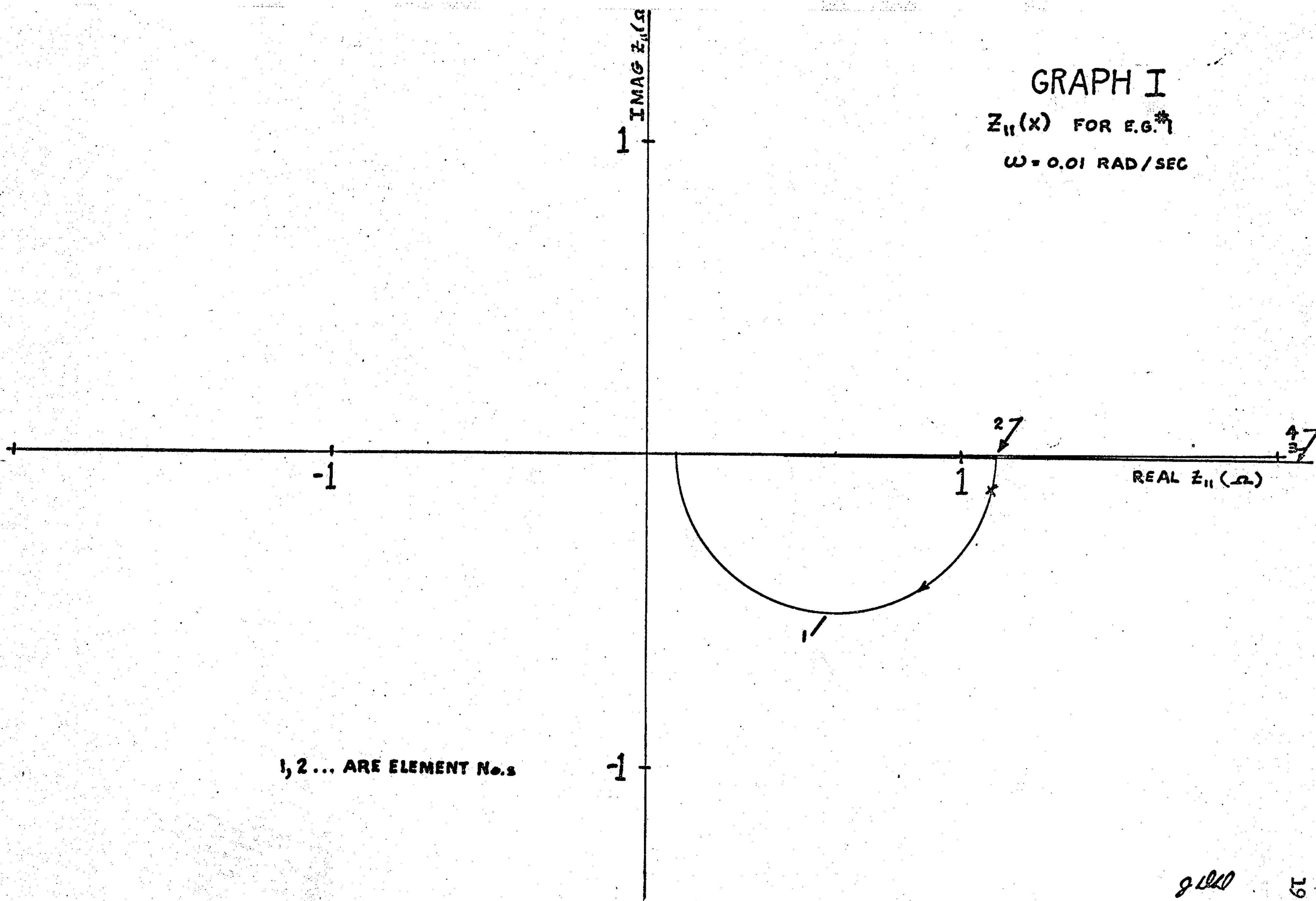


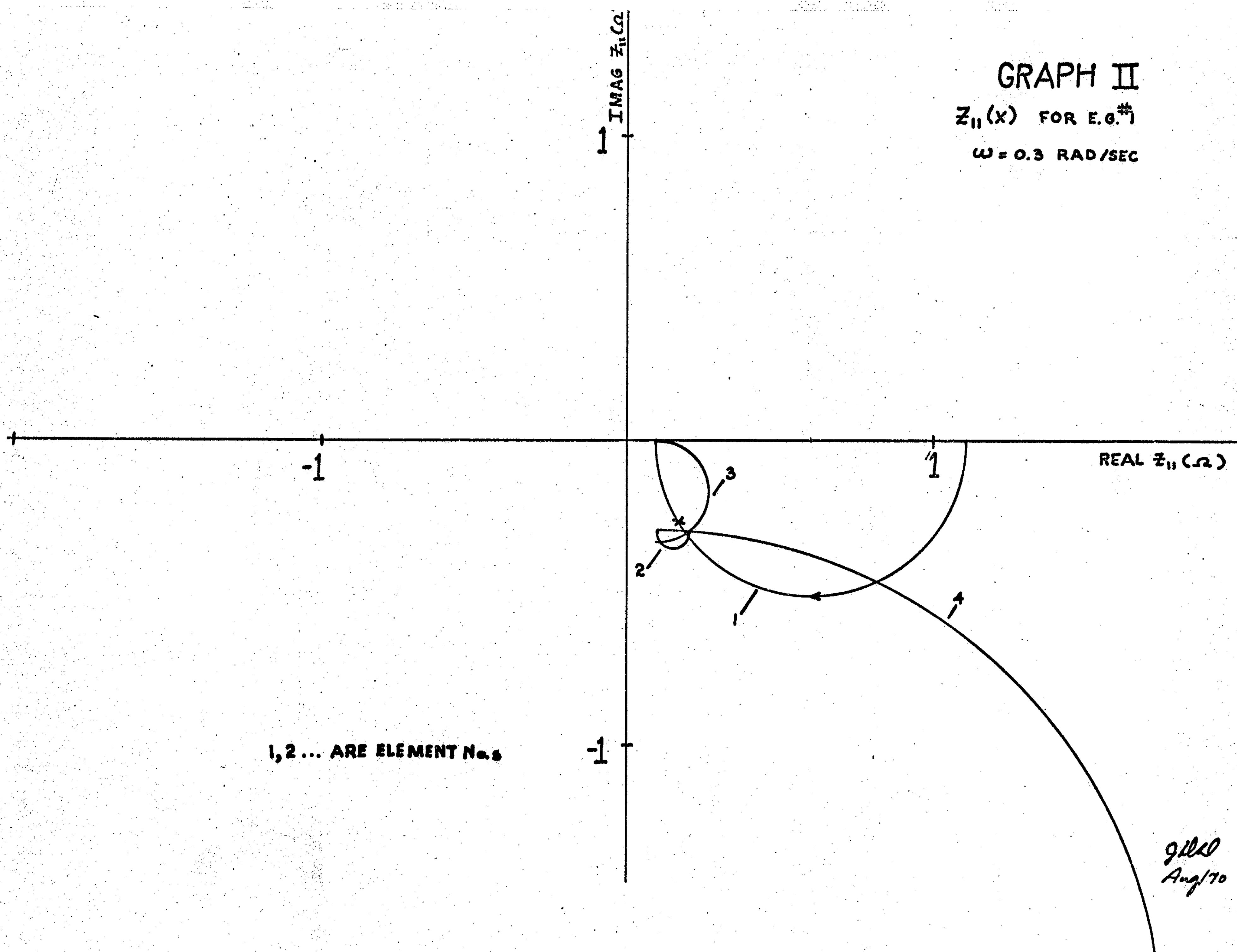
Fig. 4. Circuit for Example # 1

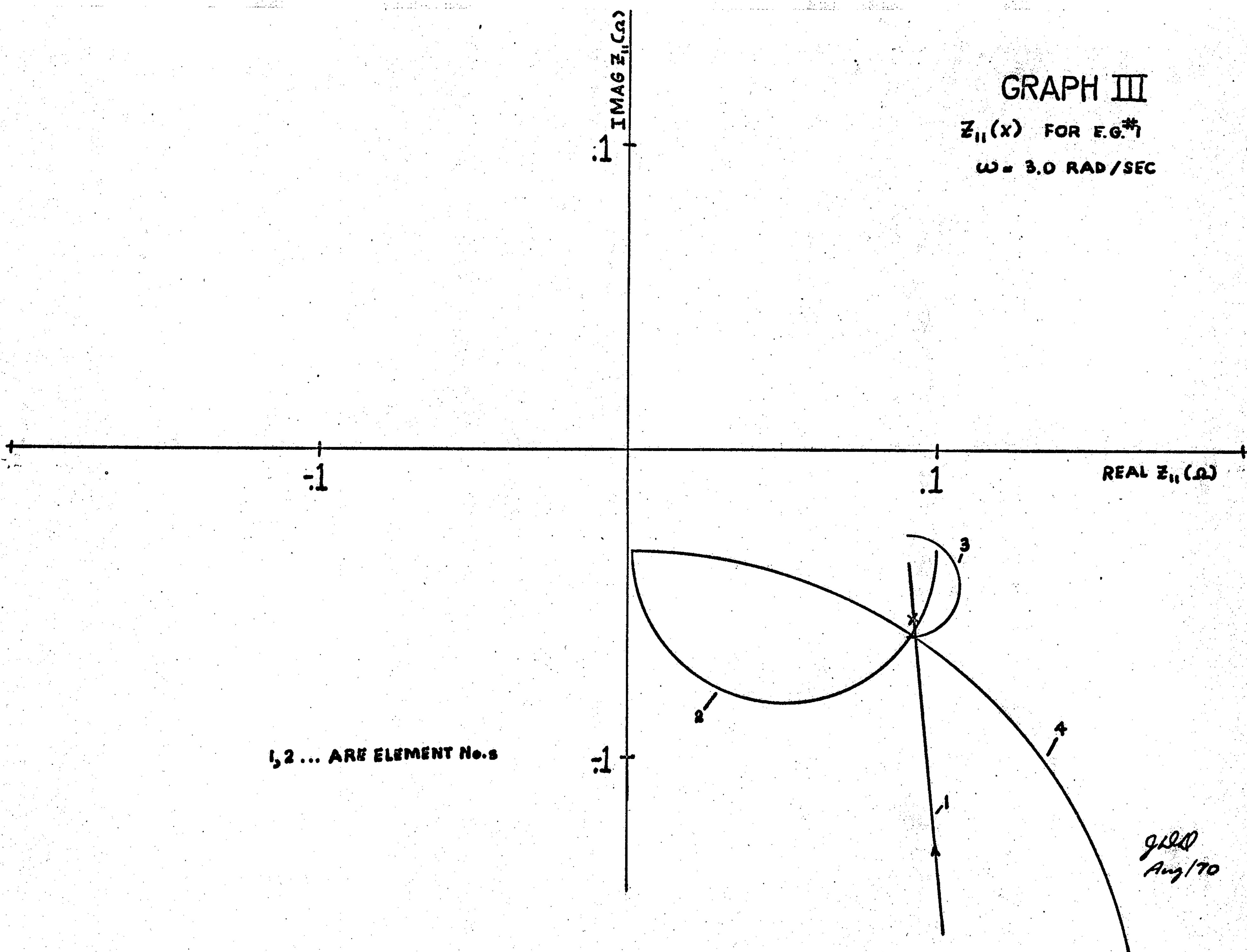
Element	No.		
1	-	C_1	= 10f
2	-	C_2	= 1f
3	-	R_3	= 1 Ω
4	-	R_4	= 0.1 Ω

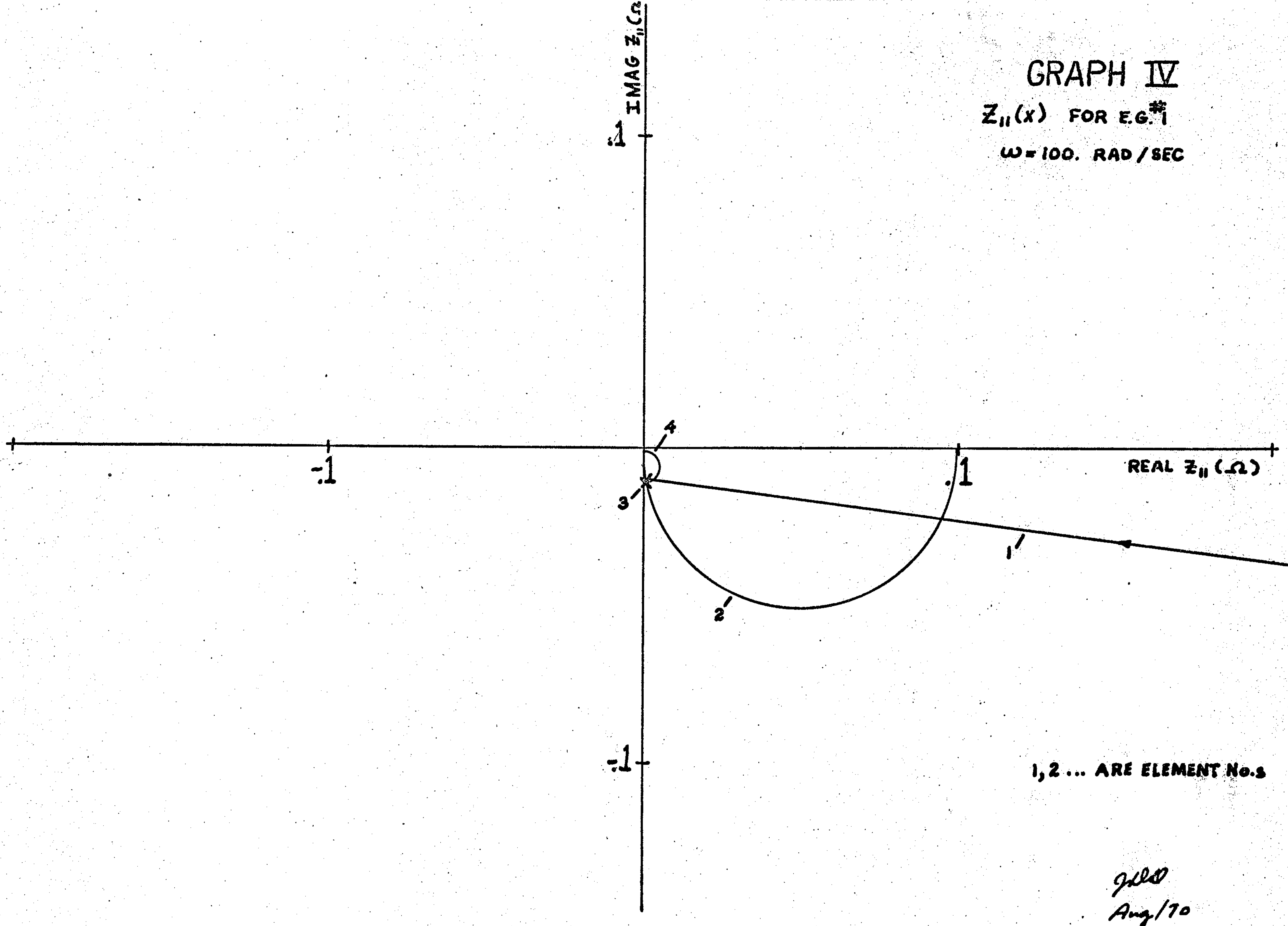
zeroes: -1.
poles: -0.1,-10.

The test frequencies of 0.01, 0.3, 3.0, and 100. radians/second were chosen, and the resulting Z_{11} loci were computed and plotted in Graphs I to IV. The Z_{11} for the simulated fault C_1 = 12f was calculated and plotted as "X" on the graphs. Graphs I, II, and III identify C_1 as be-


GRAPH I
$Z_{11}(x)$ FOR E.G. #1
ω = 0.01 RAD/SEC
IMAG Z_{11} (Ω)
REAL Z_{11} (Ω)
1
-1
1
-1
1
2
3
4
1, 2 ... ARE ELEMENT No.s
Aug/70


GRAPH II
$Z_{11}(x)$ FOR E.G.#1
ω = 0.3 RAD/SEC
IMAG Z_{11} (Ω)
REAL Z_{11} (Ω)
1
-1
1
-1
1
2
3
4
1, 2 ... ARE ELEMENT No.s
Aug/70


GRAPH III
$Z_{11}(x)$ FOR E.G.#1
ω = 3.0 RAD/SEC
IMAG Z_{11} (Ω)
REAL Z_{11} (Ω)
.1
-.1
.1
-.1
1
2
3
4
1,2 ... ARE ELEMENT No.s
Aug/70


GRAPH IV
Z11(x) FOR E.G.#1
ω=100. RAD/SEC
IMAG Z11 (Ω)
REAL Z11 (Ω)
.1
-.1
.1
-.1
1
2
3
4
1,2 ... ARE ELEMENT No.s
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ing faulty, but Graph IV is not useful since the difference between the nominal Z_{11} and the faulty Z_{11} cannot be distinguished. Graph I identifies C_1 the most distinctly since this is the test frequency at which the "X" is farthest from the other $T(x_i)$ loci. The value of C_1 is determined by solving equation (3.1) with the following data:

$$x_1 = 5.$$

$$x_2 = 15.$$

$$x_3 = 10.$$

$$T_1 = .11084 - j.13256$$

$$T_2 = .098727 - j.055678$$

$$T_3 = .10064 - j.075143$$

The solution of (3.1) is $x_F = 12.0$, as expected.

Although the four test frequencies may be useful in obtaining good definition in the fault detection, (e.g. Graph I defined the fault of C_1 most clearly), the choice of Graphs II and III would be adequate in locating all faults. Graph II would indicate faults above and below the nominal value for components C_1, R_3, R_4, but could not distinguish between one end point of the C_2 locus and the nominal Z_{11}. Similarly, Graph III is adequate for faults of C_1, C_2, and R_4 but not for R_3. Thus the test frequencies of 0.3 and 3.0 radians/second would be selected as the test frequencies for fault isolation in this circuit.

IV

CIRCUIT ANALYSIS TECHNIQUES

The n-port parameter of a circuit must be measured or computed for three values of every circuit element, if the Moebius transformation technique of fault isolation is used. Although, in theory, any method of circuit analysis can be used, hand computations are impractical for large networks. The practicality of using measurements and the digital computer analysis techniques of ECAP, topological formulas, and state equations, is examined in the following sections.

The measurement method

The n-port parameter circles can be determined by varying each of the circuit components and measuring the n-port parameters at the test frequencies. The technique of measuring the n-port parameters corresponding to three values of each component is feasible, though tedious. The calculations, that must then be performed, are simple and small in number — a small computer or even a desk calculator can be used. Stray elements are accounted for in this way and, provided that the measurements are made precisely, the coefficients of the Moebius transformation are more accurate than those obtained by analyzing a model of the circuit.

Any circuit which has only physically accessible components presents no problems in the measurement procedure. However, devices such as transistors have inaccessible internal parameters such as: transconductance, resistance, and capacitance. In order to vary these parameters, a model of the transistor must be constructed and the dependent generators must be replaced by independent sources which are regulated manually according to the dependence condition.

<u>A method for simulating the change in a transistor parameter</u>

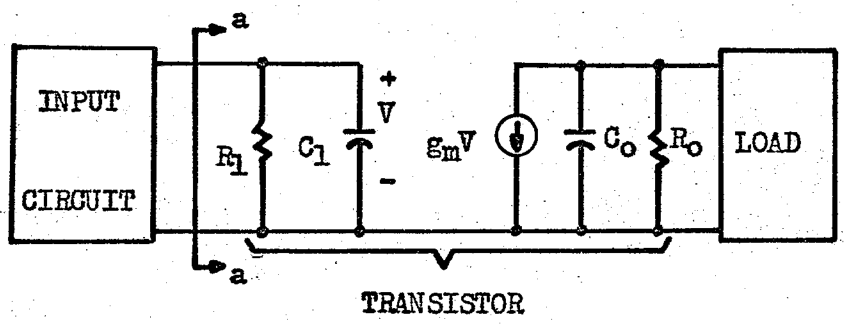


Fig. 5. Transistor equivalent circuit

The components R_1, C_1, R_o, C_o, in the transistor equivalent circuit shown above, can be varied by placing external elements in parallel with them, but g_m must be varied by modelling the transistor with accessible components. This is achieved by breaking the circuit at a-a and inserting additional components as shown below. The components C_1 and R_1 are used to load the input cir-

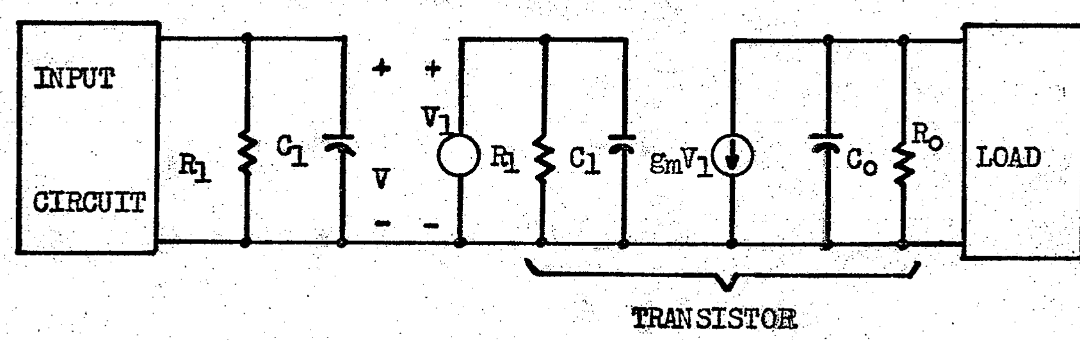


Fig. 6. Circuit for simulating a change in g_m

cuit the same way that the transistor would. The voltage V is measured and the independent source V_1 is adjusted to the value kV. Thus, if $k = 1$, the circuit is unchanged; changing k has the same effect as changing g_m. Although a more elaborate transistor model would make this technique more difficult, a similar procedure could still be applied.

For integrated circuit networks the above method would not be applicable, although the external components used in a circuit could be analyzed by the measurement method.

Computer methods

If the measurement method of obtaining the coefficients of the Moebius transformation is not used, such computer methods as ECAP, topological formaulas, and state equations must be considered as possible alternatives. Of prime importance in choosing an analysis method are: (i) the computation time used by the computer in calculating the necessary n-port parameters; (ii) the computer storage space required in the solution.

The number of n-port parameters that must be evaluated will be a factor in estimating the computation time. For the Moebius transformation method, $N_M = (2n-1)N_{TF}$ n-port parameters must be determined, where N_M is the number of times the n-port parameter is calculated, n is the number of network parameters, and N_{TF} is the number of test frequencies used. Although it was previously stated that three values of each x_i are required at each test frequency in order to obtain the coefficients of the Moebius transformation, if one value of each x_i

is the nominal circuit value, only two additional values of x_i are required. Therefore, the number of n-port parameters is $(2n-1)N_{TF}$ as previously stated, rather than $3nN_{TF}$.

The number of n-port parameters required for the fault dictionary technique of Seshu and Waxman is $N_S = hnN_{TF}$, where N_S is the number of times the n-port parameter is calculated for the fault dictionary technique, and h is the number of faults listed in the fault dictionary for each component. Since h is generally larger than two (e.g. an h of nine is typical), the fault dictionary technique requires the evaluation of more n-port parameters than the Moebius transformation technique. For large networks, h(the number of discrete faults listed per component) must be increased, because the shunting effect of many elements make the n-port parameter, $T(x_i)$, less sensitive to a change in x_i. Thus the difference between N_M and N_S increases with an increase in network size.

If ECAP is used in finding the N_M n-port parameter values, then N_M separate calculations must be made, since ECAP calculates the response at a fixed frequency only. If the network is not too large, and if the available computer storage is small, ECAP is a suitable analysis technique.

An increase in the size of the circuit increases the N_S of the fault dictionary technique considerably. The computation time may then be reduced by finding the symbolic n-port parameter, using topological formulas and making the required substitutions for circuit elements and test frequencies. The topological solution requires a large amount of storage since all trees of the network must be listed. Only one of the n-port parameters is generated per calculation, thus, if two or more n-port parameters are required for distinguishing between faults, the solution (except for the tree generation) must be repeated for each n-port parameter.

The state equation approach of analyzing a network overcomes the problem of large storage requirement and the single n-port generation feature of the topological solution. All of the n-port parameters are generated numerically in each solution, and substitution can be used for the different test frequencies. The time, required in finding the two solutions for each circuit element by the state equation method, would, generally, be less than that required in finding the symbolic n-port parameter by topological formula and then substituting the two component values. Thus, the state equation approach is superior to the topological solution in the analysis of the network as required in the Moebius transform fault identification.

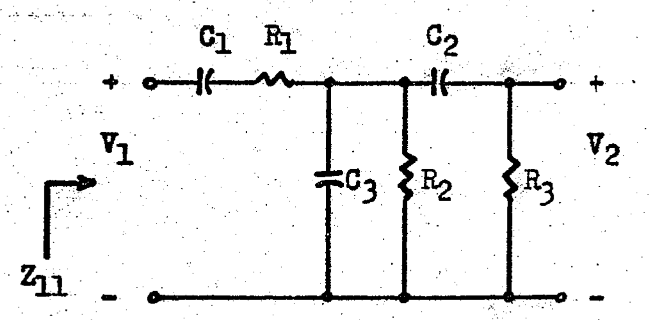
V

EXPERIMENTAL RESULTS

The response of several faulty circuits was simulated on a digital computer, and the fault diagnosis technique was applied. Laboratory measurements were also performed on several circuits. A state equation computer program, written in Fortran IV, was used for circuit analysis. Simulation of the performance of a circuit with a faulty component was done by evaluating the magnitude and phase response of the transfer (or driving point) function, which was computed by the state analysis program, at the desired test frequencies. Laboratory measurements were made using a Wavetek Model 750 phase meter, two Hewlett Packard Model 400E AC voltmeters and an Advance Instruments Model TC9A frequency meter. Component values were measured at 1 KHz., with a General Radio Type 1650-A impedance bridge.

Test Circuit # 1

The voltage transfer function, $A_V = \frac{V_2}{V_1}$, was selected for diagnosing faults in the R-C network shown below.



Element No.

1 - $C_1 = 0.0985 \mu f$
2 - $C_2 = 0.1 \mu f$
3 - $C_3 = 0.101 \mu f$
4 - $R_1 = 1000. \Omega$
5 - $R_2 = 5210. \Omega$
6 - $R_3 = 1000. \Omega$

Fig.7. Experimental Circuit # 1

The frequency was scaled by a factor of 10^{-5}. A root-finding program was used in finding the following roots of the nominal voltage transfer function:

zeroes: 0,0

poles: -0.0277, -0.00466, -0.00152

For laboratory test purposes R_3 was changed from 5210 ohms to 3820 ohms, to represent a change in R_3 of 7.5%. The voltage transfer function, A_v, was measured at the test frequencies, 18.4, 39.9, 143.6 and 2027. Hertz (which corresponds to the scaled radian frequencies, 0.00115, 0.0025, 0.009 and 0.127 radians/second). The loci of $Av(x_i)$, $i = 1, 2, \cdots 6$, at the above test frequencies were calculated, and the results were plotted on Graphs V to VIII. The measured transfer function of the faulty circuit was plotted as "X" on the graphs.

An examination of Graphs VI and VII, immediately eliminates all x_i, except x_2 and x_6, although a phase measurement error of 5% could result in the incorrect conclusion that x_1 or x_5 is the faulty element. Graph VI indicates that x_2 is the faulty component and Graph VII indicates x_6 as faulty. Since the graphs of the voltage transfer function did not provide a clear-cut fault identification, the driving point impedance, Z_{11}, was chosen for test purposes. The plot of the loci of $Z_{11}(x_i)$ at the radian test frequency of 15 radians/second, which is shown in Graph IX, identifies x_6 as the faulty component.

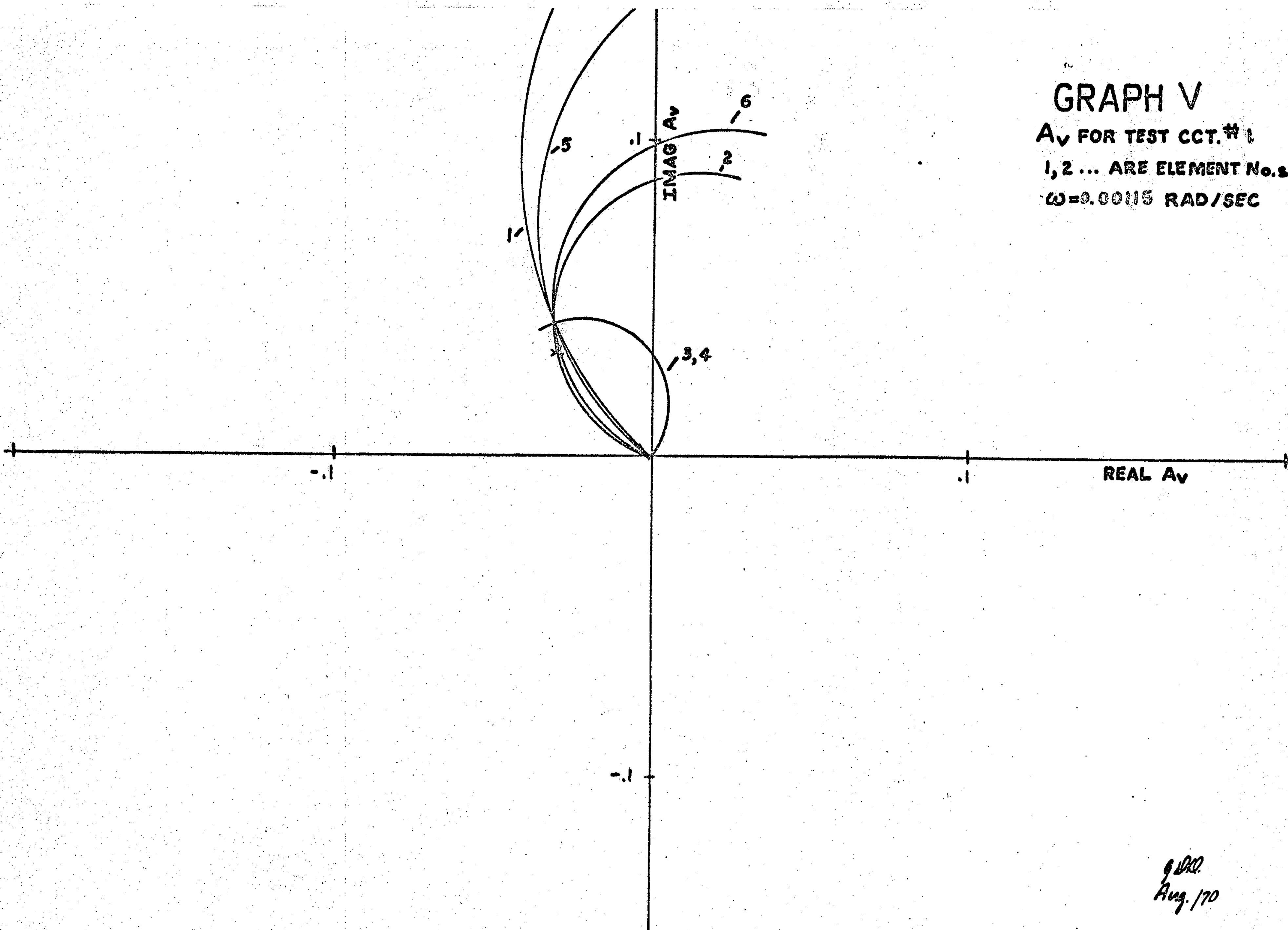
The value of x_6 was calculated using the coefficients corresponding to Graph VII, since it was on this graph that the "X" came closest to coinciding with the $T(x_6)$ locus. The computed value of x_6 is 3820. + j52. ohms, as compared to the measured value of 3820. + j0.

GRAPH V

A_v FOR TEST CCT. # 1

1, 2 ... ARE ELEMENT No.s

ω=0.00115 RAD/SEC



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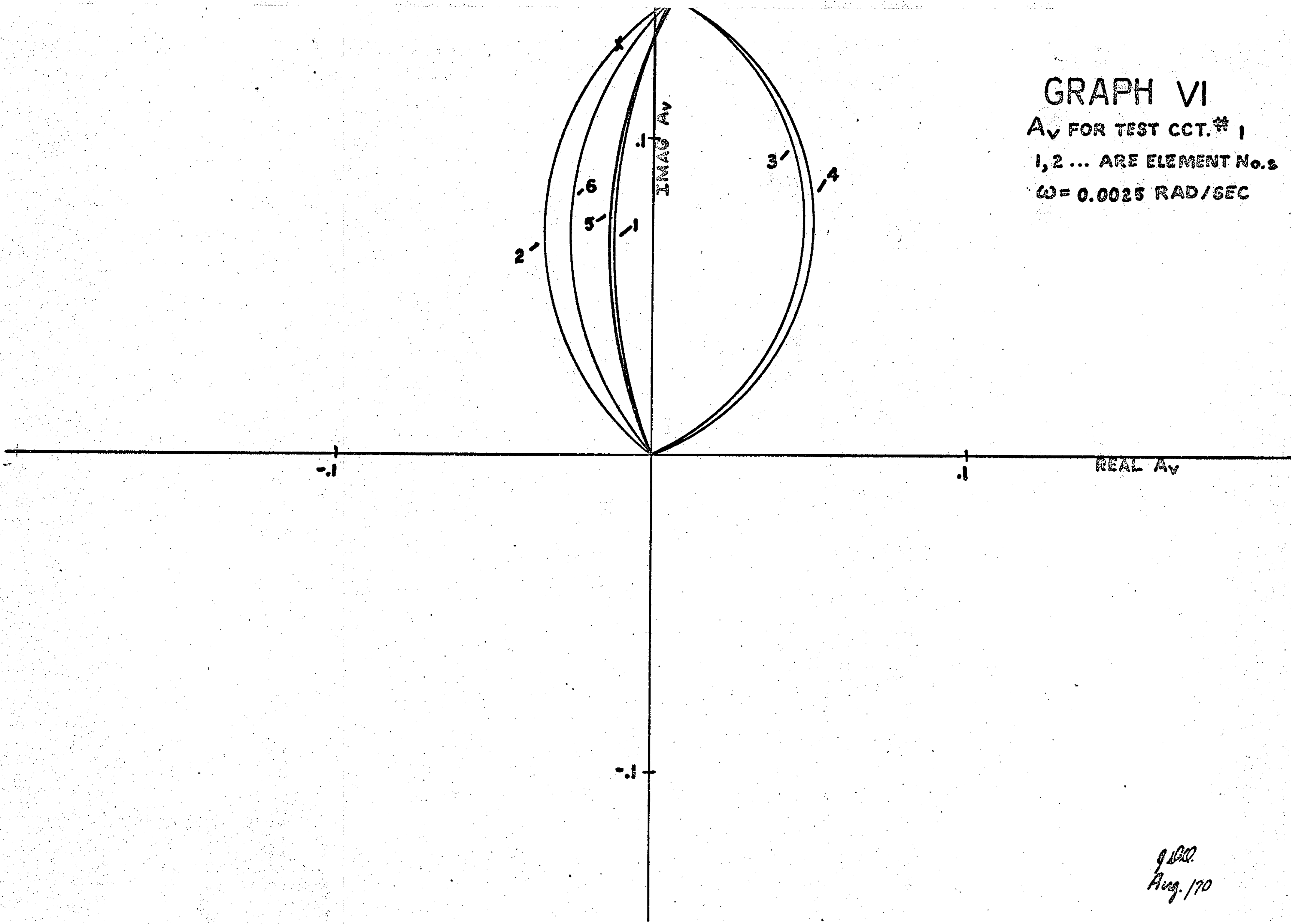
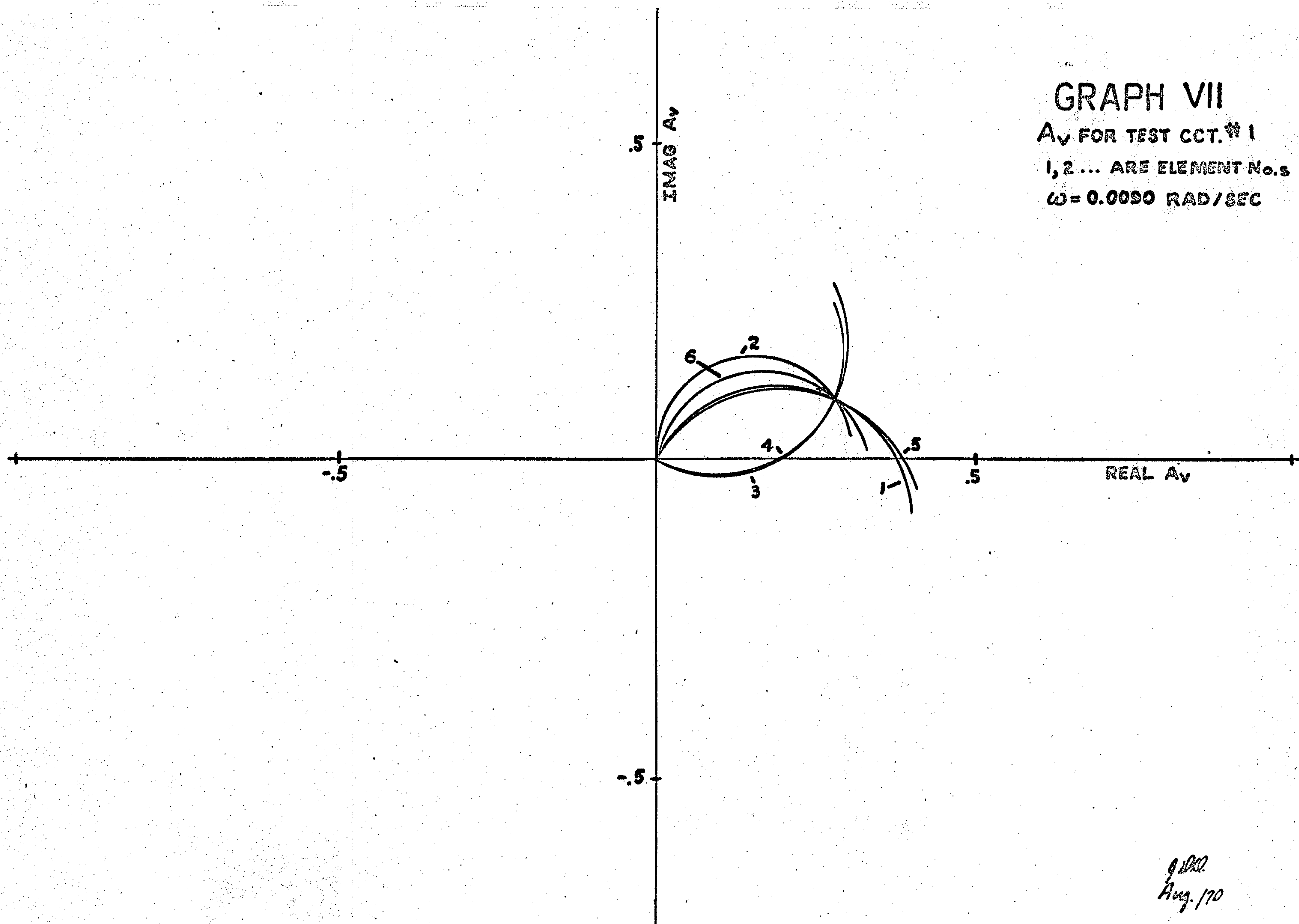


GRAPH VI

A_V FOR TEST CCT. # 1

1, 2 ... ARE ELEMENT No.s

ω = 0.0025 RAD/SEC


GRAPH VII
A_v FOR TEST CCT. # 1
1, 2 ... ARE ELEMENT No.s
ω = 0.0090 RAD/SEC
IMAG A_v
REAL A_v
.5
-.5
.5
-.5
1
2
3
4
5
6
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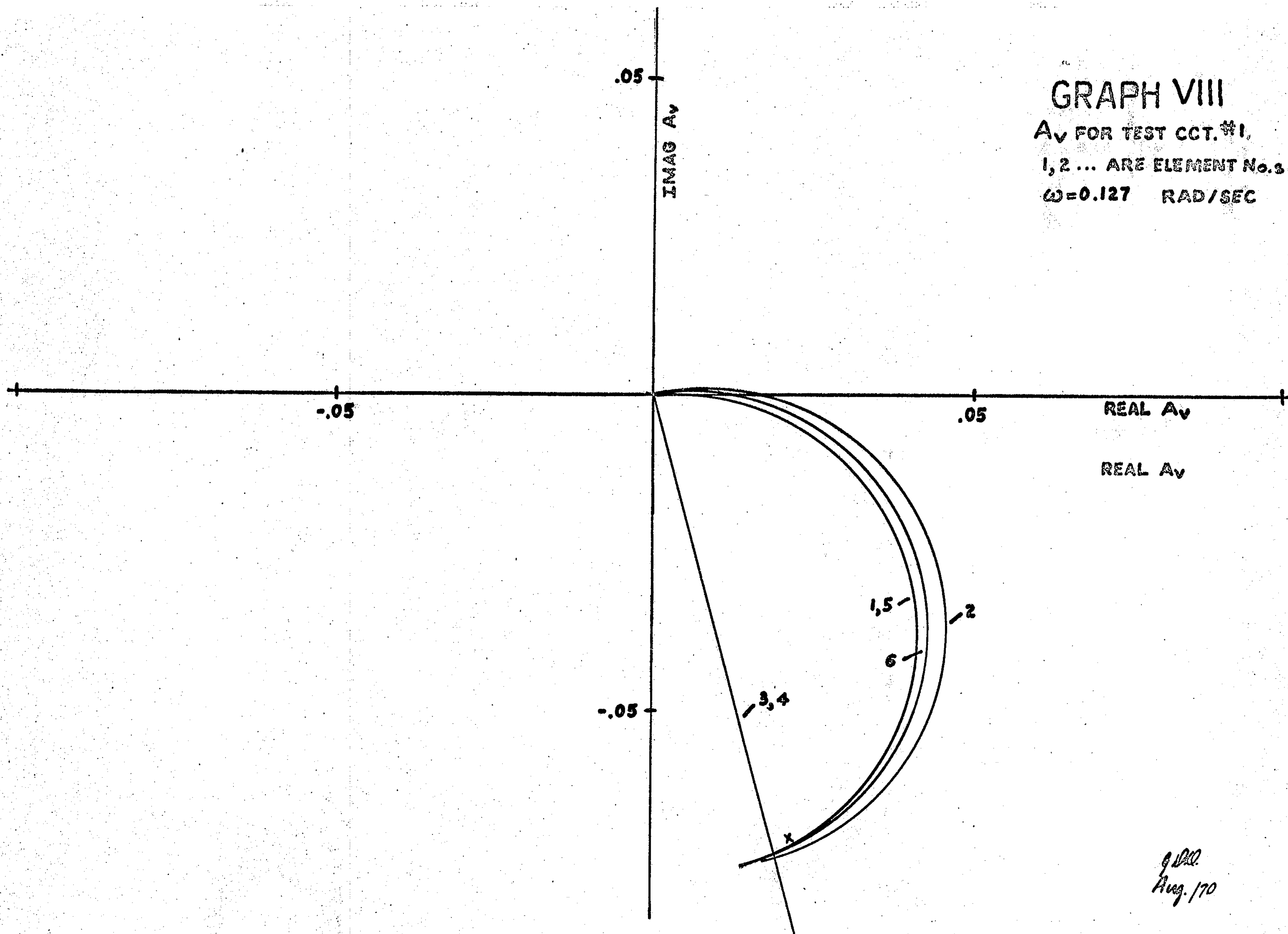
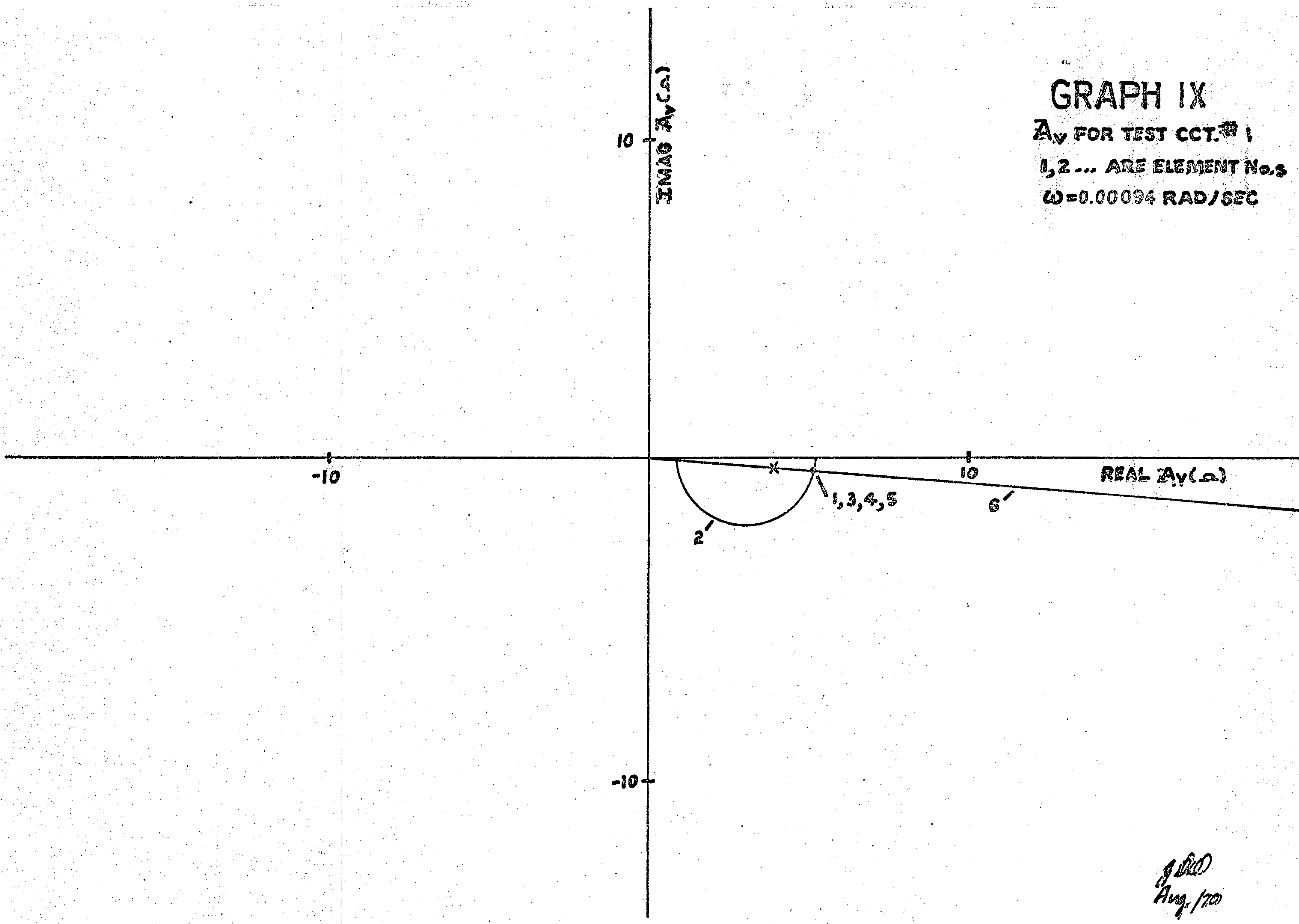


GRAPH VIII

A_v FOR TEST CCT. #1.

1, 2 ... ARE ELEMENT No.s

ω=0.127 RAD/SEC


GRAPH IX
A_v FOR TEST CCT.# 1
1, 2 ... ARE ELEMENT No.s
ω=0.00094 RAD/SEC
IMAG $A_v(\omega)$
REAL $A_v(\omega)$
10
-10
-10
10
1,3,4,5
2
6
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ohms. The non-zero imaginary part in the solution is present because some of the circuit components other than x_6 are not at their nominal value, or because the measured value of A_v does not quite lie on the $T(x_6)$ locus. If we assume the latter, we can "correct" the A_v measurement so that it is on $T(x_6)$ and recalculate the value of x_6. The values of x_6 computed at the other test frequencies are: 4184. + j118., 4042. + j162. and 2027. - j618.0. The disagreement between the computed and measured values of x_6 can be expected since the measured values of A_v do not lie on the $T(x_6)$ loci in Graphs V, VI and VIII.

The results of the computer analysis of this example are included in Appendix B. The components of this example were used in a transistor amplifier circuit which was tested more extensively; see test circuit #2.

Test Circuit # 2

The voltage transfer function, $A_v = \frac{V_2}{V_1}$, was selected for diagnosing faults in the transistor amplifier circuit shown below.

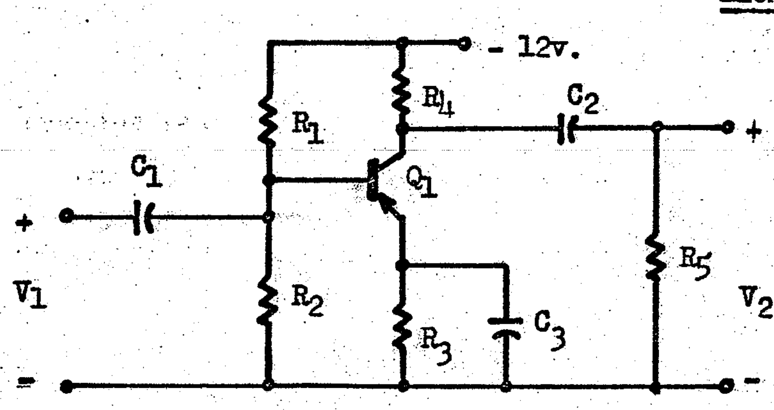


Fig. 8. Experimental Circuit # 2

Element No.

2 - C_1 = .478μf

3 - C_2 = .446μf

4 - C_3 = .683μf

5 - R_1 = 94.KΩ

7 - R_2 = 9250Ω

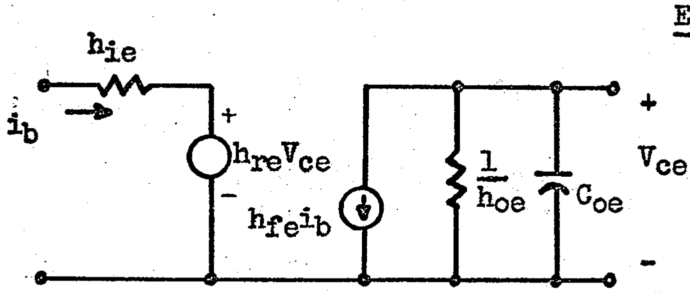
8 - R_3 = 1000Ω

10 - R_4 = 5100Ω

11 - R_5 = 5210Ω

Q1: MPS6518

The transistor equivalent circuit that was used is shown in Fig. 9.



Element No.
1 - C_{oe} = 26pf
6 - h_{ie} = 3610 Ω
9 - $1/h_{oe}$ = 35.7 KΩ
12 - h_{re} = 0.00029
13 - h_{fe} = 104.

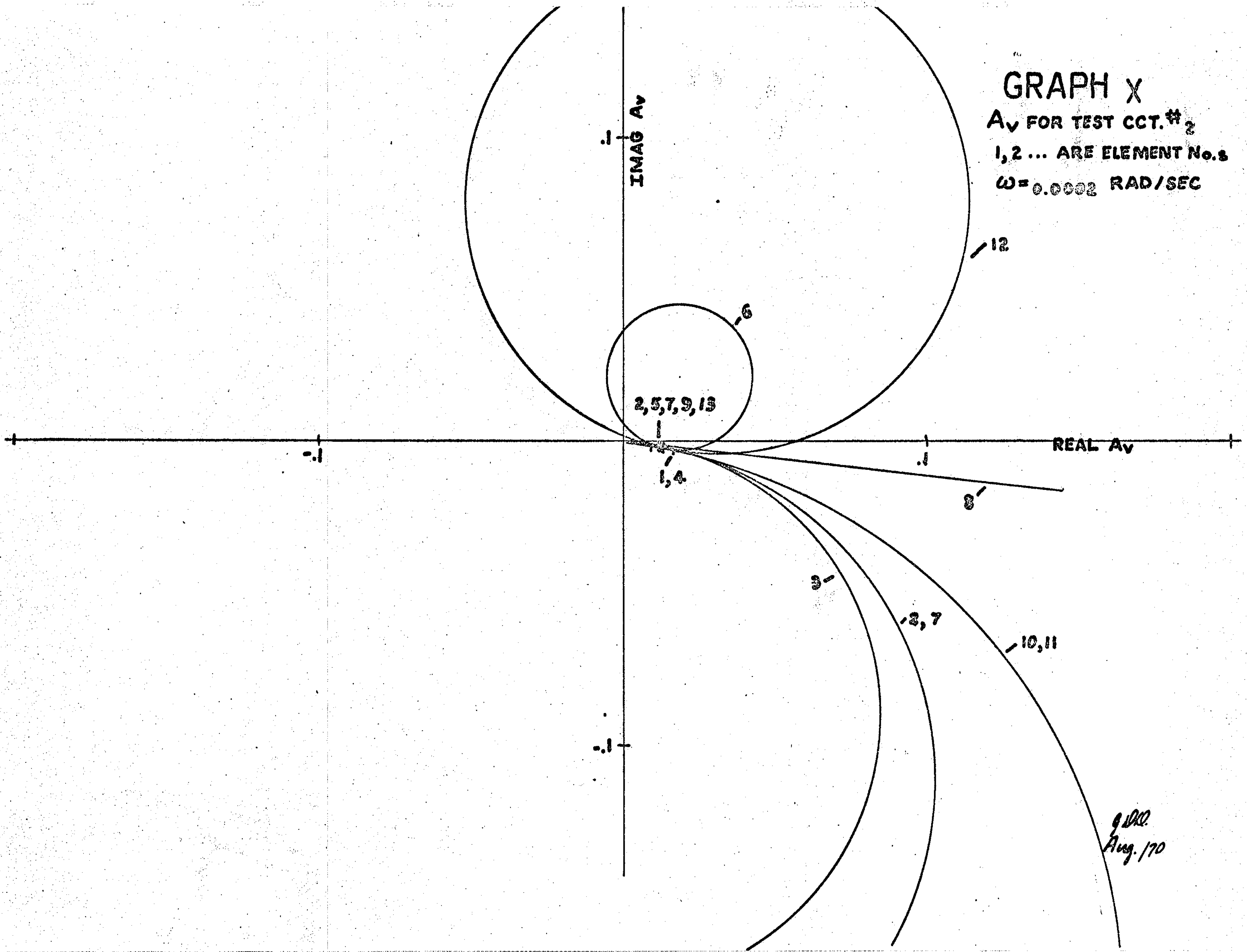
Fig. 9. Transistor equivalent circuit

The frequency was scaled by 10^{-5}. A root-finding program was used in finding the following roots of the nominal voltage transfer function:

zeroes: 0, 0, -0.01464
poles: -0.002236, -0.002604, -0.4280, -156.4.

In the digital computer simulation, R_5 was changed from 5210 ohms to 3820 ohms. The test frequencies of 3.18, 38.4, 98.5, 1270, 5.8×10^5 and 25.4×10^6 (which correspond to the scaled radian frequencies of 0.0002, 0.00242, 0.0062, 0.080, 36.4 and 1600. radians/second) were used. The loci of $A_v(x_i)$, $i = 1, \cdots 13$, at the above test frequencies were calculated, and the results were plotted on Graphs X to XIV. The transfer function of the faulty circuit was plotted as "X" on the graphs.

An examination of Graphs XII and XIII, immediately eliminate all x_i except x_{10} and x_{11}. The other graphs show that we cannot distinguish between a fault in x_{10} and x_{11} at any test frequency. This is reasonable, since the components R_4 and R_5 (x_{10} and x_{11}) of the amplifier are effectively parallel load resistors. At the test frequency of 0.08 radians/second, the two possible fault solutions are:


GRAPH X
A_V FOR TEST CCT. #2
1, 2 ... ARE ELEMENT No.s
ω = 0.0002 RAD/SEC
IMAG A_V
REAL A_V
.1
-.1
.1
-.1
12
6
2,5,7,9,13
1,4
8
3
2,7
10,11
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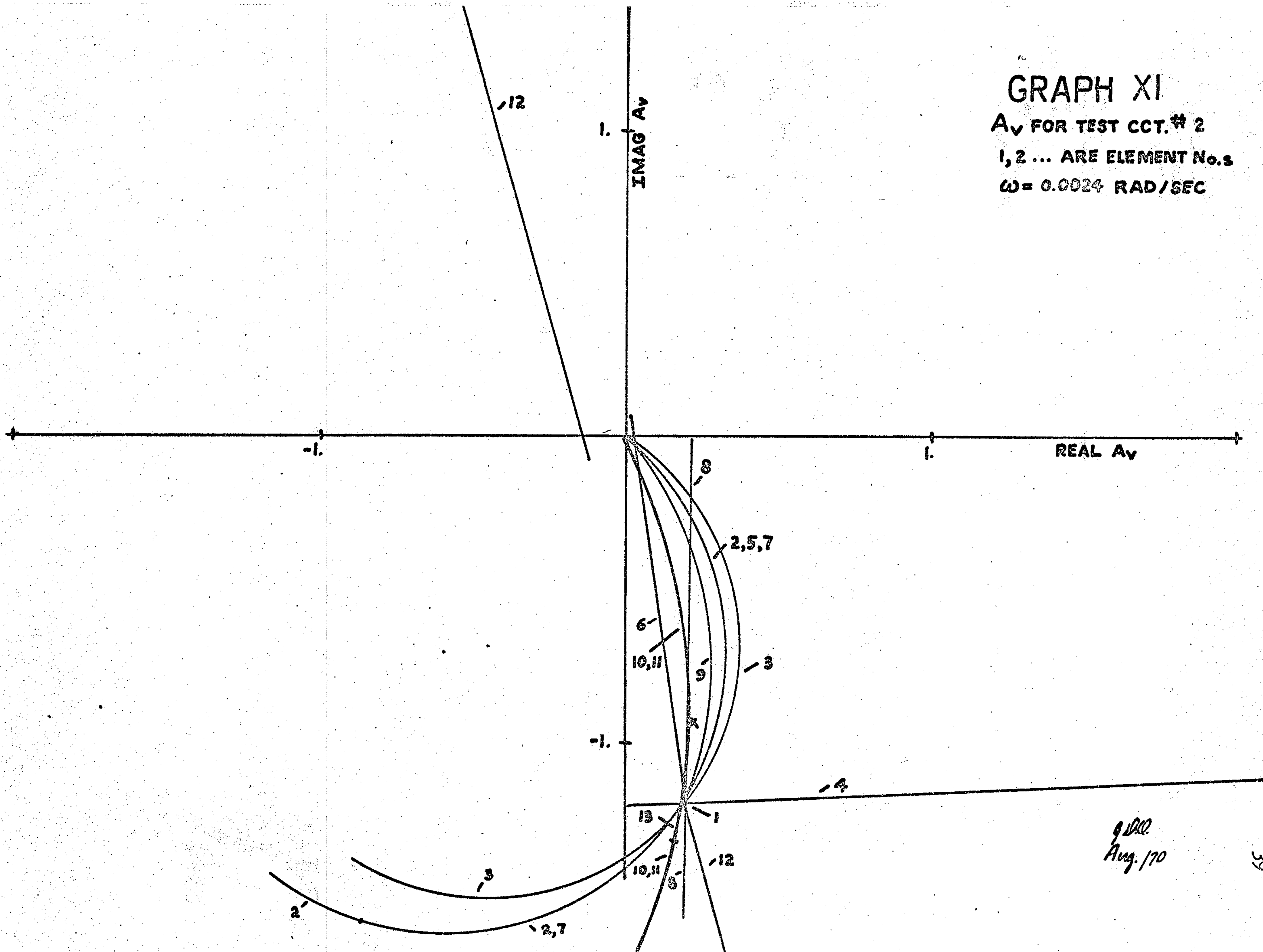
GRAPH XI

A_v FOR TEST CCT. # 2

1, 2 ... ARE ELEMENT No.s

ω = 0.0024 RAD/SEC



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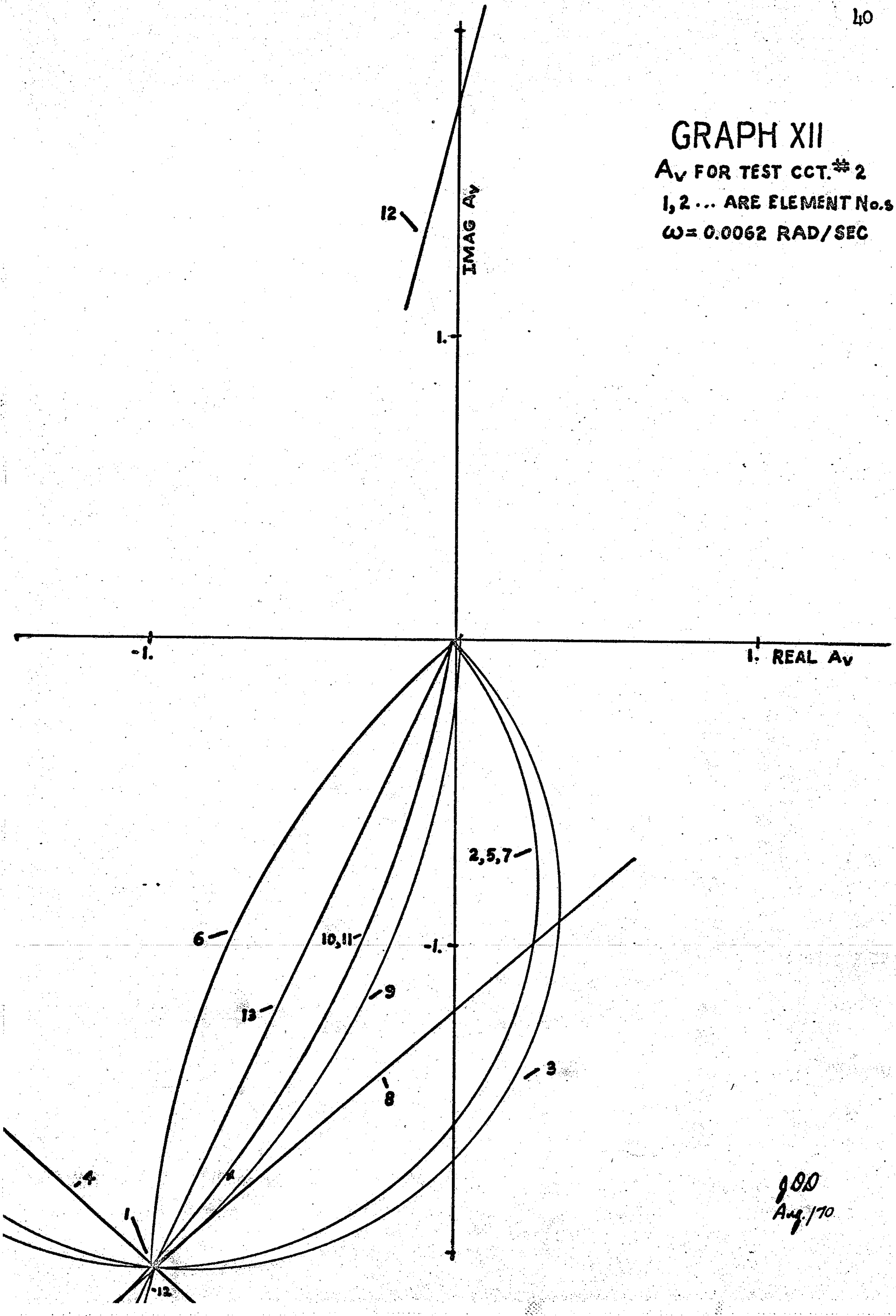
GRAPH XII
Av FOR TEST CCT. # 2
1, 2 ... ARE ELEMENT No.s
ω= 0.0062 RAD/SEC
IMAG Av
REAL Av
1.
-1.
1.
-1.
12
2,5,7
6
10,11
13
9
3
8
4
1
12
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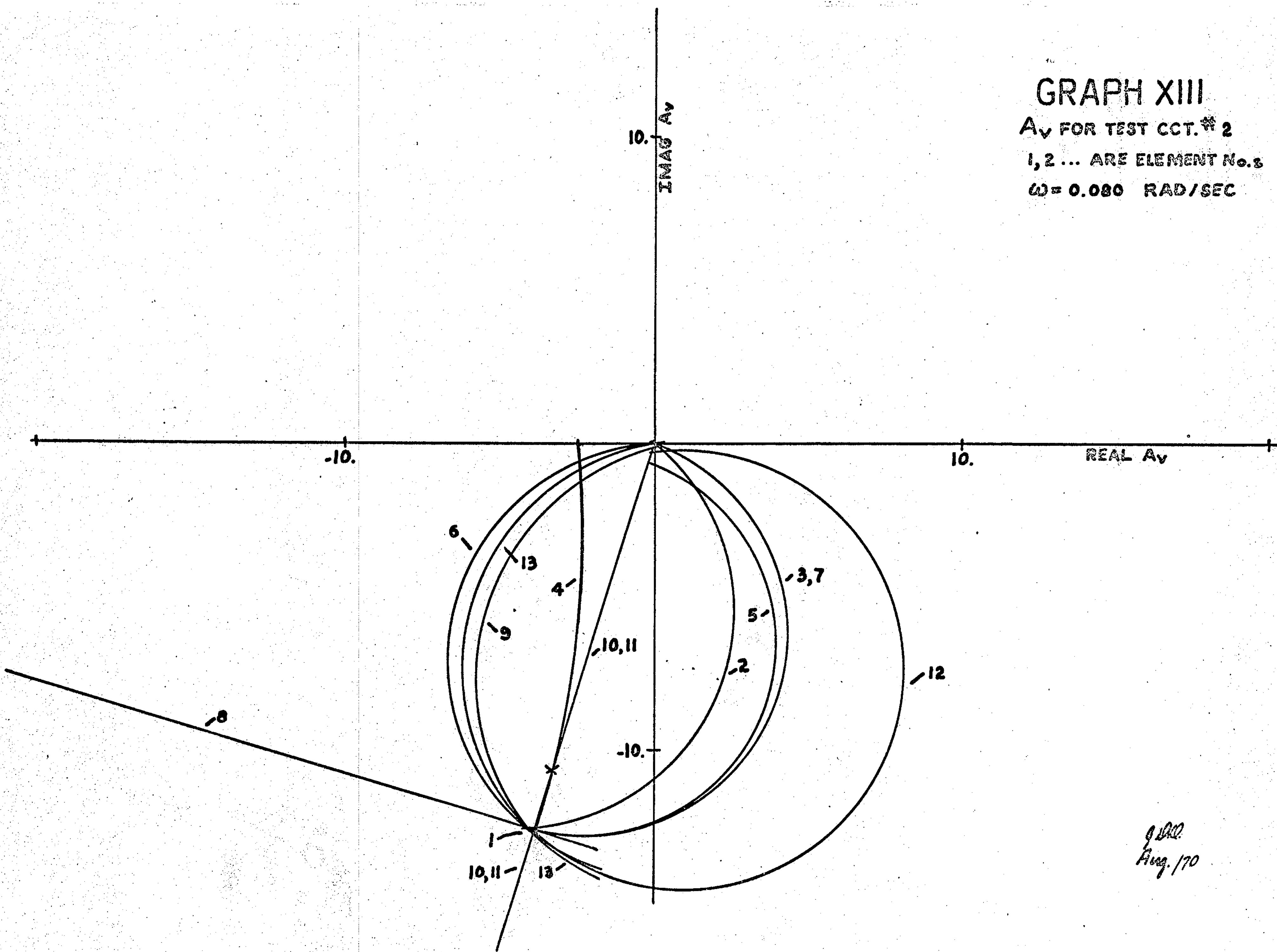
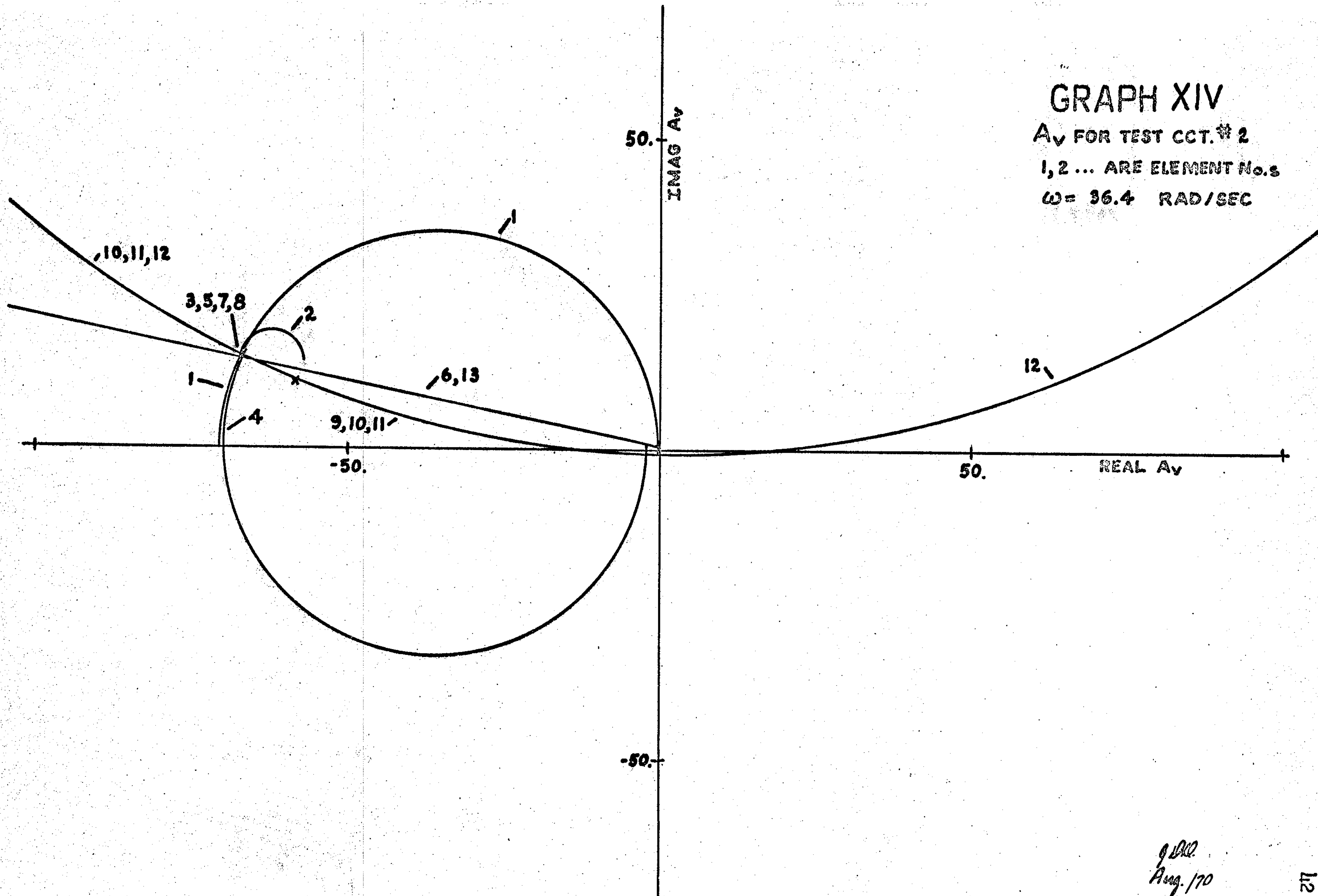


GRAPH XIII
A_v FOR TEST CCT.# 2
1, 2 ... ARE ELEMENT No.s
ω = 0.080 RAD/SEC


GRAPH XIV
Av FOR TEST CCT. # 2
1, 2 ... ARE ELEMENT No.s
ω= 36.4 RAD/SEC
50.
-50.
IMAG Av
REAL Av
1
10,11,12
3,5,7,8
2
1
4
6,13
9,10,11
12
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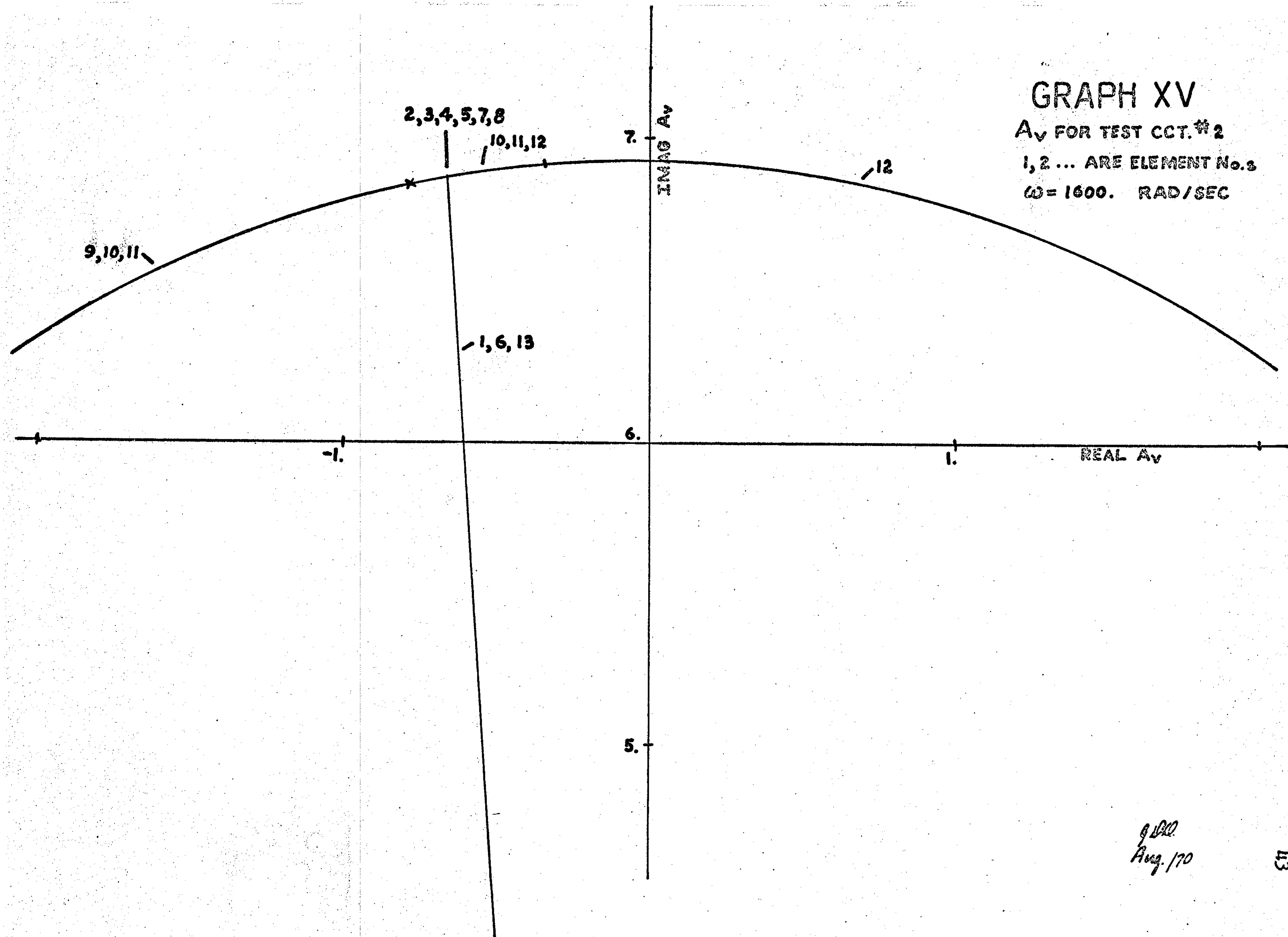
GRAPH XV

A_v FOR TEST CCT. #2

1, 2 ... ARE ELEMENT No.s

ω = 1600. RAD/SEC



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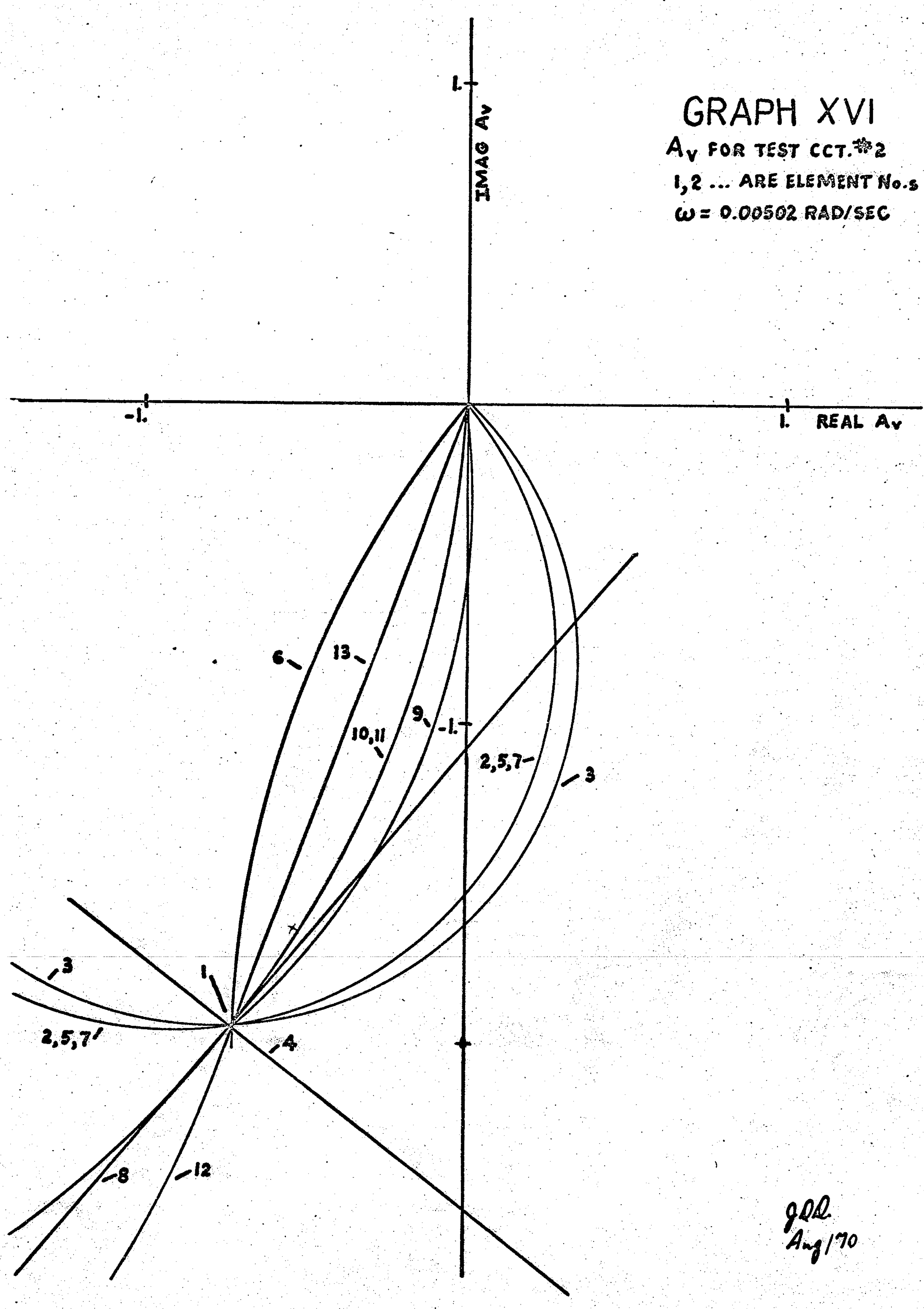
GRAPH XVI
Av FOR TEST CCT. #2
1,2 ... ARE ELEMENT No.s
ω = 0.00502 RAD/SEC
1.
IMAG Av
-1.
1.
REAL Av
6
13
10,11
9
-1.
2,5,7
3
3
1
2,5,7
4
8
12
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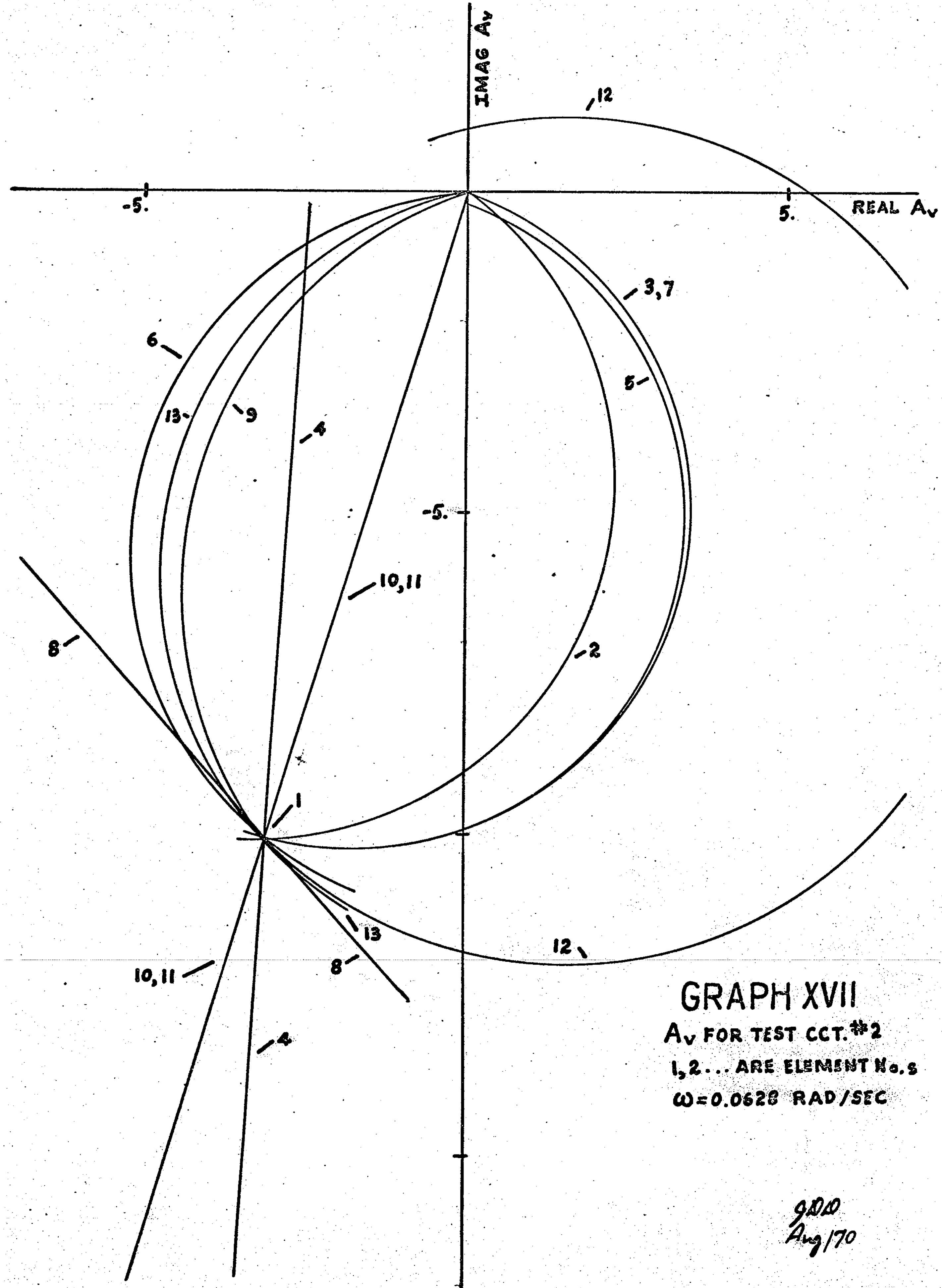

GRAPH XVII

A_v FOR TEST CCT. #2

1, 2 ... ARE ELEMENT No.s

$\omega = 0.0628$ RAD/SEC

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R_4 = 3760. ohms, and R_5 = 3820 ohms. Thus, the simulated fault is correctly identified, but the identification is not unique, and another n-port parameter must be chosen for a unique fault diagnosis.

At a low frequency (w = .0002 rad/sec), the loci of $T(x_1)$ and $T(x_4)$ are a "point", as is indicated on Graph X. The loci of $T(x_2)$, $T(x_3)$, $T(x_4)$, $T(x_5)$, $T(x_7)$ and $T(x_8)$ are "points" at the high frequency of 1600. radians/second. These results, that C_{oe} and C_3 have little effect at low frequencies, and that C_1, C_2, C_3 and consequently R_1, R_2, R_3 have little effect at high frequencies, allow the test designer to simplify the circuit model at some frequencies.

The circuit was tested in the laboratory for the faults (i) C_2 = 0.25 f, (ii) C_3 = 0.465 f, (iii) C_3 = 1.0 f and (iv) R_5 = 3820 ohms. Graphs XII and XIII, for the simulated tests, show that test frequencies(unscaled) of 79.9 and 1000. Hertz are adequate for isolating the above mentioned faults. The loci of $T(x_i)$, at the test frequencies, are plotted in Graphs XVI and XVII. Fault (iv) is marked on the Graphs with an "X". The faulty transfer functions were measured as:

Frequency	T_F for Fault No.			
	(i)	(ii)	(iii)	(iv)
79.9 Hertz	$1.88\angle{-100^\circ}$	$2.03\angle{-115.2}$	$2.24\angle{-102.7}$	$1.72\angle{-108}$
1000 Hertz	$10.96\angle{-102.9}$	$7.7\angle{-107.5}$	$15.7\angle{-104.2}$	$9.23\angle{-106}$

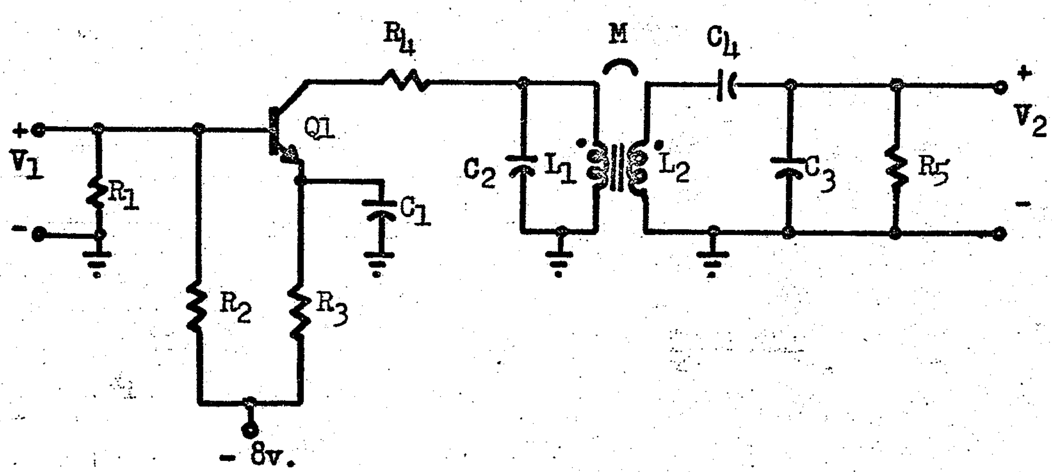
The faulty component can be uniquely identified from the plots in each of the cases. The computed values of the components are:

Frequency	Value of Fault No.			
	(i)	(ii)	(iii)	(iv)
79.9 Hertz	.289-j.001	.487+j.002	1.03-j0.043	3847-j83
1000 Hertz	.129+j.052	.485+j.021	1.05+j0.053	3954+j208

The measured and calculated results agree quite well, except that the computed value of C_2 at 1000 Hz. is .129 - j.129 instead of .25 - j0. This difference can be expected, since T_F, the measured transfer function, and A_v on Graph XVII, do not coincide.

Test Circuit # 3

The voltage transfer function, $A_v = V_2/V_1$, was selected for digital computer simulation of the fault diagnosis technique of the FM I.F. amplifier shown below. The transistor equivalent circuit that

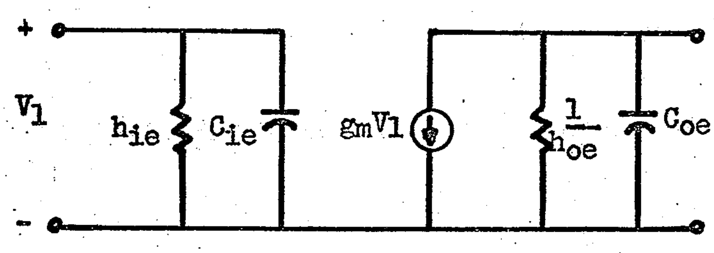


Element No.		Element No.	
	C_1 = .01 f		R_3 = 220
1 -	C_2 = 80pf	8 -	R_4 = 220
2 -	C_3 = 1008.2pf	9 -	R_5 = 892
5 -	C_4 = 80pf		L_1 = 2.7 H
6* -	R_1 = 12K		L_2 = 2.7 H
6* -	R_2 = 2.7K		M = 0.33 H
	Q1: 40245		

Fig. 10. Experimental Circuit # 3

* R_1, R_2 and h_{ie} are parallel resistors for A.C. analysis and are combined.

was used is shown in Fig. 11.



Element No.

6^* - $h_{ie} = 1.5K\Omega$

7 - $1/h_{oe} = 80K\Omega$

16 - $g_m = 70\mu\mho$

3 - $C_{ie} = 8.2pf$

4 - $C_{oe} = 1.5pf$

Fig. 11 Transistor equivalent circuit

Faults, which would affect the operation of the circuit near the I.F. frequency of 10.7 MHz, were analyzed by computer simulations. Since, as noted from Graph XV of the previous test circuit, the coupling capacitor acts as a short circuit at high frequencies, the emitter resistor is neglected in analyzing the circuit. Frequency scaling of 10^{-7} was used and the roots of the voltage transfer function were found to be:

zeroes: 0,0
poles: -.103, -.0043-j5.13, -.0043+j5.13, -.0088-j6.82,
-.0088+j6.82, -310.

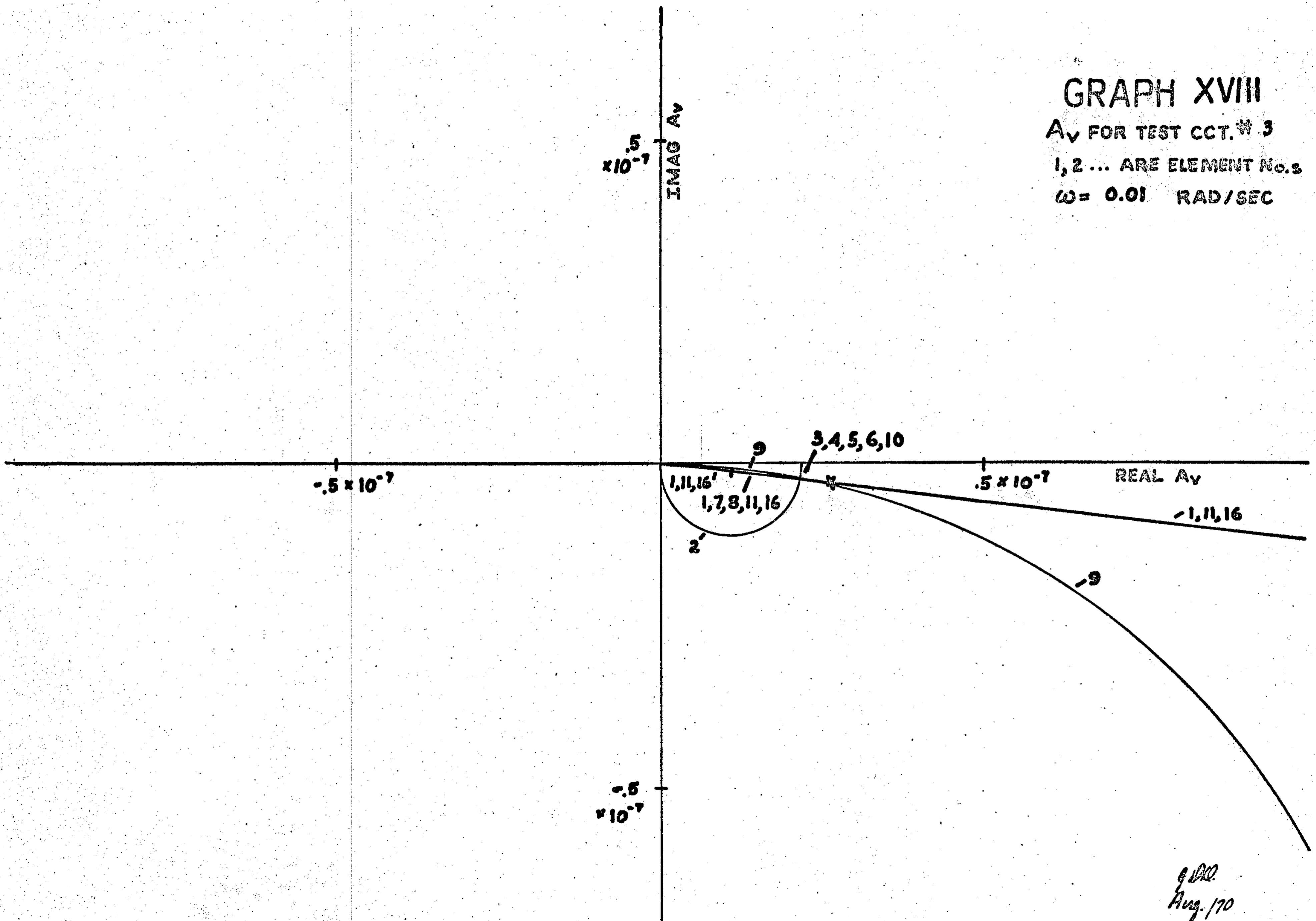
The test frequencies chosen were: 0.01, 1.0, 5.13, 6.0 and 6.82 radians/second. The loci of $T(x_i)$, which are plotted on Graphs XVIII to XXII, do not appear to isolate many faults uniquely, since the loci of two or more $T(x_i)$ are almost the same in many cases. However, if the graphs are examined systematically, the fault separation can be improved. For example, if a transfer function measurement, T_F, corresponding to $T(x_9)$ with x_9 faulty at $\omega = 5.13$ radians/second (Graph XX), is found to be near the loci $T(x_7)$, $T(x_9)$ and $T(x_{16})$, then the faulty component is either x_7, x_9 or x_{16}, but Graph XX does not indi-

GRAPH XVIII

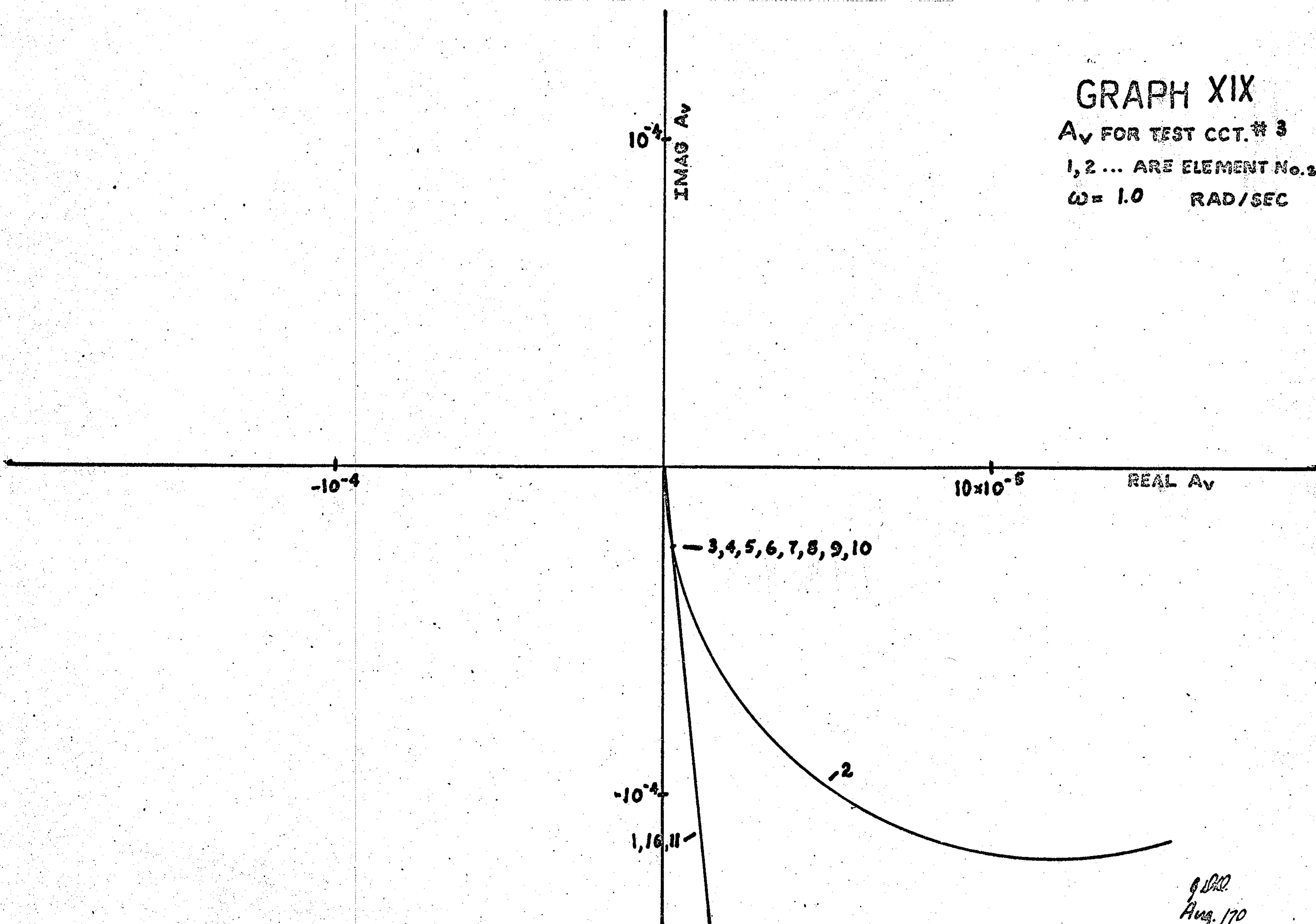
A_v FOR TEST CCT. # 3

1, 2 ... ARE ELEMENT No.s

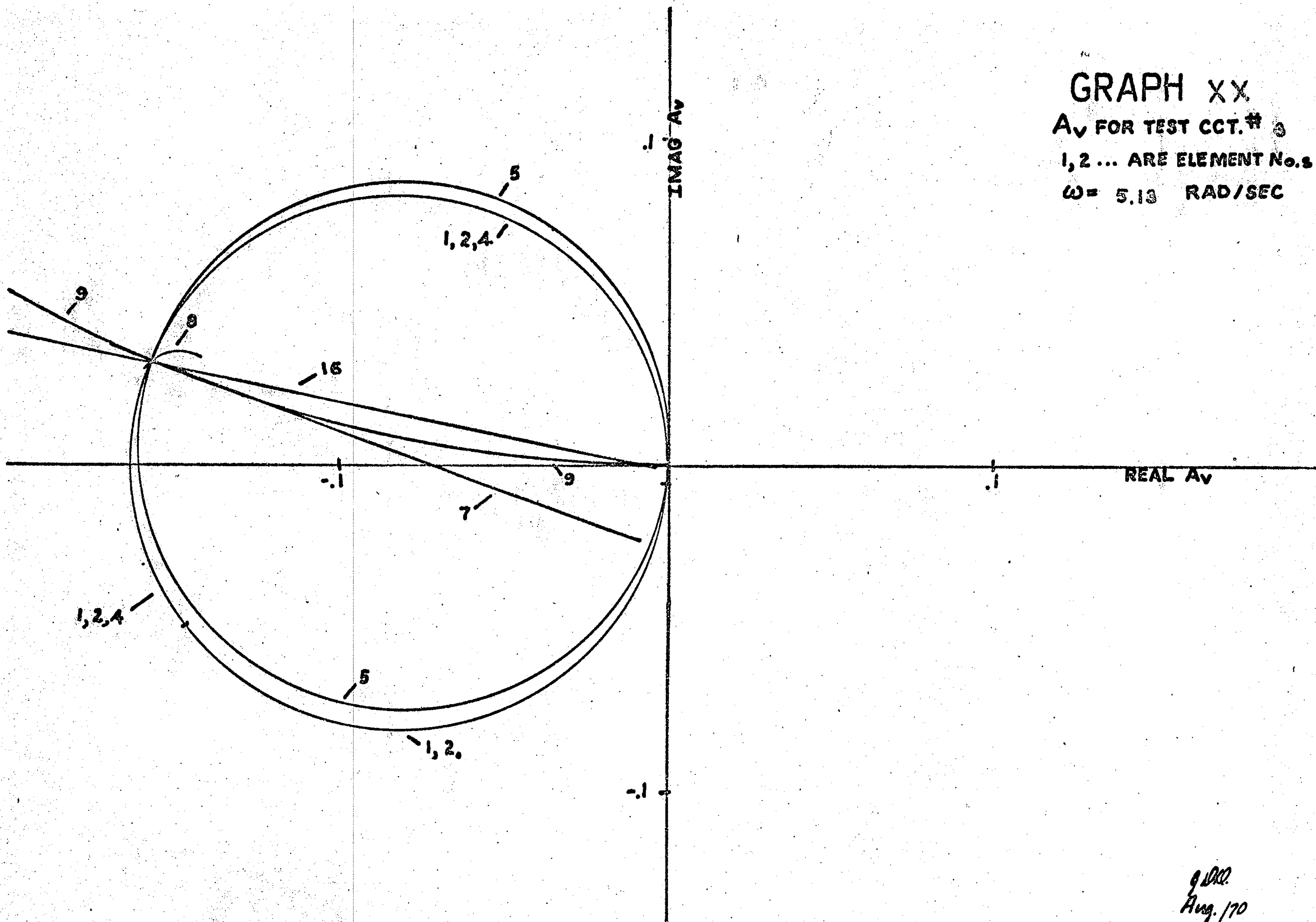
ω = 0.01 RAD/SEC

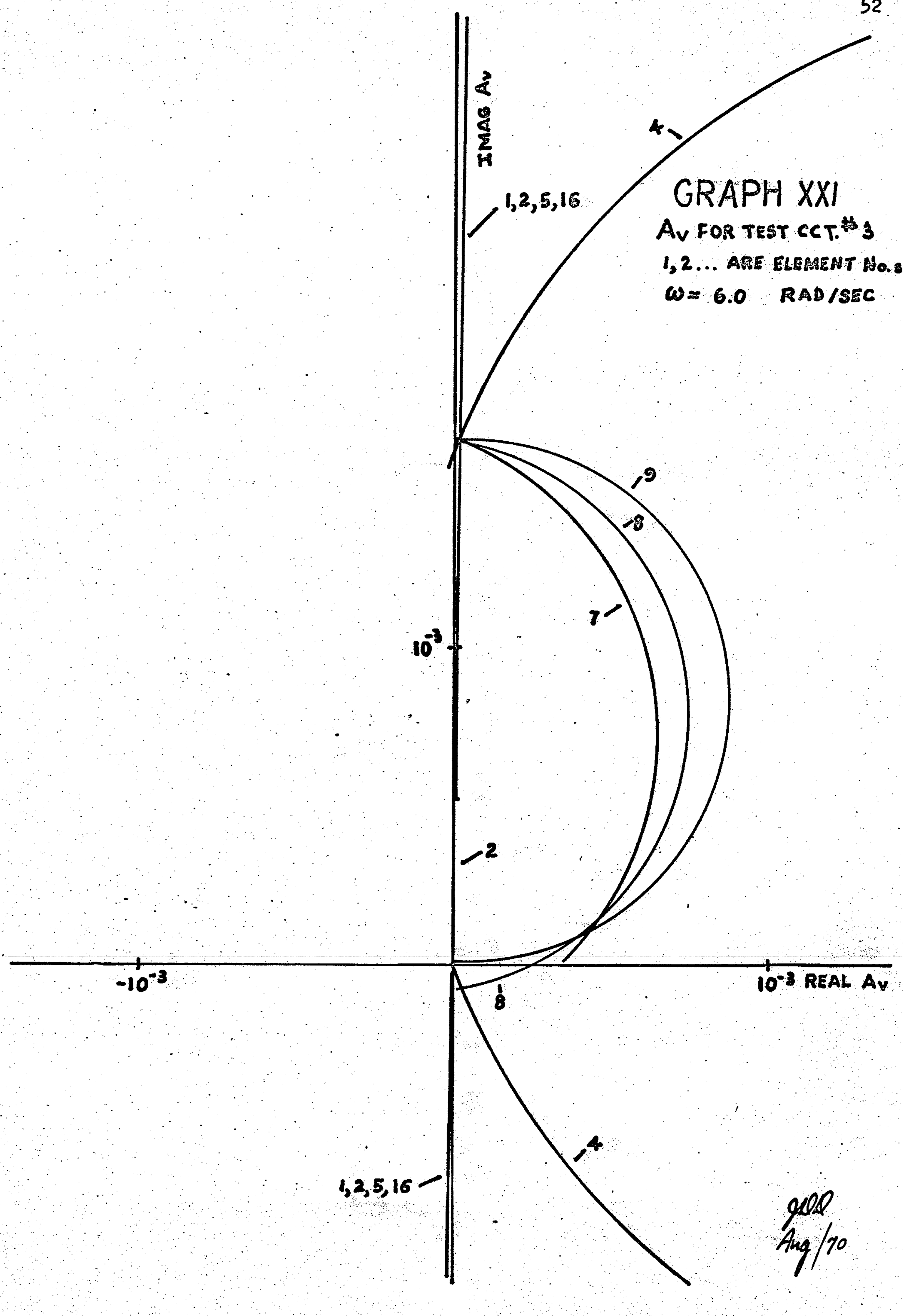


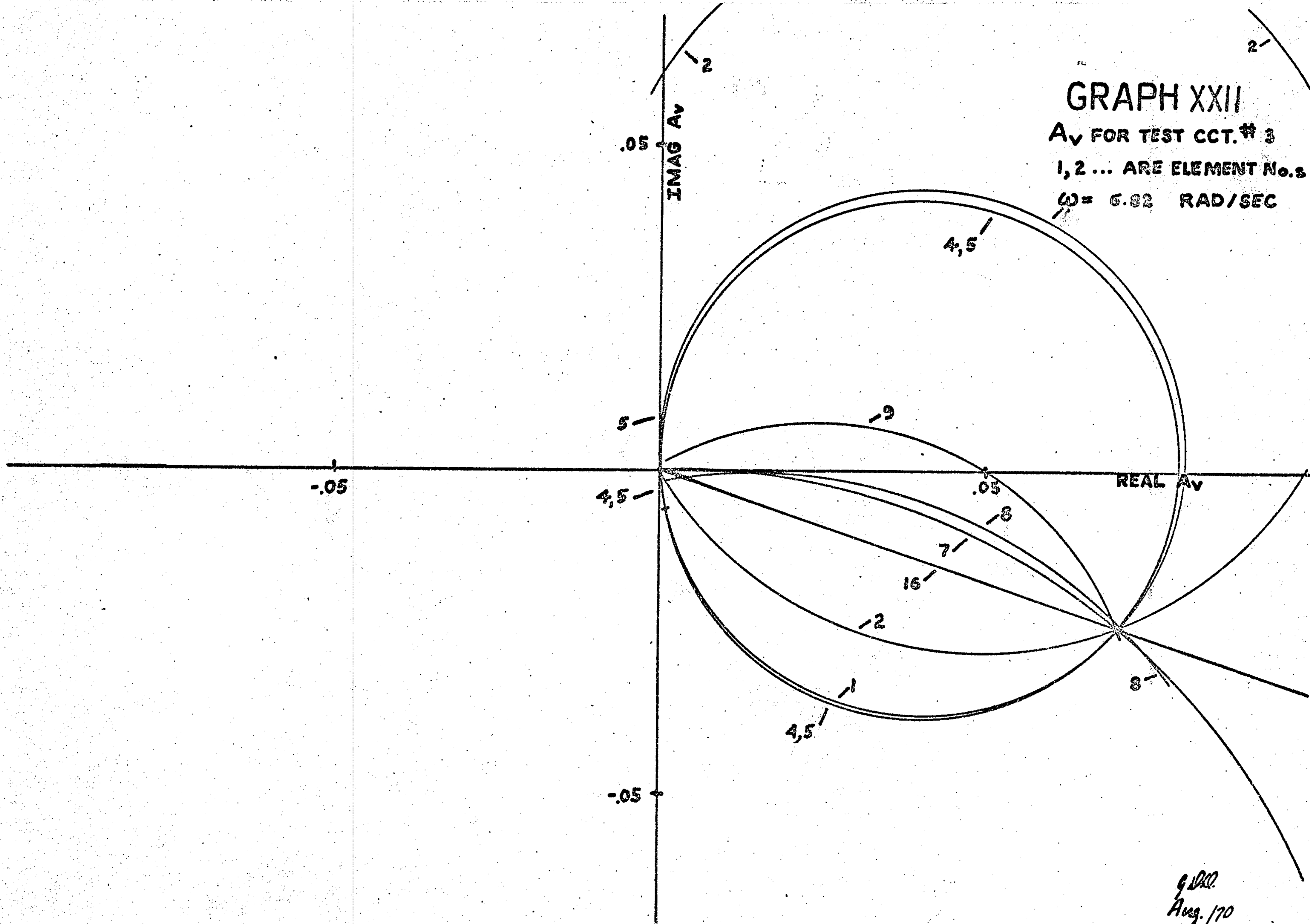
Aug./70



GRAPH XIX
Av FOR TEST CCT. # 3
1, 2 ... ARE ELEMENT No.s
ω = 1.0 RAD/SEC
IMAG Av
10^-4
-10^-4
REAL Av
10×10^-5
3,4,5,6,7,8,9,10
2
1,16,11


GRAPH XX
Av FOR TEST CCT.# 3
1,2 ... ARE ELEMENT No.s
ω= 5.13 RAD/SEC
IMAG Av
REAL Av
.1
-.1
.1
-.1
5
1,2,4
9
8
16
9
7
1,2,4
5
1,2,
Aug./70


GRAPH XXI
A_V FOR TEST CCT. # 3
1, 2 ... ARE ELEMENT No.s
ω = 6.0 RAD/SEC
IMAG A_V
REAL A_V
10^{-3}
-10^{-3}
10^{-3}
1,2,5,16
4
9
8
7
2
8
4
1,2,5,16


GRAPH XXII
Av FOR TEST CCT. # 3
1, 2 ... ARE ELEMENT No.s
ω = 6.82 RAD/SEC
IMAG Av
REAL Av
.05
-.05
.05
-.05
2
2
4,5
5
4,5
9
8
7
16
2
1
4,5
8
Aug./70

cate which one. An examination of Graph XXI allows us to eliminate x_{16}(if T_F is not near the nominal transfer function point). An examination of GraphXXII is then used to eliminate x_7, leaving us with the unique identification: x_9 is faulty. Similar methods must be employed in isolating other faults of this circuit. The computer output of this example is included in Appendix B.

VI

COMPARISON OF THE VARIOUS FAULT ISOLATION TECHNIQUES

Of the four methods of fault isolation considered, none appears to surpass the others in every aspect.

The Neu method was presented only for resistive networks, and a complete comparison with the other three methods must be based partly on conjecture, since the addition of frequency dependent elements might alter some of the procedures. The Neu method is the only method based on more than one of the n-port parameters. Generally the other methods have the advantage of being based on analysis of only one of the transfer or driving point functions, although as was pointed out, previously, additional ones may be required on occasion. The Neu method, as well as the coefficient signature and the Moebius transform methods permit the value of the fault to be calculated directly and are not subject to the interpolation problem of the faulty dictionary technique.

The coefficient signature method of Moroz[3], which was studied in some detail, did not appear as favourable as the Moebius transform method. One of the problems, encountered, was that of fitting a ratio of polynominals to the measured data. Several methods of curve fitting were examined, and, although the linear programming method appeared to be adequate, this method (or probably any curve fitting method) requires that a large computer be available after the fault diagnosis measurements have been completed. However, the method could be attractive for large firms or integrated circuit manufacturers having access to suitable computer facilities. The fault dictionary method, presented by Seshu and Waxman[6] is the only method which isolates faults on the basis of the magnitude response measurement only. This reduction in the number of test measurements is counter-balanced by an increase in the analysis requirements.

The new method presented in this thesis, is the only one which presents a graphical picture of the behaviour of the n-port parameter for all possible faults. Some of the advantages of this graphical approach include: (i) the response of the network for the extremes of parameter behaviour (short circuit or open circuit) is immediately available; (ii) the graphs indicate whether the test points which were selected are adequate in isolating all the faults or whether some of the test frequencies are redundant; this permits the test designer to choose the required test frequencies without finding the poles and zeroes of the circuit to be tested, and to reduce the number of test frequencies to a minimum by eliminating the redundant frequencies; (iii) the graphs show the test measurement error which is permissible for correct fault detection; (iv) if the fault is intermittant in such a manner that the value of the faulty component changes in value between test measurements, the faulty component could still be identified (although not evaluated) since the graphical technique is based on the comparison of the $T(x_i)$ loci (rather than a specific $T(x_i)$ points) with the faulty $T_F(x)$.

Although both phase and magnitude measurements must be used for the new method of fault diagnosis, this disadvantage is offset by a reduction in the number of test measurements that must be performed. For example, if in Example #1 (page 18) the test frequency of Graph # 1 had been selected for the first measurement, the fault $C_1 = 12f$ would have been isolated immediately and the remaining test frequencies would have been unneccessary. It was pointed out, previously, that N_M, the number of transfer or driving point functions that must be evaluated for the new technique, is typically less than 20% of N_S, the number required for the fault dictionary technique. The coefficient signature method of Moroz also requires N_M transfer functions, and it would appear that the method of Neu would require more transfer function calculations since it is based on more than a single driving point or transfer function.

The reduction in the number of transfer function calculations permits the use of the state equation technique of circuit analysis without an increase in the computation time. The previously discussed advantages of this analysis technique, namely, a decrease in computer storage, and a simultaneous solution of all n-port parameters, are of significance.

The method described in this thesis is the only one of the four which is feasible for implementation without the use of a large computer. Measurements and simple calculations can be used for the fault diagnosis precomputations.

The value of a faulty component is calculated precisely in each of the methods, except in the fault dictionary method where the value can only be determined as to the nearest discrete value listed in the dictionary. Thus the Moebius transform method avoids the interpolation problem associated with the fault dictionary method.

Although certain applications may favor the use of any one of the fault diagnosis methods considered, the versatility in the implementation of the new method presented in this thesis, and the advantages in its use, should make the Moebius transformation method the most widely used.

VII

CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER WORK

The results of simulations and laboratory measurements indicate that this new procedure of isolating faults in linear circuits is practical. An attractive feature is the variety of methods that can be used in the initial computation procedure: ECAP, topological formulas, or state equations may be used in analyzing the equivalent circuit of the network; the actual circuit may be analyzed by performing measurements if most of the elements are accessible,as in the case of discrete component networks. The calculations in the new fault isolation technique can be done with a small digital computer (or even a desk calculator) if the circuit is analyzed by laboratory measurements. However, a computer analysis of the equivalent circuit of the network is preferable if a large computer is available.

The visual indication of the behaviour of the network function with a variation of parameters is highly desirable. It provides a method of setting a confidence limit for the diagnosed faults, and provides an indication as to possible modifications to the circuit for a desired response. The graphs indicate which test frequencies are significant, and thus provide a means of eliminating redundant tests.

The precomputed coefficients of the Moebius transformation permit a direct computation of the fault value with a desk calculator or even a slide rule. This solves the interpolation problem of the fault dictionary method.

The problem of isolating a fault which also changes the value of a bias dependent parameter remains to be solved, as does the problem of those faults which cause non-linear circuit operation such as oscillation or clipping. The effect of allowing the components, which are not faulty, to assume any value within their tolerance limits, rather than holding them at their nominal value, should be investigated.

This new method for fault isolation allows the manufacturer to include a relatively simple trouble-shooting guide with a piece of eqiupment in order to aid in fault diagnosis. The calculations required in compiling the guide are fewer than those of any other known technique and can be performed in a variety of ways. This method is also applicable to the fault diagnosis in integrated circuits and could be an aid to the manufacturer for process correction. Other applications, such as circuit modification and fault analysis in remote equipment, may aid in giving this new method widespread use.

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APPENDIX A

The fault diagnosis computer program

The Fortran IV computer program, shown below, was used for the fault diagnosis computations.

page 1 of 3

```
      DIMENSION PIX(2,2),S(2,2),PI(2,2),T(20),T1(20),T2(20),T3(20),
     1 WORK(5,5),W(20),XNUM(20),XDEN(20),X(20,20),EL(20),XMAGI(20),
     1 TANGL(6),F(20)
      COMPLEX S,T1,T2,T3,X,PIX,T,QI,DTX,DEN,CENTRE,RADIUS,TZERO,TINFIN
      COMMON PIX,PI,S
      READ(5,1) NRTFCN,NCTFCN
      READ (5,2) Z,Y,TOLZER
      READ (5,1) NFREQ
1     FORMAT (2I2)
      READ (5,2) (F(I),I=1,NFREQ)
2     FORMAT (G15.8)
C     MAGN. OF TR. FCN. IS READ AS XNUM AND PHASE AS XDEN
      READ (5,40) (XNUM(I),XDEN(I),I=1,NFREQ)
40    FORMAT (2G10.4)
      DO 41 I=1,NFREQ
      W(I)=2.*3.14159*F(I)
      XDEN(I)=.01745329*XDEN(I)
41    T(I)=CMPLX(XNUM(I)*COS(XDEN(I)),XNUM(I)*SIN(XDEN(I)))
C     READING THE COMPUTED TRANSFER FCNS. FROM STATEEQ PROG. FOR ELEMENT
C     VALUES OF X1,X2,X3
      READ (3,3) NOELEM
3     FORMAT (I2)
      READ (5,4) (EL(I),I=1,NOELEM)
      DO 21 L=1,NOELEM
      X1=Z*EL(L)
      X2=Y*EL(L)
      X3=EL(L)
      DO 29 LL=1,3
      IF(L.GT.1.AND.LL.EQ.3)  GO TO 13
      READ(3,1) NR,M
      IDEG=M+1
      DO 5 K=1,IDEG
      READ(3,4) ((WORK(I,J),I=1,NR),J=1,NR)
4     FORMAT (G15.8)
5     XNUM(IDEG-K+1)=WORK(NRTFCN,NCTFCN)
      READ(3,4) (XDEN(M-K+1),K=1,M)
      XDEN(IDEG)=1.
      DO 6 I=1,NFREQ
      WI=W(I)
      CALL MAG(XNUM,XDEN,M,WI,XR,XI,XMAG,IER)
      IF(IER.EQ.1)  WRITE (6,7) LL,I
7     FORMAT (' MAGNITUDE OF T',I1,' (',I2,' )IS INFINITE')
      IF(IER.NE.1.AND.LL.EQ.1) T1(I)=CMPLX(XR,XI)
      IF(IER.NE.1.AND.LL.EQ.2) T2(I)=CMPLX(XR,XI)
      IF(IER.NE.1.AND.LL.EQ.3) T3(I)=CMPLX(XR,XI)
      XMAGI(I)=XMAG
6     CONTINUE
      WRITE(6,28)
28    FORMAT (' MAGNITUDE OF TR. FCN. IS')
      WRITE(6,12) (XMAGI(I),I=1,NFREQ)
29    CONTINUE
13    WRITE (6,14)
14    FORMAT (4X,' REAL T1',7X,' IMAG T1',7X,' REAL T2',7X,' IMAG T2',
     1 7X,' REAL T3',7X,' IMAG T3',7X,' REAL T ',7X,'IMAG T ')
      WRITE(6,12) (T1(I),T2(I),T3(I),T(I),I=1,NFREQ)
12    FORMAT (8(3X,G12.5))
      DO 21 I=1,NFREQ
C     TO SEE IF THE TR.FCN.IS INDEPENDENT OF ELEMENT(J)
      IF(ABS(REAL(T1(I)-T2(I))).LT.TOLZER.AND.ABS(AIMAG(T1(I)-T2(I))).LT
     1 .TOLZER.OR.ABS(REAL(T2(I)-T3(I))).LT.TOLZER.AND.ABS(AIMAG(T2(I)
     2 -T3(I))).LT.TOLZER)  GO TO 20
      PI(2,1)=X1-X2
      PI(2,2)=X2-X3
```

```
      PI(1,1)=PI(2,1)*X3
      PI(1,2)=PI(2,2)*X1
      S(1,1)=T2(I)-T3(I)
      S(1,2)=-S(1,1)*T1(I)
      S(2,1)=T2(I)-T1(I)
      S(2,2)=-S(2,1)*T3(I)
      CALL MULTCX
      WRITE (6,15) I,L
15    FORMAT (' PIX(W=',I2,' ) FOR ELEMENT',I2)
C    PIX(1,1)=D,  PIX(1,2)=-B,  PIX(2,1)=-C,  PIX(2,2)=A IF WE WRITE  T=
C    (AX+B)/(CX+D)
      WRITE (6,22) ((PIX(K,J),J=1,2),K=1,2)
22    FORMAT (2(3X,G15.8))
      QI=PIX(1,1)-PIX(2,1)*EL(L)
      IF (ABS(REAL(QI)).LT.TOLZER.AND.ABS(AIMAG(QI)).LT.TOLZER) GO TO 18
      DTX=(PIX(1,1)*PIX(2,2)-PIX(1,2)*PIX(2,1))/QI**2
      QI=-PIX(2,1)/QI
      GO TO 42
18    QI=CMPLX(10.**20,10.**20)
      DTX=CMPLX(10.**20,10.**20)
42    DEN=CONJG(PIX(1,1))*PIX(2,1)-CONJG(PIX(2,1))*PIX(1,1)
      RADIUS=PIX(1,1)*PIX(2,2)-PIX(1,2)*PIX(2,1)
      IF(CABS(RADIUS)*TOLZER.GT.CABS(DEN))  GO TO 30
      CENTRE=(CONJG(PIX(1,1))*PIX(2,2)-CONJG(+PIX(2,1))*PIX(1,2))/(-DEN)
      RADIUS=CMPLX(CABS(RADIUS /DEN),0.)
      TZERO=-PIX(1,2)/PIX(1,1)
      TINFIN=-PIX(2,2)/PIX(2,1)
      WRITE(6,34) CENTRE,RADIUS,TZERO,TINFIN
34    FORMAT (40X,' CENTRE AT',2(3X,G12.5),' RADIUS OF',2(3X,G12.5),/
     1       40X,' TZERO=   ',2(3X,G12.5),' TINFIN=  ',2(3X,G12.5))
      GO TO 19
C    TO GET REAL + IMAG AXIS INTERCEPTS
30    DEN=PIX(1,1)*CONJG(PIX(2,2))-PIX(2,1)*CONJG(+PIX(1,2))
      A=AIMAG(+PIX(1,2)*CONJG(PIX(2,2)))
      YINT=-A/REAL(DEN)
      XINT=-A/AIMAG(DEN)
      WRITE(6,38) XINT,YINT
38    FORMAT(/' REAL + IMAG. AXIS INTERCEPTS IN THE T-PLANE ARE:',
     1 2(3X,G12.5))
C
19    WRITE (6,17) I,QI
17    FORMAT (' THE 1/(Q+X) IN NEUS K AT W=',I2,' IS ',2(3X,G12.5))
      DTXMAG=CABS(DTX)
       WRITE (6,24) I,DTX,DTXMAG
24    FORMAT(' PARTIAL OF T W.R.T. X AT W',I2,' IS',3(3X,G12.5))
      X(L,I)=(PIX(1,1)*T(I)+PIX(1,2))/(PIX(2,1)*T(I)+PIX(2,2))
      GO TO 21
20    X(L,I)=-1.
21    CONTINUE
33    WRITE (6,16)
16    FORMAT (' THE COMPLEX SOLNS FOR X(W,ELEMENT) ARE:')
      WRITE (6,23) ((X(I,J),J=1,NFREQ),I=1,NOELEM)
23    FORMAT (2(3X,G12.5))
      WRITE (6,27) NRTFCN,NCTFCN,Z,Y,TOLZER
27    FORMAT(' TR.FCN.,Z,Y,TOLZER ARE:',2I2,3(3X,G12.5))
      WRITE (6,25)
25    FORMAT(' NOM. CCT.ELEMENTS ARE:')
      WRITE (6,26) (EL(I),I=1,NOELEM)
26    FORMAT (/10G12.5)
      WRITE(6,26) (W(I),I=1,NFREQ)
      CALL EXIT
      END
```



```
      SUBROUTINE MAG (A,B,N,OMEGA,VREAL,VIMAG,VMAG,IER)
      DIMENSION A(10),B(10)
      IER=0
C    COMPUTE REAL PART OF NUMERATOR AND DENOMINATOR
      AREAL=A(1)
      BREAL=B(1)
      S=1.
      DO 100 J=2,N,2
      JA=J+1
      S=-S
      CREAL=S*OMEGA**J
      AREAL=AREAL+A(JA)*CREAL
100   BREAL=BREAL+B(JA)*CREAL
C     COMPUTE IMAGINARY PART OF NUMERATOR AND DENOMINATOR
      AIMAG=0.
      BIMAG=0.
      S=1.
      DO 105 J=1,N,2
      JA=J+1
      S=-S
      CIMAG=S*OMEGA**J
      AIMAG=AIMAG-A(JA)*CIMAG
105   BIMAG=BIMAG-B(JA)*CIMAG
C    TEST TO SEE IF FUNCTION IS INFINITE
      DENOM=BREAL**2+BIMAG**2
      IF (DENOM) 110,115,110
115   WRITE (6,120) OMEGA
120   FORMAT (1H0,29HMAGNITUDE IS INFINITE, OMEGA=,1PE15.8/)
      IER=1
      RETURN
C    COMPUTE MAGNITUDE OF FUNCTION
110   VREAL=(AREAL*BREAL+AIMAG*BIMAG)/DENOM
      VIMAG=(AIMAG*BREAL-AREAL*BIMAG)/DENOM
      VMAG=SQRT(VREAL**2+VIMAG**2)
      RETURN
      END
      SUBROUTINE MULTCX
      COMPLEX A,C
      COMMON A,B,C
      DIMENSION A(2,2),B(2,2),C(2,2)
      A(1,1)=B(1,1)*C(1,1)+B(1,2)*C(2,1)
      A(1,2)=B(1,1)*C(1,2)+B(1,2)*C(2,2)
      A(2,1)=B(2,1)*C(1,1)+B(2,2)*C(2,1)
      A(2,2)=B(2,1)*C(1,2)+B(2,2)*C(2,2)
      RETURN
      END
```

APPENDIX B

In order to relate the computer output to the description used in this thesis, the significant output for Example # 1 will be listed. The coefficients a, b, c and d, of the Moebius transformation, are listed as real and imaginary parts under the heading PIX. For example, at test frequency # 1, the a, b, c, d corresponding to x_1 are:

$$a = 0.7225 + j2.348$$
$$b = -0.795 - j2.583$$
$$c = 0.2347 - j.0072$$
$$d = -0.2338 + j.00072$$

The centre of the arc of $T(x_1)$ is at 0.5998-j.000064; the radius of the circle is 0.50018; T(0) is 1.1-j.0000995, and $T(\infty)$ is at .0999-j.002998.

Computer output for Example # 1

The following output was obtained at the test frequencies 0.01, 0.3, 3.0 and 100. radians/second for the driving point impedance, Z_{11}, of Example # 1.

```
PIX(W= 1 ) FOR ELEMENT 1
   0.72252750        2.3483419
  -0.79501563       -2.5831099
   0.23474693E-01   -0.72252750E-02
  -0.23382902E-02    0.72312355E-03
                                        CENTRE AT    0.59983     -0.64054E-04 RADIUS OF   0.50018       0.0
                                        TZERO=       1.1000      -0.99535E-04 TINFIN=     0.99650E-01  -0.13314E-03
THE 1/(Q+X) IN NEUS K AT W= 1 IS    0.99045E-03   0.98975E-02
PARTIAL OF T W.R.T. X AT W 1 IS   -0.19606E-02   -0.97049E-02   0.99010E-02
PIX(W= 2 ) FOR ELEMENT 1
  -1.2217350        -2.5791645
   1.3515081         2.8331965
  -0.77375710        0.36651719
   0.76207280E-01   -0.38938522E-01
                                        CENTRE AT    0.59991     -0.29970E-02 RADIUS OF   0.50000       0.0
                                        TZERO=       1.0999      -0.29890E-02 TINFIN=     0.99910E-01  -0.29981E-02
THE 1/(Q+X) IN NEUS K AT W= 2 IS    0.90000E-01   0.30000E-01
PARTIAL OF T W.R.T. X AT W 2 IS   -0.18000E-01    0.24000E-01   0.30000E-01
PIX(W= 3 ) FOR ELEMENT 1
  -0.36558347E-01   -0.44927597E-02
   0.40136874E-01    0.39004087E-02
  -0.13493599E-01    0.10995150
  -0.17882560E-02   -0.10458715E-01
                                        CENTRE AT    0.59164     -0.27532E-01 RADIUS OF   0.49990       0.0
                                        TZERO=       1.0915      -0.27377E-01 TINFIN=     0.91743E-01  -0.27523E-01
THE 1/(Q+X) IN NEUS K AT W= 3 IS    0.99889E-01   0.33303E-02
PARTIAL OF T W.R.T. X AT W 3 IS   -0.22172E-03    0.33222E-02   0.33296E-02
PIX(W= 4 ) FOR ELEMENT 1
  -0.33393735E-04    0.13969839E-05
   0.33349497E-04   -0.20989683E-06
  -0.12227101E-04    0.33334456E-02
  -0.32992510E-04   -0.34215045E-05
                                        CENTRE AT    0.50012     -0.99031E-02 RADIUS OF   0.49913       0.0
                                        TZERO=       0.99719      0.35431E-01 TINFIN=     0.99010E-03  -0.99010E-02
PIX(W= 1 ) FOR ELEMENT 2
  -0.95367432E-06    0.25123358E-04
  -0.14478710E-05   -0.27481423E-04
  -0.95367432E-06    0.11920929E-06
   0.10302847E-05   -0.22446329E-06
                                        CENTRE AT    1.0914      -0.99107E-01 RADIUS OF   0.13115E-02   0.0
                                        TZERO=       1.0901      -0.99010E-01 TINFIN=     1.0927       -0.98782E-01
THE 1/(Q+X) IN NEUS K AT W= 1 IS   -0.47676E-02  -0.38141E-01
PARTIAL OF T W.R.T. X AT W 1 IS    0.19700E-06   -0.10004E-03   0.10004E-03
PIX(W= 2 ) FOR ELEMENT 2
   0.67070127E-04    0.74599883E-03
  -0.23721374E-03   -0.12907881E-03
   0.22143126E-04   -0.17583370E-05
  -0.16649719E-05    0.67919027E-05
                                        CENTRE AT    0.14946     -0.29996     RADIUS OF   0.50544E-01   0.0
                                        TZERO=       0.20000     -0.30000     TINFIN=     0.98924E-01  -0.29887
THE 1/(Q+X) IN NEUS K AT W= 2 IS    0.57022E-03   0.29647E-01
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PIX(W= 3 ) FOR ELEMENT 2
  0.49231052E-02    0.42111576E-02
 -0.63799182E-03   -0.26186882E-03
  0.12632608E-02   -0.14768764E-02
  0.47777081E-04    0.43692766E-04
                                      CENTRE AT     0.51108E-01   -0.33295E-01 RADIUS OF   0.50002E-01    0.0
                                      TZERO=        0.10111       -0.33296E-01 TINFIN=     0.11051E-02   -0.33295E-01
THE 1/(Q+X) IN NEUS K AT W= 3 IS    0.82565E-01    0.27522
PARTIAL OF T W.R.T. X AT W 3 IS   -0.15150E-01   -0.22978E-01    0.27523E-01
PIX(W= 4 ) FOR ELEMENT 2
 -0.30329777E-03   -0.11540577E-03
  0.30445153E-04    0.11238895E-C4
 -0.11542160E-02    0.30329898E-02
 -0.30320516E-05   -0.11572993E-05
                                      CENTRE AT     0.50001E-01   -0.99990E-03 RADIUS OF   0.50000E-01    0.0
                                      TZERO=        0.10000       -0.99530E-03 TINFIN=     0.99085E-06   -0.10001E-02
THE 1/(Q+X) IN NEUS K AT W= 4 IS    0.99010        0.99009E-01
PARTIAL OF T W.R.T. X AT W 4 IS   -0.19606E-02    0.97049E-02    0.99009E-02
MAGNITUDE OF TR. FCN. IS
   0.59928        0.34502        0.11186        0.10946E-01
MAGNITUDE OF TR. FCN. IS
   1.5824         0.36316        0.11070        0.10946E-01
   REAL T1         IMAG T1         REAL T2        IMAG T2        REAL T3         IMAG T3         REAL T          IMAG T
  0.59875        -0.25038E-01    1.5670         -0.22015        1.0901         -0.99110E-01    1.0858         -0.11840
  0.25376        -0.23377        0.17050        -0.32064        0.19991        -0.30300        0.17154        -0.26088
  0.93955E-01    -0.60709E-01    0.92483E-01    -0.60840E-01    0.92853E-01    -0.60819E-01    0.92514E-01    -0.55279E-01
  0.99210E-03    -0.10901E-01    0.99076E-03    -0.10901E-01    0.99110E-03    -0.10901E-01    0.99079E-03    -0.10734E-01
PIX(W= 1 ) FOR ELEMENT 3
 -0.23483467        0.72261274E-01
  0.23477197E-01   -0.72498322E-02
  0.72250366E-02    0.23483515E-01
 -0.23555696        0.69913149E-01
                                      CENTRE AT     0.99996E-01   -5.0001       RADIUS OF   5.0000         0.0
                                      TZERO=        0.10000       -0.99727E-04 TINFIN=     0.99558E-01   -10.000
THE 1/(Q+X) IN NEUS K AT W= 1 IS    0.98966E-02    0.99010E-01
PARTIAL OF T W.R.T. X AT W 1 IS    0.97049       -0.19606         0.99010
PIX(W= 2 ) FOR ELEMENT 3
  0.85972250E-02   -0.40723532E-02
 -0.84675848E-03    0.43262076E-03
 -0.12217283E-01   -0.25791734E-01
  0.98951757E-02   -0.15321933E-02
                                      CENTRE AT     0.99909E-01   -0.16966      RADIUS OF   0.16667        0.0
                                      TZERO=        0.99911E-01   -0.29950E-02 TINFIN=     0.99910E-01   -0.33633
THE 1/(Q+X) IN NEUS K AT W= 2 IS    0.90000        0.30000
PARTIAL OF T W.R.T. X AT W 2 IS   -0.80000E-01   -0.60000E-01    0.10000E 00
PIX(W= 3 ) FOR ELEMENT 3
  0.14901161E-05   -0.12211502E-04
  0.19924482E-06    0.11610391E-05
 -0.36650896E-03   -0.44964254E-C4
  0.36361016E-04   -0.18179184E-04
                                      CENTRE AT     0.91743E-01   -0.44182E-01 RADIUS OF   0.16674E-01    0.0
                                      TZERO=        0.91721E-01   -0.27508E-01 TINFIN=     0.91743E-01   -0.60856E-01
THE 1/(Q+X) IN NEUS K AT W= 3 IS    0.99891        0.33280E-01
PARTIAL OF T W.R.T. X AT W 3 IS   -0.11074E-02   -0.73893E-04    0.11098E-02
PIX(W= 4 ) FOR ELEMENT 3
  0.0               0.18626451E-08
 -0.20638158E-10   -0.18429702E-11
 -0.33341348E-06    0.37252903E-08
  0.28950264E-09   -0.36382177E-08
                                      CENTRE AT     0.99077E-03   -0.10991E-01 RADIUS OF   0.89536E-04    0.0
                                      TZERO=        0.98944E-03   -0.11080E-01 TINFIN=     0.99010E-03   -0.10901E-01
THE 1/(Q+X) IN NEUS K AT W= 4 IS    1.0000        -0.55864E-02
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PARTIAL OF T W.R.T. X AT W 4 IS   -0.10004E-05    0.36901E-08    0.10004E-05
MAGNITUDE OF TR. FCN. IS
   1.0448         0.33608        0.64435E-01    0.10788E-01
MAGNITUDE OF TR. FCN. IS
   1.1444         0.39552        0.15439        0.10976E-01
   REAL T1         IMAG T1         REAL T2        IMAG T2        REAL T3         IMAG T3         REAL T          IMAG T
  1.0401         -0.99035E-01    1.1401         -0.99235E-01    1.0901         -0.99110E-01    1.0858         -0.11840
  0.14999        -0.30075        0.24970        -0.30674        0.19991        -0.30300        0.17154        -0.26088
  0.50010E-01    -0.40631E-01    0.12585        -0.89429E-01    0.92853E-01    -0.60819E-01    0.92514E-01    -0.55279E-01
  0.19241E-02    -0.10615E-01    0.66472E-03    -0.10956E-01    0.99110E-03    -0.10901E-01    0.99079E-03    -0.10734E-01
PIX(W= 1 ) FOR ELEMENT 4
 -0.25000283E-03    0.75057011E-06
  0.24745381E-03   -0.25495945E-C4
 -0.48428774E-07    0.25063728E-05
 -0.25019050E-03   -0.17371494E-05
                                      CENTRE AT     0.99037       -49.973       RADIUS OF   49.874         0.0
                                      TZERO=        0.99010       -0.99010E-01 TINFIN=    -1.2352        -99.798
THE 1/(Q+X) IN NEUS K AT W= 1 IS   -0.21376E-03    0.10025E-01
PARTIAL OF T W.R.T. X AT W 1 IS    0.99999       -0.19997E-02    0.99999
PIX(W= 2 ) FOR ELEMENT 4
 -0.24859793E-03    0.22426364E-C4
  0.18132007E-04   -0.76822238E-04
  0.67129731E-05    0.74604061E-04
 -0.27164770E-03    0.16974984E-04
                                      CENTRE AT     0.10003       -1.9661       RADIUS OF   1.6661         0.0
                                      TZERO=        0.10000       -0.30000      TINFIN=     0.99301E-01   -3.6323
THE 1/(Q+X) IN NEUS K AT W= 2 IS    0.89292E-02    0.29983
PARTIAL OF T W.R.T. X AT W 2 IS    0.99730       -0.59893E-01    0.99910
PIX(W= 3 ) FOR ELEMENT 4
 -0.14036936E-03    0.16410521E-03
 -0.53082586E-05   -0.48559450E-05
  0.49230992E-03    0.42110542E-03
 -0.15493567E-03    0.18002954E-03
                                      CENTRE AT     0.11094E-02   -0.19996      RADIUS OF   0.16667        0.0
                                      TZERO=        0.11101E-02   -0.33296E-01 TINFIN=     0.11079E-02   -0.36663
THE 1/(Q+X) IN NEUS K AT W= 3 IS    0.82567        2.7523
PARTIAL OF T W.R.T. X AT W 3 IS    0.76593       -0.50501         0.91743
PIX(W= 4 ) FOR ELEMENT 4
  0.11542488E-06   -0.30329409E-06
  0.30313529E-09    0.11576162E-09
 -0.30329684E-04   -0.11542046E-04
  0.12699257E-06   -0.33361442E-06
                                      CENTRE AT     0.99242E-06   -0.59999E-02 RADIUS OF   0.50000E-02    0.0
                                      TZERO=        0.11438E-05   -0.99991E-03 TINFIN=     0.99899E-06   -0.11000E-01
THE 1/(Q+X) IN NEUS K AT W= 4 IS    9.9010         0.99010
PARTIAL OF T W.R.T. X AT W 4 IS   -0.97049E-02   -0.19606E-02    0.99010E-02
THE COMPLEX SOLNS FOR X(W,ELEMENT) ARE:
     12.000          0.33456E-04
     12.000          0.37037E-05
     12.000          0.23310E-04
     12.000         -0.16004E-04
     6.9380         -24.008
    -19.719         -2.7063
     0.82911        -0.11406
     1.0168         -0.33866E-02
     0.99960        -0.19991E-01
     0.73530        -0.44118
     0.27049E-01    -0.16216
     0.13236E-03    -0.11587E-01
     0.95742E-01    -0.19299E-01
     0.69862E-01     0.40151E-01
     0.96454E-01     0.47814E-02
     0.94242E-01    -0.15295E-01
TR.FCN.,Z,Y,TOLZER ARE: 1 1    0.50000         1.5000          0.10000E-11
NOM. CCT.ELEMENTS ARE:

 10.000       1.0000        1.0000      0.10000E 00

 0.10000E-01 0.30000        3.0000      100.00
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Computer output for test circuit # 1

The following output was obtained at the scaled test frequencies: 0.00115, 0.0025, 0.009, and 0.127 radians/ second for the voltage transfer function A_v of test circuit # 1.

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MAGNITUDE OF TR. FCN. IS
   0.27023E-01   0.79203E-01    0.21037        0.74800E-01
MAGNITUDE OF TR. FCN. IS
   0.76186E-01   0.19537        0.33761        0.76200E-01
MAGNITUDE OF TR. FCN. IS
   0.52628E-01   0.14499        0.29866        0.75920E-01
    REAL T1       IMAG T1       REAL T2        IMAG T2        REAL T3        IMAG T3        REAL T         IMAG T
  -0.18049E-01   0.20108E-01  -0.38599E-01    0.65684E-01   -0.30932E-01    0.42578E-01   -0.30111E-01    0.33442E-01
  -0.11068E-01   0.78426E-01   0.36263E-01    0.19159        0.65327E-02    0.14484       -0.10693E-01    0.13006
   0.17516       0.11652       0.33166        0.63111E-01    0.28288        0.95737E-01    0.26248        0.11358
   0.24909E-01  -0.70566E-01   0.17818E-01   -0.74088E-01    0.19654E-01   -0.73332E-01    0.21759E-01   -0.70729E-01
PIX(W= 1 ) FOR ELEMENT 1
   0.24539984E-04   -0.11362956E-03
   0.72759576E-11    0.15461410E-10
  -0.25690976E-03   -0.31281030E-04
   0.32133670E-04    0.56262026E-04
                                  CENTRE AT    0.83978E-01    0.93532E-01 RADIUS OF    0.12570     0.0
                                  TZERO=       0.11679E-06   -0.89255E-07 TINFIN=      0.14953     0.20079
THE 1/(Q+X) IN NEUS K AT W= 1 IS      0.63566       2.0373
PARTIAL OF T W.R.T. X AT W 1 IS   -0.20762      0.46822       0.51219
PIX(W= 2 ) FOR ELEMENT 1
  -0.19820152E-03   -0.17904514E-03
  -0.29103830E-10    0.14551915E-10
  -0.69590542E-03    0.96891820E-03
   0.39569242E-03   -0.19415686E-03
                                  CENTRE AT    0.17493        0.64680E-01 RADIUS OF    0.18650     0.0
                                  TZERO=      -0.42562E-07    0.11781E-06 TINFIN=      0.32569     0.17447
THE 1/(Q+X) IN NEUS K AT W= 2 IS      2.0376        3.4236
PARTIAL OF T W.R.T. X AT W 2 IS   0.54889       1.1530        1.2770
PIX(W= 3 ) FOR ELEMENT 1
  -0.93579409E-04    0.18701050E-03
   0.29103830E-10   -0.81229246E-10
   0.29027201E-02    0.58349431E-03
  -0.12160440E-02    0.30575329E-05
                                  CENTRE AT    0.19006       -0.95807E-01 RADIUS OF    0.21285     0.0
                                  TZERO=       0.40955E-06   -0.49528E-07 TINFIN=      0.40246    -0.81954E-01
THE 1/(Q+X) IN NEUS K AT W= 3 IS      6.3794        3.7141
PARTIAL OF T W.R.T. X AT W 3 IS   1.4228       -0.68942       1.5811
PIX(W= 4 ) FOR ELEMENT 1
   0.55409273E-06   -0.12076334E-05
  -0.45474735E-11   -0.36379788E-11
  -0.15339315E-03   -0.98988850E-04
   0.97437205E-05   -0.10920605E-04
                                  CENTRE AT    0.41192E-02   -0.38194E-01 RADIUS OF    0.38419E-01  0.0
                                  TZERO=      -0.21039E-06    0.39161E-05 TINFIN=      0.14003E-01 -0.75320E-01
THE 1/(Q+X) IN NEUS K AT W= 4 IS      10.030        0.78448
PARTIAL OF T W.R.T. X AT W 4 IS   -0.55130E-01  -0.24374E-01   0.60278E-01
MAGNITUDE OF TR. FCN. IS
   0.30586E-01   0.19455        0.28036        0.75878E-01
MAGNITUDE OF TR. FCN. IS
   0.66158E-01   0.15999        0.30297        0.75930E-01
    REAL T1       IMAG T1       REAL T2        IMAG T2        REAL T3        IMAG T3        REAL T         IMAG T
  -0.23905E-01   0.19080E-01  -0.27918E-01    0.60011E-01   -0.30932E-01    0.42578E-01   -0.30111E-01    0.33442E-01
  -0.25372E-01   0.10062       0.32755E-01    0.15660        0.65327E-02    0.14484       -0.10693E-01    0.13006
   0.24574       0.13497       0.29253        0.78841E-01    0.28288        0.95737E-01    0.26245        0.11358
   0.23742E-01  -0.72988E-01   0.19290E-01   -0.73439E-01    0.19654E-01   -0.73332E-01    0.21759E-01   -0.70729E-01
PIX(W= 1 ) FOR ELEMENT 2
  -0.40167783E-04   -0.72002818E-04
   0.63664629E-11    0.90949470E-11
  -0.59201439E-03    0.30325470E-03
   0.40466155E-04    0.35924139E-04
                                  CENTRE AT    0.15216E-01    0.43578E-01 RADIUS OF    0.46158E-01  0.0
                                  TZERO=       0.13395E-06   -0.13692E-07 TINFIN=      0.27386E-01  0.88103E-01
THE 1/(Q+X) IN NEUS K AT W= 1 IS      3.4118        4.5714
PARTIAL OF T W.R.T. X AT W 1 IS   -0.91440E-02   0.42192       0.42202
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PIX(W= 2 ) FOR ELEMENT 2
  -0.10940909E-03    0.22350680E-04
  -0.29103830E-10    0.0
   0.43409667E-03    0.16230715E-02
   0.19273537E-03   -0.23049086E-03
                                        CENTRE AT     0.55825E-01    0.70051E-01 RADIUS OF    0.89575E-01    0.0
                                        TZERO=       -0.25535E-06   -0.52166E-07 TINFIN=      0.10289        0.14627
THE 1/(Q+X) IN NEUS K AT W= 2 IS      6.8349         4.3613
PARTIAL OF T W.R.T. X AT W 2 IS     0.65239         0.42996         0.78133
PIX(W= 3 ) FOR ELEMENT 2
   0.20455234E-04    0.28645198E-04
  -0.14551915E-10   -0.23283064E-09
   0.13742819E-02   -0.11166951E-02
  -0.46522124E-03    0.28492743E-03
                                        CENTRE AT     0.15396        0.10888E-01 RADIUS OF    0.15434        0.0
                                        TZERO=        0.56234E-05    0.35076E-05 TINFIN=      0.30537        0.40801E-01
THE 1/(Q+K) IN NEUS K AT W= 3 IS      9.5126         1.8642
PARTIAL OF T W.R.T. X AT W 3 IS     0.31635        -0.48065         0.57541
PIX(W= 4 ) FOR ELEMENT 2
   0.18580067E-07   -0.54388693E-07
   0.24556357E-10    0.81854523E-11
  -0.36121681E-04   -0.11828518E-04
   0.15413689E-05   -0.24406081E-05
                                        CENTRE AT     0.88106E-02   -0.37165E-01 RADIUS OF    0.37757E-01    0.0
                                        TZERO=       -0.33489E-05   -0.45035E-03 TINFIN=      0.18556E-01   -0.73643E-01
THE 1/(Q+X) IN NEUS K AT W= 4 IS      9.9958         0.15111
PARTIAL OF T W.R.T. X AT W 4 IS    -0.10930E-01   -0.32772E-02     0.11411E-01
MAGNITUDE OF TR. FCN. IS
   0.53354E-01    0.15081         0.34137         0.14507
MAGNITUDE OF TR. FCN. IS
   0.51791E-01    0.13860         0.25970         0.51114E-01
    REAL T1         IMAG T1         REAL T2         IMAG T2         REAL T3         IMAG T3         REAL T          IMAG T
  -0.33526E-01    0.41504E-01    -0.28321E-01    0.43362E-01    -0.30932E-01    0.42578E-01    -0.30111E-01    0.33442E-01
  -0.67977E-02    0.15066         0.17681E-01    0.13747         0.65327E-02    0.14484        -0.10693E-01    0.13006
   0.29958        0.15548         0.25503        0.49023E-01     0.28288        0.95737E-01     0.26248        0.11358
   0.58934E-01   -0.13256         0.10502E-01   -0.50024E-01     0.19654E-01   -0.73332E-01     0.21759E-01   -0.70729E-01
PIX(W= 1 ) FOR ELEMENT 3
  -0.13358090E-04   -0.32553244E-05
  -0.61231475E-06    0.41873109E-06
  -0.82002953E-06    0.14670717E-04
   0.82764018E-10    0.17644197E-09
                                        CENTRE AT    -0.21752E-01    0.16731E-01 RADIUS OF    0.27429E-01    0.0
                                        TZERO=       -0.36058E-01    0.40134E-01 TINFIN=     -0.11675E-04    0.62940E-05
THE 1/(Q+X) IN NEUS K AT W= 1 IS      0.29501        0.99985
PARTIAL OF T W.R.T. X AT W 1 IS     0.51687E-01     0.18356E-01     0.54850E-01
PIX(W= 2 ) FOR ELEMENT 3
  -0.51298281E-04    0.41579566E-04
   0.52790047E-05    0.88224897E-05
   0.11018524E-03    0.78694429E-04
  -0.13387762E-08    0.43655746E-10
                                        CENTRE AT    -0.32293E-01    0.74021E-01 RADIUS OF    0.80767E-01    0.0
                                        TZERO=       -0.22024E-01    0.15413     TINFIN=      0.78587E-05   -0.60089E-05
THE 1/(Q+X) IN NEUS K AT W= 2 IS      0.84164        1.7140
PARTIAL OF T W.R.T. X AT W 2 IS     0.24278        -0.13310         0.27687
PIX(W= 3 ) FOR ELEMENT 3
   0.17302886E-03    0.17953315E-03
  -0.29832518E-05   -0.94645889E-04
   0.61353203E-03   -0.11629947E-02
   0.13242243E-08   -0.30267984E-08
                                        CENTRE AT     0.98813E-01    0.17381     RADIUS OF    0.19994        0.0
                                        TZERO=        0.28161        0.25479     TINFIN=     -0.25059E-05    0.18333E-06
THE 1/(Q+X) IN NEUS K AT W= 3 IS      2.7577         3.0970
PARTIAL OF T W.R.T. X AT W 3 IS    -0.48360        -1.1401          1.2384
PIX(W= 4 ) FOR ELEMENT 3
  -0.30147625E-04   -0.27278162E-04
   0.13007716E-04    0.13194496E-04


  -0.15214032E-02    0.18139379E-02
   0.69849193E-09    0.23283064E-09
                                        CENTRE AT     0.22700        0.21542E-01 RADIUS OF    0.22802        0.0
                                        TZERO=        0.45498        0.25987E-01 TINFIN=      0.11425E-06    0.28925E-06
THE 1/(Q+X) IN NEUS K AT W= 4 IS      9.5655         1.6150
PARTIAL OF T W.R.T. X AT W 4 IS    -0.30643         0.66972         0.73649
MAGNITUDE OF TR. FCN. IS
   0.53159E-01    0.15049         0.34987         0.14578
MAGNITUDE OF TR. FCN. IS
   0.51961E-01    0.13858         0.25432         0.51068E-01
    REAL T1         IMAG T1         REAL T2         IMAG T2         REAL T3         IMAG T3         REAL T          IMAG T
  -0.33517E-01    0.41261E-01    -0.28282E-01    0.43590E-01    -0.30932E-01    0.42578E-01    -0.30111E-01    0.33442E-01
  -0.86553E-02    0.15025         0.19273E-01    0.13723         0.65327E-02    0.14484        -0.10693E-01    0.13006
   0.30345        0.17416         0.24997        0.46842E-01     0.28288        0.95737E-01     0.26248        0.11358
   0.57743E-01   -0.13386         0.10700E-01   -0.49935E-01     0.19654E-01   -0.73332E-01     0.21759E-01   -0.70729E-01
PIX(W= 1 ) FOR ELEMENT 4
  -1340.9768        -429.18970
  -65.270462         37.715363
  -0.32252312E-01    0.15302849
   0.47683716E-06    0.10728836E-05
                                        CENTRE AT    -0.20025E-01    0.17982E-01 RADIUS OF    0.26906E-01    0.0
                                        TZERO=       -0.35986E-01    0.39643E-01 TINFIN=     -0.60840E-05    0.43983E-05
THE 1/(Q+X) IN NEUS K AT W= 1 IS      0.22852E-04    0.10676E-03
PARTIAL OF T W.R.T. X AT W 1 IS     0.52520E-05     0.23288E-05     0.57452E-05
PIX(W= 2 ) FOR ELEMENT 4
  -5758.3967         4361.3281
   516.96582          990.06250
   1.2237282          1.1070366
  -0.12397766E-04    0.28610229E-05
                                        CENTRE AT    -0.27290E-01    0.73796E-01 RADIUS OF    0.78688E-01    0.0
                                        TZERO=       -0.25701E-01    0.15247     TINFIN=      0.44084E-05   -0.63260E-05
THE 1/(Q+X) IN NEUS K AT W= 2 IS      0.83275E-04    0.19737E-03
PARTIAL OF T W.R.T. X AT W 2 IS     0.28045E-04    -0.13351E-04     0.31061E-04
PIX(W= 3 ) FOR ELEMENT 4
   19534.141          17065.766
  -734.13579         -10293.621
   6.1663361         -14.763718
  -0.15258789E-04   -0.45776367E-04
                                        CENTRE AT     0.94316E-01    0.18710     RADIUS OF    0.20953        0.0
                                        TZERO=        0.28215        0.27994     TINFIN=     -0.22725E-05    0.19827E-05
THE 1/(Q+X) IN NEUS K AT W= 3 IS      0.32513E-03    0.33028E-03
PARTIAL OF T W.R.T. X AT W 3 IS    -0.60352E-04    -0.12455E-03     0.13841E-03
PIX(W= 4 ) FOR ELEMENT 4
  -2806.3789        -2416.3047
   1307.3806         1274.7839
  -14.566968          18.564072
   0.66757202E-05    0.38146973E-05
                                        CENTRE AT     0.24537        0.26465E-01 RADIUS OF    0.24680        0.0
                                        TZERO=        0.49213        0.30515E-01 TINFIN=      0.47463E-07    0.32236E-06
THE 1/(Q+X) IN NEUS K AT W= 4 IS      0.96942E-03    0.15090E-03
PARTIAL OF T W.R.T. X AT W 4 IS    -0.30119E-04     0.68123E-04     0.74485E-04
MAGNITUDE OF TR. FCN. IS
   0.27964E-01    0.84451E-01     0.21678         0.74764E-01
MAGNITUDE OF TR. FCN. IS
   0.73414E-01    0.18585         0.33501         0.76218E-01
    REAL T1         IMAG T1         REAL T2         IMAG T2         REAL T3         IMAG T3         REAL T          IMAG T
  -0.19352E-01    0.20186E-01    -0.35796E-01    0.64096E-01    -0.30932E-01    0.42578E-01    -0.30111E-01    0.33442E-01
  -0.12764E-01    0.83481E-01     0.34916E-01    0.18254         0.65327E-02    0.14484        -0.10693E-01    0.13006
   0.18480        0.11332         0.32753        0.70426E-01     0.28288        0.95737E-01     0.26248        0.11358
   0.24643E-01   -0.70586E-01     0.17875E-01   -0.74092E-01     0.19654E-01   -0.73332E-01     0.21759E-01   -0.70729E-01
PIX(W= 1 ) FOR ELEMENT 5
   753.07031        -10540.387
   0.73242188E-03    0.24414063E-03
  -3.3580017          0.43705750
   0.34023911         0.51459378
                                        CENTRE AT     0.56662E-01    0.73688E-01 RADIUS OF    0.92954E-01    0.0
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                                             TZERO=         0.18105E-07    -0.70781E-07 TINFIN=        0.80021E-01     0.16366
THE 1/(Q+X) IN NEUS K AT W= 1 IS      0.13539E-03     0.25520E-03
PARTIAL OF T W.R.T. X AT W 1 IS    -0.15878E-04     0.44708E-04     0.47443E-04
PIX(W= 2 ) FOR ELEMENT 5
   -16463.695         -12931.305
  -0.12451172E-01     0.14648438E-02
   -4.5435181          11.833649
    3.5091782          -1.8883371
                                             CENTRE AT      0.15078         0.65769E-01 RADIUS OF      0.16450         0.0
                                             TZERO=        -0.42451E-06     0.42240E-06 TINFIN=        0.23830         0.20505
THE 1/(Q+X) IN NEUS K AT W= 2 IS      0.31626E-03     0.33569E-03
PARTIAL OF T W.R.T. X AT W 2 IS     0.53090E-04     0.96842E-04     0.11044E-03
PIX(W= 3 ) FOR ELEMENT 5
   -8969.7695          14586.926
   0.31250000E-01    -0.14892578E-01
    26.710709           3.8626099
   -11.119814         -0.38230795
                                             CENTRE AT      0.19520        -0.11097     RADIUS OF      0.22453         0.0
                                             TZERO=         0.16967E-05     0.10990E-05 TINFIN=        0.40981        -0.44949E-01
THE 1/(Q+X) IN NEUS K AT W= 3 IS      0.65674E-03     0.30565E-03
PARTIAL OF T W.R.T. X AT W 3 IS     0.12636E-03    -0.53598E-04     0.13726E-03
PIX(W= 4 ) FOR ELEMENT 5
    87.298584         -115.97583
   0.33569336E-03     0.18310547E-03
   -1.6047840         -0.99262524
   0.97543359E-01    -0.10685271
                                             CENTRE AT      0.36377E-02    -0.38326E-01 RADIUS OF      0.38496E-01     0.0
                                             TZERO=        -0.38297E-06    -0.26062E-05 TINFIN=        0.14175E-01    -0.75351E-01
THE 1/(Q+X) IN NEUS K AT W= 4 IS      0.98732E-03     0.75109E-04
PARTIAL OF T W.R.T. X AT W 4 IS    -0.52585E-05    -0.24060E-05     0.57828E-05
MAGNITUDE OF TR. FCN. IS
   0.29738E-01     0.97382E-01     0.26040         0.75653E-01
MAGNITUDE OF TR. FCN. IS
   0.68357E-01     0.16712         0.31050         0.75994E-01
    REAL T1          IMAG T1          REAL T2          IMAG T2          REAL T3          IMAG T3          REAL T          IMAG T
  -0.22428E-01     0.19527E-01    -0.30077E-01     0.61384E-01    -0.30932E-01     0.42578E-01    -0.30111E-01     0.33442E-01
  -0.22797E-01     0.94676E-01     0.33953E-01     0.16364         0.65327E-02     0.14484        -0.10693E-01     0.13006
   0.22501         0.13108         0.30109         0.75870E-01     0.28288         0.95737E-01     0.26248         0.11358
   0.21719E-01    -0.72469E-01     0.18953E-01    -0.73593E-01     0.19654E-01    -0.73332E-01     0.21759E-01    -0.70729E-01
PIX(W= 1 ) FOR ELEMENT 6
   -75102.063        -226428.50
   0.78125000E-02     0.23437500E-01
   -24.377823          11.058014
    2.0131350           2.1105528
                                             CENTRE AT      0.23410E-01     0.49531E-01 RADIUS OF      0.54785E-01     0.0
                                             TZERO=         0.10356E-06    -0.15407E-09 TINFIN=        0.35918E-01     0.10287
THE 1/(Q+X) IN NEUS K AT W= 1 IS      0.52850E-04     0.76167E-04
PARTIAL OF T W.R.T. X AT W 1 IS    -0.10592E-05     0.82781E-05     0.83456E-05
PIX(W= 2 ) FOR ELEMENT 6
   -359201.06         -42193.875
  -0.27734375        -0.78125000E-02
    4.9723816          81.726257
    12.527688         -11.293242
                                             CENTRE AT      0.78657E-01     0.69022E-01 RADIUS OF      0.10465         0.0
                                             TZERO=        -0.76412E-06     0.68009E-07 TINFIN=        0.12838         0.16110
THE 1/(Q+X) IN NEUS K AT W= 2 IS      0.10934E-03     0.79349E-04
PARTIAL OF T W.R.T. X AT W 2 IS     0.12033E-04     0.11446E-04     0.16607E-04
PIX(W= 3 ) FOR ELEMENT 6
    21966.063          164645.75
   0.68750000        -0.36328125
    103.31097         -40.304855
   -34.915527          10.853562
                                             CENTRE AT      0.16728        -0.28498E-01 RADIUS OF      0.16969         0.0
                                             TZERO=         0.16205E-05     0.43918E-05 TINFIN=        0.32889         0.23254E-01
THE 1/(Q+X) IN NEUS K AT W= 3 IS      0.16819E-03     0.43979E-04
PARTIAL OF T W.R.T. X AT W 3 IS     0.10927E-04    -0.10167E-04     0.14926E-04
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PIX(W= 4 ) FOR ELEMENT 6
    292.30078         -528.65625
   0.46142578E-01     0.78125000E-02
   -3.5472908         -1.5658989
   0.17818272         -0.23531759
                                             CENTRE AT      0.59815E-02    -0.37733E-01 RADIUS OF      0.38134E-01     0.0
                                             TZERO=        -0.24768E-04    -0.74057E-04 TINFIN=        0.17531E-01    -0.74076E-01
THE 1/(Q+X) IN NEUS K AT W= 4 IS      0.19135E-03     0.56481E-05
PARTIAL OF T W.R.T. X AT W 4 IS    -0.40213E-06    -0.15445E-06     0.43078E-06
THE COMPLEX SOLNS FOR X(W,ELEMENT) ARE:
   0.82102E-01     0.61341E-02
   0.93423E-01     0.73939E-02
   0.83898E-01     0.44614E-02
   0.53308E-01    -0.10144E-01
   0.79737E-01     0.59532E-03
   0.75598E-01     0.10569E-04
   0.67165E-01    -0.41592E-02
   0.23404E-01    -0.10599E-01
   0.32747E-01    -0.19331
   0.58357E-01    -0.83491E-01
   0.92107E-01    -0.21051E-01
   0.10292        -0.42476E-02
    217.21        -1695.5
    579.52        -694.60
    929.95        -192.11
    1019.6        -41.752
    821.65         53.495
    825.56         73.438
    836.58         56.411
    632.94        -100.62
    4184.0         117.77
    4042.1         162.35
    3820.5         52.483
    2027.0        -617.78
TR.FCN.,Z,Y,TOLZER ARE: 1 2    0.50000         1.5000         0.10000E-08
NOM. CCT.ELEMENTS ARE:

0.98500E-01 0.10000E 00 0.10100       1000.0       1000.0       5210.0

0.11561E-02 0.25070E-02 0.90226E-02 0.12736
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The following output was obtained at the scaled test frequencies: 0.00015, 0.0021, 0.0037, 0.009, 0.022, and 0.28 radians/second for the driving point impedance , Z_{11}, of test circuit # 1.

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MAGNITUDE OF TR. FCN. IS
     3890.0
     2894.3
     2246.6
     1301.3
     691.75
     70.841
MAGNITUDE OF TR. FCN. IS
     5188.1        3368.2        2475.7        1424.0        720.98        70.814
MAGNITUDE OF TR. FCN. IS
     5188.2        3424.3        2486.9        1321.3        700.41        70.872
MAGNITUDE OF TR. FCN. IS
     5188.1        3401.3        2492.6        1359.2        703.99        70.858
     REAL T1       IMAG T1       REAL T2       IMAG T2       REAL T3       IMAG T3       REAL T        IMAG T
     5172.4        -403.53       5172.4        -403.44       5172.4        -403.48       3883.3        -226.82
     2567.6        -2180.0       2585.3        -2245.4       2582.1        -2213.9       2409.8        -1603.1
     1529.3        -1863.9       1550.1        -1944.8       1591.2        -1918.6       1618.0        -1558.6
     702.44        -1238.7       627.44        -1162.8       646.22        -1195.7       701.48        -1096.0
     245.53        -677.88       270.40        -646.11       260.94        -653.84       282.80        -631.30
     3.4290        -70.731       3.4446        -70.789       3.4412        -70.774       3.7632        -70.741
PIX(W= 1 ) FOR ELEMENT 1
  -0.56849007E-04   -0.20015569E-03
   0.37482351        1.0123472
   0.0               0.72143972E-04
  -0.28943062E-01   -0.37324524
                                   CENTRE AT     5177.2        -404.48    RADIUS OF    4.8796        0.0
                                   TZERO=        5172.4        -403.57    TINFIN=      5173.6        -401.18
THE 1/(Q+X) IN NEUS K AT W= 1 IS     0.32373       0.88794E-01
PARTIAL OF T W.R.T. X AT W 1 IS      0.17705       0.84827       0.86655
PIX(W= 2 ) FOR ELEMENT 1
   0.12264002E-01    0.14674503
  -346.64961        -345.27637
   0.5?056363       -0.11968613
  -1082.9922         1606.6719
                                   CENTRE AT     2472.7        -2240.8    RADIUS OF    112.64        0.0
                                   TZERO=        2539.4        -2150.0    TINFIN=      2431.0        -2345.5
THE 1/(Q+X) IN NEUS K AT W= 2 IS     1.5971        3.1054
PARTIAL OF T W.R.T. X AT W 2 IS      167.11        -679.31       699.56
PIX(W= 3 ) FOR ELEMENT 1
   0.20684034        0.57789326E-01
  -441.33130         273.39428
   0.14839935       -1.4343207
   2444.7031         2199.5081
                                   CENTRE AT     1492.9        -1809.4    RADIUS OF    146.97        0.0
                                   TZERO=        1636.6        -1779.0    TINFIN=      1367.0        -1885.3
THE 1/(Q+X) IN NEUS K AT W= 3 IS     3.2738        3.9656
PARTIAL OF T W.R.T. X AT W 3 IS      -866.03       -780.16       1165.6
PIX(W= 4 ) FOR ELEMENT 1
   0.31262536E-03   -0.13516206
   164.93269         113.59562
  -1.8436508         0.49728489
   632.20947        -2264.0430
                                   CENTRE AT     754.93        -1111.9    RADIUS OF    137.30        0.0
                                   TZERO=        836.95        -1222.0    TINFIN=      628.42        -1058.5
THE 1/(Q+X) IN NEUS K AT W= 4 IS     6.3723        3.7169
PARTIAL OF T W.R.T. X AT W 4 IS      -623.36       808.15        1020.6
PIX(W= 5 ) FOR ELEMENT 1
  -0.31397905E-01    0.20282269E-02
   7.2564240        -24.423380
   0.29373469        0.80296844
  -597.70361        -51.622070
                                   CENTRE AT     306.74        -700.18    RADIUS OF    65.148        0.0
                                   TZERO=        290.18        -759.67    TINFIN=      296.84        -635.79
THE 1/(Q+X) IN NEUS K AT W= 5 IS     8.3103        2.6943
PARTIAL OF T W.R.T. X AT W 5 IS      249.69        246.77        351.06
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PIX(W= 6 ) FOR ELEMENT 1
  0.49548699E-05 .  0.17764978E-05
 -0.13709068E-03    0.34770789E-03
  0.43473952E-03   -0.14007858E-02
  0.97701192E-01    0.35620645E-01
                                        CENTRE AT      2.8395        -70.922     RADIUS OF     0.61951       0.0
                                        TZERO=         2.2220        -70.972     TINFIN=       3.4503        -70.818
THE 1/(Q+X) IN NEUS K AT W= 6 IS    10.123        0.36790
PARTIAL OF T W.R.T. X AT W 6 IS   0.10846       -0.44274        0.45583
MAGNITUDE OF TR. FCN. IS
     5204.5        4457.2        3691.3        2204.5        1115.2        106.52
MAGNITUDE OF TR. FCN. IS
     5161.3        2685.7        1912.0        1051.5        566.25        58.970
     REAL T1       IMAG T1       REAL T2       IMAG T2       REAL T3       IMAG T3       REAL T        IMAG T
     5209.5        -203.10       5126.4        -598.66       5172.4        -403.48       3883.3        -226.82
     3982.1        -2002.3       1847.2        -1949.6       2582.1        -2213.9       2409.8        -1603.1
     2872.9        -2317.9       1149.1        -1528.2       1591.2        -1918.6       1618.0        -1558.6
     1189.7        -1855.9       512.40        -918.18       646.22        -1195.7       701.48        -1096.0
     399.89        -1041.1       228.45        -518.12       260.94        -653.84       282.80        -631.30
     4.6550        -106.42       3.1452        -58.886       3.4412        -70.774       3.7632        -70.741
PIX(W= 1 ) FOR ELEMENT 2
   0.27482384       0.96289486
  -1431.8333       -5016.6875
   0.89531136      -0.26025391
  -744.06641        236.81250
                                        CENTRE AT      3023.6        0.73604E-01 RADIUS OF     2186.4        0.0
                                        TZERO=         5210.0        0.0         TINFIN=       837.21        -21.137
THE 1/(Q+X) IN NEUS K AT W= 1 IS   0.90365E-01    0.92227
PARTIAL OF T W.R.T. X AT W 1 IS   -744.38       -3963.7        4033.0
PIX(W= 2 ) FOR ELEMENT 2
   2.0121536       -2.5119055
  -10483.301        13086.465
  -33.251831       -23.799088
   30363.438        9294.2500
                                        CENTRE AT      2963.5        -0.14625E-01 RADIUS OF    2246.5        0.0
                                        TZERO=         5210.0        0.47587E-02 TINFIN=       730.14        -243.07
THE 1/(Q+X) IN NEUS K AT W= 2 IS    6.1161        4.6101
PARTIAL OF T W.R.T. X AT W 2 IS   -20413.        3516.1         20713.
PIX(W= 3 ) FOR ELEMENT 2
   0.11184788      -1.9299231
  -582.69312        10054.887
  -41.975708        0.44425964
   24866.188       -13547.563
                                        CENTRE AT      2892.0        -0.64654E-02 RADIUS OF    2318.0        0.0
                                        TZERO=         5210.0        -0.17276E-01 TINFIN=      595.74        -316.44
THE 1/(Q+X) IN NEUS K AT W= 3 IS    8.0897        3.6032
PARTIAL OF T W.R.T. X AT W 3 IS   -13826.        9373.9         16704.
PIX(W= 4 ) FOR ELEMENT 2
  -0.35508060      -0.43106937
   1849.9756        2245.8647
  -20.483337        19.132980
   1409.8438       -12937.656
                                        CENTRE AT      2771.5        -0.12501E-03 RADIUS OF    2438.5        0.0
                                        TZERO=         5210.0        -0.16290E-01 TINFIN=      351.83        -302.98
THE 1/(Q+X) IN NEUS K AT W= 4 IS    9.5106        1.8681
PARTIAL OF T W.R.T. X AT W 4 IS   -4467.5        7940.5         9111.0
PIX(W= 5 ) FOR ELEMENT 2
  -0.10366040      -0.49839973E-01
   540.06982        259.64307
  -5.3226156        12.575718
  -2029.3320       -3617.5664
                                        CENTRE AT      2692.2        -0.14939E-01 RADIUS OF    2517.7        0.0
                                        TZERO=         5209.9        -0.17708     TINFIN=      186.04        -240.11
THE 1/(Q+X) IN NEUS K AT W= 5 IS    9.8905        0.82877
PARTIAL OF T W.R.T. X AT W 5 IS   -1083.7        4030.0         4173.1
PIX(W= 6 ) FOR ELEMENT 2
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  -0.81378927E-03  -0.36716461E-04
   4.2397003        0.19237202
  -0.45883983E-01   1.1880522
  -41.582504       -4.8373566
                                        CENTRE AT      2606.2        -0.16886E-02 RADIUS OF    2603.7        0.0
                                        TZERO=         5209.9        1.3311       TINFIN=      2.7159        -35.105
THE 1/(Q+X) IN NEUS K AT W= 6 IS    9.9991        0.68506E-01
PARTIAL OF T W.R.T. X AT W 6 IS   -9.6967        356.61         356.74
MAGNITUDE OF TR. FCN. IS
     5188.1        3363.6        2439.6        1340.1        740.86        105.11
MAGNITUDE OF TR. FCN. IS
     5188.2        3435.4        2530.2        1353.9        661.00        59.210
     REAL T1       IMAG T1       REAL T2       IMAG T2       REAL T3       IMAG T3       REAL T        IMAG T
     5172.4        -403.53       5172.4        -403.44       5172.4        -403.48       3883.3        -226.82
     2556.8        -2185.6       2601.4        -2243.7       2582.1        -2213.9       2409.8        -1603.1
     1592.9        -1847.7       1576.5        -1979.1       1591.2        -1918.6       1618.0        -1558.6
     725.55        -1126.6       572.49        -1226.9       646.22        -1195.7       701.48        -1096.0
     391.21        -629.16       184.60        -634.70       260.94        -653.84       282.80        -631.30
     11.832        -104.45       1.7787        -59.183       3.4412        -70.774       3.7632        -70.741
PIX(W= 1 ) FOR ELEMENT 3
  -0.59771424E-04  -0.22102956E-03
   0.39836383       1.1191359
   0.0              0.36988407E-04
  -0.14741898E-01  -0.19139099
                                        CENTRE AT      5182.7        -404.63     RADIUS OF     10.292        0.0
                                        TZERO=         5172.4        -403.57     TINFIN=       5174.3        -398.55
THE 1/(Q+X) IN NEUS K AT W= 1 IS   0.15370       0.40872E-01
PARTIAL OF T W.R.T. X AT W 1 IS   0.93663E-01    0.83611        0.84134
PIX(W= 2 ) FOR ELEMENT 3
  -0.83069205E-01   0.15584338
  -126.89156       -573.02710
   0.30444241       0.74010849E-01
  -913.93311        614.47656
                                        CENTRE AT      2392.5        -2357.8     RADIUS OF     237.98        0.0
                                        TZERO=         2525.4        -2160.3     TINFIN=       2371.2        -2594.8
THE 1/(Q+X) IN NEUS K AT W= 2 IS   0.67691        1.5327
PARTIAL OF T W.R.T. X AT W 2 IS   440.98        -581.12        729.50
PIX(W= 3 ) FOR ELEMENT 3
   0.19838479       0.28213477
  -670.28906       -252.70654
   0.65771979      -0.52351761
   388.29688        1978.9766
                                        CENTRE AT      1283.6        -1875.7     RADIUS OF     310.59        0.0
                                        TZERO=         1575.8        -1770.4     TINFIN=       1104.7        -2129.6
THE 1/(Q+X) IN NEUS K AT W= 3 IS    1.2965        2.1257
PARTIAL OF T W.R.T. X AT W 3 IS   -182.34       -1307.8        1320.4
PIX(W= 4 ) FOR ELEMENT 3
   0.36173970       0.62675238E-01
  -346.37598        314.82666
  -0.29318691      -1.9117250
   2095.9141        132.32031
                                        CENTRE AT      495.57        -942.60     RADIUS OF     294.56        0.0
                                        TZERO=         783.23        -1006.0     TINFIN=       226.65        -1062.8
THE 1/(Q+X) IN NEUS K AT W= 4 IS    2.7527        3.0939
PARTIAL OF T W.R.T. X AT W 4 IS   -1566.3       -932.18        1822.7
PIX(W= 5 ) FOR ELEMENT 3
   0.25187236      -0.20944273
  -35.045364        225.55469
  -2.7229261       -2.2135887
   1105.7595       -1141.1953
                                        CENTRE AT      281.91        -408.47     RADIUS OF     246.26        0.0
                                        TZERO=         522.50        -461.03     TINFIN=       39.366        -451.11
THE 1/(Q+X) IN NEUS K AT W= 5 IS    5.2768        4.0597
PARTIAL OF T W.R.T. X AT W 5 IS   -1992.3        170.24         1999.5
PIX(W= 6 ) FOR ELEMENT 3
  -0.86786076E-02  -0.28074384E-02
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    4.0676889          0.95865250
  -0.33977419          1.1151047
  -39.740189         -12.407211
                                     CENTRE AT     228.46        -32.583      RADIUS OF     228.24        0.0
                                     TZERO=        456.65        -37.260      TINFIN=       0.24479      -35.713
THE 1/(Q+X) IN NEUS K AT W= 6 IS    9.8291        0.76036
PARTIAL OF T W.R.T. X AT W 6 IS   -58.077        342.19         347.09
MAGNITUDE OF TR. FCN. IS
    5188.1         3405.0        2514.6         1378.2         664.96         70.549
MAGNITUDE OF TR. FCN. IS
    5188.1         3396.8        2473.2         1358.9         731.91         70.928
    REAL T1        IMAG T1       REAL T2        IMAG T2        REAL T3        IMAG T3        REAL T        IMAG T
    5172.4        -403.48        5172.4        -403.48         5172.4        -403.48         3883.3        -226.82
    2589.1        -2211.5        2575.3        -2215.0         2582.1        -2213.9         2409.8        -1603.1
    1608.2        -1933.1        1580.0        -1902.8         1591.2        -1918.6         1618.0        -1558.6
    611.84        -1235.0        679.24        -1177.0         646.22        -1195.7         701.48        -1096.0
    217.00        -628.56        277.84        -677.12         260.94        -653.84         282.80        -631.30
    4.6092        -70.399        3.0375        -70.863         3.4412        -70.774         3.7632        -70.741
PIX(W= 2 ) FOR ELEMENT 4
    3364257.0          209899.19
  -0.91961958E 10    0.68825825E 10
   -68.117188         -654.41895
    1580160.0          1515342.0
                                     CENTRE AT     2572.4        -2174.3      RADIUS OF     40.815        0.0
                                     TZERO=        2595.8        -2207.7      TINFIN=       2539.4       -2150.3
THE 1/(Q+X) IN NEUS K AT W= 2 IS    0.63810E-04   0.17459E-03
PARTIAL OF T W.R.T. X AT W 2 IS   -0.13840E-01  -0.34034E-02   0.14252E-01
PIX(W= 3 ) FOR ELEMENT 4
    4187317.0         -8237549.0
   0.91790664E 10    0.21573247E 11
   -2875.8516         -666.01563
    5891543.0         -4026112.0
                                     CENTRE AT     1654.2        -1861.9      RADIUS OF     84.723        0.0
                                     TZERO=        1631.0        -1943.4      TINFIN=       1636.6       -1779.0
THE 1/(Q+X) IN NEUS K AT W= 3 IS    0.14242E-03   0.24697E-03
PARTIAL OF T W.R.T. X AT W 3 IS   -0.28008E-01   0.31093E-01   0.41847E-01
PIX(W= 4 ) FOR ELEMENT 4
   -16166192.         -4230271.0
   0.15105348E 11   -0.18536825E 11
    683.47266          10267.699
   -13118752.         -7758208.0
                                     CENTRE AT     728.68        -1302.6      RADIUS OF     135.00        0.0
                                     TZERO=        593.68        -1302.0      TINFIN=       836.93       -1222.0
THE 1/(Q+X) IN NEUS K AT W= 4 IS    0.32458E-03   0.33009E-03
PARTIAL OF T W.R.T. X AT W 4 IS    0.70564E-01   0.54436E-01   0.89121E-01
PIX(W= 5 ) FOR ELEMENT 4
   -1689252.0          11133361.
  -0.69927690E 10   -0.25596355E 10
    13521.367         -1006.9570
   -3024018.0          10553696.
                                     CENTRE AT     193.13        -720.86      RADIUS OF     95.337        0.0
                                     TZERO=        131.58        -648.06      TINFIN=       280.22       -759.65
THE 1/(Q+X) IN NEUS K AT W= 5 IS    0.57529E-03   0.39297E-03
PARTIAL OF T W.R.T. X AT W 5 IS    0.52668E-01  -0.53296E-01   0.74929E-01
PIX(W= 6 ) FOR ELEMENT 4
    10818.500         -27126.313
    1444436.0          603488.00
   -382.11548         -143.22662
    11013.516         -26806.543
                                     CENTRE AT     1.3626        -62.304      RADIUS OF     8.7215        0.0
                                     TZERO=        0.87206       -53.596      TINFIN=       2.2160       -70.984
THE 1/(Q+X) IN NEUS K AT W= 6 IS    0.99344E-03   0.70974E-04
PARTIAL OF T W.R.T. X AT W 6 IS   -0.12023E-02  -0.29506E-03   0.12380E-02
MAGNITUDE OF TR. FCN. IS
    5191.1         3401.4        2372.4         1218.0         631.72         70.678
MAGNITUDE OF TR. FCN. IS
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    5195.2         3456.8        2614.4         1434.7         731.61         70.903
    REAL T1        IMAG T1       REAL T2        IMAG T2        REAL T3        IMAG T3        REAL T        IMAG T
    5175.4        -404.15        5169.5        -402.69         5172.4        -403.48         3883.3        -226.82
    2449.4        -2360.0        2714.5        -2140.2         2582.1        -2213.9         2409.8        -1603.1
    1370.7        -1936.4        1731.1        -1959.1         1591.2        -1918.6         1618.0        -1558.6
    544.91        -1989.3        677.26        -1264.8         646.22        -1195.7         701.48        -1096.0
    252.35        -579.13        255.50        -685.54         260.94        -653.84         282.80        -631.30
    4.6513        -70.525        3.0327        -70.839         3.4412        -70.774         3.7632        -70.741
PIX(W= 1 ) FOR ELEMENT 5
    1441407.0         -430297.89
  -0.72900116E 10    0.28115615E 10
   -23.437500         -63.720703
    138891.00          324430.00
                                     CENTRE AT     5188.8        -337.93      RADIUS OF     67.570        0.0
                                     TZERO=        5178.4        -404.69      TINFIN=       5190.9       -270.39
THE 1/(Q+X) IN NEUS K AT W= 1 IS    0.48127E-05   0.44704E-04
PARTIAL OF T W.R.T. X AT W 1 IS   -0.58609E-02   0.14658E-02   0.60414E-02
PIX(W= 2 ) FOR ELEMENT 5
   -66132080.         -18748464.
   0.20546611E 12   -0.12711979E 12
    139.68750          36194.438
   -87295232.        -0.12226861E 09
                                     CENTRE AT     2907.5        -2642.8      RADIUS OF     538.32        0.0
                                     TZERO=        2371.4        -2594.5      TINFIN=       3387.4       -2398.8
THE 1/(Q+K) IN NEUS K AT W= 2 IS    0.26960E-03   0.32264E-03
PARTIAL OF T W.R.T. X AT W 2 IS    0.27673       0.20997        0.34737
PIX(W= 3 ) FOR ELEMENT 5
   -49770928.          34844832.
  -0.19221250E 11   -0.14448873E 12
    40326.563          29163.086
  -0.15168610E 09      37412352.
                                     CENTRE AT     1520.9        -2422.8      RADIUS OF     509.06        0.0
                                     TZERO=        1104.8        -2129.6      TINFIN=       2029.3       -2395.2
THE 1/(Q+K) IN NEUS K AT W= 3 IS    0.42568E-03   0.35052E-03
PARTIAL OF T W.R.T. X AT W 3 IS    0.35345      -0.49270E-01   0.35687
PIX(W= 4 ) FOR ELEMENT 5
    2049184.0          25206800.
  -0.27253871E 11   -0.35353027E 10
    35135.453         -18678.938
    3540208.0          64386496.
                                     CENTRE AT     361.73        -1365.0      RADIUS OF     331.06        0.0
                                     TZERO=        226.65        -1062.8      TINFIN=       674.33       -1474.0
THE 1/(Q+K) IN NEUS K AT W= 4 IS    0.65623E-03   0.30587E-03
PARTIAL OF T W.R.T. X AT W 4 IS    0.10358      -0.17403        0.20252
PIX(W= 5 ) FOR ELEMENT 5
    6235484.0          5096544.0
  -0.25443901E 10    0.26125394E 10
    7022.3438         -21505.859
    14644128.          10068773.
                                     CENTRE AT     72.172        -637.71      RADIUS OF     189.46        0.0
                                     TZERO=        39.327        -451.12      TINFIN=       222.15       -753.48
THE 1/(Q+X) IN NEUS K AT W= 5 IS    0.81551E-03   0.23985E-03
PARTIAL OF T W.R.T. X AT W 5 IS   -0.77376E-02  -0.90555E-01   0.90885E-01
PIX(W= 6 ) FOR ELEMENT 5
    3852.8125         -13996.090
    499047.00          141104.00
   -400.78833         -92.399597
    7440.4531         -28227.230
                                     CENTRE AT     0.48815       -53.373      RADIUS OF     17.650        0.0
                                     TZERO=        0.24757       -35.724      TINFIN=       2.2100       -70.939
THE 1/(Q+K) IN NEUS K AT W= 6 IS    0.99728E-03   0.35116E-04
PARTIAL OF T W.R.T. X AT W 6 IS   -0.12221E-02  -0.20740E-03   0.12396E-02
MAGNITUDE OF TR. FCN. IS
    2601.5         2187.6        1837.7         1184.7         665.68         70.804
MAGNITUDE OF TR. FCN. IS
    7748.5         4017.0        2749.6         1413.3         715.30         70.873
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  REAL T1        IMAG T1        REAL T2        IMAG T2        REAL T3        IMAG T3        REAL T          IMAG T
  2599.5         -101.45        7696.1         -899.99        5172.4         -403.48        3883.3          -226.82
  1986.7         -915.82        2569.2         -3088.0        2582.1         -2213.9        2409.8          -1603.1
  1513.1         -1042.9        1452.5         -2334.6        1591.2         -1918.6        1618.0          -1558.6
  760.41         -908.52        570.92         -1292.8        646.22         -1195.7        701.48          -1096.0
  318.37         -584.61        236.66         -675.01        260.94         -653.84        282.80          -631.30
  4.3983         -70.668        3.1213         -70.804        3.4412         -70.774        3.7632          -70.741
PIX(W= 1 ) FOR ELEMENT 6
 -0.33916330E 11   0.80582369E 10
  0.23488102E 09   0.0
   128398.00          506594.00
 -0.33916211E 11   0.80582328E 10
                                   CENTRE AT   -0.19429E-01   -33356.      RADIUS OF    33356.        0.0
                                   TZERO=       0.65553E-02   0.15575E-02 TINFIN=       997.78        -66697.
THE 1/(Q+X) IN NEUS K AT W= 1 IS    0.13835E-05   0.14864E-04
PARTIAL OF T W.R.T. X AT W 1 IS   0.97964      -0.15377        0.99163
PIX(W= 2 ) FOR ELEMENT 6
   0.43046625E 10   0.89850225E 10
   0.15099494E 09   0.13421773E 09
   1584914.0        -1104660.0
   0.43047281E 10   0.89851208E 10
                                   CENTRE AT   -0.14620E-01   -2612.7      RADIUS OF    2612.8        0.0
                                   TZERO=      -0.18698E-01   0.78473E-02 TINFIN=       831.37        -5089.7
THE 1/(Q+X) IN NEUS K AT W= 2 IS    0.96813E-04   0.81561E-04
PARTIAL OF T W.R.T. X AT W 2 IS   0.65057E-01  -0.42121        0.42620
PIX(W= 3 ) FOR ELEMENT 6
   0.33534723E 10   0.25257206E 10
  -26214400.        -19922944.
   564740.63        -1197647.0
   0.33534653E 10   0.25257108E 10
                                   CENTRE AT    0.25037E-02   -1619.2      RADIUS OF    1619.2        0.0
                                   TZERO=       0.78428E-02   0.34088E-04 TINFIN=       645.11        -3104.2
THE 1/(Q+X) IN NEUS K AT W= 3 IS    0.13332E-03   0.70682E-04
PARTIAL OF T W.R.T. X AT W 3 IS  -0.42329E-01  -0.22494        0.22889
PIX(W= 4 ) FOR ELEMENT 6
   0.75838614E 09     28379648.
  -19922944.          14811136.
  -101307.56         -495147.19
   0.75806310E 09     28376768.
                                   CENTRE AT    0.13203E-01   -772.49      RADIUS OF    772.47        0.0
                                   TZERO=       0.25513E-01  -0.20493E-01 TINFIN=       355.66        -1458.2
THE 1/(Q+X) IN NEUS K AT W= 4 IS    0.16813E-03   0.44052E-04
PARTIAL OF T W.R.T. X AT W 4 IS  -0.37288E-01  -0.56933E-01    0.68057E-01
PIX(W= 5 ) FOR ELEMENT 6
   0.10475955E 09   -38875648.
    10813440.         4194304.0
   -86317.313        -125212.38
   0.10476360E 09   -38858048.
                                   CENTRE AT    0.30634E-01   -379.02      RADIUS OF    378.95        0.0
                                   TZERO=      -0.77668E-01  -0.68860E-01 TINFIN=       180.61        -712.18
THE 1/(Q+X) IN NEUS K AT W= 5 IS    0.18233E-03   0.24090E-04
PARTIAL OF T W.R.T. X AT W 5 IS  -0.13241E-01  -0.12571E-01    0.18258E-01
PIX(W= 6 ) FOR ELEMENT 6
    18346.000        -116904.13
   -14528.000          3328.0000
   -1659.7529        -199.81882
    18274.891        -117105.81
                                   CENTRE AT   -0.90711E-02   -35.413      RADIUS OF    35.530        0.0
                                   TZERO=       0.46817E-01   0.11693      TINFIN=      2.4804        -70.855
THE 1/(Q+X) IN NEUS K AT W= 6 IS    0.19181E-03   0.26031E-05
PARTIAL OF T W.R.T. X AT W 6 IS  -0.18410E-03  -0.17963E-04    0.18497E-03
THE COMPLEX SOLNS FOR X(W,ELEMENT) ARE:
  -2.7710          0.79284
 -0.63135E-01      0.18376
 -0.64289E-01      0.50337E-01
 -0.15262E-02     -0.15948
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```
   0.26890         0.17428
   0.85970E-03     0.17336E-01
   0.11329        -0.45970
   0.11169        -0.36519E-01
   0.10961        -0.23450E-01
   0.10597        -0.12190E-01
   0.10359        -0.69815E-02
   0.10006        -0.90587E-03
  -5.9634          1.6384
 -0.15347          0.28447
 -0.75395E-01      0.67429E-01
  0.45586E-01     -0.16594E-01
  0.39956E-01     -0.10976E-01
  0.10093         -0.92977E-03
  -1.0000          0.0
  -1324.4           5609.2
  -1041.2           5348.0
   1696.7           1134.7
   922.80           342.45
   794.14           25.874
   903.55           22119.
   900.18           1193.4
   758.73           659.73
   732.20           317.71
   775.77           151.82
   793.24           12.399
   3900.0          0.89585E-03
   3900.0          0.72100E-03
   3900.0          0.16840E-02
   3900.0         -0.84028E-02
   3900.0         -0.18717E-01
   3900.0         -0.52120E-01
TR.FCN.,Z,Y,TOLZER ARE: 1 1    0.50000          1.5000           0.10000E-11
NOM. CCT.ELEMENTS ARE:

0.98500E-01 0.10000E 00 0.10100      1000.0      1000.0      5210.0

0.15000E-03 0.21000E-02 0.37000E-02 0.90000E-02 0.22000E-01 0.28000
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Computer output for test circuit # 2

The following output was obtained at the scaled test frequencies: 0.0002, 0.00242, 0.0062, 0.080, 36.4, 1600, 0.00502 and 0.0628 radians/-second for the voltage transfer function A_V of test circuit # 2.

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                                  D( 4) =  0.38980D-03
MAGNITUDE OF TR. FCN. IS
   0.16540E-01
    1.2190
    2.2850
    13.239
    69.096
    6.9019
MAGNITUDE OF TR. FCN. IS
   0.16333E-01     1.2189          2.2850          13.238          70.379          13.611
MAGNITUDE OF TR. FCN. IS
   0.16544E-01     1.2190          2.2850          13.239          67.054          4.6134
MAGNITUDE OF TR. FCN. IS
   0.16540E-01     1.2190          2.2850          13.239          69.096          6.9019
    REAL T1        IMAG T1         REAL T2         IMAG T2         REAL T3         IMAG T3         REAL T          IMAG T
   0.16144E-01    -0.24808E-02     0.16352E-01    -0.25157E-02     0.16347E-01    -0.25150E-02     0.16347E-01    -0.25150E-02
   0.19702        -1.2029          0.19701        -1.2030          0.19702        -1.2030          0.19702        -1.2030
  -0.97503        -2.0665         -0.97506        -2.0665         -0.97506        -2.0665         -0.97506        -2.0665
  -3.9847         -12.624         -3.9857         -12.625         -3.9852         -12.625         -3.9852         -12.625
  -69.997          7.3225         -63.548          21.397         -67.473          14.889         -67.473          14.889
  -2.6181          13.357         -0.30082         4.6036         -0.67323         6.8690         -0.67323         6.8690
PIX(W= 1 ) FOR ELEMENT 1
   0.32190725E-15   -0.54024913E-16
  -0.51322769E-17    0.16947781E-17
   0.25902103E-09   -0.43481455E-10
  -0.41271717E-11    0.13630139E-11

                                        CENTRE AT   -0.44118E-02   -0.12568      RADIUS OF    0.12491        0.0
                                        TZERO=       0.16366E-01   -0.25181E-02 TINFIN=      0.16356E-01   -0.25165E-02
THE 1/(Q+X) IN NEUS K AT W= 1 IS     0.10000E 21    0.10000E 21
PARTIAL OF T W.R.T. X AT W 1 IS     0.10000E 21    0.10000E 21    0.14142E 21
PIX(W= 2 ) FOR ELEMENT 1
   0.43314684E-17   -0.16600604E-15
   0.19885593E-15    0.37915603E-16
  -0.23245815E-12   -0.13265607E-09
   0.15963371E-09    0.25855276E-10
                                        CENTRE AT    0.19694       -1.2030       RADIUS OF    0.94985E-04,   0.0
                                        TZERO=       0.19701       -1.2030       TINFIN=      0.19701       -1.2030
THE 1/(Q+X) IN NEUS K AT W= 2 IS     0.10000E 21    0.10000E 21
PARTIAL OF T W.R.T. X AT W 2 IS     0.10000E 21    0.10000E 21    0.14142E 21
PIX(W= 3 ) FOR ELEMENT 1
  -0.17930258E-16   -0.29010759E-16
   0.42466831E-16   -0.65339883E-16
  -0.23665687E-10   -0.17356866E-10
   0.12597197E-10   -0.66242220E-10
                                        CENTRE AT   -0.97505       -2.0665       RADIUS OF    0.17337E-04    0.0
                                        TZERO=      -0.97507       -2.0665       TINFIN=     -0.97506       -2.0665
THE 1/(Q+X) IN NEUS K AT W= 3 IS     0.10000E 21    0.10000E 21
PARTIAL OF T W.R.T. X AT W 3 IS     0.10000E 21    0.10000E 21    0.14142E 21
PIX(W= 4 ) FOR ELEMENT 1
   0.17196939E-14    0.13522241E-14
  -0.10218910E-13    0.27097021E-13
  -0.12396750E-11   -0.11157963E-10
   0.13568879E-09   -0.61209704E-10
                                        CENTRE AT   -4.0295        -12.570       RADIUS OF    0.70467E-01    0.0
                                        TZERO=      -3.9842        -12.624       TINFIN=     -4.0342        -12.614
THE 1/(Q+X) IN NEUS K AT W= 4 IS     0.10000E 21    0.10000E 21
PARTIAL OF T W.R.T. X AT W 4 IS     0.10000E 21    0.10000E 21    0.14142E 21
PIX(W= 5 ) FOR ELEMENT 1
  -0.15628693E-10   -0.20209084E-10
  -0.10894845E-08   -0.14426478E-08
  -0.18193768E-05    0.13757672E-05
   0.13969839E-08   -0.23283064E-08
                                        CENTRE AT   -35.383        -0.22223E-01 RADIUS OF    35.384         0.0
                                        TZERO=      -70.759        -0.81085      TINFIN=      0.11042E-02   -0.44478E-03
THE 1/(Q+X) IN NEUS K AT W= 5 IS     18839.         85121.
PARTIAL OF T W.R.T. X AT W 5 IS     0.25385E 07    0.54629E 07    0.60239E 07
PIX(W= 6 ) FOR ELEMENT 1
   0.13987743E-11    0.52048643E-12
   0.98279658E-10    0.36860154E-10
   0.20442421E-05   -0.54895991E-05
   0.29103830E-10    0.29103830E-10
                                        CENTRE AT   -35.384        -0.52243E-03 RADIUS OF    35.384         0.0
                                        TZERO=      -70.769        -0.18617E-01 TINFIN=      0.29222E-05   -0.63898E-05
THE 1/(Q+X) IN NEUS K AT W= 6 IS     0.38097E 06    37333.
PARTIAL OF T W.R.T. X AT W 6 IS     0.51292E 06   -0.25917E 07    0.26420E 07
MAGNITUDE OF TR. FCN. IS
   0.92853E-02     0.74594         1.8721          13.046          69.096          6.9019
MAGNITUDE OF TR. FCN. IS
   0.24722E-01     1.4685          2.4004          13.301          69.096          6.9019
    REAL T1        IMAG T1         REAL T2         IMAG T2         REAL T3         IMAG T3         REAL T          IMAG T
   0.82298E-02    -0.95768E-03     0.24281E-01    -0.46508E-02     0.16347E-01    -0.25150E-02     0.16347E-01    -0.25150E-02
   0.33641        -0.66577        -0.53396E-01    -1.4675          0.19702        -1.2030          0.19702        -1.2030
  -0.26695        -1.8530         -1.2985         -2.0253         -0.97506        -2.0665         -0.97506        -2.0665
  -3.4933         -12.570         -4.1532         -12.636         -3.9852         -12.625         -3.9852         -12.625
  -67.476          14.873         -67.471          14.894         -67.473          14.889         -67.473          14.889
  -0.67326         6.8690         -0.67321         6.8690         -0.67323         6.8690         -0.67323         6.8690
PIX(W= 1 ) FOR ELEMENT 2
  -0.89584728E-05    0.27704609E-05
  -0.66449957E-10    0.22627233E-10
   0.43939799E-05    0.13826706E-04
  -0.30259434E-05    0.12043174E-05
                                        CENTRE AT   -0.88452E-02   -0.11190      RADIUS OF    0.11224        0.0
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                                       TZERO=      -0.74830E-05    0.21163E-06 TINFIN=      -0.15943E-01    -0.22391
THE 1/(Q+X) IN NEUS K AT W= 1 IS    0.12561          1.5370
PARTIAL OF T W.R.T. X AT W 1 IS   0.33623      -0.77440E-01    0.34503
PIX(W= 2 ) FOR ELEMENT 2
   0.34948974E-03    0.14632009E-03
   0.23283064E-08   -0.61118044E-09
   0.26533306E-02   -0.65193474E-02
   0.12443196E-01   -0.37163754E-02
                                       CENTRE AT   -0.58491        -0.71343    RADIUS OF    0.92255         0.0
                                       TZERO=      -0.50455E-05    0.38612E-05 TINFIN=      -1.1555         -1.4384
THE 1/(Q+X) IN NEUS K AT W= 2 IS    9.2356           10.285
PARTIAL OF T W.R.T. X AT W 2 IS  -10.070        -16.083          18.976
PIX(W= 3 ) FOR ELEMENT 2
   0.13257493E-03   -0.19261033E-03
  -0.69849193E-09   -0.46566129E-08
  -0.94350949E-02   -0.60879290E-02
  -0.76503828E-02   -0.27236234E-01
                                       CENTRE AT   -0.96868        -0.80622    RADIUS OF    1.2603          0.0
                                       TZERO=      -0.14711E-04    0.13752E-04 TINFIN=      -1.8876         -1.6687
THE 1/(Q+X) IN NEUS K AT W= 3 IS    17.431           7.4932
PARTIAL OF T W.R.T. X AT W 3 IS  -18.887         0.95512E-01     18.887
PIX(W= 4 ) FOR ELEMENT 2
   0.46722278E-05   -0.12761258E-04
   0.15110709E-06   -0.59604645E-07
  -0.77500939E-02   -0.10561268E-02
  -0.21476064E-01   -0.10275251
                                       CENTRE AT   -4.5318         -5.5172     RADIUS OF    7.1287          0.0
                                       TZERO=      -0.85300E-02   -0.73812E-02 TINFIN=      -4.4943         -12.646
THE 1/(Q+X) IN NEUS K AT W= 4 IS    20.624           0.73756
PARTIAL OF T W.R.T. X AT W 4 IS  -10.485        -0.80627         10.516
PIX(W= 5 ) FOR ELEMENT 2
  -0.17433194E-07    0.59872036E-08
  -0.91013499E-06    0.58324076E-06
   0.53243944E-04    0.24882890E-03
   0.73010139E-02    0.15994683E-01
                                       CENTRE AT   -62.284          13.764     RADIUS OF    5.3090          0.0
                                       TZERO=      -56.977          13.888     TINFIN=      -67.469          14.905
THE 1/(Q+X) IN NEUS K AT W= 5 IS    20.924           0.31483E-01
PARTIAL OF T W.R.T. X AT W 5 IS   0.70873E-01    0.32673         0.33433
PIX(W= 6 ) FOR ELEMENT 2
  -0.10658141E-13    0.54474691E-09
   0.37408867E-08    0.36675374E-09
   0.56981946E-06    0.68378313E-07
   0.85328725E-06   -0.38680546E-05
                                       CENTRE AT   -0.67326         6.8681     RADIUS OF    0.89668E-03     0.0
                                       TZERO=      -0.67312         6.8672     TINFIN=      -0.67319         6.8690
THE 1/(Q+X) IN NEUS K AT W= 6 IS    20.962           0.41427
PARTIAL OF T W.R.T. X AT W 6 IS   0.74230E-03   -0.30595E-04     0.74293E-03
MAGNITUDE OF TR. FCN. IS
   0.82946E-02    0.79333        1.9809         13.221          69.095          6.9019
MAGNITUDE OF TR. FCN. IS
   0.24680E-01    1.4093         2.3582         13.243          69.096          6.9019
    REAL T1        IMAG T1       REAL T2        IMAG T2         REAL T3         IMAG T3         REAL T          IMAG T
   0.92469E-02   -0.88757E-03    0.24199E-01   -0.48479E-02     0.16347E-01    -0.25150E-02     0.16347E-01    -0.25150E-02
   0.37536       -0.69891       -0.43762E-01   -1.4087          0.19702        -1.2030          0.19702        -1.2030
  -0.32674       -1.9538        -1.2304        -2.0118         -0.97506        -2.0665         -0.97506        -2.0665
  -3.6350        -12.712        -4.1010        -12.592         -3.9852         -12.625         -3.9852         -12.625
  -67.473         14.884        -67.473         14.891         -67.473          14.889         -67.473          14.889
  -0.67324        6.8690        -0.67322        6.8690         -0.67323         6.8690         -0.67323         6.8690
PIX(W= 1 ) FOR ELEMENT 3
  -0.76861324E-05    0.26711305E-05
  -0.37346126E-10    0.12562396E-10
   0.55336859E-05    0.15733051E-04
  -0.27974329E-05    0.11694738E-05
                                       CENTRE AT   -0.55895E-02   -0.90729E-01 RADIUS OF    0.90901E-01     0.0
                                       TZERO=      -0.48421E-05   -0.48331E-07 TINFIN=      -0.10495E-01    -0.18150




THE 1/(Q+X) IN NEUS K AT W= 1 IS    0.19328          2.0312
PARTIAL OF T W.R.T. X AT W 1 IS   0.35837       -0.89117E-01     0.36928
PIX(W= 2 ) FOR ELEMENT 3
   0.27051987E-03    0.56096585E-04
   0.13969839E-08   -0.24738256E-09
   0.13922751E-02   -0.66558234E-02
   0.10440912E-01   -0.40628389E-02
                                       CENTRE AT   -0.44841        -0.69108    RADIUS OF    0.82380         0.0
                                       TZERO=      -0.47694E-05    0.19035E-05 TINFIN=      -0.89921        -1.3806
THE 1/(Q+X) IN NEUS K AT W= 2 IS    12.255           11.182
PARTIAL OF T W.R.T. X AT W 2 IS  -11.449        -14.434          18.423
PIX(W= 3 ) FOR ELEMENT 3
   0.58508711E-04   -0.13766113E-03
  -0.20954758E-08   -0.93132257E-09
  -0.87636709E-02   -0.37331081E-02
  -0.84881857E-02   -0.21451429E-01
                                       CENTRE AT   -0.85048        -0.86198    RADIUS OF    1.2109          0.0
                                       TZERO=      -0.25044E-06    0.15328E-04 TINFIN=      -1.7024         -1.7226
THE 1/(Q+X) IN NEUS K AT W= 3 IS    19.953           7.0271
PARTIAL OF T W.R.T. X AT W 3 IS  -16.928         1.7508          17.019
PIX(W= 4 ) FOR ELEMENT 3
  -0.14091638E-05   -0.63906336E-05
   0.15739352E-06   -0.58440492E-07
  -0.52274801E-02    0.11971761E-02
  -0.37626427E-01   -0.60256675E-01
                                       CENTRE AT   -2.2178         -6.2545     RADIUS OF    6.6110          0.0
                                       TZERO=      -0.35417E-02   -0.25410E-01 TINFIN=      -4.3308         -12.519
THE 1/(Q+X) IN NEUS K AT W= 4 IS    22.400           0.61271
PARTIAL OF T W.R.T. X AT W 4 IS  -7.8064         2.1659          8.1013
PIX(W= 5 ) FOR ELEMENT 3
   0.75879143E-09   -0.28503564E-08
   0.48345828E-06   -0.30596857E-06
   0.23818902E-05    0.68139052E-04
   0.11755447E-02    0.45620352E-02
                                       CENTRE AT   -68.380          14.903     RADIUS OF    0.90783         0.0
                                       TZERO=      -69.109          14.363     TINFIN=      -67.472          14.894
THE 1/(Q+X) IN NEUS K AT W= 5 IS    22.402          -0.56553E-01
PARTIAL OF T W.R.T. X AT W 5 IS   0.16205E-02    0.10267         0.10268
PIX(W= 6 ) FOR ELEMENT 3
  -0.56319038E-09    0.94850039E-09
   0.61358598E-08    0.45070010E-08
   0.25919002E-06    0.85067484E-07
   0.75881610E-06   -0.17231077E-05
                                       CENTRE AT   -0.67320         6.8689     RADIUS OF    0.12054E-03     0.0
                                       TZERO=      -0.67325         6.8688     TINFIN=      -0.67321         6.8690
THE 1/(Q+X) IN NEUS K AT W= 6 IS    21.825           1.8944
PARTIAL OF T W.R.T. X AT W 6 IS   0.44804E-03    0.88479E-04     0.45669E-03
MAGNITUDE OF TR. FCN. IS
   0.16519E-01    1.2110         2.1636         7.0713          69.250          6.9021
MAGNITUDE OF TR. FCN. IS
   0.16529E-01    1.2347         2.4778         19.153          69.043          6.9019
    REAL T1        IMAG T1       REAL T2        IMAG T2         REAL T3         IMAG T3         REAL T          IMAG T
   0.16311E-01   -0.26106E-02    0.16352E-01   -0.24142E-02     0.16347E-01    -0.25150E-02     0.16347E-01    -0.25150E-02
   0.10623       -1.2063         0.28724       -1.2009          0.19702        -1.2030          0.19702        -1.2030
  -1.2628        -1.7569        -0.69668       -2.3778         -0.97506        -2.0665         -0.97506        -2.0665
  -2.6504        -6.5558        -6.2679        -18.098         -3.9852         -12.625         -3.9852         -12.625
  -67.778         14.199        -67.368         15.117         -67.473          14.889         -67.473          14.889
  -0.67330        6.8692        -0.67320        6.8689         -0.67323         6.8690         -0.67323         6.8690
PIX(W= 1 ) FOR ELEMENT 4
   0.25119949E-07   -0.24126075E-06
   0.24027247E-09    0.39879886E-08
   0.10663443E-05   -0.17852744E-06
  -0.16336379E-07    0.53203166E-08
                                       CENTRE AT    0.16028E-01   -0.24491E-02 RADIUS OF    0.32599E-03     0.0
                                       TZERO=       0.16250E-01   -0.26878E-02 TINFIN=       0.15715E-01    -0.23583E-02
THE 1/(Q+X) IN NEUS K AT W= 1 IS    0.18232         -4.6172
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