UNIVERSITY OF MANITOBA

LAMINAR FREE CONVECTIVE HEAT TRANSFER FROM AN ISOTHERMAL HORIZONTAL CYLINDER TO WATER NEAR $4^{o}C$

by

Peter J. Weekes

A Thesis
Submitted to the Faculty of Graduate Studies
in Partial Fulfillment of the Requirements for the Degree
of Master of Science

Department of Mechanical Engineering

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ABSTRACT

This thesis is an experimental study of laminar free convective heat transfer from an isothermal cylinder to water near 4°C. The free convection process in the vicinity of a bulk water temperature of 4°C is greatly influenced by the maximum density of water at this temperature. Under certain circumstances, both positive and negative buoyancy forces can exist simultaneously in different parts of the boundary layer. The presence of these two opposing forces complicates both the nature of the flow patterns and the distribution and magnitude of the surface heat fluxes.

Several investigators have studied the general problem of free convection from horizontal cylinders and analytical and empirical models have been constructed to describe this process. Much work has also been done on the melting of ice in different configurations in water near 4°C, as well as on free convection from vertical flat plates to water in this temperature region. Models have been proposed to describe the nature of the bidirectional flow phenomenon in this area and to locate the boundaries between different flow regions. No such work has been performed for an isothermal horizontal cylinder in water near 4°C.

The first objective of this experiment was to measure

the local heat fluxes on an isothermal horizontal cylinder in water near 4°C, for the four different flow regimes which exist in this temperature region. A horizontal cylinder was constructed, with twelve separately controlled heaters mounted around the periphery. By individually adjusting the power to each heater, an isothermal surface could be maintained.

The results of fifty-six tests, performed for a range of surface temperatures from 4.64°C to 17.94°C and bulk water temperatures from 1.07°C to 16.18°C, verified that the boundaries between different flow regions, defined for a vertical flat plate, applied to the horizontal cylinder problem as well. In addition, the form of the local heat flux distributions, in unidirectional flow regions, coincided with those derived by previous investigators. The nature of the local heat flux distributions in regions of bidirectional flow provided more insight into the complex relationship between the two opposing buoyancy forces in this region.

The second major objective was to try to correlate the average heat transfer results with one or more of the variables of physical importance in the two unidirectional and two bidirectional flow regions. The correlating equations of an earlier experimental investigation were modified to apply to the average heat transfer results from a horizontal cylinder and were fitted to the experimental data of the present work. Although there was a good deal of experimental scatter in the results, in general, they agreed with the correlations. With the aid of these correlations, accurate estimations of the rates of heat transfer from horizontal cylinders to water can be made in this temperature range.

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NOMENCLATURE

a - a constant used by Hermann (Appendix I), dimensionless

A - constants used by Yuill, $^{o}C^{-1}$ or dimensionless

b - Langmuir's film diameter, cm.

- a constant used by Hermann (Appendix I), dimensionless

B - Langmuir's film thickness for a plane surface, cm.

- constants used by Yuill, dimensionless, $^{o}C^{-1}$ or $^{o}C^{-2}$

- buoyancy force per unit mass, cm./sec^2

B^* - Rice's film thickness for a cylinder, (b-D)/2, cm.

c - a constant used by Hermann (Appendix I), dimensionless

c_p - specific heat of a fluid, Joule/gm-^{o}C

C - various correlating constants, dimensionless

C_I - a heat transfer constant used in region I, dimensionless

C_{II-N} - a heat transfer constant used in region II-N, dimensionless

C_{II-S} - a heat transfer constant used in region II-S, dimensionless

C_{III-IV} - a heat transfer constant used in regions III and IV, dimensionless

C_i - a heat transfer constant used in region II-N, dimensionless

C_o - a heat transfer constant used in region II-S, dimensionless

d	- a constant used by Hermann (Appendix I), dimensionless
D	- cylinder, wire or sphere diameter, cm.
	- constants used by Yuill, dimensionless, $^oC^{-1}$ or $^oC^{-4}$
$D_{1,2,3}$	- constants in the expression for density (Appendix IV), $^oC^{-1,-2,-3}$
E	- a constant used by Yuill, $^oC^{-1}$
f	- Hermann's azimuth function (Appendix I), dimensionless
F	- Hermann's azimuth function (Appendix I), dimensionless
	- Merk and Prins' azimuth function (Appendix II), dimensionless
	- a constant used by Yuill, dimensionless
g	- gravitational acceleration, cm/sec^2
	- Hermann's azimuth function (Appendix I), dimensionless
g^*	- Rayleigh number function (equation 1-22), dimensionless
g_c	- gravitational constant, $gm\text{-}cm/N\text{-}sec^2$
g_o	- a constant used by Hermann (Appendix I), dimensionless
G	- Hermann's azimuth function (Appendix I), dimensionless
	- Merk and Prins' azimuth function (Appendix II), dimensionless
	- constants used by Yuill, dimensionless, $^oC^{-1}$ or $^oC^{-3}$
Gr	- Grashof number, $g\beta\theta_p D^3/\nu^2$, dimensionless
Gr^*	- Grashof number modified by Yuill (equation 1-54 or V-10), dimensionless

Gr'	- modified Grashof number used by Dyer, $g\beta\dot{q}D^4/k\nu^2$, dimensionless
	- non-Newtonian Grashof number used by Gentry and Wollersheim, $(\rho/\kappa)^2 \cdot D^{n+2} \cdot (g\beta\theta_p)^{2-n}$, dimensionless
h	- surface conductance (heat transfer coefficient), W/cm^2-^{o}C
h_1,h_2,etc.	- coefficients used in Merk and Prins' azimuth function (Appendix II), dimensionless
H	- Merk and Prins' azimuth function (Appendix II), dimensionless
k	- thermal conductivity of fluid, W/cm-^{o}C
L	- length of cylinder or flat plate, cm.
m	- an exponent used by Yuill, dimensionless
m_1,m_2	- constants used by Koh, dimensionless
M	- Prandtl number function(Appendix II), dimensionless
n	- flow behaviour index for non-Newtonian fluids, dimensionless
	- an exponent used by Yuill, dimensionless
n_1,n_2	- constants used by Koh and Price, dimensionless
Nu (or $\overline{Nu}$)	- average Nusselt number, $\bar{h}D/k$, dimensionless
p	- Hermann's velocity function (Appendix I), dimensionless
P	- a density function defined by Yuill (Appendix V), dimensionless
$P_{0,1,2,3,4}$	- constants used in the expression for the Prandtl number (Appendix IV), dimensionless, $^oC^{-1,2,3,4}$
Pr	- Prandtl number, $\mu c_p/k$, dimensionless

Pr' - Prandtl number for non-Newtonian fluids, $[\rho c_p/k] \cdot [\kappa/\rho]^{2/(n+1)} \cdot D^{(1-n)/(1+n)} \cdot [g\beta\theta_p D]^{3(n-1)/2(n+1)}$, dimensionless

q - Hermann's radial coordinate (Appendix I), dimensionless

$\dot{q}$ - specific heat transfer rate, W/cm^2

Q - a density function defined by Yuill (Appendix V), dimensionless

$\dot{Q}$ - heat transfer rate, Watt

r - radius of cylinder, cm.

Ra - Rayleigh number, Gr$\cdot$Pr, dimensionless

Re - Reynolds number, uD/ν, dimensionless

R_x - local thermal resistance, cm^2-$^oC/W$

s - Langmuir's shape factor (equation 1-3), cm.

s_1, s_2, etc. - coefficients in Merk and Prins' sine function (Appendix II), dimensionless

t - Hermann's temperature function (Appendix I), dimensionless

t^* - time required for heated layer to descend (or rise) to level of cylinder (equation 1-46), sec.

T - temperature, ^oC

T_e - equivalent temperature ratio, θ_p/T_∞, dimensionless

u - tangential velocity, cm/sec.

u_1 - Merk and Prins' tangential velocity function (Appendix II), cm/sec.

v - radial velocity, cm/sec.

$V_{1,2,3}$ - constants used in the expression for dynamic viscosity (Appendix IV), $^oC^{-1,-2,-3}$

W - width of tank in the plane normal to the longitudinal axis of the cylinder (Figure 4b), cm.

x - tangential coordinate, cm.

y - radial coordinate, cm.

Y - distance from the centerline of the cylinder to the free surface (Figure 4b), cm.

Z - Yuill's buoyancy function, α - 0.02825, dimensionless

GREEK SYMBOLS

α - thermal diffusivity of fluid, cm^2/sec.

- Yuill's buoyancy function (Appendix V), dimensionless

β - thermal expansion coefficient, $^{o}C^{-1}$

γ - a constant used by Hermann (Appendix I), dimensionless

- Yuill's angle of rotation, radians

δ - boundary layer thickness, cm.

ε - a constant used by Hermann (Appendix I), dimensionless

ζ - stream function, dimensionless

η - radial coordinate, dimensionless

θ - temperature difference, $T - T_\infty$, ^{o}C

κ - specific heat ratio for a gas, dimensionless

- consistency index for non-Newtonian fluids, $gm/cm\text{-}sec^{2-n}$

λ - mean free path of a gas, cm.

μ	- dynamic (or absolute) viscosity, gm/cm-sec.
ν	- kinematic viscosity of a fluid, cm^2/sec.
ξ	- tangential coordinate, x/r, dimensionless
	- angle, degrees
ρ	- density, gm/cm^3
σ	- root mean square deviation, varying dimensions
	- accommodation coefficient for a gas, dimensionless
Σ	- non-dimensional point in the boundary layer where $\rho = \rho_\infty$, dimensionless
τ	- shear stress, N/cm^2
	- temperature function, θ/θ_p, dimensionless
ϕ	- Langmuir's conductivity function (equation 1-2), W/cm.
	- Yuill's temperature function, ^oC
Φ	- Madden and Piret's heat transfer function (equation 1-25), dimensionless
ψ	- stream function, cm^2/sec.
ω	- Hermann's tangential coordinate (Appendix I), dimensionless
Ω	- point in the boundary layer where $\rho = \rho_\infty$, cm.

SUBSCRIPTS AND SUPERSCRIPTS

crit.	- at the point of transition to turbulence
D	- evaluated for a cylinder of diameter D
f	- evaluated at the film temperature

i	- evaluated at the film temperature of the inner boundary layer(or ΔT across inner boundary layer)
k	- evaluated at the absolute temperature (^{o}K)
L	- evaluated for a plate of length L
o	- evaluated at the film temperature of the outer boundary layer(or ΔT across outer boundary layer)
	- evaluated at $\xi = 0$
p	- evaluated at the surface temperature(except c_p)
r	- evaluated using r (instead of D)
x	- evaluated using x (instead of D or L)
δ	- evaluated using δ (instead of x or D or L)
η	- differentiation with respect to η
ξ	- differentiation with respect to ξ
Σ	- evaluated at $\eta = \Sigma$
Ω	- evaluated at $y = \Omega$
∞	- evaluated at the temperature of the bulk fluid
—	- average value of a property
'	- denotes differentiation (except for Gr'and Pr')

CHAPTER 1

INTRODUCTION

1.1 Introduction to the Problem

The density of most fluids decreases with temperature in a linear manner. However, four fluids, water, antimony, gallium and bismuth exhibit a density anomaly. They all possess a maximum density at some temperature above their freezing points. In the particular case of water, which is the most abundant liquid on this planet, the maximum density occurs at 4^oC.

Fluid flows which are primarily a result of forced convection are not affected to any great extent by this density phenomenon, since density gradients are relatively unimportant. Free convective flows are initiated and maintained by density differences within the fluid. Hence, the peculiar density distribution of water is bound to have a significant effect both on the nature of the flow pattern and on the magnitude of the free convection heat transfer in this region.

The temperature at which the maximum density occurs may not, at first, seem to be significant but it must be remembered that in moderate to high latitudes, all natural bodies of water pass through this temperature twice yearly. With the advent of arctic and sub-arctic resource development, any process which occurs in a body of water may be close to this temperature for the entire year. For example, heat transfer from heated underwater pipelines will be greatly influenced by this phenomenon. Since the horizontal cylinder is a common engineering configuration, it is important to increase our knowledge of various aspects of free convection heat transfer from isothermal horizontal cylinders to water near $4^{o}C$.

1.2 Description of the Phenomenon

The effect of the density gradient on the flow regimes about a flat plate has been studied previously. Vanier [43] solved the differential equations for the flow field in the regions where similarity solutions were valid. Yuill [48] extended this analysis both analytically and experimentally to include the entire range of temperatures. The flow field around a horizontal cylinder was not expected to be radically different, especially for a large radius cylinder.

The map of different modes of free convective flow, shown in Figure 1, resulted from these analyses. It clearly shows regions of pure downflow, bidirectional flow and pure upflow. With the bulk temperature at $0^{o}C$, the zones may be physically interpreted as follows. For a surface temperature between $0^{o}C$ and $8^{o}C$, the heated fluid in the boundary layer

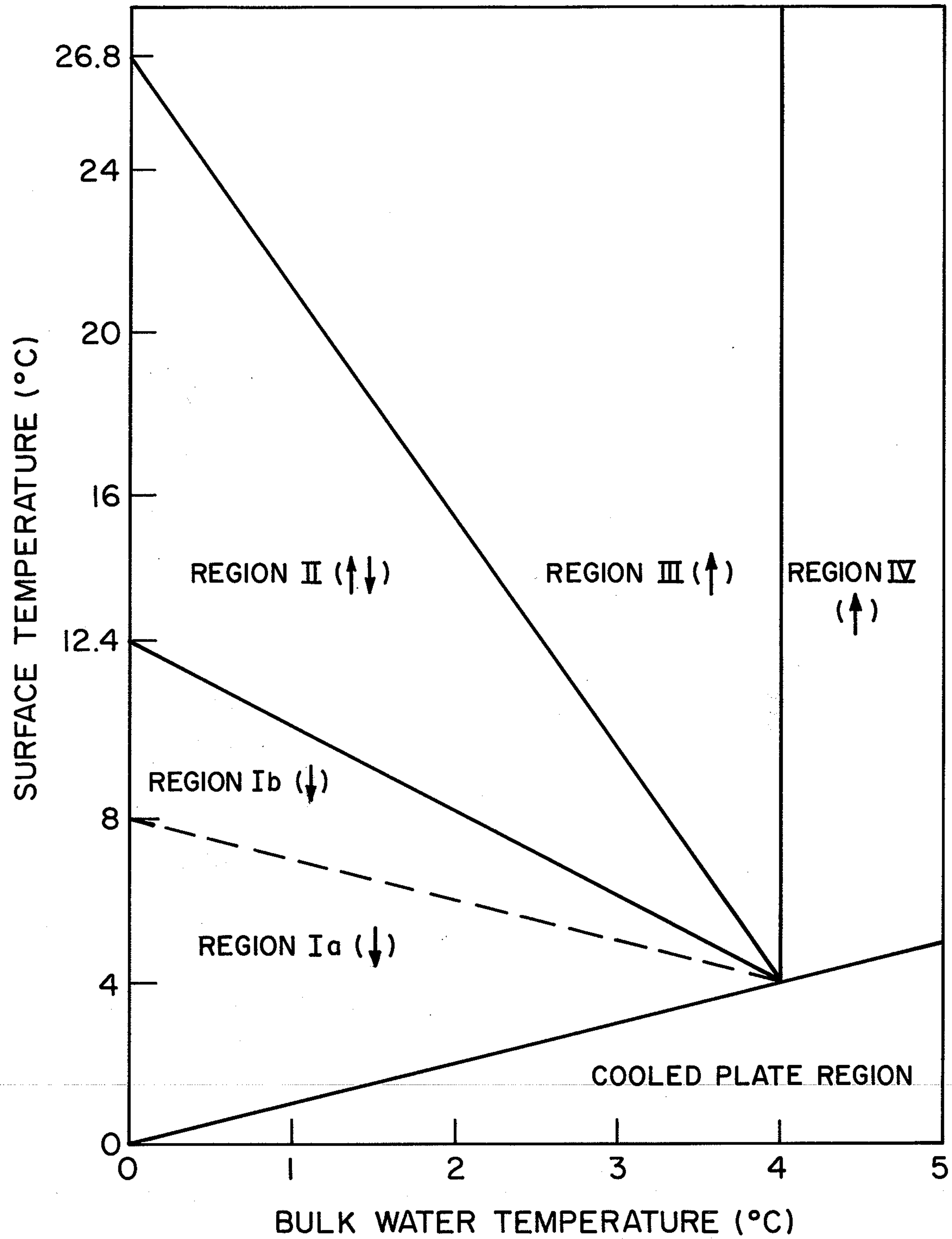

Fig. 1. Map of Free Convection Zones in Low Temperature Water.

is denser than the bulk fluid and descends. Between $8^{o}C$ and $12.4^{o}C$, the fluid adjacent to the surface is lighter than the bulk fluid but the net motion is still downward since the shear stress exerted by the outer part of the boundary layer is enough to overcome the difference in density. At $12.4^{o}C$, the velocity gradient at the surface is zero, which means that the buoyancy and shear forces are balanced. At temperatures between $12.4^{o}C$ and $26.8^{o}C$, bidirectional flow will occur; upflow near the surface and downflow towards the outer edge of the boundary layer. The relative magnitude of these two regions change as the temperature increases. At temperatures above $26.8^{o}C$, the inner upward flowing part of the boundary layer predominates. The shear force it exerts on the heavy outer part of the boundary layer is great enough so that the flow is purely upward above this point. At bulk temperatures above $0^{o}C$, the transition points between flow regimes wil change but the same general behaviour persists. Naturally, at bulk temperatures above $4^{o}C$, in the heated plate region, the flow will be purely upward, since the boundary layer must, by definition, be lighter than the bulk fluid.

1.3 Review of Previous Work

1.3.1 Free Convection From Horizontal Cylinders

The first analytic investigation of heat transfer from horizontal cylinders was carried out by Langmuir [22] in 1912. By studying free convective heat transfer from high temperature wires to different gases, he concluded that the dominant heat transfer mechanism was radial conduction through a stationary layer, or film, surrounding the wire. This he deduced from the fact that in gases, even at high temperatures, the velocity of the gas was very small while the thermal conductivity became appreciable. As a verification of this hypothesis, he presented the results of his experiments which showed that the heat loss was independent of the orientation of the wire, which would not have been the case if most of the heat was being carried away by convective currents. The analytic solution of this problem then became:

$\dot{Q}$ = heat transfer rate

$$= s(\phi_p - \phi_\infty) \quad \text{(1-1)}$$

where ϕ = conductivity function

$$= \int_0^T k \cdot dT \quad \text{(1-2)}$$

s = shape factor

$$= 2\pi L/\ln(b/D) \quad \text{(1-3)}$$

$$\text{and } b \cdot \ln(b/D) = 2B \quad \text{.......................} (1\text{-}4)$$

The subscripts p and ∞ referred to the conductivity function evaluated at the surface and bulk water temperatures respectively, k was the thermal conductivity of the fluid, L was the cylinder length, D was the cylinder diameter and b was Langmuir's film thickness for a cylinder. B was the equivalent film thickness for a plane surface and was constant for a specific gas. For air at room temperature and pressure, Langmuir found B = 0.43.

Davis [4], in the early 1920's, looked at the problem from a dimensional analysis point of view. He utilized Boussinesq's hypothesis which stated that:

$$\dot{q}D/k\theta_p = F(g\beta\theta_p D^3 \rho^2 c_p{}^2/k^2) \cdot f(\rho c_p \nu/k) \quad \text{....} (1\text{-}5)$$

where $\dot{q}$ was the heat transfer rate per unit area, θ_p was the difference between the surface and bulk water temperatures, g was the acceleration of gravity, β was the thermal expansion coefficient of the fluid, ρ was the density of the fluid, c_p was the specific heat of the fluid and ν was the kinematic viscosity of the fluid. He dropped the second function, assuming that the Prandtl number, $\rho c_p \nu/k$, was constant for all fluids and experimentally determined the following relationship;

$$\dot{q}D/k\theta_p = C\cdot(Gr\cdot Pr^2)^{0.233} \qquad (1\text{-}6)$$

where Gr was the Grashof number, $g\beta\theta_p D^3/\nu^2$ and Pr was the Prandtl number, $\mu c_p/k$. Davis [5] later discovered that omitting this function was not valid, especially for fluids, and so determined that:

$$\dot{q}D/k\theta_p = F(Gr\cdot Pr) \qquad (1\text{-}7)$$

Around the same period of time, Rice [32] combined Langmuir's film theory with Davis' dimensional analysis to find a relationship for B^*, the film thickness for cylinders:

$$B^* = 3.4(\mu/\rho)^{1/2}\cdot(D/g)^{1/4} \qquad (1\text{-}8)$$

where μ was the dynamic viscosity of the fluid . The constants and exponents were determined experimentally. He found that for long cylinders and very large film thicknesses:

$$s = 2\pi L/\ln(2B^*/D + 1) \qquad (1\text{-}9)$$

He also found that for large cylinders with comparatively small film thicknesses:

$$s = \pi DL/B^* \qquad (1\text{-}10)$$

The experimental results of Langmuir (air), Ayrton and Kilgour (air), Kennelly (air), Petavel (air, hydrogen, oxygen and carbon dioxide) and Davis (toluene, carbon tetrachloride, anilene, olive oil and glycerine) were in good agreement with equations (1-8) and (1-9).

The theory was later extended by Rice [33] to take into account the variation of the convective heat losses at low temperatures with the 5/4 power of the temperature difference, by including β and θ_p in the dimensional analysis of the expression for the film thickness:

$$B^*/D = 2.12(Gr \cdot Pr)^{-1/4} \qquad (1\text{-}11)$$

Up until this point in time, all of the work on free convection from horizontal cylinders had either been confined to empirical correlations of experimental data or to very simplified theoretical considerations. No attempt had been made to solve the full Navier-Stokes equations because of their extreme complexity . A major breakthrough in the analytic investigation of the free convective flow phenomenon came in 1930, when Schmidt and Beckmann [38] first suggested applying the approximations of boundary layer theory to simplify the full Navier-Stokes equations. The