THE RELATIONSHIP BETWEEN

THE WAYS CHILDREN REPRODUCE BASIC FACTS

AND

THEIR LATER ALGORITHMIC PROFICIENCY

IN

MATHEMATICS

by

HUANG CHENG TAY

A thesis
presented to the Faculty of Graduate Studies,
University of Manitoba,
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Abstract

This study investigated the relationship between the ways children reproduce basic facts and their subsequent algorithmic proficiency. Seventy-four grade 5 and 6 students who obtained at least a 90% score on a speed-independent basic fact quiz were identified. Data on the four variables were then accumulated for these students. By individual interviews, each student was subsequently assigned a CR ratio, measuring her/his conditioned response ratio; the proportion of basic facts s/he could reproduce without evidence of delay of a statement of a question.

The experimental group then completed as many as possible of the examples from a set of multiplication exercises. It was found in a pilot test that the variables NDONE (The number of exercises completed), NRIGHT (The number of excercises completed correctly), and PERIGHT (NRIGHT/NDONE) were most likely be productive.

In the main study, each student's CR ratio was determined by interview and each of the above variables was correlated with CR.

The Pearson correlation coefficients between CR vs NDONE, CR vs NRIGHT, and CR vs PERIGHT are 0.54, 0.64, and 0.51 respectively. Using Bonferroni tests these correlations are found to be statistically significant at an overall alpha of 0.1. It is concluded that the possession of conditioned responses is important for later algorithmic performances.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

CHAPTER 1

INTRODUCTION

Whether the mathematics curriculum emphasizes rituals or understanding, basic facts are important in arithmetic.

Facts must be reproduced with reasonable speed and accuracy in many computational procedures, and it is common knowledge that children vary in their ability to do so. The causes of these variations can be analyzed in various ways, but there has been little detailed work on what may be a salient factor; the relationship between the manner in which facts are reproduced and proficiency with algorithms that use them. This may be because it seems obvious that having basic facts under control makes calculations easier and allows attention to be directed to complex processing.

It appears, however, that the manner in which students recall basic facts may well interact with their performance of algorithms. This study is designed to examine what may be an important distinction in students' manner of recall. It is supposed that given a question like 6x5, students may respond "30" (or "25") having paid no cognitive attention to the question. Their behavior is then at least analogous to what has been called a conditioned response. Another student may find it necessary to repeat the question: "6x5" before being able to provide the answer "30". In short, this student would say, " 6x5 is 30." Other students may work out an answer in any one of a variety of ways, all of which entail some cognitive attention. They may count by fives ("5, 10, 15, 20, 25"), distribute ("5 fives are 25, and another one makes 30"), double ("3 fives are 15, so 6 fives are 30"), and so on. We will refer to these procedures as cognitive by-passes, and assume that any response requiring more than one second involves some cognitive by-pass.

Paying cognitive attention to facts may, in fact, enhance the learning of complex processes, but the need to pay attention to the means by which a fact is recalled may also interfere with the performance of algorithms.

Operationally, conditioned responses (henceforth CR) are defined to be responses provided within one second of the presentation of the question. All the other responses (henceforth CB) are defined as involving cognitive by-passes.

This analysis leads to the question motivating this study:

- Is there any relationship between students' manner of recalling basic facts and their proficiency with subsequent algorithms?

The central pedagogical issue is whether or not understandings in arithmetic are continuous with conditioned responses. It is sometimes assumed that sufficiently rich understandings will lead students to conditioned responses with no particular attention being paid to them. Charles(1986) claims that with understanding, students will come to recall basic facts quickly and accurately and retain them for a longer period of time then they otherwise would. He and others assume that when children understand arithmetic processes, they will come to be able to reproduce facts efficiently.

Other researchers and practitioners doubt that conditioned response are continuous with understanding. They doubt that any amount of understanding will lead to conditioned responses of basic facts. They believe that whether children understand algorithms or not, facts must be taught independently if it is desired that children have them.

Weaver (1955) was one of the first to explicitly distinguish between children who fail to respond to multiplication facts instantly and those who can respond instantly. He observed that some elementary school children he interviewed did not use memorized multiplication facts. They prefer to use number patterns like 2, 4, 6, . . . and tallying on their fingers or mentally. He speculated that these practices could have negative consequences and suggested they might have little difficulty finding multiples of 2 and 3 in this way but might have difficulty with larger factors. He provided no data to support his speculation.

If the possession of conditioned responses is irrelevant to later performances with algorithms, then the above is a moot question. If they are relevant, however, then a study of the question is of considerable pedagogical consequence.

We are led to:

(a) identifying children's ways of reproducing facts,

(b) examining possible correlations between identified ways of reproducing facts and subsequent performances in algorithms,

(c) determining whether or not students' conditioned responses densities are related to their performance levels in algorithms.

Because timing is not otherwise sufficiently controlled, it is necessary to use interviews to obtain evidence regarding students' means of response.

CHAPTER 2

BACKGROUND

This review of the literature has six parts:

2.1) An Overview
2.2) The Associationist Perspective
2.3) The Gestalt Perspective
2.4) The Electic Perspective
2.5) The Question
2.6) The Nature of Learning Facts

2.1. AN OVERVIEW

Memory, learning, and recall were discussed as early as in Aristotle's time, but the scientific study of those phenomena is generally taken to begin with Ebbinghaus' studies in the 1880s.

Since then, the construction of models, theories, research and recommendations for practice have clustered around two poles. At one extreme, it is assumed that learning begins with the creation of simple stimulus-response (S-R) associations and that all later learning entails the creation of a hierarchy of those associations that may become complex but is not different in kind. In the paradigms near that pole, notions like 'understanding' and 'insight' are allowed only as they can be interpreted in terms of those hierarchies.

At the other pole, we find those who reject reductionism. Some grant that S-R learning exists, but argue that there is no good evidence that it is continuous with 'higher level' learning. They prefer to study 'higher order' learning as a phenomenon in its own right, and are prepared to use words like 'insight' and 'understanding' without interpreting them in reductionist terms.

Those who have designed instructional sequences related to learning facts seem to have gravitated to three positions, whether explicitly or not deriving them from the above positions;

a) Given sufficiently rich understandings, the 'facts' will follow,

b) Given sufficient hierarchies of 'facts', the 'understanding' will follow, and

c) They are independent and both are needed: facts are not continuous with understandings, but both are necessary for later learning.

Clearly, there are significant implications of adopting one or the other position. This study attempts to provide data that can contribute to making a decision as to which to use as a basis for curriculum construction.

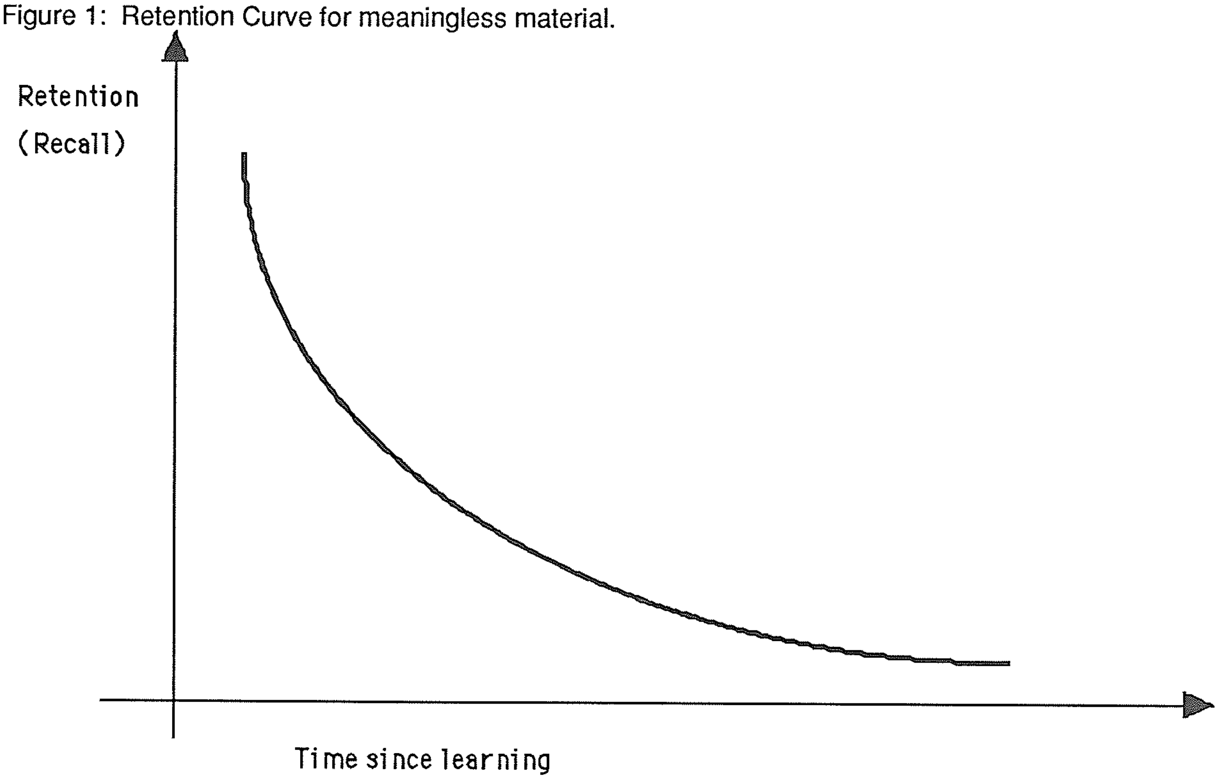
2.2 THE ASSOCIATIONIST PERSPECTIVE

Ebbinghaus (1878-1880) carried out some of the initial quantitative laboratory studies of memory and learning. In order to make learning tasks free of previous experience, he composed two thousand three hundred nonsensical syllables, and observed the number of repetitions necessary to memorize lists up to criterion levels. He found forgetting to be extensive and rapid, especially immediately after learning.

It was later found, however, that when facts have meaning for learners, one can expect higher levels of retention (Biggs & Reeds 1943) than with nonsense material. The learning curves in figures 1 and 2 (McGeogh & Irion 1952) summarize the relationship between retention curves and different levels of meaningful material.

Ebbinghaus also considered rate of learning. He believed that we learn serially, and that we learn faster and have a better grasp of material near or at the end of a series than at the beginning.

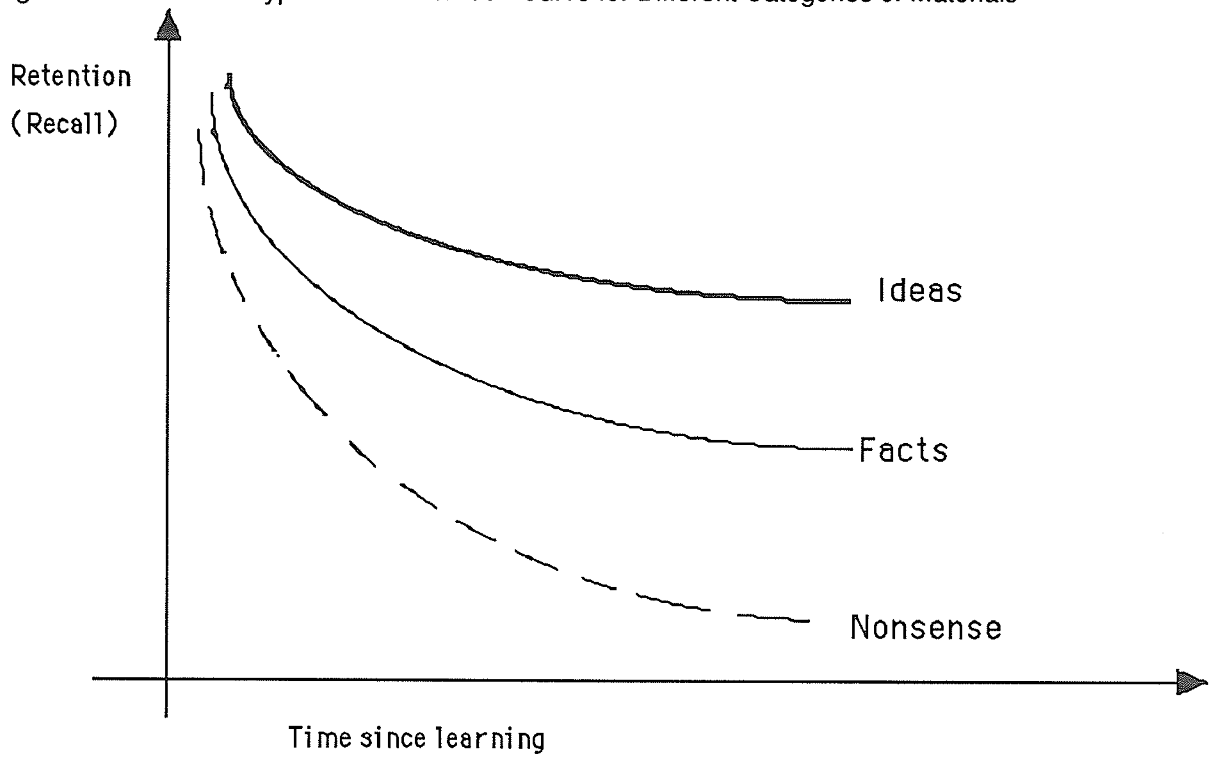
Figure 1: Retention Curve for meaningless material.



Pavlov (1927) conducted the fundamental studies in what is now called classical conditioning. The **conditioned response** was held to be the unit of learning. Pavlov showed that systematic environmental manipulation could produce new associations between stimuli and responses. He discovered that repeatedly presenting a novel stimulus immediately before a previously learned unconditioned stimulus would finally elicit a 'conditioned response'. A new association would eventually be learned. Therefore, according to Pavlov, learning comprised new associations via contiguity and repetition.

E. R. Gutherie (1886-1959) was an early convert to the associationist view; but directed more attention to responses. He concluded that when a stimulus and a response occur close together or simultaneously, an association between them forms. In his words, "What is being noticed becomes the signal for what is being done." (Koch, 1959).

Figure 2: Traditional Hypothetical Retention Curve for Different Categories of Materials



In the first quarter of the twentieth century. Edward L. Thorndike (1913) brought the associationists' views to Education in his "S-R bond theory." He asserted that learning was the formation of bonds between stimuli and responses. Repeating stimulus-response connections would gradually "stamp them in." The bond or connection would be strengthened until the desired response invariably and quickly followed the given stimulus.

According to Thorndike, the teacher's task was to present drill and practice so as to facilitate the right connections. All learning, at least in mathematics, would become a hierarchy of S-R bonds. For example, in The Complete Arithmetic, published in June 1900 by an anonymous author and antedating Thorndike's formal explanation by a decade, there is no instruction, meaningful or otherwise, before the student is intended to 'learn' 1x6 and 6x1 as unrelated basic facts.

In the 1930s and the 1940s, Clark Hull carried associationism further and had a considerable influence on later psychologists. He continued to focus on rote learning, discrimination learning, and trial-and error learning.

Hull stated that " . . psychology's primary laws are expressible quantitatively, . . . all the complex behavior of a single individual will ultimately be derivable as secondary laws from [1] these primary laws together with [2] the conditions under which behavior occurs; and that all the behavior of groups as a whole, . . . , may similarly be as quantitative laws from the same primary equations." (Hull, 1959) His theory attempted to define variables that may intervene between stimuli and responses. His most significant new variable was **reaction potential** whose strength depended on the drive, stimulus intensity, incentive motivation, and the habit strength of the subject.

In research similar to Pavlov's, he explained how a word like 'sweet' could come to elicit salivating in a **conditioned reflex**. Multiplication tables were intended to be learned in an analogous way.

Returning to a position close to Gutherie's, Skinner (1938) emphasized reinforcing 'operants' or emitted behavior and, like Thorndike, strongly influenced educational practice.

In 1958, British psychologist Donald Broadbent centred on short-term memory (STM) and long-term memory (LTM). That dichotomy is not necessarily tied to the associationist position, but has certainly provoked further associationist research. Broadbent, for example, reported that with repetition, STM incidents are passed on to the LTM.(Anderson, 1980)

In the late 1950's, Peterson and Peterson (1959), and Sperling (1960) claimed that there was transfer between STM and LTM. Miller (1956) asserted that the STM is a temporary storage system with a capacity about seven plus or minus two 'chunks' for only a limited time. By maintenance, rehearsal information can remain in STM indefinitely (Peterson and Peterson ,1959), but unrehearsed information remains in STM for less than twenty seconds. Peterson and Peterson (1959) concluded that without constant repetition, information is lost after three seconds and forgotten after eighteen seconds.

The hierarchical nature and specific nature of at least some of mathematics made it a popular candidate for applying behavioral models. Those models provided instructional guidelines, allowed for short-span measures of progress and seemed well-suited to pressures for accountability .

2.3. THE GESTALT PERSPECTIVE

At the same time, German psychologists Max Wertheimer, Kurt Koffka, and Wolfgang Kohler developed theories that dealt with the organization of perceptions, knowledge and the thinking processes in general. They only gradually came to influence American Psychology because the behavioristic (empiricist) approach was well entrenched. *Pragnanz*, their fundamental principle, states that people recognize (and remember) patterns or configurations by recognizing a few key stimuli. Patterns therefore become simpler, more complete, and more regular than they actually are (Woolfolk and Nicolich, 1984). Gestaltists concluded that perceptions tend to be organized into meaningful configurations structured holistically, and not developed by marrying or merging discrete elements. The classical Gestalt theories dealt with such qualities of perception as proximity, similarity, figure-ground discriminations, and contrast-effects.

They concluded that learning or problem solving (Wertheimer, 1959) could be achieved via sudden insight. Sometimes cognitive structures show up suddenly and solutions become instantly apparent. They raised doubts as to the behaviorists' interpretation of the learning process - a gradual accumulation of associations by trial and error or the shaping of response through reinforcement. A student, for instance, may appear to mull over a problem, culminating in an "aha!" experience in which the solution comes all at once. The student is said to have insight when s/he perceives the 'stick as the rake', as the chimpanzee, Sultan, 'saw' the way to reach the food outside the cage by joining available short sticks (Kohler 1925, cited in Rosser and Nicholson, 1984).

In general, the Gestaltists viewed learning as the process of identifying relationships and of developing insights by synthesizing and organizing bits and pieces of information into meaningful wholes and not as compartmentalized segments.

Since the 1940s, the Gestaltist's view has merged with current cognitive theories that deal with perception, organization, and memory. It has also led to the guided discovery approach in classroom instruction.

A. W. Brownell (1941, 1948, 1954, Kilpatrick and Weaver, 1977) held that children develop understandings of basic facts in a series of stages that are characterized by meaning and thinking skills. He viewed mathematics as a system of ideas, principles and processes. He suggested that teachers take advantage of that structure so as to make arithmetic less a challenge to the pupil's memory and more a challenge to her/his intelligence.

From this point of view, a mathematics program should not and probably cannot be constructed solely as a hierarchy of S-R bonds. Facts, for example, should be linked to a whole. The comprehension of large-scale properties is a prerequisite for the learning and retaining them.

Contemporary education psychologist David Ausubel (1960, 1968, 1978) promoted efficient meaningful reception and discovery learning via well-structured presentations. Mayer (1975a,1975b, 1979a, 1979b) and many others (like Luiten, Ames, and Ackerson, 1980) supported Ausubel's emphasis on meaningful learning and the use of advance organizers to relate new materials to existing knowledge. Students are challenged to think about how new laws, principles or ideas relate to others. They concentrate on each student's past experiences as represented in their cognitive structures, their intentions, and the use of techniques needed to help students discard misconceptions and replace them with correct conceptions.

Ausubel (1968, p vi) proposed this general prescription:

> If I had to reduce all of educational psychology to just one principle, I would say this: The most important single factor influencing learning is what the learner already knows. Ascertain this and teach him accordingly.

Jerome Bruner (1960, 1966) wrote in the same tradition. He strongly influenced the New Mathematics (see Ulrich Heisser's Cognitive Psychology, Glover and Ronning, 1987) and revived the spiral curriculum. He espoused Jean Piaget's four intellectual developmental stages: the sensorimotor, the preoperational, the concrete operational, and the formal operational stages. Bruner believed that "any subject may be taught to anybody at any age in some form."(Bruner, 1960 p. 12) and "any subject can be taught effectively in some intellectually honest form to any child at any stage of development" (ibid. p. 23).

Bruner felt that some aspects of the structure of any subject matter can be presented so as to be true to the spirit of the discipline from which it is drawn and be meaningful to students. He advocated the spiral curriculum, in which students are exposed to the same general topic several different times. Somewhat foreshadowing the eclectic position to follow, he supported the School Mathematics Society Group's assertion that 'experience of computational practice may be a necessary step toward understanding conceptual ideas in mathematics' (ibid p. 30). In the main, however, he proposed learning by discovery based on three modes of thought: enactive, ikonic, and symbolic.

This view remains current in Mathematics Education. In a recent article, Charles (1986) claims that his fact strategies, based on understandings, will lead students to know the facts. He has faith that once students understand the relationships among numbers, the facts will just follow.

2.4. AN ELECTIC PERSPECTIVE

A few theoreticians like Robert Gagné and Leslie Briggs appear to be prepared to work from the above views simultaneously, but that kind of electic perspective is most common among practitioners.

Gagné (1983) is probably the best known eclectic theorist. He claims that improving understanding will enhance correct performances, but also emphasizes that computational skills should be automated.

In Conditions of Learning, Gagné (1965) discussed learning by categorizing "distinctive conditions for different kinds of learning". He distinguishes between classical conditioning-signal learning; stimulus-response learning; chaining; connecting S's to R's; oral association; discrimination learning; concept learning; rule learning; and problem solving. He suggested that differing combinations of these ways of learning may be prerequisites to higher levels of learning. He also identified eight co-requisites of instruction and learning: motivation, apprehension, acquisition, retention, recall, generalization, performance, and feedback.

Gagné and Briggs (Gagné and Briggs, 1979) elaborated on several 'events of instruction' that depend on the domains of learning. They emphasized the hierarchy of skills involved in task analysis, choosing the component parts of tasks, and deciding on the content and the sequence of instruction. From their point of view, understanding is an important element of learning basic facts and algorithms, but students must also store information for future use.

Weinsten and Mayer (1983) insisted that good instruction should include teaching students how to remember as well as how to learn and how to think.

Not all practitioners have assumed that understanding should precede skills. Willoughby (1970), for example, claimed that for many children , understanding may come more appropriately after or during acquisition of the skills and the ability to verbalize what has been learned. He said, "Even though there are strongly held opinions on this view there seems to be little clear evidence regarding which children should learn which mathematics in which ways" (ibid p. 264). Wheatley (1976) cites situations in which understandings seem to be detrimental to student's operational skills.

Even Robert B. Davis (Davis 1986), a leader in the 'understanding' movement, repeated the conventional doctrine that all children should learn by understanding instead of by rote and ritual, but admitted that sometimes a few children force him to teach them by ritual (ibid p. 8).

2.5 THE QUESTION

A study of this magnitude is not going to resolve the question of which of these fundamental views of learning is the best guide to practice. It may, however, contribute to refining the question.

If we take it to be obvious that basic facts must be reproduced somehow if the following algorithms are to be performed, we are led to the question posed in Chapter 1:

"Is there any relationship between students' way of retrieving basic facts and their proficiency with later algorithms?"

If it is sufficient that students reproduce facts by means of cognitive by-passes, then protocols of the kind suggested by Charles (1986) should be supported. If, on the other hand, it is necessary that students have facts as conditioned responses, then alternative instructional sequences tending to enhance that kind of response should be encouraged.

2.6 THE NATURE OF LEARNING FACTS

As Wheatley noted (1976) teaching for understanding could create cognitive by-passes to the exclusion of conditioned responses -- The question is, "Does it matter?" If it **does not** matter, then those who adopt the Gestalt view will have a stronger case. If it **does** matter, both the Association and the Electic perspectives can take comfort.

So the task is to find out whether or not there is a distinction between the effects of conditioned responses and cognitive by-passes in reproducing basic facts.

CHAPTER 3

DESIGN AND PROCEDURE

This chapter describes both the pilot study and the main study.

3.1. The Pilot Test

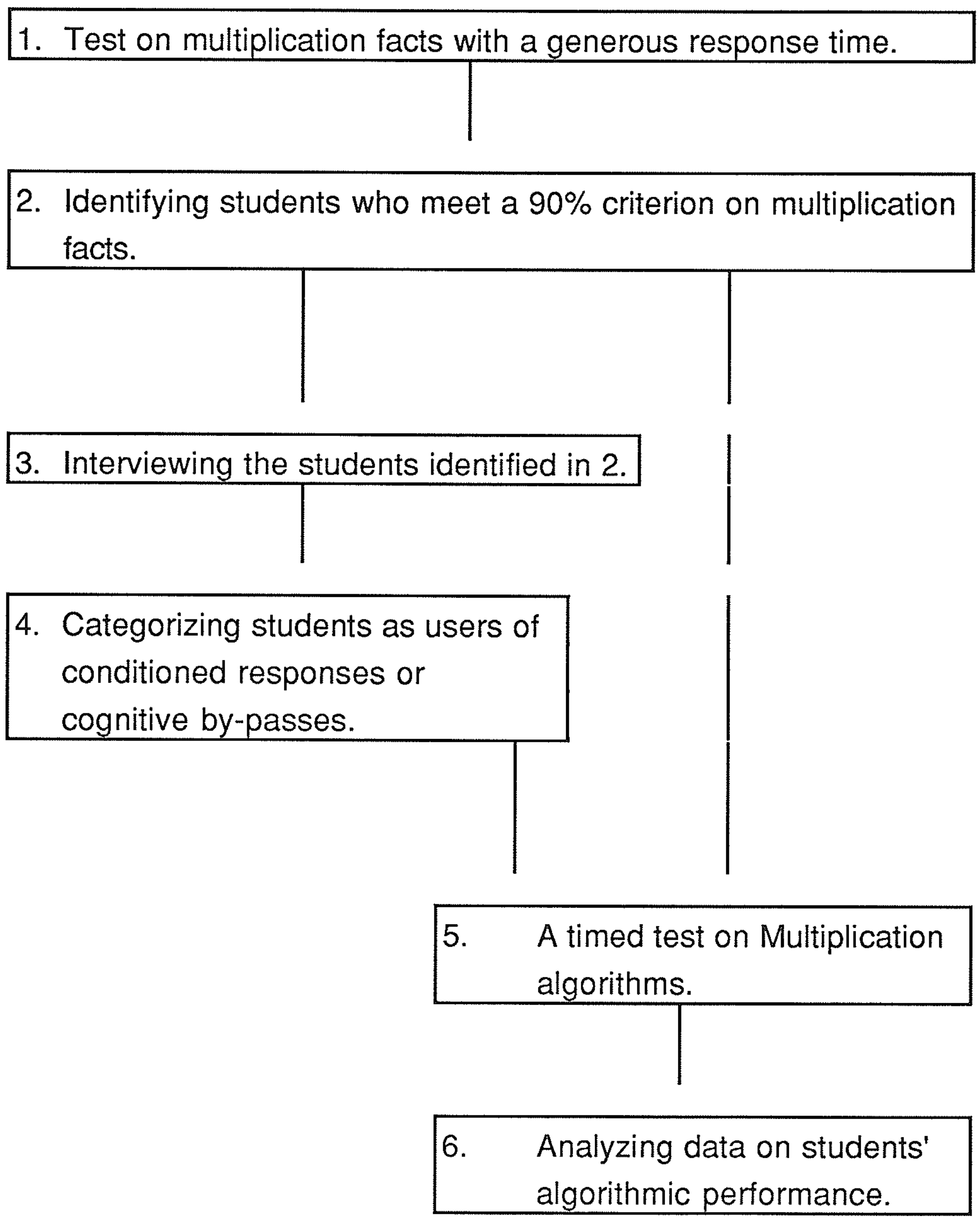
The Sample

Twenty-two grade 6 and twenty-six grade 5 students from the same elementary school in a Winnipeg-area school division participated in the pilot study. The ages of the subjects ranged from eleven years to thirteen years.

The Method and Instrument

The first flowchart (see Figure 3) outlines the procedure of the pilot study. Each classroom teacher gave the subjects the test of 60 randomly sequenced basic multiplication facts provided in Appendix I. The teacher read the sixty questions with an interval of five seconds between questions and did not repeat any questions during or after the quiz. Students scoring at least 90% on this test were identified. At this point, no attempt was made to distinguish students' modes of responses. A twenty-minute, speed-dependent test of multiplication algorithms (provided in Appendix II) was then administered to all students. Before writing that test, they were told that there were more questions than they could complete in the fifteen minutes provided; and to complete the questions in sequence. If a student was found to have skipped questions, then that student's results were deleted from the study. The remaining responses were then analyzed in an attempt to identify the

Figure 3: Detailed Flowchart for pilot test



most cogent variables for the main study. The potential variables studied included:

(a) The number of examples completed (NDONE),

(b) The number of examples completed correctly (NRIGHT),

(c) The fraction of examples completed that were completed correctly (PERIGHT),

(d) The number of errors (ERROR),

and (e) The errors per example completed (ERRPEG).

DEFINITION: The above variables are defined as follows:

(a) The number of examples completed (NDONE) is defined, as the label suggests, as the number of examples completed in the allowed time, whether the answers were correct or not;

(b) The number of examples completed correctly (NRIGHT) is operationally defined as the number of examples in (a) that were completed correctly;

(c) The PERIGHT is derived from (a) and (b).

$$\text{PERIGHT} = \frac{\text{NDONE}}{\text{NRIGHT}} \times 100\%;$$

(d) The number of errors (ERROR) is defined to be

$$\text{NDONE} - \text{NRIGHT};$$

and (e) The errors per example completed (ERRPEG) is defined as the number of individual mistakes (for example, an addition facts, a carry-digit, and the like) in an example done incorrectly. It is the ratio of the sum of the individual mistakes made to the total number of multiplication examples attempted.

At the same time, each student was interviewed according to the protocol in Appendix III and assigned a "Conditioned Response Ratio" defined as

$$\text{CR} = \frac{\text{No. of multiplication facts answered in} \leq 1 \text{ second}}{\text{No. of multiplication facts presented}} \times 100\%$$

In short, the CR ratio is a measure of the proportion of a student's responses that are conditioned responses. The CR ratio will also be referred to as the CR density.

During the interview, it was noticed that some interviewees found it necessary to repeat each question, but followed those repetitions by correct responses within one second. That behavior had not been anticipated. Other interviewees, as expected, took either more or less than one second to reproduce answers.

On account of the behavior of the 'Repeat-Respond'(RR) group, it was necessary to reexamine the dichotomy proposed for the main study. RR students did not seem to fit with the CR group because they had to repeat the question before providing a response. Yet they did not seem to belong with the CB group because they were able to provide what seemed to be conditioned responses within one second following a repetition of the question . Perhaps the RR students used very fast cognitive methods mentally while repeating the question to provide a response to the presented fact.

For purposes of the main study, it was decided to categorize responses within one second as conditioned responses(CR) and all others, including those from the RR group, as involving cognitive by-passes.

Then in a subsequent analysis that may be useful to future researchers, RR students are distinguished.

Anticipating Chapter 4, it should be noted that the variables identified in the pilot test as likely being the most significant were: NDONE, NRIGHT, PERIGHT (see p. 27).

3.2. <u>THE MAIN STUDY</u>

The Sample

44 fifth graders and 51 sixth graders participated in the main study. Forty-eight of the students were from grade six and the rest were fifth graders. Their ages ranged from eleven to thirteen years. They were all from the same school division.

The Method and Instrument

The modified design of the study is shown in Figure 4. The respective class teachers, as in the pilot test, gave the subjects a test of 60 randomly sequenced basic multiplication facts (as in the Appendix I(b)). Seventy-four students scored at least 90% on this test and were identified for the main study. A fifteen-minute, speed-independent test of multiplication questions(as shown in Appendix II) was then administered to all students.

Interviews were used to find each student's density of conditioned response.

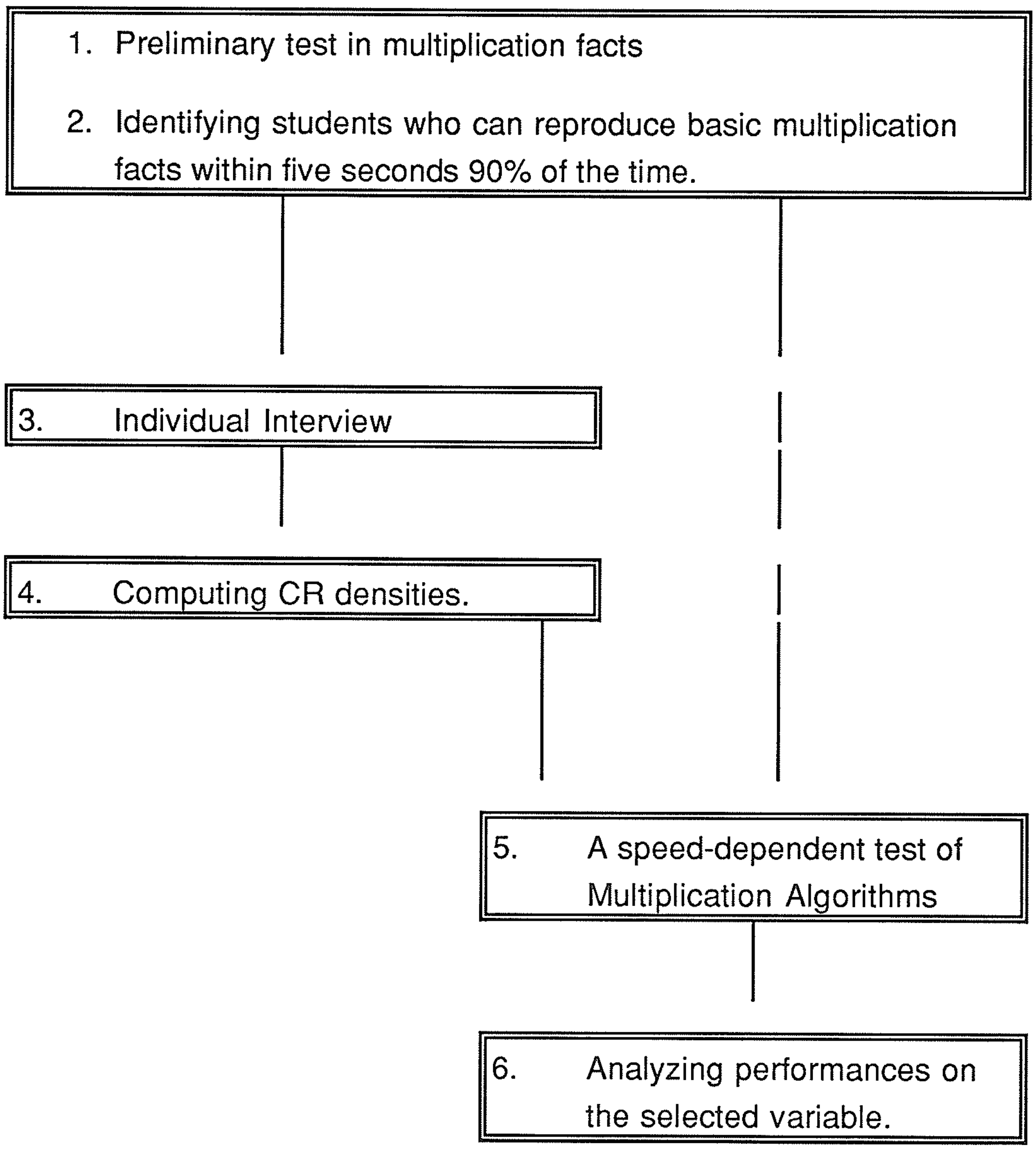
3.3. <u>THE STATISTICAL TREATMENT OF DATA</u>

In the pilot test, correlations were found between all pairs of CR ratios, NDONE, NRIGHT, PERIGHT, ERROR and the ERRPEG. Since the aim was to identify those comparisons providing greatest interest for the main study, simple tests of significance were used in all cases as though the variables were independent.

It was found that the most promising relationships seemed to be the correlations between the CR ratio and each of the variables: NDONE, NRIGHT, PERIGHT.

In the main study, Pearson Correlation Coefficients and the Bonferroni are used to analyze the data. The Bonferroni technique is used because the correlations found are not independent.

Figure 4: Detailed Flowchart for the Main study



An overall alpha of 0.1 is partitioned as shown in Table 1.

Table 1: Division of alpha for the Main study.

Variable	Alpha
CR vs NDONE	0.025
CR vs NRIGHT	0.05
CR vs PERIGHT	0.025
Total	**0.10**

The correlation between CR and NRIGHT is allocated the largest component of the overall alpha because NRIGHT seems to be the fundamental target variable.

This design is likely to detect results of any practical significance in a conservative way.

The data is analyzed in the next chapter.

CHAPTER 4

RESULTS

4.1 THE PILOT TEST

The Sample

Twenty-two grade 6 and twenty-six grade 5 students, from the same elementary school in a Winnipeg-area School Division participated in the pilot study. The ages of the subjects ranged from eleven years to thirteen years.

Pilot Test Data

TABLE 2: Descriptive Statistics of the Variables for Grade 5 and **Grade 6** students

Variable	Mean	Median	Std Deviation	Std Error of Mean	Minimum	Maximum
NDONE	23.81 **27.32**	25.00 **27.50**	9.23 **5.75**	1.81 **1.23**	7 **15**	42 **41**
NRIGHT	17.85 **23.45**	17.50 **24.00**	9.91 **6.34**	1.94 **1.35**	2 **10**	41 **41**
PERIGHT (%)	71.21 **85.19**	74.76 **85.05**	18.90 **10.42**	3.71 **2.22**	25 **64.29**	97.62 **100.00**
ERROR	5.96 **3.86**	5.5 **4.5**	3.75 **2.64**	0.73 **0.56**	1 **0**	15 **10**
ERRPEG	1.19 **0.95**	1.15 **1**	0.21 **0.42**	0.04 **0.09**	1.00 **0**	1.80 **1.67**
CR (%)	42.23 **50.68**	38 **45**	28.87 **25.27**	5.66 **5.39**	0 **10**	100 **100**

Table 2 shows that Grade 5 students' mean examples completed were 4 examples fewer, and a mean of 4 examples fewer on the number of examples completed correctly than the Grade 6 students. The older students, sixth graders, have smaller means on ERROR and ERRPEG than the younger ones in Grade 5. The latter also have a slightly lower mean CR ratio than the sixth graders.

Grade 5

Fifth grade students completed questions on the multiplication algorithm test with a mean of 18 correct answers. The average proportion of correct answers(PERIGHT) of examples completed was 71.2%.

Since this was a pilot study, correlations between variables were tested as though they are independent. Intercorrelations were computed between the CR ratios, the numbers of questions completed (NDONE), numbers of questions completed correctly (NRIGHT), the proportion of examples answered correctly (PERIGHT), the numbers of errors of those completed (ERROR), and the errors per question completed (ERRPEG). The correlation coefficients are as shown in Table 3.

Table 3: Pearson Correlation Coefficients and their **p**-values for twenty-six Grade 5 students.

Variable	NDONE	NRIGHT	PERIGHT	ERROR	ERRPEG	©®
NDONE	1.000	0.926	0.533	0.013	-0.519	0.256 ***0.21***
NRIGHT		1.000	0.770	-0.366	-0.594	0.370 ***0.06***
PERIGHT			1.000	-0.724	-0.615	0.356 ***0.07***
ERROR				1.000	0.295	-0.350 ***0.08***
ERRPEG					1.000	-0.425 **0.03**

The matrix displayed in Table 3 indicates the coefficients of correlation between pairs of variables. From Table 3, the CR (Conditioned Response) and the NDONE (the numbers of questions the students completed) had a correlation coefficient of +0.26, showing a moderate association. This correlation is not statistically significant at alpha = 0.10. The Pearson correlation coefficient of CR (the conditioned response students) and PERIGHT was above +0.36. This Pearson correlation coefficient is statistically significance at alpha $\leq$ 0.06. CR and ERROR's had a negative correlation coefficient of 0.36, and a p-value of 0.0744. Similarly, the correlation coefficient between ERRPEG and the CR was also negative with r = -0.42526.

Grade 6

These children not only completed more questions with fewer mistakes, but also had a higher proportion of right answers than the fifth graders. They had a mean score of 85.19% on the examples they completed correctly (PERIGHT) as compared to 71.21%, for fifth graders.

Table 4: Pearson Correlation Coefficients and their **p**-values for Grade 6 students.

Variable	NDONE	NRIGHT	PERIGHT	ERROR	ERRPEG	CR
NDONE	1.000	0.909	0.319	-0.003	-0.106	0.433 ***0.04***
NRIGHT		1.000	0.672	-0.420	-0.359	0.436 ***0.04***
PERIGHT			1.000	-0.917	-0.585	0.266 ***0.31***
ERROR				1.000	0.631	-0.106 ***0.64***
ERRPEG					1.000	0.185 **0.03**

There were moderate correlation between CR and NDONE, and between CR and NRIGHT with a p-value of 0.04. In Table 4, the correlation coefficients between each of the pairs (CR vs NDONE and CR vs NRIGHT) were 0.433 and 0.436 respectively. There was a strong positive association between the NDONE and NRIGHT, and an equally strong negative correlation between ERROR and PERIGHT.

The Sample (Grade 5 + 6)

TABLE 5: Descriptive Statistics of the Variables for the forty-eight Grade 5 + 6 students

Variable	Mean	Median	Std Deviation	Std Error of Mean	Minimum	Maximum
NDONE	25.42	26	7.95	1.15	7	42
NRIGHT	20.42	22	8.84	1.28	2	41
PERIGHT (%)	77.61	81.37	3.42	0.49	0	15
ERROR	5	5	3.42	0.49	0	15
ERRPEG	1.08	1	0.34	0.05	0	1.8
CR (%)	46.10	42.5	27.33	3.94	0	100

From Table 5, the means of the forty-eight students' NDONE was 26 examples, NRIGHT was 21 examples, a ratio of 77.61% of the completed examples correct, and 5 ERROR with 2 mistakes per incorrect examples.

Table 6: Pilot test: Fifth and sixth grades combined.
Pearson Correlation Coefficients and their **p**-values.

Variable	NDONE	NRIGHT	PERIGHT	ERROR	ERRPEG	CR
NDONE	1.000	0.922	0.522	-0.031	-0.317	0.331 **0.0217**
NRIGHT		1.000	0.776	-0.442	-0.472	0.412 **0.0036**
PERIGHT			1.000	-0.792	-0.568	0.344 **0.0165**
ERROR				1.000	0.483	-0.297 **0.0402**
ERRPEG					1.000	-0.108 **0.4661**

The correlation coefficients in Table 6, indicate that CR and the variables NDONE, NRIGHT, PERIGHT are moderately associated.

Given these pilot study results, it was decided that in the main study the correlation coefficients between the CR ratios and each of the variables: NDONE, NRIGHT, and PERIGHT would be identified for close study using rigorous statistical tests.

4.2 *The Main Study*

The Sample

Seventy-four participants met the ninety percent criteria for the basic multiplication test. Forty-eight of these students were from grade 6 and the rest were fifth graders.

The Main Study Results

Seventy-four grade 5 and 6 students from the same Winnipeg-area school division participated in the main study. Their ages ranged from eleven years to thirteen years.

The following tables summarize the data from of the main study.

TABLE 7: Descriptive Statistics of the Variables for the seventy-four Grade 5 + 6 students in the main study.

Variable	Mean	Median	Std Deviation	Std Error of Mean	Minimum	Maximum
NDONE	19.55	18	7.98	0.93	4	36
NRIGHT	14.16	12	7.43	0.86	2	30
PERIGHT (%)	70.50	72.73	17.45	2.03	12.5	100
ERROR	5.32	5	3.25	0.38	0	14
CR (%)	66.94	70	21.33	2.48	0	100

Table 8: Pearson correlation coefficient, the alpha, the Bonferroni test-statistic, and the critical value for the seventy-four students in the main study.

Variable	CR	Alpha	Test-statistic	Critical Value
NDONE	0.540	0.025	5.444	1.99*
NRIGHT	0.64492	0.050	7.160	1.67*
PERIGHT	0.5144	0.025	5.090	1.99*

* Significant at its respective levels where $H_0 : r = 0$.

Table 7 showed that the mean of NDONE was 20 examples with variance of 63.757. 75% of the subjects completed 27 examples in the given fifteen minutes.

The correlation coefficients (r), from Table 8 indicate a significant relationship between the respective pairs of variables earmarked by the pilot test. The correlation between CR density and NRIGHT was the strongest among the identified couples of correlations. The correlation coefficient for this pair is 0.645 and following the Bonferroni, is statistically significant at alpha = 0.10. These results provide evidence in support of the hypothesis motivating this study -- there exists an association between these students' CR and their proficiency in algorithms.

4.3 Further Explorations

Out of curiosity, the sample was divided into three groups -- a conditioned response group, a repeated response group, and a cognitive by-pass group -- at each grade level and at a combination of both grades. The conditioned response ratios and the descriptive statistics for each of the other variables are reported at each grade level and for a combination of both grade levels. The conditioned response group of students were interviewees who responded correctly within one second to at least 90% of the basic multiplication facts presented. Interviewees who responded within one second 90% of the time after they had repeated the question were categorized as the repeated response group(RR) or Group II. All interviewee-responses who did not qualify for Group I or II were placed in Group III, known as the cognitive by-pass group.

The results are summarized in Tables 9 -- 11. 26.92% of the fifth graders and 62.54% of sixth graders were classified in the conditioned response group. From Table 11, CB students completed only **half** as many examples as their peers in CR group (or Group I), and two-thirds as many as those in the RR group (or Group II) on NDONE score. Students in Group I outperformed their counterparts in the other two groups with regard to the variables obtained from the multiplication algorithm and the interview.

Association(s) may exist among the variables but this study was not designed to test for them. It seems that the verdict of such a study would be the same -- students with conditioned responses will be more successful in their later algorithmic performances.

Table 9: Descriptive statistics for grade 5 students in each group for the variables: NDONE, NRIGHT, PERIGHT, and CR.

Group	Variable	N	Mean	Median	Std Deviation	S_e
I	NDONE	7	18.71	15	7.41	2.80
	NRIGHT		16.14	13	6.99	2.64
CB	PERIGHT		86.12	86.21	11.78	4.45
	CR		84.29	80	12.72	4.81
II	NDONE	7	16.57	15	8.73	3.30
	NRIGHT		12.00	9	6.58	2.49
RR	PERIGHT		72.03	72.22	10.86	4.11
	CR		82.86	90	11.13	4.21
III	NDONE	12	11.50	11	4.21	1.22
	NRIGHT		5.75	5.5	2.96	0.85
CB	PERIGHT		51.83	52.28	20.16	5.82
	CR		38.33	45	15.86	4.58

Table 10: Descriptive statistics for grade 6 students in each group for the variables: NDONE, NRIGHT, PERIGHT, and CR.

Group	Variable	N	Mean	Median	Std Deviation	S_e
I	NDONE	30	24.87	25	6.61	1.21
	NRIGHT		19.23	20	6.22	1.
CR	PERIGHT		76.68	80.03	11.97	2.19
	CR		76.78	76.65	13.43	2.45
II	NDONE	11	19.64	19	5.99	1.81
	NRIGHT		13.09	12	5.74	5.04
RR	PERIGHT		66.15	63.16	16.70	5.04
	CR		67.27	70	10.09	3.02
III	NDONE	7	14.29	14	5.15	1.95
	NRIGHT		8.71	8	2.06	0.78
CB	PERIGHT		65.67	71.43	17.53	6.63
	CR		40.000	40	11.55	4.36

Table 11: Descriptive statistics for students (at both grade levels) in each group for the variables: NDONE, NRIGHT, PERIGHT, and CR.

Group	Variable	N	Mean	Median	Std Deviation	S_e
I	NDONE	37	23.70	25	7.09	1.17
	NRIGHT		18.65	18	6.39	1.05
CR	PERIGHT		78.47	81.25	12.35	2.03
	CR		78.20	80	13.46	2.21
II	NDONE	18	18.44	15	7.10	1.67
	NRIGHT		12.67	11	5.91	1.39
RR	PERIGHT		68.44	69.34	14.65	3.45
	CR		73.33	75	12.83	3.02
III	NDONE	19	12.53	13	4.65	1.07
	NRIGHT		6.84	7	2.99	0.69
CB	PERIGHT		56.93	55.56	19.95	4.58
	CR		38.95	40	14.10	3.24

CHAPTER 5

CONCLUDING REMARKS

5.1 SUMMARY

Students who met a 90% criterion level on a speed-independent test of basic facts were identified. The researcher categorized students' responses as CR or CB using one-on-one interviews. Their conditioned response densities and multiplication algorithm scores were analyzed. The data showed that students with high level of conditioned response ratios were most proficient in the performance of algorithms. The Bonferroni test supported the significance of the respective correlations at each predetermined level of significance.

There is, therefore, strong evidence to conclude that it is preferable for students to use conditioned responses, and the latter should be taught. Two questions remain to be resolved, (i) what is the best way to teach conditioned responses? (ii) Should understanding the basic facts and/or algorithms precede the learning of conditioned responses?

5.2 OTHER OBSERVATIONS

As a possible guide to future research, it was decided to partition the sample into three groups particularly so as to see if there are any qualities of the RR (repeated response) group that suggest a need for closer study of them.

The data provided may be of interest for further study, but no statistically valid conclusions can be drawn from that data.

Some of the errors found may be attributable to illegible handwriting. For instance, the digits four and nine, and zeroes and sixes, may have been commonly confused. It is possible that clarity of handwriting may be a significant factor in algorithmic proficiency.

Some grade 5 and 6 students do not seem to know their basic addition facts and addition algorithms. This may also be a significant factor.

It was noted that as they worked on multiplication algorithms, some students scratched their heads, tapped their pencils or indulged in other kinetic behaviors. In what may be a similar phenomenon, some college students seem to need to vocalize while they are thinking, as when they are keying data into a computer or writing a paper. And some teachers assert that they think best in front of a chalkboard or an audience. All of these observations suggest that these kinetic and/or vocal behaviors may have some connection with the manner in which we recall mathematical facts, thoughts and knowledge, and may warrant more attention than they have received.

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APPENDICES

Appendix 1(a)

Appendix 1(a): 45 Pairs of Multiplication Facts.

x	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3		9	12	15	18	21	24	27	30
4			16	20	24	28	32	36	40
5				25	30	35	40	45	50
6					36	42	48	54	60
7						49	56	63	70
8							64	72	80
9								81	90
10									100

Appendix 1(b)

Appendix 1(b): The Basic Facts Test

1	8 x 2 =	21	4 x 4 =	41	7 x 7 =
2	10 x 10 =	22	3 x 2 =	42	2 x 5 =
3	7 x 6 =	23	6 x 3 =	43	4 x 6 =
4	9 x 2 =	24	6 x 9 =	44	6 x 7 =
5	6 x 9 =	25	2 x 8 =	45	5 x 6 =
6	5 x 8 =	26	3 x 7 =	46	7 x 5 =
7	2 x 4 =	27	6 x 2 =	47	8 x 7 =
8	3 x 5 =	28	8 x 4 =	48	2 x 3 =
9	9 x 6 =	29	9 x 7 =	49	5 x 3 =
10	5 x 7	30	3 x 6 =	50	5 x 9 =
11	8 x 2 =	31	9 x 3 =	51	7 x 4 =
12	2 x 7 =	32	3 x 4 =	52	4 x 8 =
13	8 x 6 =	33	9 x 5 =	53	9 x 8 =
14	6 x 5 =	34	6 x 6 =	54	2 x 4 =
15	4 x 9 =	35	7 x9 =	55	4 x 5 =
16	8 x8 =	36	5 x4 =	56	4 x 2 =
17	7 x3 =	37	7 x 2 =	57	2 x 6 =
18	8 x 5 =	38	8 x9 =	58	3 x 9 =
19	2 x 9 =	39	5 x 5 =	59	5 x 2 =
20	3 x 8 =	40	9 x 4 =	60	6 x 4 =

Appendix II

Multiplication Algorithm

Name: ____ **Grade :**______

[Last first middle]

1 7 9 (x) 8 3	3 1 9 (x) 3 5	2 0 8 (x) 4 7
6 2 (x) 7 8	5 3 4 (x) 9 8	3 1 2 (x) 1 6
2 0 9 (x) 2 4	1 8 1 (x)5 2 3	5 9 5 (x) 7 6

7 9 9 (x) 4 5	6 9 4 (x) 5 8	3 3 4 (x) 3 3
5 2 2 (x) 6 7	1 3 9 (x) 2 5	1 6 3 (x)9 4 8
1 9 8 (x) 8 5	4 1 (x) 4 6	1 4 5 (x) 6 3
6 6 (x) 9 1	3 2 1 (x) 8 6	1 0 4 (x) 2 5
4 5 3 (x) 7 7	4 5 (x) 3 6	8 2 7 (x) 5 4

8 2 6 (x) 3 7	9 1 6 (x) 4 6	6 9 3 (x) 7 9
1 4 8 (x) 2 4	6 2 5 (x) 9 6	3 6 2 (x) 3 9
2 3 9 (x) 6 9	9 9 5 (x) 7 6	9 1 2 (x) 9 9
9 0 7 (x)5 6 4	2 2 3 (x) 5 8	4 6 5 (x) 7 3
1 9 4 (x) 7 4	3 4 6 (x) 9 5	2 7 9 (x) 8 2

6 8 9 (x) 3 5	7 4 8 (x) 8 7	3 9 9 (x)1 7 2
8 9 7 (x) 6 8	6 6 5 (x) 6 6	6 0 9 (x) 5 2
4 5 8 (x) 3 4	9 4 3 (x) 4 2	7 6 4 (x)2 6 8
9 0 2 (x) 2 9	7 6 9 (x) 8 8	8 3 1 (x) 4 9
1 8 5 (x) 9 4	5 5 3 (x) 7 2	6 0 7 (x) 5 6

4 7 5 (x) 6 4	3 6 3 (x) 2 7	1 8 6 (x) 9 7
5 6 8 (x) 6 5	3 8 8 (x)4 3 2	5 7 7 (x) 4 6
2 5 3 (x) 1 9 6	3 5 1 (x) 2 6	8 8 9 (x) 7 2
5 6 3 (x)9 0 7	3 8 6 (x)4 2 5	2 5 9 (x)1 4 2
1 8 3 (x) 3 7	9 9 7 (x) 3 2	6 4 2 (x)1 5 6

6 4 4 (x) 4 8	2 7 3 (x) 5 4	5 8 7 (x) 2 7
2 0 5 (x) 3 6	4 5 6 (x) 2 7	9 9 (x) 2 3
8 2 6 (x) 5 4	9 2 4 (x)2 2 8	5 9 9 (x) 3 9
8 6 (x) 2 5	5 1 3 (x) 9 4	9 1 8 (x) 7 3

Appendix III

Interview Protocol

The interviewer will

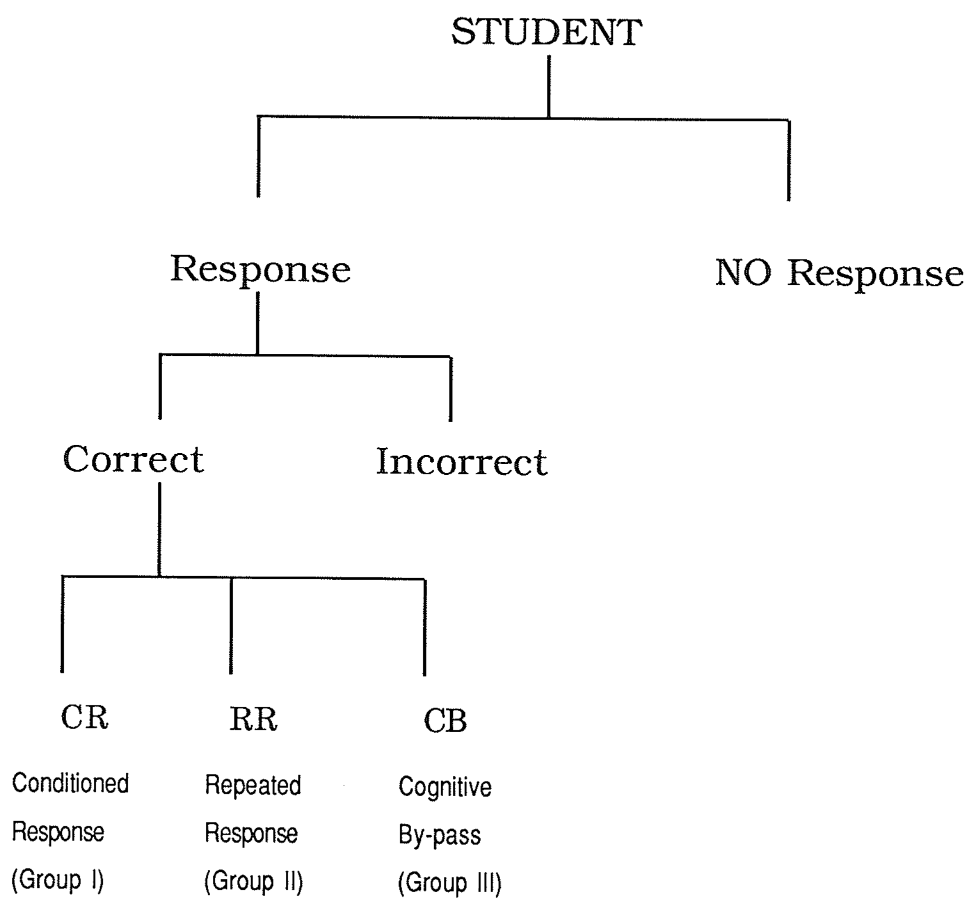
1. ask each student to reply as quickly as possible to the basic multiplication facts presented.

2. inform the interviewee that s/he can use the paper and pencil provided, or use their fingers or any other desired strategies to produce facts.

3. present the child with ten to fifteen basic facts, depending on how the interview proceeds.

4. categorize responses as resulting from cognitive bypasses(CB), conditioned responses(CR), or repeated responses(RR). Students who responded within one second with the correct answer are in Group I (the CR group) if such behavior is noted 90% of the time. If they respond correctly within one second after repeating the question 90% of the time they are classified in Group II (the RR group). All other patterns of response assign the student to Group III (the CB group). For the main study, the RR group of interviewees are combined with the CB group.

The birds' eyes view of the interview protocol including partition of sample in the main study.



5. When a child has provided sufficient responses to permit the interviewer to categorize her/his mode of response, the interview is terminated. Each interview entails three to six minutes of student time.