THE UNIVERSITY OF MANITOBA

KINETIC ENERGY BALANCE IN A CONICAL DIFFUSER

by

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A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE

DEGREE OF MASTER OF SCIENCE

DEPARTMENT OF MECHANICAL ENGINEERING

WINNIPEG, MANITOBA

1980

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ABSTRACT

The present work evaluates the turbulent and mean kinetic energy balance for a diffuser flow. The diffuser under study had a total included angle of 8 with an area ratio of 4:1 and was fed by fully developed pipe flow. Measurements of mean and fluctuating velocity correlations were taken by Arora (1978) using Pitot-static tube and Hot-wire anemometer. His results are used, in the present study, to examine the energy balance for diffuser flow.

Locally, in the mean energy budget, the source term $[(\bar{U}/2)(\partial \overline{Q^2}/\partial x)]$ is of the same order as the pressure work term throughout the diffuser. The major part of the mean energy is used to increase the pressure. The remaining mean energy either produces turbulent kinetic energy or is directly dissipated. This process of mean energy conservation, however, is carried out by transport in mean and turbulent flow fields.

The general picture which emerges by integrating the turbulent energy terms over the cross-section area, reveals production and dissipation are of the same order. Similarly, mean flow convection and total transfer are of the same order. In the entry and intermediate regions (roughly from the exit plane to 44 centimeters upstream from the exit plane) the average turbulent kinetic energy is increasing, and mean flow convection is balanced by convective diffusion due to pressure effects together with the difference of the production and dissipation. How-

ever, in the exit part of the diffuser, mean flow convection is balanced by convective diffusion due to kinetic effects, and production balances the dissipation.

ACKNOWLEDGEMENT

The author would like to thank his advisor Professor R. S. Azad for the guidance, direction and helpful discussions during the course of this work. Sincere thanks are due to Dr. A. C. Trupp and Dr. R. M. Mathur for their comments and careful review of the thesis.

NOMENCLATURE

aa	Constant in power series representation of (u')
bb	Constant in power series representation of (v')
cc	Constant in power series representation of (w')
D	Pipe diameter (10.16 centimeter)
H_1	Correction coefficient for Reynolds stress $(\overline{u^2})$
H_2	Correction coefficient for Reynolds stress $(\overline{v^2})$
H_3	Correction coefficient for Reynolds stress $(\overline{uv})$
H_4	Correction coefficient for Reynolds stress $(\overline{w^2})$
i	Subscript, i = 1, 2, 3
j	Subscript, j = 1, 2, 3
k	Constant in hot-wire response equation (section 2.5)
M	Distance from the wall where $\overline{uv}_{max.}$ or $\overline{q^2}_{max}$ occur
P	Instantaneous static pressure, $P = \overline{P} + p'$
$\overline{P}$	Mean static pressure
p'	Fluctuating component of the pressure field
Pm	Measured and normalized mean static pressure (Fig. 5)
$\overline{q^2}$	Turbulent kinetic energy, $\overline{q^2} = \overline{u^2} + \overline{v^2} + \overline{w^2}$
$\overline{Q^2}$	Mean kinetic energy, $\overline{Q^2} = \overline{U^2} + \overline{V^2}$
R	Pipe radius (5.08 centimeter)
R_0	Local diffuser or pipe radius
R_1	Reynolds number· $R_1 = U_b R/\nu$
r	Radial distance from center line of the diffuser
s	Small distance from the wall

t	Time
U	Instantaneous axial velocity, $U = \bar{U} + u'$
$\bar{U}$	Mean axial velocity
U_0	Mean center line velocity
U_b	Pipe bulk average velocity (18.32 ms^{-1})
u'	Fluctuating velocity component in x-direction
u_i	Fluctuating velocity (i = 1, 2 or 3)
$u_i u_j$	correlation of the fluctuating velocities
$\bar{V}$	Instantaneous velocity in radial direction, $\bar{V} = V + v'$
V	Mean radial velocity
v'	Flucutating velocity component in r-direction
W	Instantaneous velocity in z-direction, $W = \bar{W} + w'$
$\bar{W}$	Mean circumferential velocity
w'	Fluctuating velocity component in circumferential direction
x	Axial distance from the entry of the diffuser
y	Radial distance from the wall
z	Circumferential direction
δ	Boundary layer thickness
ε	Turbulent energy dissipation rate per unit mass
ν	Kinematic viscosity of the air
ξ_1	Non-dimensionalised axial distance, $\xi_1 = x/R$
ξ_2	Non-dimensionalised radial distance, $\xi_2 = r/R$
ξ_3	Non-dimensionalised radial distance, $\xi_3 = y/R_0$
ρ	Density of the air
ϕ	Angle of the inclined hot-wire
-	Overbar: denotes time average

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Chapter I

INTRODUCTION

The main activity of science is observation,interpretation and prediction. For a fluid mechanics modeler, prediction is of the utmost importance. But for others it may be one of the means by which the correctness of the interpretation can be checked by comparision with further observations. There is still no hope of fully predicting turbulent energy phenomenon, numerically, by rigorous deductions from Navier-Stokes equation. Therefore a crucial part of prediction remains unsolved, and experimental work remains a necessity.

The turbulent flow of real fluids is of a dissipative nature. Because of this dissipation of turbulent kinetic energy, a continuous supply of energy is necessary to maintain the turbulence, for steady flow. At the same time, owing to this turbulent motion, diffusion of fluid particles together with their kinetic energy takes place. Thus an average steady state can only exist, if there is equilibrium between the energy supplied to the turbulent motion, the transport by the mean motion, and the diffusion plus dissipation of turbulence energy.

A diffuser as a pressure recovery device, is an important tool which may throw some light on the turbulence research study. A diffuser converts kinetic energy into flow energy with a positive pressure gradient which makes it different from pipe flow which has a negative pressure gradient. The flow field may be divided into two parts i.e. turbulent flow field and mean flow field. The present work discusses the turbulent and mean kinetic energy balance for a diffuser flow.

The diffuser flow is under investigation at the University of Manitoba for the last one decade. The diffuser under study has a total included angle of 8° with an area ratio of 4:1 and was fed by fully developed pipe flow. Okwuobi (1972) studied turbulent kinetic energy balance for the diffuser flow at Reynolds numbers 152,000 and 293,000. Okwuobi and Azad (1973) and Hummel (1978) have reported the existence of Reynolds number similarity for turbulent quantities in a diffuser flow. Based on this finding, Arora (1978) choose a single pipe Reynolds number of 58,000. The present study investigate mean and turbulent kinetic energy balance and uses measurements taken by Arora.

Ruetenik and Corrsin (1955) have investigated energy budget in a fully developed, equilibrium plane diffuser flow with a total divergence angle of 2° at Reynolds number of 12,200. Cockrell and King (1967); Razinsky et al. (1967); Livesey and Turner (1964) and Robertson et al. (1957) are few other researchers who have studied diffuser flow.

To study the adverse pressure gradient type of flow some researchers [e.g. Bradshaw (1967), Shishov et al. (1978)] have examined two dimensional boundary layer flow. Their studies are used in this text to compare to the diffuser results. Similar studies has been made by other researchers for different types of flows e.g., in a boundary layer with zero longitudinal pressure by Townsend (1951): in a boundary layer by Bradshaw, Ferriss and Atwell (1967); in a jet by Rodi and Spalding

(1970); in a channel flow by Hanjalic and Launder (1972); in a wall jet by Irwin (1973); in a fully developed pipe flow with wall suction by Schildknecht, Miller and Meier (1979).

Velocity gradients were estimated by fitting polynomials (using least square techniques) to the measured data. The degree of polynomial was selected by considering regression coefficient (sum of the squares of difference between measured and computed value), and the behavior of first and second derivatives. Further details are provided in section 2.6.

Chapter II

BASIC CONSIDERATIONS

2.1 MEAN KINETIC ENERGY EQUATION

The mean energy equation for an axi-symmetry flow may be written as:

$$\mathrm{I} + \mathrm{II} + \mathrm{III} + \mathrm{IV} + \mathrm{V} + \mathrm{VI} + \mathrm{VII} = 0$$

where the different terms have the following meanings:

Mean energy source:

$$(\mathrm{I}) = -[(\bar{U}/2)\partial \overline{Q^2}/\partial x][R/U_b^3]; \qquad (2.1A)$$

Pressure work:

$$(\mathrm{II}) = -[(\bar{U}/\rho)\partial \bar{P}/\partial x][R/U^3]; \qquad (2.1B)$$

Mean energy advection:

$$(\mathrm{III}) = -[(\bar{V}/2)\partial \overline{Q^2}/\partial r][R/U_b^3]; \qquad (2.1C)$$

Transport by fluctuating velocities:

$$(IV) = -[\partial(\bar{U}\overline{u^2} + \bar{V}\overline{uv})/\partial x - \partial[r(\bar{U}\overline{uv} + V\overline{u^2})]/r\partial r][R/U_b^3]; \qquad (2.1D)$$

Turbulent energy production·

$$(V) = [\overline{u^2}\partial\bar{U}/\partial x + \overline{uv}\partial\bar{V}/ x + \overline{uv}\partial\bar{U}/\partial r + \overline{v^2}\partial V/\partial r + \bar{V}(\overline{w^2}/r)][R/U_b^3];$$

$$(2.1E)$$

Viscous Diffusion:

$$(VI) = ([\nu/2][\nabla^2\bar{U}^2 + \nabla^2\bar{V}^2])(R/U_b^3); \qquad (2.1F)$$

Direct dissipation:

$$(VII) = -\nu([(\partial\bar{U}/\partial x)^2 + (\partial\bar{V}/\partial x)^2 + (\partial\bar{U}/\partial r)^2 + (\partial\bar{V}/\partial r)^2 +$$

$$\bar{V}^2/r^2])(R/U_b^3); \qquad (2.1G)$$

2.2 TURBULENT KINETIC ENERGY EQUATION

The turbulent kinetic energy equation, which is obtained by combining the Navier-Stokes equations and the continuity equation (detailed derivation is given in Appendix A), for axi-symmetry flow may be written as:

$$I + II + III + IV + V = 0$$

where the different terms have the following meanings:

Mean flow convection:

$$(\text{I}) = -[(\bar{U}/2)\partial \overline{q^2}/\partial x + (\bar{V}/2)\partial \overline{q^2}/\partial r][R/U_b^3]; \qquad (2.2A)$$

Convective diffusion due to kinetic and pressure effects:

$$(\text{II}) = [-1/2]([\partial(\overline{uq^2} + 2\overline{p'u}/\rho)/\partial x] + [\partial r(\overline{vq^2} + 2\overline{p'v}/\rho)/r\partial r])[R/U_b^3];$$

$$(2.2B)$$

Production:

$$(\text{III}) = -[\overline{uv}\partial\bar{U}/\partial r + \overline{uv}\partial\bar{V}/\partial x + \overline{u^2}\partial\bar{U}/\partial x + \overline{v^2}\partial\bar{V}/\partial r + \overline{w^2}\bar{V}/r][R/U_b^3];$$

$$(2.2C)$$

Viscous work·

$$(\text{IV}) = [(\nabla^2\nu/2)(\overline{u^2} + \overline{v^2} + \overline{w^2})][R/U_b^3]; \qquad (2.2D)$$

Dissipation:

$$(\text{V}) = - R\varepsilon/U_b^3; \qquad (2.2E)$$

2.3 BOUNDARY CONDITIONS

One of the important task of any engineering problem is to define the boundary conditions. A similar problem, was faced in the begining of the present study, to describe the flow very close to the wall of the diffuser and at the center-line of the diffuser.

Schildknecht, Miller and Meier (1979) studied a similar problem while examining the influence of suction on the structure of turbulence in fully developed pipe flow. Their pipe flow study was extended to diffuser flow in order to define the following boundary conditions:

At the wall

$$\bar{U} = \bar{V} = u' = v' = w' = \overline{uv} = 0$$

At the center-line

$$\bar{V} = \overline{uv} = \partial \bar{U}/\partial r = 0$$

According to Schildknecht, Miller and Meier (1979) the components of the fluctuating velocity may be represented by a power series in terms of the distance from the wall

$$u' = aa(x,r,z)s + \cdots\cdots$$

$$v' = bb(x,r,z)s^2 + \cdots\cdots$$

$$w' = cc(x,r,z)s + \cdots\cdots$$

Substituting these series representations into the turbulent kinetic energy equation, Derksen and Azad (1980) showed that only viscous work (gradient diffusion rate of the turbulent energy) and dissipation are non-vanishing quantities at the wall. Non-vanishing components at the center-line are mean flow convection, convective diffusion, dissipation and production. In the mean energy budget, viscous work and direct dissipation are zero at the wall, mean energy advection is zero at the center-line.

2.4 FLOW SPECIFICATION

The present study deals with an adverse pressure gradient type of flow in a diffuser. The physical quantities used in this study were taken from Arora (1978). Briefly, fully developed pipe air flow enters into a diffuser with 8° total included angle and area ratio 4:1. Mean static pressure variation were measured at six different Reynolds number (based on pipe bulk velocities) along the diffuser. The results are shown in Figure 5. Mean and fluctuating quantities (up to fourth order) were measured with standard DISA hot-wires, at twelve different axial locations with the following distances from the exit in centimeters:

67 65 61 57 50 40 30 24 18 12 6 0

Henceforth, those axial locations will be known as axial stations, according to their distance from the exit. For example, station 18 corresponds to the axial location which is 18 centimeters from the exit. Since measurements very close to the wall were not done, all quantities in that region are extrapolations. Time derivatives of $\overline{u^2}$ measurements were used to estimate dissipation, with the assumption that isotropy is valid throughout the cross-section.

Okwuobi (1972) studied the same flow and his measurements were used to estimate axial derivatives of the turbulent kinetic energy $(\overline{q^2})$ in the mean flow convection term of the energy balance.

2.5 CORRECTIONS TO HOT-WIRE MEASUREMENTS

The possible sources of error in a hot-wire system may be:

1. Effect of high intensity turbulence
2. Effect of prong interference
3. Errors due to the wall effects
4. Longitudinal cooling of hot-wire
5. Using hot-wire in a different orientation as compared to the orientation for which it is calibrated
6. Error due to non-linearity of the response equation

In addition, there may be some errors associated with electronic instruments. Several analytical methods had been suggested, to correct hot-wire response, by different experimentalist. Rose (1962) suggests high intensity corrections to the linearized constant temprature response but he limits the correction term up to third order correlations e.g. $\overline{uv^2}$. Heskestad (1965) corrects this by including fourth order correlations e.g. $\overline{u^4}$, but the resulting correction to the Reynolds stresses are not given. Sandborn (1967) discusses the problem of high intensity and presents some experimental measurements that indicates error of less than 2% in non-linear normal wire measurement of $\overline{u^2}$ with up to 50% turbulence intensity. But Sandborn does not discuss the effect of a three-dimensional turbulent field on the normal wire response. Wichner and Peebles (1963) suggest a method of determining the Reynolds stresses which requires an accurate knowledge of the mean velocity.

Guitton (1974) discusses some corrections which may be applied to mean velocity and Reynolds stresses being measured in a two-dimensional mean flow of high turbulence intensity by means of linearized constant temperature hot-wire anemometer. The wire response is taken to fourth order in the fluctuating velocity and includes corrections due to deviation from normal cooling. Guitton conducted some experiments in high intensity flows: two-dimensional plane and curved turbulent wall jets in still surroundings. According to Guitton , in order to apply these corrections, all that is required is to multiply the measured quantity by a correcting coefficient in order to get the corrected quantity. Thus

$$\overline{u_c^2} = H_1 \overline{u_m^2}$$

$$\overline{v_c^2} = H_2 \overline{v_m^2}$$

$$\overline{uv} = H_3 \overline{uv_m}$$

$$\overline{w_c^2} = H_4 \overline{w_m^2}$$

Subscript m indicates measured quantities and subscript c indicates corrected quantities. H_1, H_2, H_3, and H_4 are correcting coefficients defined as:

Correction coefficient for $\overline{u^2}$ (H_1):

$$H_1 = 1 + \bar{V}^2/\bar{U}^2 - \bar{V}^2\,\overline{v^2}/\overline{u^2}\,\bar{U}^2 - 2\,\bar{V}\,\overline{uv}/\bar{U}\,\overline{u^2} - \overline{uv^2}/\overline{u^2}\,\bar{U}$$

$$+ \overline{u^2v^2}/\overline{u^2}\,\bar{U}^2 - (\,\overline{v^4} - (\overline{v^2})^2\,)/4\,\bar{U}^2\,\overline{u^2}; \qquad (2.5A)$$

Correction coefficient for $\overline{v^2}$ (H_2):

$$H_2 = 1/[\,1 - 2k^2/\mathrm{Sin}^2\,\phi] - \overline{uw^2}\,(\,1 + \mathrm{Cot}\,\phi\,\bar{V}/\bar{U}\,)/\mathrm{Cos}^2\phi\,\overline{v^2}\,\bar{U}$$

$$-\,(\,H_1 - 1\,)\overline{u^2}/\mathrm{Cot}^2\phi\,\overline{v^2} + \overline{vw^2}\,\bar{V}/\mathrm{Sin}^2\phi\,\overline{v^2}\,\bar{U}^2 + \overline{u^2w^2}/\mathrm{Cos}^2\phi\,\overline{v^2}\,\bar{U}^2$$

$$+ \overline{v^2w^2}/\mathrm{Sin}^2\phi\,\overline{v^2}\bar{U}^2 - [\,\overline{w^4} - (\overline{w^2})^2\,]/\mathrm{Sin}^2 2\phi\,\overline{v^2}\bar{U}^2; \qquad (2.5B)$$

Correction coefficient for $\overline{uv}$ (H_3):

$$H_3 = 1/[\,1 - k^2/\mathrm{Sin}^2\,\phi] - (\,1 + \mathrm{Cot}\phi\,\bar{V}/\bar{U}\,)\overline{vw^2}/2\,\mathrm{Sin}^2\phi\,\bar{U}\,\overline{uv}$$

$$+ \overline{vw^2}\,\bar{V}/2\,\mathrm{Sin}^2\phi\,\overline{uv}\,\bar{U}^2 + \overline{uv}\,\overline{w^2}/\mathrm{Sin}^2\phi\,\overline{uv}\,\bar{U}^2; \qquad (2.5C)$$

Correction coefficient for $\overline{w^2}$ (H_4):

$$H_4 = 1/[1 - 2k^2/\mathrm{Sin}^2\phi] + \bar{V}^2/\mathrm{Sin}^2\phi\bar{U}^2 - (H_1 - 1)\overline{u^2}/\mathrm{Cot}^2\phi\,\overline{w^2}$$

$$+ \bar{V}^2\,\overline{u^2}/\mathrm{Cos}^2\phi\,\bar{U}^2\,\overline{w^2} - \bar{V}^2\,\overline{v^2}/\mathrm{Sin}^2\phi\,\mathrm{Cos}^2\phi\,\bar{U}^2\,\overline{w^2} - \bar{V}\,\overline{v^3}/\mathrm{Sin}^2\phi\,\mathrm{Cos}^2\phi\,\bar{U}^2\,\overline{w^2}$$

$$- \overline{uv^2}/\mathrm{Cos}^2\phi\,\bar{U}\,\overline{w^2} + 2\,\bar{V}\,\overline{u^2v}/\mathrm{Cos}^2\phi\,\bar{U}^2\,\overline{w^2} - 2\,\bar{V}\,\overline{vw^2}/\mathrm{Sin}^2\phi\,\bar{U}^2\,\overline{w^2} +$$

$$\overline{u^2v^2}/\mathrm{Cos}^2\phi\,\bar{U}^2\,\overline{w^2} + \overline{v^2w^2}/\mathrm{Sin}^2\phi\,\bar{U}^2\,\overline{w^2} - [\,\overline{v^4} - (\overline{v^2})^2\,]/4\,\mathrm{Sin}^2\phi\,\mathrm{Cos}^2\phi\,\bar{U}^2\,\overline{w^2} \tag{2.5D}$$

where ϕ is the inclination angle of the wire (its value in the present study being 45°) and k is taken as zero because of high intensity turbulence effects.

The proposed correcting coefficients were evaluated for all available measurements. The measured Reynolds stresses were multiplied by correction coefficient in order to get the corrected Reynolds stresses. Corrected and uncorrected Reynolds stresses were plotted for all stations; typical Reynolds stresses are shown in Figures 1 to 4 for three diffuser stations. From their studies the following conclusions could be drawn for the correcting coefficients.

1. Correction coefficients H_2, H_3 and H_4 behave similar to one another i.e. each of them increases in the downstream direction.

2. Coefficient H_1, correction coefficient for Reynolds stress ($\overline{u^2}$), has its maximum value at the center-line ($\xi_2 = 0.0$) and decreases towards the wall.

3. Radial direction Reynolds stress ($\overline{v^2}$) correction factor (H_2) has its maximum value roughly at about ξ_2=0.6 and decreases towards the center line and the wall side.

4. H_3, correction coefficient for fluctuating shear stress, has a maximum value close to the wall and decreases toward the center-line. However a different behavior was observed for station 12 as shown in figure 3.

5. Z-direction Reynolds stress ($\overline{w^2}$) correction coefficient (H_4) radially remains the same throughout a cross-section except for having a slightly higher value in the wall region. The maximum observed value of H_4 was 1.173 at station 0 for ξ_2=1.87.

All available measurements were multiplied by the correction coefficients. Corrected datas were employed to evaluate mean and turbulent kinetic energy balances.

2.6 DATA ANALYSIS

Mean and turbulent kinetic energy balance equations involve radial and axial derivatives of mean and fluctuating velocity correlations. Therefore the correctness of energy balance depends upon the degree of accuracy with which different derivatives are obtained.

Three types of curve fitting (fourier series, cubic spline and polynomial) were studied, during the course of the present study, before selecting the polynomial curve fitting technique. The accuracy of fourier series curve fitting depends upon how many terms have been considered to fit the experimental measurements. By considering more terms, however, one may expect improved curve fitting. But, the gradient of the function will have more oscillations which may be far from the real logical picture. A good fit can only be expected if the function is periodic and continuous [Churchill (1963) and Oberhettinger (1973)]. In the present study different mean and turbulent quantities are not periodic, therefore fourier series technique can not be use to calculate the derivatives.

From the cubic spline curve fitting one may expect an improved regression coefficient (sum of the squares of difference between measured and computed value). A spline passes through all given points, but its behavior between two given points is unpredictable [Spath (1974)]. As a result fluctuations in the derivatives will be high, therefore cubic spline curve fitting was discarded for the present study.

Polynomial curve fitting was selected, which doesn't have the demerits of fourier and cubic spline curve fitting. In general, the regression coefficient reduces with an increase in the degree of polynomial. However the best degree of polynomial was selected by considering regression coefficient and observing the behavior of the derivatives logically. Sometime, to improve the accuracy in results, curves were splitted into two or more sections. For example, for axial plot of mean velocity field seprate polynomials were fitted to the different (core and wall regions) sections. Before finalising the degree of polynomial it was ascertain that both had the same function value and derivative at the joining point of the two sections.

Chapter III

RESULTS AND DISCUSSION

3.1 MEAN STATIC PRESSURE

Figure 5 shows variation of mean static pressure along the diffuser at six different Reynolds numbers. These mean static pressures were measured at the wall of the diffuser. In Figure 5, mean static pressures are non-dimensionalised by dynamic pressure, and fall on the same curve thereby showing the universality of the measurements. Also shown in the same diagram is the average axial derivative of the mean static pressures represented by a solid line.

The curve of dPm/d(x/D) show how does the pressure changes from a fully developed pipe flow, to a higher pressure at the exit of the diffuser. This phenomenon of pressure recovery is very important from the turbulent energy point of view. Due to the pressure recovery process, turbulent kinetic energy balance from a fully developed pipe flow (where production is balanced by dissipation and mean flow convection is zero) changes to diffuser flow thereby giving turbulent energy balance for adverse pressure gradient type of flows. It is evident from the curve of pressure gradient, that there is more pressure recovery in the entry region (roughly 72-52 centimeters from the exit) and pressure gradient curve is roughly linear. In the intermediate region (approximately a region of 52-28 centimeters from the exit) there is less pressure recovery as compared to entry region. Finally in the exit region (28-0 centimeters from the exit) pressure recovery is least and pressure gradient curve is linear.

Turbulent kinetic energy balance as a function of pressure gradient will be discussed later in this chapter.

3.2 MEAN ENERGY BALANCE

In the present study an evaluation of mean energy balance was carried out, the regions of concentraion were the entry region (station 57) and exit region (station 12). Section 2.1 describes the different terms involved in the mean energy balance.

Due to mean velocity flow field (in x-direction) and mean energy $(\overline{Q^2})$ derivative (in x-direction), mean energy is produced. Therefore, $(\bar{U}/2)\partial\overline{Q^2}/\partial x$ is termed as a source of mean energy in equation (2.1A). Because of diffuser flow and its high efficiency, most of the mean energy produced is utilized in increasing the pressure directly.

A part of the remaining mean energy produces turbulent energy. The rest of the energy is dissipated directly from the mean flow field in the form of heat due to viscosity. However, this process of mean energy conservation is carried out by three transport processes. One of them is due to interaction of mean velocity field in radial direction with radial derivatives of mean energy $(\overline{Q^2})$ i.e. mean energy advection. Therefore mean energy advection is responsible for transporting the mean energy in radial direction due to mean flow field. The second transport term is due to fluctuating velocities. This transport term has two components one of which involves radial derivatives of $r(\bar{U}\overline{uv} + \bar{V}\overline{u^2})$ and another with axial derivative of ($\bar{U}\overline{u^2} + \bar{V}\overline{uv}$). In the core region radial transport by fluctuating velocities is an order of magnitude higher than the axial transport. However, in the wall region they are of the same order of magnitudes. Sign-wise, total transport due to fluctuating velocities takes out the mean energy (loss of mean energy) in the core region and it supply the mean energy in the wall region. The third transport term is transport of mean energy by viscous diffusion which is significant only near the wall.

Figure 6 - 9 shows mean energy balance for two diffuser stations under consideration. In these figures viscous diffusion and direct dissipation are not shown because these two terms are very small in magnitude compared to the other terms except very near the wall. Figure 6 and 8 shows pressure work term and source term with other terms of the mean energy balance for stations 12 and 57 respectively. It is evident from these figures that other terms are small in magnitude as compared to source and pressure work. Therefore a net value of source term and pressure work was obtained and it is shown in figure 7 and 9 for stations 12 and 57 respectively with other terms.

Turbulent energy production approximately balances the mean energy advection throughout the cross-section for station 12. However, this is not true for station 57. For this station $\partial \overline{Q^2}/\partial r$ is relatively large. Therefore mean energy advection is more compared to turbulent energy production. The net term (pressure work + source) does not balances transport term. By conservation law of energy all terms of mean energy equation should balance. Taking this into consideration a difference term was estimated, which includes the viscous diffusion and direct dissipation. If this difference term is added to the net term a mean energy balance could be obtained. Since net term was obtained as a sum of two major terms, a deviation of $\pm$ 10 - 15 % in each of these terms could give a net term which is wrong enough to show inbalance in mean energy budget.

3.3 PRODUCTION

Production represents the phenomenon of taking out energy from the mean flow field and supplying it to the turbulent field. Production is the product of each Reynolds stress with its corresponding mean rate of strain and therefore it represents the rate at which the mean flow does work on the turbulence.

$$\text{Production} = -[\overline{uv}\partial\bar{U}/\partial r + \overline{uv}\partial\bar{V}/\partial x + \overline{u^2}\partial\bar{U}/\partial x + \overline{v^2}\partial\bar{V}/\partial r + \overline{w^2}\,\bar{V}/r]$$

Term $\overline{uv}\partial\bar{U}/\partial r$ denotes the work done by Reynolds stress ($\overline{uv}$) against the mean strain ($\partial\bar{U}/\partial r$). The maximum value for this term decreases toward the exit of the diffuser. At station 67 its magnitude is four times that of its magnitude at station 0 at $\xi_2 = 1.0$. The behavior of $\overline{uv}\partial\bar{U}/\partial r$ is very similar to that of total production. The radial position (ξ_2) at which $\overline{uv}\partial\bar{U}/\partial r$ is maximum is very close to the pipe radius (ξ_2=1.0) for all stations. Roughly $\overline{uv}$ is also maximum at same value of ξ_2 for all stations.

Different components of production are given in the Table 1 for each of the three regions. Figures 10-13 show the behavior of total production at various stations located in three regions.

TABLE 1

Production and its components (Re=58,000)

Station	Radial distance in cm. from CL	$\overline{uv}\partial\bar{U}/\partial r$ m^2s^{-3}	$\overline{w^2}(\bar{V}/r)$ m^2s^{-3}	$\overline{u^2}\partial\bar{U}/\partial x$ m^2s^{-3}	$\overline{v^2}\partial\bar{V}/\partial r$ m^2s^{-3}	$\overline{uv}\partial\bar{V}/\partial x$ m^2s^{-3}
12	2.0	106.0	-6.5	25.0	-6.0	0.0
	4.5	229.0	-10.3	28.0	-3.7	0.0
	8.5	64.0	-0.7	-11.0	2.6	-0.1
40	2.0	29.0	-3.5	15.5	-2.8	0.2
	5.5	271.0	-8.9	36.0	-4.0	1.6
	6.5	140.0	-4.1	-57.0	16.0	-0.6
65	2.0	25.0	-12.3	49.0	-9.5	2.44
	4.0	110.0	-21.6	102.5	-14.2	10.8
	5.0	950.0	-22.8	-208.0	63.4	9.3

$\overline{uv}\,\partial\bar{V}/\partial x$ also represents work done by Reynolds stress against the mean strain $(\partial\bar{V}/\partial x)$. In the exit region, in the range $0.0 < \xi_2 < 1.38$,

$\overline{uv}\partial\bar{V}/\partial x$ is zero because $\partial\bar{V}/\partial x$ is zero in that range. Whereas, $\overline{uv}\partial\bar{V}/\partial x$, beyond $\xi_2 = 1.38$ is roughly 0.01 times $\overline{uv}\partial\bar{U}/\partial r$ for the same radial locations. But in the intermediate and entry regions, the magnitude of $\overline{uv}\partial\bar{V}/\partial x$ is higher than in the exit region, even though it is not comparable to $\overline{uv}\partial\bar{U}/\partial r$.

$\overline{u^2}\partial\bar{U}/\partial x$ is one of the non-vanishing term at the center-line of the diffuser. Due to this term, the turbulent flow field accepts energy from the mean flow field in the core region and supply to the mean flow field in the wall region. In the exit region , this is the second important term (first being $\overline{uv}\partial\bar{U}/\partial r$) in the whole production. Whereas in the intermediate region and roughly up to 61 cm. from the exit it is approximately of the same order as $\overline{uv}\partial\bar{U}/\partial r$. However, in the 61-67 centimeter range from the exit plane (region where pressure gradient curve is very steep and linear) this term becomes more important than $\overline{uv}\partial\bar{U}/\partial r$ in the core region.

$\overline{v^2}\partial\bar{V}/\partial r$ and $\overline{w^2}(\bar{V}/r)$ are of same order of magnitude. Sign-wise, $\overline{v^2}\partial\bar{V}/\partial r$ contributes negatively to the production in the core region, while in the wall region it contributes positively. But, $\overline{w^2}(\bar{V}/r)$ contributes negatively to the production throughout the cross-section.

3.4 MEAN FLOW CONVECTION

Mean flow convection is also known as advection [Tennekes and Lumley (1977)]. It describe how the mean flow moves the turbulence energy in the flow field.

$$\text{Mean flow convection} = -\left[(\bar{U}/2)\,\partial\,\overline{q^2}/\partial x + (\bar{V}/2)\,\partial\,\overline{q^2}/\partial r\right]$$

It will be shown later that Arora's (1978) data is inaccurate to estimate mean flow convection in the exit region. Therefore, to estimate this important quantity in the energy balance equation, Okwuobi's (1972) study on a diffuser was used. It was particularly necessary to use Okwuobi's (1972) data to estimate axial derivatives of turbulent energy ($\partial \overline{q^2}/\partial x$) in the exit region.

It was found that there exist a line [henceforth referred to as the 'Energy peak line' (Ep-line)] at about 2° angle to the diffuser axis where turbulent shear stress ($\overline{uv}$), turbulent kinetic energy ($\overline{q^2}$),and radial derivative of mean velocity ($\partial\bar{U}/\partial r$) all attain their same maximum value. In addition mean velocity $\bar{U}$ is same .

$\bar{U}\partial \overline{q^2}/\partial x$ has the major contribution to mean flow convection in the core region but not in the wall layer. In the exit region from the center-line to Ep-line this has a negative contribution (loss of turbulent energy) and beyond that has a positive contribution (gain of turbulent energy). However, in the intermediate and entry regions maximum of $\bar{U}\partial \overline{q^2}/\partial x$ occurs roughly at $\xi_2 = 0.89$, and its behavior is similar to that in the exit region as mentioned above.

$\bar{V}\partial \overline{q^2}/\partial r$ is a part of mean flow convection which dominates over $\bar{U}\partial \overline{q^2}/\partial x$ in the wall region. After Ep-line (toward the wall side) $\bar{V}\partial \overline{q^2}/\partial r$ become negative with a very high value close to wall because $\partial \overline{q^2}/\partial r$ is large in that region. For any radial location, ξ_2, its magnitude increases downstream and this phenomenon is also noticed for total mean flow convection.

3.5 CONVECTIVE DIFFUSION

This is also known as eddy transport (Hinze (1975)). In a real turbulence the motion of the fluid particles are randomly distributed. This random distribution is such that when two arbitrary fluid particles move, statistically the distance between them increases with time. If we consider a number of neighboring particles at one instant and if we observe the position of various particles at subsequent instants, we observe a gradual spread throughout the space. This is the basic idea of diffusion. Taylor (1921) extended the above consideration to the diffusion in turbulent flows, taking into account continuous movement of the fluid particles, by considering the path of a marked fluid particle during its motion through the flow field.

In the turbulent kinetic energy balance this term is counter to mean flow convection (which represents transport by mean flow field) and it represents transport by turbulent flow field.

Convective diffusion due to kinetic and pressure effects:

$$[-1/2]([\partial(\overline{uq^2} + 2\overline{p'u}/\rho)/\partial x] + [\partial r(\overline{vq^2} + 2\overline{p'v}/\rho)/r\,\partial r])$$

Convective diffusion may be divided into two parts.

1. Convective diffusion due to kinetic effects.

2. Convective diffusion due to pressure effects.

In the present study convective diffusion due to kinetic effects is estimated from measured triple velocity correlations. Convective diffusion due to pressure effects, which is a result of pressure-velocity correlation, was estimated as a closing term of the turbulent kinetic energy balance equation.

Convective diffusion due to kinetic effects has two components one of which involves the radial derivatives of $\overline{rvq^2}$, and a second part which is associated with axial derivatives of $\overline{uq^2}$. Most researchers, neglect the axial derivative component of convective diffusion. It was found during the present study that $\partial(r\overline{vq^2})/\partial r$ is an order of magnitude greater than $\partial(\overline{uq^2})/\partial x$ throughout the diffuser. Convective diffusion due to kinetic effects given in this study consists of both radial and axial components. In the core region $\partial(r\overline{vq^2})/r\partial r$ and $\partial(\overline{uq^2})/\partial x$ both increases downstream for the same radial locations. Maximum value of total convective diffusion due to kinetic effects, however, increases upstream of the diffuser exit plane. At any cross-section, the integral value of convective diffusion due to kinetic effects for exit region is same, within the experimental errors. But in intermediate and entry regions its value is negligible.

Total transfer term can be estimated, from the law of conservation of energy, by the expression

Production + Mean Flow Convection + Dissipation + Total Transfer = 0

By subtracting convective diffusion due to kinetic effects from the total transfer term, convective diffusion due to pressure effects term can be obtained, which will also include viscous work. However viscous work is small throughout the cross-section except close to the wall. Therefore this estimation will be alright except in the wall region. In the exit region, convective diffusion due to pressure effects is almost negligible Whereas in the entry region, because of high pressure gradient, its magnitude is higher than the kinetic diffusion and it dominates total transport term.

3.6 DISSIPATION

It can be interpreted as the mean rate at which the turbulence does work against viscous stresses. If a body is placed in a turbulent boundary layer, the work done by the friction drag of the body, is converted into heat by viscous dissipation.

For high Reynolds number, the small eddies are not correlated with large ones which implies the motion of small eddies is isotropic. The later means that the fine structure is invariant under rotation of the axes of the reference. This was first suggested by Komogoroff (1962) who introduced the concept of local isotropic turbulence. Prandtl (1942), V. Weizsacker (1948) and Onsager (1945) came independently to the same conclusion. When the motion of the part of the turbulence that is responsible for the viscous dissipation is isotropic, the turbulent energy dissipation can be simplified. Using the continuity equation and the concept of isotropy the following relation could be obtained, details of which are given by Hinze (1975).

$$\varepsilon = 15 \; (\overline{u^2} / \; x)^2$$

Hence, for a homogenous turbulence, viscous dissipation per unit of mass is equal to the viscosity times the mean-square of the rate of strain or to the viscosity times the mean square vorticity.

Similar to the pipe flow, dissipation is of the same order of magnitude as production in diffuser flow. This is one of the important term of turbulent energy balance which is non-vanishing at the wall according to the boundary conditions mentioned in chapter 2. For the present study, dissipation has been calculated assuming isotropy is valid throughout the cross-section. In the entry region dissipation become very high near the wall. But in the intermediate region this is not true, rather it is almost constant after attaining a maximum value. However, in the exit region, after achieving a maximum value, its magnitude starts decreasing even in the wall layer. Because of the diffuser geometry wall layer expands in the downstream direction. As a result, dissipation distributes more evenly at any cross-section in the exit region after achieving a maximum value. Figure 10-13 shows the behavior of dissipation in three different regions of the diffuser.

In the core region, its value increases from its center line dissipation value. But the peak value starts shifting away from the wall in the downstream.

3.7 TURBULENT KINETIC ENERGY

Figure 14 shows distribution of cross-sectional average values of turbulent kinetic energy in the pipe and diffuser. Pipe data was taken from Laufer's (1954) study on a fully developed pipe flow. Turbulent kinetic energy ($\overline{q^2}$) was plotted against area (r^2) for all twelve stations and for the pipe. After measuring the area under the curve, with the help of a planimeter, net kinetic energy was divided by R^2_0 and an average balue was obtained. This average kinetic energy was non-dimensionalised by U_b^2 (18.32 ms^{-1}) and plotted in figure 14. As it is evident from this Figure, there is an increase in average turbulent kinetic energy from entry to up to roughly 28 centimeters from the exit beyond which it attains a definite value asymptotically. Hence, there is a correlation between the pressure recovery process and the kinetic energy. When the air enters the diffuser it contains certain amount of energy consisting of kinetic and pressure energy components. But due to pressure recovery process, more and more kinetic energy is converted into the pressure energy in diffuser's entry and intermediate regions, where pressure gradient curve (fig. 5) has a large slope. But in the final region the process of pressure recovery relaxes and pressure gradient curve is linear with a small slope. Although there is a rise in the net value of kinetic energy in the exit region but because area of the diffuser is also increasing in the downstream the average value is same within experimental errors.

3.8 CONSEQUENCES OF THE PRESENT STUDY

After careful review of turbulent energy balance, term by term, a general picture to the turbulent energy balance can be given for a flow subjected to adverse pressure gradient. In the present study, all terms of the energy balance has been weighted by area at each station and a mean value at each of these locations has been obtained. It is hoped, that it will clarify any doubts and will help readers to visualize a more clear physical picture.

For the above mentioned reason, every term of the turbulent kinetic energy balance was plotted against square of radial distance. Such a typical plot is shown in Figure 15 for station 12. After measuring area, with the help of a planimeter, under each term of the turbulent energy balance an average value of them was calculated by dividing total area under the curve to the local area (square of local radius).

Figure 16 shows axial plot of average turbulent energy balance terms for all twelve axial stations and pipe. From figure 16 it is evident that in the diffuser, production is balanced by dissipation in the region where average kinetic energy is the same (exit region). However , in those regions where average turbulent kinetic energy is increasing and pressure gradient curve have a large magnitude of slope (entry and intermediate regions) average production is more than the average dissipation. Similarly, average mean flow convection is more than the average total transfer in entry and intermediate regions; whereas average mean flow convection balances the average total transfer in the exit region.

In a fully developed pipe flow, mean flow convection is zero, production balances dissipation , and total transfer term balances the viscous gradient diffusion as shown in Figure 16 (corresponding to station 75). Although diffuser's length is 72 cms., a recent study has shown that at station 72 mean radial velocity ($\bar{V}$) exist. But, in a fully developed pipe flow mean radial velocity ($\bar{V}$) should be zero. Therefore station 75 was choosen to represent pipe flow conditions.

In Figure 16 components of turbulent kinetic energy balance are shown by their respective symbols for the diffuser flow and pipe. Since pressure diffusion term is calculated as the closing term of the energy equation, its unexpected value in the exit region of the diffuser may be due to the experimental errors associated with other terms of the turbulent kinetic energy balance. Figure 17 shows trend lines of the turbulent kinetic energy budget terms in diffuser and pipe.

Locally in the intermediate and entry regions, roughly, production balances the dissipation and mean flow convection balances the convective diffusion due to pressure kinetic effects in the core region. However, in the wall region, production does not balances the dissipation exactly. Therefore the excess of turbulent energy drained is balanced by the mean flow convection and convective diffusion due to pressure effects. In the exit region production and dissipation have more difference, locally, than other two regions. Mean flow convection is positive (gain of turbulent energy) wherever dissipation exceeds production and negative (loss of turbulent energy) wherever production is more than the dissipation except in the $0.0 < \xi_2 < 0.2$ range.

3.9 PREVIOUS DIFFUSER WORK AT U OF M VS. PRESENT STUDY

The present study is a re-evaluation of Arora's (1978) data. Arora took measurements for mean static pressure, mean velocity, various moments up to 4th order, and the first and second derivatives of u′ signal for pipe Reynolds number of 58,000 based on the pipe average velocity and the pipe radius. The conical diffuser was machined from cast aluminum and air was blown through an 89:1 contraction cone and 74 diameters long steel pipe of 10.16 inside diameter before entering the diffuser. His experimental measurement were corrected by applying Guitton's (1974) corrections to hot-wire measurements,. The turbulent and mean energy balance were analysed based upon physical interpretation of each and every term involved. There was good agreement with Arora's (1978) representation of production, dissipation and convective diffusion due to kinetic effects in the turbulent kinetic energy balance.

But Arora's (1978) work definitely had two defects:

1. Wrong estimation of mean flow convection in the exit region of the diffuser

2. Sign-wise wrong presentation of the mean flow convection for all diffuser stations. Arora plotted mean flow convection with the wrong sign.

As mentioned in section 3.4 mean flow convection has two parts, and one of them contains axial derivatives of turbulent velocity ($\overline{q^2}$). The most delicate part of the exercise was to get consistent axial derivative. Arora's turbulent kinetic energy ($\overline{q^2}$) profiles, while plotted axially for different radial positions can be divided into three sections approximately corresponding to entry, intermediate and exit regions. In the intermediate region turbulent kinetic energy ($\overline{q^2}$) profiles are not linear, and he had only two stations between 24 and 50 centimeters from the exit palne. Axial derivatives of kinetic energy, in the exit region, does not match with other two regions. Therefore author used Okwuobi's (1972) data to estimate $\partial\overline{q^2}/\partial x$ in the exit region only.

Sign-wise representation of mean flow convection was wrong in Arora's work. He plotted mean flow convection, for all diffuser stations, in opposite sign. The argument is convective diffusion due to pressure effects, calculated as a closing term, comes to be very large even for some stations it is more than production as given by Arora (1978). Physically convective diffusion due to pressure effects, in the region where pressure gradient is not large in magnitude, should be very small as compared to other terms and its integrated value should be zero. But, in regions with large magnitude of pressure gradient, convective diffusion due to pressure effects is an important quantity in turbulent energy balance.

Figure 18 and Figure 19 shows how does wrong axial derivatives of turbulent kinetic energy can change mean flow convection term in turbulent energy balance. Mean flow convection as estimated from Arora's (1978) data, is shown in Figure 18 (for station 12) along with other terms of turbulent energy balance. Assuming that, the convective diffusion is only due to kinetic effects, author estimates (as a closing term) dissipation term and is shown along with measured dissipation. A large difference, between measured dissipation and dissipation by difference, indicates there is some error with energy balance terms. Therefore, author re-estimated mean flow convection using Okwuobi's (1972) data. Taking axial mean velocity ($\bar{U}$), radial mean velocity ($\bar{V}$), radial derivatives of turbulent kinetic energy ($\partial \overline{q^2}/\partial r$) from Arora's (1978) data and axial derivatives of turbulent kinetic energy ($\partial \overline{q^2}/\partial x$) from Okwuobi's (1972) data, mean flow convection was re-estimated. It is shown in Figure 19 (for station 12) along with other terms of turbulent energy balance (similar to Figure 18). Again under the same assumptions, shown in Figure 19, dissipation from measurements and dissipation as a difference term was estimated. From Figure 18 and Figure 19 it is evident that mean flow convection as given in Figure 19 is more reliable than given in Figure 18 also it agrees with Bradshaw (1967) and Shishov et al. (1978) representations, qualtatively.

Okwuobi (1972) also studied turbulent energy balance in a diffuser, details of which are given in Okwuobi (1972). The following basic disagreement were found with Okwuobi and Azad (1973) regarding conclusions about the turbulent energy balance.

1. They claimed in the regions $0.8 < \xi_2 < 1.0$ that dissipation is balanced by mean flow convection; but it has been proved by the present study that there is no region in the diffuser where dissipation is balanced by mean flow convection. The possible source for this biased interpretation is wrong estimation of mean flow convection. As pointed-out by Arora (1978) they subtracted the two components of mean flow convections i.e. $\bar{U}\partial\overline{q^2}/\partial x$ and $\bar{V}\partial\overline{q^2}/\partial r$ instead of adding them.

2. Their conclusion, was that in the $0.2 < \xi_2 < 0.8$ region, production of turbulent energy is balanced by the total convective diffusion Truly the picture which emerges, in $0.2 < \xi_2 < 0.8$ range, is that production roughly balances the dissipation locally. However excess of production energy present in that region is convected by mean flow convection and turbulent diffusion. The possible cause of this wrong interpretation,

could be their method of evaluating energy balance, where they estimate convective diffusion term as a closing term of turbulent energy balance. So any error in other energy term, as they did in estimating mean flow convection, will give a wrong convective diffusion (closing term).

3. Their interpretation that dissipation is negligible in diffuser is far away from the actual physical picture. For any shear flow, dissipation of turbulent kinetic energy is as important as the production of turbulent kinetic energy. It was also noticed by Arora (1978).

3.10 COMPARISION OF THE PRESENT STUDY WITH THE BOUNDARY LAYER RESULTS

After studying the turbulent energy balance in a diffuser it can be concluded that diffuser flow is similar to other wall bounded flows Azad and Hummel (1979). In a nutshell, a general picture of turbulent energy balance emerges (on integral-area scheme) where production is balanced by dissipation and mean flow convection is balanced by turbulent transport term. The present work was compared with two other published boundary layer works namely 'Experimental investigation of the turbulent kinetic energy balance in the retarded boundary layer' by Shishov et al.(1978) and 'The turbulent structure of equilibrium boundary layers' by Bradshaw (1967). In order to compare the present results with the other two similar flows, the turbulent energy balance for station 12 was non-dimensionalised by local radius at station 12 and mean center-line velocity in order to put them (more or less) on the same basis as the other two flows.

Figure 20 shows the results at Shishov et al.(1978) (retarded boundary layer) and the diffuser results of station 12. Similarly, Bradshaw's (1967) study of a turbulent boundary layer and the station 12 results are plotted in Figure 21. From these figures it is evident that the non-dimensionalised turbulent energy balances for a two-dimensional boundary layer with adverse pressure gradient and the axi-symmetric diffuser flow do not show the same behavior. Therefore as a second excercise, Shishov et al.(1978) and Bradshaw's (1967) result of a turbulent boundary layer were plotted on figure 22. Again different non-dimensionalised turbulent energy terms are not identical although they are related to the same type of flow. Therefore, it can be concluded from this excercise that it is difficult to compare turbulent kinetic energy balance, quantitatively, studied by different researchers. As a result this section will deal only qualitative comparision of the turbulent kinetic energy balance in the diffuser and boundary layer with an adverse pressure gradient.

Shishov et al.(1978) studied a retarded equilibrium boundary layer at one axial station. The mean velocity variation, within the boundary layer, is govern by the law $U_0 \alpha x^{-0\ 255}$, and the boundary layer thickness (δ) varied as $\delta \alpha x^{0.851}$, where x is axial distance. Bradshaw did a more detailed study of the equilibrium turbulent boundary layer. In his study, he consider three boundary layers; one with a constant free stream velocity and two with power-law variation of free stream velocity giving a 'moderate' and 'strong' adverse pressure gradients. Value of constant 'a' in the expression $U \alpha x^a$, were 0.00, -0.15 and -0.255 for constant free stream velocity, moderate and strong adverse pressure gradients respectively. Similar, to Shishov et al.(1978) and Bradshaw (1967), the author tried to find a relation between the mean velocity and axial distance. The center-line mean velocity, U_0, was plotted on a log-linear graph for 0,6,12,18 and 24 stations. The exponent constant 'a' in the relation $U_0 \alpha x^a$ is the gradient of this plot. The calculated constant found to be -0.33. Hence, $U_0 \alpha x^{-0\ 33}$, is the governing law for mean velocity in the equilibrium region (0 to 24 centimeters from exit plane). So, diffuser flow has more severe adverse pressure gradient than Bradshaw's (1967) flow (which has exponential constant 'a' of -0.255).

In the present study an equilibrium flow was observed in the exit region of the diffuser, roughly 0-24 centimeters from the exit. This was proved by observing behavior of turbulent quantities. Figure 23 shows fluctuating kinetic energy $(\overline{q^2})$ divided by maximum value of fluctuating kinetic energy, plotted against distance from the wall divided by M, where M is the distance from the wall where $\overline{q^2}_{max}$ occur. Figure 24 shows turbulent shear stress $\overline{uv}$ divided by $(\overline{uv})_{max}$ plotted against distance from the wall divided by M, where M is distance from the wall where $(\overline{uv})_{max}$ take place. The collapsing of these two quantities on the same curve at different stations indicates that there exist a similiarity of the flow for those stations. The flow field is homogenous and in equilibrium. Curves of $\overline{uv}/(\overline{uv})_{max}$ do not collapse exactly in the center core, as shown in Figure 24, but this is thought due to experimental errors. If there this behavior were true, the author would expect a similar character to be shown in other quantities as well. For example, there is no such pecularity shown by turbulent energy $(\overline{q^2})$ in

Figure 23. Because of this reason a mean turbulent kinetic energy balance, as shown in Figure 20 and 21, at station 12 was taken as a representative picture of equilibrium region.

3.10.1 Shishov et al.'s (1978) Study vs. Present Study

The turbulent energy balance is given by the expression from their work as:

$$\bar{U}\ \partial\overline{q^2}/\partial x + \bar{V}\ \partial\overline{q^2}/\partial r = -\overline{uv}\ \partial\bar{U}/\partial y - \partial\ [\ \overline{pv}/\rho + \overline{vq^2}\ ]/\partial y + \varepsilon$$

On the basis of above turbulent kinetic energy balance the following points can be made.

1. They approximated total turbulent production by $\overline{uv}\partial U/\partial r$, neglecting the rest of the terms as given in equation (3.3A).

2. Convective diffusion, as given by equation (3.5A), has been approximated to $\partial\ [\ \overline{pv}/\rho + \overline{vq^2}\ ]/\partial y$, which means they have neglected axial-derivative component of convective diffusion due to kinetic and pressure effects.

Figure 25 and 26 shows their approximations for turbulent energy ($\overline{q^2}$) and triple velocity correlations

$$\overline{q^2} = 3/2\ (\ \overline{u^2} + \overline{v^2}\ ) \qquad \text{and} \qquad \overline{vq^2} = 3/2\ (\ \overline{u^2v} + \overline{v^3}\ )$$

respectively.

For the present study, these two approximations are also valid not only in the equilibrium flow region of the diffuser (as shown by station 12 in both Figures 25 and 26), but also near the entry region of the diffuser as shown by station 57. The important implications of these approximations are, in future experimental work, to estimate $\overline{q^2}$ approximately, only $\overline{u^2}$ and $\overline{v^2}$ measurements are necessary. A similar approximation is true for triple velocity correlations $(\overline{vq^2})$. Therefore only one set-up is required to measure $\overline{q^2}$ and $\overline{vq^2}$. In triple velocity correlations the, $\overline{vw^2}$ term is very diffcult to measure.

From Figure 20, it can be concluded that different terms of turbulent kinetic energy balance in a diffuser are similar, qualitatively, to those of adverse pressure gradient boundary layer type of flow. However, Shishov et al. (1978) work does not agree very well with $0.75 < \xi_3 < 1.0$ region of diffuser. The possible reason for which could be the flow studied by Shishov et al. (1978) may be of intermittent type. That means flow is sometime turbulent and sometime it is non-turbulent. Another region of disagreement, between the present study and Shishov et al. (1978) result, was close to the wall where present data itself is doubtful to make any firm decision about any quantity of turbulent energy balance.

Shishov et al. (1978) have shown that production of turbulent energy increases close to the wall. This interpretation may be based on their supposition that just outside the sublayer production of turbulent energy is high. However, radial locations where production is maximum, take place roughly at the same place for both diffuser flow and boundary layer flow. They approximate total mean flow convection by

$$\bar{U}\partial\overline{q^2}/\partial x \;+\; \bar{V}\partial q^2/\partial r \;=\; 2(\;\overline{q^2\bar{U}}/U_0\;)\;dU_0/dx \;-\; [\;dU_0/U_0 dx$$

$$+\;0.851/x\;]\;dq^2\;[\;\int_0^1\;Udy\;]\;/dy$$

and they claim that the above expression represents mean flow convection more close to its experimental determination. But when the author tried to calculate mean flow convection on the basis of above expression, mean flow convection was completely in disagreement with the actual estimated value of mean flow convection. It could have been better, if Shishov et al. (1978) given some more explaination about their mean flow convection expression. Mean flow convection as given in Figure 20, is in agreement with mean flow convection of Shishov et al. (1978) boundary layer flow. But the maximum of mean flow convection does not take place at the same location for two flows. Both of them show mean flow transport to the turbulent flow in wall region and mean flow transport from the turbulent flow in the center core. Convective diffusion, for diffuser flow and boundary flow with adverse pressure gradient, are roughly in agreement, qualitatively, except when $0.85 < \xi_3$ and in the region very close to the wall. Shishov et al.'s (1978) convective diffusion approach to a negligible value at the edge of the boundary layer. But in the case of diffuser flow, there is a definite value at the center line of the diffuser with its magnitude equal to the sum of dissipation and mean flow convection. Physically, that means total transport due to mean and turbulent flow is equal to dissipation at the center-line of the diffuser. Dissipation behavior for both flows is similar to eachother, also the position of maximum dissipation is roughly at the same radial locations for both the flows.

Shishov et al. (1978) are well satisfied with the reliability of turbulent energy balance $0.0 < \xi_3 < 0.6$ range. Beyond that, as they have accepted, their results are doubtful and as a result they get a large difference term in that region. The reason for large difference term could be the flow is of intermittent type or it may be the result of neglecting different components of turbulent energy balance terms.

3.10.2 Bradshaw's (1967) Results vs. Present Study

Production and mean flow convection for two-dimensional boundary layer flow and axi-symmetry diffuser flow are qualitatively similar to a certain extent as shown in Figure 21. The maxima of production and mean flow convection take place approximately at the same radial distance from the wall. Bradshaw estimates the dissipation term of the turbulent energy balance as a closing term. Therefore dissipation as given by Bradshaw may have errors since possible errors can occur in any of the other term of energy balance. Since Bradshaw claims, that for a boundary layer flow, diffusion term should integrate to a zero value. But his diffusion term doesn't integrate to a zero value that implies his diffusion term is not so accurate to estimate any other unmeasured quantity in turbulent energy balance by difference. Bradshaw calculate diffusion term, by just taking into account radial derivative of $\overline{vq^2}$ and neglecting axial derivative of $\overline{uq^2}$. He may be right in doing so, because axial derivative component does not play an important role in total convective diffusion. Over all, sign-wise, convective diffusion in a diffuser and boundary flow are in agreement.

Unlike diffuser flow, in a boundary layer flow, dissipation is zero wherever shear stress is zero and so the total production$[\overline{uv}\partial\bar{U}/\partial r$, as given by Bradshaw (1967)]. This happens at the edge of the boundary layer. Therefore it results, at the edge of a boundary layer, mean flow convection equal to convective diffusion. In a diffuser flow it is not necessary that dissipation be zero if production is zero. Hence at the center line of the diffuser mean flow convection is equal to the dissipation plus convective diffusion.

3.11 COMPARISION OF THE PRESENT STUDY WITH RUETENIK ET AL.(1955)

RESULTS

Ruetenik and Corrsin (1955) were probably the first to measure turbulence data for diffusers, and to report the turbulent kinetic energy balance for a diffuser. They investigated turbulence intensities for fully developed, plane diffuser flow at a divergence half-angle of 1°.

According to their study, in a plane diffuser mean flow convection is almost constant with its contribution to the turbulent flow field (gain of turbulent energy). However, according to the present study mean flow convection does changes sign with its contribution to the turbulent flow field (gain of turbulent energy) and the mean flow field (loss of turbulent energy) in the wall and core regions respectively. Their conclusion that production balances dissipation, integrally at a cross-section area, was found to be valid for the present study of axi-symmetry diffuser flow. They indicated dissipation and viscous work are zero at the wall of the diffuser, was completely in disagreement to the present results and the boundary condition discussed in section 2.3.

Chapter IV

CONCLUSION

After rigorous study of the turbulent and mean kinetic energy balance for adverse pressure gradient type of flows the following conclusions can be drawn for the energy balance in a diffuser.

1. Diffuser flow can be divided into three regions, axially, on the basis of pressure gradient and kinetic energy.

    a) An entry region roughly from 72 to 48 centimeters from the exit, with turbulent energy terms having the same average value (when integrated at a cross-sectional area). The pressure gradient curve is approximately linear and has a large slope, while the kinetic energy is increasing very rapidly from its pipe average value.

    b) Intermediate region roughly from 48 to 28 centimeters from the exit. Pressure gradient curve in this region is not linear and rate of average kinetic energy increase is less as compared of its rate in the entry region.

    c) Third region, which is known as exit region, extends roughly from 28 centimeters from exit to the exit plane. Pressure gradient in this region is linear with comparatively very small slope than in the other two regions, and the average kinetic energy is constant.

2. Similarly, at any station, the diffuser can be divided into two regions radially.

   a) Core region, which can be considered to be equivalent to the extension of pipe with expansion in the downstream direction.

   b) Wall layer, a region which lies close to the wall and expands toward the diffuser exit; a very important and challenging region to study turbulence.

3. There exist a line in the diffuser, at about 2° to the diffuser axis where mean velocity ($\bar{U}$) has the same value for all stations, are turbulent shear stress ($\rho\overline{uv}$) and turbulent kinetic energy ($\overline{q^2}$) attains maximum and the same value.

4. We can pointout the difference in the final condition (given by station 0) of turbulence and entry conditions (as given by station 75) for a flow subjected to adverse pressure gradient. Production and dissipation in both the cases balances each other and are of same order of magnitudes. In negative pressure gradient i.e. fully developed pipe flow gradient diffusion balances total transfer. Whereas in adverse pressure gradient flow i.e. in exit region of the diffuser the mean flow convection is balanced by kinetic diffusion.

5. Due to the imbalance of the production and dissipation in the entry and intermediate regions, as mentioned above, mean flow convection should exceed total transfer in accordance with

conservation law of energy and this is true for the present study. But in the exit region mean flow convection exactly balances total transfer.

6. Mean flow convection is the complement of the total transfer term, which consists of convective diffusion due to kinetic and pressure effects. But this comparision can be divided into two parts.

   a) In the entry and intermediate regions, convective diffusion due to pressure effects is more important. Because the fluid in the entry region undergoes a severe adverse pressure gradient process, this results in convective diffusion due to pressure effects being larger than the kinetic effect part.

   b) After the intermediate region, the flow becomes an equilibrium and homogenous flow in the exit region. Here mean flow convection is sizable and is balanced by the convective diffusion due to kinetic effects.

Chapter V

RECOMMENDATIONS

With the conclusion of this study, now we understand physically the meaning of turbulent kinetic energy balance and relation among all quantities involved. This work in itself is complete to study turbulent kinetic energy balance, qualatively, and to make any interpretation based on that. However, the author feels it will be helpful to carry out future work according to the following guidelines in order to understand turbulent and mean kinetic energy mechanism quantitatively.

1. A thorough study is needed from Ep-line to the wall region. Therefore more measurements, precisely done, are needed in that region.

2. More axial stations should be examined in order to improve axial gradients for the turbulent and mean kinetic energy balance.

3. While taking measurements, the experimentalist should keep in mind that it is necessary to obtain the correct axial and radial derivatives from the data. Therefore measurements done on different days should be repeated as required to provide continuity of the data.

4. Effort should be make to measure all the terms of the dissipation.

5. Turbulent kinetic energy $(\overline{q^2})$ should be measured in the wall layer, and from this viscous work (involving double derivatives of turbulent kinetic energy) should be estimated quantitatively.

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Appendix A

ENERGY EQUATION DERIVATION

Let U,V and W be the velocity components in the direction of the three cylindrical co-ordinates x,r and z. The Navier-Stokes equation for constant properties fluid flow in cylindrical polar co-ordinates is given by:

$$\frac{\partial U}{\partial t} + V\frac{\partial U}{\partial r} + \frac{W\partial U}{r\partial z} + U\frac{\partial U}{\partial x} = -\frac{1\partial P}{\rho\partial x} + \nu\left[\frac{\partial^2 U}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 U}{\partial z^2} + \frac{\partial^2 U}{\partial x^2} + \frac{1\partial U}{r\partial r}\right] \quad (A.1)$$

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial r} + \frac{W\partial V}{r\partial z} + U\frac{\partial V}{\partial x} - \frac{W^2}{\partial r} = -\frac{1\partial P}{\rho\partial r} + \nu\left[\frac{\partial^2 V}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 V}{\partial z^2} + \frac{\partial^2 V}{\partial x^2} + \frac{1\partial V}{r\partial r} - \frac{2}{r^2}\frac{\partial W}{\partial r} - \frac{V}{r^2}\right] \quad (A.2)$$

$$\frac{\partial W}{\partial t} + V\frac{\partial W}{\partial r} + \frac{W\partial W}{r\partial z} + U\frac{\partial W}{\partial x} + \frac{VW}{\partial r} = -\frac{1\partial P}{\rho r\partial z} + \nu\left[\frac{\partial^2 W}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 W}{\partial z^2} + \frac{\partial^2 W}{\partial x^2} + \frac{1\partial W}{r\partial r} + \frac{2}{r^2}\frac{\partial V}{\partial z} - \frac{W}{r^2}\right] \quad (A.3)$$

By defining ∇^2 as:

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2}{\partial z^2} + \frac{1\partial}{r\partial r}, \quad (A.4)$$

substituting for ∇^2 in the above three equations yields:

$$\frac{\partial U}{\partial t} + V\frac{\partial U}{\partial r} + \frac{W\partial U}{r\partial z} + U\frac{\partial U}{\partial x} = -\frac{1\partial P}{\rho\partial x} + \nu\nabla^2 U \quad (A.5)$$

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial r} + \frac{W\partial V}{r\partial z} + U\frac{\partial V}{\partial x} - \frac{W^2}{r} = -\frac{1\partial P}{\rho\partial r} + \nu\left[\nabla^2 V - \frac{2\,\partial W}{r^2\partial z} - \frac{V}{r^2}\right] \quad (A.6)$$

$$\frac{\partial W}{\partial t} + V\frac{\partial W}{\partial r} + \frac{W\partial W}{r\partial z} + U\frac{\partial W}{\partial x} + \frac{VW}{\partial r} = -\frac{1\,\partial P}{\rho r\partial z} + \nu\left[\nabla^2 W + \frac{2\,\partial V}{r^2\partial z} - \frac{W}{r^2}\right] \quad (A.7)$$

For steady flow, all the time-derivative quantities in the above equations (A.5), (A.6) and (A.7) will be zero i.e.

$$\frac{\partial U}{\partial t} = \frac{\partial V}{\partial t} = \frac{\partial W}{\partial t} = 0 \quad (A.8)$$

Substituting these values in the above equations (A.5), (A.6) and (A.7) the Navier-Stokes equations, for a steady flow, become:

$$V\frac{\partial U}{\partial r} + \frac{W\partial U}{r\partial z} + U\frac{\partial U}{\partial x} = -\frac{1\partial P}{\rho\partial x} + \nu\nabla^2 U \quad (A.9)$$

$$V\frac{\partial V}{\partial r} + \frac{W\partial V}{r\partial z} + U\frac{\partial V}{\partial x} - \frac{W^2}{r} = -\frac{1\partial P}{\rho\partial r} + \nu\left[\nabla^2 V - \frac{2\,\partial W}{r^2\partial z} - \frac{V}{r^2}\right] \quad (A.10)$$

$$V\frac{\partial W}{\partial r} + \frac{W\partial W}{r\partial z} + U\frac{\partial W}{\partial x} + \frac{VW}{r} = -\frac{1\,\partial P}{\rho r\partial z} + \nu\left[\nabla^2 W + \frac{2\,\partial V}{r^2\partial z} - \frac{W}{r^2}\right] \quad (A.11)$$

The continuity equation in cylindrical co-ordinate is

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial r} + \frac{\partial W}{r\partial z} + \frac{V}{r} = 0 \quad (A.12)$$

Multiplying equations (A.9), (A.10) and (A.11) by 2U, 2V and 2W respectively and using continuity equation (A.12) gives·

$$\frac{\partial U^3}{\partial x} + \frac{\partial U^2 V}{\partial r} + \frac{1\partial U^2 W}{r\partial z} + \frac{U^2 V}{r} = -\frac{2U\partial P}{\rho\partial x} + 2\nu[-\frac{1}{2}\nabla^2 U^2 - (\frac{\partial U}{\partial x})^2 - (\frac{\partial U}{\partial r})^2 - (\frac{1\partial U}{r\partial z})^2] \quad (A.13)$$

$$\frac{\partial UV^2}{\partial x} + \frac{\partial V^3}{\partial r} + \frac{1\partial V^2 W}{r\partial z} + \frac{V(V^2 - 2W^2)}{r} = -\frac{2V\partial P}{\rho\partial r} + 2\nu[-\frac{1}{2}\nabla^2 V^2 - (\frac{\partial V}{\partial x})^2 - (\frac{\partial V}{\partial r})^2$$

$$- (\frac{1\partial V}{r\partial z})^2 - \frac{V^2}{r^2} - \frac{2V\partial W}{r^2\partial z}] \quad (A.14)$$

$$\frac{\partial UW^2}{\partial x} + \frac{\partial VW^2}{\partial r} + \frac{1\partial W^3}{r\partial z} + \frac{3VW^2}{r} = -\frac{2W\partial P}{\rho r\partial z} + 2\nu[-\frac{1}{2}\nabla^2 W^2 - (\frac{\partial W}{\partial x})^2 - (\frac{\partial W}{\partial r})^2 - (\frac{1\partial W}{r\partial z})^2$$

$$+ \frac{2W\partial V}{r^2\partial z} - \frac{W^2}{r^2}] \quad (A.15)$$

Adding equations (A.13), (A.14) and (A.15) and introducing co-ordinates x=x, r=r and rz=z, gives:

$$\frac{\partial}{\partial x}[U^3 + UV^2 + UW^2] + \frac{1\partial}{r\partial r}r[U^2V + V^3 + VW^2] + \frac{\partial}{\partial z}[U^2W + V^2W + W^3] + \frac{2}{\rho}[U\frac{\partial P}{\partial x} + V\frac{\partial P}{\partial r}$$

$$+ W\frac{\partial P}{z}] = 2\nu[-\frac{1}{2}\nabla^2 (U^2 + V^2 + W^2) - (\frac{\partial U}{\partial x})^2 - (\frac{\partial U}{\partial r})^2 - (\frac{\partial U}{\partial z})^2 - (\frac{\partial V}{\partial x})^2 - (\frac{\partial V}{\partial r})^2$$

$$- \left(\frac{\partial V}{\partial z}\right)^2 - \left(\frac{\partial W}{\partial x}\right)^2 - \left(\frac{\partial W}{\partial r}\right)^2 - \left(\frac{\partial W}{\partial z}\right)^2 - \frac{1}{r^2}(V^2 + W^2) - \frac{2}{r}\left(V\frac{\partial W}{\partial z} - W\frac{\partial V}{\partial z}\right)\Big] \qquad (A.16)$$

Decomposing the instantaneous quantaties into mean and fluctuating parts.

$$U = \bar{U} + u \ ; \quad V = \bar{V} + v \ ; \quad W = \bar{W} + w \ ; \quad P = \bar{P} + p \qquad (A.17)$$

The prime (′) over the fluctuating parts are not shown in the following text (Appendix A). However prime was retained over the fluctuating pressure, p, in order to avoid any confusion between P and p. Introducing (A.17) into equations (A.13), (A.14) and (A.15); averaging and adding them, results in the total energy equation:

$$\frac{\partial}{\partial x}[\bar{U}^3 + \bar{V}^2\bar{U} + \bar{W}^2\bar{U} + 2\bar{V}\,uv + 2\bar{W}\,uw] + \frac{\partial}{\partial x}[3\bar{U}\,u^2 + u^3 + \bar{U}\,v^2 + uv^2 + \bar{U}\,w^2 + uw^2]$$

$$+ \frac{1}{r}\frac{\partial}{\partial r}r[vu^2 + vw^2 + v^3] + \frac{1}{r}\frac{\partial}{\partial r}r[\bar{U}^2\,\bar{V}$$

$$+ \bar{V}^3 + \bar{W}^2\,\bar{V} + \bar{V}\,u^2 + \bar{V}\,w^2 + 3\bar{V}\,v^2 + 2\bar{U}\,uv + 2\bar{W}\,vw] + \frac{\partial}{\partial z}[\bar{W}\,\bar{U}^2 + \bar{W}\,\bar{V}^2 + \bar{W}^3] +$$

$$\frac{\partial}{\partial z}[2\bar{U}\,uw + 2\bar{V}\,vw + 3\bar{W}\,w^2 + \bar{W}\,u^2 + \bar{W}\,v^2 + w^3 + v^2w + u^2w] + \frac{2}{\rho}[\bar{U}\frac{\partial\bar{P}}{\partial x} + \bar{V}\frac{\partial\bar{P}}{\partial r} + \bar{W}\frac{\partial\bar{P}}{\partial z}]$$

$$+ \frac{2}{\rho}[u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial r} + w\frac{\partial p}{\partial z}] \quad = \quad 2\nu[\frac{1}{2}\nabla^2(u^2 + v^2 + w^2) + \frac{1}{2}\nabla^2(\bar{U}^2 + \bar{V}^2 + \bar{W}^2) - \left(\frac{\partial u}{\partial x}\right)^2$$

$$
-\overline{\left(\frac{\partial u}{\partial r}\right)^2}-\overline{\left(\frac{\partial u}{\partial z}\right)^2}-\overline{\left(\frac{\partial v}{\partial x}\right)^2}-\overline{\left(\frac{\partial v}{\partial r}\right)^2}-\overline{\left(\frac{\partial v}{\partial z}\right)^2}-\overline{\left(\frac{\partial w}{\partial x}\right)^2}-\overline{\left(\frac{\partial w}{\partial r}\right)^2}-\overline{\left(\frac{\partial w}{\partial z}\right)^2}-\left(\frac{\partial \bar{U}}{\partial x}\right)^2-\left(\frac{\partial \bar{U}}{\partial r}\right)^2
$$

$$
-\left(\frac{\partial \bar{U}}{\partial z}\right)^2-\left(\frac{\partial \bar{V}}{\partial x}\right)^2-\left(\frac{\partial \bar{V}}{\partial r}\right)^2-\left(\frac{\partial \bar{V}}{\partial z}\right)^2-\left(\frac{\partial \bar{W}}{\partial x}\right)^2-\left(\frac{\partial \bar{W}}{\partial r}\right)^2-\left(\frac{\partial \bar{W}}{\partial z}\right)^2-\frac{1}{r^2}\left[\bar{V}^2+\bar{W}^2+\overline{v^2}+\overline{w^2}\right]
$$

$$
-\frac{2}{r}\left[\bar{V}\frac{\partial \bar{W}}{\partial z}-\bar{W}\frac{\partial \bar{V}}{\partial z}+\overline{v\frac{\partial w}{\partial z}}-\overline{w\frac{\partial v}{\partial z}}\right]] \qquad (A.18)
$$

Mean momentum equation for steady flow of an incompressible fluid without a body force in cylindrical co-ordinate is given by

$$
\bar{U}\frac{\partial \bar{U}}{\partial x}+\bar{V}\frac{\partial \bar{U}}{\partial r}+\frac{\bar{W}\partial \bar{U}}{r\partial z}+\frac{\partial \overline{u^2}}{\partial x}+\frac{\partial \overline{uv}}{\partial r}+\frac{1\partial \overline{uw}}{r\partial z}+\frac{\overline{uv}}{r}+\frac{1\partial \bar{P}}{\rho\partial x}=\nu\nabla^2\bar{U} \qquad (A.19)
$$

$$
\bar{U}\frac{\partial \bar{V}}{\partial x}+\bar{V}\frac{\partial \bar{V}}{\partial r}+\frac{\bar{W}\partial \bar{V}}{r\partial z}-\frac{\bar{W}^2}{r}+\frac{\partial \overline{uv}}{\partial x}+\frac{\partial \overline{v^2}}{\partial r}+\frac{1\partial \overline{vw}}{r\partial z}+\frac{\overline{v^2}-\overline{w^2}}{r}+\frac{1\partial \bar{P}}{\rho\partial r}=\nu\left[\nabla^2\bar{V}-\frac{\bar{V}}{r^2}-\frac{2\partial \bar{W}}{r^2\partial z}\right] \qquad (A.20)
$$

$$
\bar{U}\frac{\partial \bar{W}}{\partial x}+\bar{V}\frac{\partial \bar{W}}{\partial r}+\frac{\bar{W}\partial \bar{W}}{r\partial z}+\frac{\bar{V}\bar{W}}{r}+\frac{\partial \overline{uw}}{\partial x}+\frac{\partial \overline{vw}}{\partial r}+2\frac{\overline{vw}}{r}+\frac{1\partial \overline{w^2}}{r\partial z}+\frac{1\partial \bar{P}}{\rho r\partial z}=\nu\left[\nabla^2\bar{W}-\frac{\bar{W}}{r^2}+\frac{2\partial \bar{V}}{r^2\partial z}\right] \qquad (A.21)
$$

Multiplying equations (A.19), (A.20) and (A.21) by $2\bar{U}$, $2\bar{V}$ and $2\bar{W}$ then using continuity equations, gives:

$$\frac{\partial \bar{U}^3}{\partial x} + \frac{\partial \bar{U}^2\bar{V}}{\partial r} + \frac{1}{r}\frac{\partial \bar{U}^2\bar{W}}{\partial z} + \frac{\bar{U}^2\bar{V}}{r} + 2\frac{\bar{U}\partial \bar{P}}{\rho\,\partial x} + 2\bar{U}\left[\frac{\partial \overline{u^2}}{\partial x} + \frac{\partial \overline{uv}}{\partial r} + \frac{1}{r}\frac{\partial \overline{uw}}{\partial z} + \frac{\overline{uv}}{r}\right] = 2\nu\left[\frac{1}{2}\nabla^2\bar{U}^2 - \left(\frac{\partial \bar{U}}{\partial x}\right)^2 - \left(\frac{\partial \bar{U}}{\partial r}\right)^2 - \left(\frac{1}{r}\frac{\partial \bar{U}}{\partial z}\right)^2\right] \quad (A.22)$$

$$\frac{\partial \overline{UV^2}}{\partial x} + \frac{\partial \bar{V}^3}{\partial r} + \frac{1}{r}\frac{\partial \bar{V}^2\bar{W}}{\partial z} + \frac{\bar{V}}{r}(\bar{V}^2 - 2\bar{W}^2) + 2\frac{\bar{V}\partial \bar{P}}{\rho\,\partial r} + 2\bar{V}\left[\frac{\partial \overline{uv}}{\partial x} + \frac{\partial \overline{v^2}}{\partial r} + \frac{1}{r}\frac{\partial \overline{vw}}{\partial z} + \frac{\overline{v^2} - \overline{w^2}}{r}\right]$$

$$= 2\nu\left[\frac{1}{2}\nabla^2\bar{V}^2 - \left(\frac{\partial \bar{V}}{\partial x}\right)^2 - \left(\frac{\partial \bar{V}}{\partial r}\right)^2 - \left(\frac{1}{r}\frac{\partial \bar{V}}{\partial z}\right)^2 - \frac{\bar{V}^2}{r^2} - \frac{2\bar{V}\partial \bar{W}}{r^2\,\partial z}\right] \quad (A.23)$$

$$\frac{\partial \overline{UW^2}}{\partial x} + \frac{\partial \overline{VW^2}}{\partial r} + \frac{1}{r}\frac{\partial \bar{W}^3}{\partial z} + 3\bar{V}\frac{\bar{W}^2}{r} + \frac{2\bar{W}\partial \bar{P}}{\rho\, r\,\partial z} + 2\bar{W}\left[\frac{\partial \overline{uw}}{\partial x} + \frac{\partial \overline{vw}}{\partial r} + \frac{1}{r}\frac{\partial \overline{w^2}}{\partial z} + \frac{2\overline{vw}}{r}\right]$$

$$= 2\nu\left[\frac{1}{2}\nabla^2\bar{W}^2 - \left(\frac{\partial \bar{W}}{\partial x}\right)^2 - \left(\frac{\partial \bar{W}}{\partial r}\right)^2 - \left(\frac{1}{r}\frac{\partial \bar{W}}{\partial z}\right)^2 + \left(\frac{2\bar{W}\partial \bar{V}}{r^2\,\partial z}\right) - \frac{\bar{W}^2}{r^2}\right] \quad (A.24)$$

By adding equations (A.19), (A.20) and (A.21) the following mean energy equation is obtained:

$$\frac{\partial}{\partial x}[\bar{U}^3 + \overline{UV^2} + \overline{UW^2}] + \frac{1}{r}\frac{\partial}{\partial r}[r(\bar{U}^2\bar{V} + \bar{V}^3 + \overline{VW^2})] + \frac{\partial}{\partial z}(\bar{U}^2\bar{W} + \bar{V}^2\bar{W} + \bar{W}^3) + \frac{2}{\rho}\left(\bar{U}\frac{\partial \bar{P}}{\partial x} + \right.$$

$$\bar{V}\frac{-\partial \bar{P}}{\partial r} + \bar{W}\frac{-\partial \bar{P}}{\partial z}) = -2\frac{\partial}{\partial x}(\bar{U}\,\overline{u^2} + \bar{V}\,\overline{uv} + \bar{W}\,\overline{uw}) + 2\overline{u^2}\frac{\partial \bar{U}}{\partial x} + 2\overline{uv}\frac{\partial \bar{V}}{\partial x} + 2\overline{uw}\frac{-\partial \bar{W}}{\partial x} - \frac{2\partial}{r\partial r}[r(\bar{U}\,\overline{uv}$$

$$+ \overline{v^2}\,\bar{V} + \bar{W}\,\overline{vw})] + 2\overline{uv}\frac{\partial \bar{U}}{\partial r} + 2\overline{v^2}\frac{\partial \bar{V}}{\partial r} + 2\overline{vw}\frac{\partial \bar{W}}{\partial r} - 2\frac{\partial}{\partial z}(\bar{U}\,\overline{uw} + \bar{V}\,\overline{vw} + \bar{W}\,\overline{w^2}) + 2\overline{uw}\frac{\partial \bar{U}}{\partial z}$$

$$+ 2\overline{vw}\frac{\partial \bar{V}}{\partial z} + 2\overline{w^2}\frac{\partial \bar{W}}{\partial z} + 2\overline{w^2}\frac{\bar{V}}{r} - 2\bar{W}\frac{-\overline{vw}}{r} + 2\nu[-\frac{1}{2}(\nabla^2\bar{U}^2 + \nabla^2\bar{V}^2 + \nabla^2\bar{W}^2) - (\frac{\partial \bar{U}}{\partial x})^2 - (\frac{\partial \bar{U}}{\partial r})^2$$

$$- (\frac{\partial \bar{U}}{\partial r})^2 - (\frac{\partial \bar{U}}{\partial z})^2 - (\frac{\partial \bar{V}}{\partial x})^2 - (\frac{\partial \bar{V}}{\partial r})^2 - (\frac{\partial \bar{V}}{\partial z})^2 - (\frac{\partial \bar{W}}{\partial x})^2 - (\frac{\partial \bar{W}}{\partial r})^2 - (\frac{\partial \bar{W}}{\partial z})^2 - \frac{\bar{V}^2}{r^2} - \frac{2\bar{V}\partial \bar{W}}{r\partial z}$$

$$+ 2\frac{\bar{W}\partial \bar{V}}{r\partial z} - \frac{\bar{W}^2}{r^2}] \qquad (A.25)$$

Subtracting equation (A.25) from the equation (A.18) the turbulent kinetic energy equation results

$$\frac{\partial}{\partial x}(\bar{U}\,\overline{u^2} + \overline{u^3} + \bar{U}\,\overline{v^2} + \overline{uv^2} + \bar{U}\,\overline{w^2} + \overline{uw^2}) + 2\overline{u^2}\frac{\partial \bar{U}}{\partial x} + 2\overline{uv}\frac{\partial \bar{V}}{\partial x} + 2\overline{uw}\frac{\partial \bar{W}}{\partial x} + \frac{1\partial}{r\partial r}[r(\overline{vu^2}$$

$$+ \overline{vw^2} + \overline{v^3})] + \frac{1\partial}{r\partial r}[r(\bar{V}\,\overline{u^2} + \bar{V}\,\overline{w^2} + \bar{V}\,\overline{v^2})] + 2\overline{uv}\frac{\partial \bar{U}}{\partial r} + 2\overline{v^2}\frac{\partial \bar{V}}{\partial r} + 2\overline{vw}\frac{\partial \bar{W}}{\partial r} + \frac{\partial}{\partial z}(\bar{W}\,\overline{w^2}$$

$$+ \bar{W} u^2 + \bar{W} v^2 + w^3 + v^2 w + u^2 w) + 2uw\frac{\partial \bar{U}}{\partial z} + 2vw\frac{\partial \bar{V}}{\partial z} + 2w^2\frac{\partial \bar{W}}{\partial z} + 2\bar{V}\frac{-w^2}{r} - 2\bar{W}\frac{-vw}{r}$$

$$= -\frac{2}{\rho}\left(u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial r} + w\frac{\partial p}{\partial z}\right) + 2\nu\left[\frac{1}{2}\nabla^2(u^2 + v^2 + w^2) - \left(\frac{\partial u}{\partial x}\right)^2 - \left(\frac{\partial u}{\partial r}\right)^2 - \left(\frac{\partial u}{\partial z}\right)^2 - \left(\frac{\partial v}{\partial x}\right)^2\right.$$

$$\left. - \left(\frac{\partial v}{\partial r}\right)^2 - \left(\frac{\partial v}{\partial z}\right)^2 - \left(\frac{\partial w}{\partial x}\right)^2 - \left(\frac{\partial w}{\partial r}\right)^2 - \left(\frac{\partial w}{\partial z}\right)^2 - \frac{1}{r^2}(v^2 + w^2) - \frac{2}{r}\left(v\frac{\partial w}{\partial z} - w\frac{\partial v}{\partial z}\right)\right] \qquad (A.26)$$

Re-arranging equation (A.26) and dividing both sides by 2, gives

$$\frac{1}{2}\frac{\partial}{\partial x}[\bar{U}(u^2 + v^2 + w^2)] + \frac{1}{2r}\frac{\partial}{\partial r}[r\bar{V}(u^2 + v^2 + w^2)] + \frac{1}{2}\frac{\partial}{\partial z}[\bar{W}(u^2 + v^2 + w^2)] + \frac{1}{2}\frac{\partial}{\partial x}[u(u^2 +$$

$$v^2 + w^2) + \frac{1}{2r}\frac{\partial}{\partial r}[rv(u^2 + v^2 + w^2)] + \frac{1}{2}\frac{\partial}{\partial z}[w(u^2 + v^2 + w^2)] + u^2\frac{\partial \bar{U}}{\partial x} + v^2\frac{\partial \bar{V}}{\partial r} + w^2\frac{\partial \bar{W}}{\partial z}$$

$$+ uv\left(\frac{\partial \bar{V}}{\partial x} + \frac{\partial \bar{U}}{\partial r}\right) + uw\left(\frac{\partial \bar{W}}{\partial x} + \frac{\partial \bar{U}}{\partial z}\right) + vw\left(\frac{\partial \bar{V}}{\partial z} + \frac{\partial \bar{W}}{\partial r}\right) + \bar{V}\frac{-w^2}{r} - \bar{W}\frac{-vw}{r} + \frac{1}{\rho}\left[\frac{\partial pu}{\partial x} + \frac{1}{r}\frac{\partial}{\partial r}(rvp)\right.$$

$$\left. + \frac{\partial}{\partial z}(wp)\right] - \frac{\nu}{2}\nabla^2(u^2 + v^2 + w^2) + \nu\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial r}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial r}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right.$$

$$+ \left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial r}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2 \Big] + \frac{\nu}{r^2}(v^2 + w^2) + \frac{2\nu}{\rho r}\left(\frac{\partial vw}{\partial z} - 2w\frac{\partial v}{\partial z}\right) = 0 \qquad (A.27)$$

For axi-symmetry flow

$$\frac{1\partial}{r\partial z}(\;) = \frac{\partial}{\partial z}(\;) = \overline{W} = 0$$

From equation (A.25) the mean energy equation for a steady axi-symmetry and incompressible fluid flow without a body force is given by

$$\frac{1\partial}{2\partial x}(\overline{U}^3 + \overline{UV^2}) + \frac{1\partial}{2r\partial r}[r(\overline{U}^2\overline{V} + \overline{V}^3)] + \frac{1}{\rho}\Big[\frac{\partial}{\partial x}(\overline{UP}) + \frac{1\partial}{r\partial r}(r\overline{PV})\Big] + \frac{\partial}{\partial x}(\overline{U}\,\overline{u^2} + \overline{V}\,\overline{uv}) +$$

$$\frac{1\partial}{r\partial r}[r(\overline{U}\,\overline{uv} + \overline{V}\,\overline{v^2})] - \Big[\overline{u^2}\frac{\partial \overline{U}}{\partial x} + \overline{uv}\Big(\frac{\partial \overline{V}}{\partial x} + \frac{\partial \overline{U}}{\partial r}\Big) + \overline{v^2}\frac{\partial \overline{V}}{\partial r}\Big] - \overline{V}\frac{\overline{w^2}}{r} - \frac{\nu}{2}\nabla^2(\overline{U}^2 + \overline{V}^2) +$$

$$+ \Big[\Big(\frac{\partial \overline{U}}{\partial x}\Big)^2 + \Big(\frac{\partial \overline{U}}{\partial r}\Big)^2 + \Big(\frac{\partial \overline{V}}{\partial x}\Big)^2 + \Big(\frac{\partial \overline{V}}{\partial r}\Big)^2\Big] + \frac{\overline{V}^2}{r} = 0 \qquad (A.28)$$

Using continuity equation, (A.12), equation (A.28) can be written as:

$$\frac{U\partial}{2\partial x}(\overline{U}^2 + \overline{V}^2) + \frac{V\partial}{2\partial r}(\overline{U}^2 + \overline{V}^2) + \frac{U\partial P}{\rho\,\partial x} + \frac{V\partial P}{\rho\,\partial r} + \frac{\partial}{\partial x}(\overline{U}\overline{u^2} + \overline{V}\overline{uv}) +$$

$$\frac{1}{r}\frac{\partial}{\partial r}[r(\bar{U}\,\overline{uv} + \bar{V}\,\overline{v^2})] - [\overline{u^2}\frac{\partial \bar{U}}{\partial x} + \overline{uv}(\frac{\partial \bar{V}}{\partial x} + \frac{\partial \bar{U}}{\partial r}) + \overline{v^2}\frac{\partial \bar{V}}{\partial r}] - \bar{V}\frac{\overline{w^2}}{r} - \frac{\nu}{2}\nabla^2(\bar{U}^2 + \bar{V}^2) +$$

$$+\nu[(\frac{\partial \bar{U}}{\partial x})^2 + (\frac{\partial \bar{U}}{\partial r})^2 + (\frac{\partial \bar{V}}{\partial x})^2 + (\frac{\partial \bar{V}}{\partial r})^2] + \frac{\bar{V}^2}{r} = 0 \qquad (A.29)$$

From equation (A.27) the turbulent kinetic energy equation for a steady axi-symmetry flow may be written as:

$$\frac{1}{2}\frac{\partial}{\partial x}[\bar{U}(\overline{u^2} + \overline{v^2} + \overline{w^2})] + \frac{1}{2r}\frac{\partial}{\partial r}[r\bar{V}(\overline{u^2} + \overline{v^2} + \overline{w^2})] + \frac{1}{2}\frac{\partial}{\partial x}[\overline{u(u^2 + v^2 + w^2)}] +$$

$$\frac{1}{2r}\frac{\partial}{\partial r}[\overline{rv(u^2 + v^2 + w^2)}] + \overline{u^2}\frac{\partial \bar{U}}{\partial x} + \overline{v^2}\frac{\partial \bar{V}}{\partial r} + \overline{uv}(\frac{\partial \bar{V}}{\partial x} + \frac{\partial \bar{U}}{\partial r}) + \bar{V}\frac{\overline{w^2}}{r} + \frac{1}{\rho}[\frac{\partial}{\partial x}(\overline{pu}) + \frac{1}{r}\frac{\partial}{\partial r}(r\overline{pv})]$$

$$- \nabla^2\frac{\nu}{2}(\overline{u^2} + \overline{v^2} + \overline{w^2}) + \nu[\overline{(\frac{\partial u}{\partial x})^2} + \overline{(\frac{\partial u}{\partial r})^2} + \overline{(\frac{\partial u}{\partial z})^2} + \overline{(\frac{\partial v}{\partial x})^2} + \overline{(\frac{\partial v}{\partial r})^2} + \overline{(\frac{\partial v}{\partial z})^2} + \overline{(\frac{\partial w}{\partial x})^2}$$

$$+ \overline{(\frac{\partial w}{\partial r})^2} + \overline{(\frac{\partial w}{\partial z})^2}] + \frac{\nu}{r^2}(\overline{v^2} + \overline{w^2}) - 4\frac{\nu}{r}(\overline{w\frac{\partial v}{\partial z}}) = 0 \qquad (A.30)$$

Using the continuity equation, (A.12), equation (A.30) can be written as:

$$\frac{\bar{U}}{2}\frac{\partial}{\partial x}(\overline{u^2} + \overline{v^2} + \overline{w^2}) + \frac{\bar{V}}{2}\frac{\partial}{\partial r}(\overline{u^2} + \overline{v^2} + \overline{w^2}) + \frac{1}{2}\frac{\partial}{\partial x}[\overline{u(u^2 + v^2 + w^2)}] +$$

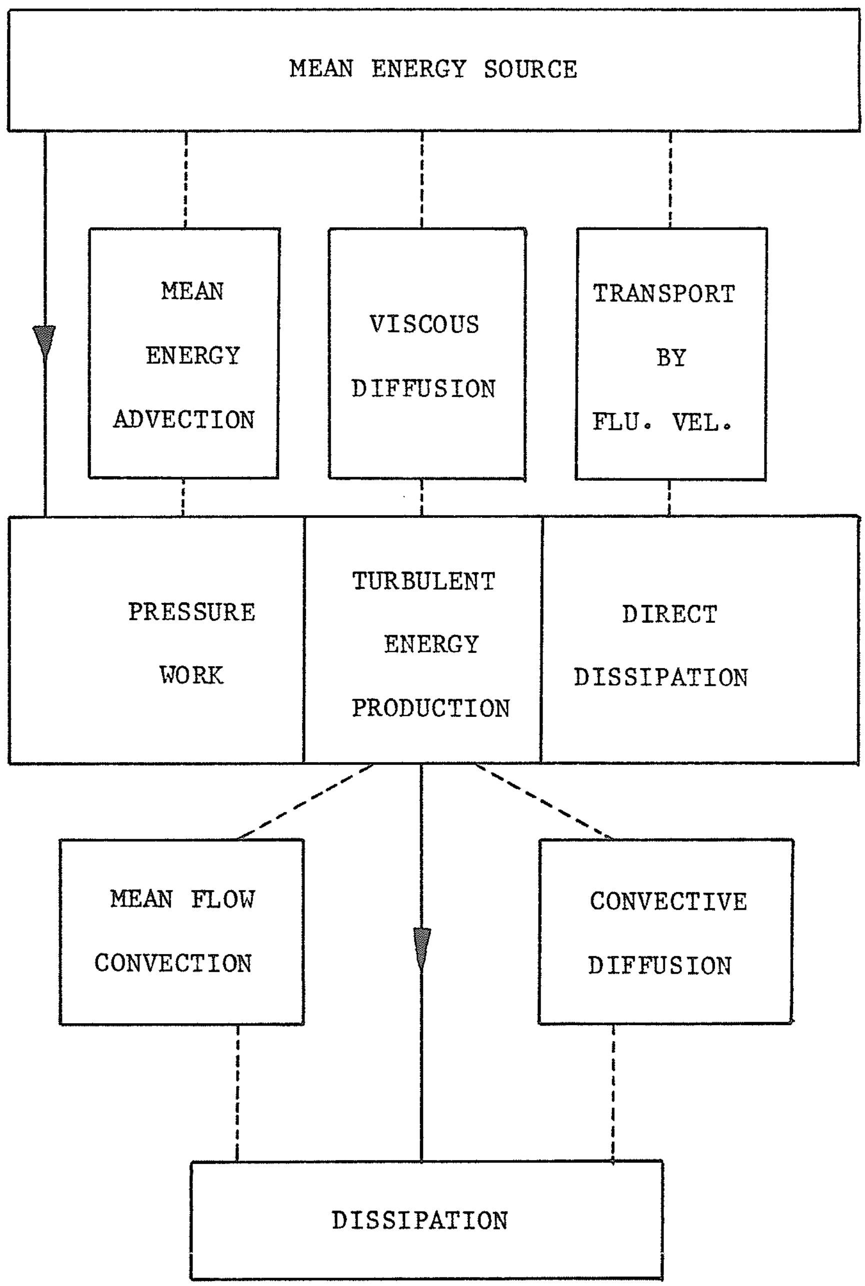
$$\frac{1}{2r}\frac{\partial}{\partial r}[r\overline{v(u^2 + v^2 + w^2)}] + \overline{u^2}\frac{\partial \bar{U}}{\partial x} + \overline{v^2}\frac{\partial \bar{V}}{\partial r} + \overline{uv}\left(\frac{\partial \bar{V}}{\partial x} + \frac{\partial \bar{U}}{\partial r}\right) + \frac{-\overline{w^2}\,\bar{V}}{r} + \frac{1}{\rho}\left[\frac{\partial}{\partial x}(\overline{pu}) + \frac{1}{r}\frac{\partial}{\partial r}(r\overline{pv})\right]$$

$$- \frac{\nu}{2}\nabla^2(\overline{u^2 + v^2 + w^2}) + \nu\left[\overline{\left(\frac{\partial u}{\partial x}\right)^2} + \overline{\left(\frac{\partial u}{\partial r}\right)^2} + \overline{\left(\frac{\partial u}{\partial z}\right)^2} + \overline{\left(\frac{\partial v}{\partial x}\right)^2} + \overline{\left(\frac{\partial v}{\partial r}\right)^2} + \overline{\left(\frac{\partial v}{\partial z}\right)^2} + \overline{\left(\frac{\partial w}{\partial x}\right)^2}\right.$$

$$\left. + \overline{\left(\frac{\partial w}{\partial r}\right)^2} + \overline{\left(\frac{\partial w}{\partial z}\right)^2}\right] + \frac{\nu}{r^2}(\overline{v^2 + w^2}) - 4\frac{\nu}{r}\left(\overline{w\frac{\partial v}{\partial z}}\right) = 0 \qquad (A.31)$$

Appendix B

ENERGY BLOCK DIAGRAM



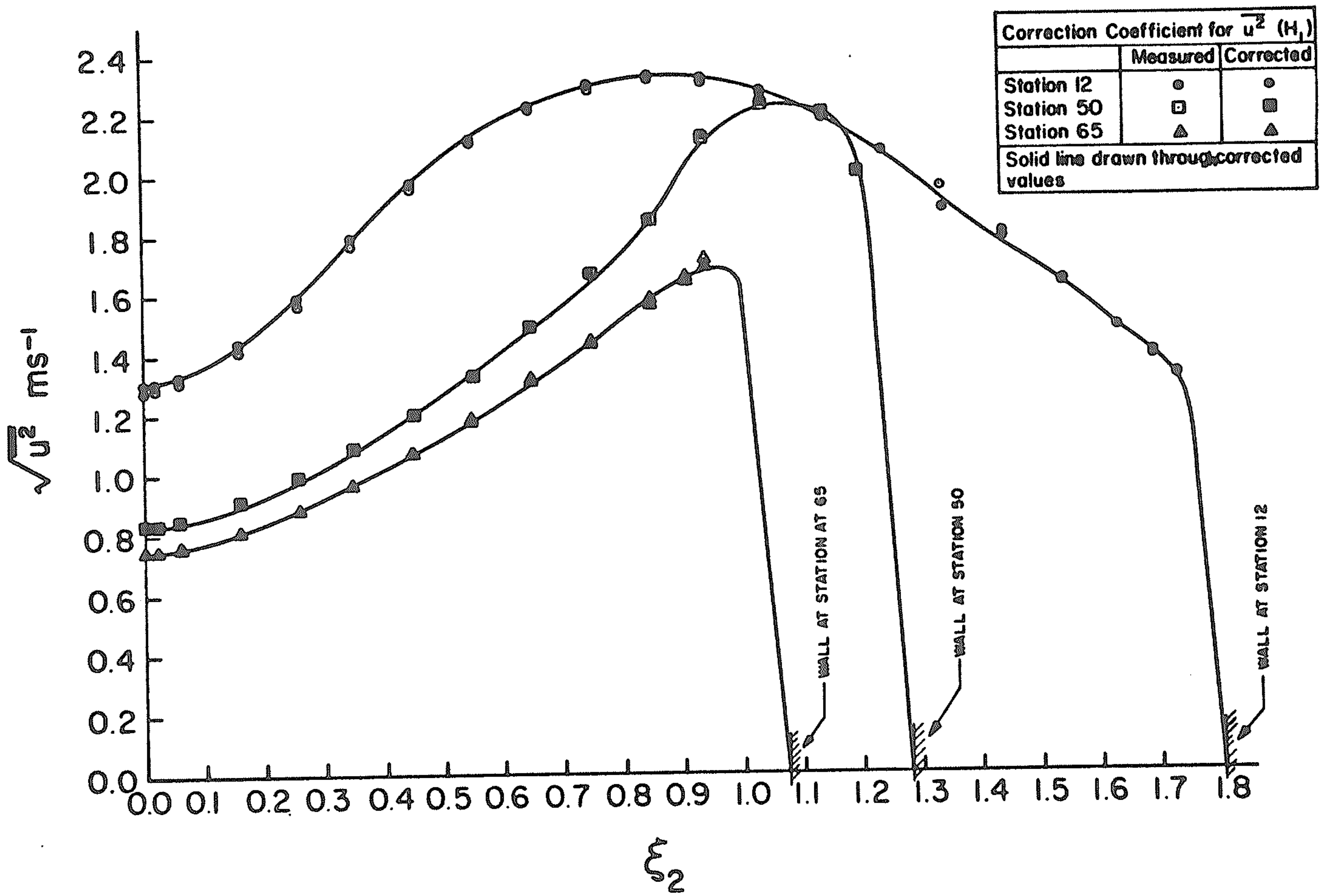


Figure 1· Corrected and un-corrected x-direction fluctuating velocity

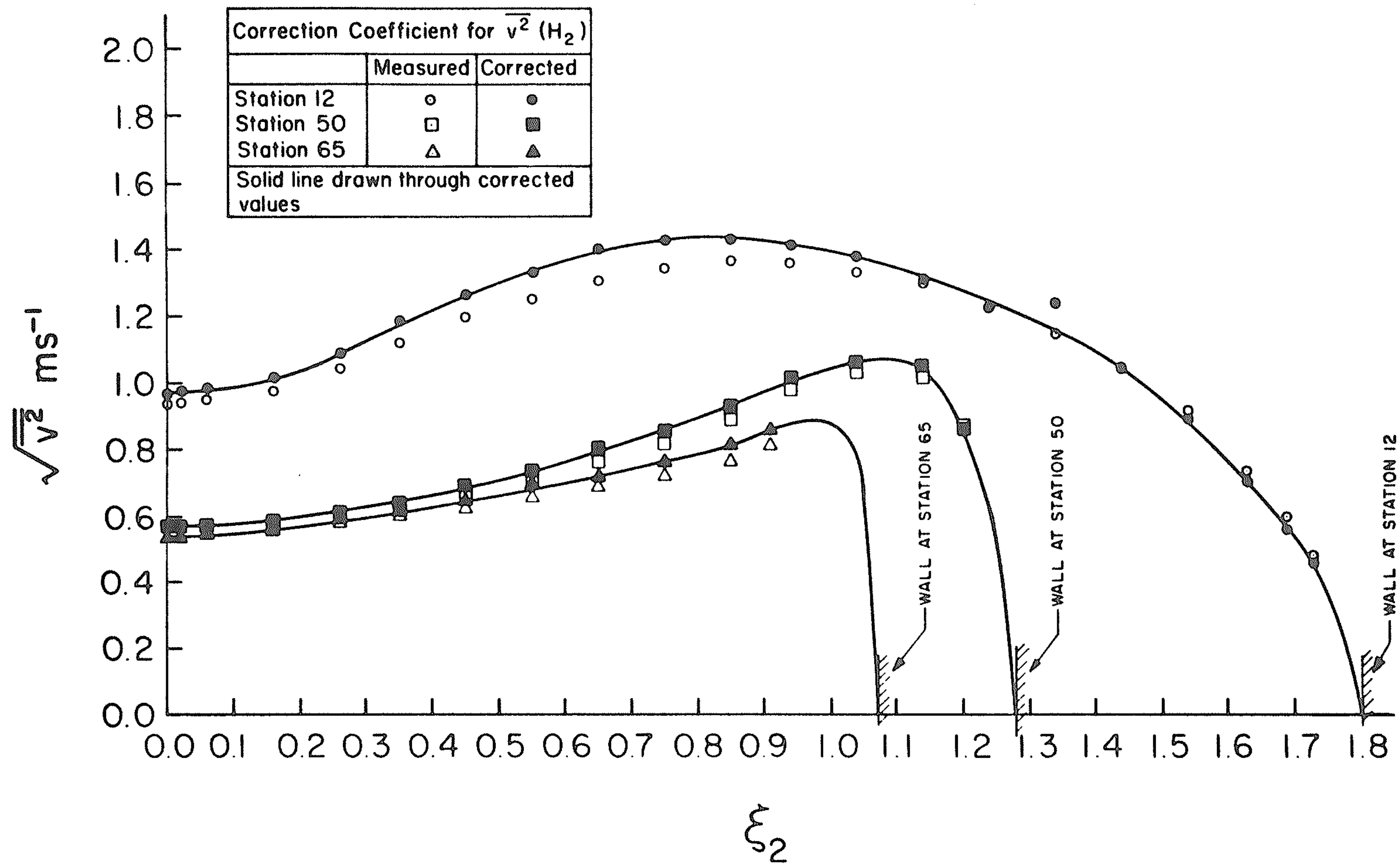


Figure 2: Corrected and un-corrected r-direction fluctuating velocity

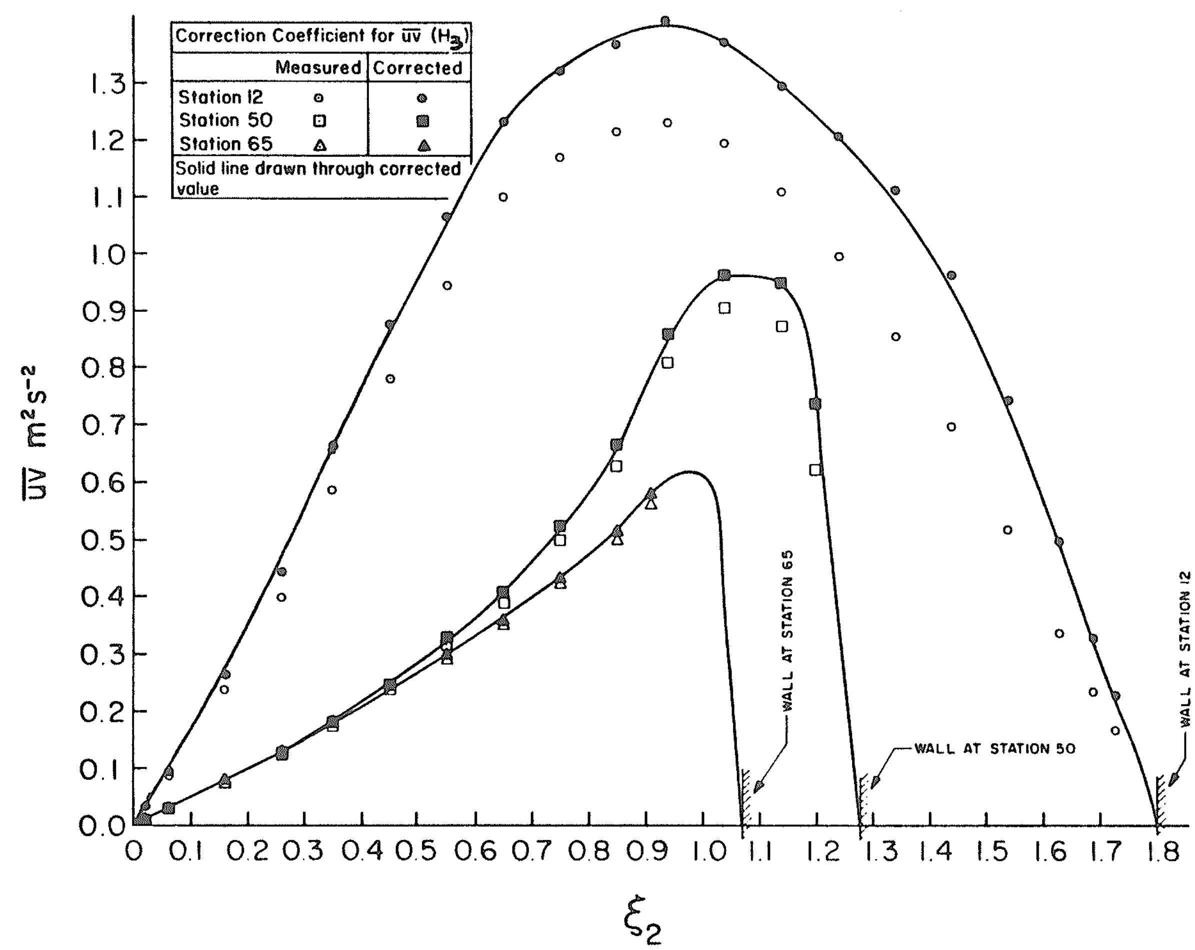


Figure 3: Corrected and un-corrected turbulent shear stress

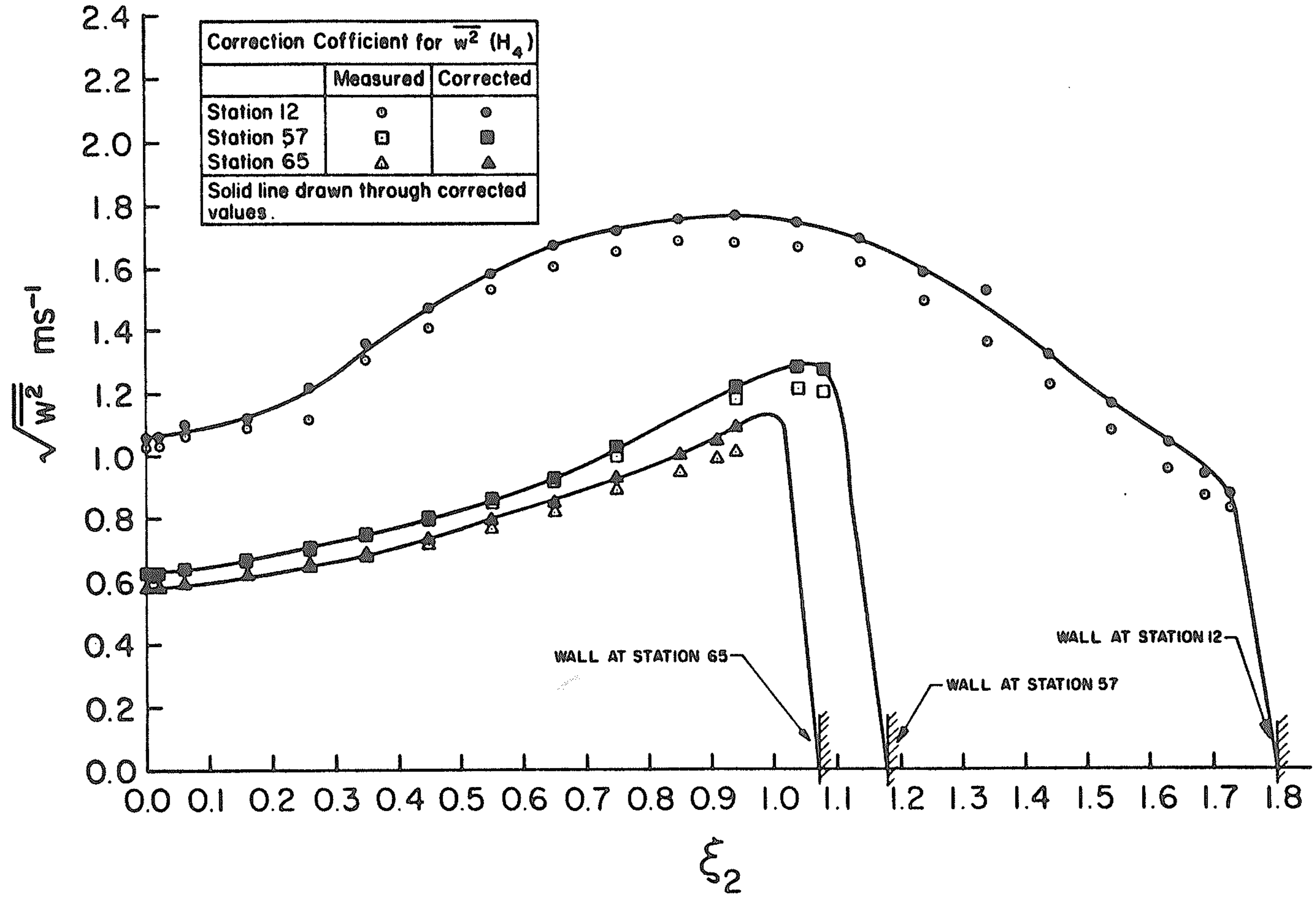


Figure 4: Corrected and un-corrected z-direction fluctuating velocity

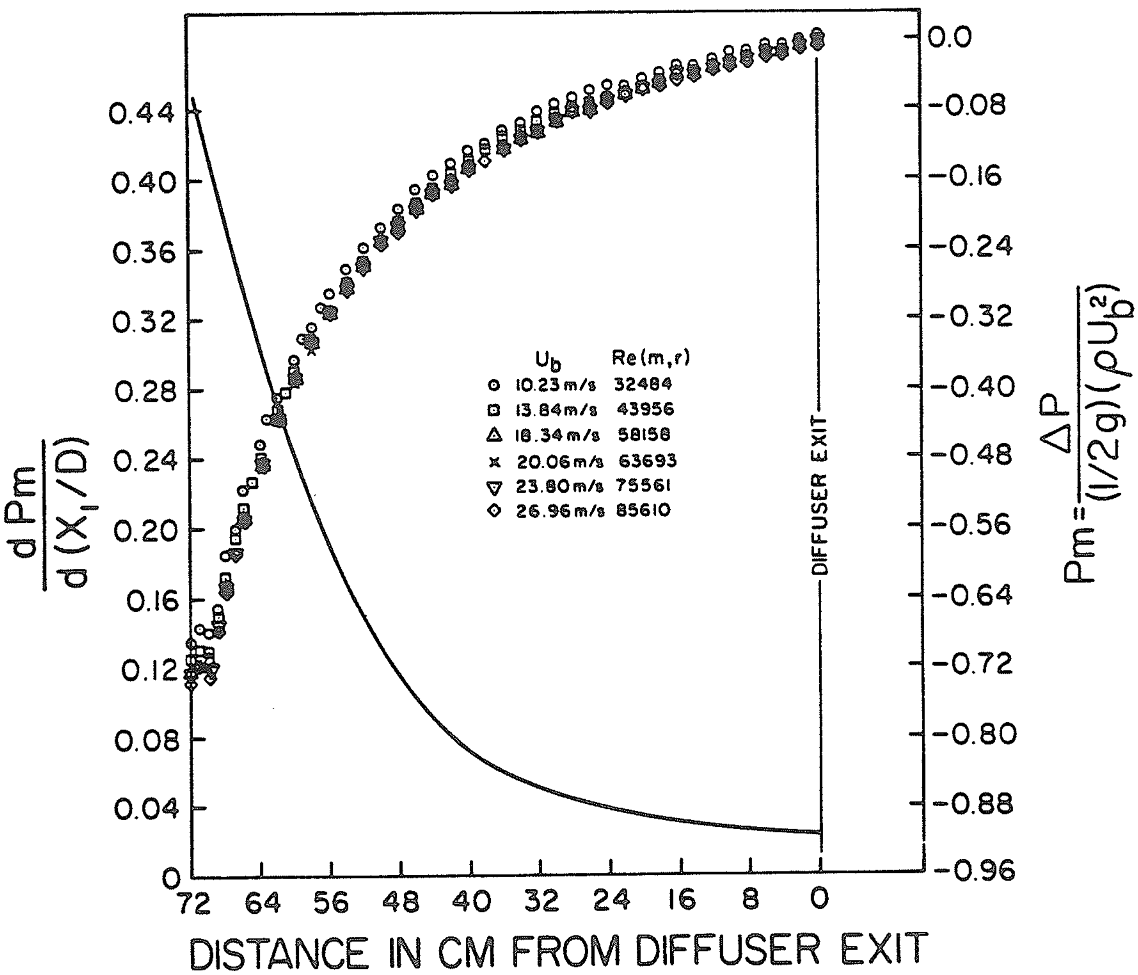


Figure 5: Mean static pressure and pressure gradient distribution

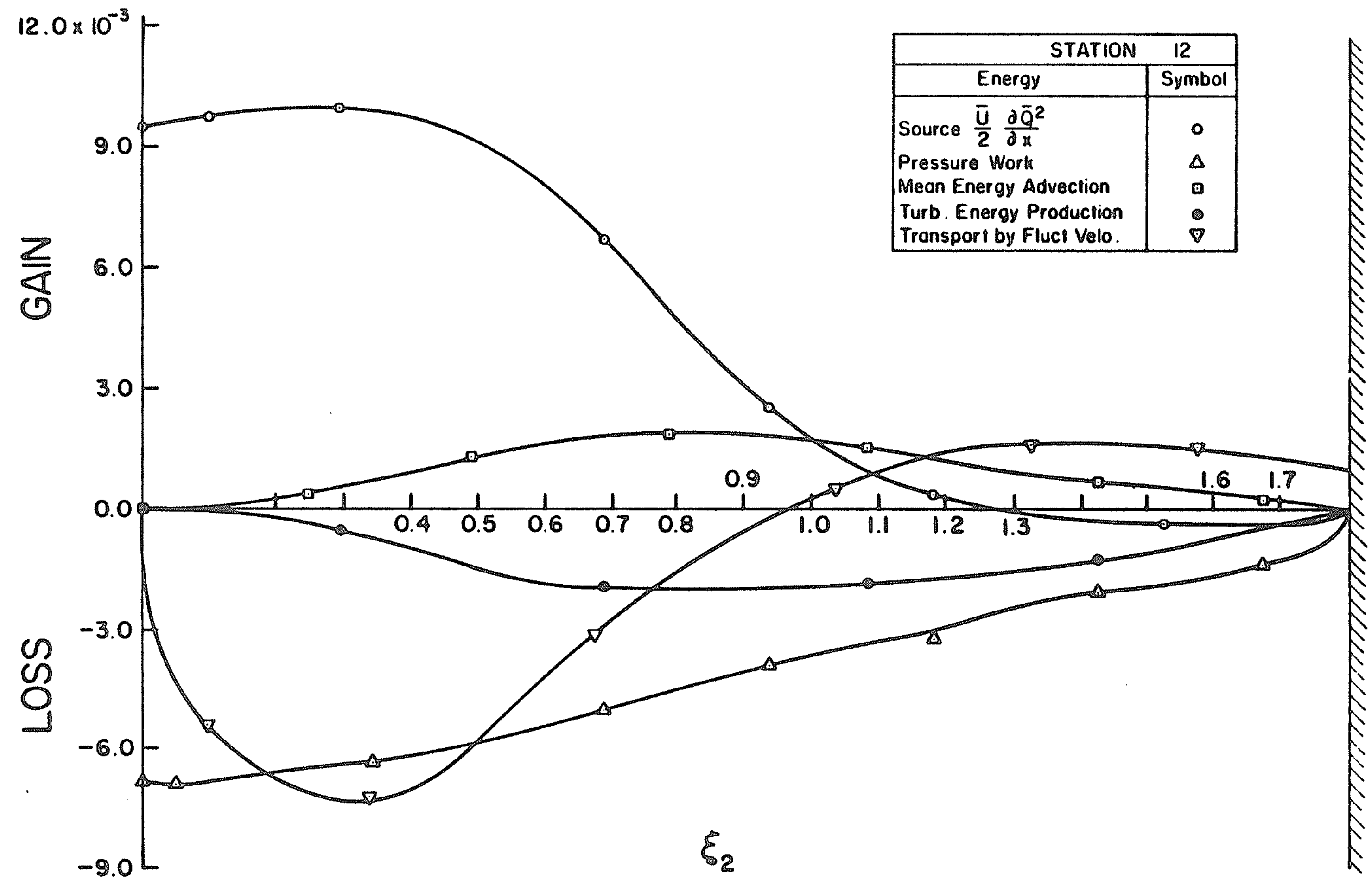


Figure 6: Mean kinetic energy balance at station 12

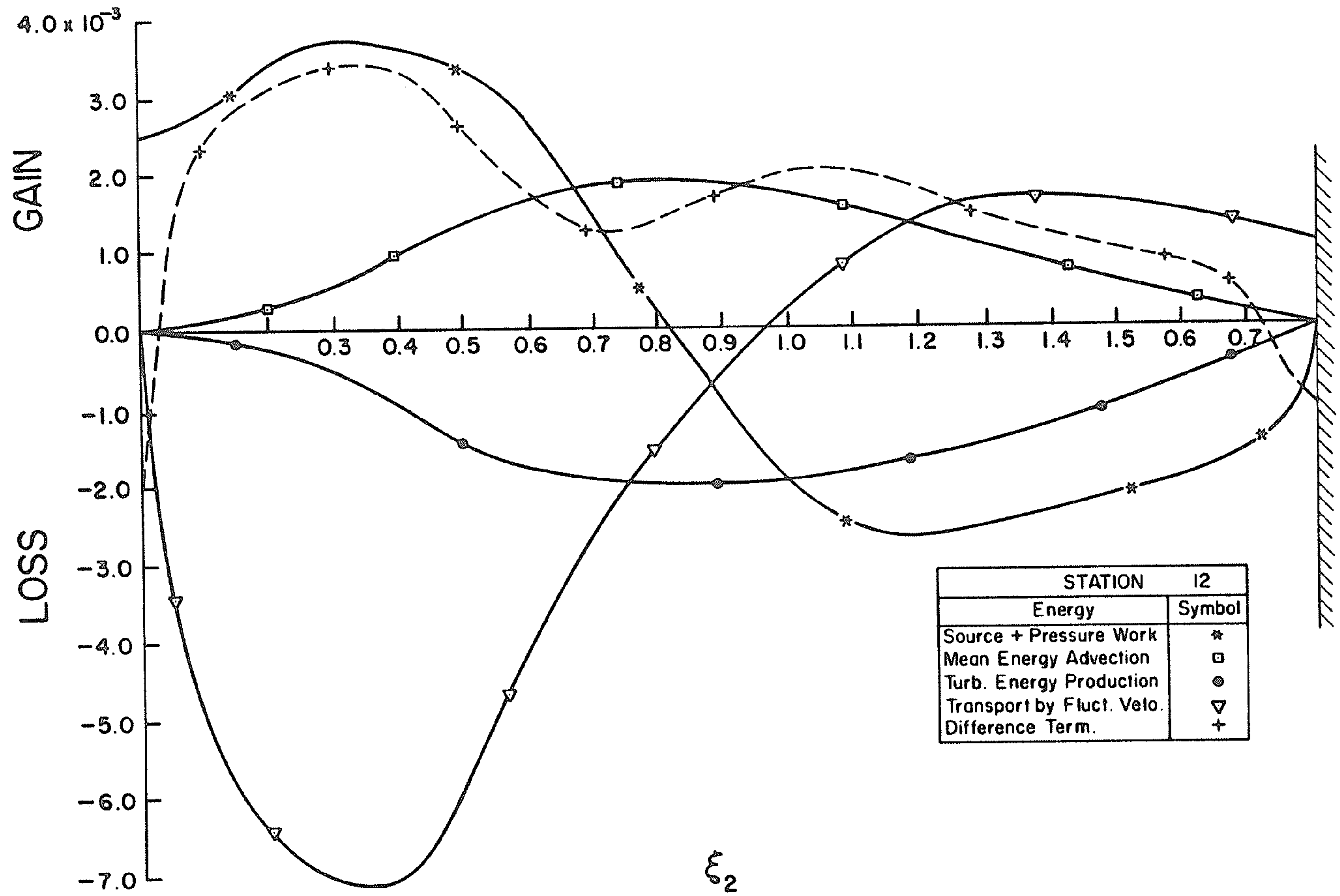


Figure 7: Mean kinetic energy balance at station 12

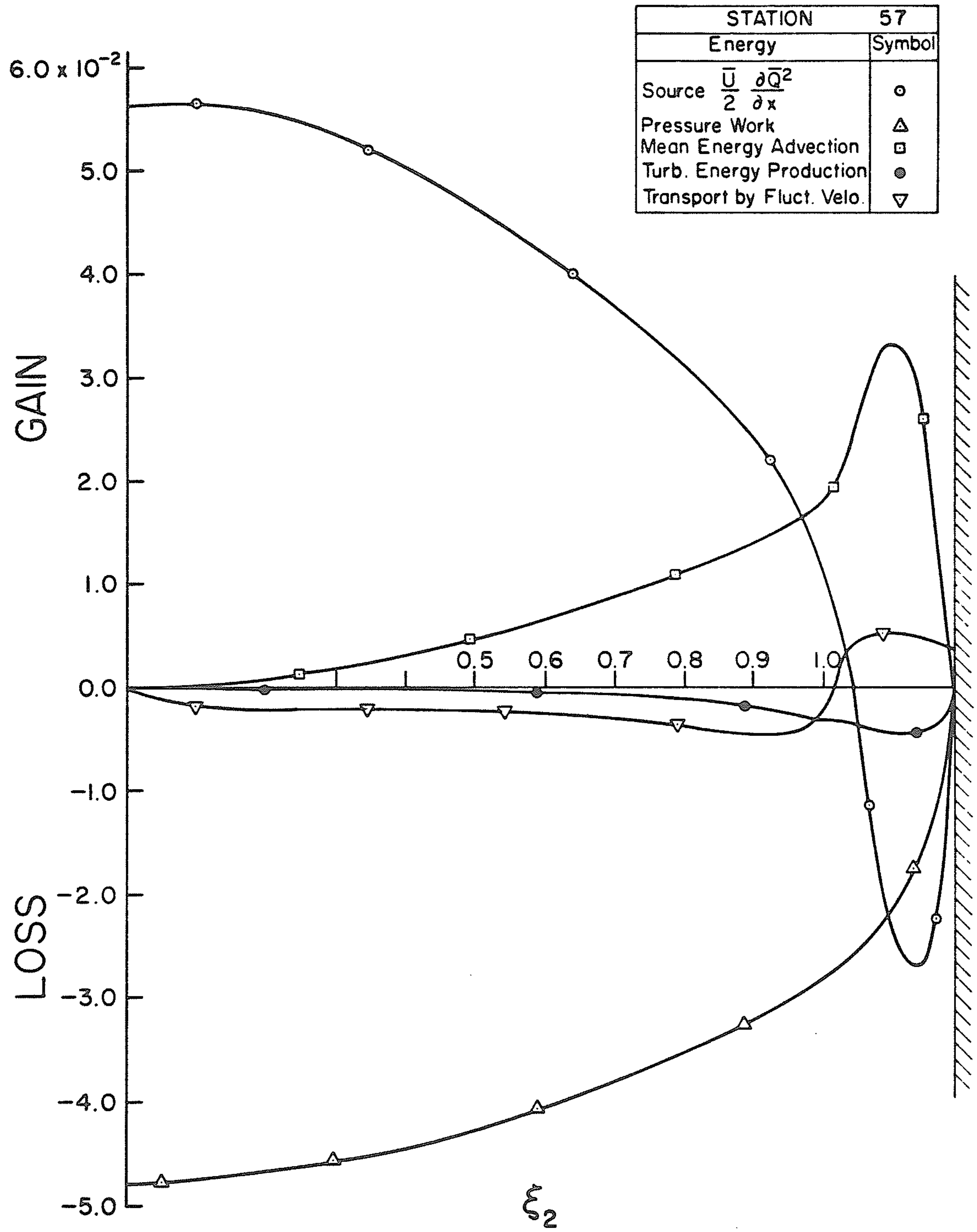


Figure 8: Mean kinetic energy balance at station 57

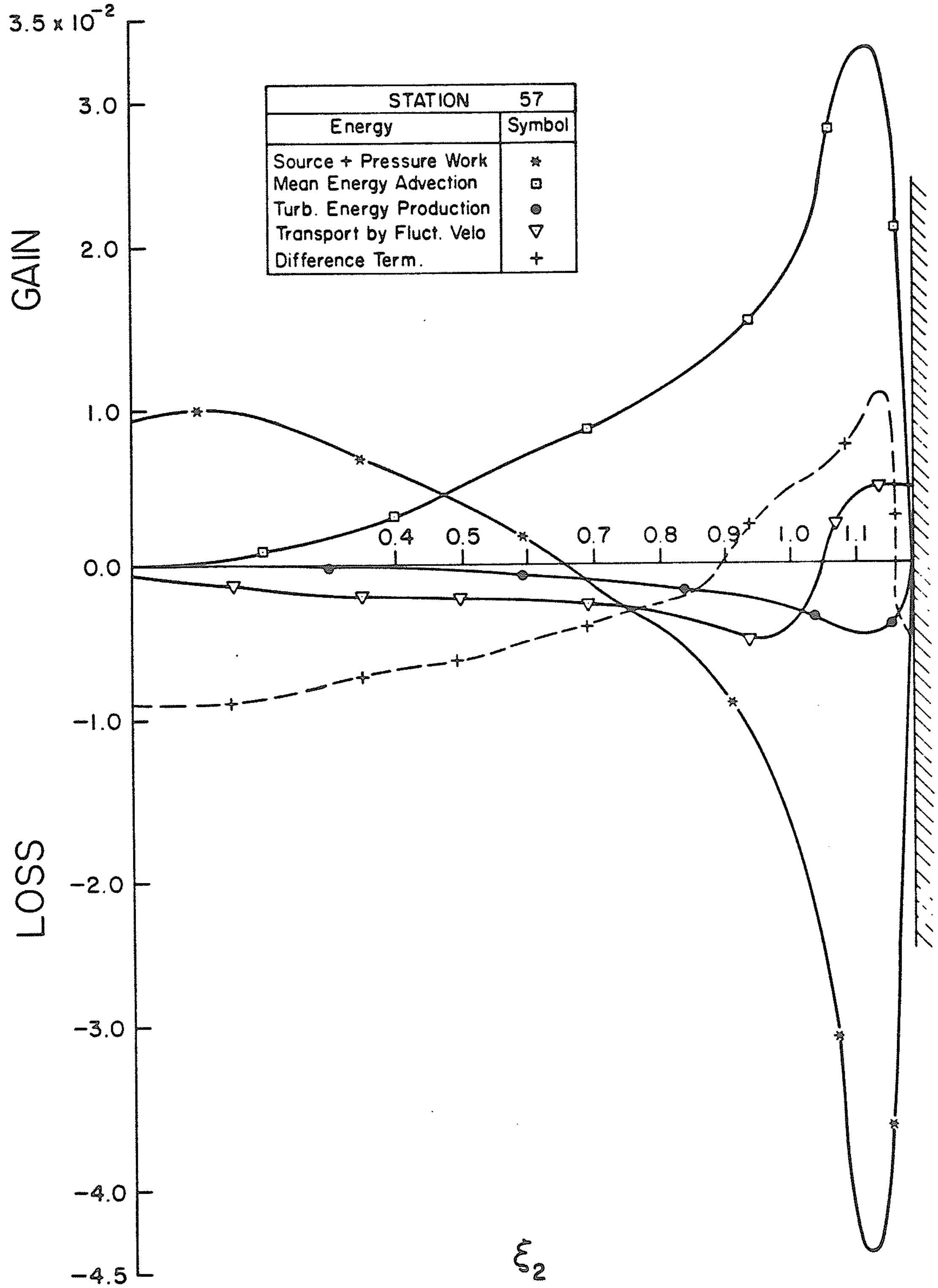


Figure 9: Mean kinetic energy balance at station 57

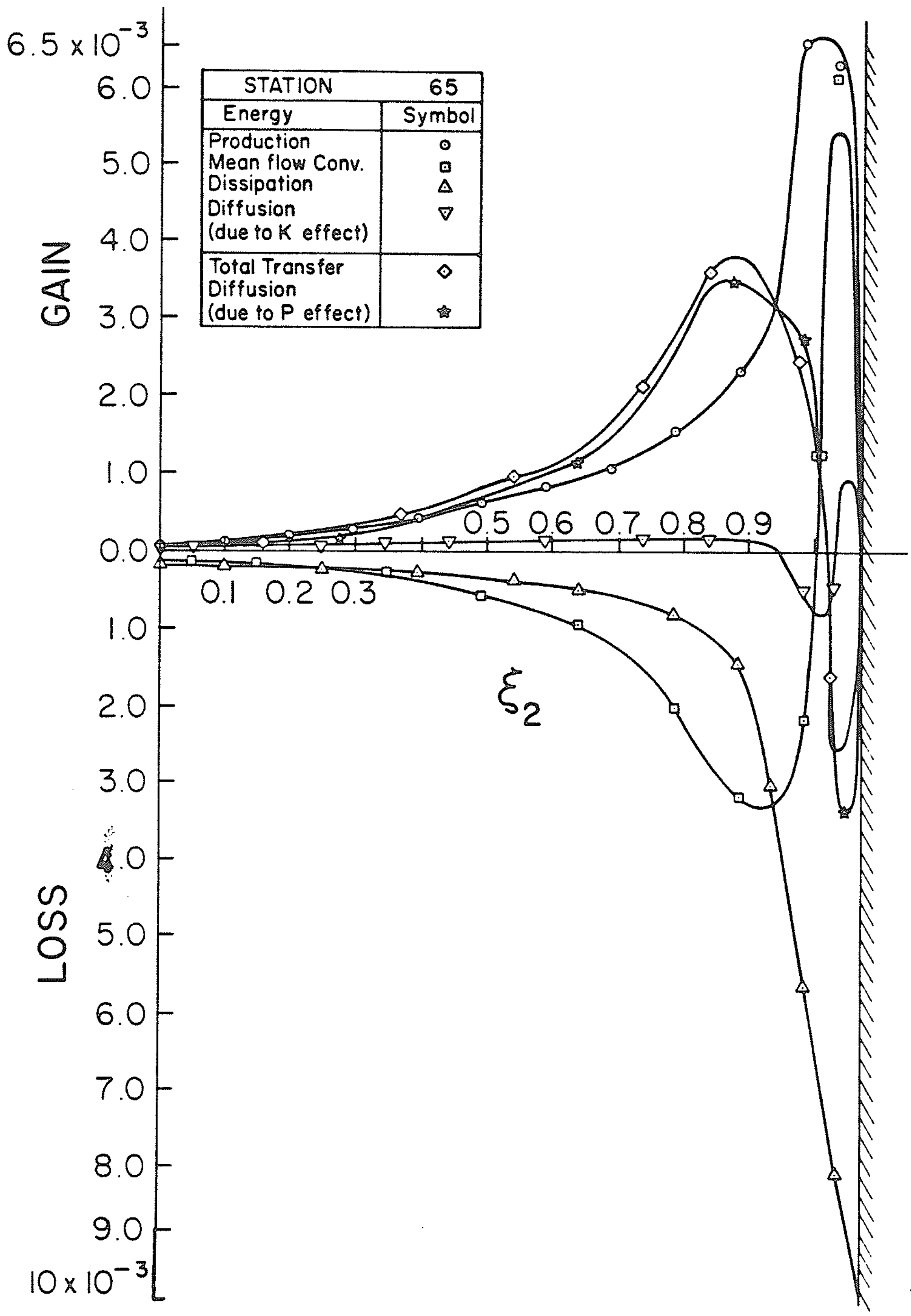


Figure 10: Turbulent kinetic energy balance at station 65

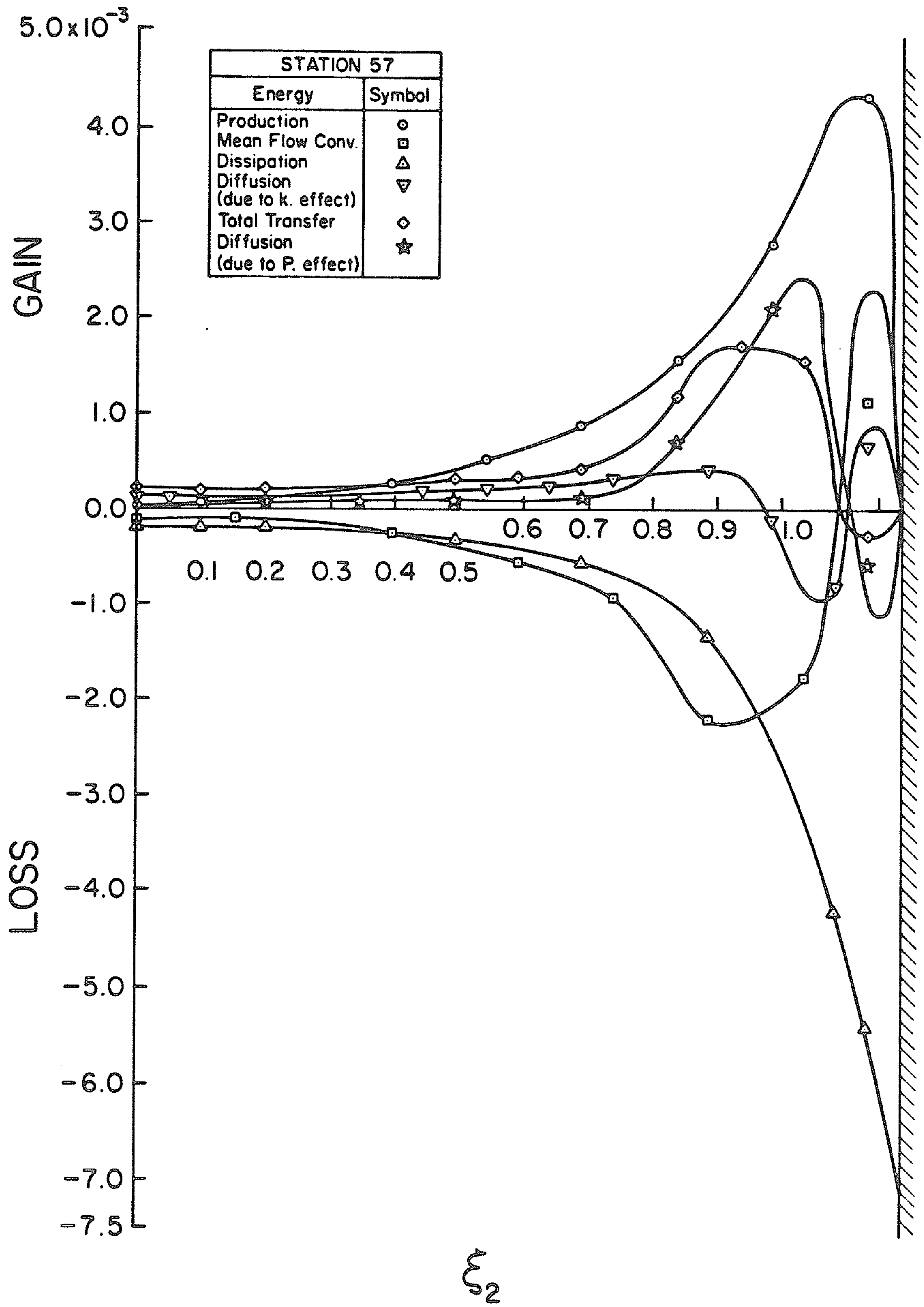


Figure 11: Turbulent kinetic energy balance at station 57

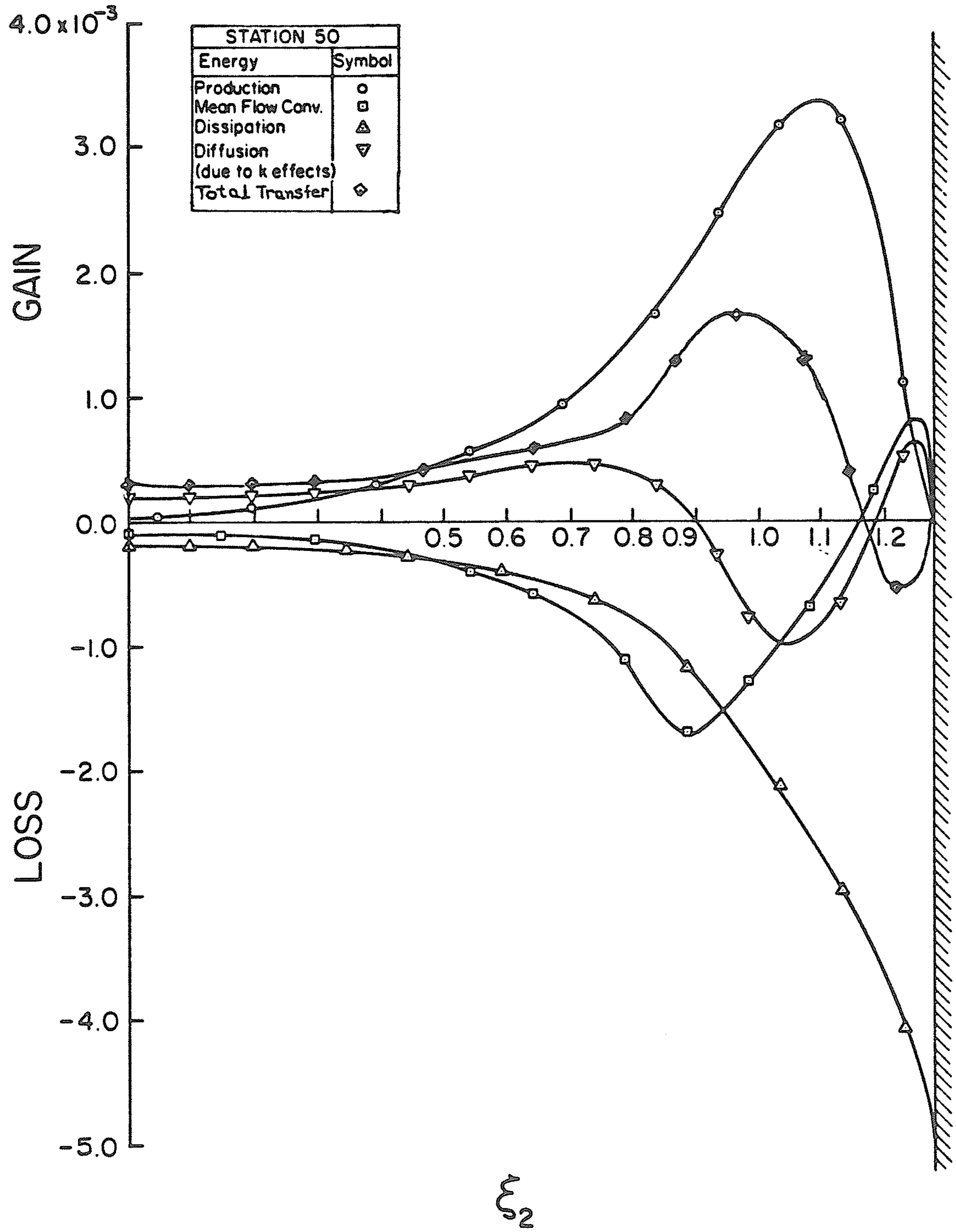


Figure 12: Turbulent kinetic energy balance at station 50

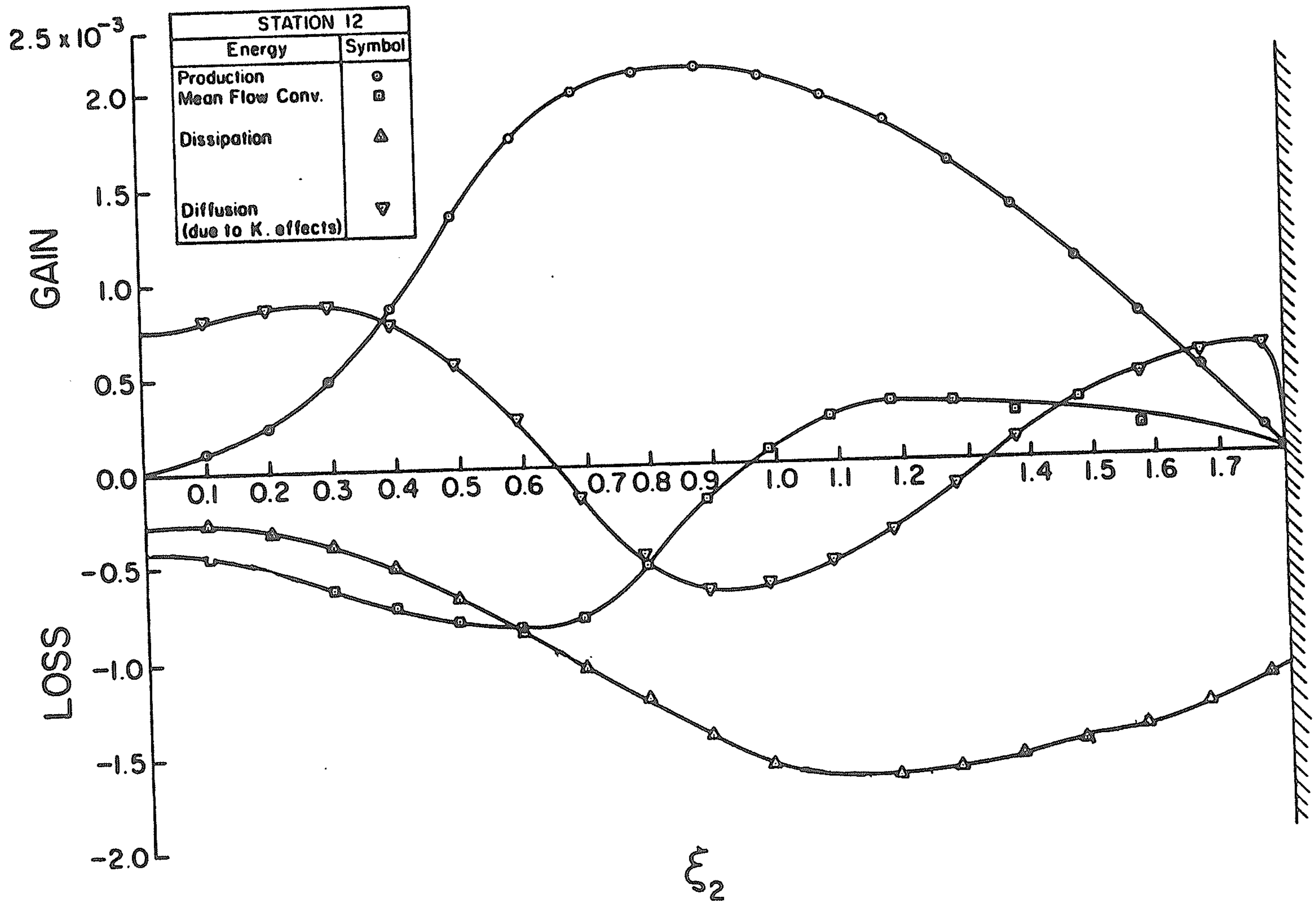


Figure 13: Turbulent kinetic energy balance at station 12

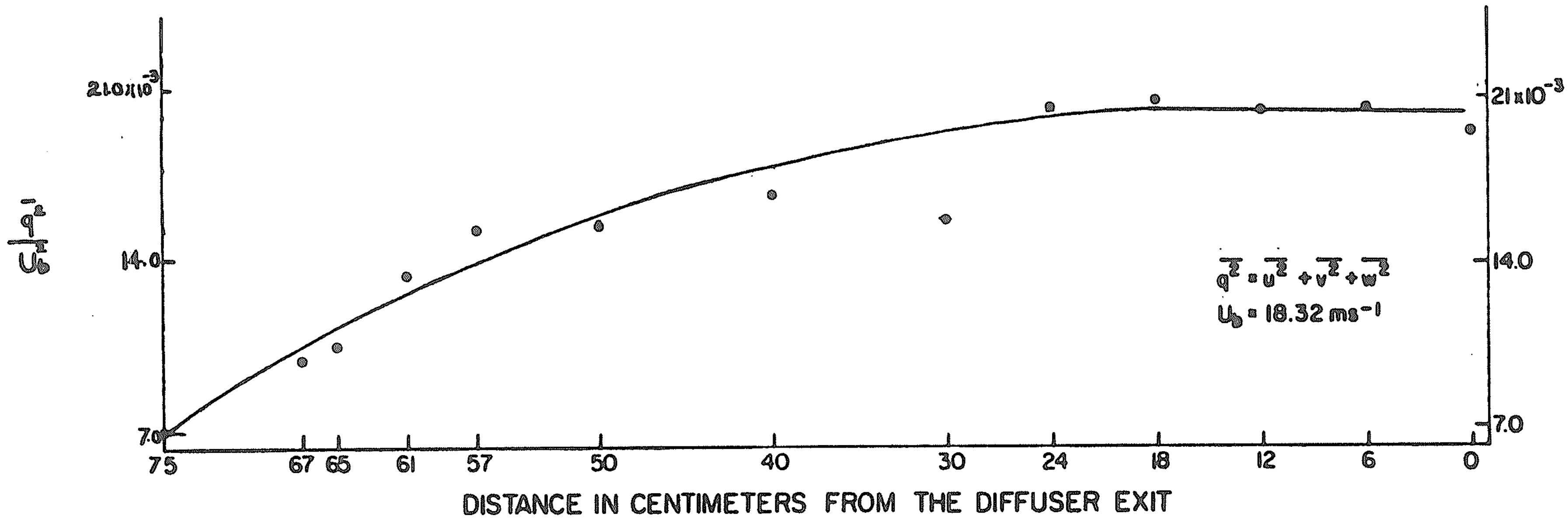


Figure 14: Average turbulent kinetic energy (q^2) distribution

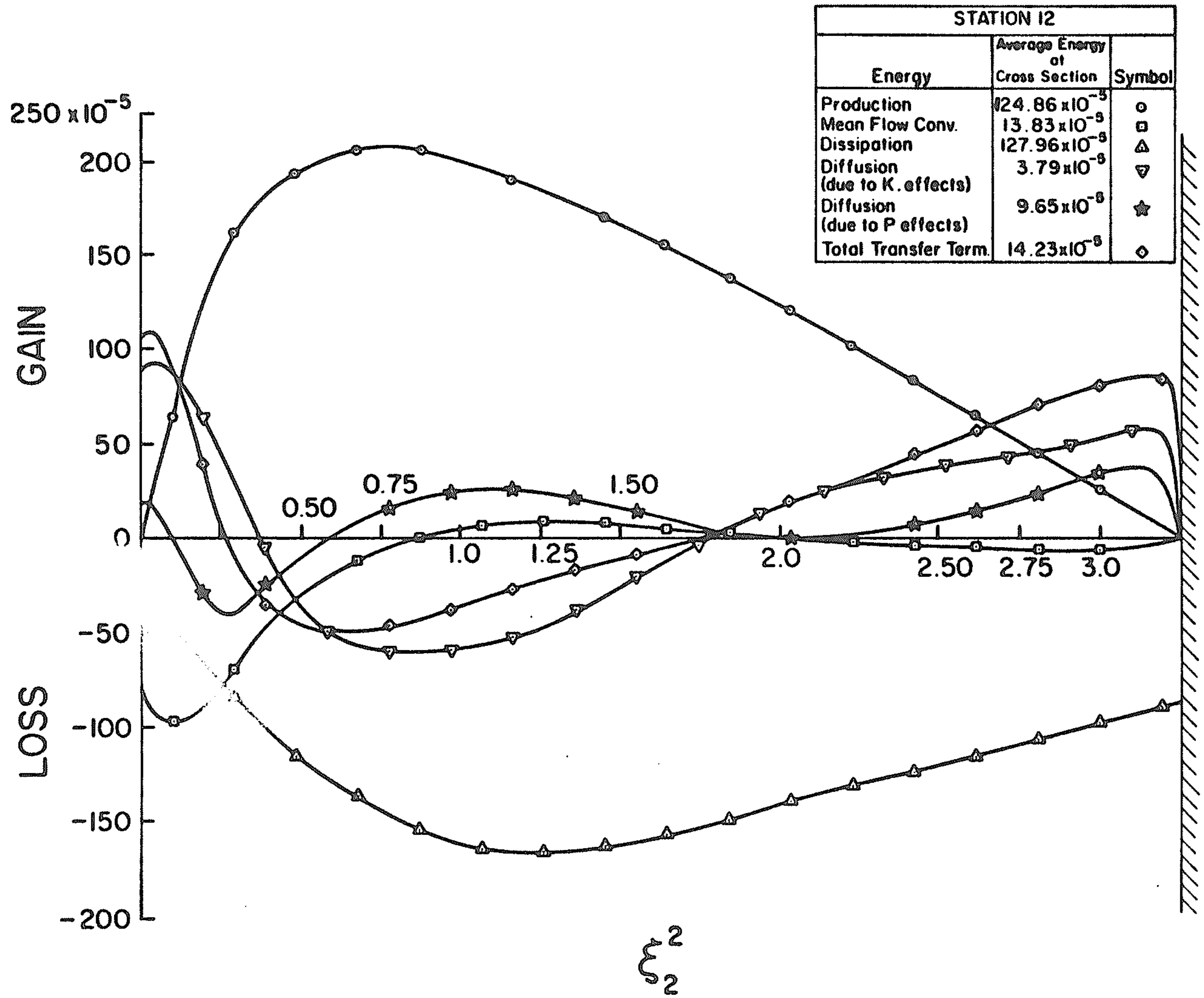


Figure 15: Turbulent kinetic energy balance plotted against area (r^2) for station 12

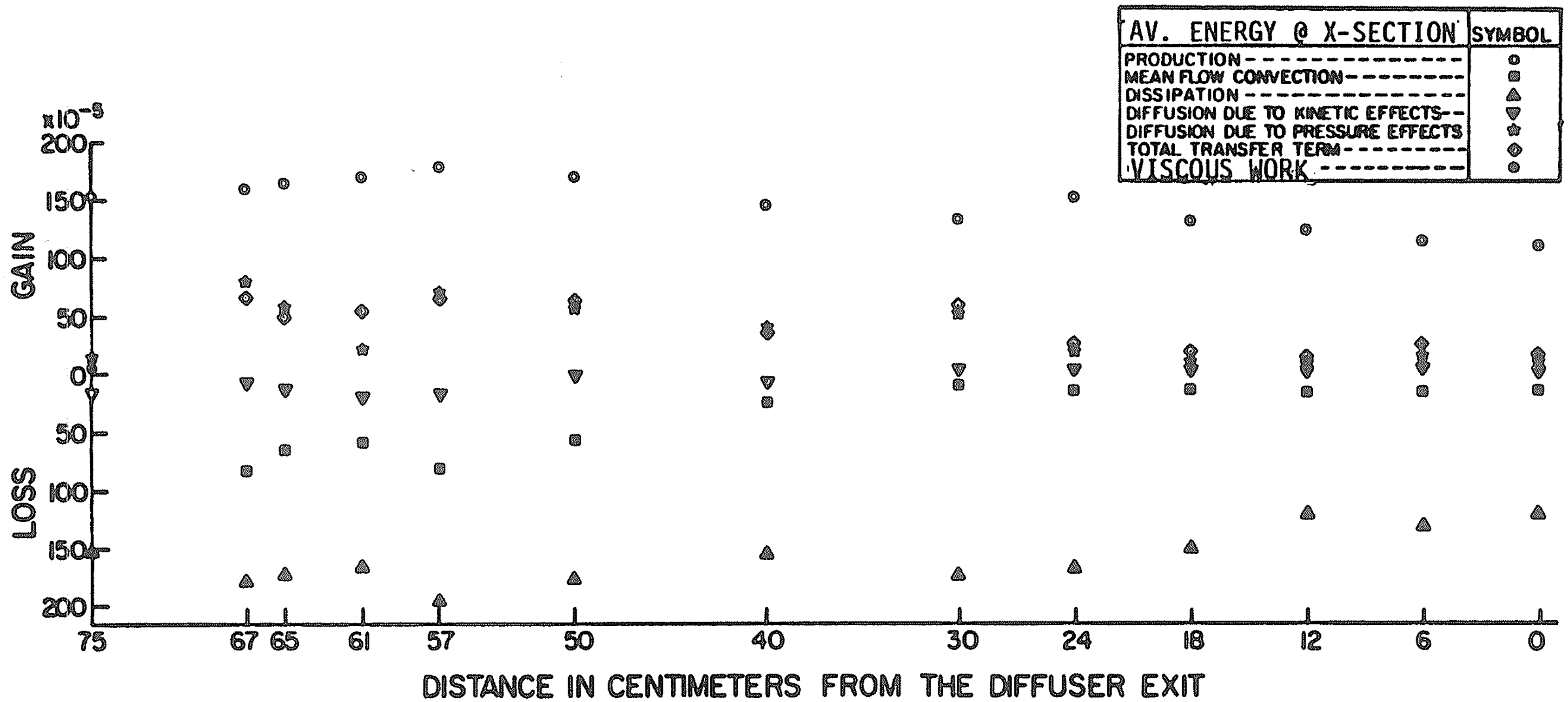


Figure 16: Turbulent kinetic energy balance along the diffuser and pipe

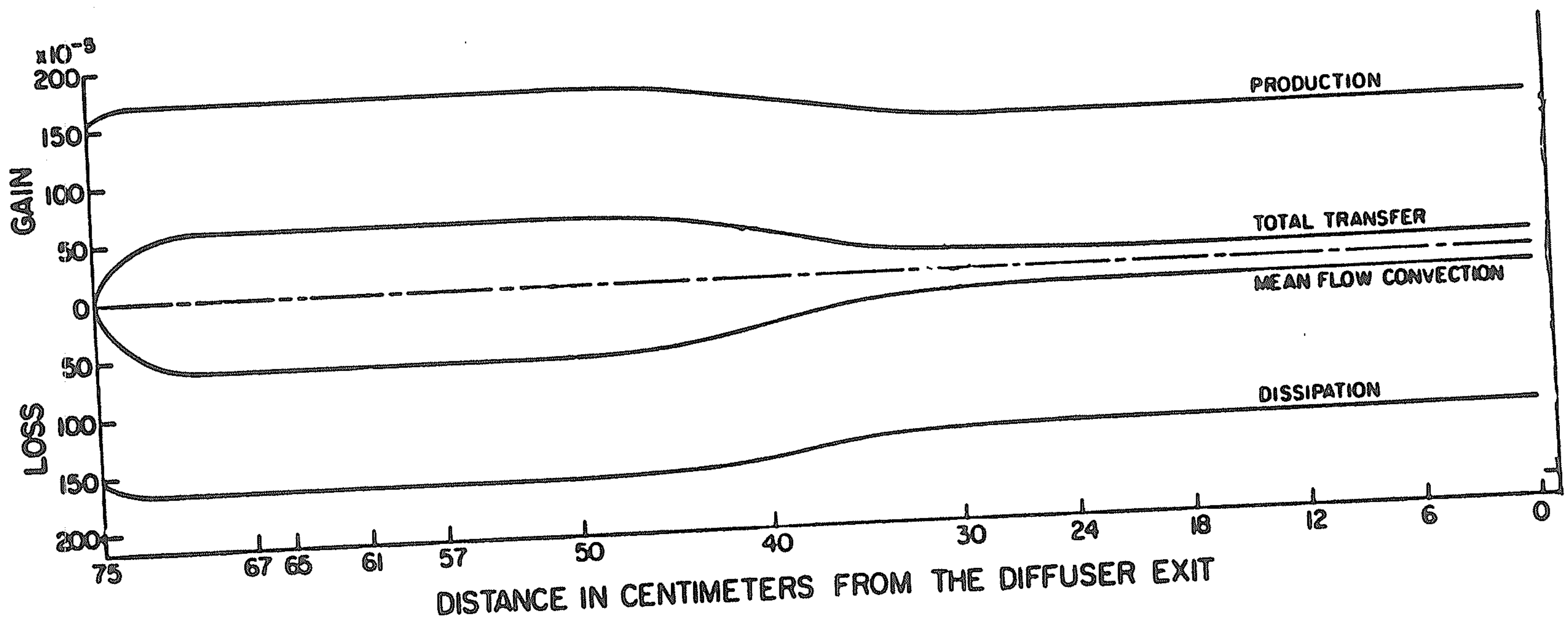


Figure 17: Trend lines for turbulent kinetic energy budget terms along the diffuser and pipe

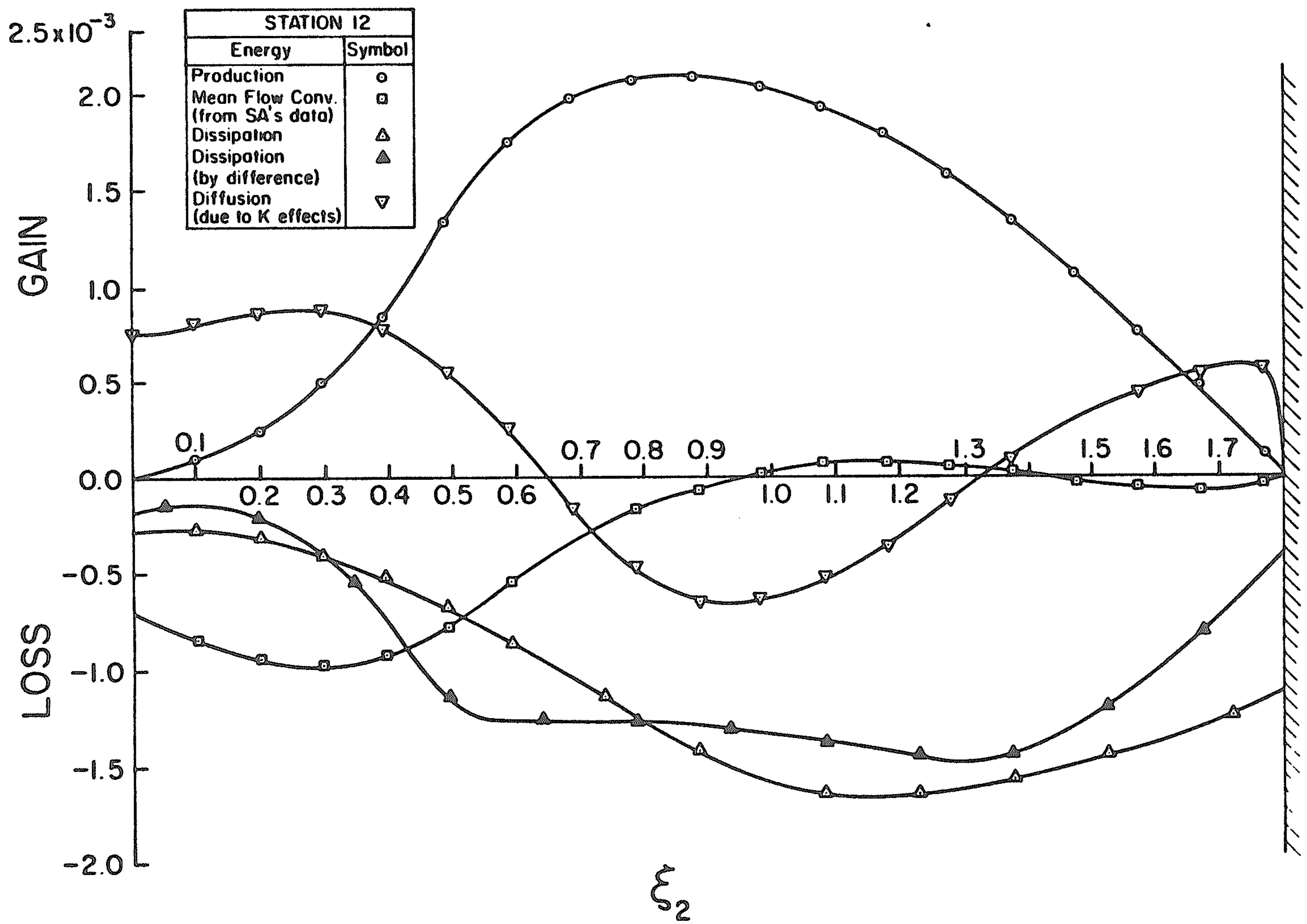


Figure 18: Turbulent kinetic energy balance at station 12 with mean flow convection from Arora s (1978) data

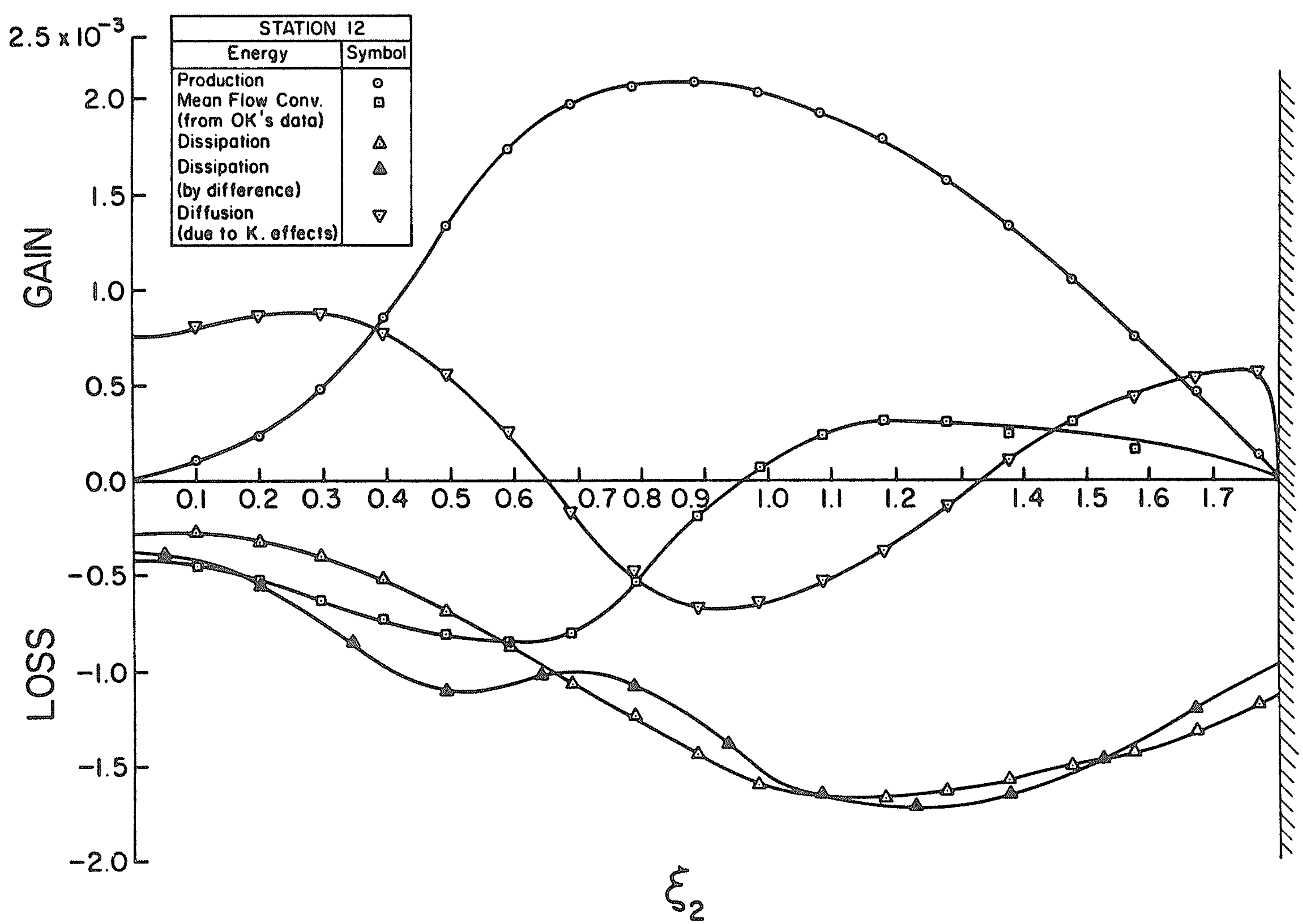


Figure 19: Turbulent kinetic energy balance at station 12 with mean flow convection from Okwuobi s (1972) data

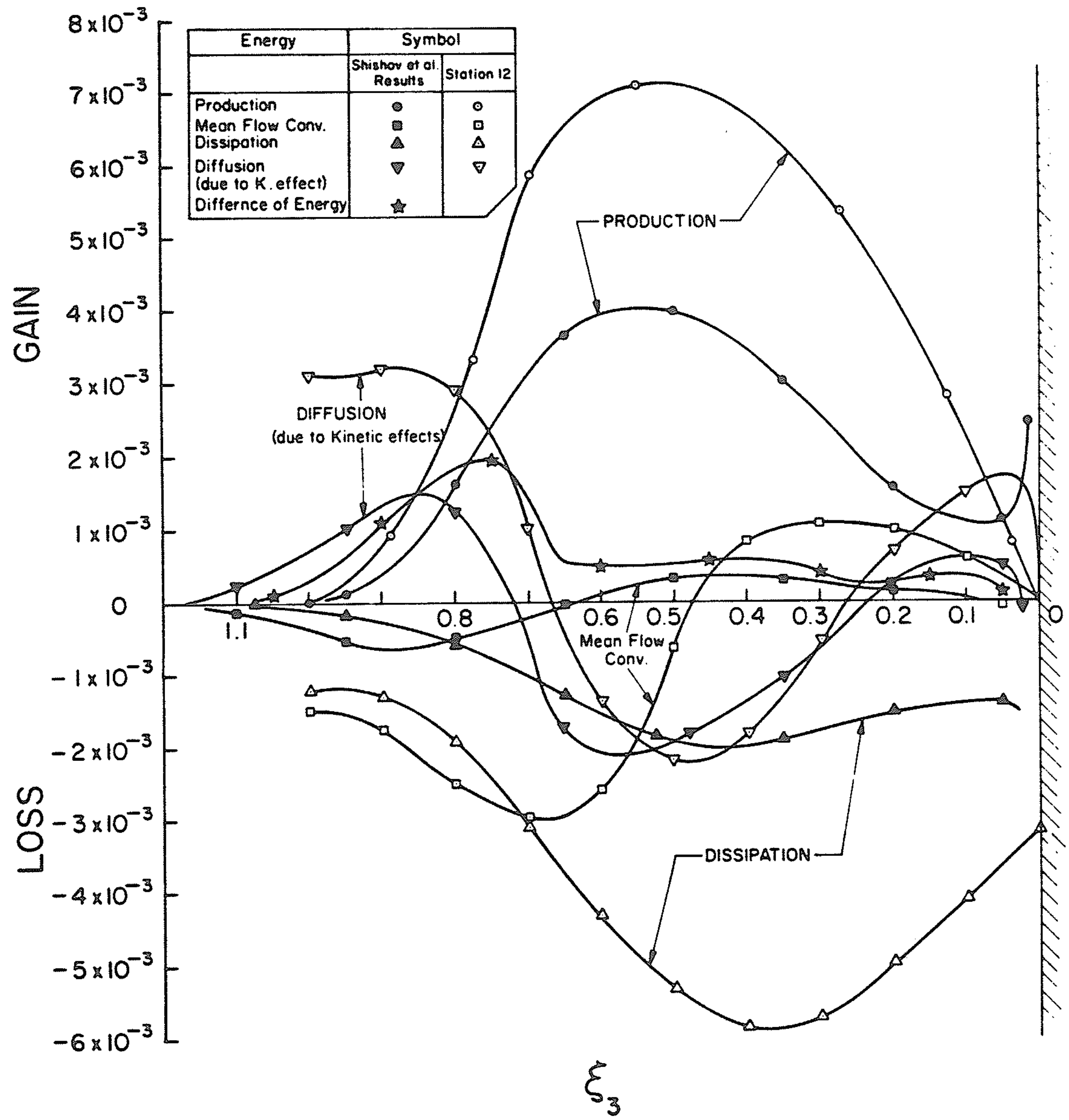


Figure 20: Turbulent kinetic energy balance at station 12 vs. Shishov et al. s (1978) boundary layer results

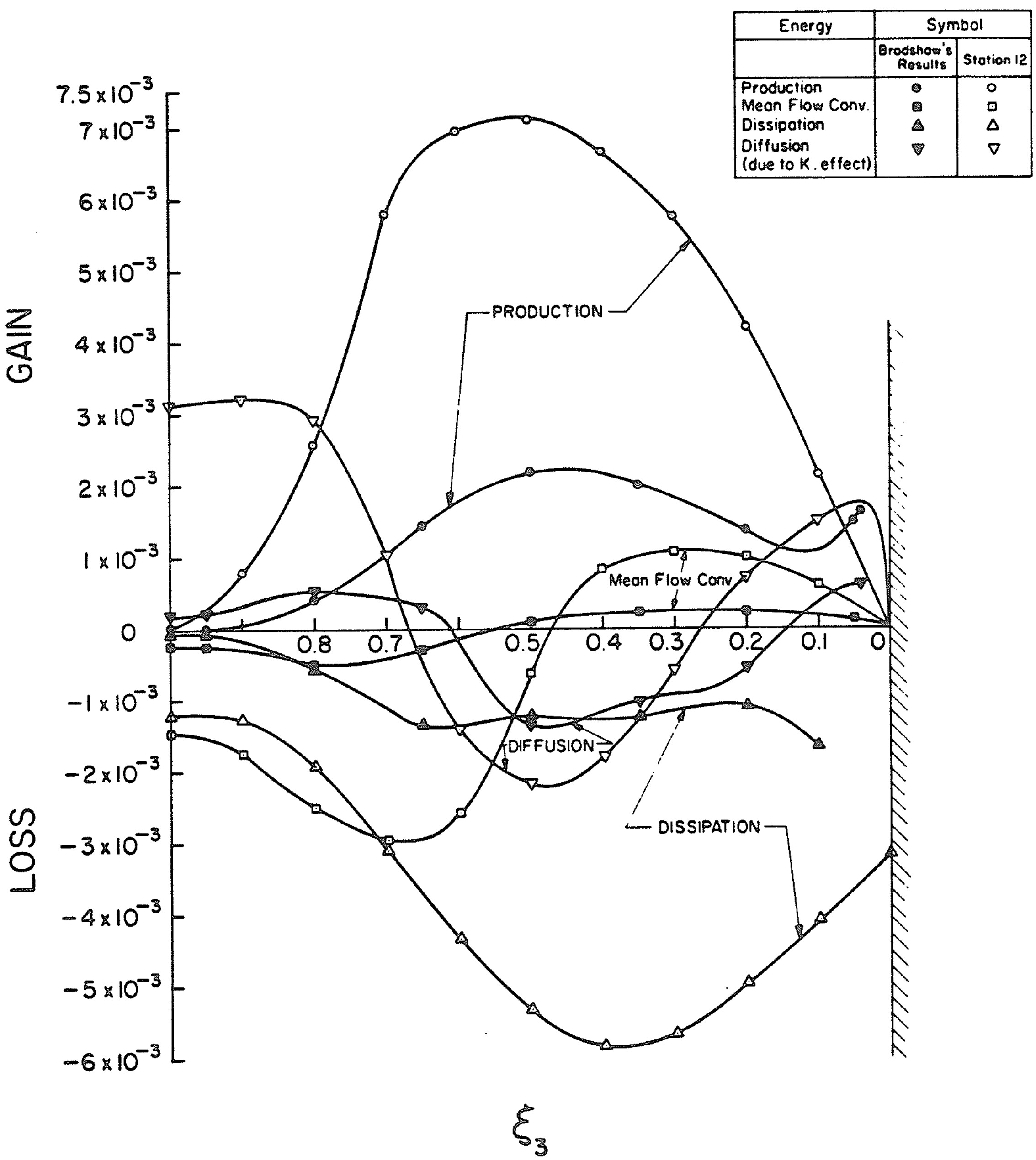


Figure 21: Turbulent kinetic energy balance at station 12 vs. Bradshaw s (1967) boundary layer results

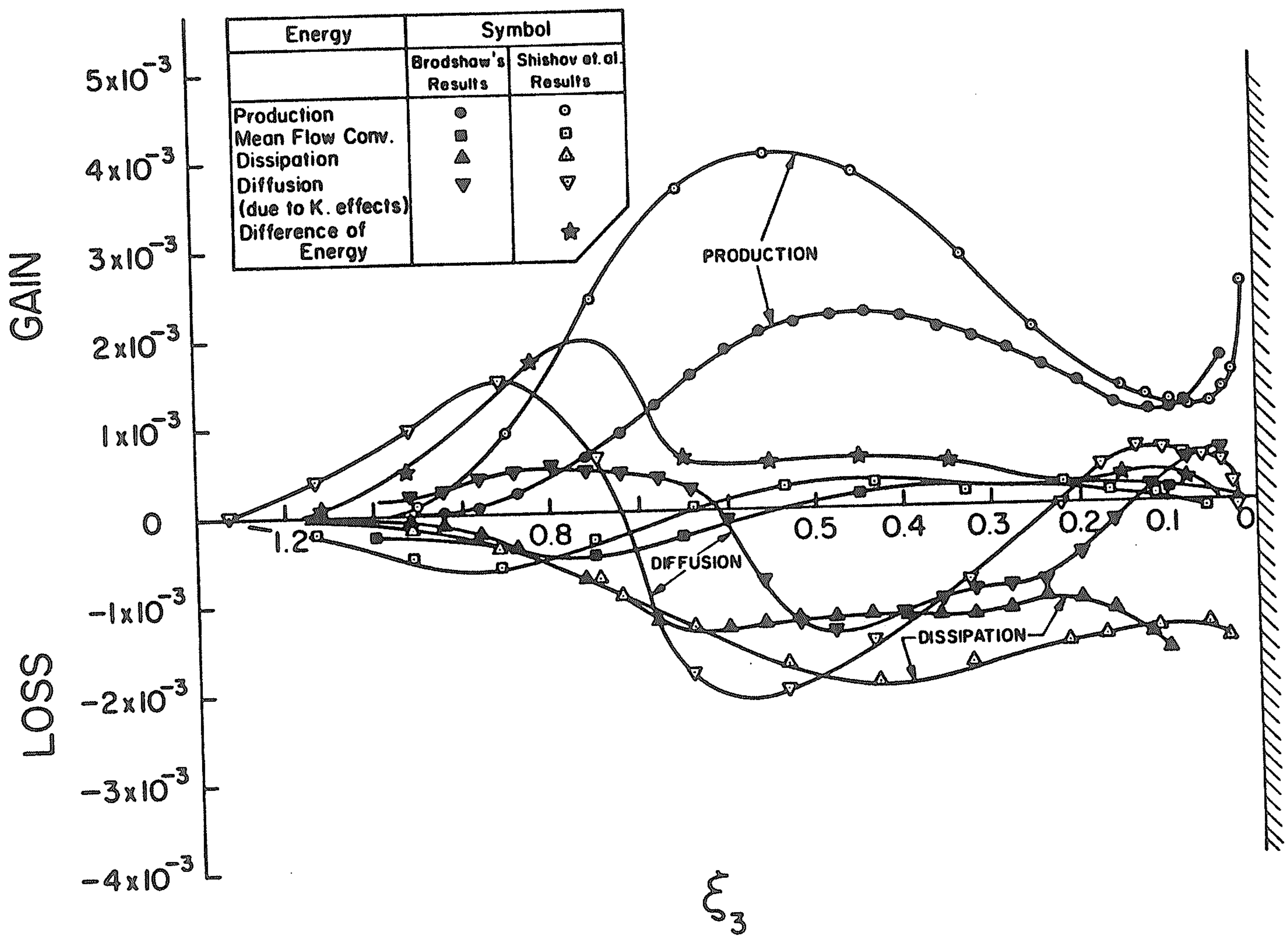


Figure 22: Turbulent kinetic energy balance for Bradshaw s (1967) result vs. Shishov et al. s (1978) results

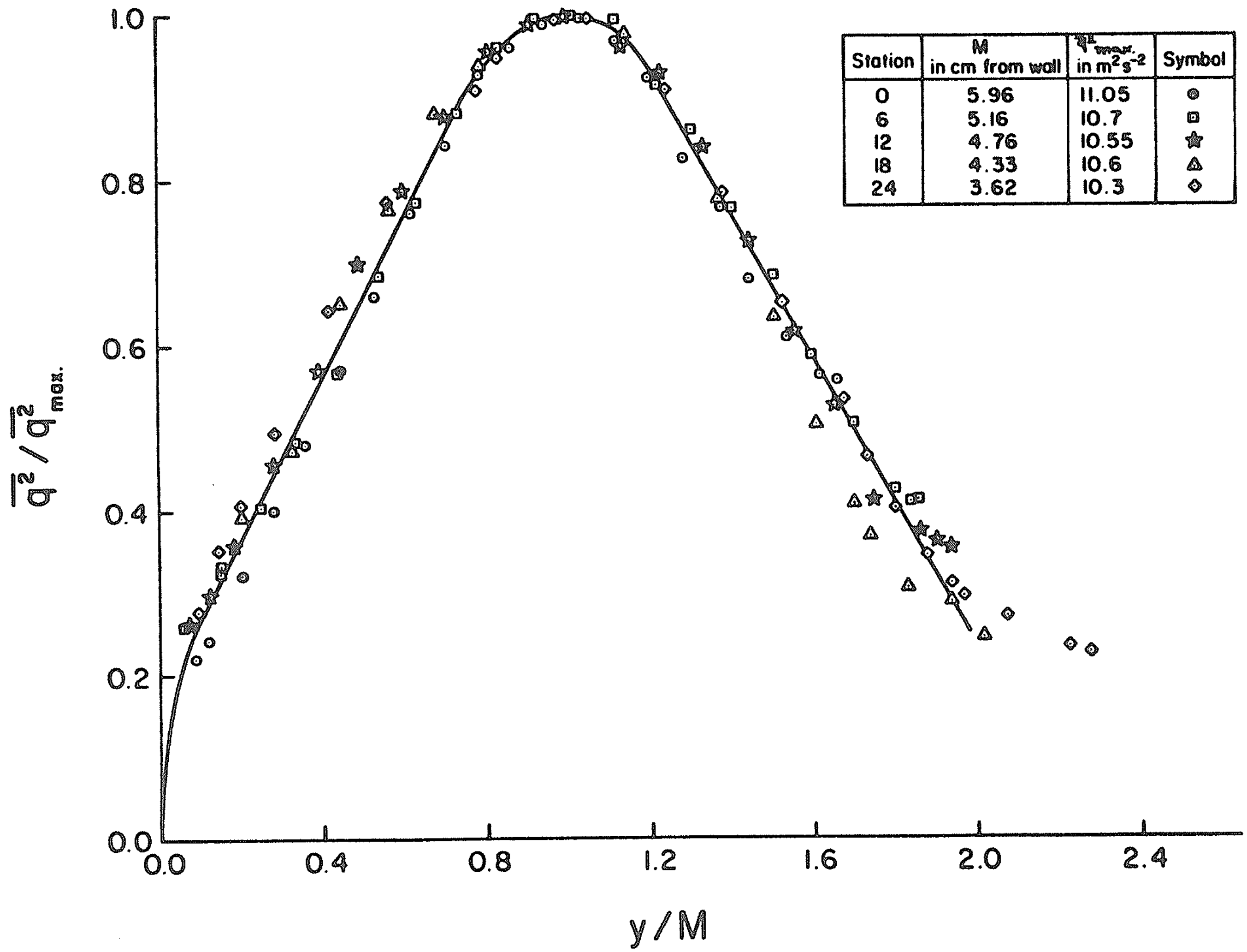


Figure 23: Distribution of q^2/q^2max in the exit region

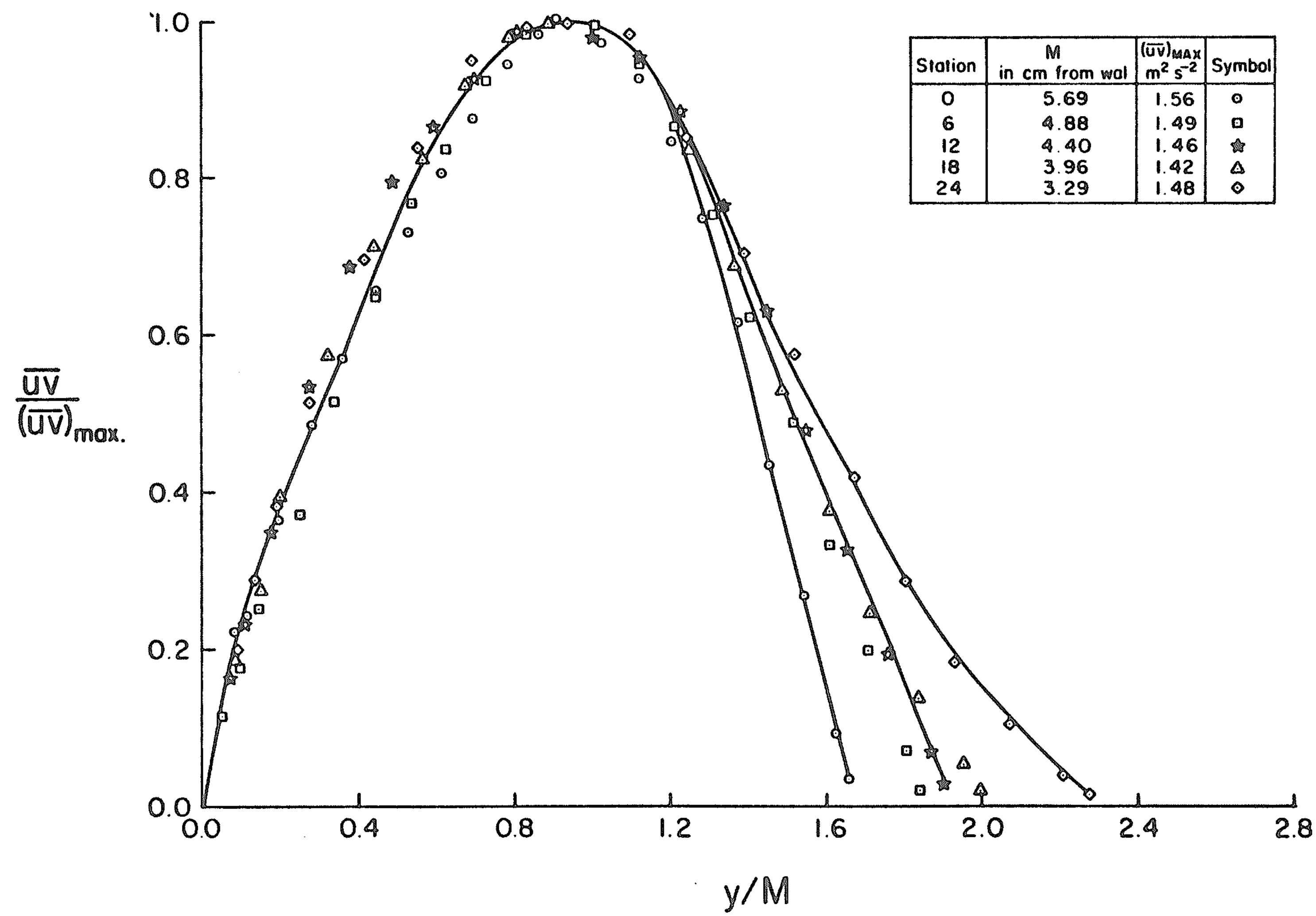


Figure 24: Distribution of uv/uv_{max} in the exit region

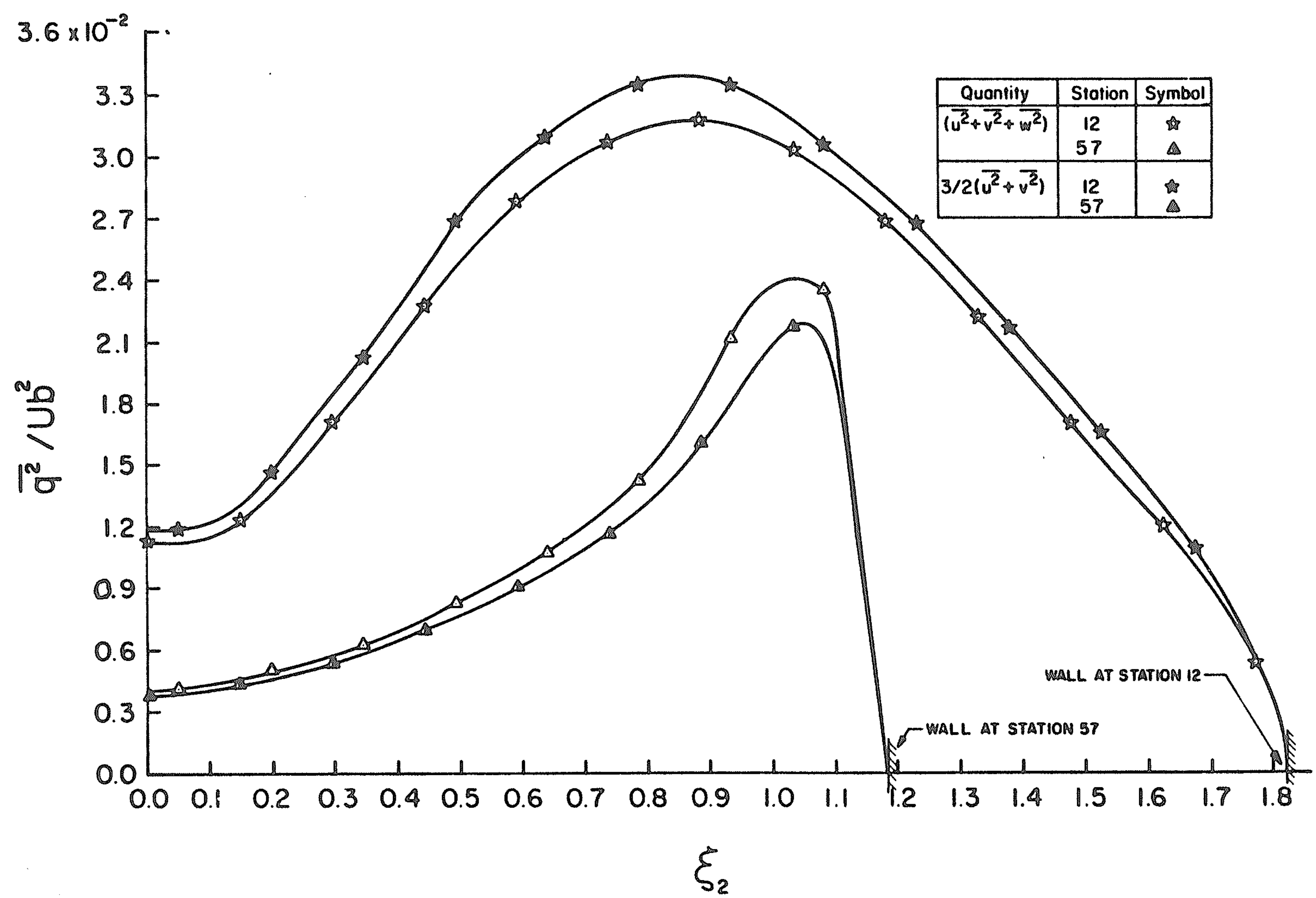


Figure 25: Turbulent kinetic energy distribution, showing the validity of the approximation $q^2 = 3/2(u^2 + v^2)$

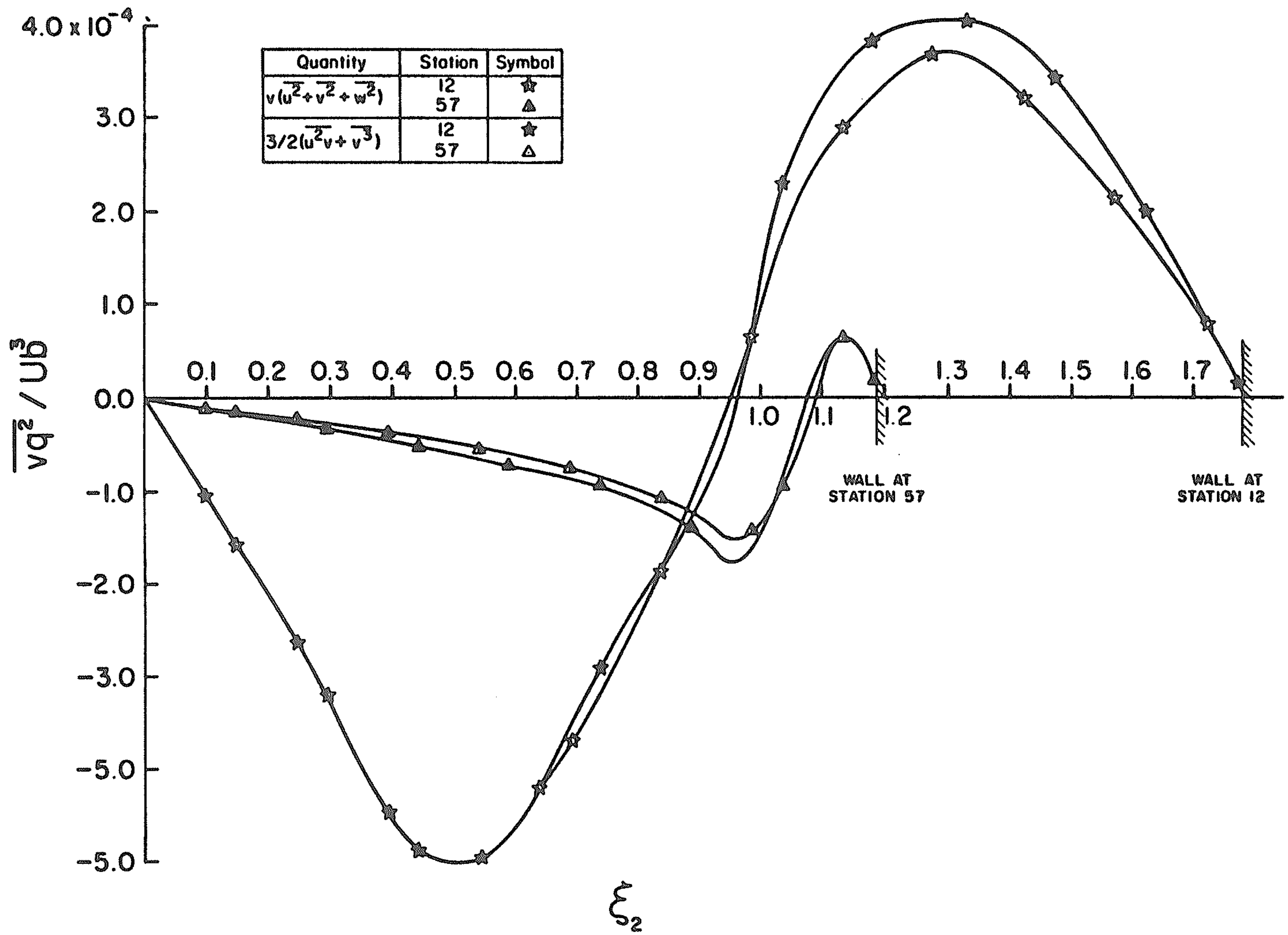


Figure 26: Triple velocity correlation distribution, showing the validity of the approximation $vq^2 = 3/2(u^2v + v^3)$