THE DEVELOPMENT OF A SEMI-AUTOMATIC POTENTIAL ANALOG APPARATUS AND ITS APPLICATION TO SOME FILTER APPROXIMATION PROBLEMS

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ABSTRACT

This thesis presents the development of a semi-automatic potential analog apparatus which employs carbon-impregnated conducting "Teledeltos" paper and an X-Y recorder, gives a derivation---by means of the potential analogy---of functions which yield solutions to low-pass approximation problems in image parameter (and insertion loss lossless filter theory) and shows how the apparatus may be used as an aid in their practical determination. Techniques of measurement using quarter planes are investigated and the results presented. Conformal transformations (of the complex plane) are studied in order to find some which have special practical utility with the apparatus, and examples of recordings (using these planes) which yielded suitable locations for points of infinite loss are presented and compared. These---and a few other recordings---show the feasibility of using the apparatus for that purpose as well as others of passive filter (and other network) synthesis problems.

The feasibility of applications of the potential analog apparatus as an aid in solving low-pass approximation problems of lossless filter theory is studied, and the practicability of the apparatus for this purpose is evaluated and compared with templates and digital computer methods. It is found that the apparatus is more practical and sufficiently accurate in many cases. One such case, which is emphasized in this thesis, is the determination of locations of points of infinite loss so as to give prescribed attenuation characteristics.

Formulae for measurement of coefficients in partial fraction

expansions of rational functions with single and double poles are derived, and examples of actual measurements (performed manually) are presented. These illustrate the necessity for the semi-automatic analog apparatus.

PREFACE

This thesis originated with the desire to investigate applications of a potential analog apparatus originally built at the University of Manitoba by Professor E. Bridges under the supervision of Professor R. A. Johnson and with the sponsorship of the National Research Council of Canada (Grant NRCG584-1956)*. This apparatus was not suitable for most practical applications, so extensive modifications were undertaken to make it semi-automatic. Manual measurements, presented in this thesis, also showed the need for such an apparatus. After the development of this apparatus had been completed, a study of the literature on network synthesis--especially filter theory--was undertaken in order to find possible applications of the apparatus in solving approximation problems. A few of the possible applications of the potential analog apparatus are presented in this thesis. The suitability of conformal mappings for use with this apparatus are studied. Actual recordings obtained with the aid of the apparatus are presented, and its use is compared with template and computer methods.

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*E. Bridges, "A Network-Function Simulator" (Unpublished Master's Thesis, The University of Manitoba, Winnipeg, September 1958).

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CHAPTER I

THE AIMS, THEIR VALIDATION, DEFINITION OF IMPORTANT TERMS AND THE ORGANIZATION

This thesis describes the evolution of a new semi-automatic potential analog apparatus suitable for network and control system synthesis applications, gives the derivation of functions suitable for approximation of low-pass image-parameter (and, in well known ways, of low-pass insertion attenuation) functions, and presents applications of the potential analog apparatus as a design aid in determining these functions. In this thesis the presentation and/or derivation, by means of the potential analogy, of functions needed for lumped-element low-pass image parameter (and insertion loss) lossless filter synthesis and the use of the potential analog apparatus as a practical design aid in this work, is given greatest emphasis.

The functions desired are functions of a complex variable, $p=\sigma+j\omega$, usually normalized to $\omega_c=1$. In chapter V Ω is sometimes used instead of the normalized ω. The matter of independent complex variables should, however, cause no trouble.

Because of the nature of the topics, the complete method of their presentation, and their length, a historical background is not presented here, but is given in each chapter which deals with a particular topic. Further help in placing this work in proper perspective and relation to that previously done, and in showing the status of the topics up until

the present time, is given by footnote references and validations and comparisons presented in each chapter of this thesis. The scope of the undertaking for each topic is also defined in the introductory section(s) of each chapter, so extra verbiage is not presented here, except that the specific aims of this thesis will be stated at the end of this chapter. These aims, when considered in relation to the historical background and justifications of this study--presented in the introductory sections of the chapters--should give the reader an initial insight into the importance and relations of the topics covered in this thesis.

I. REASONS FOR THE STUDY, AND ORGANIZATION OF THE THESIS

One of the reasons for this study was the need for a potential analog apparatus more suitable for network and control systems than any that had been previously developed. Thus, most previous analog devices were too inaccurate and tedious to work with, and yielded recordings or plots which were too small to be of practical value, or were restricted, in that only one type of conformal mapping could be used. With this need there was the desire to investigate various types of measurement techniques for potential and stream functions, and to try out the new potential analog apparatus to be described herein, in order to compare its effectiveness and practicality against point-by-point measurement techniques, as well as to investigate its over-all accuracy and modes of operation. Before the apparatus was built it was desirable to search for those features which a semi-automatic potential analog apparatus

should have in order to be most useful in the contemplated fields--especially network synthesis. After the apparatus was built its errors had to be considered, its over-all accuracy assured, and further improvements suggested.

The next three chapters are concerned with the topics of the previous paragraph. Chapter II gives point-by-point measurements of phase and magnitude functions on quarter planes, and discusses and illustrates the use of various types of measurements using these quarter planes. Chapter III deals with measurement of residues of rational functions using a half plane. Equations for such measurements are derived, and point-by-point measurements carried out. Both chapters II and III demonstrate the tediousness and the rather mediocre accuracy achievable with these methods. They also give a physical appreciation of the analog. Chapter IV deals with some of the historical background of potential analog devices for network synthesis and analysis, a study of desired features of such an apparatus, with the development of a new apparatus, and with a study of its errors and over-all accuracy. Many details of the development, the calibration, and the errors, are presented in the appendices. Further improvements are suggested in chapter IV.

Another need seemed to be a thorough investigation of possible practical applications of the apparatus to solving approximation problems in filter synthesis.

The theory of the potential analogy was used in this thesis because it leads to application of the analog apparatus, and provides a physical concept for the rather complicated mathematics of approximation

theory.

Chapter V is devoted to the derivation of approximating functions for image parameter filter theory by means of the potential analogy, to the investigation of possible applications of the potential analog apparatus as a design aid, to the actual implementation (with high accuracy) of the new potential analog apparatus to determination of points of infinite loss for low pass filters, to determinations and comparisons of conformal mappings to be used with the apparatus for low pass filter synthesis. Chapter VI gives a comparison of the use of the apparatus to the use of other methods yielding the same results. Although image-parameter filters were dealt with specifically, it is well-known that the approximation of image attenuation is usually used as a high quality approximation to insertion loss in the stop band(s).

II. THE DEFINITION OF THE IMPORTANT FILTER THEORY FUNCTIONS TO BE APPROXIMATED

Before the specific aims are given, a few words should be said about some important functions used in dealing with filter approximation theory.

Firstly, in dealing with image parameter filter theory one encounters the following functions of p, which are defined and described in a book by Cauer:[1]

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[1] W. Cauer, Theory of Linear Communication Networks, Vols. I and II (McGraw-Hill Book Company, Inc., Toronto, 1958), pp.121-142 and 221-291.