THE PSYCHOMETRIC INVARIANCE OF THREE WECHSLER INTELLIGENCE SCALE SUBTESTS

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ABSTRACT

Within a measure there are various sources or factors contributing to the variance of the measure. These sources of variability may be determined by factor analysis. If the measure in question is administered, along with other tests which measure the abilities hypothesized to contribute to the common factor variance of the given measure, to a heterogeneous group of subjects, then the factors which emerge when the subsequent matrix of correlation coefficients are factor analyzed should determine the reliable variance of the measure. When the sources of variability of a measure are known, comparisons across time, tests or groups may be legitimately made. Psychometric invariance was defined at this point as the degree of constancy of the sources of variability of a measure under certain conditions of change.

The present study attempted to determine the sources of reliable variance of three Wechsler Intelligence Scale subtests; WAIS Arithmetic, WAIS Similarities and WAIS Block Design. Following the procedure outlined above, six factors were identified in the analysis, accounting for large proportions of the reliable variance of the three WAIS subtests.

Within the complete group of subjects, two separate age groups were re-factor analyzed to examine any changes in factor structure of the three subtests across a condition of age change. The results for a 18-19 year and 20-24 year group indicated that similar factors contributed to WAIS Arithmetic variance in both age groups, but that the sources of variance changed with age for WAIS Similarities and WAIS

Block Design. It was concluded that both the Similarities and Block Design subtests lack psychometric invariance under at least one change of age, but that the Arithmetic subtest is a much more stable measure under the age change condition.

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INTRODUCTION

The Factor Analytic Method

Factor analysis is a mathematical technique employed to reduce a given set of data to a simpler form without losing any of the information inherent in the original data. The principal concern of factor analysis is the linear resolution of a set of variables in terms of a small number of categories or "factors". This resolution can be accomplished by the analysis of the correlations among the variables. A satisfactory solution will yield factors which convey all the essential information of the original set of variables. Thus, the chief aim is to attain scientific parsimony or economy of description. (Harmon, 1960).

At the heart of the factor analytic approach is the matrix of correlations between the set of variables. The strength or degree of correlation between two variables indicates the amount of common variance. Variables with low or zero correlations share little common variance and are basically dissimilar in nature. Variables which correlate highly, on the other hand, have a great deal of overlapping variance. The higher the correlation between any two variables the greater the amount of common variance. Certain factors common to both variables are then postulated to underlie this common variance.

A simple linear model is used to represent a variable, z_j, in terms of underlying factors:

$$z_j = a_{j1} F_1 + a_{j2} F_2 + \ldots + a_{jm} F_m + a_j U_j$$

where $F_1, F_2, \ldots F_m$ are the common factors

U_j is the unique factor for variable j

$a_{j1}, a_{j2} \ldots a_{jm}$ are the coefficients of the common factors (called factor loadings)

a_j is the factor loading for unique factor U_j

m is the number of common factors

Several types of factors may be distinguished;

(1) common factors involved in more than one variable. There are two kinds of common factors.

(a) general factor - present in all variables

(b) group factor - present in more than one but not in all variables.

(2) unique factor which is involved in a single variable of a set.

Common factors are necessary to account for the intercorrelations among the set of variables, while each unique factor represents that portion of a variable not ascribable to its correlations with other variables in the set.

For a particular individual i on variable j the equation is:

$$z_{ji} = a_{j1} F_{1i} + a_{j2} F_{2i} + \ldots + a_{jm} F_{mi} + a_j U_{ji}$$

where it is assumed there is a value of each factor for each of the N individuals of the sample. The basic problem of factor analysis is to determine the coefficients, $a_{j1}, \ldots, a_{jm}$, of the common factors.

The total variance of a variable may be expressed in terms of factors:

$$\sigma_j^2 = \sum (z_{ji}^2/N) = a_{j1}^2 \sum (F_{1i}^2/N) + a_{j2}^2 \sum (F_{2i}^2/N)$$

$$+ \ldots + a_{jm}^2 \sum (F_{mi}^2/N) + a_j^2 \sum (U_{ji}^2/N)$$

$$+ 2\,(a_{j1}\, a_{j2} \sum F_{1i}F_{2i}/N + \ldots + a_{jm}\, a_j \sum F_{mi}U_{ij}/N)$$

Both the variables and factors are assumed to be in standard form so that,

$$\sum (z_{ji}^2/N) = 1$$

$$\text{and} \quad \sum F_{ki}^2/N) = 1 \text{ for each factor } k$$

Therefore the equation may be rewritten

$$\sigma_j^2 = 1 = a_{j1}^2 + a_{j2}^2 + \ldots + a_{jm}^2 + a_j^2$$

$$+ 2\,(a_{j1}\, a_{j2}\, r_{F1F2} + \ldots + a_{jm}\, a_j\, r_{FmUj})$$

For uncorrelated factors the last term drops away, leaving,

$$\sigma_j^2 = 1 = a_{j1}^2 + a_{j2}^2 + \ldots + a_{jm}^2 + a_j^2$$

where each factor loading a_{jk}^2 represents the contribution of factor k to the variance of z_j

Two concepts are derived from the above expression

(1) the communality of a variable which is given by the sum of the squares of the common-factor coefficients

$$h_j^2 = a_{j1}^2 + a_{j2}^2 + \ldots + a_{jm}^2$$

(2) the uniqueness, which is the contribution of the unique factor and indicates the extent to which the common factors fail to account for the total variance of the variable.

The unique factor can be further decomposed into specific and error factors. The error variance is due to measurement inaccuracies and represents the unreliability of the variable.

The variable may now be written:

$$z_j = a_{j1} F_1 + a_{j2} F_2 + \dots + a_{jm} F_m + b_j S_j + c_j E_j$$

where S_j and E_j are the specific and error factors, respectively, and b_j and c_j their coefficients.

It follows that

$$\sigma_j 2 = 1 = a_{j1}^2 + a_{j2}^2 + \dots + a_{jm}^2 + b_j^2 + c_j^2$$
$$= h_j^2 + b_j^2 + c_j^2$$

The reliability of a variable then, is

$$r_{jJ} = h_j^2 + b_j^2$$

The factor analysis obtains the communality, h_j^2 and the uniqueness, $b_j^2 + c_j^2$, but splitting the uniqueness into specific and error components is not a part of the factor analytic solution. However the communality does represent a lower bound estimate of the reliability of the variable:

$$h_j^2 + r_{jJ} - b_j^2 < r_{jJ}$$

There does not exist in factor analysis a best or unique solution. Harmon (1960, p.21) emphasizes this fact:

The factor problem is indeterminate in the sense that, given the correlations of a set of variables, the coefficients of a factor pattern are not uniquely determined. That is, systems of orthogonal, or uncorrelated, factors may be chosen, consistent with the observed correlations, in an infinity of ways.

However despite the theoretical infinitude of solutions, certain particular types of solutions have been developed in the course of factor analytic research. The present study utilized a principal components solution conforming to the principles of simple structure.

In the principal components solution to the factor problem, all the variance is analyzed by placing unities in the principal diagonal of the correlation matrix. The principal -axes or principal components method allows each factor to extract the maximum amount of variance and give the smallest possible residuals. The correlation matrix is condensed into the smallest number of orthogonal factors by this method (Fruchter, 1954).

The factors extracted by the principal components method must be rotated to a position where they are psychologically meaningful when interpreted. The Varimax method of rotation (Kaiser, 1958) accomplishes the task of rotation in order to meet the requirements of simple structure as outlined by Thurstone (Thurstone, 1947). Basically, Varimax simplifies the description of each column or factor of the factor matrix by maximizing the following function, called the "normalized Varimax criterion":

$$\max \left\{ V = 1/n \sum_{p=1}^{m} \sum_{j=1}^{n} (a_{jp/hj})^4 - \sum_{p=1}^{m} \left(\sum_{j=1}^{n} a_{jp}^2/h_{jp}^2 \right)^2 \right\}$$

where a_{jp}^2 = factor loading of variable j on factor p

h_j^2 = communality of variable j

m = number of factors

n = number of variables

When this criteria is met, the Varimax rotation defines mathematically the intuitive notion of some aspects of simple structure. Kaiser, (1960) also claims that a Varimax solution is invariant under changes in the composition of the test battery and that Varimax factors obtained in a sample will have a greater likelihood of portraying the universe Varimax factors.

The Varimax rotation to arrive at simple structure is an attempt to reduce the complexity of the variables. The term 'simple structure', in Thurstone's sense, refers to a simple configuration in the underlying order of a given set of factors. A factor output matrix conforming to these principles of simple structure should lead to the most psychologically meaningful and valid interpretation.

The Concept of Psychometric Invariance

Within the factor analytic framework it is possible to determine the sources of variability (i.e. factors) associated with a given measure. Variability in a test score may be presumed to be a linear combination of error variance, specific variance and common factor variance. Error variance and specific variance are unique sources of variability for any one given test, but common factor variance is contributed by a number of different sources or factors. These sources of variability for a measure can be determined by utilizing the factor analytic model. If a number of reference or marker tests (tests which measure abilities involved in the given test of interest) were administered along with the test in question to a heterogeneous group of subjects, the measure of interest would be found to correlate to different degrees with the various reference variables. The extent of the correlation between the test and the reference test reflects the importance of the ability which the reference test measures in contributing to score variance in the given test. The reference or marker tests should be relatively pure measures of the abilities they measure. The subsequent correlation matrix can be factor analyzed to reveal the factor composition of the test in question in which each factor represents a particular source of variability.

This reasoning may be represented by the following factor analytic equation:

$$z_{ij} = a_{j1} z_{i1} + a_{j2} z_{i2} + \ldots + a_{jm} z_{im} + a_{js} z_{is} + e_{ij}$$

where z_{ij} = standard score of individual i on test j.

a_{jk} = factor loading of test j for common factor k

z_{ik} = score for individual i on common factor k

m = number of common factors

a_{js} = factor loading on the specific factor

z_{is} = score for individual i on the specific factor

e_{ij} = error variance of individual i on test j

It can easily be shown (Harman, 1960) that the reliable variance of test j is the sum of the squared factor loadings of the m common factors, or:

$$\alpha_j^2 = a_{j1}^2 + a_{j2}^2 + \ldots + a_{jm}^2$$

In other words the reliable variance of a test (total variance minus specific variance and error variance) is contributed by a number of sources of variability which are represented by the factor loadings. The greater the weight of factor loading, the greater the contribution of the particular factor to the variability of the test.

Determination of the sources of variability of a measure allows meaningful comparisons between two individuals from the same population or from different populations. For two individuals from the same population the equations would be:

$$z_{1j} = a_{j1} z_{11} + a_{j2} z_{12} + \ldots + a_{jm} z_{1m} + a_{js} z_{1s} + e_{1j}$$

and

$$z_{2j} = a_{j1} z_{21} + a_{j2} z_{22} + \ldots + a_{jm} z_{2m} + a_{js} z_{2s} + e_{2j}$$

Differences between the two individual scores are given by differences in z_{ik}, z_{is} and e_{ij} values. Any difference remaining after the specific factor (z_{is}) and error (e_{ij}) contributions have been eliminated would presumably reflect differences in abilities important for performance on that particular test.

Aftanas (1971) has introduced the concept of psychometric invariance to define the degree of constancy of the sources of variability of a measure under certain conditions of change. He defines the term as follows:

"A psychometrically invariant measure is one in which scores obtained can be meaningfully and legitimately compared even when a measure is obtained on persons from different populations, different age groups, different cultures, etc., and when different methods of measurement are utilized, different tests measuring the same thing are used, different methods of administration are used, etc. The assessment of psychometric invariance consists of determining the degree to which the sources of variability associated with a measure are invariant under conditions of change as indicated above. If the sources of variability basic to the measure of interest are invariant then comparisons across populations or methods of measurement are meaningful."

Determining whether or not a measure is psychometrically invariant yields three types of information:

(1) knowledge of the sources of variation within a measure. This gives the psychometrician the basis of the measure's score differentiation and hence the validity and meaningfullness of this differentiation.

(2) changes in the factor loadings of the common factors of the measure between groups. Changes in the factor loadings represent changes in the relative importance of each source of variability. Thus the source of differences between groups may be determined.

(3) changes in the actual sources of variability between different groups. In this case the actual factors contributing to group differences change.

An example is useful to illustrate these points. Suppose that for a certain age group the sources of variability of an arithmetic test have been identified as numerical ability, general reasoning and perceptual speed, and that these three factors account for all the reliable variance of this test for the age group. Comparisons between different groups on this test would be meaningful and valid only if these sources of variability are the same for the groups compared. If a memory factor was also an important source of differentiation for an older group, then individuals from the different groups would differ in a meaningful sense even if their test scores were identical. Furthermore the sources of variability should be valid for the particular test. If the perceptual speed factor was the largest contributor of variance to the scores on the arithmetic test, the test would seem to lack basic construct validity.

The determination of psychometric invariance would seem to have a useful application in psychometrics in three different cases:

(1) across tests. Do tests purporting to measure the same construct or variable in fact measure the same thing? If it can be shown that similar abilities account for the variation in both tests it would be reasonable to conclude that the two tests are psychometrically similar. The same type of assessment applies to a test with different forms or different methods of administration.

(2) across time. Does the same test administered on different occasions show psychometric invariance? The demonstration of psychometric invariance in this case indicates reliability (test-retest agreement) and possibly extends the concept of reliability in that not only do the scores across time agree but the factors entering into the scores are consistent across time. Aftanas (1971) has argued that for this reason, psychometric invariance is a more useful and general measure of a test's worth than reliability.

(3) across groups. Do people from different cultures, age groups, etc. perform well or poorly on a test for the same reason? Unless a test has psychometric invariance across groups, the groups cannot legitimately be compared, since different sources of variability enter into the test performance. Psychometric invariance in this case implies test consistency or hardiness in Cattell's sense of the term (Cattell, 1964).

This last case is of particular interest in a developmental context, in which age changes in abilities are examined. A meaningful discussion of developmental changes in abilities and functions requires a measure in which the sources of variability entering into the measure are known for different ages -- a measure in which the psychometric invariance of the measurement instrument has been determined.

One such ability in which age changes are of interest is intelligence. The following sections discuss the factors involved in intelligence and some of the age related changed in these intellectual factors.

Factorial Studies of Intelligence

The factor analytic procedure has been applied very successfully to intelligence and mental ability tests to identify the underlying abilities which are presumed to encompass the concept intelligence. Throughout the literature, certain factors have been consistently reported, while other factors have been reported with less frequency.

In one of the earliest reported studies Burt (1909; 1911) found numerical, verbal and general group factors. Later, Burt (1924) he found support for a memory span factor and a "manual" and a "scholastic" group-factor. Studies which followed identified a spatial factor, a perceptual or mental speed factor and a mechanical reasoning or visualization factor (Alexander, 1935; Brown, 1933; El Koussy, 1935).

Thurstone (1938) attempted, in quite an exhaustive manner, to discover the "primary mental abilities" and succeeded in locating nine factors: (1) verbal, (2) numerical, (3) spatial, (4) perceptual speed,

(5) visualization or mechanical reasoning, (6) word fluency, (7) inductive reasoning, (8) general reasoning and (9) deductive reasoning.

Testing of large samples of British Army and Navy recruits by Vernon (1950) yielded five group factors (or "sub-factors" in Vernon's hierarchical model): a verbal, a number, a mechanical information, a spatial and a manual factor.

A fair degree of consistency in the factors identified can be noted in these early attempts at a systemization of mental abilities. Verbal, numerical and spatial factors appear repeatedly. Other discovered factors are less dependable.

The development of the Wechsler Intelligence Scales (W-B and later WAIS) prompted further research into the nature of mental factors. Balinsky (1941) attempted to isolate and analyze the mental factors of six different age groups who were administered the W-B scale. By sampling different age groups, Balinsky hoped to discover any changes in the factors and their organization and the stability of the factors from age to age. Balinsky found evidence for a general (G) factor, a verbal factor, a performance factor, a memory factor and three speculative factors (one involving reasoning, a factor called restriction in solution and a factor involving seeing relationships in social situations). However the same factors did not always appear at each age level. The verbal and performance factors appeared most consistently, but the remaining factors appeared and disappeared in the different age groups without pattern. Thus the individual subtests changed their factorial composition from age to age, prompting Balinsky to state that:

"The same test, given to a person of a certain age, may not be measuring the same abilities in him that it would measure when given to an older or younger person. Even though the whole intelligence scale may yield the same factors for a wide span of years, the separate tests that compose the scale may not necessarily be described in terms of the same factors from age to age."

The most detailed analysis of the factorial structure of the WAIS for different ages was performed by Jacob Cohen (Cohen, 1957). He determined five factors, all but one of which appeared in each of the four age groups considered. The first factor, verbal comprehension, involved the Information, Comprehension, Similarities and Vocabulary subtests of the WAIS and showed considerable consistency across ages. A perceptual organization factor largely defined by the Block Design and Object Assembly subtests was the second factor extracted by Cohen. Cohen felt this factor was equivalent to one found in other studies of the W-B but with a different identification; performance (Balinsky, 1941), spatial-perceptual (Hammer, 1950), and closure (Birren, 1952). A memory factor, with substantial loadings on the Arithmetic and Digit Span sub-tests was also identified. Two other factors were quasi-specific and were left uninterpreted other than labeling them as a picture completion factor and a digit symbol factor. Study of the factor loading patterns revealed a remarkable degree of similarity among the youngest three groups with regard to intellectual organization. Cohen stated that the evidence was impressive that the organization of intellectual functioning (as defined by the WAIS) is essentially invariant between the ages of 18 and 54. The same generalization could not be extended to the 60-over 75 group, at least on the basis of Cohen's findings. A real change in intellectual organization in the elderly was noted, with memory playing a far more important role in determining individual differences in test performance.

Cohen's results are consistent with an earlier study of the Wechsler subscales by Birren (Birren, 1952) in which three unambiguous factors were located. The first was a verbal comprehension factor which involved the vocabulary, information, similarities and comprehension subtests. A second factor was identified which involved block design, digit span, arithmetic and digit symbol. The third factor, involving block design, object assembly and picture completion was interpreted as spatial visualization.

Guilford (1967) points out that most factor analyses of the WAIS have been inadequate in determining the number and nature of intellectual abilities represented by the WAIS. Guilford states that an adequate analysis would add about twice as many marker tests to the test battery as there are common factors represented in the Wechsler scale. In this respect the only suitable analysis of the Wechsler tests was the Davis (1956) study.

Davis went a step further in the analysis of the factor structure of the Wechsler subscales by including in the intercorrelation matrix not only the W-B subtests but also reference tests of known factor content. He prepared the following reference or marker tests, two verbal comprehension tests, one numerical facility test, one visualization test, one perceptual speed test, one reasoning test and one mechanical knowledge test. Davis interpreted the rotated factors in the subsequent analysis as: (a) verbal comprehension, (b) visualization, (c) numerical facility, (d) mechanical knowledge, (e) doublet (similarities), (f) general reasoning, (g) fluency, (h) perceptual speed, (i) education of conceptual relations, and (j) information.

Considerably more factors emerged in the Davis study than had in previous analyses of the Wechsler scale. Davis felt that this outcome indicated that the complexity of the Wechsler scale had been underestimated, primarily because of the lack of reference tests in earlier test batteries. In addition Davis suspected that space relations and memory factors were involved in the W-B and that the failure of these two factors to appear in the solution reflected the absence of strong reference variables for them.

Statement of the Problem

The purpose of the present study was to determine the sources of variance for certain intellectual tests and examine any changes in the sources of variance across age groups. Presumably such a determination would demonstrate the presence or lack of psychometric invariance in the chosen tests under one change of condition.

It was decided to examine three subtests of the WAIS for psychometric invariance. WAIS Arithmetic, Similarities and Block Design were chosen for the following reasons:

(1) Arithmetic and Similarities represented the Verbal section of the WAIS and Block Design the non-verbal or Performance half of the WAIS. Thus both major subscales (Verbal and Performance) of the WAIS were represented.

(2) The three subtests are very different in nature and comprise a fairly diverse and wide range of abilities. Thus a heterogeneous sample of tests was selected.

(3) Previous factor analytic work (Cohen, 1957; Davis, 1956) had indicated a multiple factor structure for these subtests indicating that a large proportion of common factor rather than specific variance contributed to subtest score differentiation.

(4) Standardized reference tests to measure the abilities hypothesized to contribute to WAIS Arithmetic, Similarities and Block Design variance were readily available and easy to administer.

(5) An earlier investigation (Staley, 1971) had succeeded in identifying a good proportion of the reliable variance of the WAIS Arithmetic and Similarities subtests. This information was utilized in choosing reference tests for these two subtests.

A fairly low percentage of the variance entering into the WAIS Arithmetic, Similarities and Block Design subtests has been identified by factor analytic work, largely because of the failure to include reference tests in the test battery (The Davis (1956) study is the exception).

Wechsler (1958) discussed the factorial composition of each WAIS subtest, largely basing his presentation on the factor analytic work of Cohen (1951). Table 1 presents this data for the 18-19 age group.

Table 1. Factor loadings on the five oblique factors extracted by Cohen (1951) for the WAIS Arithmetic, Similarities and Block Design subtests (18-19 age group).

	Verbal factor	Non-Verbal factor	Memory factor	Picture Completion factor	Digit Symbol factor
Arithmetic	.09	.10	.32	-.05	.00
Similarities	.23	-.07	.05	.21	.00
Block Design	-.03	.34	.12	.03	.00

These five factors, the last two undifferentiated factors, account for only 12.30%, 10.44% and 13.18% of the reliable variance of the Arithmetic, Similarities and Block Design subtests respectively.

By including reference tests in the test battery, Davis (1956) substantially increased the proportion of identified variance for these three subtests. Table 2 shows the major sources of variance for each subtest in the Davis study.

Table 2, Major factor loadings and proportion of variance by each factor for the W-B Arithmetic, Similarities and Block Design subtests (Davis, 1956).

	Arithmetic		Similarities		Block Design	
Factor	a	a^2	a	a^2	a	a^2
General Reasoning	.571	.33				
Numerical Facility	.359	.13				
Mechanical Knowledge	.340	.12			.411	.17
Information	.318	.10	.282	.08	.305	.09
Verbal Comprehension			.310	.10		
Visualization			.290	.08	.439	.19
Similarities doublet			.470	.22		
Perceptual Speed					.385	.15
Other		.13		.10		.10
Total		.81		.58		.70

For each of the subtests, variance was distributed among four factors, without a clearly dominant factor. However, comments on some of the factors are warranted. The Similarities doublet, the most important factor in the Similarities subtest, is a specific rather than a common factor and the variance contributed by this factor cannot be included as part of the identified reliable variance of the Similarities subtest. The Mechanical Knowledge factor, Davis noted, is more accurately described by the term general technical sophistication, suggesting that it is an experiential factor rather than an ability factor. The Information factor was difficult to identify and rationalize satisfactory, Davis reported. It appeared to be a measure of familiarity with facts, objects and symbols of the world. The remaining factors were clear and unambiguous abilities.

In view of the factors identified in the W-B and WAIS (Balinsky, 1941; Cohen, 1957; Birren, 1952 and Hammer, 1950) and the factors involved in the WAIS Arithmetic, Similarities and Block Design subtests (Davis, 1956; Staley, 1971) the following factors were hypothesized to contribute to WAIS Arithmetic, Similarities and Block Design variance; (1) verbal comprehension, (2) perceptual speed, (3) memory, (4) numerical facility, (5) reasoning, (6) visualization, (7) spatial relations and (8) organization.

In line with these hypothesized factors, the following reference or marker tests were planned to identify the reliable variance of the three WAIS subtests: one memory test, one perceptual speed test, two verbal comprehension tests, two numerical ability tests, one abstract reasoning test, two logical reasoning tests, one general reasoning test, two space relations tests and one mechanical reasoning test.

CHAPTER II
METHOD

Subjects: The test battery was administered to 150 male and female University of Manitoba students. Subjects were sampled to ensure an equal (n = 50) number of subjects in each of three age groupings: 18-19 years, 20-24 years and 25-34 years. These particular age groupings were chosen because they correspond to the way Wechsler grouped ages in the standardization of the WAIS. Efforts were made to obtain an even distribution of ages within any given age bracket and this goal was reached with the exception of the 25-34 age group where the lower age half of this bracket was somewhat overrepresented. Because all ages within the 25-34 bracket were not equally represented, this grouping cannot be properly compared to 25-34 age groups in other studies. Although no attempt was made to control for equal male and female representation within age brackets, the proportion of each was approximately equal.

Tests and Testing Procedure: All subjects were tested in a group session and an individual session, with the group session preceding the individual testing. There was a period of a week between the group session and the individual session. All tests were administered in strict accordance with the instructions for each test, but for practical purposes the time limits for some of the group tests were shortened. The tests with their time limits in the order of presentation were:

GROUP TESTS	TIME LIMIT
1. Vocabulary	6 min.
2. Number Series	5 min.

GROUP TESTS	TIME LIMIT
3. MAT Spatial Relations	5 min.
4. General Reasoning	8 min.
5. Visuo-Spatial Logical Reasoning	10 min.
6. Quantitative	8 min.
7. DAT Mechanical Reasoning	8 min.
8. DAT Space Relations	10 min.
9. DAT Verbal Reasoning	7 min.
10. DAT Numerical Ability	8 min.
11. DAT Abstract Reasoning	7 min.
12. DAT Clerical Speed and Accuracy	6 min.

INDIVIDUAL TESTS

13. WAIS Digit Span
14. WAIS Arithmetic
15. WAIS Similarities
16. WAIS Block Design

A brief description of each test and the rationale for inclusion is discussed below.

1. Vocabulary: This test of verbal comprehension consisted of the "Verbal Scale" test items from the Henmon-Nelson Test of Mental Ability (College Level). Items required matching a word with its synonymn among four alternatives.
2. Number Series: This test of reasoning ability consisted of twenty-five items in which the subject completed a sequence of numbers by supplying the last or last two numbers of the sequence.

3. MAT Spatial Relations: In this Space Relations test, subjects were asked to choose, among four alternatives, the group of pieces which correctly formed a given two-dimensional figure.
4. General Reasoning: In this twenty item test, subjects were required to find logical relationships between statements or determine the relationship between two-dimensional figures.
5. Visuo-Spatial Logical Reasoning: This test was derived from intelligence test items developed by Eysenck (Eysenck, 1960). Subjects were required to supply the missing figure, from among five alternatives, which logically completed a 3 x 3 array of figures. There were thirty items of this nature.
6. Quantitative: Twenty items of general numerical reasoning were adapted from the "Quantitative Scale" of the Henmon-Nelson Test of Mental Ability (College Level).
7. DAT Mechanical Reasoning: This Differential Aptitude Test examined the subjects understanding of mechanical principles applied to common situations.
8. DAT Space Relations: In this test subjects were given a pattern which could be folded into figures and were asked to decide which figures among five choices could be made from the given pattern.
9. DAT Verbal Reasoning: This test required subjects to choose from among five alternatives the pair of words which completed a sentence relating four words in the format: ______ is to ______ as _____ is to _____.

10. DAT Numerical Ability: This test consisted of numerical problems involving simple addition, subtraction, multiplication, division, square and cube roots, percentages and ratios.
11. DAT Abstract Reasoning: Each item on this test required the subjects to choose from among five alternatives the figure or design which completed a series of four other figures or designs.
12. DAT Clerical Speed and Accuracy: This test measured how quickly subjects could identify and mark given letter or number combination.
13. WAIS Digit Span: Subjects were presented sequences of digits of increasing length and asked to repeat the sequence either forward (same order) or backward (reverse order).
14. WAIS Arithmetic: This subtest consisted of simple arithmetic problems presented orally. Time bonuses were given on some problems for quick correct responses.
15. WAIS Similarities: Subjects were asked how two words were alike (example: orange and banana).
16. WAIS Block Design: The materials of this test were blocks with all white, all red and red and white sides. With these blocks, subjects were asked to arrange the blocks to form two-dimensional patterns shown to them. Time bonuses were given for correct solutions within a given time.

The four WAIS subtests were administered as instructed in the WAIS Manual (Wechsler, 1955).

Data Analysis: All tests were scored following the instructions for scoring contained in the test manuals. There was no conversion or transformation of the test scores.

The raw data for the 150 S's on the 16 variables was submitted directly to a Principle Components solution factor analysis program on the University of Manitoba IBM 360/65 computer. In keeping with the purpose of the study - to account for the maximum variance of the test variables - communalities of 1.0 were introduced into the diagonals of the correlation matrix of variables by the factor analysis program. The use of communalities of 1.0 has been argued on the basis of practical considerations by Kaiser, (1960).

The computer program output contained the mean and standard deviation of each variable, the correlation matrix, eigenvalues and the Principle Components solution factor matrix. The factor matrix representing the Principle Components solution was rotated by the computer program to a Varimax solution. The Varimax method of rotation simplifies the columns or factors of the factor matrix in order to meet the requirements of simple structure (Thurstone, 1947).

The data was also analyzed by submitting the raw scores for the 50 S's in each of the three age groups to the computer program. For each of these groups, means and standard deviations of the 16 variables, correlation matrix, eigenvalues and the rotated factor matrix were obtained.

CHAPTER III
RESULTS

The mean and standard deviation of each test for the complete group of subjects (Full Group) is presented in Table 10 in the Appendix. Table 11 in the Appendix lists the correlation coefficients of the 16 variables for the Full Group.

The means and standard deviations of the test variables indicates that the subject sample was above average in the abilities measured by the variables. A comparison of the means and standard deviations of the four WAIS subtests with the means and standard deviations of the original standardization group (Wechsler, 1958) reveals the pattern: test means are higher and standard deviations are smaller.

Factors were extracted and interpreted from the complete group of subjects in order to determine the reliable variance of the three WAIS subtests. The Full Group was then broken down into separate age groups and factors extracted and interpreted from the separate age groups in order to analyze the psychometric invariance of the three WAIS subtests across age change conditions.

In order to determine the number of factors to be retained for interpretation, a Scree test (Cattel, 1952) was performed. The Scree test involves plotting a graph of the eigenvalues against the number of factors and examining the graph for the point where the slope becomes constant. This point on the graph corresponds to the number of factors to be retained. This procedure was followed and indicated that 6 to 8 factors should be retained. Visual examination of the 6 factor, 7 factor and 8 factor output matrices revealed that the 6 factor matrix would be clearest to interpret. Table 3 lists the factor loadings of the 6 retained factors for the 16 variables.

Table 3. The matrix of factor loadings after varimax rotation.

No.	Variable	Factor I	II	III	IV	V	VI
1.	Vocabulary	-0.157	0.593	0.496	-0.061	-0.278	-0.095
2.	Number Series	0.243	0.154	0.189	-0.805	-0.081	-0.039
3.	MAT Spatial Relations	0.855	0.008	-0.022	-0.168	-0.040	0.044
4.	General Reasoning	0.016	0.148	0.535	-0.175	-0.625	0.194
5.	Visuo-Spatial Logical Reasoning	0.271	0.023	-0.032	-0.183	-0.850	-0.049
6.	Quantitative	0.184	0.116	0.537	-0.553	-0.145	-0.127
7.	DAT Mechanical Reasoning	0.668	-0.034	0.316	-0.027	-0.435	-0.148
8.	DAT Space Relations	0.599	0.422	0.286	-0.122	-0.225	0.041
9.	DAT Verbal Reasoning	0.031	0.532	0.273	-0.274	-0.481	-0.201
10.	DAT Numerical Ability	0.024	0.070	0.162	-0.843	-0.261	-0.106
11.	DAT Abstract Reasoning	0.404	0.312	0.145	-0.353	-0.587	-0.017
12.	DAT Clerical Speed and Accuracy	-0.007	0.492	-0.261	-0.470	-0.440	0.162
13.	WAIS Digit Span	-0.005	0.095	0.071	-0.108	-0.004	-0.961
14.	WAIS Arithmetic	0.214	0.113	0.865	-0.206	0.019	-0.085
15.	WAIS Similarities	0.097	0.813	0.099	-0.048	0.073	-0.093
16.	WAIS Block Design	0.339	0.526	-0.044	-0.374	-0.286	0.106

Interpretation of Factors for the Full Group

Rotated factors may be interpreted in terms of those tests which show reasonable loadings on the factor in question. In the present study, tests which showed factor loadings of ± .300 or greater on a given factor were retained for examination, with maximum weight for the interpretation assigned to variables with loadings higher than this minimum. A variable which shows a factor loading of at least ± .30 has about 10% or more of its variance accounted for by the factor in question.

Those variables which show insignificant loadings on the factor are still useful in the interpretation of the factor, as the particular factor must be absent or present only to a limited extent in these low-loading variables. Thus interpretation of factors involves the scrutinization of both high loading and low loading variables.

FACTOR I

The variables with loadings of ± .30 or greater on Factor I are:

3.	MAT Spatial Relations	+ .855
7.	DAT Mechanical Reasoning	+ .668
8.	DAT Space Relations	+ .599
11.	DAT Abstract Reasoning	+ .404
16.	WAIS Block Design	+ .339

Factor I was interpreted as a spatial-visualization factor. The MAT Spatial Relations test, which loads very highly on this factor, is a test of spatial completion involving two-dimensional figures. Similarily the DAT Mechanical Reasoning and DAT Space Relations variables require three-dimensional and 2D visualization respectively of objects and forms. In the DAT Abstract Reasoning test various figures and designs must be visualized in altered positions and orientations, and a fairly substantial loading on a spatial-visualization factor is reasonable.

The manipulation of blocks in order to complete a given design was the required task of the WAIS Block Design test and the presence of a spatial-visualization factor is certainly indicated, although to a limited degree.

The only other variable in the test battery in which spatial figures are involved is the Visuo-Spatial Logical Reasoning test which loads + .271 on Factor I.

The remaining tests in the battery, all of which load insignificantly on Factor I, were marked by an absence of spatial and dimensional content.

FACTOR II

Seven variables had factor loadings of ± .30 or greater on Factor II.

15.	WAIS Similarities	+ .813
1.	Vocabulary	+ .593
9.	DAT Verbal Reasoning	+ .532
16.	WAIS Block Design	+ .526
12.	DAT Clerical Speed and Accuracy	+ .492
8.	DAT Space Relations	+ .422
11.	DAT Abstract Reasoning	+ .312

The interpretation of Factor II was considerably more ambiguous than Factor I. Three tests of a verbal nature loaded heavily on this factor, (Vocabulary, + .593, DAT Verbal Reasoning, + .532 and WAIS Similarities, + .813). Interpreting this factor as a verbal ability was not justifiable under the light of significant loadings by the distinctly non-verbal WAIS Block Design, DAT Clerical Speed and Accuracy, DAT Space Relations and DAT Abstract Reasoning tests.

This factor appears to be most accurately interpreted as a higher-order reasoning factor. Each of the significantly loading variables has symbolic content, whether verbal spatial-figural or abstract. In addition those tests with concrete, quantitative content had minimal loadings on this factor (Number Series, .154; Quantitative, .116; DAT Numerical Ability .070; WAIS Digit Span, .095; WAIS Arithmetic, .113). The nature of the tests defining this factor imply that Factor II is symbolic, higher-order reasoning.

FACTOR III

The following variables had factor loadings ± .30 or greater on Factor III.

14.	WAIS Arithmetic	+ .865
6.	Quantitative	+ .537
4.	General Reasoning	+ .535
1.	Vocabulary	+ .496
7.	DAT Mechanical Reasoning	+ .316

Factor III is identified as a concrete reasoning factor. Variables with high loadings on Factor III were generally characterized by a nonabstract, pragmatic nature in which specific or concrete reasoning ability was required. The items on the WAIS Arithmetic subtest are composed of arithmetic reasoning problems. A large portion of the General Reasoning test consisted of items requiring a solution to specific (logical) problems or questions. The content of the Quantitative test was similar to the WAIS Arithmetic subtest, although the form of the test varied. In the Vocabulary test, one word was matched with a word with similar meaning, a task which required a certain amount of specific or concrete reasoning ability. The last test with a significant loading, DAT Mechanical Reasoning, required the application of functional reasoning to common mechanical situations or problems. Again the test content is specific and concrete.

Variables with high abstract content had low factor loadings on Factor III: (MAT Spatial Relations, - .022, Visuo-Spatial Logical Reasoning, + .032, DAT Abstract Reasoning, + 0.145), offering further support that Factor III is indeed a concrete reasoning or "problem solving" factor.

FACTOR IV+

Variables with loadings of ± .30 or greater on Factor IV were:

10.	DAT Numerical Ability	- .843
2.	Number Series	- .805
6.	Quantitative	- .553
12.	DAT Clerical Speed and Accuracy	- .470
16.	WAIS Block Design	- .374
11.	DAT Abstract Reasoning	- .353

The three high loading tests on this factor (Number Series, Quantitative and DAT Numerical Ability), involve the manipulation and operation of quantitative content. The highest loading test, DAT Numerical Ability, is basically a measure of facility of simple numerical operations such as addition, multiplication, etc. The Number Series test, which also has a very high loading on Factor IV, has as its basis the relationship between sequences of numbers. The quantitative nature of these two tests, in addition to the Quantitative variable (with a factor loading of - .553) warrants an interpretation of Factor IV as a quantitative factor.

Further support for this interpretation is derived from examination of the loadings of distinctly non-quantitative tests on Factor IV. Variables with verbal content (Vocabulary, General Reasoning, DAT Verbal Reasoning, WAIS Similarities), or with specific spatial content (Mat Spatial Relations, Visuo-Spatial Logical Reasoning, DAT Mechanical Reasoning, DAT Space Relations), had non-significant loadings on Factor IV.

Factor II, which was interpreted as a non-quantitative factor, and Factor IV, a quantitative factor, show opposing and generally mutually exclusive factor loading patterns, strengthening the validity of their interpretation.

<u>FACTOR V</u>

The following tests loaded ± .30 or greater on Factor V.

5.	Visuo-Spatial Logical Reasoning	- .850
4.	General Reasoning	- .625
11.	DAT Abstract Reasoning	- .587
9.	DAT Verbal Reasoning	- .481
12.	DAT Clerical Speed and Accuracy	- .440
7.	DAT Mechanical Reasoning	- .435

The pattern of significant factor loadings appear to establish Factor V as a logical/abstract reasoning factor. The content of the tests which load on Factor V are of an abstract, non-concrete, non-quantitative nature. Logical reasoning applied to this abstract subject matter is the common requirement of these tests. The high loading Visuo-Spatial Logical Reasoning test is the best illustration of this point: test items require the S to logically complete arrays of abstract figures in which the last figure is missing in order that a logical relationship be fulfilled. Other significantly loading tests have differing content but require some logical/abstract reasoning ability, (DAT Clerical Speed and Accuracy is the exception). The DAT Abstract Reasoning, DAT Verbal Reasoning and DAT Mechanical Reasoning tests require the identification of the logical relationship between abstract figures, words and mechanical objects respectively.

Logical reasoning with either two-dimensional figures or sentence structures is a major feature of many of the items on the General Reasoning test, a variable with a high factor loading on Factor V.

FACTOR VI

Only one factor loading was above ± .30 on Factor VI.

13. WAIS Digit Span - .961

Unlike the previous five common factors, Factor VI is involved in only a single test in the battery and is properly defined as a specific factor. It can only be interpreted with respect to the WAIS Digit Span subtest - a measure of attention and short term memory.

The purpose of the Full Group analysis was to determine the sources of variance entering into the reliable variance of the WAIS Arithmetic, Similarities and Block Design subtests. The Full Group included S's from 18 to 34 years of age - giving a more heterogeneous sample for determining the reliable variance of the subtests than groups of narrower age range would have allowed.

Within the complete group of S's, three different age groups were separated and analyzed. The three groups, 18-19 years, 20-24 years and 25-34 years, consisted of fifty S's each. Inspection of the data for the three groups led to the decision to examine the psychometric invariance of the WAIS subtests across one change of age, from 18-19 to 20-24, rather than across the three age groups. The decision not to consider the oldest age group was made because of the uncertain and ambiguous interpretation of factors from this group and the rather uneven age distribution within this group. The lower half of the age range was overrepresented and the sample lacked hetrogeneity with respect to age across the 25-34 age range. In addition the interpretation of factors for the group was less certain than for the 18-19 and 20-24 age groups. The data for the 25-34 year group, however, is included in Appendix 2. Table 15 lists the mean and standard deviation of each variable, table 16 contains the matrix of correlation coefficients and table 17 lists the factor loadings after rotation of the 16 variables on the six factors retained for projected interpretation.

Table 12 in Appendix 1 lists the means and standard deviations of the 16 variables for the 18-19 and 20-24 age groups. Table 13 gives the correlation coefficients between the test variables for the 18-19 group and table 14 lists this same information for the 20-24 age group. Both tables appear in Appendix 1.

There is very little difference with respect to means and standard deviations of the 16 variables between the two groups. The younger age group scored somewhat higher on the DAT Clerical Speed and Accuracy test ($\bar{X}$ = 116.800, S.D. = 16.147 for 18-19 years; $\bar{X}$ = 111.420, S.D. = 19.200 for 20-24 years), Number Series ($\bar{X}$ = 15.460, S.D. = 4.016 for 18-19 years; $\bar{X}$ = 14.320, S.D. = 4.838 for20-24 years) and DAT Verbal Reasoning ($\bar{X}$ = 23.720, S.D. = 5.653 for 18-19 years; $\bar{X}$ = 22.380, S.D. = 4.852 for 20-24 years). The older group showed an advantage on the DAT Mechanical Reasoning test ($\bar{X}$ = 25.800, S.D. = 6.931 for 18-19 years; X = 27.280, S.D. = 6.905 for 20-24 years). None of these differences were significant at the .01 probability level when t-tests were performed.

Interpretation of Factors for the 18-19 age group

Table 4 lists the factor loadings after Varimax rotation for the 18-19 year age group.

Table 4. The matrix of factor loadings after Varimax rotation for the 18-19 age group.

No.	Variable	I	II	III	IV	V
1.	Vocabulary	0.056	-0.190	-0.173	0.883	-0.121
2.	Number Series	-0.007	-0.390	-0.622	-0.281	-0.462
3.	MAT Spatial Relations	0.354	-0.214	-0.104	-0.357	-0.552
4.	General Reasoning	0.729	0.380	-0.370	0.062	0.054
5.	Visuo-Spatial Logical Reasoning	0.888	-0.033	-0.031	-0.016	-0.219
6.	Quantitative	0.431	-0.276	-0.713	0.098	-0.088
7.	DAT Mechanical Reasoning	0.792	-0.158	-0.006	0.080	-0.375
8.	DAT Space Relations	0.359	0.040	-0.408	0.135	-0.702
9.	DAT Verbal Reasoning	0.528	-0.321	-0.134	0.544	-0.151
10.	DAT Numerical Ability	0.399	-0.647	-0.285	0.073	-0.157
11.	DAT Abstract Reasoning	0.745	-0.235	-0.265	0.094	-0.172
12.	DAT Clerical Speed and Accuracy	-0.021	-0.762	0.078	0.135	-0.022
13.	WAIS Digit Span	0.069	-0.553	-0.583	0.193	0.103
14.	WAIS Arithmetic	0.324	0.012	-0.777	0.078	-0.165
15.	WAIS Similarities	-0.097	0.187	-0.635	0.207	-0.308
16.	WAIS Block Design	0.242	-0.012	-0.140	0.349	-0.766

For the 18-19 group it was decided to retain five factors for interpretation. This was the number of factors retained by the computer program for the usual 1.0 eigenvalue cutoff criteria. In addition, examination of the four factor output matrix revealed poor factor identification, while the six factor and seven factor outputs preserved the factor loading pattern of the five factors from the five factor output, but introduced uninterpretable singlet factors. For these reasons it was felt that the five factor output was the clearest case to interpret.

FACTOR I

Tests with loadings of ± .30 or greater on Factor I were:

5.	Visuo-Spatial Logical Reasoning	.888
7.	DAT Mechanical Reasoning	.792
11.	DAT Abstract Reasoning	.745
4.	General Reasoning	.729
9.	DAT Verbal Reasoning	.528
6.	Quantitative	.431
10.	DAT Numerical Ability	.399
8.	DAT Space Relations	.359
3.	MAT Spatial Relations	.354
14.	WAIS Arithmetic	.324

The four high loading tests on Factor I, Visuo-Spatial Logical Reasoning, DAT Mechanical Reasoning, DAT Abstract Reasoning and General Reasoning, largely measure abstract and logical reasoning. The remaining significantly loading tests also require some amount of logical reasoning ability. Factor I was identified, therefore, as a logical reasoning factor.

FACTOR II

Tests with factor loadings of $\pm$ .30 or greater on Factor II were:

12.	DAT Clerical Speed and Accuracy	-.762
10.	DAT Numerical Ability	-.647
13.	WAIS Digit Span	-.553
2.	Number Series	-.390
4.	General Reasoning	.380
9.	DAT Verbal Reasoning	-.321

Factor II was difficult to interpret. This factor primarily involved three seemingly unrelated tests; DAT Clerical Speed and Accuracy, DAT Numerical Ability and WAIS Digit Span. These three tests, as well as two lesser loading tests (Number Series and DAT Verbal Reasoning), do however all involve familiar and common symbols - either numbers, letters or words. The task required by each test is different, but familiar rather than abstract and unfamiliar figures are involved in each test. Variables with high abstract content (Visuo-Spatial Logical Reasoning, DAT Space Relations, DAT Abstract Reasoning) loaded poorly on this factor. Those tests which did load significantly on Factor II can be characterized by their noticeable lack of an abstract content or need for abstract reasoning.

Tests decreased in their importance in defining this factor as the level of reasoning ability required by a test increased. Clerical Speed and Accuracy required only quick identification of letter combinations - no reasoning ability was involved. The short term memory of a sequence of numbers, the task of the Digit Span test, requires no reasoning. The DAT Numerical Ability test is a measure of the ability to quickly perform basic arithmetic rather than a measure of arithmetic reasoning. It is not unreasonable to assume that speed or quickness played a large role in this test's performance.

Three functions appeared to compose this factor; basic and familiar content rather than abstract content, very little reasoning ability required, and a quickness or speed component.

Perceptual identification and quickness appeared to be the most accurate interpretation of Factor II. This interpretation includes the three functions involved in the factor and emphasizes the major role of perceptual speed (DAT Clerical Speed and Accuracy was afterall the highest loading test on Factor II).

FACTOR III

Tests loading ± .30 or greater on Factor III were:

14.	WAIS Arithmetic	-.777
6.	Quantitative	-.713
15.	WAIS Similarities	-.635
2.	Number Series	-.622
13.	WAIS Digit Span	-.583
8.	DAT Space Relations	-.408
4.	General Reasoning	-.370

Tests of reasoning and problem solving dominated the list of tests with significant factor loadings on Factor III. The two highest loading tests (WAIS Arithmetic and Quantitative) are composed of items of arithmetic problems. Number Series and WAIS Similarities involve seeking relationships between numbers and words respectively, and require reasoning ability. The general characteristic of these four tests is general reasoning applied to concrete material - words, numbers or problems. Tests of an abstract nature had very low loadings on this factor. It was concluded that Factor III was a concrete reasoning factor.

WAIS Digit Span had a fairly high loading on this factor, presumably because the test involved numbers or sequences of numbers. The General Reasoning test was composed almost equally of abstract and concrete items and its quite low factor loading (-.370) appears reasonable on a concrete reasoning factor.

FACTOR IV

Four tests had loadings greater than $\pm$ .30 on this factor:

1.	Vocabulary	.883
9.	DAT Verbal Reasoning	.544
3.	MAT Spatial Relations	-.357
16.	WAIS Block Design	.349

Factor IV was interpreted as a verbal, non-quantitative factor. The Vocabulary and DAT Verbal Reasoning tests essentially define this factor and both tests measure verbal ability. The MAT Spatial Relations test, which has a negative loading on Factor IV, is distinctly non-verbal in nature. The WAIS Block Design loading may be spuriously high. In the six factor and seven factor outputs for this age group the Block Design loading on the same factor was only .198 and .212.

FACTOR V

Tests with ± .30 or greater factor loadings on Factor V were:

16.	WAIS Block Design	-.766
8.	DAT Space Relations	-.702
3.	MAT Spatial Relations	-.552
2.	Number Series	-.462
7.	DAT Mechanical Reasoning	-.375
15.	WAIS Similarities	-.308

Factor V was interpreted as a visualization factor primarily because the three highest loading tests (Mat Spatial Relations, DAT Space Relations and WAIS Block Design) have a common requirement: the ability to visualize and manipulate objects and patterns. DAT Mechanical Reasoning is also a test requiring the ability to visualize relations and outcomes. The same sort of ability (although applied to numbers) is needed for the Number Series test. The wider context of this factor necessitated the interpretation as visualization, rathern than spatial-visualization.

Interpretation of Factors for the 20-24 age group

Five factors were retained for interpretation for the 20-24 age group. The rationale for retaining five factors was the same as for the 18-19 group. Table 5 lists the factor loadings on the test variables for the 20-24 age group.

Table 5. The matrix of factor loadings after Varimax rotation for the 20-24 age group.

No.	Variable					
1.	Vocabulary	0.107	0.733	0.013	-0.384	0.113
2.	Number Series	0.317	0.049	-0.169	-0.724	0.096
3.	MAT Spatial Relations	0.145	-0.084	-0.825	-0.088	-0.104
4.	General Reasoning	0.307	0.257	-0.188	-0.641	-0.163
5.	Visuo-Spatial Logical Reasoning	0.722	-0.079	-0.370	-0.195	0.021
6.	Quantitative	0.041	0.210	-0.233	-0.727	0.100
7.	DAT Mechanical Reasoning	0.169	0.044	-0.791	-0.228	0.145
8.	DAT Space Relations	0.227	0.370	-0.644	-0.239	0.042
9.	DAT Verbal Reasoning	0.473	0.525	-0.162	-0.381	0.175
10.	DAT Numerical Ability	0.432	-0.062	-0.038	-0.751	0.148
11.	DAT Abstract Reasoning	0.573	0.202	-0.476	-0.361	0.050
12.	DAT Clerical Speed and Accuracy	0.742	0.250	0.005	-0.116	-0.406
13.	WAIS Digit Span	-0.030	0.163	-0.033	-0.137	0.925
14.	WAIS Arithmetic	-0.354	0.374	-0.385	-0.664	0.050
15.	WAIS Similarities	0.103	0.754	-0.049	0.018	0.026
16.	WAIS Block Design	0.613	0.310	-0.241	-0.192	0.108

FACTOR I

The following tests had factor loadings of $\pm$ .30 or greater on Factor I:

12.	DAT Clerical Speed and Accuracy	.742
5.	Visuo-Spatial Logical Reasoning	.722
16.	WAIS Block Design	.613
11.	DAT Abstract Reasoning	.573
9.	DAT Verbal Reasoning	.473
10.	DAT Numerical Ability	.432
14.	WAIS Arithmetic	-.354
2.	Number Series	.317
4.	General Reasoning	.307

Factor I appeared to be an ability to quickly perceive relationships. The perceptual speed test (DAT Clerical Speed and Accuracy) had the highest loading on this factor, implying a quickness or speed component to the factor. The second highest loading test, Visuo-Spatial Logical Reasoning, involves discovering logical relationships between abstract figures.

WAIS Block Design and DAT Abstract Reasoning are also tests which involve patterns and relationships between objects or figures. DAT Verbal Reasoning is a test which measures relationships between words. A common requirement of perceiving relationships was present in each of these significant tests, indicating that Factor I was the ability to quickly perceive relationships.

FACTOR II

The following tests had factor loadings of ± .30 or greater on Factor II:

15.	WAIS Similarities	.754
1.	Vocabulary	.733
9.	DAT Verbal Reasoning	.525
14.	WAIS Arithmetic	.374
8.	DAT Space Relations	.370
16.	WAIS Block Design	.310

Three tests of verbal comprehension, Vocabulary, WAIS Similarities and DAT Verbal Reasoning, essentially defined Factor II. For this reason Factor II was interpreted as a verbal, non-quantitative factor. Two other WAIS subtests, Arithmetic and Block Design had low, but significant loeadings on Factor II, perhaps because the tests involved understanding verbal instructions. The .370 loading of the DAT Space Relations test presumably emphasized the non-quantitative nature of the factor.

FACTOR III

Six tests had loadings of ± .30 or greater on Factor III:

3.	MAT Spatial Relations	-.825
7.	DAT Mechanical Reasoning	-.791
8.	DAT Space Relations	-.644
11.	DAT Abstract Reasoning	-.476
14.	WAIS Arithmetic	-.385
5.	Visuo-Spatial Logical Reasoning	-.370

Factor III was identified as a spatial-visualization ability. Three tests had central importance in defining Factor III and each of these tests requires the ability to visualize the relationship between objects and figures, (MAT Spatial Relations, DAT Mechanical Reasoning and DAT Space Relations). Two other tests with lower factor loadings, (DAT Abstract Reasoning and Visuo-Spatial Logical Reasoning), have figural-spatial content and require a certain amount of visualization for solution.

FACTOR IV

Tests with loadings of $\pm$ .30 or greater on Factor IV were:

10.	DAT Numerical Ability	-.751
2.	Number Series	-.724
6.	Quantitative	-.727
14.	WAIS Arithmetic	-.664
4.	General Reasoning	-.641
1.	Vocabulary	-.384
9.	DAT Verbal Reasoning	-.381
11.	DAT Abstract Reasoning	-.361

Tests involving numbers had the highest factor loadings on Factor IV. The ability to reason with and manipulate numbers was an important requirement in each of these high loading tests. Although neither Vocabulary nor DAT Verbal Reasoning were numerical tests, they did involve concrete, familiar items - words and not abstract items, forms or figures. Factor IV was given a slightly more global interpretation than numerical reasoning and was identified as concrete reasoning since this interpretation accounted for the loadings on the verbal tests as well as the numerical tests.

FACTOR V

Two tests loaded $\pm$ .30 or greater on this factor:

13.	WAIS Digit Span	.925
12.	DAT Clerical Speed and Accuracy	-.406

The Digit Span subtest essentially defined Factor V. The Digit Span test apparently measures short term memory and attention. Because the perceptual speed variable had a negative loading on this factor, Factor V may have been more an attention or carefulness factor than an immediate memory factor.

Six factors were extracted from the whole group and five factors from the 18-19 and 20-24 age groups. Table 6 lists these factors:

Table 6. Factors identified in the present study for the whole group and two separate age groups (18-19 years and 20-24 years).

Whole Group	18-19 Group	20-24 Group
spatial-visualization	visualization	spatial-visualization
higher-order reasoning	verbal/non-quantitative	verbal/non-quantitative
concrete reasoning	concrete reasoning	concrete reasoning
quantitative	perceptual identification and quickness	ability to quickly perceive relationships
logical/abstract reasoning	logical reasoning	attention/short term memory
short term memory/attention		

It is not surprising that the three analyses yielded many of the same factors. It would be expected that the Full Group analysis would include the factors identified in the separate age group analysis, as the subjects in the 18-19 and 20-24 groups are subsets of the larger Full Group subject sample. Although the same factors appeared in the different analyses, somewhat different tests were often responsible for defining the factors from analysis to analysis. This implies that slight differences in the meaning of factors with the same name may exist.

CHAPTER IV
DISCUSSION

It was noted in the Results section that the test means were somewhat higher than found in normal standardization groups and that standard deviations were smaller. The major effect of decreased score variation is an attenuation of the correlation coefficients between tests, (Gulliksen, 1950). This decrease in correlation magnitude is a property of the restriction in test score range and is a loss of information which decreases the accountable reliable variance of each test (since in the factor analytic procedure, reliable variance derives from the correlation coefficient matrix).

Generally, similar factors emerged in the Full Group, 18-19 and 20-24 age group analyses. Identified factors were (1) spatial-visualization, (2) verbal/non-quantitative, (3) concrete reasoning, (4) quantitative, (5) logical reasoning, (6) perceptual identification and (7) memory/attention.

How do the factors compare with previously identified intellectual factors? The spatial-visualization factor has been repeatedly determined in factor analytic studies under the name visualization (Alexander, 1935; Thurstone, 1938; Birren, 1952; Davis, 1956) or closure (Hammer, 1950; Vernon, 1950). Cohen's perceptual organization factor, (Cohen, 1957), may be similar in nature to this spatial-visualization, involving as it does an ability to organize elements and visualize their completed outcome. This is largely the meaning of the spatial visualization factor in the present study.

The verbal/non-quantitative factor is similar, but does not correspond exactly to the well-known verbal factor extracted in many studies (Balinsky, 1941; Thurstone, 1938; Vernon, 1950; Cohen, 1957; Birren, 1952; Davis, 1956). The present factor is not a pure verbal ability because it is adulterated with some non-verbal variance. Besides the obvious verbal tests, (Vocabulary, Verbal Reasoning and Similarities), tests such as Space Relations, Abstract Reasoning, Clerical Speed and Accuracy and Block Design frequently had significant, although low, factor loadings on this factor. For this reason the definition of the factor was extended to non-quantitative ability, although the verbal variance is still the largest contributor to the factor. This factor is a higher-order reasoning ability.

The concrete reasoning factor is a problem solving factor, involving the ability to reason with words and numbers. The general reasoning factor in the Davis (1956) study was also defined by significantly loading verbal and numerical reasoning tests and appears to be a factor similar to concrete reasoning. Thurstone (1938) found evidence for a general reasoning factor and Balinsky (1941) determined a reasoning factor in his analysis of the W-B. That a concrete reasoning factor should emerge in the present study is not surprising.

A Quantitative factor, principally involved in the tests of numerical ability (DAT Numerical Ability, Number Series and Quantitative), was identified in the Full Group analysis. This Numerical factor has been consistently identified in many factor analyses of intelligence tests; (Thurstone, 1938; Vernon, 1950; Davis, 1956; Hammer, 1950).

A Logical Reasoning factor appeared in the Full Group analysis and both age group analyses. Other factor analyses have not identified a factor named Logical Reasoning, although Thurstone (1938) located an Inductive Reasoning factor among the "primary mental abilities". Induction is a corollary of logic and the Logical Reasoning and Inductive Reasoning factors may have much in common.

The outcomes of the factor analyses contained three other factors, two of which have been previously identified, and one unfamiliar factor. The Memory or Attention factor, defined by the WAIS Digit Span subtest, appeared in both the Cohen (1957) analysis of the WAIS and Balinsky's factor analysis of the W-B (Balinsky, 1941). A Perceptual Identification and Quickness factor emerged in the 18-19 age group analysis. Both Thurstone (1938) and Davis (1956) have identified such a factor. The final factor in the study, the ability to quickly perceive relationships, has no known antecedent in studies of intellectual abilities, although its meaning may be similar to the Perceptual Organization factor in Cohen's study (Cohen, 1957) or Birren's Closure factor (Birren, 1952).

How do these factors contribute to the variance of the WAIS Similarities, Arithmetic and Block Design subtests? Table 7 lists these three tests and the factor loadings of the six factors for each test.

Table 7. Factor loading on the six factors extracted from the Full Group analysis for WAIS Similarities, WAIS Arithmetic and WAIS Block Design.

	Spatial Visualization factor	Higher-Order Reasoning factor	Concrete Reasoning factor	Quantitative factor	Logical Reasoning factor	Memory/ Attention factor
Arithmetic	.214	.113	.865	-.206	.019	-.085
Similarities	.097	.813	.099	-.048	.073	-.093
Block Design	.339	.526	-.044	-.374	-.286	.106

Both Arithmetic and Similarities subtests have a relatively simple factor structure, while Block Design is rather more complex.

Concrete Reasoning is the dominant factor in the WAIS Arithmetic test, accounting for almost 75% of the variance. Spatial-visualization and Quantitative factors make slight contributions to the test, but the effect of other factors is negligible. Davis (1956) found General Reasoning the principal factor in W-B Arithmetic. Numerical facility, Mechanical Knowledge and Information factors also appeared, with less importance, in the factor structure of this test. Davis's Numerical Facility factor matches the Quantitative factor and his Mechanical Knowledge factor may be subsumed under the more general Concrete Reasoning, Wechsler argues that Arithmetic has a substantial Memory factor and that the importance of reasoning ability has been overestimated (Wechsler, 1958). His argument is contradicted by both Davis's results and the present investigation. Both studies included reference variables - Cohen's factor analysis (Cohen, 1957), failed to include reference test and Wechsler based his discussion of the factorial composition of his subtests largely on the basis of the Cohen analysis.

WAIS Similarities had one major source of variability, the Higher-Order Reasoning factor which accounted for 66% of the reliable variance. No other factor in the study made a contribution to Similarities variance. Wechsler (1958) stated that the test shows conspicuous loadings in Verbal Comprehension in practically all the factorial analyses with normal subjects, but he suspected that other contributing abilities would emerge in more extended factorializations. Davis (1956) found low but significant Information and Visualization loadings on the Similarities test, as well as a Similarities doublet factor which he could not interpret. The present investigation does not establish that any degree of factorial complexity exists in the Similarities test.

WAIS Block Design variance was distributed over four factors, with the Higher-Order Reasoning factor major in importance. This particular factor was characterized by both verbal and spatial content and it is probably the spatial component which accounts for the Block Design loading on the factor. Both Quantitative and Spatial-Visualization factors were contributors to about 10% of the total Block Design variance. The Spatial-Visualization factor, also found in the Davis study, is an ability to visualize and organize an outcome. Both this factor and the Higher-Order Reasoning factor appear to offer one solution to the Block Design problems by way of familiarity with spatial configuration and organization. Another type of solution seems to be implied by the factor loadings on Quantitative and Logical Reasoning factors. Such a solution to the Block Design problems would proceed step by step, logically to the conclusion or solution rather than by the global visualization of the required outcome.

The various sources of variance contribute 85.6%, 69.6% and 63.2% of the reliable variance of the WAIS Arithmetic, Similarities and Block Design subtests. The remaining variance would represent specific and error sources.

The psychometric invariance of the three WAIS subtests was examined under a condition of age difference by comparing the sources of variance of the 18-19 and 20-24 age groups. Table 8 lists these factors or sources for both groups and the factor loadings on the three WAIS tests.

Table 8. Factors and factor loadings on the WAIS Arithmetic, Similarities and Block Design subtests for the 18-19 and 20-24 age groups.

18-19 Age Group

	Logical Reasoning factor	Verbal Non-Quantitative factor	Visualization factor	Perceptual Identification and Quickness factor	Concrete Reasoning factor
Arithmetic	.324	.078	-.165	.012	-.777
Similarities	-.097	.207	-.308	.187	-.635
Block Design	.242	.349	-.766	-.012	-.140

20-24 Age Group

	Perceiving Relationships factor	Verbal Non-Quantitative factor	Spatial-Visualization factor	Memory factor	Concrete Reasoning factor
Arithmetic	-.354	.374	-.385	.050	-.664
Similarities	.103	.754	-.049	.026	.018
Block Design	.613	.310	-.241	.108	-.192

To assess psychometric invariance, the changes in factors and factor loadings between the two age groups must be measured. There are a number of problems in performing such a measurement.

The initial problem is determining which factors correspond between the two conditions. The order of factors outputed in a factor analysis does not guarantee that, for example, Factor I in one condition is the same factor as Factor I in the second condition. Factors in both conditions must first be identified before they are matched. Factors with equivalent interpretations may differ still in factor loading patterns. Determining the extent to which factors obtained in different studies are the same is the problem of factorial invariance. This important problem has not been adequately resolved in factor analysis (Nesselroade and Baltes, 1970; Crawford, 1964).

Factorial invariance is usually assessed by a method which determines the degree to which two columns of factor loadings approach a proportional relationship. Harmon (1960), Merideth (1964), Penneau and Newhouse (1964), reviewed different coefficients of congruence for measuring the degree of factor matching and were unable to find a completely adequate measure. Because a proper index to measure factorial invariance has not been developed, factors from the two conditions in the present study were matched on the basis of their identification.

Psychometric invariance is an easier concept to work with than factorial invariance because the factor loadings of a row (variable) are a linear combination while the factor loadings of a column do not form a linear combination. The sum of squares of the factor loadings across a variable equals the reliable variance. Each squared factor loading represents the proportion of the reliable variance of the test that the factor or source of variability contributes. Psychometric invariance can be measured by comparing the proportions of variance contributed by the same factor in the two different age conditions. The lack of an index to make this comparison empirically valid is a further problem in operationally defining the concept psychometric invariance. Although the precision of such an index is welcome, the lack of an index does not prevent the assessment of psychometric invariance of a test variable.

Similar factors emerged in the Analyses of the 18-19 and 20-24 age groups. Both groups contained a Verbal, Non-Quantitative factor, a Visualization factor and a Concrete Reasoning factor. The other two factors in both groups were interpreted differently, but the differences between the factors were less than the similarities. The Logical Reasoning factor in the 18-19 group is not substantially different in nature than the Perceiving Relationships factor from the older age group. Similarily the Perceptual Identification factor (from the 18-19 group) and the Memory factor (from the 20-24 group) share much in common. Both are an ability to identify and use symbols drawn from the present situation or from immediate memory. It is reasonable to conlude that generally the same sources of variability entered into the WAIS subtests for both age groups.

The following table presents the proportion of test variance contributed by each factor in the two age group analyses for the WAIS tests.

Table 9. Squared factor loadings (which yield the proportion of variance contributed by each factor) in the 18-19 and 20-24 age groups for WAIS Arithmetic, Similarities and Block Design subtests.

Factor	Arithmetic 18-19	Arithmetic 20-24	Similarities 18-19	Similarities 20-24	Block Design 18-19	Block Design 20-24
Logical Reasoning	.11	.13	.01	.01	.06	.38
Non-Quantitative	.01	.14	.04	.57	.12	.10
Visualization	.03	.15	.10	.00	.59	.06
Perceptual Identification	.00	.00	.00	.00	.00	.00
Concrete Reasoning	.60	.44	.40	.00	.02	.04
reliable variance identified	.75	.86	.59	.58	.79	.58

The same factors contributed variance to the WAIS tests in both age groups, but the importance of each factor often changed from the younger to older subjects.

In the WAIS Arithmetic, Concrete Reasoning is the most important ability for solving the WAIS Arithmetic problems in both age groups. Logical reasoning ability contributes about equally to test performance for both groups. Non-Quantitative and Visualization factors make contributions to the test for the older, but not the younger age group.

The two age groups differed strikingly on the WAIS Similarities test. The Verbal, Non-Quantitative factor was the major and almost only source of variance for the 20-24 age group, but the same factor was unimportant in the 18-19 age group. This group relied mainly on Concrete Reasoning and to a much lesser extent Visualization to solve the WAIS

Similarities questions. Psychometric invariance is clearly not a property of WAIS Similarities under changes of age condition at least.

Block Design variability differed in composition between the two groups. Younger subjects relied on Visualization and Non-Quantitative factors. Logical Reasoning was a relatively unimportant source of variability for the younger group, but it was the major source of variance in the 20-24 age group analysis. The dominant Visualization factor in the 18-19 group assumed relative unimportance in the older group. Such shifts in the major sources of variance of the Block Design test precludes the possession of psychometric invariance by WAIS Block Design.

It would appear that certain subtests of the WAIS lack psychometric invariance. This judgment is tempered though by the fact that all the reliable variance is not accounted for yet in the subtests. The study was most successful in determining the sources of variance of the WAIS Arithmetic and least successful with WAIS Similarities. Block Design reliable variance fell between the figures of the other two tests. Unless all the reliable variance of a test can be identified, it is unfair to unequivocally state that a measure is or is not psychometrically invariant. However the study does account for a large part of the reliable variance of each subtest, and the argument is compelling that WAIS Similarities and Block Design lack psychometric invariance under at least one condition of age change. WAIS Arithmetic appears to be a more stable measure.

Changes in factor structure of the Wechsler subtests between older and younger subjects have been reported in earlier factor analyses. The earliest analysis of the W-B (Balinsky, 1941) determined that individual subtests changed their factorial composition from age to age and that different abilities (factors) were present at different ages. The Cohen (1957) analysis revealed a similar pattern of factor loadings among the three youngest groups (18-19, 25-34 and 45-54), but a definite change in intellectual organization for the oldest group, as the Memory factor became much more prominent. A later study by Berger, Bernstein, Klein, Cohen and Lucas (1964), re-analyzed Cohen's 1957 data and found that for normal age groups considerable similarity was demonstrated only for the Perceptual Organization factor. Verbal and Memory factors showed age-related changes. For normal adults between 25-54 years, Verbal and Memory skills existed as relatively separate factors but for 18-19 and 60+ age groups a separate Memory factor did not appear but rather coalesced with the Verbal factor. In the Green and Berkowitz (1964) study, the number of factors and magnitude of factor loadings showed age-related changes. In the WAIS Arithmetic subtest, three factors are required to describe the test variance for the age group up to age 29, but four factors are required for the 45-54 age group and two factors are needed for the 60-65+ age group. Reigel and Reigel (1962) examined changes in the factor structure of the Wechsler scale with age and argued that there was not sufficient evidence one way or the other for changes in factor structure with advancing age. The only deviation in factor structure of the WAIS (older group) reported by Cohen they attributed to the rotation procedures applied (oblique) and to differences in opinion as to when simple structure is obtained. They concluded that:

> "Generally the failure to find differences in factor structure between different groups and conditions may be due to restrictions in the range of abilities sampled by the Wechsler tests."

Reference tests were not included in the factor analyzed test batteries and all the sources of variability were not identified in all of the above age comparison studies. Unless all the sources of variability for a test have been identified, age comparisons of factor structure are not very meaningful.

The present study did determine much of the reliable variance of certain WAIS subtests and the age comparisons in this investigation are perhaps more meaningful than comparisons in earlier studies. The results show that the abilities required for some Wechsler subtests undergo wide change within even a narrow age range. It is possible that age changes in mental ability factors have been underestimated and that when all the sources of variability of a test are accounted for the evidence indicates that different sources of variability enter into mental test performance at different ages and that the importance of different sources of variability change with age. Such a conclusion questions the validity of comparing the intellectual abilities of different ages, when these intellectual abilities are measured by the Wechsler Intelligence Scale.

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APPENDICES

APPENDIX 1

Table 10. Mean and standard deviation of the 16 variables for the Full Group.

No.	Variable	Mean	S.D.
1.	Vocabulary	19.453	5.152
2.	Number Series	14.420	4.753
3.	MAT Spatial Relations	14.573	3.984
4.	General Reasoning	11.827	2.811
5.	Visuo-Spatial Logical Reasoning	16.700	4.405
6.	Quantitative	13.047	4.403
7.	DAT Mechanical Reasoning	26.687	6.935
8.	DAT Space Relations	30.833	12.474
9.	DAT Verbal Reasoning	22.187	5.028
10.	DAT Numerical Ability	15.293	4.618
11.	DAT Abstract Reasoning	23.180	4.035
12.	DAT Clerical Speed and Accuracy	112.033	20.762
13.	WAIS Digit Span	11.747	1.980
14.	WAIS Arithmetic	12.800	2.809
15.	WAIS Similarities	18.673	2.630
16.	WAIS Block Design	37.093	6.880

Table 11. Intercorrelation matrix of the 16 variables for the complete group.

No.	Variable	1	2	3	4	5	6	7	8
1.	Vocabulary								
2.	Number Series	0.239							
3.	MAT Spatial Relations	-0.048	0.325						
4.	General Reasoning	0.402	0.385	0.127					
5.	Visuo-Spatial Logical Reasoning	0.150	0.325	0.275	0.539				
6.	Quantitative	0.310	0.473	0.232	0.421	0.303			
7.	DAT Mechanical Reasoning	0.256	0.294	0.533	0.350	0.496	0.340		
8.	DAT Space Relations	0.320	0.341	0.418	0.375	0.374	0.426	0.452	
9.	DAT Verbal Reasoning	0.606	0.373	0.188	0.498	0.404	0.479	0.320	0.436
10.	DAT Numerical Ability	0.315	0.660	0.177	0.352	0.346	0.560	0.276	0.275
11.	DAT Abstract Reasoning	0.355	0.373	0.362	0.525	0.616	0.476	0.513	0.603
12.	DAT Clerical Speed and Accuracy	0.305	0.341	0.164	0.291	0.458	0.257	0.118	0.268
13.	WAIS Digit Span	0.160	0.175	-0.038	-0.042	0.065	0.207	0.110	0.071
14.	WAIS Arithmetic	0.434	0.414	0.167	0.456	0.112	0.576	0.411	0.415
15.	WAIS Similarities	0.354	0.242	0.106	0.234	0.113	0.228	0.070	0.294
16.	WAIS Block Design	0.359	0.472	0.226	0.294	0.378	0.311	0.339	0.546

No.	Variable	9	10	11	12	13	14	15	16
1.	Vocabulary								
2.	Number Series								
3.	MAT Spatial Relations								
4.	General Reasoning								
5.	Visuo-Spatial Logical Reasoning								
6.	Quantitative								
7.	DAT Mechanical Reasoning								
8.	DAT Space Relations								
9.	DAT Verbal Reasoning								
10.	DAT Numerical Ability	0.479							
11.	DAT Abstract Reasoning	0.555	0.496						
12.	DAT Clerical Speed and Accuracy	0.467	0.432	0.480					
13.	WAIS Digit Span	0.235	0.174	0.101	-0.057				
14.	WAIS Arithmetic	0.318	0.290	0.321	-0.008	0.193			
15.	WAIS Similarities	0.370	0.111	0.259	0.337	0.166	0.227		
16.	WAIS Block Design	0.442	0.463	0.608	0.444	0.015	0.161	0.298	

Table 12. The means and standard deviations of the 16 variables for the 18-18 and 20-24 age groups.

No.	Variable	18-19 group Mean	18-19 group S.D.	20-24 group Mean	20-24 group S.D.
1.	Vocabulary	20.180	5.017	19.820	5.013
2.	Number Series	15.460	4.016	14.320	4.838
3.	MAT Spatial Relations	15.220	3.900	14.540	4.072
4.	General Reasoning	12.160	2.706	11.980	2.803
5.	Visuo-Spatial Logical Reasoning	16.700	4.339	17.080	4.379
6.	Quantitative	12.880	4.059	13.080	4.593
7.	DAT Mechanical Reasoning	25.800	6.931	27.280	6.905
8.	DAT Space Relations	32.200	12.064	31.140	12.643
9.	DAT Verbal Reasoning	23.720	5.653	22.380	4.852
10.	DAT Numerical Ability	16.440	4.581	15.300	4.678
11.	DAT Abstract Reasoning	23.880	3.734	23.340	4.034
12.	DAT Clerical Speed and Accuracy	116.800	16.147	111.420	19.200
13.	WAIS Digit Span	11.920	2.088	11.880	2.007
14.	WAIS Arithmetic	12.820	2.529	12.680	2.952
15.	WAIS Similarities	18.960	2.555	18.560	2.612
16.	WAIS Block Design	37.900	6.535	37.300	6.843

Table 13. Intercorrelation matrix of the 16 variables for the 18-19 age group.

No.	Variable	1	2	3	4	5	6	7	8
1.	Vocabulary								
2.	Number Series	0.037							
3.	MAT Spatial Relations	-0.039	0.481						
4.	General Reasoning	0.136	0.102	0.244					
5.	Visuo-Spatial Logical Reasoning	0.018	0.103	0.347	0.567				
6.	Quantitative	0.288	0.505	0.339	0.411	0.434			
7.	DAT Mechanical Reasoning	0.219	0.206	0.509	0.521	0.740	0.380		
8.	DAT Space Relations	0.274	0.462	0.435	0.373	0.482	0.491	0.518	
9.	DAT Verbal Reasoning	0.608	0.195	0.332	0.370	0.456	0.504	0.499	0.351
10.	DAT Numerical Ability	0.290	0.508	0.238	0.160	0.434	0.571	0.454	0.356
11.	DAT Abstract Reasoning	0.193	0.284	0.308	0.440	0.721	0.587	0.651	0.522
12.	DAT Clerical Speed and Accuracy	0.206	0.130	0.094	-0.244	0.058	0.123	0.103	-0.009
13.	WAIS Digit Span	0.333	0.452	0.079	0.028	0.076	0.541	0.232	0.234
14.	WAIS Arithmetic	0.226	0.493	0.171	0.505	0.341	0.646	0.301	0.619
15.	WAIS Similarities	0.252	0.334	0.172	0.240	0.080	0.403	0.080	0.368
16.	WAIS Block Design	0.339	0.350	0.196	0.158	0.426	0.333	0.487	0.699

No.	Variable	9	10	11	12	13	14	15	16
1.	Vocabulary								
2.	Number Series								
3.	MAT Spatial Relations								
4.	General Reasoning								
5.	Visuo-Spatial Logical Reasoning								
6.	Quantitative								
7.	DAT Mechanical Reasoning								
8.	DAT Space Relations								
9.	DAT Verbal Reasoning								
10.	DAT Numerical Ability	0.449							
11.	DAT Abstract Reasoning	0.516	0.471						
12.	DAT Clerical Speed and Accuracy	0.210	0.360	0.201					
13.	WAIS Digit Span	0.366	0.447	0.326	0.261				
14.	WAIS Arithmetic	0.266	0.395	0.488	0.039	0.395			
15.	WAIS Similarities	0.136	0.019	0.166	0.036	0.214	0.390		
16.	WAIS Block Design	0.381	0.357	0.413	0.015	0.134	0.393	0.266	

Table 14. Intercorrelation matrix of the 16 variables for the 20-24 age group.

No.	Variable	1	2	3	4	5	6	7	8
1.	Vocabulary								
2.	Number Series	0.299							
3.	MAT Spatial Relations	-0.024	0.319						
4.	General Reasoning	0.409	0.520	0.191					
5.	Visuo-Spatial Logical Reasoning	0.087	0.358	0.333	0.507				
6.	Quantitative	0.356	0.466	0.245	0.439	0.256			
7.	DAT Mechanical Reasoning	0.233	0.339	0.583	0.373	0.482	0.330		
8.	DAT Space Relations	0.333	0.356	0.457	0.393	0.374	0.432	0.470	
9.	DAT Verbal Reasoning	0.583	0.441	0.217	0.514	0.396	0.478	0.326	0.501
10.	DAT Numerical Ability	0.315	0.666	0.238	0.472	0.377	0.563	0.299	0.314
11.	DAT Abstract Reasoning	0.376	0.483	0.415	0.539	0.593	0.445	0.534	0.598
12.	DAT Clerical Speed and Accuracy	0.276	0.295	0.180	0.383	0.528	0.166	0.151	0.237
13.	WAIS Digit Span	0.249	0.193	-0.037	0.061	0.068	0.225	0.185	0.124
14.	WAIS Arithmetic	0.488	0.452	0.210	0.480	0.079	0.562	0.421	0.454
15.	WAIS Similarities	0.346	0.207	0.092	0.227	0.073	0.212	0.087	0.219
16.	WAIS Block Design	0.369	0.451	0.220	0.322	0.414	0.297	0.313	0.525

No.	Variable	9	10	11	12	13	14	15	16
1.	Vocabulary								
2.	Number Series								
3.	MAT Spatial Relations								
4.	General Reasoning								
5.	Visuo-Spatial Logical Reasoning								
6.	Quantitative								
7.	DAT Mechanical Reasoning								
8.	DAT Space Relations								
9.	DAT Verbal Reasoning								
10.	DAT Numerical Ability	0.475							
11.	DAT Abstract Reasoning	0.580	0.505						
12.	DAT Clerical Spped and Accuracy	0.407	0.326	0.419					
13.	WAIS Digit Span	0.242	0.180	0.091	-0.241				
14.	WAIS Arithmetic	0.322	0.282	0.297	-0.084	0.238			
15.	WAIS Similarities	0.335	0.088	0.171	0.254	0.181	0.198		
16.	WAIS Block Design	0.524	0.406	0.597	0.393	0.062	0.174	0.203	

APPENDIX 2

Table 15. The means and standard deviations of the 16 variables for the 25-34 age group.

No.	Variable	Mean	S.D.
1.	Vocabulary	18.720	5.387
2.	Number Series	13.760	5.263
3.	MAT Spatial Relations	14.200	4.081
4.	General Reasoning	11.320	2.896
5.	Visuo-Spatial Logical Reasoning	16.640	4.720
6.	Quantitative	13.480	4.599
7.	AT Mechanical Reasoning	26.640	6.916
8.	DAT Space Relations	29.680	12.863
9.	DAT Verbal Reasoning	21.040	4.477
10.	DAT Numerical Ability	14.760	4.689
11.	DAT Abstract Reasoning	22.480	4.112
12.	DAT Clerical Speed and Accuracy	108.400	24.522
13.	WAIS Digit Span	11.560	1.786
14.	WAIS Arithmetic	12.920	2.968
15.	WAIS Similarities	18.400	2.799
16.	WAIS Block Design	36.560	7.506

Table 16. Intercorrelation matrix of the 16 variables for the 25-34 age group

No.	Variable	1	2	3	4	5	6	7	8
1.	Vocabulary								
2.	Number Series	0.284							
3.	MAT Spatial Relations	-0.124	0.202						
4.	General Reasoning	0.592	0.506	0.019					
5.	Visuo-Spatial Logical Reasoning	0.240	0.471	0.273	0.597				
6.	Quantitative	0.287	0.511	0.214	0.488	0.249			
7.	DAT Mechanical Reasoning	0.295	0.373	0.604	0.308	0.327	0.321		
8.	DAT Space Relations	0.349	0.218	0.363	0.403	0.330	0.433	0.441	
9.	DAT Verbal Reasoning	0.598	0.442	0.020	0.626	0.399	0.542	0.226	0.538
10.	DAT Numerical Ability	0.287	0.783	0.148	0.427	0.269	0.583	0.172	0.221
11.	DAT Abstract Reasoning	0.423	0.579	0.376	0.642	0.566	0.456	0.457	0.680
12.	DAT Clerical Speed and Accuracy	0.367	0.532	0.150	0.556	0.621	0.399	0.122	0.430
13.	WAIS Digit Span	-0.098	-0.094	-0.223	-0.233	-0.019	-0.177	-0.145	-0.193
14.	WAIS Arithmetic	0.586	0.362	0.166	0.426	-0.034	0.541	0.535	0.255
15.	WAIS Similarities	0.411	0.159	0.014	0.251	0.152	0.112	0.058	0.237
16.	WAIS Block Design	0.346	0.524	0.259	0.437	0.365	0.372	0.273	0.437

No.	Variable	9	10	11	12	13	14	15	16
1.	Vocabulary								
2.	Number Series								
3.	MAT Spatial Relations								
4.	General Reasoning								
5.	Visuo-Spatial Logical Reasoning								
6.	Quantitative								
7.	DAT Mechanical Reasoning								
8.	DAT Space Relations								
9.	DAT Verbal Reasoning								
10.	DAT Numerical Ability	0.489							
11.	DAT Abstract Reasoning	0.547	0.491						
12.	DAT Clerical Speed and Accuracy	0.676	0.494	0.621					
13.	WAIS Digit Span	-0.079	-0.193	-0.221	-0.255				
14.	WAIS Arithmetic	0.409	0.210	0.221	-0.016	-0.076			
15.	WAIS Similarities	0.601	0.138	0.338	0.511	0.052	0.141		
16.	WAIS Block Design	0.512	0.472	0.763	0.691	-0.210	0.042	0.364	

Table 17. The matrix of factor loadings after Varimax rotation for the 25-34 age group.

No.							
1.	Vocabulary	0.400	-0.084	0.746	0.042	-0.073	0.266
2.	Number Series	0.094	0.155	0.149	0.827	0.075	0.335
3.	MAT Spatial Relations	0.016	0.897	-0.135	0.122	-0.092	0.024
4.	General Reasoning	0.208	-0.021	0.514	0.291	-0.206	0.660
5.	Visuo-Spatial Logical Reasoning	0.134	0.214	-0.020	0.170	0.085	0.898
6.	Quantitative	0.141	0.211	0.413	0.658	-0.145	0.027
7.	DAT Mechanical Reasoning	-0.032	0.789	0.403	0.111	0.026	0.186
8.	DAT Space Relations	0.462	0.518	0.247	0.039	-0.241	0.227
9.	DAT Verbal Reasoning	0.680	-0.003	0.436	0.340	-0.028	0.244
10.	DAT Numerical Ability	0.163	0.014	0.066	0.904	-0.106	0.117
11.	DAT Abstract Reasoning	0.464	0.421	0.130	0.354	-0.188	0.494
12.	DAT Clerical Speed and Accuracy	0.637	0.020	-0.063	0.387	-0.206	0.506
13.	WAIS Digit Span	0.007	-0.110	-0.043	-0.094	0.946	-0.019
14.	WAIS Arithmetic	-0.020	0.257	0.891	0.240	0.036	-0.136
15.	WAIS Similarities	0.872	-0.024	0.145	-0.003	0.161	-0.014
16.	WAIS Block Design	0.590	0.243	-0.085	0.425	-0.207	0.292