

Comments on McGee's 'Counterexample to Modus Ponens'

by

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of Manitoba in partial fulfillment of the requirements of the degree**

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Abstract. Vann McGee published a paper where he claimed that modus ponens is not a generally valid rule of inference for the natural language conditionals. Although he spoke only about the indicative conditionals, the counterexample of his that is considered in my thesis can be easily applied to subjunctive (counterfactual) conditionals also. After explaining what is paradoxical about the counterexample, and commenting upon McGee's solution, I consider various remarks to McGee's point that might be or already are made by some philosophers. I try to show that none of them helps us to remove the problem. I argue that in fact the 'counterexample' points mainly to the need of distinguishing the truth-functional and intensional disjunctions.

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Introduction

Sometimes, it is possible to derive a contradiction using the rules that appear to be in accordance with common use of natural language. There are many examples of that kind from the Ancient times to our days. If all the rules that lead to the contradiction are supported by a strong intuition, we call it a case of paradox. The usual way of solving such paradoxes is to list all the rules used in deriving the contradiction, and then to reject some of those that we find less supported by our intuition than the others. On one hand, justification for such rejection can be the paradox itself. But this seems to be an act of violence against the language (that is what we usually call ad hoc solution). On the other hand, the justification can be provided in a way that does not depend on the paradox. The latter approach is certainly a better tactic, because the paradox itself is potential counterexample to every rule used, not just to the one that we reject with the least regret. If the justification provided by the second way turns out to be more intuitively acceptable than the rejected rule (and where the latter becomes less intuitively acceptable), then it can be said that the paradox is solved in a satisfactory way; this also assumes that the second approach would give us not only the means to avoid the paradox, but also the explanation why the mistake was made, why were we inclined to make it and why it was hard to see. If all this does not happen to be the case, the solution commits violence against the language, and leaves us with the feeling of discontent, as if the question is still open.

Having these remarks on my mind, I will discuss the problem imposed by the following argument. In his paper "A counterexample to modus ponens"¹ Vann McGee says:

Opinion polls taken just before the 1980 election showed the Republican Ronald Reagan decisively ahead of the Democrat Jimmy Carter, with the other Republican in the race. John Anderson, a distant third. Those apprised of the poll results believed, with good reason:

Mc1 If a Republican wins the election, then if it's not Reagan who wins, it will be Anderson.

Mc2 A Republican will win the election.

Yet they did not have reason to believe:

McC If it's not Reagan who wins, it will be Anderson.

In what follows I will give some general remarks to make the problem raised by McGee explicit. Then I will expose his solution. Next I will consider various remarks meant to remove the problem and save modus ponens. For some of them I will give what I find a conclusive reason to reject them. For the others I will give a hint of what's wrong with them, showing that the opposite view appears to be equally well supported. After that I will give what I think is the solution, which will also enable me to explain the previous contradictions.

¹ *Journal of Philosophy* LXXXII, 9 1985, p 462

General Remarks

McGee's example has the following form:

$$\frac{A \quad A \rightarrow (B \rightarrow C)}{B \rightarrow C}$$

What is ' \rightarrow '? Whatever it might be, it cannot be material implication. There is no sensible way to deny that modus ponens holds for that connective, and McGee is well aware of that.² The sufficient explanation for that would be just to point out that classical propositional calculus, which makes material implication what it is, is consistent. But since my main concern in this paper will be the meaning of various logical devices, I will give some more remarks to make it clear.

We may say, following Hilbert, that what material implication is, that is, its meaning, is determined by the axioms and the rules of inference. In a very similar context Belnap would say that this approach "... is equally well illustrated ... by aspects of Wittgenstein and those who learned from him to treat the meanings of words as arising from the role they play in the context of discourse."³ Here this means that the meaning of material implication is determined by the role it plays in the context of formal inference. In that case modus ponens (mp) is obviously the basis of the meaning of material implication, and if we make a system with a conditional where mp is not generally valid, then the conditional is no longer material.

² cf.p.464

³ Nuel Belnap "Tonk, Plonk and Plink" in *Philosophical Logic*, ed. P.F. Strawson 1962 p.133.

Instead of Hilbert's way, one may want to do the logic differently, not starting with the formal system, that is, with syntax, but putting forward the definitions of the meaning of primitive notions, as Euclid would do. In that case one would define the meaning of material implication by truth conditions, that is, by the truth tables. Now, mp is an immediate consequence of the truth tables. So, again, material implication is not what it is without mp.

If we replaced ' \rightarrow ' by material implication in the McGee argument above, we would have true premises and true conclusion. So classical calculus does not register any problem in McGee's example. But we clearly see that there is some problem.

Thus, material implication has nothing to do with McGee's argument. He said that ' \rightarrow ' is an English indicative conditional. But the very same argument can be applied to subjunctive conditionals (counterfactuals), so I will take them both into account. The relation between material implication and mp was easy to determine, because this connective is well defined. But this is not the case with the natural language conditionals. We still don't have a logical device which can represent them in a way that makes no controversies.

The first difference relevant in this context between these conditionals and material implication is that at least the second of the two following principles holds for natural language conditionals:

$$(1) \quad A \rightarrow B \rightarrow \neg(A \rightarrow \neg B)$$

$$(2) \quad \frac{A \rightarrow B}{\neg(A \rightarrow \neg B)}$$

If the theorem of deduction doesn't hold for ' \rightarrow ', what is very likely to be the case, then we need also (3), in addition to (1)-(2):

$$(3) \quad A \rightarrow B \vdash \neg(A \rightarrow \neg B)^4$$

(1) - (3) (or at least 2 and 3) are generally accepted for indicative conditionals and counterfactuals. They are obviously in accordance with the use of natural language, and, as far as the contexts like the one described by McGee are concerned, their application raises no controversies. We might not want to accept their converse:

$$(4) \quad \neg(A \rightarrow B) \rightarrow (A \rightarrow \neg B)$$

$$(5) \quad \frac{\neg(A \rightarrow B)}{A \rightarrow \neg B}$$

$$(6) \quad \neg(A \rightarrow B) \vdash A \rightarrow \neg B$$

A can be irrelevant for B, so we would regard neither $A \rightarrow B$ nor $A \rightarrow \neg B$ as true, but we would regard both $\neg(A \rightarrow B)$ and $\neg(A \rightarrow \neg B)$ true for the same reason. That is, I think.

$$^4 \quad a \quad \frac{. A .}{B}$$

means that if A is a theorem, B is also.

$$b \quad A \vdash B$$

means that B is a logical consequence of (or is deducible from) A, no matter whether A is a theorem or not.

$$c \quad A \rightarrow B$$

means 'if A then B'. ' \rightarrow ' is a connective (not a predicate) and c, unlike a and b, belongs to object language.

a and b differ, for example, in classical predicate and modal calculus. The rules of generalisation and necessitation must be expressed in the form of a:

$$\frac{. A .}{\forall x A x} \quad \frac{. A .}{\Box A}$$

The expressions $A \vdash \forall x A x$ and $A \vdash \Box A$ are invalid, because by the deduction theorem they imply $\vdash A \rightarrow \forall x A x$ and $\vdash A \rightarrow \Box A$, the formulae which would make the calculus inconsistent.

why we reject 4-6. D. Lewis⁵, who does not like relevance logic, just points to some examples from the natural language which are not in accordance with 4-6, and rejects these principles without further explanation. Stalnaker⁶, on the contrary, accepts both 1-3 and 4-6, which is an immediate consequence of his 'conditional excluded middle':

$$(7) \quad (A \rightarrow B) \vee (A \rightarrow \neg B)$$

Since for McGee's argument (3) would be enough, we do not need to be involved in the discussion on the conditional excluded middle. I'll just add that both Lewis's and Stalnaker's approach are compatible with the relevance approach, provided that A is relevant for B. But in any case, 1-7 cannot be generally valid. Consider the case when $A = B \wedge \neg B$. Then both $A \rightarrow B$ and $A \rightarrow \neg B$ are true. Obviously 1-7 do not hold when A implies a contradiction. What propositions imply a contradiction depends on what logic we use. Lewis and Stalnaker follow the classical approach that any impossible proposition implies a contradiction. In relevance logic it is not the case. But since in McGee's argument we have only contingent antecedents, this would not be a problem either.

Let us see now what 1-3 have to do with McGee. He said that we reject the conclusion McC obtained by mp from Mc1 and Mc2, and it sounds reasonable. But we should make the reason explicit. Given the situation described by McGee, we surely believe that if it's not Reagan who wins, it will be Carter ($\neg W_r \rightarrow W_c$). Given the principles 1-3, it is easy to derive from $\neg W_r \rightarrow W_c$ that it is not the case that if it's not

⁵ Cf., for example, **Counterfactuals** Basil Blackwell 1973. p 79.

⁶ "A Theory of Conditionals" in **Causation and Conditionals** ed. Ernest Sosa Oxford University Press 1975 pp 165-79

Reagan who wins, it will be Anderson ($\neg W_r \rightarrow W_a$)⁷. The latter is the conclusion McC.

And conversely, from $\neg W_r \rightarrow W_a$ by 1-3 we can derive the negation of $\neg W_r \rightarrow W_c$.

Thus what is paradoxical in McGee's example is that we have both $\neg W_r \rightarrow W_c$ and $\neg W_r \rightarrow W_a$ which contradict each other. The first is obviously acceptable, and the second follows from Mc1 and Mc2 which are obviously acceptable by the rule of inference that is also obviously acceptable.

Nevertheless, one may complain that pointing to the fact that $\neg W_r \rightarrow W_c$ is obviously acceptable is still not a sufficient explanation of why we reject the conclusion $\neg W_r \rightarrow W_a$. It makes little difference whether we start with the acceptability of $\neg W_r \rightarrow W_c$ or with the unacceptability of $\neg W_r \rightarrow W_a$. In fact, we need an explanation for both. We might find the conditional probability the reason for rejecting/accepting those propositions. Assuming that W_r , W_c , W_a have their absolute probabilities, and that they are likely to be different from 1 and 0, we could accept $\neg W_r \rightarrow W_c$ and reject $\neg W_r \rightarrow W_a$ because $P(W_c/\neg W_r)$ is close to 1 and $P(W_a/\neg W_r)$ close to 0.

I do not think that probabilities of conditionals and conditional probabilities are the same thing (in fact, I think they are not). I do not think that they always differ either. Since in this paper I will not give a full determination of ' \rightarrow ', i.e. since I do not give a system, I cannot say that I know what ' \rightarrow ' is, and hence I do not know what its probability is either. But although we can, following Lewis⁸ or for some other reasons, make a distinction between $P(B/A)$ and $P(A \rightarrow B)$, there seem to be simple and

⁷ If Carter will win, Anderson will not ($W_c \rightarrow \neg W_a$). So by transitivity we have $\neg W_r \rightarrow \neg W_a$. And by 1-3 we have $\neg(\neg W_r \rightarrow W_a)$.

unproblematic cases where the two probabilities are the same. For example, 'If I cast a die, I'll get 5' seems to have the probability 1/6. Since there are many such examples and we very often accept conditionals because of conditional probability, I would say that a theory that *always* separates the two probabilities would not be good. In a word, it seems to me that $P(\neg W_r \rightarrow W_c)$ is $P(W_c/\neg W_r)$ and that $P(\neg W_r \rightarrow W_a)$ is $P(W_a/\neg W_r)$, I do not see what else they could be, and that is why I think that conditional probabilities are the reason why we reject $\neg W_r \rightarrow W_a$ and accept $\neg W_r \rightarrow W_c$.

There may be another explanation. One may be inclined to think about $\neg W_r \rightarrow W_a$ and $\neg W_r \rightarrow W_c$ not in terms of probability but in terms of truth. If 'just before' the election the great majority would vote Reagan, then they would do the same on the election day, assuming the principle that many people would not change their minds in a short period. In that case W_r is not just highly probable, but true. The mentioned principle tells us also that many more people will vote Carter than Anderson. So, one can say that $\neg W_r \rightarrow W_c$ is true, not just probable, and that $\neg W_r \rightarrow W_a$ is false.

Although I think that most people think about McGee's example in terms of probability, I see nothing that can be said against the approach from the last paragraph. And since both lead to McGee's paradox, I'll have to take them both into account. But please note that the two approaches do not presuppose different understandings of conditionals or probabilities of conditionals. They just presuppose different interpretation of the election case described by McGee, that is, they assume different premises (for example, in the first case W_r is highly probable but $P(W_r) < 1$; in the second case $P(W_r) = 1$

⁸ "Probabilities of Conditionals and Conditional Probability" in *Philosophical Review*, 1976. pp581-9

since W_r is true, etc.). For our purposes here it is of little importance which approach better depicts the historical situation as it really was. From the logical point of view, all that matters is that **both** describe a possible situation which leads to the paradox.

McGee's Solution

Let us see now how McGee solves the problem he discovered. After exposing his counterexamples, he argues that the law of exportation ($A \wedge B \rightarrow C \vdash A \rightarrow .B \rightarrow C$) should be valid for the conditionals we are now dealing with, because, as he says, it is an important feature of English usage. Since the converse of exportation (the law of importation: $A \rightarrow .B \rightarrow C \vdash A \wedge B \rightarrow C$) is not problematic, what McGee asserts is that in natural language we do not distinguish between the sentences of the form $A \wedge B \rightarrow C$ and $A \rightarrow .B \rightarrow C$. Why is he talking about this becomes clear from the last part of his paper where he proposes the solution.

His first solution would be this: do the semantics in such a way that $A \wedge B \rightarrow C$ and $A \rightarrow .B \rightarrow C$ have the same truth conditions, essentially the same as the formula $A \wedge B \rightarrow C$ has in Stalnaker's semantics.

The second one (McGee thinks that there is no important difference between the two) goes like this: when you say something of the form 'if ... then if --- then ___' render it as $A \wedge B \rightarrow C$; more accurately, rule out of the syntax the formulae of the form $A \rightarrow .B \rightarrow C$.

It seems that McGee is not himself quite satisfied with these solutions, because he is aware that they can be said to be ad hoc.⁹ But there is no doubt that they do remove the problem: if we replace $A \rightarrow .B \rightarrow C$ by $A \wedge B \rightarrow C$, we could not derive the contradiction from McGee's examples. I will try latter to explain why McGee could decide to resort to such radical solutions, but now let us see first what they mean.

McGee supports the opinion that the conditionals are to be found somewhere between the strict and the material implication, in the sense that the strict one entails the conditional, which in turn entails the material one. However, if we think that the rule of exportation also holds for such conditionals, then they are equivalent to the material implication (as McGee also proved, p 465-6). But this is no good, for many of us, and for McGee also. We don't want conditionals to be material implication. McGee needs exportation to get rid of his paradoxes, and in order to save the difference between conditionals and material implication he denies general validity of modus ponens for conditionals. More accurately, he puts the restriction on this rule so that it can be applied only to those conditionals whose consequent does not contain another conditional as a subformula.¹⁰ Although he did not give a formal system, he gave semantics for such conditionals. The easiest way to describe the semantics is by its relation to Stalnaker's. The function '*' reduces McGee's formulae to Stalnaker's. Stalnaker uses '>' as his symbol for conditionals, McGee uses '⇒'. '[' and ']' are Quine's 'quasi quotes'¹¹. ϕ^* and ϕ are logically equivalent:

⁹ op. cit. p 471

¹⁰ ibid p 469f

¹¹ W.V.O. Quine **Mathematical Logic** 1951. pp. 33-37.

$\phi^* = \phi$, if ϕ is an atomic formula

$\perp^* = \perp$ (\perp stands for a logically false sentence)

$\lceil \phi \vee \psi \rceil^* = \lceil \phi^* \vee \psi^* \rceil$

$\lceil \phi \wedge \psi \rceil^* = \lceil \phi^* \wedge \psi^* \rceil$

$\lceil \neg \phi \rceil^* = \lceil \neg \phi^* \rceil$

$\lceil \phi \Rightarrow \psi \rceil^* = \lceil \phi^* \supset \psi \rceil$ if ψ is in atomic sentence or \perp

$\lceil \phi \Rightarrow \psi \vee \theta \rceil^* = \lceil (\phi \Rightarrow \psi)^* \vee (\phi \Rightarrow \theta)^* \rceil$

$\lceil \phi \Rightarrow \psi \wedge \theta \rceil^* = \lceil (\phi \Rightarrow \psi)^* \wedge (\phi \Rightarrow \theta)^* \rceil$

$\lceil \phi \Rightarrow \neg \psi \rceil^* = \lceil (\phi \Rightarrow \perp)^* \vee \neg (\phi \Rightarrow \psi)^* \rceil$

$\lceil \phi \Rightarrow (\psi \Rightarrow \theta) \rceil^* = \lceil (\phi \wedge \psi) \Rightarrow \theta \rceil^*$

McGee's second solution leaves modus ponens alone, but it assumes that we have to change our informal rules of translating natural language sentences into logical formulae in such a way that when we say 'if A then if B then C' it should be rendered $A \wedge B \rightarrow C$. That way there is no need to put any restriction on mp.

McGee thinks that the two solutions are equal, so that it is not important which one we choose. It is true that both look equally violent, since both commit us to regard $A \wedge B \rightarrow C$ to be the logical structure of 'if A then if B then C' in every possible case. Also, McGee's logic greatly reduces our ability to derive a conditional as a conclusion of an inference. Let us see an example to illustrate this. The rule

$$8 \quad \frac{\begin{array}{l} \square A \\ \square(A \rightarrow B) \end{array}}{\square B}$$

holds in the classical modal logic. It can be derived by mp and the axiom

$$9 \quad \Box(A \rightarrow B) \rightarrow \Box A \rightarrow \Box B$$

But it cannot be derived in McGee's logic. Since he forbade mp for nested conditionals, he cannot derive $\Box A \rightarrow \Box B$ from the premises $\Box A$ and $\Box(A \rightarrow B)$ by 9, because the consequent of 9 is the conditional $(\Box A \rightarrow \Box B)$. This feature of McGee's logic – that it reduces our ability to derive a conditional as a conclusion of an inference – is often a disadvantage, because there are many cases when we do want to have a conditional as a conclusion. For example we want to derive $\Box A \rightarrow \Box B$ and 8 from 9 if the box ' \Box ' means 'necessity'. (And McGee surely would like the same, at least if A and B are atomic formulae.) It seems that in McGee's logic a conditional as a conclusion cannot be informative. We can get it as a trivial instantiation of an axiom or a theorem (e.g. law of identity) or we can derive it from hypotheses which already contain it. In fact, we cannot conclude a conditional if we don't know it in advance.

Still it seems that McGee's second approach has as an additional problem that its expressive power is lower. It provides no means for expressing the law of identity $(A \rightarrow A)$ when A is a conditional. If A is $B \rightarrow C$ then McGee cannot render this instantiation of the law of identity as $(B \rightarrow C) \rightarrow (B \rightarrow C)$, but as $(B \rightarrow C) \wedge B \rightarrow C$. There is nothing wrong here from the point of view of truth values – the law of identity can never have a false instantiation in McGee's logic – but it is unnatural not to distinguish the logical structure of 'if (if B then C) then (if B then C)' and 'if (if B then C) and B then C'.

Other Solutions

Soon after McGee published his paper, there appeared critiques defending mp. The first of them were short and they showed in a few words the 'mistake' in McGee's argument. Every subsequent critic was dissatisfied with his predecessors, and the papers on the subject became longer, showing more respect to McGee's puzzle, at least by admitting that it requires more space for the problem to be solved. I will either mention or comment upon most of them.

Katz & Lowe

Bernard Katz¹² focused his critique on arguing that exportation is not an acceptable rule of inference for the conditional we are dealing with here. He also argued that "...McGee's examples rely for their plausibility on the law of exportation"¹³, meaning by this that the only reason why we are inclined to find the first premise Mc1 true is the plausibility of exportation. This means that the way we come to believe Mc1 would be the following. First, we believe:

if a Republican will win and it's not Reagan, then it will be Anderson.

what is obviously true and nobody denies that. Then we conclude Mc1, and this, Katz says, is the mistake.

¹² "On a Supposed Counterexample to Modus Ponens" *The Journal of Philosophy* 1999. 404-15

¹³ *ibid* p. 404-5

The weak point of this argumentation, no matter whether exportation is acceptable or not, is that it seems that we never use this rule of inference in McGee's example.

Lowe¹⁴, I think, is well aware of that. He objects that McGee's argument has not the form

$$\frac{A \quad A \rightarrow (B \rightarrow C)}{B \rightarrow C} \quad \text{but rather} \quad \frac{A \quad A \rightarrow (B \supset C)}{B \rightarrow C}$$

where ' \supset ' is material implication. The second rule is not, of course, modus ponens and is not valid. Lowe's argument goes like this. Since we believe that a Republican will win (i.e. $\exists x(Wx \wedge Rx)$), we believe:

Reagan will win or Anderson will win (i.e. $Wr \vee Wa$)

which is equivalent to the material implication whose antecedent is the negation of the first disjunct and whose consequent is the second disjunct:

If it's not Reagan who wins, it will be Anderson ($\neg Wr \supset Wa$)

Hence if we accept, as Lowe suggests, that McGee's first premise is equivalent to 'If a Republican will win, then Reagan will win or Anderson will win', then this premise is equivalent to $\exists x(Wx \wedge Rx) \rightarrow (\neg Wr \supset Wa)$ and is not equivalent to $\exists x(Wx \wedge Rx) \rightarrow (\neg Wr \rightarrow Wa)$, and the counterexample fails.

Lowe is right that the proposition 'If it's not Reagan who wins, it will be Anderson' does not follow directly from 'A Republican will win'. Rather it is the case that from the latter it follows first that 'Reagan will win or Anderson will win', and from this we conclude finally 'If it's not Reagan who wins, it will be Anderson'.

This also means that when we infer the first premise Mc1 from McGee's story about elections, we never use exportation. We know from the story who is Republican

¹⁴ E.J. Lowe "Not a counterexample to modus ponens", *Analysis* 47, 1987. pp. 44-7

and we know the rule that there can be only one winner and that there must be one. So we first believe $\exists x(Wx \wedge Rx) \rightarrow (Wr \vee Wa)$ and then we infer Mc1, that is $\exists x(Wx \wedge Rx) \rightarrow (\neg Wr \rightarrow Wa)$. That is, I would say, what is going on in our heads. Exportation would be redundant here and it would require a few more steps in reasoning, which actually do not happen.

Let us turn now to Lowe's point. He thinks he gave us sufficient and convincing reasons for interpreting the first premise as $A \rightarrow (B \supset C)$ and that it is up to McGee to justify his interpretation $A \rightarrow .B \rightarrow C$. He also considers two further questions: is it possible to interpret Mc1 as $A \rightarrow .B \rightarrow C$, and would it then be something reasonable to believe in? Lowe does not think that we can give the positive answer to the first question, but even if we can, the answer to the second one, he is certain, is negative. But those claims are given in his paper almost without explanation.

Note that Lowe assumes that the disjunction used here $(Wr \vee Wa)$ is truth-functional inclusive disjunction of the classical propositional calculus. Although he says nothing about it, it is clear from the fact that the disjunction is equivalent to the material implication $\neg Wr \supset Wa$. Obviously Lowe does not find the question about the nature of disjunction an issue. But he is wrong here. Without an analysis of the 'or' in the above statement his argumentation is not conclusive. McGee could use the very same argument to claim that the disjunction is not truth-functional – because Mc1 has the form $A \rightarrow .B \rightarrow C$ and not $A \rightarrow .B \supset C$. And both arguments would have the same strength.

Three Belgrade Objections

Next I will mention three objections to McGee's argument which were given during the discussions I had with my colleagues in Belgrade. None of them, as it happens, especially dealt with logic, so they made their arguments solely by the natural language intuition. In a sense, this may be sometimes their advantage. Too much dealing with the logic we learned from Frege and Russell might make us blind to some subtleties of natural language. In fact, all these objections point to some conflict between natural language intuition and classical logic.

I will refer to the three objections as O_1 - O_3 . The first two will remain unsolved here. I will just try to show that the opposite view is also well supported, but no decision will be made. They will be discussed again later in the text.

O_1 . The first objection raises some doubts about the second premise (a Republican will win). We believe that Reagan will win, and that's why we also believe that a Republican will win. In a sense, 'a Republican will win' can be *misleading* in this context, or *deceiving*, and *hence false*. The quoted sentence is equivocal. If we understand it as 'a certain Republican will win', then it can safely be inferred from 'Reagan will win'. But it seems not to be the case if we understand it as 'some (of the) Republican(s) will win' or 'a Republican (no matter which one) will win', assuming that every Republican can win.

Let us take another example. Imagine a competition in running 100 meters. Half of the runners are the best white runners in the world; the others are the best black runners. One does not need to know their names to believe 'a black runner will win'.

Now imagine that among the black runners there is the best black runner in the world, but the others are some elderly black people, while the white runners remain the same. It seems that in this situation we normally would not say 'a black runner will win', although nobody doubts that the best black runner will win.

On the other hand, the answer to the question 'will a Republican win?' seems to be trivial: yes, because Reagan will win. That is what the axiom of the predicate calculus tells us: $A \vdash \rightarrow \exists x A x$. If $\exists x A x$ means 'a Republican will win', then it follows from 'Reagan is a Republican and Reagan will win'. And this is in accordance not just with the classical calculus but with the natural language intuition also. Similarly, in both versions of the example with the runners, it seems that faced with the question "Will a black runner win?" we would have to answer "Yes".

Thus it seems that the first objection is disposed of. But I think I gave a hint that somebody could still find that something might be confusing here. I hope this will be more clear later.

O₂. The second objection concerns the first premise (if a Republican will win, then if it's not Reagan, it's Anderson). We believe that a Republican will win (which is the antecedent of Mc1) and we believe that it is false that if it's not Reagan who wins, it will be Anderson (the consequent of Mc1). The two beliefs seem to be perfectly compatible. But Mc1 tells us that if the first is true, the second is not. So Mc1 is false. This can be expressed more clearly like this: Mc1 is false because it has true antecedent and false consequent.

Now I have to make a digression to avoid some possible misunderstandings. One may wonder why don't I stop here: once we noticed that $Mc1$ has true antecedent and false consequent, there is no need for further discussion. This surely means that $Mc1$ is false and hence the McGee paradox is disposed of. But such reasoning would not be good.

First, it leaves us with no explanation of why $Mc1$ seems *trivially* true, and why we use such propositions so often. A satisfactory solution of the paradox should be able to explain this, if what I said at the very beginning of this paper is correct.

Second, McGee could complain: "Of course it has true antecedent and false consequent. That's exactly what I said when I argued that mp is not generally valid." Would he be right to say so? Absolutely, I think. The only reason for rejecting a rule of inference is that it is not truth preserving. In view of the form of modus ponens, the only possible case when it can fail to be truth preserving is when there is a true conditional with a true antecedent and a false consequent. So denying the general validity of mp and claiming that a conditional can be true with a true antecedent and a false consequent are not two separate things. If we don't immediately reject McGee's argument because he denies mp , we cannot reject it now just because we noticed that $Mc1$ has true antecedent and false consequent. For, again, one cannot reject mp without accepting that a conditional can be true with true antecedent and false consequent. I am not sure why McGee keeps quiet about this. But I believe that he is well aware of what he's doing, and that there is nothing in this passage he does not already know.

I'd like to make more remarks that I hope will cast some more light both on this issue and on the situation McGee involved himself in. It is not easy when one has almost

two and a half millenniums of tradition against him. Most of the critiques of McGee's paper rely more on that tradition than on their own arguments.

Hereafter I will refer to the claim that no conditional can be true with a true antecedent and a false consequent as 'the Thesis'.

I will recall here the two approaches in determining the meaning of a connective, which were mentioned on page 3 (Hilbert's and Euclid's way). Can we, following either of these approaches, state that the Thesis is beyond doubt? Or, what is the same, is it beyond doubt that the Thesis is part of the meaning of ' \rightarrow '?

If we adopt Hilbert's holistic approach, and if we determine the meaning of ' \rightarrow ' by what we think are its formal properties, then the Thesis will be the consequence of these formal properties only if they include mp. In the case of material implication such questions were not the problem, because we know exactly what material implication is. But in the case of ' \rightarrow ', after there appeared the first theories of conditionals in the middle of this century, almost everything we can say about conditionals is an issue. We have to proceed empirically, by investigation of our use of natural-language conditionals, in order to determine their meaning. (And it turned out to be too hard a task for two generations of philosophers.) From the standpoint of natural-language usage, McGee could refuse to accept the Thesis, saying that the determination of ' \rightarrow ' described above assumes mp, which, considering his example, is no good.

If we try to determine ' \rightarrow ' in the 'Euclidian' way by defining first its truth conditions, then the Thesis could be part of these conditions, and mp an immediate consequence. McGee could complain again that such conditions are no good, because

they imply mp. Or he could directly reject the Thesis saying that he gave us an example of a true conditional with a true antecedent and a false consequent (that is, Mc1).

So in any case McGee would not be too worried if we point to the fact that he does not obey the Thesis. Even if he is wrong, something more is needed to convince him to change his view.

All of the main arguments Katz proposed against McGee's counterexample rely on the Thesis¹⁵. I will continue this digression with a few comments on his paper. As I noticed above (pp. 13-14), he first argued that the law of exportation is the only reason to accept Mc1. He also reminds us that exportation and mp cannot go together (if we don't want our conditionals to be material implication) and that McGee accepts both exportation and importation. Then he says (in all citations from Katz's paper italics are mine):

We can show, however, that the law of exportation and the law of importation cannot both apply to strong conditionals; for in the presence of several *standard assumptions* about conditionals, the law of importation is *inconsistent* with the view that Mc1 and Mc2 are both true and McC is false. One assumption is that an indicative conditional of the form [if ϕ , then ψ] is logically true just in case ϕ logically implies ψ .¹⁶

Katz then points to an instantiation of the law of identity ("every sentence logically implies itself"):

If (if ϕ then ψ), then (if ϕ then ψ)

¹⁵ Katz op.cit..

¹⁶ Katz 1999, p.409. 'Strong conditional' means 'conditional, as I use it here. Signs 'Mc1' etc, are different in Katz' paper.

By importation we can infer:

(*) If (if ϕ then ψ) and ϕ , then ψ

from which Katz concludes:

It should be clear, however, that, if the antecedent of (*) logically implies the consequent of (*), then modus ponens is a valid rule of inference.¹⁷

An instantiation of (*) is:

(**) If $\lceil \text{Mc1} \rceil$ and $\lceil \text{Mc2} \rceil$ then $\lceil \text{McC} \rceil$.

Since (**) is logically true and since a conditional is logically true only if its antecedent logically implies its consequent, it follows that it is not possible for the antecedent of (**) ... to be true and the consequent of (**) ... to be false.¹⁸

...

This piece of reasoning shows that one cannot *consistently* hold (as McGee does) both that importation is valid and that the inference from Mc1 and Mc2 to McC is not. It also shows that, if importation is valid for strong conditionals, then exportation is not.¹⁹

This argument, so far, is not enough to decide between the exportation and the importation, and Katz is aware of this. He then argues that we should reject exportation, because

If (either ϕ or ψ) and not- ϕ , then ψ

which is certainly true, yields by exportation

¹⁷ *ibid.* p.410. This reminds very much of Lewis Carroll's "What the Tortoise Said to Achilles", cf.

http://www.mathacademy.com/platonic_realms/encyclop/articles/carroll.html

¹⁸ *ibid.*

¹⁹ *ibid.*

(***) If either ϕ or ψ , then if not- ϕ , then ψ .

From the latter sentence, by appealing to the Thesis, Katz concludes that the consequence of exportation is that whenever a disjunction ' ϕ or ψ ' is true, the conditional 'if not- ϕ , then ψ ' is also true. But this reduces the conditionals to material implication, so we should reject exportation.²⁰

I tried to find a sense, other than literal, in which Katz thinks that one cannot 'consistently' hold that both importation and exportation are valid, but I couldn't. Later²¹, he said that there is a 'decisive' reason for thinking that McI is false and "...it is an essential feature of any conditional, indicative or otherwise, that it is false if it has a true antecedent and false consequent." So he means it literally that McGee is inconsistent.

Katz's claim that McGee is not consistent can be interpreted in three ways.

First, he might have thought that McGee's logic is inconsistent, which is not likely to be the case, since it is obviously not true.

Second, Katz's point might be that McGee failed to make a logic where mp is not valid, and

Third, if McGee did succeed in constructing such a logic, then the logic is no good, because it is not in accordance with some 'standard assumptions about conditionals'.

Before commenting upon this, I'd like to say again that the claim that mp is valid and the Thesis are immediate consequences of each other, and are equivalent. Thus if one

²⁰ *ibid.* p.410-11

²¹ *ibid.* p.412.

rejects one of them, he must reject the other also. And if somebody wants to refute McGee's claim that mp is not valid, he cannot do so only using the Thesis.

Nevertheless, one might still find that our intuition in favour of the Thesis might be stronger than our intuition in favour of mp, and use the former to defend mp. McGee, I believe, would not be convinced by such an argument. I tried to show this when I applied 'Hilbert's' and 'Euclid's' way to conditionals (page 19 above). The point was that McGee could directly attack the Thesis, as well as mp, using the same example.

If the second interpretation of Katz's argument is correct, then he might have had something like this in his mind. As Katz showed,

(**) If $\lceil \text{Mc1} \rceil$ and $\lceil \text{Mc2} \rceil$ then $\lceil \text{McC} \rceil$.

is logically true. Then, by appealing to the Thesis, we can conclude that, assuming Mc1 and Mc2 are true, it is not possible that McC is not. Or, more generally, since

(*) If (if ϕ then ψ) and ϕ , then ψ

is logically true, then mp is valid, because it is not possible that the antecedent of (*) is true and the consequent is not. But, as we saw, appealing to the Thesis is not an argument against McGee, because he made a semantics where there are true conditionals with a true antecedent and a false consequent. His logic provides no means for ψ to be derived if it is itself a conditional. So in his logic (*) can be (and it is indeed) true, but there is no way mp can be derived from it²². Katz makes the same mistake in arguing that exportation reduces conditionals to material implication. From (***) (If either ϕ or ψ , then if not- ϕ ,

²² We discussed this feature of McGee's logic on pages 11-12 where we showed that the rule 8 (from $\vdash \Box A$ and $\vdash \Box(A \rightarrow B)$ to infer $\vdash \Box B$) cannot be derived even if 9 ($\Box(A \rightarrow B) \rightarrow \Box A \rightarrow \Box B$) holds.

then ψ) and ' ϕ or ψ ' one cannot infer 'if not- ϕ , then ψ ' without the usage of mp which is forbidden by McGee's semantics.

The third interpretation of Katz's argumentation can be, I think, expressed in its strongest form like this: McGee made a logic where it is possible to find a proposition B which is a logical consequence of a proposition A (or A logically implies B), but A is true and B is not. Proof: an indicative conditional of the form $A \rightarrow B$ is *logically* true just in case A logically implies B (i.e. if $\vdash A \rightarrow B$ then B is a logical consequence of A; the latter is usually rendered as $A \vdash B$). The passage from $\vdash A \rightarrow B$ to $A \vdash B$ is called the converse of the deduction theorem). (***) is an example of a conditional which is logically true in McGee's logic. Hence $\lceil \text{McC} \rceil$ is a logical consequence of $\lceil \text{Mc1} \rceil$ and $\lceil \text{Mc2} \rceil$. But $\lceil \text{Mc1} \rceil$ and $\lceil \text{Mc2} \rceil$ is true and $\lceil \text{McC} \rceil$ false in McGee's logic. Q.E.D.

What could McGee say to that? The very notion of logical consequence assumes that it is not possible that a proposition is true but its logical consequence is not. Let us suppose that McGee would not go so far to deny this. He might simply say that it is ' \rightarrow ' that he understands in different way, not the logical consequence. He could say that it is not true that an indicative conditional of the form $A \rightarrow B$ is logically true just in case A logically implies B. (***) would be an example. If it follows from the notion of logical consequence that it is not possible that a proposition is true but its logical consequence is not, and if we represent this notion by ' \vdash ', then the converse of the deduction theorem is a metatheorem claiming that mp holds for ' \rightarrow '. McGee obviously could not accept a metatheorem claiming that mp is valid if he made a system where mp is not valid.

Thus we saw that McGee is not obliged to accept the third interpretation in its strongest form. In other words, he is not obliged to violate the usual notion of logical

consequence. He is just obliged to deny that "an indicative conditional of the form $A \rightarrow B$ is logically true just in case A logically implies B ", that is, to deny the converse of the deduction theorem. And he really has to deny this theorem, since it claims that mp is valid. But we cannot use the converse of the deduction theorem, any more than we can use the Thesis, against McGee's logic. Modus ponens, the Thesis and the converse of the deduction theorem pass or fail together. Once we accept one of the three, the other two *trivially* follow. And once we reject one of the three, the other two are automatically invalid. On the formal level, using one of the three principles to defend one of the two remaining would be the case of a circular reasoning.

Still, this does not necessarily mean that our intuitions supporting the three principles have the same strength. But, again, this does not work against McGee. He can directly attack the converse of the deduction theorem also, using (***) as an example where $\vdash A \rightarrow B$ holds but $A \nvdash B$ does not.

From Katz' paper I couldn't conclude whether the second or the third interpretation is correct. Sometimes it looks like he thinks the Thesis to be his main argument, sometimes it seems rather that the third interpretation is right. Anyway, neither of them helps us to remove McGee's problem.

Let us go back to the second objection O_2 . It says that, according to the Thesis, Mc1 is false. We saw that *mere* appealing to the thesis is not a conclusive argument against McGee. But it does not mean that O_2 has no strength at all. Its strength stems from our intuition in favour of mp and the Thesis. And it points to the way we can cast some doubt on Mc1. At first sight it seems that Mc1 is trivially true. Also, it seems that Mc2 logically follows from the facts we know about the election case. Thus, at first sight, it

seems that both premises Mc1 and Mc2 are undoubtedly true. If this were the case, McGee would have a very powerful, and even conclusive argument against mp. But O₂ says that the truth of Mc1 already presupposes that mp (and the Thesis) does not hold. Thus the strength of McGee's argument stems from the fact that Mc1 appears to be trivially true.

Although our intuition in favour of mp and the Thesis is very strong, one could say something like this in favour of McGee's opinion. If a Republican will win, then if it's not Reagan, it will be Anderson. Who else?

I will put this in the form of an argument. It seems that one (and only one) of the following three propositions must be true:

If a Republican will win, then if it's not Reagan, it will be Reagan.

If a Republican will win, then if it's not Reagan, it will be Carter.

If a Republican will win, then if it's not Reagan, it will be Anderson.

The first two are nonsense. So the third – Mc1 – must be true.

Thus we are so far left with the puzzle, rather than solution: both O₂ and its opposite find support in our natural language intuition.

O₃. I turn now to the third objection that I promised several pages ago. It goes like this: we have the premise 'a Republican will win'. Hence Carter is ruled out of consideration, and we cannot any more believe that it is a non-Republican Carter who will win if it's not Reagan. So, with this premise, the conclusion McC is o.k. and there is no problem at all.

D.E. Over gave a similar argument against McGee's counterexample²³, using more developed terminology. He makes a distinction between beliefs and assumptions. The relevant difference between the two notions is that 'a belief *may* be suspended or set aside when an assumption by its very nature cannot be'. His critique is based on the following arguments:

The first important point to notice is that McGee speaks of what we *believe* in the above quotation, and not of what we *assume*. In fact, he never speaks of modus ponens as the rule which allows us to infer a conclusion ψ from *assumptions* of the form ϕ and $\phi \rightarrow \psi$... and yet this is how the rule is standardly stated in systems of natural deduction.²⁴

Let us return to McC above, and imagine that we are presented with this conditional on its own and asked whether we should believe it. To answer this question, we suppose that the antecedent of McC is correct and, to avoid inconsistency, suspend some of our present beliefs, including, of course, our belief that Reagan will win the election. But by suspending that belief, we see that we no longer have any reason to believe McC above, that a Republican will win (since we have presumably based this belief on our belief that Reagan will win). And as a result of these changes and what we know about the polls, we see that we should not believe McC ...

The matter is very different in a context in which we infer McC from the assumptions Mc1 and Mc2 by modus ponens. Here we cannot 'suspend' Mc2 – in this context Mc2 is an assumption, and the rule modus ponens does not permit us to discharge its assumptions ... And so McC does follow given the assumptions Mc1 and Mc2.²⁵

²³ D. E. Over "Assumptions and the Supposed Counterexamples to Modus Ponens" *Analysis* 1987 pp. 142-6

²⁴ *ibid.* p.143

²⁵ *ibid.* p. 143-4

However, Over's objection does not help us to remove the problem. McGee could easily give a reformulation of his example, which would bring back the same puzzle. He could define a set X of propositions depicting the situation about the elections, and say that it is this set that we are starting with, not the premises $Mc1$ and $Mc2$. Both the negation of the conclusion and the premises follow directly from that set, independently one of another (that is, it appears that both ($Mc1$ and $Mc2$) and $\neg McC$ follow from X). So we still have a contradiction, no matter whether X is a set of beliefs or assumptions.

Formalization of the argument

I will now determine the set X , and after that I will derive the contradiction, in order to see what logical rules must be used for that. Let Wx stand for 'x wins', Dx for 'x is a Democrat', Rx for 'x is a Republican'. r, c, a are Reagan, Carter, Anderson. $P(A)$ is the probability of the proposition A . p is a probability big enough so that it is rational to believe A if $P(A) \geq p$. $p < 1$. $\square A$ means that it is rational to believe A . X contains:

- | | | | |
|-----|---|-------------|--|
| i | $\exists_1 x Wx$ | or instead: | $Wr \rightarrow \neg Wc \wedge \neg Wa.$ |
| | | | $Wc \rightarrow \neg Wr \wedge \neg Wa$ |
| | | | $Wa \rightarrow \neg Wr \wedge \neg Wc$ |
| ii | $\forall x (Rx \leftrightarrow \neg Dx)$ | or instead | $Rr \leftrightarrow \neg Dr$ |
| | | | $Dc \leftrightarrow \neg Rc$ |
| | | | $Ra \leftrightarrow \neg Da$ |
| iii | $\forall x (Rx \leftrightarrow (x=r \vee x=a))$ | or instead | Rr, Ra |
| iv | $\forall x (Dx \leftrightarrow x=c)$ | or instead | Dc |
| v | $P(Wr) = p_r \in [p, 1)$ | | |
| vi | $P(Wc) = p_c \in [p(1 - p_r), 1 - p_r)$ | | |

- vii $P(Wa) = 1 - P(Wr) - P(Wc)$
viii $P(\neg Wr \rightarrow Wc) = P(Wc / \neg Wr) \in [p, 1]$
ix It is rational to believe the facts from the hypotheses i-iv

101	$\Box(\neg Wr \rightarrow Wc)$	from (viii) by a supposed def. of rational belief
102	$\Box Wr$	from (v) by a supposed definition of rational belief
103	$\Box Rr$	from (iii) and (ix)
104	$\Box Rr \wedge \Box Wr$	from 102, 103 by $A, B \vdash A \wedge B$
105	$\Box(Rr \wedge Wr)$	from 104 by $\Box A \wedge \Box B \leftrightarrow \Box(A \wedge B)$
106	$\Box(Rr \wedge Wr \rightarrow \exists x(Rx \wedge Wx))$	instantiation of the axiom $A_t \rightarrow \exists x A_x$, necessitation
107	$\Box(Rr \wedge Wr) \rightarrow \Box \exists x(Rx \wedge Wx)$	from 106 by $\Box(A \rightarrow B) \rightarrow \Box A \rightarrow \Box B$
108	$\Box \exists x(Rx \wedge Wx)$	from 105 and 107 by MP (this is Mc2)
109	$\Box(\exists x(Rx \wedge Wx) \leftrightarrow \exists x((x=r \vee x=a) \wedge Wx))$	from iii and ix. substitution of equivalent formulae (SEF)
110	$\Box(\exists x((x=r \vee x=a) \wedge Wx) \leftrightarrow \exists x((x=r \wedge Wx) \vee (x=a \wedge Wx)))$	distribution of \wedge over \vee , necessitation
111	$\Box(\exists x((x=r \wedge Wx) \vee (x=a \wedge Wx)) \leftrightarrow Wr \vee Wa)$	from 110 by the theorem $\exists x(x=a \wedge Ax) \leftrightarrow Aa$
112	$\Box(\exists x(Rx \wedge Wx) \leftrightarrow Wr \vee Wa)$	from 109 -111 by SEF
113	$\Box(Wr \vee Wa \rightarrow \neg Wr \rightarrow Wa)$	necessitation of $A \vee B \rightarrow \neg B \rightarrow A$
114	$\Box(\exists x(Rx \wedge Wx) \rightarrow \neg Wr \rightarrow Wa)$	from 112 and 113 by SEF (this is Mc1)
115	$\Box(\neg Wr \rightarrow Wa)$	from 108 and 114 by $\Box(A \rightarrow B) \rightarrow \Box A \rightarrow \Box B$ and MP
116	$\Box(Wa \rightarrow \neg Wc)$	from i and ix by $A \rightarrow B \wedge C \rightarrow (A \rightarrow B) \wedge (A \rightarrow C)$, $A \wedge B \rightarrow B$
117	$\Box(\neg Wr \rightarrow \neg Wc)$	from 115 and 116 by transitivity of implication
118	$\Box \neg(\neg Wr \rightarrow Wc)$	from 117 by $A \rightarrow B \rightarrow \neg(A \rightarrow \neg B)$ and $A \leftrightarrow \neg \neg A$
119	$\neg \Box(\neg Wr \rightarrow Wc)$	from 118 $\Box A \rightarrow \neg \Box \neg A$

119 is a negation of 101.

As was mentioned on page 8 above, we can give two interpretations of the election case, both of which lead to the paradox. The first one, presented and formalized above, thinks about McGee's example in terms of probability; the second one, in terms of truth. According to the first one, the problem is that it turns out that it is both rational and not rational to believe $\neg W_r \rightarrow W_c$ (or $\neg W_r \rightarrow W_a$). According to the second one, it turns out that both $\neg W_r \rightarrow W_c$ and its negation are true. Also, it was mentioned that the two approaches do not presuppose different understandings of conditionals or probabilities of conditionals. They just presuppose different premises. Instead of the premises stating the probabilities (v-viii) and ix about the rational belief, the new set of premises (call it X_1) would contain the following: three statements about the percentage of potential voters for each candidate; statement that the results of the election will be the same as that percentage; and the statement that the candidate with the highest percent of votes is the winner. The derivation from X_1 would not differ from the sequence 101-119, except that the modal operator and some redundant steps would be dropped. For example, the without the operator 104 ($\Box R_r \wedge \Box W_r$) and 105 $\Box(R_r \wedge W_r)$ would become the same $R_r \wedge W_r$ – so we would not need the step from 104 to 105. Since the new derivation would differ only in consisting fewer steps, I will not bother the reader with a new formalization for X_1 .

Modal principles

I will now list all the theorems and rules used above and comment upon them.

First the modal ones.

201 $\Box A \wedge \Box B \leftrightarrow \Box (A \wedge B)$

202 $\Box (A \rightarrow B) \rightarrow \Box A \rightarrow \Box B$

203 $\Box A \rightarrow \Box \neg A$

204 necessitation

203 is obviously a characteristic of rational belief.

204 says that it is rational to believe theorems of logic. It is also in accordance with the notion of rational belief.

The lottery paradox casts some doubt on 201. For each ticket holder, it is rational to believe that he will lose. But it is not rational to believe that all ticket holders will lose. Hence, there is a long conjunction such that each conjunct is rationally believed but the conjunction is not rationally believed.

As I will repeat few more times in this paper, if we find a counterexample to a rule of inference or a theorem which is strongly supported by our intuition and which we use often in natural language (e.g. 201, transitivity of implication, contraposition and the similar), this does not mean that the rule or the theorem usually or never holds. It is more probable that it usually does hold, except in some rare occasions. Otherwise very often what we speak would be nonsense.

The lottery paradox points to the failure of Barcan Formula (BF) ($\forall x \Box Ax \rightarrow \Box \forall x Ax$) in the contexts where the box represents rational belief. $\forall x Ax$ can be represented as the conjunction $Aa_1 \wedge Aa_2 \wedge \dots \wedge Aa_n$ where $a_1, a_2 \dots a_n$ are all the individuals from the domain. Thus BF can be replaced by $\Box Aa_1 \wedge \Box Aa_2 \wedge \dots \wedge \Box Aa_n \rightarrow \Box (Aa_1 \wedge Aa_2 \wedge \dots \wedge Aa_n)$. In the above formalisation the formula $\Box A \wedge \Box B \leftrightarrow \Box (A \wedge B)$ is used to infer the belief that Reagan is a Republican *and* that he will win from the two

beliefs that he is a Republican and that he will win. Obviously the inference has nothing to do with BF. Also the probability of the conjunction is not lower than the probability of the less probable conjunct. The inference is trivial, and the logic of belief where it is not valid would not be good.

202 seems to be a more serious problem. Fred Dretske:²⁶

You take your son to the zoo, see several zebras, and, when questioned by your son, tell him they are zebras. Do you know they are zebras? Well, most of us would have little hesitation in saying that we did know this. We know what zebras look like, and, besides, this is a city zoo and the animals are in a pen clearly marked "Zebras". Yet, something's being a zebra implies that it is not a mule, and, in particular, not a mule cleverly disguised by the zoo authorities to look like a zebra. Do you know that these animals are not mules cleverly disguised by the zoo authorities to look like zebras? If you are tempted to say "Yes" to this question, think a moment about what reasons you have, what evidence you can produce in favour of this claim. The evidence you *had* for thinking them zebras has been effectively neutralized, since it does not count toward their *not* being mules cleverly disguised to look like zebras.

Dretske's argument is meant to point to the failure of 202. If we use the standard interpretation of propositional constants as true or false, the proposition 'I know A' would be rendered as $\Box A$. Let A be 'a is a zebra' and B 'a is not a mule cleverly disguised to look like zebra'. The argument has the form

$$\begin{array}{l}
 \Box A \\
 205 \quad \underline{\Box(A \rightarrow B)} \\
 \hline
 \Box B
 \end{array}$$

²⁶ Dretske, "Epistemic Operators", *Journal of philosophy* vol.LXVII, no. 24. 1970. pp. 1007-1023.

By 202 this argument is reduced to an instantiation of modus ponens²⁷:

$$206 \quad \begin{array}{l} \neg A \\ \neg A \rightarrow \neg B \\ \hline \neg B \end{array}$$

If we use the non-standard interpretation of propositional constants, not as true-false but as 'I know' - 'I don't know', then the operator and 202 are redundant, and in that case Dretske's argument points to the failure of modus ponens. The very same argument is, of course, applied to rational belief.²⁸

Dretske's argument, if it is correct, shows that our justification for knowing (rationally believing) a proposition can fail to be sufficient justification for some of the logical consequences of that proposition. In other words, there are cases where the set of propositions representing the justification for knowing (believing) A, can fail to be sufficient or can be irrelevant for knowing B, which is a logical consequence of A. For justifying the knowledge of B we need a *different* set of propositions.

Whether Dretske has a valid point, whether such a notion of knowledge is acceptable and the similar questions can be left to epistemologists. Here it is enough to mention that 202 can be questioned²⁹. I'll just add that often we do need 202, because it is obvious that there are cases when we must know a logical consequence of a proposition in order to know it. For example, we cannot say that we know that this is zebra if we do not know that it is an animal.

²⁷ We saw on pages 11-12 that this reduction could not be derived in McGee' logic.

²⁸ This is usually called the problem of the closure of implication in epistemic contexts.

²⁹ The formula 202 is questioned for the first time, as far as I know, not in epistemic but in moral contexts. cf. Chisholm "Contrary-to Duty Imperatives and Deontic Logic", *Analysis* vol 24 1963, 33-36.

There is something different in McGee's example: it is not the case that we need a *different* set of propositions for evaluation of the rationality of our beliefs in the premises on the one hand, and the conclusion on the other. The very same set applies both to the premises and the conclusion (that is, the set X). This means one of the two following.

First, it can be that Dretske-like argument does not apply to McGee's example. In other words, using 202 is not problematic in the context of McGee's example.

Second, if it does apply, that is if it's 202 that makes the problem, then McGee's example points to a conclusion much more radical than Dretske's – it points to the case where the same set of propositions justifies both the knowledge (belief) of the premises and the knowledge (belief) of the *negation* of their logical consequence.

I argue for the first. If the second is correct, maybe it points not to the failure of 202, but rather to the unacceptability of the presupposed notion of knowledge and rational belief. Further, note that 201-4 belong to many *consistent* systems of modal logic, that is, they are mutually consistent. But we still have a contradiction. And it is not possible that modal formulae become inconsistent when put together with non-modal formulae. This means that on the formal level something is wrong with the non-modal formulae, not with modal ones. Finally, the same contradiction can be derived from X_1 without use of the modal operator. So the problem is not removed by rejecting 202 or some of 201-204. And it would be very unconvincing to say that in the case where we start with X the problem is 202, but in the case of X_1 the problem is something else.

These are my reasons to think that it is misleading to point to the modal principles to reject McGee's argument. The reasoning expressed by 201-4 in this case is not

problematic; moreover, it is trivial in this context, and, again, the logic of belief where it is invalid would not be in accordance with any sensible notion of rational belief.

These are also my grounds to reject the critique of McGee's argument proposed by Sinnott-Armstrong, Moor and Fogelin.³⁰ They summed up their critique like this:

First, modus ponens preserves truth, not grounds for believing or probabilities. A real counterexample would have to use modus ponens to go from true premises to a false conclusion. Second, an analogue of modus ponens for grounds or probabilities must not confuse good grounds or high probability for the premises separately with good grounds or high probability for the conjunction of the premises. Finally, the probability of a conditional must not be confused with a conditional probability. With these confusions removed, McGee's case against modus ponens loses its force. Modus ponens lives!³¹

The first 'confusion' points to the problem of 202. Reducing the problem raised by McGee to the preservation of rationality or probability is, as we saw, misleading. This is not so just because the modal principles used are not problematic in this context, but also because McGee's argument can be rendered without use of the operator, that is, not in terms of rationality or probability, but in terms of truth and falsity (X_1). The three authors failed to see this, because they think that ' \rightarrow ' is material implication.

Their second point is also not of importance here because we have equally good grounds to believe the premises separately and to believe their conjunction.

³⁰ W. Sinnott-Armstrong, J. Moor and R. Fogelin, "A Defense of Modus Ponens" *The Journal of Philosophy*, May 1986 pp. 296-300.

³¹ *ibid.* p.300

As for the third, be it true or false, it solves nothing, because we still have the same contradiction in the X_1 -case, where the problem of the probability of conditionals is avoided.

I will finish the discussion on 201-4 with the following remark. It may seem that rendering the McGee argument in modal logic is redundant because it makes the formalization more complicated than necessary. For example, instead of asserting that it is rational to believe or that it is probable that if it's not Reagan who wins it will be Carter, we can simply say that the conditional is true. We can do so in the X_1 -case, but not in the case of X where we have the premises stating the probabilities for each of the candidates to win. The conditional 'If it's not Reagan who wins it will be Carter' is not true in the X -case according to such theories as metalinguistic, Lewis's and Warmbrod's³² ones. As long as it is not *necessary in the given situation* that the consequent is true if the antecedent is, the conditional is false according to these theories. The metalinguistic ones say that a conditional $A \rightarrow C$ is true iff A , together with a relevant set of further true premises, logically implies C , by laws of logic and some further principles, e.g. laws of nature. This means that the connection between A plus further premises plus relevant principles on the one side, and B on the other, is *necessary*. Possible-worlds semantics say the same thing using another terminology. $A \rightarrow C$ is true at a world i iff C holds at all the worlds that differ minimally from i .³³ These are the worlds throughout which the 'set of further true premises' hold, and this is the connection between the metalinguistic and

³² D.Lewis, **Counterfactuals**. Ken Warmbrod, "Counterfactuals and Substitution of Equivalent Antecedents" *Journal of Philosophical Logic* 1981. pp. 267-89.

the possible-worlds semantics. According to the latter theories, if $A \rightarrow C$ is true, then C holds in *all* the worlds where A holds, which means that it is *necessary* (in a given situation) that C holds in *every* A -world of a certain kind. A -worlds of a 'certain kind' are those that are most similar to i (Lewis) or accessible from i (Warmbrod) and where the 'given situation' holds.

Now, if the probability of each candidate's chance to win is between 0 and 1, the connection between, for example, the propositions 'Reagan will not win' and 'Carter will win' is *not* necessary and hence 'If it is not Reagan who wins it will be Carter' is false. Among the relevant antecedent-worlds there must be some where it's Anderson who wins, since the probability for that is higher than 0. According to the metalinguistic pattern, the argument from antecedent plus relevant propositions to consequent cannot be valid, since the connection is not logical, but only highly probable.

This feature of the mentioned theories of conditionals is, I think, good. There must be a necessary connection between the antecedent and the consequent. Otherwise, the conditionals that we use in the ordinary language would be nonsense. This does not mean that conditionals must be necessary propositions. The modal status of conditionals depends on the modal status of the set of relevant propositions that make the connection between antecedent and consequent logical. For example, 'If the match m were struck, it would light' is contingently true (if it is true) because the propositions stating the facts that m is dry, well-made and the similar propositions that would belong to the set of relevant propositions, are contingent. 'If this were bigger, it would also be heavier' does

³³ Lewis call these worlds 'the most similar' to i , Warmbrod calls them 'accessible' from i . There are important difference between Warmbrod's and Lewis's semantics, but it is not

not require any other facts, but it could be considered contingent as well if the same status is ascribed to physical laws, otherwise it would be physically necessary. 'If you were taller than me. I would be shorter than you' relies only on logical laws, and doesn't require any supplementary facts, hence it is logically necessary.

Two kinds of disjunction and the solution

Now we can turn to non-modal formulae used in the above formalization of McGee's argument. It turns out to be useful to consider first the propositional formulae without predicate ones. The whole argument can be rendered in pure propositional calculus and the contradiction can also be derived. This indicates that it is here that we should focus our first attention. I will derive the contradiction again without predicate formulae. In order to make the argument easier to follow, I will still use the same predicates, but I will not use quantifiers. The set of premises is the same set X, since for every quantified formula that belongs to X I gave the propositional formulae that can replace it. The only new formula will be A, which is meant to replace $\exists x(Rx \wedge Wx)$. $A \stackrel{\text{def}}{=} W_r \vee W_a$, that is, A means that either Reagan or Anderson will win. So A stands for 'a Republican will win'. From X it follows:

301	$\Box(\neg W_r \rightarrow W_c)$	from (viii) by a supposed def. of rational belief
302	$\Box W_r$	from (v) by a supposed definition of rational belief
303	$\Box(W_r \vee W_a)$	from 302 by the necessitation of the theorem $A \rightarrow A \vee B, \Box(A \rightarrow B) \rightarrow \Box A \rightarrow \Box B$ and mp

necessary to explain them for my present point.

304	$\Box A$	from 303 by def. of A and SEF.
305	$\Box(Wr \vee Wa \rightarrow \neg Wr \rightarrow Wa)$	necessitation of $A \vee B \rightarrow \neg B \rightarrow A$
306	$\Box(A \rightarrow \neg Wr \rightarrow Wa)$	from 305 by def. of A and SEF.
307	$\Box(\neg Wr \rightarrow Wa)$	from 304 and 306 by $\Box(A \rightarrow B) \rightarrow \Box A \rightarrow \Box B$ and mp
308	$\Box(Wa \rightarrow \neg Wc)$	from i and ix, by $A \rightarrow B \wedge C \rightarrow (A \rightarrow B) \wedge (A \rightarrow C)$, $A \wedge B \rightarrow B$
309	$\Box(\neg Wr \rightarrow \neg Wc)$	from 307 and 308 by transitivity of implication
310	$\Box \neg(\neg Wr \rightarrow Wc)$	from 309 by $A \rightarrow B \rightarrow \neg(A \rightarrow \neg B)$ and $A \leftrightarrow \neg \neg A$
311	$\neg \Box(\neg Wr \rightarrow Wc)$	from 310 by $\Box A \rightarrow \neg \Box \neg A$ and $A \leftrightarrow \neg \neg A$

311 contradicts 301. I will list now the rules and theorems from the pure propositional calculus used in both formalizations.

The rules:

$$A, B \vdash A \wedge B$$

transitivity of implication

SEF

mp

The theorems:

$$A \leftrightarrow \neg \neg A$$

$$A \wedge B \rightarrow B$$

$$A \rightarrow B \wedge C \rightarrow (A \rightarrow B) \wedge (A \rightarrow C)$$

$$A \rightarrow B \rightarrow \neg(A \rightarrow \neg B)$$

$$(A \vee B) \wedge C \rightarrow (A \wedge C) \vee (B \wedge C)$$

$$401 \quad A \rightarrow A \vee B$$

$$402 \quad A \vee B \rightarrow \neg A \rightarrow B$$

I will comment just upon the last two, and I'll give a brief remark on transitivity.

For the others, I do not see what can be said against them.

For most philosophers and logicians transitivity is not an acceptable rule of inference for indicative and subjunctive conditionals, and it is no longer an open question for them. However, even if there are some counterexamples to transitivity, it is one of those rules strongly supported by our intuition and very common in ordinary language, so, as it was mentioned earlier, it is more likely that the rule (at least) usually does hold. The use of transitivity in the above formalizations is trivial and nobody can sensibly argue that it was misleading or wrong. I think the reader would agree that one could hardly say something less interesting about McGee's example than that it in fact points to the failure of transitivity.

By 401 and 402 we can derive the rule

$$\frac{A}{\neg A \rightarrow B}$$

or, if the transitivity of implication holds, the theorem

$$A \rightarrow \neg A \rightarrow B,$$

which reduces ' \rightarrow ' to material implication. Note that 402 does not hold in any 'normal' system (i.e. system where mp holds) of conditionals. The converse of 402 –

$\neg A \rightarrow B \rightarrow A \vee B$ – is trivial in all such systems, because this is in fact the Thesis:

$\neg A \rightarrow B \rightarrow \neg(A \wedge \neg B)$. 402 and its converse together make $\neg A \rightarrow B$ equivalent to $A \vee B$ and

hence to material implication $\neg A \supset B$ also. That is why 402 is false in all systems of conditionals except McGee's. Note that this means that Mc1 (306 in the 'propositional'

formalization) is also false in these systems, and that is the reason why the contradiction cannot be derived in them.

On the other hand, we can doubt whether it is good that 402 and Mc1 are false. The reasoning expressed by 402 is very often used in natural language, and Mc1 appears to be trivially true. This requires an explanation. Mere appealing to the Thesis, as we saw in the discussion of Katz's paper, solves nothing. The true solution would have to explain what it is in McGee's example that makes the confusion.

The fact that 402 is common and the apparent triviality of Mc1 might mean one of the following. First, natural language is obviously inconsistent. Second, ' \rightarrow ' is material implication. Third, ' \vee ' in 401 and in 402 are not the same. (There might be fourth objection that, since ' \rightarrow ' and ' \neg ' also appear in 401 and 402, something can be wrong about the use of these connectives. But it is hard to see what that 'something' could be, so I will not take this into account.)

The first means that the whole human community is highly irrational, which is very unconvincing. I explained my reasons to reject the second on pages 3-4.

The third is, I believe, the explanation of what's going on. In ordinary situations, disjunctions in natural language require a connection between the disjuncts. When we know that 'Johns owns a Ford', we do not say that 'Johns owns a Ford or Brown is in Barcelona', if we don't know where Brown is or even who he is. When we say: 'Johns owns a Ford or Brown is in Barcelona', our listener understands that we claim that there is a connection between the two facts (the disjuncts). In this non-truth-functional sense the disjunction 'Johns owns a Ford or Brown is in Barcelona' is false (and so is 401). But 402 – $A \vee B \rightarrow \neg A \rightarrow B$ – and the similar formula $A \vee B \rightarrow \neg B \rightarrow A$ are true, because the

consequents of the two formulae claim that there is a connection of a certain kind between A and B. On the other hand, if the 'or' is truth-functional, 401 is obviously true, and 402 false.

The other disjunction that I have on my mind is called 'intensional'. Let us see how Anderson and Belnap justify the introduction of the intensional disjunction. In their explanation of why they reject disjunctive syllogism and a comment upon an argument by C. I. Lewis, they said³⁴:

... in rejecting the principle of the disjunctive syllogism, we intend to restrict our rejection to the case in which the 'or' is taken truth functionally. In general and with respect to our ordinary reasoning this would not be the case: perhaps always when the principle is used in reasoning one has in mind an intensional meaning of 'or', where there is a relevance between the disjuncts. But for the intensional meaning of 'or', it seems clear that the analogues of $A \rightarrow A \vee B$ are invalid, since this would hold only if the simple truth of A were sufficient for the relevance of A to B: hence, there is a sense in which the real flaw in Lewis's argument is not a fallacy of relevance but rather a fallacy of ambiguity. The passage from b to d [use of 401 – $A \rightarrow A \vee B$] is valid only if the ' \vee ' is read truth functionally, while the passage from c to e [use of 402 – $A \vee B \rightarrow \neg A \rightarrow B$] is valid only if the ' \vee ' is taken intensionally.³⁵

³⁴ The Lewis argument discussed by Anderson and Belnap goes like this

a	$A \wedge \neg A$
b	A
c	$\neg A$
d	$A \vee B$
e	B

³⁵ Alan Ross Anderson and Nuel D. Belnap, **Entailment (The Logic of Relevance and Necessity)**, Princeton University Press 1975. p.165f

[It can be objected that] to deny the principle of the disjunctive syllogism surely goes too far: "from $\neg A$ and $A \vee B$ to infer B", for example, is surely valid. For one of the premises states that at least one of A and B is true, and since the other premise, $\neg A$, says that A isn't the true one, the true one must be B ... Our reply is to remark again that this argument commits a fallacy of ambiguity. There are indeed important senses of "or", "at least one" etc., for which the argument from not-A and A-or-B ["... to B" is obviously omitted] is perfectly valid, namely, senses in which there is a true relevance between A and B, for example, the sense in which "A-or-B" means precisely that $\neg A$ entails B. However, in *this* sense of "or", the inference from A to A-or-B is fallacious, and therefore this sense of "or" is not preserved in the truth functional constant translated by the same word. As Lewis himself argued in some early articles, there are *intensional* meanings of "or", "not both", "at least one is true", etc., as well as of "if ... then ---".³⁶

The following argument concerns subjunctive conditionals, but the same can be applied to indicative conditionals also:

The truth of A-or-B, with truth functional "or", is not a sufficient condition for the truth of "If it were not the case that A, then it would be the case that B". Example: It is true that either Napoleon was born in Corsica or else the number of the beast is perfect (with truth functional "or"); but it does not follow that had Napoleon *not* been born in Corsica, 666 would equal the sum of its factors. On the other hand the intensional varieties of "or" which *do* support the disjunctive syllogism *are* such as to support corresponding (possibly counterfactual) subjunctive conditionals. When one says "that it is either *Drosophila melanogaster* or *D. virilis*, I'm not sure which", and on finding that it wasn't *Drosophila melanogaster*, concludes that it was *D. virilis*, no fallacy is being committed. But this is precisely because "or" in this context *means* "if it isn't one, than it is the

³⁶ *ibid.* p. 166

other". ... it should be ... clear that it is not simply the truth functional "or" ... from the fact that a speaker [and a listener, we could add] would naturally feel that if what he said [heard] was true, then if it *hadn't* been *D. virilis*, it *would* have been *Drosophila melanogaster*. And in the sense of "or" involved [intensional] it does not follow from the fact that it is *D. virilis*, that it is either *Drosophila melanogaster* or *D. virilis*, – any more than it follows solely from the fact that it was *D. virilis*, that if it hadn't been, it would have been *Drosophila melanogaster*.³⁷

Anderson and Belnap defined the intensional disjunction in the following way:

$$A+B =_{\text{def}} \neg A \rightarrow B$$

The definition assumes that '+' is commutative, which is very reasonable to suppose, and it also assumes that the law of contraposition holds for '→'. Systems where contraposition does not hold, D. Lewis's for example, would have to modify the definition like this:

$$A+B =_{\text{def}} (\neg A \rightarrow B) \wedge (\neg B \rightarrow A)$$

In the context of McGee's example, at least so far as propositional formalization is concerned, this means that his argument commits the fallacy of equivocation. In order to be true, the premises Mc1 and Mc2 must assume different disjunctions. Mc1 ($W_r + W_a \rightarrow \neg W_r \rightarrow W_a$) assumes the intensional, disjunction, because $W_r \vee W_a \rightarrow \neg W_r \rightarrow W_a$ is false. Mc2 $W_r \vee W_a$ assumes the truth-functional disjunction, since $W_r + W_a$ is false. Both premises are trivial for the appropriate disjunction. And both disjunctions are pronounced the same in the ordinary language – that's why the mistake was easy to overlook. So we should, I believe, make sure not to mix the two disjunctions:

³⁷ *ibid.* p.176

Truth-functional

$$\vdash A \rightarrow A \vee B$$

$$\sim \vdash A \vee B \rightarrow \neg A \rightarrow B$$

$$\sim \vdash A \vee B \rightarrow \neg B \rightarrow A$$

Intensional

$$\sim \vdash A \rightarrow A + B$$

$$\vdash A + B \rightarrow \neg A \rightarrow B$$

$$\vdash A + B \rightarrow \neg B \rightarrow A$$

We may turn now to the formalisation in the predicate calculus. The following formulae have been used

$$501 \quad A_t \rightarrow \exists x A_x$$

$$502 \quad \exists x(x=a \wedge A_x) \leftrightarrow A_a$$

Obviously, these formulae cannot lead to contradiction. Thus the error is the same as in the propositional formalization, that is, the two disjunctions are mixed. Either $A \rightarrow A + B$ is used instead of $A \rightarrow A \vee B$, or $A \vee B \rightarrow \neg B \rightarrow A$ is used instead of $A + B \rightarrow \neg B \rightarrow A$. Since among these four formulae only $A \vee B \rightarrow \neg B \rightarrow A$ is used in the formalization (113), this suggests that the error lies here.

However, more explanation is needed. The distinction between ' \vee ' and '+' gives us, I believe, the full explanation of what's going on the level of propositional calculus. But saying simply that 113 is false is not an explanation for the predicate level. We do not think about McGee's example only in terms of propositional formulae. Both premises Mc1 and Mc2 are sentences that we would render in terms of quantified formulae.

I believe that on the predicate level we have again the problem of equivocation. It is 'A Republican will win', $\exists x(Rx \wedge Wx)$, that we understand in two different senses. When we conclude $\exists x(Rx \wedge Wx)$ from 'Reagan will win', we understand it in a sense different from the one that leads us to infer that $\exists x(Rx \wedge Wx)$ implies $\neg W_r \rightarrow W_a$. This argument is parallel to the one that points to two kinds of disjunction: the ' W_r or W_a ' that

follows from W_r is not the same one from which $\neg W_r \rightarrow W_a$ follows. This similarity stems from the fact that 'A Republican will win' is equivalent to 'Reagan *or* Anderson will win'. Thus it can be equivalent to either intensional or truth function 'or'. When inferred from W_r , $\exists x(Rx \wedge Wx)$ is assumed to be equivalent to $W_r \vee W_a$. When $\neg W_r \rightarrow W_a$ is inferred from $\exists x(Rx \wedge Wx)$, the latter is assumed to be equivalent to $W_r + W_a$. The equivocation stems from the hypothesis iii: once the hypothesis is understood as $\forall x(Rx \leftrightarrow (x=r \vee x=a))$ (let us call this iii \vee); the other time it is understood as $\forall x(Rx \leftrightarrow (x=r + x=a))$ (call it iii $+$). $\exists x(Rx \wedge Wx)$ plus iii \vee yields $\exists x(Rx \wedge Wx) \leftrightarrow W_r \vee W_a$. $\exists x(Rx \wedge Wx)$ plus iii $+$ yields $\exists x(Rx \wedge Wx) \leftrightarrow W_r + W_a$. In order to be true, Mc1 must assume iii $+$, because $W_r + W_a \rightarrow \neg W_r \rightarrow W_a$ is true and $W_r \vee W_a \rightarrow \neg W_r \rightarrow W_a$ is false. However, Mc2, in order to be true, must assume the other sense of $\exists x(Rx \wedge Wx)$, the one in which $\exists x(Rx \wedge Wx)$ is equivalent to $W_r \vee W_a$. But in that case the inference from Mc1 and Mc2 to McC is not mp.

Recapitulation

Let us consider McGee's argument again:

Mc1	$\exists x(Rx \wedge Wx) \rightarrow \neg W_r \rightarrow W_a$	If a Republican wins the election, then if it's not Reagan who wins, it will be Anderson.
Mc2	$\exists x(Rx \wedge Wx)$	<u>A Republican will win the election</u> .
McC	$\neg W_r \rightarrow W_a$	If it's not Reagan who wins, it will be Anderson.

The following valuations for the two premises and the conclusion have been proposed³⁸:

	V₁	V₂	V₃	V₄
	Sinnott-Armstrong etc. Over O ₃	M ^c Gee	Stalnaker D. Lewis Lowe Katz O ₂	O ₁
premise 1	T	T	⊥	T
premise 2	T	T	T	⊥
conclusion	T	⊥	⊥	⊥

The objection given by Sinnott-Armstrong, Moor and Fogelin concerns mainly the use of the modal operator; but they also think that ' \rightarrow ' is material implication. That's why I mentioned them under v_1 . Rendered in the classical propositional calculus, both premises and the conclusion are true. There is no need to repeat here the discussion on material implication.

The same valuation was proposed by Over and the third objection on the page 26 above. The objection was that with the premises taken as *assumptions*, the conclusion is no more problematic. On the page 27 we saw that this proposal does not remove the problem.

As far as I know, the valuation v_2 is proposed by McGee and nobody else. Such a solution denies the Thesis and the general validity of mp. McGee's logic has a problem in expressing correctly the logical structure of certain sentences, which does not appear as a

³⁸ On page 11 ' \perp ' is used as a propositional constant. Here it is, of course, the negative truth value.

problem for 'normal' logics, i.e. ones where mp holds and exportation does not hold (see p. 11). Also, McGee's logic greatly reduces our ability to obtain a conditional as a conclusion of an inference (pages 11-12). However, who knows, although it is now hard to imagine, maybe McGee's logic can be developed so that it does not have more such problems than the 'normal' logics with mp do. But my point now is that there is no need to resort to such a logic to solve the problem raised by the example about the elections. This does not mean that another example in favour of McGee's logic cannot be imagined. This means that *this* example does not favour McGee's solution.

The valuation v_3 would be ascribed to the premises and the conclusion according to the systems for conditionals proposed by Stalnaker and D. Lewis. Mc1 is false because its antecedent is true and its consequent false. This is also said by the second objection (p.17). With the distinctions drawn above we can show in what sense v_3 is correct, that is, in what sense v_3 follows from the set X of premises. On the propositional level, Mc2 is 'Reagan or Anderson will win' ($W_r \vee W_a$), and it is certainly true, since Reagan will win. But then $W_r \vee W_a$ cannot imply $\neg W_r \rightarrow W_a$ (what is said by Mc1). $W_r \vee W_a$ (the antecedent of Mc1) implies $\neg W_r \supset W_a$ (as Lowe noticed), where ' \supset ' is material. $\neg W_r \rightarrow W_a$ (the consequent of Mc1) asserts something more than $\neg W_r \supset W_a$. But the truth-functional disjunction cannot provide the 'something more', so Mc1 is false. On the predicate level, 'A Republican will win' ($\exists x(Rx \wedge Wx)$) is by iiiiv ($\forall x(Rx \leftrightarrow (x=r \vee x=a))$) reduced to $W_r \vee W_a$. Thus, again, Mc2 is true because W_r is true, and Mc1 false because $W_r \vee W_a$ does not imply $\neg W_r \rightarrow W_a$.

The error in Lowe's argument is that he thought that Mc1 must have the form $A \rightarrow (B \supset C)$ in order to be true, that he 'doubts' that Mc1 can be interpreted as $A \rightarrow (B \rightarrow C)$

and that 'even if it can', then it would not be something reasonable to believe. The latter claim follows from the Thesis, although Lowe didn't explain that. Mc1, understood as $A \rightarrow (B \rightarrow C)$, is false, he would say, because it has true antecedent and false consequent. But Lowe didn't notice that Mc1 could have both antecedent and consequent false. This leads us to the explanation of v_4 .

Unlike v_4 , v_3 assumed the truth-functional disjunction and iii_{\vee} , that is, it assumed $\exists x(Rx \wedge Wx) \leftrightarrow W_r \vee W_a$. v_4 , on the contrary, assumes the intensional disjunction and $iii_{+} (\forall x(Rx \leftrightarrow (x=r + x=a)))$, that is $\exists x(Rx \wedge Wx) \leftrightarrow W_r + W_a$. That is the explanation of the first objection (page 16 above) and the sense in which Mc1 is true and Mc2 false: $W_r + W_a$ does not follow solely from W_r , and $W_r + W_a$ does imply $\neg W_r \rightarrow W_a$.

Most of us, faced for the first time with McGee's argument, ascribed the valuation v_2 to the three propositions, as McGee did. But not for the same reason. As I tried to show, we understood each premise in the sense in which it is trivially true; and these senses are not the same. The error was that we understood Mc1 in the sense of '+' and iii_{+} , and Mc2 in the sense of ' \vee ' and iii_{\vee} .

Relation between intensional disjunction and counterfactuals

The question might arise whether the solution presented above may be applied to both indicative and counterfactual conditionals. This is important because McGee's example can be applied to both. McGee never mentioned this, but I think that it was

implicitly present in his paper. Stalnaker's semantics makes no formal difference between the two kinds of conditionals, and McGee's semantics inherited this feature from Stalnaker's. For those who do make the difference, the following question may arise concerning my solution.

One might agree with me that intensional disjunction supports indicative conditionals, in the sense that an indicative $A \rightarrow B$ can be inferred from $\neg A + B$. But, as professor Warmbrod pointed out to me, one might reject a counterfactual $A \rightarrow B$ even if he accepts $\neg A + B$. In the citation on page 43 Anderson and Belnap argued that intensional disjunction supports ('possibly counterfactual') subjunctive conditionals. From:

It is either *Drosophila melanogaster* or *D. virilis*

they inferred

601 If it *hadn't* been *D. virilis*, it *would* have been *Drosophila melanogaster* and this inference looks good. At least it is not as problematic as the following one³⁹. The intensional disjunction:

Either Oswald shot Kennedy, or someone else did

seems to be equivalent to the indicative

If Oswald didn't shoot Kennedy, then someone else did

but not to the counterfactual

602 If Oswald hadn't shot Kennedy, then someone else would have.

³⁹ This argument was pointed out to me by Professor Warmbrod, as a modification of a similar argument proposed by Ernest Adams in "Subjunctive and Indicative Conditionals," *Foundations of Language* 6 (1970), pp. 89 - 94.

The point of the argument is that intensional disjunction does not support counterfactuals. The question then arises why would Mc1 in its counterfactual form be true? If the antecedent of Mc1 is $W_r + W_a$, why would it (counterfactually) imply

603 If Reagan hadn't been the winner, it would have been Anderson.

Now the puzzle is this. 601 and 603 seem to be supported by corresponding disjunctions. 602 obviously is not. What can be said about this?

First, it might be said that there are two kinds of intensional disjunction, one that can be defined in terms of indicative, the other in terms of counterfactual conditionals as $A+B =^{\text{def}} \neg A \rightarrow B$. But this not plausible. It is hard to see what it is in natural language that corresponds to that distinction.

Second, what I also do not find interesting, is to negate the difference between indicatives and counterfactuals. Adams's argument that 602 is likely to be false while the corresponding indicative conditional 'If Oswald didn't shoot Kennedy, then someone else did' is true, suggests that we should make the difference.

Third, it might be that intensional disjunction does sometimes support counterfactuals (and sometimes not). Without much elaboration this sounds ad hoc and not useful. And I will argue that it is wrong.

What I believe is that, as professor Warmbrod said, '+' does not support counterfactuals. And it makes no sense to define a disjunction in terms of counterfactuals as $A+B =^{\text{def}} \neg A \rightarrow B$. Nevertheless, I believe that 601 and 603 do follow from the corresponding disjunctions, i.e., that 601 and 603 *would* be true, *if* the corresponding disjunctions were true. Let us find the relevant difference between 602 on the one side and 601 and 603 on the other. 602 is not true because it is *not* the case that someone *must*

have killed Kennedy. If 601 is true, it is because it *must* have been either *Drosophila melanogaster* or *D. virilis*. 603 would be true if $W_r + W_a$ were true, because there *must* have been a winner.

What is going on is, I think, this. If a disjunction supports a counterfactual, it is never because of the relation of identity between the disjunction and the counterfactual (as it is the case where the conditional is indicative). A disjunction cannot support a counterfactual *alone*, but in conjunction with some other proposition that is true in a given situation. For example, 603 ($\neg W_r \rightarrow W_a$) does not follow solely from $W_r + W_a$, but from the fact that there must have been a winner *and* the disjunction together. In the case of indicative conditionals, McI was true because it could be reduced to a simple instantiation of the law of identity: $W_r + W_a$ is equivalent to (indicative) $\neg W_r \rightarrow W_a$. In its counterfactual form McI is not true because of the relation of identity between $W_r + W_a$ and $\neg W_r \rightarrow W_a$, but because *in the given situation* there is a relation between $W_r + W_a$ and $\neg W_r \rightarrow W_a$ which makes the counterfactual $W_r + W_a \rightarrow \neg W_r \rightarrow W_a$ true. But, in the *same* context, this relation does *not* exist between $\neg W_r \rightarrow W_a$ and the truth-functional $W_r \vee W_a$ (and the counterfactual $W_r \vee W_a \rightarrow \neg W_r \rightarrow W_a$ is false). So I conclude that, although '+' cannot be defined in terms of counterfactuals and does not support them, the distinction between '+' and ' \vee ' still helps us to solve McGee's puzzle, both for indicative and counterfactual conditionals.

Conclusion

In the conclusion I'd like to make clear what is done and what is left to be done.

McGee wanted to point to the problem of the meaning and truth of conditionals that have embedded conditionals as consequents, and to the problem of exportation. Nothing above has been said about these problems. And nothing will be, since I am now unable to say anything useful about them. I don't say that these problems do not exist. On the contrary, they are maybe the most interesting issue that can be raised from McGee's story. But we do not have to deal with them in order to solve the election example.

In the case of indicative conditionals, the problem of embedding was avoided because Mc1, which has an embedded conditional as a consequent, was reduced to a simple instantiation of the law of identity. In the case of counterfactuals it was slightly different. We had an intensional disjunction (that is an indicative conditional) as the antecedent of Mc1, and a counterfactual as a consequent. An indicative conditional asserts that there is a connection of a certain kind between its antecedent and its consequent. A counterfactual asserts a connection of a different kind. But, *in the given situation* of the election case, *if* the 'indicative connection' between $\neg W_r$ and W_a existed, the 'counterfactual connection' *would* also hold. And that is all we needed to establish the truth of Mc1, without solving the general problem of embedded conditionals.

The problem of exportation is avoided, as I believe, because we never use that rule when we think about McGee's example. As a claim about a psychological process, based on introspection and intuition, it is not conclusive. McGee could complain that he does use exportation in deriving Mc1 (which, by the way, he never said explicitly, as Katz did). But he could not give a conclusive argument either.

Although our arguments of this kind cannot be conclusive, they may be more or less convincing. Both Katz's claim that we do use exportation and Lowe's and mine that

we do not are much more convincing than the claim that we use ten more rules that were not mentioned in this paper. If an argument asserts that we use some steps which are redundant according to another argument, the first is usually less convincing.

To establish the truth of Mc1 we need first to establish that the antecedent of Mc1, $\exists x(Rx \wedge Wx)$, is equivalent to $Wr + Wa$. Then the consequent of Mc1 ($\neg Wr \rightarrow Wa$) follows directly from the antecedent, because the relation between $Wr + Wa$ and $\neg Wr \rightarrow Wa$ is a trivial relation of identity. We do not need an *additional* step (or steps) to establish first the truth of $\exists x(Rx \wedge Wx) \wedge \neg Wr \rightarrow Wa$ and after that to conclude Mc1 by exportation. Thus I think the exportation is redundant here. We need it only if we fail to see that there are two kinds of disjunction in natural language, and use only the truth-functional one.

To conclude, I tried to show that the 'counterexample' to modus ponens discussed above points mainly to the need of distinguishing the truth-functional and intensional disjunctions. It has nothing to do with the notion of rational belief, probability or any other problem related to the use of modal operators. And, finally, the example fails to point to the problem of exportation and general validity of mp.