A STUDY OF TEACHERS' BELIEFS

ABOUT THE NATURE OF MATHEMATICS

BY

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in Partial Fulfillment of the Requirements
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Abstract

This study investigated teachers' beliefs about the nature of mathematics and the alignment of these beliefs with the vision of mathematics presented in the Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989), (Standards). A three phase beliefs protocol was developed by the author in order to determine whether teachers held one of three views of the nature of mathematics: a Problem Solving view, an Instrumentalist view, or a Platonist view. The protocol was used during interviews with five Senior 1 (formerly grade 9) mathematics teachers in the province of Manitoba. The protocol consisted of four classroom scenarios posed for discussion, a list of questions to which participants were invited to respond, and a rating scale of twelve statements.

For the teachers whose beliefs about the nature of mathematics fell within the Problem Solving view, which is the one most congruent with the view presented in the Standards, it was found that the teachers' beliefs were narrow with respect to the Standards. Three teachers held a predominantly Problem Solving view, while one teacher held a Platonist view and one teacher's beliefs about the nature of mathematics fell within the Instrumentalist view.

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Chapter One

INTRODUCTION

There is a movement underway to reform mathematics education across North America and in other parts of the world. Not since the mid - 1960's when the "new math" hit the educational system has there been such a strong sense of the coming of change for mathematics education. According to Usiskin (1985), the stirrings in the mathematical community which have led to the present state began in 1975 when the National Advisory Committee on Mathematical Education (NACOME) called for a repudiation of a curriculum dominated by manipulative skills. This recommendation was closely followed by the National Council of Teachers of Mathematics' (NCTM) An Agenda For Action (1980). This document echoed earlier concerns about the focus on computation and algorithms in school mathematics programs and advocated that problem solving be the new focus of mathematics curriculum in the 1980's. The results of the work of these professional groups and many others discussed by Usiskin, culminated in the publishing of the Curriculum and Evaluation Standards for School Mathematics (Standards) by the NCTM in 1989. Within this document lies a new vision for mathematics education - a vision which could radically alter the way mathematics is taught and learned in our schools.

Much of the change recommended in the Standards is directed toward a broader view of mathematics than has been held in the past. This new view of mathematics is

one in which students will develop the mathematical power to be able to reason logically, to solve complex problems, and to communicate in a society which is becoming increasingly technological in its orientation. This view of mathematics is shared by many. Ernest (1989), Hersch (1986), Lakatos (1986), and Putnam (1986) all depict mathematics as a mental activity, one which is not infallible; one which is ever - changing and is the product of human creativeness. Thompson (1992) describes mathematics as "in the making" and points out that this notion is well represented in documents such as Mathematics Counts (Cockcroft, 1982), Everybody Counts (National Research Council, 1989) and the Standards.

The implications of this new view of mathematics are that the mathematics classrooms of the future should not look much like those of the past. Historically, there has been a striking similarity in the routine of mathematics classrooms which is described in Everybody Counts (1989) and documented by Fey (1979), Goodlad (1983) and Romberg and Carpenter (1986). The routine is typically one in which the previous day's work is corrected, some explanation of new material is given, most likely in broadcast form, problems are assigned, and students work individually as the teacher circulates around the room to offer help. The current wave of curriculum reform seeks to change this routine. In the future the classrooms will contain groups of students working together to solve a problem, computers being used to assist students as they analyze data and solve problems, the teacher operating as a facilitator not a broadcaster; posing problems for students to investigate and guiding their explorations of the mathematical nature of their world.

Purpose of the Study

There is no doubt that reform is in the wind for mathematics education and that the reform advocated is radical and significant. According to Ernest (1989) pressure for reform has overlooked a key factor: the teacher's knowledge, beliefs and attitudes about mathematics, mathematics teaching, and mathematics learning. The study will address this factor in that its purpose will be to examine the beliefs held by teachers about mathematics and to determine whether the beliefs held represent an obstacle to the implementation of a Standards based curriculum. To this end, the investigation will seek to answer the following two questions:

1. What are teachers' beliefs about the nature of mathematics?
2. How are teachers' beliefs aligned with the vision of mathematics of the Standards?

Rationale for the Study

The picture of the new mathematics classroom presented earlier is not one with which teachers are familiar. Teachers have operated within the broadcast metaphor for a very long time. The expectation for change advocated by the Standards is significant. Implicit in the recommendations put forward by the NCTM is the idea that the vision is not just of curriculum, but is also of instruction. Cooney (1988) suggested that there is undeniably a need for a swing in the philosophical orientation of mathematics instruction. He stated that in order to accomplish this swing it is

imperative to take into account the conceptions teachers hold about mathematics. Romberg and Carpenter (1986) expressed concern over the fact that current instructional practices may indicate that a large number of teachers believe that "mathematics is a rigid, static, bounded discipline ... that is divorced from science and other disciplines and then separated into subjects such as arithmetic, algebra, geometry, trigonometry, and so on" (p. 851). If teachers hold this traditional view of mathematics it may be very difficult for them to administer a curriculum which is based on the very different view of mathematics in the Standards.

In order for teachers to proceed with the recommended innovations they will need to reflect on their present knowledge, beliefs, and attitudes about all aspects of mathematics education; the nature of the discipline itself and the learning and teaching of mathematics.

A number of researchers (Arsac, Balacheff, & Mante, 1992; Conroy, 1987; Cooney, 1988; Nespor, 1987; Schmidt & Kennedy, 1990; Thompson, 1984; Underhill, 1988) have underscored the fact that teachers' beliefs about mathematics influence their instructional practices. Since teachers will be asked to change their teaching methods, research into the beliefs held by teachers could provide insight into the kinds of preservice and inservice training which will be needed to insure the successful implementation of a Standards based curriculum.

Limitations of the Study

This study will be limited by the fact that it is very difficult to assess what teachers are thinking. Feiman-Nemser & Floden (1986) discussed the problems in basing descriptions of teachers' beliefs on data gathered through observations or interviews. They noted that it may be difficult for teachers to articulate their beliefs. As well there is considerable evidence that the influence of beliefs on instructional practices is quite complex (Clark & Peterson, 1986; Conroy, 1987; Nespor, 1987; Thompson, 1984) and therefore difficult to document.

According to Pajares (1992) research into teachers' beliefs has proved difficult due to the fact that beliefs are not easy to define, conceptualize, or measure. He stated that researchers must first decide what belief means before attempting a study in this area. Various terms are used in relation to studies of beliefs and this makes a review of the literature difficult and confusing. Pajares found that

> attitudes, values, judgements, axioms, opinions, ideology, perceptions, conceptions, conceptual systems, preconceptions, dispositions, implicit theories, explicit theories, personal theories, internal mental processes, action strategies, rules of practice, practical principles, perspectives, repertoires of understanding, and social strategy (p. 309)

were a few of the terms that he found in the literature when reviewing research on teachers' beliefs.

Further to this, and more directly related to this study, is the problem that individual teachers may hold conflicting views about the nature of mathematics and that their views may not be well defined. This problem surfaced early in the study when an attempt was made by a panel of experts to verify a number of statements

which made up a rating scale used in the study. There was limited agreement from the members of the panel on the categorization of the statements. It seemed probable that this would affect the validity of the results. It was expected that the study would offer limited generalizability in that only five teachers took part in the study and they were all from a small geographic area within and close to the city of Winnipeg.

Significance of the Study

This study could be considered as a pilot project for the study of mathematics teachers' beliefs in the province of Manitoba. The results of this study could be influential in the development of a research project to determine to a greater extent the relationship between teacher beliefs about the nature of mathematics and the success of curriculum reform in Manitoba. It is possible that the conclusions will point to some specific areas where teachers' beliefs are not in line with those of the Standards. This information could then be used to help design preservice and inservice teacher training programs aimed at helping teachers adjust to the new curriculum.

Chapter Two

REVIEW OF THE LITERATURE

The review of the related literature has been organized into three sections. The first deals with beliefs about the nature of mathematics. The next section focuses on the research into teachers' beliefs about mathematics. The importance of considering teachers' beliefs in relation to curriculum reform in mathematics education will be reviewed in the third section.

Beliefs About Mathematics

It is important to note that researchers and reviewers use a range of terminology when referring to beliefs. The terms ideologies, views, perceptions, conceptions, and epistemologies are used by different authors in discussing teachers' beliefs.

The beliefs that individuals hold about mathematics are underpinned by different philosophies of mathematics. Ernest (1989) used the Perry Development Scheme (Perry, 1970) as did Copes (1988) and Cooney (1988) to distinguish different ideologies of the nature of mathematics. Ernest considered three of Perry's stages which would suffice to distinguish different philosophies. *Dualism* is a simple dichotomy whereby views are characterized as right or wrong, good or bad, we and others. From the dualistic perspective a person believes that every question has an answer and that the role of authority (a teacher) is to know and deliver these answers.

Mathematics is viewed as fixed and exact; it is concerned with facts, rules, and procedures. Within a *multiplistic* view of mathematics a person will believe that there exists a plurality of answers or opinions, that these are all equally good, but there is no rational choice between alternatives. Multiple answers or routes to answers are seen as equally valid and it is possible to be creative within mathematics. The third stage considered by Ernest is *relativism.* A person at this stage of development accepts a plurality of points of view but believes that justification can be made through the recognition of set standards and contexts or rule governed systems. Multiple solutions and approaches are acknowledged but evaluated on the basis of the rules and principles of the accepted mathematical system.

Copes (1988) adds one more stage, that of *commitment.* Holding this view a person is aware that knowledge is available but is something the individual must build alone.

Ernest (1989) presented a simplified version of the varying philosophies of mathematics which involved three views of mathematics to which teachers may subscribe. He noted that teachers may not fit neatly into one of the categories but that it is somewhat of a framework in which to study beliefs. There is the *problem-solving* view, in which mathematics is seen as a dynamic, problem-driven discipline. Secondly, there is the *Platonist* view, where mathematics is seen as a static, bounded body of knowledge. The third view is termed the *instrumentalist* view. Here mathematics is seen as set of useful facts, rules, procedures and skills.

Nespor (1987) discussed six structural features of beliefs that distinguish them

from knowledge. She stated that beliefs contain propositions or assumptions about the existence of entities such as students' ability to learn. According to Nespor, beliefs contain "alternative realities" which may cause a teacher to establish instructional formats which are based on early experiences and not on knowledge or training. She stated that belief systems rely heavily on affective components such as respects, feelings, and moods and are organized on the basis of personal experiences. As well she suggested that beliefs are recognized as being disputable and they are loosely bounded with no clear rules for determining their relevance.

O'Shea (1981) identified three models for mathematics curriculum which were based on views of the nature of the subject. The *pure mathematics* model emphasizes mathematics as the science of abstract structures and properties; the *applied mathematics* model stresses the relationship between mathematics and other disciplines; and the *basic mathematics* model centers on consumer and career applications. O'Shea found that teachers could be placed into one of the categories described and that there was significant diversity between demographic variables of the teachers as grouped. It is interesting to note that he found little difference in instructional variables between the three groups.

Collier (1972) divided preservice teachers into two categories with respect to their beliefs about mathematics: *formal* and *informal.* The formal group saw mathematics as a body of knowledge to be transmitted whereas teachers in the informal group focused on the creative and investigative nature of the discipline.

A study done by Dionne in 1984 and reported by Underhill (1988) attempted to

determine whether teachers held a *traditional* (rules, procedures, and formulas) a *formalist* (proofs and unifying theories) or a *constructivist* (developing thinking strategies) view of the nature of mathematics. He found that teachers were spread across the three categories with a slightly higher number being included in the constructivist group.

Lerman (1983) provided two views of the nature of mathematics. In his discussion he emphasized that the underlying philosophy of mathematics held by the teacher determines to a great extent the choices of instructional practices. The first view suggested by Lerman is that mathematics is a body of knowledge. A teacher holding this view would adopt an approach in which the rules and methods of mathematics would be presented followed by some applications. The alternative view of mathematics he presented is a problem-solving view. A teacher who holds this view will pose problems for students to work on which will lead them to test their own hypotheses and to try to express generalizations.

Research Into Teachers' Beliefs About Mathematics

In a study involving a beginning teacher Cooney (1985) found that although the teacher articulated a firm belief in the importance of problem solving in mathematics he did not act on this belief in the classroom. Rather he fell back on traditional teaching methods, relying heavily on textbook exercises for students.

Thompson (1984) conducted three case studies to examine the relationship between teachers' beliefs and perceptions about mathematics and teaching and their

instructional practices. She found that teachers' beliefs about mathematics play a subtle role in determining instructional practices. Two of the teachers conceived of mathematics as a rather static body of knowledge and tended to present content as a "finished product". The other teacher held a view described by Thompson as dynamic and tended to use a more conceptual approach to her teaching.

Collier (1972) used two Likert-type scales to measure the beliefs of elementary education students about mathematics and mathematics instruction. He found that entering students had neutral views and did not view mathematics as formal or informal. After two content and two methods courses there was a shift towards a slightly more informal view of mathematics with high achievers having a more informal view than low achievers.

In a study examining teachers' beliefs about mathematics, Schmidt and Kennedy (1990) found a wide diversity in the beliefs held. They contended that beliefs are highly personal, are formed in isolation and that many teachers held conflicting beliefs about the nature of mathematics. This led to the teachers contradicting themselves when attempting to articulate their beliefs. They suggested that this may lead to a tendency to teach numerous topics but to teach none of them well.

Taylor (1990) was involved in a collaborative research project with an inservice teacher. In order to help the teacher develop a more constructivist approach to his teaching, episodes of "cognitive perturbation" were employed in which the co-researchers analyzed classroom events and engaged in critical dialogue. Even after this extensive, reflective process, the teacher made only minor adaptations to his

traditional, centralized role in the classroom.

In a review of research on mathematics teachers' beliefs Underhill (1988) summarized his findings by stating that secondary teachers' beliefs were dominated by formalism and that nearly all the teachers studied in the case studies he reviewed adhered to a transmission role in the classroom.

The Role of Beliefs in Curriculum Reform

Nespor (1987) claimed that teachers' beliefs are important in that they play a significant role in how teachers define teaching tasks and organize knowledge. She stated that teachers tend to rely more on their beliefs than on knowledge or academic theory. This has implications for teacher education but she cautioned that beliefs seem to be particularly resilient to change and more research must be done to determine how beliefs develop and how they can be transformed.

An attempt was made by Arsac, Balacheff, & Mante (1992) to have two teachers teach exactly the same lesson each to two different classes. Before the teaching episodes occurred, it appeared as though the teachers were in agreement as to what would transpire. They had been thoroughly immersed in the theoretical analysis and debates which took place prior to the experiment. After the classes the researchers were surprised by the way the individual teachers made decisions outside the planned scenario. These decisions seemed to be unconscious and on analysis it was felt that these decisions were made on the basis of the teachers' beliefs about mathematics.

Cooney (1988) expressed concern over the fact that the new wave of reform in

mathematics education may be as unsuccessful as those that came before it. He stressed that the Standards imply a different philosophical orientation toward instruction and that educating teachers to teach in this new way is the "ultimate challenge" for teacher educators. Cooney made a strong case for considering teachers' beliefs and conceptions about mathematics when planning for the implementation of a new curriculum. As he puts it, "one cannot conjecture, explore, transform, and all the other action verbs used throughout the Standards and still reasonably maintain the notion that mathematics is a subject handed down from on high." (p.359)

In a chapter which provides a review of some of the research on teachers' perceptions of mathematics, Conroy (1989) stated that "it would be a mistake to pursue curriculum change in the belief that teachers' perceptions will change simply through the adoption of new texts, materials, and the like." (p.77) He stressed the important role that perceptions play in teachers' pedagogical decision making and urged that changing the way teachers perceive mathematics must be a top priority when implementing a new curriculum.

By examining research on teacher education, Lampert (1988) found that much of the knowledge and beliefs held by teachers is a product of their experience as learners of the subject. She indicated that research in teacher cognition has definite implications for teacher education. The beliefs about mathematics that have been allowed to develop in prospective teachers since they were in kindergarten cannot be casually challenged in a few methods courses with any hope for change.

As did Lampert (1988), Pajares (1992) stated the importance of studying teachers' beliefs. In his review of research in this area he found strong support from many researchers to indicate the connection between a teacher' beliefs and their practice. He reported that Pintrich suggested that a teacher's beliefs may be the most important psychological construct which influences their teaching and is therefore critical to teacher education. Kagan, as reported by Pajares, would likely concur as he suggested that teachers' beliefs "lie at the very heart of teaching." (p. 85)

Zollman and Mason (1992) developed an instrument to assess teachers' beliefs about the Standards. The authors felt that wide acceptance of the Standards would depend on teachers' beliefs. Their results showed that teachers tended to agree with some aspects of the Standards, but that numbers of teachers seemed to hold traditional, yet unfounded beliefs. Notably, a group of graduate students who had spent several months studying the Standards showed a much higher agreement than did the other group. The researchers concluded that it was possible to effect change in teachers' beliefs.

Thompson (1984) found that a relationship existed between teachers' beliefs about mathematics and their instructional practices. In her case studies of three teachers she noted that even though there was consistency between prevailing beliefs and the way they presented the content, the teachers said that they rarely thought about their beliefs or encountered situations which would cause them to examine their conceptions of mathematics. Of note is the fact that some inconsistencies did exist between the teachers professed beliefs and their instructional practices. This lead Thompson to

speculate that the teachers may not have been fully cognizant of or be able to articulate their beliefs.

Ernest (1989) referred to the missing program in research on teacher cognitions. As others do, he stressed the need to study teachers' beliefs about the nature of mathematics in light of the drive for change in mathematics education. He stated that the implementation of a new curriculum will require major role changes on the part of teachers and that a fundamental understanding of how teachers' beliefs manifest themselves in classrooms is needed.

Referring to recent reform movements in mathematics education Ernest noted that they have been based on a particular view of the nature of mathematics. The "new math" movement of the 1960's was built on a Platonist view of structure, laws, and unifying concepts. He stated that a parallel can be drawn between the "back to the basics" movement of the 1970's and the instrumentalist view of mathematics as a set of unrelated acts, rules, and procedures. Ernest contended that the problem-solving view has been promoted throughout the 1980's and is at the heart of the vision of mathematics recommended in the Standards. Here mathematics is seen as a continually expanding field of human inquiry.

Ernest stressed the importance of the relationship between beliefs and practice. He also noted that there can be inconsistencies between a teacher's professed beliefs and their practices. This may be due to the teachers level of consciousness in relation to his or her own beliefs and that the practice of reflection can lead to a greater integration of beliefs and practices. Ernest also made an important point about the

underpinnings of beliefs; that they are embedded in personal experiences with the teaching and learning of mathematics.

Summary of the Review of the Literature

Although the study of teachers' beliefs about mathematics is a relatively new area, significant research has been done. A number of researchers have attempted to provide a theoretical framework for the study of beliefs by developing categories of beliefs about the nature of mathematics. Researchers looking directly at teachers' beliefs about mathematics have found that many teachers hold very traditional views about the subject and that these perceptions may not be rationally founded and are rooted in personal experience. There is some agreement on the notion that the beliefs that teachers hold may not be easily challenged or changed and that this may be partly due to the fact that teachers are not necessarily aware of their beliefs about mathematics. Various researchers have indicated that they believe teachers' beliefs must be studied and taken into account before embarking on an attempt to implement a new mathematics program, particulary a Standards based one which involves a view of mathematics that is not commonly shared by the teachers who will be using the new program in the field. There is some consensus in the literature that teachers' beliefs are a critical element to consider with respect to teacher education.

Chapter Three

METHODOLOGY

The purpose of the study was to examine teachers' beliefs about mathematics and to determine to what degree their beliefs are congruent with the assumptions of the Standards.

Theoretical Framework for the Study

A model of beliefs about mathematics.

Ernest (1989) proposed an analytic model of the different types of knowledge, beliefs, and attitudes of the mathematics teacher. He stated that although there are a number of variations, and that these are not mutually exclusive, three philosophies of mathematics are distinguished because of their occurrence in the teaching of mathematics. He noted that teachers may combine elements from more than one of these views. Ernest believes that teacher beliefs about mathematics affect the way curriculum reform takes hold through the way mathematics is taught in the classroom.

First, there is the Problem-Solving view. Here mathematics is seen as an expanding field of knowledge of human inquiry and not as a finished product. Mathematics is not a body of knowledge to be handed on or transmitted but something which can be created. A variety of methods or solutions are welcome within this view and it is accepted that not all mathematical truths are known.

Secondly, there is the view of mathematics as a static but unified body of true knowledge. This is the Platonist view. Within this view mathematics is seen as absolute; something which can be discovered but not created. There is an important emphasis on the structure of mathematics and on analysis and proof.

Thirdly, there is the view of mathematics as a useful but unrelated collection of rules, facts, and procedures, which is referred to as the Instrumentalist view. Mathematics is something one does in quite precise ways. There is one correct way to obtain an answer and this is usually done by applying some formula or following a rule which has been previously laid down. From an Instrumentalist point of view there is little value in discovering the rules and structure of mathematics; it is more a case of making use of these rules to achieve a solution to a problem.

In this study, the data gathered will be used to try and show which view of mathematics a particular teacher holds. Ernest drew a parallel between the current wave of curriculum reform in mathematics education and the problem solving view of mathematics. For the purpose of this study a teacher whose beliefs about mathematics fall within the problem solving view will be assumed to hold beliefs in line with those expressed in the Standards.

The assumptions of the Standards.

The Standards were prepared on the assumption that the commonly held notion that "mathematics is a set of rules and formalisms invented by experts, which everyone else is to memorize and use to obtain unique, correct answers, must be

changed." (Romberg, 1992, p.433)

The result of several years of work on the part of classroom teachers, educational researchers and university mathematicians, the Standards provides a framework for changes in the learning and teaching of mathematics. Underlying this framework is a new vision of mathematics as a dynamic product of human inventiveness. This view calls for redefining the roles of both teachers and learners.

Within the Standards document are new goals for students of mathematics:

1. Learning to value mathematics.
2. Becoming confident in one's own ability.
3. Becoming a mathematical problem solver.
4. Learning to communicate mathematically.
5. Learning to reason mathematically. (p. 6)

These five goals stated for students imply radical changes in the instructional practices of teachers. In order to assist students in learning to value mathematics and to communicate mathematically, in becoming confident in their ability to do mathematics and to solve problems, and in learning to reason mathematically, teachers will need to shift from being transmitters of knowledge to facilitators of learning. How well teachers make this shift will depend on many things, not the least of which is their own beliefs about the nature of mathematics. If teachers do not see the same vision of mathematics that is presented in the Standards, they may have difficulty in adapting their instructional practices to encompass this view.

The Subjects

Letters were sent to a strategic sample of 12 experienced individuals currently teaching Senior 1 (formerly grade nine) mathematics in English speaking schools within a 100 km radius of Winnipeg. The letter (Appendix A) was an invitation for teachers to participate in this study. Senior 1 teachers were chosen since they will be the first ones to implement the revised Senior mathematics curriculum. This revised curriculum is, at the time of this writing, in the final draft stages and has been strongly influenced by the Standards. From those who received letters, five responded with interest and those teachers were scheduled for interviews.

Instrumentation

A three phase beliefs protocol was developed to be used during interviews with the teachers. The protocol was designed to differentiate between the three views of mathematics presented by Ernest (1989) and adapted as a model for this study. The views of mathematics that make up this model are the Problem Solving view, the Instrumentalist view, and the Platonist view. These terms are capitalized when they refer to the Ernest framework.

The first phase of the protocol involved four scenarios (Appendix B) which were posed for the purpose of discussion. The first three scenarios involved situations which might occur in a Senior 1 mathematics classroom. These three scenarios were adapted from vignettes presented in The Professional Standards for Teaching Mathematics (NCTM, 1989). The fourth scenario was a problem which was selected

from Flying Aha's and Slippery Eureka's: A Sourcebook of Mathematical Challenges for Elementary and Middle Years Curriculum (Ameis, 1993). The specific scenarios were described in such a way as to be open-ended, allowing teachers who hold different views of mathematics to respond in their own way. For example, in responding to the first scenario in which a student wants to know why she has to do all this "stuff", a teacher who holds an Instrumentalist view of the nature of mathematics might try to point out the usefulness of mathematics in other areas such as construction, banking, or science. A teacher who holds a Problem Solving view might focus the student on the important thinking skills that are being developed, while a Platonist might be inclined to tell the student that mathematics is important for its own sake.

The second phase of the protocol consisted of seven questions (Appendix C) about the nature of mathematics which were posed in an interview format. These questions were developed through a series of informal interviews held with five master teachers of mathematics in the Province of Manitoba. Using the Ernest framework as a basis for discussion a number of questions were asked of the master teachers and then the relationships between the questions and the responses were discussed. In this way certain questions were found to be more useful in helping teachers express their views about the nature of mathematics than others.

The third and final phase of the protocol was a rating scale (Appendix D) which contained twelve statements about the nature of mathematics. The statements used by Collier (1972) and Zollman & Mason (1992) were adapted and were written in such a

way as to give equal representation to the three views of mathematics outlined in the model. Statements 1, 2, 4, & 8 represent the Instrumentalist point of view in that each one in some way illustrates the usefulness of mathematics. Statements 3, 6, 7, & 11 are indicative of the Platonist view. Here mathematics is depicted as an unchanging body of knowledge. Statements 5, 9, 10, & 12 reflect the Problem Solving view since each one points in some way to the importance of creative approaches to the study of mathematics. After the statements were developed they were scrutinized by a panel of experts to verify the match of each individual statement with the view it was supposed to represent. The panel of experts consisted of three mathematics education consultants who familiarized themselves with the model by reading about its development (Ernest, 1989) and then attempted to place each statement within one of the three views of mathematics.

This exercise proved to be difficult for the panel and raised some considerations about the rating scale. As pointed out by Schmidt & Kennedy (1990) teachers hold conflicting views about the nature of mathematics. This appeared to be true of the members of the panel. Upon discussion of the Ernest model, each expert revealed areas where they felt the three categories of beliefs overlapped. Thus, there was a blurring of the edges between, for example, the Problem Solving view and the Instrumentalist view. One expert held on to the idea that problem solving meant applying mathematics to the solving of word problems. Thus it was difficult for her to decide which statements represented the Problem Solving as opposed to the Instrumentalist view.

There was agreement by the panel of experts on the categorization of some of the items in the rating scale but not others. An attempt was made to alter the wording of the statements to make the distinction between the views of mathematics more definite. It is likely that the researcher' own beliefs entered into the construction of the rating scale thus reducing the usefulness of this phase of the protocol in measuring the beliefs of the participants.

Procedure

Five teachers were exposed to the three phase beliefs protocol during an individual interview which was taped and later transcribed for analysis. All tapes were destroyed upon the completion of the study. Prior to the interview the teachers received a copy of the statements to be rated and the scenarios. They were asked to review the scenarios and to complete the rating scale before their interview.

At the start of their interview each teacher was invited to share any thoughts they had about the scenarios which represent the first phase of the protocol. This was done to put the teachers at ease by having them start the interview with their own ideas rather than by responding to a question. While the teachers talked about the scenarios, questions were raised by the interviewer which arose spontaneously out of their comments. After the discussion about the scenarios seemed to be completed, the teachers were asked the seven specific questions which made up the second phase of the beliefs protocol. When these questions had been dealt with the third phase, the rating scale of statements about the nature of mathematics, was collected. Scenario

four, the problem, was dealt with by each teacher in a slightly different way depending on their interest and level of comfort with the problem itself or the posing of such a problem in an interview setting.

Interpretation of the Data

The three phases of the beliefs protocol were analyzed separately. Each one was used to determine which view of mathematics a particular teacher holds; the Instrumentalist view, the Platonist view, or the Problem Solving view. Teachers who held a Problem Solving view of mathematics were said to have their beliefs in line with those of the Standards. Teachers who held the Platonist or Instrumentalist views were defined as holding beliefs conflicting with those of the Standards. It was to be expected that teachers would not fall neatly into one of the three suggested categories of beliefs, but that one of the views may be dominant. Since it has been shown in the review of the literature that a relationship does exist between a teachers' beliefs about the nature of mathematics and the way they teach mathematics in their classroom, it was be assumed that teachers who hold the problem solving view of mathematics are the ones who will be best able to implement a Standards based curriculum.

Chapter Four

ANALYSIS OF THE PROTOCOL RESULTS

In this chapter the three phase beliefs protocol will be analyzed individually for each teacher. The analysis will be in narrative form and will be used as a basis to determine whether each teacher holds an Instrumentalist, Platonist, or Problem Solving view of the nature of mathematics. To protect the identity of the teachers the names used here are fictitious. As will be apparent in the discussion to follow, it was very difficult for the teachers to talk about mathematics without making reference to their teaching. Thus it is possible that when I asked them to comment about aspects of mathematics they were really relating things to their teaching.

To get an idea about a teacher's beliefs about mathematics from their rating of the statements a scoring system was used. The statements were rated from strongly agree (4 points) to strongly disagree (1 point). Statements 1, 2, 4, & 8 represented the Instrumentalist view; statements 3, 6, 7, & 11 the Platonist view; and statements 5, 9, 10, & 12 the Problem Solving view.

Ellen

Ellen has been teaching mathematics and science at the junior high level for about 5 years. She has been teaching in a suburban junior high school and is presently a member of the mathematics leadership team in her school division. Ellen was the first

teacher I met with. We met in her home as she was, at the time of the interview, on leave from her school to care for her newborn child. Since all the teachers had seen the scenarios prior to the interview, I felt that this would be a good place to start.

Ellen sees mathematics somewhat as a unified body of knowledge. In discussing the third scenario she felt that the teacher shouldn't worry about spending time reviewing every day. In fact

> it should lead up to what you're doing today. Most of the time what you've done yesterday is leading up ... to what you're doing today. You just extend what you were doing yesterday. That way kids see a connection in what you've been doing.

Ellen makes frequent references to problem solving in mathematics in her responses to the questions and her comments on the scenarios:

> (a) To me, it's just teaching the child how to think in a logical manner of how to come up with some sort of solution to a problem;
>
> (b) some of it is memorized but that just helps them along the way with different problems;
>
> (c) if it's a problem solving question then as long as they go through a reasonable process for solving and they get a reasonable answer then that's OK.

She says she believes that teaching kids to think and to solve problems is the most important part of teaching mathematics. Probing further, it can be seen that her view of problem solving is rather narrow. All of the examples she used related to computational problems. She talked about "consumer skills" and "having enough money" and "it's a thinking skill but it involves using numbers" is how she described the discipline of mathematics. She definitely referred to the idea that mathematics is a

useful tool. She mentioned "using algorithms", "solving science problems", "an engineer building a bridge", and "a percentage sale". And she stated that mathematics is "useful, definitely, certainly, it's definitely useful." She was most emphatic about that point.

Ellen was creative in her approach to solving the problem I had given her. She immediately started to jot things down and said she was going to organize things and look for a pattern. She was able to employ a range of strategies in attempting to solve this problem and even called me at home later to give me the answer and then again to correct that answer. This seemed to back up her expressed commitment to the importance of problem solving within mathematics.

When responding to the statements about the nature of mathematics, Ellen scored 15 points for the Problem Solving view, 12 points for the Instrumentalist view, and 6 points for the Platonist view. This was consistent with her comments and discussion during the interview. Although Ellen does exhibit aspects of the Instrumentalist viewpoint she is basically a Problem Solver. She stated repeatedly that she thinks problem solving and thinking processes are very important to the study of mathematics and even though her view of problem solving may be rather narrow it does tend to be predominant. Ellen would say that she holds a problem solving view of the nature of mathematics and this is why she scored high on those statements. Her view of problem solving may not be totally congruent with the view presented in the Standards but the two views do share some commonality. Her views of mathematics are represented in the Standards but are only a small part of the way mathematics is

presented within the Standards .

Ted

Ted is an experienced teacher of junior high mathematics and social studies. He is presently teaching in a K - 9 school in a neighbourhood with a high percentage of families with low socio-economic status. Ted is an enthusiastic teacher, who spends a considerable amount of time coaching volleyball and track and field.

> Ted saw a common thread to all the scenarios. He felt that
>
> all of these would relate to giving students a reason for needing math; and perhaps justifying why we do things and not just having students know rules or formulas that work but being able to problem solve - which is the most important thing.

So Ted started out by stating that he believes problem solving is the most important thing. He continued to reinforce this throughout our meeting. In discussing Scenario 3 he stated that he doesn't like to start the class with review rather he tries to set the tone by presenting an interesting problem for his students to try. Later in the interview he said he would describe the discipline of mathematics as "solving problems ... I think solving problems would be it." In attempting scenario 4 which is the problem experience, Ted was very enthusiastic - "I think it's a tremendous problem and I used it with my grade 8's." He explained how he tried looking for a pattern and by the process of elimination he was able to narrow the possibilities down and then finally come up with a solution. He didn't mention looking for any sort of algorithm or rule that would help. This seemed to reinforce his interest in and

commitment to problem solving.

As Ellen did, Ted at times seemed to limit his view of problem solving by mentioning the notion of a "word problem", or "going banking or shopping - you sort of need math. Even work around the house; construction. We use it all the time in our daily lives." He seemed to focus here on a more Instrumental view of mathematics. When asked how important he felt procedures and algorithms are he was hesitant. He felt they were "less important now than I used to think. Certainly they are important rather I would like to concentrate on reasonable thinking. But showing people these formulas and algorithms that help you solve problems is good." Ted believes that by allowing children to discover mathematics for themselves that a greater level of understanding may be reached.

> I think that's an ideal and I think that happens. I thinks that's the most exciting part of teaching when they do that. I think they achieve a greater understanding when they do that than when you present them with algorithms that work or equations.

In rating the statements Ted scored 12 points on both the Problem solving and Instrumentalist areas. He scored 8 points on the statements referring to the Platonist viewpoint. Although he came across very much in support of the problem solving view of mathematics he does tend to believe to some extent in the Instrumentalist view as well. His view of mathematical problem solving is likely not as broad as the view presented in the Standards as he tended to limit his discussion to problems which are numerical in nature. This occurred when he attempted to use the examples to illustrate a point he was trying to make. On the other hand he said that he " would

like to concentrate on reasonable thinking". These comments and examples, along with his rating of the statements lead me to believe that Ted is somewhere in between being a Problem Solver and an Instrumentalist with a definite desire to become a stronger supporter of the Problem Solving view of the nature of mathematics.

Jennifer

Jennifer is a mathematics teacher with over ten years experience in the field. She has taught middle years mathematics in Great Britain for more than half of her career and is now employed at a private school in Winnipeg. She was involved in a curriculum materials production group in Great Britain and is currently serving on a Manitoba Education and Training Curriculum Revision committee.

Jennifer had a most interesting response to the students who would ask: How do I do this stuff? She said that it clearly

> depended on the stuff. Sometimes it's really for the methodologies - to give them a tool to do more complicated things. And sometimes it's because it's abstract and you just have to get your mind wrapped around it.

This tended to sound as though Jennifer had a bent towards an Instrumentalist view of mathematics. As the interview progressed this became more pronounced. In discussing Scenario 3 Jennifer stated that she felt there was no point in "going over the same material - usually it's rare that the whole class is having problems." By responding to the notion that children might be able to discover some mathematical relationships on their own she reinforced the idea that in mathematics we tend to go

over material. She thought that discovering relationships might be possible in geometry but "it's difficult in algebra - those are the skills that have to be taught in some cases." There appeared to be a difference in her mind about algebra and geometry. This was another indication that she has an Instrumentalist belief in the nature of mathematics. We discussed the possibility that something called mathematical ability exists in people to varying degrees. She is very much in agreement with this but tended to relate her notion of mathematical ability to something she called "basic number bonding." She expanded this to say that "being good at math means being good at number."

When asked how she would describe the discipline of mathematics Jennifer had absolutely no ideas. She just said "I don't know - I really haven't any idea." I suggested that we come back to it later in the session which we did. At that time she stated that she'd "really have to think about that."

Until this point problem solving had not really come up. She had mentioned a few things she had tried with her students but it didn't seem to be a focus. I tried to probe this area a little with some questions. I asked her to tell me what the term problem solving meant to her. "Well say just looking at a problem and maybe starting with a smaller version of the problem and then building up and looking for a pattern." Then she said, "I try to build everything from patterns." This was in the context of discussing how she does geometry and she didn't relate it to anything else.

She hadn't tried the problem I had given her and when we began to talk about it she said "I thought first of all could I use algebra". So her very first idea was to try

and find an algorithm that might work. Then she went on a bit to say she might try some basic number ideas and then just trial and error for a while. She ended the discussion on the problem by stating that "You'd probably only want to use it with gifted students." It is interesting to note that she wasn't particularly interested in doing the problem - it didn't seem to spark her enthusiasm at all which reinforced her lack of commitment to the Problem Solving view of the nature of mathematics.

On the rating of the statements Jennifer scored 14 points on the Instrumentalist statements, 12 points on Problem Solving and 8 points for the Platonist viewpoint. Even though her scores for both the Instrumentalist and Problem Solving views were high, I feel that Jennifer leans more to the Instrumentalist view of the nature of mathematics because of both her high score in that category and her comments throughout the interview. She sees mathematics very much as a tool and believes that rules or formulas are important in the solving of "number problems." More so than Ellen and Ted, Jennifer's view of mathematical problems is narrow and does not seem to include much of a focus on the development of mathematical thinking as opposed to mathematical skills. It would seem from the information gathered that Jennifer's beliefs about the nature of mathematics are not particularly congruent with those of the Standards.

Jan

Jan has been teaching mathematics to students in grades 5 to 8 for about 10 years. She is also currently teaching computer science and after having been away from

computers for quite some time felt the need to go back to school to learn her content again. Jan had just recently completed first year computer science and said that she really enjoyed the course and feels her knowledge is much more current and therefore she feels better prepared to teach computer science in the private school setting in which she is employed. This seemed to indicate that Jan has a real commitment to her teaching and her students.

Jan was fascinated by the problem I had provided in scenario 4. She tried a number of different approaches, including using the Sieve of Eratosthenes. She felt that the "brighter kids could make a sieve that goes up to 1000 or even 2000." She was very keen on giving this problem to her computer science students as she felt it would lend itself nicely to a solution using a Pascal program. Throughout the interview Jan displayed a real interest in mathematical problems. She saw mathematical problem solving as "the ability to take a problem and break it down in individual steps in order to solve it - that's a life skill."

It was interesting to note that Jan could see that her view of mathematics was narrow. Her thoughts were that "mathematics in some ways is very broad. We just take narrow views on it in high school." When I asked her to expand on this idea she explained that her thinking was somehow rigid. "It's more than just symbols and algorithms - it's approaches. How you approach problems. There's more to it than that..." She really couldn't put her finger on what she thought might be missing from school mathematics yet she was dissatisfied with herself and the program. At one point she said "I'm a high school teacher, I should know all this."

Throughout the interview and the discussion of the scenarios Jan seemed to be showing a distinct problem solving view of mathematics. Although this view was not completely developed in her mind, at least she was able to articulate her feelings of conflict about mathematics and what it meant to her. Upon analyzing the third part of the protocol, the rating of the statements, the conflict between views was quite apparent. Jan scored 10 points for the Instrumentalist view and 9 points each for the other 2 views. Again she recognized the difficulty as she stated "even though I was answering the agree ones I was thinking I'm not giving very consistent answers here."

Like the others she expressed her narrow view of problem solving in the examples she used to illustrate some points. She talked about the importance of right answers in science and business; and "needing to know the quadratic formula." I asked her to tell me how mathematics was useful. To this she replied that "you need it for science and business, you need to be able to do some calculations, for life skills and problem solving." Here is the notion of mathematical problems somehow involving numbers. She talks about an approach to problems but never actually discusses the ideas of thinking or reasoning nor does geometry ever come up.

Thinking about why mathematics is important (question 2) prompted Jan to support the notion that children could invent their own mathematics. "Sometimes children want to have their own mathematical symbolism and their own language for math and I think this is great." She also indicated that when she is teaching she tries not to emphasize a certain method for solving types of problems but tries to show students that different problems may require different approaches. In this respect Jan is a

supporter of the Problem Solving view of mathematics. An important aspect of Jan's beliefs is that she knows she isn't sure what she thinks. Her responses and comments frequently showed her interest and commitment to the Problem Solving view and yet she comes from a strong Instrumentalist background. This may be what has set up the conflict in her mind as to what mathematics is all about. Even though Jan's beliefs are not totally in line with the Standards it appears that she will have an open mind about this new philosophy as she is not quite satisified with the way things are at present.

Bill

Bill is a very experienced mathematics teacher. He has been teaching secondary school mathematics for about 20 years in a small town outside of Winnipeg.

Bill was interested in discussing the scenarios first so we proceeded in that way. He found the description of how to divide by zero in scenario 2 to be interesting but he said he wouldn't bother with it. He would just tell the students that they have to "remember it is undefined. If they forget which one - multiplying or dividing by zero is undefined - they can put it in their calculator and see the error message. So basically I don't explain it." This appeared to be a very strong Platonist viewpoint and Bill continued throughout the interview to show his belief in this view of mathematics. Bill felt that the teacher who spends too much time reviewing will not "be able to finish the course." He came back to this point again and seemed to stress the importance of finishing the course when he said that "we have a curriculum to get

through and I run out of time so I just don't have time to be really creative. You can cover a lot more ground and learn a lot by following the instructor's directions."

In discussing the important aspects of mathematics Bill thought that mathematics was important "in varying degrees depending on what the student will end up doing later on." So he saw the mathematics learned in school as being a prerequisite for later learning. He didn't seem to offer any ideas about the importance of mathematics for its own sake or as a vehicle for developing problem solving skills. In this respect Bill seemed to be an Instrumentalist in that he was trying to get across the idea that mathematics might be useful for something else. To get at this idea further I asked Bill to describe the basics of mathematics. He said he thought percentages were pretty basic and maybe number systems. He said that there were no parts of Geometry that he considered basic.

When the discussion turned specifically to problem solving Bill revealed a very narrow view of this aspect of mathematics. He felt that problem solving was only important "for '00' students because theoretically they are headed for a technology course. Right now there is a very good section on problem solving and it leads right into the problem solving unit in Grade 10." So, Bill seemed to see problem solving as content rather than as a process.

From the interview and discussion of the scenarios it was apparent that Bill's views of mathematics were very much outside of the Problem Solving area. This was supported clearly with the rating of the statements where Bill scored 12 points on the Instrumentalist statements, 11 points on those relating to the Platonist view and only 8

points on the statements that represented the Problem Solving view of mathematics. Bill did seem to have conflict with his beliefs as he seemed to fluctuate between an Instrumentalist viewpoint and that of a Platonist.

It is interesting to note that Bill had little to say about the problem in Scenario 4. He said he didn't know what formula to use and that he "couldn't use this with my students because I don't know the answer." This reinforced the fact that his beliefs lie on the Platonist side since he seemed interested in the formula or the answer but not really in any strategies or routes to the answer.

Bill said that he had never heard of the Standards. This was even in light of the fact that he had recently attended a two day regional NCTM conference in Winnipeg at which the focus was to raise awareness among teachers of the importance of the Standards. Bill's view of mathematics is very much in conflict with the views outlined in the Standards.

Summary

The three phase beliefs protocol was analyzed individually for each of the five teachers who participated in this study. The three teachers who seemed to hold views of mathematics most in line with those of the Standards were all junior high teachers who teach Senior 1 along with grades 7 and 8. The other two teachers held either an Instrumentalist or a Platoninst viewpoint and they were both senior high teachers who teach Senior 1 along with Senior 2, 3, & 4. This may indicate that the junior high teachers have had exposure to a broader view of mathematics than the senior high

teachers.

The three phases of the protocol seemed to complement each other in giving a glimpse into the way teachers think about the nature of mathematics. In each case, if a teacher scored a high rating on the problem solving statements, they reinforced this score during the interview with ideas which seemed to support a Problem Solving conception of mathematics. These teachers also showed quite a bit of interest in attempting the problem in scenario 4 and were most creative in their approaches to the solution of this problem. In contrast, the two teachers who had a stronger belief in either the Instrumentalist or the Platonist view of the nature of mathematics scored lower on the rating of the problem solving statements and were inclined to be vague in their discussions of the idea of problem solving. Both of these teachers found the problem difficult since they could not find an equation right away that would give them the answer and they did not seem particularly interested in the problem.

Chapter Five

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

Summary of the Study

Questions and procedures

The purpose of this study was to examine the beliefs held by teachers about the nature of mathematics and to determine whether the beliefs held represent an obstacle to the implementation of a Standards based curriculum. Two questions were posed for investigation:

1. What are teachers' beliefs about the nature of mathematics?
2. How are teachers' beliefs aligned with the vision of mathematics of the Standards?

In order to answer these questions a three phase beliefs protocol was developed and administered to five Senior 1 (grade 9) Mathematics teachers during an interview. The protocol consisted of four classroom scenarios posed for discussion, a list of questions to which participants were invited to respond, and a rating scale of twelve statements.

Results

The three phase beliefs protocol was analyzed separately for each teacher. The complete analysis can be found in narrative form in Chapter Four.

The results of the study show that some teachers do hold problem solving views

of the nature of mathematics but these views are narrow with respect to the views of mathematics outlined in the Standards. The three teachers in this study whose views were predominantly Problem Solving in nature talked about this view and about the importance of thinking and reasoning in mathematics. However, their discussions tended to be limited to number problems and problems from business and consumer mathematics.

There was a difference between junior high and senior high teachers. All three junior high teachers held Problem Solving views while one of the senior teachers fell into the Instrumentalist category and the other one was primarily a Platonist. The beliefs of these two teachers are not well aligned with those of the Standards.

Implications of the Results

The results of this study indicate the need for teachers' views of problem solving to be broadened. As Cooney (1985) stated, teachers tend to teach the way they were taught. Even though they may articulate a particular belief in the Problem Solving view of the nature of mathematics they may not be able to act on this in their classroom. Taylor (1990) has indicated that talking and thinking about a different way to teach and learn mathematics did not make a difference in the way the teacher in the study conducted his class. It seems likely that teachers of mathematics would benefit from preservice and inservice training designed to help them investigate and experience the views of mathematics presented in the Standards.

A number of researchers (Arsac, Balacheff, & Mante, 1992; Ernest, 1989; Nespor,

1987, & Thompson, 1984) have indicated that a relationship does exist between teachers' beliefs about mathematics and how they teach. If teachers do not receive further training in this area it is possible that the conflict between their beliefs and the Standards will limit the success of the implementation of the revised mathematics program in Manitoba.

Further Research

The three phase beliefs protocol did provide an indication of individual teachers' beliefs about the nature of mathematics. This protocol could be used in a large scale study to determine to what degree teachers' beliefs are aligned with those of the Standards. Since a difference emerged between senior high and junior high teachers it would be important to see if this difference would be confirmed in a large scale study. Teacher educators could then take this difference into account when planning appropriate experiences for groups of teachers at both the preservice and inservice levels.

If the three phase protocol were to be used again it would require some modifications. The most striking difficulty was found with the scenarios. It is difficult for mathematics teachers to discuss mathematics without referring to their teaching. The inclusion of scenarios related to classroom experiences complicated this further by directing discussion towards the teaching of mathematics. This was compounded by the fact that the scenarios were a natural place to start the interviews in that the teachers had been able to look them over prior to the interview. All the

teachers who participated in the study had some initial comments to make with respect to the scenarios. This started the interviews off with a focus on mathematics teaching and it became difficult to bring the focus around to mathematics itself.

Another problem area involved the statements used in the rating scale. It was difficult to compose statements which represented only one of the views of mathematics from the Ernest model. Even though a panel of experts was consulted in the design of the rating scale there was a lack of agreement between them on which view certain statements represented.

The three phase beliefs protocol could be used to study the relationship between teachers' beliefs about the nature of mathematics and their current instructional practices. Application of the protocol in combination with intensive classroom observation could yield some useful information. Teacher educators could gain insight into the kinds of instructional practices that teachers need to explore and to experience in order to implement a Standards based curriculum.

Conclusions

Senior 1 teachers of mathematics interviewed hold views of mathematics that are, to varying degrees, in conflict with the vision of mathematics put forward in the Standards. Thus implementation of the proposed Senior 1 Mathematics program in Manitoba may not be successful unless this factor is addressed through preservice and inservice training.

In order for teachers to broaden their view of mathematics they may need to be

exposed to a variety of experiences in which a Problem Solving view of the nature of mathematics is presented. This might include inservice training sessions which allow teachers to be involved in the kind of mathematical problem solving that is advocated throughout the Standards. Rather than teachers receiving training in the teaching of mathematics, the results of this study and others (Cooney, 1987; Conroy, 1989; Ernest, 1989; & Lampert, 1988) imply that it would be worthwhile for teachers to participate in activities that would have them experience the kind of mathematics learning that is to be expected of students.

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Appendix A

Letters to Teachers

Barbara E. Vukets
14 Queensway Crescent
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R2J 3P4 (204) 255-3155

April, 1993

Dear Senior 1 Teacher,

The mathematics program in Manitoba is presently undergoing reform. As a graduate student and instructor in mathematics education at the University of Manitoba, I have become interested in the reform process. For my Master's thesis I have chosen to study teachers' beliefs about the nature of mathematics and how this impacts on the implementation of new curriculum.

To this end I would like to interview a number of teachers to find out what their beliefs are about mathematics. The purpose of this letter is to ask you if you would be willing to assist me by agreeing to be interviewed on this topic. I think the interviews would take between 20 and 40 minutes. they could be done at any time that is convenient for you. I plan to tape all the interviews and then transcribe the tapes for analysis. No data will be collected about individual school divisions, schools, or teachers.

In addition to the interviews, teachers will be asked to rate a number of statements and to respond to four classroom scenarios. From these experiences I hope to get an idea of the beliefs the teachers holds with respect to the nature of mathematics.

In all, I hope to interview five teachers. I will follow this letter up with a telephone call in a few days to determine whether you would be interested in participating in this study. All the teachers who participate in the study will be provided with a final report of the project. Any tapes made during this study will be destroyed upon its completion. You would be free to withdraw from the study at any time.

Yours truly,

Barb Vukets

Barbara E. Vukets
14 Queensway Crescent
Winnipeg, Manitoba
R2J 3P4 (204) 255-3155

May, 1993

Dear Senior 1 Teacher,

Thank you for agreeing to participate in the study I am conducting for the purpose of a Master's thesis. I appreciate your support of my endeavour. I sincerely hope that you will find the experience interesting and informative.

Prior to the interview it would be helpful if you could find a few minutes to complete the attached rating scale Statements About the Nature of Mathematics. I will collect this from you at the interview. In preparation for the interview you may want to read over and think about the scenarios as I am interested in your reactions to them. I will bring a tape recorder to the interview so that I may tape our discussion. I am anticipating that the interview process will take around a half hour to complete.

I am looking forward to meeting with you and thank you again for your help.

Yours truly,

Barb Vukets

Appendix B

Scenarios

Scenarios

1. A student in your mathematics class asks why she has to do this "stuff". Explain how you would respond to her.

2. The text book says that division by zero is "undefined". A student wants to know what this means so his teacher explains it in the following way: "Let's look at an example - say 16. $16 \div 8 = 2$, $16 \div 4 = 4$, $16 \div 2 = 8$, $16 \div 1 = 16$, $16 \div \frac{1}{4} = 64$ and so on. So you see, when you divide by zero, which is the smallest number you can have, then the answer gets so large we say it is undefined." What do you think of this explanation? How would you help a student understand why we say that division by zero is undefined?

3. An experienced Senior 1 teacher has videotaped a number of her classes over the last few months. She notices that she frequently spends 25 of the 65 allotted minutes reviewing the previous day's assignment. She decides to try and reduce this time by collecting students homework on Monday and Thursday and by giving a quick quiz on Wednesday. Comment on her problem and her solution.

4. Consider the following problem which could be given to Senior 1 students. Attempt to solve the problem and while you are doing so talk about the strategies you are using and the things that come to mind as you work on this problem.

> Ten bandits held up a stagecoach. The loot from the robbery consisted a bag of gold nuggets. The ten bandits decide to split the nuggets into 10 equal piles. But when they tried to do that, there were 9 nuggets left over. Then, they tried splitting the loot into 9 equal piles, but now there were 8 nuggets left over. Next, they tried splitting the loot into 8 equal piles, but there were 7 nuggets left over. This pattern continued until their final attempt. When they tried splitting the loot into 2 equal piles, lo and behold, there was 1 nugget left over. To say they were rather frustrated would be an understatement.
>
> Matters looked serious for a while. Suddenly, one of the bandits said, "Eureka! I know what to do. First, add a small rock to the loot and pretend that it is a gold nugget. Now the loot can be divided into 10 equal piles. Unfortunately, one of us will receive a rock as part of his share but we'll draw straws to determine who that is." The bandits adopted the solution.
>
> What is the smallest number of gold nuggets that made up the loot?

Appendix C

Interview Questions

Questions About the Nature of Mathematics

1. Describe the discipline of mathematics.
2. Why is mathematics important?
3. How important are facts, algorithms, and procedures in mathematics?
4. In what ways is mathematics useful?
5. Is mathematics an ancient discipline or has much of it been created in fairly recent times?
6. What does the term "mathematical problem solving" mean to you?
7. Are you familiar with the Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989)?

Appendix D

Statements About the Nature of Mathematics

Statements About the Nature of Mathematics

For the statements given below, please attempt to indicate the degree to which you agree or disagree with each one by marking the appropriate response with a check mark.

	Strongly Agree	Agree	Disagree	Strongly Disagree
1. Mathematics is a tool to be applied to the sciences.	—	—	—	—
2. The study of mathematics must include opportunities to use mathematics in other disciplines.	—	—	—	—
3. To be good at mathematics you need only to be able to remember rules, facts, and procedures.	—	—	—	—
4. Mathematicians are only hired to make measurements and calculations for scientists.	—	—	—	—
5. Students should feel free to use any method for solving a problem that they find suitable.	—	—	—	—

	Strongly Agree	Agree	Disagree	Strongly Disagree
6. The main benefit from studying mathematics is developing the ability to follow directions.	—	—	—	—
7. Mathematics is a set of facts and skills.	—	—	—	—
8. Solving a mathematics problem usually involves finding a rule or formula.	—	—	—	—
9. Many of the basic concepts and procedures of mathematics can be discovered by students.	—	—	—	—
10. Students should be encouraged to invent their own mathematical symbolism.	—	—	—	—
11. Mathematics is a rigid discipline which functions strictly according to inescapable laws.	—	—	—	—
12. The average mathematics student, with a little guidance, should be able to discover the basic ideas of mathematics for him or herself.	—	—	—	—