# AN ABSOLUTE MEASUREMENT OF THE PAIR PRODUCTION CROSS SECTION OF LEAD AT 2.76 MEV.

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# PREFACE

The work which comprises the subject matter of this thesis was carried out at the University of Manitoba between May 1955 and May 1956.

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## ABSTRACT

A value has been obtained for the absolute pair production cross section of lead for the 2.76 Mev. gamma rays of $Na^{24}$. A collimated beam of these gamma rays was made to fall on a specially constructed target. The positrons produced in the target were detected by counting 2 quanta annihilation events by means of two scintillation spectrometers in coincidence. The strength of the $Na^{24}$ source was measured by a coincidence counting technique and the counter detection efficiency for annihilation radiation was measured with a calibrated $Na^{22}$ source. The effect of absorption of the 0.511 Mev. annihilation radiation in the target was measured in a separate experiment. The value obtained for the cross section was 2.38 ± 0.62 barns.

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# INTRODUCTION

The existence of a particle of charge +e, where -e is the charge on the electron, and mass equal to that of the electron was first observed experimentally by Anderson in 1932[1]. Dirac, in his relativistic quantum-mechanical theory of the electron (1928), showed that the possible energy states of a free electron consist of a set of closely spaced positive levels along with a complimentary set of states of negative total energy which are mirror images of the positive ones.[2] As the possibility of an electron existing in one of these negative energy states can not be excluded on quantum mechanical grounds, there was for a time some doubt as to their correct physical interpretation. With Anderson's discovery it was possible to associate the negative energy states with the positron. This was done in the well known "Dirac hole theory".[3]

According to this theory the "electron vacuum" is represented by the case in which all the positive states are empty and all the negative states are filled (by the Pauli exclusion principle a state containing one electron is "filled"). A filled positive state is observed as a negatron and an empty negative state as a positron. Although an electron in a negative energy state is not experimentally observable, it is possible for an external electromagnetic field to cause

1. C. D. Anderson, Phys. Rev. 44, 406 (1933)
2. P.A.M. Dirac, Proc. Roy. Soc. A, 117, 610, (1928)
3. P.A.M. Dirac, "The Principles of Quantum Mechanics" 3rd Ed. Chap. XI.

an electron in a negative energy state to make a transition to an unoccupied positive energy state. The result of this process, which is called "pair production", is the appearance of a pair of electrons, one positive and one negative. In order for momentum AND ENERGY to be conserved the presence of a third particle (e.g. a nucleus) is required.

Although pair production can take place in a variety of ways, the process of interest for the work of this thesis is the creation of pairs in the coulomb fields of atomic nuclei by photons. In order that it be able to produce a pair, the energy of the photon must be at least enough to account for the rest energy of the two electrons (1.022 Mev.). Any surplus appears as kinetic energy of the electrons. This energy (1.022 Mev.) is generally called the "threshold energy" for pair production.

The process of pair production by photons in nuclear fields is closely allied to the Bremsstrahlung process. In the case of the former a photon interacts with an electron in a negative energy state causing a transition to an unoccupied positive state. In the latter case an electron in a positive state undergoes a transition to a second positive state of lower energy with the emission of a photon. Because the transitions involved in the two processes are either reciprocals of one another (i.e. absorption or emission of a photon), or differ only in initial and final states, having worked out the various transition probabilities involved in one case, one can apply them in a straightforward fashion to the other. For this reason in theoretical papers the two are often treated together.

The first relativistic quantum mechanical treatment of the Bremsstrahlung - pair production problem was given by Bethe and Heitler in 1934.[4] This theory was based on the "Born approximation", i.e. plane wave functions were used for the electrons. This amounts to the condition that

$$\frac{Ze^2}{\hbar v} << 1 \qquad \text{i.e.} \quad \frac{Z}{137}\cdot\frac{c}{v} << 1$$

where Z is the charge number of the nucleus, e is the electronic charge, $\hbar$ is Planck's constant divided by $2\pi$, c is the velocity of light, $\frac{e^2}{\hbar c} = \frac{1}{137}$ is the fine structure constant and v is the electron velocity. (The inequality must be satisfied for both the positron and negatron). Since this will not be satisfied for elements of high Z (e.g. for Pb, Z = 82, Z/137 = 0.6), this theory therefore would be expected to break down in the region of high Z and low v, i.e. low incident photon energy. For low energies they considered only the simple coulomb field of the nucleus and for high energies took account of the effect of the orbital electrons by means of a screening correction. The expression obtained for the differential cross-section as well as a graph showing the values of the total cross-section, $\sigma$pair, for various materials over a wide range of energies are given in Heitler's book.[5] They obtain the result that the cross-section for fixed photon energy is proportional to $Z^2$ for the case in which screening is neglected.

4. H. Bethe & W. Heitler, Proc. Roy. Soc. A, 146, 83, (1934)
5. W. Heitler, "The Quantum Theory of Radiation" 3rd Edition, Oxford 1954, p. 258, p 262.

For low photon energy and high Z the Born approximation, as pointed out above, is no longer valid. Jaeger and Hulme have developed a theory of pair production making use of exact (coulomb) wave functions and have made numerical computations of $\sigma$pair for various high Z materials at low photon energies.[6] Values of $\sigma$pair for Z = 82(Pb) for the two theories are given in the following table;

| Photon Energy | $\sigma$pair in barns ($10^{-24}$ cms$^2$) | |
|---|---|---|
| Mev | J.H. | B.H. |
| 1.53 | o.67 | o.34 |
| 2.66 | 3.1 | 2.6 |

According to the Jaeger and Hulme theory the dependence of the cross-sections on Z is of the form: $aZ^2 + bZ^4$ where a and b are constants.

The process in which an electron pair is formed is called pair production: the process in which a negatron and positron interact and disappear is called pair annihilation. In terms of the Dirac hole theory the annihilation process involves the transition of an electron in a positive energy state to a negative energy state, the energy lost by the electron appearing as one or more photons.

In the most probable annihilation process the positron is reduced to thermal velocity by interaction with the atomic particles of the material through which it passes. It then interacts with a free electron, producing two photons. This process isreferred to as two quanta annihilation. Since the energy due to the electrons' motion is very small compared to their rest energy, the

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6. H. Hulme and J. Jaeger, Proc. Roy. Soc. 153, 443 (1936)

energy carried by the two photons must be very nearly 1.022 Mev. To satisfy conservation of momentum the energy must be shared equally between them. Thus the product of this mode of annihilation is two oppositely directed 511 Kev photons. These are usually referred to as the "annihilation radiation." Precision experiments have verified that this process does in fact occur.[8]

If the annihilation interaction occurs while the positron is moving with an energy which is not negligible compared to its rest energy, then the above statements are still true in the Lorentz system in which the centre of mass of the two electrons is at rest. However, when transformed to the system in which the negatron is at rest, resulting pairs of photons need no longer be oppositely directed or even of the same energy. The theory for this process has been given by Bethe and predicts that approximately 4% of a beam of 1 Mev positrons passing through lead will annihilate in flight.[9]

If the negatron is bound to a nucleus it is possible for a positron to annihilate with the emission of a single photon, the nucleus taking up the extra momentum. The theory for this process is given by Heitler who obtains a value for the cross-section proportional to $Z^5$.[10] Hence it is most important in heavy elements in which it may amount to as much as 20% of the two quanta annihilations.

There is one more significant process; that in which the annihilation produces three photons. This can

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8. D. E. Muller, H. C. Hoyt, D. J. Klein and J.W. DuMond, Phys. Rev. <u>88</u> (1952) 775
9. H. Bethe, Proc. Roy. Soc. A, <u>150</u> (1935) 129, 3rd Ed.

occur for positrons either in flight or at rest. From the point of view of perturbation theory this is a higher order process than the two quanta case, and hence much less likely to occur. The theoretical ratio of two to three quanta annihilations is given by Heitler as 1/370.[11] That the ratio in favour of the two quanta process is at least this great has been verified by De Benedetti.[12]

From the above it may be seen that it is feasible in an experiment to determine the rate at which positrons annihilate by determining the rate at which they annihilate at rest producing two quanta. The total rate of annihilation may then be obtained by accounting for the other processes through small corrections.

10. W. Heitler, Quantum Theory of Radiation (3rd Ed. Oxford 1954) PPS (272-4)
11. Ibid Page 278
12. S. De Benedetti and R. Siegel, Phys. Rev. 94 (1954) 955.

## THE EXPERIMENT

Although several experimental measurements of relative pair production cross sections have been made at low energy in order to determine its Z dependence, at the time this work was begun no absolute measurements had been made. The results of the relative measurements of Hahn et al[1] at 2.62 Mev. and Dayton[2] at 1.33 and 2.76 Mev. both agree with the Z dependence predicted by Jaeger and Hulme rather than that of Bethe and Heitler. As a further check on the theory an absolute measurement in this energy region was desirable.

The experiment which is the subject of this thesis is perfectly straightforward in that a target of the material being investigated is bombarded with an essentially monochromatic beam of gamma rays, for which the photon flux (no. of photons per square centimeter per second perpendicular to the beam) is known at the position of the target. The rate at which positrons annihilate in the target with the production of two oppositely directed 0.511 Mev. photons is measured by a coincidence counting technique using a pair of scintillation counters. From these data, along with certain subsidiary data discussed below, it is possible to calculate $\sigma_{pair}$.

The experiment is divided into a series of subexperiments, each of which provides part of the data required. The first of these is an experiment in which the "strength" (no. of disintegrations per second) of the source is

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1. Hahn, Baldinger and Huber, Helv. Phys. Acta. 25, 505 (1952)
2. Irving E. Dayton, Phys. Rev. 89, 544, (1953)

measured. Once this is known it is possible to calculate the photon flux at the target from the experimental geometry. Next, the "coincidence rate" i.e. the number of two quanta annihilations detected per second must be measured for each target used. It is then necessary to determine the "annihilation rate" i.e. the number of two quanta annihilations per second occuring in the target. This is necessary since not all such annihilation events are detected by the counter. The ratio of the number of events detected to the number of events occuring is called the efficiency of the counter. Finally it was necessary to perform an experiment in order to determine a correction to be applied to the annihilation rate as abtained by the above procedure because of absorption of the 0.511 Mev photons in the target itself. Each of these sub-experiments will be described separately in detail in the order given above, following a brief description of the counting equipment.

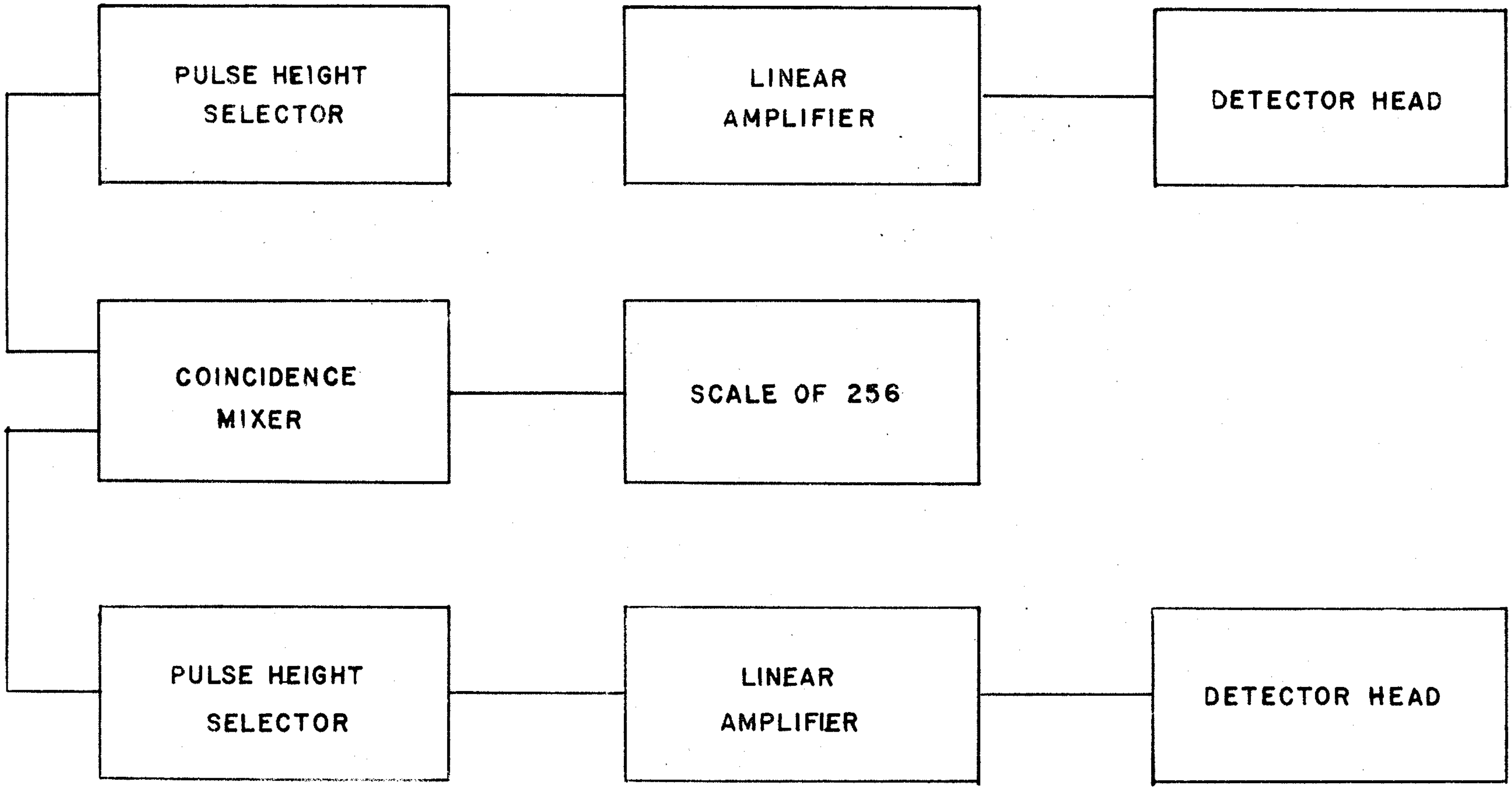
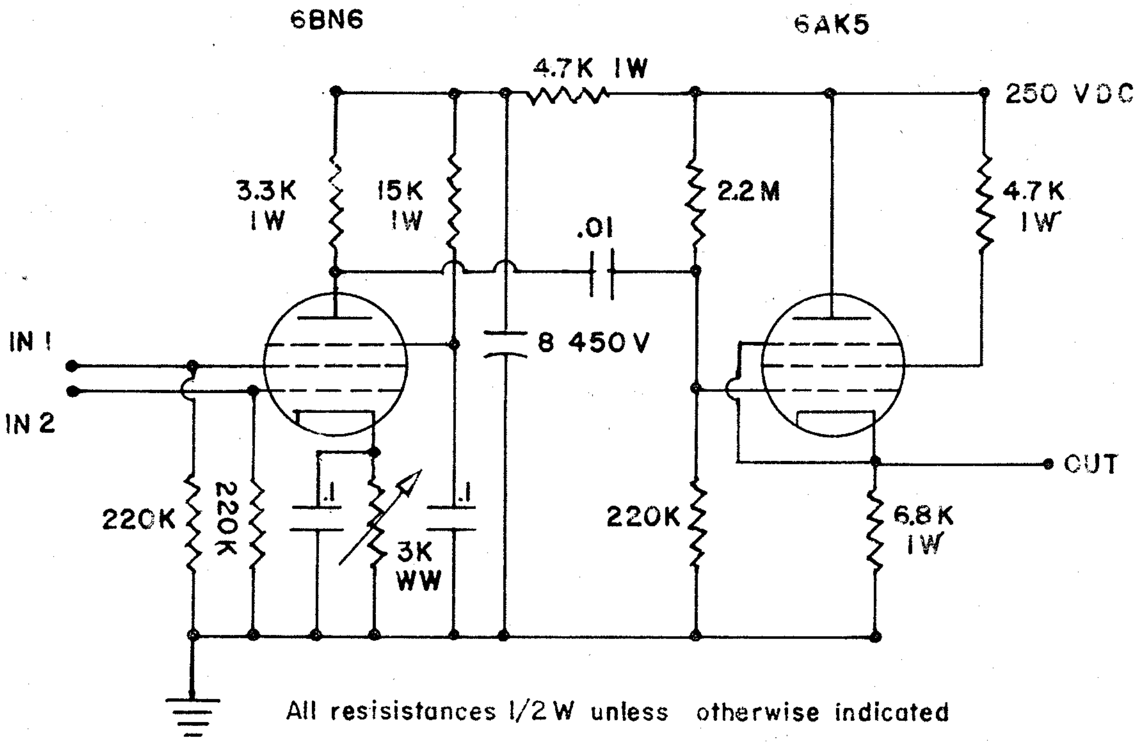


FIG. I BLOCK DIAGRAM

FIG. 2

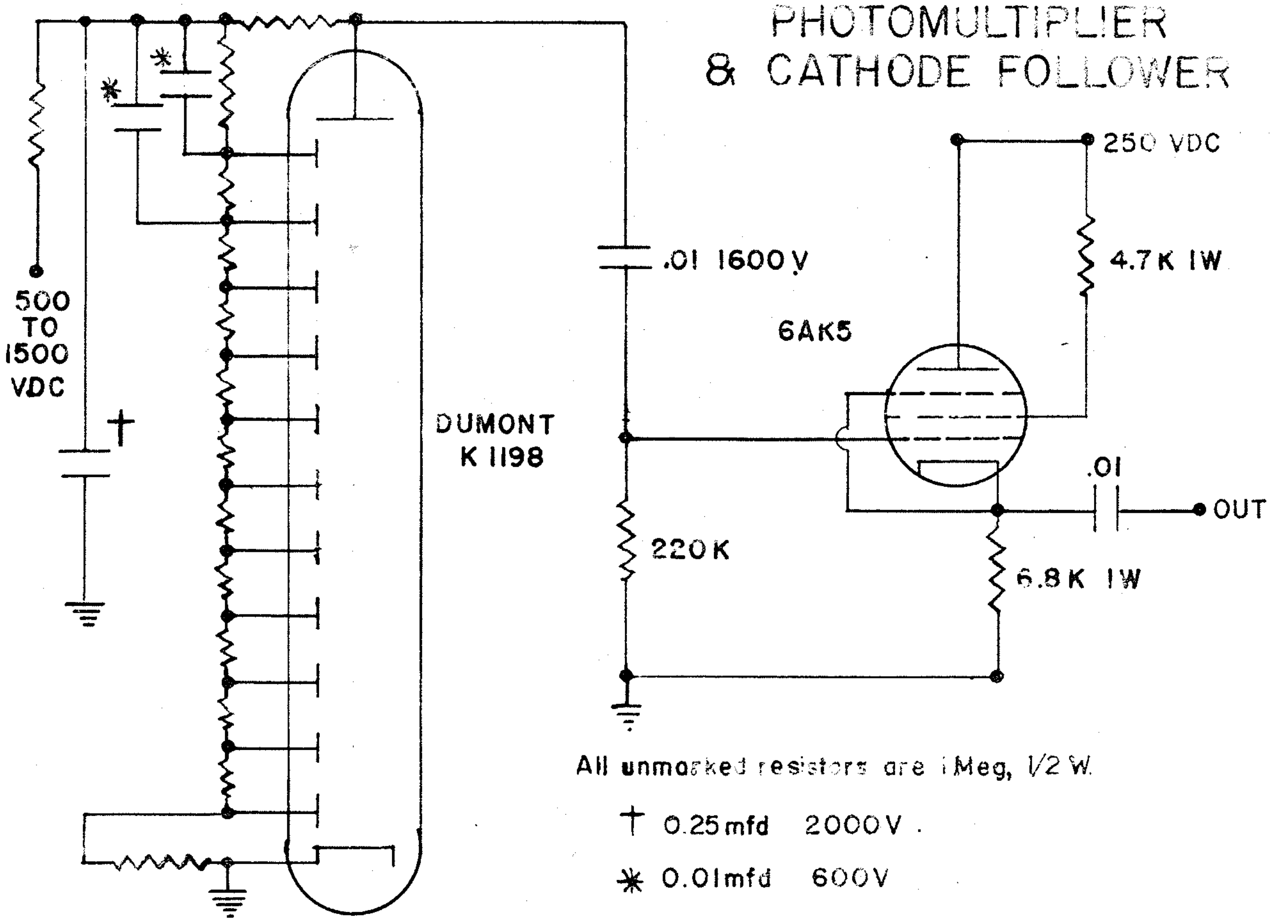
## COINCIDENCE MIXER



All resisistances 1/2 W unless otherwise indicated

All capacities in mfd unless otherwise indicated

## PHOTOMULTIPLIER & CATHODE FOLLOWER



All unmarked resistors are 1 Meg, 1/2 W.

† 0.25 mfd 2000V.

✱ 0.01 mfd 600V