THE UNIVERSITY OF MANITOBA

AN APPROACH TO THE CANONICAL REALIZATION OF RC NETWORKS THROUGH STATE VECTOR TRANSFORMATION

BY

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A THESIS

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ABSTRACT

This thesis presents a method of realizing RC equivalent canonical networks based on the proper transformation of the state vector. The proposed method is amenable to a class of networks whose driving-point impedance Z(s) has the property that Z(s) = 0 for s = ∞. Although techniques of deriving element values exist, this work takes a different approach so that one can also obtain the transformation matrices among all canonical forms.

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CHAPTER I

INTRODUCTION

In 1929 Cauer [1] first suggested that a large number of equivalent networks could be obtained by subjecting the network variables to suitable linear transformations. Since then, this idea has been explored further by many researchers, [1]-[9]. The transformation that Cauer proposed led to congruent transformations of the parameter matrices. He showed how the transformations could be constrained to keep desired driving point and transfer functions of a network invariant and pointed out that these transformations would preserve the positive semidefinite character of the parameter matrices. Since this latter property is both necessary and sufficient for realizability if ideal transformers are allowed, he thereby established a fundamental and powerful method for generating equivalent networks.

Guillemin [7],[8] was the first to realize the great importance of the normal form in generating equivalent two-element-kind networks. The normal form ties together the so-called methods of direct transformation and matrix synthesis. The former refers to Cauer's method of transforming a given network directly into an equivalent one, while the latter refers to Guillemin's procedure of generating equivalent networks from the normal-coordinate transformation procedure. Since direct transformation can always be viewed as a conversion from the network to normal form followed by conversion from the normal form to an equivalent network, it is clear that the normal form is the common feature of equivalent networks produced by linear transformations.

Duda, [10] extended Guillemin approach to accomplish a continuous transformation between two Foster canonical forms, but he failed to establish similar results for Cauer forms.

The main aspect of this thesis is based on the transformation of the state vector of the single-input, single-output networks described by $\dot{x} = Ax + bu$, $y = c'x$ through standard form [11]. The transformation between the canonical realizations and the standard form is established so as to link all canonical forms through transformation matrices. Element values in each realization will be derived directly.

In Chapter II, we introduce the standard form. In Chapter III we derive the transformation between the canonical realizations and the standard form together with the element values for each realization. Two examples are provided to illustrate the proposed method.

The detailed algebraic manipulation involved in the derivation of various transformation matrices are included in the Appendix.

CHAPTER II

THE STANDARD FORM

2.1 STATE EQUATION FOR MINIMAL SYSTEM

Consider the single-input, single-output, linear, time invariant, controllable, observable system described by the state equations

$$\dot{x}(t) = Ax(t) + bu(t) \qquad (2.1)$$
$$y(t) = c'x(t)$$

where $x(t)$ is the state vector of order n, $u(t)$ and $y(t)$ are input and output respectively, and c' denotes the transpose of c. The constant matrix A is of (nxn) and b and c are constant vectors. The transfer function of the system is given by

$$W(s) = \frac{Y(s)}{U(s)} = c'(sI_n - A)^{-1} b \qquad (2.2)$$

where I_n is the (nxn) identity matrix and s is the Laplace transform variable.

Theorem 1 [11]

Consider $W(s)$ of the form

$$W(s) = \frac{R_0 + R_1 s + \ldots + R_{n-1} s^{n-1}}{p_0 + p_1 s + \ldots + p_{n-1} s^{n-1} + s^n} \qquad (2.3)$$

of which Taylor series expansion about $s = \infty$ yields

$$W(s) = \frac{L_1}{s} + \frac{L_2}{s^2} + \ldots + \frac{L_n}{s^n} + \ldots \qquad (2.4)$$

Then $\{A,b,c\}$ of (2.2) can be represented by

$$A_0 = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ 0 & 0 & 0 & \cdots & 1 \\ -p_0 & -p_1 & -p_2 & \cdots & -p_{n-1} \end{bmatrix}, \quad b_0 = \begin{bmatrix} L_1 \\ L_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ L_n \end{bmatrix}, \quad c_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix} \qquad (2.5)$$

The realization based on $\{A_0, b_0, c_0\}$ of (2.5) is known as the standard controllable realization [11]. We represent the standard form by $S:\{A_0, b_0, c_0\}$.

The relation between the Taylor Coefficients and the coefficients of the numerator and denominator polynomial of $W(s)$ is given by the recursive formula [11]

$$L_i = R_{n-i} - \sum_{k=1}^{i-1} p_{n+k-i} L_k \qquad i=1,2,\ldots n \qquad (2.6)$$

Theorem 2 [11]

Given any two time invariant realizations of the same frequency response, say

$$\dot{x} = A_0 x + b_0 u \quad ; \quad y = c_0' x$$

and

$$\dot{z} = Az + bu \quad ; \quad y = c'z$$

there exists uniquely a constant nonsingular matrix T of $z = Tx$ such that

$$TA_0T^{-1} = A \quad , \tag{2.7a}$$

$$Tb_0 = b \quad , \tag{2.7b}$$

$$c_0'T^{-1} = c' \quad . \tag{2.7c}$$

CHAPTER III

THE TRANSFORMATION OF CANONICAL REALIZATIONS TO THE STANDARD FORM

In this chapter, we will derive the transformation matrices between the standard form of (2.5) and the canonical realizations in order to relate all canonical forms through transformation matrices. The transformation matrix between the standard forms is derived in the Appendix. In the following, we will obtain the transformation matrices T_1, T_2, T_3 and T_4 that, operating on the standard form S, yields two Foster forms and two Cauer forms, respectively, as:

$$T_1 S = F_1$$

$$T_2 S = F_2$$

$$T_3 S = C_1$$

$$T_4 S = C_2$$

Throughout the analysis we take the capacitor voltages as the state variables and the current to be the input.

3.1 FOSTER IMPEDANCE FORM

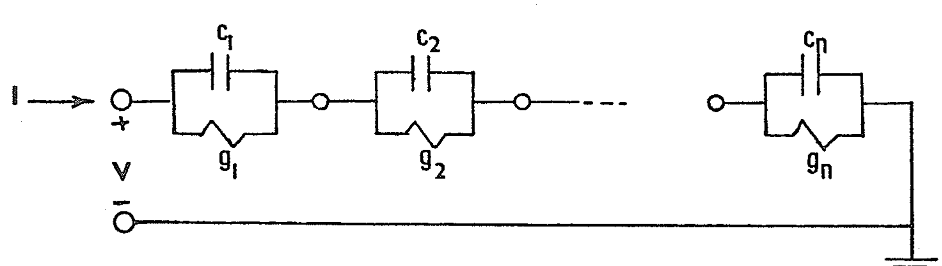

Fig. 1

3.1.1 The Transformation Matrix

Let $W(s)$ be the driving-point impedance $Z(s)$ for the linear lumped passive RC networks with $Z(\infty) = 0$, and let the n capacitor voltages be the n state variables.

$$c_i \dot{v}_i + g_i v_i = I \qquad i=1,2,\ldots n \qquad (3.1)$$

$$\sigma_i = -\frac{g_i}{c_i}$$

$$\begin{bmatrix} \dot{v}_1 \\ \dot{v}_2 \\ \cdot \\ \cdot \\ \dot{v}_n \end{bmatrix} = \begin{bmatrix} \sigma_1 & 0 & \cdots & 0 \\ & \sigma_2 & & \\ \cdot & \cdot & \cdots & \cdot \\ 0 & 0 & & \sigma_n \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \cdot \\ \cdot \\ v_n \end{bmatrix} + \begin{bmatrix} \frac{1}{c_1} \\ \frac{1}{c_2} \\ \cdot \\ \cdot \\ \frac{1}{c_n} \end{bmatrix} I \qquad (3.2)$$

$$Z_{in} = \frac{V}{I} = \frac{1}{I}(1 \; 1 \; \ldots \; 1) \begin{bmatrix} v_1 \\ v_2 \\ \cdot \\ \cdot \\ v_n \end{bmatrix}$$

from which we identify

$$c' = [1 \; 1 \; \ldots \; 1] \qquad (3.3)$$

If we represent Foster impedance form by $F_1 : \{A, b, c\}$ then from (3.2) and (3.3) we obtain

$$A = \begin{bmatrix} \sigma_1 & & & & \\ & \sigma_2 & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & \sigma_n \end{bmatrix}, \quad b = \begin{bmatrix} \frac{1}{c_1} \\ \frac{1}{c_2} \\ \cdot \\ \cdot \\ \frac{1}{c_n} \end{bmatrix}, \quad c = \begin{bmatrix} 1 \\ 1 \\ \cdot \\ \cdot \\ 1 \end{bmatrix} \tag{3.4}$$

The transformation of S to F_1 is symbolically written as

$$T_1 S = F_1 \tag{3.5}$$

where

$$T_1^{-1} = [\alpha_{ij}] \qquad i,j=1,2,\ldots n$$

From (2.7c) and (2.7a) we have $c_0' T^{-1} = c'$ and $T_1^{-1} A = A_0 T_1^{-1}$ which gives the entries of T_1^{-1} as

$$\alpha_{ij} = \sigma_j^{i-1} \qquad i,j=1,2,\ldots n \tag{3.6}$$

3.1.2 Element Values

Let

$$Z(s) = \frac{R(s)}{P(s)}$$

$$= \frac{R(s)}{(s-\sigma_1)(s-\sigma_2)\ldots(s-\sigma_n)}$$

The transformation matrix T_1 written in terms of the poles of $Z(s)$, is the inverse of the Vandermonde matrix. From (2.7b) i.e., $b = T_1 b_0$ we write

$$\begin{bmatrix} \frac{1}{c_1} \\ \frac{1}{c_2} \\ \frac{1}{c_3} \\ \cdot \\ \frac{1}{c_n} \end{bmatrix} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ \sigma_1 & \sigma_2 & \cdots & \sigma_n \\ \sigma_1^2 & \sigma_2^2 & \cdots & \sigma_n^2 \\ \cdot & \cdot & \cdots & \cdot \\ \sigma_1^{n-1} & \sigma_2^{n-1} & \cdots & \sigma_n^{n-1} \end{bmatrix}^{-1} \begin{bmatrix} L_1 \\ L_2 \\ L_3 \\ \cdot \\ L_n \end{bmatrix} \qquad (3.7)$$

and since $g_i = -c_i\sigma_i$, element values are found from (3.7).

3.2 FOSTER ADMITTANCE FORM

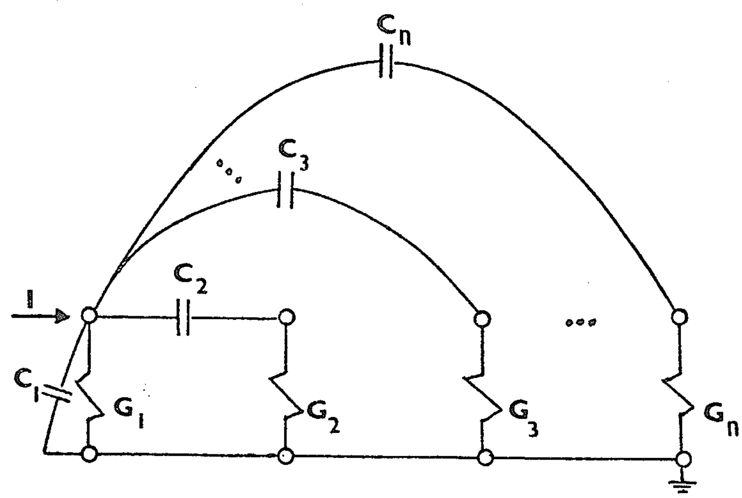

Fig. 2

3.2.1 The Transformation Matrix

We can write the state equations for Foster admittance form as $F_2:\{A,b,c\}$, with A,b and c given by

$$A = \begin{bmatrix} -\sum_{i=1}^{n} \frac{G_i}{c_1} & \frac{G_2}{c_1} & \cdots & \frac{G_n}{c_1} \\ \frac{G_2}{c_2} & -\frac{G_2}{c_2} & 0 & 0 \\ \frac{G_3}{c_3} & 0 & -\frac{G_3}{c_3} & 0 \\ \cdots & \cdots & \cdots & \cdots \\ \frac{G_n}{c_n} & 0 & \cdots & -\frac{G_n}{c_n} \end{bmatrix}, \quad b = \begin{bmatrix} \frac{1}{c_1} \\ 0 \\ 0 \\ \cdot \\ 0 \end{bmatrix} \text{ and } c = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \cdot \\ 0 \end{bmatrix} \tag{3.8}$$

Since $Y_{in} = \frac{1}{Z_{in}} = \frac{P(s)}{R(s)}$, it is clear that

$$\sigma_i = -\frac{G_i}{c_i} \tag{3.9}$$

is the poles of Y_{in} . If we consider the transformation

$$T_2 S = F_2 \tag{3.10}$$

from (2.7c) we find the first row of T_2 and from (2.7a) we obtain the other entries of the transformation matrix as

$$T_2 = [\alpha_{ij}] \qquad i\ j=1,2,\ldots n \tag{3.11}$$

$$\alpha_{11} = 1$$

$$\alpha_{1j} = 0 \qquad j=2,\ldots n$$

$$\alpha_{ij} = \frac{\sum_{k=j}^{n} p_k\, \sigma_j^k}{\sum_{k=0}^{n} p_k\, \sigma_j^k} \qquad i=2,\ldots n,\ j=1,\ldots n \tag{3.12}$$

where $p_n = 1$. Hence the transformation matrix can be constructed from the zeros and denominator polynomials coefficients of $Z(s)$.

3.2.2 Element Values

From $T_2 b_0 = b$ with b_0 and b defined in (2.5), and (3.8) respectively, we have $c_1 = \frac{1}{L_1}$, and from (2.7a) namely $T_2 A_0 T_2^{-1} = A$, we can find c_i and G_i.

3.3 CAUER FIRST FORM

The method of deriving the transformation matrix and the element values is based on the transformation of the state vector associated with each canonical form to the standard forms. For every canonical form we have,

$$TA_I T^{-1} = A_{II}$$

$$Tb_I = b_{II}$$

$$c_I' = c_{II}' T$$

which yields $n^2 + 2n$ equations in the n^2 entries of T and the n capacitances and the n conductances. For Foster forms we first obtained the transformation matrices and then the element values. In Cauer forms, the order will be reversed, we assume first that the element values are known, we derive the transformation matrix in terms of the element values, then, explicit formulae will be obtained for the elements in terms of Taylor coefficients of $Z(s)$.

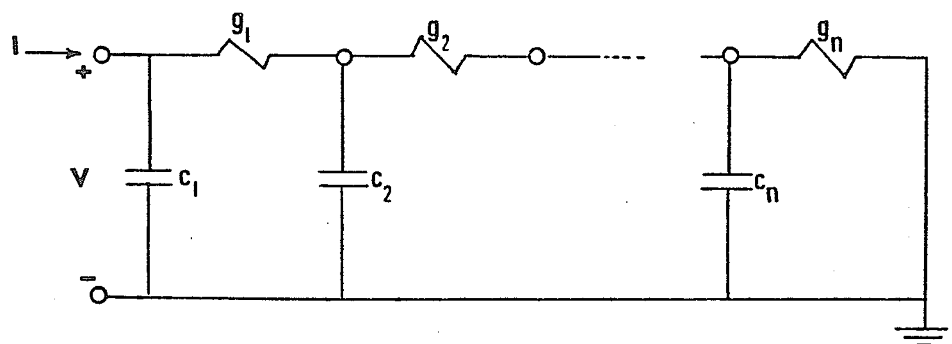

Fig. 3

3.3.1 The Transformation Matrix

Here we use the same definitions for the variables as before. Let us represent Cauer first realization by $C_1:\{A,b,c\}$

$$A = \begin{bmatrix} -\frac{g_1}{c_1} & \frac{g_1}{c_1} & 0 & \cdots & . & 0 \\ \frac{g_1}{c_2} & -\frac{g_1+g_2}{c_2} & \frac{g_2}{c_2} & \cdots & . & 0 \\ 0 & \frac{g_2}{c_3} & -\frac{g_2+g_3}{c_3} & \frac{g_3}{c_3} & . & 0 \\ . & . & . & \cdots & . & . \\ 0 & 0 & 0 & \cdots & \frac{g_{n-1}}{c_n} & -\frac{g_{n-1}+g_n}{c_n} \end{bmatrix}, \quad b = \begin{bmatrix} \frac{1}{c_1} \\ 0 \\ 0 \\ . \\ 0 \end{bmatrix} \text{ and } c = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{3.13}$$

The transformation of S to C_1 is of the form

$$T_3 S = C_1$$

$$T_3 = [\alpha_{ij}] \qquad i,j=1,\ldots n \tag{3.14}$$

$$\alpha_{ij} = 0 \qquad i < j$$

$$= 1 \qquad j=1,\ i=1,2,\ldots n$$

$$= \frac{M_{2i-4}M_{2i-3}}{M_{2i-5}M_{2i-2}}\,\alpha_{i-1,j-1} + \left\{1 + \frac{M_{2i-6}M_{2i-3}^2}{M_{2i-2}M_{2i-5}^2}\right\}\alpha_{i-1,j}$$

$$- \frac{M_{2i-6}M_{2i-3}^2}{M_{2i-2}M_{2i-5}^2}\,\alpha_{i-2,j} \quad i \geq j \qquad (3.15)$$

where

$$M_k = \left[(-1)^{i+j}L_{i+j-1}\right] \ i,j=1,2,\ldots\frac{k+1}{2} \quad \text{for } k \text{ odd}$$

$$= \left[(-1)^{i+j+1}L_{i+j}\right] \ i,j=1,2,\ldots\frac{k}{2} \quad \text{for } k \text{ even} \qquad (3.16)$$

and $\qquad M_0 = M_{-1} = M_{-2}\ldots = 1$.

Entries to the transformation matrix T_3 are found first from Taylor coefficients and then successively by (3.15).

3.3.2 Element Values

In deriving the transformation matrix, n^2 equations were used. $c_0' = c'T_3$ gives the first row of T_3 . The rest is obtained from $T_3A_0T_3^{-1} = A$. The special form of the A-matrix associated with the standard realization, simplifies the problem of solving the n^2 equations. Since in each equation only one new unknown appears, the entries of T_3 can be determined one-by-one. The solution of the remaining 2n equations of (3.14) yields the element values as:

$$g_k = \frac{M_{2k-2}M_{2k}}{M_{2k-1}^2} \qquad k=1,2,\ldots n \tag{3.17}$$

$$c_k = \frac{M_{2k-2}^2}{M_{2k-3}M_{2k-1}}$$

3.4 CAUER SECOND FORM

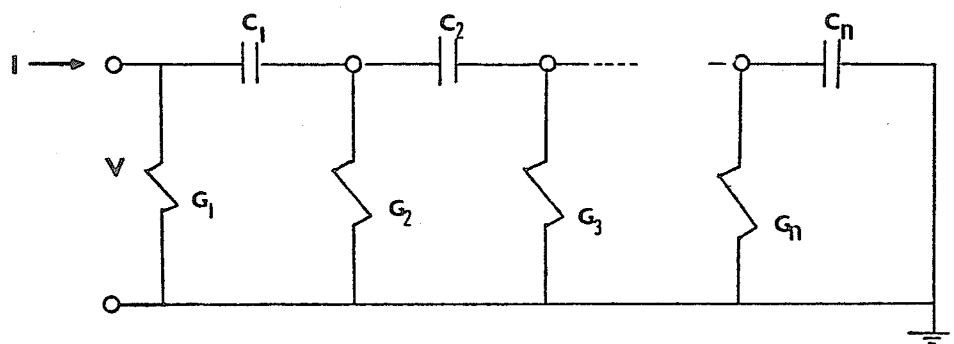

Fig. 4

3.4.1 The Transformation Matrix

Representing Cauer second form by $C_2:\{A,b,c\}$, we have,

$$A = \begin{bmatrix} \frac{\theta_1}{c_1} & \frac{\theta_1}{c_1} & \cdots & \frac{\theta_1}{c_1} \\ \frac{\theta_1}{c_2} & \frac{\theta_2}{c_2} & \cdots & \frac{\theta_2}{c_2} \\ \cdot & & & \\ \cdot & & & \\ \frac{\theta_1}{c_n} & \frac{\theta_2}{c_n} & \cdots & \frac{\theta_n}{c_n} \end{bmatrix}, \quad b = \begin{bmatrix} \frac{1}{c_1} \\ \frac{1}{c_2} \\ \cdot \\ \cdot \\ \frac{1}{c_n} \end{bmatrix} \quad \text{and } c = \begin{bmatrix} 1 \\ 1 \\ \cdot \\ \cdot \\ 1 \end{bmatrix} \tag{3.18}$$

where

$$\theta_k = \sum_{i=1}^{k} G_i \ .$$

The transformation between C_2 and S is

$$T_4 S = C_2$$

The same analysis as in Cauer first form will be followed to find T_4 as

$$T_4^{-1} = [\alpha_{ij}] \qquad i,j=1,\ldots n$$

$$\alpha_{ij} = 1 \qquad j=1,n$$

$$\alpha_{ij} = -\left\{\sum_{k=1}^{j-1} \theta_k \, \alpha_{i-1,k} \frac{M'^2_{2k-1}}{M'_{2k-2} M'_{2k}} + \theta_j \sum_{k=j}^{n} \alpha_{i-1,k} \frac{M'^2_{2k-1}}{M'_{2k-2} M'_{2k}}\right\}$$

$$i=2,\ldots n \ , \qquad j=1,\ldots n \qquad (3.20)$$

where

$$\theta_k = \sum_{\ell=1}^{k} \frac{M'^2_{2\ell-2}}{M'_{2\ell-3} M'_{2\ell-1}} \qquad \text{with } M'_k \text{ defined in (3.25)}$$

3.4.2 Element Values

A similar procedure as in (3.3) can be applied to obtain the formulae for element values, but we will make use of the relation between the two latter developments, namely, Cauer first and second forms to derive the elements value as follows.

For the circuit shown in Fig. 4, if we use RC:CR transformation [12], the driving-point impedance of the new network Z_2 becomes

$$Z_2 = \frac{1}{s} \; Z(\frac{1}{s}) \tag{3.21}$$

and $Z(s)$ is the given driving-point impedance of the original network. From (2.3)

$$Z_2 = \frac{R_0 s^{n-1} + R_1 s^{n-2} + \ldots + R_{n-1}}{p_0 s^n + p_1 s^{n-1} + \ldots + p_{n-1} s + 1} \tag{3.22}$$

Expanding Z_2 by Taylor series in the neighbourhood of $s = \infty$,

$$Z_2 = \frac{a_1}{s} + \frac{a_2}{s^2} + \frac{a_3}{s^3} + \ldots + \frac{a_n}{s^n} + \ldots \tag{3.23}$$

Since the new network, generated by frequency transformation, is in Cauer first form, we can use (3.17) for finding the elements value for the circuit of Fig. 4,

$$C_k = \frac{M'_{2k-2} M'_{2k}}{M'^2_{2k-1}} \;, \quad G_k = \frac{M'^2_{2k-2}}{M'_{2k-3} M'_{2k-1}} \qquad k=1,\ldots n \tag{3.24}$$

where

$$M'_k = [(-1)^{i+j} \; a_{i+j-1}] \qquad i,j=1,2,\ldots \frac{k+1}{2} \qquad \text{for k odd}$$

$$= [(-1)^{i+j+1} \; a_{i+j}] \qquad i,j=1,2,\ldots \frac{k}{2} \qquad \text{for k even} \tag{3.25}$$

$$M'_0 = M'_{-1} = M'_{-2} = \ldots = 1$$

and $$a_i = \frac{1}{p_0} (R_{i-1} - \sum_{k=1}^{i-1} p_{i-k} a_k) \qquad i=1,2,\ldots \tag{3.25}$$

3.5 THE TRANSFORMATION MATRICES IN TERMS OF THE ELEMENT VALUES

Given a network realized in one of the canonical forms, we can derive the network function through standard form and then all other canonical realizations can be obtained.

3.5.1 Cauer First Form

$$T_3 = [\alpha_{ij}] \qquad i,j=1,\ldots n \tag{3.26}$$

$$\alpha_{ij} = 0 \qquad i<j$$

$$= 1 \qquad i=1,\ldots n, \quad j=1$$

$$= \frac{g_{i-1}}{g_{i-1}} \alpha_{i-1,j-1} + \left(1 + \frac{g_{i-2}}{g_{i-1}}\right) \alpha_{i-1,j} - \frac{g_{i-2}}{g_{i-1}} \alpha_{i-2,j} \qquad i \geq j \tag{3.27}$$

g_i, c_i are the elements of C_1

3.5.2 Cauer Second Form

$$T_4^{-1} = [\alpha_{ij}] \qquad i,j=1,\ldots n \tag{3.28}$$

$$\alpha_{ij} = 1$$

$$= -\left(\sum_{k=1}^{j-1} \theta_k \frac{\alpha_{i-1,k}}{C_k} + \theta_j \sum_{k=j}^{n} \frac{\alpha_{i-1,k}}{C_k}\right) \qquad i=2,\ldots n, \; j=1,\ldots n \tag{3.29}$$

where

$$\theta_k = \sum_{\ell=1}^{k} G_\ell \qquad k=1,\ldots n$$

G_i, C_i are the elements of C_2

3.6 EXAMPLE 1

Given the network in Fig. 5(a), we can find Z_{in}, F_1, F_2, C_2 as follows. From Fig. 5(a) (C_1 Realization), we have C_1:{A,b,c}

$$A = \begin{bmatrix} -3 & 3 & 0 \\ 1 & -3 & 2 \\ 0 & \frac{1}{2} & -3 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \text{and} \quad c = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$T_3 S = C_1$, from (3.27) we obtain

$$T_3 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & \frac{1}{3} & 0 \\ 1 & 1 & \frac{1}{6} \end{bmatrix}$$

from $T_3 b_0 = b$, $L_1 = 1$, $L_2 = -3$, $L_3 = 12$

from $T_3 A_0 = AT_3$, $P_0 = 15$, $P_1 = 23$, $P_2 = 9$

from (2.6) $R_0 = 8$, $R_1 = 6$, $R_2 = 1$

Now we can construct the network function

$$Z_{in} = \frac{s^2 + 6s + 8}{s^3 + 9s^2 + 23s + 15}$$

$$= \frac{(s+2)\ (s+4)}{(s+1)\ (s+3)\ (s+5)}$$

$$T_1^{-1} = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -3 & -5 \\ 1 & 9 & 25 \end{bmatrix} ,$$

From (3.5) we have for F_1 :

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -5 \end{bmatrix}, \quad b = \begin{bmatrix} \frac{3}{8} \\ \frac{1}{4} \\ \frac{3}{4} \end{bmatrix}$$ which give Fig. 5(b)

$T_2S = F_2$, from (3.12) $T_2 = \begin{bmatrix} 1 & 0 & 0 \\ 6 & \frac{14}{3} & \frac{2}{3} \\ -4 & \frac{-20}{3} & \frac{-4}{3} \end{bmatrix}$, $A = \begin{bmatrix} -3 & \frac{3}{4} & \frac{3}{8} \\ 2 & -2 & 0 \\ 4 & 0 & -4 \end{bmatrix}$,

$b = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ which implies Fig. 5(c)

$T_4S = C_2$, from (3.20)

$T_4^{-1} = \begin{bmatrix} 11 & 1 & 1 \\ \frac{-15}{8} & \frac{-185}{38} & -9 \\ \frac{45}{8} & \frac{93}{4} & 58 \end{bmatrix}$ $\therefore$ for C_2 we have

$$A = \begin{bmatrix} \frac{-60}{47} & \frac{-60}{47} & \frac{-60}{47} \\ -\frac{3645}{6433} & -\frac{110643}{91462} & -\frac{110643}{95354} \\ -\frac{45}{1112} & \frac{1307}{5004} & -\frac{21935}{10008} \end{bmatrix}, \quad b = \begin{bmatrix} \frac{32}{49} \\ \frac{1944}{6533} \\ \frac{3}{139} \end{bmatrix}$$ from which

we have Fig. 5(d).

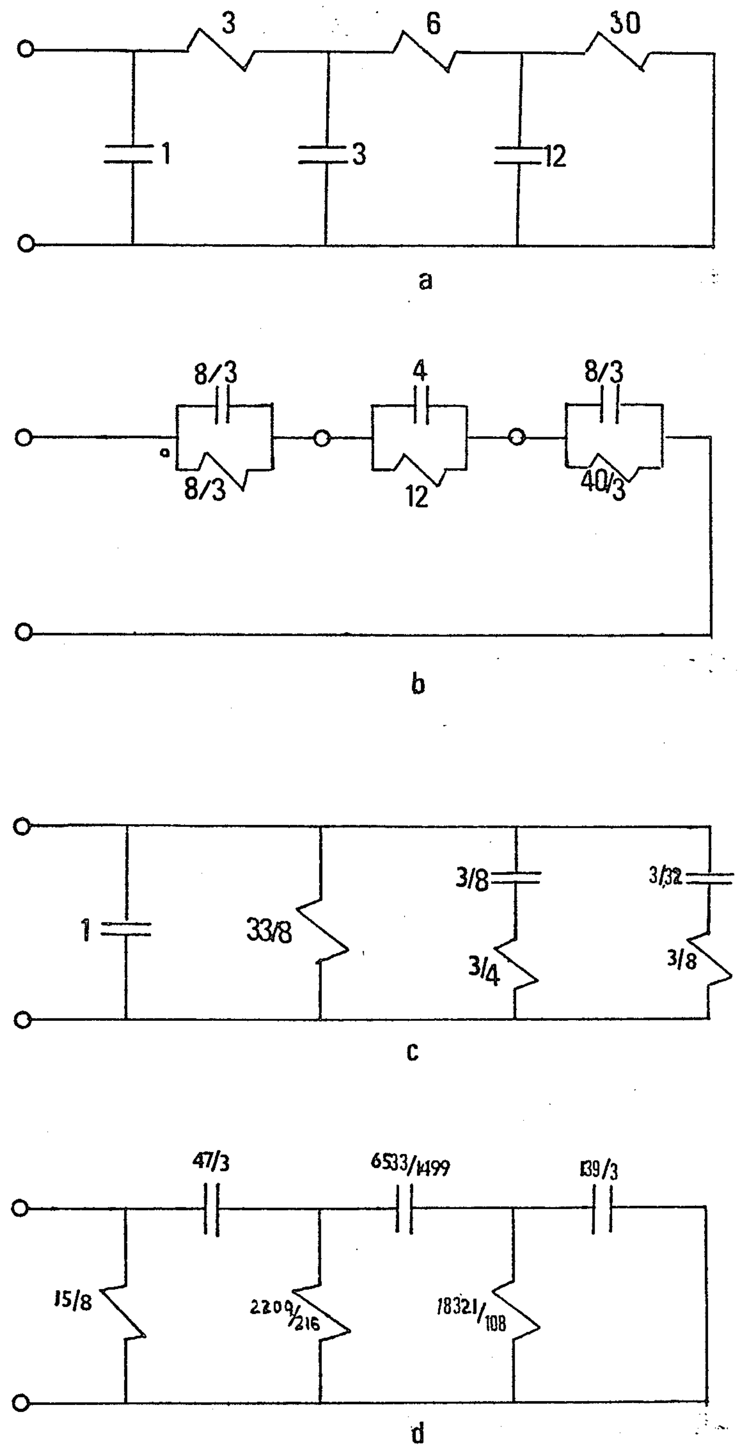
3
6
30
1
3
12
a
8/3
4
8/3
8/3
12
40/3
b
1
33/8
3/8
3/4
3/32
3/8
c
47/3
6533/1499
139/3
15/8
2209/216
18321/108
d

Fig. 5

3.7 EXAMPLE 2

$$Z(s) = \frac{s+2}{s^2 + 4s + 3}$$

$$P_o = 3, \quad P_1 = 4, \quad L_1 = 1, \quad L_2 = 2$$

$$A_o = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix}, \; b_o = \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \; c_o = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$T_1 S = F_1, \quad T_1 = \begin{bmatrix} -\frac{3}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} \end{bmatrix} \quad \therefore F_1: A = \begin{bmatrix} -1 & 0 \\ 0 & -3 \end{bmatrix}, \; b = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}, \; c = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

which implies Fig. 6(a) .

$$T_2 S = F_2, \quad T_2 = \begin{bmatrix} 1 & 0 \\ 4 & 2 \end{bmatrix} \quad \therefore F_2: A = \begin{bmatrix} -2 & \frac{1}{2} \\ 2 & -2 \end{bmatrix}, \; b = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \; c = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

from which we have Fig. 6(b).

$$T_3 S = C_1, \quad T_3 = \begin{bmatrix} 1 & 0 \\ 1 & \frac{1}{2} \end{bmatrix} \quad \therefore C_1: A = \begin{bmatrix} -2 & 2 \\ \frac{1}{2} & -2 \end{bmatrix}, \; b = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \; c = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

and we obtain Fig. 6(c).

$$T_4 S = C_2, \quad T_4 = \begin{bmatrix} \frac{8}{5} & \frac{2}{5} \\ -\frac{3}{5} & -\frac{2}{5} \end{bmatrix} \quad \therefore C_2: A = \begin{bmatrix} -\frac{6}{5} & -\frac{6}{5} \\ -\frac{3}{10} & -\frac{14}{5} \end{bmatrix}, \; b = \begin{bmatrix} \frac{4}{5} \\ \frac{1}{5} \end{bmatrix}, \; c = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

as in Fig. 6(d).

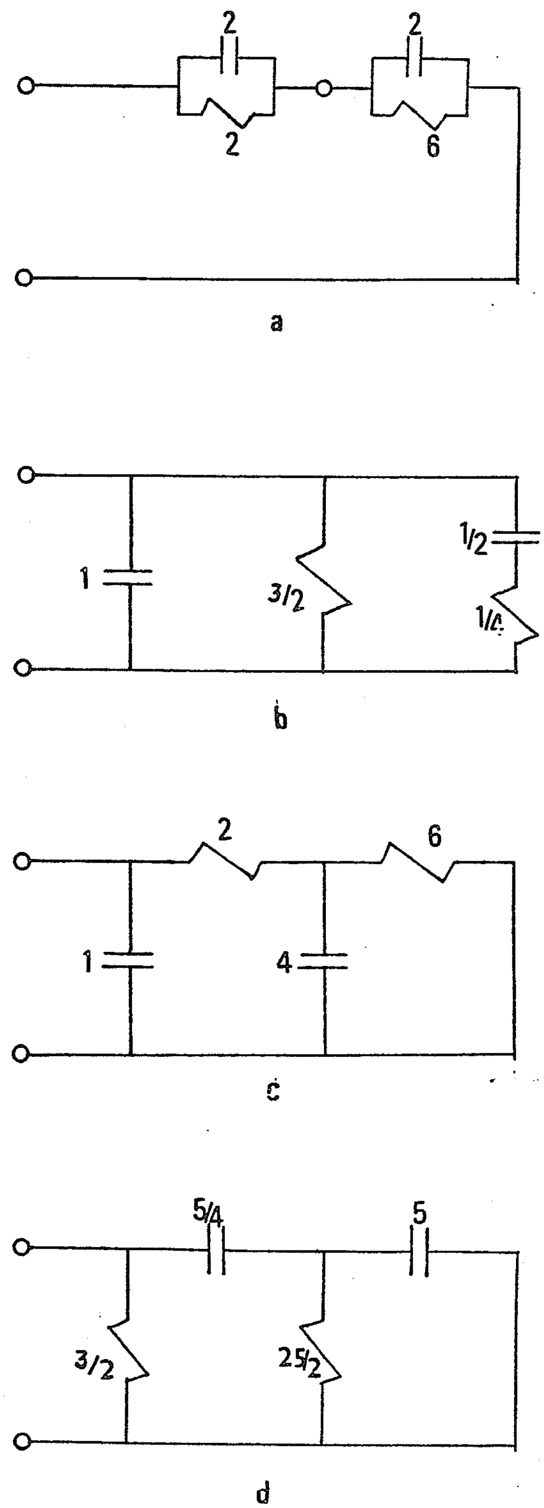

Fig. 6

CHAPTER IV

CONCLUSIONS

Given an RC driving-point function, the element values of any specific canonical network can be found through the standard form, which is uniquely determined by parameters of the given function. Conversely given a network realized in any of the canonical forms, corresponding driving-point function can be obtained by means of matrix multiplication. This thesis presents the set of transformation matrices and explicit formulae for the elements of canonical realizations.

Although the given analysis was for the RC case, analogous results can be derived for the RL and LC cases.

APPENDIX

THE DERIVATION OF THE TRANSFORMATION MATRICES

We give here the derivation of the transformation matrix between the standard forms as an example for deriving the transformation matrices. The procedure is similar for the other transformations.

The transformation between the standard controllable form S and the standard observable form S_1 [11] can be represented symbolically as

$$T_s S_1 = S \tag{A.1}$$

where $T_s = [\alpha_{ij}] \qquad i,j=1,\ldots n$ (A.2)

from (2.6)

$$T_s A_1 T_s^{-1} = A_0 \tag{A.3.1}$$

$$T_s b_1 = b_0 \tag{A.3.2}$$

$$c_0' T_s = c_1' \tag{A.3.3}$$

with $S:\{A_0, b_0, c_0\}$ and $S_1:\{A_1, b_1, c_1\}$ are given in (2.5) and [11] respectively. From (A.3.2) we write,

$$\begin{bmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1n} \\ \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \alpha_{n1} & \alpha_{n2} & \cdots & \alpha_{nn} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} = \begin{bmatrix} L \\ L \\ \vdots \\ L_n \end{bmatrix}$$

which in turn gives

$$\alpha_{in} = L_i \qquad i=1,2,\ldots n \tag{A.4}$$

and also from (A.3.3)

$$[1\ 0\ 0\ ..\ 0]\begin{bmatrix}\alpha_{11} & \alpha_{12} & .. & L_1\\ \alpha_{21} & \alpha_{22} & .. & L_2\\ . & . & .. & .\\ \alpha_{n1} & \alpha_{n2} & .. & L_n\end{bmatrix} = [R_0\ R_1 \ldots R_{n-1}]$$

we have

$$\alpha_{ij} = R_{j-1} \quad j=1,2,\ldots n \tag{A.5}$$

Now from (A.3.1)

$$T_s A_1 = A_0 T_s$$

$$\begin{bmatrix}R_0 & R_1 & .. & L_1\\ \alpha_{21} & \alpha_{22} & & L_2\\ . & . & .. & .\\ . & & & \\ \alpha_{n1} & \alpha_{n2} & .. & L_n\end{bmatrix}\begin{bmatrix}0 & 1 & 0 & . & 0\\ 0 & 0 & 1 & . & 0\\ . & . & & & \\ 0 & 0 & . & . & 1\\ -p_0 & -p_1 & . & . & -p_{n-1}\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & . & 0\\ 0 & 0 & 1 & . & 0\\ . & & & & .\\ 0 & 0 & . & .. & 1\\ -p_0 & -p_1 & & & -p_{n-1}\end{bmatrix}\begin{bmatrix}R_0 & R_1 & . & . & L_1\\ \alpha_{21} & \alpha_{22} & . & . & L_2\\ . & . & . & . & .\\ & & & & \\ \alpha_{n1} & \alpha_{n2} & . & . & \alpha_{nn}\end{bmatrix}$$

multiplying the first column of A_1 by the first n-1 rows of T_s and multiplying the first column of T_s by the first n-1 rows of A_0 and equating we have

$$\alpha_{i1} = -p_o L_{i-1} \qquad i=2,\ldots n$$

multiplying the second column by the first n-1 rows in both sides;

$$\alpha_{22} = R_0 - p_1 L_1$$

$$\alpha_{i2} = -(p_0 L_{i-2} + p_1 L_{i-1}) \qquad i=3,\ldots n$$

repeating the process, we obtain

$$\alpha_{ij} = R_{j-i} - \sum_{k=1}^{i-1} p_{k+j-i} \; L_k \qquad \text{for } i \leq j$$

$$= - \sum_{k=0}^{j-1} p_k \; L_{i-j+k} \qquad \text{for } i > j$$

Summarizing the results

$$\alpha_{ij} = R_{j-1} \tag{A.7}$$

$$= R_{j-1} - \sum_{k=1}^{i-1} p_{k+j-i} \; L_k \qquad i \leq j$$

$$= - \sum_{k=0}^{j-1} p_k \; L_{i-j+k} \qquad i > j$$

and we also obtain the recursion formula [11], which relates Taylor coefficients to the coefficients of the rational polynomials of $W(s)$ as

$$L_i = R_{n-i} - \sum_{k=1}^{i-1} p_{n+k-i} \; L_k \qquad i=1,2,\ldots n \tag{A.8}$$

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