AN INVESTIGATION ON THE EFFECTIVENESS OF SHEAR REINFORCEMENT IN COLUMN-FLAT SLAB CONNECTIONS

A Thesis
Presented to
the Faculty of Engineering
The University of Manitoba
$\qquad$

## In Partial Fulfillment <br> of the Requirements for the Degree <br> Master of Science in Engineering

by
Carl E. Mickelson April, 1965

SYNOPSIS
A laboratory investigation was undertaken to determine the effectiveness of shear reinforcing devices in thin reinforced slabs. Results obtained for ten slabs tested by the author are reported.

All specimens tested were 3 ft .2 in. square and $3 \frac{1}{2}$ in. thick. The principal variables were: the effect of concentration of tensile reinforcement across the column, the effects of a $9 \times 9 \times 3 / 8$ in. plate at the column slab interface, the effects of a shearhead of $\frac{1}{4}$ in. smooth reinforcing bar, and the effects of a shear device of bent up No. 3 deformed bars. Two slabs were tested without a shear reinforcement.

The validity of existing equations for ultimate shear strength were checked with the test results and a comparison of test results to the currently used specifications was made。

ACKNOWLEDGENENTS
I wish to acknowledge the assistance received from the National Research Council in providing the funds for the laboratory testing, and Professor G. Morris under whose direction this investigation was carried out.

TABLE OF CONTENTS
CHAPTER PAGE
I INTRODUCTION
1.1 Background ..... 1
1.2 Object ..... 2
1.3 Scope ..... 2
II REVIEN OF EXISTING RESEARCH AND DESIGN SPECIFICATIONS
2.1 General ..... 4
2.2 Existing Research ..... 4
2.3 Design Specifications ..... 6
III EXPERIMENTAL PROGRAM
3.1. Description of Test Slabs ..... 9
3.2 Materials ..... 10
3.3 Fabrication ..... 11
3.4 Support Frame ..... 12
3.5 Instrumentation and Testing ..... 12
IV EXPERTMENTAL RESULTS
4.1 General Behavior ..... 26
4.2 Individual Behavior ..... 27
4.3 Analysis and Comparison of Experimental Results ..... 30
4.4 Summary ..... 35
V SHEAR FAILURES IN FLAT SLABS
5.1 Introduction ..... 48
5.2 Evaluation of Test Results ..... 49
VI CONGLUSION ..... 53
BIBLIOGRAPHY ..... 55
APPENDIX A ..... 56

## LIST OF FIGURES

FIGURE ..... PAGE
1 Typical stress-strain curve for No. 3 bars ..... 14
2 Typical stress-strain curve for a standard
6 in. x 12 in. concrete cylinder at 7 days ..... 14
3 Detail of Slab No. 1 and 1A ..... 15
4 Detail of Slab No. 2 ..... 16
5 Detail of Slab No. 3 ..... 17
6 Detail of Slab No. 4 ..... 18
7 Detail of Slab No. 5 ..... 19
8 Detail of Slab No. 2 A ..... 20
9 Detail of Slab No. 3A ..... 21
10 Detail of Slab No. 4 A ..... 22
11 Detail of Slab No. 5A ..... 23
12. Load deflection curve for Group I ..... 39
13 Load deflection curve for Group IA ..... 40
14 Load deflection profiles for Group I ..... 41
15 Load deflection profiles for Group IA ..... 42
16 Stress distribution in the column stubs of Group I ..... 43
17. Typical flexural crack pattern as observed
in the holes of Moe's tests ..... 52
18 Schematic illustration of the arching action ..... 52

## LIST OF TABLES

TABLE PAGE
I Description of tests and test results ..... 36II Test results compared to the equations ofElstner and Hognestad37
III Test results compared to the equations ofMoe and the N.B.C.38
PLATE1 Shearhead reinforcement of Slabs 4 and 4 A
PAGE
2 Truss bar reinforcement of Slab 524
3 Loading frame and typical test assembly ..... 24
4 Form and reinforcement on vibrator beams ..... 25
5. Close-up of shearhead used in Slab 4 and 4A ..... 25
6 Exposed failure cone of slab 1 ..... 44
7. Exposed failure cone of Slab 2 ..... 44
8 Spalling effect of Slab 3 ..... 44
9. Exposed failure cone of Slab 4 ..... 45
10 Bottom view of Slab 5 after failure ..... 45
11 Exposed failure cone of Slab 5 ..... 45
12 Exposed failure cone of Slab 3A ..... 46
13 Bottom view of Slab 3A after failure ..... 46
14 Location of gauges on column stubs of Group 1 ..... 46
15 Bottom view of Slab 1 after failure ..... 47
16 Bottom view of Slab 2 after failure ..... 47
17 Bottom view of Slab 4 after failure ..... 47
a - width of slab
$A_{v}$ - area of shear reinforcement
b - perimeter of shear area at a distance d from the loaded area
$b_{0}-p e r i n e t e r$ of shear area at a distance $d / 2$ from the loaded area
d - average effective depth of compression reinforcement
Ec a modulus of elasticity obtained from standard 6 in. $x 12$ in. concrete cylinders

Es m modulus of elasticity of steel
$\mathrm{f}^{\prime} \mathrm{c}=$ compressive strength of $6 \mathrm{in} . \mathrm{x} 12 \mathrm{in}$. in concrete cylinders
$f_{s}$ - stress in the tensile reinforcement
$\mathrm{S}_{\mathrm{y}}$.. yield point of reinforcing steel
$f_{\mathrm{v}}-$ yield point of shear reinfor cement
$j$ - ratio of internal moment arm to effective depth, assumed equal to $7 / 8$
$P_{\text {calc }}$ - calculated ultimate shear load
$P_{\text {flex }}$ - calculated ultimate flexural load by "Yield Line Theory"
$P_{\text {test }}$ - measured ultimate load
p - average percent of tensile reinforcement
r - side dimension of square loaded area
$S_{V}$ - vertical component of tensile force in slab reinforcing
I - internal tensile force in slab reinforcing
V shear force
v - shear stress
$\nabla_{u}$ - ultimate shear force
$v_{u}$ ultimate shear stress
wo centre deflection of the slab
$\theta \quad \infty$ angle of inclination of shear reinforcement
$\phi_{0}$ - ratio of the ultimate strength to the allowable flexural strength as computed by the yield line theory. (taken to be 0.667.)

INTRODUCTION

### 1.1 Background

The design of reinforced concrete flat slabs with respect to shear is based on a limited amount of factual knowledge. The current A.C.I. Code does not recognize shear reinforcing devices and design methods used in handling problems of excess shear at colum slab connections differ considerably.

The shear problem has become even more pronounced with modern architecture making more use of slim columns and thin floor slabs without drop panels or column capitals. The struc. tural system has practical advantages also; the overall amount of material is reduced and forming becomes easier and less expensive.

Thus the slab column connection is a critical area for design. High bending moments and shear forces are concentrated there so that the size of all main structural members become governed by the degree to which the strength at this point can be developed and accurately predicted.

Current specifications such as the National Building Code of Canada and the A.C.I. Code, give values of shear strength that can be used for different percentages of slab reinforcement; but for connections in which the allowable shear stress is exceeded the designer is obliged to increase his member sizes or use his own judgement in making some type of reinforced connection.

Test programs reported by Elstner and Hognestad (1) in 1956 and Johannes Moe ${ }^{(2)}$ in 1961 contain pilot investigations into the effects of shear connectors to relieve excess shear in
column-slab connections. However both programs were undertaken to study the different variables which affect shear strength and to study modes of failure so that equations now used to determine allowable shear could be improved upon.

### 1.2 Object

It is the object of this thesis to investigate and compare the behaviour in shear of reinforced concrete slabs with and withe out shear reinforcement. Existing empirical approaches to determine the ultimate shearing capacity are applied and compared to the test results.

### 1.3 Scope

The effects of concrete strength, flexural capacity, eccentricity of load and column size have been documented in the reports by Elstner and Hognestad $(1)$ and by Johannes Moe. (2) No attempt has been made to present a complete and detailed review of their findings but reference is made to those aspects which are pertinent to this investigation.

To compare the shear connectors the tests were divided into two series. The slabs in each series were identical except for the shear reinforcing device.

Two slabs were tested without shear reinforcement and two slabs were tested with a concentration of tensile reinforcement across the column. Six specimens were made to test some special types of shear reinforcement.

The test specimens were all square slabs with column stubs at the centres and with simple supports at the edges. The arrangem ment approximates a flat slab extending from the column to the points of contraflexure.

All specimens tested were 3 ft . square and $3 \frac{1}{2}$ inches thick with a 6 in. square column stub loaded axially. The specimens were necessarily small because consideration had to be given to laboratory space, loading capacity and material quantities.

## REVIEN OF EXISTING RESEARCH AND DESIGN SPECIFICATIONS

### 2.1 General

Many attempts have been made to formulate a general theory of failure that will predict the strength of concrete under different combinations of stress. Despite the numerous failure theories that have been advanced, a completely rational approach to the prom blem of ultimate stress in concrete still does not exist. The problem becomes even more pronounced when considering the complexity of the mechanisms of failure of different structural components. It is evident then that a method for predicting the ultimate shear strength. of reinforced concrete slab-column connections can only be found by applying empirical approaches to the results of laboratory tests. A discussion of existing specifications and research is given below.

### 2.2 Existing Research

Elstner and Hognestad ${ }^{(1)}$ in 1956 reported on tests of thirtyeight 6 ft . square slabs which were loaded through a column stub at the centre and simply supported at the four edges.

As a result of the test program, it was concluded that;

1) The ultimate shear strength of slabs without shear reinforcement could be expressed as

$$
\begin{equation*}
\frac{\mathrm{V}_{\mathrm{u}}}{7 / 8 \mathrm{bdf} f_{\mathrm{c}}}=\frac{333}{\mathrm{f}_{\mathrm{c}}^{8}} \neq \frac{0.046}{\emptyset_{0}} \tag{1}
\end{equation*}
$$

and for slabs with shear reinforcement, as

$$
\begin{array}{r}
\frac{V_{u}}{7 / 8 b d f_{c}}=\frac{333}{f_{c}{ }_{c}}+\frac{0.046}{\not \varnothing_{0}} \neq\left(q_{v}-0.050\right)  \tag{2}\\
\quad \text { where } q_{v}=\frac{A_{v} f_{v} \sin \theta}{7 / 8 b d f_{c}}
\end{array}
$$

(By observing equation (2) it is seen that the shear reinforcement was not totally effective.)
2) A concentration of tension reinforcement directly over a column did not increase the shear capacity but did lessen the strains in the tenstion reinforcement.

In 1958 Lin, Scordelis and May (3) presented a report on shear strength in reinforced and pre-stressed lift slabs. It was observed that the equations of Elstner and Hognestad for reinforced flat plates gave good co-relation with test results.

Johannes Moe ${ }^{(2)}$ in 1961 reported tests of forty-three 6 ft . square slabs which were similar to the test specimens of Elstner and Hognestad. His report contains studies into the effects of concentration of tensile reinforcement in narrow bands across the column and the effectiveness of special types of shear reinforce. ment .

From these tests some of the important conclusions of ins terest to this report are:

1) The ultimate shear strength of slabs is predicted with good accuracy by the formula

$$
\begin{equation*}
\frac{v_{u}}{b d}=25(1-0.075 \mathrm{r} / \mathrm{d})=5.25 \phi_{0} \sqrt{f^{p} c} \tag{3}
\end{equation*}
$$

and for slabs with shear reinforcement, as

$$
\begin{equation*}
V=(6.23-1.12 \mathrm{r} / \mathrm{d}) \mathrm{bd} \sqrt{\mathrm{f}_{\mathrm{c}}}+\mathrm{A}_{\mathrm{v}} \mathrm{f}_{\mathrm{v}} \sin \theta \tag{4}
\end{equation*}
$$

(Formula. (4) should only be applied for ratios of $r / d$ less than

## 2.5.)

2) Concentration of flexural reinforcement in narrow bands across the column did not increase the shear strength. However such con-
centrations increased the load at which yielding began in the tension reinforcement.
3) Anchorage of shear reinforcement seemed to be problematical.

### 2.3 Design Specifications

The limited knowledge available regarding the mechanism of failure in shear of slabs is clearly reflected in the various specifications. Quite different rules are applied to determine the critical shear or inclined tensile stress, and the allowable stresses differ considerably.

The current Canadian National Building Code ${ }^{(8)}$ (1960) stipulates that the shear stress should be computed by

$$
\begin{equation*}
v=\frac{V}{j b d} \tag{5}
\end{equation*}
$$

in which $b$ is the periphery at a distance $d$ beyond the loaded area.
The shear stress computed by equation (5) is limited to a) $0.03 f^{8} \mathrm{c} \quad 100 \mathrm{psi}$ if more than fifty percent of the tensile reinforcement required for the bending passes through the periphery. b) $0.025 f^{\circ} \mathrm{c} \quad 85 \mathrm{psi}$ when twenty-five percent of the tensile reine foreement passes through the periphery.

The current A.C.I. Code ${ }^{(7)}$ (1963) stipulates that the critical section for shear as a measure of diagonal tension shall be perpendicular to the plane of the slab and located at a distance d/2 out from the periphery of the concentrated load or reaction area. The nominal shear stress shall be computed by

$$
\begin{equation*}
v=\frac{V}{b_{0} d} \tag{6}
\end{equation*}
$$

in which $V$ and $b_{0}$ are taken at a section $d / 2$ out from the loaded
area. The shear stress, $V$, so computed shall not exceed $2 f^{1}{ }_{c}$. unless shear reinforcement is provided, in which case $V$ shall not exceed $3 f^{\prime} c$. The use of shear reinforcement consisting of bars, rods or wires shall not be considered effective in members with a thickness less than 10 in.

In Germany a completely different approach to the design problem of shear in slabs has been practiced. In determining shear as well as flexural stress, slab strips of certain widths are assumed. The widths given for shear computations are different from those in moment, and the widths also vary with the position of load on the slab. The German Specification DIN 1045 of 1943 gives the following formula for the effective slab width in shear

$$
\begin{equation*}
b_{1}=r \neq 2 s \text { and } b_{2}=1 / 3\left(x-\frac{r \neq 2 s}{2}\right) \tag{7}
\end{equation*}
$$

where $s$ is the thickness of a load-distributing layer on top of the slab and $l$ is the span of the slab. The larger of the values $b_{1}$ and $b_{2}$ can be used.

The Norwegian Standard Specification of 1939 assumed the shearing stress to be evenly distributed around the loaded area at a distance of $2 \mathrm{~d} / 3$ from the periphery. It is however, also necessary to consider a strip of the slab of a certain specified width as a beam and check the shearing stress in this beam strip.

In the British Code of Practice (CPII4) the shear stresses in flat slabs are computed at a distance $d / 2$ from the periphery of the loaded area.

A report published by A.C.I.-A.S.C.E. Committee 326(4) was based primarily on the work of Johannes Moe. After reviewing his and other test programs they made the following recomendations:

1) The concentration of tensile reinforcement over columns in slab design should be encouraged since it increases the slab stiffness, and decreases the deflection and slab reinforcing strain.
2) With very little test results available no design procedure could be recommended for shear reinforcement, however anchorage of the reinforcement seemed to limit the effectiveness.

## EXPERIMENTAL PROGRAM

### 3.1 Description of Test Slabs

The ten test slabs were divided into two groups. All specimens were 3 ft .2 in . square with an overall thickness of $3 \frac{2}{2}$ in. The column stub in each case was 6 in. square and 8 in. high reinforced with four No. 4 bars. The slab reinforcement was in the form of mats welded at the extreme edges.
a. Group 1

The five slabs of this series are shown in figures (3, 4, 5, $6 \& 7$ ). Slabs $1,2,4$ and 5 were reinforced with thirteen No. 3 bars spaced at 3 in. c-c in each direction.

Slab I was not furnished with any type of shear reinforcing device. A detail is given in figure (3).

Slab 2 was provided with a 9 in. $x 9$ in. $x 3 / 8$ in. steel plate designed to distribute the load from the column over a larger area of slab. The plate was cast into the concrete, its top surface flush with the concrete surface as shown in figure (4). The column steel was continuous through four 1 in. diameter holes in the plate。

Slab 3 was not furnished with any shear reinforcement but the tensile steel spacing was changed to provide a concentration of 50 percent of the tensile reinforcement in the periphery of the column as shown in figure (5). This was used because of the N.B.C. specification, which provides for an increase in shear stress when 50 percent of the tensile reinforcement passes through the column periphery.

Slab 4 was reinforced with a shearhead of the type shown in figure (6) and plate (1). It was made of $\frac{1}{4}$ in. reinforcing rod welded into a sturdy grid. Such devices have been used in flat plate floor slabs to some extent in Canada and the United States.

Slab 5 was reinforced for shear with a system of bent up bars as shown in figure (7) and plate (2). Such a method is popular with some designers and especially so in European countries. The design procedure for such a device is outlined in "Shearhead Reinforcement for Flat Plate Floors" by the Portland Cement Associam tion.

The five slabs of this series are shown in figures (3, 8, 9, 10 \& 11). The reinforcing steel in slab 1A was the same as in slab 1 but in slabs $2 \mathrm{~A}, 3 \mathrm{~A}, 4 \mathrm{~A}$ and 5A were stiffened with four extra No. 3 bars as shown in the figures mentioned above.

Slab 2 A was provided with a $9 \times 9 \times 3 / 8$ in. steel plate resting on the slab as shown in figure (8).

Slab 3A did not have a special reinforcing device but was furnished with the more heavily reinforced slab of this series as shown in figure (9).

Slab 4 A had the shearhead identical to slab 4 but with the stiffer mat of this series. See figure (10).

Slab 5A had bent bars similar to slab 5. The embedment of the bent bars was extended in this case as shown in figure (11) to overcome any tendency of a bond failure.

### 3.2 Materials

High early strength Portind cement was us ed throughout the test program. The concrete mixture was designed to give a minimum compressive strength of 3000 psi at seven days. The water cement
ratio was 0.57 and the cement factor was 6.7 sacks per cubic yard of concrete. The maximum size of aggregate was $\frac{1}{2}$ in. The relative proportions by weight of cement, sand and gravel were approximately 1:3:3. This produced a $2 \frac{1}{2}$ in. to $3 \frac{1}{2}$ in. slump.

Two standard 6 in. $x 12$ in. cylinders were cast with each test slab. The cylinders were cured in the same manner as the slabs and tested on the same day as the corresponding slab. A typical stress strain curve for a $6 \times 12$ in. cylinder is shown in figure (2). An average modulus of elasticity of approximately $2.6 \times 10^{6}$ was found.

Intermediate grade deformed reinforcing bars were used throughout the tests. A typical stress-strain curve for a No. 3 bar is shown in figure (1).

### 3.3 Fabrication

Each slab was cast on a sheet of $\frac{1}{2}$ in. plywood with wooden side forms. Reinforcing bars were spot welded at the outside edges to form a sturdy mat. Tie wire was not used because it was felt that the spacing could not be controlled accurately. The tension reinforcement was supported on $3 / 8$ in. chairs at four points along the bottom bars. A photograph of the reinforced slab just prior to casting is shown in plate (4).

Two batches of concrete mixed in a 3 cubic foot mixer were necessary to cast each slab. The forms were placed on two vibrator beams and vibrated externally. The column stub was poured separately, approximately eight hours after the slab pour when the concrete had reached its initial set.

Forms were stripped twenty-four hours after casting. The specimens were cured for six days under damp burlap and left to air dry until testing at seven to nine days. The slabs were whitewashed on the bottom so that cracks could be observed more clearly.

### 3.4 Support Frame

To approximate the conditions existing in continuous structures a simple support at the perimeter of the test slabs was used. The loading frame was self contained and consisted of a welded 3 ft. square support frame fabricated from 12 inch channel section. Four $3 \times 3 \times \frac{1}{4}$ in. angles connected this frame to two 8 in. wide flange sections against which the jacking was done. Details of the frame are shown in plate (3). A one inch strip of white pine was glued to the support frame to allow some flexibility of the supports and help insure a uniform bearing at the start of each test. The slab corners were allowed to deflect upward in each case. The frome was located over a pit in the laboratory floor so that the underside of the slabs was accessible and initial cracking could be observed.

### 3.5 Instrumentation and Testing

In each test concrete strains were measured with six SR-4 type A-3 strain gauges. Gauge locations are shown on the detail drawing for each slob.

C6-141-B metafilm gauges were used on the reinforcing bars as shown in the appropriate figures. Standard practice was used in mounting all gauges.

The Budd Digital Strain recorder was used for the first four tests. Due to a malfunction in the system the full range of readings could not be obtained and the manually operated Baldwin SR-4 strain indicators were used to complete the testing program.

Deflections were measured with three 0.001 in. dial gauges at the quarter points of the slab and the column centre.

Laads were applied with a 200,000 pound hydraulic jack and the load readings were taken from a strain indicator connected to a Baldwin SR-4 load cell. The slabs were loaded in 2000 pound increments to 30,000 pounds and then in 5000 pound increments to failure. Strains and deflections were taken after each increment. Plate (3) shows the test assembly and instrumentation.

After failure, each slab was removed from the loading frame and the crack pattern observed and photographed. It was then placed back on the frame and loaded to observe the mat or membrane strength of the reinforcing steel. The failure cone was pushed out with the jack and the reinforcing bars cut with an oxy-acetylene torch so that the failure plane could be exposed.


Fig. 1 Typical stress-strain curve for intermediate grade no. 3 bars.



Fig. 3 Detail of Slab No. 1 and No. $1^{A}$


Fig. 4 Detail of Slab No. 2


Fig. 5 Detail of Slab No. 3


Fig. 6 Detail of Slab No. 4


Fig. 7 Detail of SIab No. 5


Pig. 8 Detail of Slab No. 2A


Fig. 9 Detail of Slab No. 3A


Fig. 10 Detail of Slab No. LA


Fig. 17. Detail of Slab No. 5A


## PLATE NO. 1.

Shearhead reinforcement of slabs 4 and 4 A .

PLATE NO. 2.
Truss bar reinforcement of slab 5 .


PLATE NO. 3.
Loading frame and typical test assembly.


PLATE NO. 4.
Form and tensile reinforcement seated on the vibrator beams prior to casting.


CHAPTER IV
EXPERIMENTAL RESUITS

### 4.1 General Behavior

The slabs were designed by the elastic method for an allowable load of 15 kips. The ultimate flexural capacity as computed by the Yield Line Theory ${ }^{(5)}$ was found to be 60 kips. Load-deflection profiles and centre deflection curves are shown in figures (12, 13, $14 \& 15$ ). Similar plots were made of steel and concrete strains to help analyse the test results. Pertinent data for the ultimate strength analysis are given in Table 1 . Comparisons to equations $1,2,3,4$ and 5 are given in Tables 2 and 3.

All slabs were loaded to ultimate failure which occurred by a final punch out of the column stub through the slab. In each case the sequence of events was generally as follows: 1) The first visible flexural crack appeared at approximately onehalf the ultimate load. They first formed at the centres and spread to the edges.
2) At 75 to 85 percent of the ultimate load lift in the corners was evident although not measured and the yield line patterns were distinguishable. Vertical cracks appeared randomly along the edges of the slab.
3) At ultimate load the column stub punched through violently. The load near failure was difficult to maintain.
4) The failure cone varicd in size and shape depending on the type of shear device used. Upon release of the load, noticeable rebound occurred in each case.
5) The strain measurements taken on the column stub verified the fact that the load was concentrated at the corners near the slab surface.

The strains recorded in the tensile reinforcement of the specimens are not believed to be exact measures of tensile strain in the bars for the following reasons:
a. The bending action of the slab introduces a bending stress in the bars. Since the gauges are located on the bottoms of the bars, their strain readings are slightly greater than the average strains in the bars.
b. The cracking of the concrete in the tensile zone of the slab introduces slight stress concentrations in the bars at the crack zones.

These unpredictable and unmeasureable occurances were not considered in the evaluation of the results.

### 4.2 Individual Behavior

Slab No. I (No Shear Reinforcement)
Initial hair line cracks were observed at a load of 20 kips. A distinguishable yield pattern had developed at 35 kips. Final failure occurred at 45 kips . The failure plane sloped away from the column stub at about 45 degrees. See plate (6).

Difficulty was encountered in running this first test. The loading apparatus unexpectedly interfered with the centrally located deflection dial and its readings were of no value. The Budd Digital strain recorder failed to read the strain gauges on the tension reinforcement beyond an axially applied load of 15 kips . This test was run at load increments of 5 kips and it was decided to change
in the subsequent tests to load at 2 kip increments to 30 kips and then at 5 kip increments to failure.

Sla.b No. 2 (Embedded 9 in. $x 9$ in. $x 3 / 8$ in. steel plate)
Initial cracking was observed at an applied load of 25 kips and extended toward the slab cdges as load was applied. Failure occurred at 40 kips and the steel plate punched through the slab. No bending was apparent in the steel plate, but at high load spalling was seen to develop at the stecl plate-slab interface. The failure surface sloped away from the bottom edge of the plate at an angle of approximately 20 to 30 degrees as shown in plate (7). Slab No. 3 (Concentration of Tensile Reinforcement)

No apparent cracks could be seen until the applied load had reached 25 kips. Final failure occurred at 40 kips and the failure plane sloped away from the column stub at approximately 45 degrees. Severe spalling at the column slab interface began after the applied load had reached 30 kips . This observation was unique to this test and is shown in plate (8).

Slab No. 4 (Shearhead)
First cracking appeared at 20 kips and failure occurred at 45 kips. The yield pattern was well defined, spreading from points directly under the column corners toward the slab edges. The final failure was a violent punch out, not through the shearhead, but beyond it as shown in plate (9). The failure slope was at approximately 20 to 30 degrees and began at the top tie of the shearhead. Slab No. 5 (Truss Bars)

Hair line cracks first appeared at an applied load of 14 kips and by the time failure occurred at 47 kips a somewhat random crack pattern had formed as shown in plate (10). The failure
plane sloped from the column faces at approximately 45 degrees, passing through the diagonal portion of the truss bars. See plate (11).

Slab No. IA (No Shear Reinforcement)
Slab IA was the same as Slab I and the behevior was similar. Final failure occurred at 40 kips and the failure plane sloped at 45 degrees to the plane of the slab. After failure, load was again applied and the specimen was able to support an applied force of 8 kips .

Slab No. 2 A (9 in. $x 9$ in. $\times 3 / 8$ in. steel plate)
Initial cracking formed at a load of 28 kips. The yield pattern was very irregular. Failure occurred at a load of 52 kips whe th a punch out of the plate. The failure plane sloped at approximately 20 to 30 degrees from the edge of the plate. The strength of the failed specimen was measured to be 8 kips. Slab No. 3A (Concentration of Tensile Reinforcement)

First cracking occurred at 26 kips. The yield pattern became clearly distinguishable at 35 kips and failure was at an applied load of 45 kips . The failure plane was irregular as shown in plate (12), sloping at 45 degrees on two opposite sides and approximately 20 degrees on the other sides.

Slab No. 4 A (Shearhead in Stiffer Slab)
First cracking was noticed at 24 kips. Failure occurred at 51 kips and the failure plane formed beyond the shearhead at an angle of 20 to 30 degrees with the slab surface.

Slab No. 5A (Truss Bars in Stiffer Slab)
Initial cracking was observed at 24 kins. Failure occurred at 44 kips and the failure cone sloped at 45 degrees, passing through the diagonal of the truss bars. After failure the specimen could still withstand an applied load of 17 kips.

### 4.3 Analvsis and Comparison of Experimental Results

Load deflection profiles and load deflection graphs are shown in figures (12, 13, 14 and 15). The load-deflection graphs seem to indicate that full yieldings of the specimens under load never occurred. In the elastic range, slabs IA and 5 deflected noticeably more than slabs 2, 3, and 4. It is seen that the concentration of tensile reinforcement, the steel plate and the shearm head reduced the deflection. The extra tensile reinforcement of slabs 2A, 3A, 4 A and 5A caused these specimens to behave similarily over all ranges of load.

The distribution of stress on the column stub surfaces was determined for the slabs of Group I and Slab IA. The gauges were attached to the stub on one face only, $\frac{1}{2}$ in. above the slab as shown in plate (4) 。 Figure (16) shows the stress distribution at an applied load of 24 kips . The stress concentrations were evident in all slabs through all ranges of load and did not tend to redistribute near failure as $\operatorname{Moe}{ }^{(2)}$ indicated in his report. The slabs with a shear reinforcing device had higher stress concentrations than the slabs without such reinforcoment. It was noted that the steel plate caused the highest corner concentrations. In practice such stress concentrations could cause local spalling in column corners directly under a flat plate slab which has an initially high axial load。

Tensile reinforcement strains in slab 1 A and 3 were of particular interest when compared. The concentration of tensile steel in slab 3, although reducing the deflection, did not, reduce the strain in the tensile reinforcement. The stress was consistently higher by approximately $12,000 \mathrm{psi}$ in the reinfor cement under the column stub of slab 3 than in slab IA. The concentration thus did not reduce the steel strains as Moe ${ }^{(2)}$ and Elstner-Hognestad ${ }^{(1)}$ found in their tests. It is felt that the concentration of ten sile reinforcement only causes a concentration of stress.

In no case did the shear reinforcing devices lessen the strains on the tensile reinforcement. In each case the tensile stresses in the column stub area were raised above the stresses found in a slab without a shear reinforcement.

When deflection has occurred under load, part of the load is carried through extension forces in the plane of the slab by tensile reinforcement. The resistance is known as membrane action and increases with increasing deflection. Moe ${ }^{(2)}$ developed the following equation for membrane action by assuming it equal to the total vertical component of the forces in the tensile reinforcement:

$$
\begin{equation*}
S_{v}=4 T \quad \frac{2 w_{0}}{a-r} \quad \text { where } T=\text { pydf }_{s} \tag{8}
\end{equation*}
$$

He estimated that at maximum deflection, this resistance was never larger than 6 percent of the ultimate load for the specimens of his test program.

Table I gives values of resistance obtained for the specimens of Group IA after failure. In all cases the percentage of ultimate load resisted by the membrane action of the reinforcing
mat was approximately 20 percent, well above that predicted by Moe's equation. Consideration of course must be given to the fact the deflection just prior to failure was smaller than that just after failure, but it is felt that the membrane action predicted by equation 6 is conservative.

Slab 5A with the truss bar reinforcement, resisted 17 kips after failure. This load is actually higher than the original design load. Good anchorage of the truss bars in the compression zone of the slab added to the membrane resistance. In practice, this type of shear device would probably prevent a total collapse if a local shear failure occurred.

Strain gauges were attached in two locations on the diagonal bars of the shearheads in slabs 4 and 4A. One gauge was placed in the corner area of the shearhead and the other near its centre. Figures (6) and (10) show their location. The corner diagonal bar was consistently under much higher stress than the diagonal bar located at the centre. In slab 4 at an applied load of 16 kips , the corner bar was stressed to approximately 7200 psi and the central bar to 4000 psi. In slab 4 A the same effect was recognized. However the stresses were smaller due to the extra tensile reinforcement. .The shearhead was effective in moving the periphery of failure beyond its influence, however no appreciable increase in strength was attained. The high corner stresses in the diagonal bars seem to indicate that high shear stresses were very much a local condition of the column corners.

The strain was measured on the diagonals of the truss bars in slab 5 and 5A. At an applied load of 16 kips the stress
at the gauge locations of slab 5 was approximately 12000 psi, substantially higher than in the shearhead at the same load. The length of embeddment of the truss bars in the compression zone was 6 in. more in slab 5A than slab 5, as it was first thought that the ultimate failure was precipitated by a bond failure of the truss bars. The strain readings were however so similar that this theory was discounted. It is difficult to explain the behavior of the truss bar reinforcement. It was effective, in that its load carrying capacity was developed, however the ultimate strengths of the specimens were not extended. A possible explanation would be that the concrete under the truss bar bends was crushed as the bar stresses increased, thus allowing the failure cone to develop even though a high percentage of force was resisted by the reinforcment. This explanation seems to be borne out by two observations. First, the punch out failure was not as noisy or violent as in the other tests and secondly the load carrying capacity of the specimen remained very high even after failure.

The construction of slab 2 A was revised after the results of slab 2 had been studied. The failure load of slab 2 was 40 kips, 5 kips lower than for slab 1 which had not been reinforced for shear. The plate in slab 2A was not embedded in the slab but rested on its surface. See figure (8). The ultimate load in this. case was $52 \mathrm{kips}, 12$ kips higher than for slab 2. The $3 / 8$ in. difference in shear area at the loaded section seems to be critical for developing the load capacity. Timoshenko ${ }^{(6)}$ explains that the shear near a loaded section does not follow the parabolic law but is concentrated very much at the top surface. This fact seems to be borne out for a reinforced concrete slab.

The strains in the concrete slab were measured at various locations. In all specimens the highest readings were taken in the region of the column area and the strains were proportionately smaller as the distance from the column increased. The shear devices tend to reduce the compression in the slab surface at low loads, but at high loads no trend was established from which conclusions could be drawn. The concrete surface at the column in all specimens became stressed in compression beyond 1500 psi at the design load of 12 kips. In actual flat plate floors it is possible that the bending moments at the column location are such that the allowable compressive stress in the slab is exceeded. In all cases except for the two slabs without shear reinforcement, the highest compressive stresses in the slabs were found in the immediate vicinity of the column corners at all range of applied load. The two exceptions had higher moments at the centre of the column stub and perpendicular to it.

Tables 2 and 3 have the test results compared to equations 1, 2, 3, 4 and 5. The predicted values of ultimate strength by Elstner and Hognestad (l) were higher than actually obtained by tests for all slabs with some form of shear reinforcement. For slabs with shear reinforcement in the diagonal tension zone, (4, $4 A, 5$ and 5A) the results indicate that it is not as effective as the equations predict. Close agreement between the calculated and the test ultimate loads was found for slabs without a shear device in the diagonal tension zone. Moe's equations yield higher prem dicted ultimate loads for the slabs of Group I. However for group

IA the predicted values are in good agreement with all but slab 5A. The factor of safety as determined by the current A.C.I. Code ranged from 2.65 to 3.40 . As determined by the N.B.C., the factors of safety were considerably higher, ranging from 2.80 to 4.20. 4.4 Summary

Shearing stresses were found to be a local condition of the column corner and slab surface. This effect seems to make the proper placing of shear reinforcement problematic.

The shearheads of slabs 4 and 4 A caused the failure plane to pass around rather than through the shear reinforcement. A study of the failures revealed no signs of poor anchorage.

Bearing stresses seem to be an acute problem with truss bar reinforcement in thin slabs. High bearing stresses on the concrete under the bar bends probably result in the initial crushing of the concrete and thus making the reinforcement ineffectual across the diagonal tension zone。

The only apparent advantage to concentrating the tensile reinforcement within the colum periphery is to reduce the deflection. However the steel strains in the tensile reinforcement are increased and at high loads spalling is noticeable at the column-slab interface。

Table 1

Description of tests and test results.

| $\begin{aligned} & \text { Slab } \\ & \text { No. } \end{aligned}$ | Type of Shear Device | $\mathrm{f}_{\mathrm{c}}$ | Angle of Rupture (degrees) | Center Defin. ©35 kips | $P_{\text {test }}$ kips | Strength after pailure |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | NONE | 3000 | 45 | - | 45 |  |
| 2 | $\begin{gathered} \text { PLATE } \\ 9 \times 9 \times 3 / 8 \text { in. } \end{gathered}$ | 3850 | 20-30 | $\begin{gathered} 0.307 \\ \text { in. } \end{gathered}$ | 40 |  |
| 3 | $\begin{gathered} \text { CONCBNTRA- } \\ \text { TION } \end{gathered}$ | 4000 | 45 | 0.250 | 40 |  |
| 4 | SHEARHEAD | 4450 | 20-30 | 0.212 | 45 |  |
| 5 | TRUSS BARS | 4100 | 45 | 0.300 | 47 |  |
| 1 A | NONE | 2800 | 45 | 0.320 | 40 | 8 kips |
| 2 A | $\begin{gathered} \text { PEATE } \\ 9 \times 9 \mathrm{~K} / 8 \mathrm{in} . \end{gathered}$ | 2950 | 20-30 | 0.360 | 52 | 8 |
| 3A | CONCENTRATION | 3000 | 45 | 0.225 | 45 | 9 |
| 4 A | STMARHEAD | 4100 | 20-30 | 0.270 | 52 | 8.5 |
| 5A | TRUSS BARS | 3500 | 45 | 0.258 | 4.4 | 27. |

Table 2

Test results compared to equations (1) and (2) by Elstner and Hognestad.


1) $\frac{V_{u}}{7 / 8 b d f_{c}}=\frac{333}{{ }_{f}^{i}}+\frac{0.046}{\theta_{0}}$
2) ${ }_{7}{ }_{7 / 8 b d f}=\frac{333}{{ }_{c}^{1}}+\frac{0.046}{q_{0}}+\left(q_{v}-0.050\right)$

$$
\text { where } \quad a_{V}=\frac{A_{V} f_{v} \sin \theta}{7 / 8 b d f_{c}^{f}}
$$

Table 3

Test results compared to equation (3) and (4) by Moe and to the allowable design loads.

3) $\frac{v}{b d}=\left[15\left(1-0.075 \frac{r}{d}\right)-5.250\right] \sqrt{\frac{r}{c}}$
4) $V=\left(6.23-1.12 \frac{r}{d}\right)$ bd $\sqrt{\frac{1}{c}}+A_{v}^{f} v \sin \theta$


Pig. 12 Load-deflection curves for group 1.


Fig. 13 Load-deflection curves for group lA.

$L_{\text {oad }}$ at 8 kips


Load at 12 kips


Fig. 14 Load-deflection profiles for Groupl.


Fige 15 Load-deflection profilesfor Groupla.



PLATE NO. 6.
Exposed failure cone of slab 1.

PLATE NO. 7.
Exposed failure cone of slab 2.


## PLATE NO. 8.

Spalling effect at the
column-stub interface
of slab 3 .


PLATE NO. 10.
Bottom view of slab 5 after failure.


PLATE NO. 11.
Exposed failure cone of slab 5.


PLATE NO. 12.


PLATE NO. 14.
Location of gauges on column stub and slab surface for Group 1 .


PLATE NO. 15.
Bottom view of slab 1 after failure.

PLATE NO. 16.
Bottom view of slab 2 after failure.


## CHAPTER V

SHEAR FAILURES IN SLABS

### 5.1 Introduction

While it is generally recognized that the commonly used equations for computing shearing stresses give only nominal stress values of no physical significance, it should also be realized that present knowledge of the actual behavior of slabs, subjected to concentrated load, is very limited. Elstner-Hognestad ${ }^{(1)}$ and Moe ${ }^{(2)}$ felt that shearing strength of slabs is very much related to the flexural behavior. The equations developed in both cases, " although empirical, take into account the flexural behavior of the slabs.

To properly study the failure mechanisms, Moe tested slabs with square and circular holes in them at the critical section. His explanation of the mechanism of shear failure can be stated as follows:
I) Initial cracking develops at approximately 50 percent of the ultimate load and extends rapidly to the neutral axis. See figure (17).
2) From the neutral axis cracks extend rather slowly until only a very narrow depth of compression zone remains intact at loads near ultimate.
3) The ultimate strength in shear is then governed by the strength of this very narrow compression zone above the top of the inclined cracks.
4) The periphery of the loaded area is the critical zone for failure and the vertical shearing strength of this area increases as the lateral biaxial compression increases.

### 5.2 Evaluation of Test Results

In this investigation, corresponding tensile and compressive strains were recorded at various locations in the slabs of all the test specimens. The values of strain were transposed to stress and diagramatically plotted in an attempt to approximate the position of the neutral axis.

A reasonably accurate position for the neutral axis however could not be located. At any given applied load and at different locations on the slab, the magnitudes of the tensile stress could in no way be comrelated to the corresponding compressive stress in the slab surface. The difficulty arose in trying to assume the stress distribution in the compression zone of the slab. The two way compressive stress also complicated such a co-relation.

However, when the test results were examined more generally, a definite trend in the behavior of the slab became apparent. The strains in the tensile reinforcement passing under the periphery of the loaded area, varied only slightly in each case at a given applied load. At a distance of 7 in . from the loaded area the strains in the same reinforcement were still of approximately the same magnitude as at the loaded area. In the slab surfaces however, the highest strains were found at the column corners and face, diminishing proportionately as the distance from the loaded area increased. A study of the test results indicated that this trend was established at very low applied load ( 8 kips), which seemed to eliminate the possibility of a bond failure.

Such uniform stress in the tensile reinforcement as compared to such varying stress on the slab surface, seems to indicate that although the slabs were acting in bending, they were also reacting to the applied load very much as a tied arch would resist a concentrated load at its centre. This explanation is represented schematically in figure (18).

If this was the case the flexural cracks in the slabs at the critical section, although advanced well above the neutral axis at applied loads near ultimate, did not interfere with the arching action and thus did not necessarily initiate final failure as indicated by Moe ${ }^{(2)}$.

Although too little information is available to completely justify the arching effect, the following observations of the test specimens after failure seem to indicate that the possibility does exist;

1) The failure planes were almost horizontal and very close to the slab surface at the critical section.
2) The failure planes were curved in a. slightly concave manner over their full width.
3) Flexural cracks at the slab edges in some cases extended to the slab surface, indicating tensile stresses over the complete depth of the slab along the edges.

The arching action possibly could explain the ineffectiveness of the shear reinforcement. Although in the diagonal tension zone, the shear reinforcement was not actually in the region of maximum stress. It is doubtful that the shear reinforcement could be placed
properly and be of any value in increasing the shearing strength.
It is suggested that further experimental studies be undertaken in order to determine the extent of the arching action in resisting applied concentrated loads or reactions.


Fig. 17 Typical flexural crack pattern as observed in the holes of Moe's tests.


Fig. 18 Schematic illustration of the arching action.

CHAPTER VI
CONCLUSIONS
On the basis of test results reported herein, the following conclusions are advanced:

1) The use of shear reinforcing devices in the diagonal tension zone of thin reinforced concrete slabs cannot be considered effective in raising the shearing strength.
2) Concentration of tensile reinforcement across the column periphery does not raise the ultimate shear capacity. It is questionable if this practice has any advantage other than reducing the deflection. Stress in the tensile reinforcement concentration is increased, indicating that a concentration of reinforcement only causes a concentration of stress.
3) Shear stresses are concentrated in the vicinity of the column corners and the slab surface. It is not distributed uniformly around the column periphery or in a parabolic manner over the depth of the compression zone.
4) An arching action develops internally in a slab under concentrated load. Final failure is believed to be initiated by the high shearing stress of the arching action, horizontal to, and at the slab face, and not by the continuation of the flexural cracks into the compression zone.
5) The membrane strength of the tensile reinforcement after the initial shear failure of the concrete is approximately 20 perm cent of the ultimate strength. For slabs in which bent up bars were used, the reinforcing steel will. support the design load, even after the concrete has failed in shear.
6) The equations developed by Moe and by Elstner-Hognestad for slabs without shear reinforcement, predict values of ultimate shear strength which are in good agreement with test results. However, for slabs with shear reinforcement, the equations yield values for ultimate load which are higher than actually found.
7) Values for the factor of safety varied from 2.65 to 3.40 when the test results of the slabs without shear reinforcement were compared to the allowable shear capacity as computed by the current A.C.I. Code.

## BIBLIOGRAPHY

1. Elstner, R.C. and Hognestad, E., "Shearing Strength of Reinforced Concrete Slabs," A.C.I: Journal, July 1956, V.53, pp. 29-58.
2. Noe, J., "Shearing Strength of Reinforced Concrete Slabs and - Footings Under Concentrated Loads," Portland Cement Association Research and Development Laboratories, April 1961, Bulletin D-47.
3. Lin, T.Y., Scordelis, A.C., May, H.R., "Shearing Strength of Reinforced and Prestressed Concrete Lift Slabs", State of California Department of Public Works, 1958.
4. A.C.I.-A.S.C.E. Committee 326, "Shear and Diagonal Tension", A.C.I. Journal, March 1962, V. 59, pp. 353-391.
5. Hognestad, E., "Yield Line Theory for the Ultimate Strength of Reinforced Concrete Slabs," A.C.I. Journal, March 1953, pp. 637-656.
6. Timoshenko, S., "Applied Elasticity," Westinghouse Technical Night School Press, 1925, pp. 66.
7. "American Concrete Institute Standard Building Code Requirements for Reinforced Concrete," A.C.I. 318-63.
8. "National Building Code of Canada," 1960, Part 4, Section 4.5.

APPENDIX A

IABORATORY DATA


| $5 \angle A B{ }^{\#} 2$ |  |  |  | POURED-IAN20,65 TESTED-IAN 30,65 |  |  |  | $f_{c}^{\prime}-3850 \mathrm{ps}$ |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $\begin{aligned} & \text { COAD } \\ & \text { KIPS } \end{aligned}$ | DEFCECTIONS (IN ${ }^{-3}$ ) |  |  | FOIL GNOES ON STEEC MAT (MICROIN/M) |  |  |  |  |  | WIRE GAGES ON CORITRETE (AICRO. N./N) |  |  |  |  |  |
| $\begin{array}{c\|} \hline F N G E \\ N O * \end{array}$ | 1 | 2 | 3 | 1 | 2 | 3 | 4 | $S$ | 6 | 14 | 15 | 20 | 77 | 18 | 18 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |  | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 9.7 | 15 | 15.8 | $+34$ | $+34$ | $+20$ | +10 |  |  | $-2$ | -4 | -4 | $-12$ | $-14$ | $-12$ |
| 4 | 18.4 | 25.6 | 24.2 | $+94$ | $+72$ | +68 | +44 |  |  | +2 | -4 | $-2$ | -34 | $-34$ | -26 |
| 6 | 29.4 | 38.7 | 34.1 | +204 | $+206$ | $+200$ | $+96$ |  |  | $t 1$ | $-8$ | $-32$ | $-72$ | $-78$ | -48 |
| 8 | 38.4 | 49,2 | 42.3 | +370 | $+380$ | +366 | $+328$ |  |  | +2 | -8 | $-30$ | $-1 / 2$ | -108 | $-74$ |
| 10 | 48.3 | 6.3 .3 | 53.6 | 1576 | +576 | +582 | +368 |  |  | $+2$ | -8 | $-55$ | -168 | $-166$ | $-102$ |
| 12 | 58.9 | 78.1 | 65.4 | +792 | t820 | $+796$ | +514 |  |  | $-2$ | $-12$ | $-96$ | -220 | $-256$ | $-132$ |
| 14 | 70.4 | 94.1 | 77.4 | +1004 | +1066 | 11008 | +678 |  |  | $-6$ | -28 | $-152$ | -284 | -338 | $-174$ |
| 16 | 82.6 | 110.0 | 89.4 | +1208 | +1260 | +1204 | +834 |  |  | $-20$ | $-50$ | $-216$ | -348 | $-4 / 6$ | $-214$ |
| 18 | 93.8 | 125.7 | 101.1 | 11404 | 11476 | $+1406$ | +1002 |  |  | $-38$ | $-75$ | $-292$ | -432 | $-492$ | -274 |
| 20 | 103.4 | 140.2 | 1118 | +1602 | $+1676$ | +1612 | +1168 |  |  | $-56$ | -104 | -360 | -478 | -580 | $-302$ |
| 22 | 114.7 | 155.0 | 123.0 | +1790 | +1864 | $+1802$ | $t 1328$ |  |  | $-78$ | $-136$ | $-442$ | -558 | $-676$ | -348 |
| 24 | 126.9 | 171 | 135.3 | +1996 | +2061 | $+2000$ | +1483 |  |  | $-116$ | -168 | $-556$ | -628 | $-782$ | -408 |
| 26 | 139.5 | 189 | 147.3 | +2072 | +2085 | +2460 | $+1546$ |  |  | $-150$ | $-202$ | $-662$ | -726 | -896 | -468 |
| 28 | 153.6 | 210 | 160.8 | $+2072$ | +3557 | +280\% | +1722 |  |  | -150 | $-226$ | -786 | -832 | -1038 | $-540$ |
| 30 | 208.6 | 230.8 | 174.8 | +2104 | - | $+3802$ | +1817 |  |  | -140 | -256 | $-930$ | $-840$ | -1208 | $-612$ |
| 35 |  | 307 | 219 | $+2309$ | - | $+3806$ | +1914 |  |  | -98 | -288 | -1474 | -1244 | $-1422$ | -856 |
| 40 |  | 385 |  |  |  |  |  |  |  | -124 | $-382$ | - | $-1410$ | - | $-1100$ |
| $45^{-}$ | $\rightarrow$ FAKCURE |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 50 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |





| $5 \subset A B / A$ |  |  |  | $\begin{aligned} & \text { POURED FEN 8,65 } \\ & \text { TESTED FETB17,65 } \end{aligned}$ |  |  |  |  |  | $f_{c}^{\prime}-2800$ pr |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $\begin{aligned} & \angle O A D \\ & K \mathbb{P S} \end{aligned}$ | DEFCECTION (1N ${ }^{-3}$ ) |  |  | FOLC SAGE ON STEEC (MICROMI/IN.) |  |  |  |  |  | WIRE GAGE ON TONLRETE (MIEROM/N) |  |  |  |  |  |
| $\underset{\text { GAG } \rightarrow}{\text { GAE }}$ | 1 | 2 | 3 | / | 2 | 3 | 4 | 5 | 6 | 14 | 15 | 16 | 17 | 18 | 19 |
| 0 | 0 | 0 | 0 |  | 0 | 0 | 0 |  |  | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 22 | 25 | 30 |  | $-15$ | $+10$ | 0 |  |  | $-15$ | $-30$ | $-30$ | $-90$ | $-100$ | $-90$ |
| 4 | 37 | 41 | 48 |  | -10 | $+35$ | $+30$ |  |  | -45 | $-70$ | -50 | -195 | $-210$ | $-190$ |
| 6 | 50 | 58 | 62 |  | $+15$ | +65 | $+60$ |  |  | -50 | -85 | -60 | -305 | $-330$ | -305 |
| 8 | 64 | 75 | 75 |  | 175 | +110 | +130 |  |  | -80 | -125 | -95 | $-450$ | -480 | -435 |
| 10 | 76 | 92 | 87 | : | +175 | +200 | $+200$ |  |  | -80 | -155 | $-120$ | - 570 | -605 | $-550$ |
| 12 | 88 | 110 | 99 | $\triangle$ | $+275$ | +305 | $+270$ |  |  | -80 | -195 | $-170$ | $-660$ | -725 | -660 |
| 14 | 100 | 115 | 110 | 4 | $+370$ | +420 | $+330$ |  |  | $-100$ | $-230$ | -225 | -790 | -840 | -765 |
| 16 | 110 | 140 | 120 | N | $+485$ | +570 | $+400$ |  |  | $-100$ | -250 | -280 | $-910$ | -940 | -855 |
| 18 | 120 | 153 | 130 | 0 | $+585$ | $+710$ | $+475$ |  |  | -55 | -215 | -280 | -940 | -1010 | -845 |
| 20 | 130 | 170 | 140 | रे | $+695$ | $+880$ | $+570$ |  |  | -65 | $-240$ | -345 | -1000 | $-1210$ | -880 |
| 22 | 140 | 185 | 150 |  | +815 | +1040 | +650 |  |  | +200 | $+30$ | $-100$ | -800 | -1780 | -1000 |
| 24 | 150 | 200 | 160 |  | $+945$ | $+1235$ | +760 |  |  | $t 200$ | $-10$ | -190 | -930 | -1930 | $-11 / 5$ |
| 26 | 160 | 215 | 172 |  | +1065 | 7425 | $+860$ |  |  | +320 | $+160$ | 0 | -1/30 | -2090 | $-1000$ |
| 28 | 170 | 235 | 183 |  | t1185 | +1595 | $+980$ |  |  | $+220$ | -20 | - | -1380 | -2375 | $-1330$ |
| 30 | 185 | 252 | 195 |  | +1330 | +1750 | $+1130$ |  |  | $+120$ | $-100$ | $\cdots$ | $-1460$ | $-2465$ | $-1425$ |
| 35 | 215 | 320 | 232 |  | +1555 | $+2195$ | $1+1495$ |  |  | +40 | $-230$ | $\cdots$ | $-1400$ | -2485 | -14/0 |
| 40 |  | 400 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 45 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |



|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  |  |  |  | 5\％－ | c3］ | 101 |
| 5x6－ | 0602－ | O＜\＆－ | － | － | － |  |  | 0202t | 0＜， 27 | 08127 | $0065 t$ | 022 |  | S52 | S力 |
| O06－ | 0002－ | O9乏－ | 0091－ | 028－ | －529／－ |  |  | S $\angle 1 / 1$ | 08L1t | 0161t | OE／2t | －581 | Q／2 | Sb1 | Oh |
| 028－ | OS81－ | CSE－ | O9E／－ | OOC－ | S＜E／－ |  |  | O9SH | Oのht | Oカの14 | OSL1t | $65 \%$ | 522 | 891 | S $\Sigma^{\circ}$ |
| OEL－ | $0191-$ | OIE－ | Ol1／－ | 029－ | 0511－ |  |  | SL21t | Q221t | QbElt | Oitil | 651 | Sbl | －Sth | OE |
| 019－ | OLAI－ | 082－ | 5801－ | O $\angle 5-$ | 0501－ |  |  | OS11t | $0 \varepsilon 1 /+$ | S921t | OL217 | 151 | 081 | 981 | 82 |
| 029． | S级－ | Ob2－ | 0501－ | 595－ | 066－ |  |  | 0101t | S $\angle 8+$ | 0214 | 51117 | 521 | $8 \%$ | 0221 | 92 |
| 085－ | OE21－ | Ox2－ | 056－ | OOS－ | O68－ |  |  | O06t | OSLA | 50017 | $5 \angle 8 t$ | 911 | 5.51 | L11 | bz |
| 055－ | Olll－ | 002－ | 058－ | Oカt－ | 08L－ |  |  | OLLt | 029t | 5587 | 0184 | LO1 | $1+1$ | 801 | 22 |
| S6x－ | S86－ | 081－ | SLC－ | SbE－ | 089－ |  |  | 5s9t | S8カt | 069t | S29＋ | 96 | 821 | 16 | Q2 |
| OSt－ | O18－ | 0S／－ | 001－ | S5E－ | 009－ |  |  | 0．55t | ＿ $5 \angle$ 仡 | 09St | S／St | 88 | －111 | 88 | 81 |
| OOX－ | Otb－ | O21－ | 519－ | COE－ | 025－ |  |  | Onnt | － | Oxt ${ }^{\text {O }}$ | Oltt | 8 L | 001 | 84 | 91 |
| 09\｛－ | S59－ | OO／－ | OES－ | Sx2－ | OSh－ |  |  | Shet | －SLEt | OEEt | O／Et | $\bigcirc$ | 88 | 89 | t1 |
| OOS－ | S5S－ | SO－ | OSt－ | OO2－ | 088－ |  |  | 092t | O2Et | 0227 | 061t | 99 | $2 \angle$ | $\angle .5$ | 21 |
| 052－ | Oこ力－ | O＜－ | OSE－ | S51－ | Ob2－ |  |  | O61t | 592t | $521+$ | Oह1＋ | $8 \rightarrow$ | 85 | 9\％ | 01 |
| 061－ | O1£－ | 09－ | 092－ | SO1－ | O22－ |  |  | Ollt | Oblt | S9t | $56 t$ | 65 | 56 | OE | 8 |
| OカI－ | O／2－ | O．S－ | 081－ | －S $<-$ | O91－ |  |  | OCt | OLIt | O5t | 0のt | OS | 15 | 92 | 9 |
| 08－ | Oह1－ | OS－ | S11－ | OS－ | 001－ |  |  | Ont | O2t | －S2t | Ott | $b 1$ | 02 | 41 | 力 |
| OE－ | 0．5－ | O－ | St－ | O2－ | ロカー |  |  | S／t | St | o2t | O2t | 5.8 | 6 | 5.8 | 2 |
| $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |  |  | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | 0 | 0 | 0 | 0 |
| 61 | 81 | $\angle 1$ | の1 | －S1 | bl | の | $-5$ | \＄ | $\Sigma$ | 2 | 1 | $\varepsilon$ | 2 | 7 | $\begin{aligned} & \text { acor } \\ & a 040 \end{aligned}$ |
|  |  |  |  |  |  | （N／f／n／Q2＞1w） 27315 ro 3040071 |  |  |  |  |  | $C^{\text {N }}$（1）NO4237130 |  |  | $\begin{aligned} & 5817 \\ & 06007 \end{aligned}$ |
| $\begin{aligned} & 59182 \cdot 5131031531 \\ & \text { s919/831 } 032 n 0^{6} \end{aligned}$ |  |  |  |  |  |  |  |  |  |  |  | t8 8t7s |  |  |  |


| $5 \angle A B{ }^{\#} 4 A$ |  |  |  | POURED FEB 23, 65 |  |  |  |  | $f_{c}^{\prime}-400$ ps, |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| TESTED MARCNE/G5 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $\begin{aligned} & \angle O A D \\ & \text { KIPS } \end{aligned}$ | DEFLECTION (IN ${ }^{-3}$ ) |  |  | FDIL GAGE ON STEEL (MICRO N./N) |  |  |  |  |  | WIRE GMGE ONTONCRETE (MIERO INO/N) |  |  |  |  |  |
| GAGE Mo. | $\checkmark$ | 2 | 3 | 1 | 2 | 3 | 4 | 5 | 6 | 14 | 15 | 16 | 17 | 18 | 17 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 3 | 6 | 8 | +10 | 0 | 0 | -5 | 0 | 0 | $-45$ | 0 | -45 | 0 | -55 | $-30$ |
| 4 | 8 | 13 | 14 | $+30$ | $+20$ | +20 | -5 | 0 | 0 | -95 | $-10$ | -95 | -5 | $-135$ | -60 |
| 6 | 18 | 27 | 25 | +65 | $+65$ | +50 | $-25$ | 0 | +1/5 | $-210$ | $-30$ | $-220$ | $-15$ | -280 | $-110$ |
| 8 | 28 | 41 | 37 | +155 | +95 | +105 | -20 | +15 | +45 | $-315$ | -40 | -335 | -15 | $-415$ | -205 |
| 10 | 38 | 55 | 48 | $+240$ | +115 | +145 | +5 | +25 | t85 | -430 | -50 | -450 | -20 | $-540$ | -290 |
| 12 | 51 | 74 | 65 | $+355$ | $+200$ | +195 | +45 | $+45$ | +120 | $-550$ | -60 | $-580$ | $-10$ | -680 | -375 |
| 14 | 64 | 92 | 77 | $+455$ | +275 | $+270$ | 190 | +65 | $+160$ | $-675$ | $-70$ | -710 | $-10$ | $-825$ | -450 |
| 16 | 77 | 110 | 90 | $+550$ | $+400$ | $+400$ | +145 | +85 | +195 | $-780$ | -80 | -835 | $-10$ | -980 | -525 |
| 18 | 90 | 130 | 103 | 1645 | $+535$ | $+500$ | 180 | +95 | +225 | -930 | -90 | -960 | 0 | $-1130$ | $-570$ |
| 20 | 102 | 145 | 118 | $+735$ | $+665$ | 7595 | $+240$ | +105 | +265 | -1040 | -100 | -1080 | 0 | -1290 | -650 |
| 22 | 114 | 162 | 130 | $+845$ | +785 | +700 | $+305$ | +125 | $+310$ | -1160 | $-115$ | $-1190$ | 0 | $-1430$ | -700 |
| 24 | 123 | 180 | 141 | $+955$ | $+890$ | +800 | $+370$ | +135 | $+335$ | -1250 | $-140$ | $-1285$ | -5 | -1570 | -740 |
| 26 | 134 | 192 | 153 | $+1080$ | $+1010$ | $+910$ | $+455$ | +140 | $+365$ | -1355 | $-150$ | -1380 | $-10$ | -1700 | $-770$ |
| 28 | 145 | 209 | 165 | t1225 | $+1120$ | $+1030$ | $+525$ | +165 | $+405$ | $-1450$ | $-180$ | $-1480$ | $-20$ | $-1840$ | -810 |
| 30 | 155 | 221 | 175 | $t 1365$ | +1220 | +1140 | +605 | +175 | 1430 | $-1550$ | -205 | -1580 | -35 | -1970 | -855 |
| 35 | 184 | 270 | 207 | $+1740$ | +1455 | $+1460$ | $+905$ | $+245$ | +505 | -1930 | -360 | -2050 | -180 | -2260 | -925 |
| 40 | 220 | 325 | 245 | +2245 | $+1710$ | +1845 | +1295 | $+305$ | $+565$ | -2245 | $-530$ | $-2400$ | $-285$ | -2615 | -1005 |
| 45 |  | 390 |  | 18745 | +1950 | +8700 | +1575 | +365 | +565 | -2550 | -670 | $-2700$ | -385 | -2975 | $-1110$ |
| 50 |  | 490 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| FAMEDA才 51 kIFS |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |


| $5 \angle A B H 5 A$ |  |  |  | $\begin{aligned} & \text { POURED } \rightarrow \text { FE13.24,65 } \\ & \text { TESTED } \rightarrow \text { MAEEH 3,65 } \end{aligned}$ |  |  |  | $f_{c}^{\prime}-3500 \mathrm{ps}$ |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $\begin{aligned} & \angle O A O \\ & \text { KIPS } \end{aligned}$ | DEFLERTION (IN ${ }^{-3}$ ) |  |  | FOIC GKGE ON STEEL (MIERC IN./IA) |  |  |  |  |  | WIRE GNGE ONCORICRETE (MICRO/N/NS) |  |  |  |  |  |
| GNGE $N O . \rightarrow$ | 1 | 2 | 3 | / | 2 | 3 | 4 | 5 | 6 | 14 | 15 | 16 | 17 | 18 | 19 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 10 | 13 | 14 | $+60$ | $+75$ | $+55$ | $+30$ | 0 |  | $-75$ | -20 | -80 | $-20$ | $-70$ | $-70$ |
| 4 | 20 | 24 | 24 | +135 | $+180$ | $+150$ | +80 | +15 |  | $-150$ | $-40$ | $-150$ | $-25$ | $-140$ | $-120$ |
| 6 | 27 | 33 | 32 | $+240$ | $+280$ | $+245$ | +145 | +35 |  | -220 | $-50$ | -225 | -35 | $-200$ | $-170$ |
| 8 | 38 | 47 | 42 | $+395$ | $+420$ | $+390$ | +245 | 180 |  | $-310$ | $-80$ | $-320$ | $-50$ | -285 | -235 |
| 10 | 47 | 60 | 51 | $+545$ | $+560$ | $+530$ | $+350$ | 7145 |  | $-390$ | $-100$ | -410 | $-55$ | $-355$ | $-290$ |
| 12 | 56 | 71 | 61 | $+700$ | $+695$ | $+665$ | $+460$ | $+205$ |  | $-470$ | $-130$ | $-500$ | -70 | $-430$ | $-340$ |
| 14 | 67 | 85 | 71 | +870 | $+840$ | $+810$ | $+570$ | $+280$ |  | -555 | $-150$ | -590 | -75 | $-4>0$ | $-400$ |
| 16 | 70 | 100 | 80 | $+1025$ | $+980$ | $+950$ | $+690$ | $+350$ |  | -640 | $-185$ | -685 | -90 | $-380$ | $-450$ |
| 18 | 93 | 120 | 94 | $+1220$ | +1110 | +1125 | $+850$ | $+470$ |  | $-750$ | $-220$ | -800 | -95 | -465 | $-520$ |
| 20 | 102 | 130 | 104 | +1370 | +1250 | +1265 | $+980$ | $+560$ |  | $-845$ | $-250$ | -900 | $-100$ | $-510$ | $-580$ |
| 22 | 114 | 147 | 115 | $+1525$ | $+1390$ | +1410 | $+1110$ | 1650 |  | -940 | -280 | $-1000$ | -105 | -600 | -640 |
| 24 | 123 | 160 | 123 | $+1625$ | +1500 | $+1520$ | $t 1220$ | $+740$ |  | -1030 | $-310$ | $-1100$ | $-115$ | -650 | -690 |
| 26 | 136 | 179 | 135 | $t 1770$ | +1650 | +1665 | $+1350$ | $+855$ |  | $-1150$ | -340 | $\cdot 1230$ | $-115$ | -825 | -765 |
| 28 | 149 | 195 | 187 | +1880 | $+1775$ | $+1790$ | $+1490$ | $+975$ |  | -1270 | -375 | -1360 | $-130$ | -925 | -835 |
| 30 | 160 | 211 | 157 | $+2020$ | +1905 | $+1910$ | $+1615$ | +1100 |  | -1380 | -380 | $-1490$ | -130 | -1000 | $-900$ |
| 35 | 193 | 250 | 187 | $+2240$ | $+3235$ | $+2145$ | +1910 | +1495 |  | -1805 | - | $-1880$ | $-190$ | -1160 | -955 |
| 40 | 233 | 330 | 223 | - | - | 15575 | +1930 | +1980 |  | -2600 | - | - | -220 | - | $\cdots 1010$ |
| 44 |  | 400 |  | FM1 | $C E D$ |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

