Optimal Foldover Plans for Two-Level Fractional Factorial Designs

By

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A Thesis Submitted to the Faculty of Graduate Studies
in Partial Fulfillment of the Requirements
for the Degree of
MASTER OF SCIENCE

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THE UNIVERSITY OF MANITOBA

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Abstract

Fractional factorial (FF) designs can often identify which factor effects are significant by running a fraction (subset) of a full factorial experiment (Wu and Hamada, 2000; Box, Hunter and Hunter, 2005; Montgomery, 2005; Ryan, 2007). However, there will be aliasing of effects in a FF design, which may lead to ambiguities in interpreting the results of an experiment.

One method for de-aliasing low-order effects is to run a follow-up experiment using the foldover approach. The foldover approach reverses the signs of one or more factors of the initial design to produce a follow-up design of equal size. When using the foldover approach, one attempts to minimize undesirable aliasing in the "combined" design (the initial design together with the foldover design).

The primary objective of this thesis is to select optimal foldover plans, given an initial 2^{k-p} design. Two criteria for selecting the optimal combined design are the minimum aberration (MA) criterion and the SMCE criterion, in which we sequentially maximize the number of clear effects.

In this thesis, we use the SMCE criterion to produce tables of optimal foldover plans. We then compare these to the tables of Li and Lin (2003), who used the MA criterion.

Our research concludes that there exists some noteworthy differences between the optimal combined designs selected according to the MA and the SMCE criteria. The SMCE criterion leads to the selection of combined designs that possess at least as many clear low-order effects than that obtained using the MA criterion.

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Chapter 1

Introduction and Summary

Factorial designs are used for determining which factors comprising an industrial or agricultural process have a significant influence on the response(s) of interest. As the number of factors increases, the number of runs required increases rapidly. Fractional factorial (FF) designs can identify which factor effects are significant by running a fraction (subset) of a full factorial experiment (Wu and Hamada, 2000; Box, Hunter and Hunter, 2005; Montgomery, 2005; Ryan, 2007). Therefore, FF designs are widely used in screening experiments for process design and for process improvement.

Consider 8 factors, each varied at 2 levels. A full factorial would require $2^8 = 256$ runs. In order to reduce the number of runs, consider a 16-run 2^{8-4} FF design, with generators $E = ABC$, $F = ABD$, $G = ACD$ and $H = BCD$. Here, factors E, F, G and H are called "added" factors and are generated from the "basic" factors, A, B, C and D. To construct this design, we first write down a basic design consisting of 16 runs for a full 2^4 factorial in A, B, C and D. We then add the 5^{th}, 6^{th}, 7^{th} and 8^{th} factors (E, F, G and H) by identifying their low and high levels with the plus and minus signs of their generators ABC, ABD, ACD and BCD, respectively. The list of runs for the 2^{8-4} design is shown in Table 1.1. Since $E = ABC$, if both sides are

Table 1.1: A 2^{8-4} design

Run	A	B	C	D	E	F	G	H
1	-	-	-	-	-	-	-	-
2	+	-	-	-	+	+	+	-
3	-	+	-	-	+	+	-	+
4	+	+	-	-	-	-	+	+
5	-	-	+	-	+	-	+	+
6	+	-	+	-	+	-	-	+
7	-	+	+	-	-	+	+	-
8	+	+	+	-	+	-	-	-
9	-	-	-	+	-	+	+	+
10	+	-	-	+	+	-	-	+
11	-	+	-	+	+	-	+	-
12	+	+	-	+	-	+	-	-
13	-	-	+	+	+	+	-	-
14	+	-	+	+	-	-	+	-
15	-	+	+	+	-	-	-	+
16	+	+	+	+	+	+	+	+

multiplied by E, we obtain

$$E \times E = ABC \times E,$$

$$E^2 = ABCE,$$

$$I = ABCE.$$

Note that any factor squared is equal to the identity, I, a column of +'s (or 1's). In addition to $I = ABCE$, we have $I = ABDF$, $I = ACDG$ and $I = BCDH$, from $F = ABD$, $G = ACD$ and $H = BCD$. Note that, since $I = ABDF$ and $I = ACDG$, it follows that $I = I^2 = ABDF \times ACDG = A^2BCD^2FG = BCFG$. If we multiply the four words $ABCE$, $ABDF$, $ACDG$ and $BCDH$ together two at a time, three at a time, and four at a time, then we have the complete defining relation:

$$\begin{aligned} I &= ABCE = ABDF = ABGH = ACDG = ACFH = ADEH \\ &= AEFG = BCDH = BCFG = BDEG = BEFH \\ &= CDEF = CEGH = DFGH = ABCDEFGH. \end{aligned}$$

The exponents in the products are formed by using modulus 2 arithmetic, so that any even power of a factor is equal to I, and any odd power is equal to the factor itself. There are a total of 16 elements, including the identity I, in the complete defining relation, and each element is referred to as a word. The resolution of a two-level FF design is defined to be the length of the shortest word, excluding I, in the defining relation. Based on this definition, we conclude that this 2^{8-4} design has resolution IV.

If each word in the complete defining relation is multiplied by A, we can determine which effects are aliased with A (or indistinguishable from A) in the subsequent data analysis. The "alias chain" associated with A is given by

$$\begin{aligned} A &= BCE = BDF = BGH = CDG = CFH = DEH \\ &= EFG = ABCDH = ABCFG = ABDEG = ABEFH \\ &= ACDEF = ACEGH = ADFGH = BCDEFGH. \end{aligned}$$

We can similarly obtain alias chains for other factor effects. The alias chains form what is known as the alias structure of the design. We usually omit words of length four or higher because they are negligible. Using this convention, the following is the alias structure for the 2^{8-4} design:

$$A = BCE = BDF = BGH = CDG = CFH = DEH = EFG$$

$$B = ACE = ADF = AGH = CDH = CFG = DEG = EFH$$

$$C = ABE = ADG = AFH = BDH = BFG = DEF = EGH$$

$$D = ABF = ACG = AEH = BCH = BEG = CEF = FGH$$

$$E = ABC = ADH = AFG = BDG = BFH = CDF = CGH$$

$$F = ABD = ACH = AEG = BCG = BEH = CDE = DGH$$

$$G = ABH = ACD = AEF = BCF = BDE = CEH = DFH$$

$$H = ABG = ACF = ADE = BCD = BEF = CEG = DFG$$

$$AB = CE = DF = GH$$

$$AC = BE = DG = FH$$

$$AD = BF = CG = EH$$

$$AE = BC = DH = FG$$

$$AF = BD = CH = EG$$

$$AG = BH = CD = EF$$

$$AH = BG = CF = DE.$$

We define a main effect (factor) to be clear when it is neither aliased with other main effects nor with two-factor interactions. Similarly, we call a two-factor interaction clear if it is not aliased with main effects or other two-factor interactions. From the alias structures of the preceding 2^{8-4} design, we see that all main effects are clear; however, all of the two-factor interactions are aliased with other two-factor interactions.

Definition 1.1. Let w_i denote the number of words of length i in the defining relation of a design d. The vector $W(d) = (w_1, w_2, ..., w_k)$ is called the word length pattern (WLP) of the design.

Definition 1.2. For any two 2^{k-p} designs d_1 and d_2, let r be the smallest integer such that $A_r(d_1) \neq A_r(d_2)$, where A_i denotes the number of words of length i in its complete defining relation, $1 \leq i \leq k$. Then d_1 is said to have less aberration than

d_2 if $A_r(d_1)<A_r(d_2)$. If there is no design with less aberration than d_1, then d_1 has minimum aberration.

Minimum aberration (MA) is a commonly used criterion for ranking FF designs. The MA criterion is used to select FF designs with as few short words as possible in the WLP.

The "number of clear effects criterion" is another criterion for ranking FF designs. This criterion sequentially assesses a given FF design with respect to the number of clear main effects and two-factor interactions that it possesses. An FF design that sequentially maximizes clear effects (SMCE) is then optimal according to this criterion.

An MA FF design may not be necessarily be the best design, when assessed by another criterion. Wu and Hamada (2000) provide the following example. Consider a resolution IV 2^{9-4} design. Plan A, having generators $F = ABC$, $G = ABD$, $H = ABE$ and $J = ACDE$, is the optimal plan under the MA criterion, whereas plan B, having generators $F = ABC$, $G = ABD$, $H = ACD$, and $J = BCDE$, is the optimal plan under the SMCE criterion. These plans have the following defining relations:

Plan A:

$$\begin{aligned} I &= ABCF = ABDG = ABEH = CDFG = CEFH = DEGH \\ &= ACDEJ = ACGHJ = ADFHJ = AEFGJ = BCDHJ \\ &= BCEGJ = BDEFJ = BFGHJ = ABCDEFGH \end{aligned}$$

Plan B:

$$\begin{aligned} I &= ABCF = ABDG = ACDH = AFGH = BCGH = BDFH \\ &= CDFG = ABEHJ = ACEGJ = ADEFJ = BCDEJ \\ &= BEFGJ = CEFHJ = DEGHJ = ABCDEFGHJ \end{aligned}$$

We can see that plan A has 6 words of length of four, and plan B has 7 words of length of four. Obviously, plan A is better than plan B using the MA criterion; however, if we count the number of clear low-order effects, we come to a different conclusion. Table 1.2 shows the number of clear low-order effects for both designs. In the third column of Table 1.2, plan A has values “9/9, 8/36”. This implies that the MA design has 9 out of 9 clear main effects and 8 out of 36 clear two-factor interactions. Similarly, plan B has 9 out of 9 clear main effects and 15 out of 36 clear two-factor interactions. Compared to plan A, plan B obtains 7 additional clear two-factor interactions. Therefore, using the SMCE criterion, plan B is superior to plan A.

Table 1.2: Two 2^{9-4} FF designs: A comparison of MA and SMCE

Criteria	Optimal plan	Number of clear low-order effects
MA	Plan A	9/9, 8/36
SMCE	Plan B	9/9, 15/36

Recall the previous 2^{8-4} design (Table 1.1). From the alias structure, we observed that none of the two-factor interactions are clear. Thus, after the 16-run experiment has been run and analyzed, we may be in a difficult situation. If, for example, AB appears to be significant, we will not know whether it is really AB or CE or DF or GH. If we wish to distinguish between these two-factor interactions, we may need to run a second (follow-up) experiment to “de-alias” these two-factor interactions. One method for de-aliasing low-order effects is via the foldover approach. The foldover approach reverses the signs of one or more factors of the initial design to produce a follow-up design of equal size. A number of low-order effects will be de-aliased, depending upon which factors are sign reversed and the resolution of the initial 2^{k-p} design. There are two different situations in which we may want to use the foldover approach: (1) when we want to minimize the amount of overall aliasing in the "combined" design (the initial design together with the foldover design), which is the subject of this thesis, or (2) when we want to de-alias a few low-order effects in which we are interested. Many books (for example, Wu and Hamada, 2000; Box, Hunter and Hunter, 2005; Montgomery, 2005) describe the foldover approach in detail.

We now wish to fold the MA 2^{8-4} design, and determine the optimal foldover plans under both the MA and SMCE criteria. Firstly, let us look at the optimal foldover plans under the MA criterion. Li and Lin (2003) determined the optimal foldover plans for both 16-run and 32-run initial MA FF designs using the MA criterion. They first showed that only "core" foldover plans need be considered. These are plans in which only the signs of the added factors are reversed. There are $2^4 = 16$ core foldover plans in this 2^{8-4} example. (This includes the trivial foldover plan in which none of the added factors are sign-reversed, corresponding to replicating the initial design.) Of these, according to Li and Lin (2003), seven plans are optimal with respect to MA. Folding on factors E and F is one of the 7 optimal foldover plans. Using this foldover plan we obtain a follow-up experiment with 16 runs. From the complete defining relation for the initial 2^{8-4} design, we see that, by reversing the signs of E and F, the complete defining relation for the foldover design is given

$$\begin{aligned} I &= -ABCE = -ABDF = ABGH = ACDG = -ACFH \\ &= -ADEH = AEFG = BCDH = -BCFG = -BDEG \\ &= BEFH = CDEF = -CEGH = -DFGH = ABCDEFGH. \end{aligned}$$

It follows that the complete defining relation for the optimal combined design is

Table 1.3: Optimal foldover design for an initial MA 2^{8-4} design using the MA criterion

Run	A	B	C	D	E	F	G	H
1	-	-	-	-	-	-	-	-
2	+	-	-	-	+	+	+	-
3	-	+	-	-	+	+	-	+
4	+	+	-	-	-	-	+	+
5	-	-	+	-	+	-	+	+
6	+	-	+	-	+	-	-	+
7	-	+	+	-	-	+	+	-
8	+	+	+	-	+	-	-	-
9	-	-	-	+	-	+	+	+
10	+	-	-	+	+	-	-	+
11	-	+	-	+	+	-	+	-
12	+	+	-	+	-	+	-	-
13	-	-	+	+	+	+	-	-
14	+	-	+	+	-	-	+	-
15	-	+	+	+	-	-	-	+
16	+	+	+	+	+	+	+	+
Run	A	B	C	D	$-E$	$-F$	G	H
17	-	-	-	-	+	+	-	-
18	+	-	-	-	-	-	+	-
19	-	+	-	-	-	-	-	+
20	+	+	-	-	+	+	+	+
21	-	-	+	-	-	+	+	+
22	+	-	+	-	-	+	-	+
23	-	+	+	-	+	-	+	-
24	+	+	+	-	-	+	-	-
25	-	-	-	+	+	-	+	+
26	+	-	-	+	-	+	-	+
27	-	+	-	+	-	+	+	-
28	+	+	-	+	+	-	-	-
29	-	-	+	+	-	-	-	-
30	+	-	+	+	+	+	+	-
31	-	+	+	+	+	+	-	+
32	+	+	+	+	-	-	+	+

given by:

$$
\begin{aligned}
I &= ABGH = ACDG = AEFG = BCDH \\
&= BEFH = CDEF = ABCDEFGH.
\end{aligned}
$$

Combining these two 16-run experiments (initial plus foldover) yields our optimal combined design. This 32-run design is shown in Table 1.3. Ignoring four-factor and higher interactions, the combined design has alias structure,

$$
\begin{gathered}
A = BGH = CDG = EFG \\
B = AGH = CDH = EFH \\
C = ADG = BDH = DEF \\
D = ACG = BCH = CEF \\
E = AFG = BFH = CDF \\
F = AEG = BEH = CDE \\
G = ABH = ACD = AEF \\
H = ABG = BCD = BEF \\
AB = GH \\
AC = DG \\
AD = CG \\
AE = FG \\
AF = EG \\
AG = BH = CD = EF \\
AH = BG \\
BC = DH \\
BD = CH \\
BE = FH \\
BF = EH \\
CE = DF
\end{gathered}
$$

$$CF = DE.$$

From the complete defining relation, we see that the WLP of this design is (0,6,0,0,0,1). There are a total of 8 clear low-order effects, of which all are main effects. None of the two-factor interactions are clear. We also see that the WLP is better than the initial design, which had a WLP of (0,14,0,0,0,1). However, the initial and combined designs possess the same number of clear low-order effects.

Secondly, consider the optimal foldover plans for the initial MA 2^{8-4} design using the SMCE criterion. Here, we have 8 foldover plans that are optimal with respect to our sequential ranking scheme. For example, consider the foldover plan obtained by folding on E. The combined design is shown in Table 1.4. The combined design has complete defining relation

$$\begin{aligned} I &= ABDF = ABGH = ACDG = ACFH \\ &= BCDH = BCFG = DFGH, \end{aligned}$$

with corresponding alias structure

$$\begin{gathered} A = BDF = BGH = CDG = CFH \\ B = ADF = AGH = CDH = CFG \\ C = ADG = AFH = BDH = BFG \\ D = ABF = ACG = BCH = FGH \\ F = ABD = ACH = BCG = DGH \\ G = ABH = ACD = BCF = DFH \\ H = ABG = ACF = BCD = DFG \end{gathered}$$

$$AB = DF = GH$$

$$AC = DG = FH$$

$$AD = BF = CG$$

$$AE = AE$$

$$AF = BD = CH$$

$$AG = BH = CD$$

$$AH = BG = CF$$

$$BC = DH = FG$$

$$BE = BE$$

$$CE = CE$$

$$DE = DE$$

$$EF = EF$$

$$EG = EG$$

$$EH = EH.$$

The combined design has WLP (0,7,0,0,0,0). All of the main effects and 7 two-factor interactions (AE, BE, CE, DE, EF, EG and EH) are clear. Therefore, compared to the initial design, the combined design possesses 7 additional clear low-order effects.

Both 2^{8-4} experiments use the same initial design. The difference is the manner in which we select the optimal foldover plans. Table 1.5 shows that compared with the optimal foldover plan selected via the MA criterion, we obtain 7 additional clear effects by selecting the optimal foldover plan via the SMCE criterion. Therefore, for those primarily interested in dealiasing as many low-order effects as possible, using the SMCE criterion will lead to a superior result.

There are two objectives we have met in this thesis. First, given an initial MA 2^{k-p}

Table 1.4: Optimal foldover design for an initial MA 2^{8-4} design using the SMCE criterion

Run	A	B	C	D	E	F	G	H
1	-	-	-	-	-	-	-	-
2	+	-	-	-	+	+	+	-
3	-	+	-	-	+	+	-	+
4	+	+	-	-	-	-	+	+
5	-	-	+	-	+	-	+	+
6	+	-	+	-	+	-	-	+
7	-	+	+	-	-	+	+	-
8	+	+	+	-	+	-	-	-
9	-	-	-	+	-	+	+	+
10	+	-	-	+	+	-	-	+
11	-	+	-	+	+	-	+	-
12	+	+	-	+	-	+	-	-
13	-	-	+	+	+	+	-	-
14	+	-	+	+	-	-	+	-
15	-	+	+	+	-	-	-	+
16	+	+	+	+	+	+	+	+
Run	A	B	C	D	$-E$	F	G	H
17	-	-	-	-	+	-	-	-
18	+	-	-	-	-	+	+	-
19	-	+	-	-	-	+	-	+
20	+	+	-	-	+	-	+	+
21	-	-	+	-	-	-	+	+
22	+	-	+	-	-	-	-	+
23	-	+	+	-	+	+	+	-
24	+	+	+	-	-	-	-	-
25	-	-	-	+	+	+	+	+
26	+	-	-	+	-	-	-	+
27	-	+	-	+	-	-	+	-
28	+	+	-	+	+	+	-	-
29	-	-	+	+	-	+	-	-
30	+	-	+	+	+	-	+	-
31	-	+	+	+	+	-	-	+
32	+	+	+	+	-	+	+	+

design, we select the optimal combined design according to the SMCE criterion. We subsequently compare these combined designs with those ranked according to the MA criterion in Li and Lin (2003). Second, we rank the initial FF designs according to the

Table 1.5: Two 2^{8-4} optimal foldover plans: A comparison of MA and SMCE

Criteria	SMCE of initial design	SMCE of combined design
MA	8/8, 0/28	8/8, 0/28
SMCE	8/8, 0/28	8/8, 7/28

SMCE criterion, and then we rank the combined design also using the SMCE criterion. Our research (see Tables A.1 and A.2 in the appendix) concludes that there exists some noteworthy differences between the optimal combined designs selected according to the MA and the SMCE criteria. Using the SMCE criterion leads to the selection of combined designs that possess at least as many clear low-order effects than that obtained using the MA criterion.

Chapter 2

Fractional Factorial Designs

2.1 Background

In physical experiments, we may wish to examine the effects of different factors, each varied at several levels. For example, suppose an engineer's goal is to determine how the yield of an adhesive application process can be improved by adjusting three process parameters: mixture ratio of the chemical constituents of the adhesive, curing temperature, and curing time. Suppose that each of these three parameters will be varied at two levels, (a "low" and a "high" value). Questions to address include: What are the effects of the individual factors and their interactions on the response? How do we choose the levels for the three process parameters to get maximum possible yield? Factorial designs can provide answers to questions such as these.

Factorial designs are useful for determining which factors (variables) comprising an industrial process have a significant impact on the response(s) of interest. This is achieved by varying each of the, say k, factors at a pre-determined number of levels. In many such "screening" designs all k factors are varied at only two levels—a "low" and a "high" value. Factorial designs give the user the ability to consider interactions amongst the k factors without having an exorbitant design run-size. With such designs, low-order interaction effects, such as two-factor interactions, can be assessed. Three-factor and higher interactions are typically considered negligible (Frigon and Mathews 1997, pg163; Wu and Hamada 2000, pg156; Box et. al. 2005, pg303). Without considering low-order interaction effects, incorrect conclusions may be obtained in subsequent data analysis.

As mentioned in the preceding paragraph, a common type of factorial design is a k-factor design which only involves two levels per factor. Such designs are referred to as "2^k factorial designs". A 2^k factorial design is particularly useful in the early stage of experimental work because it requires a smaller number of runs than a factorial design with several levels per factor.

As the number of factors in a 2^k factorial design increases, more runs are required for a complete replicate of the design. For example, a complete replicate of a 2^7 design requires 128 runs, and 256 runs are required if we add an 8th factor. For a 2^k design, there is a total of 2^k-1 effects, including k main effects, $\binom{k}{2}$ two-factor interactions, $\binom{k}{3}$ three-factor interactions and so on. Each effect has one degree of freedom; however, many of the degrees of freedom are associated with three-factor and higher interactions which are assumed negligible here. So, in 2^k factorial designs our emphasis is upon screening for significant main effects and two-factor interactions (that is, low-order effects).

Because of the large number of runs required for a 2^k factorial design when k is large, and also because high-order interactions are generally considered to be negligible, statisticians often conduct FF designs. A FF design can identify which low-order effects are significant by running only a fraction (subset) of the runs of a full 2^k experiment. For example, a 2^{7-2} design requires 32 runs compared to 128 runs necessitated by a 2^7 design.

2.2 The One-Half Fraction of the 2^k Design

The one-half fraction of the 2^k design is referred to as a "2^{k-1}" design. To construct a one-half fraction of the 2^k design, we first write down a full factorial design for $k-1$ factors, then add the k^{th} factor by assigning its levels according to the plus and minus signs of its corresponding generator. The one-half fraction of the 2^k design contains 2^{k-1} runs since the kth factor is generated by one or more of the columns of the $k-1$ "basic" factors.

2.2.1 2^{4-1} Fractional Factorial Designs

The eight-run 2^{4-1} FF design is a one-half fraction of the 2^4 full factorial design. This design has 4 main effects (including the added factor D) and 6 two-factor interactions, but only $8-1=7$ available degrees of freedom for estimation purposes.

Example 2.1. Consider the 2^{4-1} design having generator $D = ABC$. The complete defining relation is $I = ABCD$. Using the defining relation, we can determine which low-order effects are clear.

We say that a main effect is clear when it is not aliased with other main effects or any two-factor interactions. Similarly, we say a two-factor interaction is clear if it is not aliased any main effects or other two-factor interactions.

From Table 2.1, it is observed that this 2^{4-1} FF design is constructed by writing down a full 2^3 factorial and then adding the 4th factor D by identifying its plus and minus signs (levels) using the sign of ABC. Since $D = ABC$, if both sides are multiplied by D, then the left-hand side becomes $D^2 = I$. We see that the exponents in the products are formed by using modulus 2 arithmetic, so that any even power of a factor, is equal to I, and any odd power is equal to the factor itself. The right-hand side becomes $ABCD$. Therefore, the 2^{4-1} design has the complete defining relation

Table 2.1: A 2^{4-1} design

Run	A	B	C	$D = ABC$	Label
1	-	-	-	-	(1)
2	+	-	-	+	ad
3	-	+	-	+	bd
4	+	+	-	-	ab
5	-	-	+	+	cd
6	+	-	+	-	ac
7	-	+	+	-	bc
8	+	+	+	+	abcd

$I = ABCD$.

If both sides of the above equation are multiplied by A, we obtain

$$A \times I = A \times ABCD$$

$$A = A^2 \times BCD$$

$$A = I \times BCD$$

$$A = BCD.$$

Factor A is aliased that is, indistinguishable from, the interaction BCD. Using this approach, the alias structure is given by

$$A = BCD$$

$$B = ACD$$

$$C = ABD$$

$$D = ABC$$

$$AB = CD$$

$$AC = BD$$

$$AD = BC.$$

This design has resolution IV since $ABCD$ is a four-letter word. From the preceding alias structure, it is obvious that all main effects are clear; however, two-factor interactions are aliased with other two-factor interactions.

2.3 The One-Quarter Fraction of the 2^k Design

When the number of runs in a FF design becomes very large, we might consider further reducing the number of runs. For example, conducting a one-quarter fraction of the 2^k design is even more run-parsimonious than a 2^{k-1} design. A one-quarter fraction of the 2^k design is constructed by writing down a full 2^{k-2} factorial and then adding the k-1^{th} and k^{th} factors by assigning their levels according to the plus and minus signs of their corresponding generators.

2.3.1 2^{5-2} Fractional Factorial Designs

A 2^{5-2} design is a one-quarter fraction of the 2^5 design, in which eight runs are required. The two added factors may be labeled as D and E.

Example 2.2. Consider a 2^{5-2} FF design having added factors $D = AB$ and $E = AC$.

Table 2.2 displays the 2^{5-2} FF design. It is obtained by first writing down a 2^3 full factorial design, and then by adding the 4th and 5th factors. These factors, D and E, are determined by assigning their levels according to the plus and minus signs of their

Table 2.2: A 2^{5-2} design

Run	A	B	C	$D=AB$	$E=AC$	Label
1	-	-	-	+	+	de
2	+	-	-	-	-	a
3	-	+	-	-	+	be
4	+	+	-	+	-	abd
5	-	-	+	+	-	cd
6	+	-	+	-	+	ace
7	-	+	+	-	-	bc
8	+	+	+	+	+	abcde

corresponding generators. We obtain the following complete defining relation

$$I = ABD = ACE = BCDE,$$

where $BCDE = ABD \times ACE$. The alias structure of this resolution III design is given by

$$\begin{gathered} A = BD = CE \\ B = AD = CDE \\ C = AE = BDE \\ D = AB = BCE \\ E = AC = BCD \\ BC = DE = ACD = ABE \\ BE = CD = ABC = ADE. \end{gathered}$$

Ignoring three-factor and higher interactions, we observe that all of the main effects are aliased with two-factor interactions, and all two-factor interactions are aliased with main effects and/or other two-factor interactions. Therefore, none of the main effects and two-factor interactions are clear. Note that the shortest word in the complete defining relation is of length of 3. Thus, this is a resolution III design.

2.4 The One-Eighth Fraction of the 2^k Design

The one-eighth fraction of the 2^k design may be used when we consider further reducing the number of runs. Similar to one-half and one-quarter fractions, we construct a one-eighth fraction of the 2^k design by writing down a full factorial design in $k-3$ factors, then add the k-2^{th}, k-1^{th} and k^{th} factors by assigning their levels according to the plus and minus signs of their corresponding generators.

2.4.1 2^{7-3} Fractional Factorial Designs

The eight-run 2^{7-3} FF design is a one-eighth fraction of the 2^7 full factorial design. This design has 7 main effects (including the added factors E, F and G) and 21 two-factor interactions with only $16-1=15$ available degrees of freedom for estimation purposes.

Example 2.3. Consider a 2^{7-3} design having generators $E=ABC$, $F=ABD$ and $G=ACD$.

The design is shown in Table 2.3, and the complete definition relation is given by

$$I = ABCE = ABDF = ACDG$$
$$= CDEF = BDEG = BCFG = AEFG.$$

The defining relation is obtained by multiplying the three words $ABCE$, $ABDF$ and $ACDG$ together two at a time and three at a time in all possible combinations.

Table 2.3: A 2^{7-3} design

Run	A	B	C	D	$E=ABC$	$F=ABD$	$G=ACD$	Label
1	-	-	-	-	-	-	-	(1)
2	+	-	-	-	+	+	+	aefg
3	-	+	-	-	+	+	-	bef
4	+	+	-	-	-	-	+	abg
5	-	-	+	-	+	-	+	ceg
6	+	-	+	-	-	+	-	acf
7	-	+	+	-	-	+	+	bcfg
8	+	+	+	-	+	-	-	abce
9	-	-	-	+	-	+	+	dfg
10	+	-	-	+	+	-	-	ade
11	-	+	-	+	+	-	+	bdeg
12	+	+	-	+	-	+	-	abdf
13	-	-	+	+	+	+	-	cdef
14	+	-	+	+	-	-	+	acdg
15	-	+	+	+	-	-	-	bcd
16	+	+	+	+	+	+	+	abcdefg

The alias structure is given by

$$A=BCE=BDF=CDG=EFG$$

$$B=ACE=ADF=CFG=DEG$$

$$C=ABE=ADG=BFG=DEF$$

$$D=ABF=ACG=BEG=CEF$$

$$E=ABC=AFG=BDG=CDF$$

$$F=ABD=AEG=BCG=CDE$$

$$G=ACD=AEF=BCF=BDE$$

$$AB = CE = DF$$

$$AC = BE = DG$$

$$AD = BF = CG$$

$$AE = BC = FG$$

$$AF = BD = EG$$

$$AG = CD = EF$$

$$BG = CF = DE.$$

All of the main effects are clear. However, two-factor interactions are aliased with each other.

2.5 2^{k-p} Fractional Factorial Designs

For a 2^{k-p} FF design, we have a total of k factors where p of the factors are generated via the $k-p$ basic factors. The construction of a 2^{k-p} FF design is similar to that described for 2^{k-1}, 2^{k-2} and 2^{k-3} FF designs. First, we write down a design consisting of the runs for a full factorial in $k-p$ factors. Then we add the p factors by assigning their levels according to the plus and minus signs of their corresponding generators. The total number of words in the complete definition relation is 2^p, including the identity I.

2.6 Selecting Optimal FF Designs

Recall that the resolution of a two-level FF design is equal to the length of the shortest word, excluding I, in the complete defining relation. Designs of resolution III, IV, and V have the following properties (Montgomery, 1997, pp. 561):

1. Resolution III designs: These are designs, in which no main effects are

aliased with any other main effect, but main effects are aliased with two-factor interactions and some two-factor interactions may be aliased with each other.

2. Resolution IV designs: These are designs in which no main effect is aliased with any other main effect or with any two-factor interaction, but two-factor interactions are aliased with each other.

3. Resolution V designs: These are designs in which no main effect or two-factor interaction is aliased with any other main effect or two-factor interactions, but two-factor interactions are aliased with three-factor interactions.

In practice we typically conduct designs having resolution III, IV or V. Obviously, designs with higher resolution are more attractive to an experimenter. However, due to constraints on resources (money, time, etc.) we often have to conduct a smaller, lower-resolution FF design.

It is important to select the p generators for a 2^{k-p} FF design in such a way that we obtain the "best possible" alias structure. An intuitive is to select the generators such that the 2^{k-p} FF design has the highest possible resolution.

Consider the two 2^{7-2} FF designs in Table 2.4. We can obtain a higher resolution design choosing generators $F = ABCD$ and $G = ABDE$ instead of $F = AB$ and $G = BCDE$.

There are occasions when two or more 2^{k-p} designs have the same resolution. In this scenario, the resolution criterion is insufficient to distinguish between the designs.

The MA criterion is commonly used for distinguishing between FF designs having the same resolution. The MA criterion sequentially minimizes the number of words of shorter length in the WLP (Fries and Hunter, 1980; Wu and Hamada, 2000).

Definition 2.1. For any two 2^{k-p} designs d_1 and d_2, let r be the smallest integer

Table 2.4: Two Choices for a the 2^{7-2} Design

Generators	F=$ABCD$, G=$ABDE$	F=AB, G=$BCDE$
Complete defining relation	I=$ABCDF$=$ABDEG$ =$CEFG$	I=ABF=$BCDEG$ =$ACDEFG$
Resolution	Resolution IV	Resolution III

such that $A_r(d_1) \neq A_r(d_2)$, where A_i denotes the number of words of length i in its complete defining relation, $1 \leq i \leq k$. Then d_1 is said to have less aberration than d_2 if $A_r(d_1) < A_r(d_2)$. If there is no design with less aberration than d_1, then d_1 has minimum aberration (MA).

Suppose we have two 2^{7-2} FF designs, where "design A" has generators F=ACD and G=$ABDE$, and "design B" has generators F=ABC and G=BCD. Table 2.5 shows that the WLP for design A is (0,1,2) and that the WLP for design B is (0,3,0). There is only one four-letter word in design A, whereas design B has three four-letter words. Therefore, compared to design B, design A has less aberration.

Sometimes, both designs have the same number of minimum length words. If this is the case, the MA criterion sequentially compares the number of words of successively greater length.

For example, suppose we have two 2^{8-2} FF designs, where "design A" has generators G=ACD and H=$ABEF$, and "design B" has generators G=ABC and H=$ABEF$. In Table 2.6, we see that there is only one five-letter word in the WLP of design A, whereas there are two in the WLP of design B. Therefore, design A will be a better choice compared to design B, with respect to the MA criterion.

Table 2.5: Another Two Choices for a 2^7 Design

Design	A	B
Generators	F=ACD, G=$ABDE$	F=ABC, G=BCD
Complete defining relation	I=$ACDF$=$ABDEG$ =$BCEFG$	I=$ABCF$=$BCDG$ =$ADFG$
WLP	(0,1,2)	(0,3,0)
Resolution	IV	IV

Table 2.6: Two Choices for a 2^{8-2} Design

Design	A	B
Generators	G=ACD, H=$ABEF$	G=ABC, H=$ABEF$
Complete defining relation	I=$ACDG$=$ABEFH$ =$BCDEFGH$	I=$ABCG$=$ABEFH$ =$CEFGH$
WLP	(1,1,0,1)	(1,2,0,0)

The number of clear low-order effects that a design possesses is another possible criterion for ranking FF designs. This criterion sequentially assesses a given FF design with respect to the number of clear main effects and two-factor interactions that it possesses. An FF design that sequentially maximizes clear effects (SMCE) is then optimal according to this criterion.

Suppose that we have two 2^{7-2} FF designs, "Design A" has generators F=$ABCD$ and G=$ABDE$, and "design B" has generators F=ABC and G=ADE. Table 2.7

states that 6 additional clear two-factor interactions are obtained via design A. Therefore, using the SMCE criterion, design A is superior to design B.

Table 2.7: Two Choices for a 2^{7-2} Design

Design	A	B
Generators	$F=ABCD$, $G=ABDE$	$F=ABC$, $G=ADE$
Complete defining relation	$I=ABCDF$ $=ABDEG=CEFG$	$I=ABCF=ADEG$ $=BCDEFG$
Resolution	Resolution IV	Resolution IV
SMCE	7/7, 15/21	7/7, 9/21

2.7 Analyzing 2^{k-p} Designs

FF experiments can be analyzed using the analysis of variance (ANOVA) approach (Box and Draper, 1969; Montogmery, 2005). In this section we very briefly examine analysis issues by considering an example from Montgomery (2005, pp. 290-293).

Example 2.4. Five factors in a manufacturing process for an integrated circuit were investigated in a 2^{5-1} design with the objective of improving the process yield. The five factors were A = aperture setting (small, large), B = exposure time (20 percent below nominal, 20 percent above nominal), C = develop time (30 s, 45 s), D = mask dimension (small, large), and E = etch time (14.5 min, 15.5 min).

The 2^{5-1} design is shown, along with the response, in Figure 2.8. The complete defining relation is $I = ABCDE$, so all low-order effects are clear.

Table 2.8: A 2^{5-1} Design for the Integrated Circuit Experiment

Run	A	B	C	D	$E = ABCD$	Yield
1	-	-	-	-	+	8
2	+	-	-	-	-	9
3	-	+	-	-	-	34
4	+	+	-	-	+	52
5	-	-	+	-	-	16
6	+	-	+	-	+	22
7	-	+	+	-	+	45
8	+	+	+	-	-	60
9	-	-	-	+	-	6
10	+	-	-	+	+	10
11	-	+	-	+	+	30
12	+	+	-	+	-	50
13	-	-	+	+	+	15
14	+	-	+	+	-	21
15	-	+	+	+	-	44
16	+	+	+	+	+	63

The effect estimates of A and AB are calculated as follows:

$$A = Contrast/(n2^{k-p-1}) = (-8+9-34+52-16+22-45+60$$
$$-6+10-30+50-15+21-44+63)/8 = 89/8 = 11.125$$

$$AB = Contrast/(n2^{k-p-1}) = (8-9-34+52+16-22-45+60+6$$
$$-10-30+50+15-21-44+63)/8 = 55/8 = 6.875.$$

In this case, $n = 1$, $k = 5$ and $p = 1$, where n is the number of replicates. Similarly, the sum of squares for A and AB are obtained as follows:

$$SS_A = (Contrast)^2/n2^{k-p} = (89)^2/16 = 495.0625$$

$$SS_{AB} = (Contrast)^2/n2^{k-p} = (55)^2/16 = 189.0625.$$

Table 2.9: Estimated Effects and Sums of Squares for Example 2.4

Variable	Estimated Effect	Sum of Squares
A	11.125	495.062
B	33.875	4590.062
C	10.875	473.062
D	−0.875	3.063
E	0.625	1.563
AB	6.875	189.063
AC	0.375	0.563
AD	1.125	5.063
AE	1.125	5.063
BC	0.625	1.563
BD	−0.125	0.063
BE	−0.125	0.063
CD	0.875	3.063
CE	0.375	0.563
DE	−1.375	7.563

All of the effect estimates, as well as their corresponding sum of squares, are shown in Table 2.9. From Table 2.9, we can see that effects A, B, C and AB are large, relative to the remaining effects. This implies that aperture, exposure time and develop time, as well as the aperture by exposure time interaction, have significant effects on process yield. Mask dimension and etch time appear to have little influence. The AB (aperture by exposure time) interaction plot is shown in Figure 2.1. This plot confirms that the

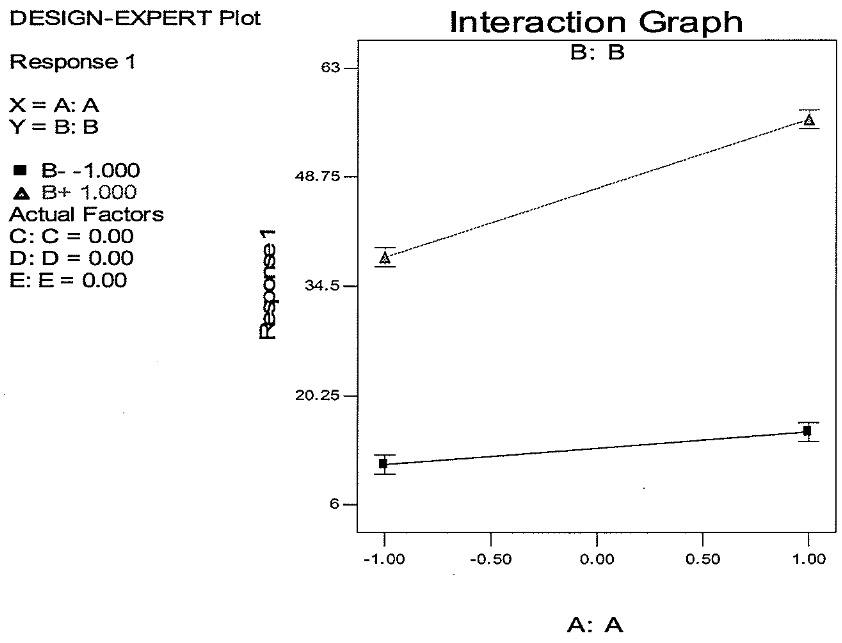

Figure 2.1: A Plot of due AB Interaction for Example 2.4

yields are higher when both A and B are at their high levels. Also, since the effect estimate of C is positive, we infer that C should be run at 45 s to ensure higher yields.

Table 2.10: Analysis of Variance for Example 2.4

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0	P-Value
A (Aperture)	495.062	1	495.0625	193.20	<0.0001
B (Exposure time)	4590.062	1	4590.0625	1791.24	<0.0001
C (Develop time)	473.062	1	473.0625	184.61	<0.0001
AB	189.0625	1	189.0625	73.78	<0.0001
Error	28.1875	11	2.5625		
Total	5775.4375	15			

Table 2.10 summarizes the ANOVA table for this experiment. Since we believe that effects other than A, B, C and AB are negligible, we can estimate the error sum of

square and the process variance by

$$SS_{Error} = SS_{Total} - SS_A - SS_B - SS_C - SS_{AB} = 28.1875$$

$$MS_{Error} = SS_{Error}/df = 28.1875/11 = 2.5625.$$

From Table 2.10, we also notice that the p-values for A, B, C and AB are highly significant (< 0.0001).

Chapter 3

Foldover Designs

We may combine a second FF of equal size with that of an original (initial) FF design to estimate additional main effects and two-factor interactions. This is known as the foldover approach and it is achieved by reversing the signs of one or more factors (columns) of the initial design (Box and Wilson, 1951). It always produces a follow-up design of equal size. Using half of the runs from a foldover design is referred to as semifolding (Mee and Peralta, 2000 and references therein). In this thesis, we focus solely upon the foldover approach.

According to Brewster and McLeod (2008), there are two different situations in which we may want to use the foldover approach: (1) when we want to minimize the amount of overall aliasing in the "combined" design (the initial design together with the foldover design), which is the subject of this thesis, or (2) when we want to de-alias a few low-order effects in which we are interested.

3.1 The Single-factor Foldover Design

By reversing the signs of one factor in a resolution III or IV initial FF design, we may de-alias its main effect and all $k-1$ two factor interactions involving that factor.

Example 3.1. Consider a 2^{5-2} FF design with added factors $D = AB$ and $E = AC$. The design is shown in Table 3.1.

Table 3.1: A 2^{5-2} Design

Run	A	B	C	$D = AB$	$E = AC$	Label
1	-	-	-	+	+	de
2	+	-	-	-	-	a
3	-	+	-	-	+	be
4	+	+	-	+	-	abd
5	-	-	+	+	-	cd
6	+	-	+	-	+	ace
7	-	+	+	-	-	bc
8	+	+	+	+	+	abcde

The complete defining relation is $I = ABD = ACE = BCDE$. The alias structure is (ignoring four-factor and higher order interactions),

$$A = BD = CE$$

$$B = AD = CDE$$

$$C = CE = BDE$$

$$D = AB = BCE$$

$$E = AC = BCD$$

$$BC = DE = ACD = ABE$$

$$BE = CD = ABC = ADE.$$

If we wish to de-alias D, as well as the four two-factor interactions involving D, we can run a follow-up experiment by folding over on factor D only. Table 3.2 shows the foldover design. The complete defining relation of the design is the following:

$$I = -ABD = ACE = -BCDE.$$

Table 3.2: A Foldover of the Initial 2^{5-2} Design, Obtained by Folding on D

Run	A	B	C	$D = -AB$	$E = AC$	Label
9	-	-	-	-	+	e
10	+	-	-	+	-	ad
11	-	+	-	+	+	bde
12	+	+	-	-	-	ad
13	-	-	+	-	-	c
14	+	-	+	+	+	acde
15	-	+	+	+	-	bcd
16	+	+	+	-	+	abce

Combining the 8-run initial and foldover experiments, we obtain a 16-run, resolution III, 2^{5-1} "combined" design. The combined design is shown in Table 3.3. It follows that the complete defining relation for the combined design is given by $I = ACE$.

This combined design has alias structure:

$$A = CE$$

$$C = AE$$

$$E = AC$$

$$AB = BCE$$

$$AD = CDE$$

$$BC = ABE$$

$$BE = ABC$$

Table 3.3: A 2^{5-1} Combined Design Obtained by Folding on D

Run	A	B	C	D	E	Label
1	-	-	-	+	+	de
2	+	-	-	-	-	a
3	-	+	-	-	+	be
4	+	+	-	+	-	abd
5	-	-	+	+	-	cd
6	+	-	+	-	+	ace
7	-	+	+	-	-	bc
8	+	+	+	+	+	abcde
Run	A	B	C	$-D$	E	Label
9	-	-	-	-	+	e
10	+	-	-	+	-	ad
11	-	+	-	+	+	bde
12	+	+	-	-	-	ad
13	-	-	+	-	-	c
14	+	-	+	+	+	acde
15	-	+	+	+	-	bcd
16	+	+	+	-	+	abce

$$CD = ADE$$

$$DE = ACD.$$

It is easy to see that main effects B and D are clear, as well as seven two-factor interactions (AB, AD, BC, BD, BE, CD and DE). Note that any two-factor interaction involving factor D is clear. This example illustrates the general result that by running a single-factor foldover of a resolution III or IV FF design, we can de-alias the main effect of that factor upon which we folded, as well as all of its two-factor interactions. However, if our goal is to obtain as many clear effects as possible, then a single-factor foldover design might not be the best choice.

From the preceding alias structure of a initial 2^{5-2} design, we can see that none of the main effects and two-factor interactions are clear. One could argue that conducting a resolution V 2^{5-1} design would be more advisable in the context, rather than folding a 2^{5-2} design. However, here we are just trying to illustrate the foldover approach.

3.2 The Full Foldover Design

From section 3.1, we know that we can run a single-factor foldover as a follow-up experiment if we are interested in a specific factor only. Now suppose that we are interested in all of the factors. Shall we run a full foldover experiment? A full foldover is the approach whereby we sign-reverse all factors (columns) in the initial design.

Consider the same initial 2^{5-2} design in section 3.1, but now we reverse the signs of all the factors in the original experiment. The initial and foldover designs are displayed in Table 3.4.

The complete defining relation of the full foldover design is $I = -ABD = -ACE = BCDE$. Combining the two eight-run designs, we obtain $I = BCDE$ as the new defining relation. The alias structure of the 16-run resolution IV combined design is

$$B = CDE$$

$$C = BDE$$

$$D = BCE$$

$$E = BCD$$

$$AD = DCE$$

$$BC = DE$$

$$BD = CE$$

$$BE = CD.$$

Table 3.4: A Full Foldover of the 2^{5-2} Design

RUN	A	B	C	D	E	Label
1	-	-	-	+	+	de
2	+	-	-	-	-	a
3	-	+	-	-	+	be
4	+	+	-	+	-	abd
5	-	-	+	+	-	cd
6	+	-	+	-	+	ace
7	-	+	+	-	-	bc
8	+	+	+	+	+	abcde
Run	$-A$	$-B$	$-C$	$-D$	$-E$	Label
9	-	-	-	-	+	e
10	+	-	-	+	-	ad
11	-	+	-	+	+	bde
12	+	+	-	-	-	ad
13	-	-	+	-	-	c
14	+	-	+	+	+	acde
15	-	+	+	+	-	bcd
16	+	+	+	-	+	abce

All five main effects are clear, as well as the two-factor interactions AB, AC, AD and AE. This example illustrates the general purpose of a full foldover design. If a full foldover experiment is conducted on a resolution III design, all main effects will become clear. Note that only a few two-factor interactions will be de-aliased. Therefore, compared to single-factor foldover designs, the full foldover design yields more clear main effects. This may be appealing since low-order effects typically are of most interest to practitioners.

3.3 Core Foldover Plans

In sections 3.1 and 3.2, we illustrated single-factor and full foldover designs. There are many other possibilities for selecting a foldover "plan", where a foldover plan is the set of factors which we sign-reverse. For example, we have 32 possible foldover plans when considering folding over a 2^{5-2} design. How do we enumerate all possible foldover plans?

Table 3.5: The 32 Foldover Plans for a 2^{5-2} Design

Number of factors to be reversed	Number of foldover plans
Reverse a single factor	$\binom{5}{1} = 5$
Reverse two factors	$\binom{5}{2} = 10$
Reverse three factors	$\binom{5}{3} = 10$
Reverse four factors	$\binom{5}{4} = 5$
Reverse five factors	$\binom{5}{5} = 1$
Replicate the initial design	$\binom{5}{0} = 1$
Total	$2^5 = 32$

Definition 3.1. If a foldover plan consists only of the added (generated) factors, then we call this foldover plan a "core foldover plan" (Li and Lin, 2003).

A core foldover plan consists of reversing the signs of one or more of the added factors only. Consider a 2^{5-2} design. Here there are $2^2 = 4$ core foldover plans obtained by folding on either D, E, D and E, or neither D nor E. In general, there are 2^k foldover plans and 2^p core foldover plans for a 2^{k-p} design. For instance, for a 2^{10-6} design, we have $2^{10} = 1,024$ foldover plans and $2^6 = 64$ core foldover plans.

3.4 Equivalent Foldover Plans

There are a total of 2^k ways to foldover the original 2^{k-p} experiment. Some of these plans are "equivalent" to one another. For instance, for the 2^{5-2} design in section 3.1, where reversing the signs of factors A and B yields the identical (that is, equivalent) foldover design as obtained by reversing the sign of factor E.

Result (Li and Lin, 2003) 3.1. For a 2^{k-p} design with p generators, there are 2^p core foldover design, and any foldover plan is equivalent to a core foldover plan.

Result 3.1 tells us that, although we have a total of 2^k ways to fold an initial design, only 2^p plans provide distinct foldover designs. Therefore, we only need to consider core foldover plans when searching for optimal plans. This result can yield considerable time savings in the search process.

Example 3.2. Consider a 2^{5-2} design, where $D = BC$ and $E = ABC$. There are 2^2 core foldover plans.

Table 3.6 displays those foldover plans that are equivalent to a given core foldover plan. The preceding table also illustrates the following result:

Result (Li and Lin, 2003) 3.2. Each core foldover plan has $2^{k-p} - 1$ foldover plans that are equivalent to it.

We see that all of the foldover plans are equivalent to one of the core foldover

Table 3.6: Core Foldover Plans for a 2^{5-2} Design

Core foldover plans	Equivalent foldover plans
Neither D nor E	AE, BC, ABD, ACD, BDE, CDE, $ABCE$
D	AB, AC, BE, CE, ADE, BCD, $ABCDE$
E	A, BD, CD, ABC, BCE, $ABDE$, $ACDE$
Both D and E	B, C, AD, ABE, ACE, $ABCD$, $BCDE$

plans. Therefore, in order to choose an optimal plan, we only need to assess core foldover plans.

3.5 Selecting an Optimal Foldover Plan

In section 3.4, we realized that we need only to consider the 2^p core foldover plans in order to choose an optimal foldover plan for a 2^{k-p} design. For instance, in order to choose an optimal foldover plan for a 2^{7-2} design, we need only to consider the 4 core foldover plans instead of the $2^7 = 128$ possible foldover plans. This reduction in the number of foldover plans to consider is especially advantageous when k is large.

Example 3.3. Using the SMCE criterion, find all optimal foldover plans for a 2^{7-2} design, where $F = ABC$ and $G = ABDE$. The complete defining relation is $I = ABCF = ABDEG = CDEFG$.

Again, there are only four core foldover plans to consider such that the optimal foldover plan(s) will be selected amongst them. For core foldover plan "1" (fold on F), the complete defining relation and alias structure are given by

$$
\begin{gathered}
I = ABDEG \\
AB = DEG \\
AD = BEG \\
AE = BDG \\
AG = BDE \\
BD = AEG \\
BE = ADG \\
BG = ADE \\
DE = ABG \\
DG = ABE \\
EG = ABD.
\end{gathered}
$$

All main effects and two-factor interactions are clear. Therefore, according to the SMCE criterion, we immediately conclude that this an optimal core foldover plan.

For core foldover plan "2" (fold on G), we have the following complete defining relation and alias structure,

$$
\begin{gathered}
I = ABCF \\
A = BCF \\
B = ACF \\
C = ABF \\
F = ABC
\end{gathered}
$$

$$AB = CF$$

$$AC = BF$$

$$AF = BC.$$

All main effects are clear; however, some of the two-factor interactions are aliased with each other. Obviously, this is not an optimal foldover plan.

For core foldover plan "3" (fold on F and G), the complete defining relation and alias structure are given by

$$I = CDEFG$$

$$CE = DFG$$

$$CF = DEG$$

$$CG = DEF$$

$$DE = CFG$$

$$DF = CEG$$

$$DG = CEF$$

$$EF = CDG$$

$$EG = CDF$$

$$FG = CDE.$$

All main effects and two-factor interactions are clear. Therefore, folding on F and G is also an optimal core foldover plan.

Finally, for core foldover plan "4" (replicate the initial design), the complete defining relation and alias structure are

$$I = ABCF = ABDEG = CDEFG$$

$$A = BCF$$

$$B = ACF$$

$$C = ABF$$

$$F = ABC$$

$$AB = CF$$

$$AC = BF$$

$$AD = BEG$$

$$AE = BDG$$

$$AF = BC$$

$$AG = BDE$$

$$BD = AEG$$

$$BE = ADG$$

$$BG = ADE$$

$$CD = EFG$$

$$CE = DFG$$

$$CG = DEF$$

$$DE = ABG = CFG$$

$$DF = CEG$$

$$DG = ABE = CEF$$

$$EF = CDG$$

$$EG = ABD = CDF$$

$$FG = CDE.$$

All main effects are clear, but a number of two-factor interactions are aliased with each other. It is clearly not an optimal core foldover plan. In fact, it is not really a foldover plan at all.It would only be considered if “replication” was of particular important.

After assessing all 4 core foldover plans, we conclude that plans 1 and 3 are the optimal foldover plans with respect to the SMCE criterion.

Chapter 4

Selecting Optimal Designs Using the SMCE Criterion

The number of low-order clear effects was proposed by Chen, Sun, and Wu (1993) as a criterion for selecting 2^{k-p} designs. The advantage of the SMCE criterion is that it is an arguably more intuitive design selection criterion than is the MA approach. Since practitioners typically conduct 2^{k-p} designs with the intent of screening for significant low-order effects, designs that are "good" in this context will have obvious appeal. In this chapter, we use the SMCE criterion to (1) select optimal foldover plans, given an MA initial 2^{k-p} design and (2) select both optimal initial and foldover designs.

4.1 Selecting Optimal Initial Designs Using the SMCE Criterion

In chapter 1, we showed an example of how to compare two designs using the SMCE criterion. From Table 2.6, we discovered that design A, having generators $F = ABCD$ and $G = ABDE$, is better than design B, having generators $F = ABC$ and $G = ADE$. It turns out that design A is the best initial design according to the SMCE criterion.

Example 4.1. Consider 3 initial 2^{7-3} designs, design 1 has generators $E = ABC$, $F = ABD$ and $G = ACD$ (see design 7-3.1 in Table A.1). Design 2 has generators $E = AB$, $F = AC$ and $G = BCD$ (see design 7-3.2 in Table A.1). Design 3 has generators $E = AB$, $F = AC$ and $G = BD$ (see design 7-3.3 in Table A.1). Find the optimal initial design.

Design 1 has SMCE entries 7/7, 0/21, whereas designs 2 and 3 have SMCE entries 2/7, 2/21 and 0/7, 4/21 respectively. Because design 1 has all main effects clear, it is superior to designs 2 and 3. According to the SMCE criterion, design 2 is superior to design 3 although both designs have the same total number of clear low-order effects.

For a given FF design, choosing the generators so as to sequentially maximize the number of clear effects is an important computing task. The majority of the computing in this thesis was accomplished using the statistical software, R. Tables A.1 and A.2 (in the Appendix) summarize the optimal initial 16- and 32-run FF designs, for $6 \leq k \leq 10$, using the SMCE criterion.

4.2 Folding the SMCE Optimal Initial Designs Using the SMCE Criterion

In section 4.1, we considered ranking the initial designs using the SMCE criterion. After we obtain the optimal initial designs, we use the foldover approach, in conjunction with the SMCE criterion, to run a follow-up experiment of equal size with the objective of sequentially de-aliasing as many low-order effects as possible.

Example 4.2. Consider the SMCE optimal initial 2^{7-2} design, having generators $F = ABC$ and $G = ABDE$ (see design 7-2.1 in Table A.2). Find the optimal foldover plan(s) using this initial design.

Table 4.1: Choosing the Optimal Foldover Plan(s) for the SMCE Optimal Initial 2^{7-2} Design Using the SMCE Criterion

Core Foldover Plans	SMCE	Optimal Foldover Plans
F	7/7, 21/21	Yes
G	7/7, 21/21	Yes
FG	7/7, 15/21	No

From Table 4.1, it is obvious that folding F or G are the optimal foldover plans according to the SMCE criterion.

For each SMCE optimal initial design, there are 2^k possible foldover plans to assess. Ranking the number of clear effects sequentially, we select the optimal foldover plan(s) for each one (see Tables A.1 and A.2 in the Appendix).

4.3 Comparison of the Optimal Foldover Plans Using the MA and SMCE Criteria

In Table A.1 and A.2, Li and Lin (2003) select the optimal foldover plans for 16- and 32-run initial FF designs using the MA criterion, where $5 \leq k \leq 11$ and $7 \leq k \leq 11$, respectively. These initial FF designs are non-isomorphic to each other. For a given 2^{k-p} designs, the first initial design given is the MA design.

We also investigate the optimal foldover plans for 16-run and 32-run initial MA FF designs, for $6 \leq k \leq 10$, and $7 \leq k \leq 10$, respectively. See Tables A.1 and A.2.

In chapter 1 (Table 1.3), we saw the different optimal foldover plans suggested by the MA and SMCE criteria when folding an initial 2^{8-4} FF design. Are there other initial 2^{k-p} designs for which the optimal foldover plans differ, when considering both the MA and SMCE criteria?

We found a total of 8 occurrences for which the optimal combined designs suggested by the MA and SMCE approaches differ. These designs are marked as " * " in Tables A.1 and A.2. In chapter 1, we provided such an example using an initial 2^{8-4} FF design (see Table 1.3). In this section, we consider an additional example.

Example 4.3. Consider a 2^{9-4} FF design with added factors $F = ABC$, $G = ABD$, $H = ACD$ and $J = BCD$. This design is denoted by "9-4.5" in Table A.2.

Table A.2 shows the number of clear effects and WLP for both plans. In the third column of Table 4.4, the MA optimal design has values "9/9, 8/36". This implies that the MA design has 9 out of 9 clear main effects, 8 out of 36 clear two-factor interactions. Similarly, the SMCE optimal design has 9 out of 9 clear main effects, 15out of 36 clear two-factor interactions. Table A.2 shows that the optimal foldover plan using the SMCE criterion obtains 7 additional clear two-factor interactions when compared to the optimal foldover plan using the MA criterion.

Differences between some of the optimal combined designs, suggested by the MA and SMCE criteria, may only be observed when sequentially comparing the number of strongly clear low-order effects that they possess.

Definition 4.1. We call a main effect or two-factor interaction strongly clear if it is not aliased with main effects, two-factor interactions, or three-factor interactions.

Example 4.4. Consider a 2^{9-5} FF design with added factors $E = ABC$, $F = ABD$, $G = ACD$, $H = BCD$ and $J = ABCD$. This desing is denoted by "9-5.1" in Table A.1.

The number of clear effects (including the strongly clear effects) and WLP for both plans are shown in Table A.1. In the seventh column of Table A.1, the MA optimal design has values "9/9, 8/36, 1/9, 0/36". This implies that the MA design has 9 out of 9 clear main effects, 8 out of 36 clear two-factor interactions, 1 out of 9 strongly clear main effects and no strongly clear two-factor interactions. Similarly, the SMCE optimal design has 9 out of 9 clear main effects, 8 out of 36 clear two-factor interactions, 1 out of 9 strongly clear main effects and 8 out of 36 strongly clear two-factor interactions. Both plans have the same number of clear low-order effects. The only difference is that the design selected by the SMCE criterion has 8 strongly clear two-factor interactions, whereas the design selected by the MA criterion has no strongly clear two-factor interactions.

In our search process we also found that, in order to get the greatest number of clear effects after folding, it is not necessary that the initial design be either an MA design or an optimal SMCE design. Consider the following example.

Example 4.5. Consider 3 initial 2^{9-4} FF designs: design A with $F = BCDE$, $G = ACDE$, $H = ABDE$ and $J = ABCE$; design B with $F = ABC$, $G = ABD$, $H = ACD$ and $J = BCDE$; design C with $F = AB$, $G = ACD$, $H = ACE$ and $J = BDE$.

Table 4.2 shows that design C is not the optimal initial design according to either the MA or the SMCE criteria, but its SMCE optimal foldover plan obtains the largest total number of clear effects. Such occurrences are denoted by a "§" in Tables A.1 and A.2.

Furthermore, our search discovered there exists a design whose optimal foldover plans are different according to the MA and SMCE criteria, but the number of clear effects are exactly the same. Consider the following example.

Example 4.6. Consider a initial 2^{10-5} design having generators $F = ABCD$, $G = ABCE$, $H = ABDE$, $J = ACDE$ and $K = BCDE$. This design is denoted by

Table 4.2: Three Possible Initial 2^{9-4} FF Designs

Design	Design A	Design B	Design C
WLP	(0 6 8 0 0)	(0 7 7 0 0)	(1 5 6 2 1)
SMCE	9/9, 8/36	9/9, 15/36	6/9, 9/36
MA initial design	Yes		
SMCE of Optimal initial design		Yes	
SMCE of Combined Designs	9/9, 24/36	9/9, 21/36	9/9, 30/36

"10-5.1" in Table A.2.

This design has 20 different optimal foldover plans under the SMCE criterion and 19 different optimal foldover plans according to the MA criterion (Li and Lin, 2003). However, both foldover plans have the same number of clear effects. This design is marked as "‡" in Table A.2.

Chapter 5

Conclusion

Box and Wilson (1951) first introduced the foldover approach as a sequential method for de-aliasing low-order effects in 2^{k-p} experiments. Foldover designs are constructed by running an equally-sized follow-up experiment with one or more of the k columns (factors) sign-reversed. The foldover approach can be used to minimize the amount of aliasing in the combined design, or simply to de-alias only several effects in which we are most interested. For a given 2^{k-p} design, there are many foldover plans to consider, and choosing an optimal design from among them may be of interest. The SMCE criterion is one approach for selecting an optimal foldover plan. This criterion sequentially assesses the number of clear low-order effects that a given FF design possesses.

Using the SMCE criterion, we select optimal foldover plans for 32-run and 64-run combined FF designs for $6 \leq k \leq 10$ and $7 \leq k \leq 10$, respectively. Our approach in this thesis has been to: (1) take an MA initial design (Li and Lin, 2003), and obtain the optimal foldover plan using the SMCE criterion, (2) select both the initial and combined designs using the SMCE criterion. Our research demonstrates that, in order to sequentially yield as many (strongly) clear low-order effects as possible, the SMCE criterion is superior to the MA criterion on a number of occasions. As such, our catalog of SMCE optimal combined designs will aid practitioners in selecting the most suitable initial and foldover designs for screening for low-order effects.

Appendices

Appendix A

Optimal 32-Run and 64-Run Combined Designs Ranked According to the SMCE Criterion

Table A.1: Optimal 32-run Combined Designs

	Initial Design			Optimal Foldover Plan	Combined Design	
$k-p$	Generators	WLP	CME, CTI [a]		WLP	CME, CTI
¶6-2.1	$E=ABC$, $F=ABD$	(0 3 0 0 0)	6/6, 0/15	E, F, EF	(0 1 0 0 0)	6/6, 9/15
§†6-2.2	$E=AB$, $F=ACD$	(1 1 1 0 0)	3/6, 6/15	EF	(0 0 1 0 0)	6/6, 15/15
§†6-2.3	$E=AB$, $F=CD$	(2 0 0 1 0)	0/6, 9/15	EF	(0 0 0 1 0)	6/6, 15/15
¶†7-3.1	E=ABC, F=ABD, G=ACD	(0 7 0 0 0)	7/7, 0/21	E, F, G, EF, EG, FG, EFG	(0 3 0 0 0)	7/7, 6/21
§7-3.2	$E=AB$, $F=AC$, $G=BCD$	(2 3 2 0 0)	2/7, 2/21	EFG	(0 1 2 0 0)	7/7, 15/21
7-3.3	$E=AB$, $F=AC$, $G=BD$	(3 2 1 1 0)	0/7, 4/21	EFG	(0 2 0 1 0)	7/7, 9/21
7-3.4	$E=AB$, $F=AC$, $G=AD$	(3 3 0 0 1)	0/7, 0/21	EFG	(0 3 0 0 0)	7/7, 6/21
†7-3.5	$E=AB$, $F=AC$, $G=BC$	(4 3 0 0 0)	1/7, 6/21	EFG	(0 3 0 0 0)	7/7, 6/21
¶†*8-4.1	E=ABC, F=ABD, G=ACD, H=BCD	(0 14 0 0 0)	8/8, 0/28	E, F, G, H, EFG, EFH, EGH, FGH	(0 7 0 0 0) [0 6 0 0 0] [c]	8/8, 7/28 [b] [8/8, 0/28]
§8-4.2	$E=AB$, $F=AC$, $G=AD$, $H=BCD$	(3 7 4 0 1)	1/8, 1/28	$EFGH$	(0 3 4 0 0)	8/8, 13/28
8-4.3	$E=AB$, $F=AC$, $G=BD$, $H=CD$	(4 5 4 2 0)	0/8, 0/28	$EFGH$	(0 5 0 2 0)	8/8, 4/28
§8-4.4	$E=AB$, $F=AC$, $G=BC$, $H=ABCD$	(4 6 4 0 0)	2/8, 0/28	EFG	(0 3 4 0 0)	8/8, 13/28
8-4.5	$E=AB$, $F=AC$, $G=BC$, $H=AD$	(5 5 2 2 1)	0/8, 2/28	$EFGH$	(0 5 0 2 0)	8/8, 4/28
†8-4.6	$E=AB$, $F=AC$, $G=BC$, $H=ABC$	(7 7 0 0 1)	1/8, 7/28	EFG	(0 7 0 0 0)	8/8, 7/28
*9-5.1	$E=ABC$, $F=ABD$, $G=ACD$, $H=BCD$, $J=ABCD$	(4 14 8 0 4)	0/9, 0/36	J	(0 6 8 0 0) [0 14 0 0 0]	9/9, 8/36, 1/9, 8/36 [9/9, 8/36, 1/9, 0/36] [d]
9-5.2	$E=AB$, $F=AC$, $G=BD$, $H=CD$, $J=ABCD$	(6 9 9 6 0)	0/9, 0/36	$EFGHJ$	(0 9 0 6 0)	9/9, 2/36
9-5.3	$E=AB$, $F=AC$, $G=BC$, $H=AD$, $J=BCD$	(6 10 8 4 2)	0/9, 0/36	$EFGH$	(0 10 0 4 0)	9/9, 2/36

Table A.1 (continued)

	Initial Design			Optimal Foldover Plan	Combined Design	
$k-p$	Generators	WLP	CME, CTI		WLP	CME, CTI
9-5.4	$E=AB$, $F=AC$, $G=BC$, $H=AD$, $J=BD$	(7 9 6 6 3)	0/9, 0/36	$EFGHJ$	(0 9 0 6 0)	9/9, 0/36
9-5.5	$E=AB$, $F=AC$, $G=BC$, $H=ABC$, $J=AD$	(8 10 4 4 4)	0/9, 0/36	$EFGJ$	(0 10 0 4 0)	9/9, 2/36
10-6.1	$E=ABC$, $F=ABD$, $G=ACD$, $H=BCD$, $J=ABCD$, $K=CD$	(8 18 16 8 8)	0/10, 0/45	JK	(0 18 0 8 0)	6/10, 0/45
10-6.2	$E=AB$, $F=AC$, $G=BC$, $H=AD$, $J=BD$, $K=ACD$	(9 16 15 12 7)	0/10, 0/45	$EFGHJ$	(0 16 0 12 0)	6/10, 0/45
10-6.3	$E=AB$, $F=AC$, $G=BC$, $H=AD$, $J=BD$, $K=CD$	(10 15 12 15 10)	0/10, 0/45	$EFGHJK$	(0 15 0 15 0)	6/10, 0/45
10-6.4	$E=AB$, $F=AC$, $G=BC$, $H=ABC$, $J=AD$, $K=BD$	(10 16 12 12 10)	0/10, 0/45	$EFGJK$	(0 16 0 12 0)	6/10, 0/45

NOTE:

a. CME denotes the number of clear main effects, CTI denotes the number of clear two-factor interactions.

b. 8 main effects and none of two-factor interaction are clear for the optimal foldover plans with respect to the MA initial design.

c. [ ] denotes the WLP or SMCE for the optimal combined designs given an initial MA design.

d. Given the initial MA design, there is 1 strongly CME and 0 strongly CTIs.

*. Implies that there exists differences between optimal combined designs selected according to the MA and the SMCE criteria.

†. Implies that the design has the max. number of clear effects among the non-isomorphic initial designs.

¶. Implies that the design sequentially maximizes clear effects among the non-isomorphic initial designs.

§. Implies that the design is not an optimal initial design according to either the MA and SMCE, but its SMCE optimal foldover plan(s) obtain(s) the largest total number of clear effects.

Table A.2: Optimal 64-run Combined Designs

Initial Design				Optimal Foldover Plan	Combined Design	
$k-p$	Generators	WLP	CME, CTI		WLP	CME, CTI
¶†7-2.1	$F=ABCD$, $G=ABDE$	(0 1 2 0 0)	7/7, 15/21	F, G	(0 0 1 0 0)	7/7, 21/21
7-2.2	$F=ABC$, $G=ADE$	(0 2 0 1 0)	7/7, 9/21	FG	(0 0 0 1 0)	7/7, 21/21
7-2.3	$F=ABC$, $G=ABD$	(0 3 0 1 0)	7/7, 6/21	F, G, FG	(0 1 0 0 0)	7/7, 15/21
†7-2.4	$F=AB$, $G=ACDE$	(1 0 1 1 0)	4/7, 18/21	FG	(0 0 0 1 0)	7/7, 21/21
7-2.5	$F=AB$, $G=CDE$	(1 1 0 0 1)	4/7, 12/21	FG	(0 0 0 0 1)	7/7, 21/21
7-2.6	$F=AB$, $G=ACD$	(1 1 1 0 0)	4/7, 12/21	FG	(0 0 1 0 0)	7/7, 21/21
7-2.7	$F=AB$, $G=CD$	(2 0 0 1 0)	1/7, 15/21	FG	(0 0 0 1 0)	7/7, 21/21
7-2.8	$F=AB$, $G=AD$	(2 1 0 0 0)	2/7, 11/21	FG	(0 1 0 0 0)	7/7, 15/21
¶†8-3.1	$F=ABC$, $G=ABD$, $H=BCDE$	(0 3 4 0 0)	8/8, 13/28	F, G, FG, FH, GH, FGH	(0 1 2 0 0)	8/8, 22/28
8-3.2	$F=ABC$, $G=ABD$, $H=ACE$	(0 5 0 2 0)	8/8, 4/28	GH	(0 1 0 2 0)	8/8, 22/28
8-3.3	$F=ABC$, $G=ABD$, $H=ABE$	(0 6 0 0 0)	8/8, 0/28	FG, FH, GH	(0 2 0 0 0)	8/8, 16/28
8-3.4	$F=ABC$, $G=ABD$, $H=ACD$	(0 7 0 0 0)	8/8, 7/28	F, G, H, FG, FH, GH, FGH	(0 3 0 0 0)	8/8, 13/28
§8-3.5	$F=AB$, $G=ACD$, $H=BCE$	(1 2 3 1 0)	5/8, 13/28	FGH	(0 0 2 1 0)	8/8, 28/28
†8-3.6	$F=AB$, $G=AC$, $H=BCDE$	(2 1 2 2 0)	3/8, 18/28	FGH	(0 1 0 2 0)	8/8, 22/28
8-3.7	$F=AB$, $G=ACD$, $H=ACE$	(1 3 2 0 1)	5/8, 10/28	FG, FH	(0 1 1 0 1)	8/8, 22/28
§8-3.8	$F=AB$, $G=CD$, $H=ACE$	(2 1 2 2 0)	2/8, 16/28	FGH	(0 0 2 1 0)	8/8, 28/28
8-3.9	$F=AB$, $G=AC$, $H=BDE$	(2 2 1 1 1)	3/8, 12/28	FGH	(0 1 1 0 1)	8/8, 22/28
8-3.10	$F=AB$, $G=AC$, $H=ADE$	(2 2 2 0 0)	3/8, 12/28	FGH	(0 1 2 0 0)	8/8, 22/28
9-4.1	$F=BCDE$, $G=ACDE$, $H=ABDE$, $J=ABCE$	(0 6 8 0 0)	9/9, 8/36	FG, FH, FJ, GH, GJ, HJ	(0 2 4 0 0)	9/9, 24/36
¶†*9-4.2	$F=ABC$, $G=ABD$, $H=ACD$, $J=BCDE$	(0 7 7 0 0)	9/9, 15/36	F, G, H, FGH, FGJ, FHJ, GHJ	(0 3 4 0 0) [0 3 3 0 0] [a]	9/9,21/36,3/9,8/36 [9/9,21/36,3/9,0/36]

Table A.2 (continued)

Initial Design				Optimal Foldover Plan	Combined Design	
$k-p$	Generators	WLP	CME, CTI		WLP	CME, CTI
9-4.3	$F=ABC$, $G=ABD$, $H=ACE$, $J=ADE$	(0 9 0 6 0)	9/9, 0/36	FJ, GH, FGH, FGJ, FHJ, GHJ	(0 3 0 4 0)	9/9, 18/36
9-4.4	$F=ABC$, $G=ABD$, $H=ACD$, $J=ABE$	(0 10 0 4 0)	9/9, 2/36	HJ	(0 3 0 4 0)	9/9, 21/36
*9-4.5	$F=ABC$, $G=ABD$, $H=ACD$, $J=BCD$	(0 14 0 0 0)	9/9, 8/36	F, G, H, J, FGH, FGJ, FHJ, GHJ	(0 7 0 0 0) [0 6 0 0 0]	9/9, 15/36 [9/9, 8/36]
§9-4.6	$F=AB$, $G=ACD$, $H=ACE$, $J=BDE$	(1 5 6 2 1)	6/9, 9/36	$FGHJ$	(0 1 4 2 0)	9/9, 30/36
9-4.7	$F=AB$, $G=ACD$, $H=ACE$, $J=ADE$	(1 7 4 0 3)	6/9, 12/36	FG, FH, FJ, FGH, FGJ, FHJ	(0 3 2 0 2)	9/9, 21/36
§9-4.8	$F=AB$, $G=CD$, $H=ACE$, $J=BDE$	(2 3 6 4 0)	3/9, 12/36	FGH, FGJ, $FGHJ$	(0 1 4 2 0)	9/9, 30/36
9-4.9	$F=AB$, $G=AC$, $H=AD$, $J=BCDE$	(3 3 4 4 1)	2/9, 15/36	$FGHJ$	(0 3 0 4 0)	9/9, 21/36
9-4.10	$F=AB$, $G=AC$, $H=BD$, $J=CDE$	(3 3 4 4 1)	2/9, 13/36	$FGHJ$	(0 2 3 1 1)	9/9, 24/36
¶‡10-5.1	$F=ABCD$, $G=ABCE$, $H=ABDE$, $J=ACDE$, $K=BCDE$	(0 10 16 0 0)	10/10, 0/45	FG, FH, FJ, GH, GJ...[b]	(0 4 8 0 0)	10/10, 24/45
10-5.2	$F=ABC$, $G=ABD$, $H=ACE$, $J=ADE$, $K=ABCDE$	(0 15 0 15 0)	10/10, 0/45	FGH, FGJ, FHJ, FJK, GHJ, GHK	(0 5 0 10 0)	10/10, 15/45
10-5.3	$F=ABC$, $G=ABD$, $H=ACD$, $J=ABE$, $K=ACE$	(0 16 0 12 0)	10/10, 0/45	GK, HJ, GHJ, GHK, GJK, HJK	(0 6 0 8 0)	10/10, 15/45
*10-5.4	$F=ABC$, $G=ABD$, $H=ACD$, $J=BCD$, $K=ABE$	(0 18 0 8 0)	10/10, 0/45	FK, GK, HK, JK, FGH, FGJ, $FHJK$, $GHJK$	(0 8 0 4 0) [0 6 0 8 0]	10/10, 18/45 [10/10, 17/45]
†*10-5.5	$F=AB$, $G=ACD$, $H=ACE$, $J=ADE$, $K=CDE$	(1 14 7 0 7)	7/10, 14/45	$FGHJ$	(0 7 7 0 0) [0 6 4 0 4]	10/10, 24/45 [10/10, 16/45]
10-5.6	$F=AB$, $G=ACD$, $H=ACE$, $J=ADE$, $K=BCDE$	(1 10 11 4 3)	7/10, 8/45	$FGHJK$	(0 6 4 0 4)	10/10, 30/45
*10-5.7	$F=AB$, $G=CD$, $H=ACE$, $J=BDE$, $K=ABCDE$	(2 7 12 7 2)	4/10, 6/45	$FGHJ$, $FGHK$, $FGJK$	(0 3 8 3 0) [0 3 6 4 2]	10/10, 30/45 [10/10, 27/45]

Table A.2 (continued)

Initial Design				Optimal Foldover Plan	Combined Design	
$k-p$	Generators	WLP	CME, CTI		WLP	CME, CTI
§10-5.8	$F=AB$, $G=AC$, $H=BCD$, $J=BCE$, $K=ADE$	(2 8 12 4 2)	5/10, 4/45	*FGHJK*	(0 2 8 4 0)	10/10, 33/45
*10-5.9	$F=AB$, $G=AC$, $H=AD$, $J=BCD$, $K=BDE$	(2 9 9 6 4)	10/10, 0/45	*FGHK*	(0 7 3 0 4) [0 3 6 4 2]	10/10, 24/45, 3/10 [10/10, 24/45, 1/10]
10-5.10	$F=AB$, $G=AC$, $H=AD$, $J=BCD$, $K=ABCDE$	(3 8 11 4 1)	3/10, 12/45	*FGHJ*	(0 3 7 4 0)	10/10, 30/45

NOTE:

a. [ ] denotes the WLP or SMCE for the optimal combined designs given an initial MA criterion.

b. The complete set is *FG*, *FH*, *FJ*, *FK*, *GH*, *GJ*, *GK*, *HJ*, *HK*, *JK*, *FGH*, *FGJ*, *FGK*, *FHJ*, *FHK*, *FJK*, *GHJ*, *GHK*, *GJK* and *HJK*.

*. Implies there exists differences between optimal combined designs selected according to the MA and the SMCE criteria.

†. Implies that the design has the max. number of clear effects among the non-isomorphic initial designs.

¶. Implies that the design sequentially maximizes clear effects among the non-isomorphic initial designs.

‡. Implies that there exists differences between the optimal foldover plans selected according to the MA and SMCE criteria.

§. Implies that the design is not an optimal initial design according to either the MA and SMCE, but its SMCE optimal foldover plan(s) obtain(s) the largest total number of clear effects.

Appendix B

Codes

B.1 R code shows the optimal foldover plan for a 2^{6-2} FF design with $E = AB$, $F = ACD$

```
##The original version of this program was written by Ben Chen, in the summer of 2007,
when Mr. Chen was a holder of an NSERC undergrade award.
## this function can be used only as a continuation on the completeDR functions
## The OFP function will condiser all possible foldover plans and pick the
optimal foldover design according to the clear effect criterion
## The inputs of this function are
##n: the number of factors
##f: the number of generators
## f1:f6 are generators, enter only "E" if the generator is E=AB
##again this function only allows a maximum of 6 generators
##if there is less than 6 generators, only enter as many as needed. (note,
please be consistent with what the inputs are for the completeDR function)
## for example, if there is 7 factors and only 3 generators, the the generators
are E=AB, F=BC, G=ACD
## to execute the program you would enter as follows
##OFP(7, 3, "E", "F", "G")


OFPCE<-function(n, f, f1=’a’, f2=’b’, f3=’c’, f4=’d’, f5=’e’, f6=’g’){


## The following if statements will take the input and determine how many generators
there are and what they are

FP<-list()
if(f==1){AG<-c(f1)}
if(f==2){AG<-c(f1,f2)}
if(f==3){AG<-c(f1,f2,f3)}
if(f==4){AG<-c(f1,f2,f3,f4)}
if(f==5){AG<-c(f1,f2,f3,f4,f5)}
if(f==6){AG<-c(f1,f2,f3,f4,f5,f6)}
```

```
## The following for-loop will generator all possible sets of foldover plans,
generally, if there is f generators, there are 2^f-1 possible sets of generators
## Since a foldover plan can be done by just folding over one generators, or any
combinations of generators, I use the if-statements to generates all possible sets
of foldover plans
## each foldover plan is store as a list, the name of the list is FP
a=1
for(h in 1:f){

if(h==1){
for(i in 1:f){
FP[[a]]<-c(AG[i])
a=a+1
}
}
if(h==2 & f>=2){
for(i in 1:(f-1)){
for(j in (i+1):(f)){
FP[[a]]<-c(AG[i], AG[j])
a=a+1
}
}
}

if(h==3 & f>=3){
for(i in 1:(f-2)){
for(j in (i+1):(f-1)){
for(k in (j+1):f){
FP[[a]]<-c(AG[i], AG[j], AG[k])
a=a+1
}
}
```

```
}
}
if(h==4 & f>=4){
for(i in 1:(f-3)){
for(j in (i+1):(f-2)){
for(k in (j+1):(f-1)){
for(l in (k+1):f){
FP[[a]]<-c(AG[i], AG[j], AG[k], AG[l])
a=a+1
}
}
}
}
}
if(h==5 & f>=5){
for(i in 1:(f-4)){
for(j in (i+1):(f-3)){
for(k in (j+1):(f-2)){
for(l in (k+1):(f-1)){
for(m in (l+1):(f)){
FP[[a]]<-c(AG[i], AG[j], AG[k], AG[l], AG[m])
a=a+1
}
}
}
}
}
}
if(h==6 & f>=6){
for(i in 1:(f-5)){
for(j in (i+1):(f-4)){
for(k in (j+1):(f-3)){
for(l in (k+1):(f-2)){
```

```
for(m in (l+1):(f-1)){
for(n in (m+1):(f)){
FP[[a]]<-c(AG[i], AG[j], AG[k], AG[l], AG[m], AG[n])
a=a+1
}
}
}
}
}
}
}
}

##CDRFP is a list, and each component of the list is the same and it is the complete
defining relation of the original fractional factorial design


CDRFP <-list()
for(h in 1:(2^f-1)){
CDRFP[[h]]<-wlpCDR[[4]]
}


##The following multiple for-loops will either a negative sign in front of th word,
or get rid of the negative sign if there is already a negative sign in front of
the word
##The method used here is using substrings, trimming and replacing


for(x in 1:length(FP)){
for(h in 1:length(FP[[x]])){
for(i in 1:length(wlpCDR[[4]])){
if(substring(CDRFP[[x]][i], 1, 1) != c("-")){
```

```
for(j in 1:nchar(wlpCDR[[4]][i])){
if(substring(wlpCDR[[4]][i], j, j)==(FP[[x]][h])){
CDRFP[[x]][i]<-c("fhjkfdfddhkhsdjkfhs")
CDRFP[[x]][i]<-strtrim(CDRFP[[x]][i], ((wlpCDR[[3]][i])+1))
substring(CDRFP[[x]][i], 2, nchar(CDRFP[[x]][i]))
<-substring(wlpCDR[[4]][i], 1, wlpCDR[[3]][i])
substring(CDRFP[[x]][i], 1, 1)<-c("-")
}
}
}
else{
for(j in 1:nchar(wlpCDR[[4]][i])){
if(substring(wlpCDR[[4]][i], j, j)==(FP[[x]][h])){
substring(CDRFP[[x]][i], 1, wlpCDR[[3]][i])
<-substring(wlpCDR[[4]][i], 1, wlpCDR[[3]][i])
CDRFP[[x]][i]<-strtrim(CDRFP[[x]][i], wlpCDR[[3]][i])
}
}
}
}
}
}

## the following for-loop is used to check out the common terms between CDRFP[[u]][z]
and wlpCDR[[4]][z]
## when there is a common term, the loop will find out the number of chars for each
common term
## and store it in CWLFP, then WLFFP


WLFFP<-list()
for(u in 1:(2^f-1)){
w=1
```

```
CWLFP <-c(0)
for(z in 1:length(CDRFP[[u]])){
if(CDRFP[[u]][z]==wlpCDR[[4]][z]){
CWLFP[w]=nchar(CDRFP[[u]][z])
w=w+1 }
}

WLFFP[[u]]=CWLFP
}

##The following for-loop is used to find out the Combined complete defining relations
##Basically, it only keeps the common terms and gets rid of the non-common terms between
 the original CDR and foldover CDR



CCDRList<-list()


for(y in 1:(2^f-1)){


u=1
CCDR<- rep("djksnjfksdkfjhskfnd", times=length(WLFFP[[y]]))
for(v in 1:length(wlpCDR[[4]])){
if(CDRFP[[y]][v]==wlpCDR[[4]][v]){
substring(CCDR[u], 1, nchar(CDRFP[[y]][v]))<-substring(CDRFP[[y]][v], 1, nchar(CDRFP[[y]][v
CCDR[u]<-strtrim(CCDR[u], nchar(CDRFP[[y]][v]))
u=u+1
}
CCDRList[[y]]=CCDR
}
}

##this assumes that all factors are to be enter by alphabetical order
a<-c("A", "B", "C", "D", "E", "F", "G", "H", "J", "K", "L", "M", "N", "O", "P",
```

```
"Q", "R", "S", "T", "U", "V", "W", "X", "Y", "Z")

##generates all single factors

fac<-list()
for(h in 1:n){
fac[[h]]<-c(a[h])
}

##generates all two factor interactions
##in this program, the order of the interactions matters AB is not the same as BA
##but later, we will only keep one of (AB, or BA) if AB is clear

twofac<-list()
b=1
for(h in 1:(n-1)){
for(i in (h+1):n){
twofac[[b]]<-c(a[h], a[i])
b=b+1
}
}

for(h in 1:(n-1)){
for(i in (h+1):n){
twofac[[b]]<-c(a[i], a[h])
b=b+1
}
}

##the following for loops will check for clear main effects as well as two factor
interactions
##the idea is basically takes any words with length 3 or 4, and determine all
possible clear main effects as well as two factor interactions
```

```
##for example,
CE<-list()
NoCE<-list()
for(y in 1:(2^f-1)){
CME<-fac
for(h in 1:(2^f-1)){
if(nchar(CCDRList[[y]][h])==3){
for(i in 1:nchar(CCDRList[[y]][h])){
for(j in 1:n){
if(substring(CCDRList[[y]][h], i, i)==fac[[j]]){
CME[[j]]<-c(" ")}
}

}
}


}

CTFI<-twofac
temp<-list()
temp2<-list()
for(h in 1:(2^f-1)){
if(nchar(CCDRList[[y]][h])<5 & nchar(CCDRList[[y]][h])>=3 ){
if(nchar(CCDRList[[y]][h])==4){
e=1
aa<-c(substring(CCDRList[[y]][h], 1, 1), substring(CCDRList[[y]][h], 2, 2),
substring(CCDRList[[y]][h], 3, 3), substring(CCDRList[[y]][h], 4, 4))
for(i in 1:3){
for(j in (i+1):4){
temp[[e]]<-c(aa[i], aa[j])
e=e+1
temp[[e]]<-c(aa[j], aa[i])
e=e+1}
```

```
}
for(l in 1:(e-1)){
for(m in 1:length(twofac)){
if(identical(twofac[[m]], temp[[l]])==TRUE){
CTFI[[m]]<-c(" ")
}
}
}
}
else{
g=1
bb<-c(substring(CCDRList[[y]][h], 1, 1),substring(CCDRList[[y]][h], 2, 2),
substring(CCDRList[[y]][h], 3, 3))
for(i in 1:2){
for(j in (i+1):3){
temp2[[g]]<-c(bb[i], bb[j])
g=g+1
temp2[[g]]<-c(bb[j], bb[i])
g=g+1}
}
for(l in 1:(g-1)){
for(m in 1:length(twofac)){
if(identical(twofac[[m]], temp2[[l]])==TRUE){
CTFI[[m]]<-c(" ")
}
}
}
}
}


}

SCME<-CME
```

```
for(h in 1:(2^f-1)){
if(nchar(CCDRList[[y]][h])==4){
for(i in 1:nchar(CCDRList[[y]][h])){
for(j in 1:n){
if(substring(CCDRList[[y]][h], i, i)==CME[[j]]){
SCME[[j]]<-c(" ")}
}
}
}
}

SCTFI<-CTFI
for(h in 1:(2^f-1)){
if(nchar(CCDRList[[y]][h])==5){
v=1
temp3<-list()
cc<-c(substring(CCDRList[[y]][h], 1, 1), substring(CCDRList[[y]][h], 2, 2),
substring(CCDRList[[y]][h], 3, 3), substring(CCDRList[[y]][h], 4, 4),
substring(CCDRList[[y]][h], 5, 5))
for(i in 1:4){
for(j in (i+1):5){
temp3[[v]]<-c(cc[i], cc[j])
v=v+1
temp3[[v]]<-c(cc[j], cc[i])
v=v+1}
}
for(l in 1:(v-1)){
for(m in 1:length(twofac)){
if(identical(CTFI[[m]], temp3[[l]])==TRUE){
SCTFI[[m]]<-c(" ")
}
}
```

```
}

}

}


ME<- list()

u=1

for(h in 1:n){

if(CME[[h]] != c(" ")){

ME[[u]]=CME[[h]]

u=u+1 }

}


TFI<-list()

w=1

for(i in 1:(n*(n-1)/2)){

if(identical(CTFI[[i]], c(" "))!=TRUE){

TFI[[w]]=CTFI[[i]]

w=w+1

}

}


SME<- list()

r=1

for(h in 1:n){

if(SCME[[h]] != c(" ")){

SME[[r]]=SCME[[h]]

r=r+1 }

}


STFI<-list()

s=1

for(i in 1:(n*(n-1)/2)){

if(identical(SCTFI[[i]], c(" "))!=TRUE){
```

```
STFI[[s]]=SCTFI[[i]]
s=s+1
}
}

NoCE[[y]]<-c(length(ME), length(TFI), length(SME), length(STFI))
CE[[y]]<-list(ME, TFI, SME, STFI)
}

MCE<-NoCE[[1]]
for(d in 2:(2^f-1)){
r=1
for(e in 1:4){
if(r==1){
if(MCE[e]!=NoCE[[d]][e]){
if(MCE[e]<NoCE[[d]][e]){
MCE<-NoCE[[d]]
r=r+1}
else{
r=r+1}
}
else{
r=1}
}
}
}

for(d in 1:(2^f-1)){
if(identical(NoCE[[d]],MCE)==TRUE){
cat("When Maximizing the no. of clear effects, the optimal foldover design is ",
 FP[[d]], "\n")
cat("The combined complete defining relation is ", CCDRList[[d]], "\n")
print(CE[[d]])
```

```
cat("The clear effect patterns are ", NoCE[[d]], "\n")
}
}

return(CE)
}

## this function can be used only as a continuation on the completeDR functions
## The OFP function will condiser all possible foldover plans and pick the optimal
foldover design according to the Minimum abberation
## The inputs of this function are
##f: the number of generators
## f1:f6 are generators, enter only "E" if the generator is E=AB
##again this function only allows a maximum of 6 generators
##if there is less than 6 generators, only enter as many as needed. (note, please
be consistent with what the inputs are for the completeDR function)
## for example, if there is only 3 generators, the the generators are E=AB, F=BC, G=ACD
## to execute the program you would enter as follows
##OFP(3, "E", "F", "G")



OFP<-function(f, f1='a', f2='b', f3='c', f4='d', f5='e', f6='g'){



## The following if statements will take the input and determine how many generators
there are and what they are

FP<-list()
if(f==1){AG<-c(f1)}
if(f==2){AG<-c(f1,f2)}
if(f==3){AG<-c(f1,f2,f3)}
if(f==4){AG<-c(f1,f2,f3,f4)}
```

```
if(f==5){AG<-c(f1,f2,f3,f4,f5)}
if(f==6){AG<-c(f1,f2,f3,f4,f5,f6)}


## The following for-loop will generator all possible sets of foldover plans, generally,
if there is f generators, there are 2^f-1 possible sets of generators
## Since a foldover plan can be done by just folding over one generators, or any
combinations of generators, I use the if-statements to generates all possible sets of
foldover plans
## each foldover plan is store as a list, the name of the list is FP
a=1
for(h in 1:f){

if(h==1){
for(i in 1:f){
FP[[a]]<-c(AG[i])
a=a+1
}
}
if(h==2 & f>=2){
for(i in 1:(f-1)){
for(j in (i+1):(f)){
FP[[a]]<-c(AG[i], AG[j])
a=a+1
}
}
}

if(h==3 & f>=3){
for(i in 1:(f-2)){
for(j in (i+1):(f-1)){
for(k in (j+1):f){
FP[[a]]<-c(AG[i], AG[j], AG[k])
```

```
a=a+1
}
}
}
}
if(h==4 & f>=4){
for(i in 1:(f-3)){
for(j in (i+1):(f-2)){
for(k in (j+1):(f-1)){
for(l in (k+1):f){
FP[[a]]<-c(AG[i], AG[j], AG[k], AG[l])
a=a+1
}
}
}
}
}
if(h==5 & f>=5){
for(i in 1:(f-4)){
for(j in (i+1):(f-3)){
for(k in (j+1):(f-2)){
for(l in (k+1):(f-1)){
for(m in (l+1):(f)){
FP[[a]]<-c(AG[i], AG[j], AG[k], AG[l], AG[m])
a=a+1
}
}
}
}
}
}
if(h==6 & f>=6){
for(i in 1:(f-5)){
```

```
for(j in (i+1):(f-4)){
for(k in (j+1):(f-3)){
for(l in (k+1):(f-2)){
for(m in (l+1):(f-1)){
for(n in (m+1):(f)){
FP[[a]]<-c(AG[i], AG[j], AG[k], AG[l], AG[m], AG[n])
a=a+1
}
}
}
}
}
}
}
}


##CDRFP is a list, and each component of the list is the same and it is the complete
defining relation of the original fractional factorial design

CDRFP <-list()
for(h in 1:(2^f-1)){
CDRFP[[h]]<-wlpCDR[[4]]
}


##The following multiple for-loops will either a negative sign in front of th word, or
get rid of the negative sign if there is already a negative sign in front of the word
##The method used here is using substrings, trimming and replacing

for(x in 1:length(FP)){
for(h in 1:length(FP[[x]])){
for(i in 1:length(wlpCDR[[4]])){
```

```
if(substring(CDRFP[[x]][i], 1, 1) != c("-")){
for(j in 1:nchar(wlpCDR[[4]][i])){
if(substring(wlpCDR[[4]][i], j, j)==(FP[[x]][h])){
CDRFP[[x]][i]<-c("fhjkfdfddhkhsdjkfhs")
CDRFP[[x]][i]<-strtrim(CDRFP[[x]][i], ((wlpCDR[[3]][i])+1))
substring(CDRFP[[x]][i], 2, nchar(CDRFP[[x]][i]))<-substring(wlpCDR[[4]][i], 1,
 wlpCDR[[3]][i])
substring(CDRFP[[x]][i], 1, 1)<-c("-")
}
}
}
else{
for(j in 1:nchar(wlpCDR[[4]][i])){
if(substring(wlpCDR[[4]][i], j, j)==(FP[[x]][h])){
substring(CDRFP[[x]][i], 1, wlpCDR[[3]][i])<-substring(wlpCDR[[4]][i], 1,
wlpCDR[[3]][i])
CDRFP[[x]][i]<-strtrim(CDRFP[[x]][i], wlpCDR[[3]][i])
}
}
}

}
}
}


## the following for-loop is used to check out the common terms between CDRFP[[u]][z]
and wlpCDR[[4]][z]
## when there is a common term, the loop will find out the number of chars for each
common term
## and store it in CWLFP, then WLFFP
```

```
WLFFP<-list()
for(u in 1:(2^f-1)){
w=1
CWLFP <-c(0)
for(z in 1:length(CDRFP[[u]])){
if(CDRFP[[u]][z]==wlpCDR[[4]][z]){
CWLFP[w]=nchar(CDRFP[[u]][z])
w=w+1 }
}

WLFFP[[u]]=CWLFP
}



##The following for-loop is used to find out the Combined complete defining relations
##Basically, it only keeps the common terms and gets rid of the non-common terms between
the original CDR and foldover CDR



CCDRList<-list()

for(y in 1:(2^f-1)){

u=1
CCDR<- rep("djksnjfksdkfjhskfnd", times=length(WLFFP[[y]]))
for(v in 1:length(wlpCDR[[4]])){
if(CDRFP[[y]][v]==wlpCDR[[4]][v]){
substring(CCDR[u], 1, nchar(CDRFP[[y]][v]))<-substring(CDRFP[[y]][v], 1,
nchar(CDRFP[[y]][v]))
CCDR[u]<-strtrim(CCDR[u], nchar(CDRFP[[y]][v]))
u=u+1
}
CCDRList[[y]]=CCDR
```

```
}
}



## the following for loop will get the word length pattern sorted out. note, it starts
with a word length of 3, if there is a wor with length one, the for loop will not count it

ALLWLP<-list()
for(t in 1:(2^f-1)){

cwlp<- rep(0, times=(max(WLFFP[[t]])-2))
for(h in 3:max(WLFFP[[t]])){
for( i in 1:length(WLFFP[[t]])){
if(WLFFP[[t]][i]==h){
cwlp[h-2]=cwlp[h-2]+1
}
}
}
ALLWLP[[t]]=cwlp
}



## for the word length pattern, where is stops is not fixed, some might
have a maximum word length of 5, others might have a maximum word length
of 7,
## therefore, the word length pattern will not match in terms of numbers
of components
## this for-loops will find out the maximum word length, and adjust the
word length patterns by adding 0 at the end of the vector, so that all
## word length pattern will have the same number of components before
comparing

MWL<-c(0)
```

```
for(a in 1:(2^f-1)){
MWL[a]=length(ALLWLP[[a]])
}
zero<- list()
Maxlength<-ALLWLP
for(b in 1:(2^f-1)){
if(length(ALLWLP[[b]]) < max(MWL)){
zero[[b]]<- rep(0, times=(max(MWL)-length(ALLWLP[[b]])))
Maxlength[[b]]<-c(ALLWLP[[b]], zero[[b]])
}
}


##the following for loop will find out the optimal word length pattern according to the MA
criterion

MA<-Maxlength[[1]]
for(d in 2:(2^f-1)){
r=1
for(e in 1:max(MWL)){
if(r==1){
if(MA[e]!=Maxlength[[d]][e]){
if(MA[e]>Maxlength[[d]][e]){
MA<-Maxlength[[d]]
r=r+1}
else{
r=r+1}
}
else{
r=1}
}
}
}
```

```
## there could be more than one optimal foldover plan, the for-loop below will search for
all the possible foldover plans.

for(d in 1:(2^f-1)){
if(identical(Maxlength[[d]],MA)==TRUE){
cat("The optimal foldover design is ", FP[[d]], "\n")
cat("The new complete defining relation is ", CDRFP[[d]], "\n")
cat("The combined complete defining relation is ", CCDRList[[d]], "\n")
cat("The word length of the combined complete defining relation is ",
ALLWLP[[d]], "\n")
}
}

cat("According the MA criterion, the optimal foldover plans word length pattern is ",
MA, "\n")
foldover<-list(wlpCDR[[2]], wlpCDR[[4]],CDRFP, WLFFP, CCDRList, ALLWLP, Maxlength, MA)
return(foldover)
}
```

B.2 ***Design Expert*** output for a 2^{10-5} design ($F = AB$, $G = ACD$, $H = ACD$, $J = ADE$, $K = CDE$) with folding on $FGHJ$

```
10 Factors: A, B, C, D, E, F, G, H, J, K

Design Matrix Evaluation for Factorial Reduced 3FI Model

    Factorial Effects Aliases

    [Est. Terms]    Aliased Terms

    [Intercept] = Intercept

    [Block 1] = Block 1 + ABF

    [Block 2] = Block 2 - ABF

    [A] = A

    [B] = B

    [C] = C + DEK + DHJ + EGJ + GHK

    [D] = D + CEK + CHJ + EGH + GJK

    [E] = E + CDK + CGJ + DGH + HJK

    [F] = F

    [G] = G + CEJ + CHK + DEH + DJK

    [H] = H + CDJ + CGK + DEG + EJK

    [J] = J + CDH + CEG + DGK + EHK

    [K] = K + CDE + CGH + DGJ + EHJ

    [AB] = AB

    [AC] = AC

    [AD] = AD

    [AE] = AE

    [AF] = AF

    [AG] = AG

    [AH] = AH

    [AJ] = AJ
```

[AK] = AK

[BC] = BC + DFG + EFH + FJK

[BD] = BD + CFG + EFJ + FHK

[BE] = BE + CFH + DFJ + FGK

[BF] = BF + CDG + CEH + CJK + DEJ + DHK + EGK + GHJ

[BG] = BG + CDF + EFK + FHJ

[BH] = BH + CEF + DFK + FGJ

[BJ] = BJ + CFK + DEF + FGH

[BK] = BK + CFJ + DFH + EFG

[CD] = CD + EK + HJ + BFG

[CE] = CE + DK + GJ + BFH

[CF] = CF + BDG + BEH + BJK

[CG] = CG + EJ + HK + BDF

[CH] = CH + DJ + GK + BEF

[CJ] = CJ + DH + EG + BFK

[CK] = CK + DE + GH + BFJ

[DF] = DF + BCG + BEJ + BHK

[DG] = DG + EH + JK + BCF

[EF] = EF + BCH + BDJ + BGK

[FG] = FG + BCD + BEK + BHJ

[FH] = FH + BCE + BDK + BGJ

[FJ] = FJ + BCK + BDE + BGH

[FK] = FK + BCJ + BDH + BEG

[ABC] = ABC

[ABD] = ABD

[ABE] = ABE

[ABG] = ABG

```
[ABH] = ABH

[ABJ] = ABJ

[ABK] = ABK

[ACD] = ACD + AEK + AHJ

[ACE] = ACE + ADK + AGJ

[ACF] = ACF

[ACG] = ACG + AEJ + AHK

[ACH] = ACH + ADJ + AGK

[ACJ] = ACJ + ADH + AEG

[ACK] = ACK + ADE + AGH

[ADF] = ADF

[ADG] = ADG + AEH + AJK

[AEF] = AEF

[AFG] = AFG

[AFH] = AFH

[AFJ] = AFJ

[AFK] = AFK


Design Matrix Evaluation Performed Omitting Aliased Terms


Factorial Effects Defining Contrast

I = CDEK = CDHJ = CEGJ = CGHK = DEGH = DGJK = EHJK = BCDFG

= BCEFH = BCFJK = BDEFJ = BDFHK = BEFGK = BFGHJ = BCDEFGHJK


Defining Contrast Word Lengths

1   2   3   4   5   6   7   8   9   10

0   0   0   7   7   0   0   0   1   0
```

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