THE UNIVERSITY OF MANITOBA

APPARENT DISTANCE IN THE POGGENDORFF ILIUSION by

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A THESIS
SUBMITTED TO THE FACULTY OF GRADUATE STUDIES
IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF ARTS

DEPARTMENT OF PSYCHOLOGY

WINNIPEG, MANITOBA
by
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A dissertation submitted to the Faculty of Graduate Studies of the University of Manitoba in partial fulfillment of the requirements of the degree of

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MASTER OF ARTS
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## ACKNOWLEDGEMENTS

The author wishes to express his appreciation to Dr.J. Brewster, Department of Statistics and Dr. M. Singer, Department of Psychology for their encouragement and advice.

Dr. A. W. Pressey merits special mention for his constant attention during all stages of the development of this thesis.

## ABSTRACT

Three predictions concerning the relationship of the apparent distance between transversals to the magnitude of illusion in the Poggendorff configuration were derived from the assimilation theory of geometric illusions. The predictions were that (a) the objective distance would be underestimated, (b) the degree of underestimation and the magnitude of Poggendorff illusion would show a similar decrease as the size of acute angle in the configuration increased, and that (c) subjects who displayed large degrees of underestimation would display large illusions.

The experiment evaluating these predictions emploved a $3 \times 5$ mixed design. The within-subject variable was task and the betweensubject variable was angle size. An equal number of subjects (36) was assigned to each of the five groups. The amount of illusion and apparent distance were obtained by the method of production.

None of the three predictions was verified. It was concluded that assimilation theory failed to provide an adequate explanation of the illusion. It was argued that serious doubts had been raised for any explanation of the illusion based on apparent distance.
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The study of illusions has maintainea the interest of psychologists for over a hundred years. This interest "was clearly recognized about 1860" (Boring, 1942) and that it has been a continuing one is indicated by the steady increase in the number of articles published on the topic over the years (Zusne, 1968)..

One of the fundamental questions concerning illusions involves their definition. Although various ones have been proposed, the one that seems to be gaining increasing acceptance argues that an illusion is a change in the attributes of the stimulus. This change may involve distance, size, direction, etc., and is the result of contextual stimuli (Zusne, 1970). Since all experience occurs in a context, it could be argued that all experience is illusory. In the words of Boring (1942) "No experience copies, reality (p. 238)." This view, that all experience is illusory, entails a philosophy which states that the study of illusions is the study of our normal perceptions.

Support for this point of view comes from Johannsen's (1972) review article on the early history of illusions. She cited the employment of optical illusions by early Greek architects. They designed buildings which contained curves introduced into horizontal lines so that the lines would appear horizontal. The columns of their buildings were irregularly spaced and shaped "in order that the to the eye they [the buildings] would appear rectilineax (Johannsen, 1971, p. 127)."

In more recent times, Zanforlin (1957) has shown the continuity between an illusion which occurs in our everyday perception and an illusion which the psychologist studies in the laboratory. This everyday illusion concerns coins. If a person is asked to make a pile of coins equal in height to the diameter of the coins, the pile will be too low. The laboratory analogue is the horizontal-vertical illusion shown in Figure 1.

Lucas and Fisher (1969) have also demonstrated the continuity between illusions occurring in nature and illusions studied in the laboratory. In their experiments, they undertook the task of determining if the degree of illusion displayed by abstract, geometric forms studied in laboratories was of the same magnitude as illusions occurring in more realistic settings. The illusion chosen for their study, the Poggendorff configuration, is shown in Figure 2. It occurs when an oblique line is interrupted by two parallel lines so that the parts of the oblique line do not appear collinear.

In their first experiment, Lucas and Fisher (1969) compared the amount of illusion found in an abstract form similar to Figure 2 with the amount of illusion found in four realistic settings. The realistic scenes Were two devils pulling a rope passing behind a column, a pulley system passing behind a steel girder, a boy with a pole placed behind a tree, and the courses of two aircraft displayed upon a radar screen. Analysis of the data failed to find statistically significant differences among the five conditions.

In the second experiment, the amount of illusion found in the


Figure 1. In the horizontal-vertical illusion the vertical line appears longer than the horizontal line, although both are objectively equal.


Figure 2. An example of the Poggendorff illusion: Line A does not appear collinear with line $B$ although both are parts of the same line.
abstract figure and in two natural concrete displays were compared. The concrete displays presented a stevedore pulling a crate with a rope placed behind a girder. The results of this experiment also indicated that there were no statistically significant differences among the abstract and concrete forms of the figure.

The experiments performed by Zanforlin (1967), Lucas and Fisher (1969), and others lend strong support to the argument that by studying illusions in the laboratory we are studying our everyday perceptions. It is this conviction that has led to the systematic study of optical illusions.

One geometric figure that has attracted considerable interest is the one studied by Fisher and Lucas (1969)--the Poggendorff illusion. Although this illusion has been studied for the past 100 years, few explanations seem satisfactory. The major purpose of this paper is to evaluate one of the more recent theories of the illusion. Before this evaluation will take place, however, the facts concerning the figure and the methods used to obtain these facts will be examined.

## Measuring the Poggendorff Illusion

Psychologists have developed a varity of methods to measure the Poggendorff illusion. Although there has been little research directed to finding the "best" method for this task the methods of adjustment and production (Pressey \& Sweeney, 1969) have been the most commonly used. This section will briefly describe the methods employed in measuring the illusion.

The method of adjustment is the classical, psychophysical method that has been used most often in measuring the Poggendorff illusion (e.g., Judd, 1899; Cameron \& Steele, 1905). The typical procedure with this method is to maintain one of the transversals, the fixed transversal, at a constant position. The second transversal, the variable transversal, is adjusted by the observer along one of the parallel lines until it appears collinear with the fixed transversal. Generally, the observer begins his adjustment of the variable transversal. from a starting position that is counterbalanced between being set obviously too high or too low. The illusion is determined by averaging the responses of the subject from these two starting positions.

The classical psychophysical methods of limits and constant stimuli have been used less often than the method of adjustment. The experiments performed by Lucas and Fisher (1969) provide an example of a study which has used these methods. With the method of limits the variable transversal is set obviously too high or too low. The experimenter moves the variable txansversal in discrete steps closer to objective collinearity. After each discrete step the observer is required to indicate if the variable transversal is set too low or too high to be collinear with the fixed transvexsal. If the subject responds too low and if the starting position was too low, the transversal is set one step closer to collinearity. The procedure is repeated until the subject responds too high. The above procedure is then repeated. The measure of the illusion is found by averaging the positions where the subject reversed his judgment from too low to too high and from too high to too low.

The method of constant stimuli has several characteristics in common
with the method of limits. The variable transversal is placed at different positions along the parallel lines. However, instead of changing the placement of the transversal in a constant manner, i.e., closer to collinearity with each discrete step, the placement of the transversal is randomly determined. The subject is required to respond if the transversal is set too high or too low to be collinear with the fixed transversal. The size of the illusion is calculated by finding the position where the subject stated that the variable transversal was either too high or too low $50 \%$ of the time.

The Poggendorff illusion has also been measured by a number of nonclassical psychophysical methods. These alternative methods are the up-and-down method (Weintraub \& Krantz, 1971), the selection method (Ellis \& Deregowski, 1972), the method of single stimuli (Restle, 1969), the method of detection (Greene \& Hoyle, 1964), and the method of production (Pressey \& Sweeney, 1969). With the up-and-down method the subject judges if the variable transversal is too low or too high to be subjectively collinear with the fixed transversal. If the transversal is judged as being too high it is lowered one discrete step. The procedure then repeats itself until the subject reverses his judgment, i.e., the observer states too low instead of too high: Then the transversal is raised one discrete step and the process is repeated. The illusion is calculated by averaging the positions where the subject changed his response.

With the method of selection an observer is presented with several figures in which the position of the variable transversal is varied between targets. The subject is required to choose the figure in which the two transversals appear collinear. The calculation of the size of the
illusion is the deviation the variable transversal is from objective collinearity.

The method of single stimuli is essentially the same as the method of constant stimuli with one exception, The subject is required to perform one additional task in his judgment. He assigns his response to one of several categories that range on a continuum from very much too high to very much too low. The magnitude of illusion is determined by averaging the mean response for each position of the transversal...

With the method of detection the target presented to the observer has the transversals objectively aligned. The subject is required to indicate if the variable transversal is too high or too low to be collinear with the fixed transversal. The strength of the illusion is determined by computing the proportion of times the illusion was found to be present.

With the method of production the observer is presented with a figure consisting of two parallel lines and the fixed transversal intersecting one of these lines. The subject is required to judge where the transversal, if extended, would intersect on the distal parallel line. The distal line is the one that the transversal does not intersect. The illusion is determined by the amount of deviation that the subject's response is from the point where the transversal would actually intersect on the distal line.

## Comparing Psychophysical Methods

In trying to establish the authenticity of experimental findings, researchers try to cross-validate results. One way that this has been attempted is by comparing results obtained with different psychophysical
methods. If different methods produce similar findings then it would decrease the likelihood that the data is an artifact of the method. Two studies that have been concerned with this issue are the ones performed by Weintraub and Krantz (1971) and Ellis and Deregowski (1972.

In the experiment performed by Weintraub and Krantz (1971) the method of adjustment and the up-and-down method were compared. The target presented to the observer had the variable transversal replaced by a dot. For the up-and-down judgment the experimenter placed a variable dot along one of the parallel lines. The subject was required to indicate if the dot was placed too high or too low to be on the continuation of the transversal. The method of adjustment trial then followed. If the subject had previously indicated that the dot was placed too low (high) the dot was placed lower (higher). The subject then moved the dot to the place where it appeared to be on the continuation of the transversal.

The results from the study indicated that the means were very similar under six different conditions but the standard errors differed considerably. They were both smaller and more reliable for the method of adjustment than for the upmand-down method. The authors noted that there were at least two possible explanations for the differences. First, the up-and-down method may be a less reliable measure of the variance of the population. Secondly, since the method of adjustment secured a response on every trial, it gave more statistical information which could lead to a smaller standard error.

Ellis and Deregowski (1972) compared responses between the method of adjustment and the method of selection. Subjects in the method of adjustment condition were required to adjust a variable transversal so
that it appeared collineax to the fixed transversal. Subjects in the method of selection task were presented a large card containing a number of Poggendorff figures with the variable transversal positioned at different places along the parallel. Subjects were required to choose the figure in which the two transversals appeared collinear. The results indicated that the correlation between the two conditions was nonsignificant which means that the subjects did not perform similarly in the two tasks.

Results from these two studies indicate that the data may depend upon the type of method that is used and this suggests that research must be performed to determine why differences occur between methods. As we shall see further on, the methodology may be a factor in the discrepancy among certain findings.

## The Choice of the Dependent Variable

One basic question concerning research with the Poggendorff illusion is the choice of the dependent variable. Traditionally, researchers have used the magnitude of linear displacement between the points of objective and subjective collinearity. In Figure 3 it is the distance b c. However, Pressey and Sweeney (1971) have argued that angular distortion could also be considered as an appropriate dependent variable. In . Figure 3 it is the acute angle labelled. The angle is defined by the point of objective collinearity, the point of intersection of the fixed transversal and the parallel line that it intersects, and the point of subjective collinearity.

Although the issue of the appropriate dependent variable has received little attention in the literature, it appears to be important


Figure 3. Angle a is the measure of angular distortion and distance $B C$ is the measure of linear displacement.
since there are at least two instances where the choice does make a difference. This occurs when the size of acute angle is varied (Robinson, 1972) and when the distance between parallels is manipulated (Pressey \& Sweeney, 1972). In later sections; we will find that differences in the interpxetation of experimental findings may result from the choice of the dependent variable.

Determinants of the Poggendorff Illusion
In order to develop and evaluate theories of the Poggendorff illusion, researchers have manipulated variables which affect the size of the illusion. The adequacy of these theories depend upon how well they explain the effects of these variables.

For ease of presentation, the discussion will first examine the effects of variables which comprise the figure. They include distance between parallels, size of acute angle, and impoverished forms of the traditional figure. The variables which have manipulated some aspect of viewing the figure will then be reviewed. These variables include orientation, tilt, fixation versus free inspection, viewing distance, and stereoscopic presentation.

## Distance Between Parallels

The variable of the distance between parallel lines has been extensively studied (Cameron \& Steele, 1905; Hill, 1971, 1974; Pressey \& Sweeney, 1972; Tong \& Weintraub, 1974; Velinsky, 1925; Weintraub \& Krantz, 1971). All the studies are in agreement if the dependent variable is measured in linear displacement. The illusion increases with an increase in distance between transversals.

This finding, however, could merely reflect a mathematical relationship which shows that as the length of two lines increases the distance between the lines increases. Fox example, consider Figure 4 in which $A B$ and $A C$ are increased in length to $D$ and $E$, respectively. Although the distance $D E$ is greater than $B C$ the difference in length is due solely to a mathematical relationship. If the increased illusion found in previous studies was due to the mathematical relationship then increasing the distance between the parallel lines would provide little information about the determining factors of the illusion. Pressey and Sweeney (1972) examined this issue by using angular distortion as their dependent variable. If the effects of increasing distance between parallels was merely a linear function when the dependent variable was linear displacement, then the amount of angular distortion should remain constant when the distance increased. Their results indicated that angular distortion also increased as the distance between parallels increased.

In order to provide comparisons with the data presented by Pressey and Sweeney (1972) this author has converted results from previous studies that presented theix findings in linear displacement. The converted data are presented in Appendix A. The majority of studies demonstrate similar trends (Cameron \& Steele, 1905; Hill, 1971; 1974; Tong \& Weintraub, 1974). Unpublished studies in which this author has participated also demonstrate the trend reported by Pressey and Sweeney (1972). Two studies (Velinsky, 1925; Weintraub \& Krantz, 1971) have presented results which conflict with the above studies. Their results may be due to the experimental procedure used. In the study performed by


Figure 4. If $A B$ and $A C$ are increased in length to include $B D$ and $C E$, respectively, the distance $D E$ is greater than $B C$.

Velinsky (1925) each subject was tested on 45 targets. It is plausible: that the effect was "masked out" by the large number of trials. In the study performed by Weintraub and Krantz (1971), the up-and-down method was used. Each subject made one judgment of the variable transversal being too high or too low and then was replaced by another subject. It may be that this procedure produces unreliable results. This interpretation has some creduality since Weintraub (Tong \& Weintraub, 1974) has concluded after an extensive series of experiments that there is not a linear increase in illusion when the dependent variable is linear displacement . The results of the studies discussed above clearly indicate that the amount of angular distortion does increase with increasing distance between parallels. This conclusion is important. As we shall see, certain theories predict that the amount of angular distortion should remain constant.

## Size of Acute Angle

At least six studies have examined the effects of changing the size of the acute angle formed between the transversals and the parallel lines (Cameron \& Steele, 1905; Novak, 1966; Velinsky, 1925; Wagner, 1969;

Walker, 1973; Weintraub \& Krantz, 1971). The conclusion derived from these studies depends upon the choice of the dependent variable. The majority of studies have reported their results in linear displacement. They demonstrate a consistent decrease in illusion as the acute angle increases.

Wagnex (1969), however, reported his results in angular distortion and found an inverted $U$-function; i.e., there was an increase and
then a decrease in illusion as the acute angle increased. In order to determine if the trend reported by Wagner (1969) was reliable, this author converted the data from the studies mentioned above (see Appendix A). They all demonstrate the same general inverted U-function reported by Wagner (1969). Although there are some discrepancies as to which size of angle produces the greatest illusion, the inverted U-shaped function appears well established.

Impovexished Forms of the Poggendorff Illusion
One question that has seemed to invoke the curiosity of researchers is "What happens if I impoverish the configuration by removing part of it?" Judd (1899) was the first researcher to examine impoverished forms of the Poggendorff illusion. His results indicated that the parts of the parallel lines that form obtuse angles with the transversal increase the illusion and the parts that form acute angles with the transversal decrease the illusion. In general, his results have been successfully replicated (Day, 1973a, 1973b; Greene \& Hoyle, 1964; Krantz \& Weintraub, 1973; Pressey \& Sweeney, 1969, 1971; Restle, 1969; Weintraub \& Krantz, 1971).

Studies that have examined three impoverished figures have generated considerable controversy. These are figures comprised solely of acute angles (Figure 5a), obtuse angles (Figure 5b), and transversals. (Figure 5c). © Day (1973a) and Judd (1899) presented results which indicated that the traditional illusion, although considerably reduced, still, occurs in the figure comprised only of acute angles: Results from other studies suggest that the illusion is reversed with this figure (Day, 1973a; Greene \& Hoyle, 1964; Restle, 1969) or with figures containing implied


Figure 5. Three impoverished forms of the Poggendorff illusion. A is formed by acute angles, $B$ by obtuse angles, and $C$ is the figure comprised of two transversals.
acute angles (Krantz \& Weintraub, 1973).
It would seem likely that the contradictory results may be due to a methodological factor. Studies which found a reversed illusion used a procedure in which the variable transversal was not moved continuously (Greene \& Hoyle, 1964; Restle, 1969; Krantz \& Weintraub, 1973). The experiments that found a positive illusion used the method of adjustment (Day, 1973a; Judd, 1899). This interpretation receives support from Day's study (1973a). With the method of adjustment he found a positive illusion, but with the method of detection he found a "tendency" towards a reversed illusion. Although it is evident that methodological differences do play a role between the discrepant findings, it is not clear. what aspects are involved.

Discrepant results can also be found among studies that compared the size of illusion between the traditional configuration and the impoverished form comprised of obtuse angles (Figure 5b). Two experiments found that the modified figures displayed the largest illusion (Judd, 1899; Day, Experiment I, 1973a) while two other studies found the opposite results (Day; 1973b, Experiment III, 1973a; Greene \& Hoyle, 1964). There does not appear to be any obvious reason for these differences since similar methods have been used by the researchers. It would seem that future experimentation will be required to clarify these findings.

The transversal figure shown in Figure 5 c has also produced controversial results. The majority of studies indicate that a positive illusion is present (Day, 1973a, 1973b; Goldstein \& Weintraub, 1972; Houck \& Mefferd, 1973; Judd, 1899); but the results from other studies
have not shown a consistent illusion. For example, Greene and Hoyle (1964) failed to find an illusion with the transversals orjented at $45^{\circ}$ or $180^{\circ}$ and Weintraub and Krantz (1971) found a reversed illusion with the transversals oriented at $31^{\circ}$.

The interesting generalization concerning these studies is that in all cases in which a positive illusion was found, the method of adjustment was employed. The other studies did not use a method in which the transversal was moved continuously. Therefore, it would seem that methodology may determine the direction of the illusion with this figure.

## Viewing Conditions

The remaining portion of this section will review studies that have examined some aspects of viewing the Poggendorff figure. It includes the effects of orientation, tilt, viewing distance, fixation, and stereoscopic presentation.

Orientation and tilt. Orientation of the transversal is one of the most powexful factors affecting the size of illusion (Greene \& Hoyle, 1964; Judd, 1899; Leibowitz \& Toffey, 1966; Velinsky, 1925; Weintraub \& Krantz, 1971). Both Judd (1899) and Velinsky (1925) suggested that the illusion was nonexistent when the transversal was oriented either vertically or horizontally. Subsequent studies by Greene and Hoyle (1964) and Weintraub and Krantz (1971) failed to verify Judd's and Velinksy's observations. In the experiment performed by Weintraub and Krantz (1971) it was found that, although the illusion was diminished substantially with horizontal and vertical orientations of the transversal, some illusion was still present. They also found that the effects of orientation
${ }^{1}$ The entire figure is rotated when studying the effects of orientation.
were not symmetrical, i.e., equal amounts of clockwise or anti-clockwise change of orientation from vertical or horizontal positions of the transversal did not produce equal amounts of change in the illusion.

Leibowitz and Toffey (1964) examined both the effects of tilt and orientation with the Poggendorff figure. There were four positions of orientation, viz., the parallel lines were placed horizontally, vertically, $45^{\circ}$ clockwise, or $45^{\circ}$ anti-clockwise. Since the acute angle was $45^{\circ}$, the transversals were positions either horizontally or vertically when the parallels were rotated from the vertical and horizontal positions. There were three positions of tilt, $0^{\circ}, 55^{\circ}$, and $80^{\circ}$. Leibowitz and Toffey (1966) found that at $0^{\circ}$ tilt, i.e.; with the figure perpendicular to the line of sight, the illusion was substantially smaller with the transversals oriented at the horizontal and vertical positions. Tilting the various orientations of the figure resulted in different effects. Tilting the figure with the parallels oriented vertically resulted in no change in the size of the illusion. With the parallels oriented horizontally, tilting the target from $0^{\circ}$ to $55^{\circ}$ decreased the illusion. However, increasing the tilt from $55^{\circ}$ to $80^{\circ}$ decreased the illusion. For the target with the parallels oriented $45^{\circ}$ clockwise, tilting the target increased the illusion. Finally, for the figure with the parallels. oriented $45^{\circ}$ anti-clockwise, tilting decreased the illusion.

Although Leibowitz and Toffey (1966) did not offer any explanation of their findings, Pressey (1970) explained their results on the basis. of the retinal image formed by the various targets. The explanation was based on three facts. First, the illusion diminishes as the distance between parallels decreases. Secondly, the illusion increases with a
decrease in the acute angles formed between the transversals and the parallel lines, and thirdly, systematic changes in the size of the retinal image occur as the figure is tilted backwards.

Pressey (1970) argued that as the target with the parallels in the vertical orientation is tilted backwards, the distance between the parallels and the distance defining the angle remains constant; thereFore, no change in the illusion should occur. Tilting the target with the parallels oriented horizontally results in a decrease in the distance between the parallels and this should result in a decrease in the distortion. But, at the same time, there is a decrease in the distance defining the acute angle, and this should produce an increase in the illusion. Consequently, these two effects work against each other which results in a minimal change in the magnitude of the illusion. For the target with the parallels oriented $45^{\circ}$ clockwise the result of tilting the figure is to keep the distance between the parallels constant but to reduce the distance defining the angle. Since smaller angles produce larger illusions, an increase in the distortion occurs. The reverse occurs for the figure with parallels oriented $45^{\circ}$ anti-clockwise. The distance between the parallels decreases as the target is tilted backwards but the distance defining the acute angle remains constant. Therefore, this results in a decrease in the size of the illusion since reduced distances between the parallels yield smaller illusions.

Although the effect of tilt seems to be explained satisfactorily, the effects of orientation have not been explained adequately. Velinsky (1925) assumed that eye movements were responsible for the effect, but it does not appear to be a likely explanation, as will be noted further on in the theoretical section. Judd (1899) believed that when the trans-
versal was at either a vertical or horizontal position the effects of the parallel lines were minimized. Although this may account for the decreased illusion at these two positions it does not at other orientations.

The majoxity of recent explanations have suggested that the orientation effect is a resuit of a factor independent of the illusion itself (Greene \& Hoyle, 1964; Pressey, 1971; Weintraub \& Krantz, 1971). Both Greene and Hoyle (1964) and Pressey (1971) have argued that both horizontal and vertical judgments are facilitated by a system of norms that an individual acquires through his interaction with his environment which results in a decreased illusion with the transversals at these positions. Weintraub and Krantz (1971) attempted to explain the effects of orientation through two principles. They suggested that, "(1) perceived orientation is affected only a little by the main Poggendorff varia'les..." and "(2) perceived orientations of the physically present segment of the transversal is deflected toward the vertical or horizontal axis of the visual field, whichever is closer (p. 262)." They have not provided independent support for their principles.

Although orientation has a pronounced effect on the illusion, few theorists, with the exception of Gillam (1971), have attempted to include its occurrence as a "prime" factor in the stimulus variables causing the illusion. Instead they have seemed willing to account for this effect by means of a subsidiary principle.

Viewing distance. Velinsky (1925), Hill (1971), and Tong and Weintraub (1974) have examined the effects of varying the distance between an observer and the target. Velinsky (1925) using a figure with
a distance between parallels equal to 20 mm and an acute angle of $30^{\circ}$ compared the size of the illusion with viewing distances between 30 to 35 cm with those between approximately 7 to 8 cm . He found that the shortest viewing distance produced the greatest illusion.

Hill (1971) used targets containing a $30^{\circ}$ acute angle and distances of $2.5,5$, and 10 mm between parallels. The viewing distances were 25 and 50 cm . He did not find a change in the magnitude of the illusion. Tong and Weintraub (1974) using 53.3 cm and 213.2 cm viewing distances, found results similar to those obtained by Hill (1971) with a number of targets.

Although it may appear that the studies have produced contradictory findings, systematic studies of the effect of this variable have yet to be performed. It may be that the choice of the viewing distances may be responsible for the apparent discrepancy among the studies.

Free inspection and fixation. Velinsky (1925), Novak (1966), and Houck and Mefferd (1973) compared the effects of fixation with free inspection. By fixation it is meant that an observer focuses on one particular point on the target. During free inspection an observer is permitted to view the entire figure.

Although Velinsky (1925) suggested that fixation results in a disappearance of the illusion, the experimental study by Novak (1966) refuted Velinsky's observation. In this studyr there were three viewing conditions, viz., free inspection, free inspection with a fixation cross, and fixation on the fixation cross: His results indicated that the illusion was significantly smaller in the fixation condition than in the two free inspection conditions. There was little difference
between the two free inspection conditions. Subsequent experimental results appear to support Novak's conclusions that fixation results in a decreased illusion (Houck and Mefferd, 1973).

A useful way to produce fixation is by stabilized images. This method eliminated the effects of the eye movements that fixation studies have attempted to control. A study by Evans and Marsden (1965) produced a stabilized Poggendorff figure by an after-image technique. Their results indicated that the illusion was still present. Of the 14 observers that were susceptible to the illusion during normal viewing, 11 continued to perceive the illusory effect in the after image. Thus, it would seem that, even with the elimination of the effects of eye movements, the illusion still occurs.

Stereoscopic viewing. Several researchers have pexformed studies requiring subjects to view the Poggendorff configuration through a stexeoscope (Day; 1965; Ohwaki, 1960; Schiller \& Weiner, 1962; Springbett, 1961). A stereoscope is designed to present different parts of a figure to each eye. Typically, the transversal is presented to one eye and the parallel lines to the other.

Stereoscopic presentation of targets has produced several controversies. The major one involves the illusion itself; is it present or not? Springbett (1961) was unable to find any evidence for the illusion and Ohwaki (1960) noted that less than half of his subjects were susceptible to the illusion. Both researchers claimed that the illusion was destroyed under stereoscopic presentation.

On the other hand, Day (1961) and Schiller and Weiner (1962) arrived at different conclusions. Subjects in Day's first experiment
were required to view the target both monocularly and stereoscopically. The subjects were required to indicate whether the illusion was equal under the two conditions or to name the condition that produced the greatest illusion. Day (1961) intexpreted his results to indicate that the illusion was still present under stereoscopic conditions.

Schiller and wiener (1961) attempted to control two confounding variables that previous researchers had noted; vizi, apparent depth and binocular rivalry. Presumably, these two effects could interfere with the judgments of subjects. They attempted to overcome these difficulties by presenting the stimuli for brief durations since pilot studies performed by the two authors indicated that these effects were greatly reduced with decreased viewing time. The stereoscopic viewing conditions in their experiment consisted of a short exposure time in which parts of the targets were presented twice for .11 seconds, a long exposure time of 1.5 seconds, and a successive presentation in which the transversal and the parallels were each presented for .055 seconds in rapid succession. The results showed that the illusion was present in all three conditions. Thus, with the elimination of binocular rivalry and depth effects, the illusion still occurs under stereoscopic viewing.

## The Influence of Organismic Variables

Several researchers have examined the possibility that organismic variables may influence the magnitude of the Poggendorff illusion. The variables that have generated the most study are practice, perceptual style, psychosis, and age.

Practice effects. The typical procedure used in studying practice effects is to present the stimulus target to the observer for a number of
trials and to record the response without providing knowledge of performance to the subject. Cameron and Steele (1905) found two interesting results. Firsc, they reported that the illusion decreased as a function of practice: and eventually disappeared with an extended number of trials. They also found that the effects of practice transferred to other Poggendorff figures. Pressey and Sweeney (1969) and Coren and Gixgus (1972), who used college students as subjects, also found that the illusion decreased with practice. The study by Pressey and Sweeney (1969) crossvalidated the practice effect with the method of production since the other two studies used the method of adjustment.

Although the practice effect seems to be well documented, several studies have not found a decrease with practice (Pressey \& Sweeney, 1970; Pressey, Bayer, \& Kelm, 1969; Vurpillot, 1957). In these studies, either children were used as subjects (Pressey \& Sweeney, 1970; Vurpillot, 1957) or adult schizophrenics and nurses (Pressey, Bayer \& Kelm, 1969). This suggests that a decrement will be found only in college students and trained observers (Pressey \& Sweeney, 1970).

An interesting observation made by Vurpillot (1957) has received Iittle attention in the literature. She found that the illusion increased over the first few trials. Examination of Table 1 in Cameron and Steele's (1905) axticle reveals a similar trend and Pressey, Bayer and Kelm (1969) reported similar findings in their study. This same trend is also evident in one of the conditions reported by Pressey and Sweeney (1970). The only study that does not display this initial increase is the one performed by Coren and Girgus (1972). The reason that is trend is interesting is because theories of practice effects
(e.g., Kohler \& Fishbach, 1950) suggest that there should be a consistent change in the illusion and not an initial increase followed by a decrease.

Pressey (1967) examined the relationship between field-dependence and the Poggendorff illusion. Measures of field-dependence were obtained for male and female observers using both the rod-and-frame and the embedded figures test. He found that the illusion was significantly correlated with the embedded figures test for both male and female subjects but that it was significantly correlated to the rod-and-frame test only for male observers. These results were interpreted as suggesting that the capacity to deal analytically with the visual display is related to the size of the illusion.

Schizophrenia. In the following study, Pressey, Bayer and Kelm (1969) put forward the hypothesis that if schizophrenics perceive in a more global, less articulated manner than normal observers; and if the size of the illusion is related to the capacity to deal analytically with a visual display, then schizophrenics should demonstrate a larger illusion than normal observers. The results supported their hypothesis. In two separate experiments schizophrenics displayed a larger illusion than normal observers.

The results from this study and the study performed by Pressey (1967) are interesting since they appear to indicate that observers who perceive in a less differentiated manner are more susceptible to the illusion.

Age. Several studies have examined the relationship between the age of a subject and the magnitude of the illusion. The consensus is
that there is an inverse relationship between age and size of the illusion. '(Hill, 1974; Leibowitz \& Gwozdecki, 1967; Pressey \& Sweeney, 1970; Vurpillot, 1957).

Leibowitz anơ Gwozdecki (1967) tested 16 different age groups on the Poggendorff illusion. The mean age of the groups ranged from 5 to 75. The results indicated a sharp decrease in the magnitude of illusion from the ages of 5 to 10 and then remained at a constant size between the other age groups. It would seem, therefore, that the greatest effect of age occurs in the early development of the individual.

Theories of the Poggendorff Tllusion
The study of illusions has generated numerous theories that have purported to explain either one particular type of illusion or a large number of illusions. These theories have suggested ways that illusions could occur at the physiological or cognitive level of the organism. Eye Movement Theory

Velinsky (1925), after noting the effects of orientation on the illusion, proposed a physiological explanation based on eye movements. He noted that the minimal illusion and the independent movement of the eye muscles occurred when the transversal was oriented either vertically or horizontally. Because of this correspondence he thought that the illusion may be due to the effects of eye movements. He suggested that the muscular system controlling eye movements were made up of two components, one controlling vertical movements and one controlling horizontal movements. He argued that the two systems act independently for vertical and horizontal movements and interdependently when the eye was
required to move obliquely. The illusion was thought to arise in the latter instance.

Velinsky (1925) suggested that when the observer is required to make a judgment with the Poggendorff target, the eye follows the transversal, enters the emoty space between the parallels and then attempts to take the shortest distance to the second parallel. As a consequence of taking the "shoxtcut", the eye arrives misplaced at the second para$11 e l$ and attempts to follow the second transversal at this position; thereby perceiving the two transversals as noncollinear.

Evidence reported by Evans and Marsden (1966) has demonstrated the inadequacy of this theory. In their study a stabilized. Poggendorff image was presented to 19 subjects. This procedure controls for the effects of eye movements by placing a negative afterimage on the retina. The afteximage varies as the eye moves thereby keeping the position of the figure on the same place on the retina. Of the 14 suojects who perceived the illusion in a non-stabilized condition, 11 continued to perceive it in the stabilized condition. Accordingly, it can be concluded that eye movements are not necessary for the illusion to occur. Angle Theories

A large number of theorists have suggested that the illusion is a result of distortion of the acute angle formed between the transversal and the parallel line. For example, if the acute angle is enlarged in a manner suggested by Figure 6 , the transversals would be perceived as non-collinear. Robinson (1972) has noted that this type of explanation was suggested as early as 1861 by Hering. Modern day exponents of this type of explanation have focused on mechanisms to explain how the acute angle could be enlarged or distorted.


Figure 6. If the acute angles defined by lines A and $C$ and the parallel lines appeared as large as the acute angles formed by lines $B$ and $D$ the lines $A$ and $C$ would not appear collinear.

Chiang (1968) suggested that the illusion was the result of the optical image formed on the retina. He believed that the perception of a figure was related to its corresponding distribution of energy on the retina. He argued that the aiscrimination of the energy distributions was influenced by abexrations of the lens and the diffraction of light within the eye. As a result of these factors, he suggested that the distributions of energy corresponding to the acute angle are perceived as a single distribution near the apex of the acute angle. The result of combining the energy distributions was thought to distort the figure in a manner similar to that shown in Figure 7.

This theory has been criticized repeatedly in the literature. Restle (1969) found that a figure comprised only of acute angles displayed a substantially decreased.illusion. Pressey and Sweeney (1969) demonstrated that a figure without any closed angles produced a statistically significant illusion. Robinson (1972) has noted that stuãies examining the effects of stereoscopic viewing pose a problem for chiang's theory. In these studies the transversal and the parallel lines are presenced to separate eyes thereby eliminating the retinal interaction required by Chiang's theory. With proper control for depth effects and binocular xivalry the illusion is still present (Schiller \& Wiener, 1962). Neurological Explanations

Recent findings by Hubel and Wiesel (summarized by Hubel, 1963) concerning types of cells in the visual pathway have stimulated neuro'Iogical explanations of the illusion. These theoxies all appear to be characterized by relying on distortion of the acute angle to explain the illusion. Included among these theories are the ones offered by Blake-


Figure 7. An exaggerated representation of the distortion suggested by Chiang (1968; after Restle, 1969).
more, Carpenter and Georgeson (1970) and Burns and Pritchard (1971). Blakemore et al. believed that the illusion displayed by figures similar to the Poggendorff target were the result of lateral inhibition among orientation detectors. An orientation detector is a cell in the visual cortex that responds to lines of pxeferred orientations. They argued that an oxientation detector would be excited by a narrow range of line orientations and would be inhibited (responds less frequently) by a larger band of line orientations. The presentation of a single Iine would produce a distribution of excitation among the population of orientation detectors. Those tuned to the same orientation as the line would be excited while those tuned to different orientations would be inhibited. Blakemore et al. suggested that if two lines of different orientation were presented to an observer they would influence the orientation detectors in the additive manner demonstrated in Figure 8. If the distribution of excitation does combine in a way suggested by these authors, the lines forming the acute angle in the Poggendorff illusion would be shifted apart and thus appear larger than it actually is.

Burns and Pritchard (1971) proposed a theory that makes very similar predictions to the one offered by Blakemore et al. These authors contended that the illusion was the result of the cortical image. The cortical image was defined as "a line across the primary visual cortex, defining the positions of neurons whose temporal patterns are most disturbed by the presence of a straight line of light across the retina (p. 613)." They suggested that the cortical image of a straight line is distorted when a second line is added to form an acute angle. With the


Figure 8. The suggested sumed distribution of excitation by orientation receptors when excited by two lines of different orientation (after Blakemore et al., 1970).
addition of the added line "...the image of the original line is shifted toward the image of the added line neax the tip of the angle pattern; further from the tip, the image of the original line is displaced from that of the added iine, while even further back from the tip there is no measurable distortion of the cortical image (p. 613).".

Both Blakemore et al. (1970) and Burns et al. (1971) have difficulty explaining several facts involving the illusion with their theories. Pressey and Sweeney (1970) measured the size of illusion found in modified forms of the configuration similar to the ones displayed in Figures 9 A and 9 B . According to the theories just reviewed, Figure 9 B should display the largest illusion. The results indicated that the figure containing the acute angle displayed the smallest amount of illusion. A second difficulty for these theories concerns the effect of increasing the distance between the parallel lines. They would predict that the amount of illusion, as measured in angular distortion, would be constant as the distance increases. However, experimental results demonstrate that there is an increase (Pressey and Sweeney, 1972). Depth Processing Theories

Traditionally, inappropriate depth processing has been offered as an explanation of the illusion. One of the clearest statements of the theory has been presented by Greene and Hoyle (1964). They suggested that the Poggendorff illusion could be found in the geometry of a room (see Figure 10). They suggest that in one sense "perceptually, the continuation of lines $A B$ and $X Y$ are $B C$ and $Y Z$. But when an observer is required to view the Poggendorff display the observer has a tendency "to continue $A B$ towards $C$ instead of $Y_{7}$ and continue $X Y$ towards $Z$ instead


Figure 9. Example of the targets used by Pressey and Sweeney (1970).


Figure 10. The Poggendorff illusion in a room (After Greene $\&$ Hoyle, 1964).
of $B$. As a result of this process a perceptual compromise is made whereby BP and YQ appear to be the required continuation (p.357)."

Gillam (1971) has provided some support for an inappropriate depth-processing explanation. In several experiments she modified apparent depth and found that the illusion was altered as predicted. However; she has also mentioned two predictions that have not been supported. One of these involves changing the distance between the parallel lines. Depth-processing theory suggests that no change in the magnitude of the angular illusion should ocur as the distance increases. As previously mentioned, the evidence indicates that it does increase (Pressey \& Sweeney, 1972). A second prediction concexns the effect of the orientation of the transversal. The prediction is that no illusion should occur with the transversal oriented either horizontally or vextically. The evidence suggests that it is present (Leibowitz \& Toffey, 1966; Weintraub \& Krantz, 1971).

## Distance Misinterpretation Theories

Two theories have attempted to account fox the poggendorff illusion by suggesting that a misinterpretation of distance determines the illusion. These are the explanations offered by Judd (1899) and Pressey (1970, 1972) and Pressey and Sweeney (1972). Judd (1899) argued that both the distance along the parallels and the distance between the transversals are perceived incorrectly while Pressey suggests that the distance between the transversals is underestimated.

Judd (1899) noted several phenomena that provided support for his theory. First, the intersection of an oblique line across a horizontal line results in the point of intersection being shifted towards the
direction of the intersecting oblique line. For example, in Figure 11 the oblique line intersects the horizontal line at its midpoint, but the distance on the obtuse angle side of the horizontal Ine appears greater than the distance on the acute angle side. In applying this effect to the Poggendorff figure it would mean that the transversals are shifted apart. The second effect received support from experimentation that examined the magnitude of illusion demonstrated by impoverished forms of the Poggendorff figure. His results indicated that the parts of the parallel lines that formed acute angles with the transversal displayed a smaller illusion than the parts forming obtuse angles with the transversal. He noted that the parts of the figure forming acute angles correspond to the outgoing form of the Mlller-wyer illusion and the parts of the figure forming obtuse angles was similax to the ingoing form of the Muller-Lyer illusion. The outgoing form of the Mbller-lyer illusion is associated with the overestimation of distances and the . ingoing form. is associated with the underestimation of distances. Juda (1899) believed that since underestimation was greatec than overestimation in this instance, the distance between the transversals was underestimated in the complete figure.

Judd (1899) attempted to explain these effects by his movement hypothesis. He suggested that with a greater sensation of movement that there would be an overestimation of distance and with a lesser sensation of movenent there would be an underestimation of distance. He thought that if the tendencies of movement were outward, then an outward movement would result in an underestimation of distance, but if an inward movement occurred it would result in an overestimation of the distance.


Figure 11. Distance $B C$ appears larger than the distance $A B$ although both are objectively equal.

He suggested that if there was an inward tendency the point where these tendencies develop will be shifted inward and if there was an outward tendency they would be shifted outward. The direction that these tendencies were supposed to travel depended upon the direction of attention. He explained this concept, thus: "By this. we mean to indicate that the particular direction of movement in any case depends on the relation of that part of space which is subjectively the most important to all other parts".

Judd's (1899) theory can be criticized. The process that Judd believed to account for the illusion is not required. His movement hypothesis requires eye movements and this is not a necessary factor in the illusion (Evans \& Marsden, 1966). In addition, Piecce (1901) found that the distance between the transversals is overestimated. It would seem, thexegore, that Judd's (1899) explanation of the illusion is not tenable.

Pressey (1971, 1972), Pressey and Sweeney, 1972) suggested that assimilation theory could explain the illusion. The theory is based upon three postulates, viz., the assimilation postulate, the attentive field postulate, and the range postulate. The assimilation postulate states that smallex magnitudes in a series of judged magnitudes will be overestimated and larger magnitudes in a series will be underestimated. The second postulate suggests that the closer a contextual magnitude is to the center of the attentive field the more effective it will be in producing an assimilative effect. An attentive field is assumed to be an approximately cjxcular region from which an observer is processing information. The range postulate states that with an increase in the range of
contextual magnitudes there will be an increase in the assimilative effect.

In applying this theory to the Poggendorff illusion, it is assumed that an observer projects a series of lines from the transversal to the distal parallel line. The way this is thought to occur is demonstrated for a simplified version of the illusion in Figure 12. The theory suggests that since the average length of projections occurring within the attentive field is shorter than the standard distance, the observer chooses a projection shorter than the distance between the transversals, thus resulting in the illusion.

The theory has success in explaining the effects of manipulation of the stimulus variables comprising the figure. The range postulate has been used by Pressey (1972) to explain two functions viz., the increase in the illusion as the distance between the parallel lines increase and the decrease in illusion as the acute angle increases.

The theory also explains the results obtained in an experiment performed by Weintraub and Krantz (1971). In this experiment the effect of rotating one transversal while keeping the second transversal constant was examined. The size of the acute angle formed between the non-rotated transversal and the parallel was kept constant at $16.7^{\circ}$. The size of the angle formed by the rotated transversal varied from $0^{\circ}$ to $130^{\circ}$. Subjects were required to position the part of the rotated transversal that intersected with the parallel line so that it appeared collinear with the non-rotated transversal. The results indicated that angles at $0^{\circ}$ or larger than the fixed angle produced larger illusions. Pressey and Sweeney (1972) argued that the addition of a transversal

[^0]

Figure 12: According to assimilation theory an observer projects a sexies of lines from the transversal to the distal Iine when judging where the transversal would intersect on the vertical line.
would result in additional lines being projected on the added transversal. Since at $0^{\circ}$ of transversal would be collinear with the parallel line in the figure, the effect of rotating the transvexsal a snall amount from the parallel would produce projections on the transvexsal longer than the distance between the transversal, thus resulting in a smaller illusion. When the transversal was rotated a large amount the effect would be to produce projections shorter than the distance between the transversals, thus increasing the illusion relative to the one found at $0^{\circ}$. On these grounds, Pressey and Sweeney (1972) argued that the magnitude of illusion should decrease and then increase as the transversal is rotated from collinearity with the parallel line. This is the function found by Weintraub and Krantz (1971).

Pressey's theory has received very little criticism in the litexature. Since the theory is based on cognitive activity, it is not contradicted by the experiments performed with stabilized images (Evans \& Marsden, 1966), and with stereoscopic viewing (Schillex \& Wiener, 1962). Further experimentation is necessary in order to decide if this theory does provide an adequate explanation of the illusion.

STATEMENT OF THE PROBLEM

Of all the theories that have been reviewed, assimilation theory (Pressey, 1971) seems to be most capable of providing a satisfactory explanation of the Poggendorff illusion. However, the theory is subject to criticisin by the resuits of a study performed by pierce (1901). In this experiment two obsexvers were required to judge the distance between the transversals. The results indicated that the distance was overestimated, i.e., the results were opposite to what would be predicted from assimilation theory.

Although Fierce's (1901) study appears to refute assimilation theory's interpretation of the illusion there is an important factor that should be considered. The subjects in the study made their judsments by adjusting the length of a line while they were observing the stimulus figure. Since the midale of the attentive field is assumed to correspond to the center of the most distant points between the standard. and the contextual magnitudes (Pressey \& Bross, 1973) the positioning of the comparison magnitude may have played an important role in determining Pierce's results. Since the position of the comparison line is not stated in Pierce's study, it may have been placed in a manner such that the position of the attentive field was altered to emphasize contextual magnitudes longer than the distance between the transvexsals. For example, if the comparison line was positioned as shown in Figure 13 , contextual magnitudes longer than the distance between the transversals would be produced. If this did occur, then Pierce's results do not


Figure 13. By placing the comparison line A below the parallel lines contextual magnitudes longer than the distance between the transversals would be produced.
reflect upon the adequacy of the assimilation theory of the Poggendorfe illusion.

## Major Aim

The main purpose of this study is to replicate pierce's experiment with the position of the attentive field controlled. In order to provide the appropriate set of conditions for testing the basic prediction of assimilation theory, the attentive field must be the same when both the distance between the transversals are judged and when the normal Poggendorff task is performed. If an observer makes his response of the distance between transversals while viewing the stimulus target, his attentive field will be altered to include the comparison magnitude. Thjs suggests that a subject should not make his response when he is viewing the stimulus figure. One way to accomplish this is by a method of successive production. This method would involve showing the subject the standard distance in the Poggendoxff display, removing the figure, and then requiring the subject to produce the distance between the transvexsals. By using this procedure the center of the attentive field would be the same in both the normal Poggendorff task and in the task that requires the subject to estimate the distance between the transversals.

With the same attentive field in both a normal Poggendorff task and in a task that requires the subject to estimate the distance between the transversals, it is predicted from assimilation theory that the distance between the transversals will be underestimated:

## Minor Aims

A subsidiary purpose of this study is to examine a second predic-
tion that can be dexived from assimilation theory. The second prediction is that if the distance between the transversals is underestimated then the magnitude of underestimation should vary as the magnitude of . the illusion varies. There are two ways to test this prediction, viz., by comparing functions and by correlation. From previous studies (e.g. . Velinsky, 1925) it is known that the magnitude of illusion decresses as the acute angle formed between the transversal and the parallel line increases. Assimilation theory would predict that the magnitude of undexestimation of the distance between transvexsals would also decrease as the size of the acute angle increased. This study will vary the sjze of the acute angle in order to determine if this prediction is upheld. The second way that the same hypothesis can be evaluated is by correlating subjects" responses in the normal Poggendorff task to their response in the task requiring judgments of the inter-transversal distance. According to assimilation theory the size of attentive fields is an organismic variable (Pressey, Bayer \& Scrivner, 1971) and thus should vary between individuals. If the size of attentive fields affects the size of the Poggendorff illusion and the perceived distance between the transversals then it would be predicted from assimilation theory that there should be a positive correlation between these two tasks. Summary of the statement of the Problem

The experiment described below is designed to investigate three predictions derived from assimilation theory. They are (1) that subjects will underestimate the inter-transversal distance in the Poggendorff illusion, (2) that the magnitude of underestimation and the magnitude of the Poggendorff will vary in a similar manner, and (3) that the degree
of underestimation and poggendorff illusion will be systematically related. If these predictions are supported by the experiment it would provide unqualified support for assimilation theory.

## Method

## Subjects

The subjects in the experiment consisted of introductory psychology students enrolled at the University of Manitoba. Only those with good vision, eithex with or without glasses, were asked to participate. Ten of the 190 subjects were eliminated from the experiment because they failed to follow instructions. Experimental Design

The experiment used a $3 \times 5$ mixed design. Tbere were 5 levels of the between-subject faction (angle), and 3 levels of the within-subject factor (the task requixed of the subject).

Tasks
The method of production (Pressey \& Sweeney, 1969) was employed in the Poggendorff task. In this task the subject was presented with a figure comprised of two vertical lines and a transversal that intersected one of the parallel lines (Figure $14 B$ ). The subject was required to judge where the transversal, if extended, would appear to intersect on the distal parallel line.

A method of successive production was used in the Poggendorfy distance täsk. In this task subjects were required to estimate the inter-transversal distance. They were shown a figure identical to the one used in the poggendorff task but with a dot placed on the point where the transversal, if objectively extended, would intersect on the
distal paxallel line (figure l4A). The target was then removed and replaced with a response sheet (Eigure 14D) on which the subject made his response.

The method of successive production used in the Poggendorff distance task is similar to a procedure used by Taylor (1961). He showed subjects a line, xemoved the line, and then required subjects to reproduce the length of the line. He found that lines between one and two inches were underestimated while lines between two and five inches were overestimated. These results suggested that a control task should be employed in the present experiment in order to control for possible differential effects of this procedure.

In the control task subjects were shown a figure consisting of two dots. The placement of the dots coincided with the positions of the dot and the point of intersection of the transversal with the left vextical line in the figure used in the Poggendorff distance task. The subjects in this task were required to juage the distance between . the pair of dots (see Figure I4C) and then produce the judged distance on the response sheet.

A study by Shipley, Nann, and Penfield. (1949) and the results of experiment $V$ in Taylor's study were taken into account when considexing the response in the distance estimation tasks. Shipley et al. (1949) found that the percejved length of lines varied as a function of orientation. Taylor's results indicated that the perceived orientation of oblique lines varied with successive reproduction although horizontal and vertical lines did not. Since the perceived orientation of Iines vaxy as a function of successive reproduction and because the
perceived length of lines change as orientation changes it was decided that subjects would make their responses horizontally in the two distance estimation tasks.

Targets
The targets were modified Poggendorff configurations consisting of two 160 man long, parallel, vertical lines placed 30 mm apart. The transversal was 40 mm long and intexsected the left vertical line 40 mu from its tip. The size of the acute angle formed between the transversal and the left vertical line was $20^{\circ}, 30^{\circ}, 40^{\circ}$, $50^{\circ}$, or $60^{\circ}$.

The dots in the targets for the Poggendorfi distance task, the control task, and the response sheets in the distance estimation tasks were 2 mm in diameter. The dot on the response sheet was placed so that when a subject produced his response of the judged distance, the xesponse would be placed through the center of the "hypothesized" attentive field. The placement of the dot on the response sheet was also determined by the distance to be judged and the size of the response sheet. An equal amount of horizontal distance was present on either side of the objective distance on each response sheet.

For experimental purposes all targets were reproduced from master drawings of the figures. The targets were centered on white: sheets of paper 21.5 cm wide and 28 cm . long. The figures were black contours .4 mm wide.

Apparatus
A white target holder was the only equipment used in the experi-


Figure 14. The targets used in the experiment.
ment. The target holdex was 385 mm high and 375 mm wide, and the part on which the target was located was tilted backwards $20^{\circ}$. A chin rest was attached to the front of the holder so that viewing distance was kept constant at about 410 mm .

## Procedure

Subjects, when they entered the experimental room, were randomly assigned to one of the five levels of the between-subject factor This resulted in 36 subjects being assigned to each group. The three tasks were then explained to the subject (see Appendix B for the complete set of instructions). The order of the explanation of the three tasks was counterbalanced between subjects.

For the Poggendorff task subjects were informed that they were required to judge where the transversal would intexsect on the right vextical line. They were asked to make their estimate by drawing a small. dot on the vertical line at the point where the transversal, in conm tinued would appear to intersect the Iine. A dummy target was usea as an aid in explaining the task.

In the Poggendorfi distance task subjects were told that they were required to judge the distance between the dot and the point where the transversal and the left vertical line intersected. These two positions were demonstrated on a dumy target. Theix estimate of the judged distance was to be performed on a second sheet of paper that contained only one dot. Their estimate was to be accomplished by drawing a second dot directly to the right of the fixst dot so that the distance between the dot they drew and the dot already present appeared equal to the judged distance.

Whe instructions for the control task was identical to the instructions for the Poggendorff distance task with two exceptions. The distance to be judged was the distance between the pair of dots and the dumay target used for demonstration purposes consisted of two dots.

Subjects wexe tested two times on each of the three tasks. The order that the tasks were performed was counterbalanced between subjects of the same group. During the testing each subject was required to view the target for 5 seconds before making his response. Subjects made their responses in the Poggendorff task on the paper containing the stimulus figure. For the tasks requiring distance estimation, subjects made their responses on the response sheet which was provided after the stimulus figure was viewed.

Measurement of Responses
Responses were measured with a straight edge scale to the nearest millimeter. In the Poggendorff task (PT) the measured response was the distance between the point of intersection of the transversal with the parallel line and the placement of the dot on the distal parallel line. The measured response in the control (C) and Poggendorff distance task (PDT) was the distance between the dot the subject produced and the dot already provided on the response sheet. Appendix $C$ presents the raw scores for individual subjects in each of the chree tasks.

## Reliability of Methods

In order to determine the reliability of the methods used in the experiment, Pearson product-moment coefficients were calculated by computing correlation coefficients between trial 1 and trial 2 for each of the three tasks within each group of subjects. The results of these calculations are sumarized in Table 1 . This Table would seem to indicate that the methods used in the experiment produce reliable results.

Raw Scores.
Table 2 presents the mean raw scores $(\bar{X})$ and the standard deviation (SD) for the PT, PDT, and the $C$ tasks. This Table shows that as the objective distance decreased the mean raw score in each of the three tasks also decreased.

## TABLE 1

Reliability Coefficients for the Poggendorff, Poggendorff Distance, and Control Tasks

| Angle | Poggendorff | Poggendorff <br> Distance | Control |
| :---: | :---: | :---: | :---: |
| 20 | .525 | .728 | .437 |
| 30 | .669 | .629 | .727 |
| 40 | .856 | .643 | .123 |
| 50 | .559 | .662 | .316 |
| 60 | .362 |  | .531 |

Required level of $r$ to be statistically significant for two-tailed tests. . 05 probability level .329 ; .01 probability level .424; . O01 probabilicy level .524.

## TABLE 2

Mean Raw Scores and Standard Deviations for the Poggendorff, Poggendorff Distance and Control Tasks

| Angle | Objective Distance | Poggendorff |  | Poggendorff Distance |  | Control |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | X | SD | $\bar{X}$ | SD | $\overline{\mathrm{X}}$ | SD |
| 20 | 87.5 | 67.29 | 10.84 | 94.11 | 19.03 | 85.81 | 13.49 |
| 30 | 59.5 | 47.19 | 6.08 | 64.33 | 13.20 | 58.44 | 9.59 |
| 40 | 46.5 | 39.00 | 3.64 | 48.63 | 9.61 | 47.04 | 6.68 |
| 50 | 39.0 | 34.29 | 1.35 | 40.58 | 6.53 | 39.46 | 3.97 |
| 60 | 34.5 | 32.19 | . 84 | 36.19 | 3.83 | 34.62 | 3.07 |

Poggendorff Distance and Poggendorff Illusion Juagments
The effect of judging the distance in the Poggendorff figure was determined by. subtracting PDr responses from C xesponses. The computational formula was $C-P D \dot{A}=P D$ (Poggendorff distance). A measure analogous to $P D$ was provided for the Poggendorif illusion by subtracting PT from the objective distance. The computational formula was objective distance - $P T=P I$ (Poggendorff illusion).

Figure 15 depicts the mean PI and PD values as a function of the size of the acute angle. The figuxe indicates that Pr is a positive vaiue and that $P D$ is a negative value. This means that when subjects performed the Poggendorif task the length of the response was less than the objective distance. However, when subjects estimated the objective distance in the Poggendorff figure, the distance was overestimated,

Figure 15 also shows the relationship between PI and PD as a function of acute angle size. It indicates that for $P$ f the amount of underestimation decreased as the acute angle increased. For PD the amount of overestimation decreased as the acute angle increased. This indicates that performance in the two tasks did not vary in a similar manner as the size of the acute angle varied. The Relationship Between PI and PD

Pearson product-moment correlation coefficients (Hays, 1973) were computed to provide an index of the relationship between PI and PD. These calculations are summarized in rable 3. Since the magnitude of the correlation coefficients failed to reach the . 05 level of confidence and since they are vaxiable in sign it cannot be concluded


Figure 15. Magnitude of underestimation in the Poggendorff (PI) and Poggendorff
Distance (PD) tasks.
that there is systematic relationships between the two tasks. Inspection of frequency distributions of the PI and PD responses (see Appendix D) indicated that the distribution of the responses for the two tasks were dissimilar. Since dissimilar distribution of responses can attenuate Pearson product-moment correlation coefficients (Gorsh, 1974), rank-order correlations (Runyon \& Haber, 1968) between PI and PD were computed. These procedures also indicated that the size of the correlation coefficients failed to attain significance at the .05 level of confidence and that the sign of the coefficients were variable (see Table 3). It would seem, then, that this analysis also failed to find a statistically significant relationship between the two tasks.

## TABLE 3

Magnitude of Correlation Between Poggendorff and Poggendorff Distance Judgments

|  | Product-Moment ${ }^{\text {I }}$ | Rank-Order |
| :---: | :---: | :---: |
| Angle |  |  |
| 20 | . 090 | .117 |
| 30 | $-.144$ | -. 126 |
| 40 | -. 011 | . 027 |
| 50 | . 176 | . 087 |
| 60 | -. 265 | -. 217 |
|  |  |  |
| ${ }^{1}$ Required product-moment correlation coefficients for |  |  |
| statistical significance. |  | ility level <br> ility level <br> bility level |
| ${ }^{2}$ In order to obtain significance at the same level of prob- |  |  |
| ability for the same degrees of freedom the magnitude of |  |  |
| rank-order correlation coefficients must be greater than |  |  |
| the product moment correlation coefficient. (c.f., Tables |  |  |
| $F$ and G, pp.259, in Runyon and Haber, 1968.) |  |  |

Assimilation Theory
This study failed to support any of the three predictions made by assimilation theory. Contrary to the predictions the intortransversal distance was overestimated; performance in the Poggendorff and. Poggendorff distance task did not vary in a similar manner as the size of the acute angle changed; and performance in the two experi-mental tasks was not systematically related.

The failure to support the predictions has at least two important implications for assimilation theory. The first imolication concerns the explanation of the illusion. According to assimilation theory, the illusion results from the underestimation of the intertransversal distance. However, these results support Pierce's (1901) findings that inter-transvexsal distance is overestinated. It would seem that the most basjc aspect of the explanation is not supported when it is put to an empirical test.

The second implication involves the attentive field construct. The size of an attentive field is postulated to be an organismic variable and thus should vary between individuals. If performance in the Poggendorff and Poggendorff distance task is affected by the size of attentive fields and if the size of attentive fields are an organismic variable then performance in the two tasks should be systematically related. The failure to find a systematic relationship between the two tasks suggests that the predictive power of this
construct may be limited.

Alternative Distance Explanations

Examination of Figure 15 suggests that apparent distance is involved in the Poggendorfe illusion. There are two ways that it could play a role. For example, the illusion could be a result of pexceiving the objective distance as too long. rhe second way is that the inter-transversal distance is perceived as greater than the objective distance as a result of some other process that determines the illusion.

The former explanation appears to be logically incorrect. If the placement of the transvexsals depends upon apparent distance and if apparent distance is greater than the objective distance chen the logical conclusion is that the transversals would be placed too far apart when they are arranged to appear collinear. In other words, this explanation would predict an illusion in a direction opposite to the classical finding.

The second suggestion, that the inter-transversal distance appears greater than the objective distance as a result of some other process that determines the illusion, could be more feasible. Part of Judd"s (1899) theory serves as an example of this type of explanation. He argued that the transversals were displaced away from each other so that distances or the acute angle side of the parallel line were shortened while distances on the obtuse angle side were lengthened. If the transversals were displaced in this manner then the intertransvexsal distance would appear greater than its objective distance since the transversals would be further apart.

Experimentation performed by Weintraub and Krantz (1971) does not support this explanation. They examined the effects of rotating the variable transversal while maintaining the fixed transversal at a fixed position. The size of the angle formed between the rotated transversal and the parallel line varied from $0^{\circ}$ to $130^{\circ}$ (see Figure 16). From Judd's explanation it would be predicted that as the angle changed from an acute angle to an obtuse angle that the illusion would decrease since the variable transversal would be displaced closer to the fixed transversal. However, the results indicated that the illusion was increased instead of diminished.

The Role of Apparent Distance
What is the role of apparent distance in the Poggendorff illusion? None of the theories, which has been discussed, can explain both the illusion and the overestimation of the intertransversal distance. As well as the lack of empirical support for these theories, the failure to find a systematic relationship between the Poggendorff and Poggendorff distance task also suggests that apparent distance does not play a role in the illusion. At this time it must be concluded that there are some serious doubts in providing a satisfactory explanation of the Poggendorff illusion on the basis of apparent distance.

Suggestions for Future Research
The Inter-Transversal Distance
One problem for future research is to determine why the intertransversal distance was overestimated. The task that demonstrated this effect, the Poggendorff distance, is the estimation of the distance


Figure 16. A is the range of angles used in the experiment performed by Weintraub and Krantz (1971).
between two points contained on a pair of parallel lines. Conceptually, the Poggendorff distance task is similax to an experiment rem ported by Piaget (1969). Subjects were required to estimate the length of the diagonal line contained in a rectangle. Therefore, in both Piaget's and the present study subjects were required to estimate distances contained between parallel lines. In Piaget's study the distance was underestimated and not overestimated as in this study. Although the taxgets used in the two studies differ in a mumber of respects, it is these differences that probably produced the change in the direction of the effect, i.e., overestimation versus underestimation. It may prove to be important to find out which part(s) of the figures determine the difference in results.

The Method of Successive Production
The method of successive production proved to be a reliable procedure in the present experiment. rhe reliability of the technigue is indicated by the magnitude of the reliability coefficients for the Poggendorff distance and the control tasks. This method should be useful when researchers do not wish subjects to produce their xesponse at the same time the stimulus to be judged is being viewed.

Summary and Conclusions
The present study examined apparent distance in the Foggendorff illusion in order to evaluate three pxedictions derived from assimilation theory. Contrary to the predictions the intex-transversal distance was overestimated, apparent distance and magnitude of illusion did not vary in a similar manner as angle size changed, and a systematic relationship between apparent distance and magnitude of

Poggendorfe illusion was not found. Since the inter-transversal. distance was overestimated it was concluded that assimilation theory does not provide an adequate explanation of the illusion. It was also -suggested that the failure to find a systematic relationship between apparent distance and magnitude of illusion indicated that the predictive power of attentive fields, an important construct in assimilation theory, may be limited.

TWo alternative explanations of the role of apparent distance in the Poggendorff illusion were considered. One was rejected on logical grounds and the second was rejected on the basis of expeximentation percomed by Weintraub and Krantz (1971). It was argued that there were some serious doubts to providing a satisfactory explanation of the Poggendorff illusion on the basis of apparent distance.

The results from judging the inter-transversal distance were compared to a study in which the length of a diagonal line in rectangle was estimated (Piaget, 1969).. Although both tasks were conceptually similar, the results indicated that overestimation occurred with the inter-transversal distance judgment and underestimation in the judgment of the diagonal line. It was suggested that systematic investigation of the differences in the targets used in the two studies may be important in detexmining the factors which produce overestimation in one instance and underestimation in the other..

Finally, it was noted that the method of successive production produced reliable results. This method should be useful in research when investigators do not wish subjects to produce their judgments at the same time they are viewing the stimulus to be judged.

## References

Blakemore, C., Garpenter, R.H., \& Georgeson, N.A. Lateral inhibition between orientation detectors in the human visual system. Nature, 1970, 228, 37-39.

Boring, $E$.G. Sensation and pecception in the history of experimental psychology. New York: Appleton-Century Crofts, 194?.

Burns, B.D., \& Pritchard, R. Geometrical illusions and the response of neurons in the cat's visual cortex to angle patterns. Joumal of Physiology, 1971, 213, 599-618.

Cameron, E.H., \& Steele; W.M. The Poggendorff illusion. Psychological Review, 1905, Monograph supplement. 7, 83-111.

Chiang, C.A. A new theory to explain geometrical illusions produced by crossing lines. perception and Psychophysics, 1968, 3, 174-176.

Goren. S.s. Girgis, J. Illusion decrement in intersecting line figures. . Psychonomic Science, 1972, 26, 108-110.

Day, R.H. On the stereoscopic observation of geometric illusions. PercepEual and Motor Skil1s, 1961, 13, 247-258:

Day, R.F. The Poggendorff illusion with obtuse and acute angles. Perception axid Psychophysics, 1973, 14, 590-596.

Day, R.H. The oblique line illusion: The Poggendorff effect without parallels. Quarterly Journal of Experimental Psychology, 1973, $25,535-54$.

Ellis, H., \& Deregowski, J. Note on methods used to determine magnitudes of geometric illusions. Perceptual and Motor Skil1s, 1972, 34, 50.

Evans, C.R., \& Marsden, R.P. A study of the effect of perfect retinal stabilation on some well-known visual illusions, using the afterimage as a method of compensating for eye-movements. British Journal of Physiological optics, 1966, 23, 242-248.

Gillam, B. A depth processing theory of the Poggendorff illusion. Perception and Psychophysics, 1971, 10, 211-216.

Goldstein, MoB., \& Weintraub, D.J. The parallel-1ess Poggendorff: Virtual contours put the illusion down but not out. Perception and Psychophysics, 1972, 11, 353-355.

Gorsch, R.L. Factor analysis. Toronto: W.B. Saunders, 1974.
Greene, R.T., \& Hoyle, I.M. The influence of spatial orientation on the Poggendorff illusion. Acta Psychologica, $1964,22,348-365$.

Hays, W.H.L. Statistics for the social sciences. New York: Folt, Rinehatt \& Winston, 1973.

Hill, A. 1. Poggendorff illusion: Effects of intelligence, viewing distance and space between the vertical lines. Psychonomic Science, 1971, 25, 71-72.

Hill, A.L. Examination of Pressey's assimilation theory of the Poggendorff illusion. Pexceptual and Motor Skills, 1974, 38, 27-35.

Houck, R,L., \& Mefford, R.B. The Poggendorff illusion: A neurophysiological based hypothesis. Amexican Journal of Psychology, 1974, 86, 283-309.

Hubals D.A. The visual cortex of the brain. Reprint from the Scientific American, Nov., 1973.

Johannsen, D.E. Early history of the perceptual illusion. Joumal of the Bistory of the Behayioral Sciences, 1971, 7, 127-140.

Judd, C.H. A study of geometrical illusions. Psychological Review, 1899, $\quad 241-261$.

Kohler, W., \& Fishback, J. The destruction of the Miller-Lyer jllusion in repeated trials: I. An examination of two theories. Journal of Experimental Psychology, 1950, 40, 276-289.

Krantr, D.M., \& Weintraub, D.J. Factors affecting perceived orientation of the Poggendorff transversal. Pexception and Psychophysics, 1973:14, 511~517.

Lefbowitz, H.W., \& Gwozdecki, J. The magnitude of the Poggendorff illusion as a function of age. Child Developments 1967, 38, 573-580.

Lefbowitz, H. W. \& Poffey, S. The effect of rotation and tilt on the magnitude of the Poggendorff illusion. Vision Research, 1966, 6, 101-103.

Lucas, A., \& Fisher, G.H, Illusion in concrete situations: II. Experimental studies of the Poggendorff illusion. Egronomics, $1964,12,395-402$.

Novak, S. Effects of free inspection and fixation on the Poggendorff illusion. Perceptual and Motor Skil1s, 1966, 23, 663-670.

Ohwaki, S. On the destruction of geometrical illusions in stereoscopic observations. Tohoku Psychologia Folia, 1960, 19, 29-36. cited by J.O. Robinson in The Psychology of Visual Tllusions. London: Hutchinson, 1972.

Piaget, J. The mechanisms of perception. Translated by G.N. Seagrin, New Yock: Basic Books, 1969.

Pierce, A.H. Scudies in auditoxy and visual space perception. New York: Longmans Green, 1901.

Pressey, A.W. Field dependence and susceptibility to the Poggendorff illusion. Perceptual and Motor Skills, 1967, 24, 309-310.

Pressey, A.W. An explanation of the effect of tilt on the Poggendorff illusion Psychonomic Science, 1970, 18, 347-348.

Pressey, A.W. An extension of assimilation theory to illusions of size, area, and direction. Perception and Psychophysics, 1971, 9. 172-176.

Pressey, A.W. The assimilation theory of geometrical illusions: An additional postulate. Perception and Psychophyics, 1972, 11, 28-30.

Pressey, A.W., Bayer, C.A., \& Kelm, H. The Poggendorff illusion in schizophrenic patients. Schizopirenia, 1969, 1, 88-94.

Pressey, A.W., \& Bross, M. Assimilation theory and the revexsed MHJler-Lyer illusion. Pexception, 1973, 2, 211-217.

Pressey, A.W., Butchard, N., \& Scrivner, L. Assimilation theory and the Ponzo illusion: Quantitative predictions. Canadian Journal of Psychology, 19 , 25, 486-497.

Pressey, A.W., \& Sweeney, O. A variation of the Poggendorff illusion. Perceptual and Mocor Skills, 1969, 28, 833-866.

Pressey, A.W., \& Sweeney, O. Age changes in the Poggendorff illusion as measured by a method of production. Psychonomic Science, 1970, 19, 99-100.

Pressey, A.W., \& Sweeney, O. Some purzling results on the Poggendorff
illusion. Perception and Psychophysics, 1972, 12, 433-437.
Pressey, A.W., \& Sweeney, O. Acute angles and the Poggendorff illusion. Quasterly Journal of Experimental Psychology, 1972, 24, 169-174.

Restle, F. Illusions of bent lines. Perception and Psychophysics, 1969 , 5, 273-274.

Robinson, J.O. The psychology of visual illusion. London: Hutchinson, 1972.

Runyon, R.P., \& Haber, A. Fundamentals of behavioral statistics, Don Mills: Addison-Wesley, 1968.

Schiller, $P_{0}$, \& Wienex, $N$. Binocular and stereoscopic viewing of geometric illusions. Perceptual and Motor Skills, 1962, 15, 739‥747。

Shipley, W. C., Nann, B.M., \& Penfield, M.J. The apparent Iength of tilted lines. Journal of Experinental Psychology, 19, 39, 548-551.

Springbett, B.M. Sone stereoscopic observations and their implications. British Journal of Psychology, 1961, 52, 105-109.

Taylor, M. M. Effects of anchoring and distance perception on the reproduction of forms. Perceptual and Motor Skills, 1961, 12, 203-230.

Tong, L., \& Weintraub, D.J. Contour displacement and tracking errors: Probing "twixt" Poggendorff parallels. Perception and Psychophysics, 1974, 15, 258-268.

Velinsky, S. Explication physiologique de l'illusion de Poggendorff. Année Psychologique, 1925, 26, 107-116.

Vurpillot, E. L'influence de la signification du materiel sur illusion de Poggendorff. Année Psychologicue, 1957, 57, 339-352.

Wagner, H. L. Simultaneous and successive contour displacement. Doctoral dissertation; University of Wales; 1969. Cited by Robinson, J.O., The Psychology of Visual Illusion, London: Hutchinson, 1972.

Weintraub, D.J., \& Krantz, D.H. The Poggendorff illusion: Amputations, rotations, and other perturbations. Perception and Psychophysics, 1971, 10, 257-264.

Zarforlin, M. Some observations on Gregory's theory of perceptual ilIusions. Quarterly Journal of Experimental Psycholory, 1967, 19, 193-197.

Zusne, L. Visual illusions: Publicationtrends. Perceptual and Motor Skil1s, 1968, 27, 175-177.

Zusne, $L$, Visual perception of form. New York: Academic press, 1970.

APPENDIX A

Conversion of Previous Results from MM to Degrees

| Investigator | Distance Between Parallels | Acute <br> Angle | Illusion in man | Illustion in deg. |
| :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \text { St. Velinsky } \\ (1925) \end{gathered}$ | 5 | 10 | 13.4 | 8.485 |
|  | 10 | 10 | 23.3 | 6.662 |
|  | 15 | 10 | 34.6 | 6.553 |
|  | 20 | 10 | 47.5 | 6.876 |
|  | 25 | 10 | 51.8 | 5.527 |
|  | 5 | 20 | 4.5 | 3.426 |
|  | 10 | 20 | 9.3 | 8.820 |
|  | 15 | 20 | 15.3 | 10.065 |
|  | 20 | 20 | 18.2 | 8.556 |
|  | 25 | 20 | 22.4 | 8.374 |
|  | 5 | 30 | 2.8 | 10.471 |
|  | 10 | 30 | 5.0 | 9.064 |
|  | 15 | 30 | 8.4 | 10.471 |
|  | 20 | 30 | 11.0 | 10.231 |
|  | 25 | 30 | 13.3 | 9.805 |
|  | 5 | 40 | 2.2 | 13.066 |
|  | 10 | 40 | 4.2 | 12.341 |
|  | 15 | 40 | 5.3 | 10.023 |
|  | 20 | 40 | 7.1 | 10.079 |
|  | 25 | . 40 | 10.5 | 12.341 |
|  | 5 | 50 | 1.5 | 11.671 |
|  | 10 | 50 | 3.0 | 11.671 |
|  | 15 | 50 | 3.6 | 9.074 |
|  | 20 | 50 | 5.2 | 9.925 |
|  | 25 | 50 | 6.4 | 9.754 |
|  | 5 | 60 | 1.2 | 11.358 |
|  | 10 | 60 | 2.3 | 10.845 |
|  | 15 | 60 | 3.3 | 10.335 |
|  | 20 | 60 | 4.8 | 11.359 |
|  | 25 | - 60 | 6.1 | 11.564 |
|  | 5 | 70 | . 90 | 9.8576 |
|  | 10 | 70 | 1.5 | 7.923 |
|  | 15 | 70 | 2.5 | 8.839 |
|  | 20 | 70 | 3.2 | 8.472 |
|  | 25 | 70 | 3.7 | 7.813 |
|  | 5 | 80 | . 23 | 2.575 |
|  | 10 | 80 | . 50 | 2.800 |
|  | 15 | 80 | 1.00 | 3.742 |
|  | 20 | 80 | 1.78 | 5.009 |
|  | 25 | 80 | 2.26 | 5.089 |



| Investisator | Distance Between Pacallels | Acute <br> Amgle | Tllusion in ma | Tllusion <br> in Deg: |
| :---: | :---: | :---: | :---: | :---: |
| Retardates, viewing distance 250 mm | 2.5 | 30 | 1.7 | 13.55 |
|  | 5.0 | 30 | 4.8 | 22.33 |
|  | 10.0 | 30 | 9.6 | 22.33 |
| Retardates, viewing distance 500 mm | 2.5 | 30 | 1.6 | 12.43 |
|  | 5.0 | 30 | 4.0 | 17.01 |
|  | 10.0 | 30 | 9.0 | 20.24 |
| Tong \& Weintraub (1973) | 8 | 45 | 1.1 | 4.22 |
|  | 16 | 45 | 2.5 | 4.84 |
|  | 24 | 45 | 4.9 | 6.49 |
|  | 32 | 45 | 6.5 | 6.45 |
| Walker (1973) | 25 | 30 | 8.75 | 5.89 |
|  | 25 | 40 | 4.80 | 5.01 |
|  | 25 | 45 | 4.25 | 5.31 |
|  | 25 | 50 | 3.75 | 5.43 |
|  | 25 | 60 | 2.80 | 5.05 |
|  | 25 | 70 | 1.80 | 3.72 |
|  | 25 | 30 | . 80 | 1.79 |

APPENDTX B

Experimental Instructions

## Experimental Instructions

Before I tell you what we are going to do I'm going to inform you of your rights as a paxticipant in this study. If at any tine you decide that you do not wish to participate or to continue to take part in what we are going to do for any reason, then you have the right to withdraw without losing your experimental credit. Are there any questions?

What we are going to do, is to see how people judge distances (the continuation of an oblique line on a vertical line) and how people judge the continuation of an oblique line on a vextical (dism tances) line. This will involve three different types of tasks.

One task will reguire you to estimate the continuation of an oblique line on a vertical line. You will be shown a figure similar to this. You are to judge where the oblique line, if continued, would intersect on the vertical line. You are to make your estimate of the point of intersection by drawing a small dot on the vextical line at the point where the oblique Ine would appear to intersect on the vertical line. Are there any questions?

One task will require you to estimate the distance between a dot (the point of intersection of an oblique line with a vertical ine) and the point of intersection of an oblique line with a vertical line (a dot). You will be shown a figure similar to this. You are to judge the distance between the dot (the point of intersection of the oblique line with the vertical line (the dot) and the point of inter-
section of the oblique line with the vertical line (the dot) on one sheet of paper, and then I'll exchange this sheet of paper for a seond one that contains only one dot. You are to make your estimate of the judged distance by drawing a second dot directly to the right of the fixst dot so that the distance between the dot you draw and the dot already present, appears equal to the judged distance. Are there any questions?

One task will require you to estimate the distance between a pair of dots. You will be shown a figure similar to this. You are to judge the distance between the pair of dots on one sheet of paper and then I'll exchange this sheet of paper for a second one that contains only one dot. You are to make your estimate of the judged distance by drawing a second dot directly to the right of the first dot, so that the distance between the dot you draw and the dot already present arpears equal to the judged distance. Are there any questions?

There are two rules that you are required to follow. One, use only your visual system in making your judgment. Use your hands only for drawing the dot. Secondly, please wait until I ask you before making your response. Are there any questions?

RAW SCORES OF 20 DEGREE GROUP

| Poggendorff | Poggendorff Distaace | Control | Poggendorff | Poggendorff <br> Distance | Control |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 47.00 | 126.00 | 107.00 | 57.00 | 107.00 | 102.00 |
| 85.00 | 99.00 | 85.00 | 60.00 | 98.00 | 99.00 |
| 66.00 | 98.00 | 39.00 | 53.00 | 104.00 | 79.00 |
| 70.00 | 104.00 | 98.00 | 66.00 | 112.00 | 95.00 |
| 54.00 | 110.00 | 110.00 | 61.00 | 92.00 | 32.00 |
| 34.00 | 42.00 | 51.00 | 81.00 | 52.00 | 64.00 |
| 53.00 | 46.00 | 90.00 | 46.00 | 69.00 | 71.00 |
| 78.00 | 103.00 | 78.00 | 70.00 | 97.00 | 79.00 |
| 48.00 | 75.00 | 71.00 | 53.00 | 79.00 | 79.00 |
| 59.00 | 116.00 | 103.00 | 63.00 | 103.00 | 62.00 |
| 65.00 | 110.00 | 88.00 | 64.00 | 93.00 | 37.00 |
| 35.00 | 120.00 | 85.00 | 67.00 | 75.00 | 38.00 |
| 54.00 | 37.00 | 47.00 | 63.00 | 36.00 | 31.00 |
| 70.00 | 94.00 | 98.00 | 53.00 | 93.00 | 76.00 |
| 73.00 | 85.00 | 89.00 | 55.00 | 93.00 | 85.00 |
| 75.00 | 103.00 | 99.00 | 74.00 | 129.00 | 30.00 |
| 45.00 | 107.00 | 86.00 | 69.00 | 93.00 | 99.00 |
| 58.00 | 97.00 | 83.00 | 46.00 | 80.00 | 71.00 |
| 62.00 | 41.00 | 30.00 | 58.00 | 77.00 | 85.00 |
| 62.00 | 85.00 | 77.00 | 60.00 | 77.00 | 74.00 |
| 65.00 | 103.00 | 95.00 | 66.00 | 103.00 | 81.00 |
| 50.00 | 133.00 | 104.00 | 65.00 | 104.00 | 103.00 |
| 53.00 | 102.00 | 94.00 | 59.00 | 106.00 | 98.00 |
| 83.00 | 96.00 | 67.00 | 73.00 | 97.00 | 95.00 |
| 95.00 | 92.00 | 102.00 | 88.00 | 98.00 | 80.00 |
| 59.00 | 105.00 | 73.00 | 59.00 | 91.00 | 74.00 |
| 72.00 | 89.00 | 76.00 | 57.00 | 89.00 | 32.00 |



| Poggendorff | Poggendorff Distance | Control | Poggendorff | Foggendorfe Distance | Control |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 87.00 | 85.00 | 100.00 | 92.00 | 93.00 | 97.00 |
| 73:00 | 117.00 | 92.00 | 73.00 | 104.00 | 33.00 |
| 93.00 | 95.00 | 76.00 | 35.00 | 92.00 | 100.00 |
| 52.00 | 117.00 | 111.00 | 74.00 | 125.00 | . 123.00 |
| 64.00 | 74.00 | 97.00 | 57.00 | 100.00 | 85.00 |
| 70.00 | 89.00 | 88.00 | 57.00 | 108.00 | 125.00 |
| 75.00 | 95.00 | 91.00 | 72.00 | 97.00 | 76.00 |
| 95.00 | 39.00 | 93.00 | 61.00 | 94.00 | 74,00 |
| 61.00 | 117.00 | 49.00 | 32.00 | 103.00 | 96.00 |

RAT SCORES OF 30 DEGREE GROIJP

| Poggendorff | Poggendorff Distance | Control | Poggendorff | Poggendorff Distance | Conerol |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 58.00 | 67.00 | 58.00 | 52.00 | 59.00 |  |
| 35.00 | 52.00 | 52.00 | 34.00. | 48.00 | 61.00 |
| 48.00 | 47.00 | 49.00 | 53.00 | 55.00 | 47.00 |
| 61.00 | 56.00 | 68.00 | 33.00 | 63.00 | . 63.00 |
| 45.00 | 72.00 | 63.00 | 43.00 | 73.00 | 54.00 |
| 52.00 | 41.00 | 32.00 | 52.00 | 29.00 | 27.00 |
| 49.00 | 98.00 | 65.00 | 49.00 | 58.00 | 64.00 |
| 45.00 | 104.00 | 59.00 | 44.00 | 35.00 | . 50.00 |
| 53.00 | 67.00 | 54.00 | 55.00 | 65.00 | 60.00 |
| 45.00 | 94.00 | 90.00 | 44.00 | 74.00 | 73.00 |
| 53.00 | 35.00 | 51.00 | 57.00 | 59.00 | 38.00 |
| 47.00 | 86.00 | 53.00 | 43.00 | 69.00 | 57.00 |
| 38.00 | 54.00 | 60.00 | 41.00 | 54.00 | 55.00 |
| 38.00 | 43.00 | 62.00 | 39.00 | 82.00 | 52.00 |
| 53.00 | 67.00 | 62.00 | 43.00 | 69.00 | .63.00 |
| 51.00 | 42.00 | 69.00 | 49.00 | 55.00 | . 60.00 |
| 36.00 | 49.00 | 45.00 | 49.00 | 39.00 | $\bigcirc 33.00$ |
| 70.00 | 64.00 | 53.00 | 56.00 | 55.00 | - 59.00 |
| 45.00 | 55.00 | 53.00 | 47.00 | 68.00 | 56.00 |
| 46.00 | 70.00 | 75.00 | 47.00 | 63.00 | 69.00 |
| 43.00 | 53.00 | 33.00 | 46.00 | 61.00 | . 55.00 |
| 43.00 | 65.00 | 78.00 | 49.00 | 65.00 | 53.00 |
| 45.00 | 77.00 | 63.00 | 45.00 | 77.00 | 72.00 |
| 53.00 | 64.00 | 63.00 | 44.00 | 71.00 | 64.00 |
| 50.00 | 76.00 | 54.00 | 52.00 | 70.00 | . 59.00 |
| 57.00 | 63.00 | 52.00 | 54.00 | 53.00 | 54.00 |
| 40.00 | 76,00 | 68.00 | 40.00 | 59.00 | 61.00 |
| 38.00 | 42.00 | 45.00 | 47.00 | 41.00 | 38.00 |


| Poggendorfs | Poggendorfi Distance | Control | Poggendorff | Poggendorff Distance | Control |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 40.00 | 77.00 | 60.00 | 37.00 | 72.00 | 60.00 |
| 48.00 | 67.00 | 61.00 | 43.00 | 68.00 | 61.00 |
| 42.00 | 58.00 | 60.00 | 43.00 | 77.00 | 68.00 |
| 45.00 | 79.00 | . 52.00 | 40.00 | 78.00 | 59.00 |
| 49.00 | 66.00 | 55.00 | 45.00 | 73.00 | 52.00 |
| 41.00 | 54.00 | 53.00 | 43.00 | 60.00 | 64.00 |
| 48.00 | 60.00 | 51.00 | 58.00 | 46.00 | 53.00 |
| 59.00 | 36.00 | 84.00 | 47.00 | 86.00 | 68.00 |

RAW SCORES OF 40 DEGREE GROUP

| Poggendorff | Poggendorff Distance | Control | Poggendorff | Foggendorff Distance | Control |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 37.00 | 44.00 | 41.00 | 35.00 | 44.00 | 41.00 |
| 43.00 | 48.00 | 44.00 | 40.00 | 55.00 | 54.00 |
| 49.00 | 34.00 | 42.00 | 45.00 | 33.00 | 52.00 |
| 37.00 | 61.00 | 54.00 | 35.00 | 67.00 | 52.00 |
| 42.00 | 50.00 | 42.00 | 41.00 | 53.00 | 44.00 |
| 37.00 | 22.00 | 46.00 | 35.00 | 57.00 | 46.00 |
| 41.00 | 44.00 | 53.00 | 41.00 | 42.00 | 47.00 |
| 43.00 | 45.00 | 45.00 | 40.00 | 43.00 | 44.00 |
| 42.00 | 37.00 | 44.00 | 40.00 | 34.00 | 33.00 |
| 36.00 | 35.00 | 56.00 | 34.00 | 39.00 | 54.00 |
| 47.00 | 58.00 | 54.00 | 43.00 | 64.00 | 52.00 |
| 38.00 | 56.00 | 56.00 | 40.00 | 59.00 | 56.00 |
| 40.00 | 70.00 | 59.00 | 36.00 | 73.00 | 71.00 |
| 36.00 | 41.00 | 74.00 | 35.00 | 29.00 | 23.00 |
| 39.00 | 48.00 | 14.00 | 38.00 | 47.00 | 55.00 |
| 37.00 | 4.4 .00 | 42.00 | 36.00 | 45.00 | 46.00 |
| 35.00 | 49.00 | 40.00 | 37.00 | 47.00 | 41.00 |
| 37.00 | 55.00 | 39.00 | 35.00 | 43.00 | 38.00 |
| 40.00 | 50.00 | 47.00 | 37.00 | 45.00 | $\cdots 46.00$ |
| 49.00 | 50.00 | 48.00 | 52.00 | 53.00 | 51.00 |
| 42.00 | 60.00 | 52.00 | 40.00 | 59.00 | 51.00 |
| 33.00 | 32.00 | 43.00 | 35.00 | 37.00 | 43.00 |
| 36.00 | 60.00 | 49.00 | 35.00 | 48.00 | 38.00 |
| 36.00 | 67.00 | 41.00 | 39.00 | 60.00 | 50.00 |
| 40.00 | 52.00 | 30.00 | 41.00 | 47.00 | 44.00 |
| 42.00 | 65.00 | 46.00 | 40.00 | 44.00 | 42.00 |
| 43.00 | 50.00 | 50.00 | 43.00 | 44.00 | 43.00 |


| Foggendorff | Poggendorff Distance | Concrol | Poggendorff | Poggendorfí Distance | Control |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 40.00 | 52.00 | 48.00 | 38.00 | 49.00 | 44.00 |
| 33.00 | 60.00 | 55.00 | 35.00 | 61.00 | 42.00 |
| 42.00 | 51.00 | 44.00 | 42.00 | 48.00 | 48.00 |
| 34.00 | 59.00 | 40.00 | 34.00 | 62.00 | 65.00 |
| 36.00 | 47.00 | 48.00 | 37.00 | 51.00 | 47.00 |
| 39.00 | 49.00 | $\therefore 41.00$ | 39.00 | 46.00 | 53.00 |
| 41.00 | 45.00 | 32.00 | 40.00 | 28.00 | 32.00 |
| 35.00 | 48.00 | 63.00 | 37.00 | 41.00 | 63.00 |
| 36.00 | 35.00 | 47.00 | 39.00 | 30.00 | 32.00 |

RAN SCORES OF 50 DEGREE GROUP

| Poggendorff | Poggendoxis Distance | Control | Poggendorif | Poggendorff Distance | Control |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 34.00 | 33.00 | 41.00 | 33.00 | 35.00 | 41.00 |
| 35.00 | 31.00 | 29.00 | 33.00 | 29.00 | 29.00 |
| 36.00 | 42.00 | 42.00 | 33.00 | 41.00 | 41.00 |
| 35.00 | 32.00 | 33.00 | 37.00 | 33.00 | 38.00 |
| 33.00 | 41.00 | 35.00 | 33.00 | 35.00 | 38.00 |
| 36.00 | 42.00 | 39.00 | 33.00 | 43.00 | 47.00 |
| 32.00 | 40.00 | . 38.00 | 33.00 | 43.00 | 39.00 |
| 37.00 | 52.00 | 41.00 | 35.00 | 41.00 | 36.00 |
| 35.00 | 39.00 | 34.00 | 33.00 | 37.00 | 35.00 |
| 34.00 | 40.00 | 48.00 | 34.00 | 46.00 | 45.00 |
| 33.00 | 40.00 | 39.00 | 33.00 | 47.00 | 43.00 |
| 34.00 | 32.00 | 42.00 | 33.00 | 38.00 | 40.00 |
| '36.00 | 35.00 | 37.00 | 34.00 | 40.00 | 35.00 |
| 37.00 | 47.00 | . 44.00 | 35.00 | 44.00 | 40.00 |
| 34.00 | 36.00 | 36.00 | 35.00 | 38.00 | 35.00 |
| 32.00 | 35.00 | 31.00 | 33.00 | 33.00 | 37.00 |
| 32.00 | 49.00 | 50.00 | 33.00 | $\triangle 3.00$ | 33.00 |
| . 31.00 | 31.00 | 36.00 | 33.00 | 38.00 | 35.00 |
| 38.00 | 40.00 | 43.00 | 35.00 | 35.00 | 34.00 |
| 34.00 | 37.00 | 39.00 | 34.00 | 28.00 | 43.00 |
| 35.00 | 40.00 | 45.00 | 33.00 | 45.00 | 47.00 |
| 35.00 | 56.00 | 49.00 | 32.00 | 42.00 | 41.00 |
| 32.00 | 35.00 | 33.00 | 33.00 | 38.00 | 33.00 |
| 33.00 | 40.00 | 39.00 | 36.00 | 35.00 | 39.00 |
| 35.00 | 45.00 | . 37.00 | 35.00 | 40.00 | 43.00 |
| 33.00 | 42.00 | 44.00 | 32.00 | 56.00 | 39.00 |
| 36.00 | 38.00 | 36.00 | 35.00 | 39.00 | 38.00 |


| Poggendorff | Poggendorff Distance | Control | Poggendorfi | Poggendorff Distance | Control |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 36.00 | 55.00 | 48.00 | 35.00 | 57.00 | 42.00 |
| 35.00 | 52.00 | 41.00 | 34.00 | 53.00 | 50.00 |
| 33.00 | 40.00 | 49.00 | 34.00 | 3700 | 36.00 |
| 35.00 | 30.00 | 20.00 | 35.00 | 35.00 | 41.00 |
| 35.00 | 47.00 | 43.00 | 36.00 | 44.00 | 42.00 |
| 38.00 | 39.00 | 37.00 | 36.00 | 41.00 | 38.00 |
| 33.00 | 40.00 | 37.00 | 33.00 | 37.00 | 35.00 |
| 37.00 | 63.00 | 48.00 | 36.00 | 50.00 | 35.00 |

RAW SCORES OR 60 DEGREP GROUP

| Poggendorff | Poggendorff Distance | Control | Poggendorfit | Poggendorff Distance | Control |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 31.00 | 37.00 | 33.00 | 32.00 | 32.00 | 34.00 |
| 32.00 | 37.00 | 38.00 | 37.00 | 37.00 | $\therefore 37.00$ |
| 32.00 | 32.00 | 31.00 | 31.00 | 37.00 | 31.00 |
| 31.00 | 33.00 | 25.00 | 31.00 | 28.00 | 28.00 |
| 33.00 | 41.00 | 30.00 | 33.00 | 43.00 | 40.00 |
| 33.00 | 37.00 | 39.00 | 33.00 | 37.00 | 35.00 |
| 32.00 | 41.00 | 38.00 | 32.00 | 45.00 | 37.00 |
| 32.00 | 31.00 | $\cdot 29.00$ | 33.00 | 31.00 | 33.00 |
| 35.00 | 33.00 | 35.00 | 34.00 | 2.8 .00 | 37.00 |
| 32.00 | . 37.00 | 35.00 | 33.00 | 38.00 | 38.00 |
| 32.00 | 41.00 | 35.00 | 32.00 | 34.00 | 29.00 |
| 32.00 | 36.00 | 36.00 | 31.00 | 38.00 | 34.00 |
| 31.00 | 37.00 | 36.00 | 31.00 | 39.00 | 36.00 |
| 32.00 | 49.00 | 34.00 | 32.00 | 37.00 | 38.00 |
| 32.00 | 38.00 | .. 37.00 | 32.00 | 34.00 | 38.00 |
| 31.00 | 43.00 | 46.00 | 31.00 | 40.00 | 40.00 |
| 32.00 | 32.00 | 33.00 | 32.00 | 32.00 | 37.00 |
| 32.00 | 38.00 | 33.00 | 32.00 | 41.00 | 38.00 |
| 32.00 | 36.00 | 33.00 | 32.00 | 33.00 | 31.00 |
| 31.00 | 29.00 | 32.00 | 33.00 | 31.00 | 31.00 |
| 33.00 | 40.00 | 32.00 | 33.00 | 35.00 | 32.00 |
| 32.00 | 38.00 | 33.00 | 32.00 | 37.00 | 34.00 |
| $: 32.00$ | 40.00 | 32.00 | 31.00 | 35.00 | 35.00 |
| 32.00 | 39.00 | 37.00 | 32.00 | 33.00 | 36.00 |
| 32.00 | 41.00 | 36.00 | 32.00 | 42.00 | 33.00 |
| 33.00 | 32.00 | 35.00 | 34.00 | 33.00 | 33.00 |
| 33.00 | 38.00 | 35.00 | 32.00 | 37.00 | 35.00 |


| Poggendorff | Poggenciorff Distance | Control | Poggendorff | Poggendorff Distance | Control |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 34.00 | 41.00 | 36.00 | 32.00 | 37.00 | 36.00 |
| 33.00 | 39.00 | 42.00 | 32.00 | 40.00 | 38.00 |
| 32.00 | 32.00 | 32.00 | 31.00 | 27.00 | 26.00 |
| 32.00 | 33.00 | 35.00 | 31.00 | 31.00 | 35.00 |
| 32.00 | 37.00 | 33.00 | 32.00 | 35.00 | 36.00 |
| 31.00 | 37.00 | 32.00 | 31.00 | 38.00 | 35.00 |
| 32.00 | 30.00 | 31.00 | 33.00 | 32.00 | 34.00 |
| 33.00 | 43.00 | 35.00 | 32.00 | 36.00 | 40.00 |
| 33.00 | 31.00 | 32.00 | 32.00 | 34.00 | 32.00 |

## APPENDIX D

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    Fxequency Distributions of Poggendoref
and Poggendorff Distance (DD) Judgments
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Midpoint of class interval (in mon) for the $20^{\circ}$ group.


Midpoint of class interval (in mm) for $30^{\circ}$ group.


Midpoint of class interval (in mm) for the $40^{\circ}$ group.


Midpoint of class interval (in mm) for the $50^{\circ}$ group.


Midpoint of class interval (in mm ) for the $60^{\circ}$ group.


[^0]:    ${ }^{2}$ See Figure 16, page 65, for a schematic representation of this manipulation.

