

THE PROCESSING AND INTERPRETATION
OF LONG WAVELENGTH AEROMAGNETIC
ANOMALIES

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BY
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by

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A thesis submitted to the Faculty of Graduate Studies of
the University of Manitoba in partial fulfillment of the requirements
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MASTER OF SCIENCE

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ABSTRACT

Digital data processing techniques are now available and widely used in the study of aeromagnetic surveys. It allows detailed analysis to be made of the shallow and deep crustal magnetic units and also enables the short and long wavelength anomalies to be separated and evaluated.

The origin and nature of the long wavelength anomalies are believed to be closely associated with the deep crustal magnetization. These deep crustal magnetic units in turn are related to crustal structure seen on surface geologic maps. The processing sequence of aeromagnetic data that yields long wavelength aeromagnetic anomalies is compiled and evaluated in this study. Different digital processing techniques are examined and employed in an attempt to put together an efficient aeromagnetic processing system.

A long wavelength aeromagnetic map portraying deep crustal magnetic anomalies in the project area (60.00 to 61.75 degrees North and 102.00 to 94.75 degrees West) is presented. The derived map shows a definite alignment of various long wavelength aeromagnetic anomalies of different sizes in a Northeast-Southwest and East-West orientation. As an aid to the interpretation of these aeromagnetic

anomalies, a method of "inverse" depth and magnetic intensity determinations is introduced. The automated interpretation method is based on the optimization of the best fit anomaly field generated by a vertical-sided prismatic model. This interpretation procedure has eliminated the use of the tedious method of type-curve fitting and manual calculations.

The long wavelength anomalies show a good correlation with the broad features of deep crustal structure. The regional geology of the surrounding areas is reviewed and a strong relation between the long wavelength anomaly system and a major structural feature is found.

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CHAPTER 1

INTRODUCTION

When the total anomaly system composed of short wavelengths anomalies are filtered out of an aeromagnetic map, there remains a relatively broad-trend, low-amplitude and long wavelength anomaly field. These anomalies are termed long-wavelength aeromagnetic anomalies.

Long-wavelength aeromagnetic anomalies show considerable correlation with broad crustal structure and provide valuable information in the study of magnetic units lying in the upper and lower layers of the earth's crust. The present study is part of a continuing program of detailed deep crustal magnetic investigations in Manitoba with extensions up to the Northwest Territories.

In recent years, there has been a rapid development in the digital processing techniques of geophysical data. This study deals with the digital processing and interpretation of aeromagnetic data located in the Northwest Territories directly north of the Manitoba provincial boundary. An attempt is made in this thesis to develop a more efficient processing sequence for regional magnetic anomalies,

including the design of a two-dimensional filter for the aeromagnetic data in the frequency (or wave-number) domain. This is achieved with the fast Fourier transformation algorithm developed by Cooley and Tukey (1965).

Two-dimensional aeromagnetic data can be converted into the frequency domain by applying the forward Fourier transformation. The procedure of using one-dimensional transform to calculate the two-dimensional transform is used so as to cut down the available storage space and computing time. Pretreatment of data is essential. The presence of undesired spikes and trends were acknowledged and a computer program package was written to carry out every step of the processing sequence including the filtering of high frequency components of the anomaly field in the frequency domain.

McGrath and Hall (1969) developed a seventeen-point numerical filter in the space domain to remove short wavelength anomalies from an aeromagnetic map. Linear filtering in the frequency domain is under investigation here and a comparison of the nature of the two types of filter is done on the aeromagnetic map in this project. Moreover, various attempts have been made to display the frequency response of the frequency domain filter.

A great deal of interest has been laid on the representation of potential field data in the frequency domain. For

this reason, spectral analysis has become a routine and essential technique in the processing of aeromagnetic maps. One dimensional power spectrum analysis was performed on a few profiles running east-west within the area of concern. Two dimensional power spectrum was also displayed on part of the area. The Maximum Entropy Method for the power spectral density estimation as developed by Burg (1967, 1968) is also reviewed in this thesis and it was experimentally investigated on a few profiles across the area to serve as a comparison to the more conventional power spectral estimation via the fast Fourier transform algorithm.

One of the major characteristics of deep crustal magnetic anomalies are their low amplitude. The amplitudes of this type of anomaly are often less than 200 nanotesla. This is because of the vertical magnetic field decreases as the depth of source increases. Hence the core generated background field plays an important role in the interpretation of the long wavelength aeromagnetic anomalies. A computer Fortran program issued by the Institute of Geological Sciences, Hailsham, Sussex was used for computing the background field values of the project area. It is based on a specified model adopted after the International Association of Geomagnetism and Aeronomy (IAGA) Symposium held in October, 1968 and is termed as the International Geomagnetic Reference Field (IGRF).

The question of uniqueness and ambiguity of potential field interpretations have been discussed by Skeels (1947), and by Roy (1962). For a given magnetization distribution, there is always a predictable magnetic field but the inverse is not always unique. A given magnetic anomaly field may be caused by an infinite number of magnetic distributions and no unique solution can be clearly defined. In the present project, a direct interpretation using the BLOCK model-fit by Coles (1973) was first used for the anomaly interpretations. Then, an inverse problem¹ was set up, with the input of the field values and using optimization processes, the configuration of the blocks that were most acceptable both physically and geologically were then determined.

-
1. The computer program "INVBLOCK" to provide an indirect iterative method to solve the inverse problem was originally derived and implemented by Mr. C. Hasselfield, programmer, Geophysics Section, University of Manitoba.

CHAPTER 2

DIGITAL PROCESSING OF AEROMAGNETIC DATA

2.1 Processing Sequence

The long-wavelength aeromagnetic anomaly system can be separated from the local anomalies due to shallow, narrow sources by eliminating the high wave number (short wavelength) components of the aeromagnetic map. This can be done in several ways; for example, by upward continuation of low-level magnetics. Alternatively, based on the difference in the frequency (wave-length) characteristics of the two major anomaly types, direct filtering can be used. The processing sequence used here for the preparation of the long-wavelength aeromagnetic maps was based on the direct wavelength filtering technique.

Figure 2.1.1 is a flow diagram of the processing sequence of the digitized aeromagnetic map. The computer program FILTER 2D in Appendix B is a FORTRAN routine based on this sequence.

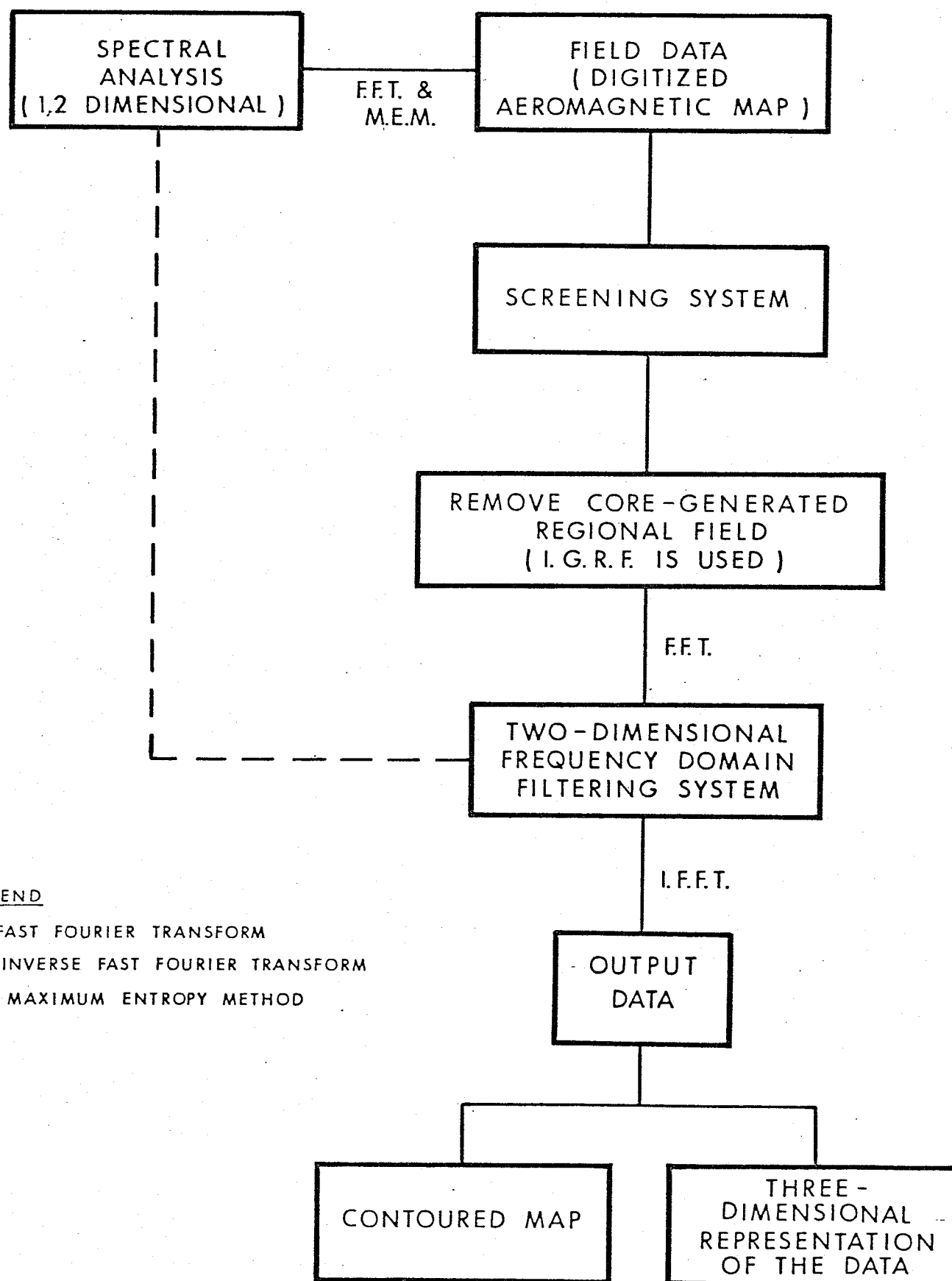


FIGURE 2.1.1. FLOW DIAGRAM OF THE PROCESSING SEQUENCE

It is assumed that the input map consists of digitized data on a regular spaced square or rectangular grid. The sampling (or digital) interval must be equal in the x and y directions.

Both one and two dimensional power spectra were constructed on the data. This analysis provides a spectral scan of the frequency contents of the input map before the filtering technique was applied. Details of the spectral analysis can be found in a later chapter.

Preconditioning of data is also very important. Different screening and smoothing operations were done on the input data before passing the map through the filter system.

After the above steps had been accomplished, the data was then passed into the filter system. Direct filtering in the frequency domain was used in this study. The filter was designed for a series of different cutoff wavelengths and impulse-response of the study map area. To aid interpretation, different displays both in contour map and three-dimensional representation forms were drawn using the IBM calcomp plotter.

2.2 Two-Dimensional Frequency Aliasing

Frequency aliasing is a well-known effect caused by the insufficient sampling of a signal frequency. It occurs when a band limited function is sampled at interval that exceeds $\frac{1}{2f_i}$ where f_i is the highest frequency component of interest recorded.

For two-dimensional data, such as the potential field data, it is necessary to digitize the data from the maps at a suitable sampling interval. A small sampling interval may be desirable since it enables us to study the shorter wavelength anomalies and throw out the aliasing problem. However, one should be aware of the computer limitations in analysing enormous amounts of data and the tedious digitization problem.

Analogous to the one-dimensional case, two-dimensional frequency aliasing rule can be obtained. In order to avoid frequency aliasing when sampling a continuous function band-limited at frequencies of K and L , the sample interval must be selected so that

$$\Delta x \leq \frac{1}{2K} \quad \text{and} \quad \Delta y \leq \frac{1}{2L}$$

In the present project, long-wavelength anomalies with wavelengths usually greater than 60 km are of major concern,

hence, K and L have very high values. The sampling interval of 6.5 km in both x and y directions were selected for the project. This sampling interval was found to be sufficient and reasonable in the computer processing viewpoints and it was done by decimating the unwanted data points in the original data.

2.3 Preconditioning of Data

Preconditioning of data is essential and necessary, especially when filtering in the frequency domain. Spikes appearing in the data, due to digitization error or other data preparation errors, produce problems in the frequency filtering process. It will produce output in the Fourier transform at all frequencies and it must be removed with a search and screening routine. One of the simpler devices for testing doubtful observations is known as "Chauvenet's Criterion of Rejection", according to which rejectability is determined as a function of deviation, dispersion, and number of measurements. This criterion was used in this study as the selection system of the input data set¹. The standard deviation was first determined and a dispersion of ± 3 standard deviations was found to be the bound of the input data

1. The "Chauvenet's Criterion of Rejection" was first designed and programmed by P. McGrath, University of Manitoba.

set. The data values which exceed the boundary limits will then be cut down to the appropriate boundary values. This not only served the purpose of removing unwanted spikes, it also removed the anomalous peaks which might result from local iron formation.

Additional rows and columns of numeric zeros were added to the data matrix during the data preparation stage in order to satisfy the prime factor assumption of the fast Fourier transform algorithm (see Section 2.4). The presence of these zeros in the data matrix introduced a step-like function at the boundaries and severe edge effects resulted if they were not eliminated. This search and elimination process was also introduced into the screening system of this project. This was done by means of replacing numeric zeros found in the input matrix by the average value (or mean) of the data set.

If the data have a regional trend, spurious components at all frequencies will be introduced in the Fourier Transform. Best-fit plane produced by a least-squares fit has been commonly used for regional trend extraction. The plane was subtracted from the data set to produce a new set of data with zero regional trend. This approach was not used in the present study. Instead, a numerical model of the sufficient complexity, including adequate secular variation correction was used as a suitable representation of the

regional field. This is the best known and most widely used field model: the International Geomagnetic Reference Field (IGRF). The IGRF values were computed at every grid point of the data matrix by means of a computer program² and it was subtracted from the field value at that grid point. This removes the regional trend from the data before the filtering process. A more detailed account of the IGRF surface can be found in Section 2.5.

2.4 The Fast Fourier Transform

The Fourier transform has long been a principle analytical tool in such diverse fields as linear systems, optics and signal analysis. The advent of the development of the fast Fourier Transform (FFT), an algorithm that efficiently computes the discrete Fourier transform which reduces the enormous amount of computation time adds even more applications to this powerful analytical technique. The fast Fourier transform algorithm was the principal tool used in this thesis to study the potential field data.

The potential field data $f(x,y)$ is considered as a mixture of random noise and aperiodic signals. The spectral

2. Computer program supplied by the Institute of Geological Sciences, Haillsham.

decomposition of the potential field data $f(x,y)$ can be represented by the following complex Fourier transform relations in the digital domain.

$$F(K_1, K_2) = \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} f(x,y) e^{\frac{2\pi i}{N}(K_1 x + K_2 y)}$$

for $K_1, K_2 = 0, 1, 2, \dots (N-1)$

and conversely

$$f(x,y) = \frac{1}{N^2} \sum_{K_1=0}^{N-1} \sum_{K_2=0}^{N-1} F(K_1, K_2) e^{-\frac{2\pi i}{N}(K_1 x + K_2 y)}$$

for $x, y = 0, 1, 2, \dots (N-1)$

where $F(K_1, K_2)$ are N complex-valued vectors. The wavelengths λ_1 and λ_2 are given by $\frac{2\pi}{K_1}$ and $\frac{2\pi}{K_2}$, where K_1 and K_2 are the wave-numbers along the x and y directions.

The fast Fourier transform was developed from these fundamental equations. The criteria for the formulation of these equations and their various properties have been dealt with thoroughly by Good (1958), Cooley and Tukey (1965) and Gentleman and Sande (1966). In general, direction computations of the Fourier transform require N^2 operations, where N is the size of the input vector. Fast Fourier transform (FFT) algorithm offers a much faster route of less than $KN \log_K N$ operations (K is a factor of N). In other words,

one can obtain the transforms $(\frac{N^2}{KN \log_K N})$ times faster. The accuracy of the results is also increased by the same ratio.

The fundamental basis of the fast Fourier transform algorithm is to decompose each data vector into simple factors. (For example, $N = K_1 \cdot K_2 \cdot K_3 \cdot \dots \cdot K_n$). Then, the elementary transformations were followed by a permutation sorting procedure to produce the required coefficients. In the same manner, a two-dimensional data set such as the aeromagnetic field data in this project can be considered as a vector (size N_1 by N_2) of length $N = N_1 N_2$ which the fast Fourier algorithm can be applied in the usual way.

2.5 Background Field Analysis

The earth's total magnetic field is composed of the field generated by the earth's core plus anomalous fields which are attributed to the magnetic properties of shallow and deep crustal sources. An important application of aeromagnetic surveys is to interpret the magnetic properties or characteristics of deep crustal magnetic sources. These deep magnetic sources generate long wavelength anomalies which are often low in amplitude, usually less than 200 nanotesla. As a result, the core generated background field plays an important role in the interpretation of the long-wavelength aeromagnetic anomalies. This core generated field

has to be more accurately defined and carefully removed in order to arrive at a set of meaningful anomalies. An ambiguous definition of the base level used will then lead to a high degree of inaccuracy and uncertainty in the interpretation of the residual anomaly maps.

During the past several years, geophysicists have expressed a need for establishing a universally recognized geomagnetic reference field so that it can provide a common datum suitable for various applications in the analysis of the earth's magnetic field. This, of course, includes the application of the removal of trends from detailed aeromagnetic surveys. As a result of combined efforts of various groups within the International Association of Geomagnetism and Aeronomy, a mathematical model: the International Geomagnetic Reference Field (IGRF) was chosen at the Symposium on Description of the Earth's Magnetic Field held October 22-25, 1968, in Washington, D.C.

The IGRF is defined by a set of spherical harmonic coefficients to degree and order 8 ($m=n=8$) for both main field and secular variation (a total of 160 coefficients). A computer Fortran program SUBROUTINE IGRF, (see Appendix B) issued by the Institute of Geological Sciences, Haillsham, Sussex, was used for computing the background field values of the project area.

A computer contour plot of the International Geomagnetic Reference Field of the study area is shown in Figure 2.5.1. The unfiltered data of the project area in three dimensions is shown in Figure 2.5.2. Figure 2.5.3 shows the map area with the IGRF removed.

Removal of the geomagnetic field from the aeromagnetic data can also be done using the polynomial approximation method³. A second-order equation can be derived using the best fitted least squares to the surface (map area) of the total magnetic intensity map published by the Dominion Observatory. The background field values can then be computed using the derived equation. This approach was tested for the project area; it was found to be tedious and suffer some form of inaccuracy in the derivation because of the limited size of the area involved. The IGRF was found to be more efficient, accurate and economical in terms of computer storage.

3. Removal of the geomagnetic field from the aeromagnetic data by polynomial approximation method was first discussed by P.H. McGrath and D.H. Hall (1969).

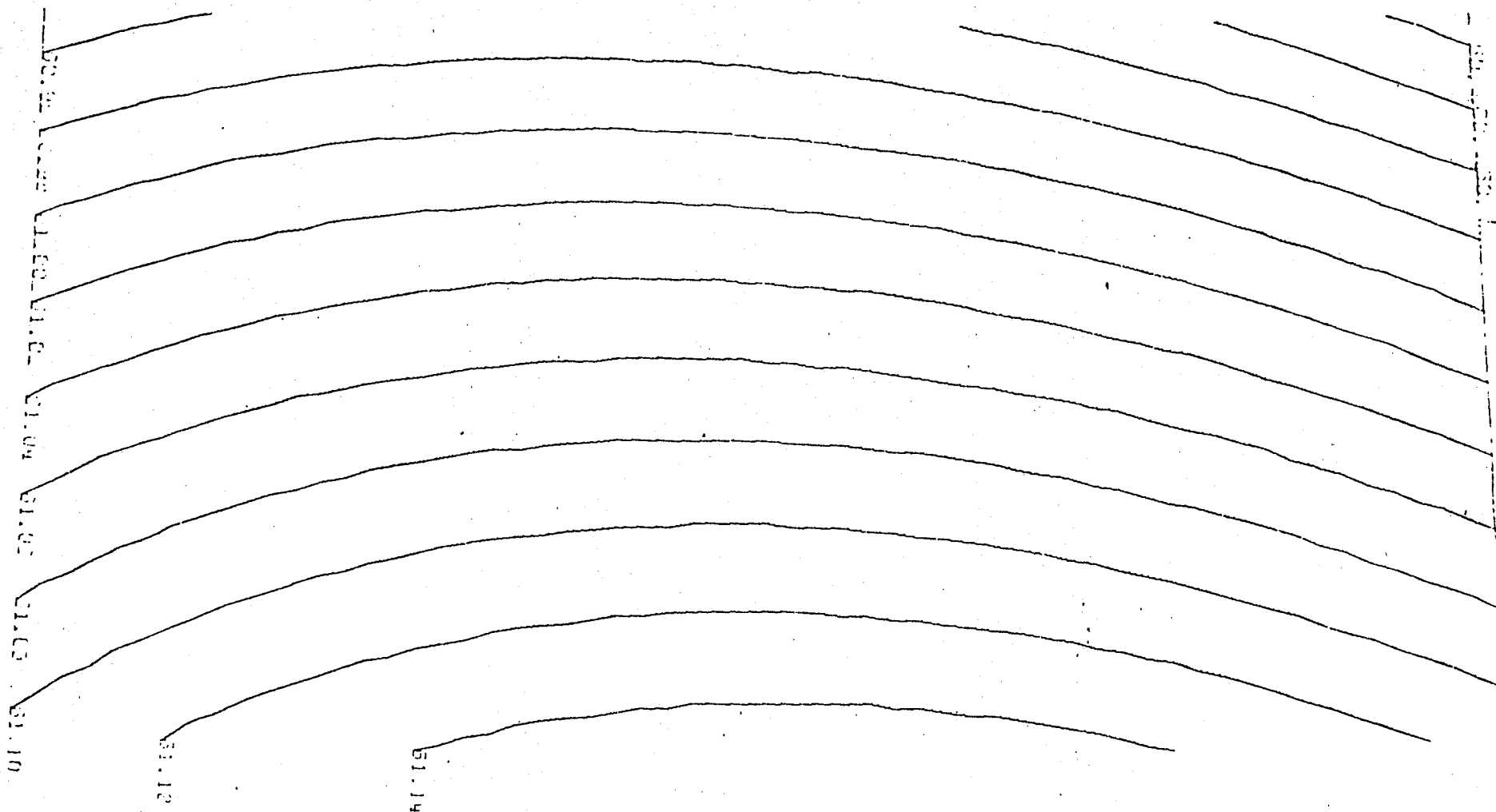
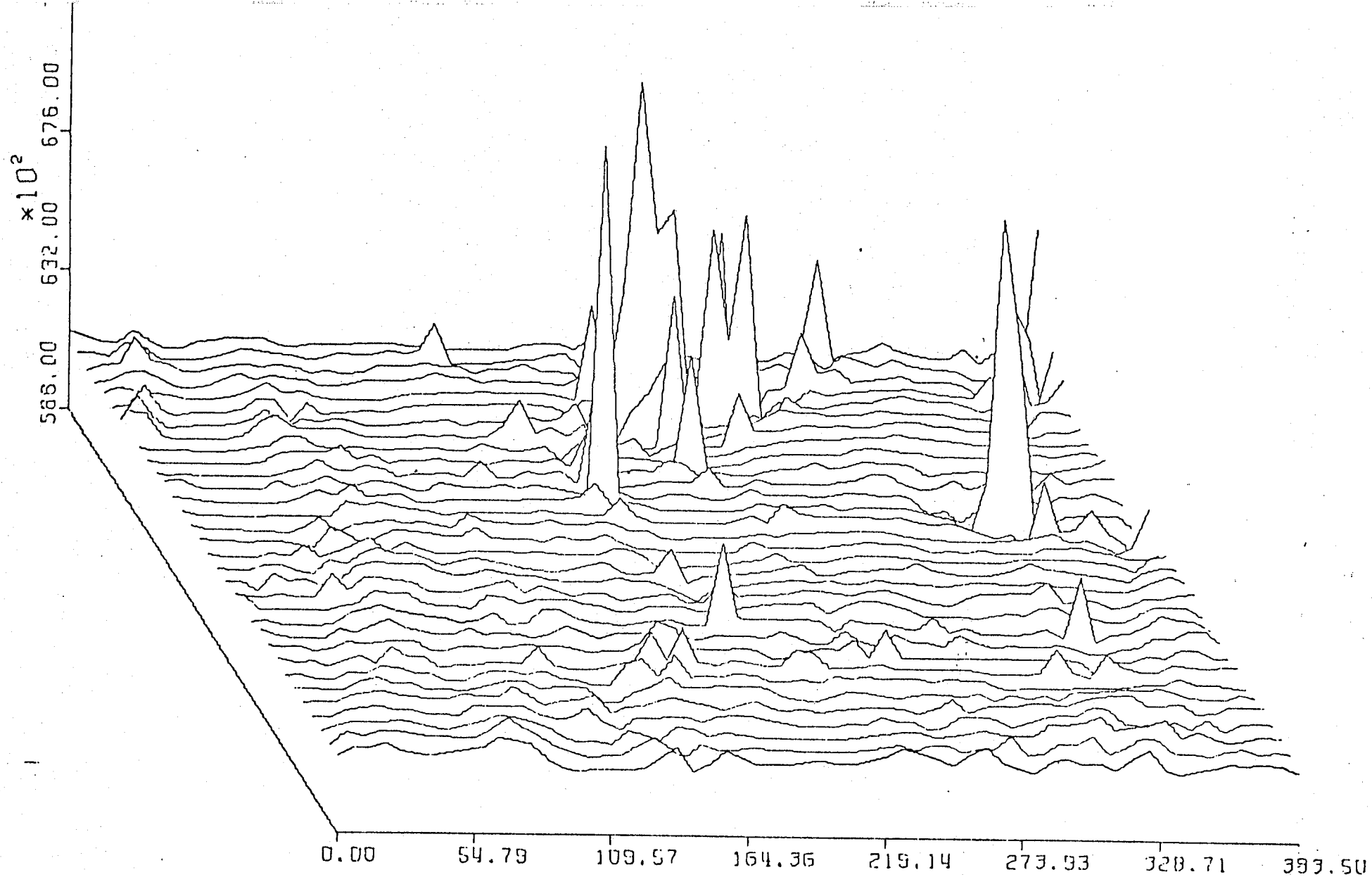
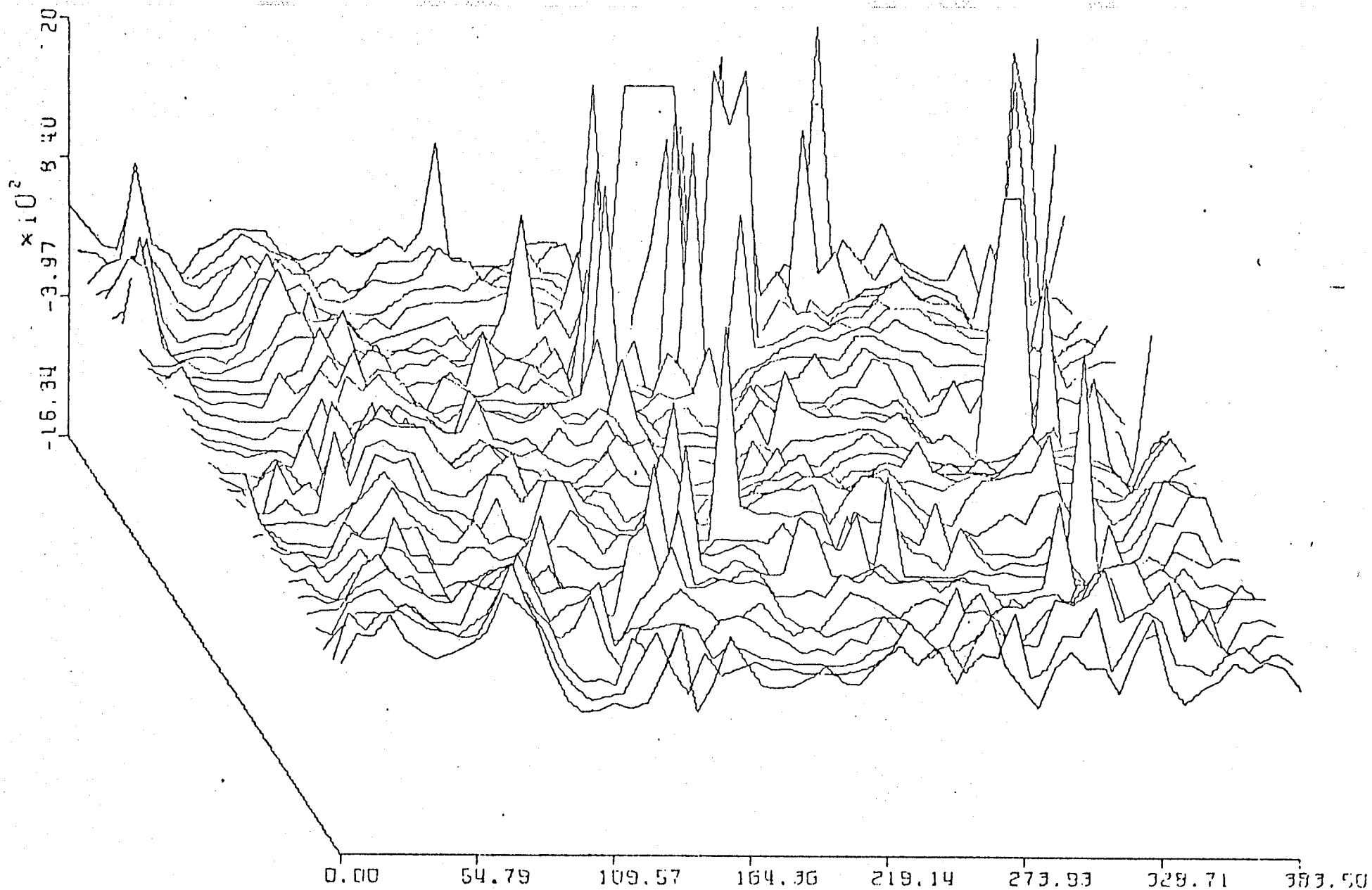


Figure 2.5.1 International Geomagnetic Reference Field
of the Project Area



3-D REPRESENTATION OF THE GEOMAGNETIC FIELD: N.W.T. 60-61.75N 102-94.75W

Figure 2.5.2



MAP AREA (N.W.T. 60-61.75N 102-94.75W) WITH I.G.R.F. REMOVED

Figure 2.5.3

CHAPTER 3

THE AREA UNDER STUDY

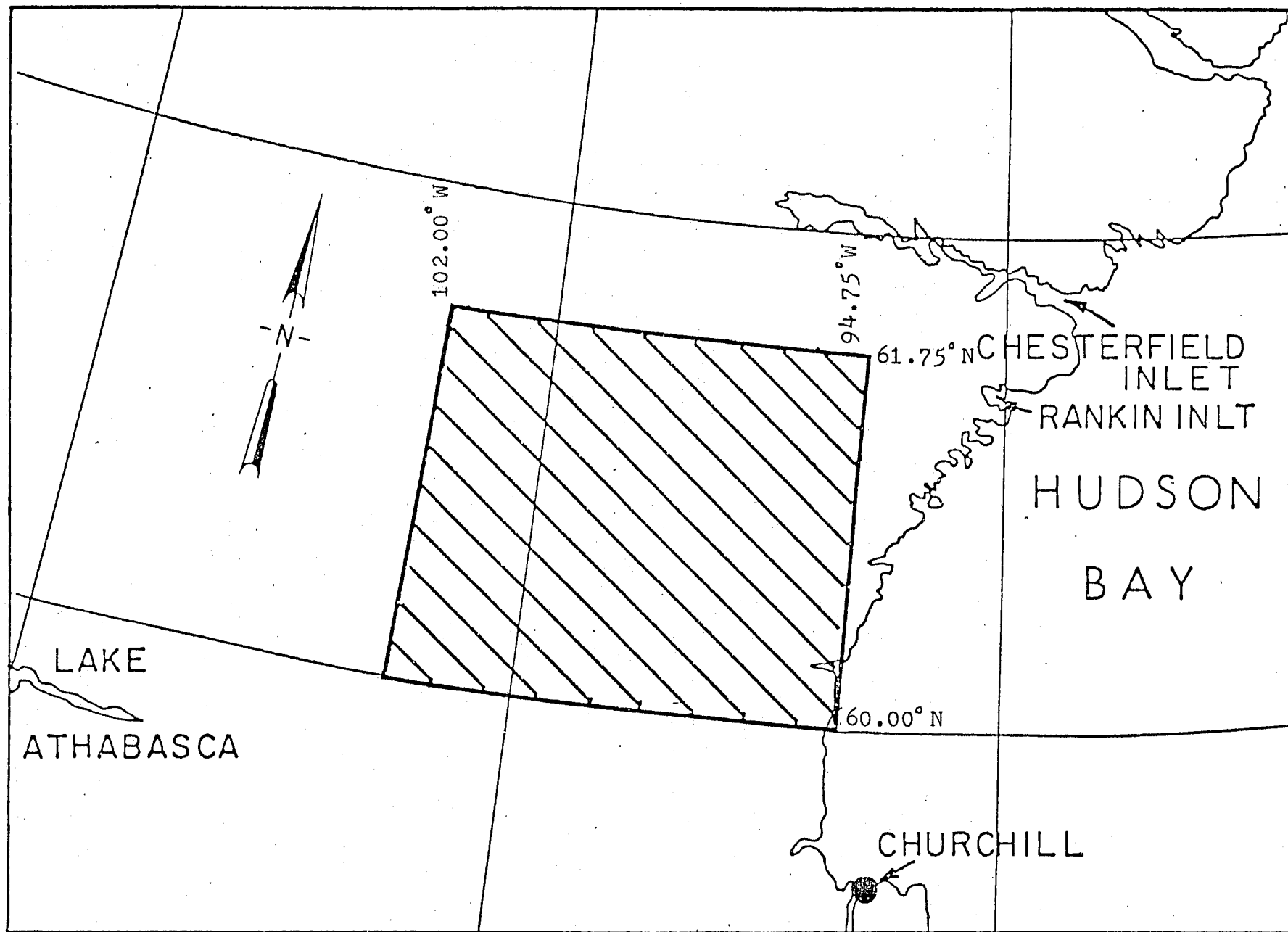
3.1 Regional Setting and General Geology

The area presently under consideration lies wholly within the Churchill Province of the Canadian Shield. Geographically, the area lies directly adjacent to the northern boundary of the Province of Manitoba in the Northwest Territories. It is bounded by 60.00 to 61.75 degrees North and 102.00 to 94.75 degrees West (see Figure 3.1.1).

The Canadian Shield is a vast region of Precambrian rocks that occupy nearly half of Canada and form the bedrock for a land area of about 4,825,000 square kilometers. The greater part of the Shield has now been geologically mapped and much of it has been covered by gravity and aeromagnetic surveys. Detailed accounts of the geology have subdivided the Shield into a number of structural provinces and sub-provinces, based mainly on the overall differences in internal structural trends and style of folding. The area under study has an aerial extent of approximately 137,000 square kilometers in the Churchill Structural province.

64°

60°



Scale: 1:5,000,000

Miles



100 0 100

100°

92°

Figure 3.1.1 Location map of the project area

Closed structures predominate the structural features of the Churchill Province, but linear structures are also well represented in the area. These linear structures form a large arc, concave to the south, around Hudson Bay, so that trends are mainly northeast in the Western part, easterly close to Hudson Bay, and southeasterly in the extreme eastern part of the province. In the interior of the province, the folded rocks are unconformably overlain by flat lying or gently folded rocks. Small bodies of nearly flat lying rocks unconformably overlying folded rocks can be seen near the border of the province.

No detailed geological reports are available on this localized area, although more detailed accounts of the geology have been published on the Churchill-Superior Province boundary in the South (Bell, C.K. 1966, Kornik, L.J. and MacLaren, A.S. 1966).

A major tectonic feature of the Churchill structural province is the Wollaston Lake fold belt. The fold belt extends from the southern edge of the Canadian Shield in Saskatchewan, north eastward into extreme northwestern Manitoba. The extension of the Wollaston Lake fold belt northeastward towards Hudson Bay is under investigation here and one of the objectives of this study is to examine the

possibility of tracing the extension of the fold belt from its magnetic expression¹.

3.2 Data Acquisition

Data for the analysis were taken from a magnetic tape (ER7792) supplied by the Aeromagnetic Survey, Earth Physics Branch, Ottawa. The magnetic tape contains digital data (in gammas) of two major project areas, the Central Canada project and the Nelson River project. The origin of these projects starts from 113 degrees west longitude and 56 degrees north latitude. The airborne magnetic surveys of these projects were done by Spartan Air Services Ltd. and Spartan Aero Limited in the years of 1964, 1969 and 1970. The flight elevation of the various surveys in the project area was kept constant at 1,000 ft. (304.8 meters) above ground elevation. The flight spacing for the surveys was set at 1/4 mile (0.402 kilometers). The airborne flight traverse and flight pattern for the surveys were carefully controlled by the photomosaic method.

To obtain the digital data for the project area under study, a computer routine was written to extract the re-

1. The magnetic expression of the Wollaston Lake fold belt was later found to be predominantly "negative" (magnetic low) in nature in this project.

quired data set from the project tape. Once the data set was obtained, a data file was created to store the data for further investigation.

Decimation were done on the data both in the east-west and north-south direction and a final rectangular array of 6.5 km sampling interval was obtained, ready to be input into the processing system. The output of the system was both written onto magnetic tape and printed as a two-dimensional array. The latter display gave a quick way of checking the output against the final machine contoured map.

CHAPTER 4

TWO DIMENSIONAL DIGITAL FILTERING

4.1 Basic Principles and Design of a Two-Dimensional Filter

The application of two-dimensional harmonic analysis and two-dimensional filtering in the analysis of potential field data has found increased acceptance in recent years. Since 1965, an increasing number of papers have appeared (Battacharyya 1965; Darby and Davies 1967; Fuller 1967; Odegard and Burg 1965; and Zurflueh 1967) concerning the application to geophysical data. The main emphasis in these papers was primarily on the design and application of two-dimensional filters.

Many periodic components are present in geologic structure. Tectonic activity, for example, particularly compressional movement involves the production of folds which viewed in cross-section are periodic in nature. Separation by filtering is based on the assumption that the near surface, presumably higher frequency components of the field can be isolated from the lower frequency trends caused by deep sources. Thus, the design of two-dimensional filters is desired. In this chapter, the design and analysis

of two-dimensional filters in the space domain (4.2) as well as in the frequency (or wavenumber) domain (4.3) are considered.

The theory presented in this section is well known and is given here to provide the basic framework and to specify a notation for the following applications.

A set of two-dimensional data $f(x,y)$ in the space domain can be represented in the wavenumber domain by means of the two-dimensional Fourier transform $F(u,v)$ where

$$F(u,v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y) e^{-2\pi(ux + vy)} dx dy$$

and the inverse transform is given by the equation

$$f(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u,v) e^{i2\pi(ux + vy)} du dv$$

The variables u and v represent wavenumber.

The process of convolution of two-dimensional data with two-dimensional operator or filter can be expressed in the space domain by the convolution integral written as

$$g(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(x-\lambda, y-\rho) k(\lambda,\rho) d\lambda d\rho$$

where:

$g(x,y)$ = output of the convolution process

$h(x,y)$ = input data

$k(\lambda,\rho)$ = linear operator or filter

and by $G(u,v) = H(u,v) K(u,v)$ in the wavenumber (or frequency) domain.

For data sampled on a square grid, the equations above then take the form:

$$F(u,v) = \Delta x \Delta y \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} f(n\Delta x, m\Delta y) e^{-i2\pi(un\Delta x + vm\Delta y)}$$

$$f(x,y) = \Delta u \Delta v \sum_{k=-\infty}^{\infty} \sum_{L=-\infty}^{\infty} F(k\Delta u, L\Delta v) e^{i2\pi(k\Delta u x + L\Delta v y)}$$

$$g(x,y) = \Delta x \Delta y \sum_{\lambda=-\infty}^{\infty} \sum_{\rho=-\infty}^{\infty} h(x-\lambda\Delta x, y-\rho\Delta y) k(\lambda\Delta x, \rho\Delta y)$$

4.2 Space Domain Filter Operator

The digitized aeromagnetic data under study was first filtered by a space-domain filter operator via the convolution process. A seventeen-point numerical filter as designed by McGrath and Hall (1969) was employed

$$F_o(x,y) = \frac{1}{259} \left[55 F_i(x,y) + 42S_1 + 29S_{\sqrt{2}} + 10S_{\sqrt{5}} \right]$$

where

$F_o(x,y)$ = the filtered value of $F_i(x,y)$

and S_r = the sum of the data falling on circles of radius r centred at $F_i(x,y)$, where r is in sampling units.

The filter response for this space domain numerical operator will be discussed in Section 4.4.

In the derivation of this formula, it was assumed that the noise to be extracted from the aeromagnetic data was randomly distributed. In examining the raw digitized data, a slightly skewed distribution was observed and this tends to degrade the quality of filtered data. However, preconditioning of the raw data before applying the filter operator was done and more effective filtering was achieved. Figures 4.2.2 and 4.2.3 show the filtered aeromagnetic map of the study area using a cutoff wavelength of 60 km.

4.3 Two-Dimensional Filtering in the Frequency Domain

Two-dimensional map data can be converted into the frequency (or wavenumber) domain by applying the direct Fourier transformation. The digitized aeromagnetic data in the study area (after preconditioning) is passed into the fast Fourier routine to be first converted into the frequency domain. Then "filtering" is achieved in the frequency domain

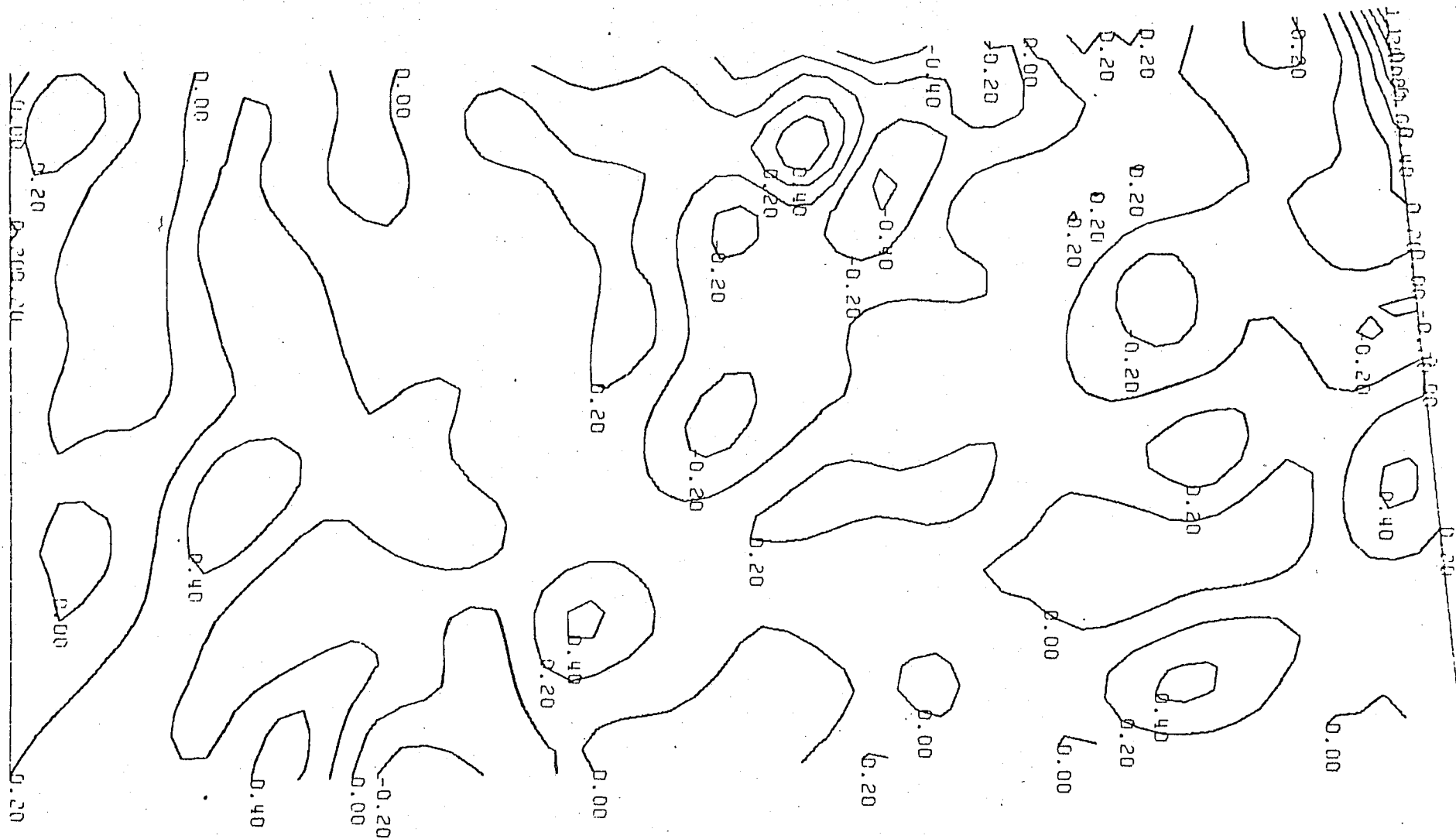


Figure 4.2.2 Filtered Aeromagnetic Map of the Project Area Using
the Space Domain Filter Operator (cutoff wavelength: 60 Km)

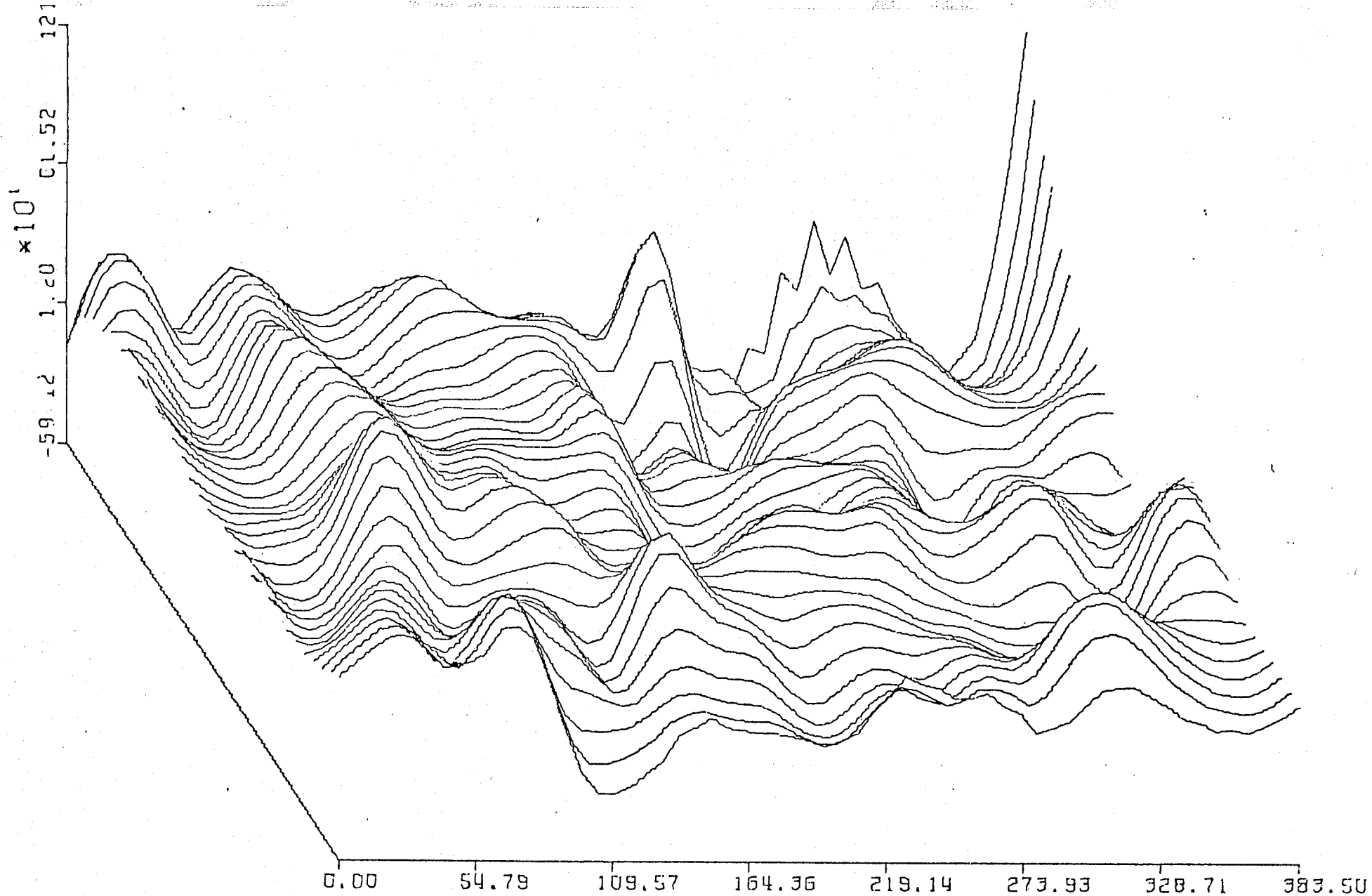


FIGURE 4.2.3

FILTERED DATA (SPACE DOMAIN FILTER APPLIED)

by means of a few computational operations and coefficient manipulations as described below. Then, an inverse Fourier transform is performed to reconstitute the map in the space domain.

Some of the properties and details of the filter in the frequency domain should be noted at this stage. After performing the direct Fourier transform, the data points are laid out in a rectangular grid with Fourier coefficients from zero to the nyquist frequency. The exact cutoffs of these Fourier coefficients are controlled and guided by an elliptical equation $(\frac{u^2}{a^2} + \frac{v^2}{b^2} = 1)$. Once a cutoff wavelength is assigned, the variables a and b of the elliptical equation are computed and grid points falling outside the elliptical quadrant are substituted by zeros, thus, retaining only the Fourier coefficients of the required frequencies. This then will be the filtered map in the frequency domain. Detailed study indicates that the filter provides sharp cutoff in terms of frequencies but some spectral leakage into adjacent bands was observed. Various cutoff wavelengths were applied (30-70 km) and filtered maps were plotted for examination (see Figures 4.3.1 - 4.3.5).

4.4 Frequency Response of Two-Dimensional Filters

The frequency characteristics of the two-dimensional filters employed here in the project is under investigation in this section.

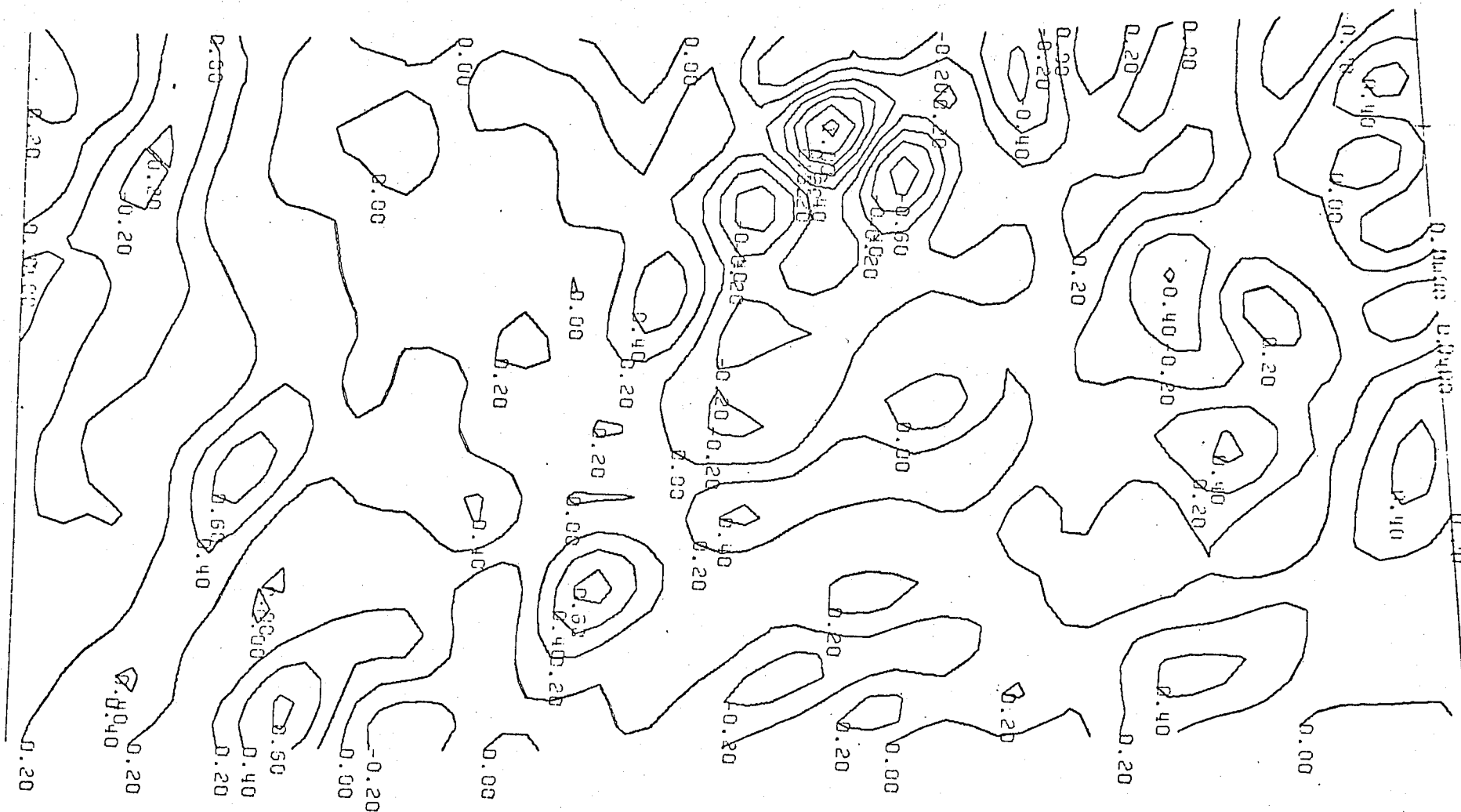


Figure 4.3.1 Filtered Aeromagnetic Map of the Project Area Using Frequency

Domain Technique (Cont. Int. = 200 nanotesla

Cutoff Wavelength = 30 Km)

0 41.3 Km

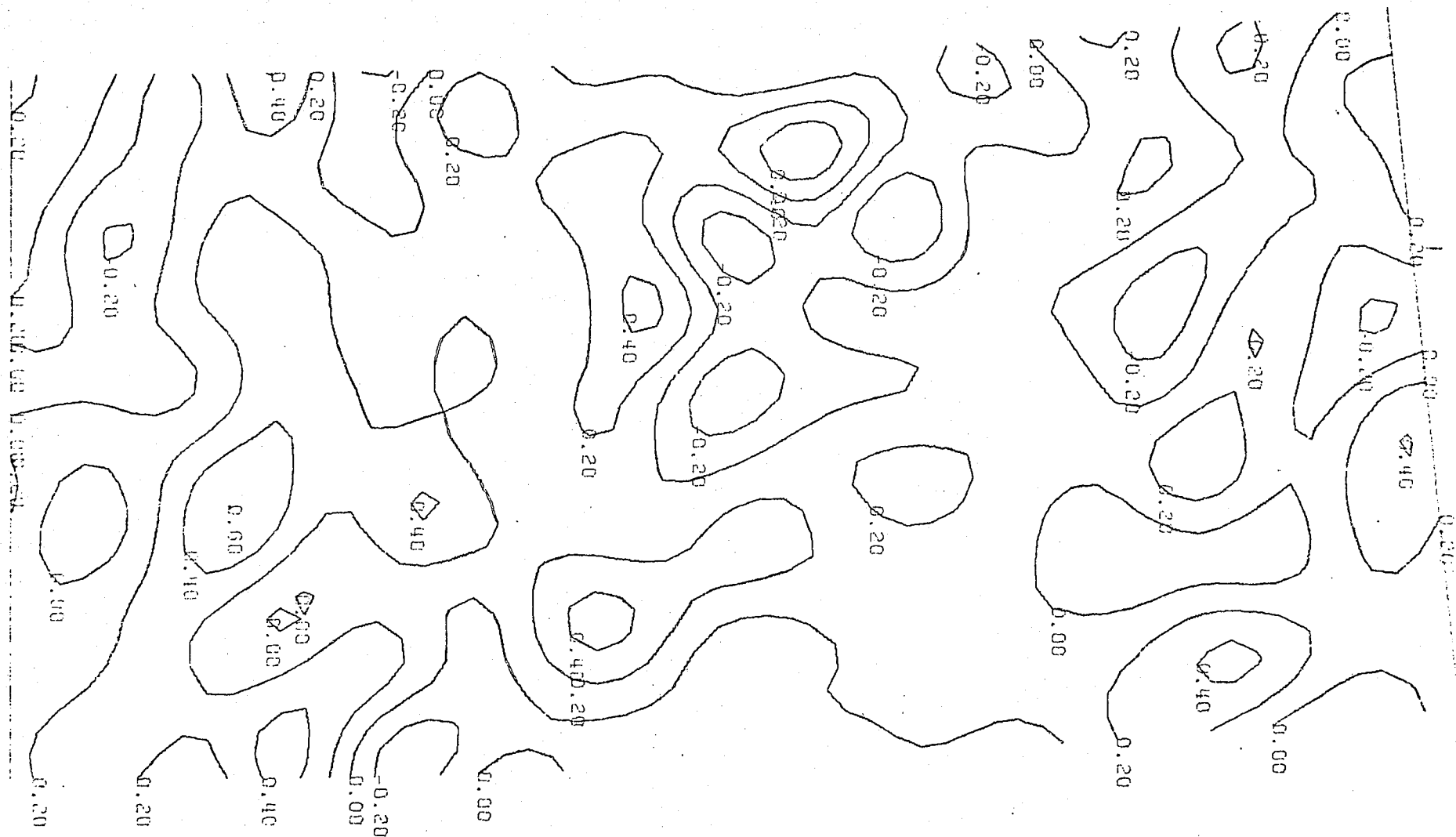
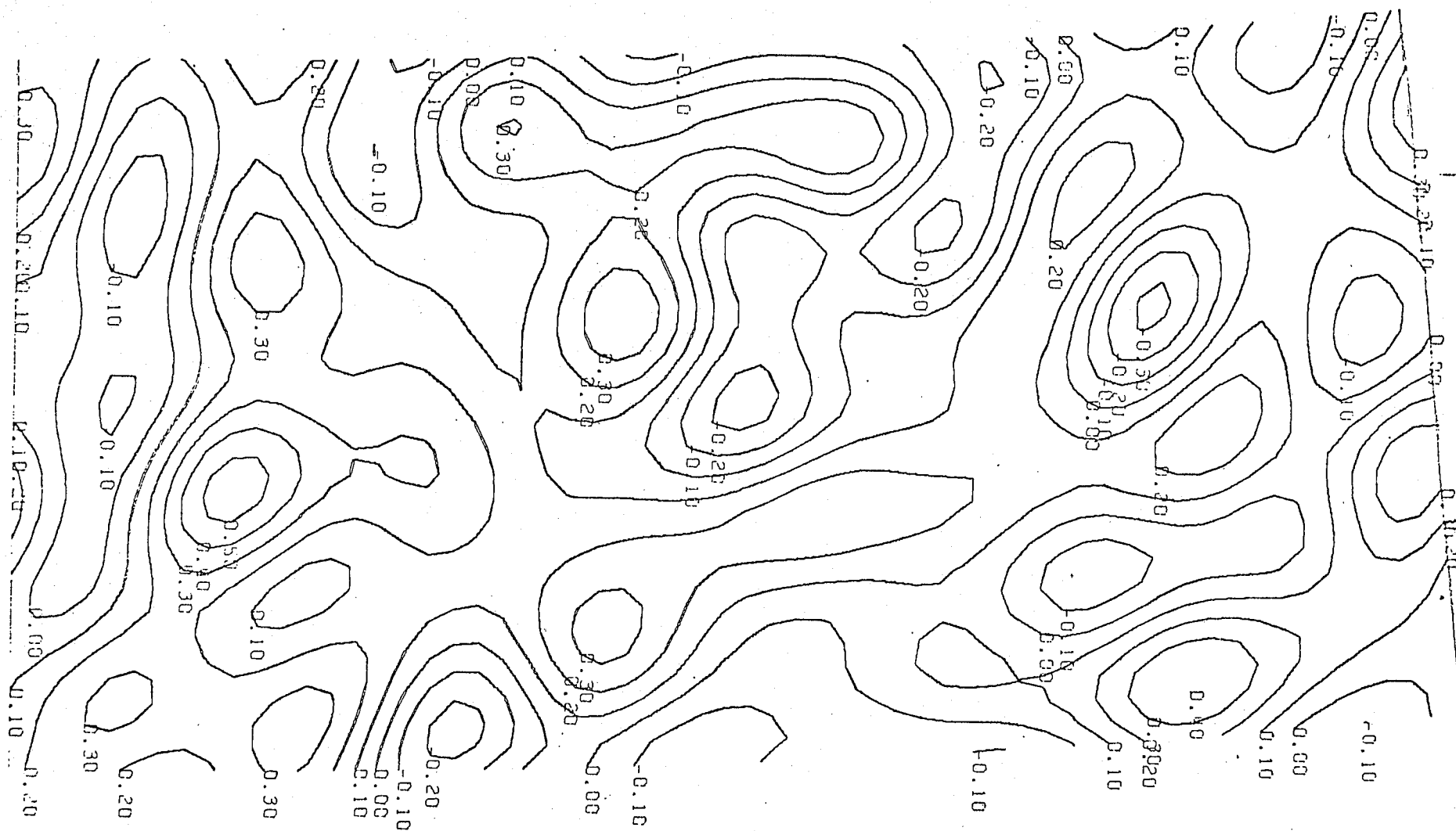


Figure 4.3.2 Filtered Aeromagnetic Map of the Project Area Using Frequency

Domain Technique (Cont. Int. = 200 nanotesla

Cutoff Wavelength = 40 Km)

0 41.3 Km



Cutoff Wavelength = 50 Km)

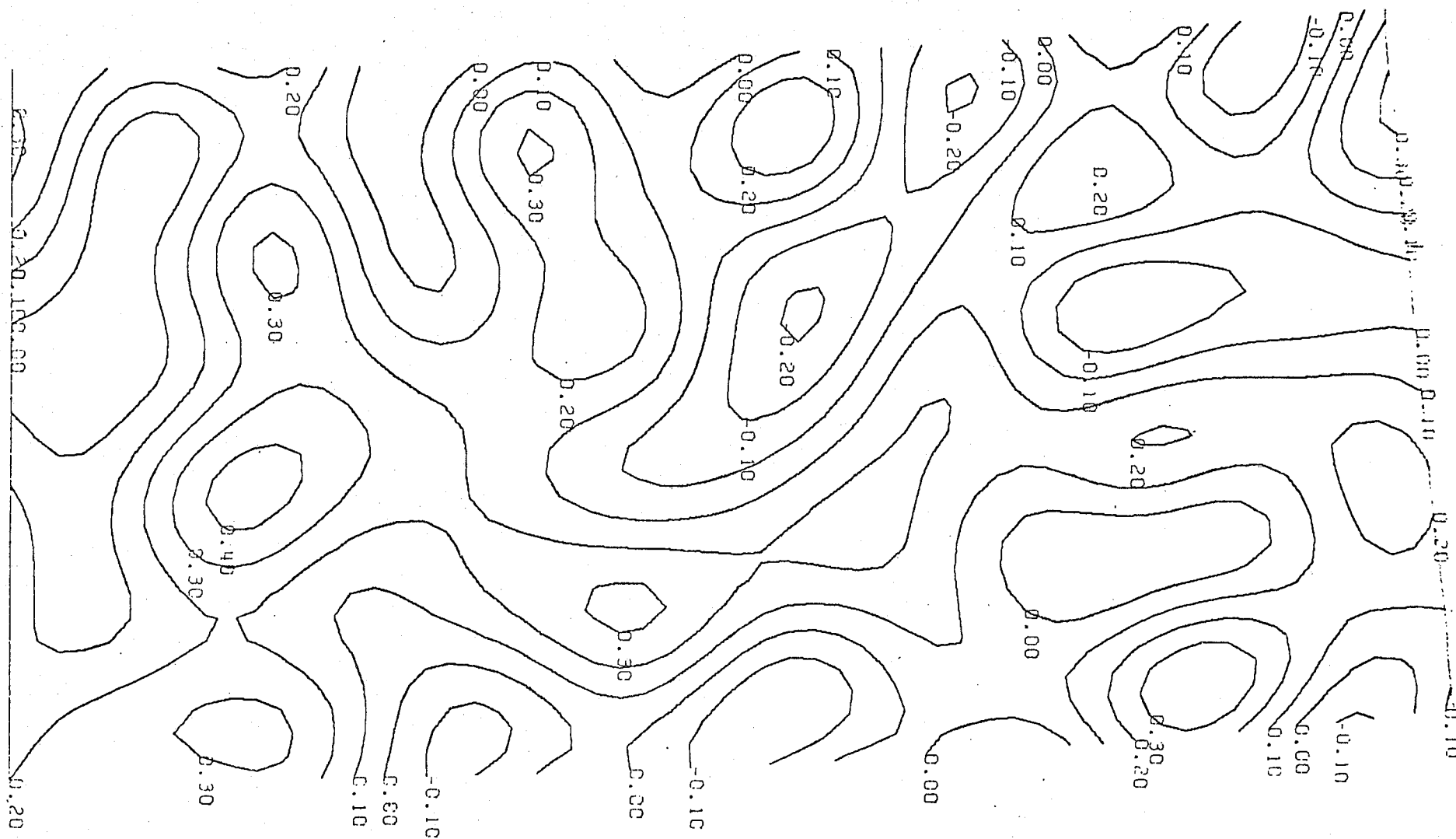


Figure 4.3.4 Filtered Aeromagnetic Map of the Project Area Using Frequency

Domain Technique (Cont. Int. = 100 nanotesla

Cutoff Wavelength = 60 Km)

0 41.3 Km

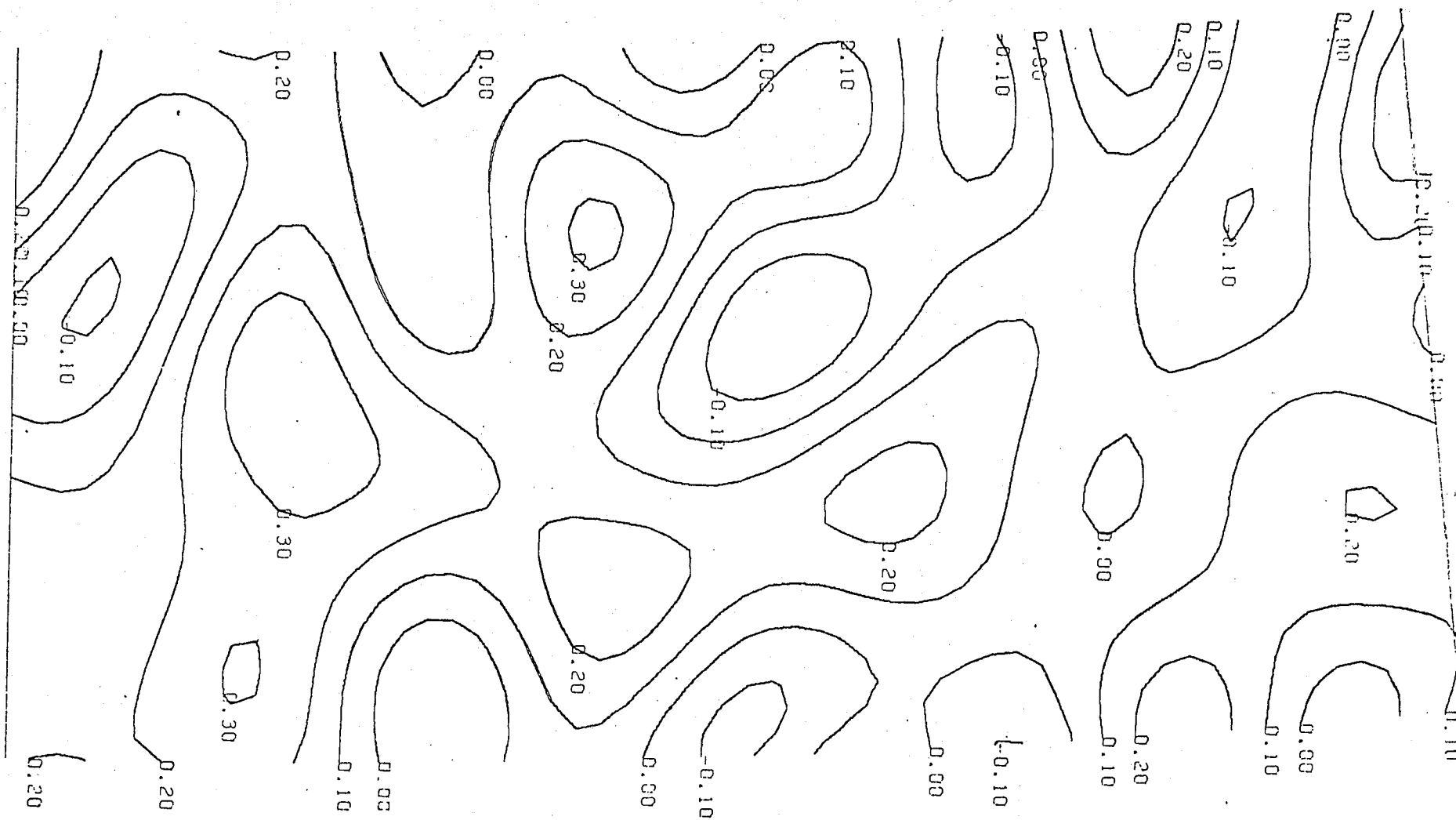


Figure 4.3.5 Filtered Aeromagnetic Map of the Project Area Using Frequency

Domain Technique (Cont. Int. = 100 nanotesla

Cutoff Wavelength = 70 Km)

0 41.3 Km

The filter response of the space domain seventeen-point numerical filter (as described in Section 4.2) was first presented by McGrath and Hall (1969). The theoretical filter response formula is given by Dean (1958):

$$H(u,v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W(x,y) e^{-i(ux + vy)} dx dy$$

where:

$H(u,v)$ = theoretical frequency response

$W(x,y)$ = the weighting function of the filter operator

u,v = angular frequencies in the x and y directions respectively.

The empirical response curve was obtained by comparing the standard deviation of digitized input data to that of the output data for a particular filter operator (as used in Section 4.2). In this study, the input data were obtained by digitizing the surface $M(x,y)$, where

$$M(x,y) = \sin 2\pi x \cdot \sin 2\pi y$$

and Percent Response is given by

$$R(u,v) \times 100$$

where

$$R(u,v) = \frac{\text{standard deviation of output data}}{\text{standard deviation of input data}}$$

As described by McGrath and Hall (1969), the response curves for the above two surfaces (theoretical and empirical) converge as the size of the sampling interval is reduced (see Figure 4.4.1). Since regional aeromagnetic surfaces have variable anomaly shapes, a sample can be selected which renders the filter operator more or less independent of anomaly shape. Therefore, with the 6.5 km sampling interval employed in this study, regional aeromagnetic anomalies having wavelengths greater than 78 km (12 sampling intervals per wavelength) were passed by this particular filter operator with little anomaly distortion. All aeromagnetic anomalies whose wavelengths are less than 26 km were strongly rejected by the numerical filter operator. For the long-wavelength anomalies ($\lambda > 60$ km) under study in this project, this seventeen-point numerical filter appears to be unsatisfactory because those anomalies that fall between 60-78 km suffered some form of distortion introduced by the filter operator. A smaller sampling interval or a sharper cutoff filter operator is needed in order to minimize the distortion of the long-wavelength aeromagnetic anomalies in the project area.

The filter characteristics of the frequency domain filter employed in this project are studied using the following technique (see Figure 4.4.2). The original data set is first converted to the frequency domain via the forward Fourier transform. The Fourier coefficients are then trun-

SAMPLE INTERVAL PER WAVELENGTH

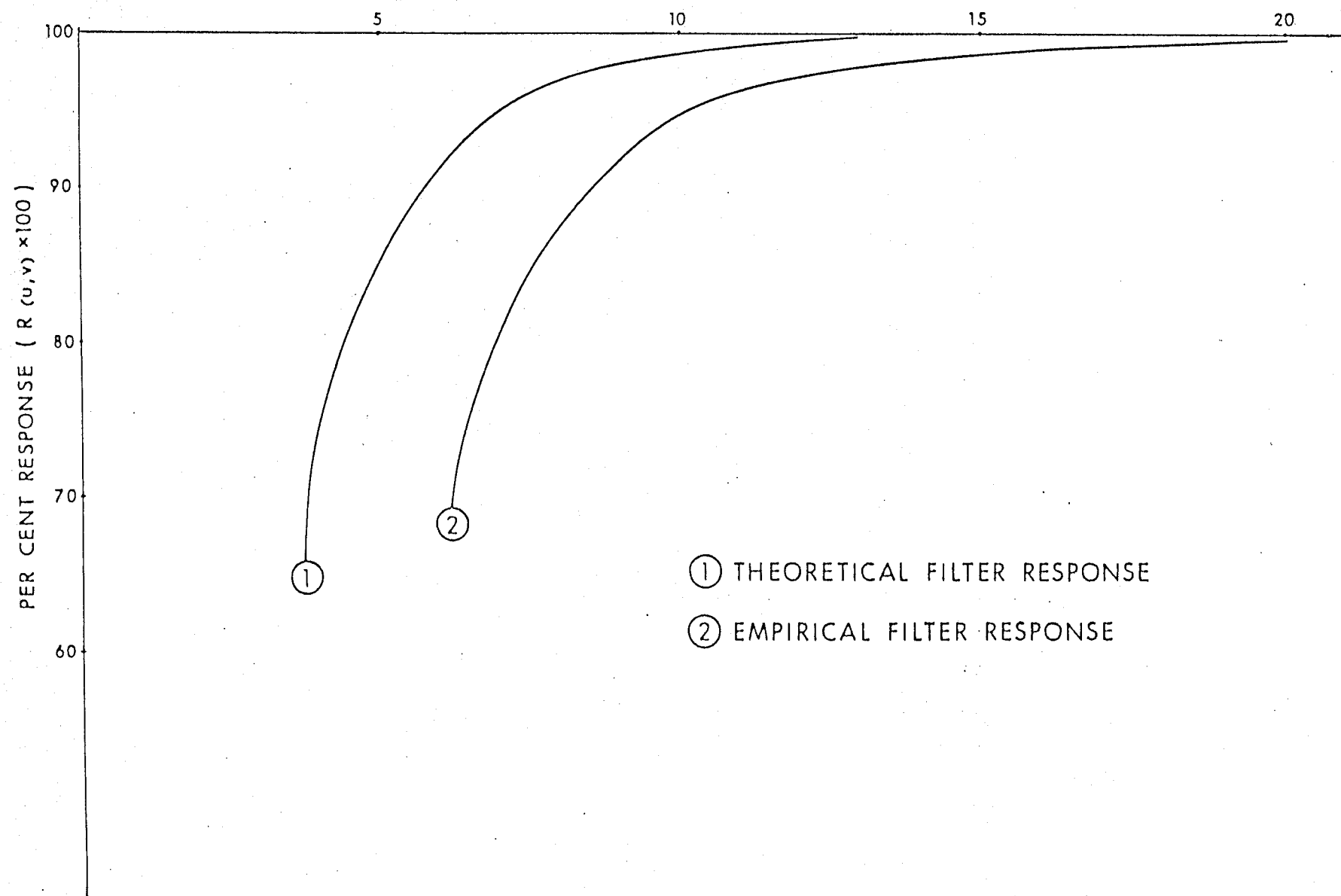


FIG. 4. 4. 1. RESPONSE CURVE OF THE SEVENTEEN POINT NUMERICAL FILTER IN THE SPACE DOMAIN

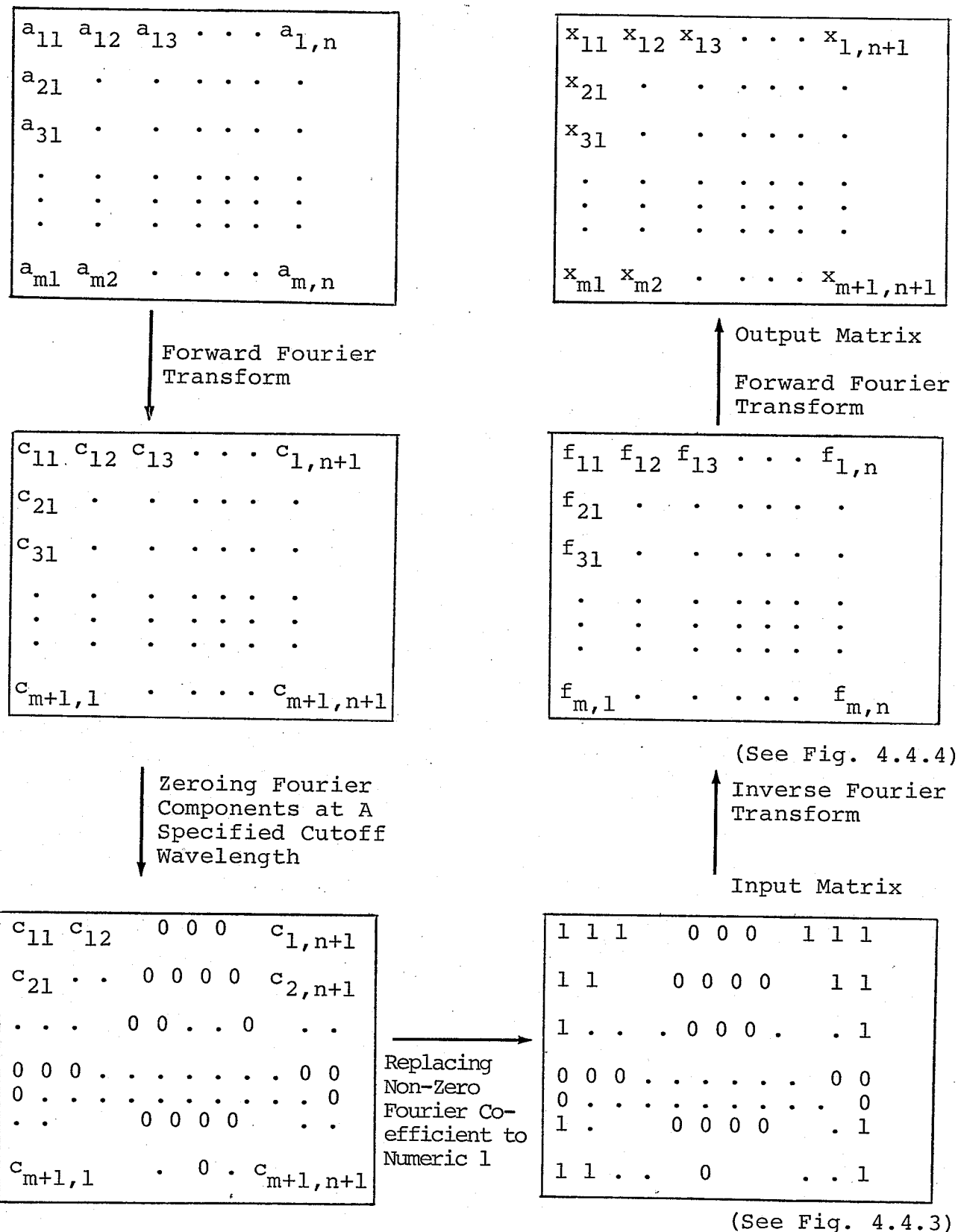


Figure 4.4.2 Flow Diagram to Outline the Technique Used to Obtain the Frequency Characteristics of the 2-D Frequency Domain Filter

cated at the desired cutoff wavelengths using the criteria described in Section 4.3. The non-zero coefficients are then replaced by ones and the data matrix becomes a set of synthetic Fourier coefficients of ones and zeros (see Figure 4.4.3). It is then converted back to the space domain by applying inverse Fourier transform (see Figure 4.4.4). Another forward transform is performed and the output coefficients are printed for comparison to the input synthetic coefficients.

The values of the output matrix varies from 0.9999955 to 0.0000011 which lies very close to ones and zeros of the original input matrix of synthetic Fourier coefficients. This indicates that there is little or no distortion introduced by this filter operation and the method is found to be very satisfactory. A very steep cutoff is obtained as compared to the tapered response curve of the space domain filter. Moreover, little or no spectral leakage into adjacent bands can be observed in this two-dimensional frequency domain filter.

4.5 A Discussion of the Filtered Aeromagnetic Maps

A series of filtered aeromagnetic maps of the study area with different cutoff wavelengths is shown in Section 4.3. Three-dimensional plots of these filtered maps are

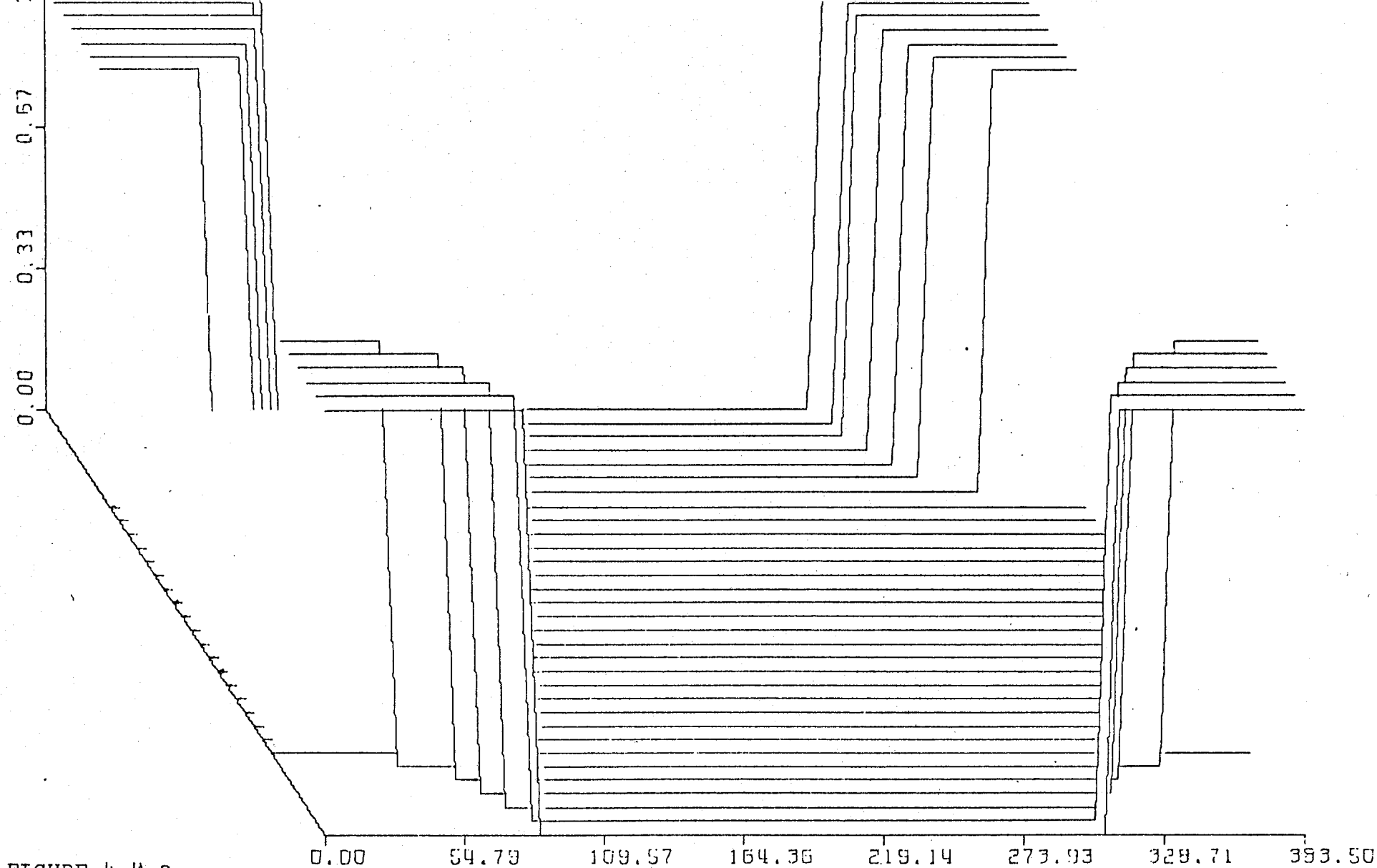


FIGURE 4.4.3

FOURIER TRANSFORM MAP WITH NON-ZERO COEFFICIENTS EQUAL TO ONES
(CUTOFF WAVELENGTH: 30Km.)

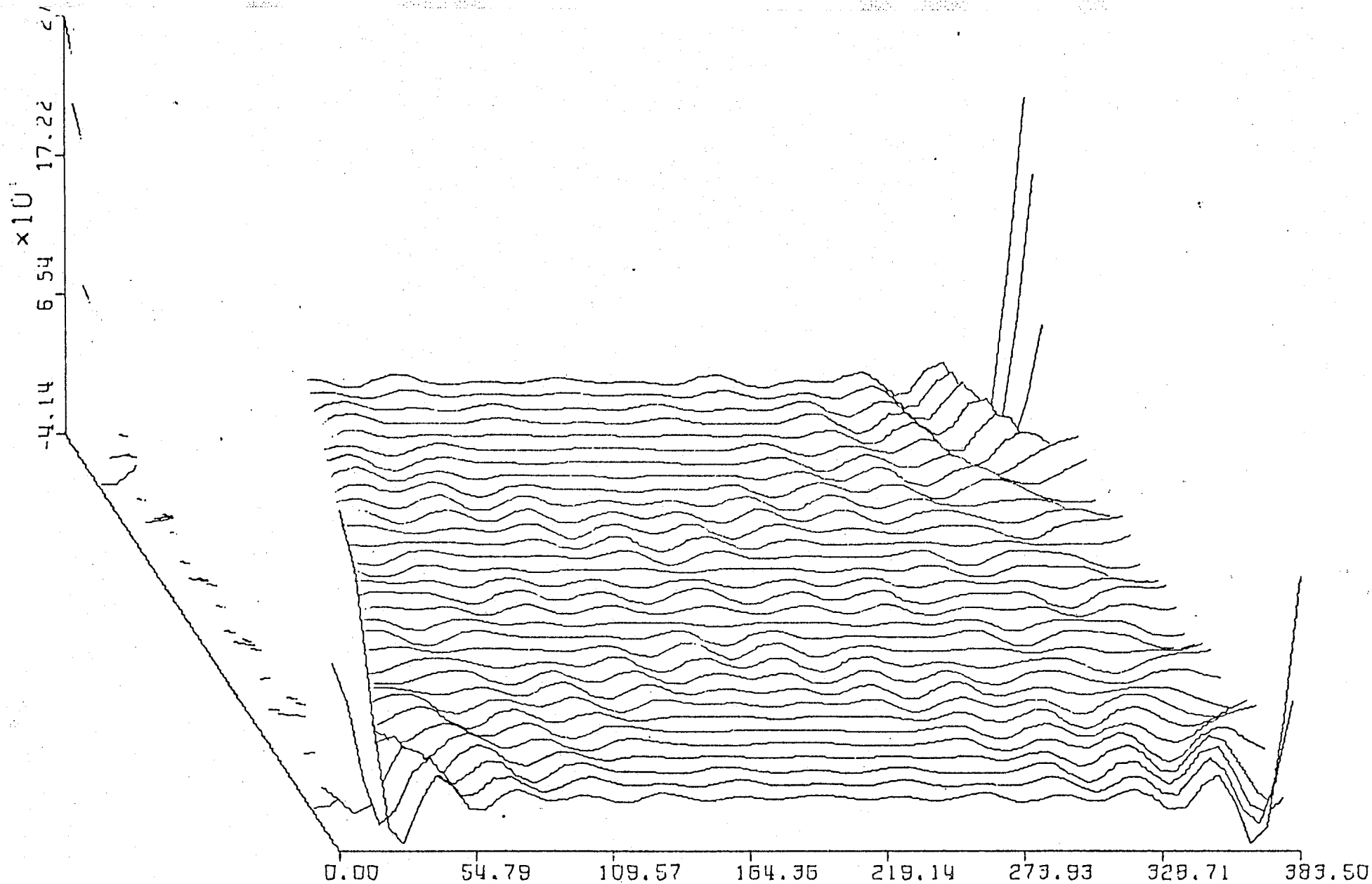


FIGURE 4.4.4

INVERSE FOURIER TRANSFORM MAP OF THE SYNTHETIC COEFFICIENTS

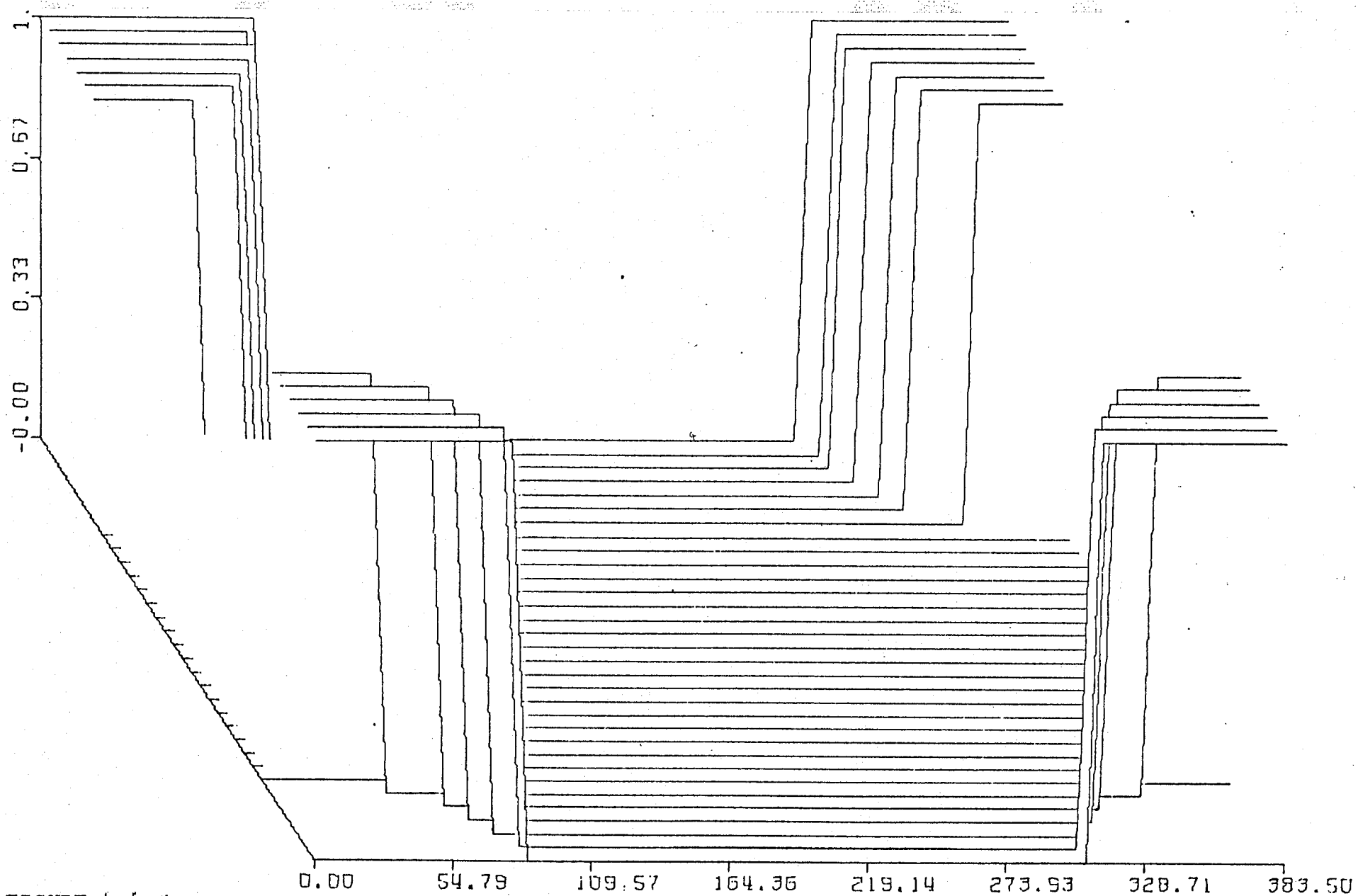


FIGURE 4.4.5

RESPONSE OF THE FREQUENCY DOMAIN FILTER (CUTOFF WAVELENGTH=30 KM)

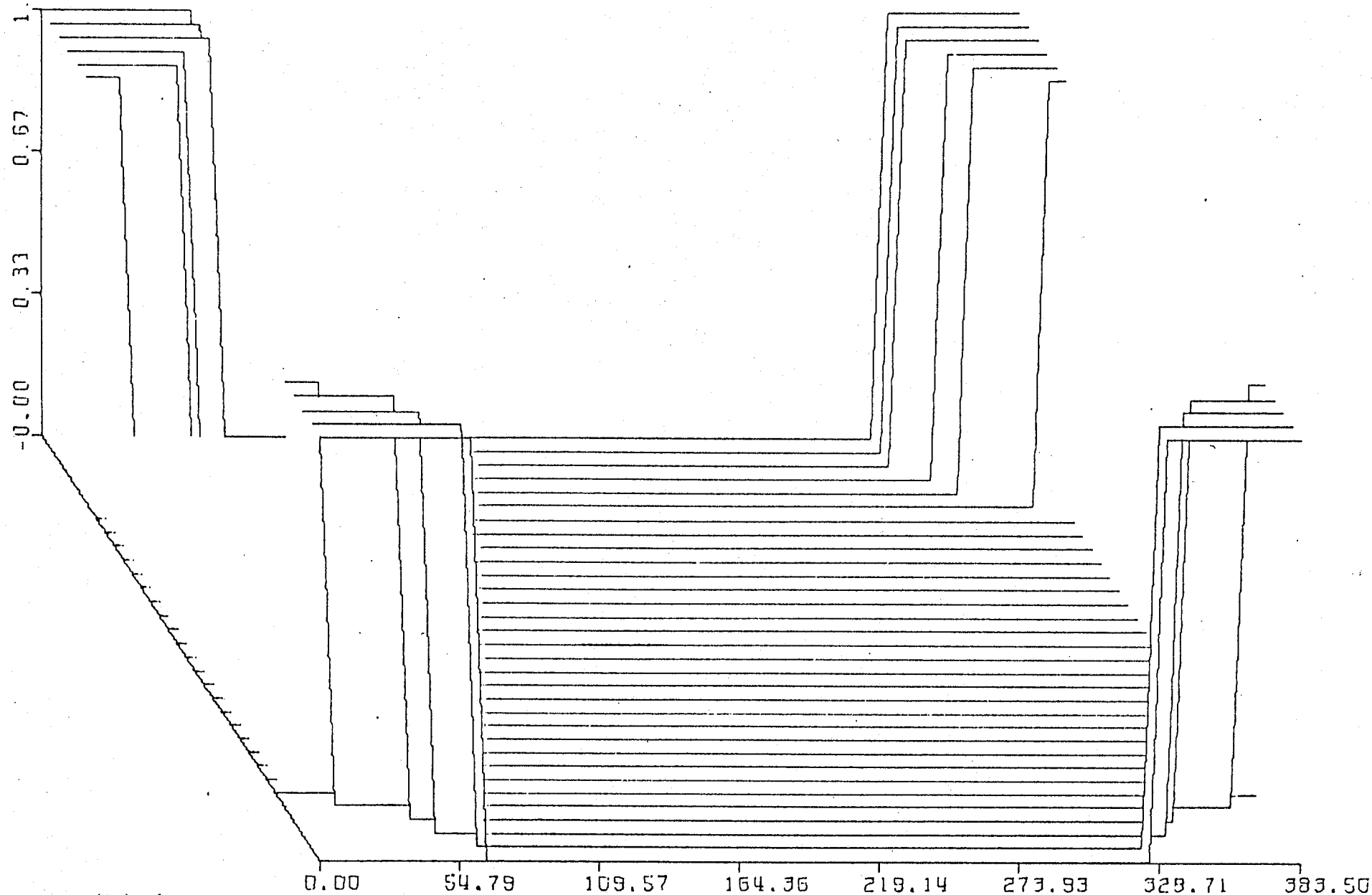


FIGURE 4.4.6

RESPONSE OF THE FREQUENCY DOMAIN FILTER (CUTOFF WAVELENGTH=40 KM)

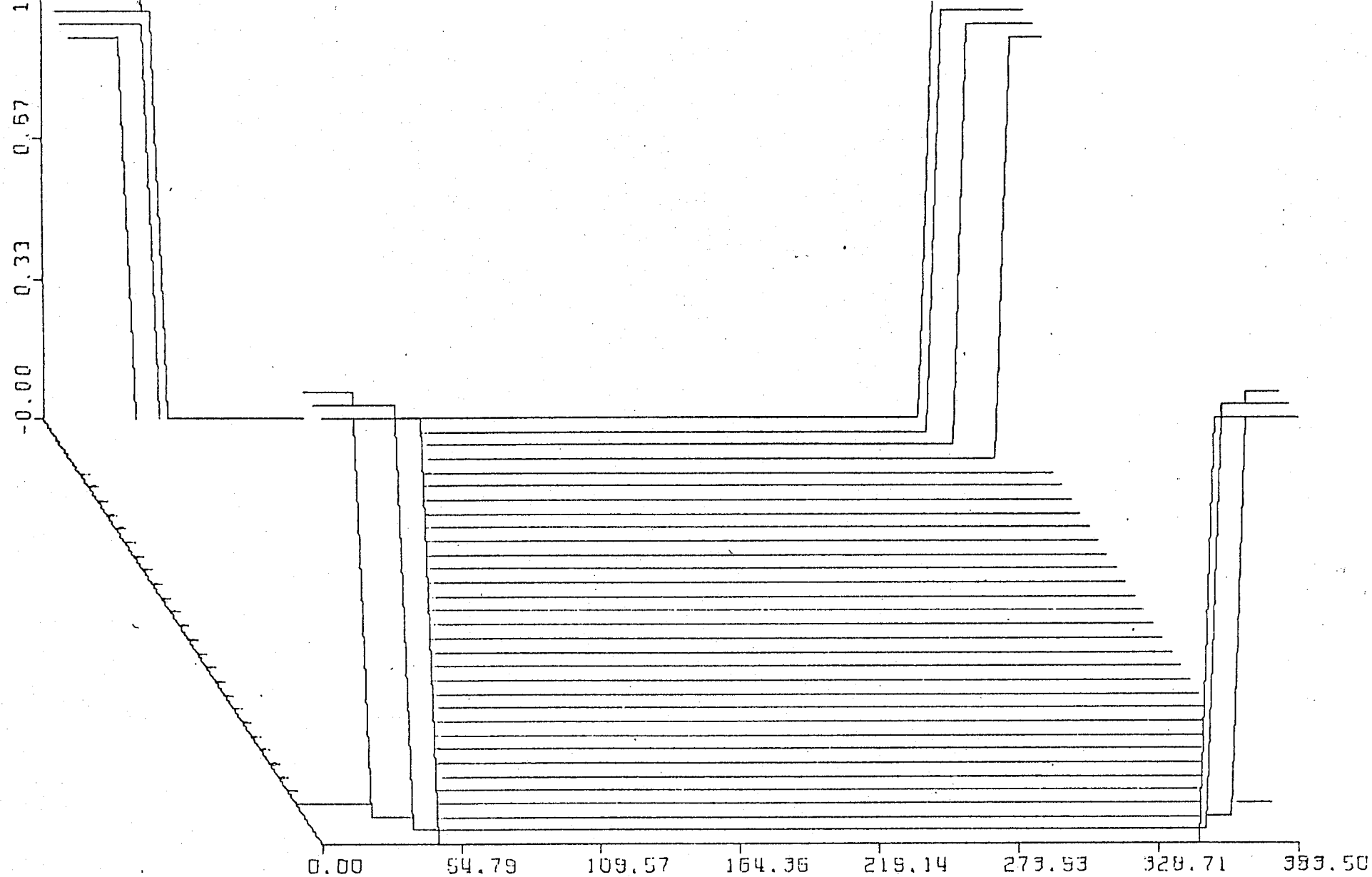


FIGURE 4.4.7

RESPONSE OF THE FREQUENCY DOMAIN FILTER (CUTOFF WAVELENGTH=60 KM)

also presented in Appendix A. The advantage of these three-dimensional plots is that they add insight and a third dimension to aid interpretation. The filtered map with a cutoff wavelength of 30 km shows a somewhat random distribution of the anomalies. As the cutoff wavelength increases, a general trend (northeast, southwest) of these aeromagnetic anomalies starts to show up. The long-wavelength features in the Northern central region and the southwest region appear to be "genuine" and they become more prominent as the cutoff wavelength increases.

A filtered map with the space domain filter operator applied (as described in Section 4.2) at a cutoff wavelength of 60 km is shown in Figure 4.2.2. Figure 4.3.4 shows an output map of the same area at the same cutoff wavelength using the frequency domain filter. A comparison of the two figures indicates that the space domain filtered map appears to be more "jagged" than the frequency domain filtered map. This could have been caused by the presence of high frequency noise in the data that leaked through the filter. The noisy appearance of the map can also be explained by the finite number of coefficients used to represent the filter operator in the space domain. The filtered map generated by the frequency domain filter operator appears to be somewhat smoother and more acceptable. Higher attenuation is also

achieved by the frequency domain filter. In addition, a much sharper cutoff can be achieved by the frequency domain filter as demonstrated in the last section.

In theory, filtering by convolution in the space domain and multiplication in the frequency domain are nothing but the different aspects of the same operation. In practice, however, the two methods do not yield identical results as observed in the project. This is because of the difference in the truncation of the coefficients of the filter series. The truncation appears to be more severe in the space domain operation than in the frequency domain and hence less efficient. The two filters will have identical results as the number of convolution coefficients tends to infinity.

For a given number of data points, the frequency domain filter was found to be more efficient than the space domain filter. The frequency domain filter operator took about three-quarters of the computer time needed to complete the filtering operations as by the convolution method in this project. The FFT method does not require the computation of extensive convolution coefficients which generally need considerable computing time. Moreover, the number of filters that needed to be applied can be handled a lot easier in the frequency domain than in the space domain. From these considerations, frequency domain filtering is particularly attractive compared to the convolution method for aeromagnetic map analysis.

CHAPTER 5

SPECTRAL STUDIES

5.1 Introduction

The use of power spectra in the analysis of potential field data is widely discussed in the literature (Spector and Grant 1970; Mishra and Naidu 1974; Horton et al. 1964) and various authors have shown that the analysis yield some additional information about the subsurface geologic structures.

In the present work, longer wavelength aeromagnetic anomaly features are of particular concern, and both one-dimensional and two-dimensional power spectral analysis were employed in the study area.

The aeromagnetic profiles of the project area were analyzed using the two techniques. Firstly, the power spectrum of the complete data in each line was computed to determine the statistical characteristics of the data. This may yield some clue to resolving regional and near-surface components of the magnetic field in the project area.

In the second approach, the data in each line were analyzed by the Maximum entropy method as suggested by Burg (1967). A more detailed discussion of this approach is dealt with in the next section.

5.2 One-Dimensional Power Spectra

Spectral analysis consists of analysis of the magnetic field data $f(x)$ described in the frequency domain.

Frequency domain representation is obtained using a numerical version of the one-dimensional Fourier transform integral:

$$F(w) = \int_{-\infty}^{\infty} f(x) e^{-jwx} dx$$

It is evident from the above equation and its inverse relation

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(w) e^{jwx} dw$$

that the aperiodic function $f(x)$ may be thought of as a synthesis of an infinite aggregate of sinusoids e^{jwx} of all angular frequencies w in the range $(-\infty, \infty)$ having a complex amplitude $F(w)$. The power spectrum of $f(x)$ is then defined as:

$$E(w) = F(w) \cdot F(w)^*$$

(* superscript implies the complex conjugate)

An estimate of the Fourier transform integral is achieved because the aeromagnetic profiles in the study area have finite lengths.

The following is a step by step procedure used in obtaining the estimate of the power spectrum of each profile in this study.

Step 1: The profiles under investigation here were digitized to a detailed interval of 2.2 km where the maximum frequency present to be $f_{Nq} = \frac{1}{2 \cdot T} = \frac{1}{4.4} = 0.2273$ cycles per km. This was done in order to give a more detailed account of the frequency characteristics of the study area. Then the data was pre-processed and the mean value was removed.

Step 2: Having the series $(x_0, x_1, x_2 \dots x_{N-1})$, the M central values of the autocorrelation function were calculated:

$$\phi_{xx}(\tau) = V(\tau) = \frac{1}{N} \sum_{t=0}^{N-1-\tau} x_{t+\tau} x_t$$

$$\tau = 0, 1, 2, 3 \dots M$$

where for stable estimates, the value for M was chosen to be roughly equal to $N/10$. The resolution in the frequency interval $(0, f_{Nq})$ increases as the value of M increases but the computing time was enormous. Hence, some compromise is necessary in practice.

Step 3: After the autocorrelation coefficients $(r_0, r_1, r_2, \dots, r_M)$ were computed, they were then passed into the fast Fourier transform algorithm for the calculation of the power spectrum. This is based on the fact that power spectrum and autocorrelation are Fourier transforms of each other.

Step 4: The Fourier transform at $f = 0, \Delta f, 2\Delta f, \dots, M\Delta f$ where $f = \frac{f_{Nq}}{M}$ were evaluated. The raw estimates $P_0, P_1, P_2, \dots, P_M$ of the power spectrum were obtained. (M is the number of lag values calculated for the autocorrelation.)

Step 5: The raw estimates found in Step 4 above were then smoothed by the following operation:

$$\begin{aligned}\hat{P}_0 &= 0.5P_0 + 0.5P_1 \\ \hat{P}_1 &= 0.25P_0 + 0.5P_1 + 0.25P_2 \\ \hat{P}_2 &= 0.25P_1 + 0.5P_2 + 0.25P_3 \\ &\vdots\end{aligned}$$

$$\hat{P}_M = 0.5P_{M-1} + 0.5P_M$$

This smoothing is called the Hanning method. If the weights of 0.23, 0.54 and 0.23 were used, it is then called Hamming window smoothing. Smoothing is necessary as it can be shown that the smoothed estimates $\hat{P}_k$ has a variability which decreases with increased record length while this is not true for the unsmoothed estimates.

After the above steps have been performed, we can then plot the power spectrum (or the log power versus frequency).

A few profiles were analyzed in the study area and an example of a typical plot of a power spectrum is shown in Figures 5.2.1 and 5.2.2. From the various power spectra obtained, the following general features with few exceptions were observed:

- a) A definite break in the general slope of the spectral curves cannot be readily identified, though a slightly sharper drop-off at low frequencies were indicated.
- b) The frequency distribution appears to be fairly evenly distributed with a slight dominance of the longer wave-

1-D POWER SPECTRUM

(ALONG LATITUDE 61.75 DEGREES N)

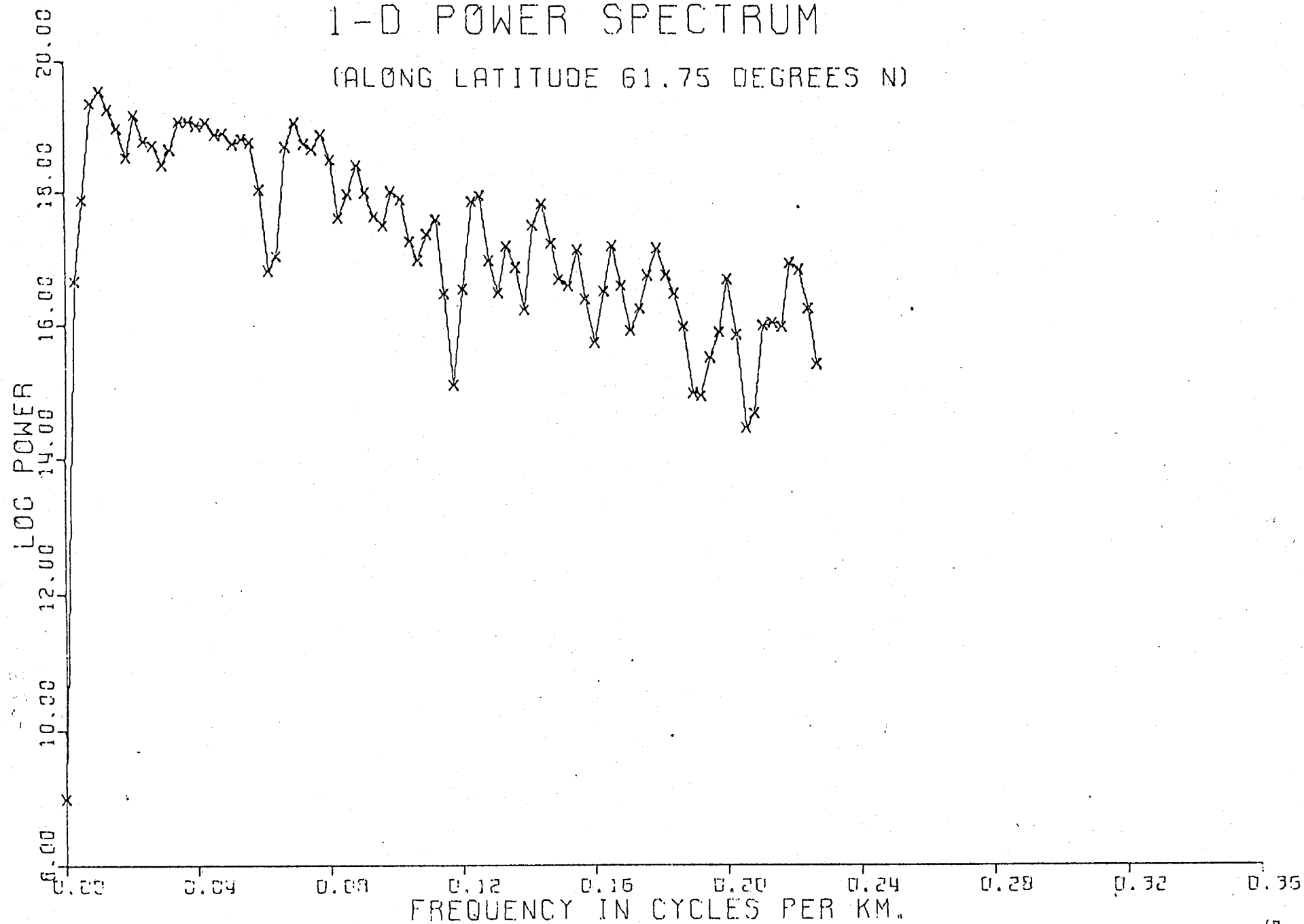


FIGURE 5.2.1

1-D POWER SPECTRUM

(ALONG LATITUDE 61.75 DEGREES N)

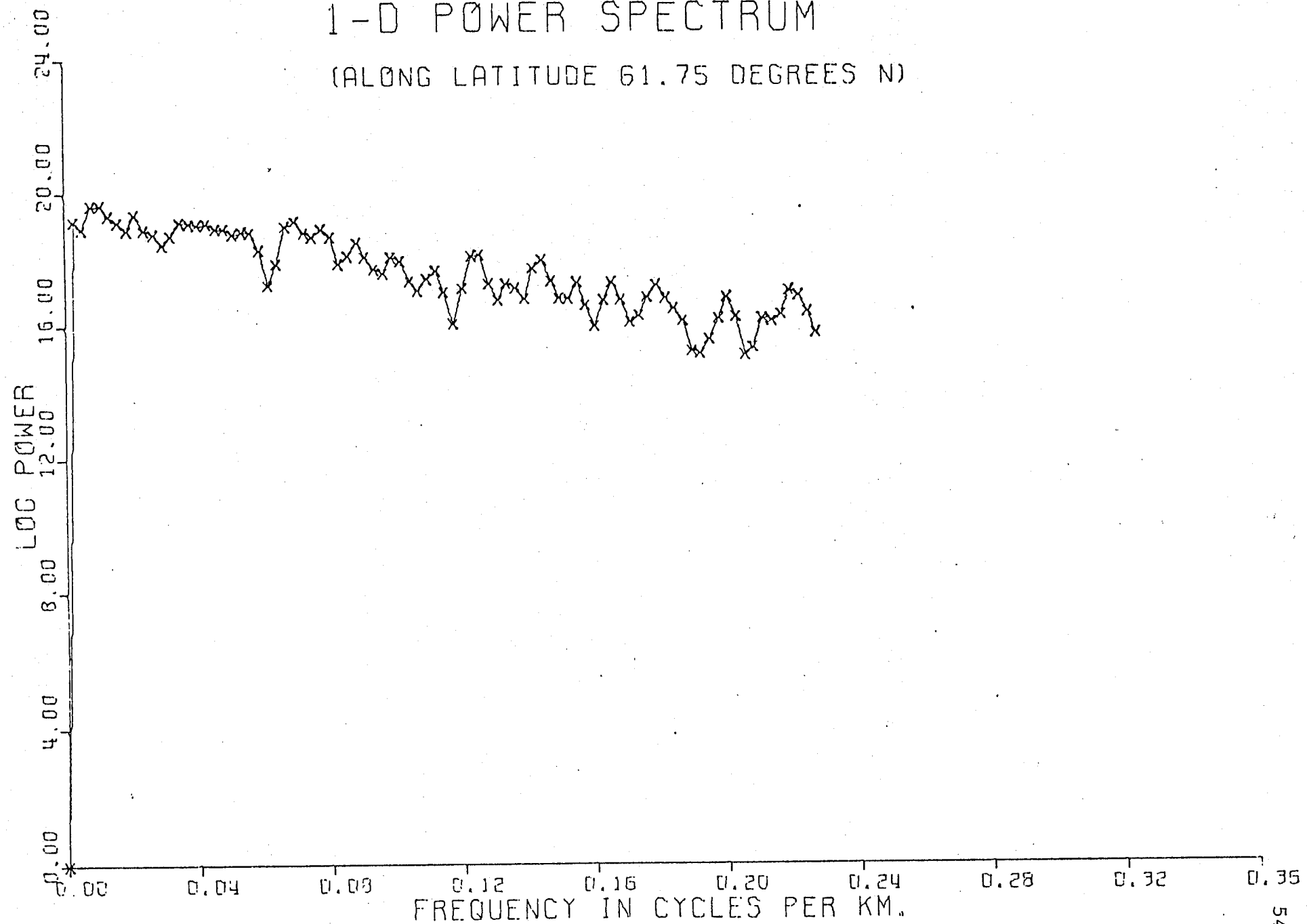


FIGURE 5.2.2 (SMOOTHING APPLIED)

lengths components. A few sharp breaks appeared in the spectral curves (at wavelength 10 km and 12 km) but it is not significant in a regional sense.

5.3 Maximum Entropy Spectral Analysis

The Maximum Entropy Method (MEM) for power spectral density estimation was originally proposed by Burg (1967). Its mathematical properties have been widely discussed in detail by Lacoss (1970), Ulrych and Bishop (1972) who showed that the newer technique (Maximum Entropy Method) is in general superior to the other conventional spectral analysis methods. These include better resolution for a given series of data and more realistic power estimates for short data lengths.

The Maximum Entropy spectral analysis was employed here to study the spectral characteristics of a few profiles in the project area. A fast and simple procedure for MEM spectral estimation was developed by Anderson (1974) and was employed here for the analysis.

Given a set of data $x_1, x_2, x_3 \dots x_N$ with sampling interval Δt , the MEM power spectrum $P(f)$ is estimated by

$$P(f) = \frac{P_m \Delta t}{\left| 1 - \sum_{n=1}^m a_{mn} e^{-2\pi i f n \Delta t} \right|^2}$$

$$\text{where: } -\frac{1}{(2\Delta t)} \leq f \leq \frac{1}{(2\Delta t)}$$

and P_m and the coefficients a_{mn} are determined by the equation:

$$\begin{pmatrix} \phi_0 & \phi_1 & \cdot & \cdot & \cdot & \phi_m \\ \phi_1 & & & & & \cdot \\ \cdot & & & & & \cdot \\ \cdot & & & & & \cdot \\ \cdot & & & & \phi_1 & \\ \phi_m & & \phi_1 \phi_0 & & & \end{pmatrix} \begin{pmatrix} 1 \\ -a_{m1} \\ \cdot \\ \cdot \\ -a_{mn} \end{pmatrix} = \begin{pmatrix} P_m \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{pmatrix}$$

is the autocorrelation with lag i , so that P_m is the output power of the $(m+1)$ long prediction error filter $(1, -a_{m1}, \dots, -a_{mn})$. A recursive procedure for P_m can then be derived. The recursive procedure in obtaining the estimates is summarized by Anderson (1974) in the form of a flow diagram. A computer program (basing on this flow diagram by Anderson) is developed for the computations of a few profiles in the present project.

The profile along Latitude 61.75 degrees North is used in the power spectral analysis in the previous section and

is first tested here to obtain a Maximum Entropy Autopower spectrum. The spectrum obtained as shown in Figure 5.3.1 gives a clear picture of the frequency characteristics of the aeromagnetic profiles under study.

Three distinct frequency bands corresponding to wavelengths 200 - 40 km, 40 - 15 km and 15 - 10 km can be easily identified from the spectrum (see Figure 5.3.1). A few smaller peaks are located on the higher frequency portion of the spectrum indicating the presence of the shorter wavelength components. The longer wavelength band (200 - 40 km) appears to be the ideal target of our study. The appropriate cutoff wavelength as indicated by the spectrum lies in the vicinity of 40 km.

The predictive error filter coefficients were then increased from 20 to 40, and from 40 to 60. A change of the spectrum characteristics was observed as the number of the predictive error filter coefficients was increased. More distinct narrow peaks appears as the filter coefficients increases indicating a clear improvement of the resolution of the spectrum (see Figures 5.3.1 to 5.3.3). A few other profiles were tested (see Figures 5.3.4 and 5.3.5) and they all appear to yield the same results.

MAXIMUM ENTROPY AUTOPOWER SPECTRUM

(ALONG LATITUDE 61.75 DEGREES N)

(NUMBER OF P.E. FILTER COEFFICIENTS = 20)

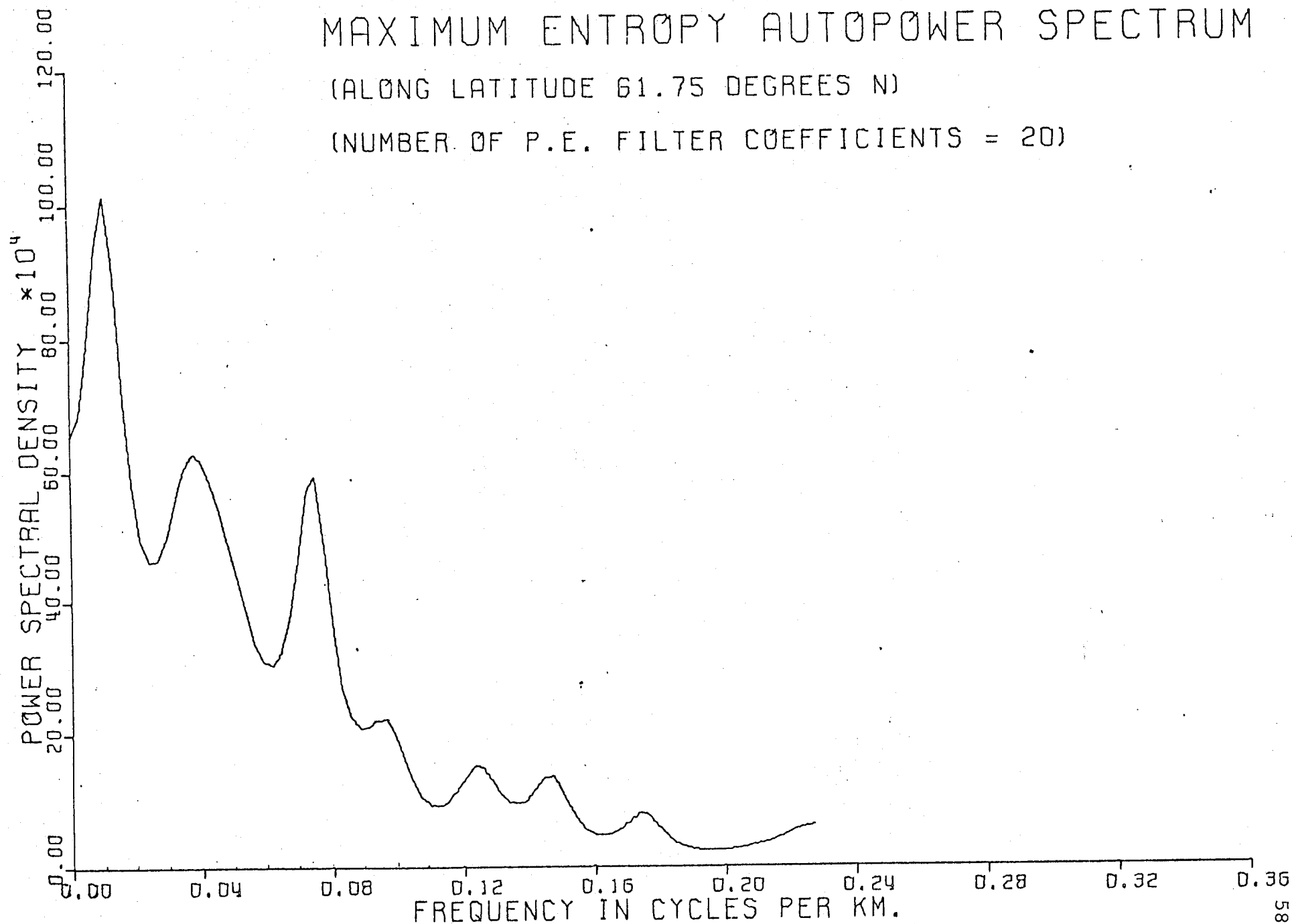


FIGURE 5.3.1

MAXIMUM ENTROPY AUTOPOWER SPECTRUM

(ALONG LATITUDE 61.75 DEGREES N)

(NUMBER OF P.E. FILTER COEFFICIENTS = 40)

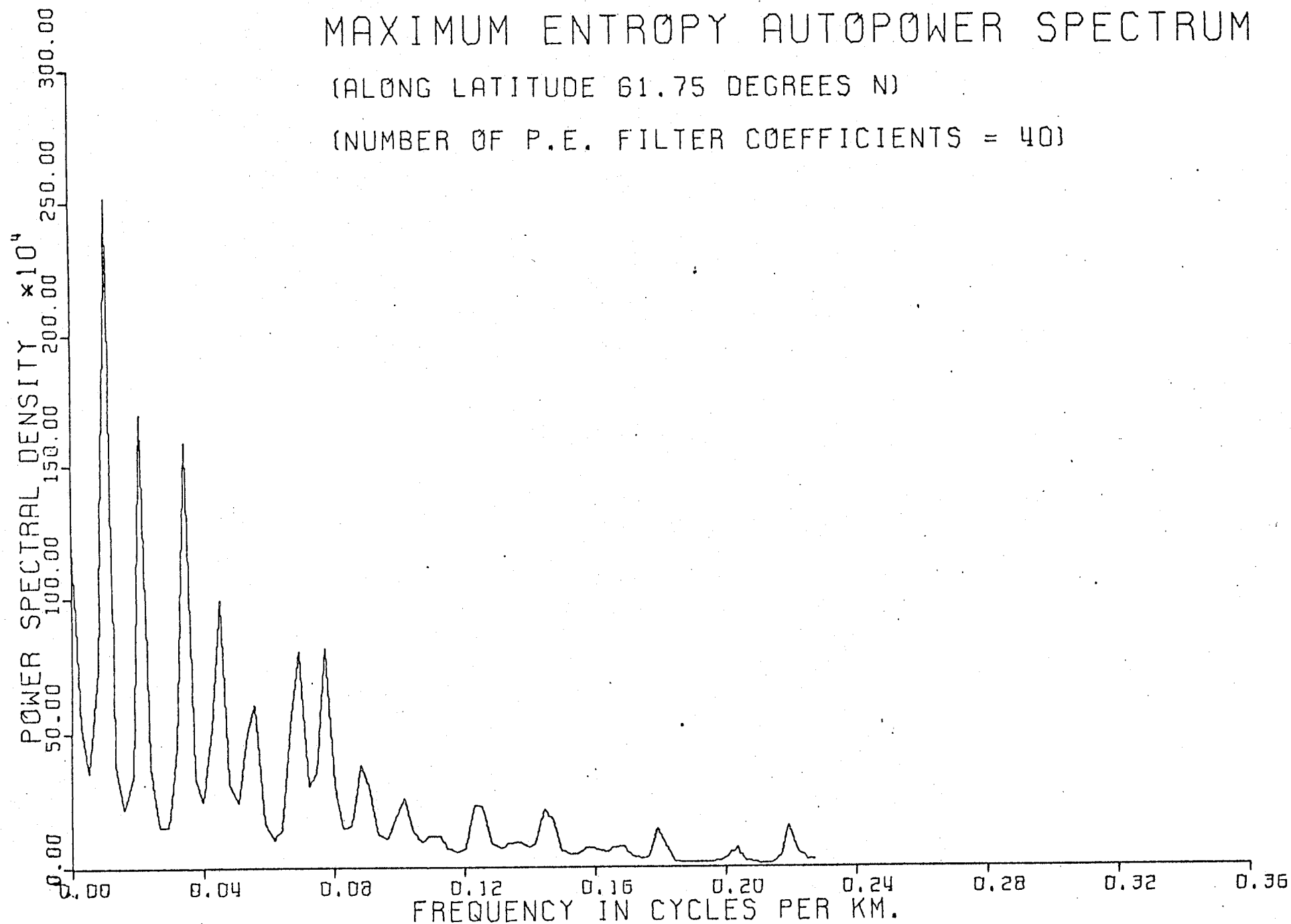


FIGURE 5.3.2

MAXIMUM ENTROPY AUTOPOWER SPECTRUM

(ALONG LATITUDE 61.75 DEGREES N)

(NUMBER OF P.E. FILTER COEFFICIENTS = 60)

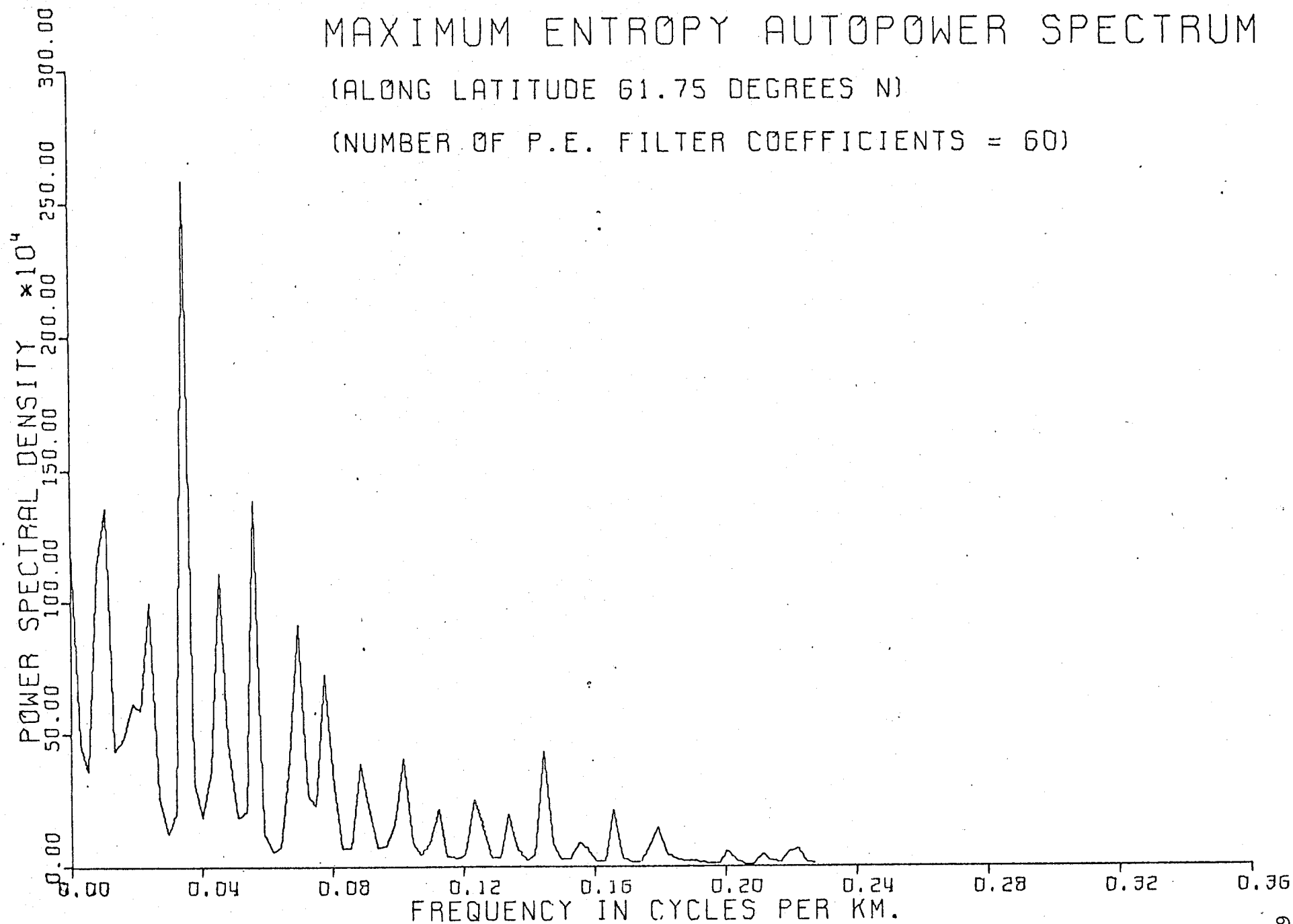


FIGURE 5.3.3

MAXIMUM ENTROPY AUTOPOWER SPECTRUM

(ALONG LATITUDE 61.08 DEGREES N)

(NUMBER OF P.E. FILTER COEFFICIENTS = 40)

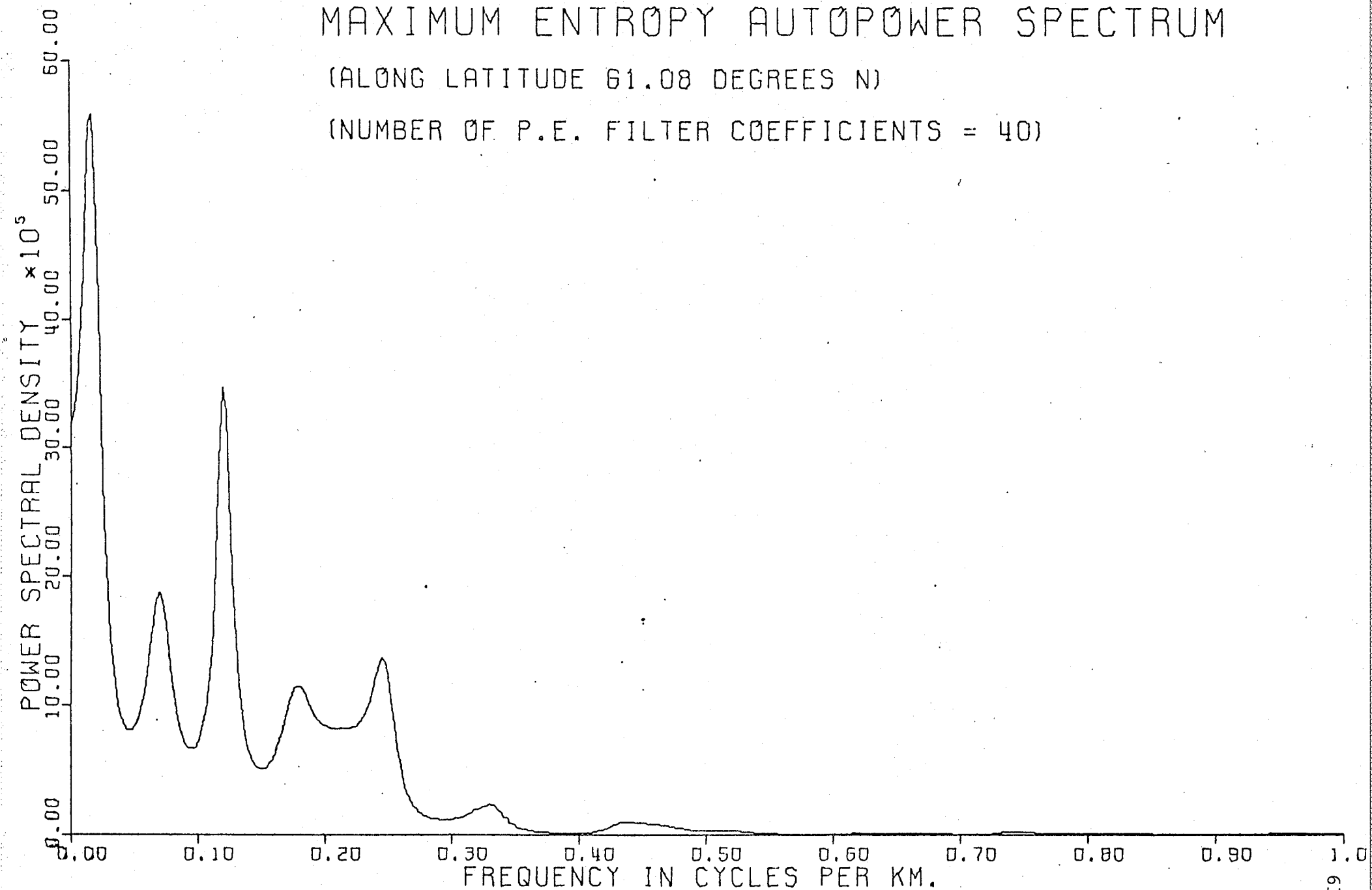


FIGURE 5.3.4

MAXIMUM ENTROPY AUTOPOWER SPECTRUM

(ALONG LATITUDE 61.08 DEGREES N)

(NUMBER OF P.E. FILTER COEFFICIENTS = 85)

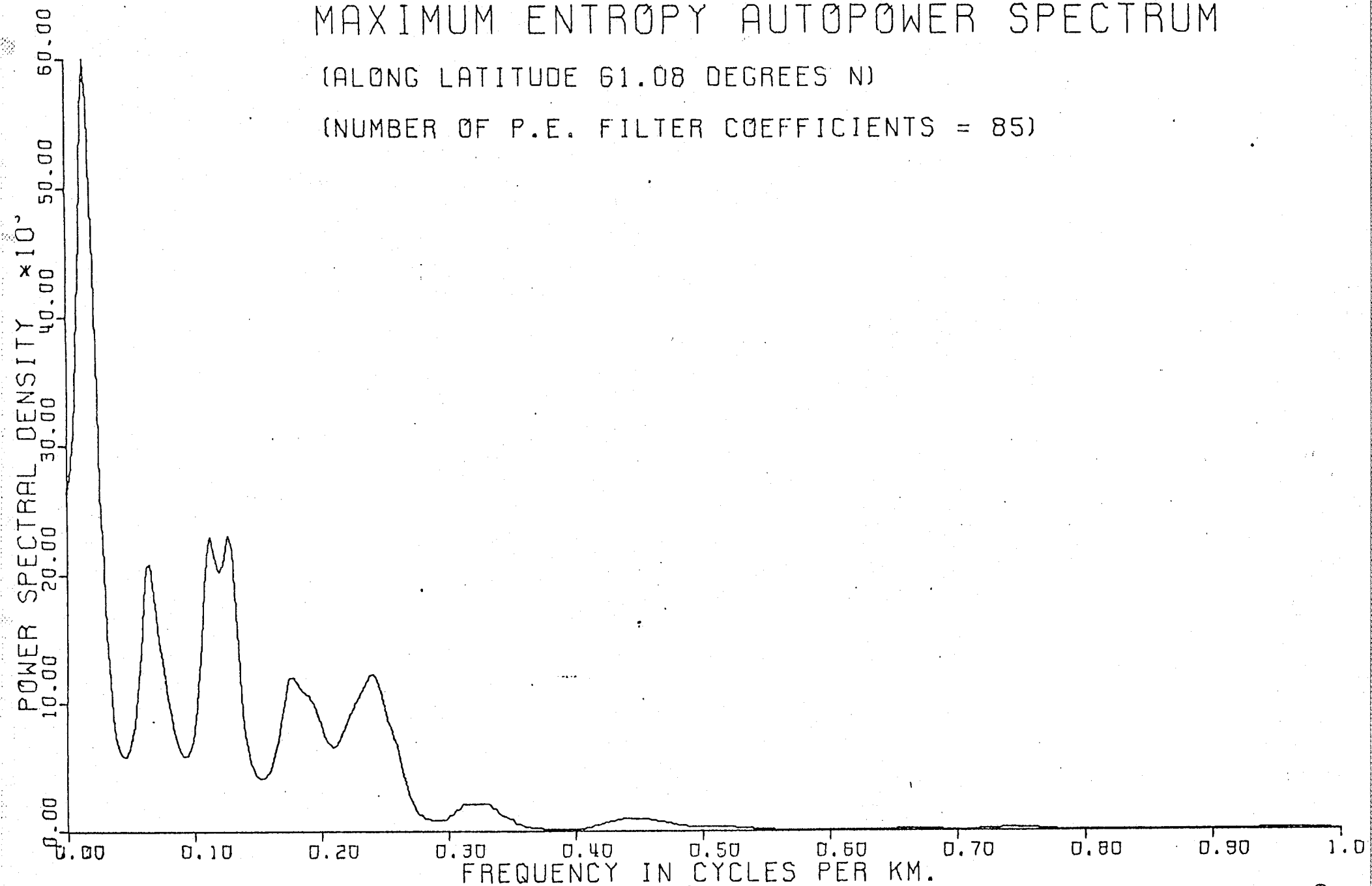


FIGURE 5.3.5

A comparison was made between the spectra obtained from the power spectral analysis and the Maximum Entropy Spectral analysis (Burg) and the following is observed.

1. For a given series of data (profile along 61.75 degrees North) the MEM bandwidth is better defined than that observed by FFT analysis (see Figures 5.2.1 and 5.3.1).
2. Higher resolution is obtained with the increase of the number of predictive error coefficients. The more conventional FFT method depends largely on the number of data samples within a series. Severe truncation of Fourier coefficients after filtering may lead to inaccuracy of the power spectral estimates if the data lengths are too short. Hence, MEM method is ideal for short data series.

5.4 Two-Dimensional Power Spectrum Analysis

One-dimensional power spectrum analysis yields information of the frequency characteristics in a profile in a particular direction. This technique of power spectral analysis when extended to two dimensions yields strikes as well as amplitudes of periodic phenomena. An attempt to display a two-dimensional power spectrum was made here in this section to further investigate the anomaly field in this study area.

Spectral estimates were determined for the study area using the computer program POWER, written originally by Coles (1973). The program uses the circular cosine bell window as the data taper and the fast Fourier algorithm for the computations (see Figures 5.4.1 and 5.4.2). A second computer program PROFIL as described by Coles (1973) was then used to select one-dimensional views of the spectra. This program, PROFIL, uses a spline interpolation routine to obtain values on the profile (see Figures 5.4.3 and 5.4.5).

5.5 Radial Spectrum and Its Interpretation

Radial spectrum as described by Naidu (1969, 1970), Mishra and Naidu (1974) and various other authors was under investigation here for a possible preliminary interpretation of the distribution and wavelength characteristics of various magnetic units in the study area.

From the two-dimensional spectrum obtained as described in the last section, radial spectrum can be computed as follows:

$$R(s) = \frac{1}{2\pi} \int_0^{2\pi} S(x) (S \cos \theta, S \sin \theta) d\theta$$

where $R(s)$ is radial spectrum, $S_x(u,v)$ is the two-dimensional spectrum, $u = S \cos \theta$ and $v = S \sin \theta$. A FORTRAN subroutine RAID

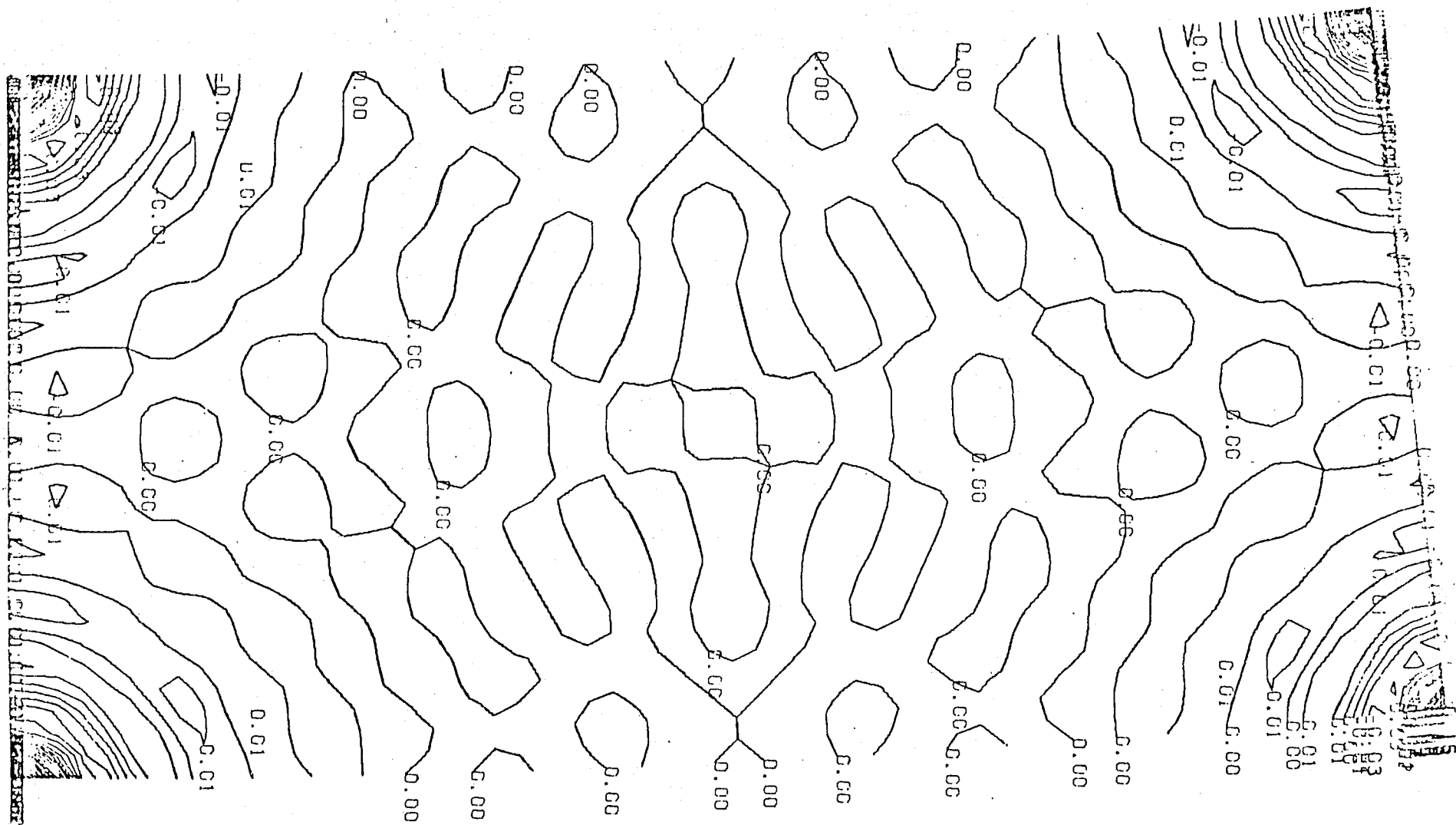


Figure 5.4.1 Two-dimensional Power Estimate of the Project Area

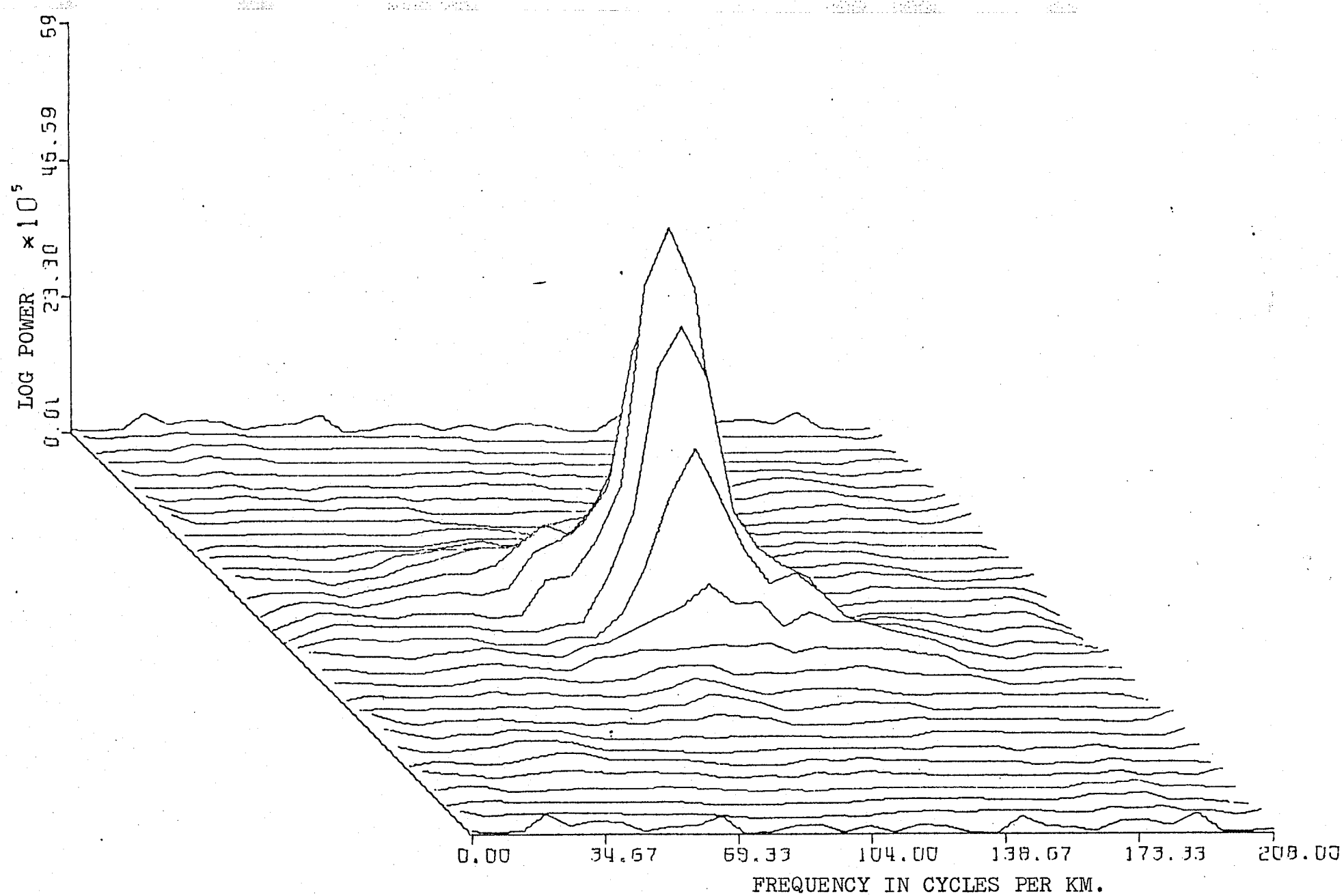


Figure 5.4.2 Three-dimensional Representation of the
Power Spectrum Estimate of the Project Area

PROFILE ACROSS A 2-D SPECTRUM
(45 DEGREES COUNTERCLOCKWISE TO +X AXIS)

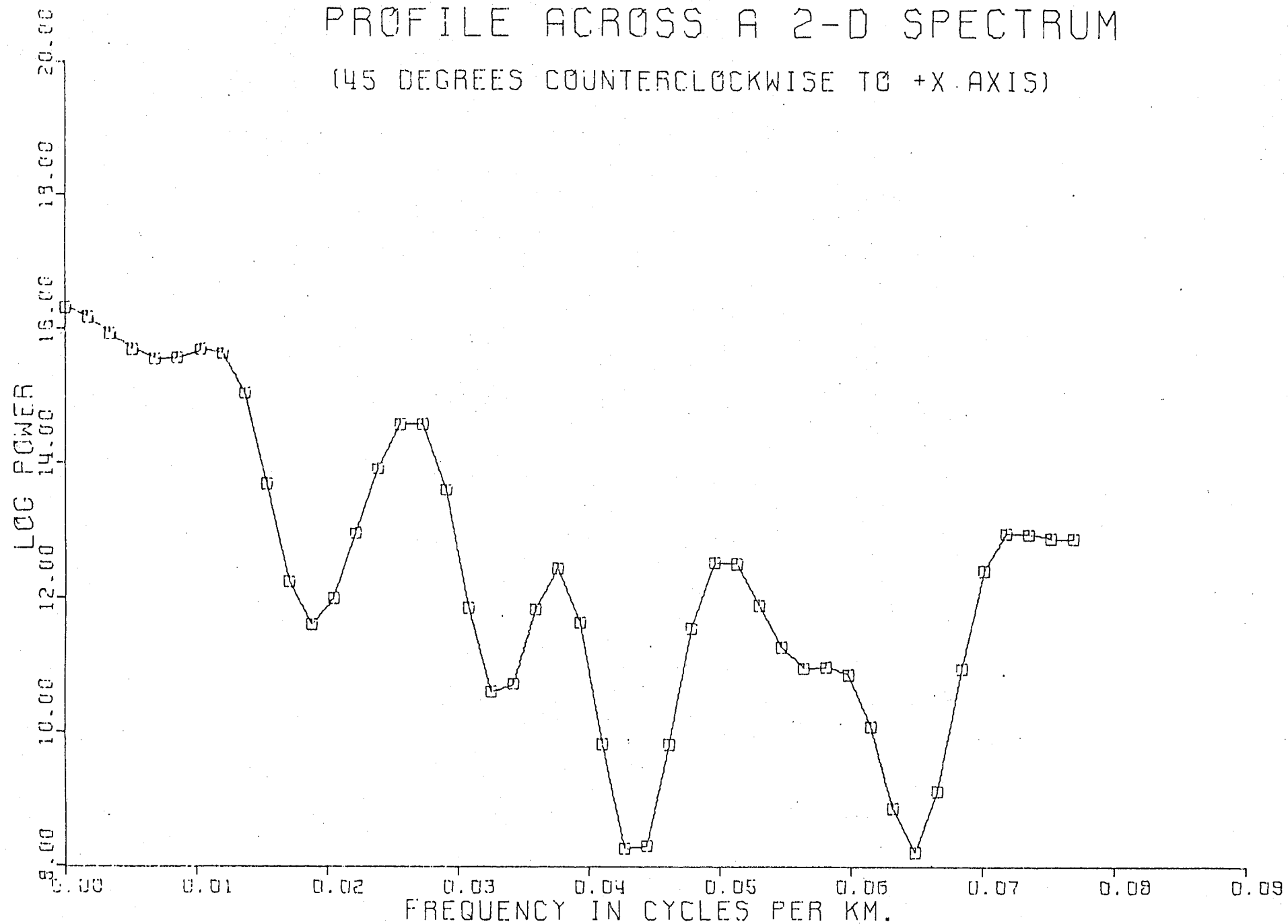


FIGURE 5.4.3

PROFILE ACROSS A 2-D SPECTRUM (90 DEGREES COUNTERCLOCKWISE TO +X AXIS)

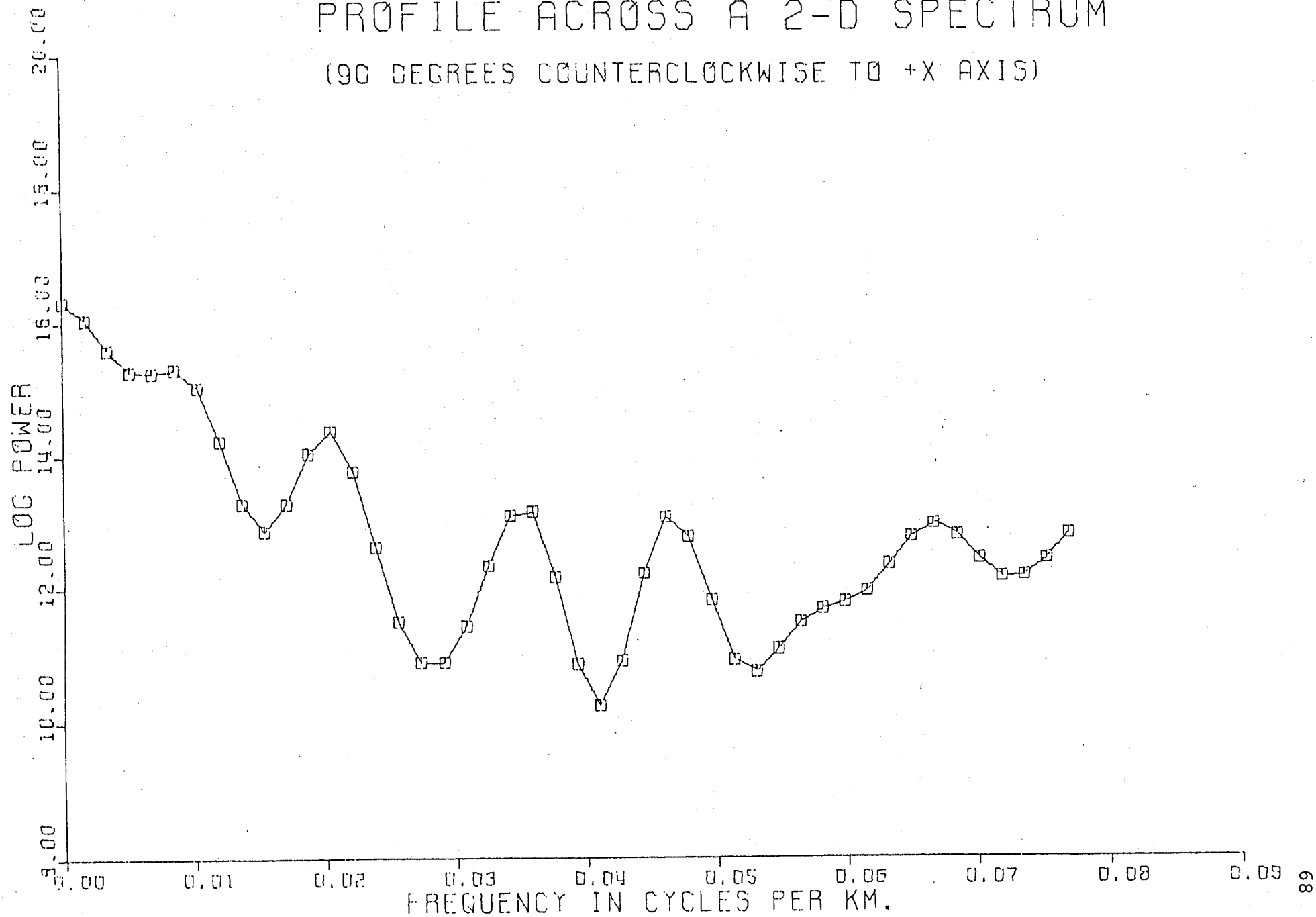


FIGURE 5.4.4

PROFILE ACROSS A 2-D SPECTRUM

(135 DEGREES COUNTERCLOCKWISE TO +X AXIS)

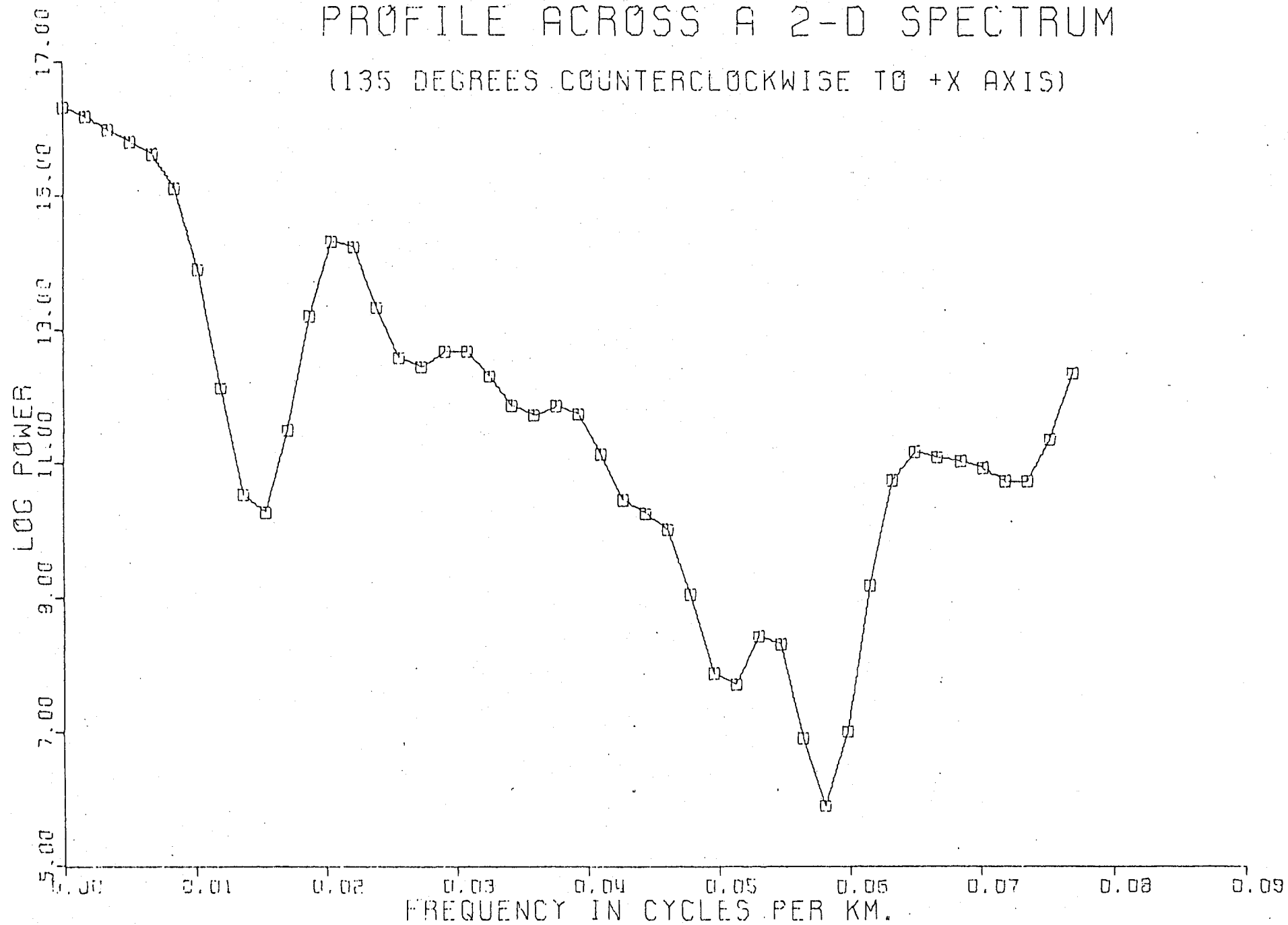


FIGURE 5.4.5

(see Appendix B) was written to perform the above step to obtain the radial spectrum (with or without numerical smoothing) for the map area for analysis. Figure 5.5.1 and Table 5.5.2 give a smoother version of the spectrum.

It is interesting to note that the radial spectrum obtained show a bimodal distribution more strikingly than the profiles obtained in the last section. A preliminary interpretation was done on the spectrum (see Figure 5.5.3) with slopes drawn as close to the best fit lines of the data points. The slopes of the straight lines can be interpreted as to correspond to the depths to two possible layers of magnetization. A definite break in the general slope of the spectral curve occurs in the region corresponding to about 32 km wavelength (0.032 cycles/km). This could indicate that two processes (or sources) of differing characteristics are contributing to the anomaly fields.

The choice of optimum filtering bandwidth for separation of sources is dependent on the nature of the magnetization distribution present. Hence, from the radial spectrum obtained, filtering at or above the wavelength of 32 km (0.032 cycles/km) will essentially remove the shallow effects, thus leaving the more desirable long-wavelength anomalies.

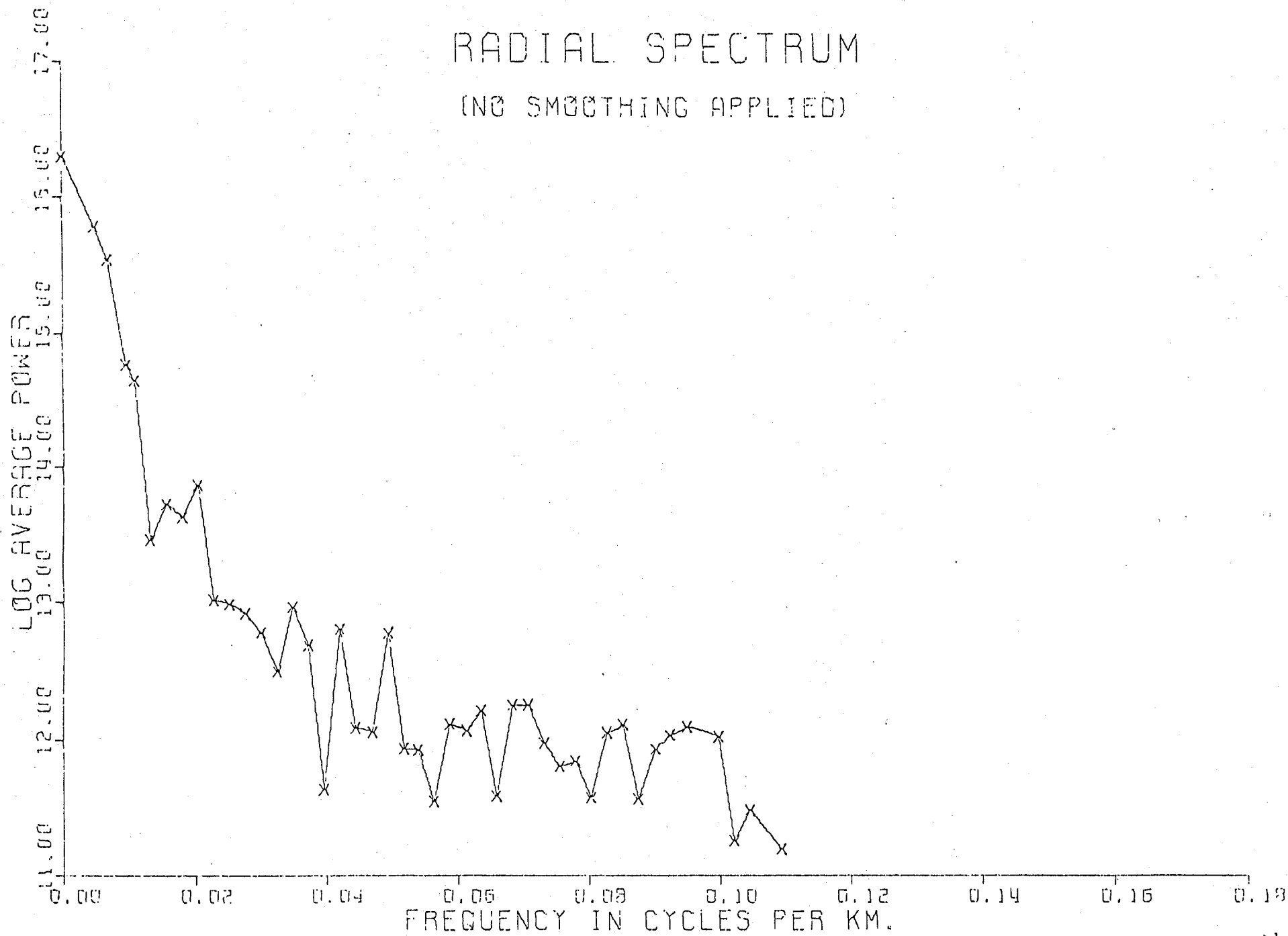


Figure 5.5.1 Radial Spectrum of the Aeromagnetic Map Under Studied

NUMBER	LOG AVERAGE POWER	FREQUENCY	SAMPLE SIZE
1	16.3040	0.0	1
2	15.7754	0.0048	4
3	15.5407	0.0068	4
4	14.7673	0.0096	4
5	14.6549	0.0108	8
6	13.4634	0.0132	8
7	13.7308	0.0156	8
8	13.6286	0.0180	12
9	13.8700	0.0204	20
10	13.0222	0.0228	12
11	12.9889	0.0252	16
12	12.9184	0.0276	16
13	12.7836	0.0300	24
14	12.5018	0.0325	12
15	12.9665	0.0349	28
16	12.6882	0.0373	20
17	11.6427	0.0397	28
18	12.8085	0.0421	28
19	12.0934	0.0445	40
20	12.0590	0.0469	24
21	12.7781	0.0493	32
22	11.9421	0.0517	28
23	11.9321	0.0541	44
24	11.5489	0.0565	20
25	12.1151	0.0589	48
26	12.0747	0.0613	40
27	12.2173	0.0637	48
28	11.5911	0.0661	36
29	12.2629	0.0685	52
30	12.2605	0.0709	44
31	11.9800	0.0733	40
32	11.8092	0.0757	48
33	11.8522	0.0781	64
34	11.5751	0.0805	36
35	12.0592	0.0829	40
36	12.1220	0.0853	24
37	11.5723	0.0877	28
38	11.9432	0.0901	16
39	12.0427	0.0925	24
40	12.1035	0.0950	20
41	12.0254	0.0998	16
42	11.2572	0.1022	12
43	11.4944	0.1046	8
44	11.1980	0.1094	4

DIGITAL INTERVAL = 6.5000 KM. NYQUIST FREQUENCY = 0.0769 CYCLES PER K

Table 5.5.2 Log Average Power vs Frequency
of the Radial Spectrum

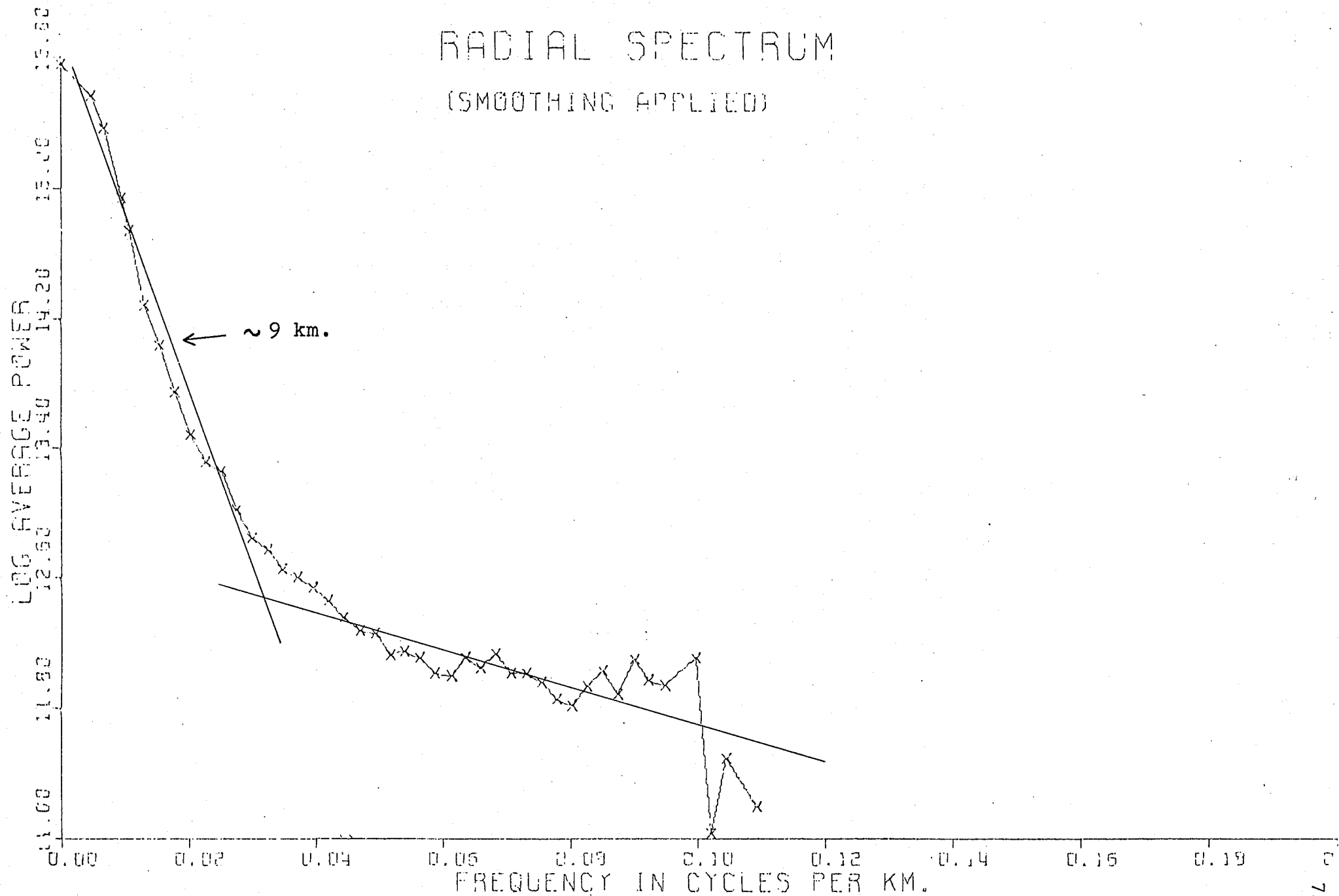


Figure 5.5.3 Radial Spectrum of the Aeromagnetic Map Under Studied
(smoothing applied)

NUMBER	LOG AVERAGE POWER	FREQUENCY	SAMPLE SIZE
1	15.7597	0.0	1
2	15.5654	0.0048	4
3	15.3707	0.0068	4
4	14.9445	0.0096	4
5	14.7465	0.0108	8
6	14.2869	0.0132	8
7	14.0397	0.0156	8
8	13.7488	0.0180	12
9	13.4908	0.0204	20
10	13.3215	0.0228	12
11	13.2660	0.0252	16
12	13.0246	0.0276	16
13	12.8513	0.0300	24
14	12.7743	0.0325	12
15	12.6553	0.0349	28
16	12.6079	0.0373	20
17	12.5448	0.0397	28
18	12.4611	0.0421	28
19	12.3571	0.0445	40
20	12.2803	0.0469	24
21	12.2630	0.0493	32
22	12.1250	0.0517	28
23	12.1509	0.0541	44
24	12.1153	0.0565	20
25	12.0133	0.0589	48
26	12.0033	0.0613	40
27	12.1136	0.0637	48
28	12.0502	0.0661	36
29	12.1330	0.0685	52
30	12.0123	0.0709	44
31	12.0193	0.0733	40
32	11.9575	0.0757	48
33	11.8597	0.0781	64
34	11.8167	0.0805	36
35	11.9378	0.0829	40
36	12.0281	0.0853	24
37	11.8895	0.0877	28
38	12.1073	0.0901	16
39	11.9790	0.0925	24
40	11.9475	0.0950	20
41	12.1127	0.0998	16
42	11.0295	0.1022	12
43	11.4944	0.1046	8
44	11.1980	0.1094	4

DIGITAL INTERVAL = 6.5000 KM. NYQUIST FREQUENCY = 0.0769 CYCLES PER K

(SMOOTHING APPLIED)

Table 5.5.4 Log Average Power vs Frequency
of the Radial Spectrum

CHAPTER 6

INTERPRETATION

6.1 Introduction

Model-fitting has played an important part in the interpretation of geophysical surveys. The arbitrary characteristic curve-fitting approaches to potential field data analysis now seem to be disappearing from use. In their place, computer modeling has become more widely used. This is mainly due to the reduction of computer cost, increase in computer core sizes and machine efficiency in recent years. At the present time, interactive computer modeling can be handled conveniently in three dimensions as well as two dimensions.

6.2 Interpretation of the Long-Wavelength Aeromagnetic Anomalies Using Prismatic Blocks

Interpretation of the long-wavelength aeromagnetic anomalies in the project area is carried out by three-dimensional modeling using the computer iterative method. Due to the inherent ambiguity in interpretation and severe complexities in analytical inversion of potential field data,

a simple geometrical unit - the finite depth vertical sided rectangular prism - is used.

The iterative method described here in this thesis assumes initially that the anomalous field values are caused by a three-dimensional distribution of homogeneously magnetized rectangular block. The expression for the total field anomaly at a point above such a prism was described by Bhattacharyya (1964). The computer program "BLOCK", based on Bhattacharyya's expression as derived and written by Coles (1973), is used in this project for the interpretation of the aeromagnetic anomalies. The program BLOCK accepts for input any number of right-angled prisms, with arbitrary dimensions, directions and intensities of magnetization. An optimization routine "INVBLOCK"¹ was then developed and implemented with BLOCK as the major subroutine to provide an indirect iterative method for the interpretation of the long-wavelength anomalies in this project.

Anomaly field chosen for input INVBLOCK is shown in the Schematic diagram Figure 6.2.1. Seven major blocks were chosen for this study and a rectangular grid was designed over each anomaly for digitization of the anomaly

1. The computer program INVBLOCK was originally derived and implemented by Mr. C. Hasselfield, programmer, Geophysics Section, University of Manitoba.

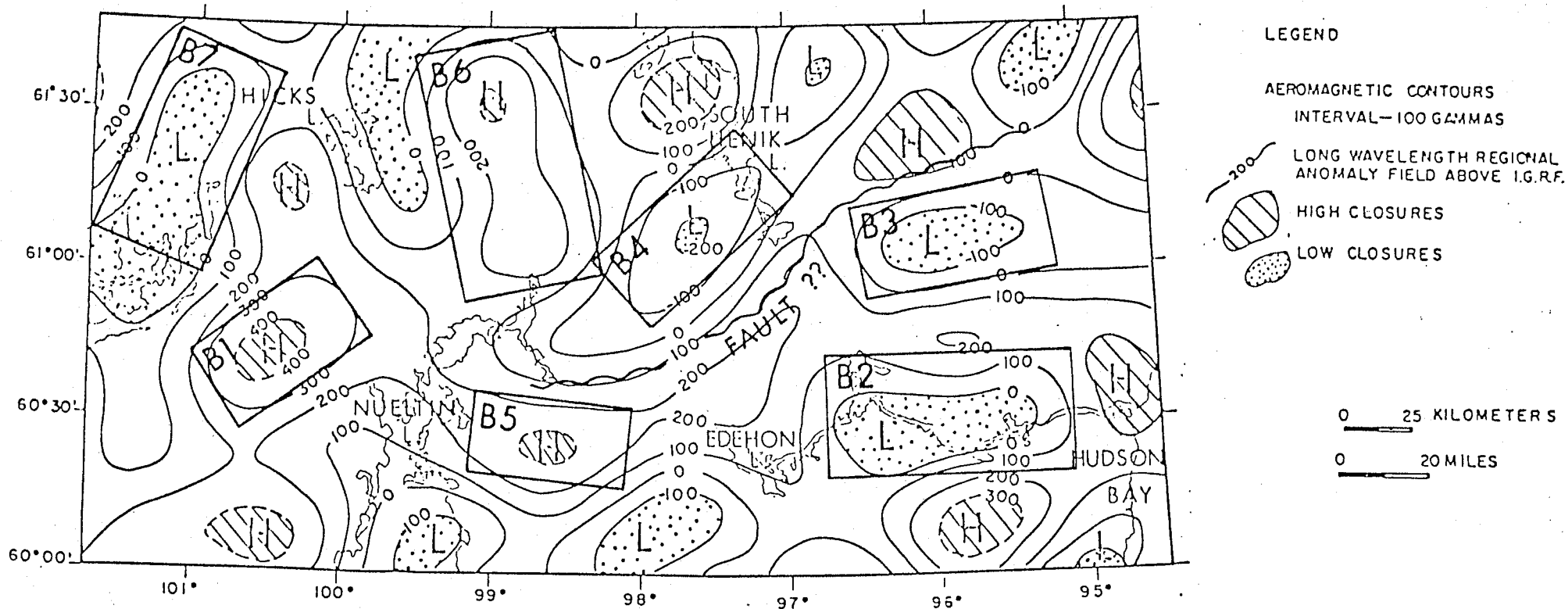


Figure 6.2.1 Schematic Diagram of the Location of Various Interpretive Blocks of the Study Area
(Detail Grids Not Shown)

fields. The digitized magnetic field values were then stored in an array and together with some additional information (e.g., declination, inclination, etc.) were passed into the routine. A summary of the results is shown in Table 6.2.2.

6.3 Discussion

The average depth to the top of the anomalies studied was found to be approximately 11 km with the exception of blocks 3 and 5. It is interesting to note that the slope of the graph on Figure 5.5.3 (Radial Spectrum of the Aero-magnetic Map Under Studied) gives an average depth to the ensemble of bodies in the map area of approximately 9 km using Spector and Grant's technique (1970). This seems to compare well with the 11 km depth obtained by computer BLOCK modelling.

The magnetic intensity of polarization of the blocks in the map area varies quite a bit from 850 emu (block 3) in the northeast corner to 9974 emu (block 6) in the central area (see Figure 6.2.1). The wide variations of magnetic inclinations and declinations of the blocks and their deviations from the present field (see Table 6.2.2) indicates that the magnetization of these blocks seems to be "remanent" rather than "induced" in nature.

BLOCK	ITENS (emu)	BASE (km)	TOP (km)	BOTTOM (km)	DECL (degrees)	INCL	RMS ERROR	ITERATIONS
1	2724.9	219.5	10.5	24.5	140.0	155.0	24.54242	335
2	4474.9	144.5	10.3	14.8	0.0	215.0	33.18250	188
3	475.0	73.7	0.5	91.8	135.0	-105.0	24.24573	283
4	2250.0	131.0	13.9	50.0	190.0	230.0	30.98927	222
5	1975.0	163.2	0.3	7.0	-10.0	215.0	42.20583	203
6	1600.0	32.0	10.3	30.0	40.0	115.0	32.66182	662
7	2725.0	144.5	10.8	23.5	50.0	215.0	24.78154	522

Table 6.2.2 Table of the results of the blocks from INVBLOCK

Two major trends can be identified readily on the interpreted map. The major trending appears to be roughly striking northeast-southwest. The northwestern four blocks (1, 4, 6 and 7) represent deeper magnetic units not aligned in a specific direction. It is estimated that the sources of these anomalies lie in the lower crust from a depth of approximately 11 km. It should be emphasized that the long-wavelength anomalies can equally well be explained by much shallower sources of larger areal extent.

The other trend in the southeastern corner of the map comprises blocks 3, 5 and 2. This trend appears to be better aligned and striking east-west. Blocks 3 and 5 were found to be of much shallower origin than block 2 but all three blocks share a common magnetic declination. Examining the relative depth of burial of the upper and lower set of interpreted blocks, it appears to show evidence of a step-like structure in this area, thus implying the possible presence of a northeast-southwest trending fault (see Figure 6.2.1). Moreover, the overall magnetic anomaly pattern shows a close and interesting connection with the fault zone of the Wollaston domain.

It is interesting to note that the major anomaly trend (northeast-southwest) as found in the long-wavelength aeromagnetic map presented in this project fits in very well with the "linear belt" as suggested by Camfield and Gough

(1977). A long, narrow belt of rocks of very high electrical conductivity has been discovered and mapped by Camfield and Gough (1977) by means of large arrays of recording magnetometers. This conductive body (see Figure 6.3.1) trends over a distance of 1400 km from southeastern Wyoming to the edge of the Canadian Shield in Saskatchewan. On the northwest side of the conductor mapped by Camfield et al. lies the Needle Falls Shear Zone. It is then followed by the Wollaston Domain immediately from the northeast. Within this Domain, the Wollaston Fold Belt crosses the Saskatchewan-Manitoba border near latitude 59°N and believed to swing eastward in northern Manitoba to Hudson Bay. The major anomaly trend as found here in the project substantiates the claim that the Wollaston Fold Belt does go into the Northwest Territories and swings eastward towards Hudson Bay. It also proves the presence of tectonic structures within the Wollaston Domain. The misalignment of the northwestern four blocks and its depth of burial (11 km) indicates disturbances at depth and can be interpreted as the magnetic signature of the intensely folded basement rocks of the Wollaston Lake Fold Belt.

The conductive body mapped by Camfield et al., the Needle Falls Shear Zone, the Wollaston Lake Fold Belt, and the anomaly trend as found in this project, does form a definite "linear belt" through Wyoming to the Northwest

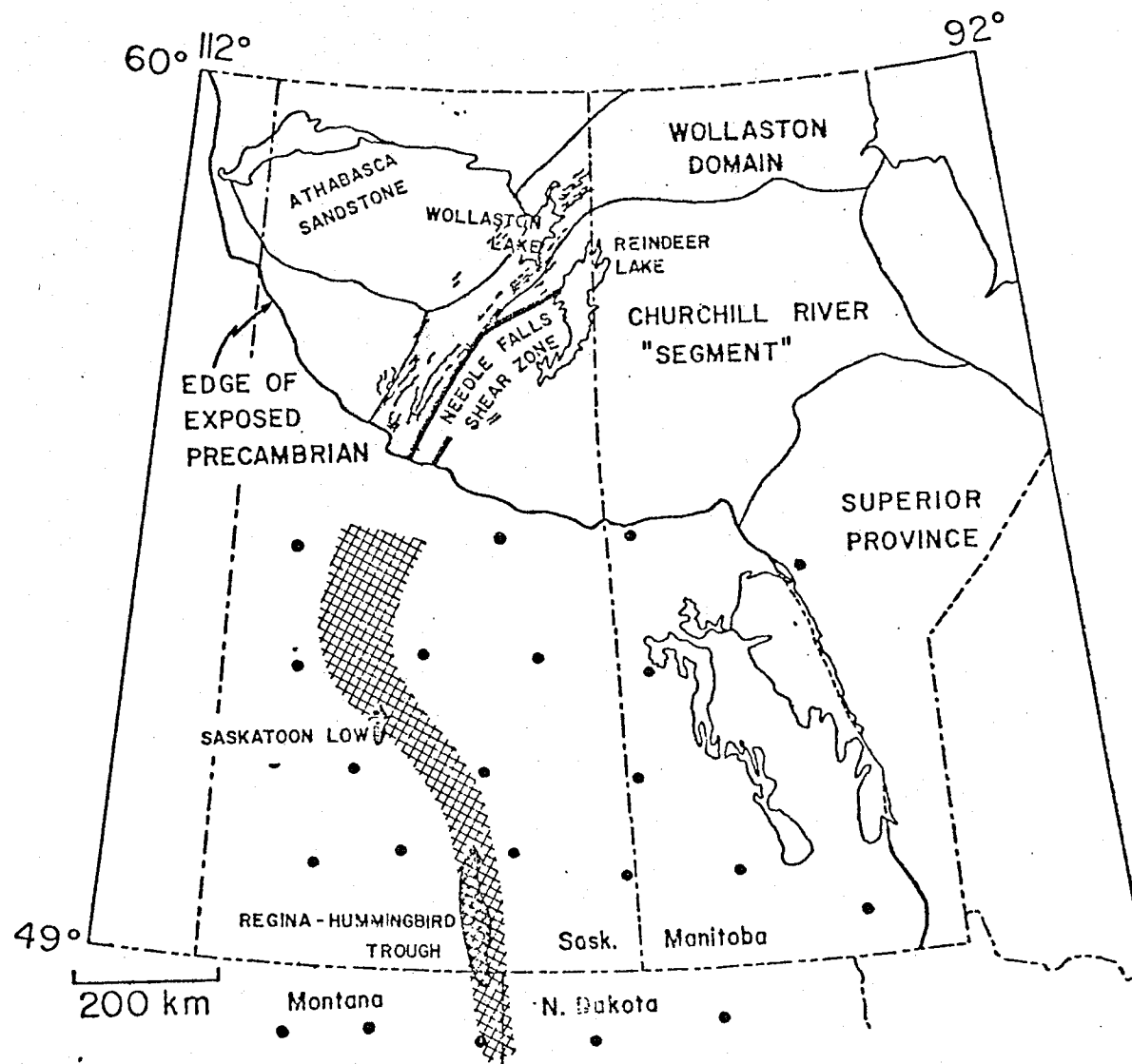


Figure 6.3.1 Location of the conductive body and the general geology of the Churchill Province

Territories with a general southwesterly strike. This linear belt has been hypothesized as an expression of the presence of a Proterozoic plate boundary by Camfield and Gough (1977).

The anomaly trend as mapped puts forward more crustal evidence of the presence of the Wollaston Fold Belt in the Northwest Territories adjacent to the northern boundary of the Province of Manitoba. It also suggests that the magnetic expression of the Wollaston Fold Belt is of "negative" (magnetic low) in nature and strikes generally northeast-southwest. This magnetic expression appears to have the opposite "sign" as Camfield and Gough stated for the Wollaston Fold Belt. Moreover, it forms part of the linear belt and adds to the proof of the presence of the Proterozoic plate boundary hypothesis in the Northwest Territories.

CHAPTER 7

CONCLUSIONS

The processing sequence and the program package FFT2D for processing aeromagnetic data to yield long-wavelength anomalies as suggested here in this study were found to be adequate, efficient and satisfactory.

The review of various filtering methods indicated that frequency domain filtering technique is more efficient and predictable than the space domain operations. This was particularly visible when the technique was applied on closely grid or larger areal extent aeromagnetic surveys.

Two different techniques in spectral analysis were tested in the project area. A comparison was made between the spectra obtained from the power spectral analysis method (via FFT) and Maximum Entropy Spectral analysis. The Maximum Entropy method was found to be ideal for short data series whereas severe truncation error introduced with the other technique. Moreover, the bandwidth of the spectra obtained using the Maximum Entropy Method was better defined than that by the Fast Fourier Transform analysis.

The International Geomagnetic Reference Field was used in this study for the removal of the geomagnetic field from the aeromagnetic data. It was found to be more efficient, accurate, consistent and readily accessible than developing equation to define the regional field from published total magnetic intensity map.

A close correspondence between regional geological features in the lower crustal layer and the final contoured map of the interpretive blocks was observed. The study presents more crustal evidence of the presence of the Wollaston Fold Belt in the Northwest Territories and the possible indication of the northern extent of the Proterozoic plate boundary as hypothesized by Camfield and Gough.

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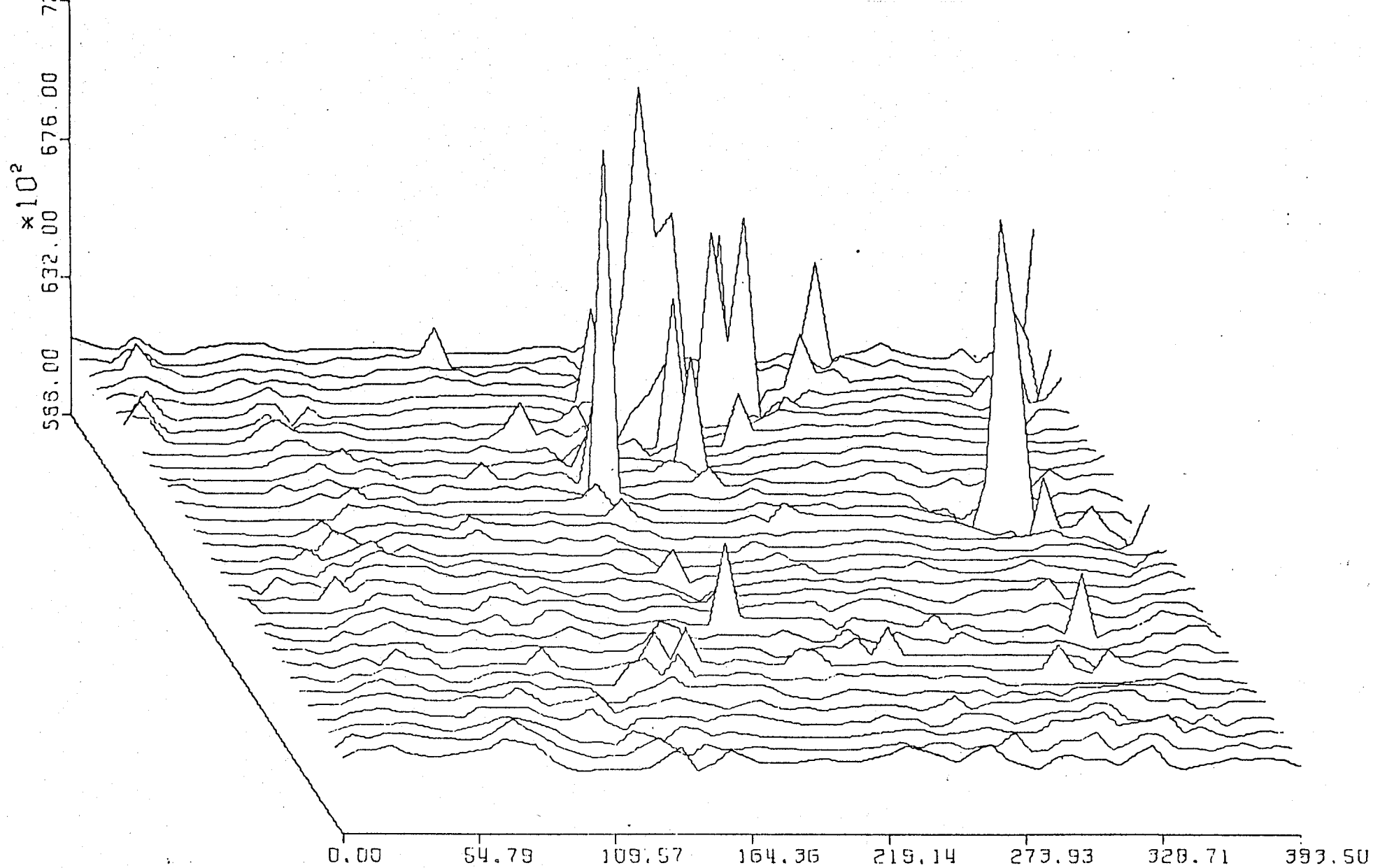
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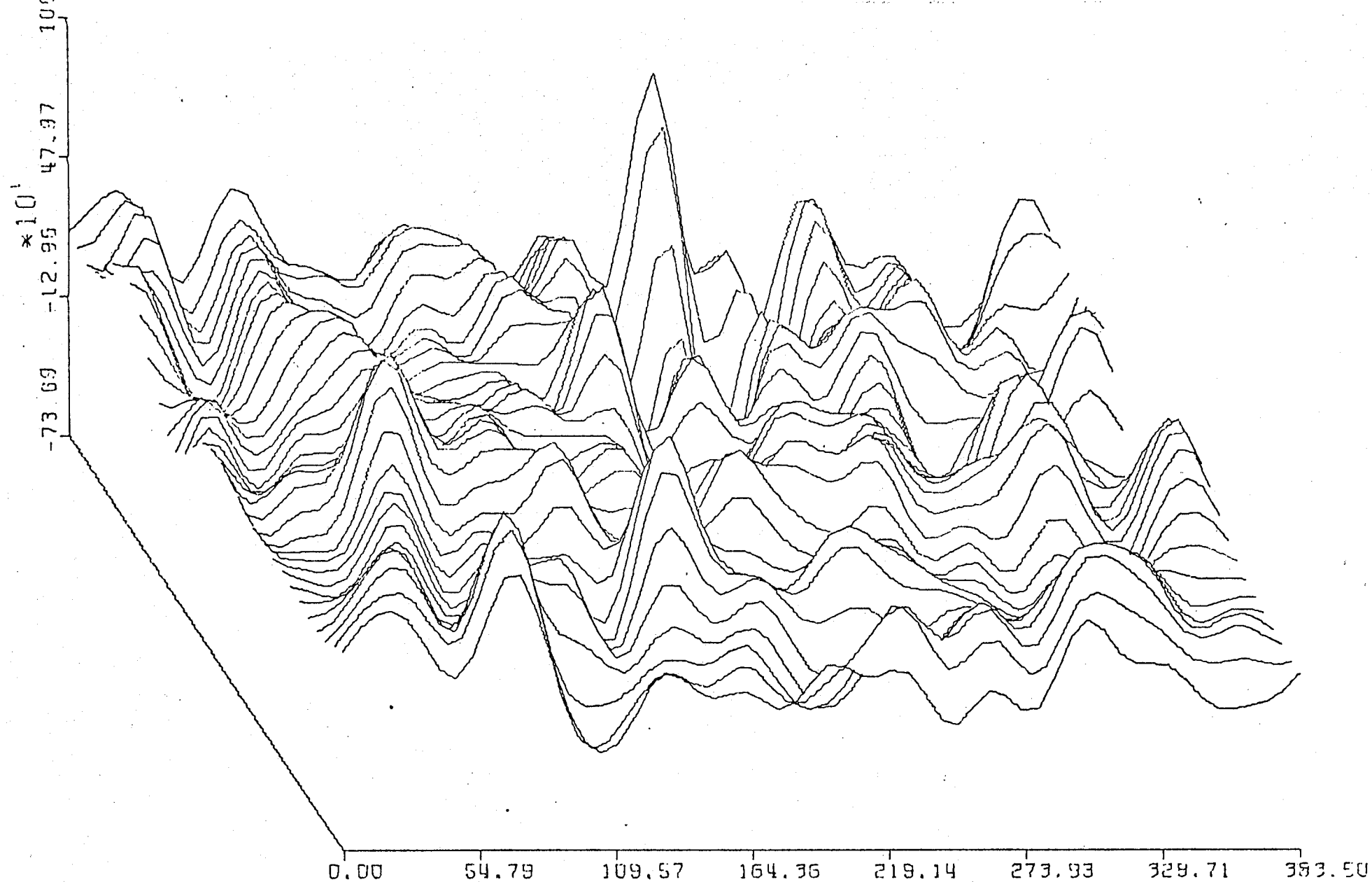
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APPENDIX A

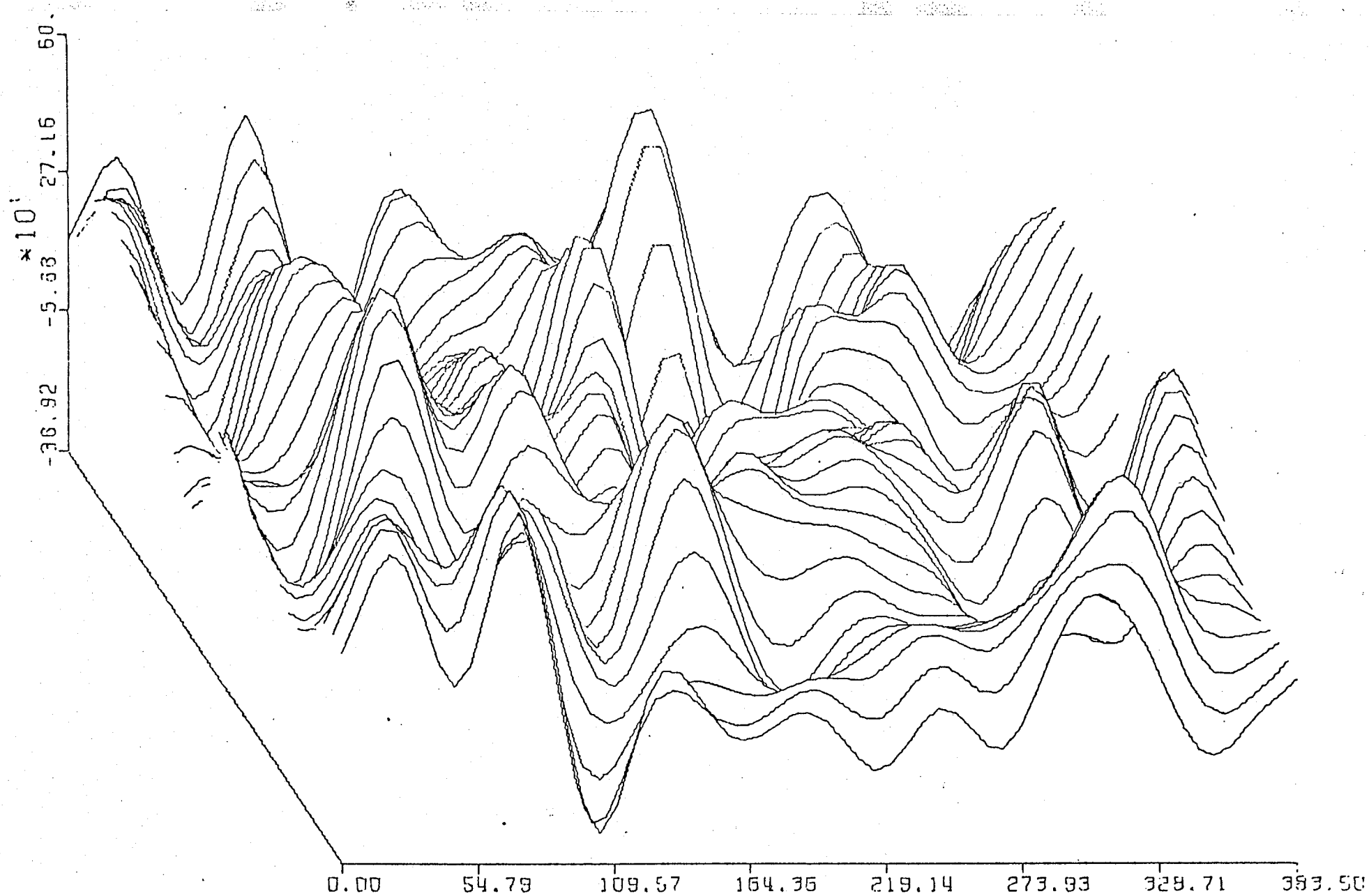
THREE-DIMENSIONAL PLOTS OF THE FILTERED MAPS



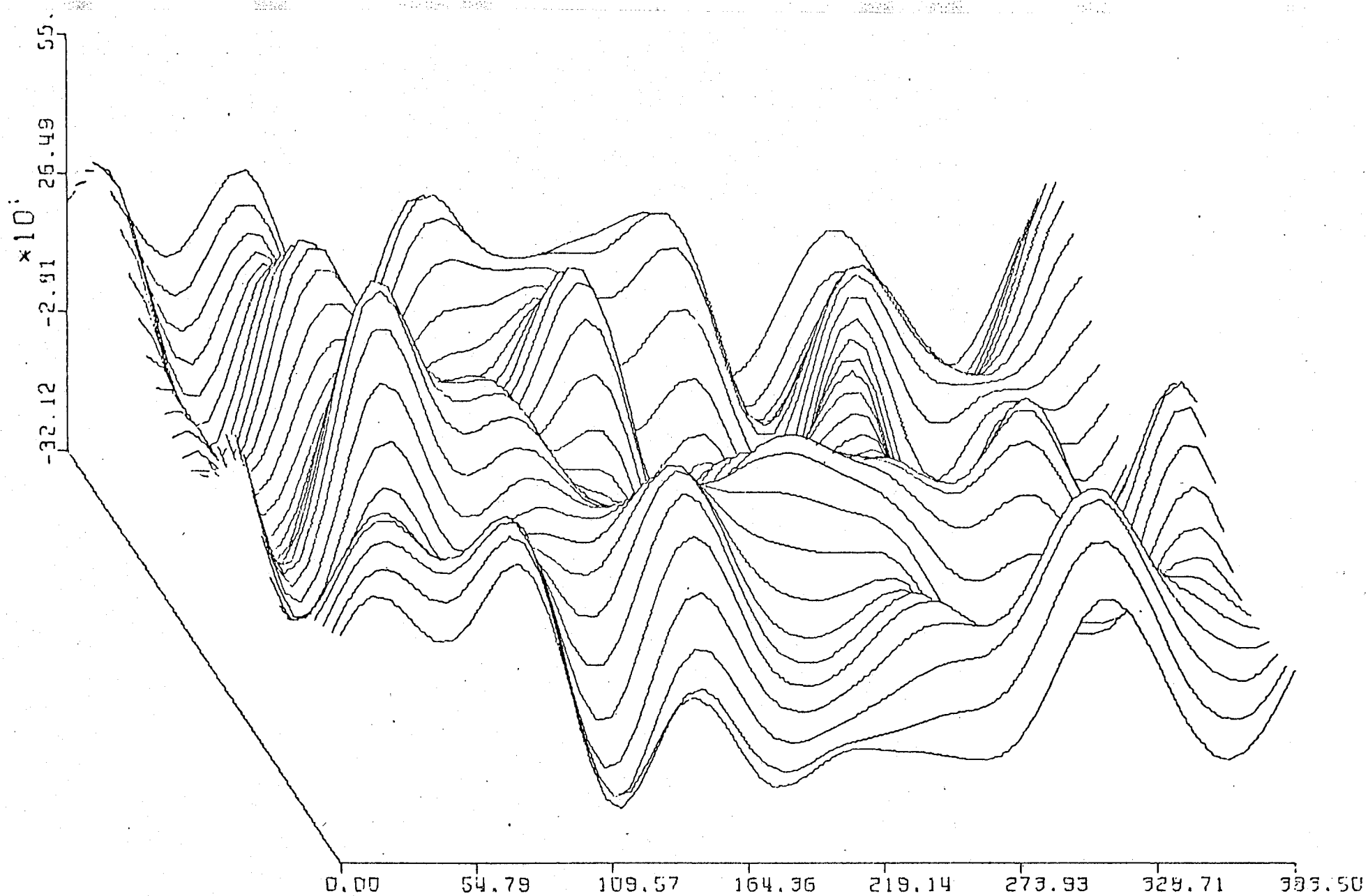
3-D REPRESENTATION OF THE GEOMAGNETIC FIELD: N.W.T. 60-61.75N 102-94.75W



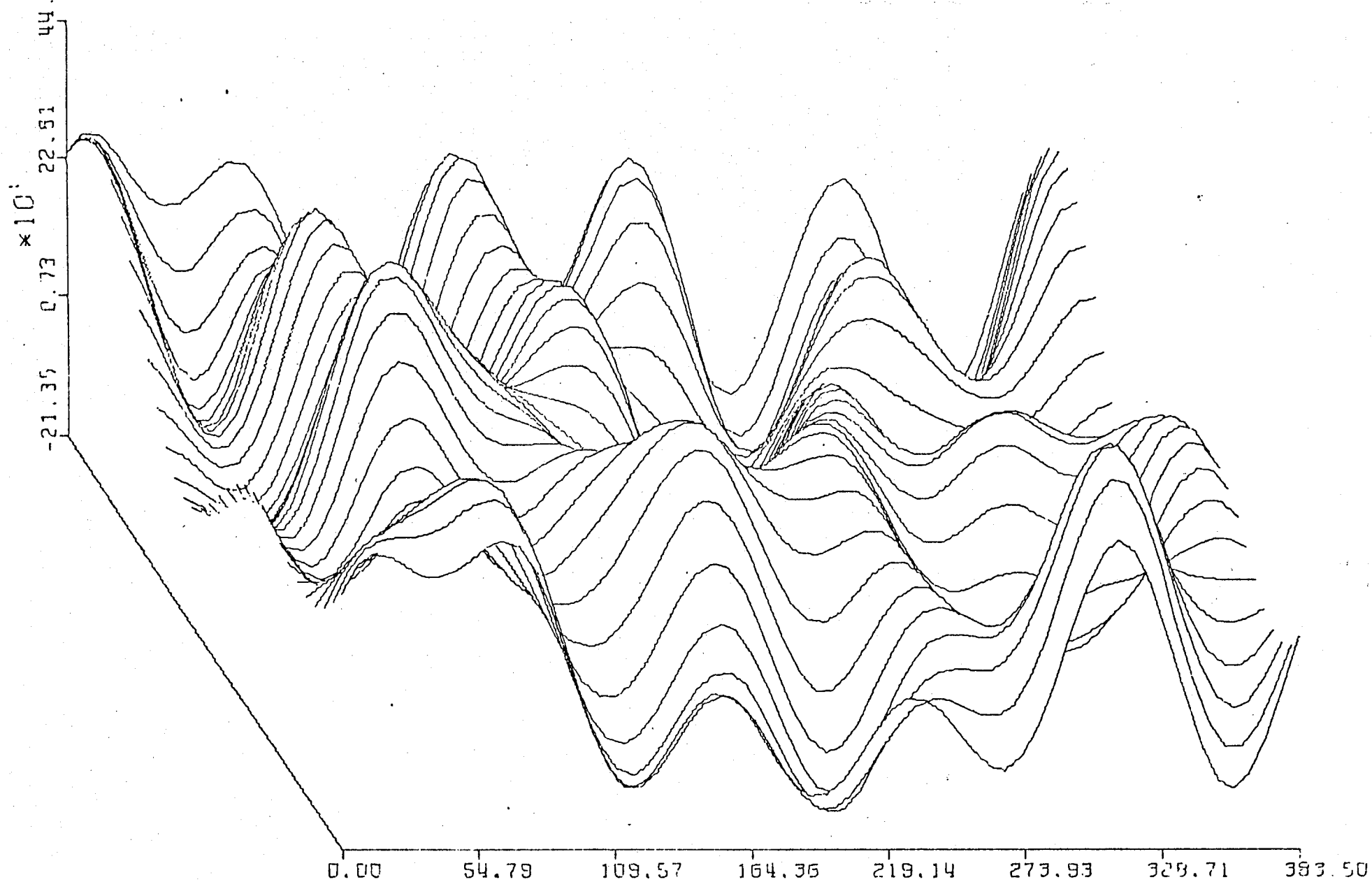
FILTERED DATA CUTOFF WAVELENGTH : 30KM. (E-W ; N-S)



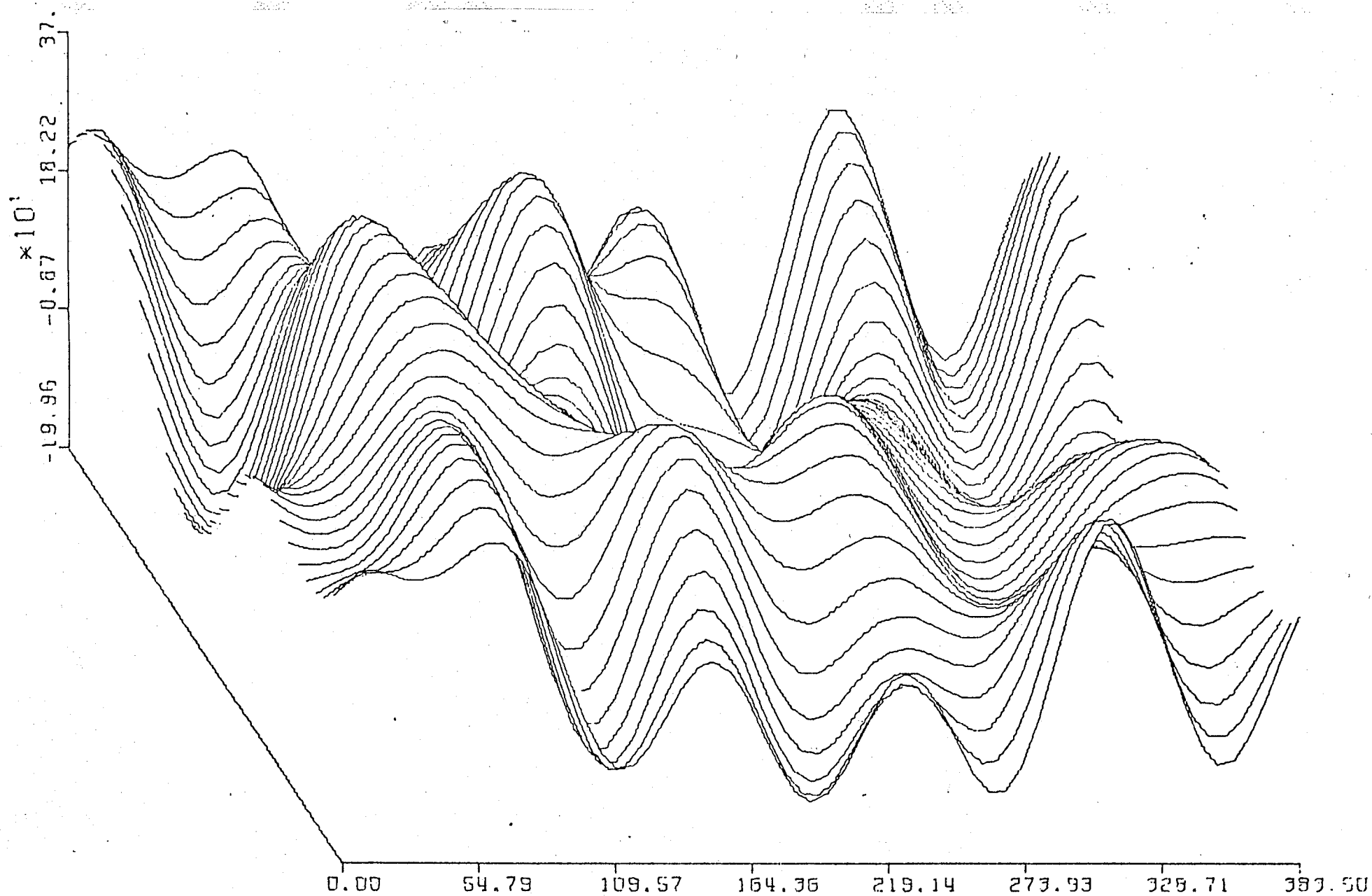
FILTERED DATA CUTOFF WAVELENGTH : 40KM. (E-W ; N-S)



FILTERED DATA CUTOFF WAVELENGTH : 50KM. (E-W ; N-S)



FILTERED DATA CUTOFF WAVELENGTH : 60KM. (E-W ; N-S)



FILTERED DATA CUTOFF WAVELENGTH : 70KM. (E-W ; N-S)

APPENDIX B

COMPUTER PROGRAMS

- FILTER 2D
- HIDE 3D
- RAID

HIDE 3D

```

C
C
C-----
C
C PROGRAM - HIDE3D
C UNIVERSITY OF MANITOBA
C DEPARTMENT OF EARTH SCIENCES ... A.TANG (NOV..1974)
C THIS PROGRAM USES THE BUILT IN SUBROUTINE HIDE TO PRODUCE
C A 3 DIMENSIONAL REPRESENTATION OF THE GEOMAGNETIC FIELD
C OF A DIGITIZED INPUT AREA .
C

```

```

C SHORT DESCRIPTION OF THE SUBROUTINE HIDE .....
C SUBROUTINE HIDE IS A FORTRAN SUBROUTINE WRITTEN BY HUGH WILLIAMSON
C OF THE TRACOR COMPUTING CORPORATION, AUSTIN, TEXAS, AND PUBLISHED
C IN THE COMMUNICATION OF THE A.C.M., FEB., 1972, VOL. 15, NUMBER 2.
C ALGORITHM 420
C HIDE PRODUCES A TWO-DIMENSIONAL REPRESENTATION OF A SURFACE
C OR FIGURE BY PLOTTING SEGMENTS OF A SUCCESSION CURVES ; EACH CURVE
C IS PLOTTED WHERE IT IS NOT HIDDEN BY ANY OF THE CURVES PREVIOUSLY
C PLOTTED (THAT IS, WHERE IT DOES NOT FALL BELOW ANY OF THEM AS THEY
C APPEAR IN THE TWO-DIMENSIONAL REPRESENTATION).
C
C-----
C
C
C

```

```

C
C INTEGER TITLE
C DIMENSION X(200),Z(200),XG(2600),G(2600),XH(2600),H(2600)
C DIMENSION XG1(2600),G1(2600),TITLE(20),IBUFF(6000)
C REAL MAP(200,100)
C EQUIVALENCE (XH(1),TITLE(1))
C CALL PLOTS (IBUFF,6000)
C CALL PLOT (0.0,-5.0,23)
C CALL PLOT (0.0,0.5,23)
C READ IN THE GRAPH TITLE
C
C READ (5,10) TITLE
10  FORMAT (20A4)
C NR   NUMBER OF ROWS OF THE INPUT MATRIX
C NC   NUMBER OF COLUMNS OF THE INPUT MATRIX
C DI   DIGITAL INTERVAL IN KM.
C READ (5,11) NR,NC,DI
11  FORMAT (2I4,F8.4)
C WRITE (6,129) NR,NC,DI
129  FORMAT ('0','NUMBER OF ROWS= ',I4,' NUMBER OF COLUMNS =',I4,
C 0' DIGITAL INTERVAL =',F8.4,' KM.')
```

```

C-----
C GENERATE THE X ARRAY
C-----
C

```

```

C DO 22 I=1,NC
C X(I)=(I-1)*DI
22  CONTINUE
C
C-----
C

```

```

C ARGUMENT LIST
C

```

```

C X IS THE ABSCISSA ARRAY-BEING GENERATED IN THE MAIN PROGRAM
C Y IS THE ORDINATE ARRAY-CONTAINING THE FIELD VALUES OF EACH ROW
C XG,G-CURRENT VISUAL MAXIMUM FUNCTION ON EACH RETURN FROM HIDE
C XH,H-WORKING ARRAYS,DIMENSIONED BY MAXDIM IN THE MAIN PROGRAM
C NG-THE # OF POINTS IN THE CURRENT MAXIMUM FUNCTION
C MAXDIM IS THE DIMENSION IN THE CALLING PROGRAM OF THE
```

```

C   APRAYS   XG,G,   XM,H
C   N1 IS THE # OF POINTS TO BE PLOTTED IN A GIVEN CALL TO HIDE 99
C   NFNS-IS THE TOTAL # OF CURVES TO BE PLOTTED FOR THE GRAPH
C   XLNTH-IS THE LENGTH IN INCHES OF THE HORIZONTAL AXIS
C   ZLNTH-IS THE LENGTH OF THE VERTICAL AXIS IN INCHES

```

```

C
C
C   XLNTH=7.
C   ZLNTH=3.00
C   N=NC
C   N1=-NC
C   NG=0
C   NG1=-3
C   NFNS=NR
C   MAXDIM=2600

```

```

C   READ IN THE MATRIX TO BE PLOTTED

```

```

C
C   DO 123 I=1,NR
C   READ (8,70) (MAP(J,I),J=1,NC)
C   WRITE (6,125) I
C   WRITE (6,124) (MAP(J,I),J=1,NC)
123  CONTINUE
70   FORMAT (5G13.6)
124  FORMAT ('0',5F16.7)
125  FORMAT ('0','ROW NUMBER =' ,14)

```

```

C   FIND THE MAXIMUM AND MINIMUM OF THE INPUT ARRAY
C   IN ORDER TO DETERMINE THE SCALE OF THE AMPLITUDE

```

```

C   ZMIN=MAP(1,1)
C   ZMAX=MAP(1,1)

```

```

C
C   DO 1234 I=1,NR
C   DO 1234 J=1,NC
C   IF (MAP(J,I).GT.ZMAX) ZMAX=MAP(J,I)
C   IF (MAP(J,I).LT.ZMIN) ZMIN=MAP(J,I)
1234 CONTINUE

```

```

C   XMIN IS A LOWER BOUND FOR X
C   DELTAX IS THE X DATA INCREMENT PER INCH FOR THE PLOT
C   XMIN AND DELTAX DETERMINE THE PLOTTING SCALE FOR X
C   ZMIN AND ZSCALE SIMILARLY,DETERMINE THE SCALE FOR Z
C   XMIN=0.
C   DELTAX=X(NC)/XLNTH
C   X(N+1)=XMIN
C   X(N+2)=DELTAX
C   ZSCALE=(ZMAX-ZMIN)/ZLNTH
C   Z(NC+1)=ZMIN
C   Z(NC+2)=ZSCALE

```

```

C
C   INVERT THE MATRIX PUTTING THE LAST ROW FIRST INTO THE Z ARRAY
C   DO 1000 I=1,NFNS
C   IX=NR+1-I
C   DO 1001 J=1,NC
C   Z(J)=MAP(J,IX)
1001 CONTINUE

```

PLOT THE PART OF THE CURVE THAT LIES ON THE UNHIDDEN PART OF THE TOP OF THE SURFACE 100

CALL HIDE (X,Z,XG,G,XH,H,NG,MAXDIM,N,NFNS,TITLE,-XLNTH,-ZLNTH)

```
WRITE (6,21) I
FORMAT ('0', 'ROW NUMBER =', I4, ' PLOTTED')
WRITE (6,27) MAXDIM
FORMAT ('0', 'MAXDIM =', I6)
```

PLOT THE PART OF THE CURVE THAT LIES ON THE UNHIDDEN PART OF
THE UNDERSIDE OF THE SURFACE

CALL HIDE (X,Z,XG1,G1,XH,H,NG1,MAXDIM,N1,O,'NONE',XLNTH,ZLNTH)

```

CONTINUE
CALL PLOT TO MOVE THE PEN OFF THE GRAPH TO THE RIGHT AND THE PLOT
CALL PLOT(16.,-2.,999)

```

STOP
END

FILTER 2D

C
C-----
C
C*****UNIVERSITY OF MANITOBA*****
C*****DEPARTMENT OF EARTH SCIENCES*****

C
C*** TWO DIMENSIONAL FREQUENCY DOMAIN FILTERING ROUTINE OF AEROMAGNETIC DATA
C*** (USING FAST FOURIER TRANSFORM SUBPROGRAMS BY R.G.AGARWAL 1968)
C PROGRAMMER : WRITTEN ORIGINALLY BY C.HASSELFIELD JULY, 1974
C
C MODIFIED BY A.TANG MAY, 1975
C-----

C DIMENSIONS OF MAP MUST HAVE HIGHEST FACTOR OF LESS THAN 19
C TITLE IDENTIFICATION TITLE
C X(I),Y(I),S(I) WORK ARRAYS FOR TRANSFORMS
C MAP(J,I) STORAGE ARRAY FOR INPUT MAP
C DELT DIGITAL INTERVAL IN KM.
C NR NUMBER OF ROWS
C NC NUMBER OF COLUMNS
C NPTS TOTAL NUMBER OF DATA POINTS
C WX CUTOFF WAVELENGTHS IN KM. (W-E)
C WY CUTOFF WAVELENGTHS IN KM. (N-S)
C
C

C
C REAL MAP(60,32),IMAG(60,32)
C REAL X(1920),Y(1920),S(1920)
C DIMENSION TITLE (80)
C EQUIVALENCE (MAP(1,1),X(1)),(IMAG(1,1),Y(1))
C INTEGER N(3)
C READ (5,99) (TITLE(I),I=1,80)
C WRITE (6,99) (TITLE(I),I=1,80)
99 FORMAT (80A1)

C
C N(1),N(2) CONTAIN THE DIMENSIONS OF THE MAP WITH ZERO IN THE THIRD
C POSITION (I.E. N(3)=0)
C N(2) IS ROWS N(1) IS COLUMNS
C READ (5,11) N(2),N(1),DELT
11 FORMAT (2I5,F10.2)
C N(3)=0
C NR=N(2)
C NC=N(1)
C PRINT 22,NR,NC,DELT
22 FORMAT(' MAP SIZE ',I5,' ROWS ',I5,' COLUMNS '
C /' DIGITIZING INTERVAL',F10.2,' KM.')

C
C INPUT THE CUTOFF WAVELENGTHS
C READ (5,44) WX,WY
44 FORMAT (2F8.2)
C NPTS=NR*NC
C WRITE (6,33) NPTS
33 FORMAT (' ', 'TOTAL NUMBER OF DATA POINTS =', I6)
C
C INITIALIZE THE VECTOR Y TO ZERO
C CALL ZERO(Y,NPTS)
C
C-----

C INPUT THE MAP TO BE PROCESSED
C
C CALL READIN(NR,NC,MAP)
C
C PRINT THE INPUT MAP
C
97 FORMAT (' ', 'ROW NO. =', I3)
C DO 200 IROW=1,NR

```

WRITE(6,97) IRCW
WRITE (6,40) (MAP (ICCL,IFOW),ICOL=1,NC)
200 CONTINUE
40  FORMAT (' ',10F10.2)
-----
C   SCREEN OFF IRRELEVANT SPIKES FROM MAP
CALL SCREEN(NR,NC,MAP)
C
C   REMOVE IGRF FROM MAP
CALL CORE(NR,NC,MAP)
C
WRITE (6,213)
WRITE (6,214)
214  FORMAT ('0', '      INPUT MAP AFTER REMOVING THE IGRF' )
WRITE (6,213)
213  FORMAT ('0', '-----')
C
C   FIND THE MEAN OF THE INPUT DATA
DC=0.0
DO 106 I=1,NR
DO 106 J=1,NC
DC=DC+MAP(J,I)
106  CONTINUE
DC=DC/FLOAT(NPTS)
WRITE (6,118) DC
118  FORMAT ('0', ' MEAN OF DATA =' ,G13.6)
DO 107 I=1,NR
WRITE (6,97) I
4    FORMAT (' ',10F10.2)
92   FORMAT (' ', 'RCW NO. =' ,I3)
107  WRITE (6,4) (MAP(J,I),J=1,NC)
C
C
C   PRINT INPUT VECTORS (X AND Y) FOR CHECKING BEFORE FORWARD TRANSFORM
WRITE (6,108)
108  FORMAT ('1', 'THE INPUT X AND Y VECTORS' )
WRITE (6,40) (X(KK),KK=1,300)
WRITE (6,40) (Y(KK),KK=1,300)
C
C   FORWARD FOURIER TRANSFORM
CALL ARMDFT(N,X,Y,S)
C
C   PRINT THE FOURIER TRANSFORM OF THE MAP
C   (THE VALUES ARE BEING NORMALIZED BY THE TOTAL NUMBER OF DATA POINTS)
CALL PRINT(NR,NC,MAP,IMAG)
C
C
-----
C
C   FREQUENCY DOMAIN FILTER
C   ZERO THE UNWANTED FREQUENCIES
C
C
PRINT 23,WX,WY
23  FORMAT(' CUTOFF WAVELENGTHS'/F10.2, ' (DISTANCE UNITS) W-E'
&  /F10.2, ' (DISTANCE UNITS) N-S')
K=0
C   FEW  CUTOFF FREQUENCY (EAST-WEST)
C   FNS  CUTOFF FREQUENCY (NORTH-SOUTH)
FEW=1.0/WX
FNS=1.0/WY
C   PEW  CUTOFF POINTS (EAST-WEST)
C   PNS  CUTOFF POINTS (NORTH-SOUTH)

```

```
C TAKING THE NYQUIST FREQUENCY =1./2*DIGITAL INTERVAL
C USING THE RATIOS CUTOFF FREQUENCY:NYQUIST FREQUENCY
C AND CUTOFF POINTS:TOTAL NUMBER OF POINTS * 0.5
PEW=FEW*DELT*FLOAT(NC)
PNS=FNS*DELT*FLCAT(NR)
PEW=PEW*PEW
PNS=PNS*PNS
C THE FOLLOWING SELECTION/REJECTION ROUTINE IS SET USING
C THE FORMULATION OF AN ELLIPSE :  $X^{**2}/A^{**2}+Y^{**2}/B^{**2} = 1$ 
DO 16 I=1,NR
IQ=I-1
SQI=FLOAT(IQ)*FLCAT(IQ)
IQ=NR-I+1
BCI=FLOAT(IQ)*FLOAT(IQ)
DO 16 J=1,NC
IQ=J-1
SQJ=FLOAT(IQ)*FLOAT(IQ)
IQ=NC-J+1
BCJ=FLOAT(IQ)*FLCAT(IQ)
K=K+1
IF((SQI/PNS+SQJ/PEW).LE.1.0) GO TO 16
IF((BCI/PNS+SQJ/PEW).LE.1.0) GO TO 16
IF((SQI/PNS+BCJ/PEW).LE.1.0) GO TO 16
IF((BCI/PNS+BCJ/PEW).LE.1.0) GO TO 16
Y(K)=0.0
X(K)=0.0
16 CONTINUE

C -----
C INVERSE FOURIER TRANSFORM
C CALL CONJU(Y,NPTS)
C CALL ARMDFT(N,X,Y,S)
C -----
C OUTPUT THE FILTERED MAP ON PRINTER AND ON DISK FOR CONTOURING
C CALL OUTPUT(NR,NC,MAP)
C
C STOP
C END
```

C
C
C

SUBROUTINE CORE(NR,NC,MAP)

C
C

C SUBPROGRAM TO REMOVE THE CORE GENERATED FIELD
 C THE INTERNATIONAL GEOMAGNETIC REFERENCE FIELD IS USED
 C CONST CONSTANT TO BE ADDED
 C DATE DATE OF SURVEY (YEAR)
 C ALT TOTAL ALTITUDE IN KM.
 C REAL MAP(NC,NR)
 C COMMON /YEAR/ DATE, ALT
 C INPUT LONG. & LAT. OF NW. CORNER & EXTENT OF MAP. ALL IN DEGRE
 C READ 1,XCO,YCO,XL,YL
 C 1 FORMAT(4F10.3)
 C PRINT 2,XCO,YCO,XL,YL
 C 2 FORMAT(' NW CORNER AND MAP SIZE',4F12.2)
 C 3 FORMAT(' CORE FIELD FOR NW CORNER IS ',G14.4)
 C
 C INPUT CONSTANT TO BE ADDED TO ALL FIELD VALUES , DATE OF SURV
 C AND ALTITUDE
 C READ 1, CONST, DATE, ALT
 C PRINT 5, CONST, DATE , ALT
 C 5 FORMAT(' CONSTANT ADDED TO INPUT ',F12.2,' YEAR',F12.2
 C 1 /' TOTAL ALTITUDE',F12.2)
 C
 C XS=F(XCO, YCO)
 C PRINT 3,XS
 C DO 4 J=1,NC
 C X=XCO-(J-1)*XL/(NC-1)
 C DO 4 I=1,NR
 C Y=YCO-(I-1)*YL/(NR-1)
 C XS=F(X,Y)
 C
 C ADD CONSTANT GAMMAS TO INPUT FIELD AND SUBTRACT CORE FIELD
 C
 C 4 MAP(J,I)=MAP(J,I) + CONST - XS
 C RETURN
 C END

C

C-----

C

001

FUNCTION F(XLGN, YLAT)

C

C-----

002

DATA ITYPE/1/

003

COMMON /YEAR/ DATE, ALT

004

COLAT=90. - YLAT

005

ELONG=360. - XLGN

C

006

CALL IGRF(DATE, ITYPE, ALT, COLAT, ELCNG, X,Y,Z, T)

007

F=T

008

RETURN

009

END

001

C

C-----

C

SUBROUTINE READIN(NR,NC,MAP)

C

C-----

C

C

C

C

C

SUBPROGRAM TO INPUT THE MAP TO BE PROCESSED
 STORAGE IN MAP MUST BE SAME AS INTERNAL STORAGE IE. FIRST
 & SUBSCRIPT VARIES FASTEST. NO SPACES ALLOWED BETWEEN COLUMNS

002

REAL MAP(NC,NR)

003

DO 1 I=1,NR

004

1 READ (8,2) (MAP(J,I),J=1,NC)

005

2 FORMAT (6G13.6)

006

RETURN

C

C

C

C-----

C

007

ENTRY OUTPUT(NR,NC,MAP)

C

C-----

C

C

008

PRINT THE FILTERED MAP

009

WRITE (6,88)

010

88 FORMAT ('0', ' FILTERED MAP')

011

WRITE (6,66)

012

66 FORMAT ('0', '-----')

013

DO 3 I=1,NR

014

WRITE(6,99) I

015

3 WRITE (6,78) (MAP(J,I),J=1,NC)

99 FORMAT (' ', 'ROW NO. =', I3)

C-----C

C

WRITE FILTERED MAP ON DISK

C

READY FOR CONTOURING (DATA FORMAT 6G13.6)

016

DO 77 I=1,NR

017

77 WRITE (9,78) (MAP(J,I),J=1,NC)

018

78 FORMAT (6G13.6)

C-----C

C

019

RETURN

020

END

```

C
C
C-----
001      SUBROUTINE PRINT (NR,NC,MAP,IMAG)
C
C-----
002      REAL MAP (NC,NR)
003      REAL IMAG(NC,NR),MAG(200)
004      NPTS=NR*NC
C      NORMALIZATION
005      DO 6 I=1,NR
006      DO 6 J=1,NC
007      MAP(J,I)=MAP(J,I)/NPTS
008      IMAG(J,I)=IMAG(J,I)/NPTS
009      PRINT 47
010      47  FORMAT ('1', '-----')
011      PRINT 10
012      10  FORMAT ('0', '      FOURIER TRANSFORM OF INPUT MAP')
013      PRINT 48
014      48  FORMAT ('0', '-----')
015      DO 7 I=1,NR
016      WRITE (6,11) I
017      DO 12 J=1,NC
018      12  MAG(J)=SQRT(MAP(J,I)**2 + IMAG(J,I)**2)
019      7   WRITE(6,8) (MAG(J),J=1,NC)
020      8   FORMAT (10F10.2)
021      11  FORMAT ('0', 'ROW NUMBER =', I3)
022      RETURN
023      END

```

SUBROUTINE SCREEN(NR,NC,MAP)

REAL MAP(NC,NR),MEAN,LOWER

NPTS=NR*NC

MEAN=0.

DO 1 I=1,NR

DO 1 J=1,NC

MEAN=MEAN + MAP(J,I)

MEAN=MEAN/NPTS

TO CALCULATE THE STANDARD DEVIATION OF DATA

SD=0.

DO 2 I=1,NR

DO 2 J=1,NC

SD=SD + (MAP(J,I) - MEAN)**2

SD=SQRT(SD/(NPTS-1))

UPPER=MEAN + 3.*SD

LOWER=MEAN - 3.*SD

PRINT 3,MEAN,SD,LOWER,UPPER

FORMAT('0MEAN=',F10.1,' SD=',F10.1,' LOWER LIMIT=',F10.3,' U
&LIMIT=',F10.3)

APPLY CHAUVENET CRITERIA

CLIP VALUES MORE THAN 3 SD FROM MEAN

DO 4 I=1,NR

DO 4 J=1,NC

IF(MAP(J,I) .LT. LOWER) MAP(J,I)=LOWER

IF(MAP(J,I) .GT. UPPER) MAP(J,I)=UPPER

CONTINUE

RETURN

END

C

110

C

C

001

SUBROUTINE ZERO(X, NOPTS)

C

C

C

002

SETS THE VECTOR TO ZERO

003

REAL X(1920)

004

DO 1 J=1,NOPTS

005

X(J)=0.0

006

1

CONTINUE

007

RETURN

END

C

C-----

C

SUBROUTINE CONJU(Y,NOPTS)

C

C-----

C

TAKES THE COMPLEX CONJUGATE OF THE IMAGINARY PART

REAL Y(1920)

DO 1 J1=1,NOPTS

Y(J1)=-Y(J1)

1

CONTINUE

RETURN

END

SUBROUTINE IGRF (DATE, ITYPE, ALT, COLAT, ELONG, X, Y, Z, T)

0000014

THIS SUBROUTINE COMPUTES VALUES OF X, Y, Z AND T FOR A GIVEN EPOCH
AND POSITION FROM THE SPHERICAL HARMONIC COEFFICIENTS OF THE
INTERNATIONAL GEOMAGNETIC REFERENCE FIELD

DATA DATE = REQUIRED EPOCH IN YEARS AND DECIMALS OF A YEAR, A.D.
ITYPE = 1 IF GEODETIC COORDINATES ARE BEING USED
ITYPE = 2 IF GEOCENTRIC COORDINATES ARE BEING USED
ALT = HEIGHT ABOVE MEAN SEA LEVEL IN KM (ITYPE=1)
ALT = RADIAL DISTANCE FROM CENTRE OF EARTH IN KM (ITYPE=2)
COLAT = COLATITUDE IN DEGREES (0.0 TO 180.0)
ELONG = EAST LONGITUDE IN DEGREES (0.0 TO 360.0)

OUTPUT X = NORTH COMPONENT OF MAGNETIC FORCE IN GAMMA
Y = EAST COMPONENT OF MAGNETIC FORCE IN GAMMA
Z = VERTICAL COMPONENT OF MAGNETIC FORCE IN GAMMA

T = TOTAL MAGNETIC FORCE IN GAMMA

N.B. THE COORDINATE SYSTEM FOR X, Y AND Z IS THE SAME AS THAT
SPECIFIED BY ITYPE

DIMENSION AGH(80), DGH(80), P(44), Q(44), CL(8), SL(8)

THE S.H. COEFFICIENTS ARE SUPPLIED BY THE FOLLOWING DATA STATEMENT

ALTERNATIVELY, THE COEFFICIENTS MAY BE SUPPLIED BY STATEMENTS OF
THE FORM: AGH(1) = -30339. AGH(2) = -2123. ETC.
OR THE COEFFICIENTS MAY BE READ IN THE MASTER PROGRAM AND MADE
AVAILABLE TO THE SUBROUTINE BY A COMMON STATEMENT

DATA AGH/	-30339.,	-2123.,	5758.,	-1654.,	2994.,	-2006.,	1567.,	130.,	0000046
1	1297.,	-2036.,	-403.,	1289.,	242.,	843.,	-176.,	958.,	0000047
2	805.,	149.,	492.,	-280.,	-392.,	8.,	256.,	-265.,	0000048
3	-223.,	357.,	16.,	246.,	125.,	-26.,	-123.,	-161.,	0000049
4	-107.,	-51.,	77.,	47.,	60.,	-14.,	4.,	106.,	0000050
5	-229.,	68.,	3.,	-32.,	-4.,	-10.,	-112.,	-13.,	0000051
6	71.,	-54.,	-57.,	0.,	-27.,	12.,	-8.,	-25.,	0000052
7	9.,	-9.,	23.,	13.,	-19.,	-2.,	-17.,	10.,	0000053
8	9.,	3.,	-3.,	-13.,	-12.,	5.,	-4.,	-17.,	0000054
9	7.,	4.,	-5.,	22.,	12.,	-3.,	6.,	-16.,	0000055
DATA DGH/	15.3,	8.7,	-2.3,	-24.4,	0.3,	-11.8,	-1.6,	-16.7,	0000056
1	0.2,	-10.8,	4.2,	0.7,	0.7,	-3.8,	-7.7,	-0.7,	0000057
2	0.2,	-0.1,	-3.0,	1.6,	-0.1,	2.9,	-2.1,	-4.2,	0000058
3	1.9,	1.1,	2.3,	2.9,	1.7,	0.6,	-2.4,	0.0,	0000059
4	0.8,	1.3,	-0.3,	-0.1,	-0.3,	-0.9,	1.1,	-0.4,	0000060
5	1.9,	2.0,	-0.4,	-1.1,	-0.4,	0.1,	-0.2,	0.9,	0000061
6	-0.5,	-0.3,	-1.1,	-0.7,	0.3,	-0.5,	0.4,	0.3,	0000062
7	0.2,	0.0,	0.4,	-0.2,	0.2,	-0.6,	0.3,	0.1,	0000063
8	0.4,	0.1,	0.6,	-0.2,	0.0,	-0.3,	0.0,	-0.2,	0000064
9	-0.1,	-0.3,	0.3,	-0.4,	-0.3,	-0.3,	-0.5,	-0.3,	0000065

SET INITIAL VALUES

T = DATE - 1965.0
R = ALT
ONE = COLAT * .0174533

```

SLAT=COS(ONE)
CLAT = SIN(ONE)
ONE = ELONG*0.0174533
CL(1)= COS(ONE)
SL(1)= SIN(ONE)
X = 0.0
Y= 0.0
Z = 0.0
CD = 1.0
SD = 0.0
L = 1
M = 1
N = 0
GO TO (1,2), ITYPE

```

CONVERSION FROM GEODETIC TO GEOCENTRIC COORDINATES

```

1 A2= 40680925.
  B2= 40408585.
  ONE = A2*CLAT*CLAT
  TWO = B2*SLAT*SLAT
  THREE = ONE + TWO
  FOUR = SQRT(THREE)
  R = SQRT(ALT*(ALT + 2.0*FCUR) + (A2*ONE + B2*TWO)/THREE)
  CD = (ALT + FOUR)/R
  SD = (A2 - B2)/FCUR*SLAT*CLAT/R
  ONE = SLAT
  SLAT= SLAT*CD - CLAT*SD
  CLAT = CLAT*CD + ONE*SD
2 RATIO = 6371.2/R

```

COMPUTATION OF SCHMIDT QUASI-NORMAL COEFFICIENTS P AND X(=Q)

```

P(1) = 2.0*SLAT
P(2) = 2.0*CLAT
P(3) = 4.5*SLAT*SLAT - 1.5
P(4) = 5.1961524*CLAT*SLAT
Q(1) = -CLAT
Q(2) = SLAT
Q(3) = -3.0*CLAT*SLAT
Q(4) = 1.7320508*(SLAT*SLAT - CLAT*CLAT)
DO 15 K=1,44
  IF (N-M) 3,4,4
3 M = 0
  N = N+1
  RR = RATIO**((N+2))
  FN = N
4 FM = M
  IF (K-5) 8,5,5
5 IF (M-N) 7,6,7
6 ONE = SQRT(1.0 - 0.5/FM)
  J = K-N-1
  P(K) = (1.0+1.0/FM)*ONE*CLAT*P(J)
  Q(K) = ONE*(CLAT*Q(J) + SLAT/FM*P(J))
  SL(M) = SL(M-1)*CL(1) + CL(M-1)*SL(1)
  CL(M) = CL(M-1)*CL(1) - SL(M-1)*SL(1)
  GO TO 8
7 ONE = SQRT(FN*FN-FM*FM)
  TWO = SQRT((FN-1.0)**2-FM*FM)/ONE
  THREE = (2.0*FN-1.0)/ONE
  I=K-N
  J=K-2*N+1
  P(K) = (FN+1.0)*((THREE*SLAT/FN*P(I) - TWO/(FN-1.0)*P(J))

```

Q(K) = THREE*(SLAT*Q(I) - CLAT/FN*P(I)) - TWO*Q(J)

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SYNTHESIS OF X, Y AND Z IN GECCENTRIC COORDINATES

```
8 ONE = (AGH(L) + DGH(L)*T)*RR
  IF (M) 10,9,10
9 X = X + ONE*Q(K)
  Z = Z - ONE*P(K)
  L=L+1
  GO TO 14
10 TWO = (AGH(L+1) + DGH(L+1)*T)*RR
  THREE = ONE*CL(M) + TWO*SL(M)
  X = X+THREE*Q(K)
  Z = Z - THREE*P(K)
  IF (CLAT) 12,12,11
11 Y = Y+(ONE*SL(M) - TWO*CL(M))*FM*P(K)/((FN + 1.0)*CLAT)
  GO TO 13
12 Y = Y+(ONE*SL(M) - TWO*CL(M))*Q(K)*SLAT
13 L = L+2
14 M = M+1
15 CONTINUE
```

CONVERSION TO COORDINATE SYSTEM SPECIFIED BY ITYPE

```
ONE = X
X = X*CD + Z*SD
Z = Z*CD - ONE*SD
T = SQRT(X*X + Y*Y + Z*Z)
```

RETURN

LIBRARY FUNCTIONS USED BY THIS SUBROUTINE ARE SIN,COS,SQRT

SAMPLE RESULTS:

```
IGRF(1965.0,1, 0.0, 0.0, 30.0, 2541., -240., 56349.,56407.)
IGRF(1965.0,1, 0.0,180.0,295.0, 12923., 9661.,-56900.,59144.)
IGRF(1969.3,1, 0.0, 57.0,195.0, 25092., 5729., 30145.,39638.)
IGRF(1965.0,2,6371.2, 45.0, 10.0, 21848.,-1052., 40417.,45956.)
IGRF(1967.5,2,9000.0, 31.0,359.0, 5750.,-1226., 17853.,18796.)
END
```

RAID

```
SUBROUTINE RAID (NN,NNN,DELT)
  DIMENSION SUM(200),FREQ(200),PROD(200),ISUM(200)
  COMMON/STORE/XP(84,84)
  JINT=2*NN
  DO 10 J=1,JINT
    SUM(J)=0.0000
    PROD(J)=0.0000
    ISUM(J)=0
10  CONTINUE
  C=FLOAT(NN)
  D=FLOAT(NNN)
  DO 100 I=1,NN
    DO 100 J=1,NNN
      A=FLOAT(I)
      B=FLOAT(J)
      RAD=(A-(C+1.0)/2.0)**2+(B-(D+1.0)/2.0)**2
      RAD=SQRT(RAD)
      DO 50 K=1,JINT
        P=FLOAT(K)
        FREQU=P/2.0
        IF (RAD.LE.FREQU) GO TO 60
50  CONTINUE
60  ISUM(K)=ISUM(K)+1
      SUM(K)=SUM(K)+XP(I,J)
      P=XP(I,J)
      P=ALOG(P)
      PROD(K)=PROD(K)+P
100  CONTINUE
    DO 110 K=1,JINT
      P=FLOAT(K)
110  FREQ(K)=(P/2.0-0.25)/(DELT*(NN-1))
      K=0
      DO 120 I=1,JINT
        IF (ISUM(I).EQ.0) GO TO 120
        K=K+1
        PROD(K)=PROD(I)/ISUM(I)
        S=SUM(I)/ISUM(I)
        SUM(K)=ALOG(S)
        FREQ(K)=FREQ(I)
        ISUM(K)=ISUM(I)
120  CONTINUE
      FREQ(1)=0.0000
      FREQ(2)=1.0/(DELT*(NN-1))
      FREQ(3)=1.414*FREQ(2)
      FREQ(4)=2.0*FREQ(2)
      FREQ(5)=2.239*FREQ(2)

  C
  WRITE (6,1)
1  FORMAT ('1',' ')
  WRITE (6,2)
2  FORMAT ('0',5X,' RADIAL SPECTRUM')
  WRITE (6,3)
3  FORMAT ('0','-----')
  WRITE (6,4)
4  FORMAT ('0',5X,'NUMBER',10X,'LOG AVERAGE POWER',10X,'FREQUENCY',
  5X,'SAMPLE SIZE')
  WRITE (6,3)
  C
  DO 150 J=1,K
    WRITE (6,5) J,SUM(J),FREQ(J),ISUM(J)
```

150 CONTINUE

C

117

WRITE (7,222) (SUM(J),J=1,K)

WRITE (7,223) (FREQ(J),J=1,K)

222 FORMAT (5F14.4)

C

C

5 FORMAT (' ',5X,I3,14X,F10.4,14X,F10.4,13X,I3)

RETURN

END