A PROGRAMMED MODEL FOR A PROVINCIAL INPUT/OUTPUT TABLE

A Thesis

Presented To

the Faculty of the Department of Economics

University of Manitoba

In Partial Fulfillment

of the Requirements for the Degree

Master of Arts

by

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September, 1966

ABSTRACT

This thesis is presented to the Faculty of Arts, Department of Economics, in partial fulfillment for the Degree of Master of Arts.

The theme is the development of a procedure whereby a relatively inexpensive, highly aggregated provincial input/output table can be formed. It is directed toward the Economist, by whom it is assumed, the table would be compiled. The compiler is also assumed to have little or no knowledge of computer procedure, and the method by which the data is transformed to a table format by the computer is given in step-by-step layman's terms.

Chapter I gives an Introduction to the study and a layman's perspective of the input/output model. The model actually used is developed and rationalized in Chapter II. The sources and methods of gathering the raw data for the table are considered in Chapter IV and is meant as essential reading for the execution of the step-by-step procedure of Chapter IV. Chapter IV contains the detailed procedure for registering the data and arranging for and executing the computer program. Finally, Chapter V reviews the thesis and considers subsequent procedures.

There are three appendices connected with the topic. The first is a note on the historical evolution of the input/output discipline up to and including the early works of Leontief. The second contains the two basic programs referred to in Chapter IV and the last appendix describes the procedure for converting the program from Fortran II to Fortran IV - a revision necessary for executing the program on the IBM 360 line of computers.

PREFACE

Understanding the theory of Input/Output Analysis is a necessary but not sufficient condition of being able to actually compile such a table. Knowledge must be had not only of data sources but also of the procedure and variations on the basic model appropriate to the data available. The purpose of this paper is to outline an input/output model appropriate to provincial availability of data and to present a computerized method of compiling the required tables.

The author is indebted to the Manitoba Economic Consultative Board for the opportunity of working on such a project, and in particular for the guidance and direction of Dr. M. Cormack, Senior Economist of the Board. The experimental method of compiling the interindustry section of the Manitoba input/output table was undoubtedly the least expensive approach in the light of the circumstances although a guideline such as this paper represents would have made the task considerably easier. The emphasis in the paper then is on procedure tempered by experience rather than by theory.

The author appreciates the guidance given to him in regard to this paper by Dr. Cormack, Dr. Chen (Economics Dept. - University of Manitoba) and, not the least, his wife, whose sense of urgency caused the thesis to be completed.

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CHAPTER I

INTRODUCTION

Input/output analysis is a means whereby the economy can be looked at as an equilibrium of micro-relationships. Taken purely as an accounting framework, it is a method of categorizing and delineating the components of the economy; taken as an economic model, it is a means of tracing and quantifying total effects from a given effect. In the first sense, input/output analysis can be termed "descriptive"; in the second, "predictive" and this distinction will be maintained throughout this paper.

The input/output accounting framework is based on two identities namely that a sector of the economy has its outputs equal to its inputs (in value terms), and that its output is equal to the uses, both industrial and final, to which it is put. The economy is thus divided into sectors in such a way as to be convenient for measurement and subsequent use. The manufacturing industry sectors normally form the bulk of an input/output table and it is this aspect with which this paper is concerned. Other sectors such as final demand categories, primary goods categories and non-manufacturing industries are treated only in passing.

In an economic sense, the substantive assumption whereby predictions can be made, is that certain quantity (not value) relationships hold constant when considered as a proportion of output. In particular, it is assumed that "produced" inputs are a constant proportion of the using sectors output. In the base year calculations of these input proportions, or coefficients, it is assumed the value relationships reflect the quantity relationships. In a strict sense, when a "prediction" is made from these calculations, the price effect should be eliminated in order to maintain the "quantity" relationships[(1)].

This paper is designed as a "handbook" for compiling a basic input/output table and in particular in collecting data and arranging it in the proper forms for the interindustry part of the table. It is written not only for the compiler but also for anyone concerned with such a project. To this end, the chapters can be looked at as a number of papers in themselves:

I - Introduction

The remainder of this chapter is directed toward the layman and is concerned with basic concepts. The compiler himself could begin at Chapter II with no loss in continuity.

II - The Accounting Framework

The model is developed in the light of available information on a provincial basis. Aggregation, transportation and trade margins, and imports, are the main points of consideration, and the model developed is reflected in the computer program of Chapter IV. The usefullness of input/output analysis is also considered. The economist considering an input/output project in the provincial context might read this to advantage; the compiler might again omit this chapter although it would probably be

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(1) See page 16

useful in putting "work-flow" of the project, in its perspective.

III - The Data Sources

The suggested sources of data are considered in detail and a method of recording data is outlined. This, together with the next chapter is the compilers "handbook".

IV - The Computer Program

In developing this chapter, the assumption was made that the compiler is not, and need not be, a computer programmer. The procedure outlined in for the compiler in order to set up the data, and the program in such a way as to allow a computer operator to "run" the program directly. Included are procedure instructions for the operator as well. Emphasis is on establishing a working relationship between the compiler and the computer centre. Once the model and the data sources are accepted, the need for developing a program itself is eliminated.

V - Conclusion

A procedure is outlined for developing the "inverse" as given both by direct inversion and by the power series approximation. Methods for deriving the model of Chapter II are considered and the procedure is gone over once again. Useful references are considered briefly.

The thesis itself is based on the "hindsight" view the author has had in working on such a project for M.E.C.B. The initial data (DATA I of Chapter IV) was entered on 10,000 keyhole cards with certain information coded on holes on the perimeter of each card. The method of sorting and arranging the data - by hand - proved inefficient in view of the availability of computer processing and the low cost of IBM cards. Again, more than twenty computer programs were developed, on an ad hoc basis, many of which proved redundant and inefficient. The program outlined in Chapter IV and V are composite programs designed for simplicity and compactness. Hopefully the compiler need know nothing more than what is outlined in Chapter IV in order to arrange the procedures with the computer centre.

The much discussed problems of joint products and secondary products have not been discussed in the paper because they have been "assumed" away in the light of the high degree of aggregation of the square (invertable) matrix. If greater detail is required or if the above problem is bothersome, the alternate Industry by Industry table (see Chapter III - section 3) may be desired. The cost of such a table would probably be considerably higher.

The remainder of this Chapter is taken up with developing the Model in basic, laymans terms.

THE LAYMAN'S PERSPECTIVE

Interindustry analysis is a relatively new field of economics. As the name would suggest, a study of this nature inquires into the interrelationships among the various productive sectors of the economy. Input/output analysis is a somewhat broader term in that the relationships of the productive with the non-productive sectors of the economy may be considered. On the model or accounting system used to display such information, the sectors (usually industry groupings) are delineated according to "likeness" of input structure.

Two basic uses of this approach to economic accounting should be made from the outset; input/output analysis can be used for either predictive or descriptive purposes, and only the former requires the traditional homogeneity-substitution assumptions. In a descriptive sense, an input/output table could be used for:

i) a method of presenting detailed statistics;

ii) a means of checking other sources of statistics;

iii) a basis for international and interregional comparisons of economic and technological structures;

iv) a data format useful for "Marketing Analysis" information. "What products in what proportion would be required to produce a given product and where, if at all, would these be available?"

In the traditional, predictive usages, an input/output table can be mathmatically manipulated so as to indicate the direct and "feedback" effects of a given change in a parameter. More precisely, such predictive uses might be:

i) tracing the total effect on resources, including labour, and on industry output, of a change in final demand (hereafter referred to as final demand);

ii) tracing the effect of a change in a production coefficient of a given industry resulting from a change in technology, a substitition for certain input(s) (say steel replacing wood) or a change in the labour-capital content of output;

iii) tracing the effects of taxing certain commodities or resources or determining change in tax revenues as a result in change (projected) in final demand via population increase, taste changes, etc.;

iv) tracing the total effects of import replacement, of export campaigns and of resource scarcity;

v) analyzing price shifts as a result of changes in the various items mentioned above.

All of the uses outlined above will be considered in detail in the next chapter. The remainder of this chapter will be taken up with the make-up of the accounting framework and with a brief consideration of the historical development of this type of analysis.

The Accounting Framework

The basic identity on which the input/output accounting framework is based is that the total production of a sector is distributed to:

i) other sectors which "consume" this output in making their own products; and

ii) final demand sectors which include private consumption, government consumption and exports, business investment.

Let

x_{ij} = Total dollar value of product from sector "i" used in production by sector "j";

y_i = Final (autonomous) demand for the production of sector "i" measured as a dollar amount;

x_i = Total dollar amount of domestic production of sector "i";

The identity, then, is:

$$x_i = x_{i1} + x_{i2} \ldots\ldots\ldots x_{ij} \ldots\ldots x_{in} + y_i \ldots\ldots\ldots (I - 1)$$

or

$$x_i = \sum_{j=1}^{n} x_{ij} + y_i$$

Thus, if industry "i" were the Steel Industry, then the identity states that the total production of steel is exactly equal to the sum of its uses. Note that the variable x_{ij} (j=1) is included and represents the amount of steel the industry uses to produce steel. This may or may not be a zero amount. The users of steel for further production were divided up into "n" sectors where the magnitude of "n" may vary according to convenience.

A series of such equations can then be laid out:

$$X_1 = X_{11} + X_{12} \;\ldots\ldots + X_{1j} \;\ldots\ldots\; X_{1n} + Y_i$$

$$X_i = X_{i1} + X_{i2} \;\ldots\ldots + X_{ij} \;\ldots\ldots\; X_{in} + Y_i$$

$$X_m = X_{m1} + X_{m2} \;\ldots\ldots + X_{mj} \;\ldots\ldots\; X_{mn} + Y_m$$

..... (I - 2)

where $j = 1 \ldots\ldots n$ as before

and $i = 1 \ldots\ldots m$

$n \lesseqgtr m$

Note that no requirement is made that the number of sectors delineated in columns (the "j"s) need be equal to the number of sectors delineated in rows (the "i"s). If $n \neq m$ then normally the k^{th} row sector is not the same as the k^{th} column sector.

These equations can now be looked at in a different way. In the above layout, they were regarded as a series of rows. They can now be looked at as a series of columns where, given a column, the "j"th one, the vector of X values for that column represents the dollar value of inputs into that (j^{th}) sector.

$$\begin{matrix} X_{1j} \\ X_{2j} \\ \cdot \\ X_{ij} \\ \cdot \\ X_{mj} \end{matrix}$$

Where X_{ij} represents the amount of input from row sector "i" used by column sector "j". But these inputs are only the "produced" inputs; there are other inputs which go into the production process which are not commonly thought of as being produced, labour being the most obvious example. Let V_j represent the dollar total amount of these primary (non-produced) inputs into the j^{th} sector. This leads to the second identity of an input/output table, namely, that total production equals the sum of the inputs, for each producing sector.

Let

x_j = total production of the j^{th} column sector.

Then

$$x_j = x_{ij} + x_{2j} \ldots\ldots + x_{ij} \ldots\ldots x_{mj} + V_j \ldots\ldots\ldots (I - 3)$$

or

$$x_j = \sum_{i=1}^{m} x_{ij} + V_j$$

Making, now, the usual assumption that the complete domestic economy is represented, it follows that any productive entity will be represented (in aggregated or disaggregated form) by both a column and a row. Whereas the steel industry was previously classified as the i^{th} row sector, it would now also have to be represented either in the same form ("j^{th}" column = the Steel Industry) or in aggregated form ("j^{th}" column = the Metal Industry) or in disaggregated form ("j^{th}" column = cold rolling Steel Mills and "k^{th}" column = other Steel Industries). As a matter of emperical convenience, most input/output tables begin as a rectangular table where the row sectors represent commodity detail and the column sectors represent industry detail in which case, in the above notations, $m > n$. A table in this form is commonly used for the "descriptive" uses outlined above; for "predictive" purposes which require an "inverse" table, a square table must be built. A square table could be made from a commodity by industry tables (as outlined above) by:

i) aggregating commodity, <u>row</u> sectors into industry, <u>row</u> sectors;

ii) making the industry row sectors correspond to the industry column sectors (the k^{th} row industry must correspond with the k^{th} column industry).

This now completes the discussion of the table as far as a descriptive framework is required. Thus for descriptive purposes, the accounting scheme is built on the following identities:

$$x_i = \sum_{j=1}^{n} x_{ij} + y_j$$

$$x_j = \sum_{i=1}^{m} x_{ij} + v_j$$

$$n \gtrless m$$

The magnitude of n and m varies according to the degree of detail desired and the degree of detail available. A convenient tabular format would be:

	Industries			Final Demand (s)	total Output
	x_{1j} ⋯	x_{1j} ⋯	x_{1n}	Y_1	X_1
	$\vdots$	$\vdots$	$\vdots$		
Commodities	x_{i1} ⋯	x_{ij} ⋯	x_{in}	Y_i	X_i
	$\vdots$	$\vdots$	$\vdots$		
	x_{mi} ⋯	x_{mj} ⋯	x_{mn}	Y_m	X_m
Primary Inputs					
	V_1	V_j	V_n		
Total Output					
	X_1'	X_j'	X_n'		

Another identity is brought into the model in order to make it convenient for predictive purposes. The individual inputs are now expressed as a proposition of the output of the column sector.

$$a_{ij} = \frac{x_{ij}}{X_j} \quad \ldots\ldots\ldots\ldots\ldots (I - 4)$$

If x_{ij} and X_i were expressed in physical units, a_{ij} would represent the technological coefficient; if they are measured in dollar amounts, a_{ij} represents the "production" coefficient. The stability or constancy of a_{ij} over time is the crucial question in input/output analysis: all the "questionable" assumptions of homogeniety of inputs and outputs, and of non-substitutability, are made for this purpose.

The identities I-1, I-2 and I-4 and the conditions of correspondence of row and column sectors result in the basic input/output table as used for predictive purposes. Note that the original balance identity I-1 can now be modified by I-4 such that:

$$x_i = \sum_{j=1}^{n} a_{ij} X_j + Y_i \quad (i = 1\ldots\ldots\ldots n) \quad \ldots\ldots\ldots\ldots\ldots (I - 5)$$

and

$$x_j = \sum_{i=1}^{n} a_{ij} X_i + V_j \quad (j = 1\ldots\ldots\ldots n) \quad \ldots\ldots\ldots\ldots\ldots (I - 6)$$

where

m now equals n and the former is dropped.

A convenient tabular format would now be:

$$
\begin{array}{ccccc}
a_{11} \cdots\cdots a_{1j} \cdots\cdots a_{1n} & & Y_1 & & X_1 \\
\vdots \qquad\quad \vdots \qquad\quad \vdots & & & & \\
a_{i1} \cdots\cdots a_{ij} \cdots\cdots a_{in} & & Y_i & & X_i \\
\vdots \qquad\quad \vdots \qquad\quad \vdots & & & & \\
a_{n1} \cdots\cdots a_{nj} \cdots\cdots a_{nn} & , & Y_n & & X_n \\
V_1 \qquad\quad V_j \qquad\quad V_n & & & & \\
X'_1 \qquad\quad X'_j \qquad\quad X'_n & & & &
\end{array}
$$

where now

$$X'_1 = X_1$$

$$X'_j = X_i \quad (i=j)$$

$$X'_n = X_n$$

and the prime is dropped.

The individual a_{ij} embody the economic law unique to input/output analysis. For predictive uses of input/output, the basic assumption made is that the a_{ij}'s are fixed over the time period considered. Any deviations from this assumption is regarded as a deviation from the norm; these matters are treated in detail in the next two chapters.

The basic predictive ability can now be shown with the help of matrix algebra and the use of superscripts - a "o" indicating a base year measurement and a "/" indicating prediction year measurement.

Then let

$$A^o = \begin{matrix} a^o_{11} & \cdots & a^o_{1j} & \cdots & a^o_{1n} \\ \vdots & & \vdots & & \vdots \\ a^o_{i1} & \cdots & a^o_{ij} & \cdots & a^o_{in} \\ \vdots & & \vdots & & \vdots \\ a^o_{n1} & \cdots & a^o_{nj} & \cdots & a^o_{nn} \end{matrix}$$

$$X^o = \begin{matrix} X^o_1 \\ \vdots \\ X^o_i \\ \vdots \\ X^o_n \end{matrix} \qquad Y^o = \begin{matrix} Y^o_1 \\ \vdots \\ Y^o_i \\ \vdots \\ Y^o_n \end{matrix}$$

$$I = \begin{matrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 1 \end{matrix}$$ n rows and n columns

By (I - 5)

$$X^o = A^o X^o + Y^o \qquad \cdots\cdots \text{(I - 7)}$$

$$X^o - A^o X^o = Y^o$$

$$(I - A^o) X^o = Y^o$$

$$X^o = (I - A^o)^{-1} Y^o$$

Now if $A^{o} = A'$ then,

$$X = A' X' + Y'$$

$$X' = A^{o} X' + Y'$$

$$\therefore X' = (I - A^{o})^{-1} Y' \qquad \text{..... (I - 8)}$$

Equation (I - 8) is no longer an identity; A^{o} does not equal A by definition. Thus, given a derived A^{o} matrix, based on base year data, and if Y' and X' are definitionally equivalent to their base year counterparts, then given Y' or X', the other can be solved for by (I - 8)'.[2]

The usual solution required is the prediction of total output needed (X') to support a bill of final demand Y'. By use of the inverse, not only are the direct requirements computed but also the indirect consumptions needed for the direct requirements. This can be nicely illustrated by assuming a final demand bill of unity and solving $(I - A^{o})'$ by the Leontief-Cornfield expansion.

Using

$$X' = (I - A^{o})^{-1} Y' \qquad \text{..... (I - 9)}$$

$$= (I - A^{o})^{-1} \quad \text{because } Y' \text{ assumed } = \begin{pmatrix} 1 \\ 1 \\ . \\ \vdots \\ \vdots \\ n \end{pmatrix}$$

Now

$$(I - A^{o})^{-1} = I + A + A^{2} + A^{3} \ldots$$

where the right hand side converges if $|A| > o$[3] which is here assumed, then for a projected increase in demand

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(2) Partial information on each of Y' and X' will allow remaining values to be solved for - See Chapter II, p. 35.

(3) See Chapter V.

of 1 unit for the production of each sector, the resultant required increase in production will be the sum of the above terms.

Thus, I (identity matrix), represents the production supplied directly to final demand.

A represents the amounts supplied to the suppliers of final demand.

A^2 represents the amounts supplied to the suppliers of the suppliers of final demand and so on.

A concrete example may make this clear. Assume a three sector economy:

	Food	Machines	Other
Food	$a_{11} = .3$	$a_{12} = .6$	$a_{13} = .1$
Machines	$a_{21} = .5$	$a_{22} = .1$	$a_{23} = .1$
Other	$a_{31} = .1$	$a_{32} = .2$	$a_{33} = .6$

where for example for every dollar's worth of food, 30 cents of Food, 50 cents of Machines and 10 cents of Other is required. Postulating an increase in final demand of $1.00 for Food, $1.00 for Machines and $1.00 for Other, the net requirements would be

$$\underline{\quad I \quad} + \underline{\quad A \quad} + \underline{\quad A^2 \quad}$$

$$\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} \quad + \quad \begin{matrix} .3 & .6 & .1 \\ .5 & .1 & .1 \\ .1 & .2 & .6 \end{matrix} \quad + \quad \begin{matrix} \begin{matrix} .3 \times .3 \\ +.6 \times .5 \\ +.1 \times .1 \end{matrix} & \cdots & \cdots \\ \vdots & \vdots & \vdots \\ \cdots & \cdots & \begin{matrix} .1 \times .1 \\ +.2 \times .1 \\ +.6 \times .6 \end{matrix} \end{matrix} \quad + \cdots$$

Although for expositary purposes, the power series $(I + A + A^2 + A^3 \ldots)$ has been used here, a number of actual inversion techniques may be used. In some cases, certain properties of the A matrix may be used for computational simplicity.(4)

The constancy of A^o over other periods is usually assumed if A^o represents technological (physical) coefficients. Since A^o is normally developed from value figures, a deflation of A^o by a price index (if available) would strengthen the constancy assumption. Indeed, in reference (11), the author shows that $a_{ij} = f\left(\bar{a}_{ij}, \frac{P_i}{P_j}\right)$

where

$\bar{a}_{ij}$ = physical input coefficient

a_{ij} = value input coefficient

P_i = price of "i"

P_j = price of "j"

Japan has used this type of relationship, weighting P_i/P_j according to a price index and has found that indeed $\bar{a}_{ij}$ is more stable than a_{ij}.(5)

In the next chapter, these accounting relationships are gone into in some detail and are looked at in view of data availability. The historical development of input/output analysis is traced in Appendix "A".

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(4) See Chapter V below.

(5) See Reference (27).

CHAPTER II

THE BASIC MODEL

This chapter is concerned with the type of model the data gathered will allow. The possible methods of coefficient determination are first considered, then a number of common problems are analyzed, and certain models are developed in the light of available information. Finally, the typical uses of input/output analysis are considered in some detail.

Data Gathering Methods

The emperical data used in estimating the production coefficients can be compiled in a number of ways:

1. The coefficients can be formed from a sample of value flows of inputs and outputs for each industry. To this end, the Industrial Census Forms on file with the Provincial Government can be used. Most of these forms will give a commodity breakdown of input and output flows over a given year for each SIC[1] industry. An individual form corresponds with an Establishment[2] classified to an industry, and these establishments can be aggregated (by commodity categories) for an industry sample. The production coefficient can then be formed for each industry by summing up all outputs and

(1) SIC stands for "Standard Industrial Classification" a D.B.S. code for industries - the basic manual (47)

(2) Establishment is defined by D.B.S. although the meaning enjoys international acceptance. An Establishment is the smallest section of a firm which keeps its own accounts.

dividing this into the sum of input values of each commodity. Usually only a selection of establishments are available for each industry and furthermore many entries on the available forms may be inaccurate or not filled out at all. In any case, most tables are compiled by this method..

2. Data on the distribution of the output of the various commodities (or industries) can be used to fill the cells of the value table. Here the row vectors are estimated, the column vectors (cost structures) following as a matter of definition. Due to lack of detail in regard to destination of commodities (or industry outputs) this method is only used as a check on method (1) above, if at all. A case in point is the use of such a procedure by Irving Stone(3) in developing his Social Accounting Matrix (SAM). A matrix of the distribution of outputs is compiled as well as cost structure matrix, and the two are mathematically averaged out.

3. Industry experts can be used to estimate the input coefficients directly. This would be done on the basis of engineering and technological studies and native experience. The feasibility of such an approach is normally not considered, although it is used for checking the reasonableness of data as well as for estimating certain non-manufacturing sectors -- e.g. mining.

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(3) See Irving Stone (39), Chapter II

4. Dorfman, Samuelson and Solow[4] present another method whereby a correlation analysis is run on final demand and corresponding output vectors from different time periods, and the inverse coefficients are directly estimated. The row input coefficients are then estimated by:

$$A = I - (I - A^{-1})^{-1}.$$

Unfortunately, the requirements that:

(a) the observed bill of final demands vary significantly and independently; and

(b) that there are at least more observations than there are divisions of final demand;

require the almost impossible condition of fine commodity breakdown and a large number of observations. In that output/demand data sets are at best available on a quarterly basis, the latter condition would require a number of years of observations, yet this would cast doubt on the constancy of the computed coefficient. The author has been unable to find reference to a practical use made of this approach.

The argument for using the first procedure as outlined above is based on the easy availability of the required data although it has the added advantage that the Hawkins-Simon[5] conditions for resolvability are satisfied. These conditions require that for any industry, a unit of its output will not require, as input, a unit of the same output or more, either directly or indirectly. These conditions can safely be assumed to hold given that lack of profit would eliminate such industries

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(4) See Dorfman, Samuelson and Solow, (6), Chapter 12

(5) See Hawkins and Simon, (26) and Hawkins (25)

and that this would be reflected in the emperical sample. On a provincial basis then, it would appear that the best procedure would be to:

1. Aggregate input data (maintaining 3-digit SCC[6] detail) and output data (in total), into industry categories.

2. Assuming that the resulting value figures represent a true picture of the proportional breakdown of inputs, forming the coefficients by dividing each input value figure by the corresponding total output value figure.

This procedure will be treated in detail in Chapter III, Part I.

The Accounting Structure and Aggregation

Let

Y = a column vector of final (autonomous) demand, with n industry divisions.

X_{ij} = the value of input from industry (commodity) i to industry j, or in other terms, the value of output from industry (commodity) i used by industry j.

V_j = value of labour used by industry j.

W_j = value of all non-labour primary factors used by industry j.

Let inputs into an industry be shown as a column vector and the distribution of outputs of an industry as a row vector.

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(6) SCC stands for Standard Commodity Classification - a D.B.S. code for commodities - the basic manuals are (44, 45, 46)

Then, the following equations hold:

$$X_{i1} + X_{i2} + \ldots\ldots + X_{ij} + \ldots\ldots + X_{in} + Y_i = X_i \qquad \ldots\ldots \text{(II - 1)}$$

and

$$X_{1j} + X_{2j} + \ldots\ldots + X_{ij} + \ldots\ldots + X_{mj} + V_j + W_j = X_j \qquad \ldots\ldots \text{(II - 2)}$$

and

$$a_{ij} = \frac{X_{ij}}{X_j} \qquad \ldots\ldots \text{(II - 3)}$$

$$i = 1, \ldots\ldots n$$

$$j = 1, \ldots\ldots m$$

The above equations would allow the compilation of a commodity by industry table from the Industrial Census Forms. A preliminary step, however, is required to aggregate the individual Establishment (one per form) into the corresponding industry category. The basic assumption made is that the cost structure of an industry is a simple average of the component Establishment cost structures.

Thus

$$\begin{array}{l}
X_{1j} = X^{1}_{1j} + X^{2}_{1j} + \ldots\ldots + X^{k}_{1j} \\
X_{2j} \\
\vdots \\
X_{ij} = X^{1}_{ij} + X^{2}_{ij} + \ldots\ldots + X^{k}_{ij} \\
\vdots \\
X_{mj} \\
\vdots \\
V_j \\
\vdots \\
W_j = W^{1}_j + W^{2}_j + \ldots\ldots + W^{k}_j
\end{array}$$

where the superscripts represent an individual establishment, there being k of them for this particular j^{th} industry. A given column of the right hand side above, would represent value data from one form. The output value figures from the k establishments would be summed and divided into the composite value input figures in order to calculate the "average" input coefficients for that industry.

The resultant commodity by industry table of coefficients (or value) may be of particular interest to market analysts in that it gives a commodity breakdown of inputs for each industry.

The commodities and industry can be conveniently aggregated into Major Groups with the result that a number of tables can be derived from the basic commodity by industry one. These are:

Major Group by Industry

Commodity by Major Group

Major Group by Major Group

The last table is perhaps the most important in that it fulfills the requirement for inversion, of equal rows and columns.

Commodity to Major Groups:(7) Unlike the problem of assigning Industries to Major Groups, commodities do not by definition belong to particular Major Groups. It was found that less ambiguity arose if the SCC commodities were assigned directly to Major Groups rather than being first assigned to Industries and then to Major Groups. Commodity input coefficients can be conveniently aggregated into Major Group input coefficients by simple summation. Thus, given a unique assignment of a

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(7) Major Group is an SIC defined category by which Industries with like outputs are grouped

commodity to a Major Group, the following holds:

Let the following commodities be assigned to Major Group 8:

Commodity in row 10

Commodity in row 11

Commodity in row 12

Then

$$a_{8,j} = a_{10,j} + a_{11,j} + a_{12,j}$$

$$j = 1, \ldots\ldots n$$

where the coefficients on the left hand side are not correspondingly subscripted to the ones on the right hand side. This formula can be expressed in value terms:

$$\frac{X_{8,j}}{X_j} = a_{8,j} = \frac{X_{10,j}}{X_j} + \frac{X_{11,j}}{X_j} + \frac{X_{12,j}}{X_j}$$

which shows that if the inputs were measured in value terms, the composite $X_{8,j}$ would still equal the simple sum of $X_{10,j}$, $X_{11,j}$ and $X_{12,j}$.

Industry to Major Group: The aggregation of Industry columns into Major Group columns is not as simple as the above. Although the Industries by SIC definition belong uniquely to a Major Group, the individual industries must be weighted in order of their importance. Thus, if one used simple summation, as above, then:

$$a_{i,8} = a_{i,10} + a_{i,11} + a_{i,12}$$

but when put in value terms, the $X_{i,8}$ composite would not equal the individual $X_{i,10} + X_{i,11} + X_{i,12}$.

Thus,

$$\frac{X_{i,8}}{X_8} = \frac{X_{i,10}}{X_{10}} + \frac{X_{i,11}}{X_{11}} + \frac{X_{i,12}}{X_{12}}$$

which expresses the last equation only in expanded terms, would disallow

$$X_{i,8} = X_{i,10} + X_{i,11} + X_{i,12}$$

unless $X_8 = X_{10} = X_{11} = X_{12}$, which is normally not the case.

Before considering the theoretical aspects of aggregation, let it suffice here that it has been found convenient to weight the coefficients by the proportion of Major Group output that the individual industry's output forms. Thus:

If we make

$$a_{i,8} = a_{i,10} \left(\frac{X_{10}}{X_{10} + X_{11} + X_{12}}\right) + a_{i,11} \left(\frac{X_{11}}{X_{10} + X_{11} + X_{12}}\right)$$

$$+ a_{i,12} \left(\frac{X_{12}}{X_{10} + X_{11} + X_{12}}\right)$$

then $$a_{i,8} = \frac{X_{i,10}}{X_{10} + X_{11} + X_{12}} + \frac{X_{i,11}}{X_{10} + X_{11} + X_{12}} + \frac{X_{i,12}}{X_{10} + X_{11} + X_{12}}$$

This shows that if the total output of the Major Group equals the simple sum of the total outputs of the industries, then the composite value of inputs will equal the corresponding sum of the input values for each industry when calculated from the coefficients.

The aggregation of these Industries into Major Groups has been done on the assumption that either 1) their input structures are the same. (By definition their outputs are similar and, therefore, their input structures are usually similar as well), or 2) that any change in output is distributed proportionately between the component industries even though they may have different input structures.

Any practical input-output table will be expressed in terms of rows and columns which represent aggregation of "ideal" divisions. This "ideal" which is normally taken to represent unique commodity divisions, cannot be attained in a practical sense because accounts of business are not normally available. In Canada, for instance, the D.B.S. Census Form is used for registering industrial information, there being one form per Establishment. An Establishment by definition is the smallest accounting sector of a firm, yet these Establishments themselves produce more than one commodity as a rule. As far as a "predictive" input-output table based on such a fine commodity detail is concerned, the assumption of no input substitution would certainly not hold. Thus, the criterion on aggregation must be something other than that it be a reflection of the "ideal" dissaggregated model. The accepted criterion has become "for all possible variations in final demand, the total output, when aggregated from the original sectors, should be equal to the total output of the aggregated sectors".(8)

This "acceptable" requirement can be met if:

1. aggregation is made of sectors having equal input coefficients -- the Chenery-Clarke(9) case of horizontal integration; or

2. aggregation is made of sectors whose outputs change in proportion -- the Chenery-Clarke(10) case of vertical integration.

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(8) See Fei, J. (18)

(9) See Chenery, H. B., and Clarke, P. H. (5), Chapter 12

(10) op. cit.

A number of articles have been written on the question of aggregation, the more interesting of which are -- Ara, K. (13), Theil, H. (40), and Fei, J. (18) gives a concise, precise account of the potential error involved in aggregation and disaggregation in regard to the inverse, and Fei (18) shows that the conditions of "acceptability" can be met if the column sums of the component column of an aggregated sector are equal -- i.e. if the proportion that primary (non-produced) commodities form of the total output of each of the original sectors are equal.

In drawing up a provincial table, it appears convenient to aggregate both row and column sectors of the basic commodity by industry table in SIC defined Major Groups because:

1. The grouping is standard and coincides closely with the U.N. International Standard Industrial Classification (I.S.I.C.);

2. The grouping is defined by D.B.S. in regard to industries on the basis of similarity of output and consequently similarity of input structure;

3. The number of sectors involved is convenient, there being 20 manufacturing sectors $\pm$ any changes;

4. The commodity detail formed on the Census Forms can be conveniently categorized in Major Groups.

Certain of the Major Groups may be aggregated or disaggregated; this is considered below in Chapter III.

In the model developed in the next section, the Industries are aggregated into the corresponding Major Groups by weighting the coefficients according to the proportion of Group output each Industry output is comprised.

The conditions of "acceptability" are met by the assumption that the resultant output levels of a Major Group, by computation of the "inverse" and a bill of final demand, equal the simple sums of the Industry output level that would result in inverting an Industry by Industry table. Granted that the above assumption is unlikely to hold very strictly, a consolation lies in the fact that by definition the input structures are close to being the same.

The following discussion of common problems of input-output tables is cursory in that references 5,7,8,11,41 are to the point and cover them thoroughly. The U.N. handbook (41) is the most up-to-date and gives a good account under all headings.

Producers vs. Purchase Prices

Computing input/output coefficients on the basis of census data requires a decision as to valuation of inputs and outputs. The D.B.S. Census Forms stipulate to the reporting firms that inputs are to be registered in "purchasers" prices while outputs are to be registered in "producers" prices.

"Producers" price valuation corresponds with F.O.B. valuation: the commodities are valued at factory cost and do not include subsequent "margins" (transportation costs, wholesale and retail trade margins, and net indirect taxes).

"Purchasers" price valuation is simply the "producers" price valuation with these margins added on.

An input/output table measured in purchasers prices would show inputs inclusive of margins and a row sector to indicate margins added on to a producers (column) output.

Thus, if:

X'_{ij} = the input from the i^{th} sector to the j^{th} sector valued at what the j^{th} sector pays (= what the i^{th} sector receives + margins;

M'_j = the margins added on to the j^{th} sectors output to show it at purchasers prices;

X'_j = the output value of the j^{th} sector inclusive of margins;

then

$$\sum_{j=1}^{n} X'_{ij} + Y'_i = X'_i \qquad \text{..... (II - 4)}$$

$$\sum_{j=1}^{n} X'_{ij} + M'_j + V'_j = X'_j \qquad \text{..... (II - 5)}$$

$$a'_{ij} = \frac{X'_{ij}}{X'_j} \qquad \text{..... (II - 6)}$$

$$\therefore \quad A'X' + Y' = X' \qquad \text{..... (II - 7)}$$

A table measured in producers prices would show inputs at F.O.B. prices -- i.e. exclusive of margins and the margins input row measures margins paid for each of the inputs by the purchasing industry.

Thus, if:

X_{ij} = the input from the i^{th} sector to the j^{th} sector valued at what the i^{th} sector receives;

M_j = the margins paid by the j^{th} sector to obtain the required inputs;

X_j = output of the j^{th} sector valued at F.O.B. prices;

then

$$\sum_{j=1}^{n} X_{ij} + Y_i = X_i \qquad \text{..... (II - 8)}$$

$$\sum_{i=1}^{n} X_{ij} + M_j + V_j = X_j \qquad \text{..... (II - 9)}$$

$$a_{ij} = \frac{X_{ij}}{X_j} \qquad \text{..... (II - 10)}$$

$$\therefore \quad AX + Y = X \qquad \text{..... (II - 11)}$$

Note that in the above case (II - 11), if a bill of final demand is given (normally at purchasers price valuation), then the margins must be subtracted before multiplying by the inverse for the resultant output levels. Given Y', one must estimate the margin M_y in order to use (II - 11).

Thus,

$$Y = Y' - M_y$$

and from (II - 11)

$$X = (I - A)^{-1} (Y' - M_y) \qquad \text{..... (II - 12)}$$

The differences between a purchasers as compared to a producers valuation system can be illustrated by the following:

1. Purchasers Prices: Marketing costs are double counted, first as inputs to a sector from the marketing cost sector and second as part of the value of output of that producing sector;

   Producers Prices: Marketing costs are only singly counted -- as inputs to the purchasing industry.

2. Purchasers Prices: Marketing costs are distributed along each row such that if these costs vary from cell to cell in a given row, and if output distribution along a row changed, the value of production may change even though actual production does not;

Producers Prices: The above effect will be absent because no marketing costs are distributed along the rows.

3. Purchasers Prices: Marketing costs are assumed to vary with the output structure of a sector but output structure, over time, is regarded as relatively unstable;

Producers Prices: Marketing costs are assumed to vary with the input structure of a sector, which is regarded as a more stable relationship than the output structure.

4. Purchasers Price: A table compiled on this basis of valuation involves less additional work in that inputs are already registered in purchasers prices (re D.B.S. Census Forms), and only the margins in sector aggregate need be computed to being output valuation from producers prices (as registered on Forms) to purchasers prices;

Producers Prices: Compiling a table under this system of valuation would require an estimate on margins for each cell if (as in the case of Census Forms), the input value figures are in purchasers prices.

It would appear then that items 1, 2 and 3 above give the advantage to producers prices to such a degree as to overbalance the disadvantage as outlined in 4. Indeed, the United Nations has recommended[(11)] standardizing all basic input/output tables in producers prices.

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(11) See United Nations, (41), Chapter 2, Section V.

A combination of producers and purchasers prices can be used which allows compilation direct from the Census Forms. As before, the prime is used to denote valuation at purchasers prices while unprimed variables are valued at producers prices. Then:

$$\sum_{j=1}^{n} X'_{ij} + Y'_i - M_i = X_i \qquad \ldots\ldots \text{(II - 13)}$$

$$\sum_{i=1}^{n} X'_{ij} + V_j = X_j \qquad \ldots\ldots \text{(II - 14)}$$

$$a'' = \frac{X'_{ij}}{X_j} \qquad \ldots\ldots \text{(II - 15)}$$

$$\therefore\ A''X + Y' - M = X$$

$$X = (I - A'') . (Y' - M) \qquad \ldots\ldots \text{(II - 16)}$$

where

A'' is defined by (II - 15)

Y' = bill of final demand inclusive of margins

M = margins - by row

The drawback to using (II - 16) is that margins are assumed autonomous yet at the same time are assumed proportional to output (through A''). This last problem can be got around if M is measured, and assumed, as a constant proportion of output. Thus:

If $\quad B_i = \dfrac{M_i}{X_i} = \dfrac{M_i}{X_j} \qquad i = j \qquad \ldots\ldots$ (II - 17)

and $\quad M_i = B_i X_i$

$$M = BX$$

where B is a diagonal matrix of the form

$$
\begin{array}{cccccc}
B_1 & 0 & 0 & \cdots & 0 \\
0 & B_2 & 0 & & \\
0 & 0 & \ddots & & \\
\vdots & & & B_i & \\
\vdots & & & & \ddots \\
0 & \cdots & \cdots & \cdots & B_n
\end{array}
$$

then

$$A'' X + Y' - BX = X$$

$$A'' X - BX - X = -Y'$$

$$Y' = X + BX - A'' X$$

$$Y' = (I + B - A'')X$$

$$\therefore X = (I - A'' + B)^{-1} Y' \qquad \ldots\ldots \text{(II - 18)}$$

Note that in the above model, all the disadvantages (and advantages) of the purchasers price system are present, with the exception of the double counting of margins aspect. Also, the resultant outputs are measured in producers prices not purchasers prices.

Imports-Exports

In input/output analysis, it is generally assumed that exports are autonomously determined (by extra-national considerations) and at least are not a function of domestic output. Normally then, exports are measured as part of final demand and do not enter in the compilation of the basic table. Thus:

$A = (a_{ij})$ are formed from X_{ij} and X_j

and

$X =$ is predicted in other than base year computations from the parameter A and the exogenous variable Y via

$$X = (I - A)^{-1} Y$$

Exports can now be considered as an exogenous variable by

$$EX_i \quad = \quad \text{exports of output of } i^{th} \text{ sector}$$

then

$$X \quad = \quad (I - A)^{-1} \quad (Y + EX)$$

Imports, on the other hand, are normally regarded as a function of the output of the corresponding sector although a distinction is made between competitive imports and non-competitive imports. Competitive imports are those commodities which are also produced domestically whereas non-competitive imports are those commodities which do not even have close substitutes produced domestically -- e.g. mangoes.

As registered on the industrial Census Forms, input items are inclusive of imports and no ready distinction between domestic and imported inputs can be made. Output items are, of course, produced domestically only. An initial compilation then would be of the form

$$\begin{array}{ccccc}
X_{11} & \cdots\cdots & X_{1n} & Y_1 & X_1^t \\
\vdots & & & \vdots & \vdots \\
X_{m1} & \cdots\cdots & X_{mn} & Y_m & X_m^t \\
\hline
V_1 & & V_n & & \\
\hline
X_1^o & \cdots\cdots & X_n^o & &
\end{array}$$

The row output totals X_i^t, $i = 1, \ldots n$, represent total requirements -- i.e. domestic production, plus input imports plus imports for final demand. In order to make these compatible with the column totals (domestic production), the total values of sector imports must either be subtracted from the row totals or added to the column totals. Considering this in model form:

Let

M_i^I = imports used for inputs $i = 1, \ldots n$

M_i^y = imports used by final demand $i = 1, \ldots n$

M_i^t = total imports

Y_i^t = total final demand (domestic and imports)

X^o = domestic production

X^t = domestic production plus all imports

i.e. $X^t = X^o + M^I + M^y$

Now if the input coefficients are formed directly from the data found on the Census Forms, then:

$$A'' = \frac{X_{ij}^t}{X_j^o} \qquad \ldots\ldots \text{(II - 19)}$$

$$\sum_{j=1}^{n} X_{ij}^t + Y_i + M_i^y = X_i^t \qquad \ldots\ldots \text{(II - 20)}$$

$$\sum_{i=1}^{m} X_{ij}^t + V_j + M_j^t = X_j^t \qquad \ldots\ldots \text{(II - 21)}$$

Taking (II - 20) and adding $(-M_i^I)$ to both sides then

$$\sum_j X_{ij}^t + Y_i + M_i^y - M_i^I = X_i^t - M_i^I$$

or

$$\sum_j X_{ij}^t + Y_i - M_i^I = X_i^t - M_i^I - M_i^y = X_D$$

and combining (II - 19) with (II - 20)

$$\sum_j A''_{ij} \quad X^o_i \quad + \quad Y_i \quad - \quad M^I_i \quad = \quad X^D_i \qquad \text{..........(II - 22)}$$

Assuming aggregation based on section I above, then

$$X^D \quad = \quad (I - A'')^{-1} \quad (Y - M^I) \qquad \text{..........(II - 23)}$$

But here, as in (II - 16), an item is included in both the autonomous section as well as the endogenous section (A) of the balance equations. This can be got around by forming a coefficient C such that:

$$C_i \quad = \quad \frac{M^{I(13)}_i}{X^D_i}$$

--i. e. M^I_i is the input import content of the output of the i^{th} industry.

or $\quad M^I \quad = \quad CX$

where M^I and X are column vectors and C is a diagonal matrix of co-efficients, then

$$A'' \quad X^D \quad + \quad Y^t \quad - \quad CX^D \quad = \quad X^D \quad + \quad M^y$$

$$\therefore \quad X^D \quad = \quad (I - A'' \quad + \quad C)^{-1} \quad (Y^t - M^y) \qquad \text{...............(II - 24)}$$

or

$$X^D \quad = \quad (I - A'' + \quad C)^{-1} \quad (Y^D) \qquad \text{...............(II - 25)}$$

As for the question of non-competing imports, they are best distributed along a separate row and are treated as primary commodities (non-produced). Only if an interregional model was desired (see (1) and (5)) would the non-competing imports be possibly included in the "invertible" part of the table. Pitts and Sawyer consider the overall question of imports in some detail - see (33).

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(13) This arrangement introduces the assumption that the input imports are a constant proportion of output.

Overall Model

On the basis of the above considerations then, the suggested provincial model based on the D.B.S. Census Forms, becomes:

$$X = (I - A + B + C)^{-1} Y^D \qquad \ldots\ldots (II - 26)$$

where

$A = \dfrac{X'_{ij}}{X_j}$ $\qquad X'_{ij}$ = input from i to j measured at purchasers prices and reflecting the simple sum of domestic production and imports of i going to j.

X_j = total domestic production of j, at producers prices.

$X = (X_j)$

I = unit matrix - identity matrix

B = a diagonal matrix with the i^{th} element in the diagonal (i^{th} row, i^{th} column), showing the proportion margins are of output valuation along a row sector of the total output of that i^{th} row.

C = a diagonal matrix with the i^{th} element in the diagonal (i^{th} row, i^{th} column), showing the proportion that the input imports into the i^{th} row are of the total output of that row.

Y^D = final demand for domestic output.

Of course, Y^D is interchangeable with $Y^t - M^y$.

Survey of Possible Uses

The following is a brief account of common uses to which an input/output table can be put. These uses are under the headings:

1. Marketing Analysis
2. Resource Base Studies
3. Statistical Checks
4. Import Replacement Studies
5. Taxation Studies
6. Price Relationships
7. Coefficient Tracing

Reference is made to the information needed in the light of the suggested model (II - 26).

1. Marketing Analysis

The basic, most detailed, non-inverted table could be of considerable use to market research people, both government and private. Working directly from the Census Forms, the most detailed table would be rectangular, listing commodities in three-digit detail (SCC classes) down the columns, and industries with four-digit detail (SIC Industries) along the rows. From such a table, an idea of the cost structure of a given industry could be obtained, or, by the same token, the distribution pattern of a given commodity class over the industries. Again, although detail would be lost in "squaring up" the table, the inverse would give an idea of what industries and resources would be "strained" given an increase in the production of a given industry.

2. Resource Base Studies

The inverse can be used to obtain resultant production levels from stipulated bills of final demand by:

$$X^k = (I - A)^{-1} (Y^k + EX^k) \qquad \text{..... (II - 27)}$$

where k signifies a "set" of observations

Y^k = predicted, k^{th}, set of final demands

X^k = computed production levels implicit in Y^k via $(I - A)^{-1}$

The computed X^k can then be used to find the different requirements placed on the primary (non-produced or base resource) commodities through:

$$AMT_i^k = \sum_{j=1}^{n} V_{ij} X_j^k \qquad \text{..... (II - 28)}$$

where

V_{ij} is the amount (value) of the i^{th} primary commodity used per dollar of output of sector j.

AMT_i^k = resultant amount of the i^{th} primary commodity required to accommodate the k^{th} postulated bill of final demand.

A direct formulation of (II - 28) would be

$$AMT_i^k = V (I - A)^{-1} (Y^k + EX^k)$$

where V = the row vector $(V_{ij})_{j=1}^{n}$

3. Statistical Checks

One of the more common uses to which national input/output tables may be put is in revealing "gaps and redundancies" in national accounts, foreign trade statistics and employment and similar compilations. The table may serve both as a quantitative check and as qualitative check on the suitability of existing classificatory schemes. In that a provincial table may be the only source for such statistics on a regional basis, the checking function is lost.

4. Import Replacement Studies

In predicting future output levels on the basis of postulated future demands, a substantial amount of induced import requirements may arise. Such information may be of value to provincial governments in suggesting areas for encouragement of industrial development. One must temper any such results with the fact that import data on a provincial basis is normally very sparse and often inaccurate due to lack of proper data. In the relatively compact import model developed above (see (II - 26)) an additional problem arises in the fact that the model by incorporating the "B" coefficients assumes that the row distribution of input imports is a fixed proportion of the output for that sector. In any case the model can be used to determine input import requirements (final demand imports are assumed exogenous) by:

$$M^I = BX$$

where

$$X = (I - A)^{-1} Y$$

5. Taxation Studies

Direct taxes imposed on factor services are not usually included directly in an input/output table although they can be predicted from the model by first determining the requisite amount (value) of factor services for a given bill of final demand. The incidence of a sales tax could be usefully analyzed in this sense although if the table were looked at in terms of prices (see next section), information could be found in regard to resultant price changes. In terms of the model, a total tax bill could be determined by:

$$BILL = P_1 X$$

where

P_1 = row vector of taxes (as a percentage) imposed on each of the sectors.

and

X = the column vector of total outputs computed from

$$X = (I - A)^{-1} Y$$

BILL = a scalar, value quantity

Personal income taxes could be computed from BILL = $P_2 \cdot AMT$

where

P_2 = the incidence of personal income tax in terms of a scalar percentage (average tax as a percent of income), OR a diagonal matrix of taxes imposed -- as a percentage of output differentiated by sectors.

AMT = the amount of labour computed via (II - 28) which would be a scalar and would correspond with the scalar P_2 or a vector of the labour requirements differentiated by sectors:

$$AMT_j = V_{ij} X_j \qquad j=1,\ldots n$$

In either case, BILL is a scalar quantity. This particular use of an input/output table may be most useful in a provincial framework where the interindustry schema represents the only approach for determining total (direct and indirect) effects.

6. Price Relationships

If the cost structures of industries are looked at as a series of price-quantity relationships, one can set up a model to determine relative prices from the input coefficients. Thus, if:

$$A_j^v = \frac{V_j}{X_j}$$

$$V_j = P_j X_j - \sum_{i=1}^{n} a_{ij} P_i X_j$$

dividing through by X_j

$$\frac{V_j}{X_j} = P_j - \sum_{i=1}^{n} a_{ij} P_i$$

then

$$A^v = P - A^t P$$

$$\therefore P = (I - A^t)^{-1} A^v \qquad \ldots\ldots \text{(II - 29)}$$

where

A^t = transpose of A

P = price vector related to sector outputs

It can be seen from (II - 29) that once the input coefficients and the value added proportions of output are given, prices (relative) are fixed. If the A^v (value added as proportion of corresponding outputs) can be assumed constant over different levels of output, then prices can be determined independently of output levels. If the A^v can not be assumed fixed over a range of output levels, then the price equation (II - 29) must be "tied-in" with the quantity equation (II - 26). Yamada(14) has done this conveniently by adding two more sets of equations, one showing the demand for a sector's output as a simple linear function of price and the other showing quantity produced as a simple linear function of price.

The crucial variable in such price equations is the hidden one of profit levels, a problem assumed away heretofore. Equation (II - 29) only holds if the sum of costs indicated exactly equals the revenue. On the grounds that most prices are fixed by non-competitive factors over a considerable range, it would appear doubtful that attempting price analysis through (II - 29) would be worthwhile. In any case, the model developed above can be used for determining such price variables, the only additional data needed being the value-added-per-output coefficients.

(14) See Yamada, (11), Chapter 1

7. Coefficient Trace

Having once computed the cell members of the inverse $(I - A)^{-1}$, it may be useful to trace the effect of a change in one (or more) of the original A coefficients on the inverse. There may be some doubt as to a given coefficient -- say, technological studies suggest it is too high -- in which case it would be useful to know how significant this margin of error might be. Again, the object might be given a postulated change in technique substitution, labour-capital switch, to find what the net effect on the inverse, and hence on computed production levels. Tracing the effects of such changes requires a re-calculation of the inverse although the author has outlined and programmed a method in Chapter IV which calculates the net change itself, to any required degree of accuracy via the Leontief-Cornfield[15] multiplier process. Having once determined the net effect on the inverse, the matrix of such net changes may be used in equations (II - 26), (II - 28) and (II - 29) to show that net resultant effects on the output levels, resource requirements and other computed variables.

(15) Represented by $I + A + A^2 \ldots\ldots$, The approximation of $(I-A)^{-1}$

It is helpful to have a good idea of the uses to which an input/output table can be put before compiling it although, for a provincial model, one would be hard pressed to accommodate the United Nations' suggestion that "the particular applications of input/output analysis govern the preparation of the statistical table of transactions".[16] It has been argued here that the availability of data severely restricts the type of table formed, and that the more feasible provincial approach is to build up the table -- rectangular -- to square -- to including non-manufacturing sectors -- keeping as much detail as convenient and adjusting the basic table later, to suit specific purposes.

In passing, it is useful to note that while an "open model" (autonomous final demand from households) has been assumed, a "closed model" may be useful in determining employment changes. If it is postulated that certain sector(s) are stimulated, then the resultant effects on employment (via (II - 28) can be thought of as putting a further demand for consumption goods and, in turn, further stimulating employment. This sensitivity of final demand to employment can be introduced into the model by including the household sector in the body of the table where the output is labour (distributed along the n + 1 row) and the inputs are final demand goods (down the n + 1th column). Inverting this augmented matrix -- assuming that the proportional breakdown of consumption goods per unit of income is fixed -- allows the determination of these consumption multiplier effects. Given an increase in autonomous final demand -- now restricted to non-household consumption -- one is able to compute the augmented effect on production levels.[17]

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(16) See United Nations, (42)
(17) See Dorfman, Samuelson and Solow (6)

In the next chapter, the possible sources of information for compiling the basic table are considered.

CHAPTER III

THE BASIC SOURCES OF DATA

In any input/output study but those of the most ambitious scale, industry and commodity classificatory schemas are set by the available data format. In the case of a provincial table using the D.B.S. Census Forms for base year data, the Standard Industrial Classification (SIC) is used in the coding of industries. Commodity detail entered on these forms does not necessarily coincide with any set classification although the guideline for setting up the form's entries was the Standard Commodity Classification (SCC). As a consequence, input (and output) entries on these forms can be conveniently coded to an SCC category. In this chapter, the emphasis will be on these sources of data and how the proposed model is adapted to availability of such information.

Although it is possible to use the Census Forms -- or even direct data gathering -- and reclassify the contents according to some arbitrary scheme, the original SIC framework in conjunction with SCC coding of detail is recommended for the provincial model under consideration because:

1. Substantial (read costly) effort would be required to keep the number of divisions down and yet satisfy the homogeneity constraints of the predictive input/output model.

2. The SIC and SCC is common to other data gathering agencies both of government and private industry. Thus, the presentation of input/output information in these standardized formats can more readily be interpreted by these agencies.

3. The SIC is a classificatory system whereby industries with like outputs and hence like inputs, are grouped together, satisfying (at least in degree) the homogeneity assumption of input/output analysis.

4. A ready-made aggregation of industries into Major Groups is available, each industry being uniquely assigned to one of these Major Groups by definition. The SCC commodity Groups (three-digit) can also be conveniently assigned to the SIC Major Group categories.

The formation of a basic commodity by industry table within the above framework, and using the D.B.S. Industrial Census Forms, requires only two basic judgements: the SCC commodity Groups must be assigned to input detail on the basis of the given description and these commodity Groups must be assigned to the SIC Major Groups.

In the initial compilation of the table then, it is suggested that the columnar divisions be based on the four-digit SIC industries and that in the subsequent, more aggregated table, these industries be taken into the SIC Major Group divisions. Again, it is suggested that the row divisions be made on the basis of SCC three-digit commodity Groups and this requires the compiler to assign each input entry to one (in some cases, more) of these Groups. In the aggregation of these commodity Groups, it is suggested that these be directly assigned to SIC Major Groups rather than an attempt being made at assigning them first to SIC Industries and then Major Groups. It was found, in working with the Manitoba compilation, that three-digit commodity detail could not be unambiguously assigned to an industry. Indeed, some commodities could not even be unambiguously assigned to a Major Group -- e.g. packaging.

If the above classificatory scheme is used together with the Census Forms, four tables can be compiled, namely:

1. ROWS = SCC COMMODITY GROUPS
   COLS. = SIC INDUSTRIES

2. ROWS = SIC MAJOR GROUPS
   COLS. = SIC INDUSTRIES

3. ROWS = SCC COMMODITY GROUPS
   COLS. = SIC MAJOR GROUPS

4. ROWS = SIC MAJOR GROUPS
   COLS. = SIC MAJOR GROUPS

In that the columns must be compatible with the rows in order that a table be inverted, the basic "predictive" input/output table is 4. above. The initial table, and the one most useful in the "descriptive" sense is 1. above. A certain aggregation of the Census Form data is required, however, before Table 1 is obtained, and is described below.

Aggregation of Establishments:

The SIC manual[1] describes, or rather defines, an Establishment as the smallest business entity that maintains a set of business accounts (profit/loss, etc.). Each Establishment in the country is uniquely categorized in an Industry, and as such, is sent a Census Form to be completed. After the initial coding of the entries to SCC categories, as described above, the data would take the form.

---

(1) See D.B.S. (47)

Industry	1011				1013			1030
Establishment	1	2	3	4	1	2	3	1
Commodity								
051	X'_{11}	X^2_{11}	X^3_{11}	X^4_{11}	X'_{12}	X^2_{12}		
065.......	.			.				
101.......	.			.				
.	X'_{i1}			X^4_{i1}				

The Establishments have been distinguished by superscripts. By definition the Establishments under a given Industry have similar outputs and on this basis it is assumed that the input structures are similar. In order to form Table 1 above, one could compute Establishment coefficients and then form an average of each for the Industry Input coefficient.

Thus:

$$a_{11} = \frac{X_{11}}{X'_1 + X^2_2 + X^3_3 + X^4_4} \neq \left(\frac{X'_{11}}{X'_1} + \frac{X^2_{11}}{X^2_2} + \frac{X^3_{11}}{X^3_3} + \frac{X^4_{11}}{X^4_4} \right) / 4$$

The Industry coefficients are computed in a slightly different manner in the computer program of Chapter IV.

Each X^k_{ij} is entered on the basic data cards but the Establishment coefficients are by-passed. Each X^k_{ij} is summed over k as is the column totals X^k_j. The industry coefficient is then computed by:

$$a_{ij} = \sum_{k=1}^{l} X^k_{ij} \Big/ \sum_{k=1}^{l} X^i_j \qquad L = \text{number of Establishments under Industry } j.$$

Working in this manner, Establishment identity is not required to be maintained although the basic deck (DATA I) does so, in order to allow checking and removal of inaccurate or unreasonable data from selected Establishments. Actually, the computer program does compute Establishment coefficients but only for checking, and they are not used again in the compilation. The revised basic deck of cards (DATA II) which is simply the original, with unwanted Establishments "weeded out" is fed into another program and the basic commodity by industry table is output in the following form:

Industry .... Commodity		1011	1013	1030
	051	a_{11}	a_{12}	a_{13}
	065	a_{21}	a_{22}	a_{23}
	101	a_{23}	⋮	⋮
	⋮	⋮	⋮	⋮

where $a_{ij} = \dfrac{X_{ij}}{X_j}$ $\quad j = 1, \ldots n$ industries

$i = 1, \ldots m$ commodities

and

$m > n$ normally

Tables 2, 3 and 4 are derived from Table 1 above with the additional of the following information:

1. List of assignments of Commodity Groups to Major Groups.

2. List of assignments of Industries to Major Groups.

3. List of weights to be given to individual Industries for aggregation into Major Groups.

Of the above, 1) must be formed by the compiler, 2) follows by SIC definition, and 3) is computed in the program. The next chapter considers the form and procedure for obtaining this information in detail.

The author would like to emphatically suggest here that great effort be taken to assign the Census Form entries to commodity Groups carefully. Changing one of these classifications after, say, the Major Group by Major Group has been compiled is tantamount to redoing the whole compilation.

Some Relevant Observations

Every Establishment is required, by law, to fill out the Census Forms, for return to the Dominion Bureau of Statistics. These Establishments are requested though not required to send a form to the Provincial Government and the latter receives roughly a 60% coverage(2). Of these, many will have to be eliminated due to obviously innaccurate content or in some cases, lack of content. The Forms relating to some Industries eg. the clothing industry, do not require input detail and as a consequence are useless for input/output work. In general, of the three types of Census Forms - Long, Medium and Short, only the latter lacks input detail.

The aspect of "confidentiality" should also be considered in compilation. The law requires that no statistics arising out of the Census Forms be published if they are given on an Industry where only three or fewer Establishments are defined, or again, if one Establishment accounts for 60% or over of the value of that Industries output. On a provincial basis, this can create a problem although not in regard to the "predictive", Major Group by Major Group table. Concerning any of the "Industry" tables (Tables 1 and 2 above) distribution may have to be limited to government agencies or else the relevent Industries must be aggregated into other Industries.

It is hoped that in the future, the quality and quantity of such Industrial information will be improved. Indeed, if the provincial "compiler" were to have access to the DBS commodity cards, which are

(2) That is, the provincial files have only a 60% coverage of the potential measured by the output totals of the forms on hand as compared to the overall output total.

distinguishable by province, not only would the amount of data be improved, but the form entries would already be classified to commodity Groups and be compatible with the national compilation at that. According to Gigantes and Robb[3], the Bureau is considering relabelling the entries on the Census Forms to coincide with the standard Commodity Classification (SCC).

Gigantes and Pitts[4] have argued that the model should not be decided on until the basic data has been compiled. This allows flexibility in accounting for joint products and secondary products. Their model differs from the one outlined in the last chapter in that the square table compiled (corresponding to Major Group by Major Group) is an Industry by Industry table. In deriving such a table the problem of secondary and joint products must be considered carefully because an Industry in many cases produces products that are in the main, produced by another Industry (s). Because the "predictive" table suggested in the last chapter has a high level of aggregation - Major Group by Major Group - this problem can be assumed away. The drawback is that the assumption of constant cost structure falls down because within a Major Group, many products are produced and if a demand change in traced, by way of the inverse (II - 26) it must be assumed that the component products are "demanded" in the same proportions as they were in the base period, as expressed in the weighting system of section 1, Chapter II.

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(3) See Gigantes, T. and Robb, M., (19), page 14

(4) See Gigantes, T. and Pitts, P., (20), Introduction

The author feels justified in suggesting such an assumption because:

1. The final demand vector used for prediction will still be in the aggregate form (Major Group) divisions such that although Industry detail (changes in output) cannot be obtained, the broad movement should be reflected.
2. The Criterion for aggregating Industries into Major Groups, according to the SIC, is similarity of products, and hence similarity of cost structure of the component Industries of a Major Group tends to offset the effects of differential demand.

Actually, if one desires to pursue the Gigantes-Pitts method, the data is available from the computer program with addition of the procedure outlined in Appendix "C" (the "Make" matrix). Following this procedure, the row commodity Groups need not be assigned to Major Groups; they are assigned to Industries directly through the model using "Make" and "Use" matrices. Irving Stone also uses this method - ref. (39). Whereas the basic data is available from the program of Chapter IV and Appendix "C", it is left up to the compiler to combine the "Make" and "Use" matrices to develop the Industry by Industry table - ref. (20).

Commodity Classification

The Canadian "Standard Commodity Classification" is to be found in three basic volumes published by the Dominion Bureau of Statistics. Volume I (44) expresses the classification in ordered code form. This comprehensive classification is made up of:

5	Sections
82	Divisions
498	Groups
5,634	Classes

The basic data cards have each entry from the Census Forms coded to the Group level of detail; these Groups, in turn, belong by definition to a Division and a Section. Volume II (45) is a classified index containing a list under each Class, of commodities which typically belong to it. Volume III (46), perhaps the most useful in the original compilation, lists commodities in alphabetical order with their assigned code following. This is the largest of the manuals in that many commodities are listed more than once -- e.g. "sugar, maple," and "maple sugar".

Commodities are classified according to principal component or, where convenient, according to use. In the last case, such a classification prevents large n.e.s.[(5)] grouping but at the same time is at odds with the "predictive" input/output requirement of input homogeneity.

The five basic Sections of the SCC are:

1. Live Animals
2. Food, Feed, Beverages and Tobacco
3. Crude Materials, Inedible
4. Fabricated Materials, Inedible
5. End Products, Inedible

The distinction between 3, 4 and 5 is not that of vertical levels of production, as the introduction to Volume III (46) is at pains to point out. A commodity is part of Section 3, if the only operation done on it is cleaning and preparing for transit. The distinction between Sections 4 and 5 is not clear-cut although the working principle is that if a commodity loses its identity in another use, it is part of Section 4, and if it retains its identity in use, it is part of Section 5.

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(5) N.E.S. - not elsewhere specified.

In finer detail, these Sections are broken down into 82 commodity Divisions, as indicated by the first two digits of the code, and 498 commodity Groups, indicated by the first three digits of the code. It is to this last level of detail that the Census Form entries are coded in order to be accepted by the program of Chapter IV. In that a commodity Group tends to encompass those Classes to which an entry (from the Form) could as easily be categorized as another, much ambiguity of coding drops away if the commodities are only distinguished at the three-digit level.

Certain entries on the Census Forms cannot be classified by the SCC, and in view of the fact that there is no "99" Division, it is suggested that, at least initially, the following arbitrary classification be followed:

99-1	Work done by others
99-2	Office supplies and expenses
99-3	Other services
99-4	Not elsewhere specified (n.e.s.)
99-5	Wages
99-6	Salaries
.	
.	
.	
.	
99-9	Must be kept a null "Group" in that the computer program requires this code for termination.

In the next chapter, a suggested format and procedure is given for registering such data.

Industrial Classification

The Canadian "Standard Industrial Classification (SIC) is to be found in the D.B.S. publication (47). In regard to the Manufacturing Section only, the levels of detail are: 20 Major Groups,

Industries given by a four-digit code and a varying number of Establishments[6] within each Industry is given. Each Industrial Census

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(6) See page 31.

Form represents an Establishment which often, but not necessarily represents a firm, and is assigned to an Industry by the D.B.S. according to its principal product. An input/output table coded according to the SIC can be used for international comparisons in that the SIC was designed to accommodate the U.N. scheme, ISIC[7].

The heading on the Census Forms have the Establishment name and address (printed), the four-digit Industry code to which it belongs, a provincial code, a "within province" area code and an Establishment code number. Unfortunately, the Establishment identification code may be duplicated with a province, for a given Industry: the D.B.S. has subdivided the provinces into areas within which, and only within which, the assigned establishment number is unique. Thus, the Establishment identity is best maintained on the data deck by using the area code as well as the Establishment number. Appendix "D" contains a sample Census Form on which is indicated the location of the above information.

The Census Form itself can be looked at as a number of basic sections:

1. Inventories - "Stock-on-hand" is listed for the beginning and the end of the reporting year -- with varying degrees of detail.

2. Fuel and Electricity - Again, with varying degrees of detail.

3. Inputs - Some forms -- e.g. the Chemical Industry -- are very detailed (>200 entries) whereas others -- e.g. the Clothing Industry -- show only the sum of the inputs. In the latter case, recourse must be made to supplementary data for determining the cost structure.

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(7) See U. N. (42)

4. Outputs - Most forms show considerable commodity detail ($>$ 100 entries) although for the basic program only the sum is needed.

5. Wages and Salaries - Usually the two are distinguished on the Form, and the degree of detail is standard.

6. Other Accounting Information - Taxes, office expenses, capital expenditures, etc. follow (on the Form) the input and output sums -- such information may be subsequently useful and it is suggested that it be recorded on "supplementary" cards as per Chapter IV.

The Census Forms are also classified according to size. There are three basic sizes known as Long, Medium and Short Forms, respectively, although the degree of detail in a given size may vary from Industry to Industry. In general, the Short Form data is not used in compiling the table because of lack of input detail, although in the initial basic data cards, all Establishments (and hence, Forms) are registered.

The SIC was revised in 1960 although Census Forms are normally available dating back to 1950. The Industrial Census Forms can be found on file with a provincial government department; in Manitoba's case, with the Department of Industry and Commerce. Descriptions of revisions, including the 1960 revision, the code itself and Major Group assignments can be found in the basic manual.[8]

The Major Groups, which form the basis of the suggested final aggregation of the table, are groupings of Industries with like products. There are 20 Manufacturing Major Groups defined in the SIC:

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8. See References (45, (46) and (47).

1. Food and Beverage Industries
2. Tobacco Products Industries
3. Rubber Industries
4. Leather Industries
5. Textile Industries
6. Knitting Mills
7. Clothing Industries
8. Wood Industries
9. Furniture and Fixtures Ind.
10. Paper and Allied Industries
11. Printing, Publishing and Allied Ind.
12. Primary Metal Industries
13. Metal Fabricating Industries
14. Machinery Industries
15. Transportation Equipment Industries
16. Electrical Products Industries
17. Non-Metallic Mineral Products Ind.
18. Petroleum and Coal Products Ind.
19. Chemical and Chemical Products Ind.
20. Miscellaneous Mfg. Industries

In the case of a given province, this particular division may not be appropriate. In Manitoba, for instance, the output value of the Industries included in Major Group 2, and again in Major Group 3, is less than $5 M. whereas that of Major Group 1 is $125 M. In this case, it is convenient to subdivide Major Group 1 into, say, Meat Products, Dairy Products, Grain Mill Products, and Other Food Processors. By the same token, Major Group 2 and Major Group 3 could be combined.

The SIC Major Groups can also be added to, to include other convenient sectors of the economy such as

Petroleum Mining

Metal Mining

Non-Metal Mining

Crops

Livestock

The Major Groups form that part of the table which is subjected to "feedback" and, as such, is the part which is inverted. Other sectors can be added on as rows, to represent "non-produced" inputs such as labour, profits, depreciation, land, taxes, non-competing imports.

Imports

There are no readily available sources of data on imports on a provincial breakdown. In the case of the Manitoba study, the base year coincided with a study on Import Replacement done for COMEF[9] from which the C matrix of (II - 25) was estimated. Lacking this, access would have to be made to way-bill documents of transportation companies for data on commodity description and destination; an ambitious task. It is left to the user to determine the method and source for determining these Imports. It is important to keep in mind that distinction must be made between input imports and final demand imports, and again, between competing and non-competing imports.

In the next chapter, the computer program is described together with a step-by-step procedure for incorporating the data considered above into the model considered in Chapter II.

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9. Committee on Manitoba's Economic Future, Government of Manitoba, Winnipeg, 1963.

CHAPTER IV

THE COMPUTER PROGRAM

This chapter is concerned with the actual procedure, step-by-step, used in compiling the basic tables. In the following section, the computer and how it is used is briefly discussed. The next section, labelled Section I, gives the detailed method of putting the required data in "computer acceptable" form. Section II describes the data and programs required under the headings of the desired outputs. Operator sheets under each object section are for the computer operator as a guide for input card sequences and for any subsequent program revisions. Section III gives a concise description of the card decks involved. The diagrams found at the end of the chapter are referred to in the text.

The Computer

A computer is a "sequence controlled calculator"[1] which, in the course of pursuing this function, serves as a temporary storage of information, a selector of sequences to follow, and a producer of answers in various forms. Instructions are "fed into" the computer, then data is "fed in", and after the computer has executed the instructions on the data, the answers, or "computed" data is "fed out" or Outputed. This is diagrammatically represented on page 103, Diag. I. The computer "thinks" only in a discrete sense in that any decisions made by it are based on given information (quantified) and/or computed information.

The use to which the computer is put in the program outlined here is a data processing function - a transformation of data with only simple calculations involved.

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1. Berkeley, E. C.; Giant Brains, Science Editions Inc., New York, 1961.

Certain cardinal rules in computer input preparation should be stressed here. In considering instructions and data, the machine takes things quite literally. If, say, a variable name is entered as P I M Ø, and, if later in the program, it is accidentally replaced by P 1 M O, the computer cannot take the common sense view that the programmer meant I instead of one, and Ø instead of zero. Again, if the computer is instructed to accept data on the first ten columns of a series of cards, and if the data, in fact, is mispunched on the third card such that it extends into the first eleven columns, only the first ten columns are read, even though to the naked eye this would appear absurd. In other words, instructions and data must be entered exactly as specified.

Geometric symbols are used to illustrate program procedures without having to consider detail. The symbols are defined on page 104 Diag. II, as based on the standard I.B.M. template, with a few convenient revisions.

Input/Output Media

Both instructions and input data are entered into the computer on I.B.M. 5081 cards. Output is either on the same type of card and/or on a printer (typewriter with continuous paper flow). The 5081 cards have 12 rows, 10 of which are numbered, and 80 columns - see Diag. III. Each column can register an alphanumeric[2] symbol; for instance, 80 digits could be entered on the card. Normally, what is entered in the form of punched holes in body of the card, is also printed in a single line along the top of the card. It is important to keep any series of cards - called a "deck" in its proper - normally original - order; this is especially true of any program deck where the computer considers the sequence of instructions as indicated by the sequence of cards.

---

2. Arabit numerals, letters and special characters.

Fortran: The computer cannot be made to understand English; there are too many possible interpretations of meaning for many words, which require a human "intuition" to distinguish. As a consequence, the programs, at least those considered in this chapter, are written in an English-like language called FORTRAN which the computer can be made to understand. The question of the advantages and disadvantages of other languages and of how the computer accepts FORTRAN need not be gone into here. The input/output programs written are, in the basic FORTRAN II which can normally be used on pre-1966 computers. The I.B.M. 360 line of computers required a slightly modified version, and these changes are outlined in Appendix "C". It is advisable that any adjustments of this nature be left to a computer programmer.

Fortran instructions can be mastered fairly easily although it is suggested that the Economist qua Economist need go no further than being able to establish a professional rapport with the computer people. To this end, one could consider the following aspects of Fortran.

1. Programs are written on coding sheets where each line represents one card. The first six columns are used for line (statement) reference.

2. The computer is told to accept data by a READ statement which stipulates the statement number which describes the layout of the data (FØRMAT) and the list of variables to be read -- e.g.

```
      READ   100,  NUMBER
  100 FØRMAT  (layout)
```

3. The computer is told to output data by

```
  PUNCH  100, (list of variables)   -   for cards
  PRINT  100, (list of variables)   -   for printer
```

4. The sequence of statements followed by the computer corresponds to the sequence of lines on the coding sheet unless the following "branch" statements are come across

   GØ TØ (Statement number)

   IF (Variable to be tested) statement numbers branched to conditional on test.

5. Repetitive operations, a series of lines or statements can be repeated by using a DØ statement -- e.g.

```
10  DØ   100   I = 1, J
.          .
.          .
.          .
.          .
.          .
.          .
100        .
```

   Here statements 10 to 100 are done with I set consecutively (from 1 to J).

6. Arithmetic symbols are:

A - B	minus, or subtract B from A
A + B	plus, or add B to A
A * B	multiply B by A
A / B	divide A by B
A = A + B	what was in A is replaced by A + B (does not mean equal)

A simple program to read three data cards and output three cards on which is the product of the two numbers on the input cards is given in Diag. IV.

The Input/Output Program

Three basic program decks are used for computing the input/output tables. These are labelled as follows:

SIFT

TABL

INVERT .

In this chapter only SIFT and TABL are considered. SIFT does a series of operations on the basic row data in preparation for input into TABL which is a composite program designed to output the various non-inverted tables. INVERT is essentially a program for inverting the Major Group by Major Group Table and is considered in the last chapter.

The procedure suggested is that the compiler oversees the coding of the Census Forms and the transferrence of this data to I.B.M. data coding forms, according to the layout described in Section I of this chapter. These forms are then given to a card-puncher -- usually at the computer centre -- who would make up the basic data deck, labelled DATA I. The compiler would then give the DATA I deck or any computed deck plus required control cards to the computer centre as per requirement sheets of Section II. Section II is indexed by the Output required and gives step-by-step instructions for working with the computer operator. Section III gives a complete description of all program and input/output decks as well as control cards used. SIFT and TABL are listed in Appendix "B".

SECTION I

PREPARING THE DATA

A. DATA I - This deck is the one which registers the entry information from the Census Forms. Each card contains the Industry code, the Establishment number and "code" number and four entries. The procedure to be followed is:

1. Obtain the manufacturing Census Forms and pencil in the SCC assignment to each entry. Also keep a list of the 4-digit Industry code (see diagram VI) and assign a unique Establishment number to each form.
2. Transfer the information on the coded forms to the IBM data coding form (diagram V)(1). Each line on these sheets represent a card and following layout is convenient.

Columns	
1 - 4	SIC
5 - 7	EST.
8	Code
9 - 11	SCC
12 - 20	Value
21 - 26	Quantity
27 - 29	SCC
30 - 38	Value
39 - 44	Quantity

---

(1) These forms are available at the computer centre.

45 - 47	SCC
48 - 56	Value
57 - 62	Quantity
63 - 65	SCC
66 - 74	Value
75 - 80	Quantity

The code for column 7 should be according to the following:

Code[(2)]	=	0	Input card - all fields filled
		1	" " - last field empty
		2	" " - two last fields empty
		3	" " - three last fields empty
		4	" " - no inputs
		5	Output card- all fields filled
		6	" " - last field empty
		7	" " - two last fields empty
		8	" " - three last fields empty
		9	" " - no output

3. Place the completed coding sheets with a card puncher in order for a card deck to be made up from them. Label the resultant deck with a marker pen, as DATA I, and keep it together with an elastic band.

---

(2) The "fields" referred to are the sets of 17 columns used for each form entry.

B. SELECT: This deck simply lists the Establishments one wishes to eliminate from the DATA I deck in order to remove unreasonable or inaccurate data. Once the decision has been made as to which ones should be removed, the SIC number with corresponding Establishment number should be listed on the I.B.M. data coding forms (Diag. V) in the following way - starting at Column 1, write in the four-digit industry code, leave a blank, two-digit Establishment code, leave a blank, next Industry code, etc. The procedure then is:

1. Run DATA I on SIFT to obtain COEF I (p. 69) and on the basis of COEF I, select the Establishments to be removed.

2. List the SIC code and corresponding Establishment code as per above.

3. Have the cards punched.

4. Label SELECT and put a rubber band around the deck.

C. SCC - MGRP ASS : This deck shows the Major Group assignments for each of the listed commodities. The procedure is:

1. Using only those SCC's listed on the SCC - LIST deck (see p. 73) assign each one to a Major Group, on the basis of the description for each SCC code, as found in the Manual (ref. (44)).

2. Register these assignments on the coding forms (Diag. V), starting at the first column of every card, as:

   SCC code, three columns

   a blank column

   Major Group assignment, two columns

   two blank columns

   and repeat until the 80 columns are filled, for each card.

3. Have the coding forms punched out into a card deck, and label SCC - MGRP ASS and put a rubber band around the deck.

D. SIC - MGRP ASS : This deck shows which Major Group each Industry belongs to. The assignment is given in the SIC Manual (47), if that code is followed, although another Major Group code may be devised if desired. The procedure here is:

1. Using the EST LIST deck (see p.73) assign each SIC code to a Major Group.

2. Enter each SIC with its corresponding Major Group assignment onto the coding forms (Diag. V), starting at Column 1, by

   SIC, four columns

   one blank column

   Major Group Assignment, two columns

   one blank column

   and repeat for all 80 columns, for each card.

SECTION II

OBJECT - REQUIREMENT SHEETS

The following sheets are categorized by the computer output desired. The description of the decks referred is to be found in Section III (p. 97). Under each object heading there are four sheets:

1. The object deck and the corresponding required decks and control cards are listed.

2. A simple flow diagram of the object-requirement as described on the previous sheet.

3. An instruction sheet for the computer operator.

4. A more detailed flow diagram of the actual program for use by a programmer for any subsequent revisions.

SHEET 1

Object: COEF I

Requirements: DATA I SIFT

CONTROL I

Procedure: Give the computer operator Sheet 3 (page 71) and the three requirement decks.

Fold under at dotted line.

p. 70

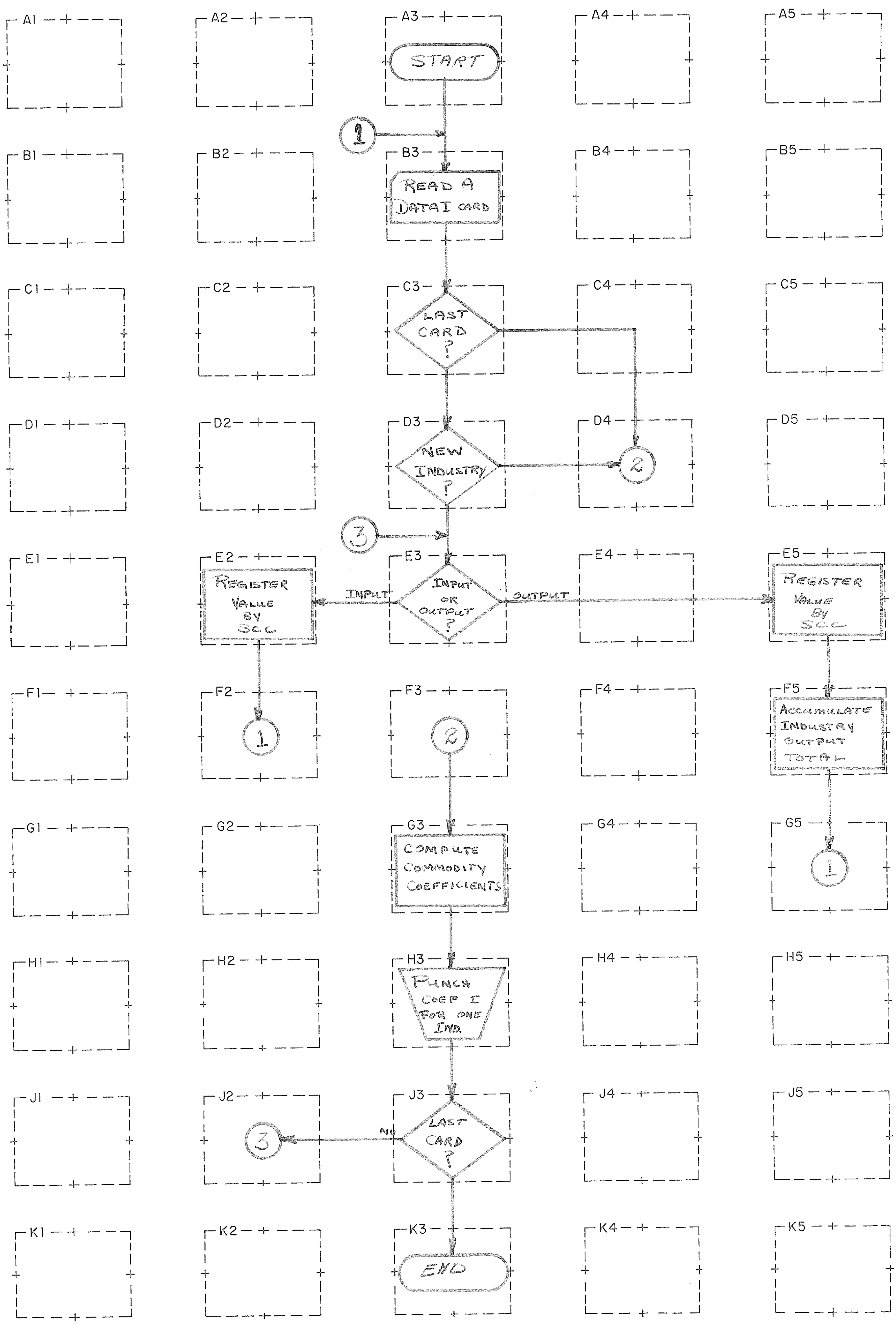
12 13 14 15 16 17 18 19 20 21 22 23 24 25

Fold under at dotted line.

IBM **Flowcharting Worksheet**

PRINTED IN U.S.A.
X20-8021-2

Programmer: D. SMEATON Program No.: ______ Date: ______ Page: ______
Chart ID: ______ Chart Name: ______ Program Name: SIFT : COEF I



Fold under at dotted line.

Fold under at dotted line.

SHEET 3

Program is written in FORTRAN II and is labelled SIFT

Program, as it stands requires approximately 40,000 bytes.

To run, the decks given to you should be in the following order

SIFT

CONTROL I

DATA I

"The missing pages ... 72, 76, 80, 84, 88, 90, 92, 96, 102, 114, 115, 124, were removed from all copies of the thesis as these were in effect more in the nature of work sheets rather than part of the thesis itself". - Extract from a letter dated January 3, 1967 from the University of Manitoba to the National Library.

SHEET 1

Object: COEF II and SCC LIST and EST LIST

Requirements: SELECT SIFT

DATA I

CONTROL II

Procedure: Give the computer operator Sheet 3 (p.75) and the four requirement decks.

IBM DIAGRAMMING AND CHARTING WORKSHEET

Application ______________________ Date ____________ Page ______ of ______

Procedure ______________________ Drawn By ____________

01 02 03 04 05 06 07 08 09 10 11 12 13 14 15 16 17 18 19

01
02
03
04
05
06
07
08
09
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25

p.74

Fold under at dotted line.

Fold under at dotted line.

IBM Flowcharting Worksheet

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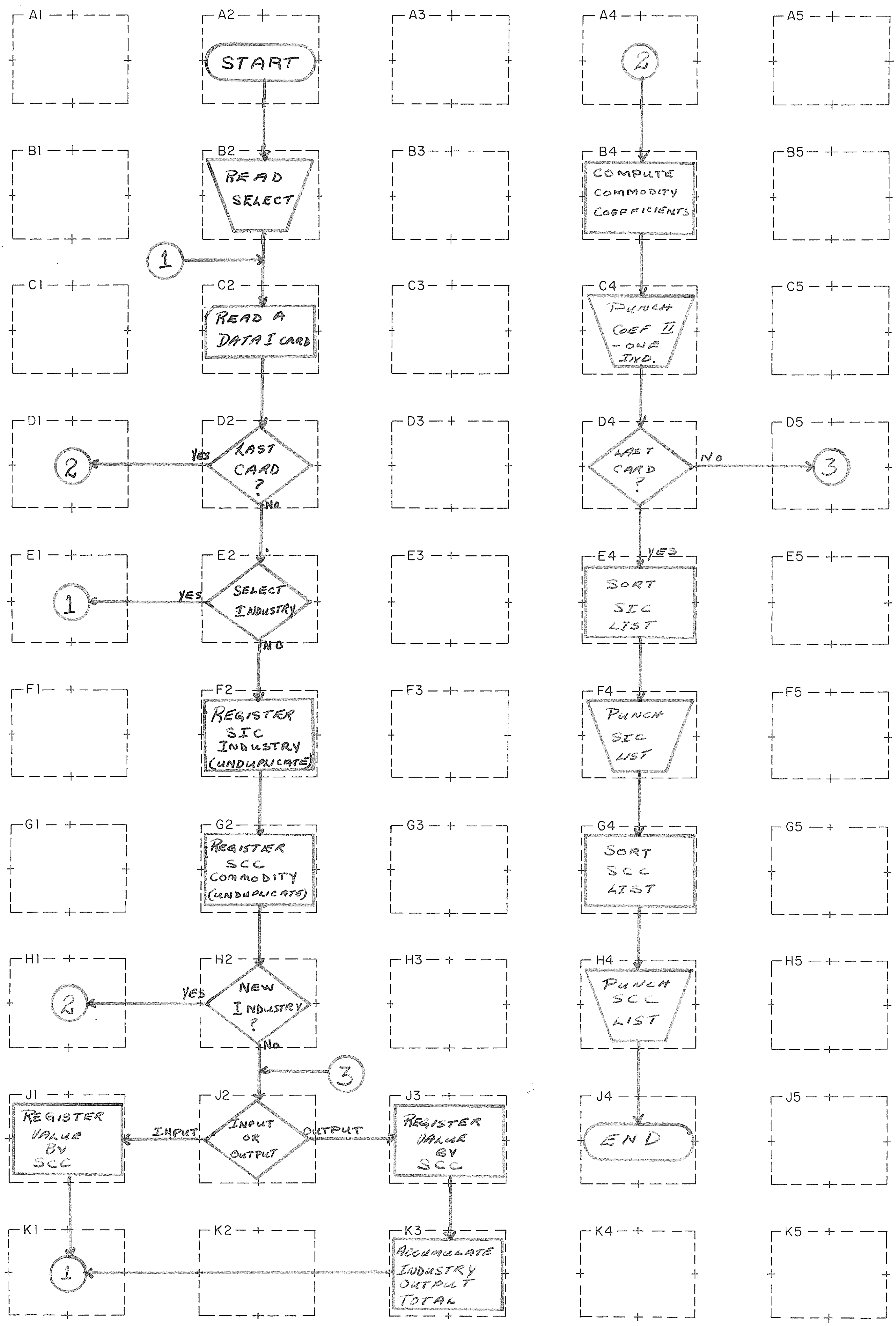
p.74

Programmer: D. SMEATON Program No.: Date: Page:

Chart ID: Chart Name: Program Name: SIFT '. COEF II, SICLIST, SCC LIST



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SHEET 3

Program is written in FORTRAN II and is labelled SIFT

Program requires approximately 40,000 bytes.

To run, the decks given to you should be in the following order.

SIFT

CONTROL II

SELECT

DATA I

"The missing pages ... 72, 76, 80, 84, 88, 90, 92, 96, 102, 114, 115, 124, were removed from all copies of the thesis as these were in effect more in the nature of work sheets rather than part of the thesis itself". - Extract from a letter dated January 3, 1967 from the University of Manitoba to the National Library.

SHEET 1

Object: WEIGHTS

Requirements: COEF II TABL
SIC MGRP ASS
SCC LIST
CONTROL V

Procedure: Give the computer operator Sheet 3 (p. 79) and the five requirement decks.

IBM DIAGRAMMING AND CHARTING WORKSHEET

Application ______ Date ______ Page ______ of ______

Procedure ______ Drawn By ______

p.78

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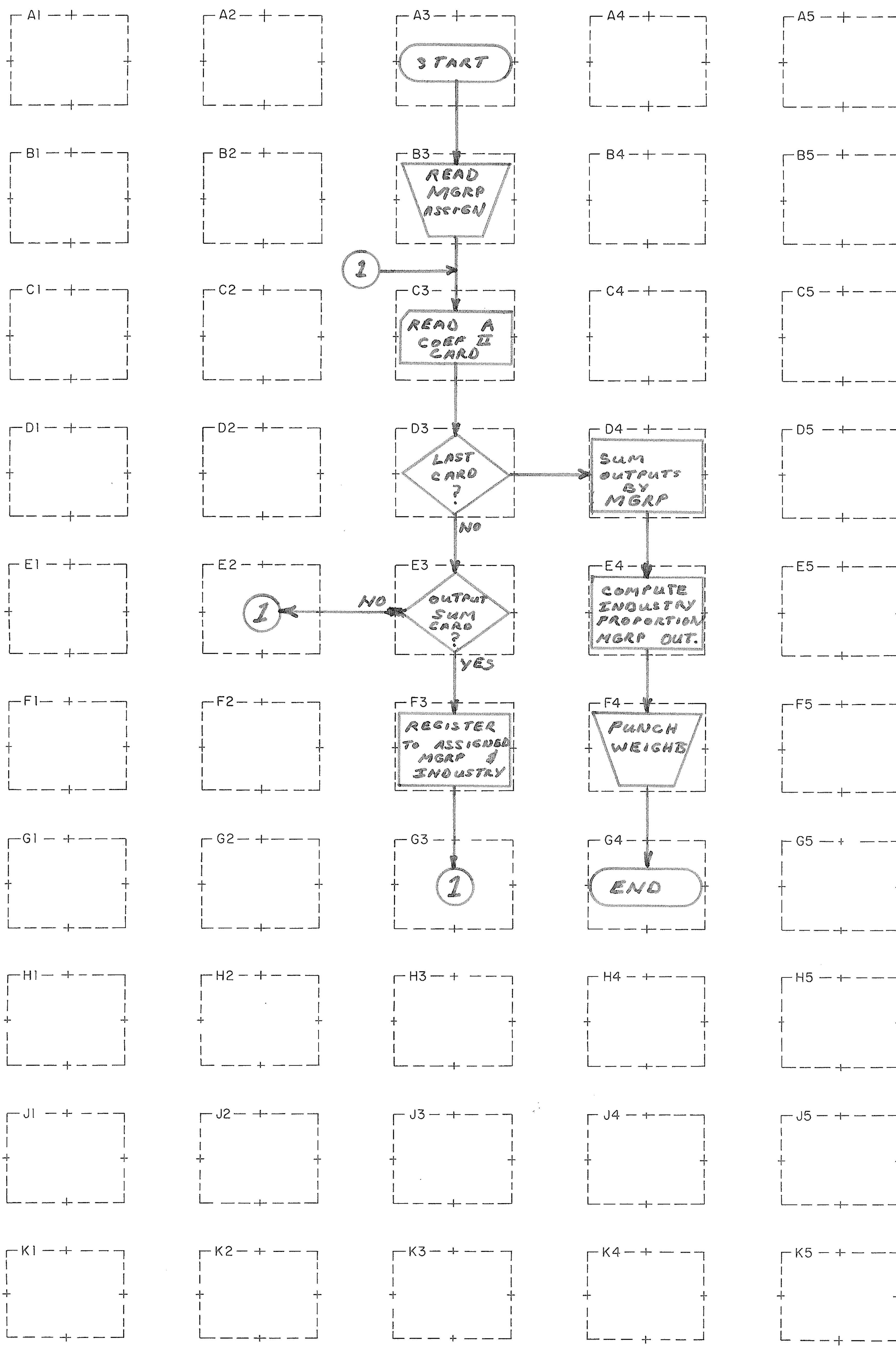
IBM Flowcharting Worksheet

PRINTED IN U.S.A. p.78
X20-8021-2

Programmer: D. SMEATON Program No.: ________ Date: ______ Page: ______
Chart ID: ____ Chart Name: ______________ Program Name: TABL: WEIGHTS



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SHEET 3

Program is written in FORTRAN II and is labelled TABL

Program requires approximately 80,000 bytes.

To run, the decks given to you should be in the following order.

TABL

CONTROL V

SCC LIST

SIC MGRP ASS

COEF II

"The missing pages ... 72, 76, 80, 84, 88, 90, 92, 96, 102, 114, 115, 124, were removed from all copies of the thesis as these were in effect more in the nature of work sheets rather than part of the thesis itself". - Extract from a letter dated January 3, 1967 from the University of Manitoba to the National Library.

SHEET 1

Object: SCC x SIC TABLE

Requirements: SCC LIST TABL

SELECT

DATA I

CONTROL VI

Procedure: Give the computer operator Sheet 3 (p.83) and the four requirement decks.

IBM DIAGRAMMING AND CHARTING WORKSHEET

Application ______ Date ______ Page ______ of ______

Procedure ______ Drawn By ______

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IBM Flowcharting Worksheet

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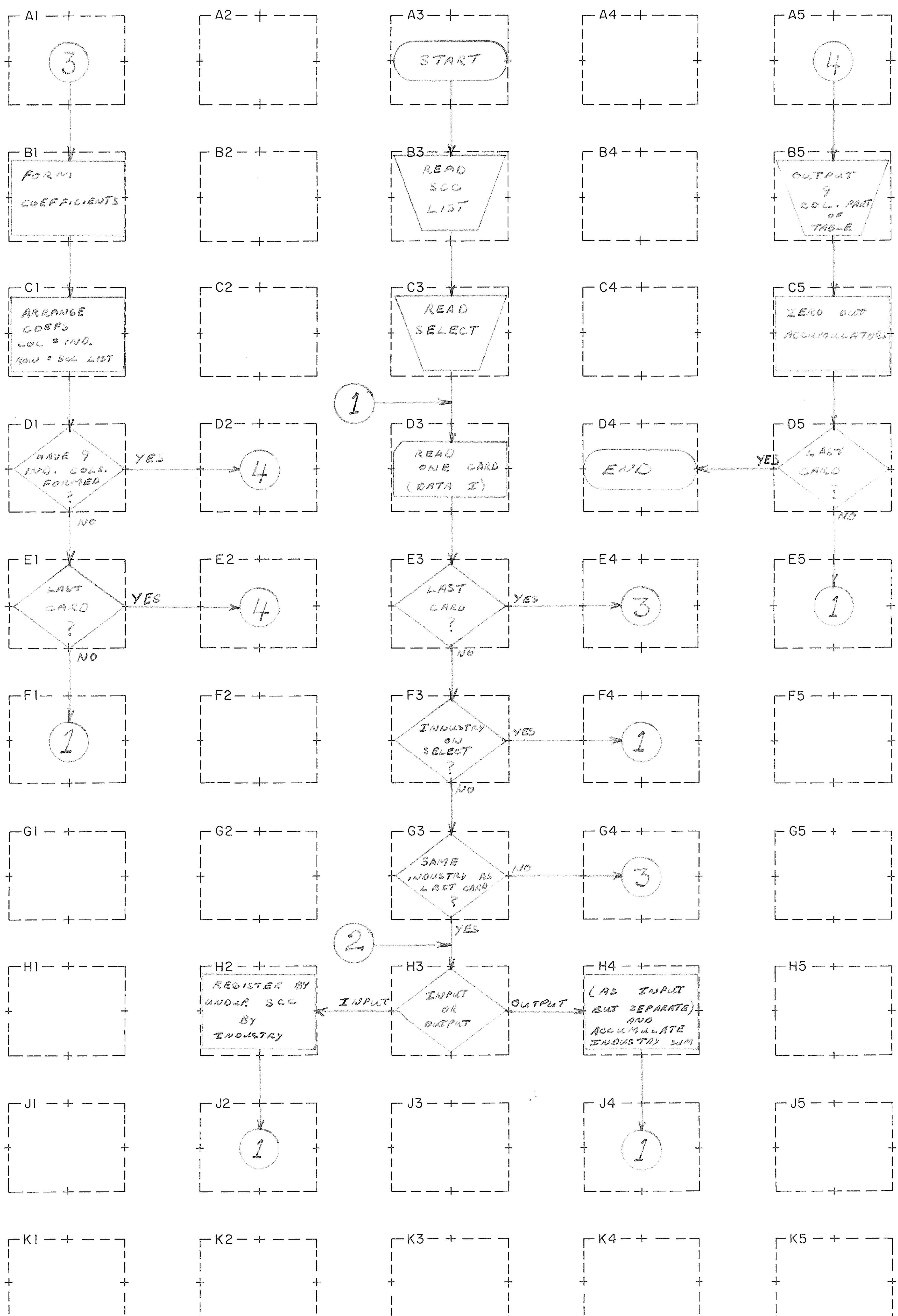
Programmer: D. SMEATON Program No.: ______ Date: ______ Page: ______

Chart ID: ______ Chart Name: ______ Program Name: TABL: SCC x SIC TABLE



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SHEET 3

Program is written in FORTRAN II and is labelled TABL.

Program requires approximately 80,000 bytes.

To run, the decks given to you should be in the following order:

TABL

CONTROL VII

SCC LIST

SELECT

DATA I

"The missing pages ... 72, 76, 80, 84, 88, 90, 92, 96, 102, 114, 115, 124, were removed from all copies of the thesis as these were in effect more in the nature of work sheets rather than part of the thesis itself". - Extract from a letter dated January 3, 1967 from the University of Manitoba to the National Library.

SHEET 1

Object:	MGRP x SIC TABLE
Requirements:	SCC MGRP ASS TABL
	SCC x SIC TABLE
	CONTROL VII

Procedure: Give the computer operator Sheet 3 (p.87) and the four requirement decks.

IBM® DIAGRAMMING AND CHARTING WORKSHEET

Application ____________ Date ____________ Page ______ of ______

Procedure ____________ Drawn By ____________

p. 86

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p. 86

IBM Flowcharting Worksheet

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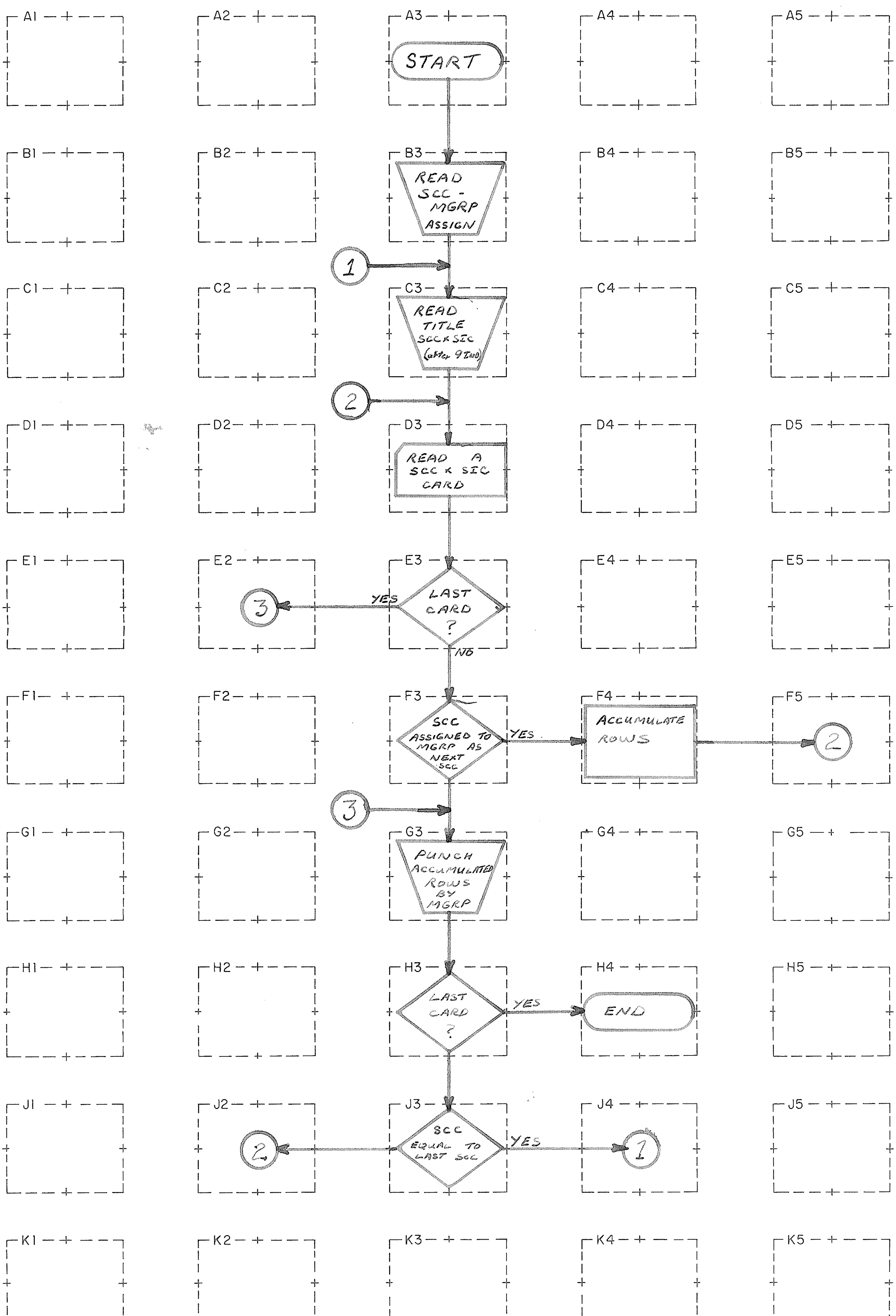
Programmer: D. SMEATON Program No.: ____ Date: ____ Page: ____
Chart ID: ____ Chart Name: ____ Program Name: TABL: MGRP x SIC



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SHEET 3

Program is written in FORTRAN II and is labelled TABL.

Program requires approximately 80,000 bytes.

To run, the decks given to you should be in the following order.

TABL

CONTROL VII

SCC MGRP ASS

SCC x SIC TABLE

"The missing pages ... 72, 76, 80, 84, 88, 90, 92, 96, 102, 114, 115, 124, were removed from all copies of the theses as these were in effect more in the nature of work sheets rather than part of the thesis itself". - Extract from a letter dated January 3, 1967 from the University of Manitoba to the National Library.

SHEET 1

Object: SCC x MGRP TABLE

Requirements: WEIGHTS TABL

SIC MGRP ASS

SCC x SIC TABLE

CONTROL VIII

Procedure: Give the computer operator Sheet 3 (p.91) and the four requirement decks.

"The missing pages ... 72, 76, 80, 84, 88, 90, 92, 96, 102, 114, 115, 124, were removed from all copies of the theses as these were in effect more in the nature of work sheets rather than part of the thesis itself". - Extract from a letter dated January 3, 1967 from the University of Manitoba to the National Library.

SHEET 3

Program is written in FORTRAN II and is labelled TABL.

Program requires approximately 80,000 bytes.

To run, the decks given to you should be in the following order.

TABL

CONTROL VIII

SIC MGRP ASS

WEIGHTS

SCC x SIC TABLE

"The missing pages ... 72, 76, 80, 84, 88, 90, 92, 96, 102, 114, 115, 124, were removed from all copies of the theses as these were in effect more in the nature of work sheets rather than part of the thesis itself". - Extract from a letter dated January 3, 1967 from the University of Manitoba to the National Library.

SHEET 1

Object: MGRP x MGRP TABLE

Requirements: SIC MGRP ASS TABL

WEIGHTS

MGRP x SIC TABLE

CONTROL IX

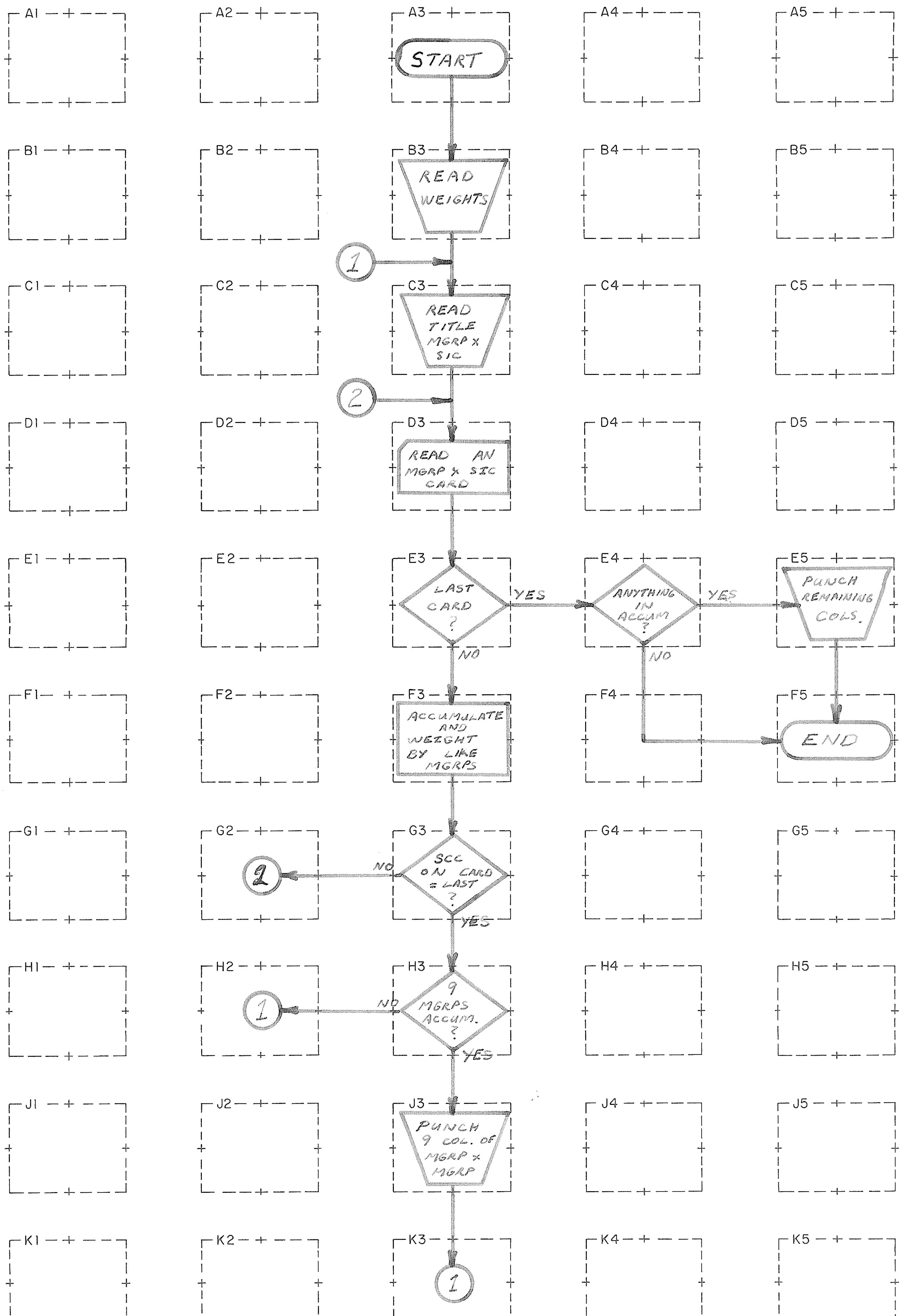
Procedure: Give the computer operator Sheet 3 (p.95) and the five requirement decks.

p.94

IBM Flowcharting Worksheet

PRINTED IN U.S.A.
X20-8021-2

Programmer: D. SMEATON Program No.: Date: Page:
Chart ID: Chart Name: Program Name: TABL: MGRP x MGRP



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IBM DIAGRAMMING AND CHARTING WORKSHEET

Application ______ Date ______ Page ______ of ______

Procedure ______ Drawn By ______

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p. 94

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SHEET 3

Program is written in FORTRAN II and is labelled TABL.

Program requires approximately 80,000 bytes.

To run, the decks given to you should be in the following order.

TABL

CONTROL IX

SIC MGRP ASS

WEIGHTS

MGRP x SIC TABLE

"The missing pages ... 72, 76, 80, 84, 88, 90, 92, 96, 102, 114, 115, 124, were removed from all copies of the theses as these were in effect more in the nature of work sheets rather than part of the thesis itself". - Extract from a letter dated January 3, 1967 from the University of Manitoba to the National Library.

SECTION III

DECK DESCRIPTION

DATA I

Register inputs and outputs from the Census Forms. Each card has the first eight columns reserved for form (establishment) identification. The Industry code is found in the first four columns, the Establishment code in the next three, and the eighth column is reserved for a code which indicates whether the entries in the columns that follow are inputs or outputs. The entries have three parts, the SCC code, the value figure and the quantity figure. Each entry uses 18 columns (3 + 8 + 7) and although the card may not be filled, normally it has 4 entries registered on it (4 x 18 = 72 + 8 (identification cds.) = 80). This deck will be the largest and will probably involve 2,000 to 4,000 cards.

COEF I

Is an output deck from the SIFT program and lists for each Establishment, the proportion that each input forms of output (value terms) by unduplicated SCC code. It also shows the output proportions and the value of output. The deck is designed in a self-explanatory way and the column by column content need not be considered.

SELECT

Is formed from on the basis of inspecting COEF I and choosing Establishments which contain unreasonable or inaccurate data. Each such Establishment is registered in column groups of eight, where the first four columns contain the Industry code, the next three, the Establishment code, followed by a blank. Thus there can be 10 such Establishments registered per card.

COEF II

Is an output deck from SIFT program and is derived from COEF I and SELECT. Thus, it corresponds exactly to COEF I except that the undesired Establishments are not included.

SCC LIST

Again an output from SIFT, using DATA I and simply lists the unduplicated SCC codes which have been entered on DATA I. These three-digit codes are outputed in ascending numerial order as 20 groups of the 3 column code and a blank.

EST LIST

Lists are unduplicated SIC codes from DATA, in ascending order, one per card, together with the number of Establishments associated with it. On each card, the first four columns are the SIC code, followed by three blank columns and the three column number of Establishments. This deck is outputed from SIFT using DATA I.

WEIGHTS

Is an output deck from the TABL program, using COEF II (or I), and the SIC MGRP ASS. These weights have on it the weights to be applied to the industry coefficients when aggregated into Major Groups. In turn, it is used as input into TABL in order to obtain the SCC x MGRP and the MGRP x MGRP tables. The information is registered on each card in eight groups of sixteen, where the first four columns indicate the Industry, two blank columns, two columns indicating the assigned Major Group, two blank columns, five columns for the weights (first column for the decimal, next four for the numbers) and finally a blank column.

SIC MGRP ASS

A small deck indicating the Major Group to which each Industry is assigned. Each Industry is entered, with its Major Group assignment in ten groups of eight columns, on each card. In each group, the Industry is registered on the first four columns, a blank column, the Major Group on the next two columns and finally another blank column.

SCC MGRP ASS

As above except a list of the commodity codes and the corresponding Major Group to which they have been assigned. Information is entered onto each card in ten groups of eight columns, where the first three columns indicate the three-digit commodity code, a blank column, the corresponding Major Group assignment in two columns and finally two blank columns.

SCC x SIC TABLE

Is the commodity by Industry tables outputed by the TABL program which aggregates the Establishment coefficients of COEF II (or I). There are nine Industry columns on each card such that the table is outputed including all commodity rows (one row = one card) for each group of nine Industries. For each such group, there is an Industry code title card followed by the SCC rows and the corresponding Industry coefficients. If a printout is made from this deck where blank cards are added for spacing purposes, care must be taken to remove these cards and leave the deck in the condition it was outputed in, in order to use it as input to obtain the other tables.

MGRP x SIC TABLE

Is a deck identical to the above table except that the commodity rows have been aggregated into Major Group Rows. It is outputed from the TABL program using the SCC x SIC TABLE and the SCC MGRP ASS deck.

SCC x MGRP TABLE

Another outputed table from the TABL program, this time using the SCC x SIC TABLE and the WEIGHTS deck. It is identical to the SCC x SIC TABLE except that the groups of nine columns are now Major Groups and the title cards indicate the Major Groups rather than the Industries.

MGRP x MGRP TABLE

Is a deck outputed from the TABL program using MGRP x SIC TABLE and the WEIGHTS deck. This is the "final" deck which by having corresponding rows and columns is suitable for inversion. Once the deck is obtained it is suggested that it be printed-out and then copied by pencil, onto columnar paper where non-manufacturing sectors can be added at the compilers convenience. Rather than use the deck as input into the INVERT program of the next Chapter, it is suggested that the table be repunched according to directions in the next Chapter, from the columnar pads.

CONTROL CARDS

These cards are used simply to indicate to the computer what it is going to receive in the way of input data and in turn what is desired as output. The function and layout on each card is listed below. Only columns 5 to 10 need be reserved for the information the computer requires, the other columns may be used for any description the compiler or programmer sees fit to enter on the card (the computer ignores these other columns). For convenience, the columns 5 to 10 are referred to as the "code zone", in the following.

CONTROL I - for obtaining COEF I -- punch 1. in code zone.

CONTROL II - for obtaining COEF II, SCC LIST and EST LIST -- Punch 2. in code zone.

CONTROL V - for obtaining WEIGHTS -- punch 5. in code zone.

CONTROL VI - for obtaining SCC x SIC TABLE -- punch 6. in code zone.

CONTROL VII - for obtaining MGRP x SIC TABLE -- punch 7. in code zone.

CONTROL VIII - for obtaining SCC x MGRP TABLE -- punch 8. in code zone.

CONTROL IX - for obtaining MGRP x MGRP TABLE -- punch 9. in code zone.

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DIAGRAM I

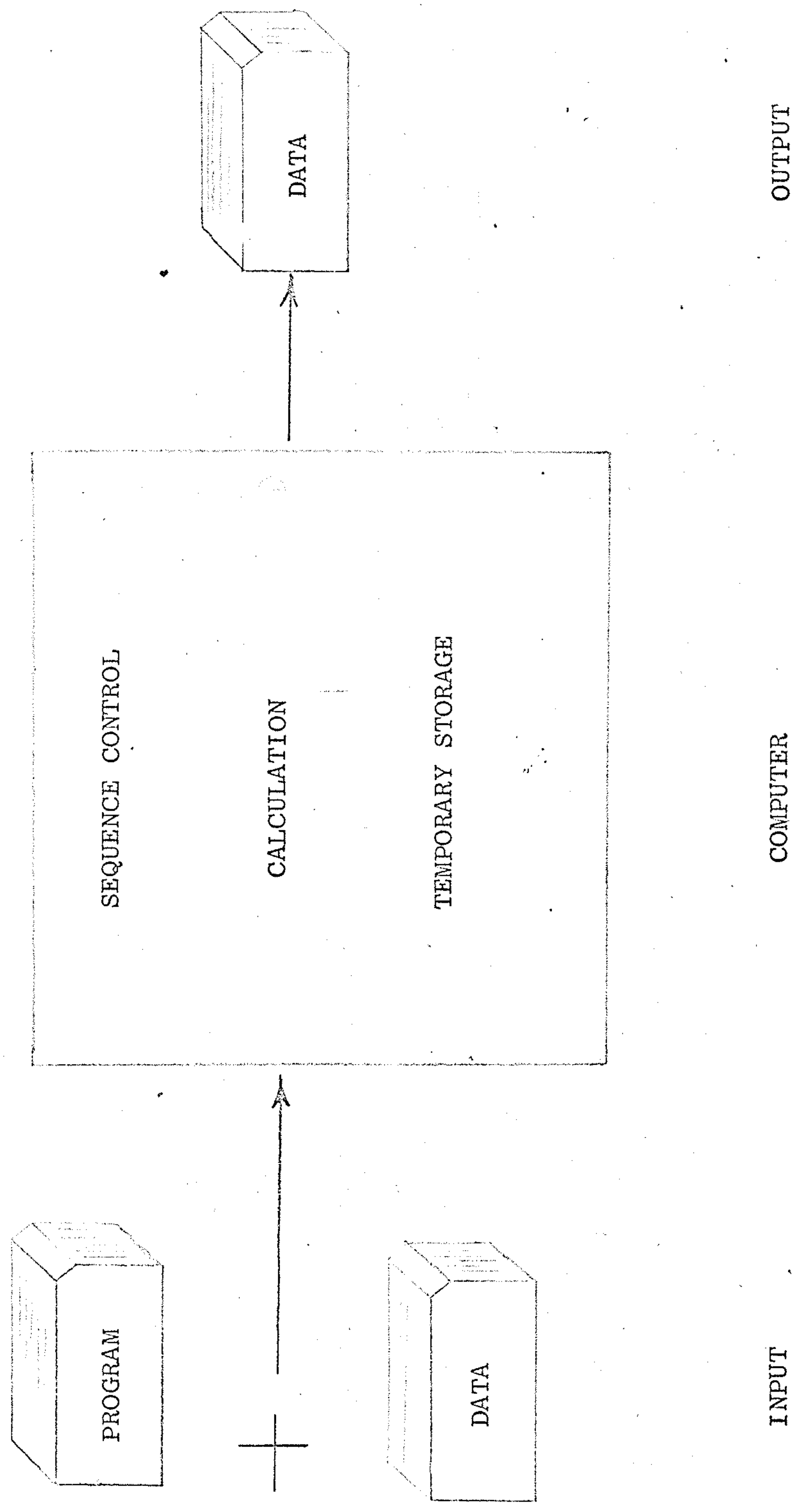
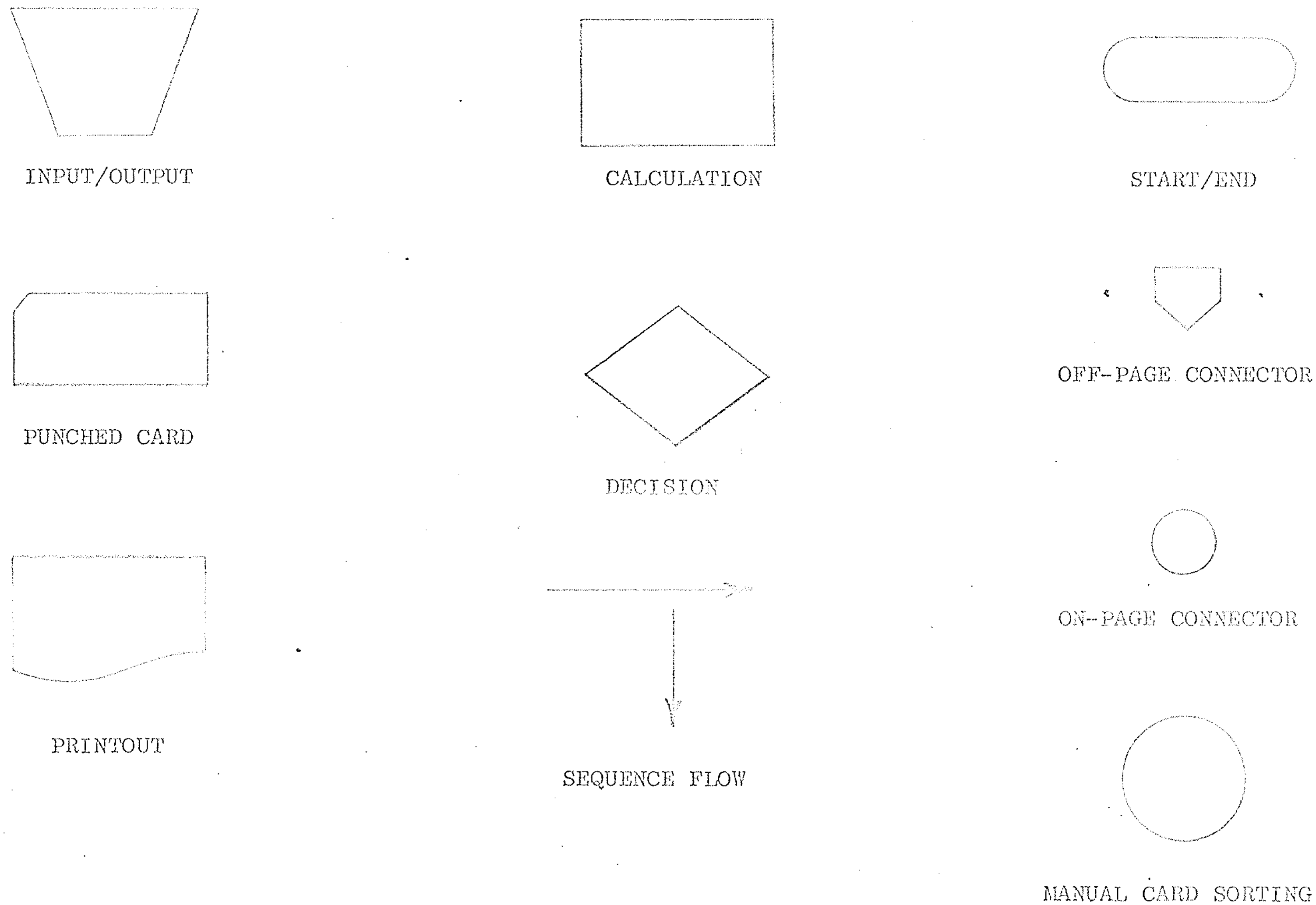


DIAGRAM I

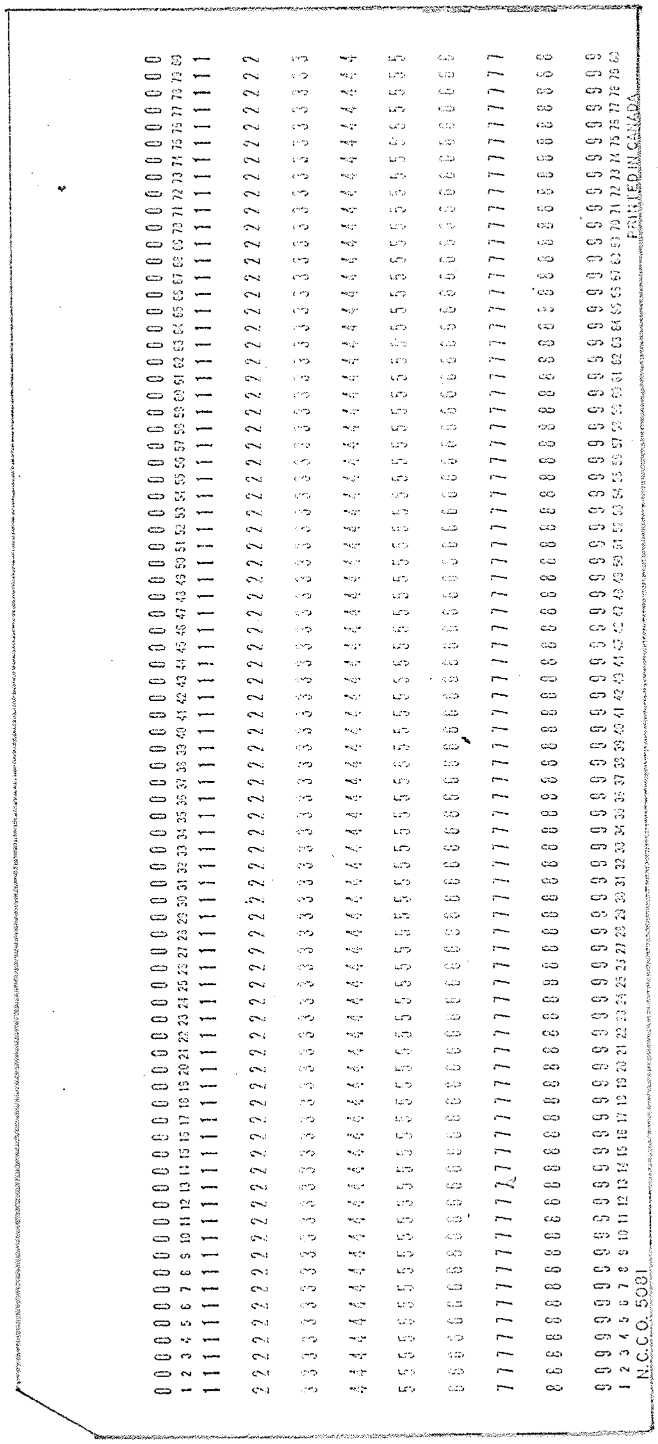
DIAGRAM II



FLOW CHART SYMBOLS

DIAGRAM II

DIAGRAM III



I. B. M. 5081 Card

DIAGRAM III

Program	D. SMEATON		Punching Instructions		Page 1 of 1
Programmer	SAMPLE	Date	Graphic / Punch	Card Form # *	Identification 73–80

C FOR COMMENT

FORTRAN STATEMENT

STATEMENT NUMBER (1–5)	Cont. (6)	FORTRAN STATEMENT (7–72)
C		THIS PROGRAM IS ONLY FOR ILLUSTRATION
		DO 1 J=1,3
		READ 100,A,B
100		FORMAT (F8.2,F7.2)
		C=A+B
		PUNCH 101,A,B,C
101		FORMAT (F8.2,F7.2,F15.0)
1		CONTINUE
		END

* A standard card form, IBM electro 888157, is available for punching source statements from this form.

80 COLUMN DATA SHEET

PROGRAM					BY		DATE
1 10	20	30	40	50	60	70	80

G-963

DIAGRAM I

CHAPTER V

CONCLUSION

Review

The argument of the thesis has been that an inexpensive input/output table, compiled on a provincial basis, may conveniently be built up from D.B.S. Census Forms and SCC and SIC classification schemes. Further, it has been argued that many routing problems (secondary and joint products) can be ignored if the predictive, invertable table (MGRP x MGRP) is of a high level of aggregation. The more detailed Industry by Industry square table is possible from the data, but involves the routing problems as well as the problem of assigning commodities to Industries (rather than to Major Groups - a less ambiguous task).

Chapter I was an introduction to the thesis as well as considering some basic economic concepts involved in input/output analysis. In particular, the concept of "sectorizing" the economy and tracing "feedbacks" was discussed.

In Chapter II, the basic suggested model was considered and a rationale given for its use in terms of data availability. The manner of including imports and trade margins was considered and a composite model taking these factors into account was developed. Finally, some possible uses of such a model were discussed in a general sense.

Chapter III assumed the use of the model developed in the previous chapter and considered data sources, coding problems and two different tables available from this data. Emphasis was on existing classification schemes and readily available sources of data.

Chapter V outlined the step-by-step procedures for gathering the data and using a computer to arrange the tabular displays of the desired layouts of this data. It was assumed that the compiler was not familiar with computer operations and computer programming. Again, it was assumed that the computer operator, card punchers and programmers were not familiar either with the economics of input/output or even with the model used.

In this final chapter, a brief survey has been taken of what has been done after which follows a discussion of preparaing the MGRP x MGRP table for inversion and procedures for inversion itself.

Adding Sectors onto MGRP x MGRP TABLE

Whereas as the column sectors only include the Manufacturing Major Groups - by definition of Industry - the column sectors are in terms of Major Group but may include Non-Manufacturing Major Groups. In Chapter III, it was suggested that "non-manufactured" commodities (rows) be classified to special Major Groups such as Crops, Livestock, Metal Mining, etc. If this format has been followed, and if, as was suggested in Chapter IV, the MGRP x MGRP table has been entered on a columnar pad, then the task remains of "squaring up" the sector by determining the cost structures of these special Groups. In this tabular form, the compiler can also enter any non-invertable sectors such as final demand and primary factors (labour, etc.) in as fine degree of detail as desired. In preparation for inversion, however, these added sectors should be omitted.

Subsequent Programming

The inversion programs are only outlined in this chapter in the light of the fact that by this time, the compiler should be able to program it himself or at least be able to explain the situation to a professional programmer. Also, the size and desired layout of the table can be specifically stipulated and any desired adjustment can be easily accommodated if the procedure is only "sketched".

Inversion

The inversion of a matrix is a common procedure in computer work and the particular computer centre may best advise on which developed or "canned" program to use in the light of the requirements. "Canned" programs are previously developed and tested programs which only require instructions as to how to read in the data. If such a program were to be used, the compiler would discuss the arrangement of the table on punched cards and have it subsequently put on these punched cards. The computer centre would then take over and supply the inversion in both printout and punched card forms.

Inversion in a direct sense, however, may involve considerable "round-off" errors. The reader is directed to reference (8) by Christ, C. on this matter, and if he deems the level of error to be significant, he may prefer to use the power series approximation.

$$(I - A)^{-1} = I + A + A^2 \ldots \quad (V-1)$$

A program for computing this is outlined on page 112. The series may be taken out to as high a power as is desired. A simple test on the accuracy is to multiply the approximate inverse by the original (I - A) matrix, and the degree to which the product differs from the identity matrix I is indicative of the error involved. This follows from the mathematical relationship:

$$(I - A)\ (I - A)^{-1} = I$$

The author would also like to mention here a method of inverting a matrix of any <u>net</u> changes in coefficients of A. It is assumed that the basic inversion $(I - A)^{-1}$ has been computed. Although it is valid, the usefulness of such a "short-cut" has not been satisfactorily determined and it is left to the compiler/programmer to make this judgement. The procedure for developing such a program is outlined on page 113 and is based on the following formula.

Let A^* = a matrix the dimensions of A, with 0's in every cell except the desired changes where the net change is entered in the proper cell.

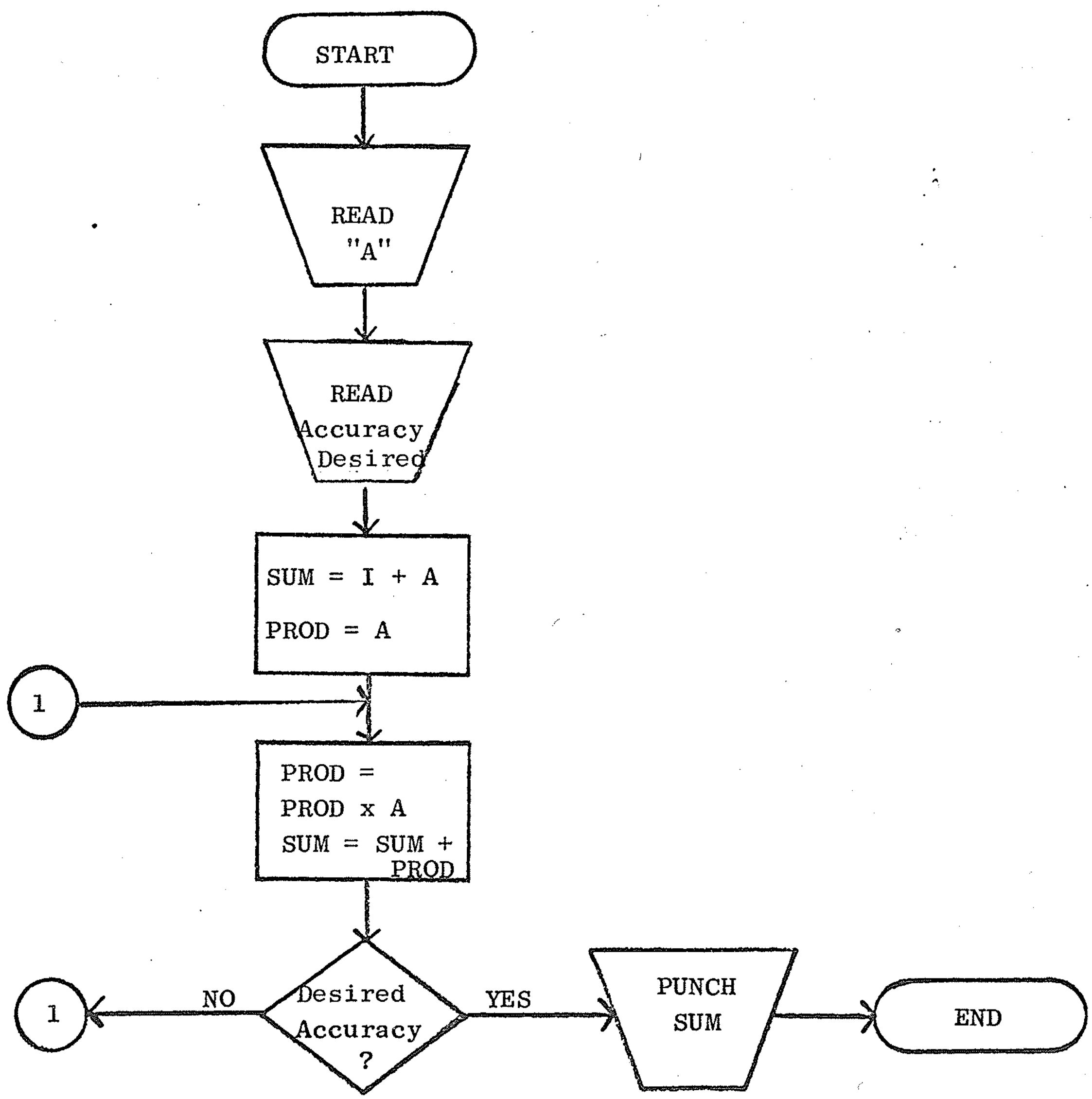
$((I - A)^{-1})^*$ = the net effect of such changes on the inversion $(I - A)^{-1}$

Then

$$((I - A)^{-1})^* = I + A^* + AA^* + (AA^*)A^* + (AA^*A^*)A^* \ldots \quad \ldots\ldots \text{(V-2)}$$

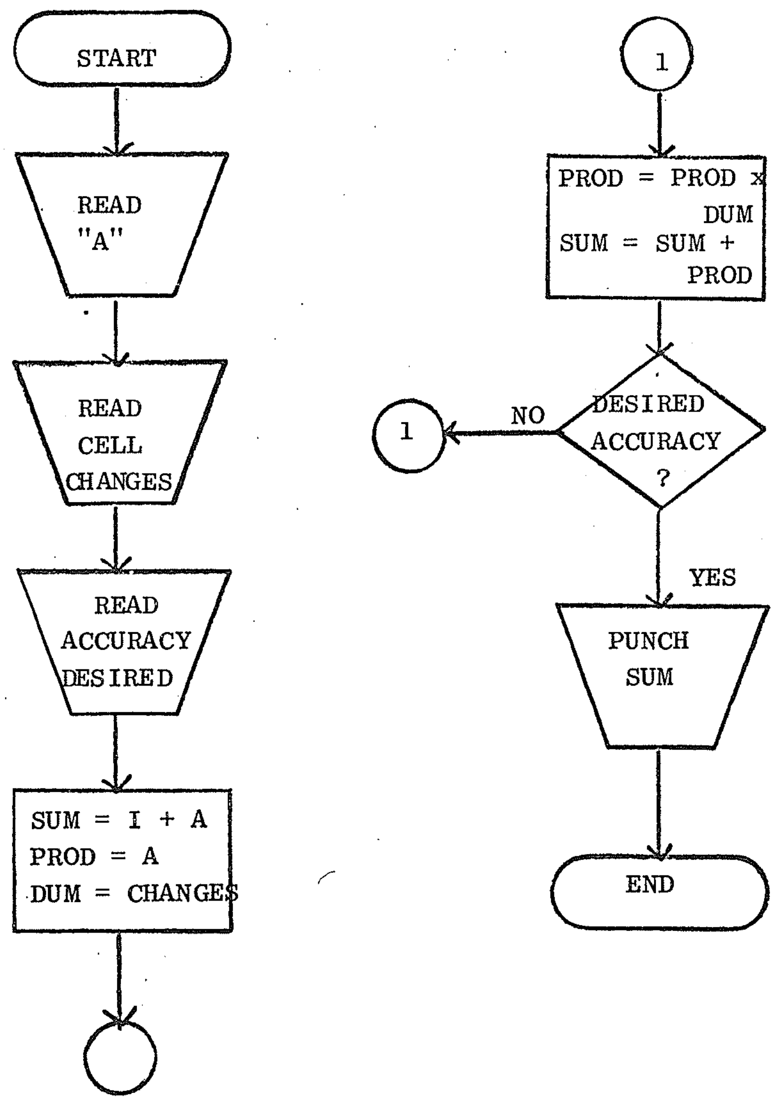
INVERSION USING THE LEONTIEF-CORNFIELD EXPANSION

If it is desired to compute the inverse according to (V-1), the following procedure can be used. Successive powers of "A" are accumulated to any degree desired, and added to the Identity Matrix; the result is an approximation to $(I-A)^{-1}$. The input required is the "A" matrix and some indication of how far (to what power) the expansion should be carried. A basic flowchart for such a program is:



COMPUTING CHANGE ON INVERSE OF A CHANGE IN "A"

The procedure here is similar to that on page 112 except that a dummy matrix is set up (call it "DUM") which has the dimension of Matrix "A" with zeroes in all cells except for the postulated changes. The suggested program is outlined in the following flowchart:



"The missing pages ... 72, 76, 80, 84, 88, 90, 92, 96, 102, 114, 115, 124, were removed from all copies of the theses as these were in effect more in the nature of work sheets rather than part of the thesis itself". - Extract from a letter dated January 3, 1967 from the University of Manitoba to the National Library.

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"The missing pages ... 72, 76, 80, 84, 88, 90, 92, 96, 102, 114, 115, 124, were removed from all copies of the theses as these were in effect more in the nature of work sheets rather than part of the thesis itself". - Extract from a letter dated January 3, 1967 from the University of Manitoba to the National Library.

A P P E N D I X

Historical Development

In most input/output writings, the Physiocrats are credited with having first "discovered" the feedback or indirect effects of production, Walras is credited with having, at least theoretically, used "feedback" in a general equilibrium model, and Leontief with having given the Walrasian system sufficient trimming as to make it emperically workable.

Prior to the Physiocrats, Economics was studied more as an Art than a Science, the Scholastics having kept the subject closely associated with ethics and religious doctrine. The Mercantilists and other pre-Physiocratic writers studied Economics as a Science only in a restricted sense-foreign trade for example, and worked out relationships through "cascade"(1) rather than "feedback" effects. The Physiocrats and, in particular, Quesnay with his "Tabuleau Economique" considered these "feedback" effects but in the rather misleading frame-work of sterile vs. productive employments, and in terms of "advances" flowing interdependently between three broad classes of economic life. It would appear that their precursing of input/output analysis was limited to their using interaction and feedback as a viable framework for considering economic workings.

Economics as a Science, after the Physiocrats, fell into two fairly distinct camps, that of micro and macro relationships. It was not until Walras that a macro view of the economy as an aggregation of micro relationships, was thought possible. Cournot, a predecessor of Walras, is found referring explicitly to a Walrasian system, namely

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(1) This type of effect can be visualized as a logic tree.

that "stimulation in production in one sector will stimulate production in supplying sectors in (even diminishing) circles"[2] but he dismissed this approach off hand with the remark that it would be impossible to enumerate such relationships.

Leon Walras is generally regarded as the father of general equilibrium. In his "Elements de Politique Economique Pure"[3], he presented what he considered was a (re) solvable and emperically measureable system by considering the economy as an equilibrium of micro relationships. As Friedman points out, this system required the measurement of utilities, or, more precisely, the assumption that individuals preference functions were additive. Commentators[4], this author has studied, regard Walras' tatonenment or "groping" process as simply begging the question. In any case, the Walrasian system was expressed in terms of four sets of equations, the correspondence to the modern input/output tables of which should be obvious:

1. An equation for each produced consumer good showing the quantity produced as a function of prices of production factors and the prices of other consumer goods.
2. An equation for each factor of production showing the total supply of that factor as divided into its various uses.
3. A cost equation for each consumer goods showing the cost of the various factor services making it up.

4. An identity equation expressing the equilibrium condition the cost of a commodity unit is equal to its selling price.

Of the above equations, (4) corresponds to (I - 2) and (1) roughly corresponds to (I - 1).

The two "extra" equations are necessary because the Walrasian purports to determine more variables under more rigid conditions than does the Leontief adaptation. A case in point is the fact that Leontief, unlike Walras, did not require his model to work within a fixed supply of primary factors. The exogenous variables in the Walrasian system then were:

1. consumer tastes and preferences
2. available supplies of factor services
3. production techniques

which, by way of the four sets of equations, determined the endogenous variables:

1. prices of finished goods
2. prices of production factors
3. quantities of finished goods produced.

In the Leontief system, the relative system of prices can be determined given the quantities of output as the quantities of output can be determined if relative prices are assumed fixed, whereas in the Walrasian model, both are simultaneously determined by virtue of a greater number of equations.

The prices and quantities can be mutually determined if other equations are added: Yamada[5] does this concisely by assuming demand as a simple linear function of prices.

W. W. Leontief is commonly regarded as the father of Input/ Output Analysis. His original accounting framework as first presented in "The Structure of the American Economy, 1919-1929" is used today with only minor modifications (most of which Leontief made himself in subsequent editions). He developed the theory as a Research Associate of the National Bureau of Economic Research (U.S.), a post he relinquished in 1931 in order to conduct statistical compilation within his newly devised framework. This work was completed by 1939 and was published under the above title in 1941. The book has undergone two subsequent revisions, the first produced in 1951, the second in 1965.

The accounting system he developed was based on the Walrasian production framework with the addition of certain simplifying assumptions in order to allow statistical valuation of the relevant variables. Leontief, as Walras, assumed a "closed" model where the household sector's consumption of goods was regarded as fixed in regard to the proportion that each type of good (sector) constituted of total final demand. In his second edition, Leontief changed this and worked with an "open" model where the final demand bill was regarded as a column vector of exogenous variables. Leontief, as Walras, assumed that inputs were a fixed proportion of output over time although it has been shown (6) that in Leontief's case this was an unnecessarily strict assumption. In contemporary tables, fixed production coefficients can theoretically be replaced by a linear function of output or even a non-linear function (along the lines of a Cobb-Douglas function), but this procedure is seldom followed because of the mathematical complexity, the burdensome increase in computations, and the great increase in necessary data. Contemporary tables normally differ from the Leontief original in a number of other ways:

1. Labour was Leontief's only primary (non-produced) commodity - corresponding to V_j of (I - 2). Now there are often between five and ten of such sectors.
2. Leontief's inclusion of the following factors have been dropped from input/output tables:
   (a) productivity factor - the "A"'s of (A -2) below.
   (b) specific savings-investment factor - the "B"'s of (A -2) below.
   (c) overall savings-investment factor - the "C"'s of (A -2) below.
3. Leontief's treatment of final demand as fixed in regard to sector demand in relation to total i normally dropped. Dorfman, Samuelson and Solow[(7)] point out that this approach can still be useful if approximate projections of changes in employment resulting from, say taxation, are desired.

Leontief's book is divided into three sections; the first being a discussion of the basic accounting relationships, the second a "Walrasian" delineation of the model and the third a description of the results of applying the model to the American economy of 1919 and 1929. In the first part he considers the relationship of double entry bookkeeping to the model, the relationship of his model to other models, and the question of homogeneity requirements on sector selection. He takes note of the absence, in his framework, of the sectors of 1) distribution charges, wholesale and retail; 2) banking and finance expenses; 3) all non-rail transportation charges; and 4) the income expenditure accounts of all public bodies. In the light of greater availability of data and the increased importance of these items, most input/output tables include these elements in one form or another. Leontief, in this section, also considers the importance of imports to the model, the distinction between competitive and non-competitive imports, and the difference between static and dynamic systems.

In the second section, the input/output balance identities are discussed. These form a close correspondence to (I-1, 2, 4) above. The first set of equations express, in physical terms, the fact that a sector's total output is equal to the sum of its uses. The second set expresses, in dollar value terms, the fact the value of inputs into a sector equals the value of that sector's output. This is written as:

$$\frac{X_j P_j}{B_j B} + \sum_{i=1}^{n} X_{ij} P_i = 0 \qquad \text{..........(A - 1)}$$

where

B_j = a saving-investment coefficient (weight) unique to the "j" th sector.

B = a saving-investment coefficient (weight) common to all sectors - reflecting interest rate, etc.

As was pointed out before, the B_j and B did not turn out to be useful parameters.

The third set of equations express the technical structure of an industry and correspond to (I - 4).

Here

$$X_{ij} = \frac{a_{ij} X_i}{A_j} \qquad \ldots\ldots\ldots\ldots\ldots (A-2)$$

A_j = a productivity coefficient (weight)

Again, the A_j has been dropped because of difficulty of measurement and lack of stability.

A significant feature is that there is no autonomous final demand sector(s) and only one primary sector (labour). By these three sets of equations, the outputs and prices are completely determined and the economy is in a "stationary" state. Unlike the questions asked of input/output analysis in today's context, Leontief considered the central object of his work - apart from the descriptive comparisons of the 1919-1929 production structures - the expression of changes in prices and quantities in terms of changing "a_{ij}"'s.

The Leontief model has been discussed above with the object of suggesting the evolution of the discipline as well as putting the original model in a modern interpretation.

FORTRAN IV - CONVERSION

The programs "SIFT" and "TABL" are written in FORTRAN II, a language which can be understood by most pre-1965 computers. If it is required that these programs be run on the S/360 I.B.M. line of computers, the language must be translated to FORTRAN IV. Care has been taken in developing "SIFT" and "TABL" to make most statements compatible to both languages - such as spacing GØ TØ statements. The input/output statements, however, must be changed, along with a number of corresponding FØRMAT statements. Thus:

1. FØRTRAN II READ 100, X, Y, Z

List of variables
Statement number of FØRMAT

FØRTRAN IV READ (2, 100) X, Y, Z

Input Device reference number
statement number of FØRMAT

Note: No comma after bracket

2. FØRTRAN II WRITE 100, X, Y, Z
or PUNCH 100, X, Y, Z
or PRINT 100, X, Y, Z

FØRTRAN IV WRITE (3, 100) X, Y, Z

Output Device reference number
statement number of FØRMAT

Note: No comma after bracket

3. FØRTRAN II FØRMAT (7 H to SAMPLE)

Number of spaces for field of literal constant
Hollerith field indicator
literal constant

FØRTRAN IV FØRMAT (' to SAMPLE')

literal constant
No need to count spaces - quote marks define field.

```
C     SIFT PROGRAM FOR COEF I, COEF II,SCC LIST, SIC LIST                 SIF
      DIMENSION ISCC(4),VAL(4),QUAN(4),IT1(200),IT2(500),IT3(200),IT4(20  SIF
     10),IT5(200),T4(200),T5(200)                                         SIF
      M1=1                                                                SIF
      M2=1                                                                SIF
      M3=1                                                                SIF
      M4=1                                                                SIF
      M5=0                                                                SIF
      M6=1                                                                SIF
      M7=0                                                                SIF
      SUM=0.0                                                             SIF
      DO 1 I=1,200                                                        SIF
      IT1(I)=0                                                            SIF
      IT3(I)=0                                                            SIF
      IT4(I)=0                                                            SIF
      IT5(I)=0                                                            SIF
      T4(I)=0.0                                                           SIF
    1 T5(I)=0.0                                                           SIF
C     READ CONTROL CARD                                                   SIF
      READ 1000,ARK                                                       SIF
 1000 FORMAT (F5.0)                                                       SIF
      MARK=ARK                                                            SIF
      GO TO (100,800,100,100),MARK                                        SIF
C     READING BASIC DATA                                                  SIF
  100 READ 1001,ISIC,IEST,ICODE,(ISCC(J),VAL(J),QUAN(J),J=1,4)            SIF
 1001 FORMAT (I4,I3,I1,4(I3,F9.0,F6.0))                                   SIF
      IF(ISIC-9999)101,500,500                                            SIF
  101 GO TO (301,102,201,104),MARK                                        SIF
  102 ITTT=(ISIC*1000)+IEST                                               SIF
      DO 103 I=1,MA                                                       SIF
      IF(IT2(J)-ITTT)103,100,103                                          SIF
  103 CONTINUE                                                            SIF
  104 DO 105 I=1,M1                                                       SIF
      IF(ISIC-IT1(I))105,106,105                                          SIF
  105 CONTINUE                                                            SIF
      IT1(M1)=ISIC                                                        SIF
      GO TO 200                                                           SIF
  106 IT3(I)=IT3(I)+1                                                     SIF
C     LISTING OF INDUSTRIES ABOVE, LISTING OF COMMODITIES BELOW           SIF
  200 GO TO (301,201,201,100),MARK                                        SIF
  201 00 203 J=1,4                                                        SIF
      DO 202 I=1,M2                                                       SIF
      IF(ISCC(J)-IT2(I))202,203,202                                       SIF
  202 CONTINUE                                                            SIF
      IT2(M2)=ISCC(J)                                                     SIF
      M2=M2+1                                                             SIF
  203 CONTINUE                                                            SIF
C     ACCUMULATION OF INPUTS AND OUTPUTS BY SCC                           SIF
  300 GO TO (301,301,100,100),MARK                                        SIF
  301 M5=4-ICODE                                                          SIF
```

```
      IF(M5)400,901,302                                                 SIF
  302 IF(M7)303,303,500                                                 SIF
  303 DO 306 J=1,M5                                                     SIF
      DO 305 I=1,M4                                                     SIF
      IF(IT4(I)-ISCC(J))305,304,305                                     SIF
  304 T4(I)=T4(I)+VAL(J)                                                SIF
      GO TO 306                                                         SIF
  305 CONTINUE                                                          SIF
      IT4(M4)=ISCC(J)                                                   SIF
      T4(M4)=VAL(J)                                                     SIF
      M4=M4+1                                                           SIF
  306 CONTINUE                                                          SIF
      GO TO 100                                                         SIF
  400 M5=M5+5                                                           SIF
      IF(M5)902,902,401                                                 SIF
  401 DO 404 J=1,M5                                                     SIF
      DO 403 I=1,M6                                                     SIF
      IF(IT5(I)-ISCC(J))403,402,403                                     SIF
  402 T5(I)=T5(I)+VAL(J)                                                SIF
      SUM=SUM+VAL(J)                                                    SIF
      GO TO 404                                                         SIF
  403 CONTINUE                                                          SIF
      IT5(M6)=ISCC(J)                                                   SIF
      T5(M6)=VAL(J)                                                     SIF
      M6=M6+1                                                           SIF
  404 CONTINUE                                                          SIF
      M7=1                                                              SIF
      GO TO 100                                                         SIF

C     FORMING COEFS FROM VALUES                                         SIF

  500 GO TO (513,600,700,600),MARK                                      SIF
      M6=M6-1                                                           SIF
      M4=M4-1                                                           SIF
      DO 501 I=1,M6                                                     SIF
  501 T5(I)=T5(I)/SUM                                                   SIF
      DO 502 I=1,M4                                                     SIF
  502 T4(I)=T4(I)/SUM                                                   SIF
C     SORTING AND PUNCHING INPUTS AND OUTPUTS                           SIF
      LX=0                                                              SIF
      MOM=M4-1                                                          SIF
  503 DO 505 I=1,MOM                                                    SIF
      IF(IT4(I+1)-IT4(I))504,505,505                                    SIF
  504 LXT=IT4(I)                                                        SIF
      CXT=T4(I)                                                         SIF
      IT4(I)=IT4(I+1)                                                   SIF
      T4(I)=T4(I+1)                                                     SIF
      IT4(I+1)=LXT                                                      SIF
      T4(I+1)=CXT                                                       SIF
      LX=1                                                              SIF
  505 CONTINUE                                                          SIF
      IF(LX)507,507,506                                                 SIF
  506 LX=0                                                              SIF
      GO TO 503                                                         SIF
  507 PUNCH 1002,ISIC,IEST,M7,(I4(I),T4(I),I=1,M4)                      SIF
 1002 FORMAT (I4,I3,I1,6(1X,I3,1X,F7.5))                                SIF
      M7=0                                                              SIF
      LX=0                                                              SIF
      MOM=M6-1                                                          SIF
  512 DO 509 I=1,MOM                                                    SIF
```

```
      IF(IT5(I+1)-IT5(I))508,509,509                                    SIF
  508 LXT=IT5(I)                                                        SIF
      CXT=T5(I)                                                         SIF
      IT5(I)=IT5(+1)                                                    SIF
      IT5(I+1)=LXT                                                      SIF
      T5(I+1)=CXT                                                       SIF
      LX=1                                                              SIF
  509 CONTINUE                                                          SIF
      IF(LX)511,511,510                                                 SIF
  510 LX=0                                                              SIF
      GO TO 512                                                         SIF
  511 PUNCH 1002,ISIC,IEST,M7,(IT5(I),T5(I),I=1,M6)                     SIF
      M7=2                                                              SIF
      PUNCH 1003,ISIC,IEST,M7,SUM                                       SIF
 1003 FORMAT (I4,I0,I1,10X,F10.1)                                       SIF
      M7=0                                                              SIF
      M4=1                                                              SIF
      M6=1                                                              SIF
      IF(ISIC-9999)303,900,303                                          SIF
C     SORTING AND PUNCHING INDUSTRY LIST                                SIF
  600 M1=M1-1                                                           SIF
      MOM=M1-1                                                          SIF
      LX=0                                                              SIF
  601 DO 603 I=1,MOM                                                    SIF
      IF(IT1(I+1)-IT1(I))602,603,603                                    SIF
  602 LXT=IT1(I)                                                        SIF
      IXT=IT3(I)                                                        SIF
      IT1(I)=IT1(I+1)                                                   SIF
      IT3(I)=IT3(I+1)                                                   SIF
      IT1(I+1)=LXT                                                      SIF
      IT3(I+1)=IXT                                                      SIF
      LX=1                                                              SIF
  603 CONTINUE                                                          SIF
      IF(LX)604,605,601                                                 SIF
  604 LX=0                                                              SIF
      GO TO 601                                                         SIF
  605 PUNCH 1005,M1                                                     SIF
 1005 FORMAT (20H  NO. OF INOUSTRIES I5)                                SIF
      PUNCH 1006,(IT1(J),IT3(J),J=1,M1)                                 SIF
 1006 FORMAT (8(2I5))                                                   SIF
      GO TO (700,900,900,700),MARK                                      SIF
  700 M2=M2-1                                                           SIF
      LX=0                                                              SIF
      MOM=M2-1                                                          SIF
  701 DO 703 I=1,MOM                                                    SIF
      IF(IT2(I+1)-IT2(I))702,703,703                                    SIF
  702 LXT=IT2(I)                                                        SIF
      IT2(I)=IT2(I+1)                                                   SIF
      IT2(I+1)=LXT                                                      SIF
      LX=1                                                              SIF
  703 CONTINUE                                                          SIF
      IF(LX)705,705,704                                                 SIF
  704 LX=0                                                              SIF
      GO TO 701                                                         SIF
  705 PUNCH 1007,M2                                                     SIF
 1007 FORMAT (21H  NO. OF COMMOOITIES I5)                               SIF
      PUNCH 1008,(IT2(J),J=1,M2)                                        SIF
 1008 FORMAT (16I5)                                                     SIF
```

```
      GO TO 900                                                         SIF
  800 DO 801 I=1,500                                                    SIF
  801 IT2(I)=0                                                          SIF
      READ 1009,A                                                       SIF
 1009 FORMAT (F5.0)                                                     SIF
      MA=A                                                              SIF
      DO 802 I=1,MA                                                     SIF
      READ 1010,ISIO,IESO                                               SIF
 1010 FORMAT (2I5)                                                      SIF
      IF(ISIO-9999)802,100,800                                          SIF
  802 IT2(I)=ISIO*1000+IESO                                             SIF
  900 CALL EXIT                                                         SIF
  901 M7=0                                                              SIF
      PUNCH 1011,ISIC                                                   SIF
 1011 FORMAT (20H  NO INPUT FOR IND. I5)                                SIF
      GO TO 100                                                         SIF
  902 M7=0                                                              SIF
      PUNCH 1012,ISIC                                                   SIF
 1012 FORMAT (21H  NO OUTPUT FOR IND. I5)                               SIF
      GO TO 100                                                         SIF
      END                                                               SIF
```

```
C     TABL  PROGRAM  FOR   WEIGHTS, SCC X SCC, SCC X MGRP, MGRP X SIC
C       AND  MGRP X MGRP.


C     WEIGHTS

      DIMENSION  ISIC(200),SUM(200)MGRP(200),PROP(200),ISCC(500),ISICT(9
     1),TABLE(500,9)
      DO 100 I=1,200
      SUM(I)=0.0
  100 PROP(I)=0.0
      READ 1000,N1
      ISIC(1)=N1
      K=1
  101 READ 1001,NSIC,M7,TSUM
      IF(NSIC-9999)102,106,106
  102 IF(M7-2)101,103,101
  103 IF(MSIC-ISIC(K))105,104,105
  104 SUM(K)=SUM(K)+TSUM
      GO TO 101
  105 K=K+1
      ISIC(K)=NSIC
      SUM(K)=TSUM
      GO TO 101
      READ,1002,NOSIC
      READ 1003,(MGRP(J),J=1,NOSIC)
      IF(NOSIC-K)900,107,900
  900 PUNCH 1050
 1050 FORMAT (34H  NO. OF S.I.C. IN DECKS NOT EQUAL)
  901 CALL EXIT
 1004 FORMAT (7H S.I.C. I4,7H M.GRP I2,7H PROP F6.4,5H SUM F9.0)
  107 DO 110 I=1,K
      PROP(I)=SUM(I)
      DO 110 J=1,K
      IF(I-J)108,110,108
  108 IF(MGRP(I)-MGRP(J))110,109,110
  109 PROP(I)=PROP(I)+SUM(J)
  110 CONTINUE
      PUNCH 1005,NOSIC
 1005 FORMAT (19H NO. OF INDUSTRIES I6)
      DO 111 I=1,K
      PROP(I)=SUM(I)/PROP(I)
      II=ISIC(I)
      SS=SUM(I)
      MM=MGRP(I)
      PP=PROP(I)
  111 PUNCH 1004,II,MM,PP,SS

C     SCC BY SIC TABLE

      READ 1010,NOSCC
 1010 FORMAT (5X,I5)
      READ 1011,(ISCC(I),I=1,NOSCC)
 1011 FORMAT (16(I3,2X))
      K=1
      ISICT(1)=9999
      DO 200I=1,NOSCC
      DO 200J=1,9
  200 TABLE(I,J)=0.0
```

```
 201 READ 1012,NSIC,M7,(ISCCT(I),PROPT(I),I=1,6)                    TAB
     IF(NSIC-9999)202,210,210                                       TAB
 202 IF(M7-1)201,203,201                                            TAB
 203 IF(NSIC-ISICT(K))206,207,204                                   TAB
 204 K=K+1                                                          TAB
     IF(K-9)205,210,210                                             TAB
 205 ISICT(K)=NSIC                                                  TAB
     GO TO 207                                                      TAB
 206 ISICT(K)=NSIC                                                  TAB
 207 DO 209 I=1,6                                                   TAB
     DO 208 J=1,NOSCC                                               TAB
     IF(ISCCT(I)-ISCC(J))208,209,208                                TAB
 208 CONTINUE                                                       TAB
 209 TABLE(J,K)=PROPT(I)                                            TAB
     GO TO 201                                                      TAB
 210 PUNCH 1013,(ISICT(I),I=1,K)                                    TAB
     DO 211 I=1,NOSCC                                               TAB
     N1=ISCC(I)                                                     TAB
 211 PUNCH 1014,N1,(TABLE(I,J),J=1,K)                               TAB
     K=1                                                            TAB
     IF(NSIC-9999)205,901,901                                       TAB
                                                                    TAB
C    COEF III                                                       TAB
                                                                    TAB
     READ 1010,NOSCC                                                TAB
     READ 1011,(ISCC(I),I=1,NOSCC                                   TAB
 300 TABLE(I,1)=0.0                                                 TAB
     N1=9999                                                        TAB
 301 READ 1020,NSIC,M7,(ISCCT(I),PROPT(I),I=1,6)                    TAB
     IF(NSIC-9999)                                                  TAB
 302 IF(M7-1)301,303,309                                            TAB
 303 IF(NSIC-N1)304,305,304                                         TAB
 304 N1=NSIC                                                        TAB
 305 DO 308 I=1,6                                                   TAB
     DO 307 J=1,NOSCC                                               TAB
     IF (ISCC(J)-ISCCT(I))307,306,307                               TAB
 306 TABLE(J,1)=TABLE(J,1)+PROPT(I)                                 TAB
     GO TO 308                                                      TAB
 307 CONTINUE                                                       TAB
 308 CONTINUE                                                       TAB
     GO TO 301                                                      TAB
 309 I=1                                                            TAB
     DO 311 J=1,NOSCC                                               TAB
     IF(TABLE(J,1))311,311,310                                      TAB
 310 SUM(I)=TABLE(J,1)                                              TAB
     MGRP(I)=ISCC(I)                                                TAB
     I=I+1                                                          TAB
 311 CONTINUE                                                       TAB
     TSUM=                                                          TAB
     DO 312 J=1,I                                                   TAB
 312 SUM(J)=SUM(J)/TSUM                                             TAB
     PUNCH 1021(N1,(MGRP(J),SUM(J),J=1,I)                           TAB
     PUNCH 1022,N1,SUM                                              TAB
     DO 313 J=1,NOSCC                                               TAB
 313 TABLE(I,1)=0.0                                                 TAB
     GO TO 301                                                      TAB
                                                                    TAB
C    MGRP BY SIC                                                    TAB
                                                                    TAB
     READ 1030,NOASS                                                TAB
```

```
      READ 1031,(MGRP(I),I=1,NOASS)                                      TAB
      DO 400 I=1,2                                                       TAB
      DO 400 J=1,9                                                       TAB
  400 TABLE(J,J)=0.0                                                     TAB
  401 READ 1013,ISICT                                                    TAB
      PUNCH 1013,ISICT                                                   TAB
      I=1                                                                TAB
  402 READ 1014,N1,(TABLE(1,J),J=1,9)                                    TAB
      IF(N1-99)403,410,410                                               TAB
  403 IF(I-NOASS)404,407,407                                             TAB
  404 IF(MGRP(I)-MGRP(I+1))407,405,407                                   TAB
  405 READ 1014,N1,(TABLE(2,J),J=1,9)                                    TAB
      DO 406 J=1,9                                                       TAB
  406 TABLE(1,J)=TABLE(1,J)+TABLE(2,J)                                   TAB
      I=I+1                                                              TAB
      GO TO 403                                                          TAB
  407 N1=MGRP(I)                                                         TAB
      PUNCH 1014,N1,(TABLE(1,J),J=1,9)                                   TAB
      DO 408 J=1,9                                                       TAB
      TABLE(1,J)=0.0                                                     TAB
  408 TABLE(2,J)=0.0                                                     TAB
      I=I+1                                                              TAB
      IF(I-NOASS)402,401,901                                             TAB
  410 CALL EXIT                                                          TAB
                                                                         TAB
C     MGRP   BY   MGRP                                                   TAB
                                                                         TAB
      READ 1005,NOSIC                                                    TAB
      DO 500 J=1,NOSIC                                                   TAB
      READ 1004,II,MM,PP,SS                                              TAB
      ISIC(I)=II                                                         TAB
      MGRP(I)=MM                                                         TAB
  500 PROP(I)=PP                                                         TAB
      K=1                                                                TAB
  501 DO 504 I=2,NOSIC                                                   TAB
      IF(MGRP(I)-MGRP(I-1))502,503,502                                   TAB
  502 K=K+1                                                              TAB
  503 MGRP(I)=K                                                          TAB
  504 CONTINUE                                                           TAB
  505 READ 1013,ISICT                                                    TAB
      DO 509 I=1,9                                                       TAB
      DO 508 J=1,NOSIC                                                   TAB
      IF  (ISIC(J)-ISICT(I))508,506,508                                  TAB
  506 ISICT(I)=MGRP(J)                                                   TAB
  507 ITEMP(I)=J                                                         TAB
      GO TO 509                                                          TAB
  508 CONTINUE                                                           TAB
  509 CONTINUE                                                           TAB
      DO 510 I=1,K                                                       TAB
      READ 1014,N1,(TEMP(J),J=1,9)                                       TAB
      DO 510 J=1,9                                                       TAB
      JL=ISICT(J)                                                        TAB
      JJL=ITEMP(J)                                                       TAB
  510 TABLE(I,JL)=TABLE(I,JL)+(TEMP(J)*PROP(JJL))                        TAB
      IF(JJL-NOSIC)505,511,511                                           TAB
  511 LLL=1                                                              TAB
      KKK=9                                                              TAB
  512 DO 513 I=1,K                                                       TAB
  513 PUNCH 1014,I,(TABLE(I,J),J=LLL,KKK)                                TAB
      IF(KKK-K)514,516,516                                               TAB
```

```
514 LLL=LLL+9                                                    TAB
    KKK=KKK+9                                                    TAB
    IF(KKK-K)512,515,515                                         TAB
515 KKK=K                                                        TAB
    GO TO 512                                                    TAB
516 CALL EXIT                                                    TAB
    END                                                          TAB
                                                                 TAB
```