CONSTANT STARTING TORQUE CONTROL
OF WOUND ROTOR INDUCTION MOTORS

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ABSTRACT

The three phase induction motor is the most popular one used in industry. Both the speed control and starting of polyphase induction motors are still among the perennial research topics in electrical engineering.

In this paper the author proposes a new starting method for the wound rotor induction motor. The relevant empirical equations are developed. The equations are good for any wound rotor motor operated at any desired starting torque under any applied voltage.

Recommended control circuits in connection with this new starting method have been designed, and the author does anticipate the proposed starting method could be used.

ACKNOWLEDGEMENT

The author wishes to thank Professor G. W. Swift of the electrical engineering department, University of Manitoba for his kind instruction and help in many respects in connection with this thesis.

LIST OF PRINCIPAL SYMBOLS

V Phase applied voltage, p.u.value/phase.

I Phase rotor current, p.u.value/phase.

r_1 Stator resistance per phase, p.u.value/phase.

x_1 Stator reactance per phase, p.u.value/phase.

r_2 Rotor resistance per phase, p.u.value/phase.

x_2 Rotor reactance per phase, p.u.value/phase.

g_c Shunt conductance per phase, p.u.value/phase.

b_m Shunt susceptance per phase, p.u.value/phase.

r External series resistance per phase, p.u.value/phase.

x External series reactance per phase, p.u.value/phase.

Re External parallel resistance per phase, p.u.value/phase.

Xe External parallel reactance per phase, p.u.value/phase.

r' Resistance component in Xe, p.u.value/phase.

R' Equivalent resistance of parallel combination, p.u.value/phase.

X' Equivalent reactance of parallel combination, p.u.value/phase.

R Total equivalent resistance per phase, p.u.value/phase.

X Total equivalent reactance per phase, p.u.value/phase.

Z Circuit impedance per phase, p.u.value/phase.

Y Circuit admittance per phase, p.u.value/phase.

T Internal electromagnetic torque per phase, p.u.value/phase.

T_{max} Maximum internal electromagnetic torque per phase, p.u.value/phase.

$\cos\theta$ Power factor.

P_{g1} Power delivered to the air gap per phase, watts/phase.

P Internal mechanical power per phase, watts/phase.

P_{12} Copper loss of rotor per phase, watts/phase.

f Frequency, Hz.

S Slip.

S_r Rated slip.

S_m Slip at T_{max}.

ω_s Synchronous angular speed, radians/second.

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CHAPTER I

THE STUDY OF WOUND ROTOR INDUCTION MOTOR STARTING

I. INTRODUCTION

The three phase induction motor is the most popular one among electrical motors. It is widely used in industry.

Its advantages, over other motors, are as follows:

(1) Its convenience in utilization of electrical supply,

(2) Its low initial cost,

(3) Its ease of control and maintenance,

(4) Its rigidity and ability to stand abuse.

There are two types of induction motor:

(1) Cage rotor,

(2) Wound rotor.

From the operation point of view, the main difference between the wound rotor and the cage rotor is that the former can be regulated in torque but the latter cannot.

The deep bar and double squirrel cage rotor can offer a good starting characteristic as shown in Fig. 1. However, the wound rotor has an advantage as stated by G.W.Heumann[10], "Wound-rotor motors are also applied to large drives, sometimes because it is desirable to control the starting torque, sometimes to reduce starting current peaks to a value which can be tolerated by the power system."

Since the torque of the wound rotor can be regulated, it is possible to operate it as a variable speed motor.

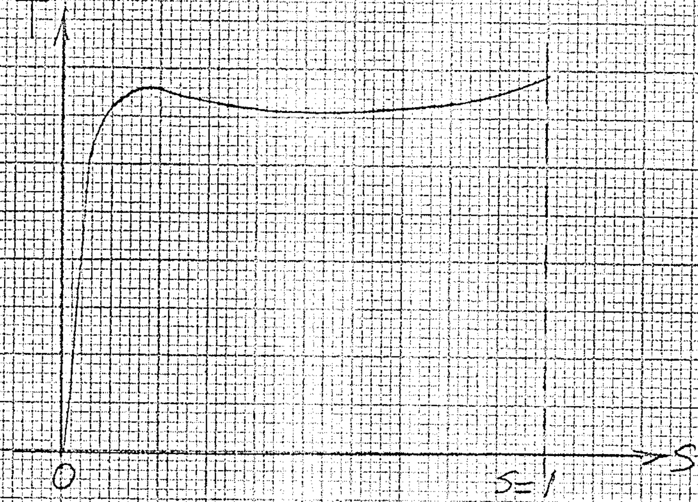
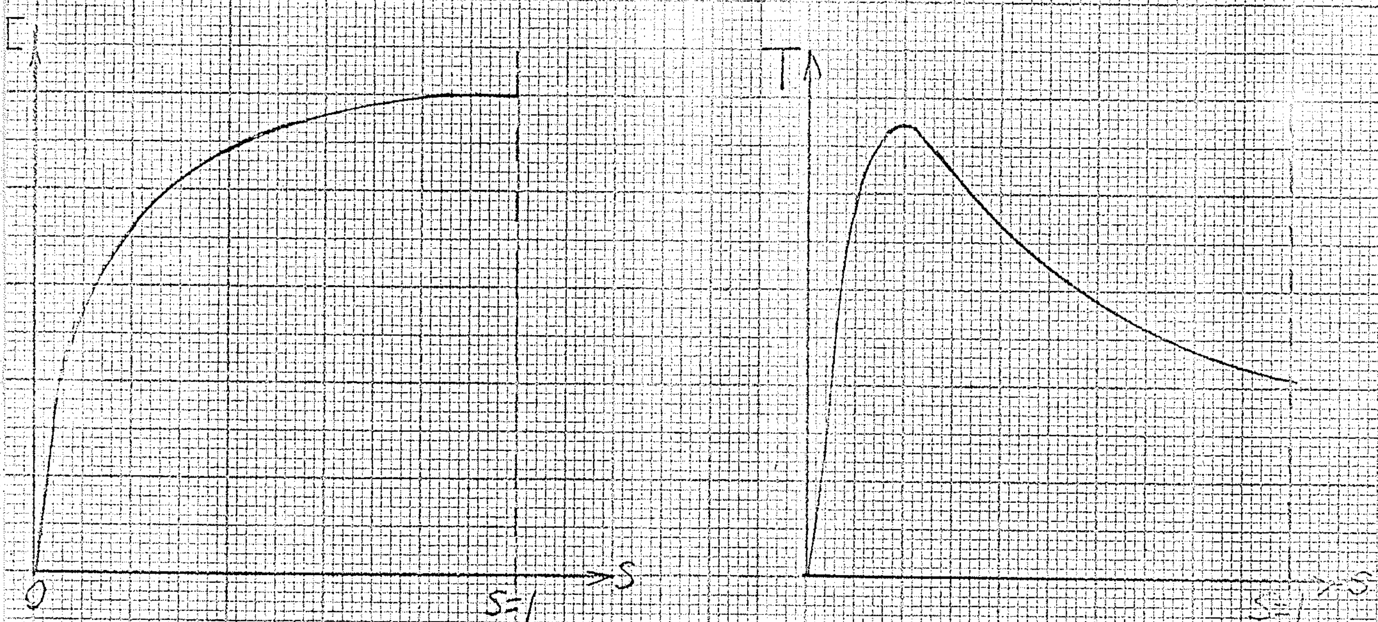


Fig. 1. Starting torque characteristic of Double Cage Rotor Motor



a. Starting current characteristic

b. Starting torque characteristic

Fig. 2. Cage Rotor Motor

The characteristic curves of current and torque versus slip for squirrel cage machines are shown in Fig. 2.

We assume in this thesis that it is desirable to control the starting torque-speed characteristic of a given wound rotor machine in a particular way, namely, that the torque be essentially constant throughout the starting period. In this paper, a new starting method is developed which attains this end.

The results, based on reasonable assumptions and simplifications, indicate that the proposed method can be used to arrive at a solution well within practical engineering tolerances.

II. PER UNIT VALUE

In the following calculations and analysis, the bases of per unit value are as follows:

Voltage: $V_{base} = V$ = rated phase voltage, volts

Current: $I_{base} = I$ = rated phase current, amperes

Speed: $\omega_{base} = \omega_s$ = synchronous angular speed, radians/second

Torque: $T_{base} = \frac{I_{base}^2}{\omega_{base}} \left(\frac{r_2}{S_r} \right)$, newton-meters/phase

Impedance: $Z_{base} = \frac{V_{base}}{I_{base}}$, ohms

III. EQUIVALENT CIRCUIT AND MOTOR PERFORMANCE

The equivalent circuit of an induction motor is shown in Fig. 3 in which the current and impedance of the rotor are defined as their values referred to stator. The exciting branch can be neglected and the results are still within the engineering tolerances.[1] The simplified equivalent circuit, which will be used in the following development, is shown in Fig. 4.

For the three phase induction motor, the per unit phase internal electromagnetic torque T in newton-meters is

$$T = \frac{1}{\omega_s} V \cdot I \cos\theta$$

and the power P delivered to the air gap by the stator is

$$P = (1 - S)\omega_s T = (1 - S)P_{g1} = I^2 r_2 \frac{1 - S}{S}$$

Therefore, $T = \frac{1}{\omega_s} I^2 (r_2/S)$

The torque T and power P are not the output values avaliabl at the shaft, because friction, windage and stray losses have to be accounted for.

Mechanical Output = Internal mechanical power - (Friction and windage losses + Stray load loss)

From the simplified equivalent circuit of Fig. 4,

$$I = \frac{V}{r_1 + r_2/S + j(x_1 + x_2)} = \frac{V}{\sqrt{(r_1 + r_2/S)^2 + (x_1 + x_2)^2}}$$

$$T = \frac{V^2}{\omega_s \left[(r_1 + r_2/S)^2 + (x_1 + x_2)^2\right]} r_2/S$$

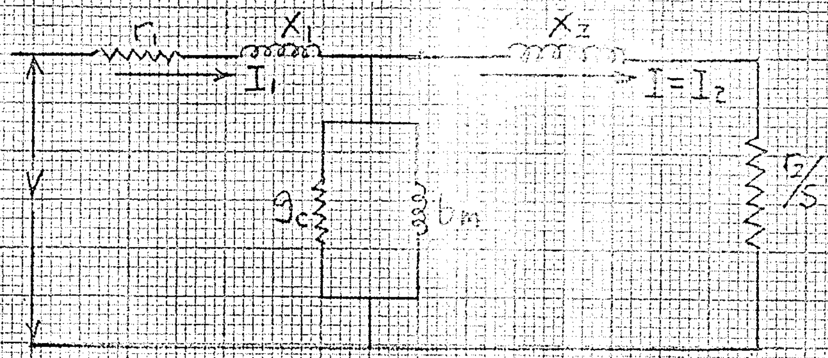


Fig. 3. Equivalent circuit

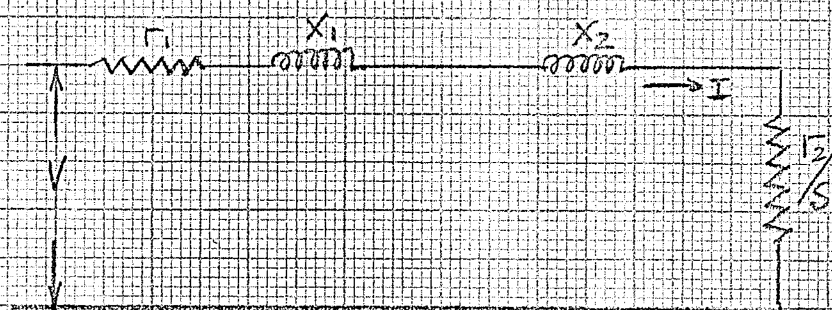


Fig. 4. Simplied equivalent circuit

IV. STUDY OF STARTING METHODS

In connection with motor starting, the following points have to be considered:

(1) Low starting current and constant starting torque with reduced values at standstill avoid damage to the mechanical parts.

(2) Smoothness of starting period.

(3) Simple control equipment and circuits.

From Fig. 2, it is obvious that high inrush starting current and non-constant starting torque are produced in the induction motor. These two characteristics can be corrected in a wound rotor induction motor using the starting method described in this paper.

The starting methods most frequently used for the wound rotor motor are as follows:

(1) Insertion of an external resistor into the rotor circuit either with a balanced or unbalanced connection,- the most popular and accepted method being used at the present time.

(2) Use of a saturable reactor or variable reactance inserted in the rotor circuit. This method has not been widely accepted since a feed back control system is required and its complication increases the initial cost of the whole control equipment.

The most common starting method for wound rotor motors is still the use of external resistance in the rotor. Fig. 5 is the circuit for a 300 HP wound rotor induction motor with five steps of acceleration during the starting period. Fig. 6 is its starting torque characteristic curve.

There are several disadvantages of using resistance starting. They are

(1) complicated control equipment and circuitry,

(2) high initial cost; for instance, the cost of acquiring the five accelerating contactors in the rotor side as shown in Fig. 5.

(3) the starting torque is not constant but rather pulsating between T_1 and T_2 as shown in Fig. 6. The difference between these two torque values depends upon many factors, such as the number of steps, maximum torque available and rotor resistance, It may be as high as 50% of the average.

Based upon the previous discussion, it is desirable to find some other starting method which will satisfy the following requirements:

(1) low starting current,

(2) desired constant torque preferably 20% tolerance or better,

(3) simplicity in control circuitry and equipment.

Constant starting torque and the simplicity of the control circuitry and equipment are the most important of the three points mentioned.

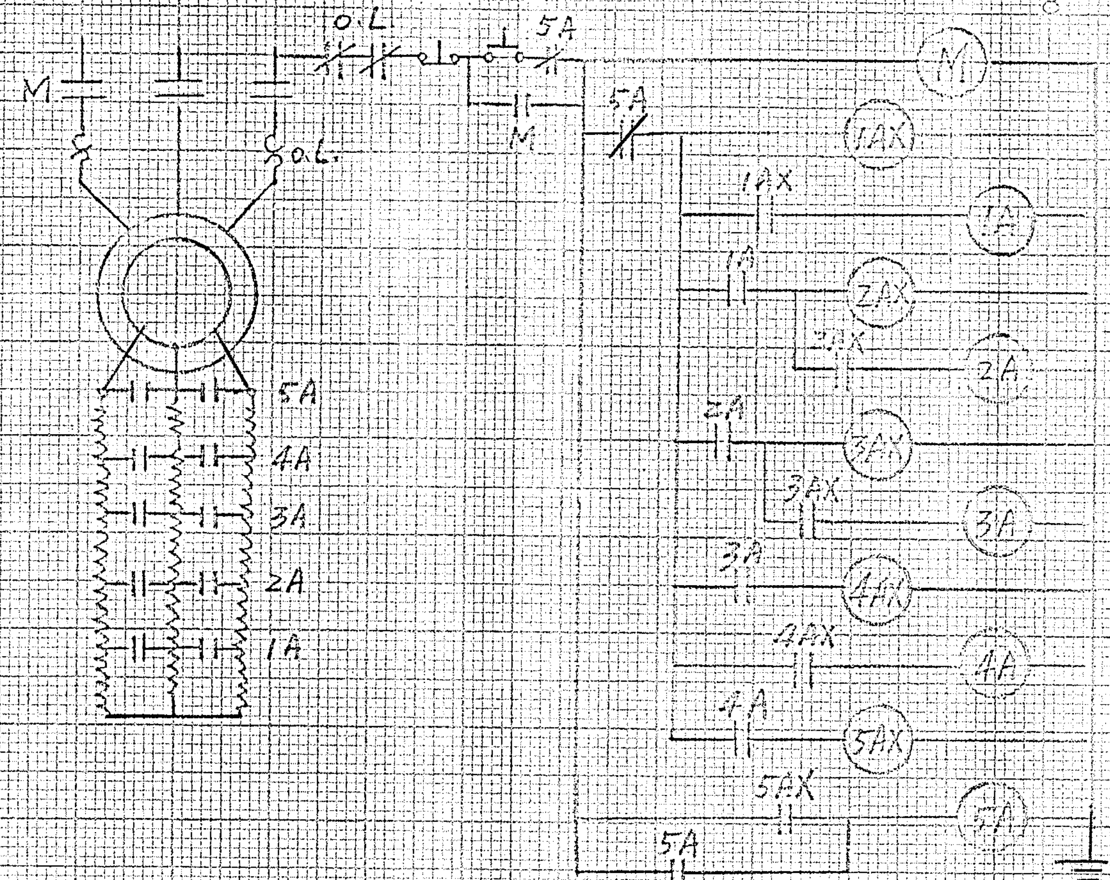


Fig. 5. Control circuit of 300 H.P. wound rotor motor

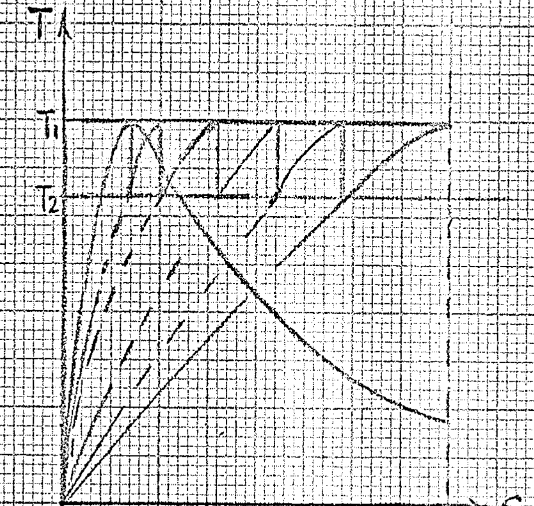


Fig. 6. Starting torque characteristic of 300 H.P. motor

CHAPTER II

THE Z PLANE AND THE CONSTANT TORQUE CIRCLE

I. Z PLANE

The locus of the impedance of any circuit, for instance the impedance of the equivalent circuit of an induction motor, can be represented graphically in a Z plane. Thus if an external parameter is connected to the rotor circuit of an induction motor the locus concerned can still be shown graphically in the Z plane.

The following three circuits are plotted graphically for purposes of illustration.

(1) R and X in series: The circuit and the locus of its impedance with variable R_e are shown in Fig. 7.

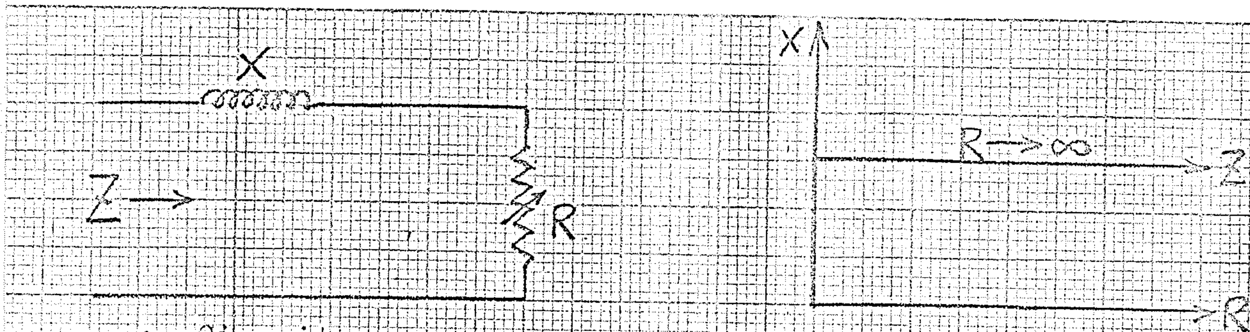
(2) R_e and X_e in parallel: The circuit and the locus of its impedance with variable R are shown in Fig. 8.

(3) R_e and X_e in parallel in series with r and x: The circuit and the locus of its impedance with resistances varied at same rate are shown in Fig. 9. Its locus is not straight.

The locus of the rotor circuit impedance of a wound rotor induction motor with external rotor elements connected in parallel is similar to that shown in Fig. 9 c.

II. CONSTANT TORQUE CIRCLE

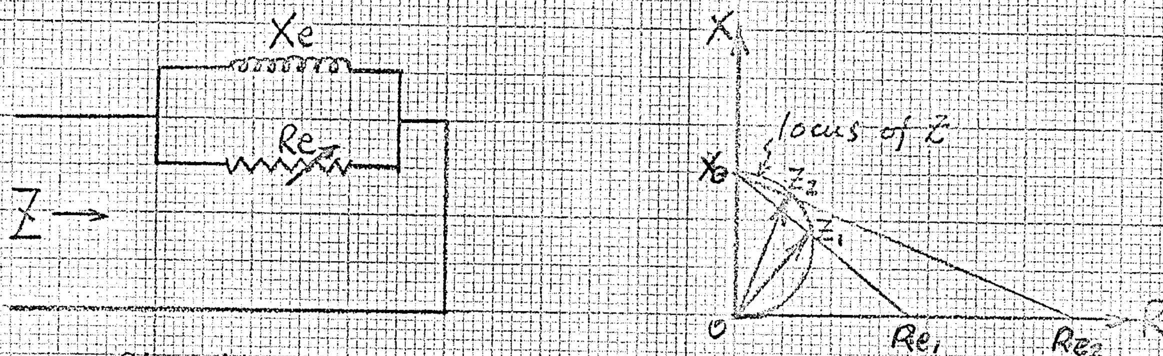
A family of performance circles for constant torque can be plotted in the Z plane.



a. Circuit

b. Locus

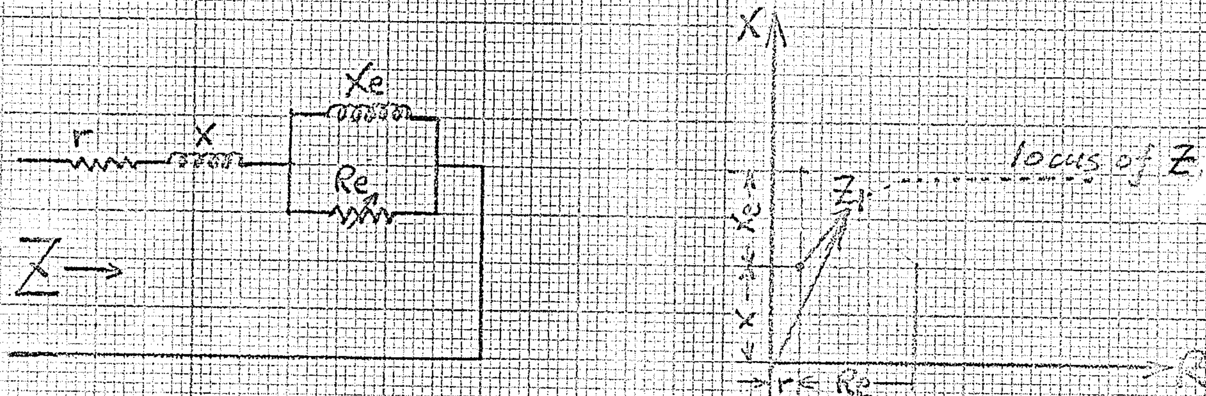
Fig. 7. R-X in series



a. Circuit

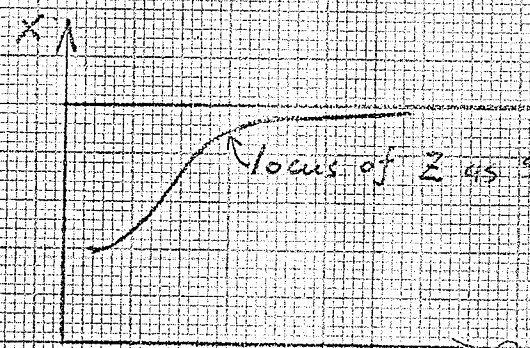
b. Locus

Fig. 8. R-X in parallel



a. Circuit

b. Z_1 in Z-plane



c. Locus of Z

Fig. 9. R-X in parallel and in series with r & x

As $T = \frac{1}{\omega_s} \frac{V^2}{(r_1 + r_2/S)^2 + (x_1+x_2)^2} \frac{r_2}{S}$

Let $R = r_1 + \frac{r_2}{S}$ and $X = x_1+x_2$

Therefore, $T = \frac{1}{\omega_s} \frac{V^2}{R^2 + X^2}(R - r_1)$

In per unit, the torque $T_{p.u.} = \frac{V^2}{R^2+X^2}(R - r_1)$

Thus $(R - \frac{V^2}{2T})^2 + X^2 = (\frac{V^2}{2T})^2 - \frac{V^2}{T} r_1$

This equation represents a family of constant torque circle with their center located at $R = \frac{V^2}{2T}$ and $X = 0$ and radii $K = \sqrt{(\frac{V^2}{2T})^2 - \frac{V^2}{T} r_1}$. Thus for a given applied voltage, a famil of constant torque circles can be constructed.

When rated voltage is applied ($V = 1$ p.u.), then

$(R - \frac{1}{2T})^2 + X^2 = (\frac{1}{2T})^2 - \frac{1}{T} r_1$

A family of constant torque circles under rated applied voltage is shown in Fig. 10.

For given motor constants r_1, x_1, r_2 and x_2 together with rated slip S_r, it is easy to locate the maximum torque available, the rated torque and the starting torque in the constant torque circle diagram.

The relevant torques are located as follows:

(1) Maximum torque: $R = X = x_1+x_2$

(2) Rated torque: $R = r_1 + r_2/S_r$ and $X = x_1+x_2$

(3) Starting torque: $R = r_1+r_2$ and $X = x_1+x_2$.

It is possible to regulate the starting torque even up to the maximum value available, by external means, along the horizont line determined by $X=x_1+x_2$ in the constant torque circle diagram.

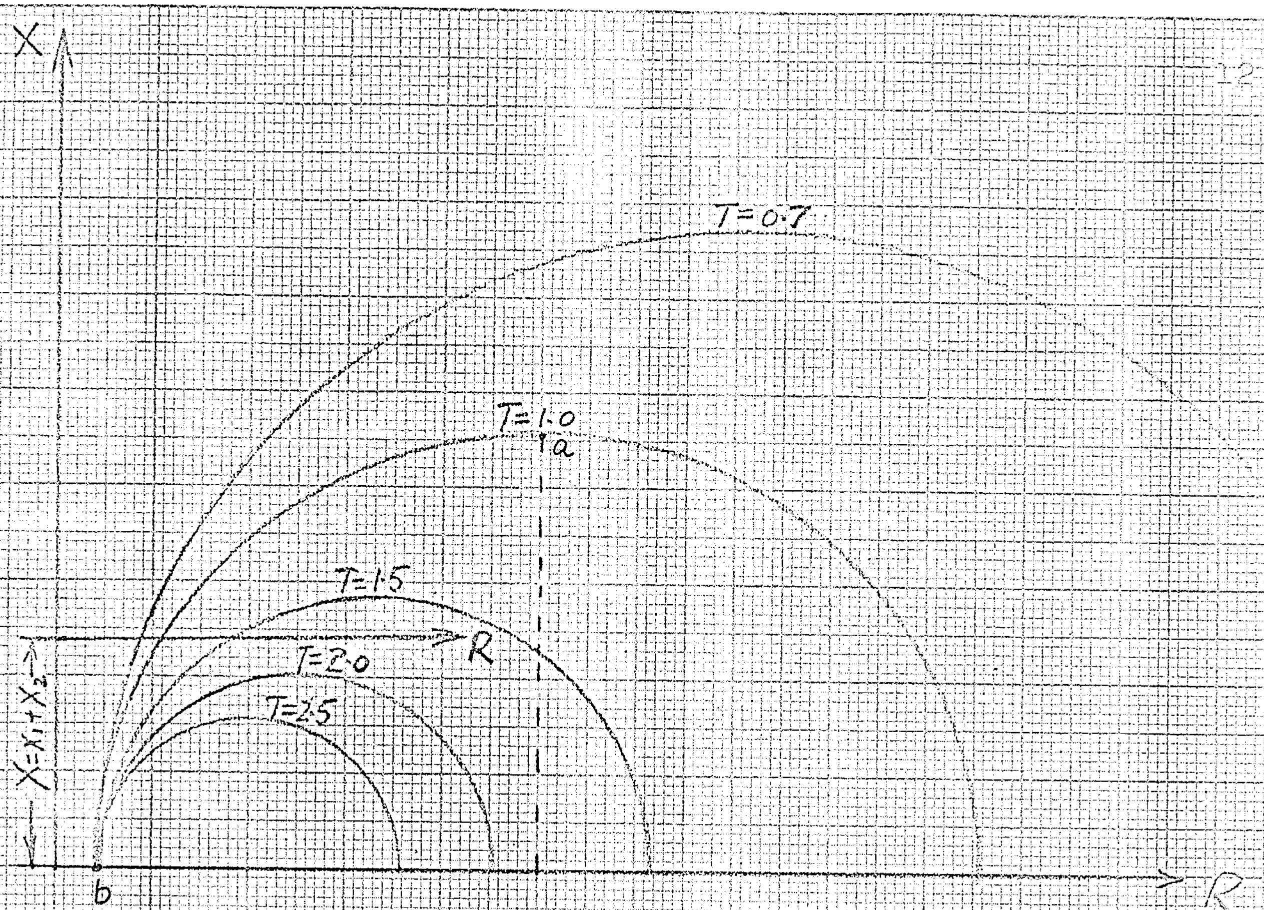


Fig. 10. Constant torque circles diagram

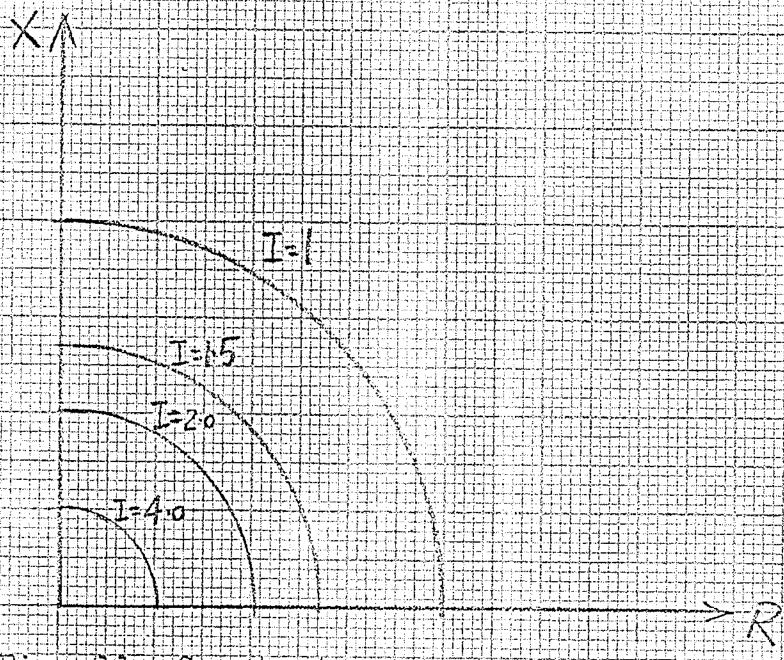


Fig. 11. Constant current circles diagram

III. CONSTANT CURRENT CIRCLE

A family of constant current circles can also be plotted in the Z plane.

$$I^2 = \frac{V^2}{(r_1+r_2/S)^2+(x_1+x_2)^2} = \frac{V^2}{R^2 + X^2} \text{ or } R^2+X^2=\left(\frac{V}{I}\right)^2$$

The above equation represents a family of concentric constant current circles with their common center located at R =0 and X =0, and their radii $K = \frac{V}{I}$ for a given applied voltage.

If V is the rated voltage, V= 1 p.u., and $R^2+X^2=\left(\frac{1}{I}\right)^2$

Fig. 11 indicates these concentric constant current circles

The current corresponding to a specified torque can be located on the constant current circle diagram. For example,

(1) Current at maximum torque: $R = X = x_1+x_2$

(2) Current at rated torque: $R= r_1+r_2/S_r$ and $X =x_1+x_2$

(3) Current at starting torque: $R = r_1+r_2$ and $X=x_1+x_2$

It is convenient to show both the constant torque and current circles in one diagram, in which the evaluation of the different performances is readily available. Moreover the starting characteristic and the calculation of the proper parameters can be studied collectively.

IV. ANALYSIS OF ROTOR CIRCUIT WITH EXTERNAL SERIES PARAMETERS

One way to acheive the starting purpose is to connect the external resistance and reactance in series with the rotor as shown in Fig. 12. Fig. 13 is its equivalent circuit.

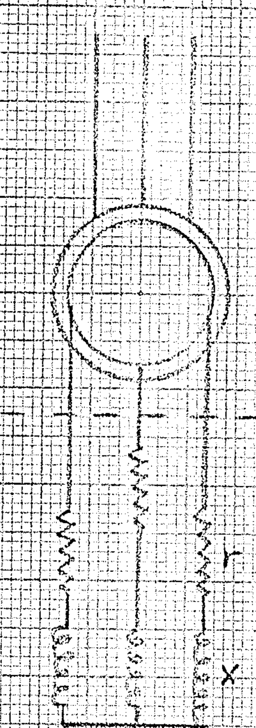


Fig. 12. Rotor circuit with external series parameters

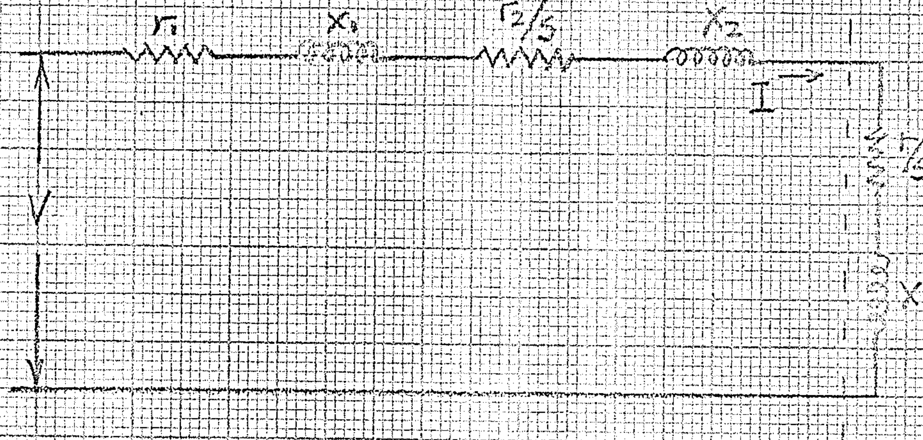


Fig. 13. Equivalent circuit of Fig. 12

The circuit impedance $Z=(r_1 + \frac{r_2+r}{S})+j(x_1+x_2)= R+jX$

The locus of the circuit impedance in the Z plane with constant torque and current circles is shown in Fig. 14. From that diagram the starting characteristic can be determined.

As the impedance locus of this specified rotor circuit is a straight line, it is impossible to obtain a constant torque during the starting period.

V. ANALYSIS OF ROTOR CIRCUIT WITH EXTERNAL SERIES AND PARALLEL PARAMETERS

The combination of external parameters in series and parallel connected in the rotor circuit is shown in Fig. 15a. Fig. 15b is its equivalent circuit. Any parallel circuit may be converted into a corresponding series circuit, so that the equivalent circuit of this specified rotor impedance is shown in Fig. 15c. The total equivalent resistance and reactance are as follows:

$$R=r_1+\frac{r_2+r}{S}+\frac{\frac{Re}{S}Xe^2}{(\frac{Re}{S})^2+Xe^2} \quad \text{and} \quad X=x_1+x_2+\frac{(\frac{Re}{S})^2Xe}{(\frac{Re}{S})^2+Xe^2}$$

Consequently, the locus of this specified circuit impedance is a curve as indicated in Fig. 9. The shape of the locus of Z can be made to approximate a desired circular shape by adjustment of the parameters r, x, Re and Xe. As the locus of this specified rotor impedance approaches approximately the constant torque circle, approximately constant starting torque will be achieved.

A graphical approach would seen to be the best method of attacking this problem. This approach is outlined in Chapter III.

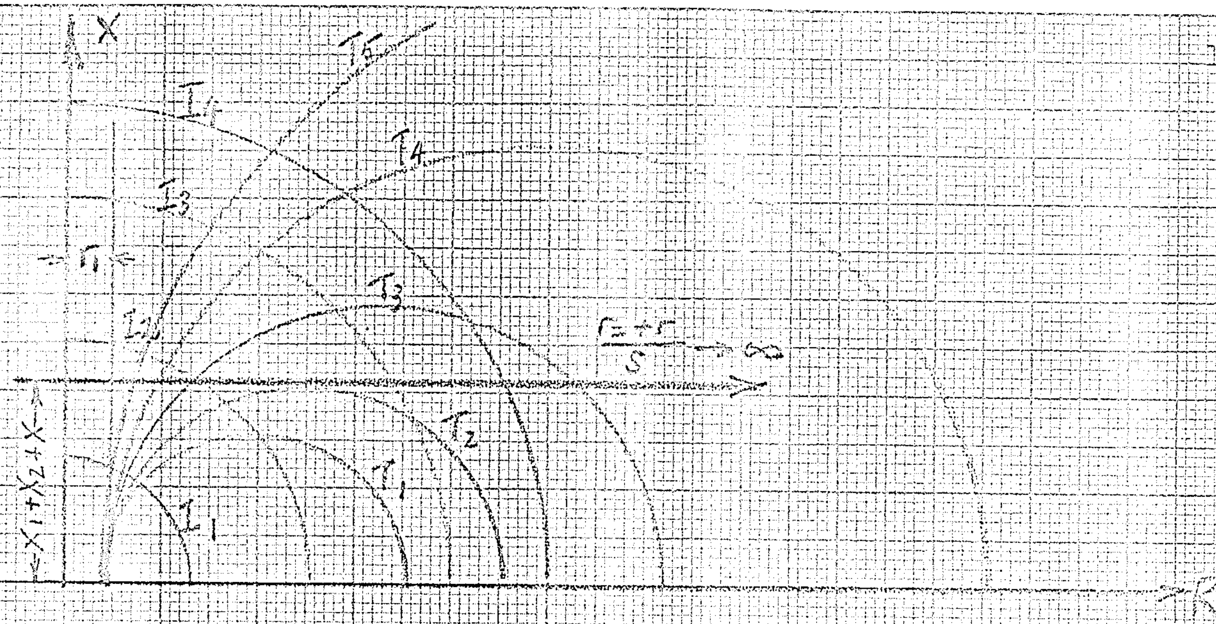
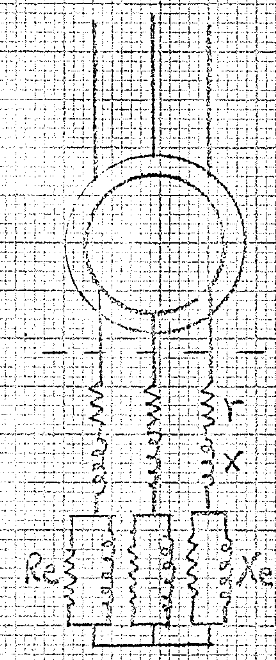


Fig. 14. Locus of circuit impedance together with constant torque and constant current circles in Z-plane



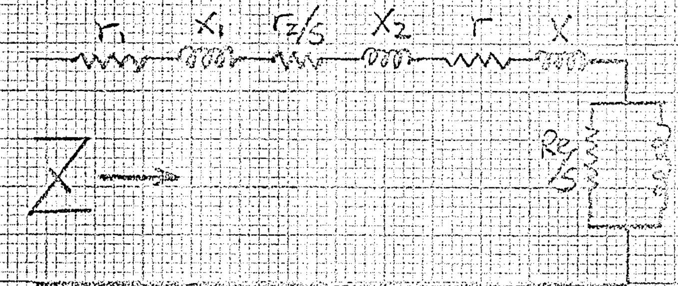


Fig. 15b. Equivalent circuit

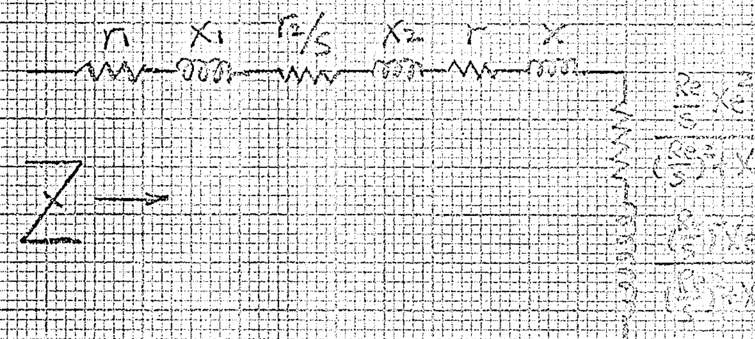


Fig. 15a. Rotor circuit with external series and parallel parameters

Fig. 15c. Equivalent circuit with equivalent series Resistance and reactance of parallel parameters

VI. SUMMARY

(1) The locus of the rotor circuit impedance with external parameters in series and parallel combination can be made to approximate a constant torque locus with judicious choice of the parameters.

(2) A graphical approach together with an iteration process is preferable to mathematical analysis.

CHAPTER III

DETERMINATION OF PARAMETERS BY GRAPHICAL APPROACH

I. GRAPHICAL APPROACH

As the starting period lies between S=1 and around S=0.1, the external parameters should be cut off at the end of the starting period, whereupon the motor will run on its natural characteristic curve. During the starting period, the starting torque is expected to be constant.

From the constant torque circle diagram, Fig. 10, the portion of the circle between "a" and "b" is of interest because the possible impedance locus of the rotor may lie along that portion "a" and "b" approximately. The point "a" is chosen to be the point of the end of starting period, S=0.1, where $R=X=\sqrt{\left(\frac{V^2}{2T}\right)^2-\frac{V^2}{T}r_1}$

The total equivalent resistance and reactance of the circuit at S=0.1 are as follows: $R=r_1+\frac{r_2+r}{0.1}+\frac{(Re/0.1)Xe^2}{(Re/0.1)^2+(Xe)^2}\approx 10(r_2+r)+\frac{10Xe^2}{Re}$ and $X=x_1+x_2+x+\frac{(Re/0.1)^2Xe}{(Re/0.1)^2+(Xe)^2}\approx x_1+x_2+x+Xe$

Therefore, $R=\sqrt{\left(\frac{V^2}{2T}\right)^2-\frac{V^2}{T}r_1}\approx 10(r_2+r)+\frac{10Xe^2}{Re}=X\approx x_1+x_2+x+Xe$.

At standstill, S=1, the total equivalent resistance and reactance are as follows: $R=r_1+r_2+r+\frac{ReXe^2}{Re^2+Xe^2}$ and $X=x_1+x_2+x+\frac{Re^2Xe}{Re^2+Xe^2}$

This specified point of the circuit impedance at S=1 should lie at the point with the torque value lower than the required constant value on the constant torque circle.

For the given motor constants and applied voltage together with the required starting torque, the external parameters, r, x, R_e and X_e, cannot be solved for by the equations listed previously.

As a result of the graphical approach, based on successive approximations and the previous equations, the optimum values in per unit of the external parameters in terms of the motor constants, applied voltage and the required starting torque are as follows:

(1) $r=(\frac{V^2}{2T})0.106 - r_2$

(2) $x=(\frac{V^2}{2T})0.592 - (x_1+x_2)$

(3) $Re=(\frac{V^2}{2T})0.145$

(4) $Xe=(\frac{V^2}{2T})0.363$ and the illustration of graphical approach is Fig.

An associated digital computer program used to verify successive graphical trials is decribed in Appendix I.

II. RESULTS FROM THE COMPUTER

The data put into the computer are as follows:

(1) $T=0.5$, $r_2+r=0.106$, $Re=0.145$, $x_1+x_2+x=0.592$, $Xe=0.363$

(2) $T=0.75$, $r_1+r=0.0707$, $Re=0.0967$, $x_1+x_2+x=0.395$, $Xe=0.242$

(3) $T=1.0$, $r_1+r=0.053$, $Re=0.0724$, $x_1+x_2+x=0.296$, $Xe=0.181$

(4) $T=1.25$, $r_1+r=0.0425$, $Re=0.058$, $x_1+x_2+x=0.237$, $Xe=0.145$

The computer output gives S, equivalent resistance R, equivalent reactance X, current I and torque T and is listed in Appendix II.

Fig. 20 to Fig. 23 are the characteristic curves of starting torque based on the computer output.

In order to check the sensitivity of the flatness of the torque-speed curve to changes in the values of the parameters used, these parameters have been allowed to vary from their optimum values and the resulting torque-speed curves are shown in Fig. 24 to Fig. 31.

III. EFFECT OF REACTOR RESIDUAL RESISTANCE

The resistance component r' in reactor should be taken into consideration. Fig. 17a shows Re, Xe, and r' and Fig 17b is an assumed equivalent circuit.

The circuit impedance $Z=\dfrac{1}{\dfrac{1}{Re/s}+\dfrac{1}{r'/s+jXe}}$

$$=\frac{\frac{Re}{s}Xe^2+jXe\frac{Re}{s}\left(\frac{Re+r'}{s}\right)}{\left(\frac{Re+r'}{s}\right)^2+Xe^2}+\frac{Re\,r'(Re+r')/s^3}{\left(\frac{Re+r'}{s}\right)^2+Xe^2}$$

If Xe=2.5Re and r'=0.1Xe(typical values), then Re=r' can be assumed to have Re in parallel with Xe in addition to a series component $\dfrac{Re\,r'(Re+r')/s^3}{\left(\frac{Re+r'}{s}\right)^2+Xe^2}$, which can be neglected when S is large and is approximately equal to Re in parallel with r' when S is small. Moreover, this component is merely a very small portion in the total equivalent resistance of the circuit.

IV. SUMMARY

The foregoing results, based upon the graphical approach and iteration process seen to be satisfactory. Consequently the following equations are available for the calculation of external parameters in obtaining approximately any constant starting torque desired with reasonable tolerance.

$r=\frac{V^2}{2T}-0.106-r_2$, $\qquad x=\frac{V^2}{2T}-0.592-(x_1+x_2)$

$Re=\frac{V^2}{2T}-0.145$, $\qquad Xe=\frac{V^2}{2T}-0.363$

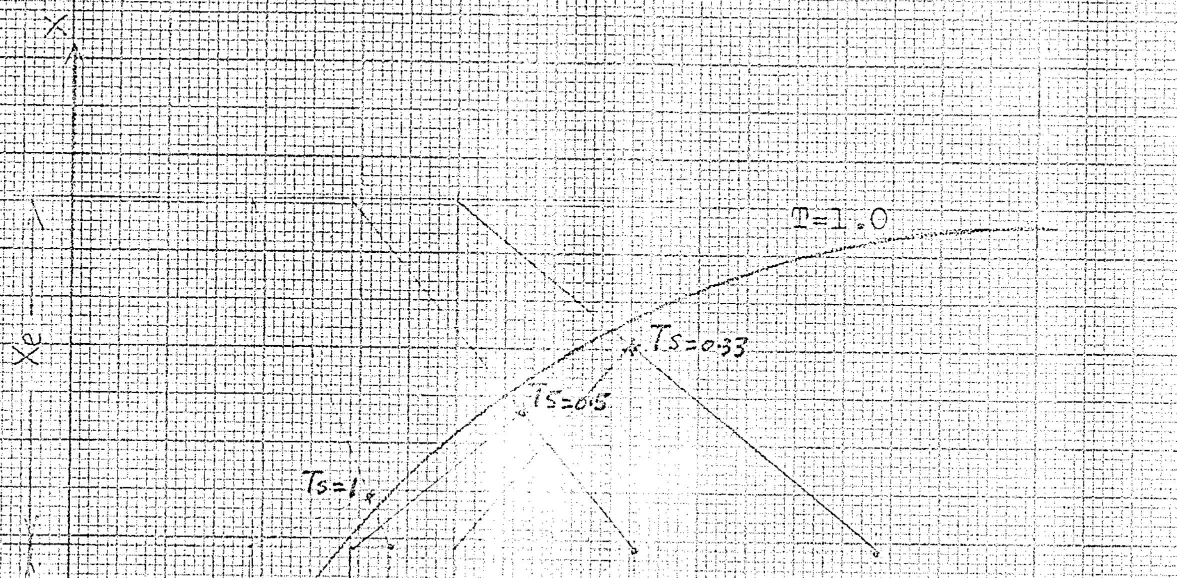
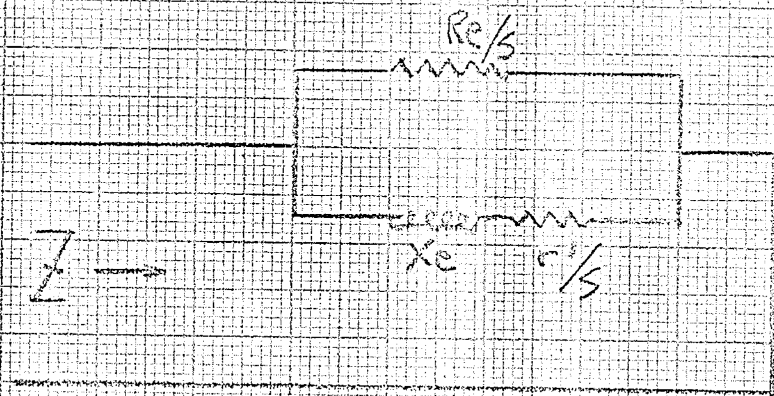
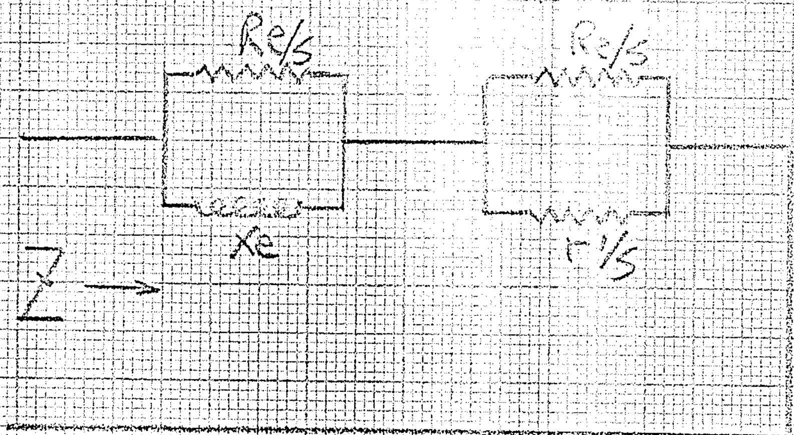


Fig. 16. Graphical Approach with $r_1=.04$, $r_2+r=.053$
$Re=.0724$, $x_1+x_2+x=.296$, $Xe=.181$



a. Equivalent circuit of Re in parallel with Xe and r' in series



b. Assumed equivalent circuit

Fig. 17. Re in parallel with Xe & r'

CHAPTER IV

STARTING PERFORMANCE OF WOUND ROTOR INDUCTION MOTOR

In starting a wound rotor induction motor, the important variables are

Torque,

Current,

Power Factor,

Power Output,

Copper Loss in Rotor.

The equations giving these variables in terms of motor constants and given conditions are as follows:

(1) Torque

$$T = \frac{\frac{V^2}{\omega_s}\left(\frac{r_2+r}{s} + \frac{\frac{Re}{s}\,Xe^2}{(\frac{Re}{s})^2+Xe^2}\right)}{\left(r_1 + \frac{r_2+r}{s} + \frac{\frac{Re}{s}\,Xe^2}{(\frac{Re}{s})^2+Xe^2}\right)^2 + \left(X_1+X_2+X+\frac{(\frac{Re}{s})^2 Xe}{(\frac{Re}{s})^2+Xe^2}\right)^2}$$

(2) Current

$$I = \frac{V}{\sqrt{\left(r_1 + \frac{r_2+r}{s} + \frac{\frac{Re}{s}\,Xe^2}{(\frac{Re}{s})^2+Xe^2}\right)^2 + \left(X_1+X_2+X+\frac{(\frac{Re}{s})^2 Xe}{(\frac{Re}{s})^2+Xe^2}\right)^2}}$$

(3) Power Factor

See the equivalent circuit of Fig. 3. Neglect g_c and r_1 since they are small compared to the other parameters. Then

$$Y = \frac{\left(\frac{r_2+r}{s} + \frac{\frac{Re}{s}\,Xe^2}{(\frac{Re}{s})^2+Xe^2}\right) - j\left(X_1+X_2+X+\frac{(\frac{Re}{s})^2 Xe}{(\frac{Re}{s})^2+Xe^2}\right)}{\left(\frac{r_2+r}{s} + \frac{\frac{Re}{s}\,Xe^2}{(\frac{Re}{s})^2+Xe^2}\right)^2 + \left(X_1+X_2+X+\frac{(\frac{Re}{s})^2 Xe}{(\frac{Re}{s})^2+Xe^2}\right)^2} - j\,b_m.$$

$$\cos\theta = \frac{Re\,Y}{|Y|} = \frac{\frac{r_2+r}{s} + \frac{\frac{Re}{s}\,Xe^2}{(\frac{Re}{s})^2+Xe^2}}{\sqrt{\left(\frac{r_2+r}{s} + \frac{\frac{Re}{s}\,Xe^2}{(\frac{Re}{s})^2+Xe^2}\right)^2 + \Big[b_m\left(\frac{r_2+r}{s} + \frac{\frac{Re}{s}\,Xe^2}{(\frac{Re}{s})^2+Xe^2}\right)^2 + b_m\left(X_1+X_2+X+\frac{(\frac{Re}{s})^2 Xe}{(\frac{Re}{s})^2+Xe^2}\right)^2 + X_1+X_2+X+\frac{(\frac{Re}{s})^2 Xe}{(\frac{Re}{s})^2+Xe^2}}}$$

(4) Power Output (Internal mechanical power developed)

$$P = (1-S)\omega_s T = (1-s)\frac{V^2\left(\frac{r_2+r}{S} + \frac{(Re/s)X_e^2}{(Re/s)^2 + X_e^2}\right)}{\left(r_1 + \frac{r_2+r}{s} + \frac{Re/s\, X_e^2}{(Re/s)^2+X_e^2}\right)^2 + \left(X_1+X_2+X+\frac{(Re/s)^2 X_e}{(Re/s)^2+X_e^2}\right)^2}\ \text{Watts.}$$

(5) Copper Loss in Rotor

$$P_{l2} = \omega_s T S = \frac{V^2 S\left(\frac{r_2+r}{s} + \frac{Re/s\, X_e^2}{(Re/s)^2+X_e^2}\right)}{\left(r_1+\frac{r_2+r}{s}+\frac{Re/s\, X_e^2}{(Re/s)^2+X_e^2}\right)^2 + \left(X_1+X_2+X+\frac{(Re/s)^2 X_e}{(Re/s)^2+X_e^2}\right)^2}\ \text{Watts.}$$

CHAPTER V

CONTROL OF INDUCTION MOTOR

I. INTRODUCTION

With respect to the control of a motor, the simplest circuit with the least number of electrical components giving the proper function is preferred. This is not only because of the initial cost but also the cost of maintenance.

As discussed previously in Chapter I, the control circuit for resistance starting of the wound rotor is rather complicated and many electrical components are needed to meet the requirements of many steps of acceleration. That is one of the main disadvantages of using resistance starting for the wound rotor motor.

In connection with motor protection, short circuit protection, over-load protection, under voltage protection and single phasing or phase unbalance protection are the most important and quite commonly practised.

The accelerating contactors installed in the rotor for the purpose of removing the external elements after the starting period can be actuated by either a time relay with preset of specified time delay or a current relay with preset limit current value. After the external elements have been completely cut off, the motor will operate on its natural characteristic. In practise, the time relay is more popular than the current relay.

II. CONTROL CIRCUIT AND ITS COMPONENTS

(1) The circuit shown in Fig. 18 is recommended for definit time control. The electrical components needed for this circuit are listed as follows:

(a) Three pole knife switch,

(b) Line contactor with overload protection,

(c) Two pole contactor,

(d) Time relay,

(e) "Start" and "Stop" push buttons,

(f) Three fuses,

(g) External R and X combination elements for starting purposes.

Pressing the "Start" push button causes the coils of the line contactor and time relay with preset time to be energized. Subsequently the contacts of the line contactor connect the motor to the source, and it starts. The motor accelerates until the preset time is reached and the time relay actuates the cut-off comtactor and the external elements are shorted out. At that time the starting period is ended and the motor goes into the natural characteristic.

Pressing the "Stop" push-button causes the electrical source of the secondary control circuit to be disconnected, and the contacts of the line contactor to open. The circuit is automatically ready for the next starting.

The above circuit has short circuit protection by "fuses", overload protection incorporated in the line contactor and low voltage protection incorporated in the line contactor.

The setting of the time relay is the time required for the whole starting period, which can be calculated.

(2) The circuit illustrated in Fig. 19 is for limit current control instead of that for time control as shown in Fig. 18. In this circuit a current relay is installed in the rotor circuit. The control circuit is otherwise identical. The current relay with preset current value actuates the contactor at the end of the starting period. The operation of and protection in this circuit are similar to that of the definite time control circuit.

The setting value of the current relay is easily determined from the constant current circle diagram or by use of the appropriate equations. A current transformer may be necessary.

In the secondary control circuit of Fig. 19, the contact of the current relay in series with the coil of the accelerating contactor should be opened before the contact of the lime contactor closes in order to have the proper function. The control equipment can usually be ordered with this requirement. Otherwise, the control circuit has to be modified to have the contact of the accelerating contactor in the rotor side remain open during the starting period.

III. SUMMARY

(1) The foregoing circuits illustrated may be adopted for application. The design of each circuit for each particular motor should be compatible with the nature of the available electrical components.

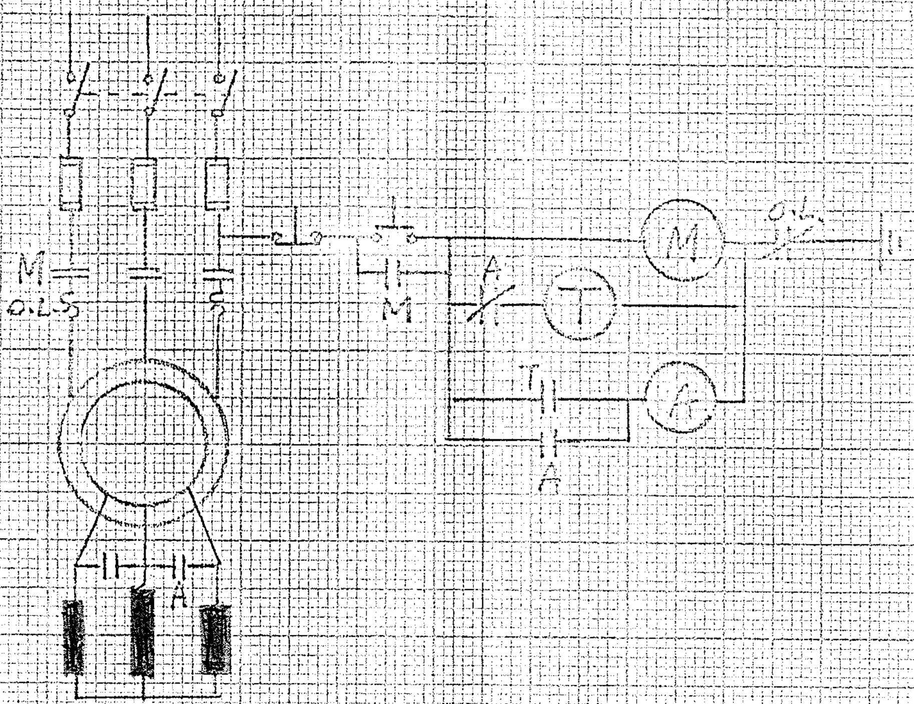


Fig. 18. Control circuit for definite time control

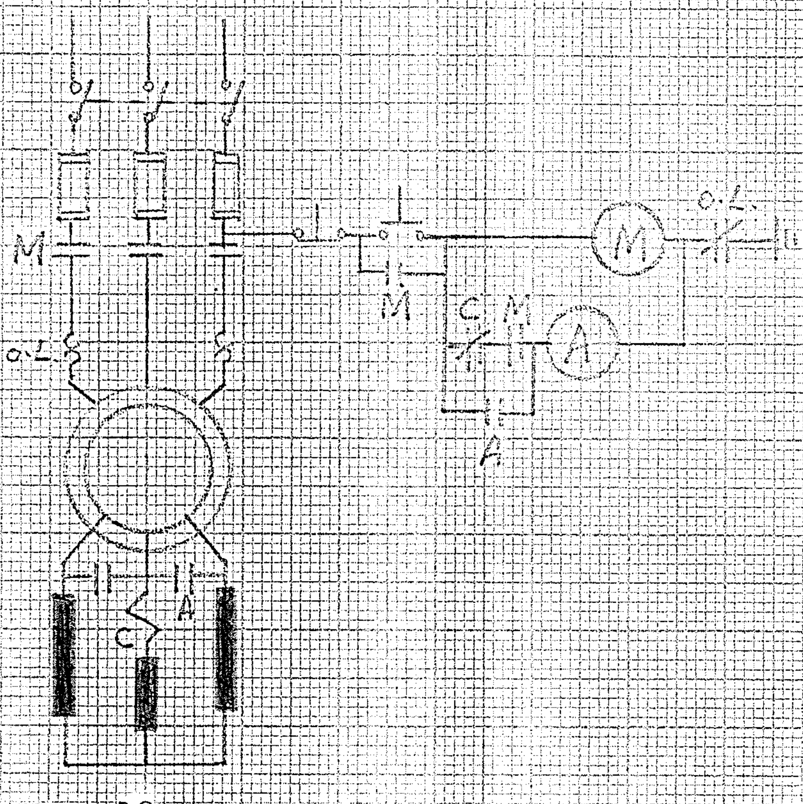


Fig. 19. Control circuit for limit current control

(2) The foregoing circuit diagrams illustrate the advantages of the method herein presented, namely, simplicity of control circuit and equipment, low initial cost and convenience in maintenance.

(3) If manual operation is desired, a manual one step switch is used for the purpose of removing the external elements. This simple switch is still much superior to the drum controller with multiple steps for resistance starting. Moreover, the characteristic resulting from the manual operation of a drum controller is difficult to obtain satisfactorily.

CHAPTER VI

CONCLUSION

(1) Using a fixed element with an R-X combination in series and in parallel installed in the rotor circuit, it is possible to achieve an approximately constant starting torque.

(2) The equations listed in Chapter III are suitable for determining the value of external parameters under any applied voltage and any desired starting torque for any wound rotor induction motor.

From the computer output, a slight variation in the specified values of the external parametere might still offer a quite satisfactory starting characteristic, well within practical engineering tolerances.

(3) As to manufacturing, there is no difficulty with these parameters. Moreover, using these parameters with an R-X combination results in a much smaller volume than resistances alone.

(4) The control circuit and equipment for this proposed starting method is simpler than that for stepped resistance starting.

APPENDIX I

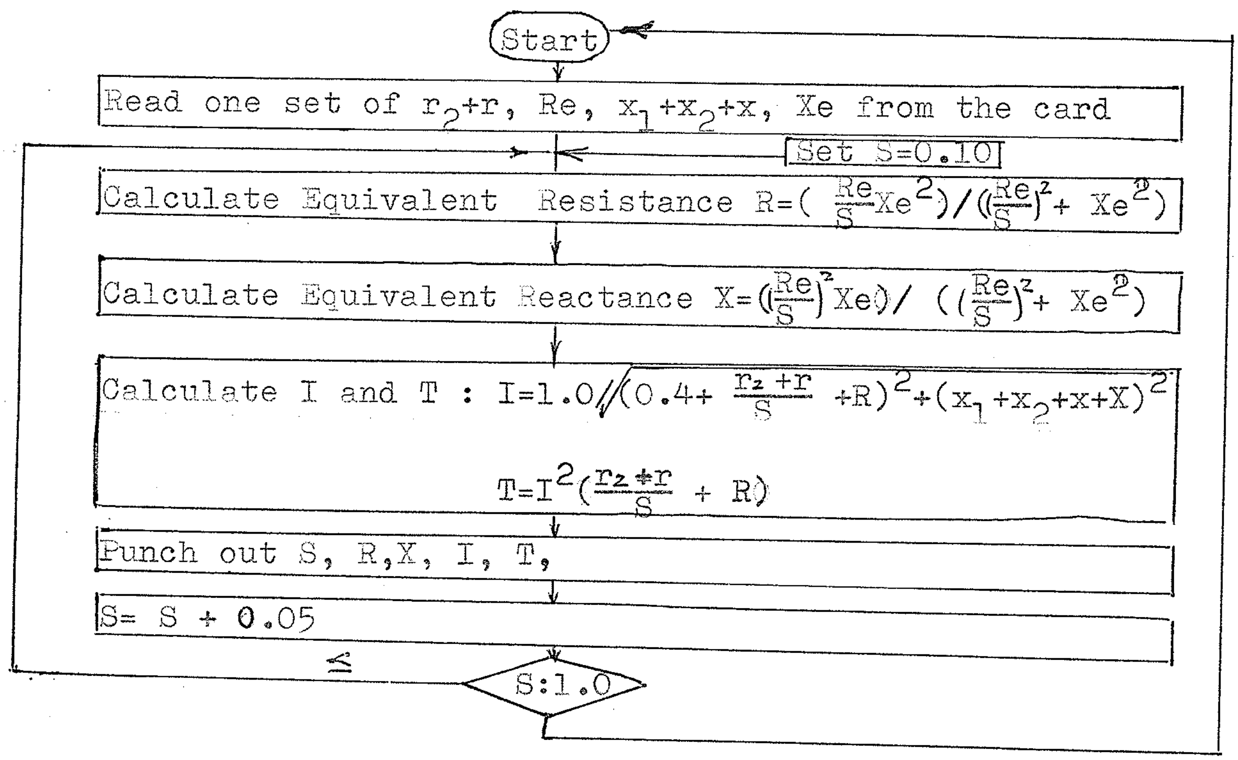
DIGITAL COMPUTER PROGRAM

(1) The external parameters, r,x,Reand Xe,are determined by the equations developed with the assumption of rated voltage being applied, V=1 p.u., and r_1 being 0.04 p.u.

(2) The unknowns of interest to be found are I,T and the equivalent resistance and reactance of the external parallel R-X combination.

(3) S is assumed to be varied from S=1 to S=0.1 with increments of 0.05 representing the starting period concerned.

(4) The flow diagram is listed as follows:



(5) Fortran statement:

```
 1 FORMAT(4F6.4)
 2 FORMAT(5XF6.3,5XF6.3,5XF6.3,5XF6.3,5XF6.3)
 5 READ 1,R2R,RE,X123,XE
   S=0.10
10 R=((RE/S)*XE**2.0)/((RE/S)**2.0+XE**2.0)
   X=((RE/S)**2.0*XE)/((RE/S)**2.0+XE**2.0)
   A=0.04+R2R/S+R
   B=X123+X
   CUR=1.0/(SQRT(A**2.0+B**2.0))
   T=CUR**2.0*(R2R/S+R)
   PUNCH 2,S,R,X,CUR,T
   S=S+0.05
   IF(S-1.0)10,10,5
   END
```

APPENDIX II

COMPUTER OUTPUT AND TORQUE CHARACTERISTIC CURVES

(S)	(R)	(X)	(CUR)	(T)
T=0.5				
1.000	.125	.049	1.435	.475
.950	.129	.054	1.418	.485
.900	.134	.059	1.399	.494
.850	.139	.065	1.379	.503
.800	.145	.072	1.357	.511
.750	.150	.080	1.333	.519
.700	.156	.089	1.307	.526
.650	.161	.099	1.278	.531
.600	.167	.111	1.247	.535
.550	.172	.125	1.213	.538
.500	.177	.141	1.176	.538
.450	.180	.159	1.137	.537
.400	.181	.181	1.094	.534
.350	.179	.205	1.048	.531
.300	.174	.232	.999	.526
.250	.163	.260	.944	.523
.200	.145	.290	.880	.523
.150	.119	.318	.795	.523
.100	.085	.341	.662	.503
T=0.75				
1.000	.083	.033	2.126	.696
.950	.086	.036	2.101	.710
.900	.089	.039	2.073	.723
.850	.093	.043	2.043	.736
.800	.096	.048	2.011	.748
.750	.100	.053	1.975	.759
.700	.104	.059	1.936	.769
.650	.107	.066	1.893	.777
.600	.111	.074	1.847	.783
.550	.115	.083	1.797	.787
.500	.118	.094	1.743	.788
.450	.120	.106	1.684	.786
.400	.120	.120	1.621	.783
.350	.119	.136	1.554	.777
.300	.116	.154	1.481	.771
.250	.108	.173	1.400	.767
.200	.096	.193	1.305	.767
.150	.079	.212	1.180	.767
.100	.056	.227	.983	.738

(S)	(R)	(X)	(CUR)	(T)

T=1.0

1.000	.062	.024	2.804	.907
.950	.064	.027	2.770	.925
.900	.067	.029	2.734	.942
.850	.069	.032	2.694	.959
.800	.072	.036	2.651	.974
.750	.075	.040	2.603	.988
.700	.077	.044	2.552	1.001
.650	.080	.049	2.496	1.011
.600	.083	.055	2.435	1.019
.550	.086	.062	2.369	1.024
.500	.088	.070	2.298	1.026
.450	.089	.079	2.221	1.024
.400	.090	.090	2.139	1.020
.350	.089	.102	2.050	1.013
.300	.086	.115	1.954	1.006
.250	.081	.130	1.848	1.002
.200	.072	.144	1.723	1.001
.150	.059	.158	1.558	1.002
.100	.042	.170	1.298	.965

T=1.25

1.000	.049	.020	3.458	1.106
.950	.051	.021	3.416	1.127
.900	.053	.023	3.371	1.148
.850	.055	.026	3.322	1.168
.800	.057	.028	3.268	1.187
.750	.060	.032	3.210	1.204
.700	.062	.035	3.146	1.219
.650	.064	.039	3.077	1.232
.600	.066	.044	3.002	1.242
.550	.068	.050	2.921	1.248
.500	.070	.056	2.834	1.250
.450	.071	.063	2.739	1.249
.400	.072	.072	2.638	1.244
.350	.071	.082	2.529	1.236
.300	.069	.092	2.411	1.228
.250	.065	.104	2.281	1.223
.200	.057	.115	2.127	1.223
.150	.047	.127	1.923	1.224
.100	.034	.136	1.604	1.181

(S)	(R)	(X)	(CUR)	(T)

T=0.5 (R=110 PERCENT)

(S)	(R)	(X)	(CUR)	(T)
1.000	.133	.058	1.403	.492
.950	.138	.063	1.385	.501
.900	.143	.069	1.366	.508
.850	.148	.076	1.345	.516
.800	.153	.084	1.322	.522
.750	.158	.092	1.297	.528
.700	.163	.102	1.270	.532
.650	.168	.113	1.241	.536
.600	.173	.126	1.210	.538
.550	.177	.141	1.176	.538
.500	.179	.158	1.140	.537
.450	.181	.177	1.102	.535
.400	.180	.198	1.061	.532
.350	.176	.222	1.017	.528
.300	.169	.247	.970	.525
.250	.156	.274	.917	.523
.200	.136	.300	.853	.523
.150	.110	.325	.766	.521
.100	.078	.345	.628	.492

T=0.5 (R=115 PERCENT)

(S)	(R)	(X)	(CUR)	(T)
1.000	.137	.063	1.387	.500
.950	.142	.068	1.369	.507
.900	.147	.075	1.349	.514
.850	.151	.082	1.327	.520
.800	.156	.090	1.304	.526
.750	.161	.099	1.279	.531
.700	.166	.109	1.252	.534
.650	.171	.121	1.223	.537
.600	.175	.134	1.192	.538
.550	.178	.149	1.159	.538
.500	.180	.166	1.124	.536
.450	.181	.185	1.086	.534
.400	.179	.206	1.046	.530
.350	.174	.229	1.003	.527
.300	.166	.254	.956	.524
.250	.152	.280	.904	.523
.200	.132	.305	.840	.523
.150	.106	.328	.752	.520
.100	.075	.346	.612	.486

T=1.0 (R=105 PERCENT)

(S)	(R)	(X)	(CUR)	(T)
1.000	.064	.027	2.772	.923
.950	.066	.029	2.738	.940
.900	.069	.032	2.701	.955
.850	.071	.035	2.660	.970
.800	.074	.039	2.616	.984
.750	.077	.043	2.568	.997
.700	.079	.047	2.516	1.007
.650	.082	.053	2.460	1.016
.600	.085	.059	2.399	1.022
.550	.087	.066	2.333	1.025
.500	.089	.074	2.263	1.025
.450	.090	.084	2.187	1.022
.400	.090	.094	2.107	1.017
.350	.089	.106	2.020	1.010
.300	.085	.119	1.926	1.004
.250	.079	.133	1.821	1.001
.200	.070	.147	1.697	1.001
.150	.057	.160	1.530	1.001
.100	.040	.171	1.267	.956

T=1.0 (R=110 PERCENT)

(S)	(R)	(X)	(CUR)	(T)
1.000	.054	.018	2.859	.924
.950	.057	.020	2.827	.946
.900	.059	.022	2.792	.968
.850	.062	.024	2.753	.990
.800	.064	.027	2.710	1.011
.750	.067	.030	2.663	1.031
.700	.070	.034	2.611	1.050
.650	.073	.038	2.554	1.067
.600	.077	.043	2.490	1.082
.550	.080	.049	2.420	1.093
.500	.083	.056	2.343	1.100
.450	.086	.065	2.258	1.103
.400	.089	.075	2.165	1.101
.350	.090	.087	2.063	1.093
.300	.089	.100	1.950	1.081
.250	.086	.116	1.826	1.066
.200	.079	.133	1.681	1.048
.150	.067	.151	1.497	1.022
.100	.049	.166	1.225	.949

(S)	(R)	(X)	(CUR)	(T)
T=1.0 (R=115 PERCENT)				
1.000	.068	.031	2.710	.952
.950	.070	.034	2.674	.966
.900	.073	.037	2.635	.979
.850	.075	.040	2.593	.991
.800	.078	.044	2.547	1.002
.750	.080	.049	2.499	1.011
.700	.083	.054	2.446	1.018
.650	.085	.060	2.390	1.023
.600	.087	.066	2.330	1.026
.550	.089	.074	2.265	1.026
.500	.090	.082	2.197	1.024
.450	.090	.092	2.124	1.019
.400	.089	.103	2.046	1.013
.350	.087	.114	1.963	1.007
.300	.082	.126	1.872	1.003
.250	.075	.139	1.769	1.001
.200	.066	.152	1.644	1.003
.150	.053	.163	1.472	.997
.100	.037	.172	1.201	.935
T=1.0 (R=95 PERCENT)				
1.000	.060	.022	2.836	.887
.950	.062	.025	2.803	.906
.900	.064	.027	2.768	.925
.850	.067	.030	2.729	.943
.800	.070	.033	2.687	.960
.750	.073	.037	2.641	.976
.700	.075	.041	2.590	.991
.650	.078	.046	2.535	1.003
.600	.081	.051	2.474	1.013
.550	.084	.058	2.408	1.020
.500	.087	.066	2.336	1.024
.450	.089	.075	2.258	1.024
.400	.090	.086	2.174	1.021
.350	.090	.098	2.084	1.014
.300	.087	.111	1.986	1.007
.250	.083	.126	1.878	1.001
.200	.074	.141	1.753	1.000
.150	.061	.156	1.590	1.002
.100	.044	.169	1.335	.975

T=1.0 (X=105 PERCENT)

(S)	(R)	(X)	(CUR)	(T)
1.000	.063	.024	2.711	.854
.950	.065	.026	2.680	.872
.900	.068	.028	2.646	.889
.850	.070	.031	2.609	.907
.800	.073	.035	2.568	.923
.750	.076	.039	2.524	.938
.700	.079	.043	2.475	.952
.650	.082	.048	2.422	.964
.600	.085	.054	2.364	.973
.550	.088	.061	2.301	.980
.500	.091	.069	2.232	.983
.450	.093	.079	2.157	.983
.400	.094	.090	2.077	.980
.350	.094	.103	1.990	.974
.300	.092	.117	1.897	.967
.250	.086	.132	1.793	.961
.200	.077	.148	1.673	.960
.150	.064	.164	1.517	.961
.100	.046	.177	1.272	.933

T=1.0 (X=95 PERCENT)

(S)	(R)	(X)	(CUR)	(T)
1.000	.061	.025	2.910	.969
.950	.063	.028	2.874	.987
.900	.066	.030	2.834	1.003
.850	.068	.033	2.791	1.019
.800	.070	.037	2.745	1.033
.750	.073	.041	2.694	1.046
.700	.075	.045	2.640	1.057
.650	.078	.050	2.581	1.065
.600	.080	.056	2.517	1.072
.550	.083	.063	2.448	1.075
.500	.084	.071	2.374	1.075
.450	.085	.080	2.295	1.072
.400	.085	.090	2.210	1.066
.350	.084	.101	2.119	1.060
.300	.081	.114	2.020	1.053
.250	.075	.127	1.910	1.049
.200	.066	.140	1.779	1.050
.150	.054	.152	1.604	1.049
.100	.038	.162	1.327	1.002

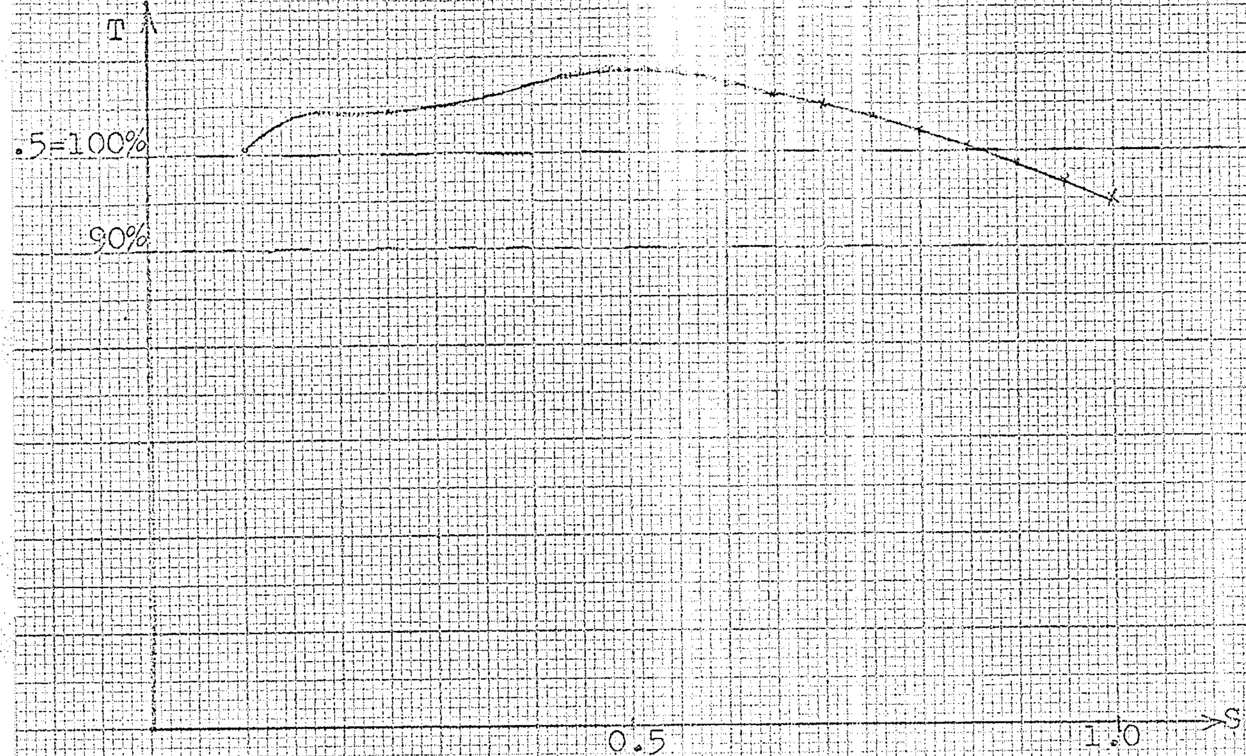


Fig. 20. Starting torque characteristic for T = 0.5

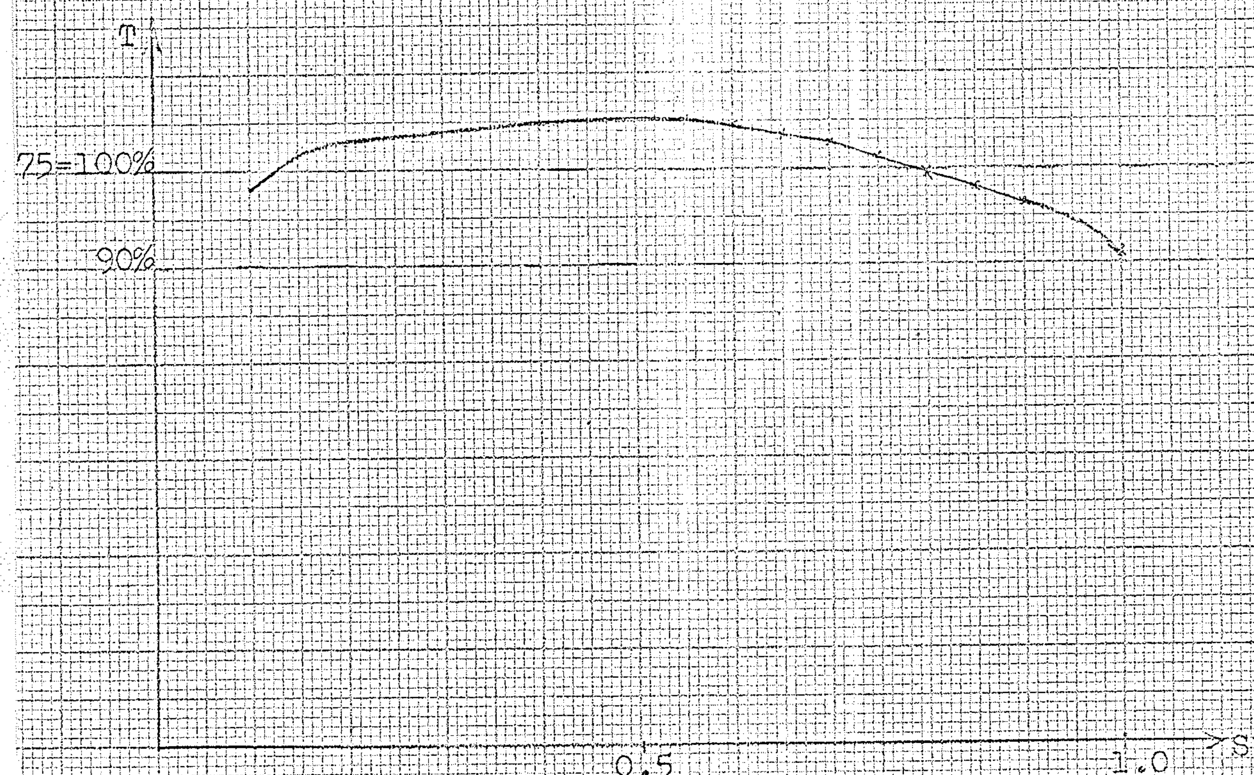


Fig. 21. Starting torque characteristic for T = 0.75

Fig. 22. Starting torque characteristic for T = 1.0

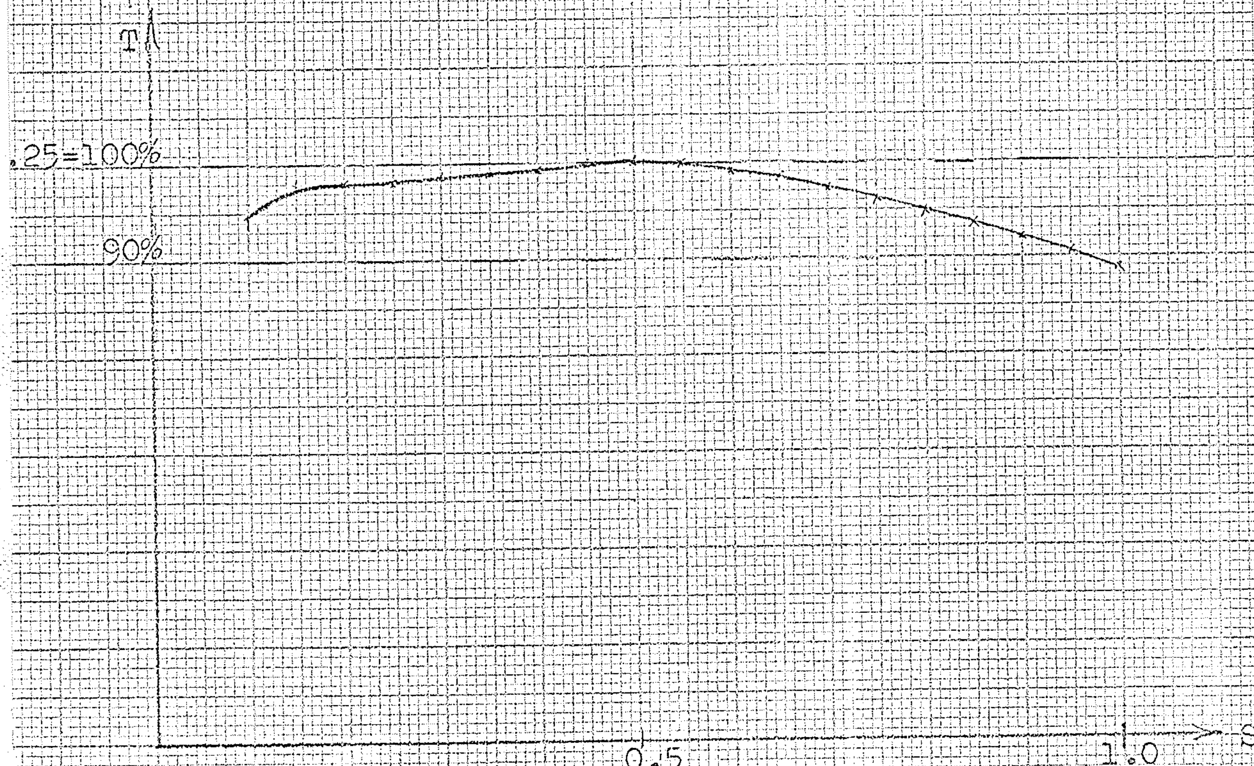


Fig. 23. Starting torque characteristic for T = 1.25

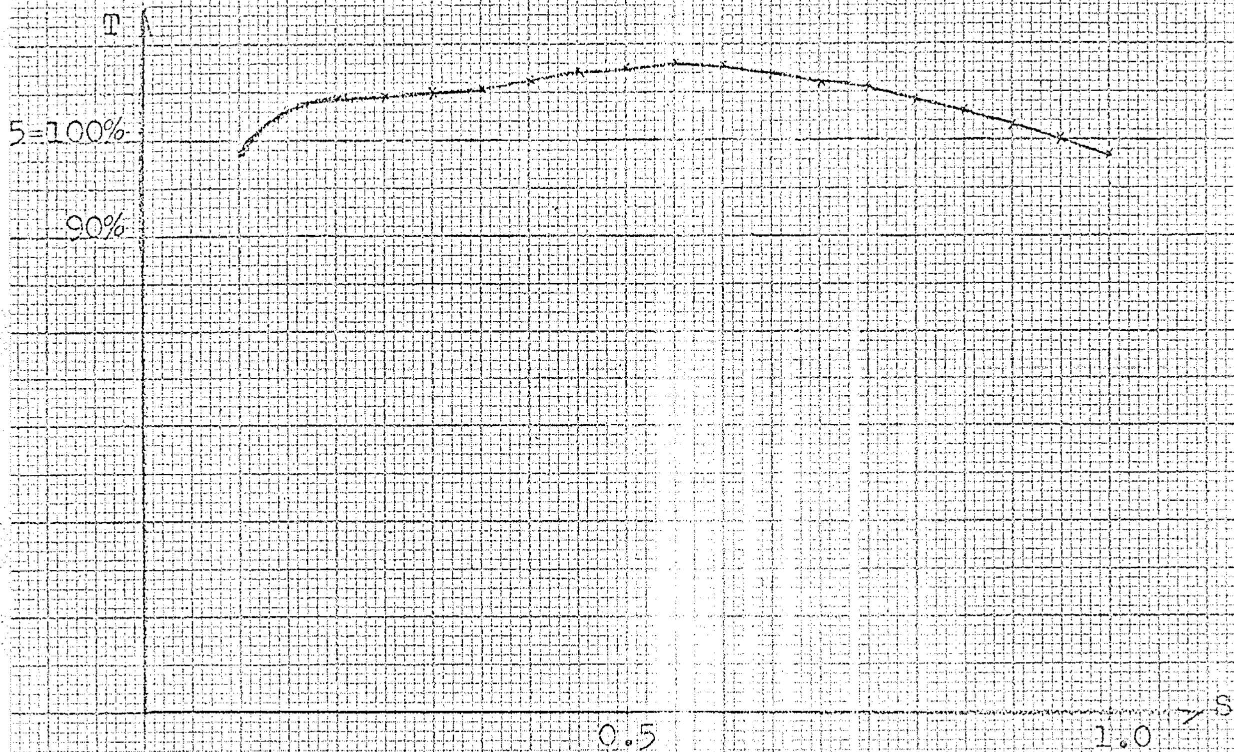


Fig. 24. Starting torque characteristic for T = 0.5 and with the values of resistance 10% greater than calculated values

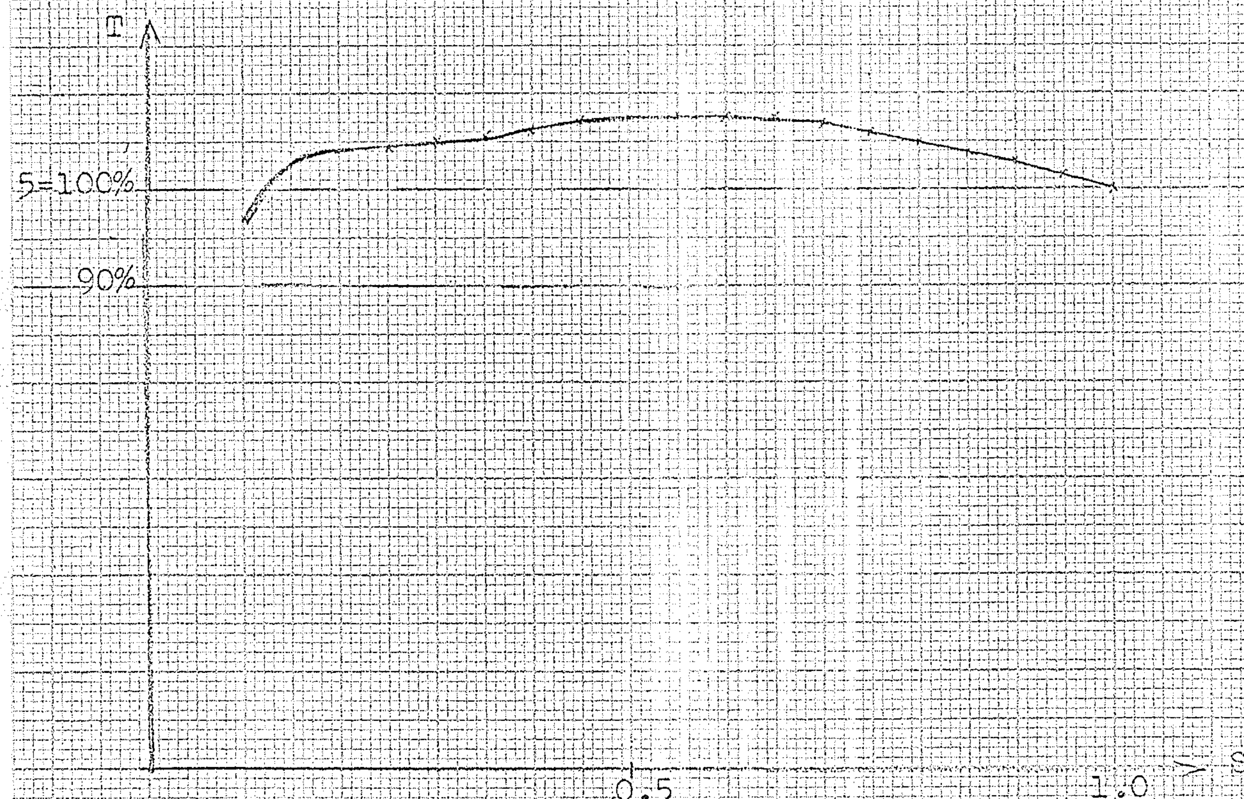


Fig. 25. Starting torque characteristic for T = 0.5 and with the values of resistance 15% greater than calculated values

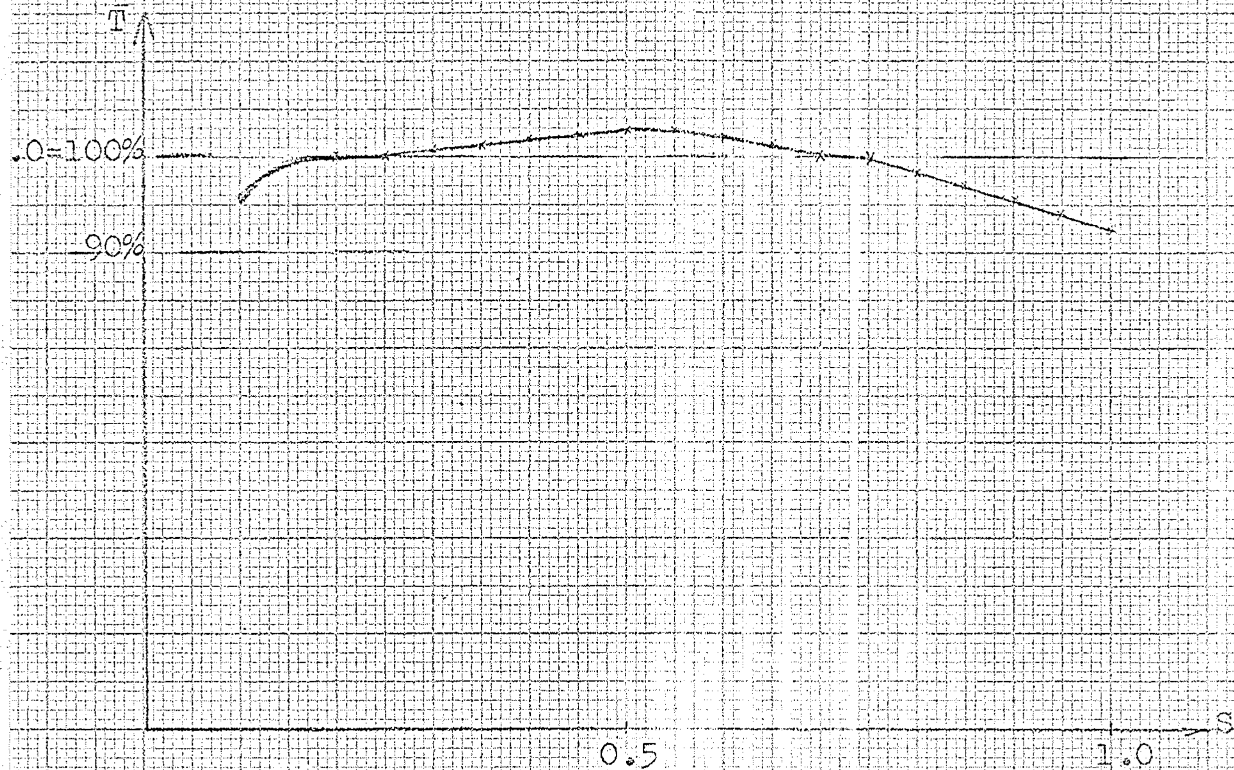


Fig. 26. Starting torque characteristic for T = 1.0 and with the values of resistance 5% greater than calculated values

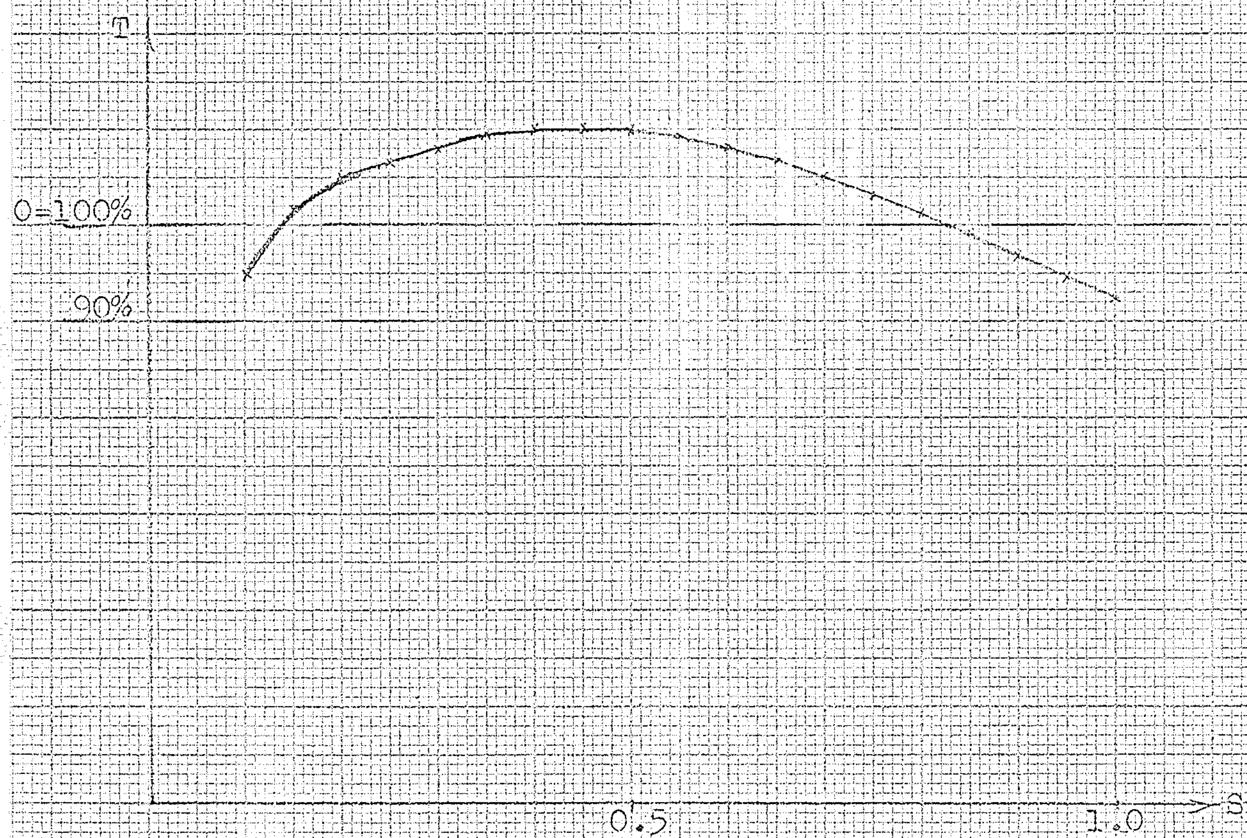


Fig. 27. Starting torque characteristic for T = 1.0 and with the values of resistance 10% greater than calculated values

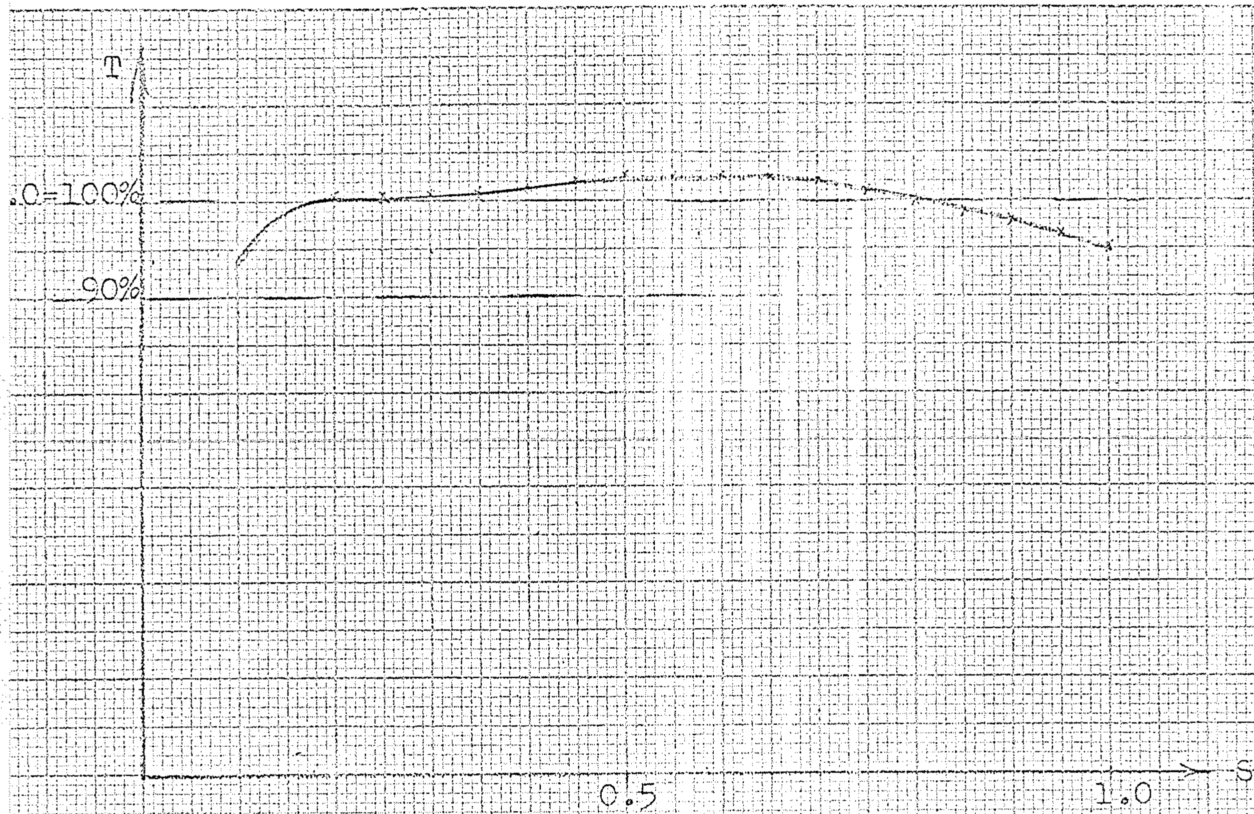


Fig. 28. Starting torque characteristic for T= 1.0 and with the values of resistance 15% greater than calculated values

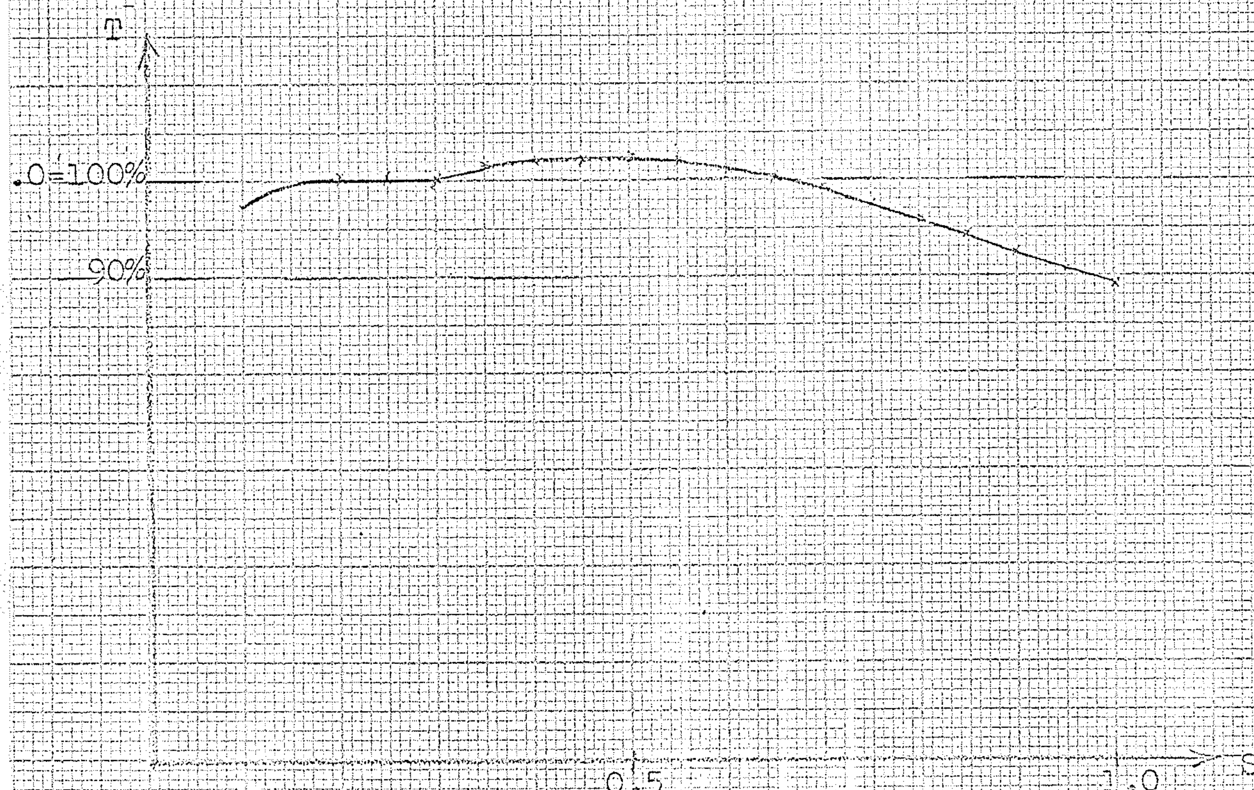


Fig. 29. Starting torque characteristic for T=1.0 and with the values of resistance 5% smaller than calculated values

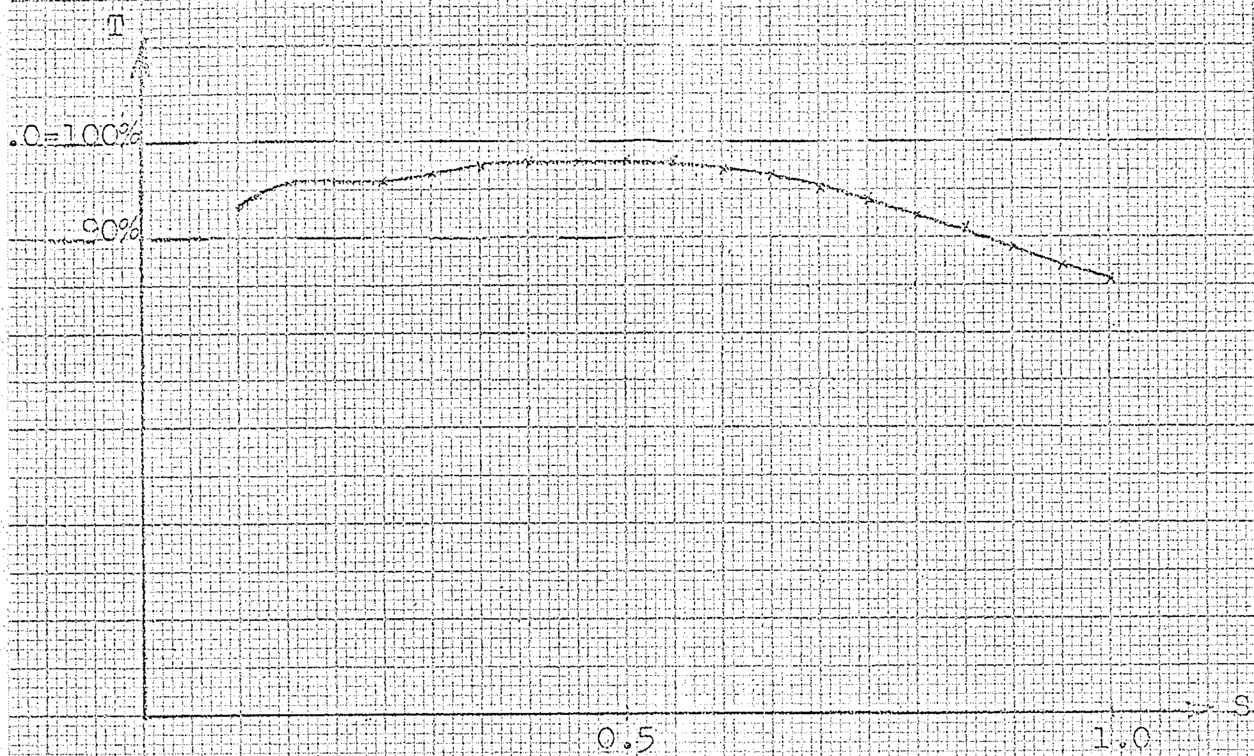


Fig. 30. Starting torque characteristic for T= 1.0 and with the values of reactance 5% greater than calculated values

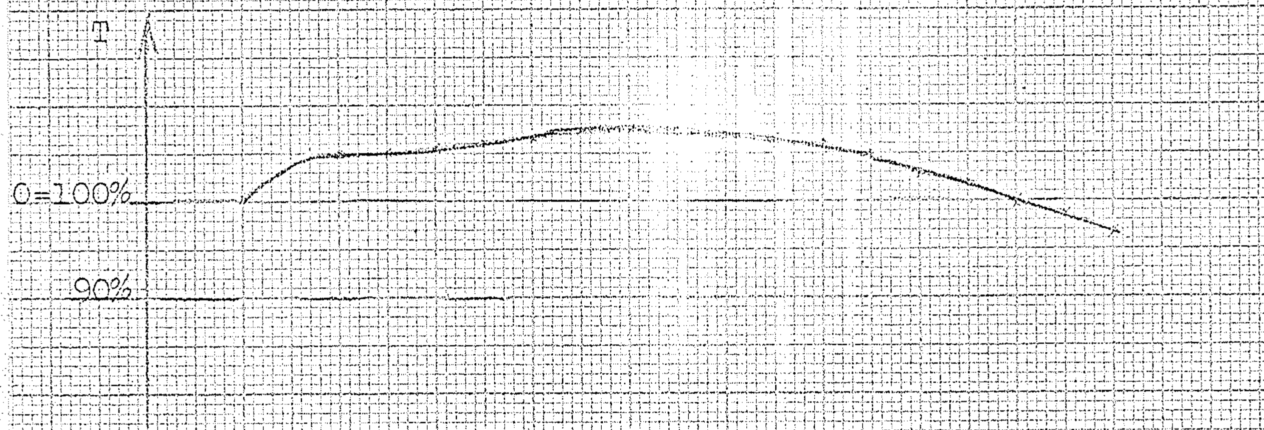


Fig. 31. Starting torque characteristic for T= 1.0 and with the values of reactance 5% smaller than calculated values

APPENDIX III

EXPERIMENTAL VERIFICATION

The motor (3 HP 1,145RPM wound rotor) used in the experiment was small in capacity and the elements available were not typical. However, the purpose of the experiment was fulfilled, namely, to observe the flatness of the torque produced during the starting period by using the proposed starting method.

The torque output was measured by a dynamometer and the average value was taken as reference (100% torque).

The values of external parameters, both calculated and actual, are listed as follows:

Calculated Value	Actual Value
r=0.118 ohms/phase	resistance in x & Xe
x=1.435 ohms/phase	Z=1.45 ohms/phase
Re=0.558 ohms/phase	Re=0.65 ohms/phase
Xe=1.400 ohms/phase	Z=1.45 ohms/phase

The torque output together with the slip is listed as follows:

RPM	S	Torque in %
1050	.125	102
990	.175	97
940	.217	97
900	.25	95
750	.375	95
600	.50	100
500	.585	100
400	.667	105
300	.75	105
200	.835	105

The above record is plotted in Fig. 32.

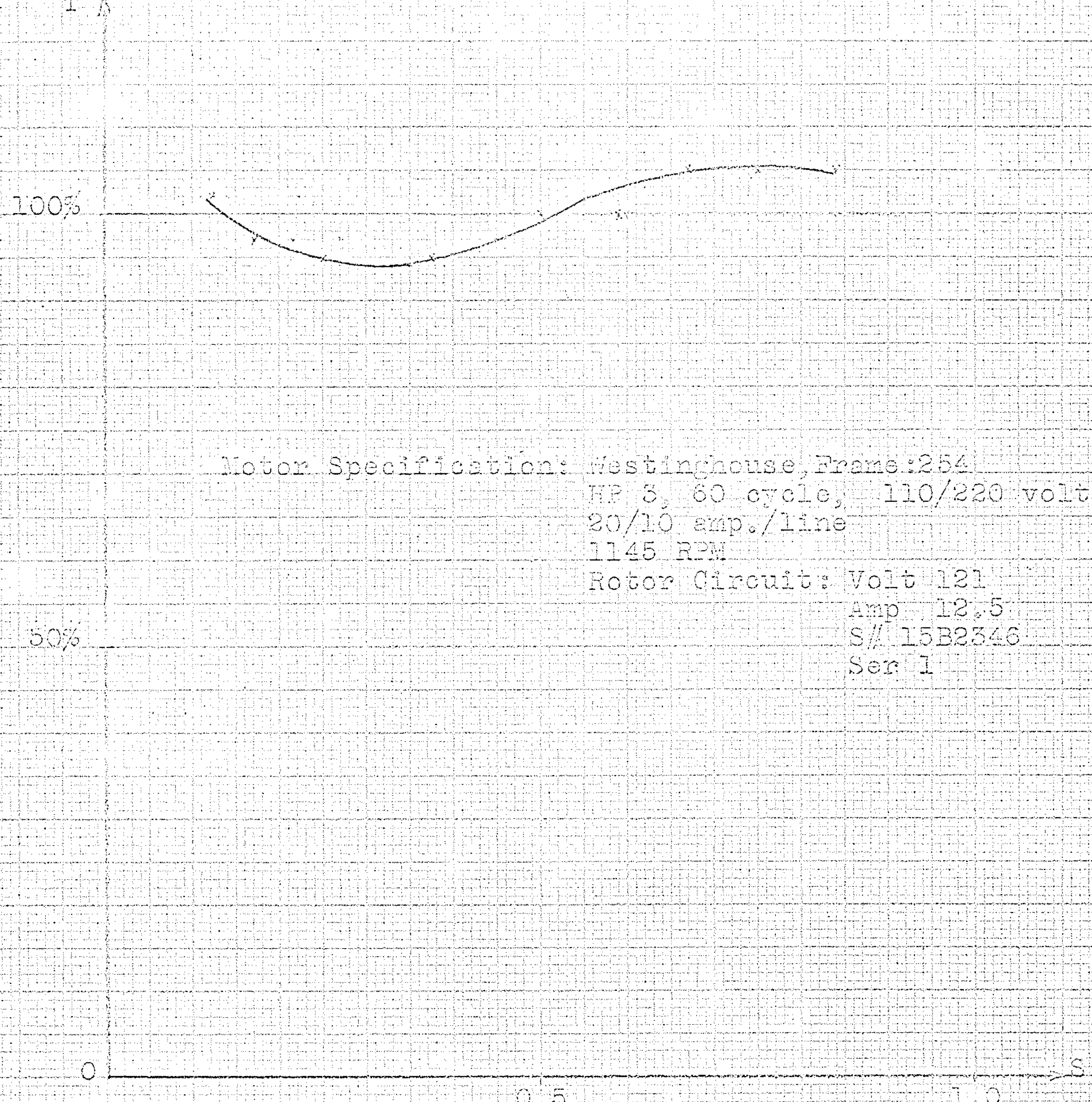


Fig. 32. Starting torque characteristic of experiment

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