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LATERAL GRAIN PRESSURES IN FLEXIBLE PLASTIC CONTAINERS

by

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ABSTRACT

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LATERAL GRAIN PRESSURES IN FLEXIBLE PLASTIC CONTAINERS

An investigation was undertaken to determine lateral pressures exerted by wheat against flexible container walls. Flexible polyethylene containers of sizes varying in diameter from 25 cm to 90 cm and varying in height from 63 cm to 147 cm were filled in layers with hard red spring wheat in order to determine the magnitude and distribution of lateral pressures. Lateral pressures in the test containers were determined by measuring percent circumferential elongation in the containers. Calibration curves for polyethylene were plotted for each test condition. These curves were used to determine stresses in the container walls and lateral pressures were calculated from these stresses.

The following physical properties of wheat were measured: bulk density (0.882 metric ton/m^3), angle of repose (21.7°), and coefficient of friction on polyethylene (0.366). These material properties were used to calculate parameters in Janssen's equation. Janssen's equation was found to be inapplicable in predicting lateral pressures in flexible containers.

A dimensional analysis approach was used to develop an equation for predicting lateral pressures in flexible containers. The equation parameters were experimentally determined. The dimensional analysis equation closely represented experimental lateral pressures. Effect of container material thickness, up to 60% variation, was not significant. Lateral pressures in the containers decreased with time. Failures of containers appeared to occur when maximum percent elongation became greater than 11%.

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LIST OF SYMBOLS AND ABBREVIATIONS

A	=	Cross-section area of container
cm	=	Centimeter
D	=	Container diameter
Do	=	Original container diameter
Eq.	=	Equation
F	=	Force dimension
Fa	=	Wall force in active stage
Fig.	=	Figure
f	=	Friction force
H	=	Height of grain
H_d	=	Horizontal distance between two points
h	=	Height of grain
hr	=	Hour
K	=	Ratio of lateral to vertical pressure
K'	=	Parameter in dimensional analysis equation
K_a	=	Rankine's coefficient of active pressure
kg	=	Kilogram
L	=	Length dimension
L_p	=	Lateral grain pressure
M.C.	=	Moisture content
m	=	Meter or metre
min	=	Minute
N	=	Normal force

n	=	Parameter in dimensional analysis equation
P_a	=	Wall pressure in active stage
R	=	Hydraulic radius
Sec.	=	Section
T	=	Container wall thickness
U	=	Container perimenter
V	=	Volume
v	=	Vertical pressure
V_1, V_2	=	Vertical heights at two points
W	=	Weight of grain
Ww	=	Weight of water
w	=	Bulk density
X	=	Angle of plane of rupture
y	=	Depth of grain
θ	=	Angle of internal friction
θ'	=	Angle between the resultant of normal force and friction force, and the normal force
σ	=	Stress in container walls
ϕ	=	Functional notation
ϕ_r	=	Angle of repose
°	=	Degrees
°F	=	Degrees fahrenheit
%	=	Percent
π	=	Pi
μ	=	Coefficient of internal friction
μ'	=	Coefficient of friction on bin walls

1. INTRODUCTION

The problem of feeding the world population is twofold. First, land and water are not being fully utilized to produce food. Secondly, when food is produced, there are problems in getting it to its destination without loss of quality and quantity. F.A.O. reports indicate that a conservative figure of annual world food loss is 5% (Wright and Southgate, 1962). In tropical countries the food loss is of the order of 25% and may be as high as 70% in many cases.

A major factor, responsible for this substantial loss, is inadequate facilities for grain storage and transportation. Storage space in some tropical countries consists of a part of the living quarters of the family or an adjacent building constructed from locally available materials. These buildings provide ideal conditions for the growth of insect pests.

The storage space in temperate countries consists mainly of open grain piles or conventional structures of steel, wood, or concrete. In open piles, precipitation provides favorable conditions for mold growth. Conventional structures do not permit successful fumigation because gaps in the eaves and badly fitting doors allow fumigant to escape. Sinha (1958) estimated that the annual economic loss in Canada, resulting from quality deterioration of grain due to heating, is nearly one million dollars. To avoid such a great loss, it would be necessary to store grain in properly designed and well constructed storage units. The grain storage capacity requirement is also increasing due to a combination of high yields and poor markets.

The transportation of grain in tropical countries consists mainly of 95-lb. jute sacks carried on human heads, bicycles, or carts. Jute sacks have lodging areas for micro-organisms and are capable of holding moisture. Thus, these sacks provide an ideal environment for the growth of bacteria, fungi, and mites with a resulting deterioration in grain quality. They are easily penetrated by insects and facilitate the spreading of infestation.

In Canada, where grain is being handled in bulk, there are a number of problems associated with it.

1. Parts of the existing railroad system consist of obsolescent freight cars and light-weight rails. The elevator companies have about five thousand elevators of wooden construction, most of them are more than forty years old. These elevators are not suitable for either rapid grain handling or proper long term storage. Grain companies are also confronted with the problem of branch line abandonment (McDonald 1968).

2. There is a shortage of labor during the critical harvest period. Lack of storage facilities on the farm

requires that grain be transported to the elevator immediately after harvesting.

3. Large quantities of grain deteriorate during transit due to improper conditions.

4. Grain from a number of small lots is mixed into large elevator bins before grading. Sometimes, this results in contamination of a large bin of grain by one small lot. Grain grading prior to mixing would eliminate this loss.

One way of solving these problems of grain transportation and storage is to apply a total system approach, i.e., transportation and storage should be considered as an integrated flow process and not two separate operations. This can be done by containerization and considering the containers as the handling units. The containers could be directly filled and graded in the field and would serve as temporary storage units on the farm. They can be moved to rail heads at any convenient time. The containers of suitable size would help grain grading since each container would have a nearly homogeneous content. These containers should be capable of providing sealed storage with effective protection up to one year and could be re-used in following years.

Containers made from flexible plastic or synthetic rubber would not absorb moisture and would serve as airtight storage units. Insects can not enter these sealed containers and those already in the grain would die due to shortage of oxygen. Therefore, it would not be necessary to add any chemical for insect control; thus the cost of the chemical as well as the risk of toxicity to human or animal would be eliminated. The probability of contamination or spread of insect infestations is greatly reduced by using sealed containers during transportation. The empty containers could be easily folded and returned to the farmer for re-use.

Because of this potential for flexible membrane containers in grain handling and storage, information should be available to allow for proper engineering design of these containers. There is information available on the design of rigid grain bins, the walls of which resist compression loads. But no work has been done on grain pressure distribution in a bin, which has walls made up of a material that can not resist compressive loads.

The objectives of the research project were:

1. To determine lateral grain pressure in relation to height and diameter of flexible plastic containers;

2. To study the effect of container wall thickness on laterial pressure in flexible plastic containers;

3. To test the applicability of existing earth pressure and rigid bin theories to flexible containers;

4. To develop an equation which would be applicable to flexible containers.

To study the effect of container diameter and height, cylindrical polyethylene containers of sizes varying from 25 cm x 62 cm to 90 cm x 147 cm were filled with wheat. These container sizes represented a height-to-diameter ratio range from 0.33 to 2.5.

2. REVIEW OF LITERATURE

2.1 Storage in Plastic Containers

Grain quality can be maintained for a longer time in air-tight containers than in non air-tight containers (Culpin 1965; Muir 1970). Flexible plastic containers serve as air-tight units. Mitsuda, Kuga, and Kawaii (1969) found that containers made from polyethylene laminated nylon film could be successfully used for storing grains under water. Muir (1970) found that wheat at 19% to 22% moisture content stored in air-tight butyl rubber bin remained free-flowing and had no visible mold growth while similar grain stored in a non air-tight steel bin became caked with visible mold. The Tropical Stored Products Centre (Southgate 1965; Hall 1968) has also developed above and below ground plastic containers for safe grain storage. A 3-ton plastic container 'Pillow Tank' is being successfully used in Nigeria (Southgate 1965). At present, these containers are being designed by trial and error method because there is no information available on grain pressures in flexible containers.

2.2 Material Properties

Pressures exerted by granular materials on bin walls are affected by material properties such as bulk density, coefficient of friction on bin walls, angle of repose, coefficient of internal friction, and moisture content.

Bulk density of a granular material is the weight of unit volume of the material.

$$w = \frac{W}{V} \qquad \ldots (2.1)$$

where:

w = Bulk density of granular material, and

W = Weight of V volume of granular material.

Coefficient of friction of granular material on bin walls is the ratio of the friction force f to the normal force N.

$$\mu' = \tan^{-1}\theta' = f/N \qquad \ldots (2.2)$$

where:

μ' = Coefficient of friction of granular material on bin walls, and

θ' = Angle between the normal force N and the resultant of the friction force f and the normal force N.

Coefficient of internal friction of a granular material is the tangent of angle of internal friction.

$$\mu = \tan\,\theta \qquad \ldots (2.3)$$

where:

μ = Coefficient of internal friction, and

θ = Angle of internal friction.

Angle of internal friction is affected by interlocking of particles, by any cohesion present, and by surface roughness.

Angle of repose is the friction angle of granular material under zero normal pressure. This angle depends on the equilibrium of the least stable particles and is smaller than the angle of internal friction.

The ratio of lateral to vertical pressure, K, has been a very controversial factor in grain pressure analysis. Janssen and Airy (Ketchum 1919) assumed this factor to be a constant, independent of grain depth. Pleissner (Ketchum 1919) found this factor to decrease with increase in grain depth. Ketchum (1919), Krammer (1944), and Jaky (1948) found that it increased with grain depth up to a certain depth and then became constant. Leczner (1963) noted a different behaviour. He reported that the value of this factor increased linearly with grain depth up to four bin diameters, increased hyperbolically till six bin diameters, and then became constant. Even when K is considered constant, there is no direct method available for its determination. It is usually calculated using the following equation:

$$K = \frac{1 - \sin \phi_r}{1 + \sin \phi_r} \qquad \ldots \; (2.4)$$

where:

ϕ_r = Angle of repose of granular material.

Equation 2.4 is similar to one used for determining Rankine's coefficient of active pressure (Section 2.4.2), except that angle of repose is substituted for angle of internal friction. Equation 2.4 estimates the value of K for minimum pressure conditions. Jaky (1948) proposed another equation for calculating the value of K for at rest conditions.

$$K = 1 - \sin\theta \qquad \text{. . . (2.5)}$$

where:

θ = Angle of internal friction of granular material.

In agricultural engineering applications, Equation 2.4 is more commonly used (Hall 1961; Canadian Farm Building Standards, N.R.C. 1970).

Moisture content of grain is the proportion of water present in the grain on weight basis. This proportion is commonly expressed in percent wet basis.

$$M.C. = \frac{Ww}{W} \times 100 \qquad \text{. . . (2.6)}$$

where:

M.C. = Percent moisture content of grain on wet basis,

W = Weight of a given volume of grain, and

Ww = Weight of water present in grain.

2.3 Grain Pressure Theories

The problems of calculating grain pressures on bin walls and earth pressures on retaining walls are similar. It is incorrect to determine grain pressure by assuming that the granular material behaves like a fluid because grain particles exert frictional forces and tend to arch. This arching creates both lateral and vertical force components on bin walls. The vertical force component at the bin wall reduces the load on the bin floor. Grain pressures do not build up with depth as much as fluid pressures. Therefore, hoop pressures exerted by grains are lower than those exerted by a fluid of the same density.

The problem of determining grain pressures has been studied for more than two hundred years. Most of the work done in the past is limited to rigid bins, whose walls take a part of the vertical load. There are solutions available for each of the two cases of shallow and deep bins. In shallow bins, the plane of rupture (Fig. 2.1) passes through the upper grain surface before it meets the opposite wall. In deep bins, the plane of rupture meets the opposite wall before passing through the upper grain surface. Airy (Ketchum 1919) proposed the following equation for calculating the critical height-to-diameter ratio for shallow and deep bins:

$$h/D = \tan X = \mu + \sqrt{\frac{\mu(1 + \mu^2)}{\mu + \mu'}} \quad . . . (2.7)$$

2.4 Shallow Bin Theories

2.4.1 Coulomb's Theory

In 1773, Coulomb (Taylor 1948) using a geometrical approach developed equations for determining lateral earth pressures on retaining walls. He considered the following two stages:

1. Active stage in which the wall moves away from the backfill; pressure conditions in this stage and in grain

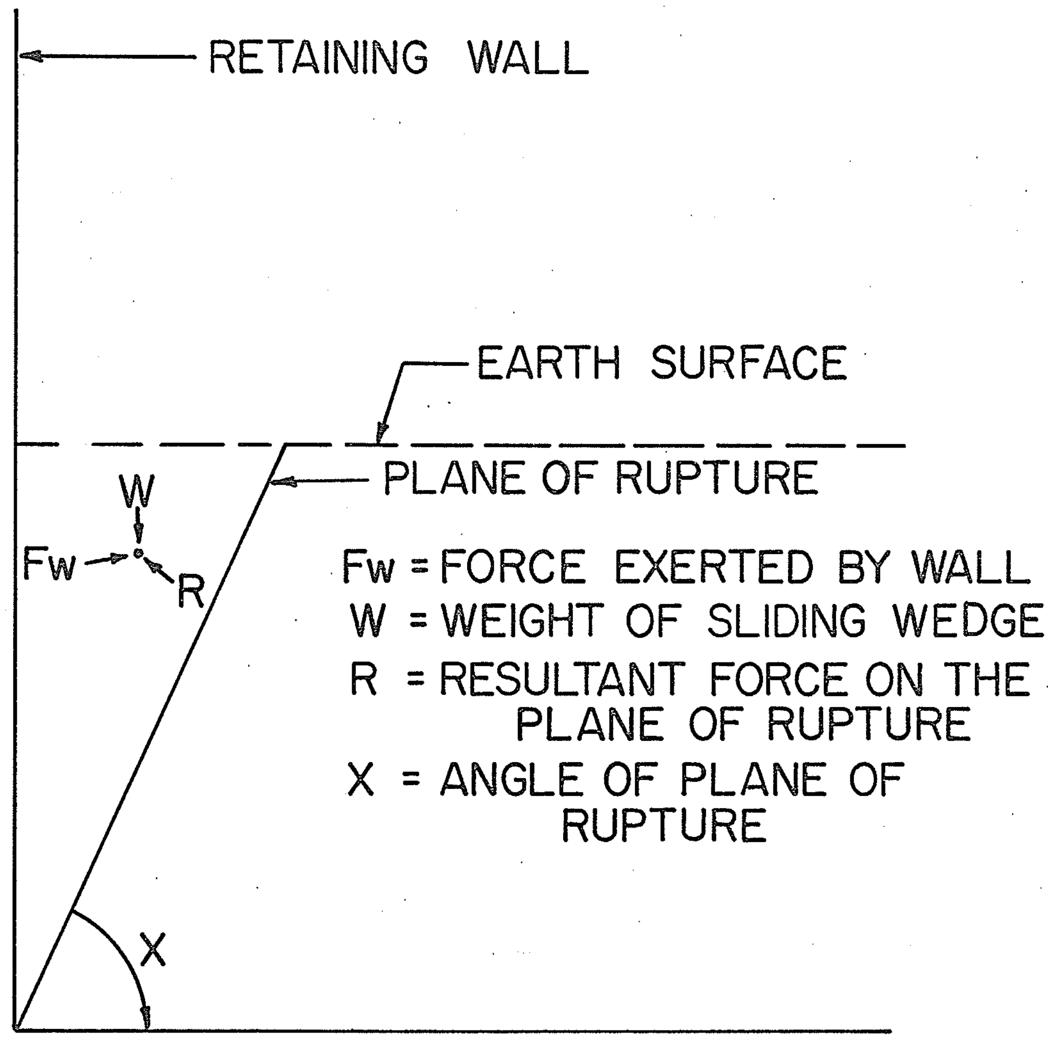


FIG. 2.1 COULOMB'S MODEL OF EARTH PRESSURE

bins are similar, and

2. Passive stage in which wall moves towards the backfill.

He made the following assumptions in the development of his equations.

1. A surface of failure exists which can be assumed to be a plane (called the plane of rupture), originating at the base of the wall.

2. Earth mass is semi-infinite.

3. Earth mass has negligible cohesion.

4. Earth surface is horizontal.

5. The active and passive stages represent the conditions of minimum and maximum pressures respectively.

Coulomb's equation for the active stage is:

$$Fa = \frac{wH^2}{2} \frac{\text{Cos } \theta}{(1 + \sqrt{2} \text{ Sin } \theta)^2} \quad \ldots (2.8)$$

where:

Fa = Force exerted by wall in active stage,

H = Height of fill, and

θ = Angle of internal friction, and

w = Bulk density of earth mass.

The point of application of the resultant force is considered at a distance 2H/3 below the earth surface. Stewart (1967) reported that Coulomb's equation was accurate for predicting active grain pressures in shallow bins. This equation is not commonly used by agricultural engineers.

2.4.2 Rankine's Theory

In 1857, Rankine (Taylor 1948) assumed that pressure at any point below earth surface was proportional to depth, and that the relationship between vertical and horizontal pressure in the earth mass was not affected by the presence of the wall. For the following conditions, he developed his well-known equations of earth pressure.

1. A semi-infinite cohesionless mass is being supported by a frictionless wall.

2. Active and passive stages represent the conditions of minimum and maximum pressures.

3. The resultant pressure on the wall acts in a horizontal direction.

4. The retaining wall is rigid.

Rankine's equation for the active stage is:

$$P_a = K_a \, wh \qquad \ldots (2.9)$$

where:

P_a = Horizontal pressure on wall in the active stage,

$$K_a = \frac{1 - \sin\theta}{1 + \sin\theta} \qquad \ldots (2.10)$$

K_a = Rankine's coefficient of active earth pressure,

w = Bulk density of granular material,

h = Depth of granular material, and

θ = Angle of internal friction.

The greatest inapplicability of Rankine's theory is the assumption that the presence of a wall does not alter the pressure distribution. It is expected that the stress conditions at the wall would be different from the stress conditions within the earth mass due to different conditions of friction and cohesion at the two places. This theory is not commonly used in grain bin design.

2.4.3 Airy's Theory

In 1897, Airy (Ketchum 1919) presented a mathematical analysis of grain pressures. His work was an expansion of Coulomb's sliding wedge-theory (Sec. 2.4.1). He considered the effect of grain friction on the wall and forces on the plane of rupture.

Airy's equation is:

$$L_p = wh \left[\frac{1}{\sqrt{\mu(\mu + \mu')} + \sqrt{1 + \mu^2}} \right] \quad ...(2.11)$$

where:

L_p = Lateral grain pressure on bin wall,

w = Bulk density of granular material,

μ = Coefficient of internal friction, and

μ' = Coefficient of friction on bin walls.

In the development of this equation, he assumed that the ratio of lateral to vertical pressure is a constant, and that bin walls take a part of the vertical load. The bin walls were assumed to be rigid. The bin cross-section was assumed to be uniform for the entire depth.

2.5 Deep Bin Theories

2.5.1 Janssen's Theory

In 1895, Janssen (Ketchum 1919) advanced his well-known theory of grain pressures. At present, his theory is the one most commonly used in grain bin design.

In the development of his equation, he made the following assumptions:

1. The bin has uniform cross-sectional area and circumference for the entire depth.
2. The ratio of lateral to vertical pressure is a constant.
3. Vertical pressure is uniform on any horizontal plane.
4. Bin walls take a part of the vertical load.
5. Grain surface is horizontal.

By a simple mathematical analysis (Append. 1) he derived the following equation:

$$L_p = \frac{wR}{\mu'} (1 - e^{-K\mu' h/R}) \quad . . . (2.12)$$

where:

L_p = Lateral pressure,

w = Bulk density of granular material,

R = Hydraulic radius (A/U),

A = Cross-sectional area of bin,

U = Perimeter of bin,

μ' = Coefficient of friction of grain on bin walls, K = Ratio of lateral to vertical pressure, and

h = Grain depth above the point of consideration.

2.5.2 Airy's Theory

In 1897, Airy (Ketchum 1919) advanced his theory of grain pressures in deep bins. This theory is an expansion of his shallow bin theory and is based on the same assumptions (Sec. 2.4.3). Airy's equation for deep bins is:

$$L_p = \frac{wD}{\mu + \mu'} \left[1 - \frac{\sqrt{1 + \mu^2}}{\sqrt{\frac{2h}{D}(\mu + \mu') + 1 - \mu\mu'}} \right] \quad . \; . \; (2.13)$$

where:

D = Bin diameter.

After 1897, most of the work done in this field has been limited to testing the applicability of Janssen's and Airy's equations (Eq. 2.12 and Eq. 2.13) to rigid bins.

2.6 Summary of Grain Pressure Research in Deep Bins

1. In 1882, Roberts (Jamieson 1904) found that the pressures on bin floors did not increase after a depth of 4.5 bin diameters.

2. Jamieson (1903), Ketchum and Williams (Ketchum 1919), and Neberhaus (1965) found Janssen's equation (Eq. 2.12) to be applicable to the static case. Prante (Jamieson 1903), Toltz (Jamieson 1903), and Leczner (1963) found that the lateral pressure in the static case was less than that predicted by Janssen's equation (Eq. 2.12).

3. Lufft (1904) found that lateral pressure decreased with time.

4. Ketchum and Varnes (Ketchum 1919) did not find any change in grain pressures during filling, emptying, or at rest conditions; whereas, Jamieson (1903) and Collins (1963) observed a 10% increase in pressures due to moving grain. Bovey (1903) found grain pressures to be 10% greater during emptying than during filling.

2.7 Applicability of Earth Pressure and Rigid bin Theories to Flexible Bins

The earth pressure theories advanced by Coulomb (Sec. 2.4.1) and Rankine (Sec. 2.4.2) assume rigid walls. Flexible bins do not satisfy this assumption. Therefore, these theories are not expected to be directly applicable.

The deep bin theories, advanced by Janssen (Sec. 2.5.1) and Airy (Sec. 2.5.2) assume that the bins have uniform cross-section and circumference and that the bin walls take a part of the vertical load and have negligible deflection. These assumptions are not justified in flexible bins, and therefore, preclude the application of these theories to flexible bins.

2.8 Wall Flexibility

Saul (1953) working with rectangular wooden bins found that wall deflection resulted in a slight decrease in lateral grain pressure. Britton (1969) also observed a similar effect for fertilizer in wooden bins, but his results were inconclusive at the 5% level of statistical significance.

Collins (1963) did some qualitative work on grain pressures in flexible paper cylinders of 30-cm diameter. Cylinders were filled in 5-cm layers of dry sand. He found that the cylinders failed due to buckling at 22.5 cm from the bottom when the height of fill was 80 cm. During unloading, he found that each cylinder failed in a like manner, irrespective of the rate of unloading. Buckling, in this case, consisted of a series of diamond-shaped indentations in a diagonal row, approximately 15 cm from the bottom when unloading from the 50-cm depth. A complete wall failure was reported when unloading from the 55-cm depth.

3. DEVELOPMENT OF PREDICTION EQUATION

The existing theories of earth pressures and grain bin pressures are not applicable to flexible containers (Sec. 2.7). A dimensional analysis approach was used to develop a prediction equation which would be applicable to flexible plastic containers. The functional relationship between lateral pressure and various factors affecting it can be expressed as:

$$L_p = \phi(w, h, D, \mu, \mu') \quad \ldots (3.1)$$

where:

- L_p = Lateral grain pressure. Its dimensions are FL^{-2},
- w = Bulk density of granular material. Its dimensions are FL^{-3},
- h = Depth of grain. Its dimension is L,
- D = Container diameter. Its dimension is L,
- μ = Coefficient of internal friction, dimensionless, and
- μ' = Coefficient of friction on bin walls, dimensionless.

By Buckingham 'π' theorem:

$$(L_p)^a = C(w)^b (h)^c (D)^d (\mu)^e (\mu')^r \quad \ldots (3.2)$$

or

$$(FL^{-2})^a = C(FL^{-3})^b (L)^c (L)^d (1)^e (1)^r \quad \ldots (3.3)$$

Since μ' and μ are dimensionless variables and if it is assumed that these would be constant for a particular combination of granular material and flexible plastic material; a new constant, B, can be defined as:

$$B = C\ (\mu)^e\ (\mu')^r \qquad \ldots(3.4)$$

Equating the powers of each dimension:

a. Powers of dimension F,

$$a = b \qquad \ldots(3.5)$$

b. Powers of dimension L,

$$-2a = -3b + c + d \qquad \ldots(3.6)$$

Substituting, a = b in (Eq. 3.6) and re-arranging,

$$a = c + d$$

or

$$c = a - d \qquad \ldots(3.7)$$

Equation 3.2 can now be re-written as:

$$L_p^a = B\ w^a\ h^{a-d}\ D^d \qquad \ldots(3.8)$$

or

$$L_p = B^{1/a}\ w\ h^{1-d/a}\ D^{d/a}$$

or

$$L_p = K'\ w\ h^{1-n}\ D^n \qquad \ldots(3.9)$$

This prediction equation is dimensionally homogeneous. Therefore, it would be applicable in any consistent system of units.

Values of parameters K' and n depend on such factors as coefficient of internal friction, coefficient of friction on bin walls, container material homogeneity, and material thickness. The parameters K' and n must be evaluated experimentally.

4. PRELIMINARY INVESTIGATIONS

4.1 Selection of Fill Material

Wheat is the main crop of Manitoba and requires improved systems of local storage and transportation to foreign countries. Most past investigators have used wheat for grain pressure research (Long 1932). Considerable information is available on its properties such as density, angle of repose, and coefficient of friction on various material surfaces. On the basis of above considerations, hard red spring wheat was selected as fill material for the research project.

4.2 Selection of Container Material

Tests were conducted on various materials to compare their suitability for making test containers from them. The criterion for selection was:

1. The material should elongate in a range, measurable by an ordinary steel measuring tape,

2. The material should not undergo excessive elongation,

3. The material should preferably not elongate into the plastic range,

4. The material properties should be affected very little by time and temperature.

The following materials were considered:

1. Butyl rubber,
2. Ethylene propylene diene monomers,
3. Paper,
4. Knitted polyethylene,
5. Polypropylene,
6. Flexible polyvinyl chloride,
7. High density polyethylene, and
8. Low density polyethylene.

Samples of these materials were tested in tension to the magnitude of stresses equal to the range expected in the test containers, predicted by Janssen's equation (Eq. 2.12). None of the materials exhibited a linear relationship between stress and percent elongation. The characteristics of the materials tested are briefly described below:

4.2.1 Butyl Rubber

Test results indicated that with the best combinations of test container sizes and available thicknesses (0.075 cm to 0.315 cm) of butyl rubber, the elongations of the container-circumference would be in the range 11.4 cm to 296 cm (15% to 105%). At constant stress, elongation increased 40% within 2 hr. Also, the material did not regain its original length on unloading. This material was not considered suitable due to its high elongation and plastic deformation.

4.2.2 Ethylene Propylene Diene Monomers

Ethylene propylene diene monomers had load-elongation characteristics similar to that of butyl rubber. It was found that the best combinations of test container sizes and available thicknesses would give circumferential elongations in the range of 2.87 cm to 421 cm (4% to 149%). At constant stress, elongation increased 15% within 2 hr and 33% in 15 days. This material was considered unsuitable for the research project due to its high elongation.

4.2.3 Paper

For paper an elongation of 0.4% was observed at the failure tensile stress of 70 kg/cm^2. This material was not considered suitable for the research purpose because its low elongation could not be measured with an ordinary steel measuring tape.

4.2.4 Knitted Polyethylene

Samples of both transparent and black knitted polyethylene were tested. The material had an elongation of 1.5% at a stress of 62 kg/cm^2 and regained its original length within 6 min after unloading. Because of the material's low percent elongation, it was not considered suitable for the research project.

4.2.5 Polypropylene

An elongation of 1.5% was observed at a stress of 87 kg/cm^2. For the propylene film thicknesses available, the maximum elongation for a 25-cm diameter container would be only 0.056 cm (0.24%). Propylene, therefore, was not considered suitable for the research project.

4.2.6 Flexible Polyvinyl Chloride

Test results indicated that circumferential elongations of test containers made from flexible polyvinyl chloride would be in the range of 6.3 cm to 85 cm (9% to 30%). This material would possibly be suitable but it was very expensive. A cheaper material which would elongate in a lower range, was a better choice.

4.2.7 High Density Polyethylene

An elongation of 5% was observed at the failure tensile stress of 63 kg/cm^2 of high density polyethylene. An elongation of 0.34 cm (0.45%) was expected for a 25-cm diameter container. Time-elongation tests indicated that at a constant stress, the material continued to elongate with time for 13 days and then it assumed a constant value of elongation. However, this material regained its original length within 1 hr of unloading. The low percent elongation made this material unsuitable for the research project.

4.2.8 Low Density Polyethylene

Test results indicated that the circumferential elongation for the test containers constructed from low density polyethylene would vary from 1.4 cm to 40.2 cm (1.95% to 8.5%). This material had the best range of elongations of all the materials tested, and therefore, was used for making the test containers. Load-elongation characteristics of low density polyethylene were slightly affected by temperature. Therefore, one stress-elongation calibration curve could not be used for all containers due to ambient temperature variation. Separate calibration curves had to be plotted for each test condition. Physical properties data of low density polyethylene supplied by the manufacturer, are reported in Table 4.1.

4.3 Time-Elongation Characteristics of Low Density Polyethylene

Tests were conducted in a temperature controlled room to study the effect of time on constant stress elongation of low density polyethylene. A constant elongation of 8.8% was obtained after 215 hr at 49 kg/cm^2. Elongation was 5.8% at the end of 30 min of loading and 6.8% at the end of 24 hr. During the first 30 min the material elongated to 86% of its elongation at 24 hr and 64% of its maximum elongation. The average rate of elongation during the 30-min to 60-min period after loading, was 0.0115%/min.

4.4 Effect of Material Thickness

To study the effect of material thickness on the stress-elongation characteristic of low density polyethylene, tests were conducted at a constant elongation rate in a temperature controlled room. Strips of this material of thickness varying from 0.01 cm to 0.31 cm were subjected to an elongation rate of 2.5 cm/min. The stress required to cause a given percent elongation was found to depend on

Table 4.1. Physical Properties of Low Density Polyethylene*

Material property	ASTM Test method	Units	Values
Colour	-	-	Clear
Specific Gravity	D1505-57 T	-	0.910-0.925
Failure Tensile Stress	D638-60 T	kg/cm^2	98.0-155.0
Ultimate Elongation	D638-60 T	%	150-600
Tensile Modulus	D638-56 T	kg/cm^2	5625-17575
Hardness Rockwell	D785	D Scale	44-48
Vicat-Softening Temperature	D1525-58 T	°F	185-214
Brittleness Temperature	D746-55 T	°F	Below-180
Coefficient of Linear Expansion	D696-44	cm/cm °F	10^{-4}

*Johnston Industrial Plastics (1970).

material thickness. Therefore, it became necessary to plot calibration curves for each material thickness used for making test containers.

4.5 Test on Commercial Containers

Tests were conducted on commercial polyethylene containers to determine their suitability for using them as test containers. Polyethylene containers, 45 cm in diameter and 90 cm in height were filled with wheat. These containers did not have a formed bottom. Lateral pressures in these containers were predicted by Janssen's equation (Eq. 2.12) and stresses in the container walls corresponding to predicted lateral pressures were calculated using the following equation:

$$\sigma = \frac{L_p D}{2T} \qquad \ldots(4.1)$$

where:

σ = Circumferential stress in container walls,

L_p = Lateral pressure on container wall,

D = Container diameter, and

T = Container wall thickness.

Calibration strips (Sec. 5.2.7) of the same thickness as the container material, were subjected to the same temperature and loading conditions as the containers. Stress-elongation curves for the strips were plotted and used to determine stresses in the container walls. From these stresses, lateral grain pressures on the container walls, were calculated using Equation 4.1. Lateral pressures in the containers were greater than those predicted by Janssen's equation (Eq. 2.12). No increase in circumferential elongation was observed with time. Measurements near the container bottom were difficult due to many wrinkles present. This problem could be reduced by using formed bottom containers.

4.6 Conditions of Maximum Lateral Pressure

To determine the plane of maximum lateral pressure for constant grain height in containers, tests were conducted in 25-cm diameter polyethylene containers with a formed bottom. Circumferential grid lines were drawn on these containers at 15-cm intervals. The containers were filled in 15-cm grain layers. To eliminate the restraining effect of the glued bottom, the lowest grain layer was not considered. All measurements were made with respect to the grid line 15 cm above the container bottom. This grid line was called the reference grid line. Circumferential elongation at each grid line was determined for each grain layer added. It was found that the circumferential elongation for a given depth from the grain surface in a container was lower than that observed at the reference line when the total depth of grain above the reference grid line was equal to the given grain depth in that container. Greater wheat density near the container bottom, due to greater height of fall was, probably, the reason for this variation. To make the test results valid for maximum lateral pressures, subsequent containers were filled in grain layers and only elongation at the reference grid line was considered.

5. EXPERIMENTAL EQUIPMENT AND PROCEDURE

5.1 Experimental Equipment

5.1.1 Test Containers

Container sizes were selected such that a container represented the case of a shallow bin when filled up to a certain height and represented the case of a deep bin when filled to a greater height. It was not possible to conduct tests in the field due to the cold climate. Therefore, maximum container size was restricted to permit laboratory testing. Smallest container size was selected such that the circumferential elongation could be measured by an ordinary steel measuring tape. Thickness of polyethylene film was chosen for each container size to obtain, at maximum expected loading, an elongation close to 10% (Table 5.1).

Containers were made by separately gluing a formed bottom to the container walls. To make a bottom, a circular piece of polyethylene was formed into a cylindrical bottom over a circular table top. Container walls were made by forming a cylinder from polyethylene film of desired size. Swift Canadian Company adhesive number 7813 was used to glue all container joints. An overlapping of 15% was provided on all container walls. Overlapping between container walls and bottom was nearly 10% of container diameter (Fig. 5.1).

Table 5.1. Test Container Sizes

Container Diameter cm	Maximum Grain Height in Containers cm	Material Thickness cm	Ratio of Maximum Grain Height to Container Diameter
25	63	0.0038	2.50
47	88	0.0100	1.88
67	105	0.0150	1.64
67	154	0.0250	2.30
90	147	0.0250	1.63

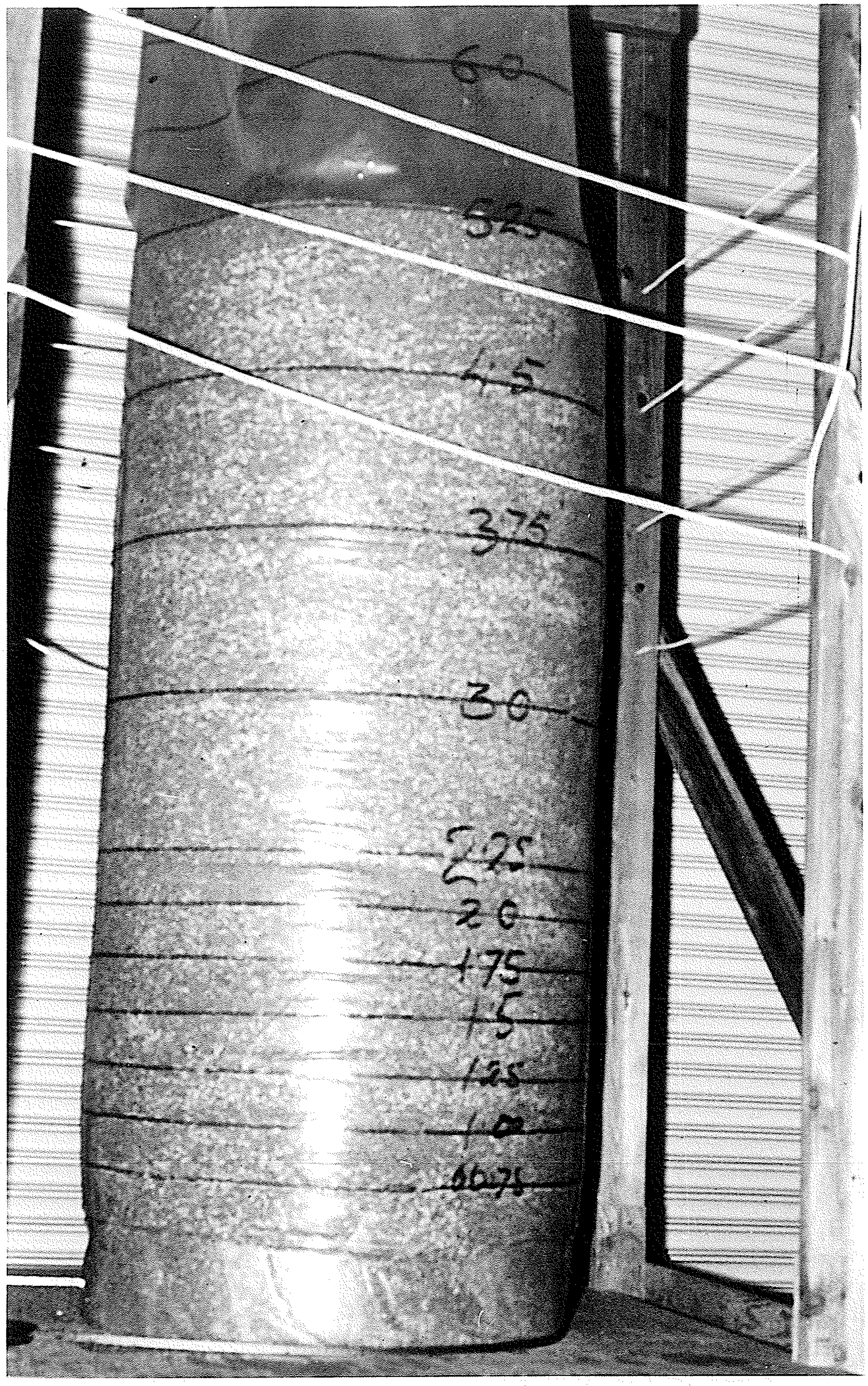


Fig. 5.1. Test Container.

5.1.2 Effect of Container Wall Thickness

To study the effect of container wall thickness on lateral pressure in flexible containers, 67-cm diameter containers were made from polyethylene films of two thicknesses 0.15 cm and 0.25 cm. The criterion in selecting material thicknesses was that the percent elongation of either thickness container should not exceed the limits of reasonable elongation (Sec. 4.1). Containers made from both thicknesses had the same percent overlapping and were filled in a similar manner.

5.1.3 Conveying Unit

The handling cycle consisted of conveying wheat from holding bin to test container and back to holding bin (Fig. 5.2). A 10-cm diameter auger was employed to convey wheat from the holding bin to the container. Another 10-cm diameter auger returned wheat to the holding bin. A hole was made in the container bottom for unloading the container and the rate of flow of grain was controlled by a slide gate mechanism provided under the platform supporting the container. A 157-cm x 112-cm x 96-cm steel hopper was used as the holding bin. A frame was constructed from 5-cm x 10-cm wood to stabilize the test container during unloading but the container was free standing throughout a test.

5.1.4 Measurement System

There are no strain gages available for measuring

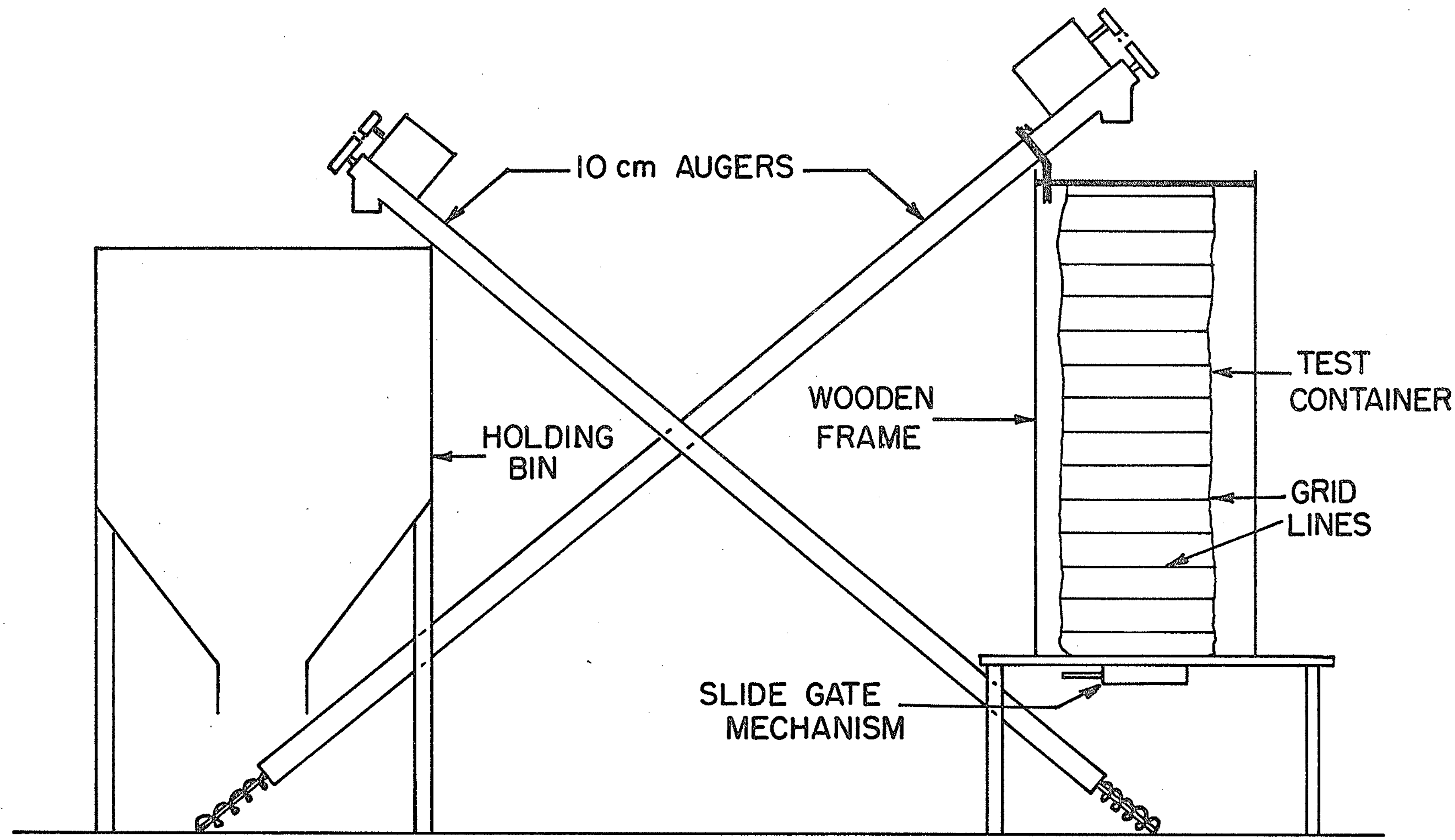


FIG. 5.2 EXPERIMENTAL SET-UP

elongation on polyethylene films. Circumferential grid lines were drawn on the test containers and elongations were measured along these grid lines. Thus, the containers themselves became the measuring elements. Measurements were also taken perpendicular to grid lines (i.e., in the vertical direction) to determine buckling of container walls.

A Lufkin steel tape, model number 146 ME, was used for measuring elongation in calibration strips (Sec. 5.2.7). Container circumference was measured by a Lufkin steel tape, model number S 263 ME. This flexible tape could be aligned with the circumferential lines, thus, accurate measurements were possible. Vertical distance between circumferential lines was measured using a Stanley steel tape, model number W2ME. All these tapes had minimum graduations of 1 mm.

5.2 Experimental Procedure

5.2.1 Material Properties

The material properties required to calculate parameters in Janssen's equation (Eq. 2.12) were determined. These properties are density, angle of repose, and coefficient of friction on the bin surface. The values of these properties for wheat given in the literature, had a wide variation. Therefore, they were determined in the laboratory for the particular wheat used in the tests.

5.2.2 Density

Most of the past investigators have reported a value of 0.769 to 0.817 metric ton/m^3 (Gray 1955; Barre and Sammet 1964; Agricultural Engineer's Yearbook 1969; and Britton 1969). These values represent a condition of minimum loading, because they have been determined by filling containers with almost zero height of fall. These investigators used rigid walled containers, and therefore, these values may not be applicable to flexible containers. Very little information is available on the variation of grain density with height of fall.

Wheat density was determined in polyethylene containers, 16.1-cm in diameter and 35.6-cm in height. The height of fall was varied from 30 cm to 150 cm. Wheat was poured into the containers through a funnel fastened at various heights, making certain that free flow was permitted. Volumes of the containers were measured by the water displacement method and the containers were weighed on a Toledo model No. 4180 platform weighing scale. Minimum graduations on the weighing scale were 0.25 kg.

For comparison, wheat density was also determined in rigid-walled containers. An aluminium container, 20 cm in diameter and 35.5 cm in height was filled with wheat. The height of fall was varied, similar to that for flexible containers. The average of three tests was used in the calculations.

5.2.3 Angle of Repose

Angle of repose of wheat was determined by measuring the angle of slope of wheat surface in an open pile obtained by emptying a wooden box. This technique is similar to that used by Britton (1969). A 30-cm x 30-cm x 30-cm wooden box with one removable side, was filled with wheat. The removable side was held in position by bar clamps while filling the box and levelling the wheat surface in the box. Then, this side was pulled away and the test sample was allowed to flow freely. A combination square was used to measure vertical heights of the wheat pile at two points along the slope, 20 cm apart, inside the box. Angle of repose was obtained in the following manner:

$$\phi_r = \tan^{-1} \frac{V_1 - V_2}{H_d} \qquad \text{. . .(5.1)}$$

where:

ϕ_r = Angle of repose,

V_1, V_2 = Vertical heights at two points, and

H_d = Horizontal distance between two points.

Tests were repeated 15 times and the average of all tests was calculated.

5.2.4 Coefficient of Friction

Coefficient of friction of grain on polyethylene was determined by measuring the slope at which wheat started moving on the polyethylene surface.

A plumb bob was fastened to an adjustable 60-cm protractor, mounted on one end of a tilting top drafting table. Before every test, the scale of the protractor was adjusted so that the zero line fell directly behind the plumb bob string when the top surface was level. The slope of the table top could be directly read on the protractor.

To conduct a test, a polyethylene sheet was fastened to the top of the table and a layer of wheat was uniformly spread over it. A 30-cm x 30-cm wooden frame was placed on the test layer and filled with wheat. The back edge of the table top was slowly raised. At the instant when the frame started to move, the angle of the slope of the table top was read from the protractor. The tangent of this angle was the coefficient of friction. An average of 15 tests was calculated.

5.2.5 Filling of Test Containers

Containers were filled in successive layers so that each container could be used for various height-to-diameter ratios. Large containers were filled in grain layers equivalent to a height-to-diameter ratio of 0.33. However, for a 25-cm diameter container this ratio represented a grain layer of 8 cm only. Therefore, this size of container was filled in grain layers corresponding to a height-to-diameter ratio of 0.625.

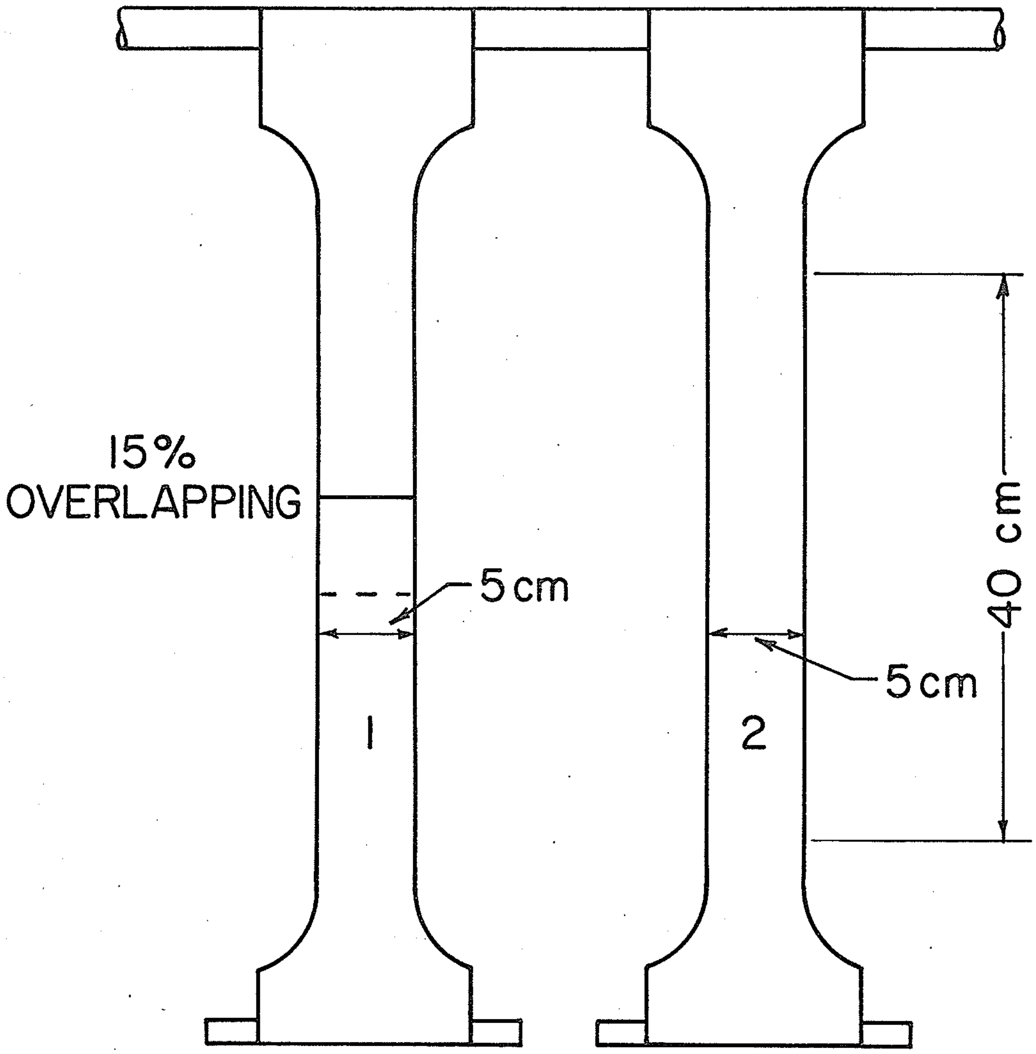
Measurements on test containers were made 30 min after each fill. Total time required for all measurements for each fill was less than 10 min. Since the rate of elongation of polyethylene with time was less than 0.0115%/min during the 30-min to 60-min period after loading (Sec. 4.3), the maximum error due to effect of time was less than 0.115%. This error was considered negligible.

5.2.6 Effect of Gluing

To correct the observations on test containers for effect of gluing and overlapping, a correction factor was determined. Polyethylene strips were made by gluing two strips together. An overlapping of 15% was provided on the strips (Fig. 5.3). Stress-elongation characteristics of these strips were determined and compared to the stress-elongation characteristics of unglued strips under the same conditions of temperature and loading. Ratios of elongations in unglued strips to that in glued strips were calculated at various stress levels. The average of these ratios was called the correction factor. Elongations measured on the containers were multiplied by the correction factor to obtain elongations corrected for the effect of overlapped joints.

5.2.7 Calibration Curves

Stress-elongation calibration curves were plotted for each test container. Calibration strips, 5 cm wide were made of the same thickness of polyethylene as the test containers. The expected lateral pressure in the test containers for each fill was predicted using Janssen's equation (Eq. 2.12). Stresses in the test containers corresponding to predicted pressures were calculated (Eq. 4.1). Loads required to produce stresses in calibration



1. GLUED POLYETHYLENE STRIP

2. UNGLUED POLYETHYLENE STRIP

FIG. 5.3 CALIBRATION STRIPS

strips corresponding to 50%, 75%, 100%, and 125% of expected pressures were calculated.

For each container, 12 calibration strips were tested. Loads corresponding to expected stresses for first grain layer were applied to the strips just when the first grain layer was loaded into the container. Measurements were taken on containers and strips at the same time,30 min after loading. Additional loads were applied to the strips for each subsequent grain layer added to the containers. Stress-elongation curves were plotted for each grain layer and stresses corresponding to corrected elongations in the containers were determined from the curves (Append. 2). Lateral pressures in the test containers for each fill were calculated using Equation 4.1. Since the calibration strips and the containers were made from the same material thickness and were subjected to the same conditions of temperature and loading, no correction for these factors was required.

5.2.8 Effect of Time

To study the variation of lateral pressure in flexible containers with time, one container was allowed to stand for a longer time after filling. Measurements on the container were taken in circumferential and vertical direction four times during the test period of 78 hr.

To plot calibration curves, strips of polyethylene were subjected to the same loading conditions as the containers and measurements were taken simultaneously.

Corrected elongations in the containers were calculated by multiplying the observed elongationsby a correction factor (Sec. 5.2.6). These corrected elongations were compared to calibration curves and stresses in containers were determined. Lateral pressures corresponding to these stresses were calculated for each time period using Equation 4.1.

6. RESULTS AND DISCUSSION

6.1 Material Properties

Experimental values of material properties, i.e., density, angle of repose, coefficient of friction on polyethylene, and ratio of lateral to vertical pressure are reported in Table 6.1. Values of these material properties available in the literature are also reported for comparison.

6.1.1 Density

Density of wheat increased with height of fall at a decreasing rate in both flexible and rigid containers, (Fig. 6.1). The difference between densities in flexible and rigid containers appeared to increase with height of fall. A paired difference t-test confirmed that the densities in flexible containers are significantly higher than in rigid containers. The lower density in the rigid containers indicated that grain kernels become re-oriented less in rigid containers than in flexible containers. A value of 0.882 metric ton/m^3 was used in the calculations.

6.1.2 Angle of Repose

Average angle of repose was found to be 21.7° with a standard deviation of 0.54°. This experimental value is considerably lower (approximately 22.3%) than the value of 28° reported by Jamieson (1904) and listed in Farm Building

Table 6.1. Material Properties of Wheat

Property	Experimental Value	Recommended Value*
Grade	No. 1 Northern	
Dockage, Percent	2.0	
Moisture Content, Percent wet basis	10.3	
Density, Metric ton/m^3	0.882	0.769 - 0.817
Angle of Repose	21.7°	21.8° - 28.0°
Coefficient of Friction on Polyethylene	0.366	0.350
Ratio of Lateral to Vertical Pressure	0.452	0.306

*Agricultural Engineer's Yearbook (1969), Britton (1969), and Brubaker and Pos (1963).

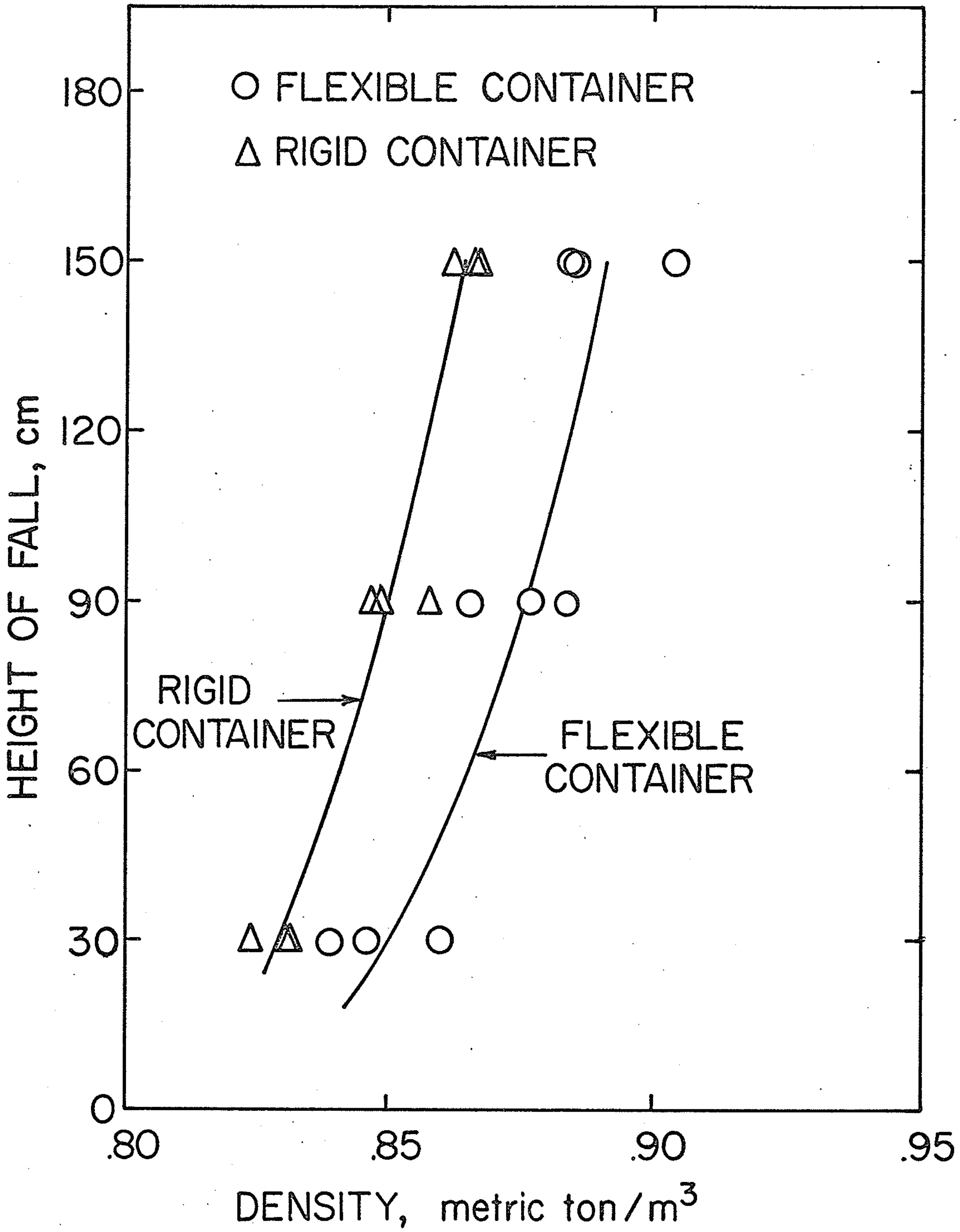


FIG. 6.1 WHEAT DENSITY IN RELATION TO HEIGHT OF FALL

Standards (N.R.C. 1970), and Agricultural Engineer's Yearbook (1969) for hard red spring wheat. But, this value of 21.7° compares very well with a value of 21.8° reported by Britton (1969). The value of 28° appears to be incorrect because Jamieson (1904) did not distinguish between angle of repose and angle of internal friction. A lower experimental value is probably due to the fact that Manitou wheat is a small-kernel variety.

A smaller angle of repose estimates higher values of K in Janssen's equation (Eq. 2.12) when K is determined by the following equation:

$$K = \frac{1 - \sin \phi_r}{1 + \sin \phi_r} \qquad \text{(Eq. 2.4)}$$

K is estimated as 0.306 when the angle of repose is 28°. This is lower than the value of 0.452 estimated for K using angle of repose equal to 21.7°. The value of K used in this thesis was 0.452.

6.1.3 Coefficient of Friction

Coefficient of friction of wheat on polyethylene was 0.366 with a standard deviation of 0.022. This experimental value compares well with the value of 0.350 for coefficient of friction of wheat on polyethylene reported by Brubaker and Pos (1963).

6.2 Maximum Height of Fill

When the height of grain in each size of container exceeded a certain value, the container failed by tilting over and falling down. The height-to-diameter ratio did not appear to control the point of failure because the maximum height-to-diameter ratio decreased from 2.50 for the smallest (25-cm diameter) to 1.63 for the largest container (90-cm diameter). Also the height-to-diameter ratio at failure was not the same for 67-cm diameter containers made from two different thickness polythylene films (0.015 cm and 0.025 cm). The height-to-diameter ratio at failure was 2.30 on the thick-walled container and 1.67 on the thin-walled container. But for all diameters and thicknesses, failure appeared to occur when the percent elongation measured at the reference grid line became greater than 11%.

6.3 Lateral Pressures

Lateral pressures of wheat determined in flexible polyethylene containers are shown in Figures 6.2 to 6.7. Lateral pressure in the test containers increased non-linearly with grain height. The nature of variation is similar for all the container sizes tested. The effect of height on lateral pressure decreased with grain height. Therefore, Coulomb's theory (Eq. 2.8) and Rankine's theory (Eq. 2.9) are not applicable to polyethylene containers because these theories assume that lateral earth pressures are directly proportional to earth depth.

No bulge of pressure, as reported by Collins (1963) was observed. Collins (1963) did not consider the restraining

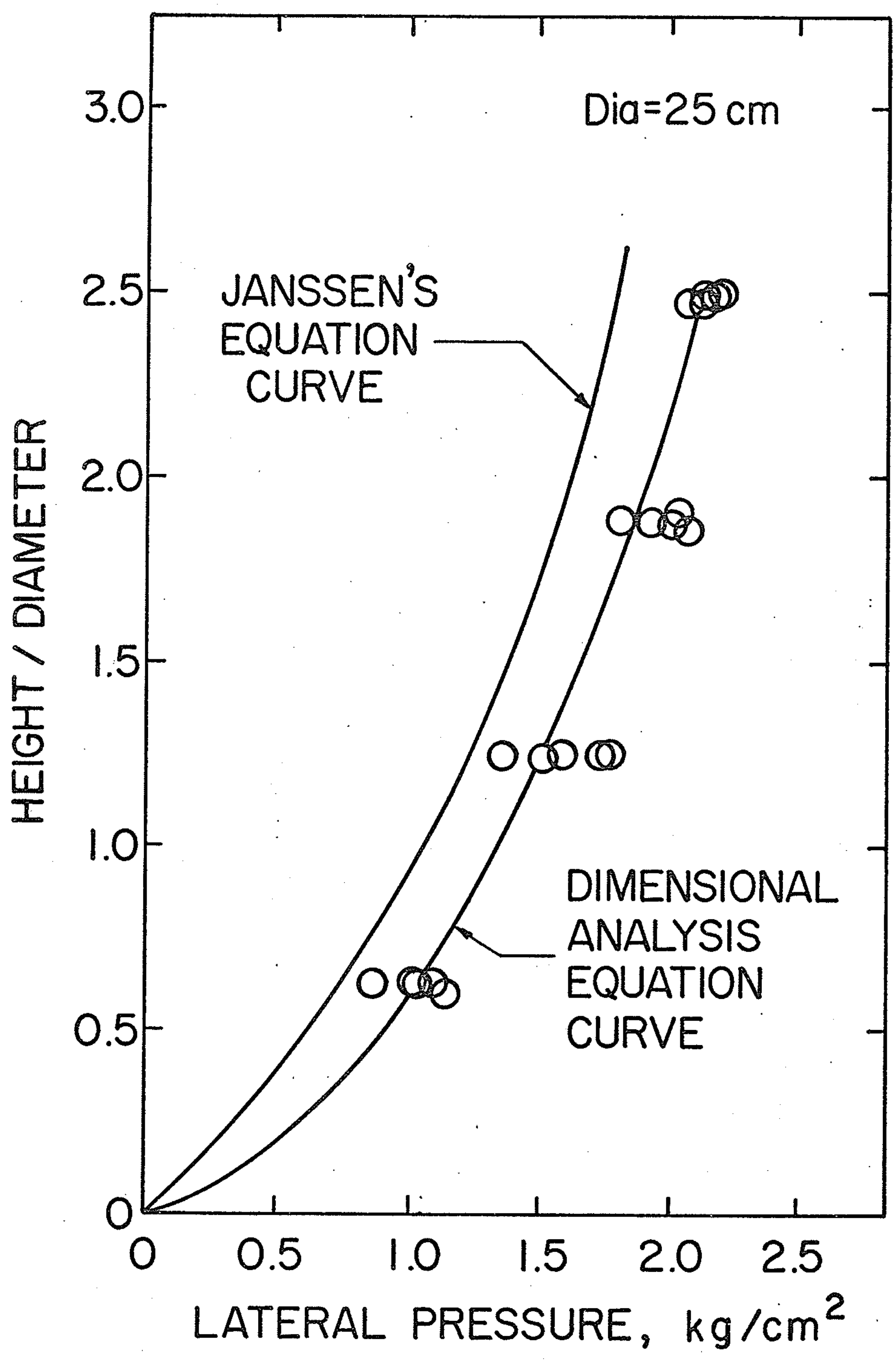


FIG. 6.2 LATERAL PRESSURE IN 25-cm DIAMETER CONTAINER

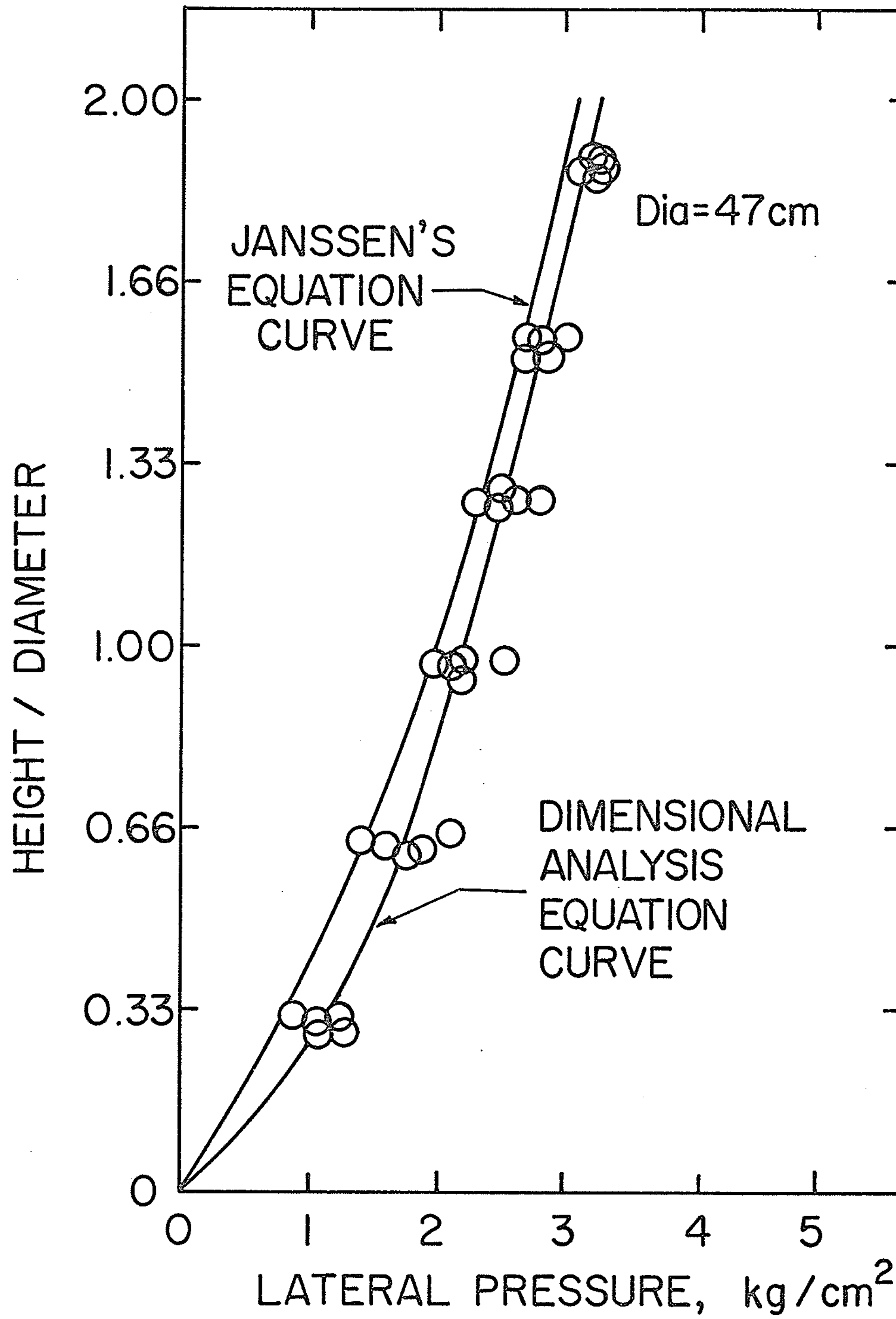


FIG. 6.3 LATERAL PRESSURE IN 47-cm DIAMETER CONTAINER

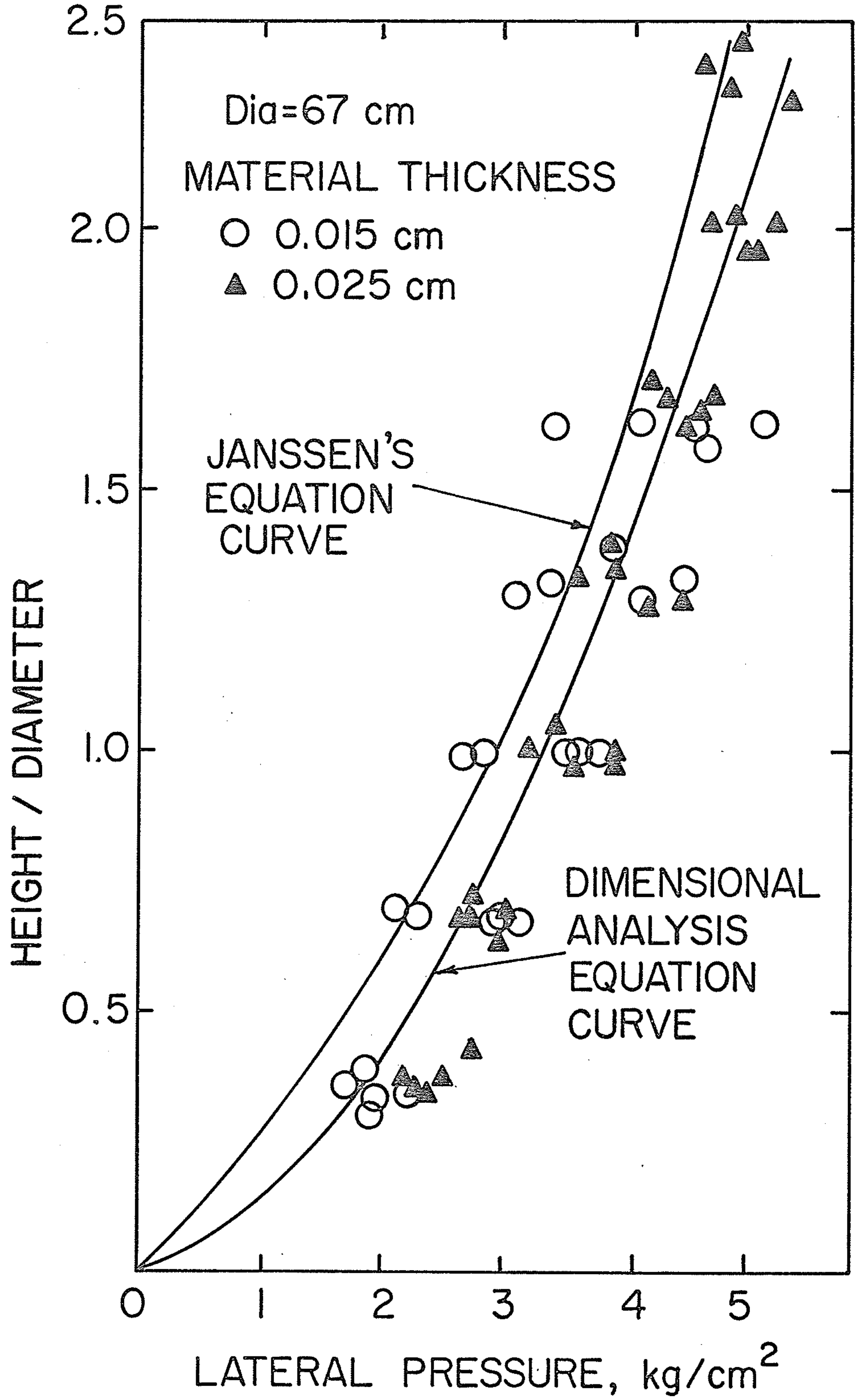


FIG. 6.4 LATERAL PRESSURE IN 67-cm DIAMETER CONTAINER

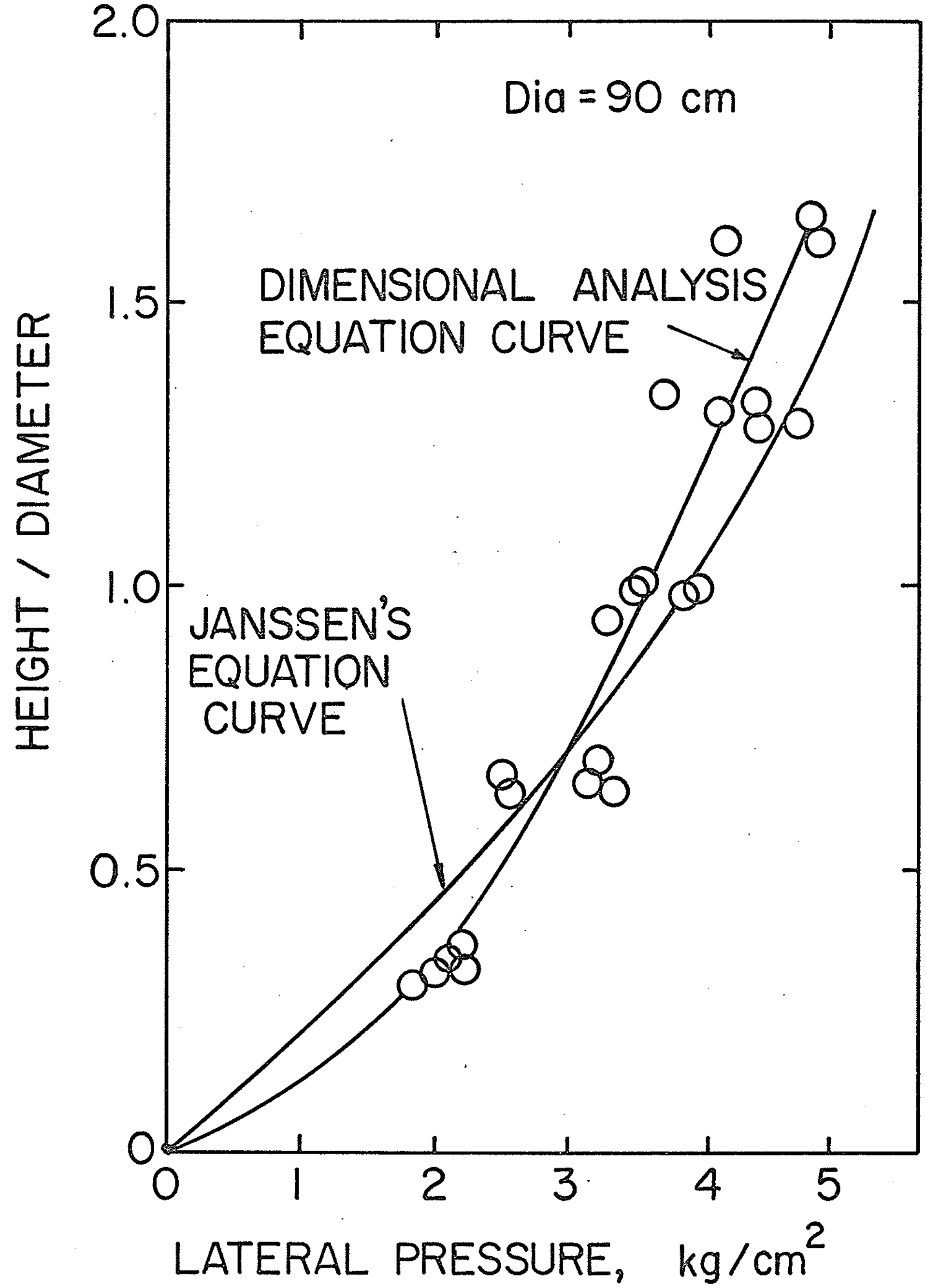


FIG. 6.5 LATERAL PRESSURE IN 90-cm DIAMETER CONTAINER

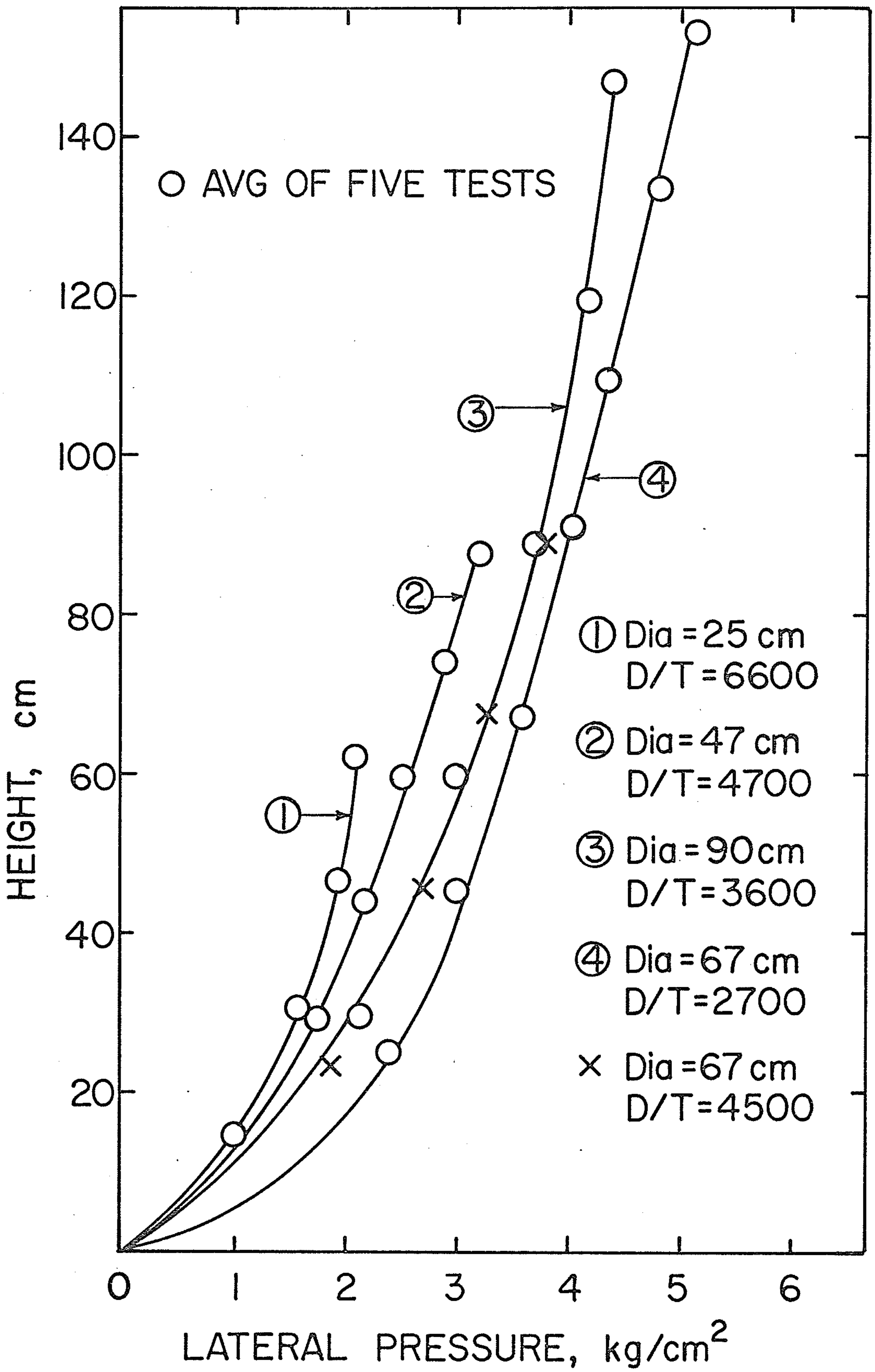


FIG. 6.6 LATERAL PRESSURE IN RELATION TO GRAIN HEIGHT

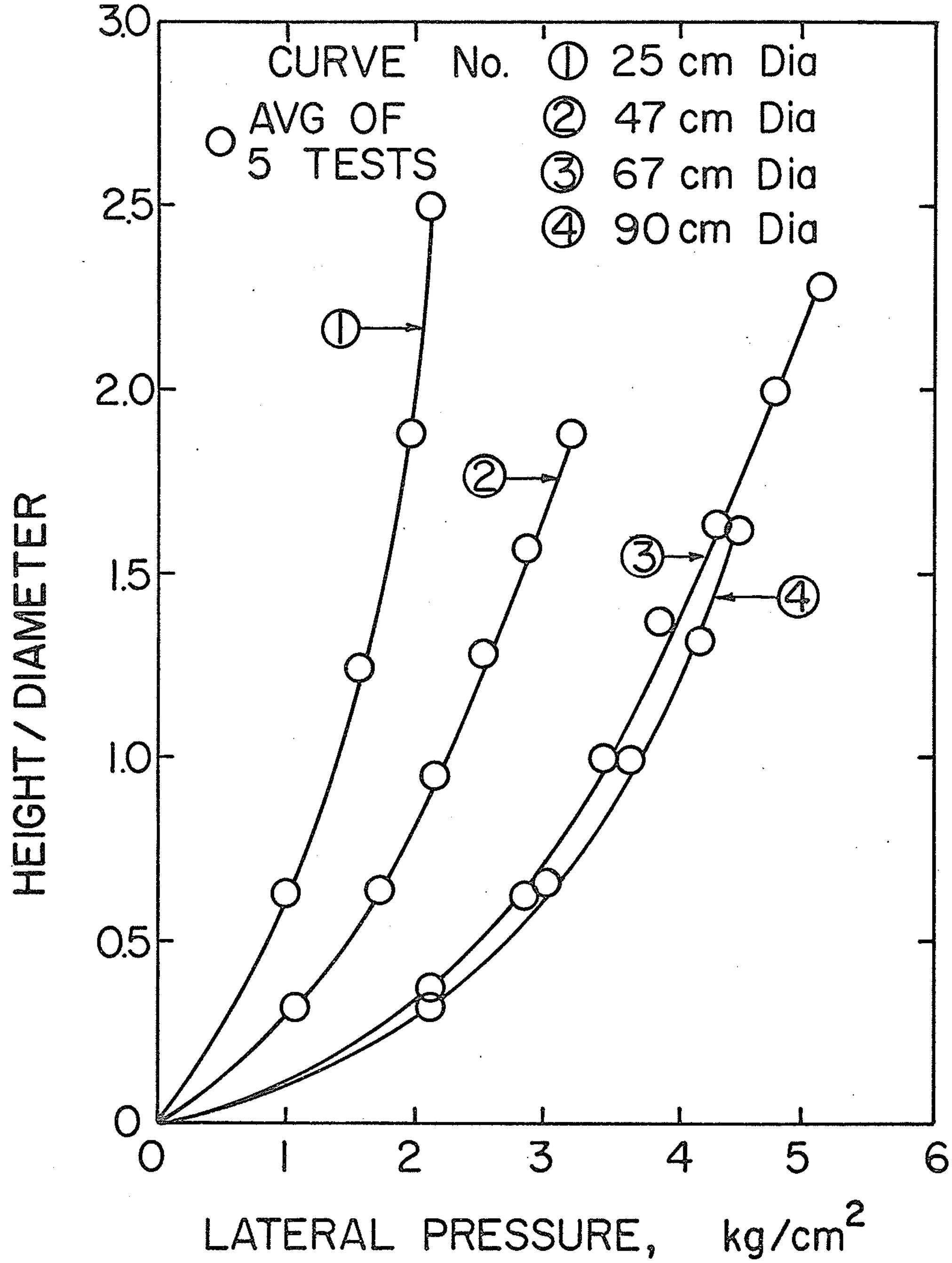


FIG. 6.7 EFFECT OF CONTAINER DIAMETER ON LATERAL PRESSURE

effect of container bottom on wall deflection and observed a bulge of pressure at a height-to-diameter ratio of nearly 0.25. Since the bottom grain layer up to a height-to-diameter ratio of nearly 0.35 was not considered in this study, the restraining effect of the formed bottom was eliminated.

Figure 6.6 shows that for the same grain height, lateral pressure increased with container diameter up to 67-cm and then decreased for 90-cm diameter containers. Therefore, lateral grain pressure is not directly proportional to container diameter. However, the container wall thickness was not the same in all the cases. Lateral pressure for the same grain height decreased with diameter-to-thickness ratio except for the 67-cm diameter container made from 0.015-cm thick polyethylene (Fig. 6.7).

6.4 Effect of Container Wall Thickness

Lateral pressure of wheat in 67-cm diameter containers made from 0.015-cm and 0.025-cm thick polyethylene film was compared. Lateral pressures in the thick-walled containers were greater than those in the thin-walled containers up to a height-to-diameter ratio of 1.33 and were equal at a ratio of 1.67 (Fig. 6.4). A paired difference t-test at the 5% level of statistical significance did not indicate any significant difference in lateral pressures in the two containers. However, percent circumferential elongation at a height-to-diameter ratio of 1.67 was 5.3% on the thick-walled container and 9.1% on the thin-walled container.

6.5 Comparison of Experimental Pressure with Janssen's Equation

Theoretical lateral pressures were predicted by Janssen's equation (Eq. 2.12) using the values of parameters experimentally determined. Experimental pressures are greater than predicted pressures in 25-cm, 47-cm, and 67-cm diameter containers (Fig. 6.2 to 6.5). Janssen's equation was developed for the case in which the container walls take a part of the vertical grain load, resulting in lower loads on the container floor. Since, flexible container walls do not take any vertical loads, vertical pressures on container floors and lateral pressures on the walls, would be expected to be greater than those predicted by Janssen's equation for rigid bins. In 90-cm diameter containers, predicted pressures were lower than experimental pressures up to a height-to-diameter ratio of 0.66 and then became greater. One reason for this variation could be that experimental pressures did not increase with container diameter as much as predicted by Janssen's equation.

Janssen's equation assumes that lateral pressure is directly proportional to container diameter at a given height-to-diameter ratio. Lateral pressures increased with container diameter for constant height-to-diameter ratio (Fig. 6.7). However, this relationship was not directly proportional, which does not agree with Janssen's equation.

An attempt was made to modify Janssen's equation (Eq. 2.12) to predict lateral grain pressures in flexible containers. Various values of the parameter K were used in the range of 0.1 to 1.0. Container diameter was modified by multiplying by a constant factor. Predicted values of lateral pressures were also multiplied by another factor. None of the changes or combination of changes modified the equation to closely predict lateral wheat pressures in flexible polyethylene containers.

6.6 Buckling of Container Walls

Measurements were taken in the vertical direction, to determine buckling of container walls. Vertical distance between any two circumferential grid lines decreased with each subsequent grain layer added to the containers. Height of grain decreased from 150 cm to 147 cm during the test in the 90-cm diameter containers. A series of wrinkles was observed all around the container circumference. The intensity of wrinkles increased as the grain depth increased. More wrinkles were observed near the container bottom. All containers fell over before tensile failure occurred.

6.7 Dimensional Analysis Equation

A dimensional analysis equation was developed to predict lateral pressures in flexible containers. The equation as developed in Chapter 3 was:

$$L_p = K' \; w \; D^n \; h^{1-n} \qquad \text{(Eq. 3.9).}$$

Value of parameter n for wheat and polyethylene calculated from the experimental pressures was 0.45

(Append. 3). Values of the parameter K' for various sets of conditions, are reported in Table 6.2. Using these values of the parameters, this equation closely represented lateral pressures of wheat in polyethylene containers (Fig. 6.2 to 6.5). Equation 3.9 is similar to Janssen's equation (Eq. 2.12) in that both assume direct proportionality between lateral pressure and grain density. However, the relationship of lateral pressure with container diameter and grain height is different in the two equations.

6.8 Effect of Time on Lateral Pressure

A continuous decrease in lateral pressure was observed with time (Fig. 6.8). Lateral pressure decreased from 3.20 kg/cm^2 to 3.07 kg/cm^2 within 17 hr of loading. The rate of decrease rapidly dropped after 17 hr. It appears from Figure 6.8 that the lateral pressure would become constant after some time. This observation is in agreement with the findings of Lufft (1904) and Saul (1953) in rigid bins.

Percent circumferential elongation in the container increased to a maximum of 6.4% from 5.6% in 42 hr and then decreased to 6.3% in the next 46 hr. Percent elongation on calibration strips continued to increase with time throughout the test. The rate of increase of elongation in the calibration strips was greater than that in the container. This is why lateral pressure on the container walls decreased with time even though the circumferential elongation

Table 6.2. Values of Parameter K' in Dimensional Analysis Equation

Container Diameter cm	K'
25	0.0609
47	0.0543
67	0.0572
90	0.0457

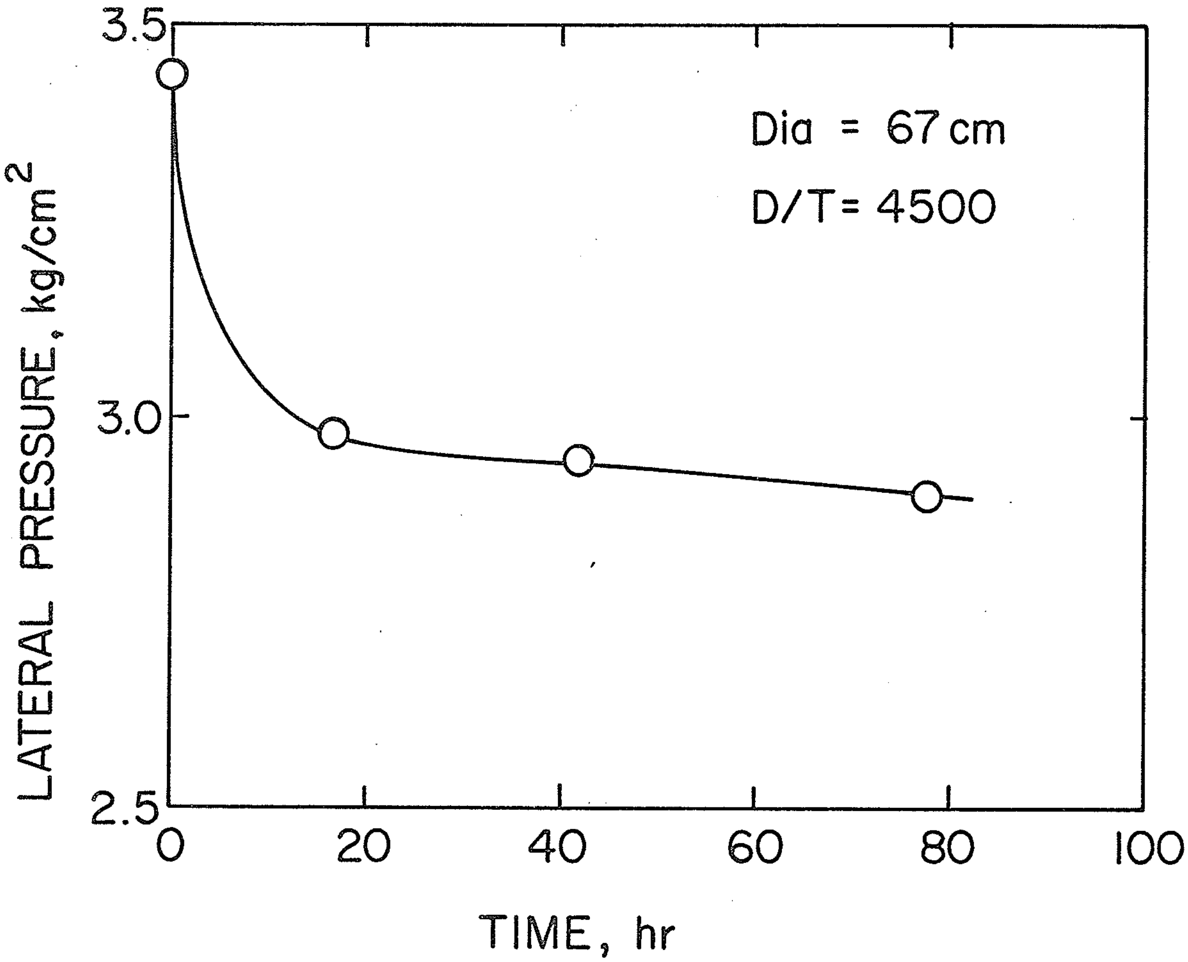


FIG. 6.8 EFFECT OF TIME ON LATERAL PRESSURE

increased.

The maximum increase in percent circumferential elongation with time was 0.75%. Therefore, it can be expected the containers will not collapse after any period of time.

7. CONCLUSIONS

On the basis of this study, the specific conclusions regarding the lateral pressures of wheat in flexible polyethylene containers of sizes varying from 25 cm x 63 cm to 90 cm x 147 cm are as follows:

1. Lateral wheat pressures in flexible polyethylene containers increase with grain depth at a decreasing rate.

2. Lateral wheat pressures in flexible polyethylene containers increase with container diameter for the same height-to-diameter ratio.

3. Janssen's equation in its present form and with the modifications attempted did not closely predict lateral pressure of wheat in flexible polyethylene containers.

4. For the test conditions used, an equation of the following form closely represented lateral wheat pressures in flexible polyethylene containers:

$$L_p = K' \, w \, D^n \, h^{1-n}$$

5. Effect of container wall thickness (up to 60% variation) on lateral pressure is not significant.

6. Lateral wheat pressures in flexible polyethylene containers decrease with time at a decreasing rate.

7. Circumferential elongation in polyethylene containers increases very little up to 17 hr and then decreases.

8. Polyethylene grain containers should be designed for a maximum percent circumferential elongation of less than 10%.

8. SUGGESTIONS FOR FURTHER STUDY

1. Lateral pressures of various granular materials should be determined in flexible containers made from various plastic and synthetic rubber materials.

2. Tests should be conducted in larger full-size bins.

3. Effect of time and container wall thickness on lateral pressure should be studied in more detail.

4. A new analytical theory should be developed for predicting grain pressures in flexible containers.

5. Numerical methods such as the finite element method, should be used to develop a general equation for predicting lateral pressures in flexible containers. A general computer program should also be developed which would be applicable to all grains and flexible containers.

6. The relationship of material density and height of fall should be studied in greater detail.

7. Economics of the system should be studied and compared to conventional systems.

8. Improved instrumentation should be developed for more accurate measurements in flexible containers.

9. Lateral grain pressures in flexible containers should be compared for the conditions of filling, emptying, and at rest.

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APPENDICES

APPENDIX 1

DERIVATION OF JANSSEN'S EQUATION

The mathematical model assumed by Janssen in developing his equation (Eq. 2.12) is shown in Figure A.1. For the static case,

$$vA + Aw\ dy = (v + dv)\ A + L_p U\ \mu'\ dy$$

or

$$dv = w\ dy - \frac{L_p U \mu'}{A}\ dy$$

substituting, $L_p = Kv$,

$$dv = w\ dy - \frac{Kv\mu' Udy}{A}$$

substituting, $R = A/U$,

$$dv = w\ dy - \frac{Kv\mu'}{R}\ dy$$

substituting, $\frac{K\mu'}{R} = n$,

$$dv = (w - nv)\ dy$$

or

$$\frac{dv}{w-nv} = dy$$

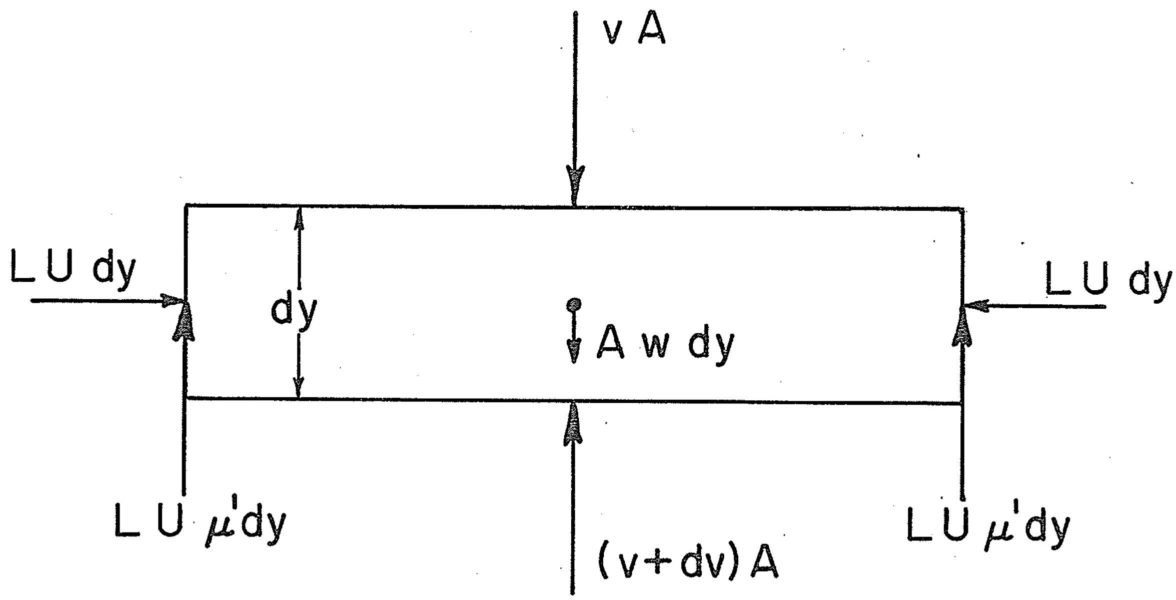
Integrating,

$$\ln (w - nv) = - ny + c$$

Boundary condition,

$$\text{If } y = o, \quad v = o$$

$$\ln w = c$$



L = LATERAL GRAIN PRESSURE

U = BIN PERIMETER

v = VERTICAL GRAIN PRESSURE

A = BIN CROSS SECTION AREA

w = BULK DENSITY OF GRAIN

μ' = COEFFICIENT OF FRICTION ON BIN WALL

FIG. A.1 JANSSEN'S MODEL OF GRAIN PRESSURES or $\ln \frac{w - nv}{w} = - ny$

or

$$\frac{w - nv}{w} = e^{-ny}$$

or

$$v = \frac{w}{n} (1 - e^{-ny})$$

Resubstituting

$$n = K\mu'/R,$$

$$L_p = Kv$$

and putting

$$h = y$$

$$L_p = \frac{wR}{\mu'} (1 - e^{-k\mu' h/R})$$

This is the most common form of Janssen's equation.

APPENDIX 2

PROCEDURE OF CALCULATING EXPERIMENTAL LATERAL PRESSURE IN TEST CONTAINERS

To calculate experimental values of lateral pressure in test containers from observed circumferential elongation, the following procedure was adopted (Table A.1):

1. Calibration Curves

Stress-elongation calibration curves were plotted for each test condition as described in Section 5.2.7 (Fig. A.2).

2. Height-to-Diameter Ratio

Ratio of grain height and container diameter was determined for each fill (Table A.1).

$$h/D_o = \frac{\text{Height of grain}}{\text{Original container diameter}} \quad \ldots (A.1)$$

3. Observed Circumferential Elongation

Circumferential elongations at the reference grid line was measured for each grain layer (Sec. 5.2.5). These elongations were expressed on percent basis.

$$\text{Percent elongation} = \frac{\text{Increase in container circumference}}{\text{Original container circumference}} \times 100 \quad \ldots (A.2)$$

4. Corrected Circumferential Elongation

Observed circumferential elongations at the reference

Table A.1. Calculation of Experimental Lateral Pressures

Height/ Diameter	Observed Elongation Percent	Corrected Elongation Percent	Container Diameter cm	Stress in Container Walls kg/cm^2	Lateral Pressure kg/cm^2
0.33	2.31	2.43	70.5	32.3	2.14
0.66	3.11	3.27	71.0	47.6	2.94
1.01	4.53	4.76	71.9	59.8	3.65
1.33	6.42	6.74	73.5	74.9	4.47
1.64	8.83	9.27	75.1	87.1	5.10

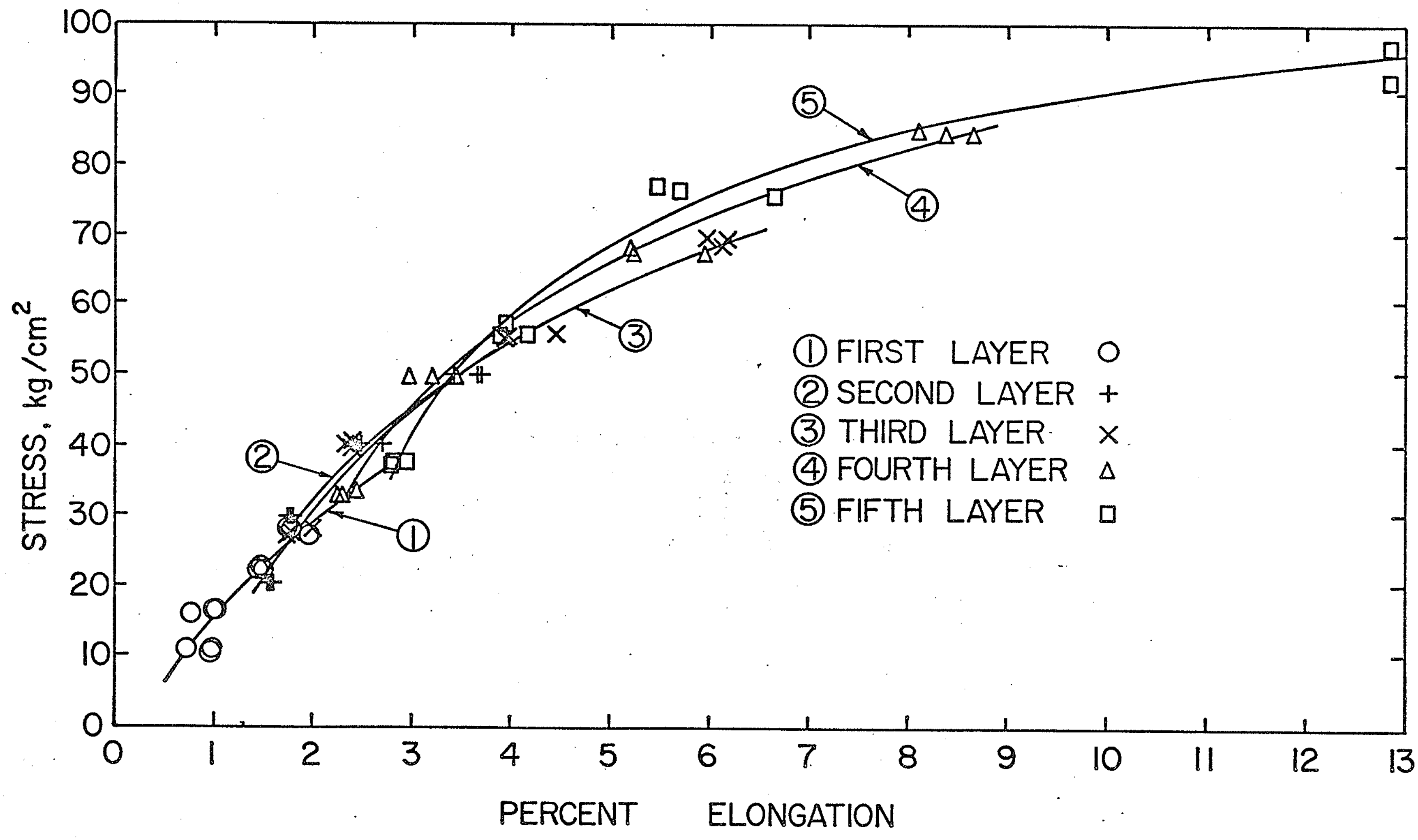


FIG. A.2 CALIBRATION CURVES

grid were corrected for the effect of gluing and overlapping (Sec. 5.2.6). The value of correction factor for the test condition was determined to be 1.05.

$$\text{Corrected percent elongation} = \text{Observed percent elongation} \times \text{correction factor} \quad \text{. . .(A.3)}$$

5. Container Diameter

Container diameter at the reference grid was calculated for each grain layer added.

$$\text{Container diameter} = \frac{\text{Observed container circumference}}{\pi} \quad \text{.(A.4)}$$

6. Stresses in Container Walls

Stresses in container wall were determined for each grain layer by comparing the value of corrected elongation (Step 4) with the corresponding calibration curve (Fig. A.2).

7. Lateral Pressure on Container Walls

Lateral pressures on container walls were calculated from corresponding stresses (Step 6) using the following equation:

$$L_p = \frac{2\sigma T}{D} \quad \text{. . .(A.5)}$$

where:

L_p = Lateral pressure on container walls

σ = Stress in container walls

T = Container wall thickness

D = Container diameter.

APPENDIX 3

EVALUATION OF THE PARAMETERS K' AND n

IN THE DIMENSIONAL ANALYSIS EQUATION

The dimensional analysis equation as developed in Chapter 3 is:

$$L_p = K' \; w \; D^n \; h^{1-n} \qquad \text{(Eq. 3.9)}$$

If it can be assumed that the values of parameters K' and n are constant for a particular size of test container, then Equation 3.9 can be re-written as:

$$\frac{(L_p)_1}{(L_p)_2} = \frac{(h_1)^{1-n}}{(h_2)^{1-n}} \qquad \text{. . . (A.6)}$$

Average values of lateral pressures at grain heights of 31 cm and 88 cm for a 47-cm diameter container are 1.77 kg/cm^2 and 3.15 kg/cm^2 respectively (Fig. 6.6). Substituting these values in Equation A.6:

$$\frac{1.77}{3.15} = \frac{(31)^{1-n}}{(88)^{1-n}}$$

or

$$0.563 = (0.352)^{1-n}$$

or

$$\ln\,(0.563) = (1-n)\,\ln\,(0.352)$$

or

$$-0.57448 = (1-n)\ (-1.04412)$$

or

$$1-n = \frac{-0.57448}{-1.04412}$$

$$= 0.55$$

or

$$n = 0.45$$

Values of parameter n, calculated for each container size, were close to 0.45. This value was used for each container size.

Substituting the value of lateral pressure at a grain height of 31 cm for the 47-cm diameter container in Equation 3.9, and using the value of density experimentally determined,

$$1.77 = K'\ (0.882)\ (47)^{0.45}\ (31)^{0.55}$$

or

$$K' = \frac{1.77}{(0.882)\ (47)^{0.45}\ (31)^{0.55}}$$

$$= 0.0537$$

Values of parameter K' were determined for each test grain height in each container and average for each container size was calculated. Average values of parameter K' for each container size are reported in Table 6.2.