

THE UNIVERSITY OF MANITOBA

MEASUREMENTS OF THERMAL DEFORMATION OF LAMINATED
GRAPHITE FIBER REINFORCED EPOXY RESIN PLATES
BY HOLOGRAPHIC INTERFEROMETRY

by

Richard Joseph Lewak

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE
OF MASTER OF SCIENCE

DEPARTMENT OF MECHANICAL ENGINEERING

WINNIPEG, MANITOBA

OCTOBER, 1975

"MEASUREMENTS OF THERMAL DEFORMATION OF LAMINATED
GRAPHITE FIBER REINFORCED EPOXY RESIN PLATES
BY HOLOGRAPHIC INTERFEROMETRY"

by

RICHARD JOSEPH LEWAK

A dissertation submitted to the Faculty of Graduate Studies of
the University of Manitoba in partial fulfillment of the requirements
of the degree of

MASTER OF SCIENCE

© 1975

Permission has been granted to the LIBRARY OF THE UNIVER-
SITY OF MANITOBA to lend or sell copies of this dissertation, to
the NATIONAL LIBRARY OF CANADA to microfilm this
dissertation and to lend or sell copies of the film, and UNIVERSITY
MICROFILMS to publish an abstract of this dissertation.

The author reserves other publication rights, and neither the
dissertation nor extensive extracts from it may be printed or other-
wise reproduced without the author's written permission.



ABSTRACT

Thermal deformation of laminated graphite fiber reinforced epoxy resin plates was measured by using laser holographic interferometry techniques for the simulation of the change of environment encountered by satellite components. Comparison was made between interferometric deformation measurements and deformation predicted by finite element analysis based on the theory of thermoelasticity. From discrepancies between theoretically predicted and measured deformations it was concluded that marked transition in material behavior with temperature is due to changes in the material properties of the epoxy resin matrix with temperature and that substantial error may be introduced into theoretical calculations of deformation by the assumption of temperature independent material properties.

ACKNOWLEDGEMENT

The author wishes to express his appreciation to Drs. T. R. Hsu and K. T. Kedward for their continuing assistance, advice and guidance during the course of this study. Acknowledgement is also due to Mr. Y. T. Yeow for his assistance in the experimental work.

This study has been done through the assistance of Bristol Aerospace, Limited, of Winnipeg. This company has provided the specimens for the experimental work in this study as well as a computer program for the use of the author in the course of this study.

The financial support provided by the National Research Council in Canada under a special PRAI grant (Project Research Applicable to Industries) No. P-7310 is also acknowledged.

TABLE OF CONTENTS

	page
CHAPTER 1 INTRODUCTION	1.
1.1 Literature Review	2.
1.2 Industrial Interaction	6.
CHAPTER 2 THEORETICAL ASPECTS OF THE THERMAL DEFORMATION OF LAMINATED COMPOSITE PLATES	8.
2.1 Basic Theory	8.
2.2 Deformation Prediction by TEANAL Computer Program	18.
2.3 Deformation Prediction by ANSYS Computer Program	21.
CHAPTER 3 HOLOGRAPHIC INTERFEROMETRY	23.
3.1 Principles of Holography	23.
3.2 Holographic Interferometry	27.
3.3 Double Exposure Holographic Interferometry Technique	29.
3.4 Multiple Exposure Holographic Interferometry Technique	30.
3.5 One Dimensional Fringe Interpretation Method	31.
3.6 Three Dimensional Fringe Interpretation Method	33.
CHAPTER 4 EQUIPMENT FOR EXPERIMENTAL INVESTIGATION	37.
4.1 Optical Bench and Holographic Accessories	37.
4.2 Gravity Convection Oven	38.

	page
4.3 Temperature Measurement Devices	39.
4.4 Film Plates	39.
4.5 Film Plate Holders	40.
CHAPTER 5 SPECIMEN DESIGN	41.
5.1 Graphite Fiber Reinforced Epoxy Resin Lamina	41.
5.2 Specimen Layup Determination	41.
CHAPTER 6 PHASE 1 EXPERIMENTS	45.
6.1 Test of Holographic Interferometry Technique	46.
6.2 Thermal Deformation of a Cantilever Specimen	48.
CHAPTER 7 PHASE 2 EXPERIMENTS	53.
7.1 Phase 2 Specimen	53.
7.2 Experimental Procedure	54.
7.3 Correlation of Experimental and Theoretical Results	55.
7.4 Discussion of Results	56.
CHAPTER 8 PHASE 3 EXPERIMENTS	59.
8.1 Phase 3 Specimen	59.
8.2 Experimental Procedure	60.
8.3 Correlation of Experimental and Theoretical Results	61.
8.4 Discussion of Results	62.

	page
CHAPTER 9 CONCLUSIONS AND RECOMENDATIONS	64.
9.1 Conclusions	64.
9.2 Recommendations	67.
REFERENCES	70.
APPENDIX A: ESTIMATE OF ERROR IN PHASE 2 TEMPERATURE MEASUREMENT	72.
APPENDIX B: SAMPLE TEANAL OUTPUT	73.
APPENDIX C: LEAST SQUARES COMPUTER PROGRAM USED IN DISPLACEMENT CALCULATION WITH 3-D FRINGE INTERPRETATION METHOD	81.
FIGURES	84.

LIST OF FIGURES

1. Finite Element Idealization of a Plate Specimen for ANSYS Computer Program
2. The Making of a Hologram
3. Holographic Recording Parameters
4. Reconstruction of a Hologram
5. Holographic Interference
6. Schematic Diagram of the 1-D Fringe Interpretation Method
7. Schematic Diagram of the 3-D Fringe Interpretation Method
8. Spectra-Physics Stabelite, Model 124-A He-Ne Laser
9. Blue M Electric Model OV-712A Gravity Convection Oven
10. Jodon MPH-45 W Immersion Type X-Y Plate Holder
11. Wooden Frame Plate Holder
12. Variation of Principle Plate Curvatures With Layer Thickness Proportions
13. Correlation of Deflection Measurements of a Cantilever Specimen
14. Arrangement of Equipment for Multiple Exposures of Cantilever Specimen
15. Fringe Pattern on a Cantilever Specimen with Maximum End Deflection of 142 micro-inches
16. Fringe Pattern on a Cantilever Specimen with Maximum End Deflection of 265 micro-inches
17. Correlation of Deflection Measurements by Multiply Exposed Hologram
18. General Arrangement of Equipment for Multiple Exposures
19. Cantilever Specimen as Clamped in Oven
20. Interference Fringes on Thermally Distorted Cantilever Specimen for a Temperature Increment of 25.5°C to 29.0°C
21. Interference Fringes on Thermally Distorted Cantilever Specimen for a Temperature Increment of 36.7°C to 39.3°C

22. Interference Fringes on Thermally Distorted Cantilever Specimen for a Temperature Increment of 100.2°C to 102.7°C
23. Interference Fringes on Thermally Distorted Cantilever Specimen for a Temperature Increment of 111.7°C to 108.9°C
24. Plate Specimen with $\frac{1}{4}$ " by $\frac{1}{4}$ " Grid
25. Phase 2 Plate Specimen Fixed in Oven
26. Arrangement of Equipment for Phase 2 Experiments
- 27(a). Correlation of Results of Phase 2 Plate Specimen Between 21.1 and 22.1°C
- (b). Hologram of Phase 2 Plate Specimen Between 21.2 and 22.1°C
- 28(a). Correlation of Results of Phase 2 Plate Specimen Between 50.4 and 51.06°C
- (b). Hologram of Phase 2 Plate Specimen Between 50.4 and 51.06°C
- 29(a). Correlation of Results of Phase 2 Plate Specimen Between 80.4 and 81.16°C
- (b). Hologram of Phase 2 Plate Specimen Between 80.4 and 81.16°C
- 30(a). Correlation of Results of Phase 2 Plate Specimen Between 92.56 and 93.24°C
- (b). Hologram of Phase 2 Plate Specimen Between 92.56 and 93.24°C
- 31(a). Correlation of Results of Phase 2 Plate Specimen Between 101.60 and 102.06°C
- (b). Hologram of Phase 2 Plate Specimen Between 101.60 and 102.06°C
32. Normalized Thermal Distortion of Phase 2 Specimen
33. Arrangement of Equipment for Phase 3 Experiment
34. Phase 3 Plate Specimen in Oven
35. Displacement Contours by Holographic Interferometry for Temperature Increment of 2.5°C
36. Displacement Contours from Thermoelastic Theory for Temperature Increment of 2.77°C
37. Correlation of Results of Phase 3 Plate Specimen Between 22.4 and 24.9°C

38. Correlation of Results of Phase 3 Plate Specimen Between 44.35 and 47.0°C
39. Correlation of Results of Phase 3 Plate Specimen Between 46.82 and 49.4°C
40. Correlation of Results of Phase 3 Plate Specimen Between 51.64 and 54.00°C
41. Correlation of Results of Phase 3 Plate Specimen Between 62.77 and 64.80°C
42. Correlation of Results of Phase 3 Plate Specimen Between 73.00 and 75.18°C
43. Correlation of Results of Phase 3 Plate Specimen Between 75.18 and 77.36°C
44. Correlation of Results of Phase 3 Plate Specimen Between 77.36 and 79.27°C
45. Correlation of Results of Phase 3 Plate Specimen Between 84.91 and 86.09°C
46. Correlation of Results of Phase 3 Plate Specimen Between 91.40 and 92.36°C
47. Correlation of Results of Phase 3 Plate Specimen Between 96.91 and 98.55°C
48. Correlation of Results of Phase 3 Plate Specimen Between 105.82 and 107.09°C
49. Normalized Thermal Distortion Along Horizontal Centerline of Phase 3 Specimen
50. Normalized Thermal Distortion Along Vertical Centerline of Phase 3 Specimen
51. Jodon HN-2 He-Ne CW Laser

NOMENCLATURE

$\sigma_{1k}^m, \sigma_{2k}^m, \sigma_{12k}^m$	= mechanical stresses in kth lamina in fiber coordinate system.
$\epsilon_{1k}^m, \epsilon_{2k}^m, \epsilon_{12k}^m$	= mechanical strains in kth lamina in fiber coordinate system.
$\sigma_{xk}^m, \sigma_{yk}^m, \sigma_{xyk}^m$	= mechanical stresses in kth lamina in arbitrary plate coordinate system.
$\epsilon_{xk}^m, \epsilon_{yk}^m, \epsilon_{xyk}^m$	= mechanical strains in kth lamina in arbitrary plate coordinate system.
$\sigma_{1k}^T, \sigma_{2k}^T, \sigma_{12k}^T$	= thermal stresses in kth lamina in fiber coordinate system.
$\epsilon_{1k}^T, \epsilon_{2k}^T, \epsilon_{12k}^T$	= thermal strains in kth lamina in fiber coordinate system.
$\sigma_{xk}^T, \sigma_{yk}^T, \sigma_{xyk}^T$	= thermal stresses in kth lamina in arbitrary plate coordinate system.
$E_{11} = E_L$	= modulus of elasticity of lamina parallel to fibers.
$E_{22} = E_T$	= modulus of elasticity of lamina perpendicular to fibers.
$G_{12} = G_{LT}$	= shear modulus of elasticity of lamina.
$\nu_{12} = \nu_{LT}$	= Poisson's ratio of lamina with direction parallel to fibers considered as major direction.
$\nu_{21} = \nu_{TL}$	= Poisson's ratio of lamina with direction perpendicular to fibers considered as major direction.
$[Q]_k$	= stiffness matrix of kth lamina in fiber coordinate system.
$[\bar{Q}]_k$	= stiffness matrix of kth lamina in arbitrary plate coordinate system.
θ_k	= orientation of kth lamina fiber coordinate system to arbitrary plate coordinate system.
Z_k	= distance from midplane of plate to midplane of kth lamina.

$[T]_k$	= Transformation matrix for transforming tensors from the arbitrary plate coordinate system to the kth lamina coordinate system.
ΔT_k	= temperature excursion of kth lamina.
ΔT_0	= temperature excursion of plate midplane.
$\Delta T/\Delta Z$	= linear thermal gradient through the plate.
α_{1k}	= coefficient of thermal expansion of kth lamina parallel to fibers.
α_{2k}	= coefficient of thermal expansion of kth lamina perpendicular to fibers.
u, v, w	= displacements of plate in x, y, z directions of arbitrary plate coordinate system.
$\epsilon_x^0, \epsilon_y^0, \epsilon_{xy}^0$	= strains at the plate geometrical midplane in arbitrary plate coordinate system.
$\kappa_x, \kappa_y, \kappa_{xy}$	= curvatures of plate in arbitrary plate coordinate system.
t	= total thickness of plate.
h_k	= distance from plate midplane to outer surface of kth lamina.
h_{k-1}	= distance from plate midplane to inner surface of kth lamina.
N_x, N_y, N_{xy}	= stress resultants of plate in arbitrary plate coordinate system.
M_x, M_y, M_{xy}	= moment resultants of plate in arbitrary plate coordinate system.
N_x^T, N_y^T, N_{xy}^T	= thermal stress resultants of plate in arbitrary plate coordinate system.
M_x^T, M_y^T, M_{xy}^T	= thermal moment resultant of plate in arbitrary plate coordinate system.
[A]	= inplane stiffness matrix of plate.
[B]	= coupling stiffness matrix of plate.
[D]	= bending stiffness matrix of plate.

$[A^*], [B^*], [C^*], [D^*]$	= matrices in partially inverted form of general constitutive equations.
$[A'], [B'], [C'], [D']$	= matrices in totally inverted form of general constitutive equations.
R_o	= distance from object beam source to origin of hologram plane.
R_r	= distance from reference beam source to origin of hologram plane.
$U_o(x)$	= amplitude of object beam at point x in hologram plane.
$U_r(x)$	= amplitude of reference beam at point x in hologram plane.
A_o	= amplitude of object beam.
A_r	= amplitude of reference beam.
ϕ_o	= phase of object beam at origin of hologram plane.
ϕ_r	= phase of reference beam at origin of hologram plane.
θ_o	= angle of inclination of object beam to normal passing through hologram plane.
θ_r	= angle of inclination of reference beam to normal passing through hologram plane.
$I(x)$	= intensity of combined object and reference beams at point x in hologram plane.
δ	= displacement vector of object.
Δ	= phase difference of light.
θ	= angle of reference beam to film plate.
α	= angle between displacement vector and reflected ray from point P on object's surface.
β	= angle between ray incident to point P and ray reflected from point P on object's surface.
λ	= wavelength of laser light.
X, Y, Z	= coordinates of point P on object's surface in general coordinate system.

- X_0, Y_0, Z_0 = coordinates of laser source in general coordinate system.
- X_K, Y_K, Z_K = coordinates of observation point K on hologram in general coordinate system.
- R_K = distance from observation point K on hologram to point P on object's surface.
- R_0 = distance from laser source to point P on object's surface.
- n_K = fringe order number as observed from observation point K on hologram.
- m = fringe order number as counted from point of zero displacement on object.
- U, V, W = components of displacement vector of point P in x, y, z directions respectively.

CHAPTER 1

INTRODUCTION

Graphite fiber reinforced plastic materials are presently used in a wide variety of aerospace applications principally due to their high stiffness-to-weight and strength-to-weight ratios. In some applications an additional feature may be utilized by recognizing the achievable low coefficient of thermal expansion which the material may exhibit in certain configurations. Satellite antennas and waveguide tubing are specific examples of such applications which, due to the requirements of dimensional stability over a wide variation of temperature encountered in the upper atmosphere, benefit from the use of both low weight and low coefficient of thermal expansion provided by graphite epoxy laminates. However, it has been realized that the directionally dependent expansion coefficients of the composite and the associated temperature dependent coefficient of thermal expansion of the epoxy resin matrix influence the performance of the component. Consequently, some additional basic information was required on the thermal distortion characteristics of graphite-epoxy resin composite materials over an appropriate temperature range.

Because of the low coefficient of thermal expansion character of the material, specialized measurement methods were required in order to measure the small deformations induced. In this investigation, holographic interferometry is used which, is of relatively low cost, is highly accurate, and can measure deformations as low as one-quarter of the wavelength of the interfering light. Holographic interferometry

also has the additional favorable factor of requiring no physical contact with the hot deforming specimen. Furthermore, holographic interferometry is able to measure the full displacement field of the specimen unlike conventional displacement transducers which can handle only point by point measurements.

The present study involves the deformation of a number of plate-like specimens of graphite fiber reinforced epoxy resin composites which simulate a satellite component undergoing a change of thermal environment above room temperature. This deformation would be measured by various holographic interferometry techniques and the measurements compared with the deformation as predicted by various theoretical methods now being used in the design of composite components. Discrepancies between the theoretical and experimental results would indicate deficiencies in the theoretical predictions which, although not corrected in this study, are explained and discussed.

1.1 Literature Review

Among the more recent literature encountered on the subject of the thermal deformation of composites is Chailleux and Ferte's account of the application of a torsional pendulum configuration to the measurement of thermally induced twisting of non-symmetrical or unbalanced Boron-Aluminum composite plates [1].* Under controlled heating rates the torsional deflection was monitored from room

*Number in brackets denote the Reference number cited in this thesis.

temperature to 150°C. Discrepancies between theory and experiment were attributed to the influence of specimen width, although no reasons for such differences were advanced. It was also noted that the plastic deformation of the aluminum matrix created difficulty.

In a study by Kalnin [2] a Netzch dilatometer Model 402/T2 distributed by the Dynatech Corporation was used to measure the in-plane thermal expansion of high elastic modulus carbon fiber epoxy matrix composites. The range of temperatures covered in the experiments was from 25 to 150°C. Both unidirectional and quasi-isotropic specimens were employed. The experimental results were reported to be somewhat higher than the predicted values.

In a study by Geiler [3], the thermoelastic properties of graphite-epoxy composite materials were examined in detail. In this work both solid laminates and aluminum honeycomb sandwich panels with laminated composite facesheets were studied. The specimens were subjected to a temperature range of -157°C to 93°C using two different heating cycles. In-plane deformations were measured in two perpendicular directions by an electro-optical extensometer manufactured by the Optron Corporation. Experimental values were compared with analytical predictions based on macromechanical elastic theory formulation. The analysis predicted a significant variation in the coefficient of thermal expansion over the temperature range. Comparison between experimental and predicted results was considered good allowing for the experimental limitations. It was also significant that results varied according to both the rate of heating and the sense, i.e., temperature increase or decrease.

Fahmy and Ragai [4] reported on the in-plane thermal expansion behavior of both unidirectional and balanced angle ply graphite-epoxy composites. Measurements were made in different directions as defined by the angle of the fibers using dilatometric methods and specimens were heated between 20°C and 180°C. From the results on unidirectional composites it was found that unidirectional composites may be treated as homogenous orthotropic bodies which are isotropic in the plane perpendicular to the fiber direction and that the in-plane coefficient of thermal expansion varies with the angle between the direction of measurement and that of the fiber as expressed by the simple formula:

$$\alpha_{\theta} = \alpha_L \cos^2 \theta + \alpha_T \sin^2 \theta$$

In further study by Fahmy and Ragai [5], the thermal expansion coefficient of laminated composites in the thickness direction was considered. Experiments performed on angle ply composites yielded a maximum coefficient of thermal expansion in the thickness direction for orientations of $\pm 45^\circ$. In a subsequent study by Pagano [6], the error in the analysis of [5] was pointed out. Pagano's analysis, and that of Fahmy and Ragai, was confined to a linear treatment.

An article by W. R. Goggin [7] represents a contribution most closely related to this study. Goggin exploited holographic interferometry to monitor thermal distortion of a 20 cm. diameter graphite-epoxy/honeycomb paraboloid reflector. He also used holography to examine the micro-yield and micro-creep characteristics of graphite-epoxy composites. However, measurement of thermal expansion coefficients of the composites between 5°C and 40°C was achieved by

using a vacuum dilatometer which had a reputed accuracy of $\pm 1 \times 10^{-8}/^{\circ}\text{C}$ with very low expansion materials. Despite similarities in materials and techniques in Goggin's work and this study it is important to note that his objectives were concerned with the evaluation of a new material for use in precision optical equipment.

Pirgon et. al. [8] tested specimens comprised of Courtaulds HTS fibers in ERLA 4617 resin in order to measure the thermal expansion coefficients. Specimens included both pure resin and unidirectional composites with measurements based on the use of Fizeau interference fringes taken in various directions to the fiber direction. The work of Pirgon et. al. positively indicated that a marked change with temperature in the transverse coefficient of thermal expansion, i.e., in the direction perpendicular to the fibers, is a manifestation of resin properties. The same trend was exhibited in the pure resin sample and suggested that the phenomenon is connected with an onset of chain mobility which heralds an approach to the glass transition. Temperatures at which the marked change in coefficient of thermal expansion occurs spanned the 10 to 80°C range. No significant change in the thermal response of the composites could apparently be attributed to structural changes of the carbon fibers in the temperature range of -150°C to $+130^{\circ}\text{C}$.

On the subject of holographic interferometry measurements, Hsu and Moyer [9] reported on the one-dimensional displacement measurement of the deflection of a mechanically loaded Poco graphite cantilever beam at temperatures up to 800°C with high accuracies. Further on the subject of holographic interferometry, Heflinger et. al.

[10] reported on the measurement of the coefficients of thermal expansion of various diffuse reflecting bodies by holographic interferometry.

1.2 Industrial Interaction

As mentioned earlier, the present investigation is geared to the applications of aerospace components such as satellite antennas. Satellite antennas, requiring a high dimensional stability over the temperature range of -100°C to $+110^{\circ}\text{C}$ encountered in the upper atmosphere, make use of the low coefficient of thermal expansion inherent in graphite fiber reinforced epoxy composite plates. Furthermore, satellite antennas use a category of laminated plate known as balanced or symmetric. A balanced plate is laminated such that at an equal distance from its midplane the lamina are identical with regards to materials, thickness and fiber orientation. Due to the symmetry of stacking of this type of plate, a uniform temperature rise will cause no bending or warpage of the plate as thermally induced moments on one side of the midplane are balanced by equal and opposite moments on the other side. However, if the temperature rise is non-uniform through the thickness, a condition often encountered in the upper atmosphere due to large temperature differences between sunlit and shadowed areas, a certain amount of bending of balanced plates would occur due to unequal moments about the midplane. This situation can be similar to a uniform temperature rise in an unbalanced plate depending upon the lamination sequence and orientation of the plate. In both situations, out of plane warping or bending would occur.

The merit of the present investigation is well recognized by the major manufacturers of satellite antenna components from graphite epoxy composites, in particular, the Bristol Aerospace, Ltd. of Winnipeg.

CHAPTER 2

THEORETICAL ASPECTS OF THE THERMAL DEFORMATION OF LAMINATED COMPOSITE PLATES

2.1 Basic Theory

Generally speaking a composite material is a material with several distinct phases present. Normally the composite will consist of reinforcing fibers supported in a matrix material. This study has been concerned with filamentary composites, i.e., continuous fiber composites as opposed to short fiber or whisker reinforced composites. The filamentary composite is made up of several plies or laminae. One lamina consists of rows of parallel fibers surrounded by a matrix. Each of the laminae can be considered to be macroscopically orthotropic because of the fibers being oriented in the same direction. Laminae are then stacked with various fiber directions between lamina to obtain a laminated composite plate with the desired physical properties.

The theory behind the thermal deformation of a laminated compositated plates is based on certain assumptions. These are:

- (1) Orthotropic lamina are comprised of isotropic or anisotropic elastic fibers in an isotropic matrix.
- (2) The fibers may have anisotropic thermoelastic properties.
- (3) Thin plate theory may be applied.
- (4) Linear thermal gradients may occur through the thickness.
- (5) The lamina is a homogenous medium.

The theory behind the thermal deformation of laminated

composite plates using these assumptions is essentially extracted from References [11] and [12]. Using this theory in a more rigorous form, consider a laminated plate composed of L homogeneous orthotropic laminae. If the k^{th} lamina at a distance Z_k from the midplane of the plate, having its fibers oriented at an angle θ_k to the general coordinate axes of the plate, is considered, the stress-strain relationships can be given in a state of plane stress as:

$$\begin{aligned}\sigma_{1k}^m &= Q_{11k}\epsilon_{1k}^m + Q_{12k}\epsilon_{2k}^m \\ \sigma_{2k}^m &= Q_{12k}\epsilon_{1k}^m + Q_{22k}\epsilon_{2k}^m \\ \sigma_{12k}^m &= Q_{33k}\epsilon_{12k}^m\end{aligned}\quad (2.1)$$

where: $\sigma_k^m = k^{\text{th}}$ ply mechanical stresses in longitudinal, transverse, and twist directions relative to the fibers.

$\epsilon_k^m = k^{\text{th}}$ ply mechanical strains in directions corresponding to the stresses.

$Q_k =$ components of the k^{th} ply elastic stiffness matrix such that:

$$Q_{11} = \frac{E_{11}}{1 - \nu_{12}\nu_{21}}$$

$$Q_{22} = \frac{E_{22}}{1 - \nu_{21}\nu_{12}}$$

$$Q_{12} = \frac{\nu_{12}E_{22}}{1 - \nu_{12}\nu_{21}}$$

$$Q_{13} = Q_{23} = 0$$

$$Q_{33} = G_{12}$$

$$\nu_{21}E_{11} = \nu_{12}E_{22}$$