THE VERIFICATION OF WEBER'S LAW IN VISION

by

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THE VERIFICATION OF WEBER'S LAW IN VISION.

Historical

Since the idea of brightness is common to all estimates of the mental impression produced by lights, colored or colorless, we should expect that early in the history of optical research, attempts would be made to find a relation between the intensity of the sensation and the intensity of the objective light used as stimulus. As there is even yet no means of measuring the absolute intensity of a sensation in terms of some definite unit, the problem has been attacked from the view-point of how the intensity of the light sensation is affected by changes in intensity of the objective light.

The smallest perceptible gradations of the light sense do not correspond to equal changes of objective luminosity, but, the higher the intensity of the stimulus, the greater must be the change in its intensity to cause a change in the value of the sensation. In general, photometric measurements show that for very different degrees of luminosity, the difference of luminosity that can be just perceived is nearly the same fraction of the total luminosity.

In 1760, Bouguer (1) tried to determine the amount of this difference by illuminating a white screen by two equal candles, one of the candles casting a shadow of a rod on the screen. He then moved this candle away from the screen until the shadow made by it just ceased to be visible. He found that this candle had to be about eight times farther than the other for the shadow to vanish, i.e., the ratio of the intensities of the shadow and the background was approximately 1:64. Thus he could just perceive a difference of about 1/64 of the total luminosity. He noted that he had not observed a change in the ratio when the brightness of the lights was varied.

Later, Arago (2) (1858), in developing Bouguer's work, noticed that when movement was involved still smaller changes of intensity could be perceived. Under the most favorable conditions he was able to recognize a difference of 1/131 of the total luminosity.

Masson's (3) experiments, published in 1845, were made with rotating white discs with small black sectors. He found that under poor conditions of vision the perceptible fraction was sometimes as low as 1/50, but that with good visual conditions it might be even less than 1/120. He further reported that, though different people give different values of $\Delta I/I$, the ratio is constant for a given person regardless of intensity or color.

Steinheil (4) (1837), with his newly-invented prism photometer, found the just perceptible difference in intensity was 1/38.

About the same time, Weber (5) (1834) discovered for the sense of touch that he could discriminate between two weights if they differed by one or two parts in thirty, and, similarly, that he could just discriminate visually between two lines if they differed by 1 part in 100, regardless of their absolute magnitudes. He concluded as a result of his investigations, that, "The just appreciable increase of stimulus bears a constant ratio to the original stimulus", or, expressed algebraically,

$$\frac{\Delta I}{I} = C$$

where ΔI denotes the just appreciable increase of stimulus, or, as it is generally termed, "the differential threshold"; I is the physical intensity of the original stimulus; and C is a constant. This statement, now known as Weber's law, sums up all the work done previous to this date.

In a paper published in 1858, Fechner (6) stated that repetition of Bouguer's experiments showed the value of $\Delta I/I$ to be uniformly 1/100. Extending Weber's work, Fechner then attempted to express sensations in terms of quantitative units. He assumed that all just-perceptible differences of sensation contain an equal number of sensation units, i.e., that ΔI always liberates the same whole number of sensation units, the "C" in Weber's relation being the measure of this. According to Parson's treatment (7), Fechner's law states that: "The sensation varies as the logarithm of the stimulus; i.e., the sensation changes in aritmetical proportion as the stimulus increases in geometrical proportion.

Stated algebraically, if S is the measure of a sensation, and δS the just appreciable difference, I the measure of the stimulus, and ΔI the small increment, then:

$$\delta S = k \frac{\delta I}{I} \qquad \text{(Weber's Law)}$$

where k is a constant; therefore, on the questionable assumption that it is permissible to integrate small finite quantities, (δS, etc.),

$$S = k \int \frac{\delta I}{I}$$

$$= k \log I + k' \qquad \text{(Fechner's Law)}$$

where k' is another constant."

Fechner realized that Weber's ratio is constant only within limits, and he set these limits at the two extremes of the intensity scale, the upper limit being due to the dazzling effect of high intensities, the lower to the intrinsic light of the retina. To take this latter into consideration, he later (1860) introduced its value I_o into Weber's equation, which then became:

$$\delta S = k \frac{\Delta I}{I + I_o}$$

Discussing these limits, Helmholtz (1866) (8) made the first criticism of the Weber-Fechner law. He had experimented with Masson's method, and had found, contrary to the latter, that the ratio $\Delta I/I$ varied at different intensities; e.g. $\Delta I/I$ was equal to 1/150 - 1/167 with outside daylight, and increased to 1/117 with interior daylight. From this he concluded that the circumstances "...........which vitiate Fechner's law at the upper and lower limits show their influence, under accurate observation, in the medium illuminations as well, which naturally does not prevent the law from being a first approximation to the truth".

The first reliable investigation of the variations in value of the ratio $\Delta I/I$ over a wide range of intensities was made by Aubert (1865) (9), using a procedure essentially the same as Bouguer's. The intensities used ranged from those barely perceptible to those obtained by admitting daylight into the dark room. He found that $\Delta I/I$ was roughly constant over a very limited range of illuminations, and beyond that range the discrimination ratio decreased steadily as the intensity increased.

Aubert's work was followed by that of Konig and Brodhun (1887) (10), "whose data have become the final statement of the behavior of $\Delta I/I$ for the eye" (Hecht).

Konig, keeping his eye in darkness adaptation, used as a sensory stimulus a patch of light viewed through a suitable ocular. The upper half of the patch was maintained at a definite intensity, the intensity of the lower half being varied by nicol prisms until it was just perceptibly brighter than the upper. The difference between the physical intensities of the two patches was then determined by the relative rotations of the nicols, and taken as the value of the differential threshold ΔI. The intensity I was then altered by the numerical value of ΔI to give a new I, and in this manner the readings repeated throughout the whole intensity range of the I. Data were obtained for various monochromatic radiations and for white light, and some of these are given in Table I. -Page 23 There is no evidence of a linear relation in these data, but as the intensity increases the ratio $\Delta I/I$ first decreases and theniincreases.

The Present Status of Weber's Law.

This law has been subjected to many experimental tests, not only in vision but in some of the other senses. The original work of observers in senses other than vision is not discussed in detail by authors, though they conclude that the law is only an approximation for any sense. A great amount of work has been done on the eye, König's, as stated above, being generally accepted as the best. His extensive researches in vision preclude any errors in his observations, and his work is consequently the most frequently quoted, (e.g. Hecht 1924) (11), as proof that the Weber-Fechner relation is not valid.

Again, Parsons states (12): "Weber's law does not hold good for very high or very low intensities of stimuli, as is only approximate at the best".

Recently this law was carefully investigated experimentally by Macdonald and Allen (13), their results proving that Weber's law does hold exactly, but that at certain intensities there are abrupt changes in the value of the constant. This change in value of the constant probably misled investigators who had expected to find a single constant holding for all ranges of intensity. The different numerical values for the ratio obtained by different observers could be accounted for on this basis, and also on an observation made by the above-mentioned authors, that the ratio varies with the area of the stimulus examined. One of the curves obtained is shown in Fig.I. It bears a close resemblance to the Ferry-Porter graphs.

How are we then to explain the opposing results of the above exact work and that of König? To quote from the paper by Macdonald and Allen: "While König's measurements are probably beyond criticism, the interpretation of them as a valid experimental examination of the Weber law appears to be unsound for the following reasons:-

In observing the patch of light, one half being at an intensity I, and the other at a greater intensity $(I + \Delta I)$, two sensations are evoked by the stimulation of two adjoining retinal areas, whereas the Weber law deals with but a single sensation at any one time.

It has been shown by Allen (14), that stimulation of a retinal area elicits complex neural reactions which modify the sensitivity of adjoining areas. The magnitude of the inductive reactions varies in some way with the intensity and duration of the stimulation, and with the wave-length of the light employed. The unequal stimulation of two adjoining areas by the same hues but of different intensities, as in König's experiments, causes mutual inductive actions which tend to equalize the sensations. Under such conditions a value of ΔI obtained by slowly increasing the brightness of one patch cannot

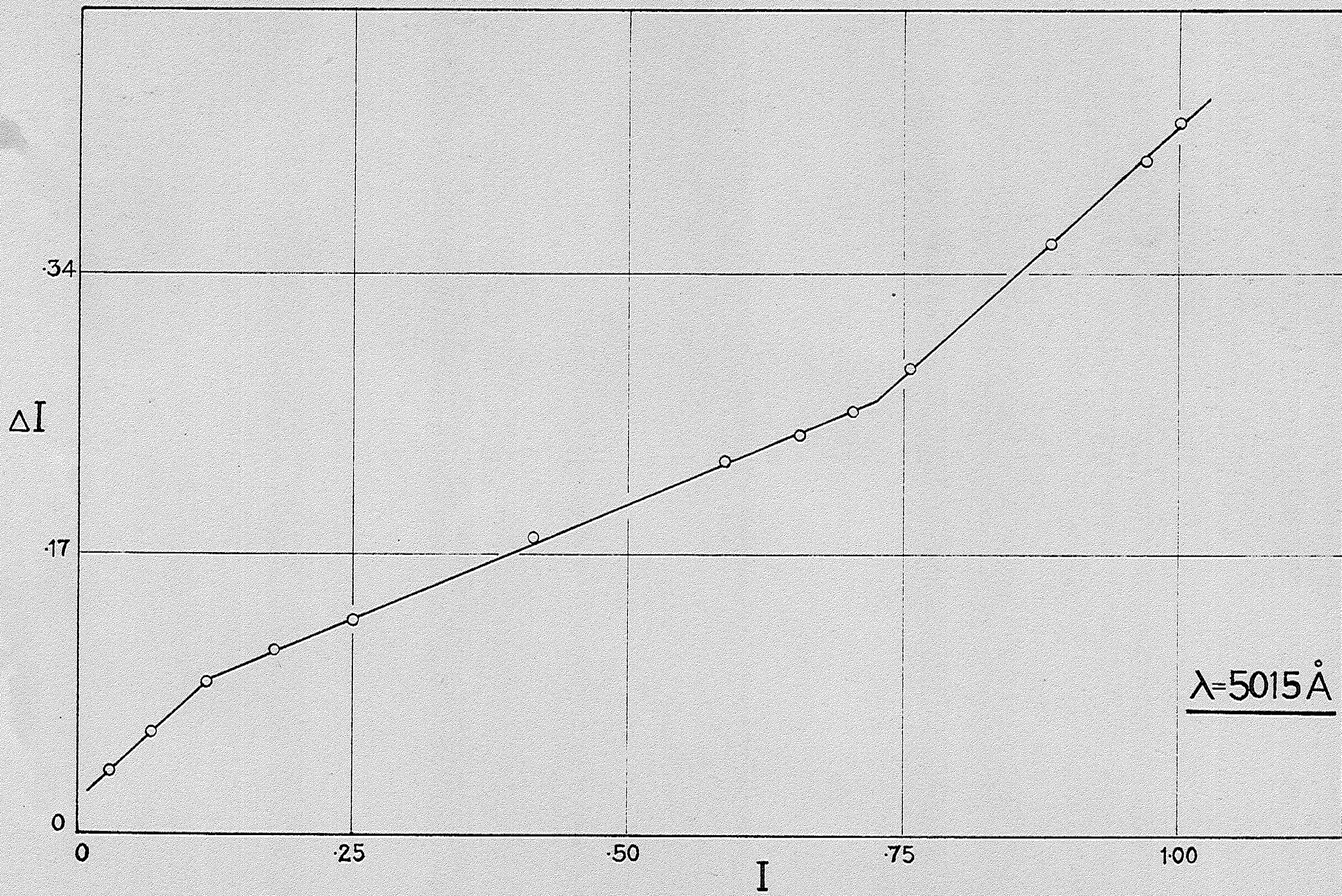


Fig. I

accurately represent the real differential threshold, and the ratio $\Delta I/I$ will consequently be either too large or too small. It has also been shown by Allen that the darkness adaptation of the unused eye further modifies the sensitivity of the receptors in the observing eye.

One of the outstanding features of the visual mechanism is its great amplitude of adaptation, or its ability to respond accurately to stimuli varying over a wide range of intensity. It might be expected that for the process of adaptation to operate in the most efficient manner, there would occur some modifications of the neural reaction processes at different ranges of intensity of stimulation. König, however, seems to have taken no precaution against the danger of concealing such possible modifications.

Consider, for example, his curve obtained with radiation of wave-length 6700 Å. There are in it only sixteen points, representing a range of intensities from the least perceptible to the greatest. The extensive graphical interpolation necessitated by such widespread points would effectively conceal any alterations in the response of the receptors except those of the greatest magnitude. Such measurements, therefore, should not be interpreted as a satisfactory experimental examination of Weber's law.

Again much work has been done with white light. It has been found by Allen (13) that each spectral wave-length is a physiological stimulus with a character peculiar to itself, differing, sometimes widely, from that possessed by every other hue. Since white light may be composed of all wave-lengths in the visible spectrum, and in varying amounts depending on the nature of the source, it would contain practically an infinite number of variables, and would, in consequence, elicit a corresponding complexity of sensations. In considering the interpretation of experimental results, it should be remembered that the essential condition of the scientific method of experimentation is the reduction to a minimum of the number of variables observed at any one time."

The work done by Macdonald and Allen was planned to eliminate all these criticisms of König's work. Monochromatic light was employed, and the use of a second patch of color as a standard of comparison was avoided by employing a small patch of light, over the whole of which a measured increment of the same color could be added uniformly by opening a shutter for a brief interval of time. The measurements were made in a room well lighted by diffused daylight, that is, both eyes were in a state of normal light adaptation. A large number of readings were made over smaller ranges of intensity than employed by König. In this way Weber's law was examined with three spectral colors - red, yellow, and green - of wave-lengths 6678 Å, 5875 Å and 5015 Å, and was found to hold exactly, as has already been mentioned. (Cf. Fig.I).

Purpose of this Research.

Since the non-validity of Weber's law is so firmly established in the minds of most psychologists and physiologists, it was decided to check the work of Macdonald and Allen. For this purpose a different apparatus and slightly altered technique were used, although the fundamental principles of the latter were unchanged.

In all the work done in this field up to the present time, the intensity and increment beams have been obtained from different sources the energy distributions of which were unknown. Consequently the energy relations between the intensity and increment beams for the various wave-lengths were unknown. In the present work, both beams were obtained from the same source, the energy distribution of which was known. All the increment and intensity measurements could therefore be computed to the same energy scale. It was hoped that in this way, some relationship might be found between the values of $\Delta I/I$ for different wave-lengths and intensities.

Experimental Apparatus

The apparatus used was designed by Dr. P. A. Macdonald under whose direction this work was carried out. A diagram of the apparatus is shown in Fig. II. The intensity and increment beams were obtained from the same source A, a 500 watt lamp having a special type of filament which gave practically a straight line source. When operated on 3.90 amps. the black-body temperature of this lamp was 2940°K. as determined by the U.S.A. Bureau of Standards. Its spectral energy distribution was computed from Planck's formula by means of special tables and graphs obtained from the U.S.A. Bureau of Standards, the energy values being correct to within 0.33%. Fig. III shows this energy distribution, the energy at 560 mμ having the value 100.

For the intensity beam I, light from A passed directly through the lens L_1 which focussed the image of the filament at the collimator slit S_1 of the spectroscope. The intensity of this beam was controlled by rotation of the polarizer of the pair of nicol prisms N_1.

The increment beam ΔI, was obtained from A by internal reflection at the four 90° prisms, P_1 P_2 P_3 P_4. The lens L_2 of long focal length, focussed the image of the filament at the slit S_1 of the spectroscope. The intensity

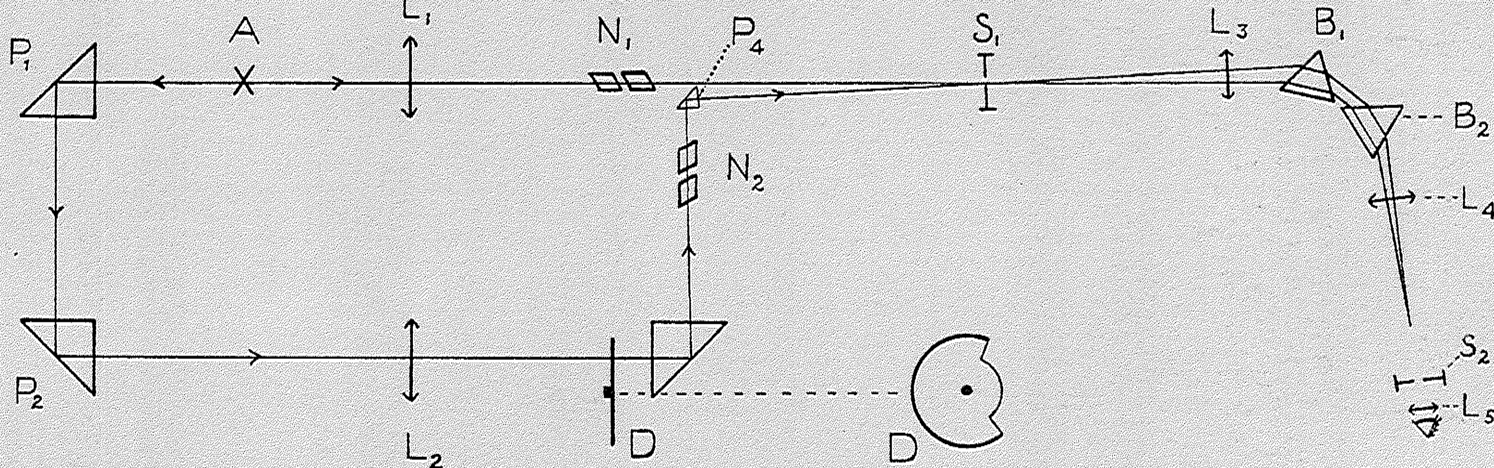


Fig.2

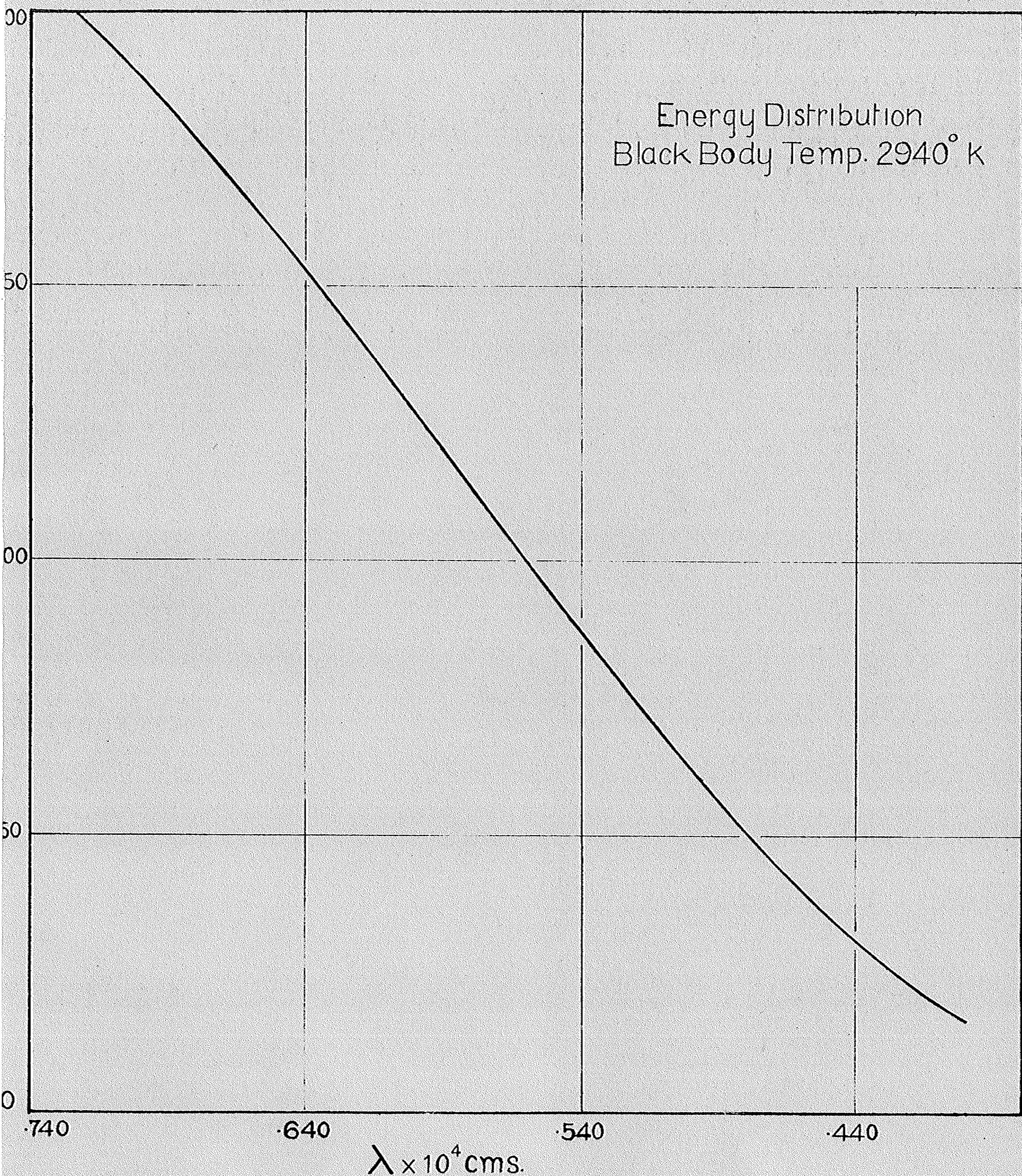

Energy Distribution
Black Body Temp. 2940° K
00
50
00
50
0
·740
·640
·540
·440
λ x 10⁴ cms.

Fig. 3

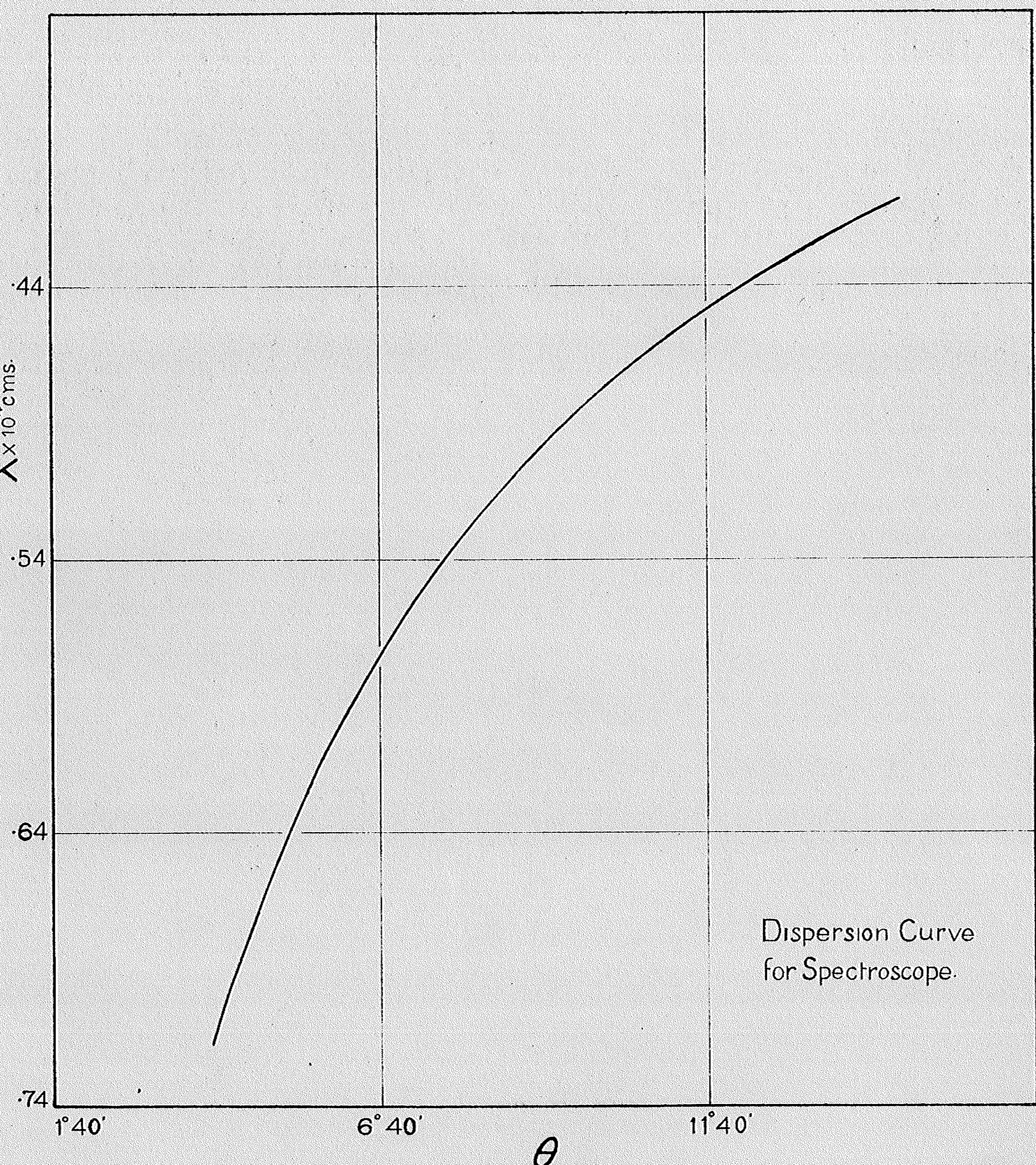
of this beam was controlled by rotation of the polarizer of the nicols N_2. The increment was cut off and on by the circular disc D rotated at a slow speed by a small synchronous motor operated from the A.C. supply. The velocity of this motor was correct to within 1%.

The spectroscope (S_1 L_3 B_1 B_2 L_4 L_5) used was a two prism type, carefully calibrated. Its dispersion curve is shown in Fig. IV. The area of the superimposed images I and ΔI viewed by the eye, was cut down by a shutter S_2 in the eyepiece, so that only the fovea centralis was used in making observations.

The apparatus was mounted in a room well-lighted by diffused daylight. Thus, both eyes were in the state of normal daylight adaptation in which they are ordinarily used.

Experimental Technique.

Before making an observation, the polarizer of the nicols N_1 was first set to give a definite intensity of stimulus I. The polarizer of the nicols N_2 was then, in making an observation, rotated till the flicker caused by ΔI being alternately cut off and on just ceased to be visible. The first reading taken at any one intensity I was generally only approximate. The polarizer of N_2 was



·44
·54
·64
·74
λ x 10⁴ cms.
1°40'
6°40'
11°40'
θ
Dispersion Curve
for Spectroscope.

Fig. 4

then set within about 30' of the value obtained, and a rest period of ten minutes observed to allow the eye to return to its normal state, after which another reading was taken.

All readings were taken using the right eye, the left eye being closed during the observation period to enable more exact fixation with the observing eye. As the time required to make these exact observations was never more than two seconds, any change in adaptation of the unused eye due to closing it would be too small to affect the sensitivity of the receptors in the observing eye.

The observations were usually made in the morning between the hours of ten and twelve, when the daylight was most constant, but occasionally readings were taken in the early afternoon as well. Readings were generally obtained one day using arbitrarily chosen values of I, ranging from the least perceptible to the greatest which the eye could observe without being dazzled (where this was less than the maximum possible from the apparatus used). From these readings an approximate curve was plotted which indicated where additional readings should be taken in order to define the character of the curves more accurately. Only rarely could all the readings necessary to obtain the complete curve for any one wave-length be taken in one day, so that two or more days were generally required. That it was justifiable to do this will be shown in the discussion of results.

The relative intensities of ΔI and I were determined by flicker photometry as used by Haycraft, Ferry and Allen, in which the critical frequency of alternation of the lights is measured, i.e. two lights are regarded as being of the same luminosity when the flicker produced by rapid alternation of such with black disappears at the same speed of alternation.

It had been hoped that the ratio $\Delta I/I$ might be determined in this manner throughout the entire spectrum but time permitted doing this for wave numbers 140 to 165 only.

Results.

The readings have been calculated in terms of relative energies, taking into account the energy distribution of the source and the dispersion of the spectroscope, the energy corresponding to 560 mμ being taken as 100. The values of ΔI and I so obtained are given on pages 16-18, and are plotted in Figs. 5-10.

N = 140

		ΔI	I
Feb.18th./30 A.M.	I	14.4	45.8
		18.3	64.0
		22.6	84.2
		26.6	106
Feb.19th./30 A.M.	II	30.0	106
		36.4	128
		43.8	150
Feb.21st./30 P.M.	III	54.8	192
		63.7	226
		69.1	248
Feb.22nd./30	III	66.3	256
		66.6	256
		64.3	248
		58.9	226

N = 145

		ΔI	I
Feb.7th./30 A.M.	I	5.68	16.2
		6.64	20.2
		9.09	31.0
		12.1	45.2
Feb.6th./30 P.M.	I	18.5	87.6
		20.0	95.0
		26.9	128
		28.4	135
Feb.7th./30 A.M.	II	30.3	147
		32.0	156
		40.1	206
		41.3	212

H = 150

Date	Group	ΔI	I
Feb.3rd./30 P.M.	I	2.42	11.8
	I	3.49	14.2
	I	4.30	16.8
	II	5.71	20.5
	II	8.22	31.4
Feb.7th/30 P.M.	II	10.9	43.9
	II	18.1	72.5
	II	25.3	103
	II	25.5	103
	II	32.3	132
	III	38.5	155
Feb.8th./30 A.M.	III	38.5	155
	III	43.9	170
	III	45.7	176

H = 155

Date	Group	ΔI	I
Feb.8th./30 P.M.	I	.424	1.18
	I	.891	3.02
	I	1.48	4.69
	I	2.54	7.88
	II	7.29	18.2
	II	11.5	27.9
Feb.10th./30 A.M.	II	7.15	18.2
	II	11.4	27.9
	II	14.3	34.3
	II	16.2	39.0
	II	21.7	51.3
	II	28.0	64.3
	II	33.8	77.9
Feb.11th./30	III	36.5	91.5
	III	38.0	99.4
	III	40.5	117
	III	40.7	117

H = 160

Date	Group	ΔI	I
Feb.11th./30 A.M.	I	.124	.391
	I	.312	.973
	I	.727	2.48
	I	1.09	3.87
Feb.12th./30 A.M.	II	2.06	6.48
	II	3.65	9.73
	II	6.11	15.0
	III	22.7	52.9
Feb.13th./30 A.M.	II	3.46	9.73
	II	3.47	9.73
	II	3.47	9.73
	II	5.95	15.0
	II	21.8	48.5
	III	22.8	52.9

H = 165

Date	Group	ΔI	I
Feb.14th./30 A.M.	I	.0841	.277
	I	.191	.768
	I	.339	1.50
	I	.605	3.05
	I / II	1.00	5.11
	II	2.72	7.68
	II / III	5.22	11.8
	III	7.40	18.1
	III	7.46	18.1
	III	10.6	25.3
	III	13.2	33.2

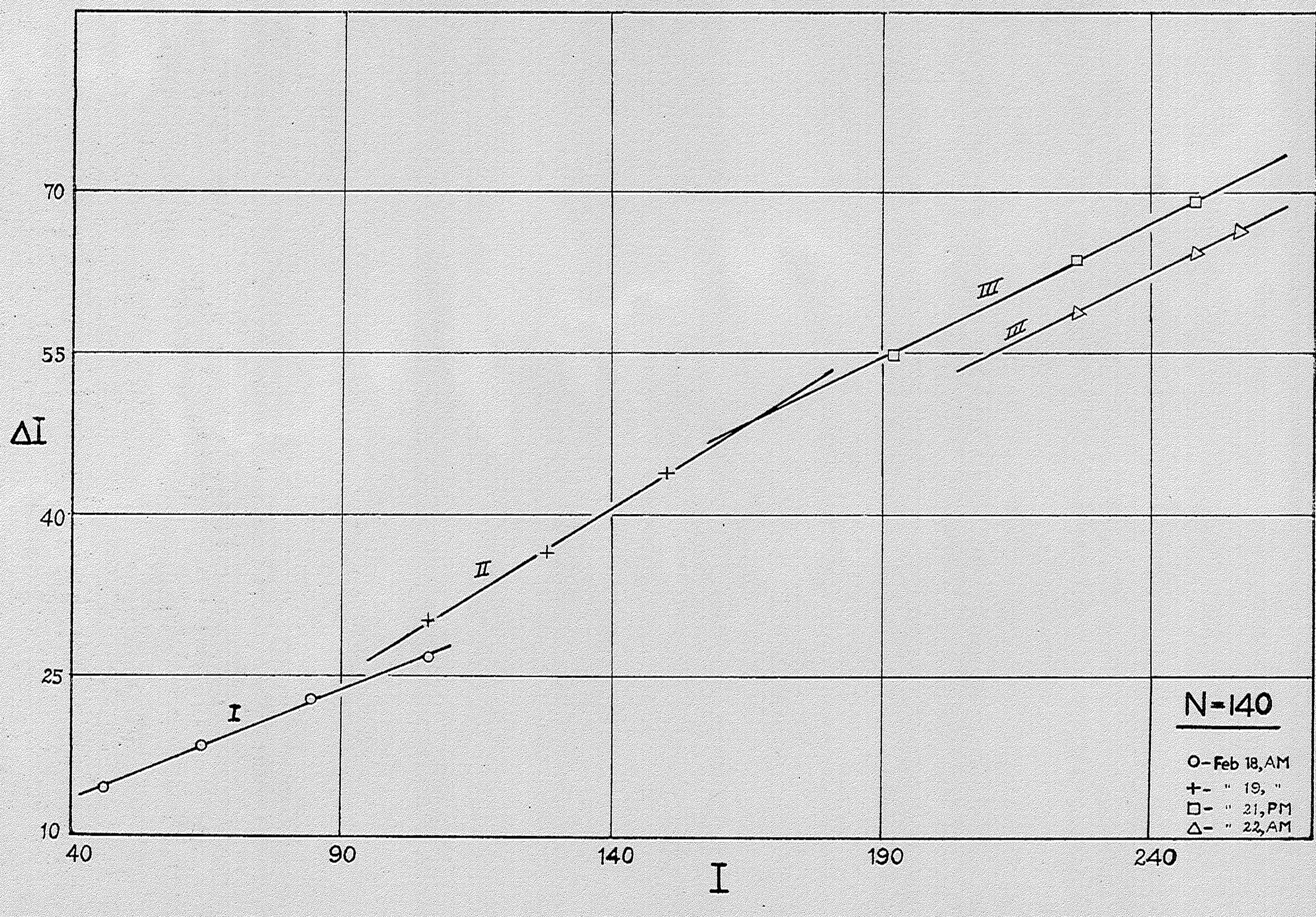

70
55
40
25
10
ΔI
40
90
140
190
240
I
I
II
III
III
N=140
O- Feb 18, AM
+- " 19, "
□- " 21, PM
△- " 22, AM

Fig. 5

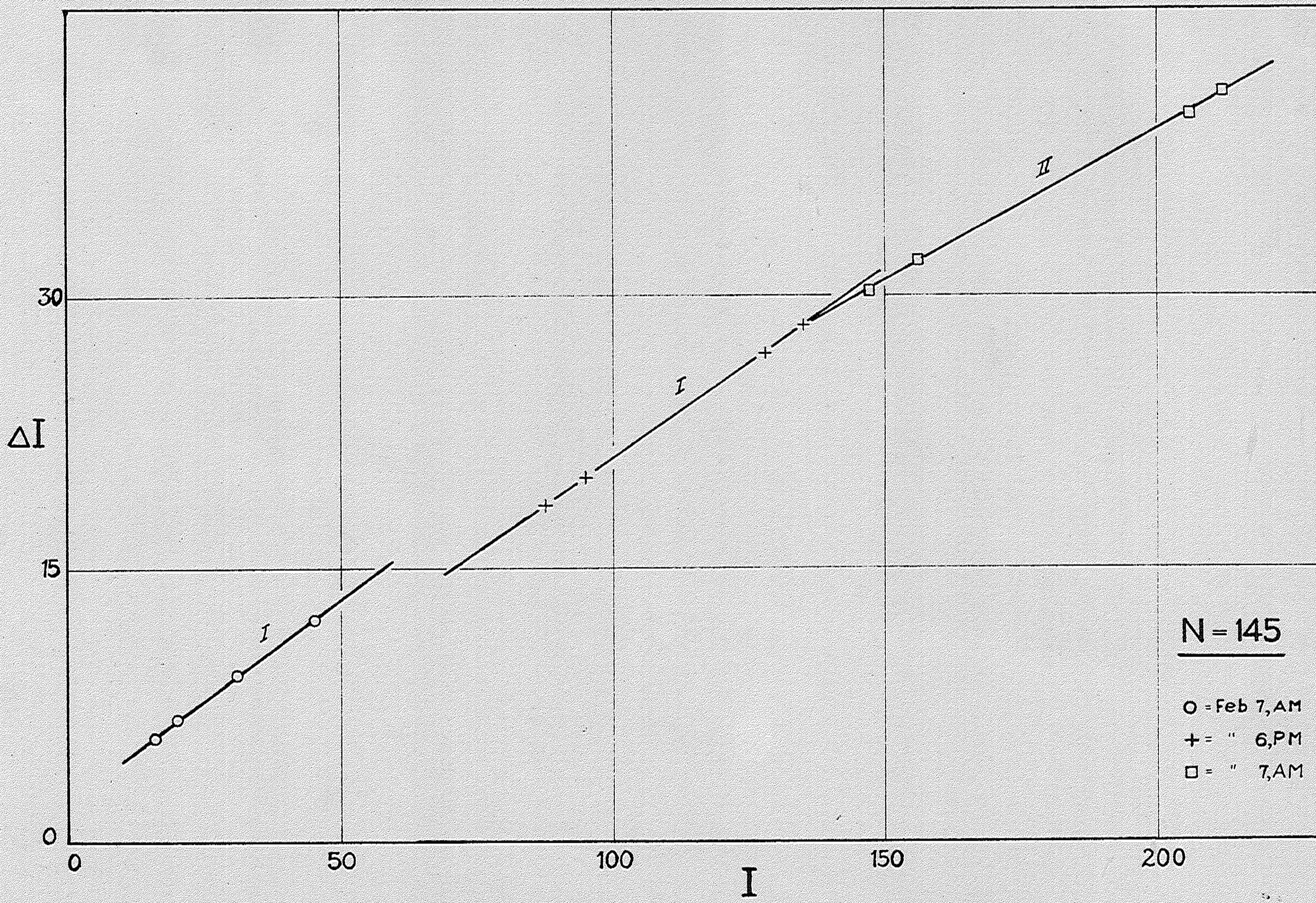


Fig. 6

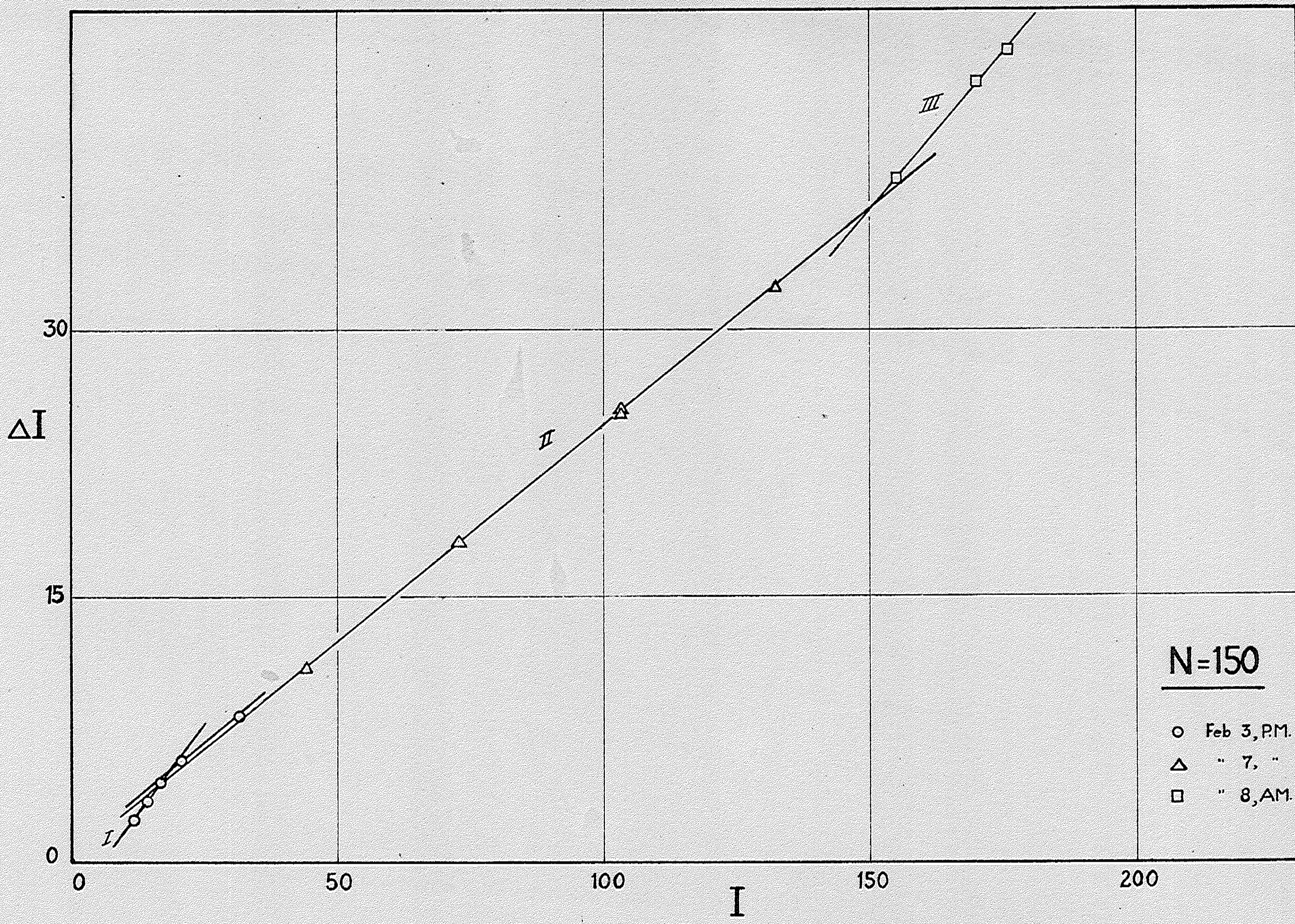


Fig. 7

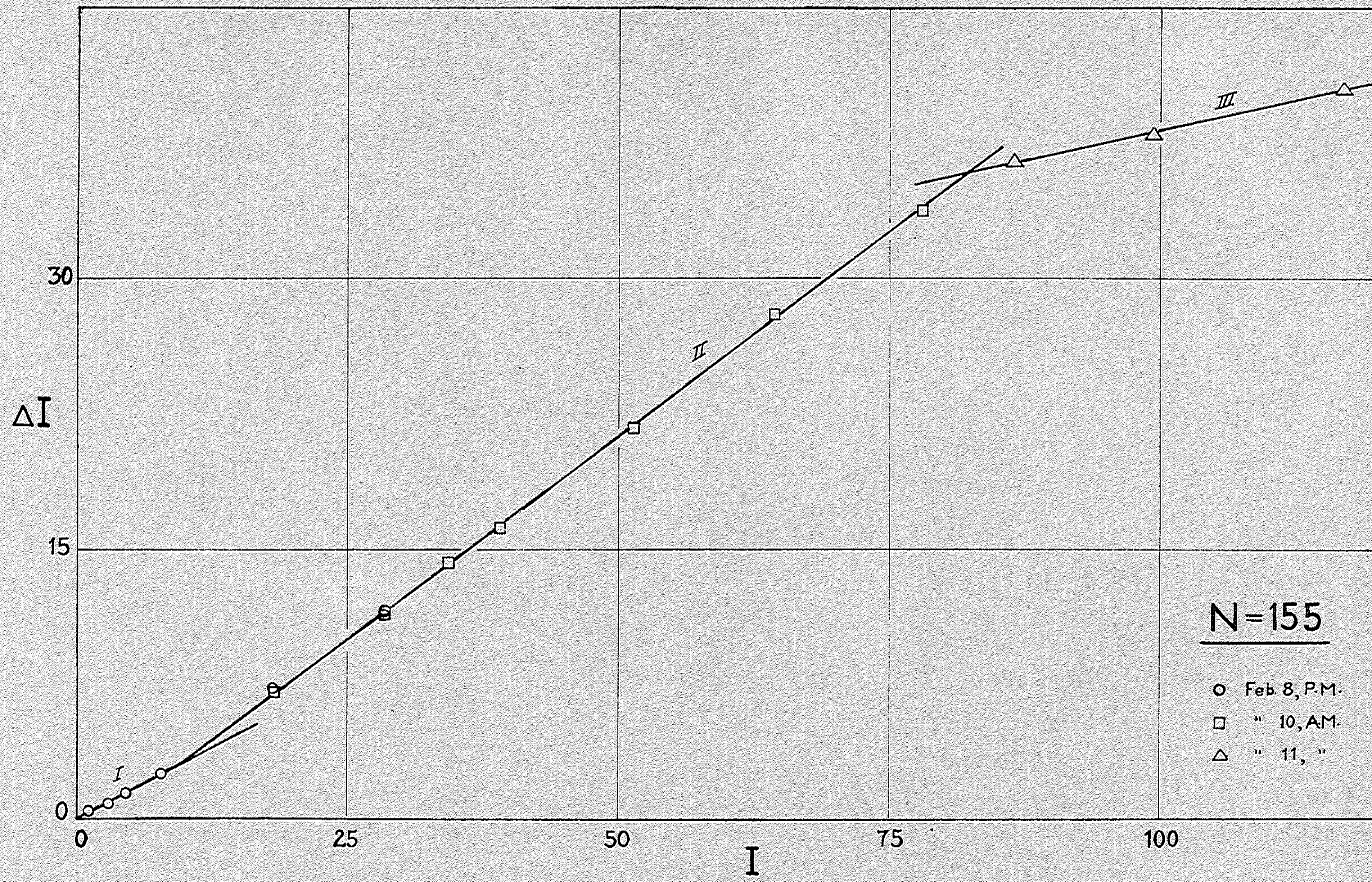


Fig. 8

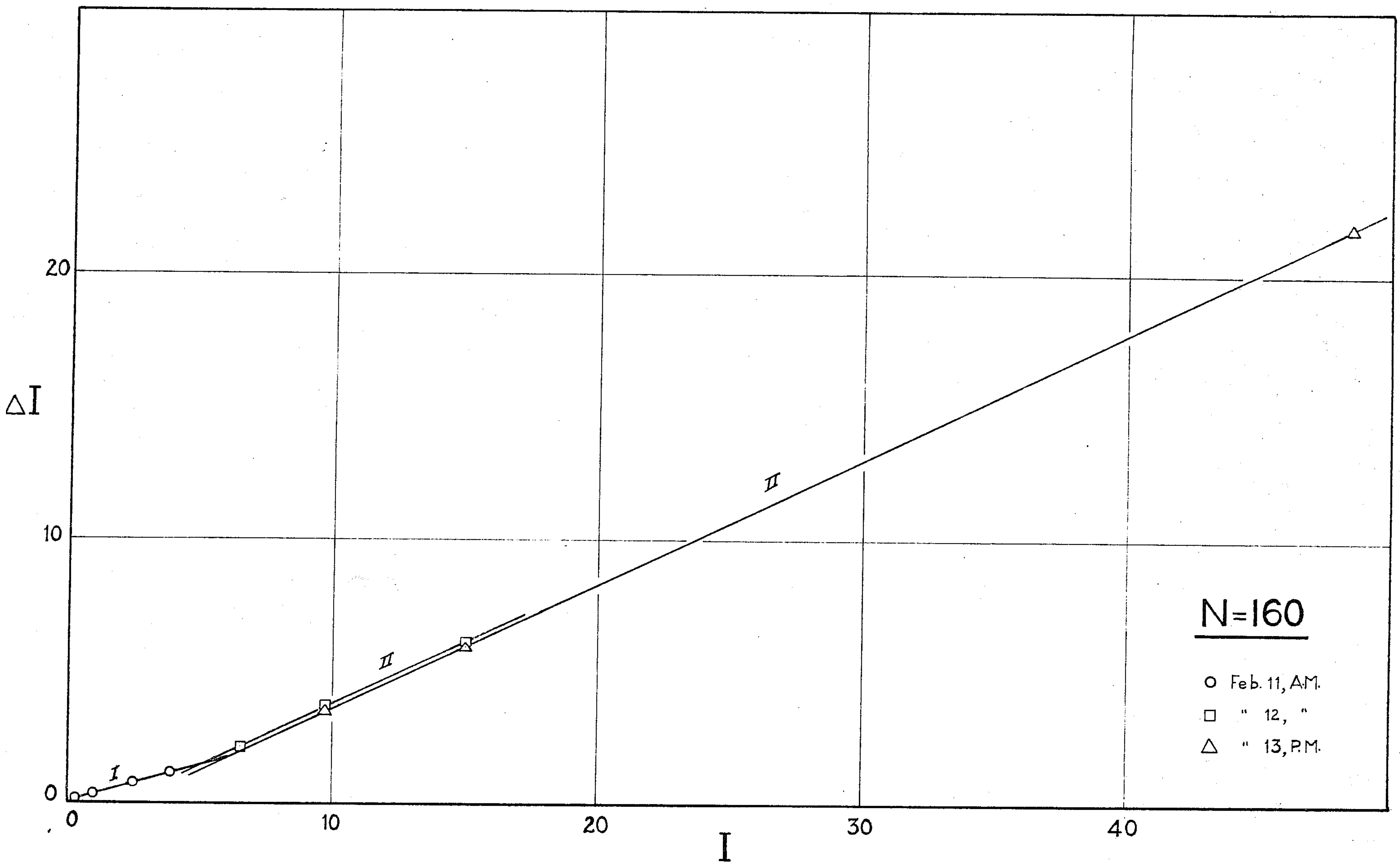


Fig. 9

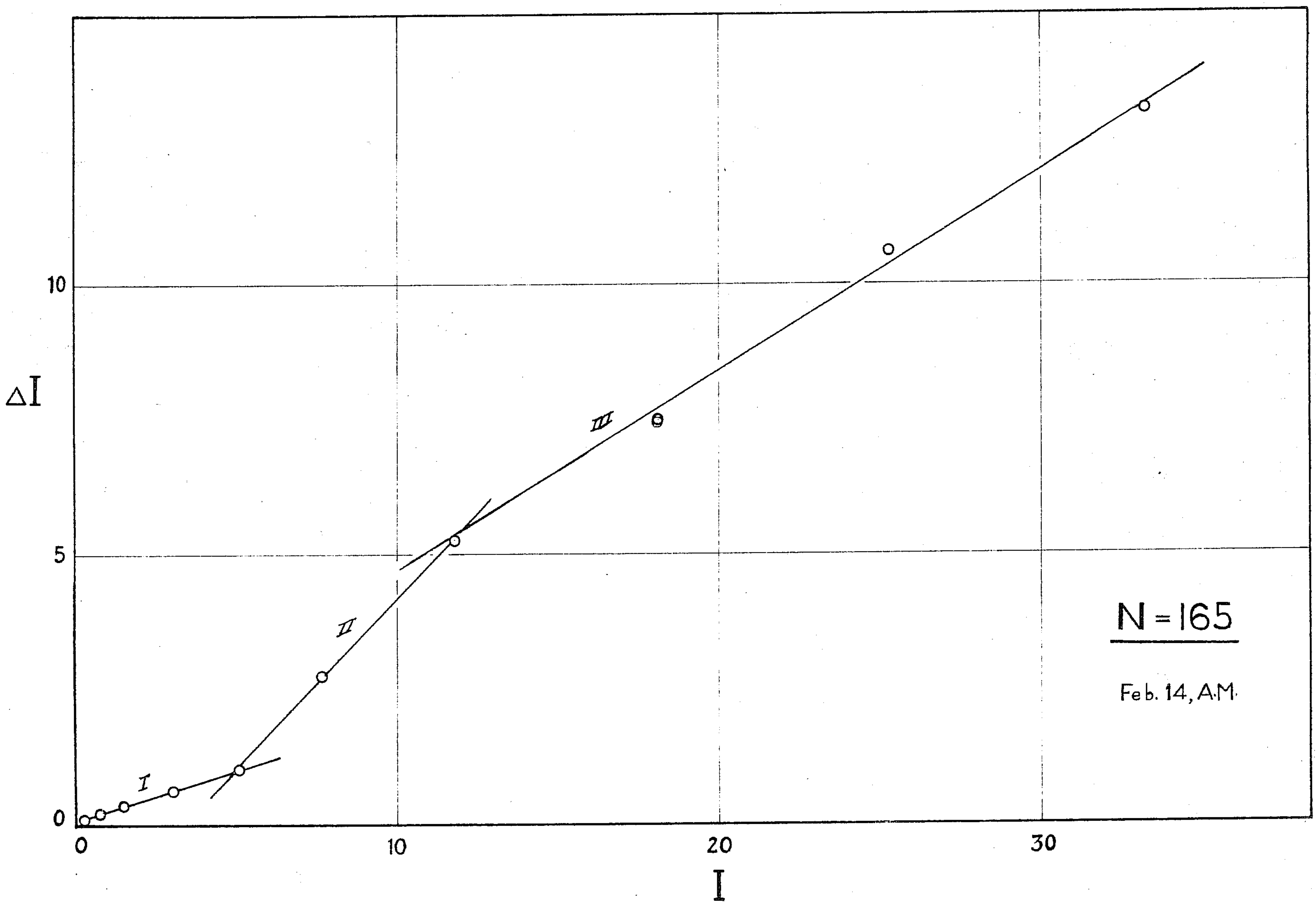


Fig.10

Calculated Slopes of Curves

Wave No.	Slope		
	I	II	III
140	.193	.317	.247
145	.217, .209, .220	.169	
150	.401	.248	.343
155	.308	.446	.158
160	.275	.470	
165	.188	.631	.373

Discussion of Results.

It will be seen from the curves that there is a linear relationship between ΔI and I but the slope is not the same throughout the entire range of intensities, that is, Weber's law, $\Delta I/I = C$, holds for the wave-lengths examined, but at definite intensities there are abrupt changes in the value of the constant "C".

Examination shows that though the values of I corresponding to given values of I may, and generally do, change from day to day, the slope of any one branch remains constant; that is, a series of parallel lines is obtained for each branch of the curve. Examples of this are shown in Fig. 5 - branch III, Fig. 6 - branch I, and Fig. 9 - branch II where the slopes are the same within error of drawing. Especially is attention drawn to the three values (of which only two are plotted) obtained at different times for the slope of branch I, N = 145 (Fig.6), namely 209, 217, and 220. The maximum deviation from the mean is only 3%, which shows the relatively high accuracy obtained in this work. This (the parallel slopes) means, of course, that the point of intersection of any two adjacent branches, i.e., the intensity at which there is a change in the value of $\Delta I/I$, may not be the same as if the branches had been determined on the same day. However, this is not of prime importance as this intensity will also vary from person to person depending on the intensity range of the individual eye. This latter, as is well-known, varies greatly with different individuals, and we should expect it to vary in the individual with slight changes in adaptation and in nervous condition. The point to be emphasized is, that there is a change in the value of $\Delta I/I$, and that this change takes place abruptly at some one intensity.

The individual branches of the curves were redrawn to a larger scale for calculation of the slopes which are given on page 19. It will be noticed that branch I of N = 140 and branch I of N = 165 have a common slope within the experimental error, as have branch 3 of N = 140 and branch 2 of N = 150. This also applies to branch I of N = 155 and branch 2 of N = 140, and to branch 2 of N = 145 and branch 3 of N = 155. No other relationship has been found between the various slopes.

The physical condition of the observer, occupation and retiring-hour the previous night, and the kind of day (e.g. dull, cloudy day, or bright day) were recorded each day, in the hope of finding some correlation between these conditions and the readings obtained. However, nothing very definite was noted in this respect. One could not predict from the conditions what the nature of the results would be. On some days, when all conditions seemed ideal, great difficulty was found in obtaining readings, whereas on other days definite readings were easily obtained even in the afternoon. Again, when the observer was in the later stages of a heavy cold, good readings were obtained on two consecutive days.

Conclusion.

From the data obtained, it is concluded that:

(1) Weber's law does hold, for the visual sense, over all ranges of intensity, but with the limitation that at certain definite intensities abrupt changes occur in the value of the constant C, i.e., the ratio of the differential threshold to the stimulating intensity depends on the intensity range.

(2) It remains to be proved whether or not a relationship exists between the various values of the ratio $\Delta I/I$ which hold over different intensity ranges throughout the spectrum. It is felt that the present data probably represents the best that can be obtained, but a complete range of values of $\Delta I/I$ throughout the entire spectrum might disclose some relationship not apparent in the limited range investigated.

TABLE I.

KÖNIG'S DATA

$\lambda = 6700$ Å

I	ΔI/I
48950	.0215
19680	.0163
9844	.0158
4912	.0180
1967	.0169
983	.0172
490	.0206
196	.0224
97.1	.0300
48.1	.0391
19.1	.0465
9.35	.0701
4.54	.101
1.66	.207
.742	.347
.312	.603

$\lambda = 5050$ Å

I	ΔI/I
19610	.0197
9819	.0184
4920	.0163
1965	.0179
982	.0188
490	.0197
196	.0222
97.6	.0250
48.7	.0258
19.4	.0306
9.64	.0375
4.76	.0513
1.87	.0701
0.920	.0874
0.454	.100
0.178	.124
0.0866	.154
0.0408	.224
0.0150	.336
0.00729	.372
0.00339	.475

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