THE UNIVERSITY OF MANITOBA

PONY TRUSS BUCKLING ANALYSIS — AN APPLICATION OF THE THREE-DIMENSIONAL GEOMETRIC STIFFNESS MATRIX

by

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A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES

IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE

OF MASTER OF SCIENCE

DEPARTMENT OF CIVIL ENGINEERING

WINNIPEG, MANITOBA

August 1978

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A dissertation submitted to the Faculty of Graduate Studies of the University of Manitoba in partial fulfillment of the requirements of the degree of

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TO MY WIFE,

SHWU-CHING

ABSTRACT

Pony Truss Bridges are also known as Low Truss Bridges. To permit vehicle clearance, these bridges have no lateral bracing of the top or compression chords.

The author used a three-dimensional geometric stiffness, which includes the effect of axial force, to investigate the top chord buckling problem and to predict the upper-bound buckling load and the load-deflection relationship. The truss bridge was treated as a three-dimensional framed structure with appropriate releases at the four supports. A linear incremental technique was used to find the load-deflection curves, while an eigenvalue analysis was used to find the upper-bound buckling problem. The effect of initial displacement on the results was also demonstrated.

A computer program was developed, which was extensively tested with planar and three-dimensional structures and found to be very successful. The computer program can handle any configuration of Pony Truss Bridge.

The divergence between certain experimental results and those from the computer program are discussed.

ACKNOWLEDGEMENTS

I wish to express my most sincere appreciation to my adviser, Dr. R. B. Pinkney for his suggestions, criticisms and encourgement throughout this thesis work, and beside these, financial support during the preparation of this thesis. Without all the help he offered, this thesis work would have not been possible.

It is my pleasure to thank Dr. G. A. Morris and Dr. D. W. Trim for their careful reading of the thesis and their invaluable suggestions. Thanks are also due the Computer Center of the University of Manitoba for providing the computer time necessary to develope and run the program developed herein.

I wish to say a big "thank you" to my wife, Shwu-Ching for her encouragement and support. I appreciate her patience and understanding at all times, especially when I was working for long hours.

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NOTATION

d: Translational Displacement

$\overline{D}$: Local Displacement Matrix

$\overline{D}_I$: Local Displacement Matrix at End I of a Member

$\overline{D}_J$: Local Displacement Matrix at End J of a Member

$\overline{D}'$, $\overline{D}''$, $\overline{D}'''$: Global Displacement Matrices

E: Elastic Modulus of Material

F: Axial Force of a Member

G: Shear Modulus of Material

$\overline{H}_{IJ}$: Transfer Matrix (from End J to End I of a Member)

$\overline{I}$: Unit Matrix

$\overline{K}$: Locally-Oriented Member Stiffness

$\overline{K}'$: Globally-Oriented Member Stiffness

$\overline{K}''$: Overall Global Stiffness Matrix of a Structure

$\overline{K}_E$: Overall Elastic Stiffness Matrix of a Structure

$\overline{K}_G$: Overall Geometric Stiffness Matrix of a Structure

m: Moment in External Force Matrix

p: Force in External Force Matrix

$\overline{P}$: Local Force Matrix of a Member

$\overline{P}_I$: Local Force Matrix at End I of a Member

$\overline{P}_J$: Local Force Matrix at End J of a Member

$\overline{P}'$: External Force Matrix

$\overline{R}$: Rotational Matrix

$\overline{\overline{U}}$: Strain Energy

X, Y, Z: Local X-, Local Y-, Local Z-axes respectively

X', Y', Z': Global X-, Global Y-, Global Z-axes respectively

$\overline{O}$: Null Matrix

θ: Rotational Displacement

CHAPTER 1

STRUCTURAL ACTION OF A PONY TRUSS

1.1 Buckling Problem of a Pony Truss

A Pony Truss (Figure 1.1) consists of two top chords supported by U-shaped cross frames (Figure 1.2) at intermediate upper joints. There are diagonal members connecting each top chord joint with the floor beam of the adjacent lower joint.

The absence of lateral bracing for the top chord increases its effective length, thus decreasing its resistance to buckling.

The U-shaped cross frames, which provide the stability and stiffness of the top chord, each consist of two verticals and a floor beam. In general, an increase in stiffness of these frames increases resistance to lateral deflection and twisting of the top chord, thus tending to increase the buckling load.

Other factors that may affect the buckling capacity are:

(1) the torsional rigidity of the web members;

(2) the distribution of the floor beam loading; and

(3) the initial eccentricities of the top chord joints.

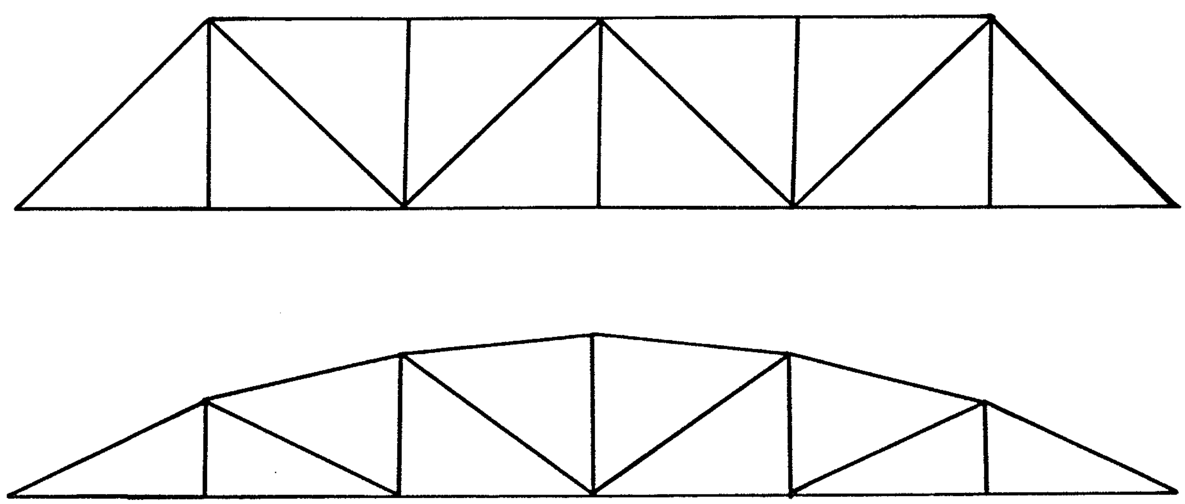

Fig. 1.1 Some Typical Pony Truss Bridges

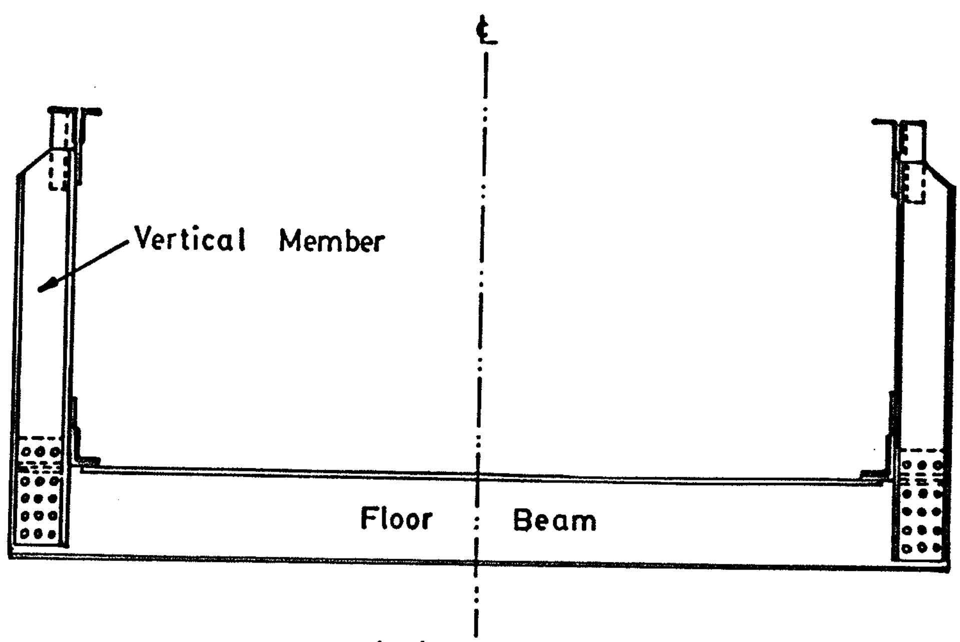

Fig. 1.2 Cross Frame

1.2 Scope of Study

The Pony Truss buckling problem was frequently discussed between the 1920's and 1960's. Some investigators, such as Holt ((10)* through (14)), Bleich (5), Timoshenko (29) and Hsi (15), etc., treated it as a continuous beam-column problem. Many other investigators (for instance Hu (16)), also discussed part of the Pony Truss problem.

In this thesis, the truss is treated as a three-dimensional framed structure, so that most of the member properties will be accounted for in the analysis. Furthermore, initial eccentricities due to crooked or mis-aligned members are included in the analysis.

In a three-dimensional system, the initial eccentricities of the top chord should be measured relative to a unique global system, thus avoiding incorrect measurements of eccentricities in reference to the line joining the two end joints, which themselves could be offset if in reference to a unique global system.

The direct stiffness method of analysis is used, incorporating the effect of large displacement.

The introduction of the geometric stiffness matrix

* Numbers in parentheses refer to references listed in the Bibliography

corrects the structural stiffness for the axial stresses existing in all of the members.

The object of this thesis is to predict the upper-bound buckling load and load-deflection relationship of a Pony Truss using a three-dimensional geometric stiffness matrix and incorporating the effect of initial eccentricities.

1.3 History of the Analysis

In 1928, Bleich (5) analysed a European Pony Truss (Figure 1.3), using the concept of the beam-column. The top chord was assumed to be elastically supported by web members at the panel points. The equilibrium equations were set up by assuming that the two end moments were equal to zero.

Beginning in 1952, Holt published a series of papers concerning the analysis of Pony Truss Bridges ((10) through (14)). In one of his papers, entitled "The Stability of Bridge Chords without Lateral Bracing",

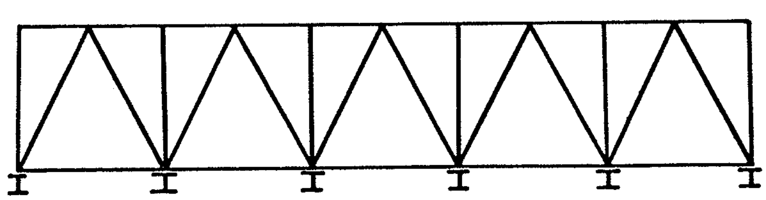

Fig. 1.3 European Pony Truss used in Bleich's Thesis

he suggested several different approaches:-

(1) Reduce the problem to one of a beam-column on an elastic foundation, with the assumption that the top chord is elastically supported.

(2) The buckling load can be determined from the simultaneous, linear, homogeneous equations being set up by using an indeterminate analysis.

(3) Given the shape of the buckling mode of the top chord, the energy method can be applied.

(4) Relaxation and numerical procedures can be employed.

In 1952, Hu (16) analysed a Pony Truss in his Ph. D. dissertation. The top chords were considered as beam-columns supported on elastic, deflectional springs which were equally spaced and subjected to axial compressive forces. An energy method was used to derive formulas that gave the relation between axial forces and spring constants of the supports. Variation of cross-section of top chords, axial forces in the chords and spring constant of both the end and intermediate joints were considered.

In 1953, Hsi (15) presented an analytical method for determining the stresses in the members of a Pony Truss due to transverse frame action of the floor-beams and verticals when the frames are subjected to concentrated loads. The moment distribution method was used.

In 1958, Lee and Clough (19) published a paper, "Stability of Pony-Truss Bridges", taken from the doctoral thesis by the former. Using the slope-deflection method (stiffness modified for axial load), a set of equations at joints along the top chord was set up. Then

(1) The solution of these equations gave the displacement components of the joints, from which the secondary stresses were calculated.

(2) By setting the determinant of the overall structural stiffness to be zero, the upper bound buckling load of the structure was found.

The following secondary effects were also taken into consideration:

(1) Varying axial compression along the chord.

(2) Non-uniform chord section.

(3) Chord curvature.

(4) Various end conditions.

(5) Torsional stiffness of the member.

(6) Support provided by the diagonals.

(7) Axial thrust in the web members.

(8) Continuity between chord and web members.

(9) Unequal joint stiffness.

(10) Bending of floor beams.

(11) Initial eccentricities.

The assumptions made in the paper were:

(1) The diagonals were assumed to be fixed at their lower ends.

(2) The lower extremities of the verticals were assumed to be fixed only in torsion but elastically restrained by the floor beam in bending.

(3) The influence of the bottom chords and the diagonals on the bending of the cross frame was neglected.

(4) The cross-section of all members was symmetrical about both principal axes.

In 1963, Pinkney (24) carried out full-scale tests of an Alberta Pony Truss Bridge and used Lee's technique to determine the buckling load and also plot the load-deflection curves of the top chord joints. As was pointed out in his thesis,

(1) Collapse of a Pony Truss may have a number of causes. Buckling of the top chord is only one of the criteria.

(2) Failure may be initiated by local overstressing; proper analysis of Pony Truss Bridges must include the evalution of secondary stresses.

In 1975, Bleich's technique was modified by Csagoly, Bakht and Ma (8). The computer program developed by them also takes into account the two end moments. The live load on the Pony Truss is gradually increased. Solutions obtained for each of the increased loadings are checked against yielding of steel. The live load that stresses the compression chord to the yield point of the steel is taken as the buckling load of the truss. The program applies only to typical, standard, symmetrical American Pony Truss (Figure 1.4).

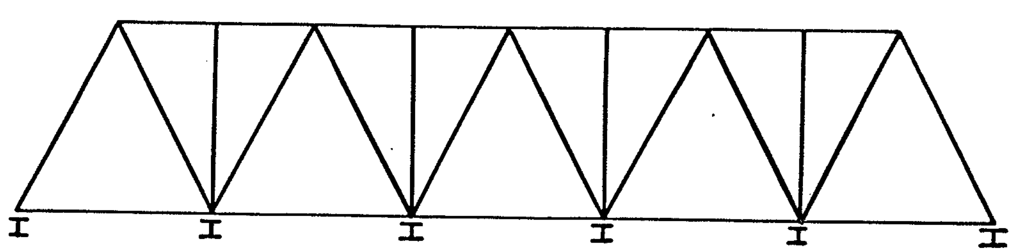

Fig. 1.4 Pony Truss used in Csagoly's Report

CHAPTER 2

DEFINITIONS AND EQUATIONS FOR THE ELASTIC LINEAR ANALYSIS

2.1 Equilibrium Equation

In structural analysis, one way to represent the relationship between external loads and resulting displacements in a unique coordinate system is

$$\overline{K}'' \quad \overline{D}' = \overline{P}' \qquad (2.1)$$

where $\overline{P}'$ is the external force matrix (including forces and moments):

$$\overline{P}' = \begin{Bmatrix} p'_1 \\ p'_2 \\ \cdot \\ \cdot \\ p'_n \end{Bmatrix} \quad ; \text{ and}$$

$\overline{D}'$ in Eq.(2.1) is the displacement matrix (including translations and rotations):

$$\overline{D}' = \begin{Bmatrix} d'_1 \\ d'_2 \\ \cdot \\ \cdot \\ d'_n \end{Bmatrix} \quad .$$

$\bar{K}''$ in Eq.(2.1) is called the overall stiffness matrix of the structure. It is the summation of all the contributing member stiffnesses.

It should be noted that any force or moment, say p'_i, in $\overline{P}'$ corresponds to only one translation or rotation, say d'_i in $\overline{D}'$ respectively. The three matrices, $\bar{K}''$, $\overline{D}'$ and $\overline{P}'$ are all expressed in one unique global coordinate system.

2.2 Global Coordinate System

A global coordinate system is one that has the same orientation throughout the structure. All joint coordinates, external loads and joint displacements are expressed in the global system.

In this work, the global X-axis, hereafter designated as X', is along the longitudinal direction of the truss. The global Y-axis, Y', is the vertical axis. Using the right-hand rule, the global Z-axis, Z', is the transverse axis (i.e. the longitudinal axis of the floor beam). See Figure 2.1. The origin of coordinates is at the position indicated in the figure.

2.3 Local Coordinate System

The local system is needed to define the properties of, and forces in, a member; these include I_{xx}, I_{yy}, I_{zz} and so on.

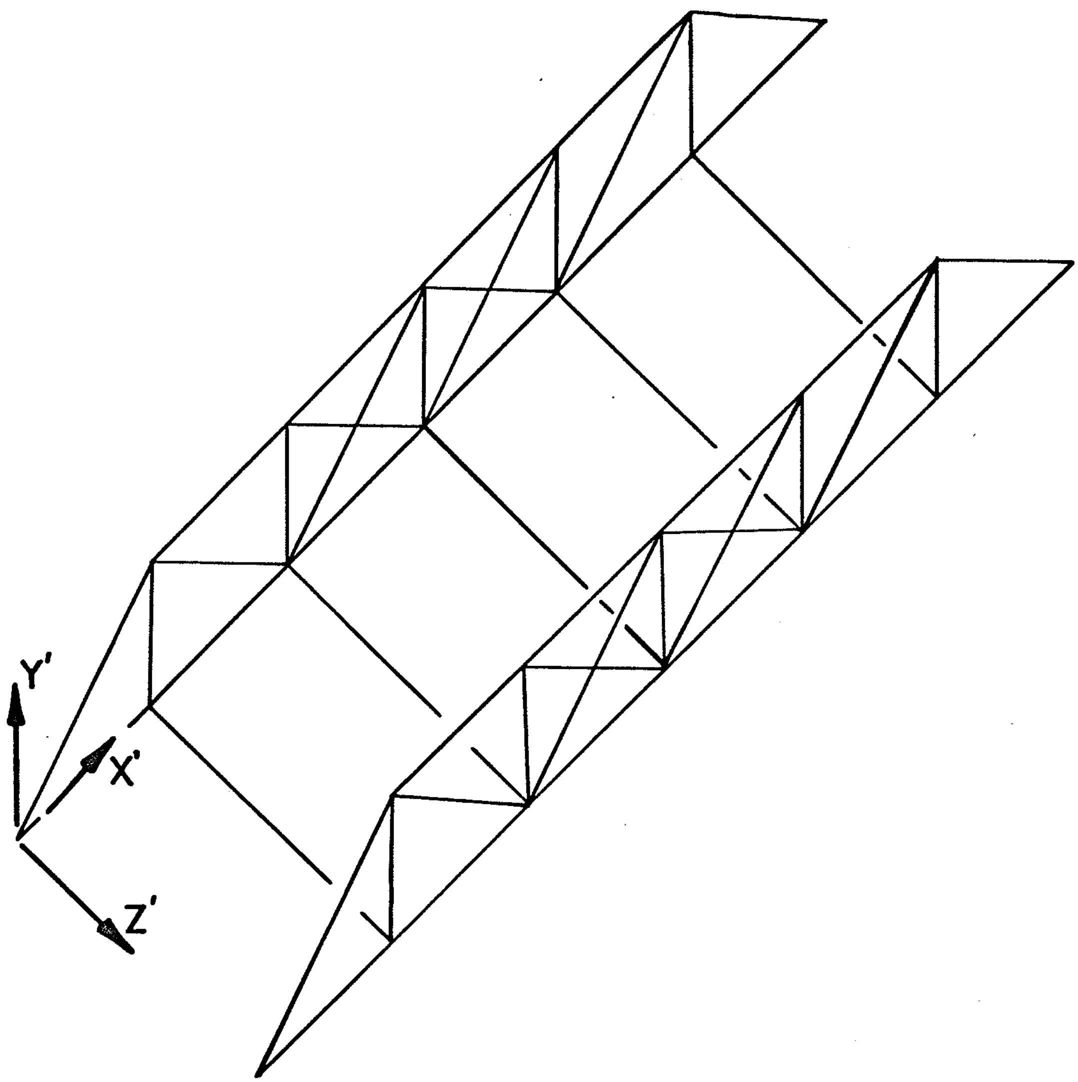

Fig. 2.1 Global Coordinate System

The three local axes are defined as below:

(1) The local X-, Y-, Z-axes are mutally perpendicular to each other.

(2) The local X-axis is the longitudinal axis of the member.

(3) The local Z-axis lies in the X'-Z' plane and thus

(4) The local Y-axis points upward from X'-Z' plan for the case of a non-vertical member.

The relationships between X, Y, Z and X', Y', Z' are shown in Figure 2.2.

For vertical members, rule (4) cannot apply. Therefore in this work, the local coordinate system for a vertical member is defined as shown in Figure 2.3.

2.4 Structural Topology

In a computer analysis, it is necessary to define the interconnectivity of members and joints of a structure. Joints and members are numbered and each member is given a direction. The 'beginning' of a member is connected to joint I and is called End I of the member; the 'end' of the member is connected to joint J and is called End J of the member. The structural topology is often described by a Member Incidence Table (MIT).

Consider a structure with 3 members and 4 joints as shown in Figure 2.4. The direction of member 1 is

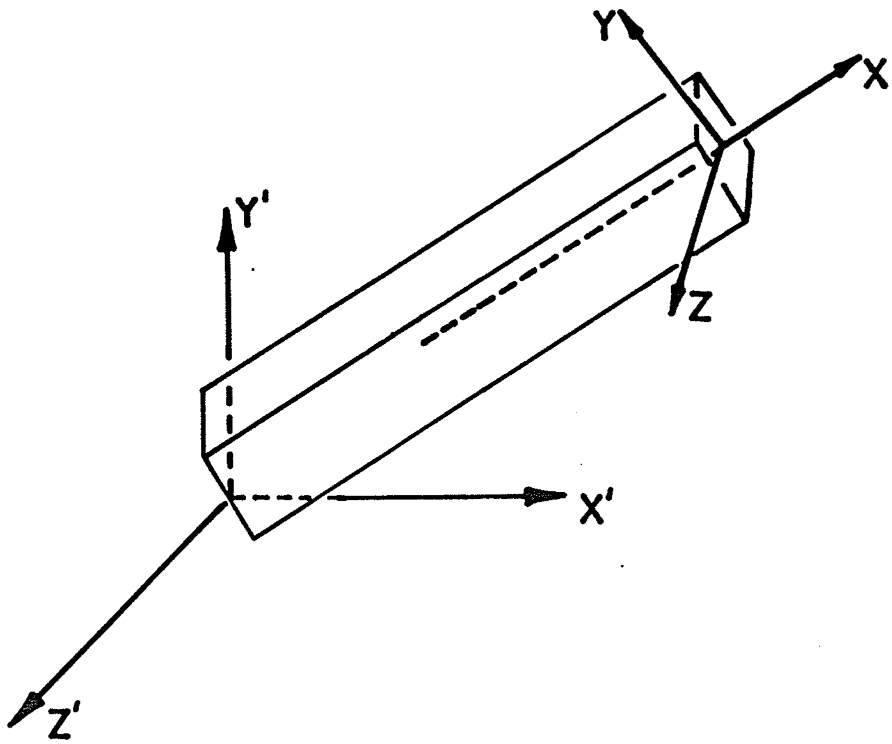

Fig. 2.2 Local Coordinate System for Non-vertical Members

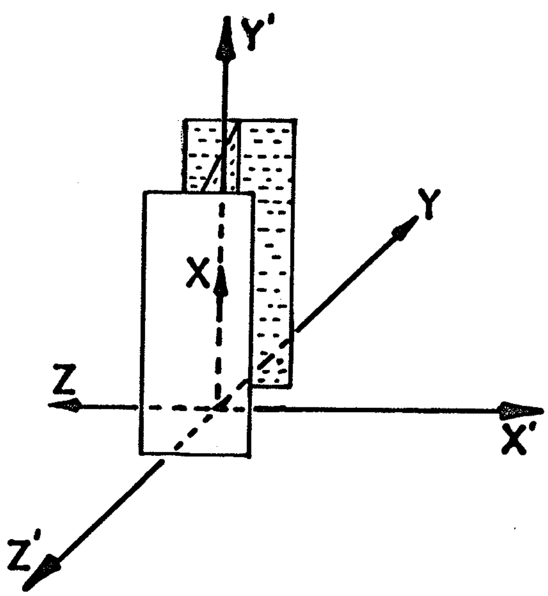

Fig. 2.3 Local Coordinate System for Vertical Members

defined as from joint 1 to joint 2, member 2 from joint 3 to joint 2, and member 3 from joint 3 to joint 4.

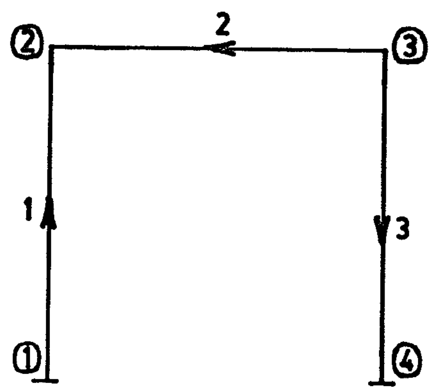

Fig.2.4

The MIT of the structure is

member	I	J
1	1	2
2	3	2
3	3	4.

2.5 Joint Identification Table

The Joint Identification Table (JIT) specifies the displacement boundary conditions. There is one entry for each joint, chosen from the following options:

Case 1	0	0	free joint
Case 2	1	1	fixed support except for X'-translation
Case 3	2	1	fixed support except for Z'-translation
Case 4	4	1	fixed support except for X', Y', Z'-rotations
Case 5	0	2	fixed support

Any combination of the above Case 2 to Case 4 inclusively, can be used.

2.6 Rotational Matrix

When two or more members are involved in a structural analysis, they may have different local coordinate systems. In order that Eq.(2.1) be valid, the contribution from various member stiffnesses to each element in the overall structural stiffness must be referred to a common global system. For example, if K''_{12} is one element in an overall structural matrix that relates external force p'_1 and deflection d'_2, then the elements from various member stiffness that relate p'_1 and d'_2 must be all expressed in the same global system before the member stiffness matrix can be summed to form K''_{12}.

Due to this requirement, a rotational matrix is introduced to transform the member stiffness matrix of the local system to the corresponding stiffness in the global system.

2.6.1 Rotational Matrix for Members other than Vertical Members

Using the definition of the local system in Section 2.2, a figure showing the relations between global system and local system can be drawn, as shown in Figure 2.5.

Let $\vec{u}_1$, $\vec{u}_2$, $\vec{u}_3$ be unit vectors along the local X-, Y-, Z-axes, respectively, and also let

$$\vec{u}_1 = \ell \vec{i} + m \vec{j} + n \vec{k}$$

where

ℓ, m and n are direction cosines and $\vec{i}$, $\vec{j}$, and $\vec{k}$ are unit vectors along the X'-, Y'-, Z'-axes respectively.

Since axes X and Y are perpendicular, OA is the projection of $\vec{u}_1$, and $\vec{u}_3$ lies in X'-Z' plane, it follows that $\angle DOA = 90^\circ$; thus

$$\angle OAB = \angle DOC$$

$$\vec{OC} = \vec{k} \; |\vec{OD}| \cos \angle DOC$$

$$= \frac{\ell}{\sqrt{\ell^2 + n^2}} \vec{k}$$

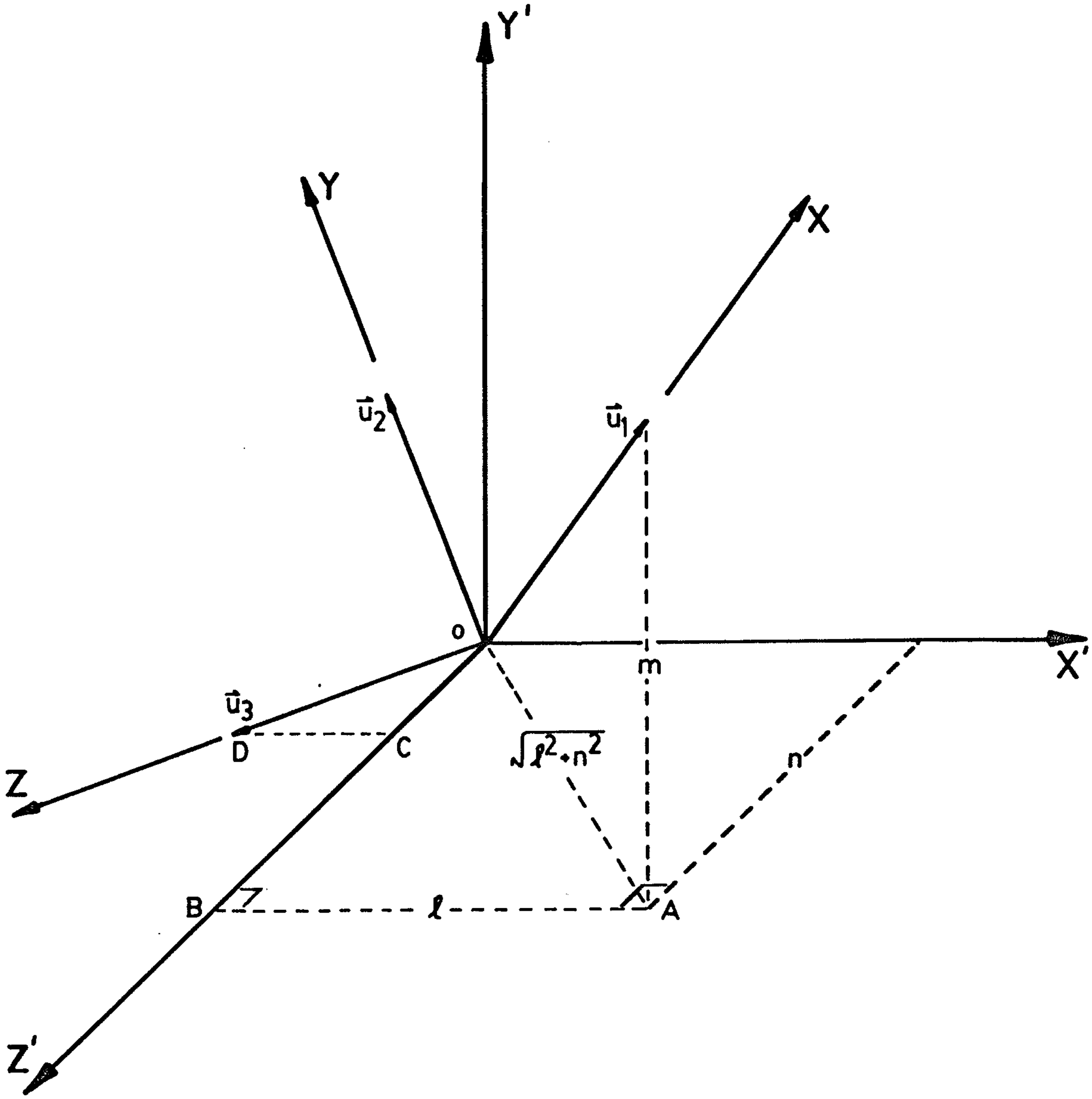

Fig. 2.5 Derivation of Rotational Matrix for Non-Vertical Members