BUCKLING OF SIMPLY SUPPORTED PLYWOOD PANELS IN COMPRESSION

A Thesis

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by

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ABSTRACT

It was the purpose of this thesis to derive expressions which could be used to predict the buckling of plywood panels.

To solve the problem the plywood sheet was considered to be an orthotropic plate. An expression was derived using the differential equation of the deflection surface of an orthotropic plate. Boundary conditions were used which considered the edge members to have both flexural and torsional stiffness.

The expression derived was solved for a 1/4 inch plywood plate with several different edge members. Special solutions such as simple support and free edge were also obtained from the expression derived. These solutions were obtained with the aid of a computer. Charts of all solutions obtained were drawn which made simple and fast solutions possible by hand for the complicated expression derived.

Good quantitative agreement between previous published solutions was obtained for the special solutions. The theoretical results showed that the local buckling of the plywood skin of the panel could be predicted by assuming that the plywood plate was simply supported on its unloaded edges.

A series of tests on full size panels were run. Quantitative agreement was not obtained from these tests but recommendations for future research were made which could lead to quantitative agreement.

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NOTATION

a - length of plywood plate

b - width of plywood plate

t - thickness of plywood plate

σ_x - Uniform compressive stress in x direction over the ends of the plywood plate.

σ_b - uniform compressive stress in x direction over the ends of the wood edge members.

σ_{cr} - critical buckling stress of plywood plate

E_x - modulus of elasticity of orthotropic plate in the x direction

E_y - modulus of elasticity of orthotropic plate in the y direction

ν_y - Poisson's ratio of an orthotropic plate associated with strain in the x direction due to stress in the y direction.

ν_x - Poisson's ratio of an orthotropic plate associated with strain in the y direction due to stress in the x direction.

G - modulus of rigidity of an orthotropic plate

w - deflection of a plate in the z direction.

M_x - moment per unit length of an orthotropic plate along the edges $x=a$, and $x=o$

M_y - moment per unit length of an orthotropic plate along the edges $y={}^b/_2$ and $y= -{}^b/_2$

E_b - modulus of elasticity in bending of the wood edge members in the x direction.

E_{bp} - modulus of elasticity in plane stress of the wood edge members in the x direction.

A_b - area of wood edge members

I_b - moment of inertia of wood edge members about the y axis

G_b - modulus of rigidity of wood edge members

d - depth of the cross section of the edge member

c - width of the cross section of the edge member

B - constant used to determine the torsional rigidity of the edge members

C - torsional rigidity of the edge members

E_L - modulus of elasticity parallel to the grain of wood

E_T - modulus of elasticity of wood perpendicular to the grain where the deformation is tangentical to the annual rings

E_R - modulus of elasticity of wood perpendicular to the grain where the deformation is radial to the annual rings

G_{LT} - modulus of rigidity of wood associated with shear deformation in the LT plane from shear stresses in the LR and RT planes

ν_{LT} - Poisson's Ratio of wood associated with lateral deformation in the T direction due to stress in the L direction

ν_{TL} - Poisson's Ratio of wood associated with lateral deformation in the L direction due to stress in the T direction.

E_{xp} - modulus of elasticity of a plywood plate in plane stress

I - moment of inertia of a one foot strip of the plywood plate

I_1 - moment of inertia of a one foot strip of the plies of a plywood plate whose grain is parallel to the x direction

I_2 - moment of inertia of a one foot strip of the plies of a plywood plate whose grain is perpendicular to the x direction

CHAPTER I

INTRODUCTION

1.1 Statement of problem.

A plywood panel when loaded in compression may fail due to buckling before the allowable compressive stress based on the ultimate compressive strength is reached. The true allowable compressive stress should therefore depend upon the buckling stress of the plywood panel.

1.2 Purpose of thesis.

It was the purpose of this thesis to derive an expression or expressions which could be used to predict the buckling of plywood panels and hence could be used to establish proper allowable compressive stresses for these panels. It was also the purpose to run a small series of tests to examine the validity of any expressions derived.

CHAPTER II

DERIVATION OF CRITICAL BUCKLING STRESS EQUATION

2.1 Introduction.

The preceding chapter stated the purpose of this thesis. In this chapter the expression required will be derived.

2.2 Panel considered.

The panel considered is shown in Figure 2.1a. The panel was composed of a single rectangular sheet of plywood with two solid wood members fastened along the edges which were parallel to the face grain of the plywood. The edge members were equal and rectangular in cross section. The plywood sheet was a units in length, b units in width, and t units in thickness.

2.3 Choice of axes and loading.

The choice of axes was as shown in Figure 2.1b. The x axis was parallel to the face grain of the plywood, and therefore parallel to the edge members.

The loading of the panel is also shown in Figure 2.1b. The panel was assumed to be loaded with uniform compressive forces parallel to the x axis over the ends $x = o$, and $x = a$.

2.4 Assumptions.

The plywood sheet was assumed to be an orthotropic plate compressed in its middle plane by uniform forces parallel to the x axis. The loaded edges of the plate were assumed to be simply supported (see

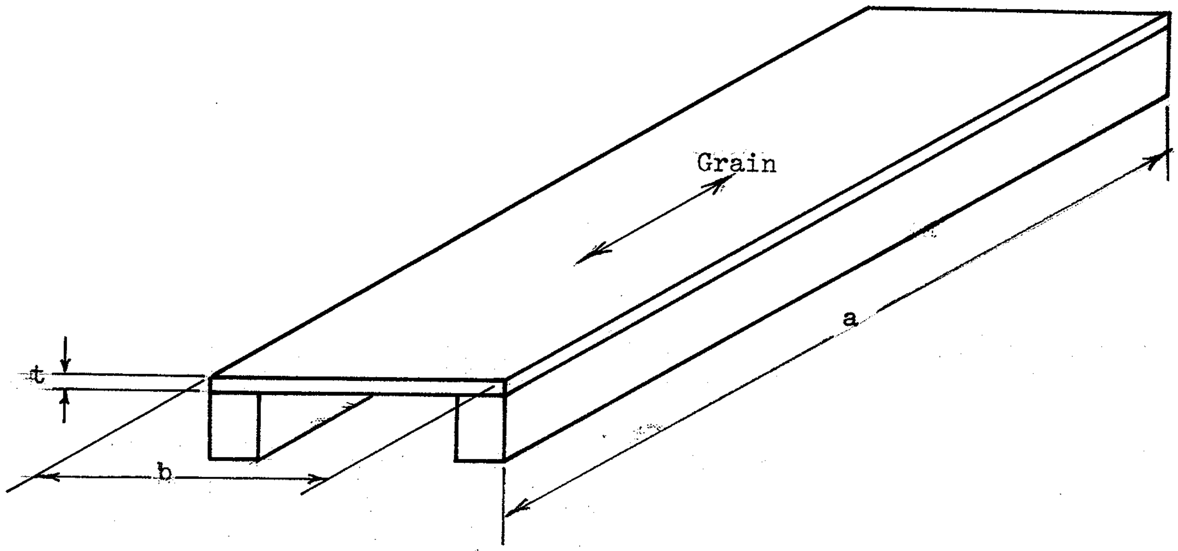


Figure 2.1(a)

Panel Considered

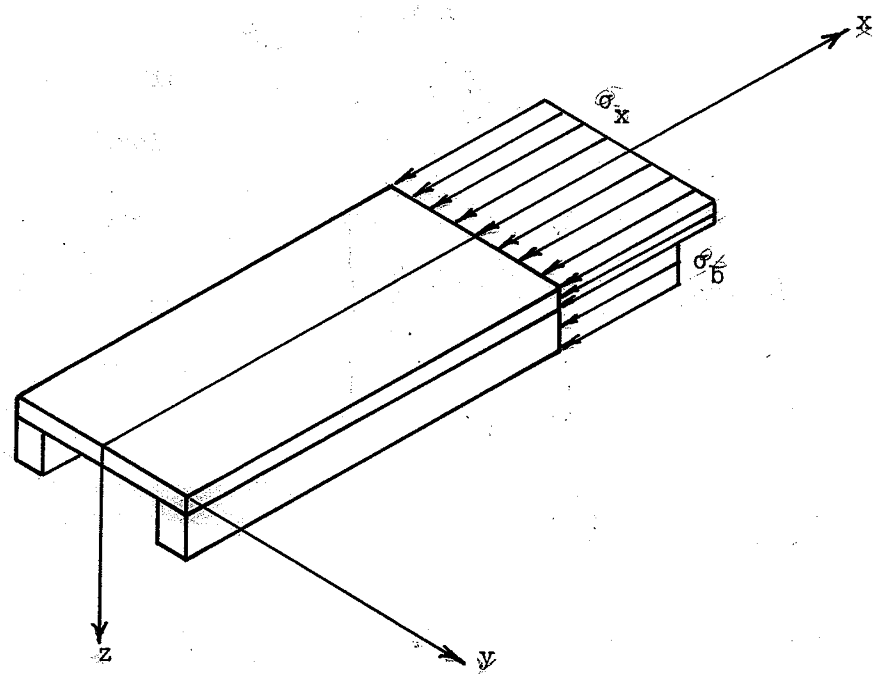


Figure 2.1(b)

Loading and Choice of Axes

Figures 5.13, 5.14 and 5.15 for loading method). The plywood plate, along its unloaded edges ($y=\pm\frac{b}{2}$), was assumed to be restrained against rotation and deflection by the two wood members. The wood edge members were assumed to be loaded by uniform compressive forces.

2.5 Method of solution.

The method of solving the stability problem was to assume that the plate buckles slightly under the action of forces applied in its middle plane and then to calculate the magnitudes that the forces must have in order to keep the plate in such a slightly buckled shape. This was done by solving the differential equation of the deflection surface of a thin orthotropic plate for the boundary conditions pertaining to the plate considered. Timoshenko shows [2-1]* that the assumed buckling of the plate is possible only for certain values of compressive stress. The smallest of these compressive stresses is the required stress for which buckling first takes place.

2.6 Differential Equation.

From equation (A-41), in Appendix A, the differential equation of the deflection surface of an orthotropic plate, loaded by compressive forces in the x direction, is

$$D_x\frac{\partial^4 w}{\partial x^4} + 2H\frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y\frac{\partial^4 w}{\partial y^4} + t\sigma_x\frac{\partial^2 w}{\partial x^2} = o \qquad (2\text{-}1)$$

where

$$D_x = \frac{E_x t^3}{12(1-\nu_y\nu_x)} \qquad (2\text{-}2)$$

* Numbers in square brackets are to the references on page 56.

$$D_y = \frac{E_y t^3}{12(1-\nu_y\nu_x)} \qquad (2\text{-}3)$$

$$D_{xy} = \frac{Gt^3}{12} \qquad (2\text{-}4)$$

$$2H = D_x\nu_y + 4D_{xy} + D_y\nu_x \qquad (2\text{-}5)$$

E_x is the Modulus of Elasticity in the x direction.

E_y is the Modulus of Elasticity in the y direction.

G is the Modulus of Rigidity.

ν_x is Poisson's Ratio associated with strain in the y direction due to stress in the x direction.

ν_y is Poisson's Ration associated with strain in the x direction due to stress in the y direction.

As stated in Section 2.5 of this chapter the differential equation (2-1) must be solved using the appropriate boundary conditions to yield the critical buckling stress.

2.7 Boundary conditions.

The panel to be studied was described in section 2.2. Now, the boundary conditions, appropriate to the panel under consideration, will be discussed and the appropriate equations developed.

a) Boundary conditions at edges x=o, and x=a.

At the edges x=o, and x=a, the plate is simply supported; hence the deflection and the moment are zero along these edges.

Thus

$$w = o \qquad (2\text{-}6)$$

And

$$M_x = -D_x\left[\frac{\partial^2 w}{\partial x^2} + \nu_y\frac{\partial^2 w}{\partial y^2}\right] = o$$

where

M_x is the moment per unit length along the edges x=o, and x=a. (equation (A-32), Appendix A.)

Since D_x does not equal zero, it follows that

$$\frac{\partial^2 w}{\partial y^2} + \nu_y \frac{\partial^2 w}{\partial y^2} = 0 \qquad (2\text{-}7)$$

Equations (2-6) and (2-7) are the two boundary conditions required along the edges x=o, and x=a.

b) Boundary conditions at edges $y = \frac{+b}{-2}$

At the edges $y = \frac{\pm b}{2}$ the plate is supported by edge members assumed to have flexural and torsional stiffness. That is, the bending and twisting of the edge member is considered.

The bending of these members is considered first. It is assumed that they are pin-ended and are compressed with a stress σ_b at the ends x=a, and x=o.

The differential equation of deflection is [2-1]

$$E_b I_b \frac{\partial^4 w}{\partial x^4} + A_b \sigma_b \frac{\partial^2 w}{\partial x^2} = q \qquad (2\text{-}8)$$

where

E_b is the Modulus of Elasticity of the edge member in the x direction.

I_b is the moment of inertia of the edge member about the y axis.

A_b is the area of the edge member.

q is the intensity of the load transmitted from the plate to the edge member.

The expression for q is [2-1] (using orthotropic plate expressions)

$$q = \pm D_y \left[\frac{\partial^3 w}{\partial y^3} + \nu_x \frac{\partial^3 w}{\partial y \partial x^2} \right] \pm 4 D_{xy} \frac{\partial^3 w}{\partial x^2 \partial y} \quad (y = \pm \frac{b}{2}) \qquad (2\text{-}9)$$

Inserting the expression for q from equation (2-9) into equation (2-8) the following equation is obtained:

$$E_b I_b \frac{\partial^4 w}{\partial x^4} + A_b \sigma_b \frac{\partial^2 w}{\partial x^2} = \pm D_y \left[\frac{\partial^3 w}{\partial y^3} + \nu_x \frac{\partial^3 w}{\partial y \partial x^2} \right] \pm 4 D_{xy} \frac{\partial^3 w}{\partial x^2 \partial y} \quad (y = \pm \frac{b}{2}) \qquad (2\text{-}10)$$

The twisting of the edge members is now considered. The angle of rotation of any cross section of the member is [2-1]

$$\frac{\partial w}{\partial y}$$

and the rate of change of this angle along the edges is

$$\frac{\partial^2 w}{\partial x \partial y}$$

Hence the twisting moment in the member is

$$C \frac{\partial^2 w}{\partial x \partial y}$$

where

C is the torsional rigidity of the member

This moment varies along the edge, since the plate is rigidly connected with the member and transmits continuously distributed moments to it. The magnitude of these applied moments per unit length is equal and opposite to the bending moments per unit length in the plate. Hence, from a consideration of the rotational equilibrium of an element of the edge member, the following equation is obtained:

$$C \frac{\partial}{\partial x} \left(\frac{\partial^2 w}{\partial x \partial y} \right) = -M_y$$

Substituting the expression for M_y (equation (A-33) Appendix A) into the above equation yields

$$C \frac{\partial}{\partial x}\left(\frac{\partial^2 w}{\partial x \partial y}\right) = D_y \left[\frac{\partial^2 w}{\partial y^2} + \nu_x \frac{\partial^2 w}{\partial x^2}\right] \qquad (2\text{-}11)$$

Equations (2-10) and (2-11) are the two required boundary conditions for the edge $y = \frac{b}{2}$

2.8 Derivation of critical buckling stress equation [2-1]

In this section the equation that gives the critical buckling stress of the panels is derived.

Assuming that under the action of compressive forces the plate buckles in m sinusoidal half waves in the x direction, the solution of the differential equation (2-1) is taken in the form,

$$w = f(y) \sin \frac{m\pi x}{a} \qquad (2\text{-}12)$$

where

f(y) is a function of y alone, which is determined by the boundary conditions along the edges at $y = \pm\frac{b}{2}$. Equation (2-12) satisfies the boundary conditions, equations (2-6) and (2-7), along the simply supported edges, x=o and x=a, of the plate.

Upon introducing equation (2-12) into the differential equation (2-1) the following ordinary differential equation of the fourth order is obtained:

$$f^{IV}(y) - \frac{2H}{D_y}\frac{m^2\pi^2}{a^2} f^{II}(y) + \left[\frac{D_x}{D_y}\frac{m^4\pi^4}{a^4} - \frac{t\sigma_{cr}}{D_y}\frac{m^2\pi^2}{a^2}\right] f(y) = o \qquad (2\text{-}13)$$

where

σ_x is replaced by σ_{cr}, the unknown critical stress at which the plate buckles.

The general solution of equation (2-13) is

$$f(y) = A_1 \cos \beta_y + A_2 \sin \beta_y + A_3 \cosh \alpha_y + A_4 \sinh \alpha_y$$

where

$$\beta = \sqrt{-\frac{H}{D_y}\frac{m^2\pi^2}{a^2} + \sqrt{\frac{m^2\pi^2}{a^2}\left[\left(\frac{H^2}{D_y^2} - \frac{D_x}{D_y}\right)\frac{m^2\pi^2}{a^2} + t\frac{\sigma_{cr}}{D_y}\right]}}$$

$$\alpha = \sqrt{\frac{H}{D_y}\frac{m^2\pi^2}{a^2} + \sqrt{\frac{m^2\pi^2}{a^2}\left[\left(\frac{H^2}{D_y^2} - \frac{D_x}{D_y}\right)\frac{m^2\pi^2}{a^2} + t\frac{\sigma_{cr}}{D_y}\right]}}$$

Thus, the general solution of the differential equation (2-1) is

$$w = \sin\frac{m\pi x}{a}\left(A_1 \cos \beta_y + A_2 \sin \beta_y + A_3 \cosh \alpha_y + A_4 \sinh \alpha_y\right) \quad (2\text{-}14)$$

Since the edge members were assumed to be equal in every respect, the deflection, w, corresponding to the smallest value of σ_{cr}, is a symmetric function of y. Therefore the terms $A_2 \sin \beta_y$ and $A_4 \sinh \alpha y$, in equation (2-14), vanish and solution reduces to

$$w = \sin\frac{m\pi x}{a}\left(A_1 \cos \beta_y + A_3 \cosh \alpha_y\right) \quad (2\text{-}15)$$

To determine the constants A_1 and A_3 in equation (2-15) the boundary conditions along the edge $y = \frac{b}{2}$ are used.

This is done by introducing the solution for the deflection, equation (2-15), into equation (2-10) and (2-11) (which are the expressinns for the boundary conditions) for a value of $y = \frac{b}{2}$.

This gives the two equations:

$$A_1 \left[-E_b I_b \frac{m^4\pi^4}{a^4} \cos \beta_y + D_y\beta^3 \sin \beta_y + \nu_x \frac{m^2\pi^2}{a^2} D_y \sin \beta_y \right.$$

$$\left. + 4 D_{xy} \frac{m^2\pi^2}{a^2} \beta \sin \beta y + A_b\sigma_b \frac{m^2\pi^2}{a^2} \cos \beta y \right] +$$

$$A_3 \left[-E_b I_b \frac{m^4\pi^4}{a^4} \cosh \alpha_y + D_y \alpha^3 \sinh \alpha_y - \right.$$

$$\nu_x \frac{m^2\pi^2}{a^2} D_y \alpha \sinh \alpha_y - 4 D_{xy} \frac{m^2\pi^2}{a^2} \alpha \sinh \alpha_y +$$

$$\left. A_b\sigma_b \frac{m^2\pi^2}{a^2} \cosh \alpha_y \right] = o \qquad (2\text{-}16)$$

$$A_1 \left[-C \frac{m^2\pi^2}{a^2} \beta \sin \beta_y - D_y\beta^2 \cos \beta_y - D_y\nu_x \frac{m^2\pi^2}{a^2} \cos \beta_y \right] +$$

$$A_3 \left[C \frac{m^2\pi^2}{a^2} \alpha \sinh \alpha_y + D_y\alpha^2 \cosh \alpha_y - D_y\nu_x \frac{m^2\pi^2}{a^2} \cosh \alpha_y \right]$$

$$= o \qquad (2\text{-}17)$$

Equations (2-16) and (2-17) are homogeneous linear equations and non-zero values for A_1 and A_3 are only possible when the determinant, Δ, of this system of equations vanishes. Therefore Δ=o is the buckling criterion, which leads to the stability condition

$$\left[-E_b I_b \; C \; \alpha \frac{m^4\pi^4}{a^4} + A_b\sigma_b \; C \; \alpha \frac{m^2\pi^2}{a^2} + D_y{}^2 B^2 \alpha^3 \frac{a^2}{m^2\pi^2} - \right.$$

$$D_y{}^2 \nu_x B^2 \alpha - D_y 4 D_{xy} B^2 \alpha + D_y{}^2 \nu_x \alpha^3 - D_y{}^2 \nu_x{}^2 \alpha \frac{m^2\pi^2}{a^2} -$$

$$D_y 4 D_{xy} \nu_x \alpha \frac{m^2\pi^2}{a^2} \Bigg] \tanh \alpha_y +$$

$$\Bigg[D_y{}^2 B^3 \frac{\alpha^2 a^2}{m^2\pi^2} - D_y{}^2 \nu_x \beta^3 + D_y{}^2 \nu_x \beta \alpha^2 - D_y{}^2 \nu_x{}^2 \beta \frac{m^2\pi^2}{a^2} +$$

$$D_y \; 4 \; D_{xy} \; \beta \; \alpha^2 - D_y \; 4 \; D_{xy} \; \nu_x \; \beta \; \frac{m^2\pi^2}{a^2} - C \; E_b I_b \; \beta \; \frac{m^4\pi^4}{a^4} +$$

$$C \; A_b \sigma_b \; \beta \; \frac{m^2\pi^2}{a^2} \Bigg] \tan \beta_y +$$

$$\Bigg[D_y \; C \; \beta^3 \; \alpha + D_y \; C \; \beta \; \alpha^3 \Bigg] \tan \beta_y \tanh \alpha_y +$$

$$\Bigg[-E_b I_b \; D_y \; \alpha^2 \; \frac{m^2\pi^2}{a^2} + A_b \sigma_b \; D_y \; \alpha^2 - E_b I_b \; D_y \; \beta^2 \; \frac{m^2\pi^2}{a^2} +$$

$$A_b \sigma_b \; \beta^2 \; D_y \Bigg] = o \qquad (2\text{-}18)$$

Equation (2-18) is the critical buckling stress equation. Its solution gives the critical buckling stress, σ_{cr}, of the panels. The next two chapters will be concerned with the solution of this equation.

CHAPTER III

TERMS IN CRITICAL BUCKLING STRESS EQUATION

3.1 Introduction.

In this chapter the constants which appear in the critical buckling stress equation will be evaluated so that a solution will be possible.

3.2 Elastic Constants of Wood.

Wood was considered to be an orthotropic material. An orthotropic material is one which has three mutually perpendicular planes of elastic symmetry. For wood, these planes are determined by the three principal directions, the longitudinal, radial and tangential directions, as shown in Figure 3.2

The modulus of elasticity of wood perpendicular to the grain is designated as E_T when the direction in which the deformation takes place is tangential to the annual rings and as E_R when the direction is radial to the annual rings. The modulus of elasticity parallel to the grain is E_L.

The modulus of rigidity is associated with shear deformation in one of the three mutually perpendicular planes, the L, T or R planes, and with shear stresses in the other two planes. The modulus of rigidity, G_{LT}, refers to shear deformation in the $_{LT}$ plane due to shear stresses in the $_{LR}$ and $_{RT}$ planes.

Because wood has three principal direction, there are six different values for Poisson's ratio. Let Poisson's ratio be ν_{ab},

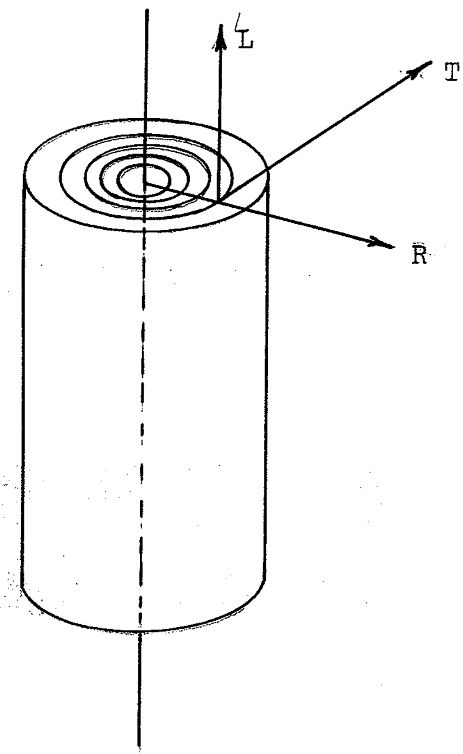


Figure 3.2

Three Principal Directions of Wood

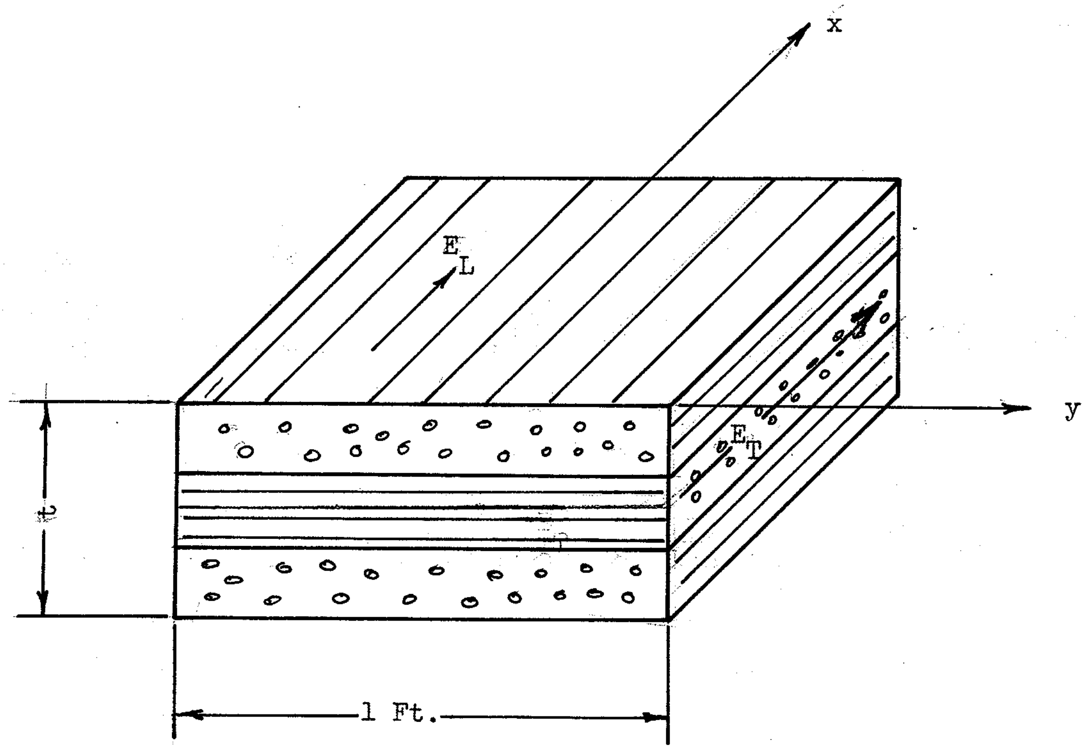


Figure 3.3

Arrangement of Plywood Veneers

where a is the direction in which the load is applied and b is the direction in which the lateral deformation, due to the applied load, occurs. Then the six combinations for a and b are the off-diagonal elements of the matrix:

	L	T	R
L		ν_{LT}	ν_{LR}
T	ν_{TL}		ν_{TR}
R	ν_{RL}	ν_{RT}	

Because wood is assumed to have three planes of elastic symmetry, the following relationship, which is called Betti's reciprocal Theorem, is obtained:

$$\frac{\nu_{ab}}{\nu_{ba}} = \frac{E_a}{E_b}$$

From this relationship the following equation is obtained:

$$\frac{\nu_{TL}}{\nu_{LT}} = \frac{E_T}{E_L} \qquad (3\text{-}1)$$

The modulus of elasticity, E_L, in bending used for Douglas Fir was 1.95×10^6 psi. It was found in reference [3-1]. It has been found [3-2] that a good average for $\frac{E_T}{E_L}$ is 0.05. Therefore, the value of E_T used was 9.75×10^4 psi.

The value [3-2] of ν_{LT} used was 0.45. Then, from equation (3-1) and the assumed $\frac{E_T}{E_L}$ value, ν_{TL} was determined to be 0.0225.

It has been found [3-2] that a good average for $\frac{G_{LT}}{E_L}$ is 0.06.

Therefore, the value of the modulus of rigidity of Douglas Fir used was 1.17×10^5 psi.

The value of 1.10×10^6 psi. was found [3-3] for the modulus of elasticity, E_b, of the spruce edge member. A value of 70×10^3 psi. was used for the modulus of rigidity of the edge member.

3.3 Plywood as an orthotropic plate.

The ordinary wood veneer can be regarded as an orthogonal, anistropic, homogeneous material to which the theory of elasticity and the theory of bending of orthotropic plates applies.

Plywood comprising single wood veneers at ninety degrees to each other also can be regarded as an orthotropic plate, but a plate heterogeneous over its thickness. Because of this heterogeneity, if the relations applicable to homogeneous material are formally retained, the modulus of elasticity of a plywood plate has different values for stresses in its plane than for bending stresses. Poisson's ratios are also different for a plywood plate compared to a single veneer. However, the modulus of rigidity, G_{LT}, for a plywood plate is not affected because the modulus of rigidity of each veneer is the same regardless of whether the grain of the veneer is in the x direction or the y direction.

3.4 Elastic and plate constants of plywood.

To take into account the effect of plywood construction on the modulus of elasticity, simple approximate equations were used. [3-1] [3-2] [3-4]

The equation for the modulus of elasticity of a plywood plate in plane stress is

$$E_{xp} = \sum_i \frac{E_{xi}\ t_i}{t} \qquad (3\text{-}2)$$

where

E_{xp} is the modulus of elasticity of a plywood plate in plane stress.

E_{xi} is the modulus of elasticity of the ith ply in the x direction.

t_i is the thickness of the ith ply.

t is the thickness of the plywood plate.

The equation for the modulus of elasticity in the x direction of a plywood plate in bending is

$$E_x I = \sum E_{xi} I_i \qquad (3\text{-}3)$$

where

E_x is the modulus of elasticity in the x direction of a plywood plate in bending.

I is the moment of inertia of a one foot strip of the plywood plate.

E_{xi} is the modulus of elasticity of the ith ply in the x direction.

I_i is the moment of inertia of the ith ply.

Since $I = t^3$ for a 1 ft. strip of thickness t inches, and referring to Figure 3.3, equation (3-3) becomes

$$E_x t^3 = E_L I_1 + E_T I_2 \qquad (3\text{-}4)$$

where

I_1 is the moment of inertia for a 1 ft. strip of the cross sectional area of the plies whose grain is parallel to the x direction.

I_2 is the moment of inertia for a 1 ft. strip of the cross sectional area of the plies whose grain is perpendicular to the x direction.

Since $\frac{E_T}{E_L}$ is 0.05, equation (3-4) becomes

$$E_x = \frac{E_L}{t^3} (I_1 + 0.05 I_2) \tag{3-5}$$

Similarly, the modulus of elasticity (E_y) in the y direction of a plywood plate in bending, is

$$E_y = \frac{E_L}{t^3} (0.05 I_1 + I_2) \tag{3-6}$$

plate constants D_y, D_x and D_{xy}

From equation (2-3) the expression for D_y is

$$D_y = \frac{E_y t^3}{12(1-\nu_y\nu_x)}$$

The values of ν_y and ν_x for 1 ply have been evaluated in Section 3.2 of this chapter. These values do not apply to the entire thickness but the product $\nu_y \nu_x$ for 1 ply can be used without serious error [3-1] to apply to the entire thickness. Therefore, using the values of 0.0225 and 0.45 from Section 3.2 for ν_{TL} and ν_{LT}, respectively, the expression for D_y becomes

$$D_y = \frac{t^3 E_y}{11.9} \tag{3-7}$$

Similarly, the expression for D_x is

$$D_x = \frac{t^3 E_x}{11.9} \tag{3-8}$$

From equation (2-4) the expression for D_{xy} is

$$D_{xy} = \frac{G\ t^3}{12}$$

As explained in Section 3.3, the modulus of rigidity (for any given direction) is not affected by the heterogeneity through the thickness. Therefore the value of 1.17 x 10^5 psi for G_{LT} found for 1 ply of Douglas Fir can be used for G in equation (2-4) and, hence, equation (2-4) becomes

$$D_{xy} = 9750\ t^3 \qquad (3\text{-}9)$$

plate constants $\nu_x D_y$ and H.

From equation (3-7) $\nu_x D_y$ is

$$\nu_x D_y = \frac{E_y \nu_x\ t^3}{11.9} \qquad (3\text{-}10)$$

From equation (3-1) $\nu_{TL}E_L$ equals $\nu_{LT}E_T$ for 1 ply. Therefore $\nu_x E_y$ evaluated for 1 ply is constant throughout the thickness and $\nu_{LT}E_T$ or ($\nu_{TL}E_L$) can be used instead of $\nu_x E_y$. Equation (3-10) therefore becomes

$$\nu_x D_y = 3690\ t^3 \qquad (3\text{-}11)$$

From equation (2-5) and noting that from explanation above $\nu_x E_y = \nu_x E_y$ the expression for H is

$$H = \nu_x D_y + 2\ D_{xy} \qquad (3\text{-}12)$$

By substituting the expressions for $\nu_x D_y$ and D_{xy}, from equations (3-11) and (3-9) respectively, equation (3-12) becomes

$$H = 23{,}200\ t^3 \qquad (3\text{-}13)$$

constants for 1/4 inch plywood.

The critical buckling stress equation was solved in this thesis for 1/4 inch plywood and tests were run using 1/4 inch plywood. Reference [3-5] tabulates values for I_1 and I_2 found in equations (3-5) and (3-6) and also values for t_i found in equation (3-2) so the solution of these equations is very simple. The other equations may be solved easily also by a simple substitution for the thickness, t.

Table I gives the values of the constants for a 1/4 inch plywood plate used in solving the critical buckling stress equation.

TABLE I

ELASTIC AND PLATE CONSTANTS FOR 1/4 INCH PLYWOOD

Exp	t	I_1	I_2	E_x	E_y	D_y	D_x	D_{xy}	H	$\nu_x D_y$
psi	in.	in.^{4}	in.^{4}	psi	psi	in. lb.	in. lb.	in. lb.	in. lb.	in. lb.
1.19×10^6	.25	.0146	.00106	1.84×10^6	2.24×10^5	294	2410	152	362	57.6

3.5 Wood edge member constants

torsional rigidity, C.

The torsional rigidity was calculated from [3-6]

$$C = \beta d c^3\ G_b \qquad (3\text{-}14)$$

where

C is the torsional rigidity of the edge member.

d is the depth of the edge member.

c is the width of the edge member.

β is a constant depending upon the d/c ratio of the cross section and is shown in Figure 3.4.

G_b is the modulus of rigidity of the edge member which is 70 x 10^3 psi as determined in Section 3.2.

Table II gives the values of the constants of the different edge member sizes used.

TABLE II

CONSTANTS FOR EDGE MEMBERS

Spruce: $E_b = 1.10 \times 10^6$ psi $G_b = 70 \times 10^3$ psi

Size	A_b	$E_b I_b$	C
nominal in.xin.	in.^{2}	lb. in.^{2}	lb. in.^{2}
2 x 2	2.64	63.9×10^4	68.1×10^3
1 x 3	1.97	12.4×10^5	20.9×10^3
2 x 4	5.89	71.0×10^5	26.0×10^4

stress in edge member σ_b.

By a consideration that the strains are equal in the wood edge member and the plywood plate the following equation is obtained:

$$\sigma_b = \frac{E_{bp}}{E_{xp}} \sigma_{cr} \qquad (3\text{-}15)$$

where

E_{bp} is the modulus of elasticity of the edge members in plane stress (equal to 0.90 times the modulus of elasticity in bending)

Substituting values for E_{bp} and E_{xp} into equation (3-15) the expression for σ_b is

$$\sigma_b = 0.833\, \sigma_{cr} \qquad (3\text{-}16)$$

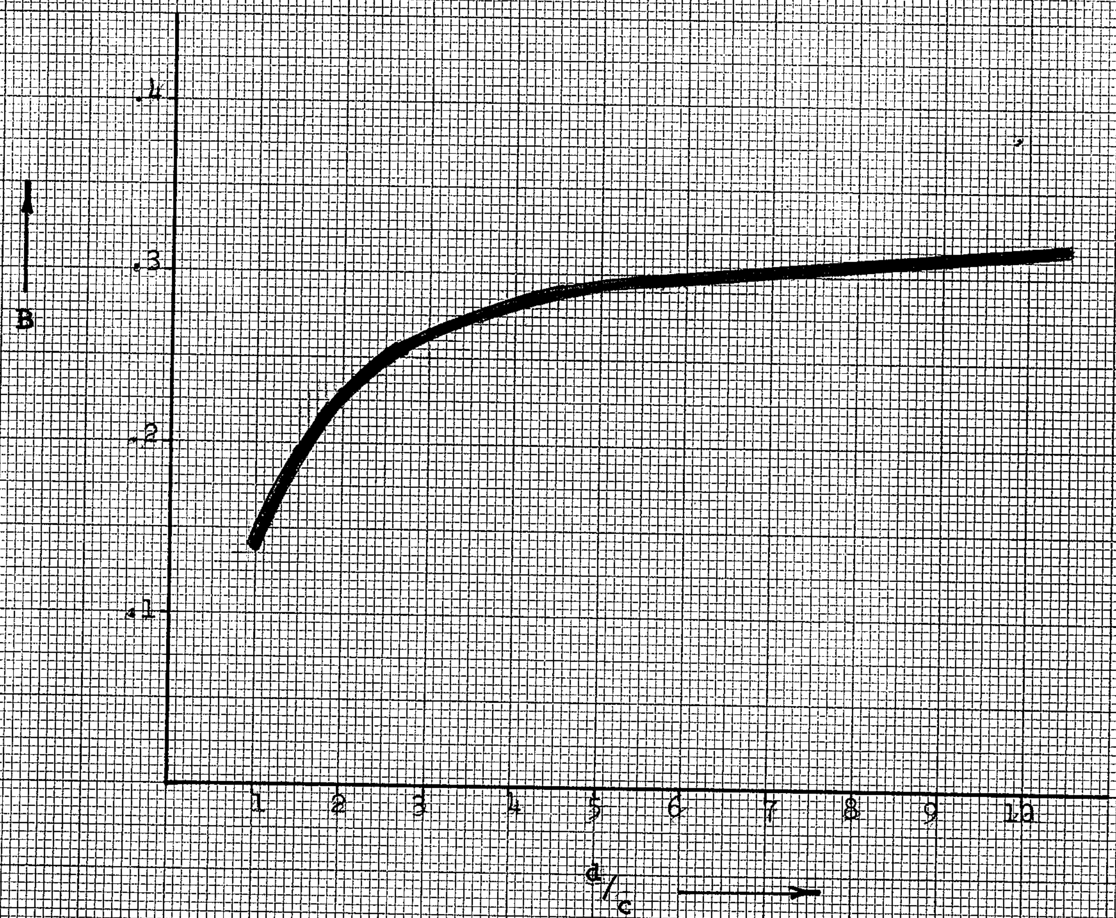


Figure 3.4

Values of Constant B Used in Finding Torsional Rigidity Constant C

CHAPTER IV

SOLUTION OF CRITICAL BUCKLING STRESS EQUATION

4.1 Introduction.

In this chapter will be presented the method of solving the buckling equation and some solutions.

4.2 Method of solution

Due to the complexity of the critical buckling stress equation (equation 2-18) a trial and error solution, utilizing a digital computer was used. The full details of the computer programs used are given in Appendix B.

In the trial and error method used, a solution for σ_{cr} was chosen and the equation evaluated. This was done on the computer by having two terms of the equation equal the other two, (these are terms SI = $B_1 + B_2$ and $S_2 = B_3 + B_4$ in the programs) and having the computer print out the difference between the two sides (DF = $S_1 - S_2$ in the programs). As the sign of the value of the difference changed it was known that a solution was present in that particular range of trials and the answer was then zeroed in by selecting appropriate trial answers within that range.

Two types of programs were written for this method of solution. The first type, called Type I in the appendix, consisted of two "Do" loops. One loop was for the number of modes of buckling (m) and the other loop was for the panel lengths (a). The first trial value of stress was read with the first mode and the loop was completed for the panel lengths required. Then the loop was completed with the next mode required but with the same stress. This procedure was followed for all values of trial stress submitted to the computer. This program was used to determine the preliminary range of stress values within which the solution lay.

The second type of program, called Type II in the appendix, was used for the zeroing in on the answer once the preliminary range was established by the first program. In the second type, values of stress within the range with the appropriate values of length and mode were read in and the difference found again. The answer could be found to any accuracy by making the difference between values of stress read in very small.

The basic program for both types was identical. There were two "If" statements present to trap imaginary numbers. When an imaginary number appeared, the calculation was halted and the program returned to the next calculation. Thus, only real solutions were calculated.

4.3 Results of computer programs

Solutions of the critical buckling stress equation were found for a 24 inch wide panel, 1/4 inch plywood, with 2 x 2's, 1 x 3's and 2 x 4's as the edge members. The critical buckling stress for these panels at various lengths are given in Tables III to V. The modes, and two constants K_{cr} and ϕ, which will be explained in the next section, are also given.

Special solutions for a 24 inch wide, 1/4 inch thick plywood plate were also found. These solutions were for simple support along the loaded edges, x = o and x = a, and simple and free support along the edges $y = \pm b/2$. These solutions are given in Tables VI and VII.

TABLE III

RESULTS FOR PANEL WITH 2 x 2 EDGE MEMBERS

BUCKLING MODE (m)	LENGTH (a) INCHES	ϕ	BUCKLING STRESS (σ_{cr}) ± 3 psi.	K_{cr}
1	40.63	1	165	2.86
1	60.95	1.5	203	3.51
1	81.26	2	178	3.08
1	101.58	2.5	115	1.99
1	121.89	3	90	1.56
1	142.21	3.5	65	1.13
1	162.52	4	44	.76
1	182.84	4.5	40	.69
1	203.15	5	28	.48
1	223.47	5.5	28	.48
1	243.78	6	23	.40
2	81.26	2	165	2.86
2	101.58	2.5	178	3.08
2	121.89	3	203	3.51
2	142.21	3.5	209	3.62

TABLE IV

RESULTS FOR PANEL WITH 2 X 4 EDGE MEMBERS

BUCKLING MODE (m)	LENGTH (a) INCHES	ϕ	BUCKLING STRESS (σ_{cr}) ± 3 psi.	K_{cr}
1	40.63	1	165	2.86
1	60.95	1.5	203	3.51
1	81.26	2	296	5.09
1	182.84	4.5	259	4.48
1	203.15	5	209	3.62
1	223.47	5.5	178	3.08
1	243.78	6	153	2.65

TABLE V

RESULTS FOR PANEL WITH 1 X 3 EDGE MEMBERS

BUCKLING MODE (m)	LENGTH (a) INCHES	ϕ	BUCKLING STRESS (σ_{cr}) ± 3 psi	K_{cr}
1	40.63	1	165	2.86
1	60.95	1.5	203	3.51
1	101.58	2.5	209	3.62
1	121.89	3	159	2.75
1	142.21	3.5	128	2.21
1	162.52	4	103	1.78
1	182.84	4.5	78	1.35
1	203.15	5	65	1.13
1	223.47	5.5	53	.92
1	243.78	6	45	.78
2	60.95	1.5	184	3.18
2	81.26	2	165	2.86
2	101.58	2.5	178	3.08
2	121.89	3	203	3.51
2	142.21	3.5	246	4.26
3	81.26	2	203	3.51
3	101.58	2.5	171	2.96
3	121.89	3	165	2.86
3	142.21	3.5	171	2.96
3	162.52	4	203	3.51

TABLE VI

RESULTS FOR PANEL WITH FREE UNLOADED EDGES

BUCKLING MODE (m)	LENGTH (a) INCHES	ϕ	BUCKLING STRESS (σ_{cr}) ± 0.5 psi	K_{cr}
1	40.63	1	58	1.00
1	50.79	1.25	37	.64
1	60.95	1.50	26	.45
1	71.10	1.75	20	.35
1	81.26	2	16	.28
1	91.42	2.25	12	.21
1	101.58	2.50	10	.17
1	111.73	2.75	8	.14
1	121.89	3	6.5	.11
1	132.05	3.25	6	.10
1	142.21	3.50	5	.09
1	152.36	3.75	4.5	.08
1	162.52	4	4	.07
1	172.68	4.25	3.5	.06
1	182.84	4.50	3	.05
1	193.00	4.75	3	.05
1	203.15	5	2.5	.04
1	213.31	5.25	2.5	.04
1	223.47	5.50	2	.04
1	233.63	5.75	2	.04

TABLE VII

RESULTS FOR PANEL WITH SIMPLY SUPPORTED UNLOADED EDGES

BUCKLING MODE (m)	LENGTH (a) INCHES	ϕ	BUCKLING STRESS (σ_{cr}) ± 0.5 psi	K_{cr}
1	30.47	.75	185	3.20
1	40.63	1	165	2.86
1	50.79	1.25	177	3.06
1	60.95	1.50	205	3.55
2	50.79	1.25	220	3.81
2	60.95	1.50	185	3.20
2	81.26	2	165	2.86
2	91.42	2.25	168	2.91
2	101.58	2.50	177	3.06
2	111.73	2.75	189	3.27
3	91.42	2.25	185	3.20
3	101.58	2.50	173	2.99
3	111.73	2.75	167	2.89
3	121.89	3	165	2.86
3	132.05	3.25	166	2.87
3	142.21	3.50	171	2.96
4	132.05	3.25	175	3.03
4	142.21	3.50	169	2.92
4	152.36	3.75	166	2.87
4	162.52	4	165	2.86
4	172.68	4.25	166	2.87
4	182.84	4.50	168	2.91
4	193.00	4.75	172	2.98
5	172.68	4.25	171	2.96
5	182.84	4.50	168	2.91
5	193.00	4.75	166	2.87
5	203.15	5	165	2.86
5	213.31	5.25	166	2.87
5	223.47	5.50	167	2.89
5	233.63	5.75	170	2.94

4.4 Graphs of results.

For comparison with published results and to calculate critical stresses for various widths, it is convenient to plot curves of what is called the critical buckling coefficient as a function of the reduced aspect ratio.

The critical buckling coefficient is defined as:

$$K_{cr} = \frac{\sigma_{cr}\, b^2\, t}{\pi 2\ D} \qquad (4\text{-}1)$$

where

K_{cr} is the critical buckling coefficient

σ_{cr} is the critical buckling stress at width b of the panel.

$$D = \frac{\sqrt{E_x E_y}\ t^3}{12\ (1-\nu_x \nu_y)} \qquad (4\text{-}2)$$

The reduced aspect ratio is defined as:

$$\phi = \sqrt[4]{\frac{E_y}{E_x}}\ \frac{a}{b} \qquad (4\text{-}3)$$

where

ϕ is the reduced aspect ratio

Values of K_{cr} and ϕ for the panels solved are given in Tables III to VII. Graphs of K_{cr} versus ϕ are given in Figures 4.5 to 4.9. For a clearer comparison the values of K_{cr} versus ϕ for all panels solved were plotted on the same graph in Figure 4.10.

4.5 Previous Solutions.

To test the validity of the critical buckling stress equation it is useful to compare with known results if possible. The equation for

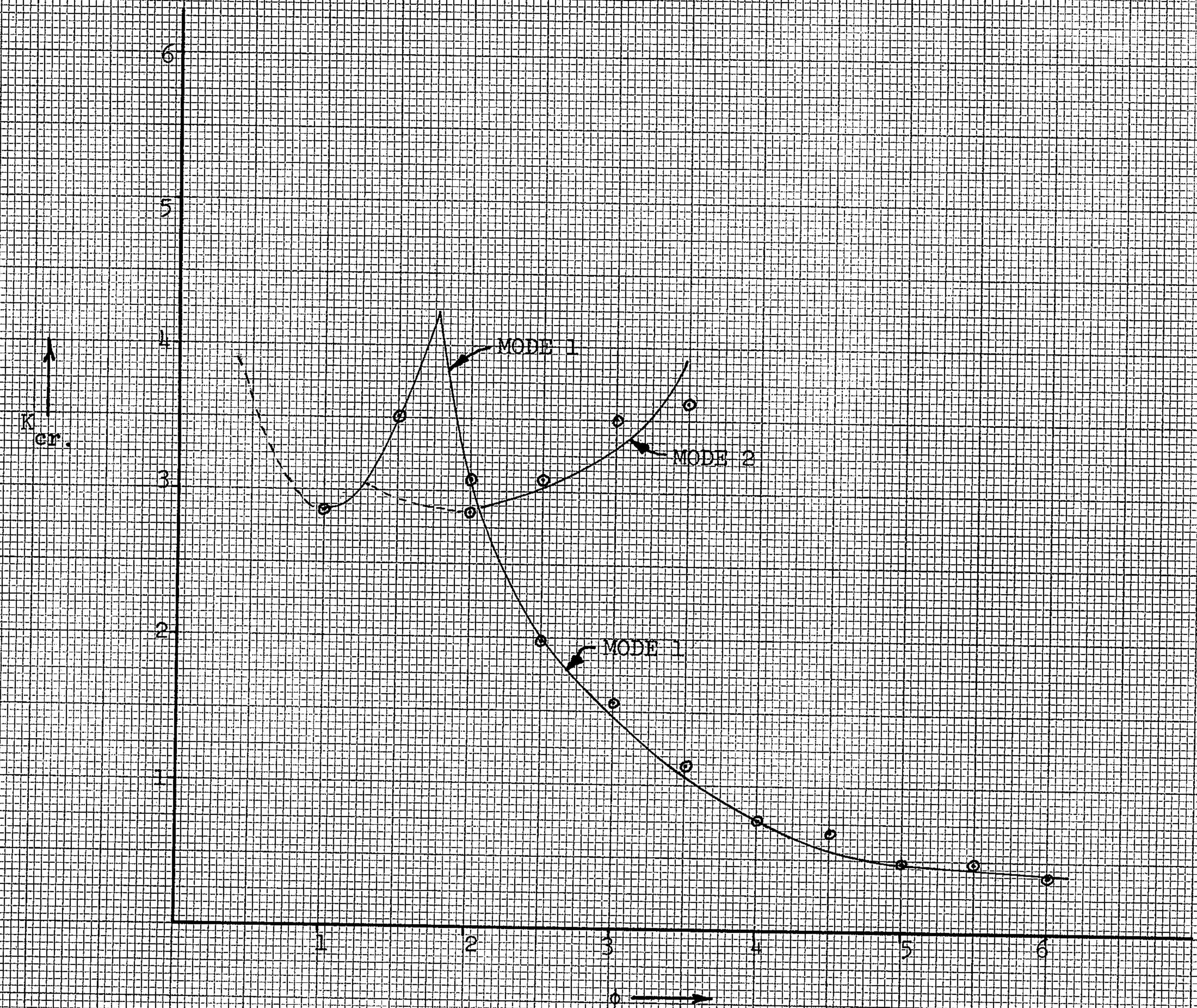


Figure 4.5
Critical Buckling Coefficient as a Function of Reduced Aspect Ratio for Panel with 2x2 Edge Members

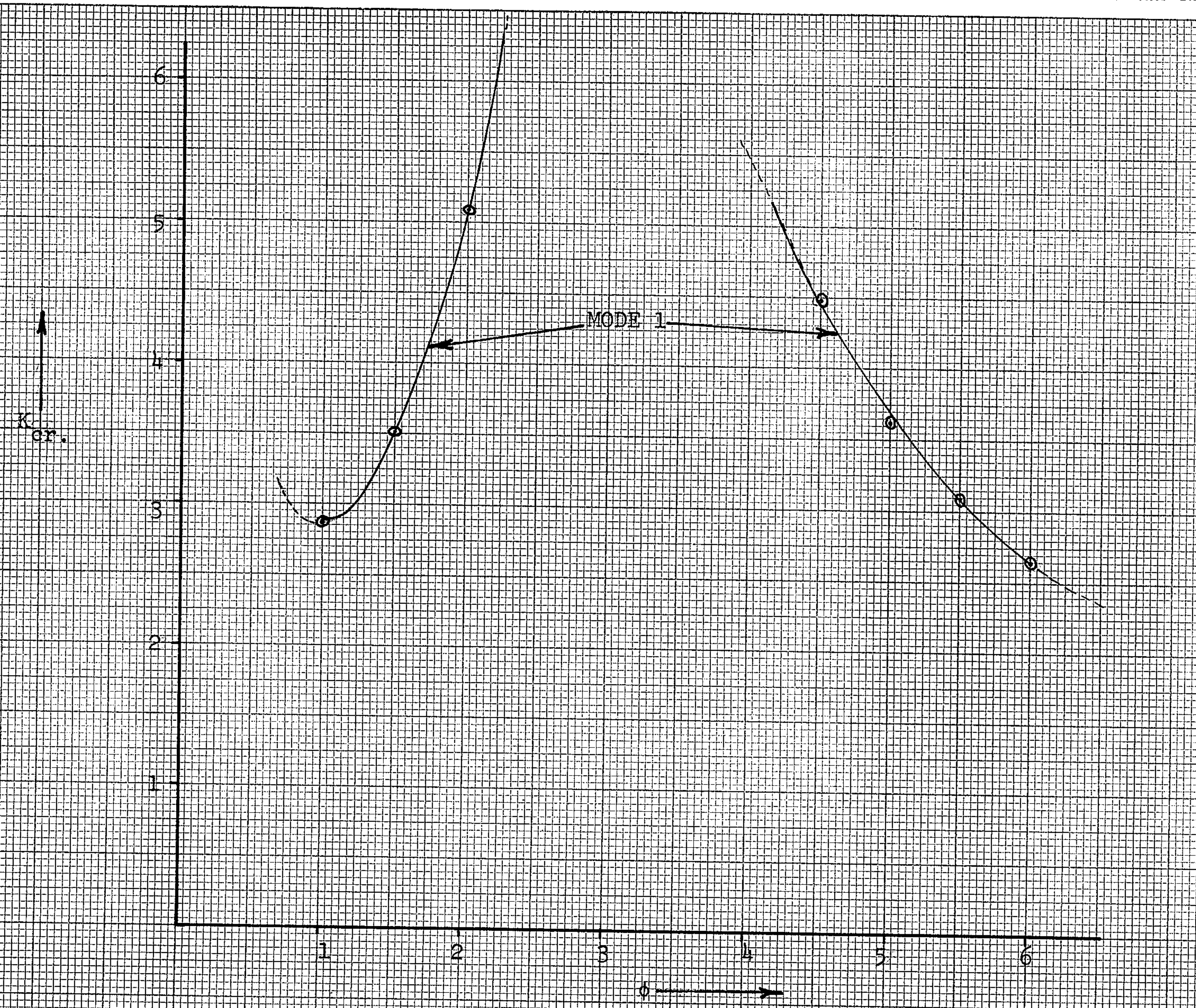


Figure 4.6
Critical Buckling Coefficient as a Function of Reduced Aspect Ratio for Panel with 2x4 Edge Members

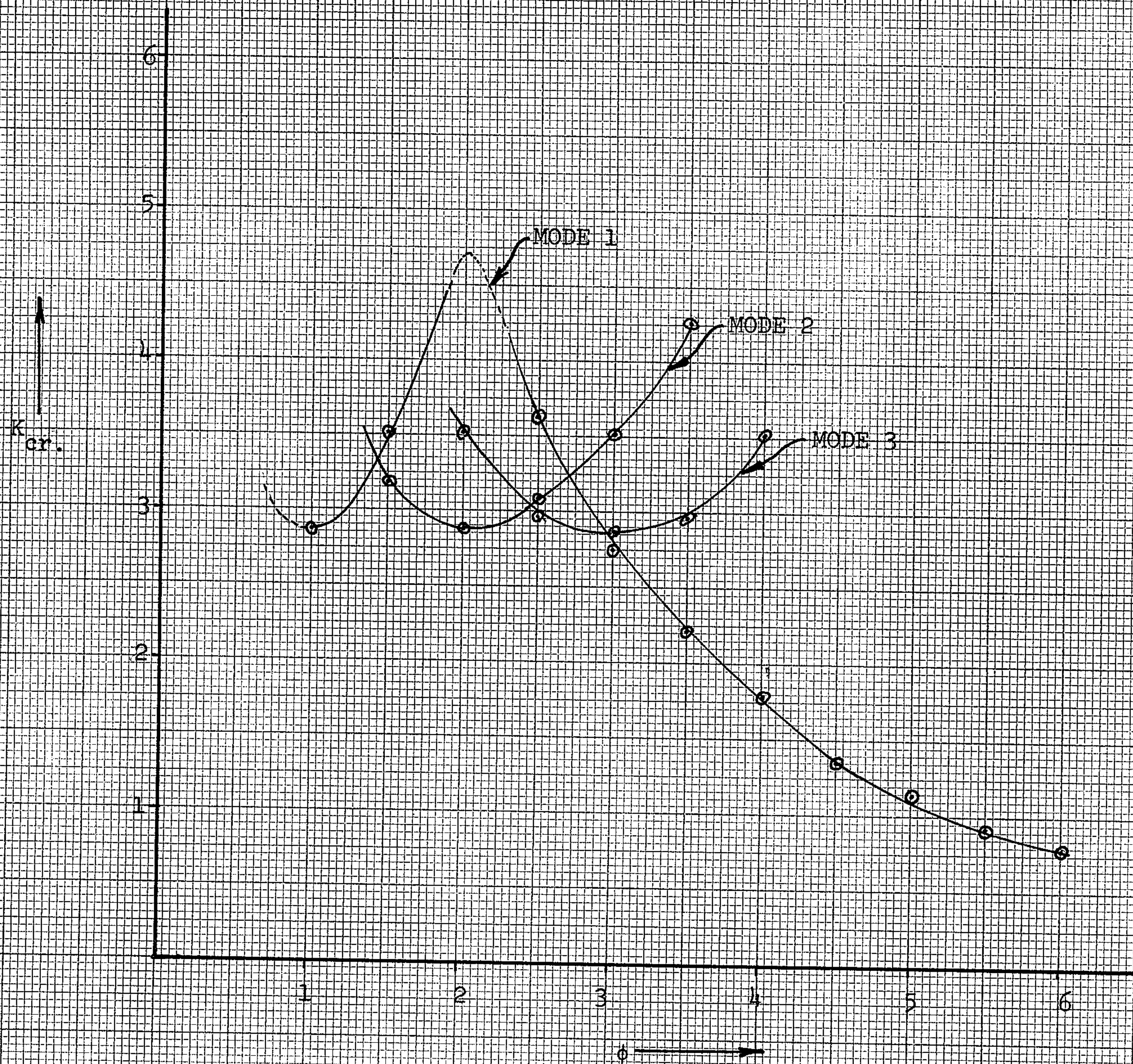


Figure 4.7
Critical Buckling Coefficient as a Function of Reduced Aspect Ratio for Panel with 1x3 Edge Members

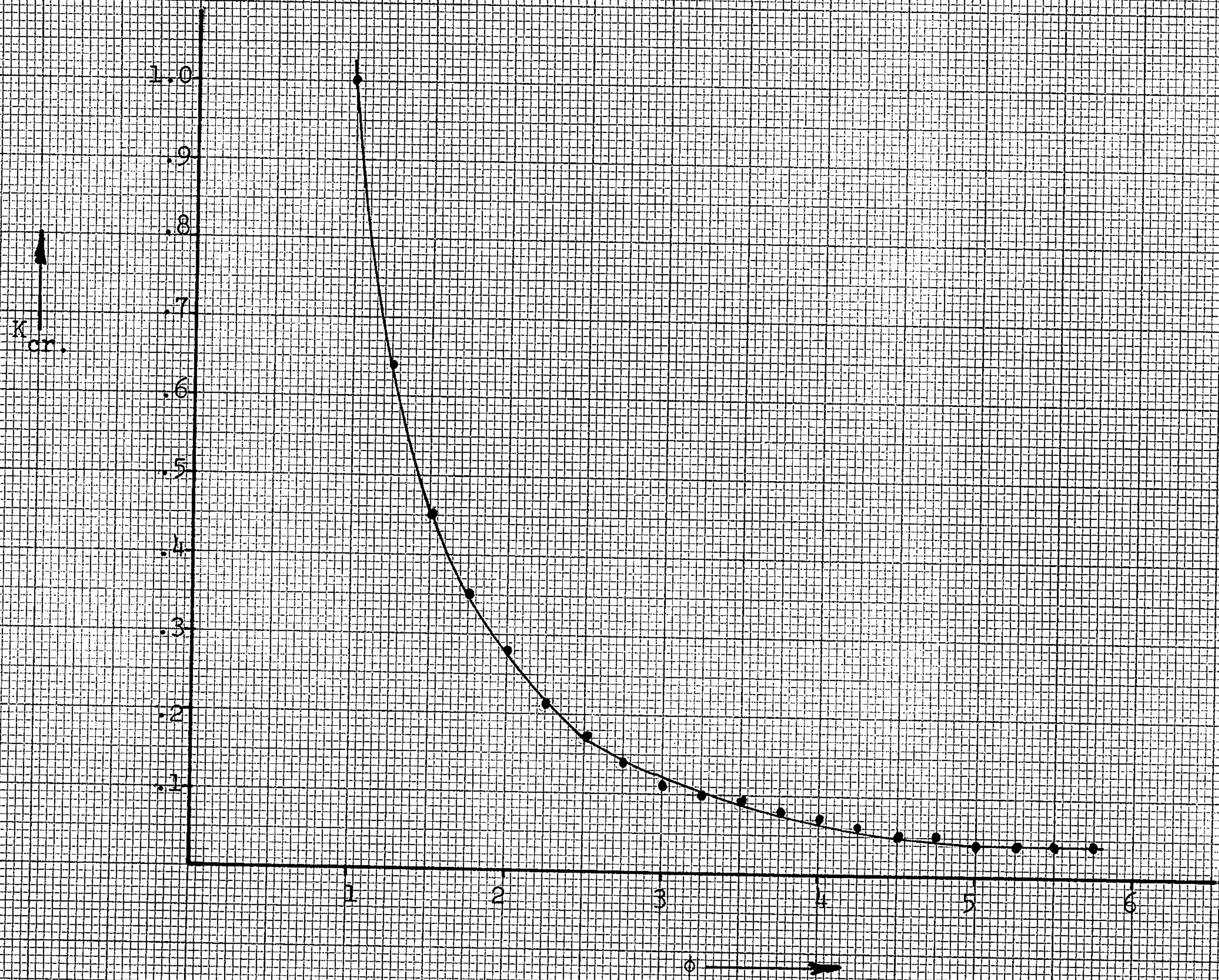


Figure 4.8
Critical Buckling Coefficient as a Function of
Reduced Aspect Ratio for Free Support Case

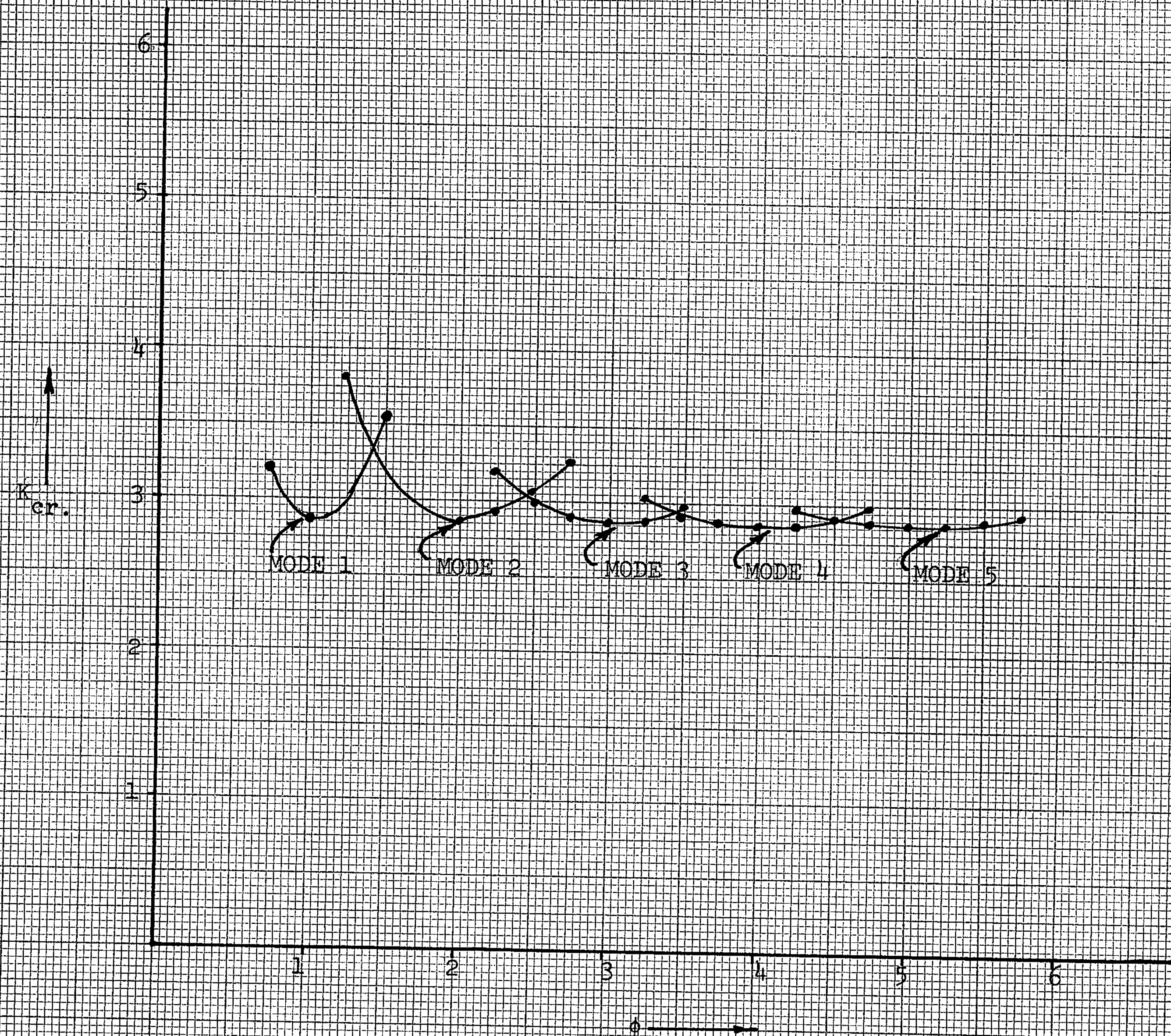


Figure 4.9
Critical Buckling Coefficient as a Function of
Reduced Aspect Ratio for Simple Supported Case

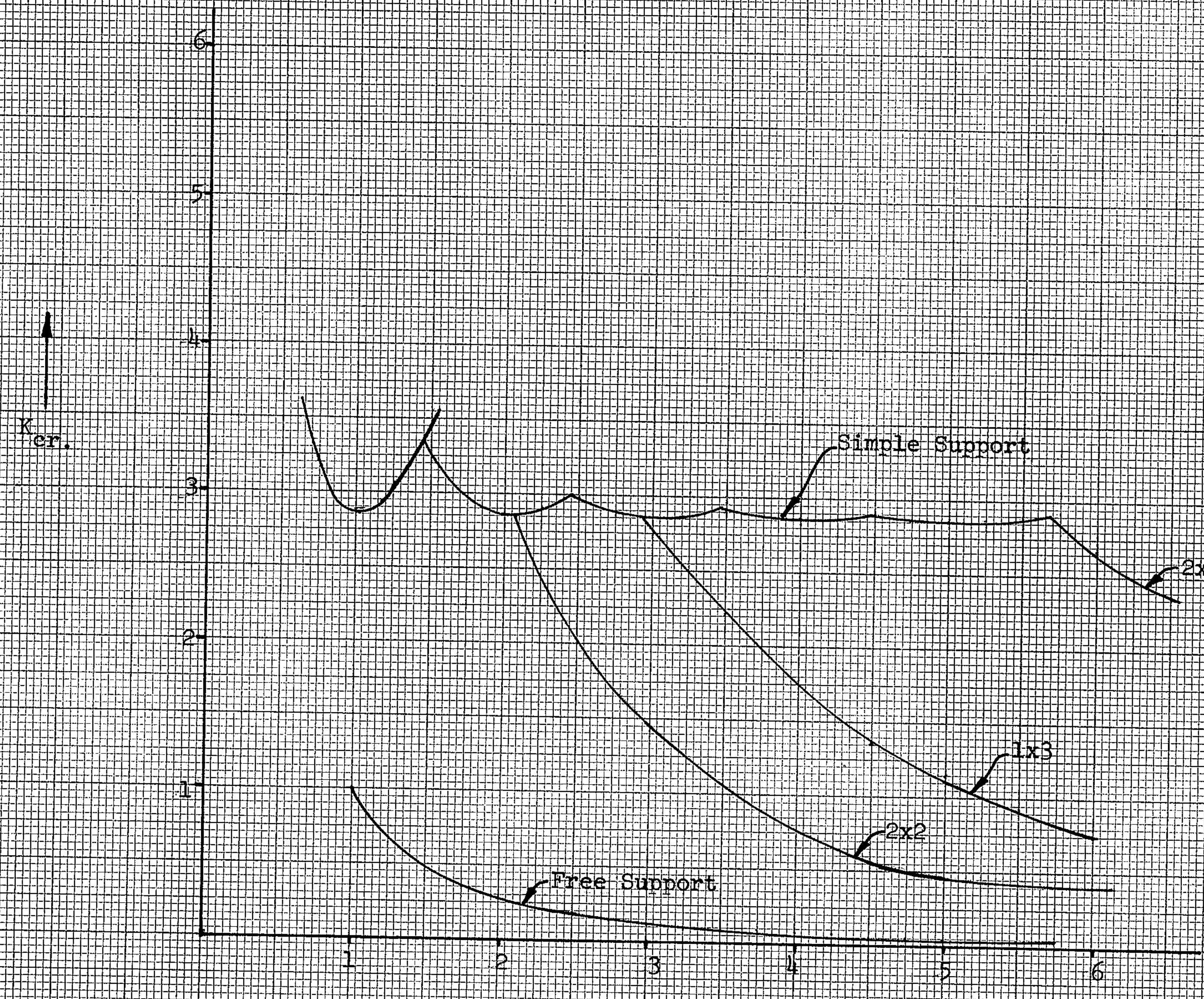


Figure 4.10
Critical Buckling Coefficient as a Function of Reduced Aspect Ratio For All Cases From Computer Program

a plywood plate, simply supported at the unloaded edges, is [4-1, 4-2, 3-1, 3-4]

$$\sigma_{cr} = \frac{\pi^2}{b^2 t}\left[\frac{D_y\ a^2}{m^2\ b^2} + 2H + D_x\frac{m^2 b^2}{a^2}\right] \qquad (4-4)$$

The symbols used in equation (4-4) are consistent with those in this thesis. Equation (4-4) may appear in a different form in the references given because of different notation being used. Table VIII gives the values for a solution of this equation for a 24 inch wide, 1/4 inch thick, plywood plate. A graph of this solution is given in Figure 4.11.

TABLE VIII

RESULTS OF SIMPLE SUPPORT CASE FROM PREVIOUS SOLUTIONS

BUCKLING MODE (m)	LENGTH (a) INCHES	ϕ	BUCKLING STRESS (σ_{cr}) psi	K_{cr}
1	10.16	.25	977	16.80
1	20.32	.50	294	5.08
1	30.47	.75	184.6	3.19
1	40.63	1	165.9	2.87
1	50.79	1.25	177.0	3.06
1	60.95	1.50	204.9	3.54
2	50.79	1.25	220.0	3.81
2	60.95	1.50	184.6	3.19
2	81.26	2	165.9	2.87
2	101.58	2.50	177.0	3.06
3	101.58	2.50	172.7	2.98
3	121.89	3	165.9	2.87
3	142.21	3.50	170.5	2.95
4	142.21	3.50	169.2	2.93
4	162.52	4	165.9	2.87
4	182.84	4.50	168.3	2.91

In considering a plywood plate with free unloaded edges, no known solution could be found. For comparison, Euler's equation for a one foot

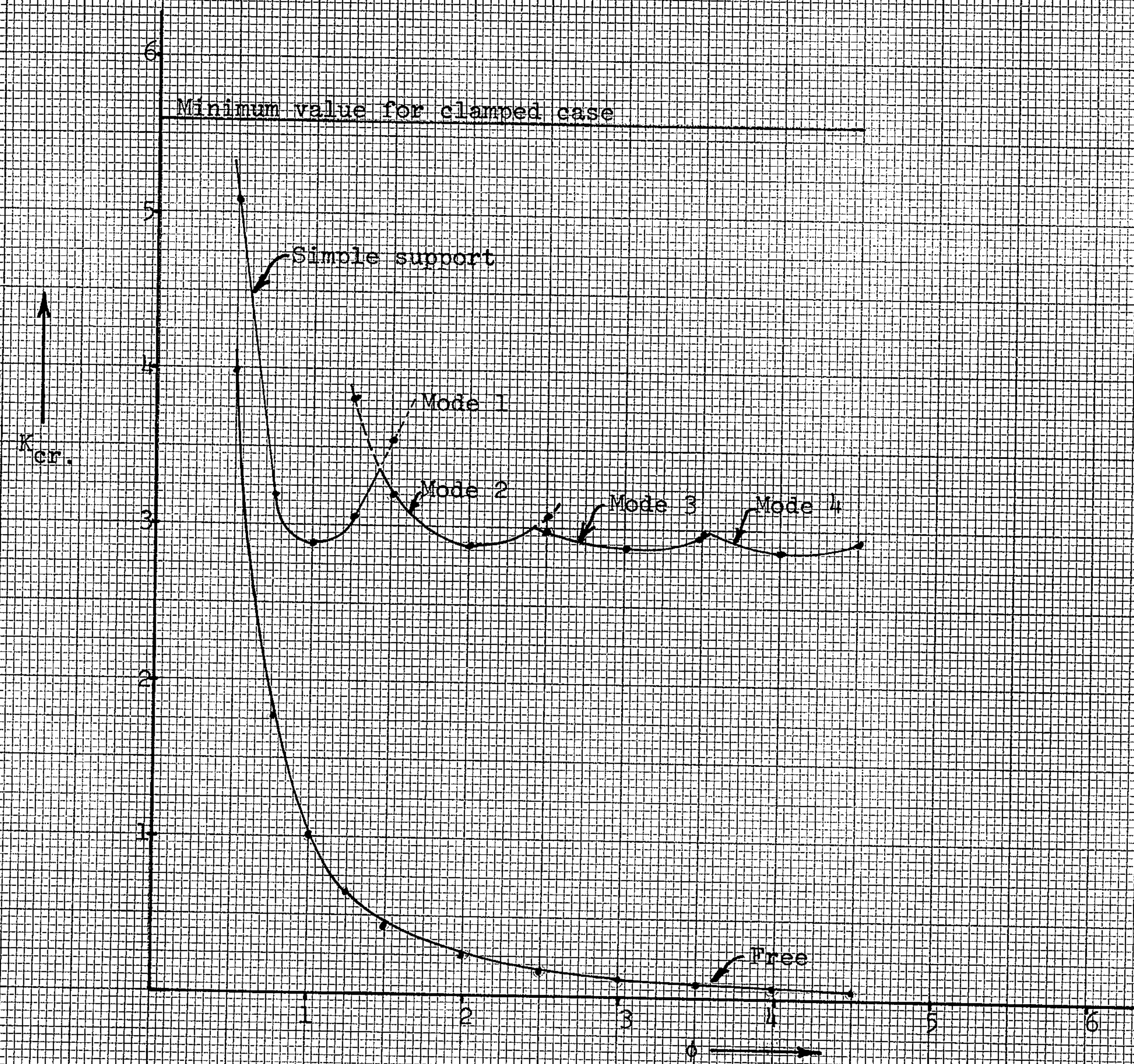


Figure 4.11
Results of Previous Solutions for Simple
Support, Free, and Clamped Cases

strip column, with the flexural stiffness of a plate (D_x) replacing the flexural stiffness of the column (E_I), was used. This equation is

$$\sigma_{cr} = \frac{\pi^2 D_x}{t\ a^2} \qquad (4\text{-}5)$$

Table IX gives the solution of this equation for a 1/4 inch plywood plate. The graph of this result is shown in Figure 4.11.

TABLE IX

RESULTS OF EULER'S EQUATION FOR A PLYWOOD PLATE

LENGTH (a) INCHES	ϕ	BUCKLING STRESS (σ_{cr}) psi.	K_{cr}
10.16	.25	924	16.00
15.24	.375	411	7.10
20.32	.50	231	3.98
30.47	.75	102.6	1.77
40.63	1	57.6	1.00
50.79	1.25	36.8	.64
60.95	1.50	25.6	.44
81.26	2	14.4	.25
101.58	2.50	9.2	.16
121.89	3	6.4	.11
142.21	3.50	4.7	.08
162.52	4	3.6	.06
182.84	4.50	2.9	.05

The equation for the minimum value of the critical buckling stress for a plywood plate with unloaded edges clamped is [4-1, 4-2, 3-4]

$$\sigma_{cr} = \frac{\pi^2}{b^2 t}\left[4.554\sqrt{D_x D_y} + 2.474\ \nu_x D_y + 4.943\ D_{xy}\right] \qquad (4\text{-}6)$$

Evaluating equation (4-6) for a 24 inch wide, 1/4 inch thick, plywood plate, the minimum value of σ_{cr} was found to be 324 psi. The critical buckling coefficient corresponding to this stress is 5.59. This critical buckling stress occurs at increments of a/b given by:

$$\frac{a}{b} = .664 \sqrt[4]{\frac{D_x}{D_y}} \qquad (4\text{-}7)$$

Evaluating equation (4-7) and using the expression for ϕ at which the minimum buckling stress occurs is 0.664. That is, the minimum buckling stress occurs at $\phi = 0.664$, $\phi = 2 \times 0.664$, $\phi = 3 \times 0.664$ and so on. This corresponds to a value of $\phi = 1$ for the case of simple support. The minimum value for the clamped case is shown in Figure 4.11.

4.6 Discussion of results.

In comparing the results obtained from the critical buckling stress equation with those from the known equation for the simple support case, it can be seen that there is excellent agreement. Also there is excellent agreement between the free edge case from the critical buckling stress equation with Euler's load for a one foot strip. This validates the critical buckling stress equation quantitatively for these special cases. In trying to validate the equation for the other cases, the results agree qualitatively with those found by Bleich [4-3] for an isotropic plate supported elastically on one unloaded side and simply supported on the other. The graph of the critical buckling coefficient as a function of the aspect ratio from Bleich's results shows the curve following the simply supported curve and then following a curve giving smaller K_{cr} values with increasing aspect ratio. This is identical with Figure 4.10 which shows the different curves. The ones marked simple support and the ones marked 2 x 2, 1 x 3 and 2 x 4.

In evaluating the results from the cases with edge members, it can be seen that there are two types of buckling. The first type is plate buckling of the plywood skin of the panel. The second type is column buckling of the entire panel.

The first type is usually classed as local buckling. In comparing the various curves for this local buckling with that of the simply supported case, it can be seen that they are identical. Therefore, for the range of edge members tested, local buckling can be predicted by considering the plywood plate to be simply supported at the unloaded edges.

There are two possible reasons for this agreement of local buckling with the case of simple support. One, is that the torsional rigidity of the edge members is negligible (which is usually assumed in calculations of this kind) and the edge members are flexurally rigid. The second possibility is that any decrease in buckling strength due to flexure of the edge members is countered by an increase in buckling strength due to torsional rigidity of the edge members. However, it can be seen from the clamped case that an increase in buckling strength over the simple support case also causes a decrease in the wave lengths ($^a/_b$ ratio) at which buckling occurs. From a comparison of the results it can be seen that there is no decrease, from the simple support case, in the $^a/_b$ ratio at which the minimum value of the buckling stress occurs for the cases with edge members. Because of this, it would appear that the local buckling case agrees with the simply supported case because the edge members have negligible torsional rigidity and are flexurally rigid.

The second type of buckling is column buckling of the entire panel. This type of buckling should be predictable by an Euler equation. This is a complex problem in itself due to the different moduli of elasticity which exist in the panel. However, if a suitable equation of the Euler type could be derived, the prediction of the two types of buckling failure could be simplified. Instead of using the critical buckling stress equation, the column type of failure could be predicted by an Euler equation and the local buckling phenomena checked by using the plate formula for the simple support case. This could, no doubt, be done by hand calculation and the difficulty in using the critical buckling stress equation overcome. If, of course, the Euler buckling prediction was not accurate; graphs of the critical buckling stress equation, as drawn in this thesis, for various edge members and thicknesses of plywood, could be used with no great difficulty.

CHAPTER V

TEST PROGRAM

5.1 Introduction

A series of tests were run on various panels. This chapter outlines the test program used.

5.2 Test specimens.

Eight panels were tested. The tests were designated as A to H, inclusive.

The panels were made of 1/4 inch thick plywood with 2 x 2 construction-grade spruce as the edge members. This is consistent with the theory derived to this point. The edge members were glued to the plywood using a casein glue and were also nailed.

The first panel made (for test A) was 42 inches wide. After this panel was tested, 6 inches were cut from one side, another edge member fastened and a 36 inches wide panel (for tests B, C and D) was made. This was done on all panels until the final panel, (for test H) which was 18 inches wide, was tested to failure. The general panel is shown in Figure 5.12. The lengths and widths of the various panels, corresponding to tests A to H, are given in Table X.

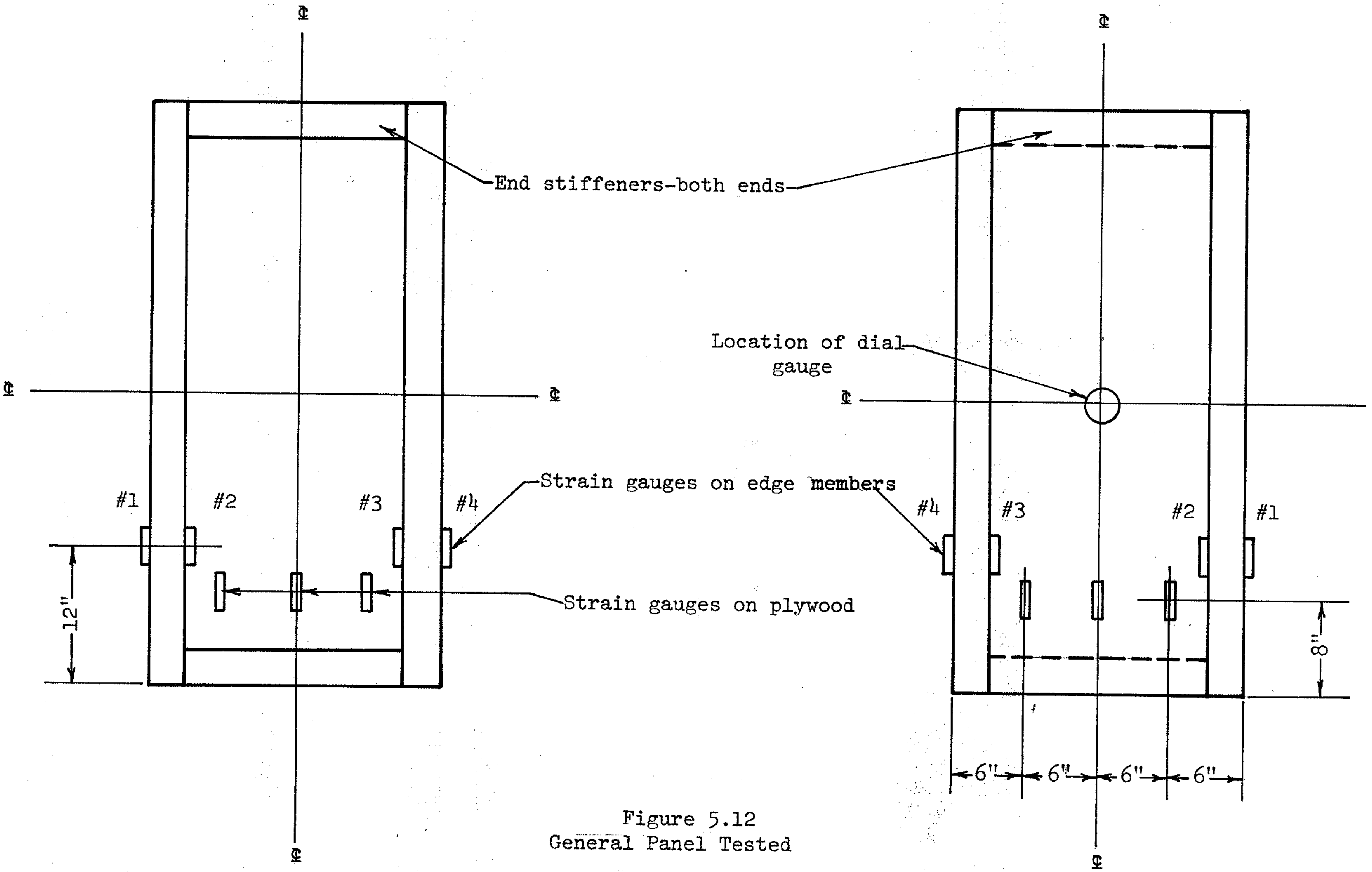


Figure 5.12
General Panel Tested

TABLE X

CHARACTERISTICS OF TEST SPECIMENS

Test	l Inches	w Inches	No. of Gauges on Plywood	End Stiffeners
A	48	42	14	Yes
B	48	36	12	Yes
C	48	36	12	Yes
D	48	36	12	Yes
E	48	30	10	Yes
F	48	30	10	No
G	46.5	24	8	No
H	46.5	18	6	No

To predict the buckling load, dial gauge readings were taken at the centre of each panel on the opposite side of the plywood to which the edge members were fastened. To give the load on the edge members at buckling, strain gauges were fastened to them. Two gauges were fastened to each member and these gauges were numbered 1 to 4 as shown in Figure 5.12.

Strain gauges were also fastened to each side of the plywood as shown in Figure 5.12. The exact number of strain gauges on each panel are given in Table X. The purposes of the strain gauges fastened to the plywood was to help in predicting when buckling occurred. This was found not to work so they were merely used as a check on the stress in the panel at buckling.

Two types of strain gauge were used. Both types were glued to the plywood or edge members. The type used on the plywood was an A-3-S6.

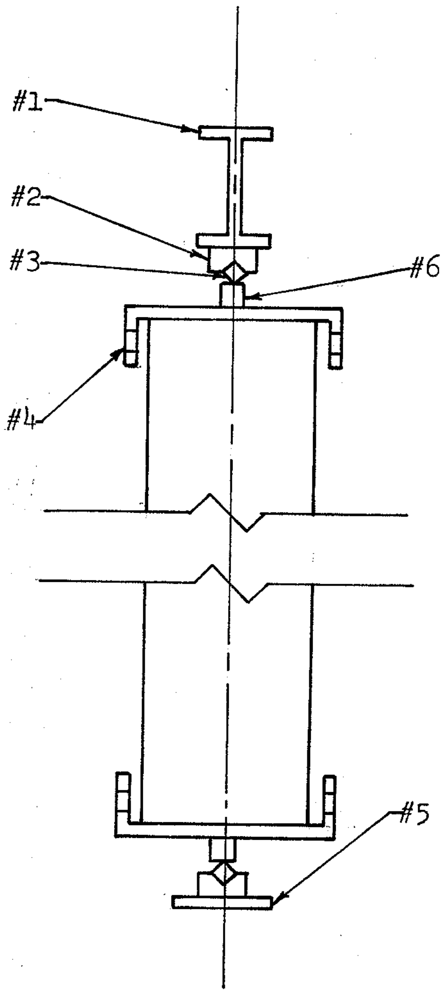
This gauge was easier to apply than the A-1-S6 type used on the edge members.

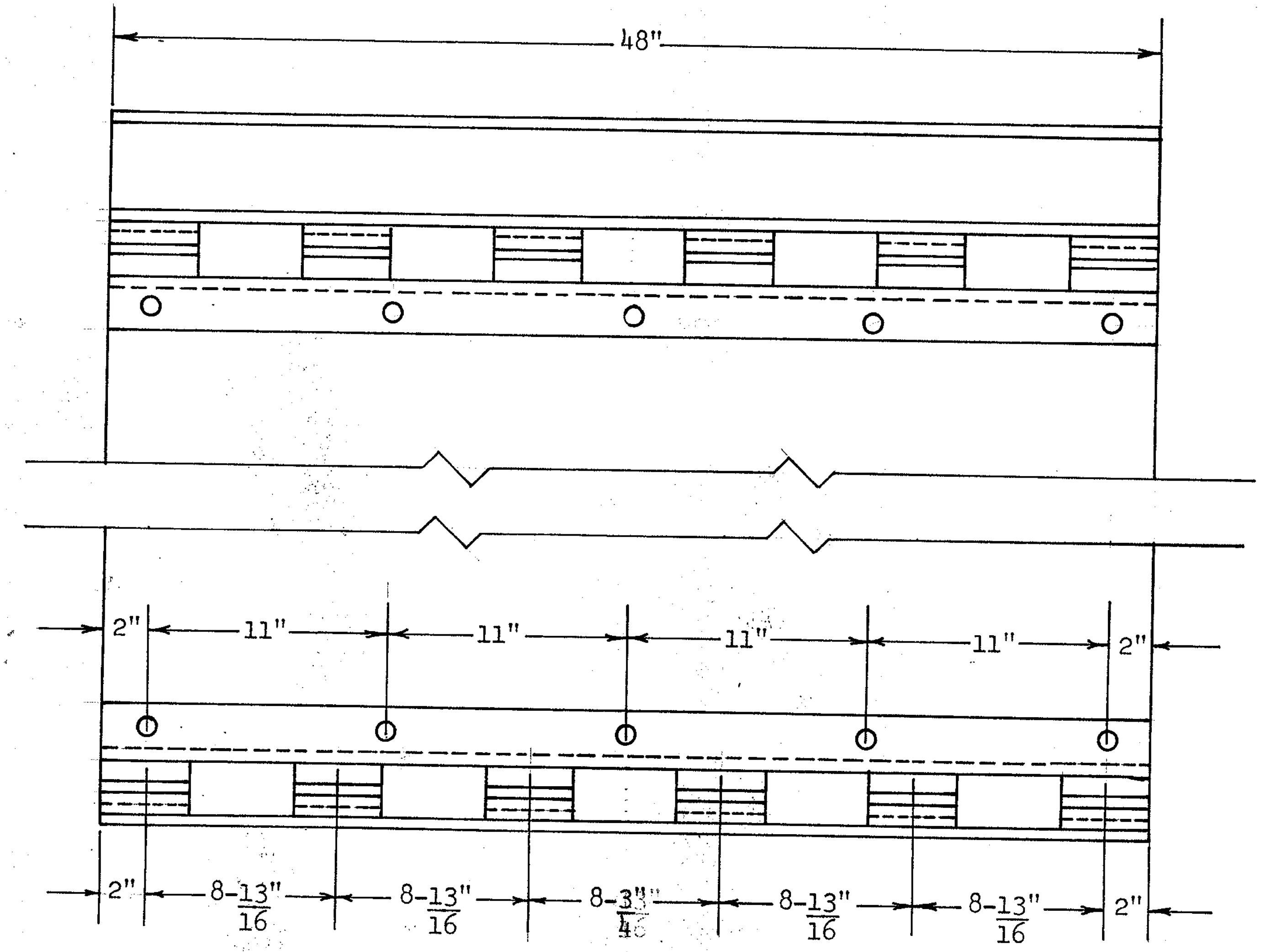
5.3 Test procedure and preparation.

In order to simulate as closely as possible end conditions of simple support and a uniform load distribution, the panels were tested in the apparatus shown in Figures 5.13 to 5.16. Two by two's were used to hold the panel in the channels, as shown in Figure 5.14, for tests A and B, while angles, as shown in Figures 5.15 and 5.16, were used for the other tests. End stiffeners, to help prevent the panel from twisting, were used for tests A to E, but removed for tests F to H. These end stiffeners are shown in Figure 5.12.

In preparing the panels for testing, the end bearing surfaces had to be as flat as possible. This was achieved by clamping two angles at each side of the panel at the ends and sanding the ends flat, using the angles as a guide. This gave a very flat and smooth surface but took a long time to do.

In testing the panels, the load was taken just beyond the estimated buckling load. This load was reached in stages with dial readings recorded at the various stages. The panels (except the last panel for test H) were not tested to failure because they were cut down and used again as explained earlier, and the purpose of this thesis was not to test the ultimate strength of these panels. Strain gauge readings were taken with a Budd digital strain indicator. This indicator is shown in Figure 5.14.





#1 - 5 I 14.75 (1 reg'd.)

#2 - 2" x 4" x 1" steel bars (12 reg'd.)

#3 - $\frac{3}{4}$" x $\frac{3}{4}$" x 4" tool steel bits (12 reg'd.)

#4 - 9 [20 channels (2 reg'd.)

#5 - 4" x 48" x $\frac{1}{2}$" steel plate (1 reg'd.)

#6 - 1" x 1" x 4" steel bars (12 reg'd.)

Scale: 1-$\frac{1}{2}$" = 1'-0"

Figure 5.13

EDGE APPARATUS USED IN TESTS

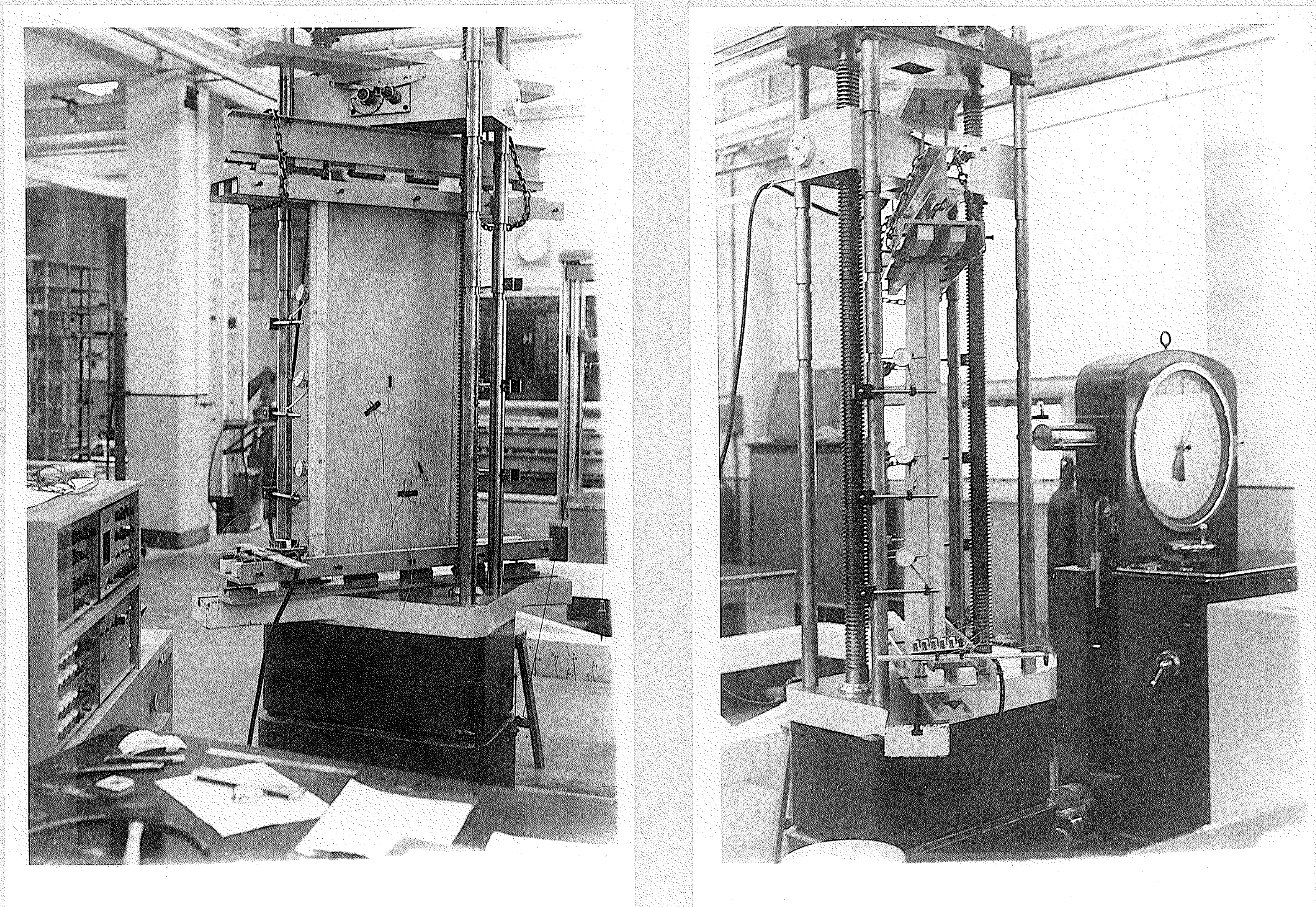


Figure 5.14 Photos of Test Apparatus and Specimens

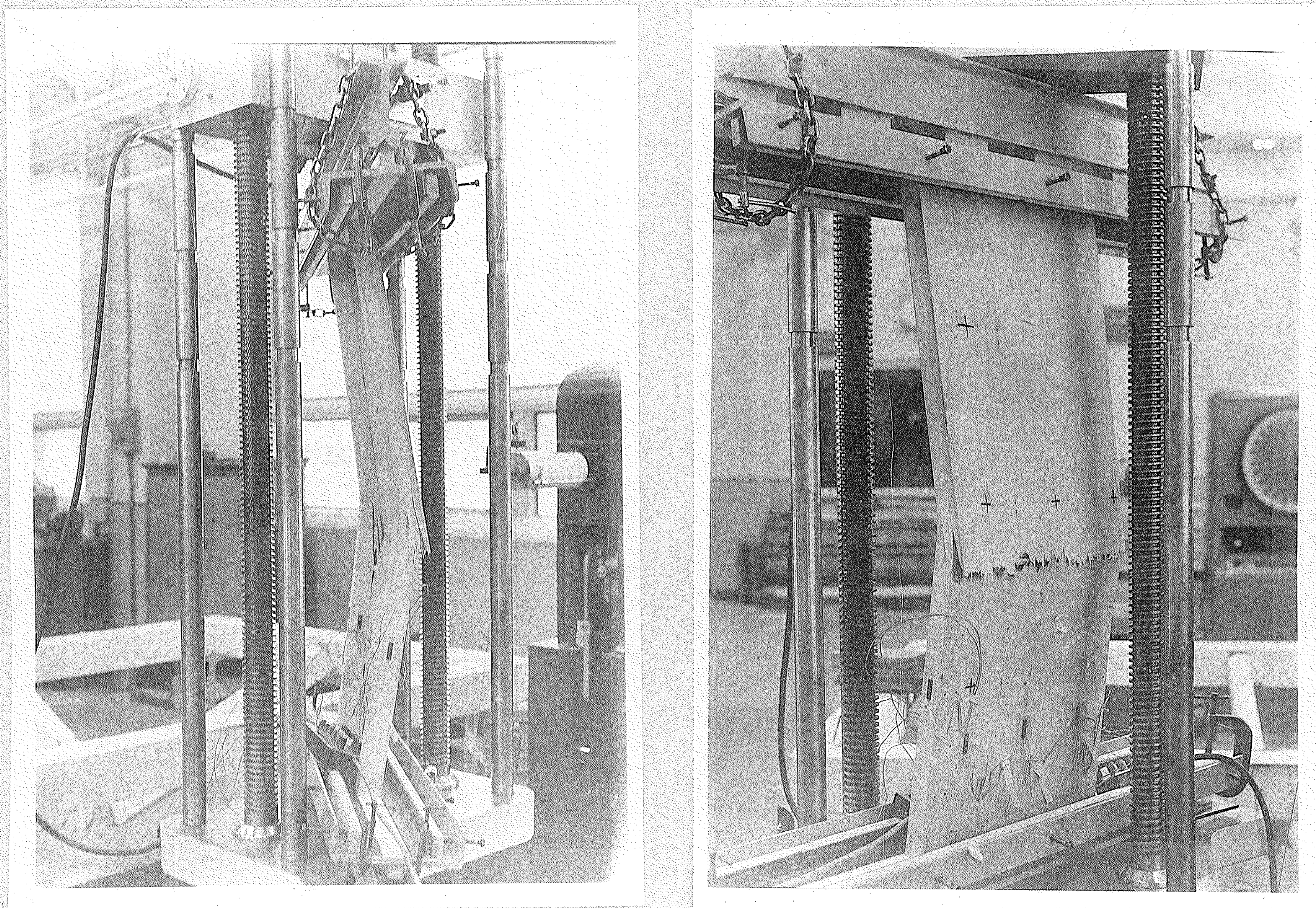


Figure 5.15 Photos of Test Apparatus and Specimens

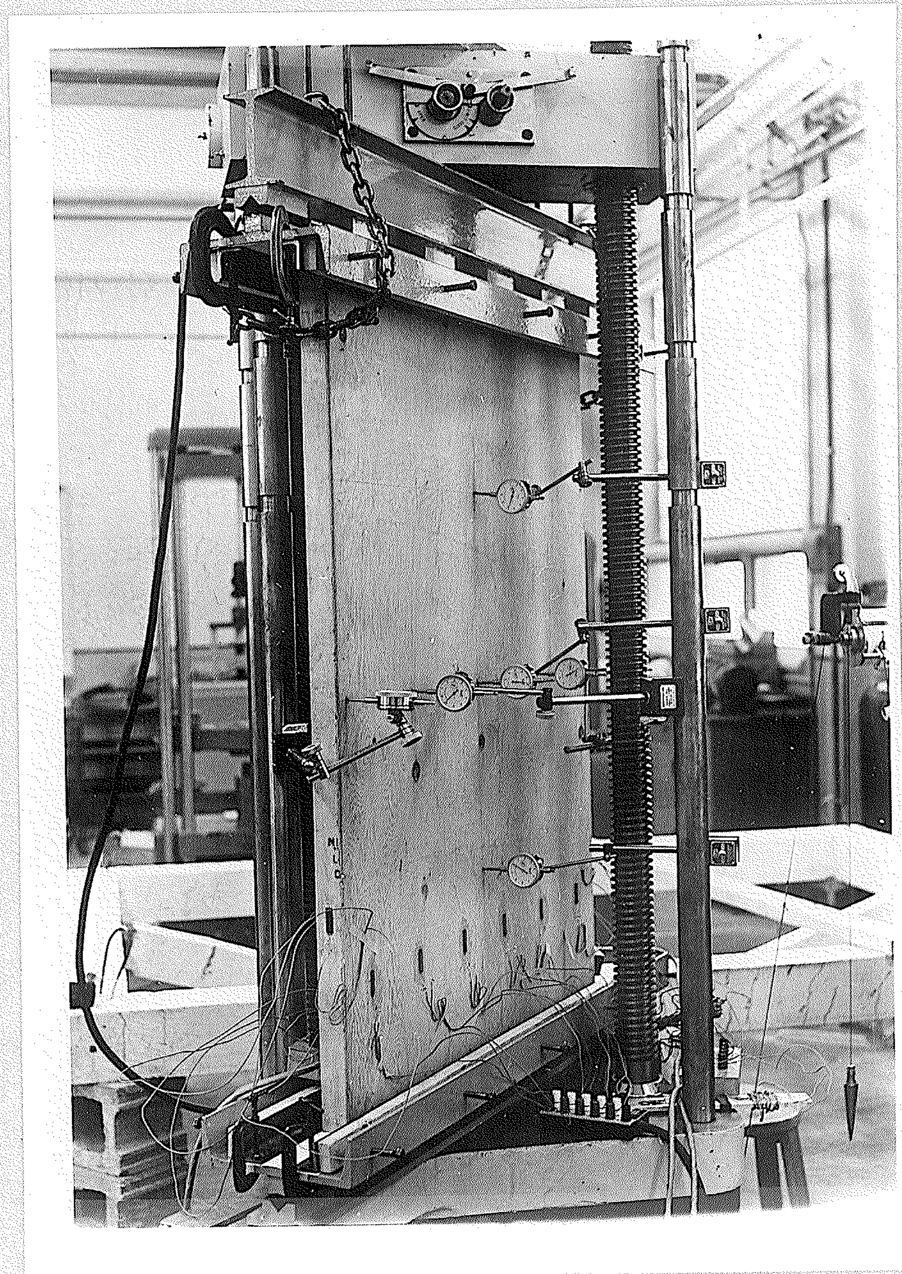


Figure 5.16 Photos of Test Apparatus and Specimens

CHAPTER VI

TEST RESULTS

6.1 Introduction

Presented in this chapter are the theoretical and experimental results of the test program outlined in Chapter V.

6.2 Theoretical results.

Given in Table XI are the theoretical values of the eight tests. Note that a and b differ from the values given for the length and width

TABLE XI

THEORETICAL RESULTS OF TEST PROGRAM

TEST	a INCHES	b INCHES	ϕ	K_{cr}	σ_{cr} psi
A	44.75	38.75	.683	3.42	75.8
B	44.75	32.75	.807	3.08	95.5
C	44.75	32.75	.807	3.08	95.5
D	44.75	32.75	.807	3.08	95.5
E	44.75	26.75	.988	2.86	133
F	48	26.75	1.06	2.86	133
G	46.50	20.75	1.32	3.11	240
H	46.50	14.75	1.86	2.86	437

of the panels in Table X. This is because the values used for a and b are the plywood dimensions less the widths of edge members or end stiffeners. The values of a, for tests F, G and H, are identical to the values of l given in Table X because there are no end stiffeners on these panels.

6.3 Experimental results.

The experimental results have been presented fully in Appendix C. Table XII gives a summary of these results.

TABLE XII

EXPERIMENTAL RESULTS OF TEST PROGRAM

TEST	σ_{cr} (psi) Edge Member	K_{cr}	σ_{cr} (psi) Plywood Readings	K_{cr}
A	165	7.45	175	7.90
B	278	8.96	214	6.90
C	427	13.78	404	13.04
D	516	16.65	528	17.05
E	412	8.86	241	5.18
F	558	12.00	326	7.01
G	501	6.50	320	4.15
H	705	4.61	261	1.71

The experimental and theoretical results of the test program are given in Figure 6.17.

6.4 Discussion.

From a comparison of the test results and the theoretical results, it can be seen that there is not very good agreement between them. The

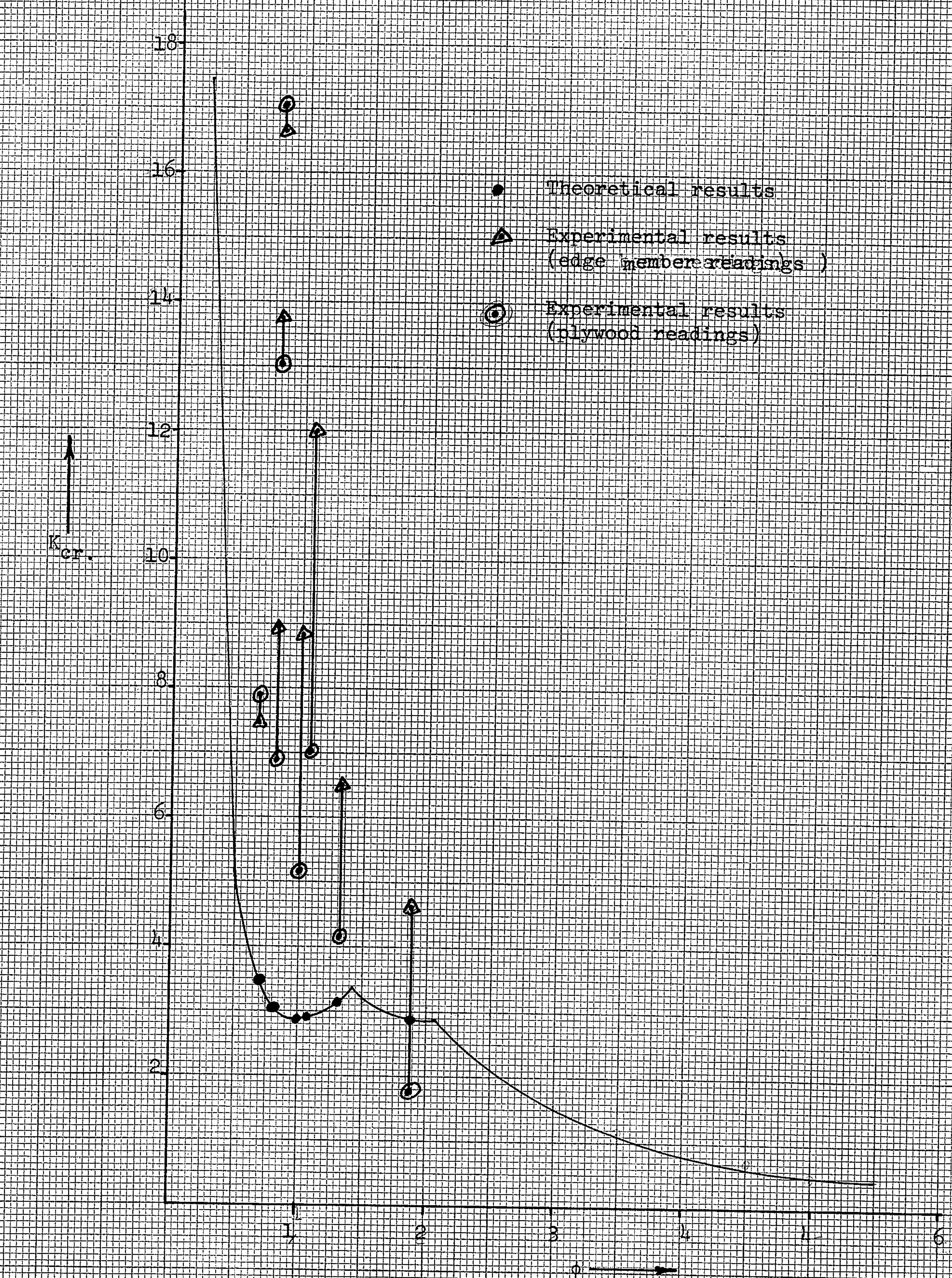


Figure 6.17

Theoretical and Experimental Results Of Test Program experimentally determined buckling loads are generally very much higher than the theoretical values.

One of the reasons for this is that there was greater restraint on the edges during the test than was assumed for the theoretical analysis. That is the reason the 2x2's were replaced by the angles, and the end stiffeners were removed. However, there doesn't seem to have been much effect on the test results by these actions. Tests C and D gave higher values than test B when the angles were used after test B. Also, when the end stiffeners were removed after test E, test F gave a higher buckling load than test E. This would seem to imply that these factors did not give as much restraint as was thought. The loaded edges could be restrained by the channel and knife edge arrangement. It was noted during the tests that, in fact, the panel did rotate under this arrangement, but under very high loads. No rotation was noted at the lower loads. It is believed that considerable restraint is inherent in the end loading arrangement used. Also the edge members seem to give considerable restraint to the plywood, contrary to what was mentioned previously. This is one area that could be investigated in any future tests.

Another major problem in evaluating the results was that it was very hard to tell when the panels had buckled. With the exception of tests C and D, no extreme buckling deflection was noted. This makes the evaluation of reliable buckling loads difficult. Strain gauge readings were taken initially to help to determine when the panels buckled. These were discarded because they were of no help.

It is suggested that, in any future tests, the panels be made smaller and with heavier plywood. The large panels used in these tests of 1/4 inch plywood are too flexible and are readily susceptible to the effects of initial curvature and twisting. Smaller panels of heavier plywood would be more rigid and would probably give a better buckling behaviour without the effects of initial curvature. These effects of initial curvature are great and present many complications. A comparison of the test results, obtained by using the edge member readings, with those using the plywood readings, shows the difficulties in obtaining reliable results.

One useful piece of information obtained from the tests is that the buckling (if, in fact, the panels can be considered to have buckled) did not destroy the load-carrying capacity of these panels. In all cases the load supported by the panel was in excess of the theoretical buckling load. The panel for test H, which was loaded to failure, sustained a load of 14,750 pounds. This compares with a total theoretical load at buckling of 3900 pounds.

CHAPTER VII

CONCLUSION

7.1 Introduction

Most of the conclusions and recommendations for future research have been presented already, but a summary of these points will be presented here for completeness and compactness.

7.2 Conclusions.

1. The critical buckling stress equation was validated quantitatively for special cases and qualitatively for the general case of edge members.
2. Local buckling of the plywood skin of the panels can be predicted, based on the theoretical results, by considering the plywood plate to be simply supported.
3. The panels tested had considerable post-buckling strength, if, in fact, the panels buckled as indicated from the results available.
4. The test apparatus used did not satisfy, closely enough, the assumed end conditions of simple support in the theoretical analysis.

7.3 Recommendations for future research.

1. The possibility of an Euler-type equation being used to predict column buckling of the panel could be investigated. This would then replace the use of the critical buckling stress equation for predicting this type of buckling.
2. For future tests different end apparatus should be investigated.
3. The torsional stiffeners of the edge members should be thoroughly investigated.

4. For future tests smaller panels with thicker plywood sheets should be used to help eliminate the effects of initial curvature.

5. A theoretical analysis could be made with $\sigma_b = o$ and, then, tests run loading only the plywood. This would eliminate some complications of end bearing in the tests and would help to validate the critical buckling stress equation using a simpler test than used here.

6. If smaller panels are used, strain gauges should be placed at the quarter points of the diagonal on the plywood sheet. It has been found that these give excellent results in predicting when the plywood buckled.

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APPENDIX A

ORTHOTROPIC PLATE THEORY

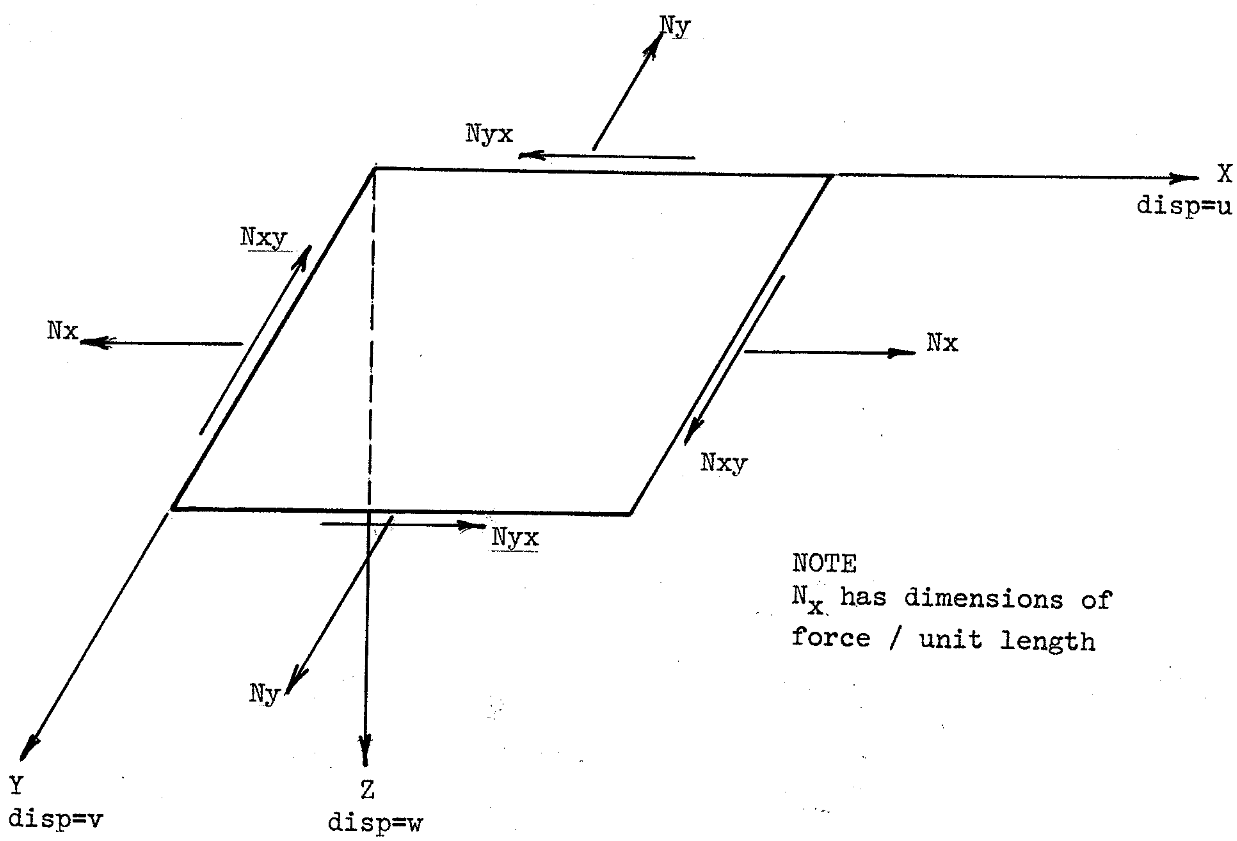
Introduction.

In considering the stability of plywood panels appropriate equations of a thin orthotropic plate in buckling are needed. The equations needed are the differential equation of the deflection surface, and the moment-displacement relations. Isotropic plate theory is well established and known, and has been thoroughly treated by many authors [A-1] and [4-3]. Orthotropic plate theory has not been dealt with so thoroughly in the literature and, even though its development follows closely that of isotropic plate theory, its full development will be presented here for completeness.

The development of orthotropic plate theory presented in this appendix closely follows the development of isotropic plate theory used by G. Gerard [A-2]. Equations pertaining to orthotropic plate theory used are from references [A-1], [3-3], [4-1] and [A-3].

Orthotropic plate considered.

Figure A.18 [A-2] shows an orthotropic plate loaded by various forces in the plane of the plate. The thickness of the plate is small in comparison with the dimensions of the plate in its plane. At the boundaries it is assumed that the edges of the plate are free to move in the plane of the plate. The deflection of the plate is assumed small in comparison with its thickness.



NOTE
N_x has dimensions of force / unit length

Figure A.18
LOADING AND DISPLACEMENTS OF ORTHOTROPIC PLATE

(positive as shown)

Equilibrium equations.

When the plate buckles it deflects laterally out of its plane into a deflection surface depending upon the external loads and restraints at its edges. This deflection causes internal bending moments and shears in the plate. The forces in the plane of the plate produce components which act in equilibrium with the shears. Equilibrium of the middle-surface forces will be considered first. Then the equilibrium of the bending and shear forces will be considered for simplification.

middle-surface forces

The forces and displacements of a differential element $d_x d_y$ of the plate after buckling are shown in Figure A.19 [A-2]. In considering the z-direction components of these forces, it is assumed that

$$\sin \frac{\partial w}{\partial x} = \frac{\partial w}{\partial x}$$

since the deflections are small. The projection of the N_x forces on the z axis is

$$\left[N_x + \frac{\partial N_x}{\partial x} d_x\right] \left[\frac{\partial w}{\partial x} + \frac{\partial^2 w \; d_x}{\partial x^2}\right] dy - N_x \frac{\partial w}{\partial x} dy$$

Retaining only the lower - order terms involving N_x, the z component of the N_x forces becomes

$$N_x \frac{\partial^2 w}{\partial x^2} \; d_x d_y$$

Similarly, the z component of the N_y forces is

$$N_y \frac{\partial^2 w}{\partial x 2} \; d_x d_y$$

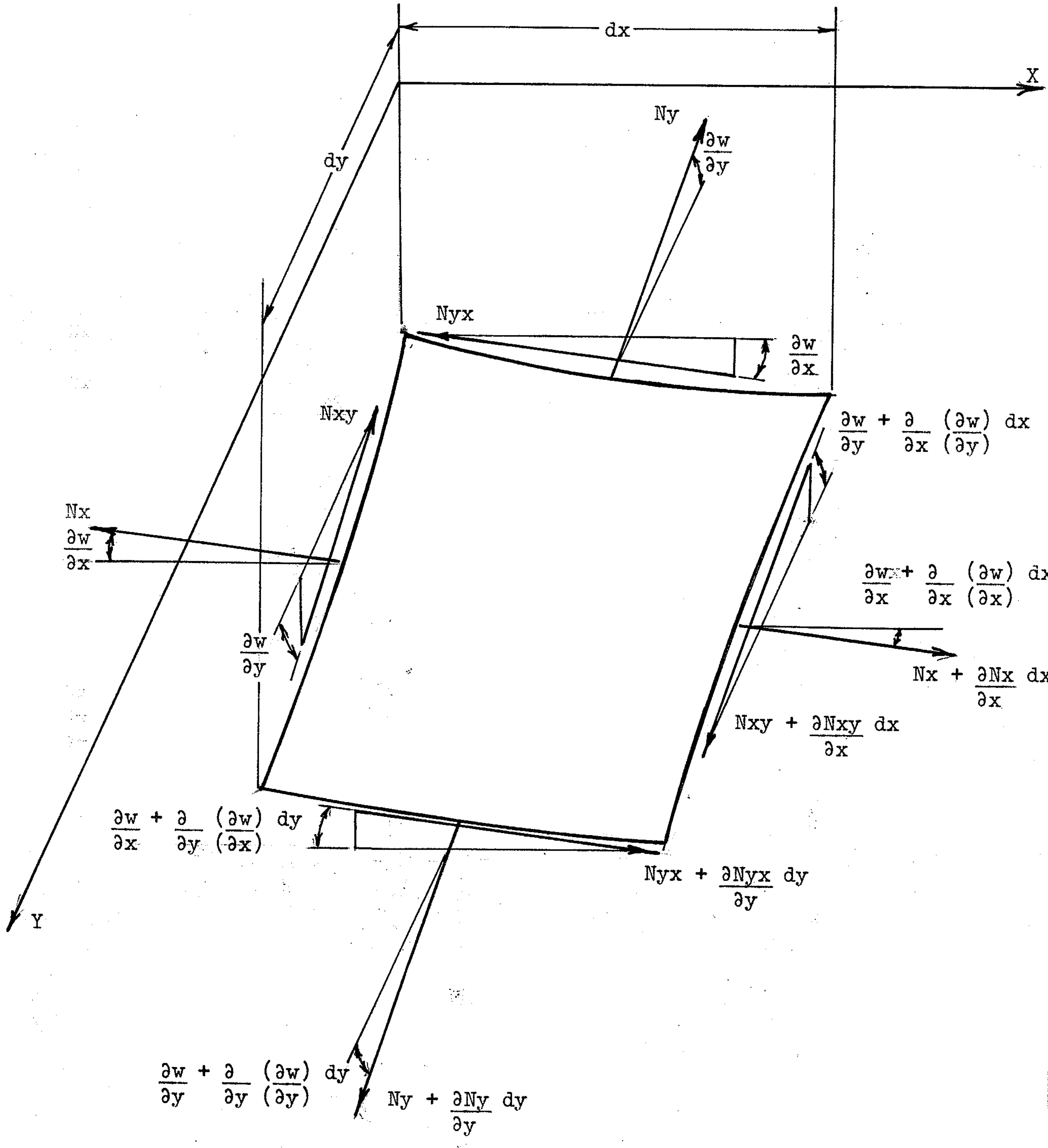


Figure A.19

FORCES AND DISPLACEMENTS OF
ORTHOTROPIC PLATE AFTER BUCKLING

The z component of the N_{xy} forces is

$$N_{xy} \frac{\partial^2 w}{\partial x \partial y} d_x \, d_y$$

The z component of the N_{yx} forces is

$$N_{yx} \frac{\partial^2 w}{\partial x \partial y} d_x d_y$$

Summing the four z components of force, and noting that, by taking moments about the z axis, $N_{yx} = N_{xy}$, the z components of the middle-surface forces are

$$\left[N_x \frac{\partial^2 w}{\partial x^2} + 2 \, N_{xy} \frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2} \right] d_x \, d_y \tag{A-1}$$

bending moments and shears

The bending moments and shears of a differential element $d_x \, d_y$ of the plate after buckling are shown in Figure A.20 [A-2]. The z component of these shears is

$$\left[\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} \right] d_x \, d_y \tag{A-2}$$

The equation for equilibrium of the z-component forces is the sum of equations (A-1) and (A-2):

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + N_x \frac{\partial^2 w}{\partial x^2} + 2 \, N_{xy} \frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2} = o \tag{A-3}$$

By taking moments about the x axis of Figure A.20, and using the right-hand rule for positive moments, the following equation is obtained:

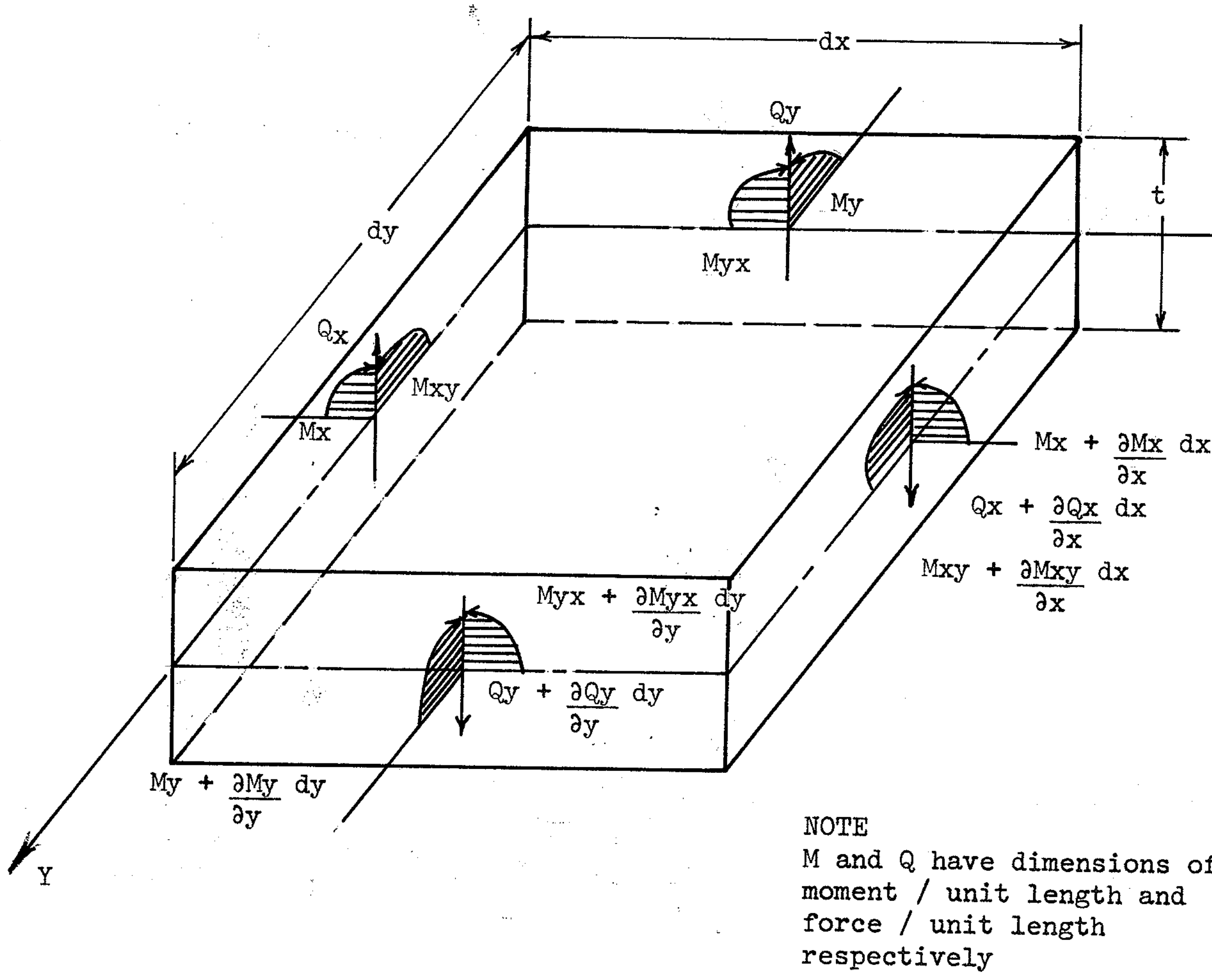


Figure A.20

MOMENTS AND SHEARS OF ORTHOTROPIC
PLATE AFTER BUCKLING

(positive as shown)

$$- \left[M_{xy} + \frac{\partial M_{xy}}{\partial x} d_x \right] d_y + M_{xy}\, d_y + M_y\, d_x$$

$$- \left[M_y + \frac{\partial M_y}{\partial x} d_y \right] d_x + \left[Q_y + \frac{\partial Q_y}{\partial y} d_y \right] d_x\, d_y = o \qquad \text{(A-4)}$$

By simplifying and neglecting the term containing the higher-order derivative of Q_y, equation (A-4) becomes

$$\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} - Q_y = o \qquad \text{(A-5)}$$

Similarly by taking moments about the y axis, the following equation is obtained:

$$\frac{\partial M_x}{\partial x} + \frac{\partial M_{yx}}{\partial y} - Q_x = o \qquad \text{(A-6)}$$

By performing the operation $\partial/\partial y$ on equation (A-5) and ∂/∂_x on equation (A-6), adding the two resulting equations, and noting that $M_{yx} = M_{xy}$, the following equation is obtained:

$$\frac{\partial M_x}{\partial x2} + 2\,\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y2} = \frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} \qquad \text{(A-7)}$$

By combining equation (A-3) and equation (A-7), the equation for the equilibrium of the z-component forces is

$$\frac{\partial^2 M_x}{\partial x2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} = - \left[N_x \frac{\partial^2 w}{\partial x^2} + 2\, N_{xy} \frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y2} \right] \qquad \text{(A-8}$$

MOMENT - Displacement equations

As stated in the introduction to this appendix it is necessary to have certain appropriate equations of a thin orthotropic plate in buckling in order to solve the stability problem of plywood panels.

To obtain one of these equations, the differential equation of the deflection surface, it is necessary to express the force-equilibrium relation [Eq. (A-8)] in terms of the lateral displacement. To do this the moments in equation (A-8) must be expressed in terms of the displacement, w. The moment-displacement relations are also needed for writing boundary condition equations. The equations derived in the preceding part of this appendix are valid for isotropic as well as orthotropic plates. In the following part of this appendix the expressions which pertain specifically to orthotropic plates appear.

The stresses distributed on the x and y faces of a differential element, dxdy, are shown in Figure A.21 [A-2]. The z-component stresses are small in comparison to σ_x, σ_y and τ_{xy} since the plate is thin, and can be neglected.

From Hooke's Law, the relations between stress and strain are:

$$\epsilon_x = \frac{\sigma_x}{E_x} - \frac{\nu_y \sigma_y}{E_y} \qquad \text{(A-9)}$$

$$\epsilon_y = \frac{\sigma_y}{E_y} - \frac{\nu_x \sigma_x}{E_x} \qquad \text{(A-10)}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} \qquad \text{(A-11)}$$

where

ν_y is Poisson's Ratio associated with strain in the x direction due to stress in y direction.

ν_x is Poisson's Ratio associated with strain in the y direction due to stress in x direction.

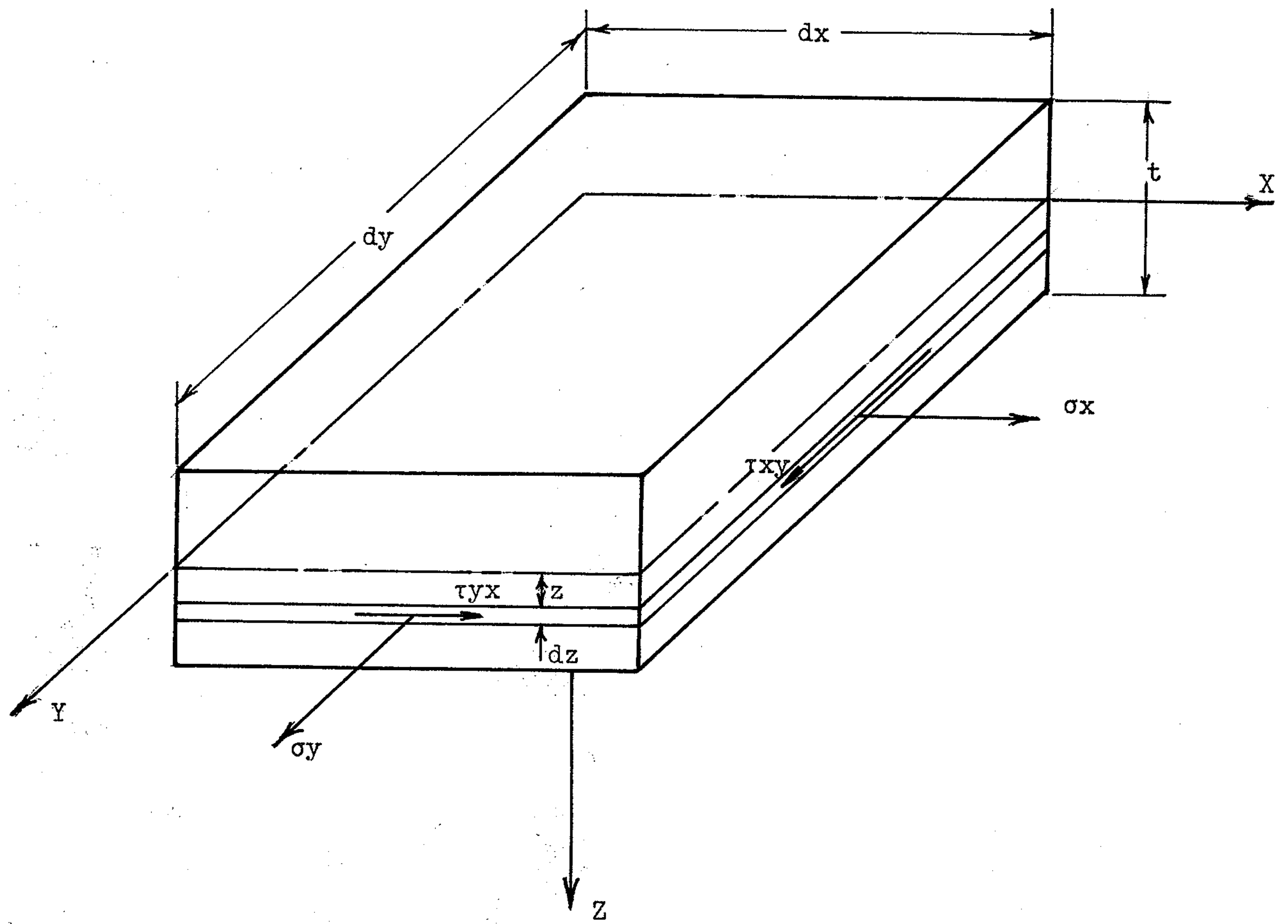


Figure A.21

STRESSES ON X AND Y FACES OF DIFFERENTIAL ELEMENT OF BUCKLED ORTHOTROPIC PLATE

E_x is the Modulus of Elasticity in the x direction.

E_y is the Modulus of Elasticity in the y direction.

G is the Modulus of Rigidity.

In terms of stresses, equation (A-9) to (A-11) are:

$$\sigma_x = \frac{E_x}{1-\nu_y\nu_x} (\varepsilon_x + \nu_y \varepsilon_y) \qquad \text{(A-12)}$$

$$\sigma_y = \frac{E_y}{(1-\nu_y \nu_x} (\varepsilon_y + \nu_x \varepsilon_x) \qquad \text{(A-13)}$$

$$\tau_{xy} = G \gamma_{xy} \qquad \text{(A-14)}$$

These stresses distributed over the sides of the element can be reduced to couples, which are equal to moments on the sides, as follows:

$$M_x = \int_{-t/2}^{t/2} \sigma_x \, z \, d_z \qquad \text{(A-15)}$$

$$M_y = \int_{-t/2}^{t/2} \sigma_y \, z \, d_z \qquad \text{(A-16)}$$

$$M_{xy} = \int_{-t/2}^{t/2} \tau_{xy} \, z \, d_z \qquad \text{(A-17)}$$

strain-displacement relations

$$\varepsilon_x = \frac{\partial u}{\partial x} \tag{A-18}$$

$$\varepsilon_y = \frac{\partial v}{\partial y} \tag{A-19}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \tag{A-20}$$

The following assumptions are made

1. plane sections remain plane
2. The middle surface remains unrestrained because the plate is thin.
3. The plate can move in its plane
4. The deflection is small in comparison with its thickness.

Therefore, from Figure A.22 [A-2], the following relations result:

$$u = -z \frac{\partial w}{\partial x} \tag{A-21}$$

$$v = -z \frac{\partial w}{\partial y} \tag{A-22}$$

By substituting equations (A-21) and (A-22) into equations (A-18) to (A-20) the relations between strain and lateral displacement are:

$$\varepsilon_x = -z \frac{\partial^2 w}{\partial x^2} \tag{A-23}$$

$$\varepsilon_y = -z \frac{\partial^2 w}{\partial y^2} \tag{A-24}$$

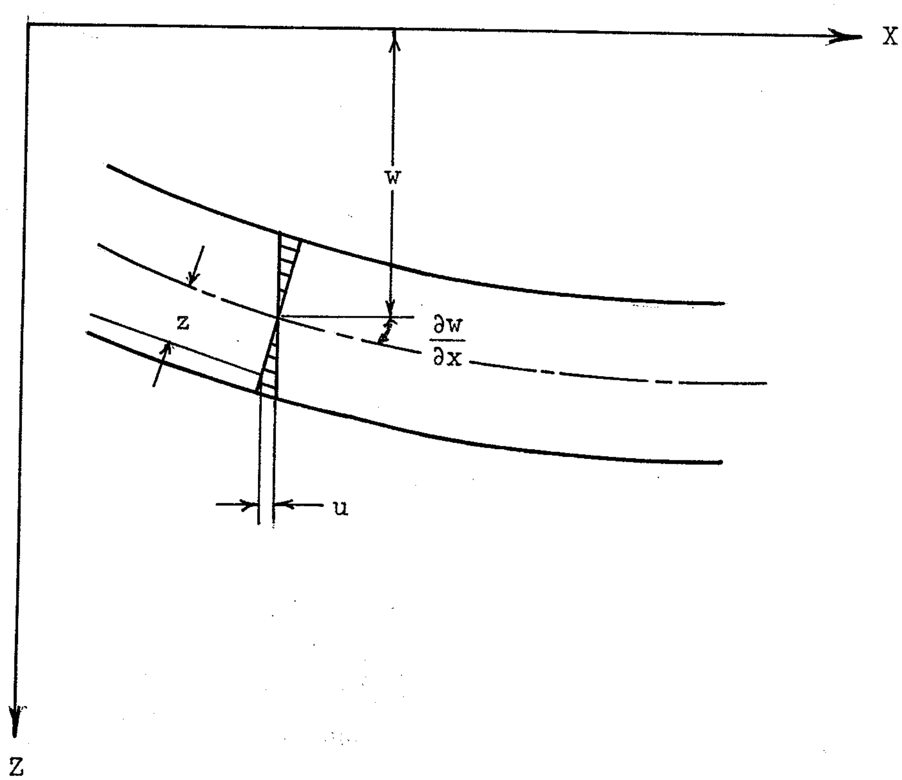


Figure A.22

DISPLACEMENTS FOR A SLIGHTLY BENT ORTHOTROPIC PLATE

$$\gamma_{xy} = -2z \frac{\partial^2 w}{\partial x \partial y} \tag{A-25}$$

By substituting equations (A-23) and (A-24) into equation (A-12), the equation for σ_x becomes

$$\sigma_x = \frac{-E_x}{1-\nu_y\nu_x} \left[\frac{\partial 2w}{\partial x2} + \nu_y \frac{\partial^2 w}{\partial y2}\right] z \tag{A-26}$$

Similarly, by substituting equations (A-23) and (A-24) into equation (A-13), the equation for σ_y becomes

$$\sigma_y = \frac{-E_y}{1-\nu_y\nu_x} \left[\frac{\partial^2 w}{\partial x2} + \nu_x \frac{\partial 2w}{\partial y2}\right] z \tag{A-27}$$

By substituting equation (A-25) into equation (A-14) the equation for

$$\tau_{xy} = \tau_{xy} = -2z\left(\frac{\partial^2 w}{\partial x \partial y}\right) G \tag{A-28}$$

Now, by substituting equations (A-26), (A-27) and (A-28) into equations (A-15), (A-16) and (A-17) respectively and performing the integrations, the following equations for moments are obtained:

$$M_x = -\frac{E_x t^3}{12(1-\nu_y\nu_x} \left[\frac{\partial 2w}{\partial x2} + \nu_y \frac{\partial^2 w}{\partial y2}\right] \tag{A-29}$$

$$M_y = -\frac{E_y\ t^3}{12(1-\nu_y\nu_x} \left[\frac{\partial 2w}{\partial y2} + \nu_x \frac{\partial 2w}{\partial y2}\right] \tag{A-30}$$

$$M_{xy} = \frac{-2\ G\ t^3}{12} \frac{\partial 2w}{\partial x \partial y} \tag{A-31}$$

Rewriting equations (A-29) to (A-31), introducing new notation, the following equations are obtained:

$$M_x = -D_x \left[\frac{\partial 2w}{\partial x2} + \nu y \frac{\partial 2w}{\partial y2}\right] \tag{A-32}$$

$$M_y = - D_y \left[\frac{\partial 2w}{\partial y2} + \nu_x \frac{\partial 2w}{\partial x2}\right] \quad (A-33)$$

$$M_{xy} = - 2 D_{xy} \frac{\partial 2w}{\partial x \partial y} \quad (A-34)$$

where

$$D_x = \frac{E_x t3}{12(1-\nu y \nu x)} \quad (A-35)$$

$$D_y = \frac{E_y t^3}{12(1-\nu y \nu x)} \quad (A-36)$$

$$D_{xy} = \frac{G t^3}{12} \quad (A-37)$$

The quantities D_x and D_y represent the flexural rigidities of the plate in the x and y direction respectively and are analogous to the flexural rigidity EI of a beam.

Equations (A-32) to (A-34) are the moment-displacement relations needed in solving stability problems of orthotropic plates.

Differential equation of deflection surface of orthotropic plate in compression.

Now, substituting into equation (A-8) the expressions for M_x, M_y and M_{xy} of equations (A-32) to (A-34) respectively to obtain the force-equilibrium relation [Eq. (A-8)] in terms of lateral displacement w, the following equation is obtained:

$$D_x \frac{\partial 4w}{\partial x4} + (D_x \nu_y + 4 D_{xy} + D_y \nu_x) \frac{\partial 4w}{\partial x2 \partial y2} + D_y \frac{\partial 4w}{\partial y4}$$

$$= N_x \frac{\partial 2w}{\partial x2} + 2 N_{xy} \frac{\partial 2w}{\partial x \partial y} + N_y \frac{\partial 2w}{\partial y2} \quad (A-38)$$

Introducing new notation into equation (A-38), the following equation is obtained:

$$D_x \frac{\partial^4 w}{\partial x^4} + 2H \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4}$$

$$= N_x \frac{\partial^2 w}{\partial x^2} + 2 N_{xy} \frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2} \qquad \text{(A-39)}$$

where

$$2H = D_x \nu_y + 4 D_{xy} + D_y \nu_x \qquad \text{(A-40)}$$

Equation (A-39) is the differential equation of the deflection surface of a thin orthotropic plate in buckling under the action of forces shown in Figure A.18.

If the same orthotropic plate is loaded only by compressive forces in the x direction, $N_y = N_{xy} = o$, and $N_x = \sigma_x t$, where N is now positive compression, equation (A-39) reduces to

$$D_x \frac{\partial^4 w}{\partial x^4} + 2H \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} + t\,\sigma_x \frac{\partial^2 w}{\partial x^2} = o \qquad \text{(A-41)}$$

APPENDIX B

COMPUTER PROGRAMS

Introduction.

Given in this appendix are the details of the computer programs written in Fortran IV to solve the critical buckling stress equation (2-18). Given herein are the programs written out, the flow charts, the symbols used and a user's guide to input and output data.

Symbols

The symbols used in the programs and their corresponding symbols from the critical buckling stress equation are given in Table XIII.

Flow Charts

The flow charts for programs Type I and Type II are given in Figure B.23 and Figure B.24 respectively.

Programs

Each program consists of two parts. The basic part is identical for all programs. The part which precedes the basic part is different for the Type I and Type II programs. There are two different programs for each of the Type I and Type II parts. These are designated "E Spec." and "F Spec." Following are the different parts which make up any one program.

TABLE XIII

SYMBOLS USED IN PROGRAMS

SYMBOL USED IN CRITICAL BUCKLING STRESS EQUATION	SYMBOL USED IN FORTRAN IV PROGRAMS.
$E_b I_b$	EI
C	C
a	ALONG (LONG)
b	SPAN
t	THK
A_b	AREA
σ_b	ST
σ_{cr}	z
D_x	DX
D_y	D
Dy2	D2
$D_y 2 \nu_x$	DVX
$D_y 2 \nu_x 2$	DVX2
D_y 4 D_{xy}	D4D
Final number of modes	N
D_y 4 D_{xy} ν_x	D4DVX
Initial length of panel	INITIL
H/Dy	H

Figure B.23

Program Flowchart Type I

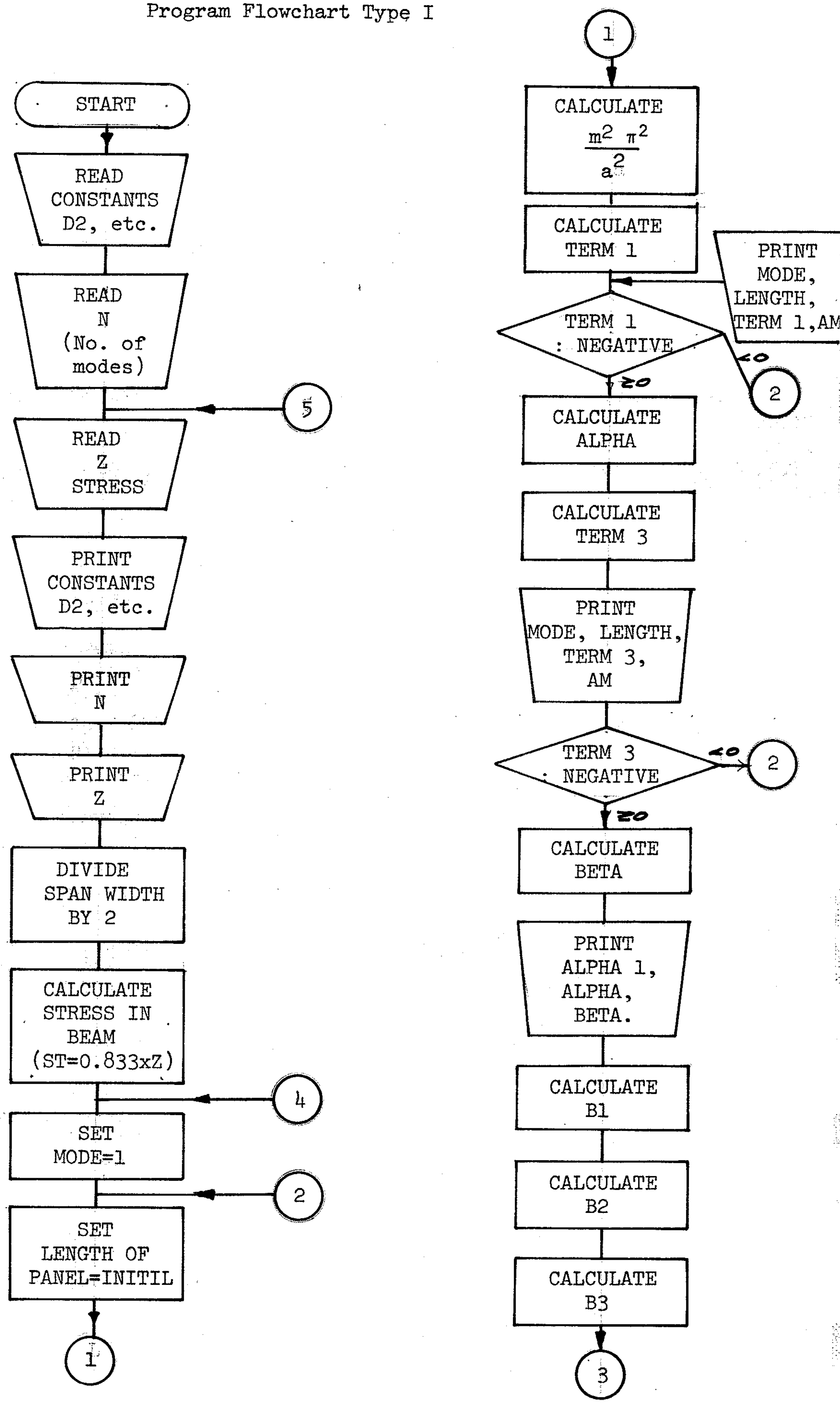


Figure B.23 Contd.

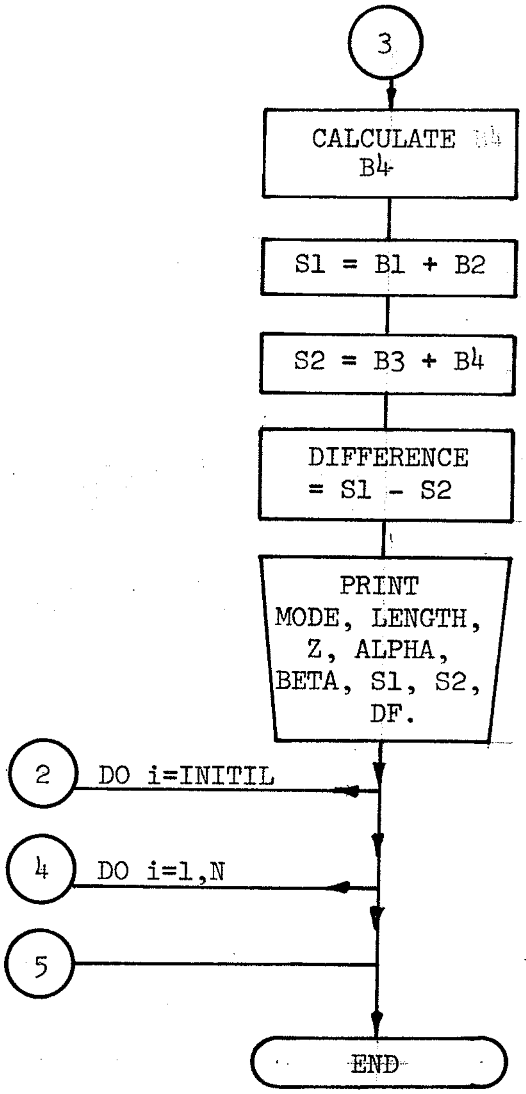
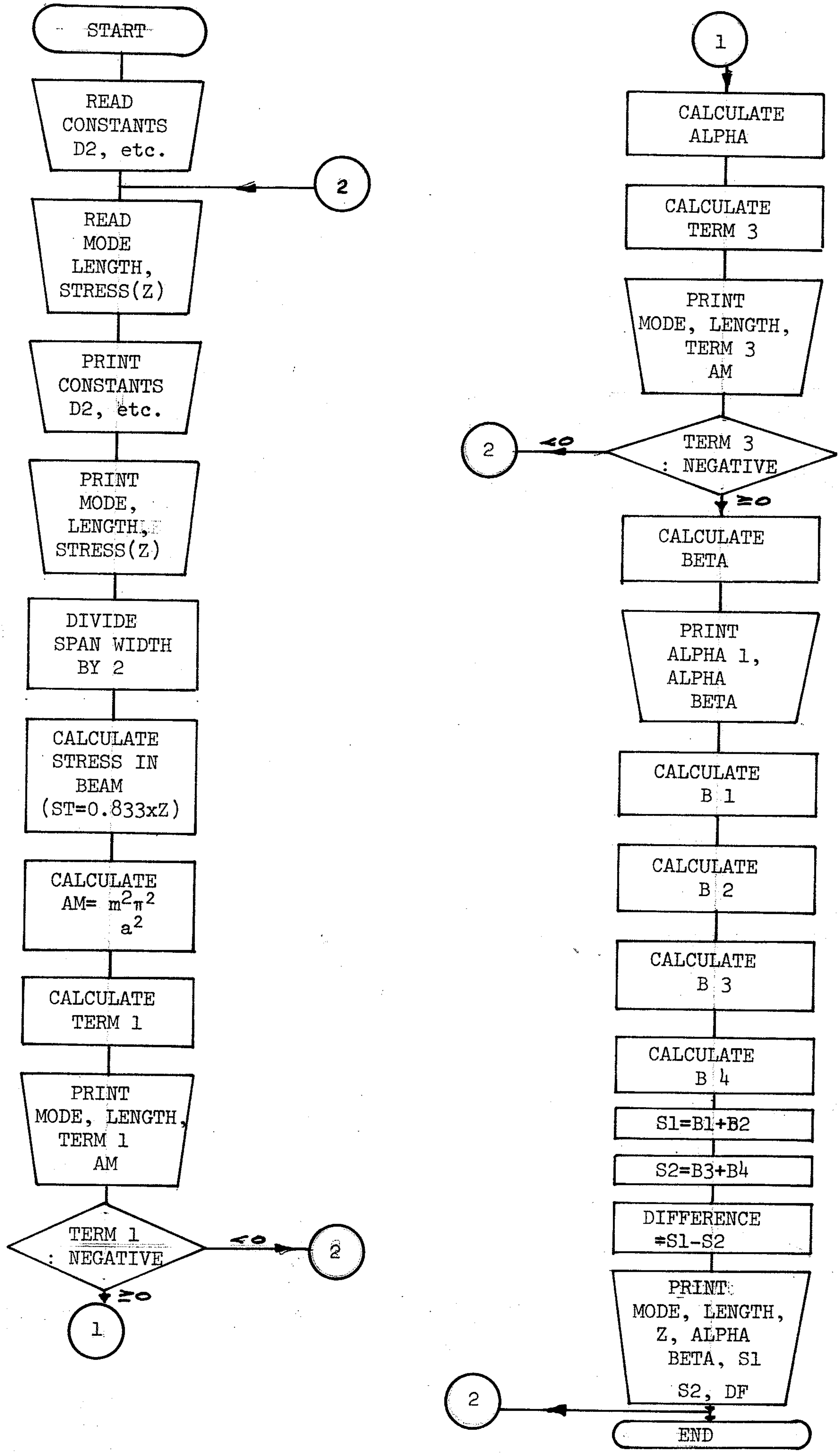


Figure B.24
Program Flowchart Type II



BASIC PART

```
      AM=AMØDE**2*9.87/ALØNG**2
      T1=H**2-DX/D
      T2=T1*AM
      T3=THK*Z/D
      T4=T2+T3
      TERM1=AM*T4
      WRITE(3,25)MØDE,LØNG,TERM1,AM
      IF(TERM1)12,7,7
    7 CØNTINUE
      ALPHA1=SQRT(TERM1)
      X1=H*AM
      X2=X1+ALPHA1
      ALPHA=SQRT(X2)
      TERM2=-X1
      TERM3=TERM2+ALPHA1
      WRITE(3,25)MØDE,LØNG,TERM3,AM
      IF(TERM3)12,8,8
    8 CØNTINUE
      BETA=SQRT(TERM3)
      WRITE(3,23)ALPHA1,ALPHA,BETA
      B1=((-EI*C*ALPHA*AM*AM)+(AREA*ST*C*ALPHA*AM)+(D2*BETA*BETA*ALPHA*
     1ALPHA*ALPHA*1./AM)-(DVX*BETA*BETA*ALPHA)-(D4D*BETA*BETA*ALPHA)+
     2(DVX*ALPHA*ALPHA*ALPHA)-(DVX2*ALPHA*AM)-(D4DVX*ALPHA*AM))*(TANH
     3(ALPHA*Y))
      B2=((D2*BETA*BETA*BETA*ALPHA*ALPHA*1./AM)-(DVX*BETA*BETA*BETA)+
     1(DVX*BETA*ALPHA*ALPHA)-(DVX2*BETA*AM)+(D4D*BETA*ALPHA*ALPHA)-
     2(D4DVX*BETA*AM)-(C*EI*BETA*AM*AM)+(C*AREA*ST*BETA*AM))*(SIN(BETA*Y
     3)/CØS(BETA*Y))
      B3=-((D*C*BETA*BETA*BETA*ALPHA)+(D*C*BETA*ALPHA*ALPHA*ALPHA))*
     1(SIN(BETA*Y)/CØS(BETA*Y))*(TANH(ALPHA*Y))
      B4=-((-EI*D*ALPHA*ALPHA*AM)-(EI*D*BETA*BETA*AM)+(AREA*ST*BETA*
     1BETA*D)+(AREA*ST*D*ALPHA*ALPHA))
      S1=B1+B2
      S2=B3+B4
      DF=S1-S2
      WRITE(3,9)MØDE,LØNG,Z,ALPHA,BETA,S1,S2,DF
   12 CØNTINUE
      GØ TØ 5
      END
```

TYPE I "F SPEC."

```
 1 FØRMAT(8F10.0,/I10,5F10.0)
 2 FØRMAT(I10)
 3 FØRMAT(F16.1)
 9 FØRMAT(1H ,2I5,4E15.7,/2E15.7)
20 FØRMAT(1H ,8F10.0,/I10,5F10.0)
21 FØRMAT(1H ,I10)
22 FØRMAT(1H ,F16.1)
23 FØRMAT(1H ,3E15.7)
25 FØRMAT(1H ,2I5,2E15.7)
   READ(1,1)D2,DVX,D4D,DVX2,D4DVX,THK,DX,D,INITIL,SPAN,EI,C,AREA,H
 4 READ(1,2)N
 5 READ(1,3)Z
   WRITE(3,20)D2,DVX,D4D,DVX2,D4DVX,THK,DX,D,INITIL,SPAN,EI,C,AREA,H
   WRITE(3,21)N
   WRITE(3,22)Z
   Y=SPAN/2
   ST=0.833*Z
 6 DØ 12 MØDE=1,N
   AMØDE=MØDE
   DØ 12 LØNG=INITIL,240,20
   ALØNG=LØNG
```

TYPE I "E SPEC."

```
 1 FØRMAT(8F10.0,/I10,F10.0,2E15.7,2F10.0)
 2 FØRMAT(I10)
 3 FØRMAT(F16.1)
 9 FØRMAT(1H ,2I5,4E15.7,/2E15.7)
20 FØRMAT(1H ,8F10.0/I10,F10.0,2E15.7,2F10.0)
21 FØRMAT(1H ,I10)
22 FØRMAT(1H ,F16.1)
23 FØRMAT(1H ,3E15.7)
25 FØRMAT(1H ,2I5,2E15.7)
   READ(1,1)D2,DVX,D4D,DVX2,D4DVX,THK,DX,D,INITIL,SPAN,EI,C,AREA,H
 4 READ(1,2)N
 5 READ(1,3)Z
   WRITE(3,20)D2,DVX,D4D,DVX2,D4DVX,THK,DX,D,INITIL,SPAN,EI,C,AREA,H
   WRITE(3,21)N
   WRITE(3,22)Z
   Y=SPAN/2.
   ST=0.833*Z
 6 DØ 12 MØDE=1,N
   AMØDE=MØDE
   DØ 12 LØNG=INITIL,240,20
   ALØNG=LØNG
```

TYPE II "F SPEC."

```
 1 FØRMAT(8F10.0,/5F10.0)
 2 FØRMAT(I10,2F10.0)
 3 FØRMAT(1H ,I10,2F10.4)
 9 FØRMAT(1H ,2I5,4E15.7,/2E15.7)
20 FØRMAT(1H ,8F10.0,/5F10.0)
23 FØRMAT(1H ,3E15.7)
25 FØRMAT(1H ,2I5,2E15.7)
   READ(1,1)D2,DVX,D4D,DVX2,D4DVX,THK,DX,D,SPAN,EI,C,AREA,H
 5 READ(1,2)MØDE,ALØNG,Z
   WRITE(3,20)D2,DVX,D4D,DVX2,D4DVX,THK,DX,D,SPAN,EI,C,AREA,H
   WRITE(3,3)MØDE,ALØNG,Z
   Y=SPAN/2.
   ST=0.833*Z
   LØNG=ALØNG
   AMØDE=MØDE
```

TYPE II "E SPEC."

```
 1 FØRMAT(8F10.0,/F10.0,2E15.7,2F10.0)
 2 FØRMAT(I10,2F10.0)
 3 FØRMAT(1H ,I10,2F10.4)
 9 FØRMAT(1H ,2I5,4E15.7,/2E15.7)
20 FØRMAT(1H ,8F10.0,/F10.0,E15.7,F5.0,2F10.0)
23 FØRMAT(1H ,3E15.7)
25 FØRMAT(1H ,2I5,2E15.7)
   READ(1,1)D2,DVX,D4D,DVX2,D4DVX,THK,DX,D,SPAN,EI,C,AREA,H
 5 READ(1,2)MØDE,ALØNG,Z
   WRITE(3,20)D2,DVX,D4D,DVX2,D4DVX,THK,DX,D,SPAN,EI,C,AREA,H
   WRITE(3,3)MØDE,ALØNG,Z
   Y=SPAN/2.
   ST=0.833*Z
   LØNG=ALØNG
   AMØDE=MØDE
```

TYPE I INPUT DATA

The input data are submitted according to three Format statements (1, 2 and 3). The constants D2, DVX, D4D, DVX2, D4DVX, THK, DX and D are read in on the first card. The 8 constants are punched in fields of 10 columns each. These numbers must have decimal points punched but may be punched anywhere in the field and may have any number of decimal places. The second card contains the constants INITIL, SPAN, EI, C, AREA, and H and are read according to the last part of Format statement number 1. The first constant, INITIL, must be punched in the right-justified position with no decimal. The last five constants may be punched in the same manner as the numbers on the first card, having a decimal point. The third card is read according to the Format statement number 2. On this card is punched the number of modes that are required to be run through in the calculations. This number must be right-justified in the ten-column field and punched with no decimal point. The fourth card is read according to the Format statement number 3. On this card is punched the stress (z) which is assumed to be the solution of the equation. This number may be punched anywhere within the first 16 columns, with any number of decimal places and with a decimal point present. If more stresses are desired to be submitted, simply punch them out as in card 4 and submit them as cards 5, 6, etc.

There are two types of Format statements for statement number 1. The so-called "E Spec." and "F Spec." These Format statements are for the first two cards of the data deck. The change between the "F-" and "E-" type specification occurs in the second card (i.e. the field specification after the slash (1) in Format statement 1). The constants EI and C are sometimes very large and thus the "F Spec." is inadequate for these large numbers. The "E Spec." is used for these large numbers.

In punching out the input data for the "E Spec." on the second card the numbers are the same as before for the first two fields. For the constants, EI and C, two fields of fifteen columns each are used. The numbers are punched with 7 decimal places present. Then E is punched, denoting that the exponential form is present. After the letter E, the power to which the number is being raised, is punched. For example, .9999999 x 10^{40} is punched as .9999999 E 40.

The constants, AREA and H, are punched in columns 50 to 60 and 61 to 70, respectively. These are punched with decimal points and are right-justified.

A "Do" statement must be punched out with each set of data and placed in the program. The statement is Do 12 LONG = INITIL, _____, _____. The final length of the panel that is to be dealt with in the calculations is punched in the first blank space. In the second blank is punched the increment in which the length of the panel is to be increased from the initial to the final length.

TYPE I OUTPUT DATA

The output data are printed according to six Format statements, (9,20,21,22,23 and 25). The first four input data cards are printed out first according to statements 20, 21 and 22. This enables the user to determine if the computer is reading the input data correctly.

The next line is printed out according to statement 25. This line gives the number of modes (MODE), the length of the panel (LONG), TERM 1 and AM. If TERM 1 is negative, the next line consists of the next length of panel, TERM 1 and AM, and so on. But, if TERM 1 is positive, the calculation continues and the next line of print consists of MODE, LONG, TERM 3 and AM. If TERM 3 is negative, the calculation is halted and the next length is run through. But, if TERM 3 is positive, the calculation continues and the next line of print-out consists of the values for ALPHA 1, ALPHA, and BETA according to statement 23. The next two lines of print-out data consist of MODE, LONG, Z., ALPHA, BETA and S1, and then S2 and DF.

Then the program continues with the next length and so on. After all the lengths of panel have been run through with the first mode, they are run through again with the next mode of buckling.

The pattern, which is followed in the print-out of the output data, was arranged so that the user could analyze the calculations of the equation easily and quickly (i.e. the user could cut up the output data according to stresses so that at the top of each sheet the stress being used was printed out.) This was made possible by simply having the first three data cards printed out again, and then the next stress, which was read in, printed out at the end of the cycle of the outer "Do" loop.

TYPE II INPUT DATA

The input data are submitted according to Format statements 1 and 2. The first card is identical to that of the Type I program. The second card contains the constants SPAN, EI, C, AREA and H. These constants are punched in the same manner as those on the first card. The third card is read according to Format statement 2. On this card is punched the number of modes of buckling (MODE), the length of the panel (ALONG), and the stress (z) which is assumed to be the solution of the equation. These three numbers are punched in fields of ten columns each. The first mode must be punched in the right-justified position with no decimal point within columns 1 to 10. The next two fields (11 to 20 and 21 to 30) contain ALONG and z respectively. These numbers are punched in the same manner as the numbers on the first two cards. If more stresses are desired to be submitted, simply punch them out as in the third card with ALONG and MODE and submit them as cards 4, 5 etc.

There is also an "E Spec." and "F Spec." for Type II program. They are similar to those of the Type I programs.

TYPE II OUTPUT DATA

The output data are printed according to the Format statements 3, 9, 20, 23 and 25. The first three input data cards are printed out first, according to statements 20 and 3. This enables the user to determine if the computer is reading the input data correctly. The next line is printed out according to statement 25. This line gives the number of modes, the length of the panel, TERM 1 and AM. If term 1 is negative, the next three lines are the same as the first three lines. But, if TERM1 is positive, the calculation continues and the next line of print consists of MODE, LONG, TERM 3 and AM. If TERM 3 is negative, the calculation is halted, and the next lines consist of the data from the first three input cards. But, if TERM 3 is positive, the calculation, continues and the next line of print-out consists of the values for ALPHA 1, ALPHA and BETA, according to statement 23. The next two lines of print-out data consist of MODE, LONG, Z, ALPHA, BETA, and S1, and then S2 and DF. Then the program continues with the next mode, length and stress and prints out the first two input cards once again.

APPENDIX C

TEST RESULTS AND SAMPLE CALCULATIONS

Introduction.

Presented in this appendix are the results of the tests that were out lived in Chapter V. Sample calculations are presented for test A. All other tests are calculated the same way.

TEST A.

Given in Table XIV are the values of the deflection, taken from the dial gauges, for the corresponding value of the total load on the panel. By total load is meant the load on the plywood plus the load on the edge members.

From a plot of the values given in Table XIV, as shown in Figure C.25, the value of the total buckling load can be seen to be about 3000 lbs.

The readings of the gauges (1-4) on the edge members are given in Table XV.

TABLE XIV

TOTAL LOAD VS DEFLECTION FOR TEST A

TOTAL LOAD (lbs.)	DEFLECTION (ins.)
200	0
500	.026
1000	.056
1500	.086
2000	.122
2500	.152
2700	.164
2900	.177
3000	.184
3100	.192
3200	.225
3300	.235
3500	.251
4000	.310
4500	.376

TABLE XV

EDGE MEMBER READINGS AT 3000 LBS. FOR TEST A

Gauge No.	1	2	3	4
Strain Reading in/in x 10^{-6} (-ve. COMP.)	-276	-254	-188	-159

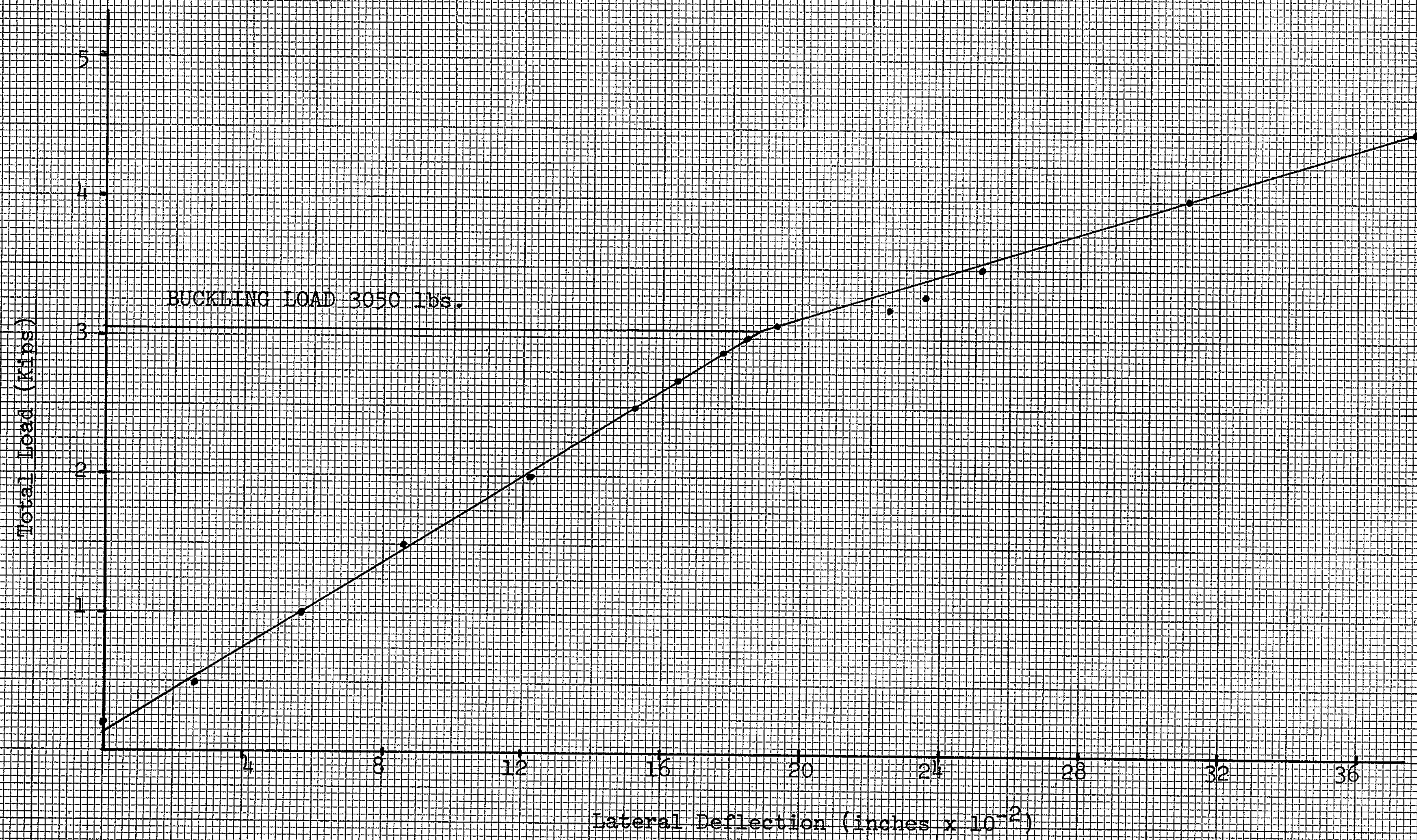


Figure C.25
TOTAL LOAD ON PANEL AS A FUNCTION OF LATERAL DEFLECTION OF PANEL FOR TEST A

The average of the readings from Table XV is

219×10^{-6} in/in

Therefore the stress in the edge members is

219×10^{-6} in/in $\times 1.10 \times 10^{6}$ psi = 241 psi

Therefore the force in the edge members is

241 psi x 5.28 in.^{2} = 1272 lbs.

Load on plywood at buckling is

3000 - 1272 = 1728 lbs.

Therefore σ_{cr} is

$$\frac{1728}{42x.25} = 165 \text{ psi.}$$

Table XVI gives the values of the strain readings for the gauges on the plywood.

TABLE XVI

PLYWOOD READINGS AT 3000 lbs FOR TEST A

Gauge No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Strain Rdg.	-126	-125	-186	-76	-41	62	64	32	-48	-149	-223	-253	-156	-100

in/in x 10^{-6}

(-ve. comp.)

(tve tens.)

The average of the readings from Table XVI is

-95×10^{-6} in/in.

Therefore σ_{cr} is

-95×10^{-6} in/in $\times 1.84 \times 10^{6}$ psi. = 175 psi.

TESTS B TO H

TABLE XVII

TOTAL LOAD VS DEFLECTION FOR TESTS B TO H.

TOTAL LOAD (lbs.)	DEFLECTION (INCHES) TEST B	TEST C	TEST D	TEST E	TEST F	TEST G	TEST H
300	0						
400		0	0	0	0	0	0
800	.006	-.009					
1000			-.019				
1200				.008	-.004	.004	.023
1400	.027	-.022					
1600			-.041				
2000	.038	-.032		.017	-.009	.009	.032
2200			-.057				
2400					-.011		
2600	.054	-.040					
2800			-.072	.028		.018	.046
3000	.062	-.044					
3400			-.085				
3600	.078	-.046		.044	-.015	.029	.063
4000			-.098				
4200	.098	-.043					
4400				.063	-.015	.046	.081
4600			-.106				
4800	.129	-.035					
5200		-.025	-.112	.088	-.010	.070	.103
5400	.171						
5800		.002	-.112				
6000	.203			.121	.001	.102	.128
6200		.033					
6400		.069	-.106				
6600	.241 End of Dial						
6800				.163	.022	.143	.160
7000			-.084				
7200	.324						
7600			0	.214	.056	.197	.197
7800	.378		.160				
8200				.262			
8400					.113		.239
9200					.203		
9600					.256		

Note that in Table XVII positive values of deflection mean deflection in the direction opposite to that side of the plywood which has the wood members.

From a plot of the values given in Table XVII as shown in Figures C.26 to C.32 the values of the critical buckling load are as given in Table XVIII.

TABLE XVIII

CRITICAL BUCKLING LOAD FOR TESTS B TO H.

TEST	B	C	D	E	F	G	H
Pcr.(lbs.)	4200	6400	7400	5200	7200	5000	5800

In Table XIX are given the strain readings of the gauges on the edge members.

TABLE XIX

EDGE MEMBER READINGS AT CRITICAL BUCKLING LOAD
FOR TESTS B TO H.

TEST	GAUGE NO. (RDGS. in/in x 10^{-6}, -ve COMP)			
	1	2	3	4
B	-310	-292	-276	Damaged
C	-450	-409	-464	Damaged
D	-482	-465	-481	Damaged
E	-364	-366	Damaged	360
F	-494	-497	-567	-512
G	-339	-344	-359	-335
H	-684	-473	-311	-342

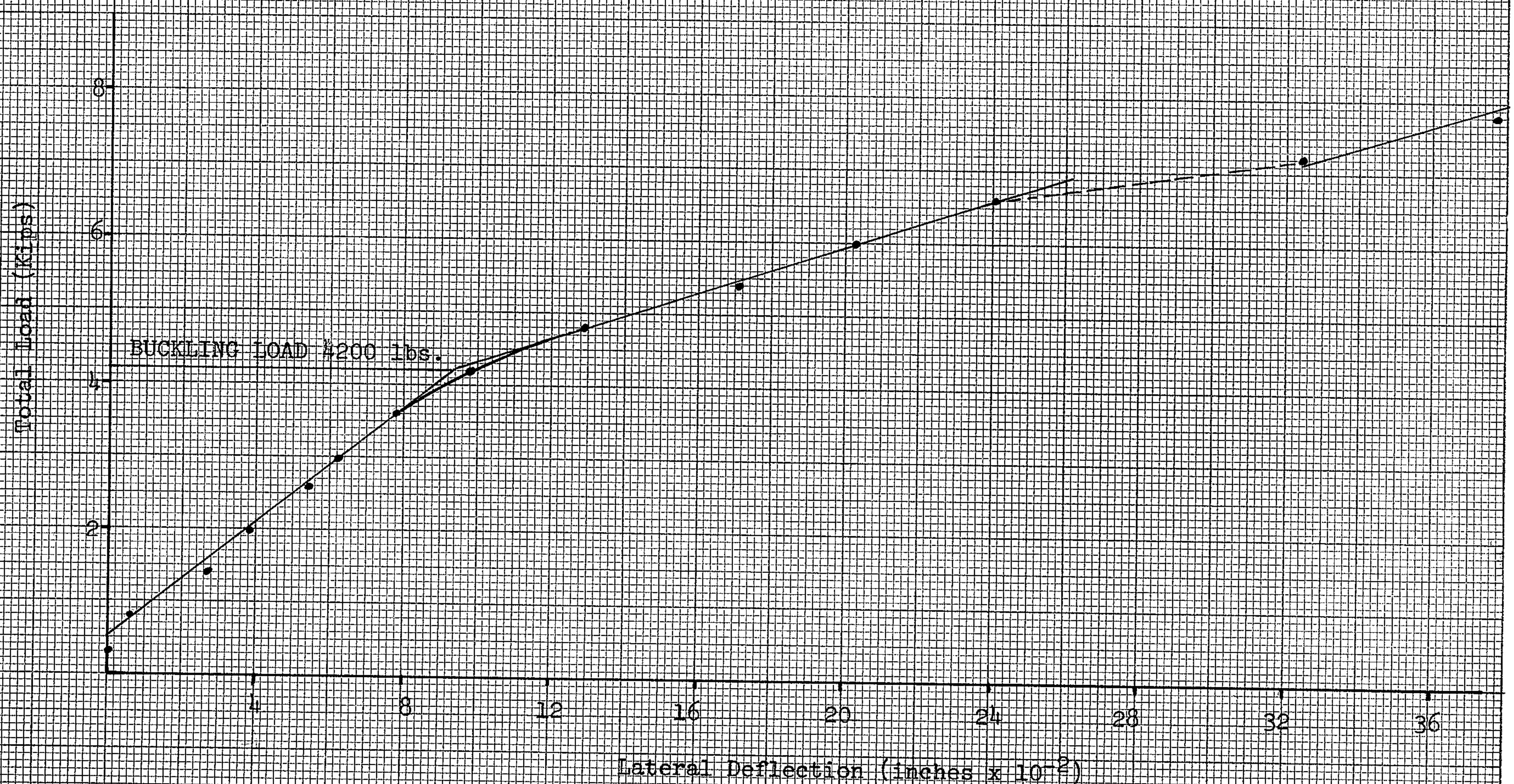


Figure C.26
TOTAL LOAD ON PANEL AS A FUNCTION OF LATERAL DEFLECTION OF PANEL FOR TEST B

THIS MARGIN RESERVED FOR BINDING.

IF SHEET IS READ THIS WAY (HORIZONTALLY), THIS MUST BE TOP.

IF SHEET IS READ THE OTHER WAY (VERTICALLY), THIS MUST BE LEFT-HAND SIDE.

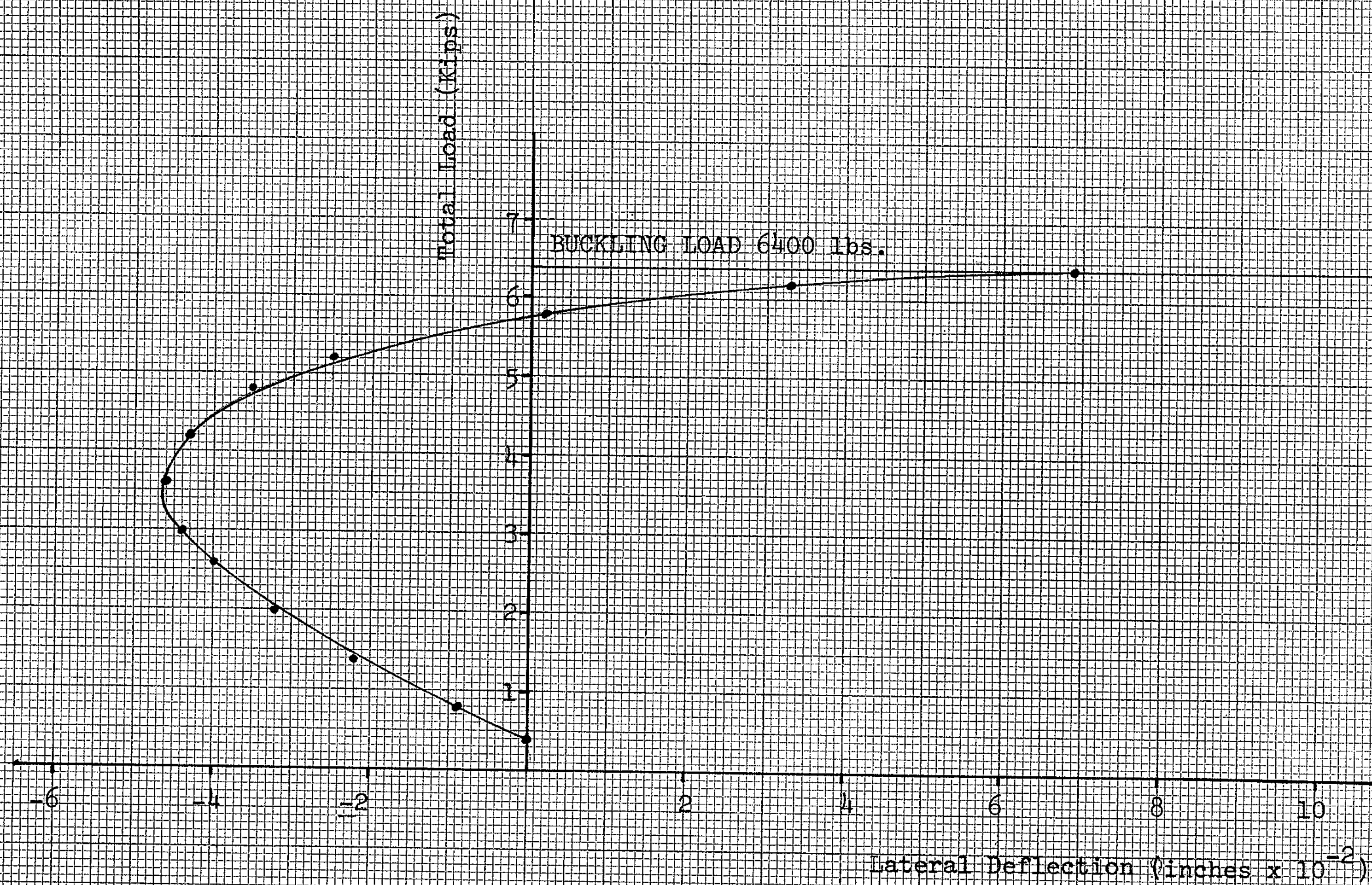


Figure C.27
TOTAL LOAD ON PANEL AS A FUNCTION OF LATERAL DEFLECTION OF PANEL FOR TEST C

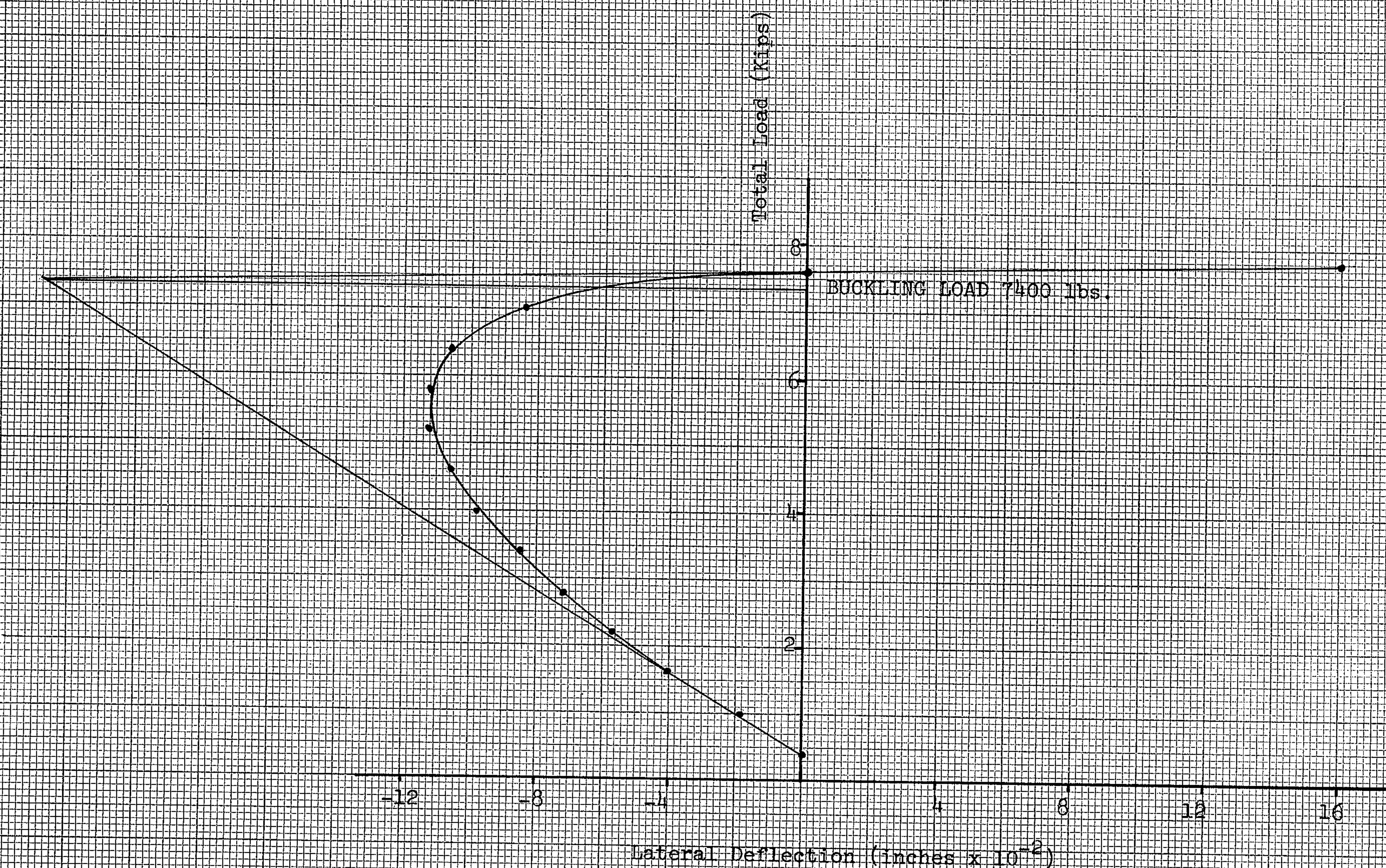


Figure C.28
TOTAL LOAD ON PANEL AS A FUNCTION OF LATERAL DEFLECTION OF PANEL FOR TEST D

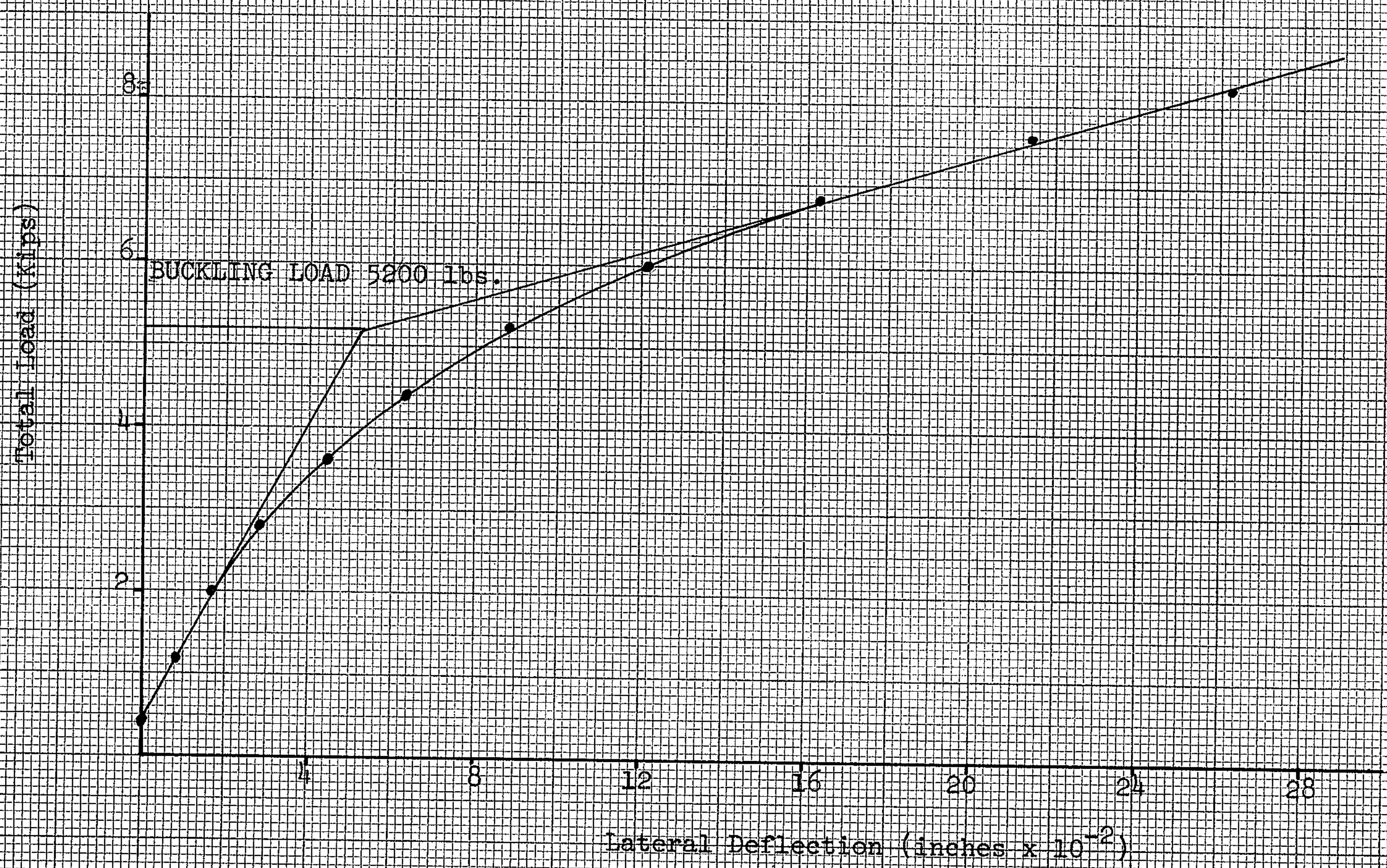


Figure C.29
TOTAL LOAD ON PANEL AS A FUNCTION OF LATERAL DEFLECTION OF PANEL FOR TEST E

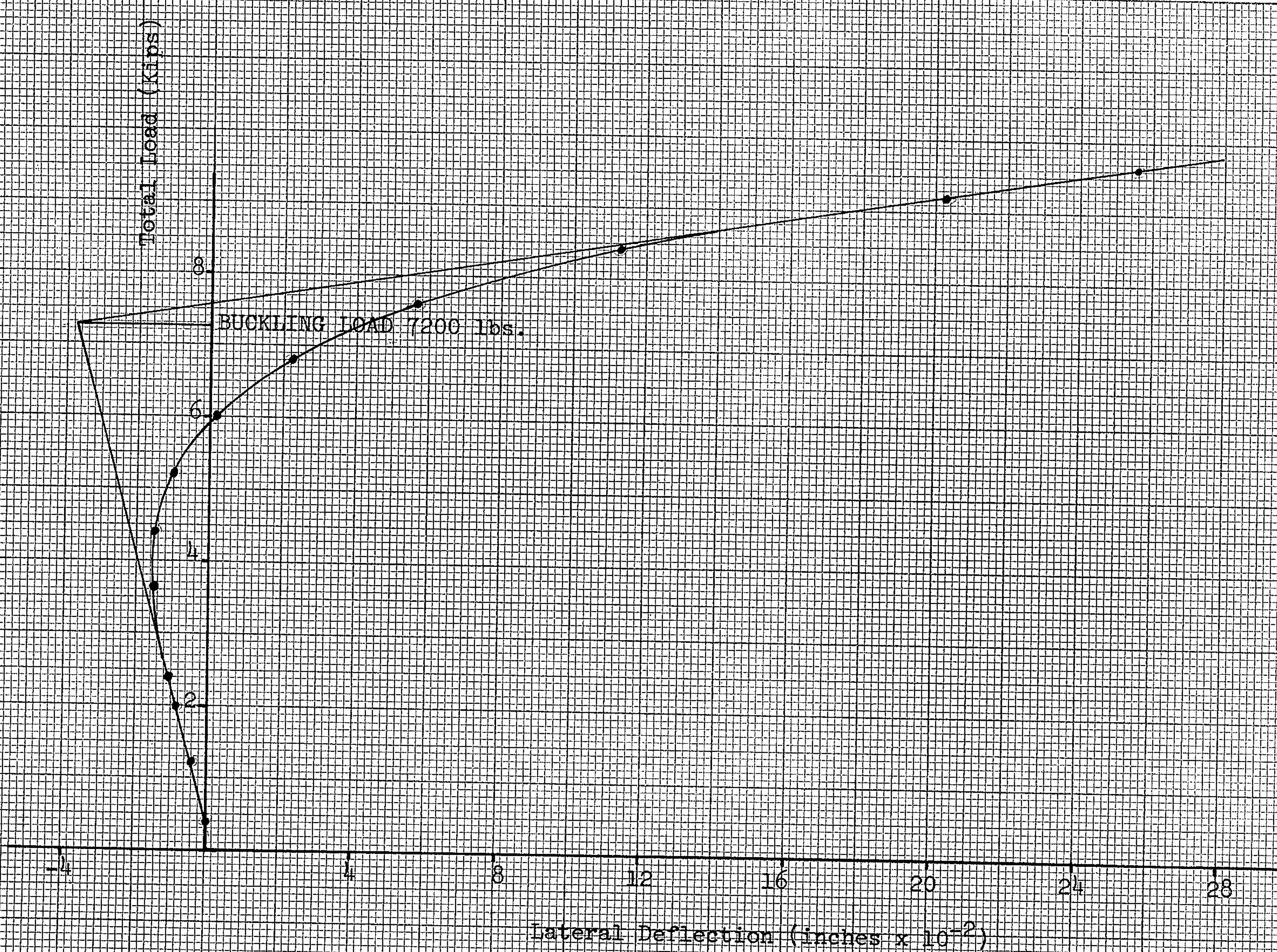


Figure C.30
TOTAL LOAD ON PANEL AS A FUNCTION OF LATERAL DEFLECTION OF PANEL FOR TEST F

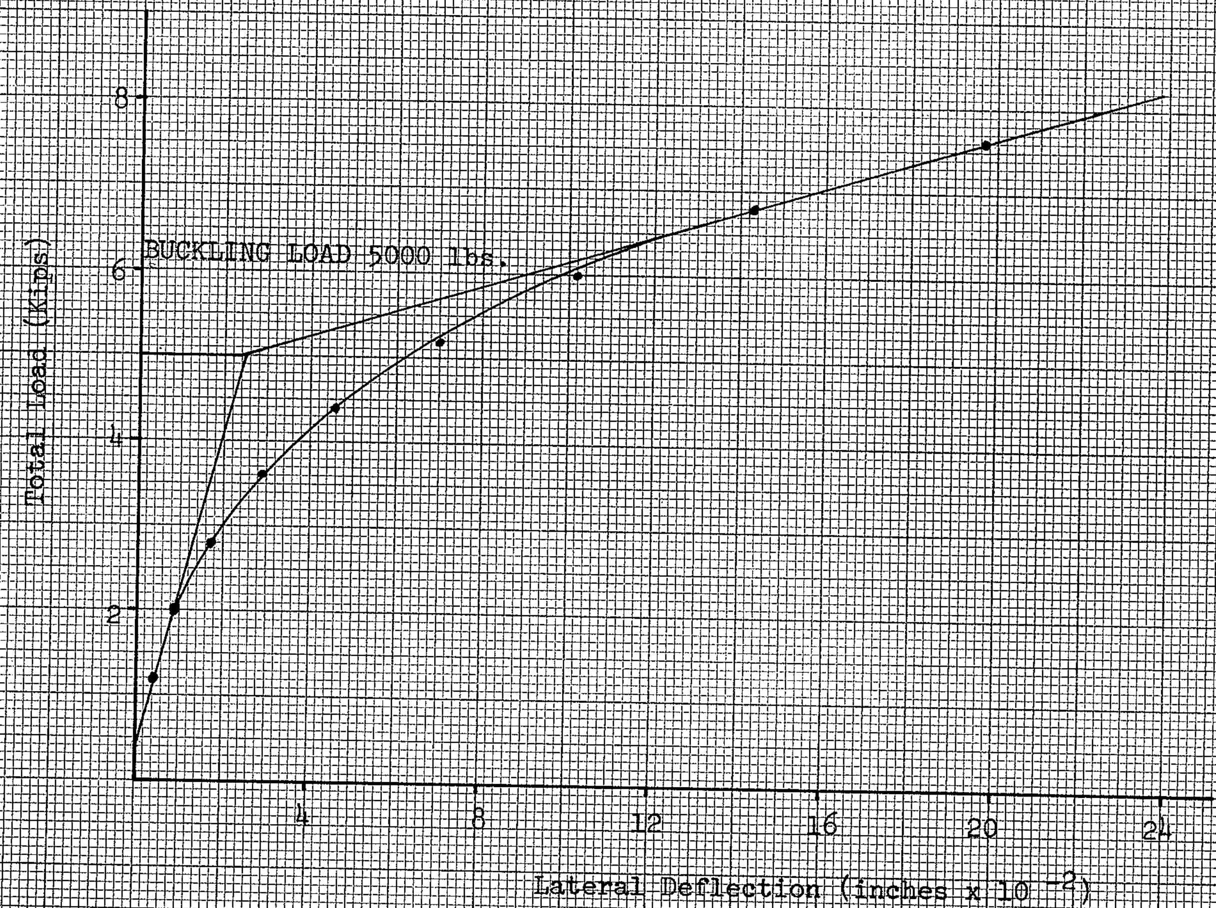


Figure C.31
TOTAL LOAD ON PANEL AS A FUNCTION OF LATERAL DEFLECTION OF PANEL FOR TEST G

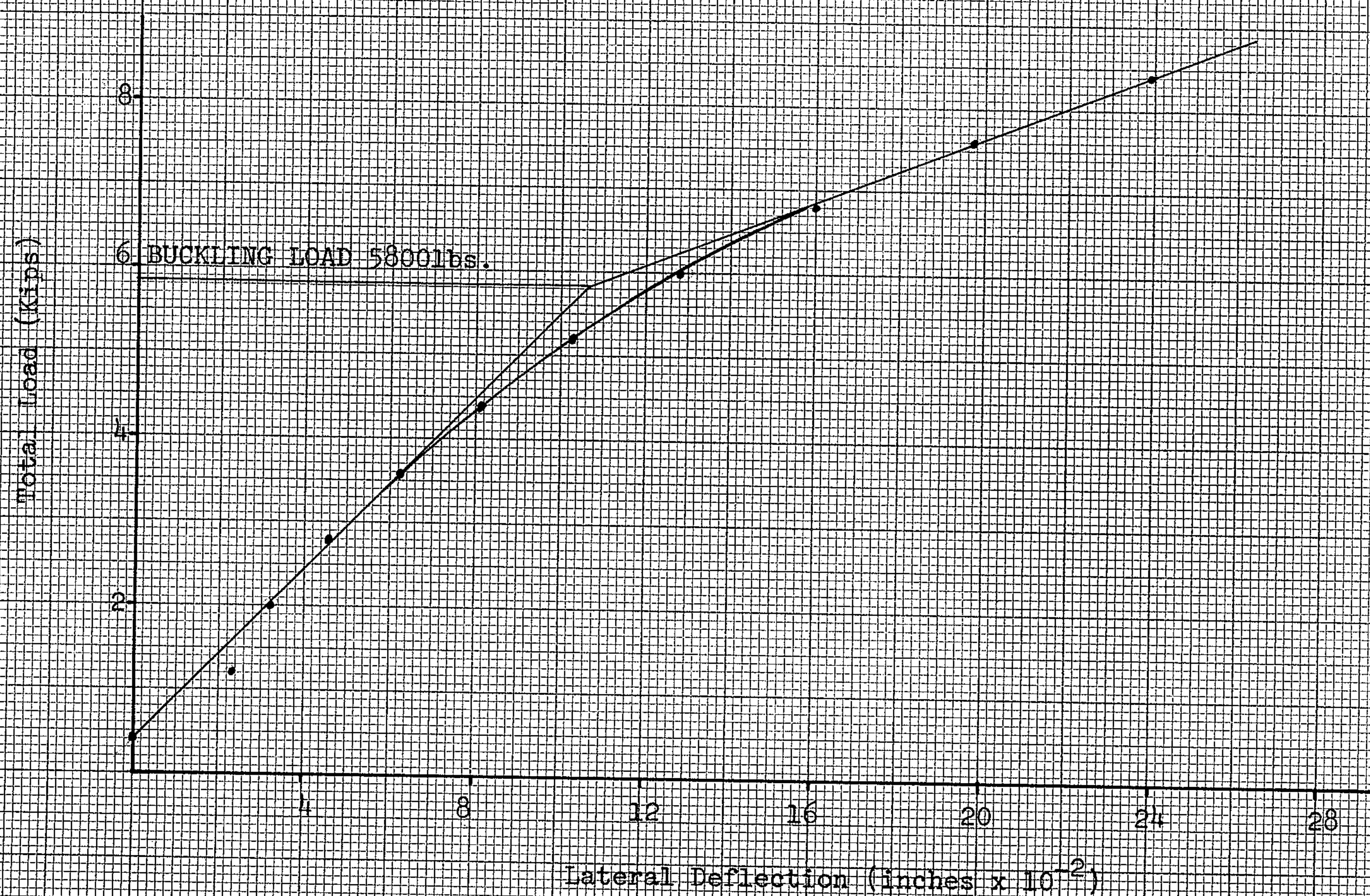


Figure C.32
TOTAL LOAD ON PANEL AS A FUNCTION OF LATERAL DEFLECTION OF PANEL FOR TEST H

Table XX gives the values of the critical buckling stress calculated from the critical buckling load and the edge member readings.

TABLE XX

CRITICAL BUCKLING STRESS FOR TESTS B TO H

FROM EDGE MEMBER READINGS

TEST	B	C	D	E	F	G	H
σ_{cr} (psi)	278	427	516	412	558	501	705

Table XXI gives the values of the gauge readings attached to the plywood.

TABLE XXI

PLYWOOD READINGS AT CRITICAL BUCKLING LOAD

FOR TESTS B TO H

TEST	GAUGE NO. RDGS. $^{in}/in \times 10^{-6}$ (-ve COMP, + ve TENS.)											
	1	2	3	4	5	6	7	8	9	10	11	12
B	87	74	31	-126	-88	13	-28	-361	-603	-225	-151	-20
C	-2	-28	-121	-279	-270	7	-53	-387	-672	-423	-284	-119
D	-93	-446	-613	-811	-706	-89	-54	-21	-366	-131	56	-167
E	-188	-6	3	-41	-30	-143	-218	-164	-224	-303		
F	-262	-225	-210	-291	-145	-179	-172	123	-43	-361		
G	-321	-268	-230	44	-33	-142	-190	-255				
H	171	-391	-648	-500	-115	37						

Table XXII gives the critical buckling stress values calculated from the plywood gauge values.

TABLE XXII

CRITICAL BUCKLING STRESS FOR TESTS B TO H FROM PLYWOOD GAUGE READINGS

TEST	B	C	D	E	F	G	H
σ_{cr} (psi)	214	404	528	241	326	320	261