THE UNIVERSITY OF MANITOBA

OPTIMATIZATION OF TRANSIENT RESPONSE
OF PULSE NETWORKS

BY

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ABSTRACT

This thesis is concerned with the transient response characteristic of low pass filters designed to be fast with little or no overshoot.

Methods are proposed to find the pole configuration for an all-pole transfer function which yields a desirable transient response. The first method is the pole-shifting method. The radius vectors of the poles are allowed to move on a parabolic contour by varying the angles between the radius vectors and the negative real axis. Freqency and time responses are investigated to locate several optimal positions corresponding to some figures of merit.

Another approach to the problem is by the extremum method. Some error criteria of the actual and ideal responses are defined for the time and the frequency domains. A performance index which is a weighted sum of the integrals of these errors is then minimized by a direct search program. Remarkable results are obtained for the third and fourth order cases.

The transient responses of the proposed filters are found to be superior to all other filters appeared in literature. In the case of the third order case, for example, a rise time of 1.761 seconds is obtained compared to 2.291 seconds for a Butterworth filter. To make fair comparisons, the dc gain as well as the 3-db bandwidth are normalized to unity. The results obtained are very useful in the design of filters for pulse application.

AKNOWLEDGEMENTS

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CHAPTER I

INTRODUCTION

I.1 General introduction

For a low-pass filter designed for pulse applications, it is required that the rise time, delay time and overshoot be as small as possible.

The problem of designing a network is really that of finding a good approximation H(s) using a realizable rational function of s. Since Butterworth [1] proposed the maximally flat magnitude charecteristic, numerous methods have been suggested. However, the problem of obtaining the least rise time for a given overshoot or least overshoot for a given rise time has not been solved. This thesis is an attempt to improve the transient response and to show such results.

I.2 Transient Response

It has been a common practice to compare the various low pass filters by comparing their responses to a unit-step excitation. The quantities associated with a step response are rise time(t_r), delay time(t_d) and overshoot(γ). By far the most commonly used criterion is the 50% delay time and 10-90% rise time as depicted in Figure I.1.

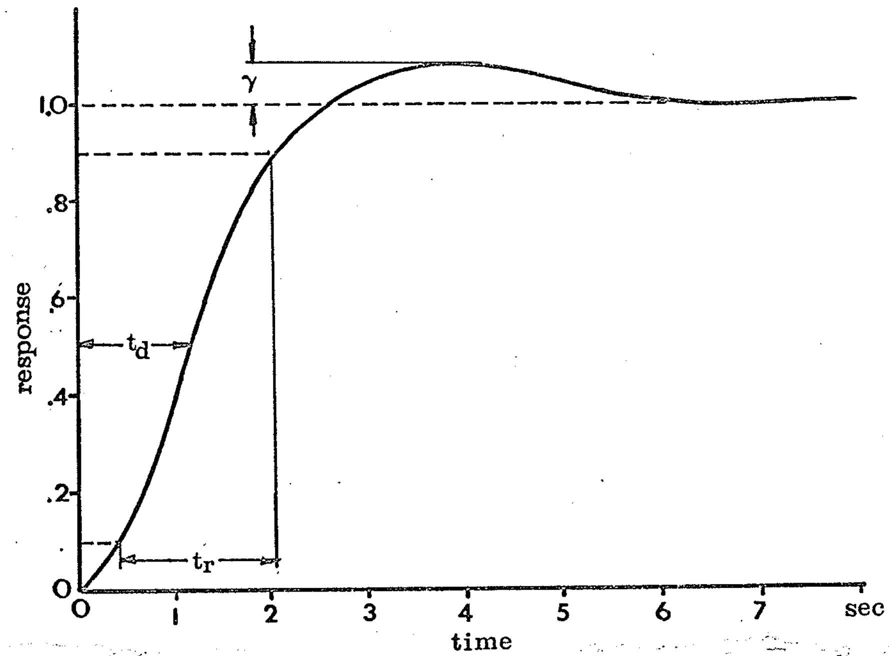


Fig. I.1 Unit step response.

Elmore [20] defined rise time and delay time as follows.

$$t_d = \int_0^\infty h(t)\, t \cdot dt \qquad \text{I.1}$$

$$t_r = \left[2\pi \int_0^\infty (t - t_d)^2\, h(t)\, dt\right]^{\frac{1}{2}} \qquad \text{I.2}$$

where h(t) is the impulse response of the network.

Elmore's definitions are analytically convenient for networks with no overshoot or very small overshoot. Several other definitions have also been proposed and utilized. However, in the following discussion the conventional definition is used.

I.3 Formulation of the problem

The problem is to find the location of poles for a all-pole transfer function H(s) which yields a desirable transient response. If the dc response is normalized to unity, then the filter function is

$$H(s) = \prod_{i=1}^{n} \frac{|p_i|}{(s - p_i)} \qquad \text{I.3}$$

where $p_i = -\sigma_i \pm j\omega_i$ is the ith pole in the left half of the s-plane.

The unit step response of this filter is given by

$$f(t) = \mathcal{L}^{-1} \left[\frac{1}{s} H(s)\right] \qquad \text{I.4}$$

In order to evaluate and to compare the performance of the filters, it is necessary to normalize the bandwidth to 1 rad/ sec [8]. The requirement imposes the following constraint,

$$\prod_{i=1}^{n} \left(1 + \frac{2\omega_i + 1}{\sigma_i^2 + \omega_i^2}\right) = 2 \qquad \text{I.5}$$

For the first order filter, in order to satisfy all the requirements, the pole must be at s=-1. The step response is monotonic with t_d =0.693 sec. and t_r =2.197 seconds.

For the second order filter, the poles are $p_{1,2} = -\sigma_1 \pm j\omega_1$ where σ_1 and ω_1 are real and postive.

$$H(s) = \frac{\sigma_1^2 + \omega_1^2}{s^2 + 2\sigma_1 s + \sigma_1^2 + \omega_1^2} \qquad \text{I.6}$$

Let $\omega_o = (\sigma_1^2 + \omega_1^2)^{\frac{1}{2}}$; $\xi = \sigma_1/\omega_o$

Then $f(t) = 1 - \dfrac{e^{-\omega_o \xi t}}{(1 - \xi^2)^{\frac{1}{2}}} \sin\left(\omega_o (1-\xi^2)^{\frac{1}{2}} + \cos^{-1} \xi\right)$

And $\gamma = e^{-\pi\xi(1-\xi^2)^{-\frac{1}{2}}} \times 100\%$ I.7

$$t_r = (t_{90\%} - t_{10\%}) \text{ sec} \qquad \text{I.8}$$

$$t_d = t_{50\%} \text{ sec} \qquad \text{I.9}$$

For calculating delay and rise time it is necessary to solve three transcendental equations. Thus it is obvious that a direct method of minimizing t_r and t_d is not feasible.

In chapter II a comparison is made of the various known systems, which leads to the development of the present work. In chapter III the suggested method and results are discussed. In chapter IV another method of optimizing the transient response is briefly described, followed by the discussion of the two methods presented.

CHAPTER II

PREVIOUS CONTRIBUTIONS

II.1 Introduction

The problem of determining the suitable transfer functions of lowpass filters has attracted much attention in recent years. Much of the work was involved in finding a locus for the poles on the left half of the s-plane. The poles were located at the intersection of the curve with radial straight lines from the origin with fixed Butterworth angles; or they were located on the curve with equally spaced imaginary parts [7]-[12]. The following is a comparison of various known systems, which leads to the work of this thesis.

In comparing various systems, the following aspects are studied:

(1) pole locations,

(2) rise time and delay time,

(3) overshoot.

In order that these comparisons to be valid, normalization is carried out according to the description in chapter I.

II.2 Various systems

(1) Butterworth Configuration [1]

The poles lie symmetrically on the left half of a unit circle in the complex plane. It has the advantage of fast transient responses. However the large overshoot is very undesirable for pulse networks. Furthermore the overshoot increases with the increase in the order of the filter.

(2) Thomson configuration [2]

This class of filters has negligible overshoot for low-order and no overshoot at all for high-order filters. Quite contrary to the anticipation, the rise time and delay time are much smaller than Butterworth response. For example the third order filter has a rise time and delay time of 2.181 and 1.681 sec. compared to Butterworth's 2.291 and 2.135 seconds. This is true only after bandwidth normalization.

(3) Transitional Butterworth-Thomson Configuration [3]

This class of filter can be made to attain response characterstics between those of Butterworth and Thomson by choosing poles which lie on paths passing through the Butterworth pole $p_B(r_B, \theta_B)$ and Thomson pole $p_T(r_T, \theta_T)$. The path is defined by

$$p = re^{j(\pi - \theta)} = -re^{-j\theta} \qquad \text{II.1}$$

Where $r = r_T^{\,m}$

$\theta = \theta_B - m(\theta_B - \theta_T)$

m = variable parameter.

This entails a trade off between Butterworth and Thomson characteristics rather than providing the advantages of both simultaneously.

(4) Elliptic Configuration [9]

This class of filters was proposed by J. O. Scanlan. Poles are located with equal spacing on the imaginary axis and are generated by varying the eccentricity of the ellipse.

The equation of the ellipse is given by

$$\sigma_i^2 = a^2 (1 - \omega_i^2) \qquad \text{II.2}$$

(5) Catenary Configuation [10]

This class of filters was proposed by M.S. Ghausi and

M. Adamowicz. Poles lie on a catenary curve given by

$$\sigma_i = a_c(\cosh \omega_i - \lambda) \qquad \text{II.3}$$

where a_c, λ are real positive constants such that the curve passes through the point $(-1, j0)$ on the s-plane. Thus a_c and λ are related by

$$a_c = \frac{1}{\lambda - 1}$$

and pole locations are determined by the intersection of the curve with radial straight lines from the origin given as

$$\omega_i = \sigma_i \tan \theta_i \quad . \qquad \text{II.4}$$

(6) Parabolic Configuration [7]

This configuration was suggested by Mullick. The poles lie on a parabolic arc defined by

$$4(a_p + b)\ (\sigma_i + a_p) = \omega_1^2 \qquad \text{II.5}$$

where a_p is the distance of the focus from the vertex; b is the shift of the focus to the right from the origin.

The poles are located at the intersection of radial lines of Butterworth angles and the parabolic contour.

Figure II.1 shows some of the loci and pole locations for third order filters. Table II.1, II.2 and II.3 compare the rise time, delay time and overshoot of the various systems described.

II.3 Observation

The following observations are made. For Butterworth and Thomson filters, the pole locations are fixed, whereas for parabolic, elliptic and other recent filters, a curve is defined in the left half plane. By varying the parameter associated with the curve, the shape of the curve can be changed. With a carefully chosen value for the parameter certain improvements are made. An example is given for the fourth order filter:

	t_r(sec)	t_d(sec)	γ(%)
Butterworth	2.432	2.820	10.830
Parabolic b=0	2.374	2.557	2.560
Parabolic b=3	2.058	1.858	0.218

Both the rise time and delay time for b=3 are much smaller than the corresponding values for b=0. However, this is not the 'best' filter, further optimizations are to be made.

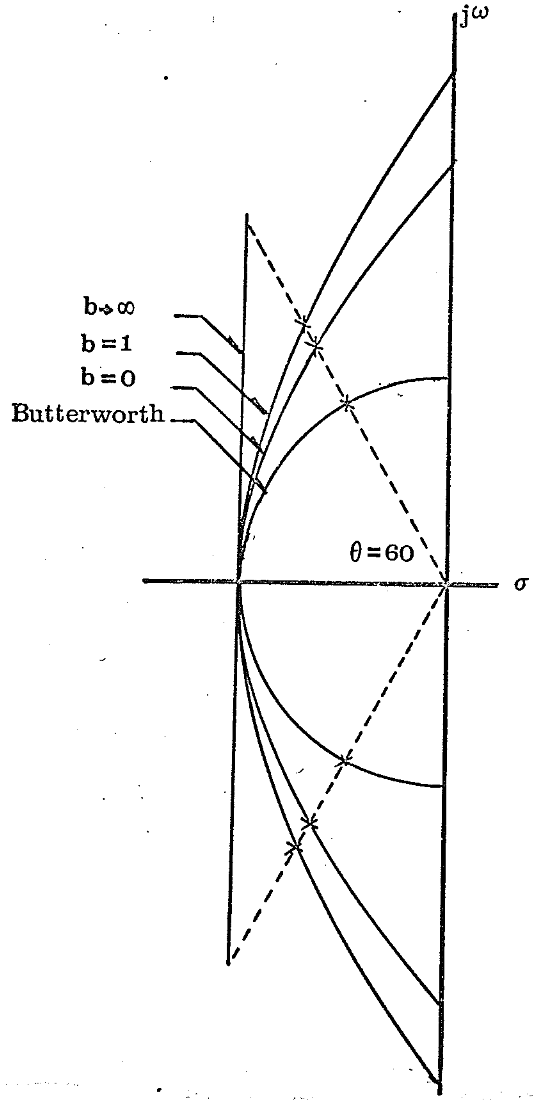


Fig. II.1 Parabolic distribution of poles;
n=3.

n=3

		t_r (sec)	t_d (sec)	γ (%)
Butterworth		2.2907	2.1352	8.1462
Thomson		2.1808	1.6808	0.7541
Transitional-	m=.2	2.2472	1.9766	3.8731
Butterworth-	m=.4	2.2472	1.9766	3.8731
Thomson	m=.6	2.2217	1.8420	2.4684
	m=.8	2.1946	1.7552	1.4411
Elliptic	a=.4	2.4461	2.2070	1.0922
	a=.6	2.2697	1.9434	2.2161
	a=.8	2.2047	1.7756	1.5007
Catenary	λ=2.	2.3000	2.1311	7.2401
	λ=2.5	2.3148	2.1148	5.6061
	λ=5.0	2.3186	2.0318	2.0458
Parabolic	b=0.0	2.3641	2.2804	8.4485
	b=1.25	2.3647	2.1288	2.8141
	b=3.0	2.2883	1.9733	0.0000
	b=8.0	2.2528	1.8498	0.0000

Table II.1 Comparison of rise time, delay time and overshoot for 3rd order filters.

n=4

		t_r (sec)	t_d (sec)	γ (%)
Butterworth		2.4324	2.8203	10.8297
Thomson		2.2039	2.0762	0.8810
Transitional-	m=0.20	2.3996	2.6672	7.5790
Butterworth-	m=0.40	2.3437	2.4961	5.0323
Thomson	m=0.60	2.2847	2.3320	3.1210
	m=0.80	2.2355	2.1883	1.7485
Elliptic	a=0.40	2.3730	2.6816	4.1321
	a=0.60	2.2683	2.3308	1.8400
	a=0.80	2.1547	1.8000	0.0731
Caternary	λ=2.00	2.4543	2.8008	8.3523
	λ=2.50	2.4496	2.7308	5.5799
	λ=5.00	2.2031	2.2539	0.1263
Parabolic	b=0.00	2.3742	2.5574	2.5598
	b=1.25	2.1078	2.0640	0.3370
	b=3.00	2.0578	1.8578	0.2182
	b=8.00	2.1070	1.6957	0.0534

Table II.2 Comparison of rise time, delay time and overshoot for 4th order filters.

n=5

		t_r (sec)	t_d (sec)	γ (%)
Butterworth		2.5624	3.4955	12.7093
Thomson		2.2015	2.3996	0.7725
Transitional-	m=0.20	2.5139	3.2807	8.8386
Butterworth-	m=0.40	2.4200	3.0180	5.7581
Thomson	m=0.60	2.3243	2.7680	1.8340
	m=0.80	2.2496	2.5609	1.8340
Elliptic	a=0.40	2.2006	2.7132	0.9299
	a=0.60	2.2345	2.6262	1.3618
	a=0.80	2.4401	3.4180	16.9381
Caternary	λ=2.00	2.2483	1.9346	1.0726
	λ=2.50	2.4985	3.2186	5.1115
	λ=5.00	2.1230	2.4587	0.4193
Parabolic	b=0.00	2.2313	2.7262	1.0785
	b=1.25	2.0745	2.1762	0.3423
	b=3.00	2.1322	2.0381	0.1325
	b=8.00	2.1728	1.8651	0.0136

Table II.3 Comparison of rise time, delay time and overshoot for 5th order filters.

CHAPTER III

OPTIMIZATION BY POLE SHIFTING [19]

III.1 Introduction

From the observation in chapter II, it is realized that the transient response characteristics can be further optimized. A method of accomplishing this, based on the shifting of poles along the parabolic contour to locate the optimal transient response, is presented. The technique, although discussed here with reference to parabolic filters, is equally applicable to other types of pole distributions.

III.2 The Method

The radius vectors of the poles are allowed to move on the parabolic contour by varying the angles between the radius vectors and the negetive real axis . Both the frequency and time responses are investigated for each combination of angles. With the help of a digital computer, this method offers a systematic search strategy, and is practical up to fifth order filters.

III.3 Computer Subroutines

Several computer subroutines are written to facilitate the investigations. These subroutines are

ROOT..to locate the poles and associated angles.

NORM..to find the unnormalized bandwidth and the normalized new pole positions.

PAFA..to calculate the residues of the poles for the inverse Laplace Transform.

TRTD..an interpolation for t_r and t_d.

OSHT..to find % of overshoot.

COEF..to obtain coefficients (a_i) for the characteristic polynomial.

The flow charts for the subroutines are shown in Appendix A. The main program calls these subprograms for each combination of angles and calculates the responses.

The selected responses are plotted by a Calcomp Plotter which is an off-line plotter in the University of Manitoba Computer Centre.

III.4 Second Order

Before normalization, poles with larger value of b will be further away from the $j\omega$ axis; however when the bandwidth is normalized to unity, the pole locations are found to be independent of b. The only workable parameter is the angle θ.

Rise time remains constant for small angle θ at values of approximately 2.16 seconds. It decreases steadily to 1.60 seconds as θ approaches 90 degrees. Delay time, on the other hand, increases from 1.08 seconds to maximum 1.62 seconds which corresponds to angle θ of 75 degrees. Overshoot increases rapidly as angle increases. The new bandwidth, rise time, delay time, overshoot, pole locations ($\alpha \pm j\beta$) and the coefficients of the characteristic polynomial are summarized in Table III.1 and Figures III.1 and III.2.

Angle	0°	20°	40°	60°	80°
Bandwidth*	0.644	0.725	1.039	1.696	2.591
t_r (sec)	2.162	2.155	2.150	2.084	1.788
t_d (sec)	1.081	1.140	1.355	1.646	1.705
γ (%)	0.000	0.000	2.365	16.302	57.467
α	-1.552	-1.337	-0.835	-0.393	-0.114
β	0.000	0.487	0.701	0.681	0.648
a_1	3.103	2.675	1.670	0.786	0.228
a_0	2.410	2.026	1.189	0.618	0.433

Table III.1 Summary of 2nd order filters with increasing angles.

(α, β are the real and imaginary parts of the conjugate poles)

(a_1, a_0 are the coefficients of the characteristic polynomials).

*Unless otherwise stated, the bandwidth values given are referred to those before normalization. The value of t_r and t_d are values obtained after normalization.

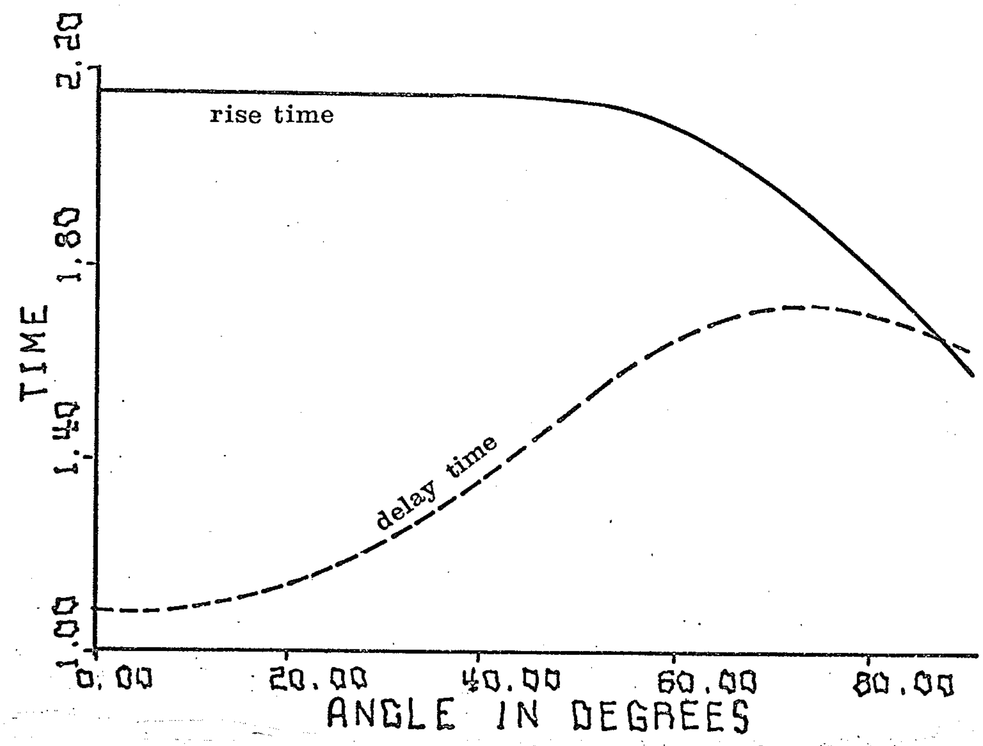


Fig. III.1 Rise time and delay time vs. angle (n=2).

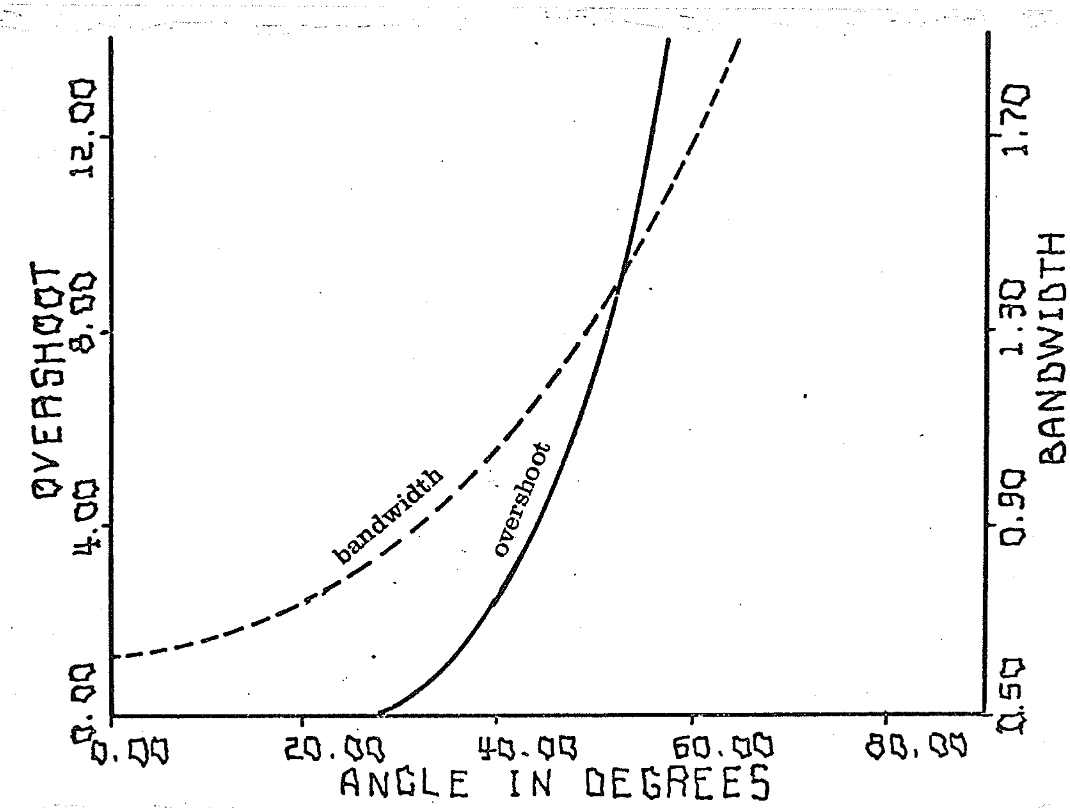


Fig III.2 Overshoot and bandwidth vs. angle (n=2).

III.5 Third Order

In the case of the third order filters, one pole is on the negative real axis and the other two poles form a conjugate pair. The complex pole makes an angle θ with the negative real axis. As θ increases, the bandwidth of the filter increases and the effect of b becomes more apparent.

The bandwidth, overshoot, rise time and delay time versus angle are plotted in Figures III.3, III.4, III.5 and III.6. The rise time is almost constant but the delay time gradually increases as the pole angle increases from zero to forty degrees; within this interval the overshoot is negligible. The rise time, delay time and overshoot become larger as the angle continues to increase except in the region close to the imaginary axis where the trend is reversed. In this region, the rise time drops sharply but points of inflexion are observed in the transient response which results in the distortion of the output pulse shape. In general, the poles with angles smaller than Butterworth angles and with large focus distance correspond to the favorable response. For example, one such angle would be 22 degrees, with rise time and delay time of 2.151 and 1.428 seconds respectively. The frequency response is reasonably good.

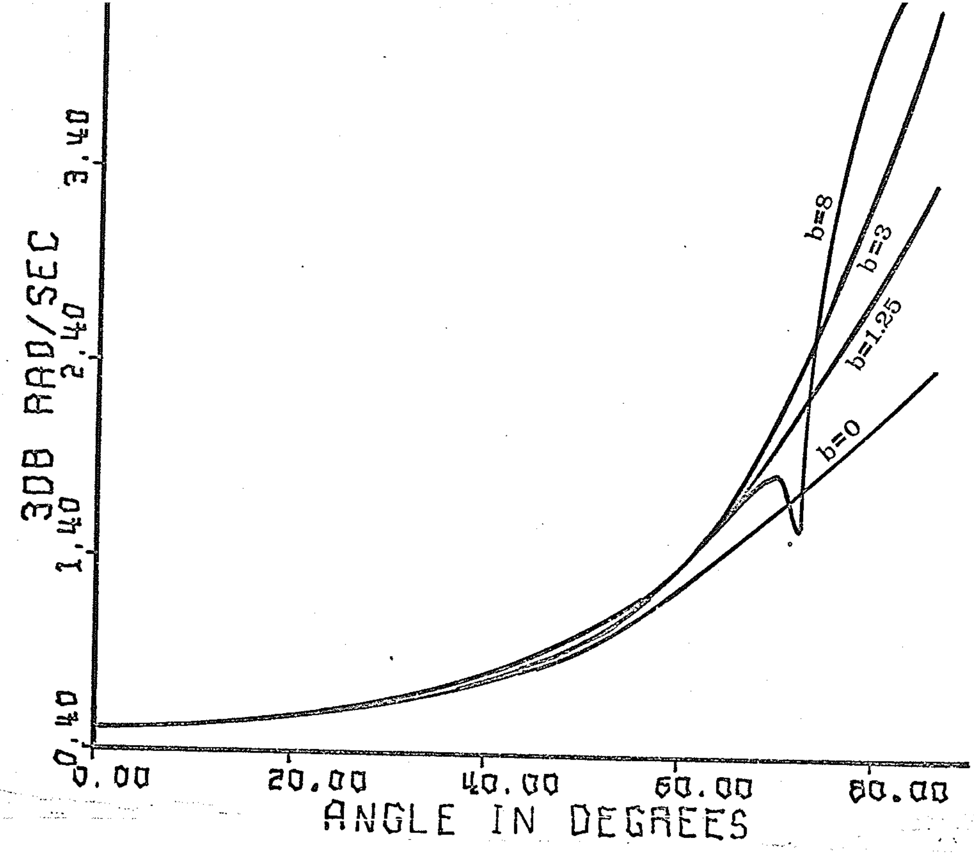


Fig. III.3 Bandwidth vs. angle (n=3).

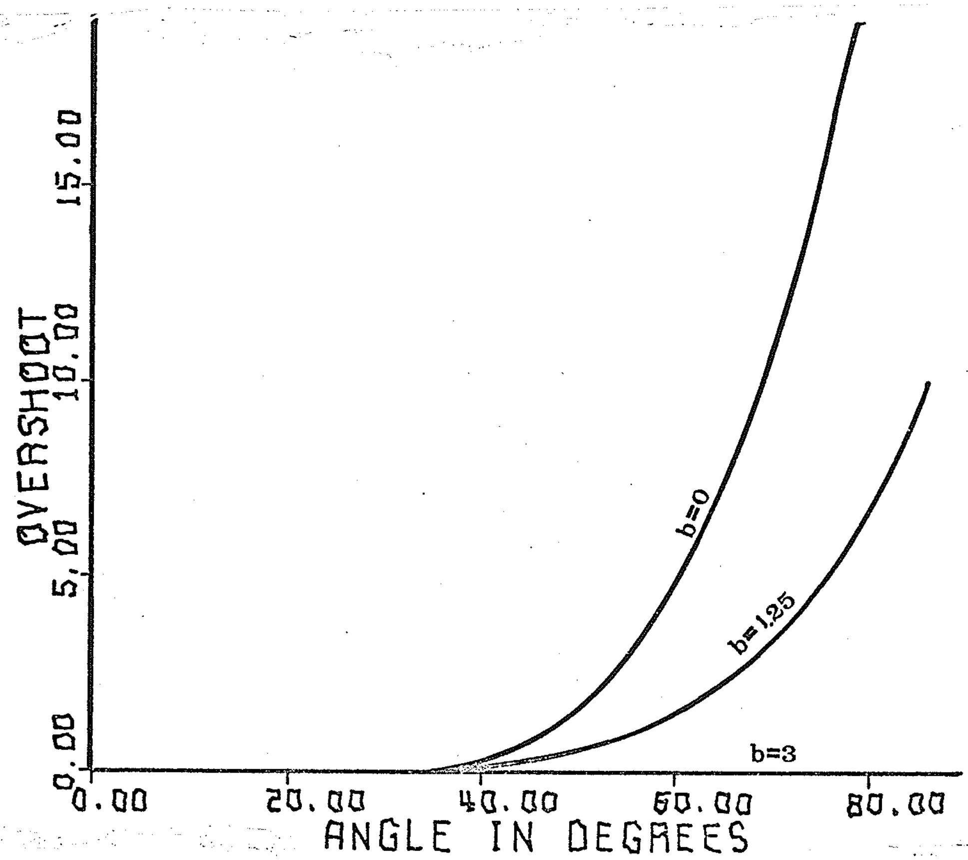


Fig. III.4 Overshoot vs. angle (n=3).

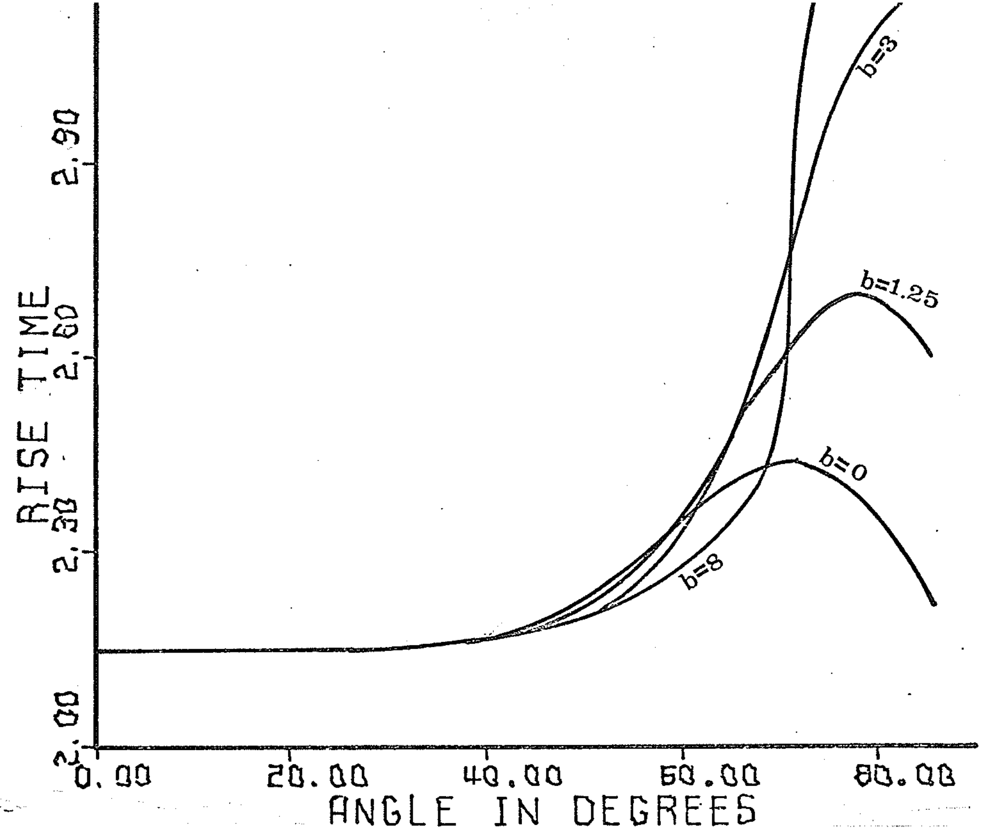


Fig. III.5 Rise time vs. angle (n=3).

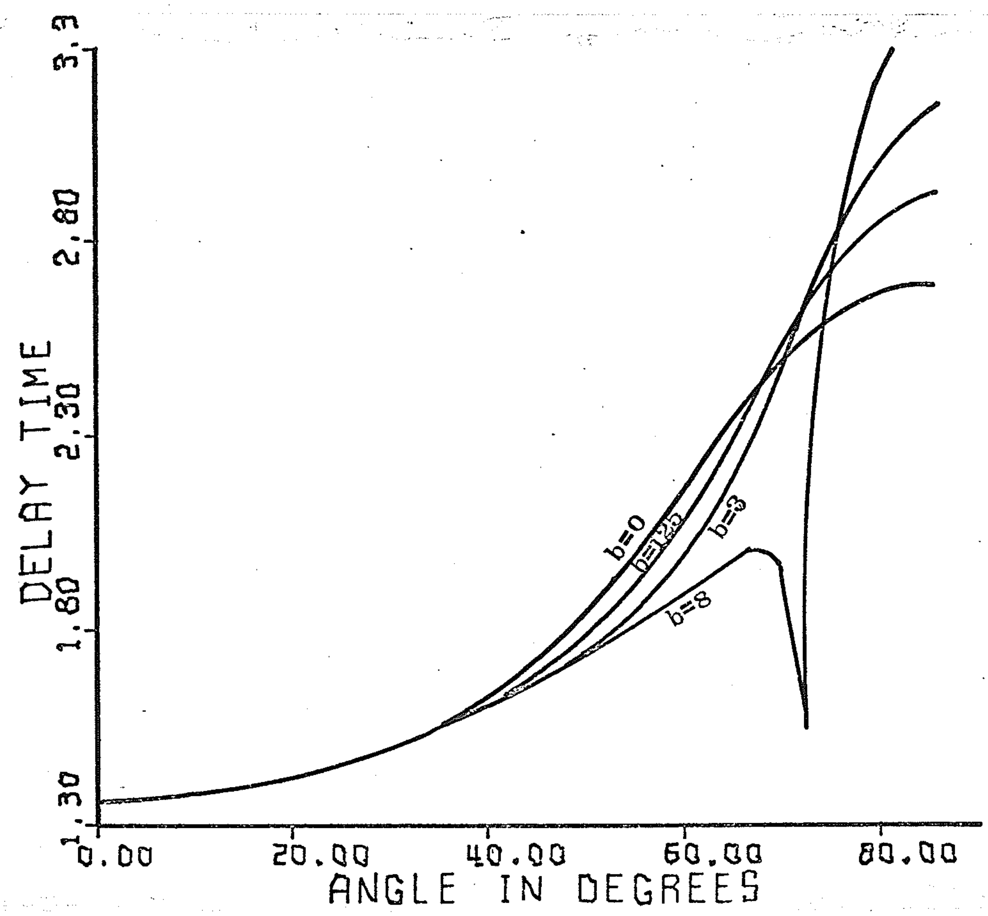


Fig. III.6 Delay time vs. angle (n=3)

III.6 Fourth and higher orders

The analysis of higher order cases is similar to the third order case. The 4th and 5th order results are plotted in Figures III.7 to III.14.

As the order n increases, the rise time and delay time increase but the bandwidth and overshoot decrease. On the other hand, the effect of the variation of the angles and the parameter b on the rise time and delay time becomes less as the order increases.

The curves of rise time versus angle show a noticeable decrease in rise time when angle θ_2 is between 60 and 70 degrees. Following a local minimum, c.f. Figure III.9 and III.13, the rise time increases considerably. The local minimum is smaller for greater angle θ_1. However, increasing both angle θ_1 and angle θ_2 will give rise to undesirable distortion of the output pulse shape. For general application, therefore, the angles should not be greater than 70 degrees.

The following are some results when a pattern search program [17] is employed to find the local minimum.

n	b	θ_1	θ_2	t_r(sec)	t_d(sec)	γ(%)
4	0.9	10.7	70.0	2.014	1.814	0.003
4	2.0	10.0	10.0	2.144	1.621	0.000
5	2.8	5.0	40.0	2.147	1.920	0.000
5	0.3	27.6	70.0	2.065	2.185	0.218

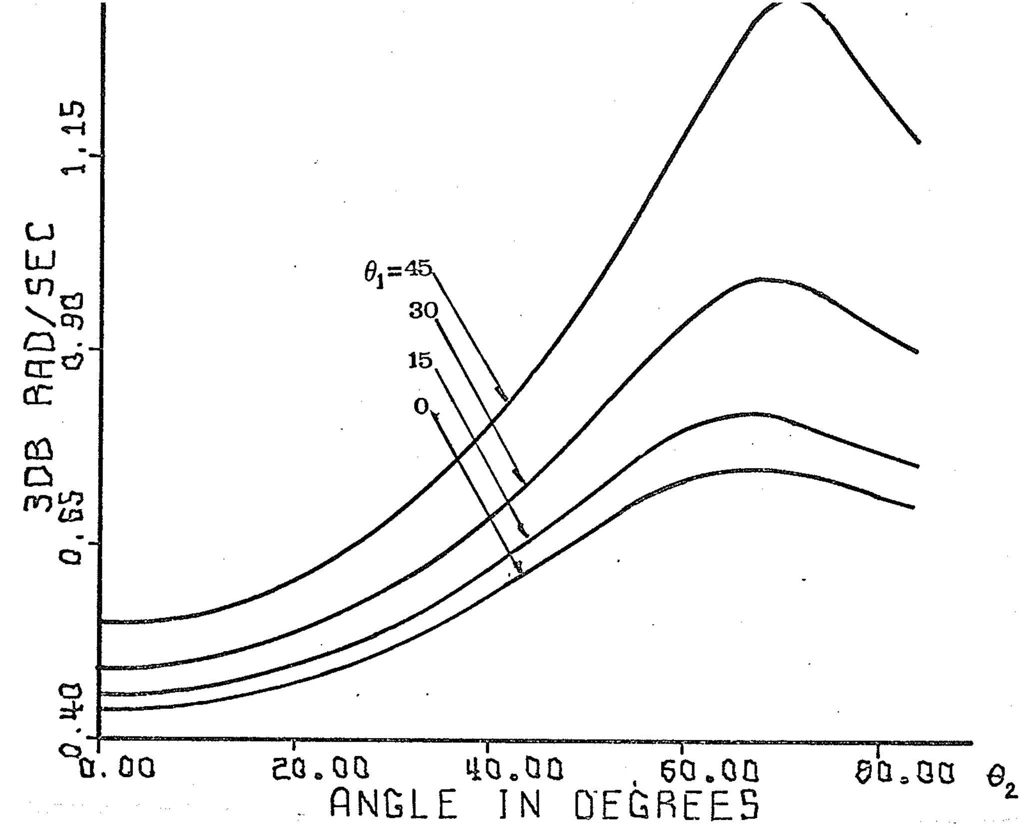


Fig. III.7 Bandwith vs. angle θ_2(n=4, b=0).

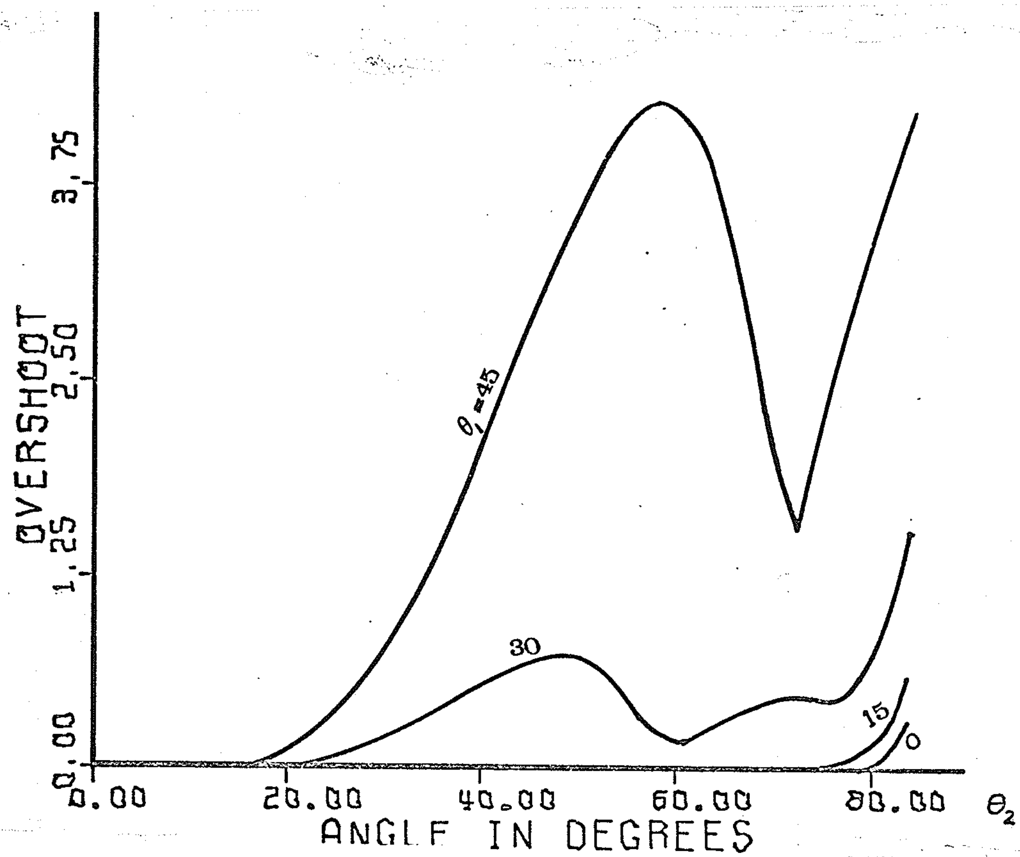


Fig. III.8 Overshoot vs. angle θ_2 (n=4, b=0).

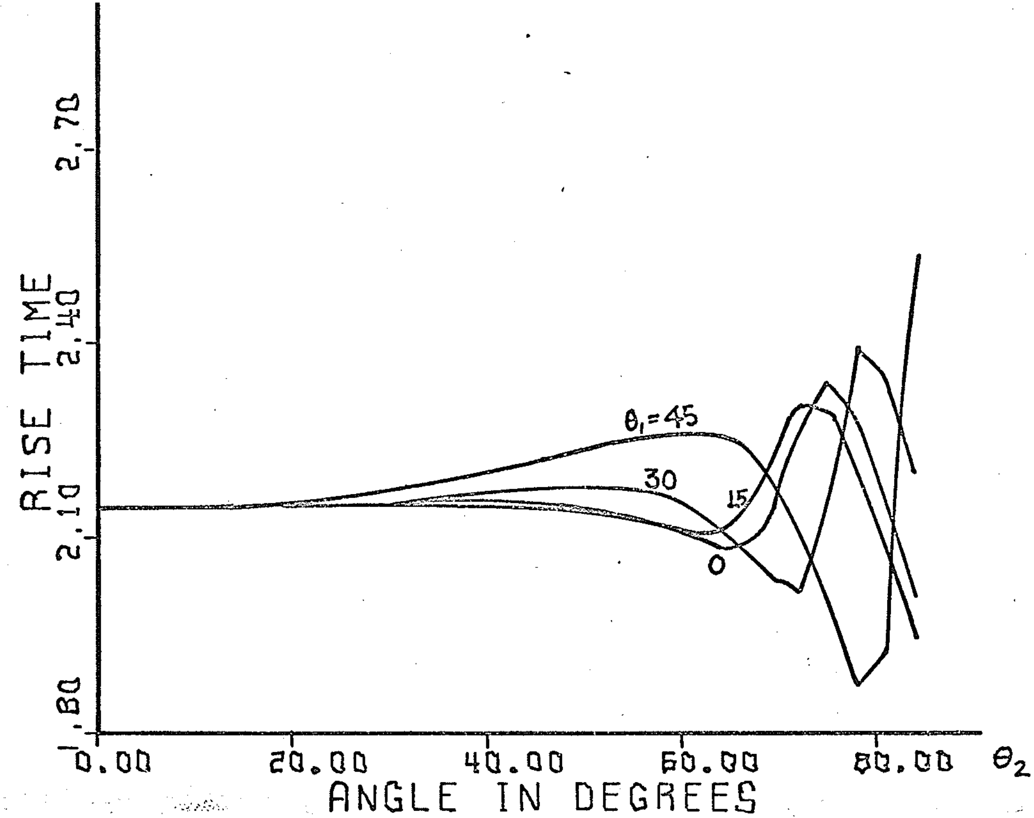


Fig. III.9 Rise time vs. angle θ_2 (n=4,b=0).

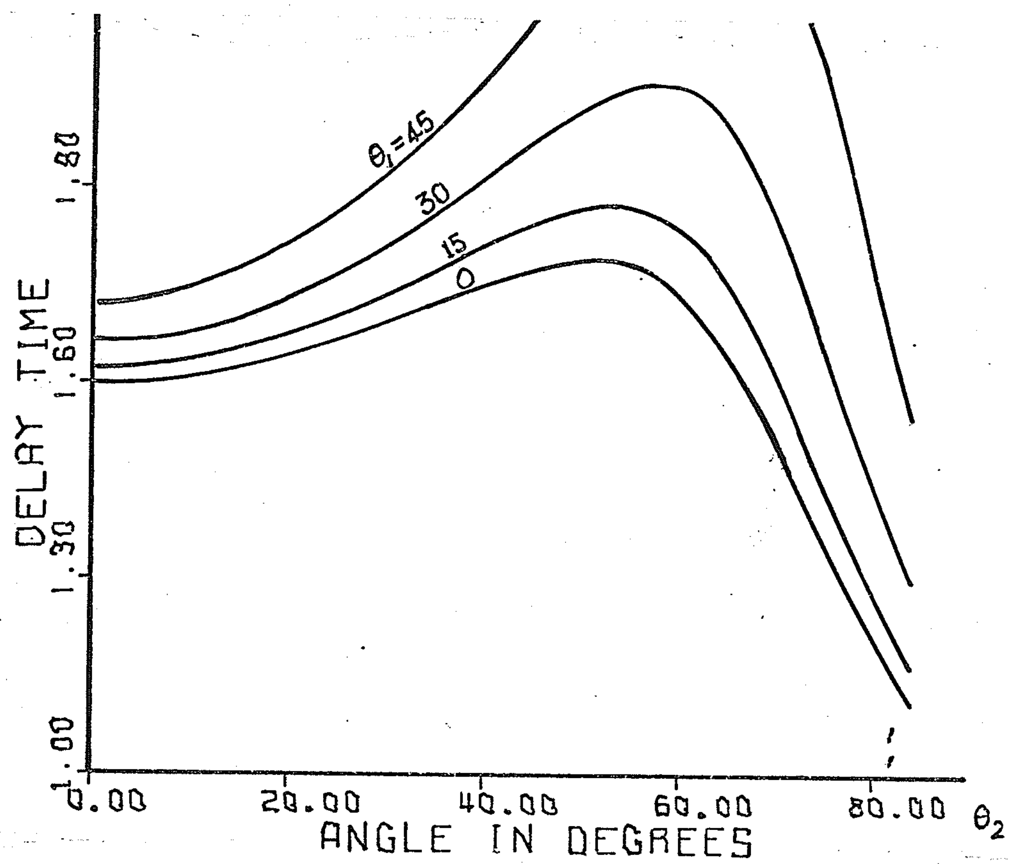


Fig. III.10 Delay time vs. angle θ_2 (n=4, b=0).

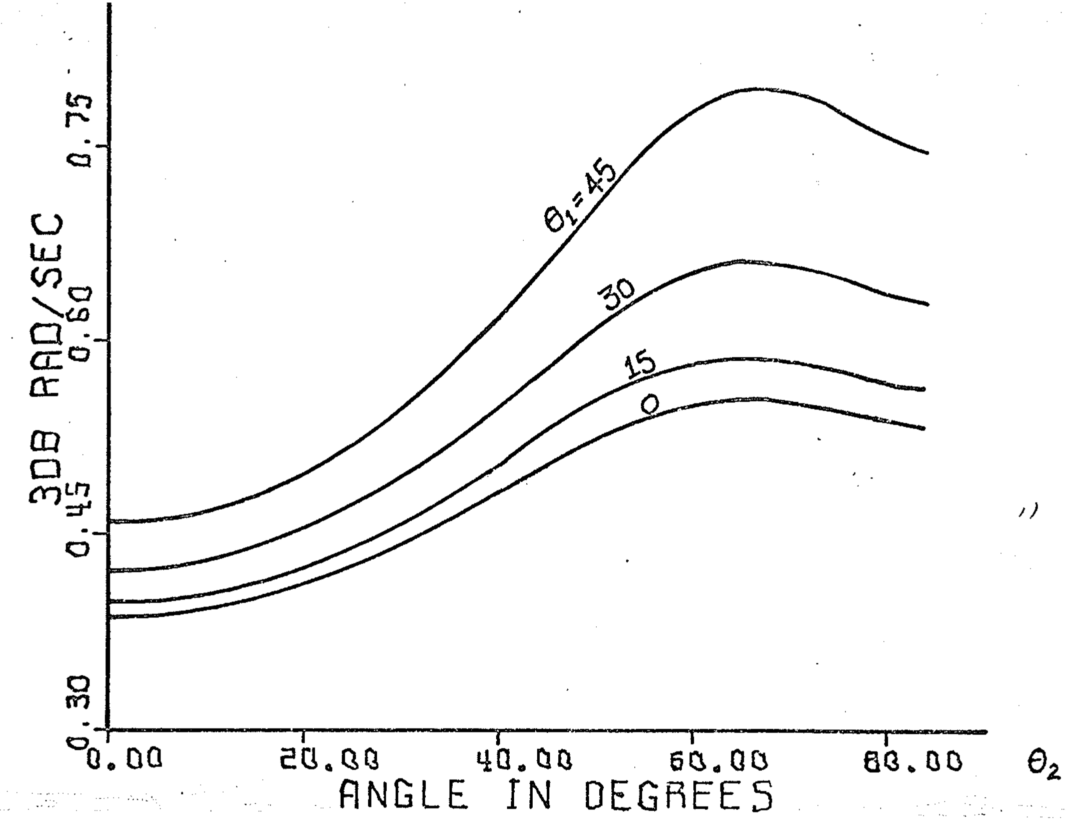


Fig. III.11 Bandwith vs. angle θ_2(n=5, b=0).

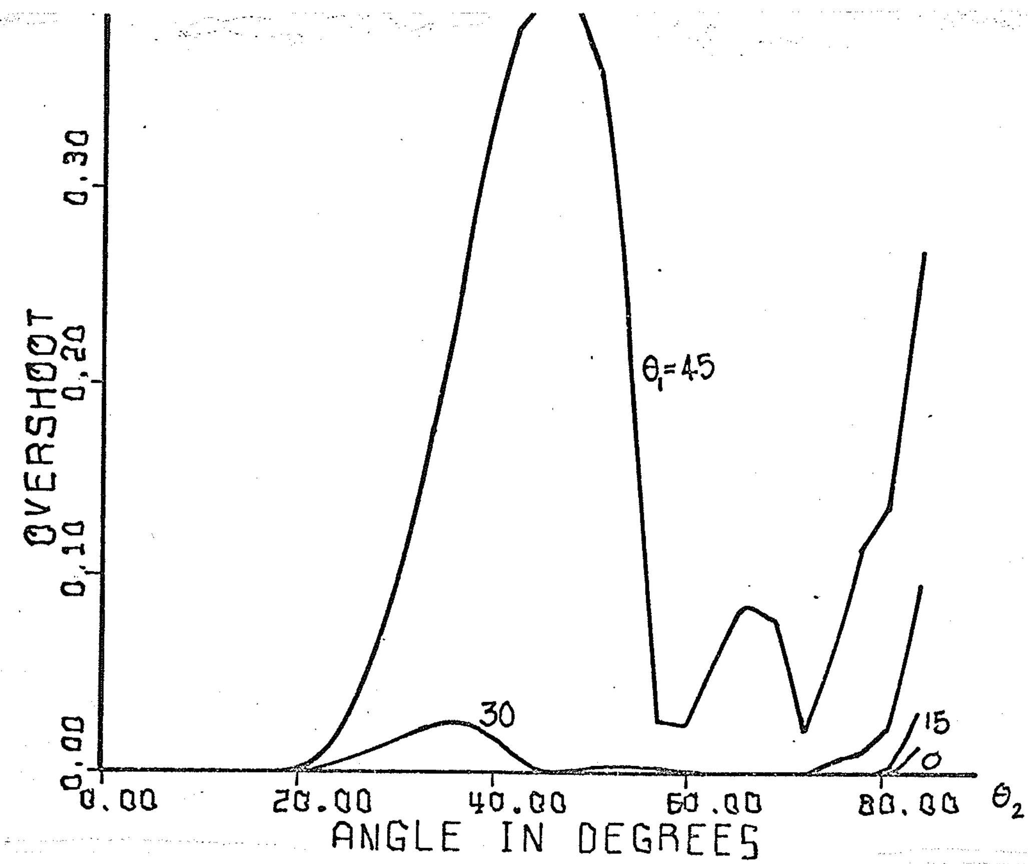


Fig. III.12 Overshoot vs. angle θ_2 (n=4, b=0).

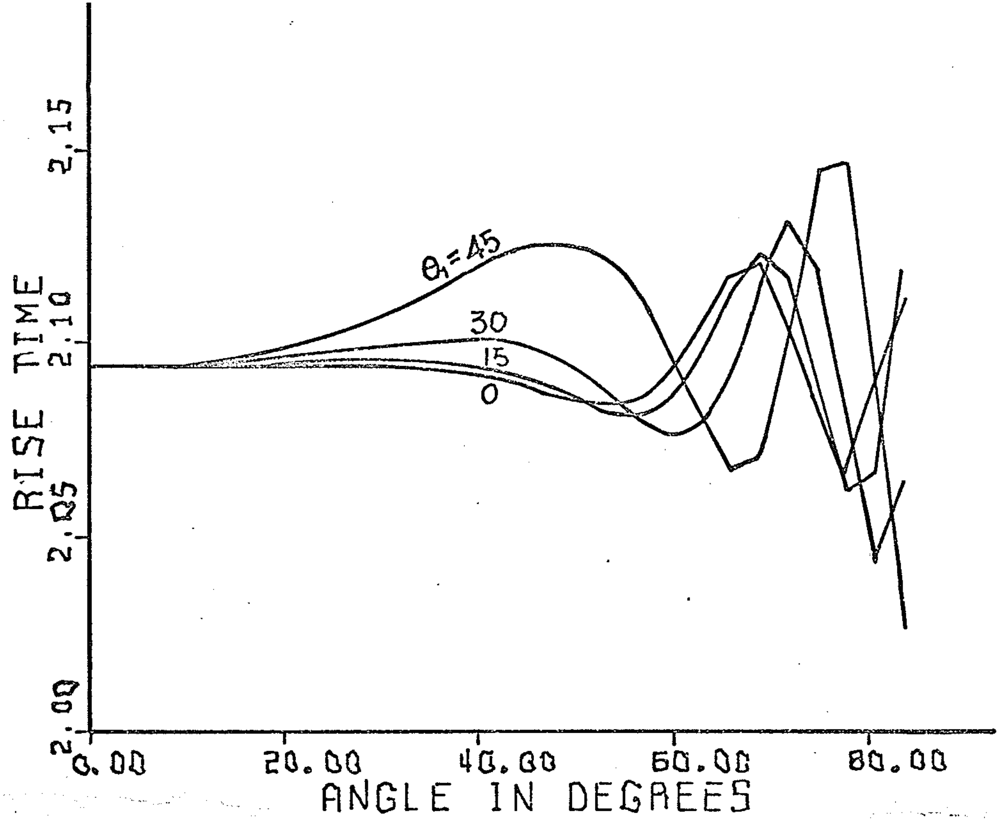


Fig. III.13 Rise time vs. angle θ_2 (n=5,b=0).

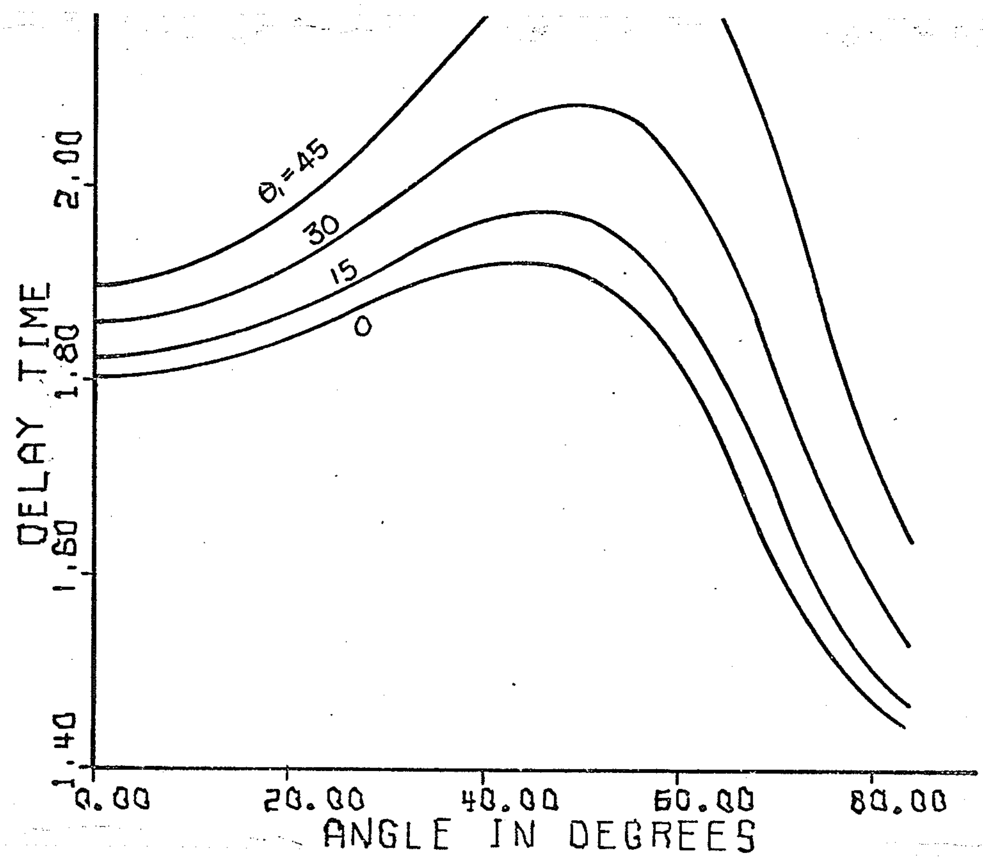


Fig. III.14 Delay time vs. angle θ_2 (n=5, b=0).

III.7 Discussion

The curves on Figure III.7 to III.14 show how the rise time, delay time and overshoot vary with the angles. For the third order case, an optimal point is located at an angle of 22 degrees. The rise time and delay time are 2.151 seconds and 1.428 seconds respectively, as compared to 2.364 seconds and 2.280 seconds of the parabolic filter with an angle of 60 degrees.

For the fourth and fifth order cases, several optimal points are located depending on the relative importance of the rise time delay time, overshoot or the frequency responses. For example, two optimal points are located for the fourth order case. The first point has a minimum rise time of 2.014 seconds and the second point has a minimum delay time of 1.621 seconds. Both points have negligible overshoots and reasonable frequency responses. If the requirement on the output pulse shape is relaxed, faster responses can be obtained by increasing the angles.

CHAPTER IV

OPTIMIZATION BY EXTREMUM METHOD

IV.1 Introduction

For an nth order filter, there are n degrees of freedom in choosing the position of poles. A required constraint of fixed bandwidth removes one degree of freedom result in n-1 degrees of freedom. For a complete optimization, there should be n-1 parameters to work with. Table IV.1 shows the number of parameters available for the pole shifting technique.

ORDER OF FILTER	DEGREES OF FREEDOM	PARAMETERS AVAILABLE
2	1	θ_1
3	2	b, θ_1
4	3	b, θ_1, θ_2
5	4	b, θ_1, θ_2
6	5	b, θ_1, θ_2, θ_3
7	6	b, θ_1, θ_2, θ_3
8	7	b, θ_1, θ_2, θ_3, θ_4

Table IV.1 Available parameters for pole-shifting method, where θ_i is the angle of the ith pole vector.

Therefore, instead of confining the poles on a parabolic, elliptic or any other loci, the designer has complete freedom to choose the pole locations from the left half plane, subjected only to the bandwidth constraint.

IV.2 Pole distribution under bandwidth constraint

Consider the third order case, and let p_1, p_2, p_3 be the poles on the left half plane such that

$$p_1=(-b+jc);\; p_2=(-a+j0);\; p_3=(-b-jc)$$

where a,b,c are real positive numbers.

The transfer function of the filter is

$$H(s) = \frac{a\,(b^2 + c^2)}{(s + a)\,(s^2 + 2bs + b^2 + c^2)} \qquad \text{IV.1}$$

Let $$q = b^2 + c^2$$

$$H(s) = \frac{aq}{(s + a)\,(s^2 + 2bs + q)}$$

The bandwidth is set equal to unity.

$$|H(j1)|^2 = \frac{a^2 q^2}{(1+a^2)\ ((q-1)^2 + 4b^2)} = \tfrac{1}{2} \qquad \text{IV.2}$$

Equation IV.2 is rearranged as a quadratic equation of q.

$$(a^2 - 1)\ q^2 + 2q(1 + a^2) - (1 + 4b^2)\ (1 + a^2) = 0 \qquad \text{IV.3}$$

Let $$g = \frac{(a^2 - 1)}{(1 + a^2)} \qquad \text{IV.4}$$

then $$q = \frac{-1 \pm \sqrt{1 + (1 + 4b^2)\,g}}{g} \qquad \text{IV.5}$$

The sign before the square root is chosen to make the expression positive.

$$c^2 = q - b^2 = \frac{-1 \pm \sqrt{1 + (1 + 4b^2)\ g}}{g} - b^2 \qquad \text{IV.6}$$

provided $$a^2 \geq \frac{2b^2}{1 + 2b^2} \qquad \text{IV.7}$$

Similarly, for the fourth order case, let p_1, p_2, p_3 and p_4 be the poles of the transfer function.

$$p_1=(-A+jB); \quad p_2=(-C+jD);$$

$$p_3=(-C-jD); \quad p_4=(-A-jB);$$

$$Q_1=A^2+B^2; \qquad Q_2=C^2+D^2.$$

where A, B, C and D are real positive numbers.

The transfer function of the filter becomes

$$H(s) = \frac{Q_1 Q_2}{(s^2 + 2As + Q_1)\ (s^2 + 2Cs + Q_2)} \qquad \text{IV.8}$$

The bandwidth is set to unity.

$$\tfrac{1}{2} = \frac{Q_1^2\ Q_2^2}{[(Q_1 - 1)^2 + 4A^2]\ [(Q_2 - 1)^2 + 4C^2]} \qquad \text{IV.9}$$

Equation IV.9 is rearranged as a quadratic equation of Q_1.

$$(Q_2^2 + 2Q_2 - 1 - 4C^2)\ Q_1^2 + 2\ (Q_2^2 - 2Q_2 + 1 + 4C^2)\ Q_1$$

$$- (Q_2^2 - 2Q_2 + 1 + 4C^2)\ (4A^2 + 1) = 0 \qquad \text{IV.10}$$

Let
$$m = \frac{Q_2^2 + 2Q_2 - 1 - 4C^2}{Q_2^2 - 2Q_2 + 1 + 4C^2}$$

$$Q_1^2\, m + 2Q_1 - (4A^2 + 1) = 0 \qquad \text{IV.11}$$

then

$$Q_1 = \frac{-1 \pm \sqrt{1 + (4A^2 + 1)\, m}}{m} \qquad \text{IV.12}$$

the sign before the square root is chosen to make the expression positive.

$$B^2 = Q_1 - A^2 = \frac{-1 \pm \sqrt{1 + (4A^2 + 1)\, m}}{m} - A^2 \qquad \text{IV.13}$$

provided

$$1 + (4A^2 + 1)\, m \geq 0 .$$

Thus, for the third order case, the two independent variables 'a' and 'b' are the real parts of poles p_1 and p_2 respectively. The imaginary part of p_1 is calculated from equation (IV.6). The imaginary part of the second pole is zero because it lies on the negative real axis. For the fourth order case, the three independent variables are 'A','C' and 'D'. The value of 'B' is obtained from equation (IV.13). The transfer function thus obtained for the third and fourth order filters will satisfy the bandwidth constraint.

IV.3 Minimizing the performance index

The simplest performance index is defined as follows

$$\mathcal{J} = w_1 t_r(x) + w_2 t_d(x) + w_3 \gamma(x) \qquad \text{IV.14}$$

where w_i are the weighting functions and x is the design parameter vector. For the third order case, the elements of the vector are 'a' and 'b'; whereas, for the fourth order case, the elements are 'A', 'C' and 'D'. A direct search program, namely, the pattern search [19] program, is employed to minimize the function $\mathcal{J}$. Very small values of rise time and delay time can be obtained from this simple performance index by decreasing 'a' and increasing 'c'. The situation corresponds to moving the poles towards the imaginary axis in chapter III. This index is discarded because the frequency response is not acceptable.

A new performance index is defined to enable the control of the frequency response as well as the shape of the output pulse in the process of optimization.

$$\mathcal{J} = \int_{0}^{\infty} w(t)\,|e(x,t)|^{k}\,dt + \phi P(x,\omega) \qquad \text{IV.15}$$

where $P(x,\omega)$ is a penalty function, ϕ is a weighting factor.

The function $e(x,t)$ is the error between the actual and the desired network responses. It is defined as follows

$$e(x,t)=u(t)-f(x,t) \qquad \text{IV.16}$$

where $u(t)$ is a unit-step function, $f(x,t)$ is the network response due to a unit-step excitation and x is the adjustable parameter vector. A unit-step function has zero rise time, delay time and overshoot. Therefore, when $f(x,t)$ is closest to the ideal response, the rise time, delay time or overshoot will be the least. The weighting function $w(t)$ is a time function to emphasize or deemphasize certain parts of the response to suit the designer's requirement. If $w(t)=t$, the error is magnified as t increases. The integer k raises the power of the error $e(x,t)$, and in this case is set to 2.

$P(x,\omega)$ is a penalty function defined as follows

$$P(x,\omega) = \int_{o}^{1} |(1 - |H(x,j\omega)|)|\,d\omega + \int_{1}^{\infty} |H(x,j\omega)|\,d\omega \qquad \text{IV.17}$$

It is the frequency integral of the error between the actual and the ideal low-pass frequency responses. The penalty function is a minimum when the frequency response is closest to the ideal low-pass response. Thus by minimizing the function $\mathcal{J}$ it is possible to minimize the rise time and delay time as well as to keep the frequency response close to the ideal response.

In carrying out the evaluation of the function $\mathcal{J}$, equation IV.15 is approximated as follows

$$\mathcal{J} = \sum_{i=1}^{N} W(t_i)\,[e(x,t_i)]^2\,\Delta t + \phi\Big(\sum_{i=1}^{m} \big|1 - |H(x,\omega_i)|\big|\Delta\omega + \sum_{j=m+1}^{N} |H(x,\omega_j)|\,\Delta\omega\Big) \qquad \text{IV.18}$$

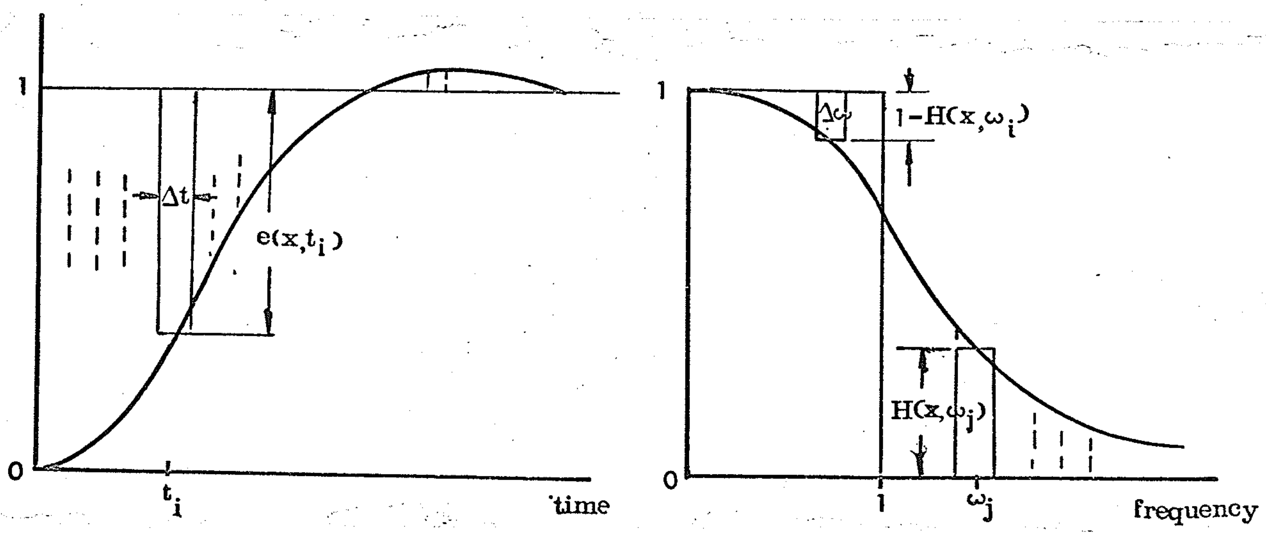


Fig. IV.1 Area approximation in the time and frequency domains.

Figure IV.1 depicts the summation process and it is found sufficient to carry the summation up to 10 seconds for the time domain term and up to 3 rad/sec for the frequency domain term. After the index is defined, it is minimized by the pattern search program to locate the optimal point.

IV.4 Result of optimization

The minimum points of the function I are different with different weighting function ϕ. When frequency response is deemed more important, the weighting factor ϕ is to assume a large value. This weighting process is very subjective because strictly objective criteria do not exist. Therefore several optimal points are located with different weightings. The results for the third and fourth order are summarized in Table IV.2. For filter I, the weighting on time response is heavy, giving a rise time of 1.761 seconds and 1.966 seconds for the third and the fourth orders respectively. Filter III has the heaviest weighting on frequency response, thus the rise time increases to 2.097 seconds and 2.138 seconds. Figure IV.2 and IV.4 show that although filter I has a smaller rise time, the frequency response has a ripple effect. Filter II, however, is a favorable medium, having a rise time of 2.04 seconds for the third order. All the three filters are superior to the parabolic filter (IV) which is included for comparison.

n=3	a	b	c	t_r(sec)	t_d(sec)	γ(%)
I	1.974	0.525	0.678	1.761	1.395	0.000
II	1.916	0.737	0.737	2.041	1.430	0.000
III	1.698	0.743	0.845	2.097	1.565	0.000
IV	0.976	0.563	0.845	2.320	2.087	4.075

n=4	A	B	C	D	t_r(sec)	t_d(sec)	γ(%)
I	0.709	2.123	1.224	0.042	1.996	1.743	0.000
II	0.685	7.293	1.493	0.132	2.075	1.158	0.004
III	2.210	0.380	2.220	0.396	2.138	1.614	0.000
IV	0.426	1.171	0.805	0.325	2.447	2.685	2.631

Table IV.2 Result of optimization.

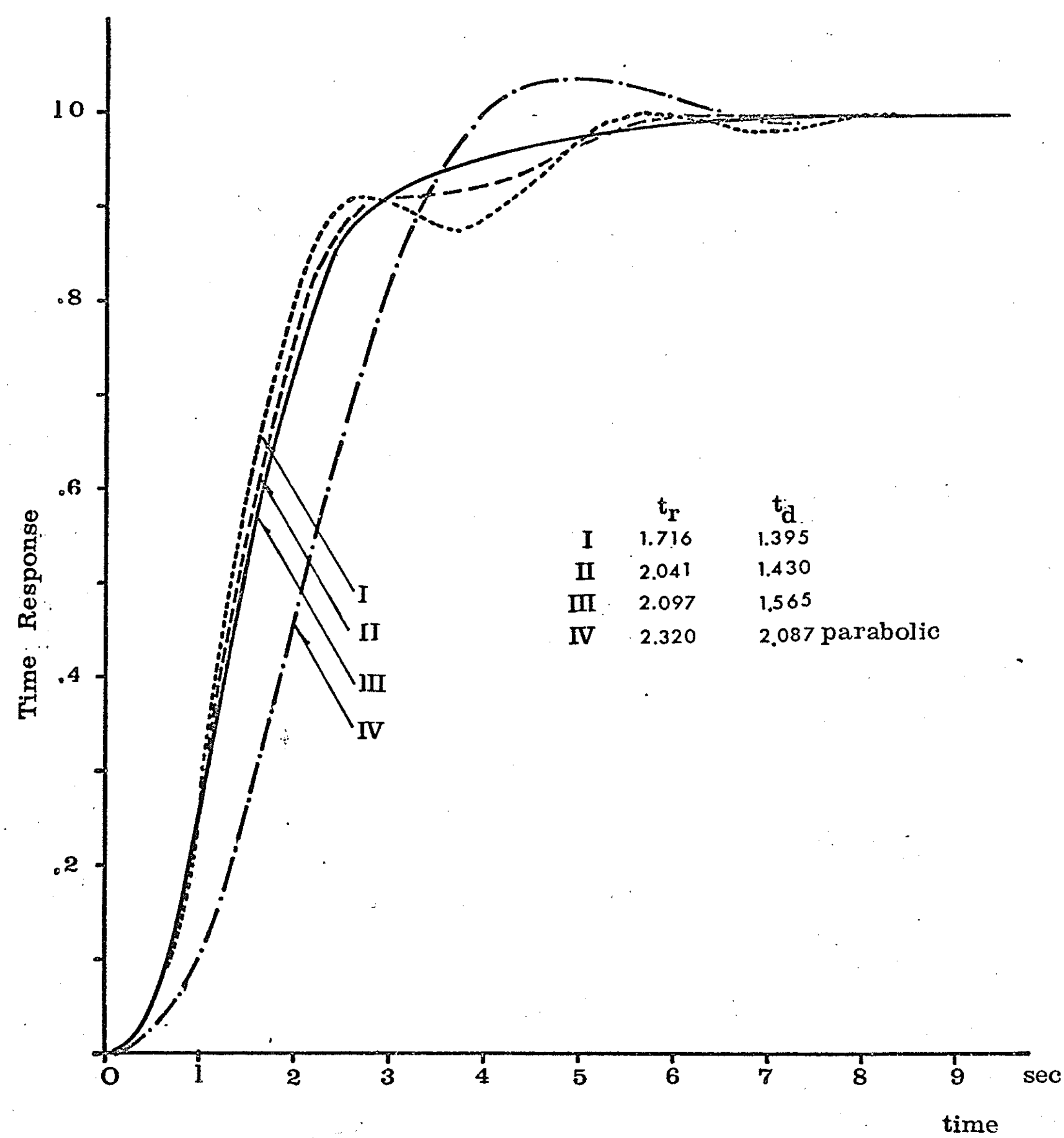


Fig. IV.2 Unit-step response for the third order filters.

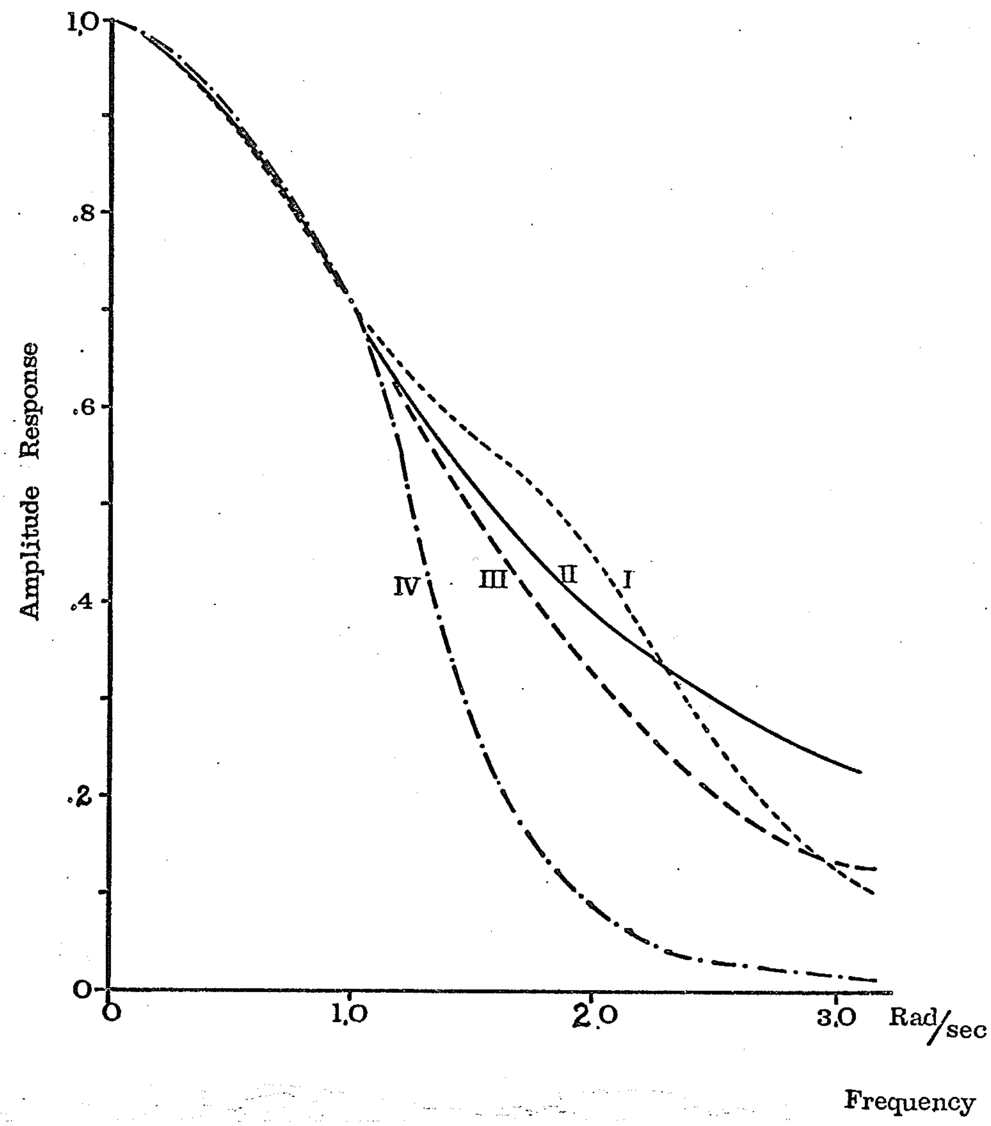


Fig. IV.3 Frequency response for the third order filters.

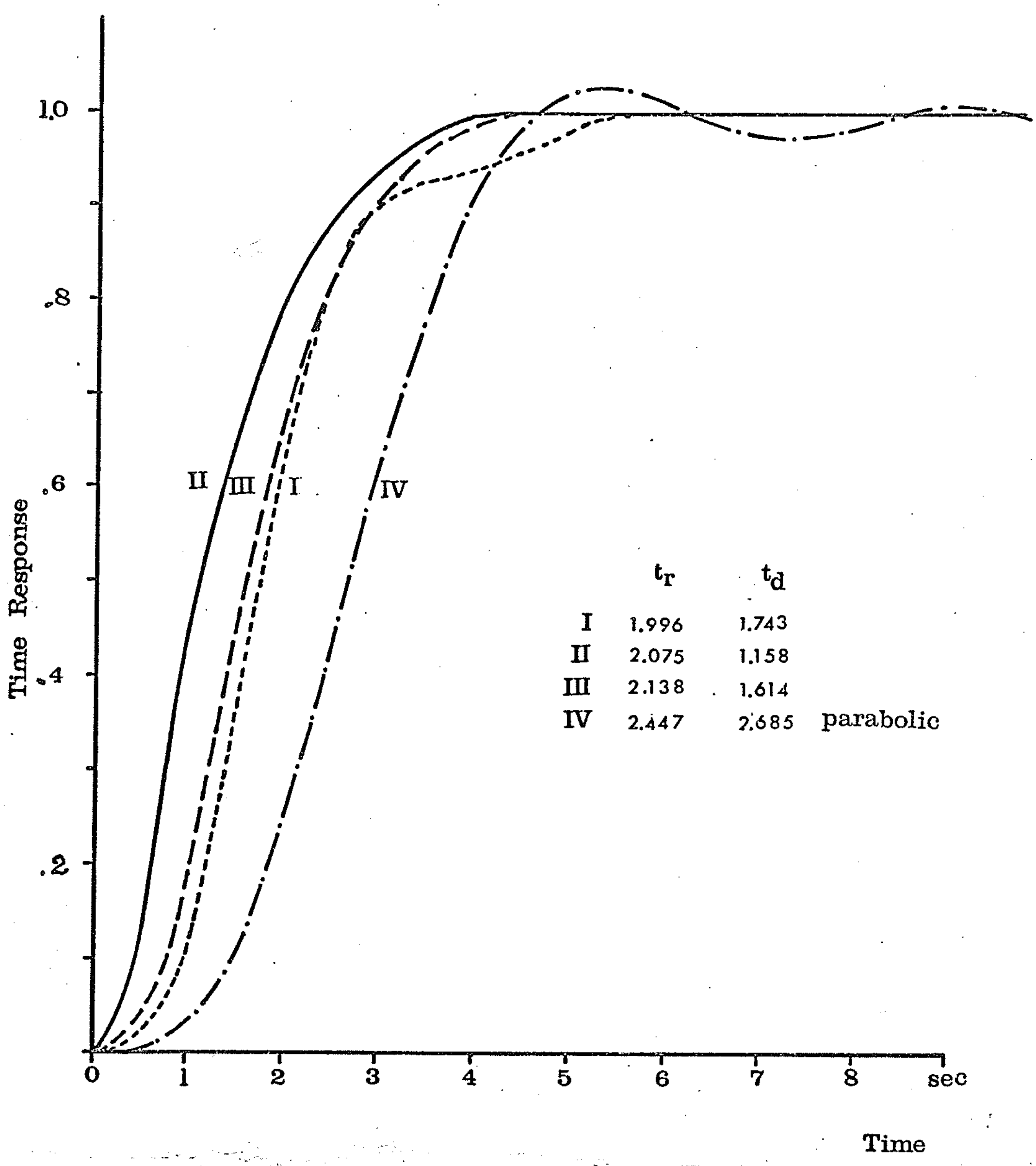


Fig. IV.4 Unit-step response for the fourth order filters.

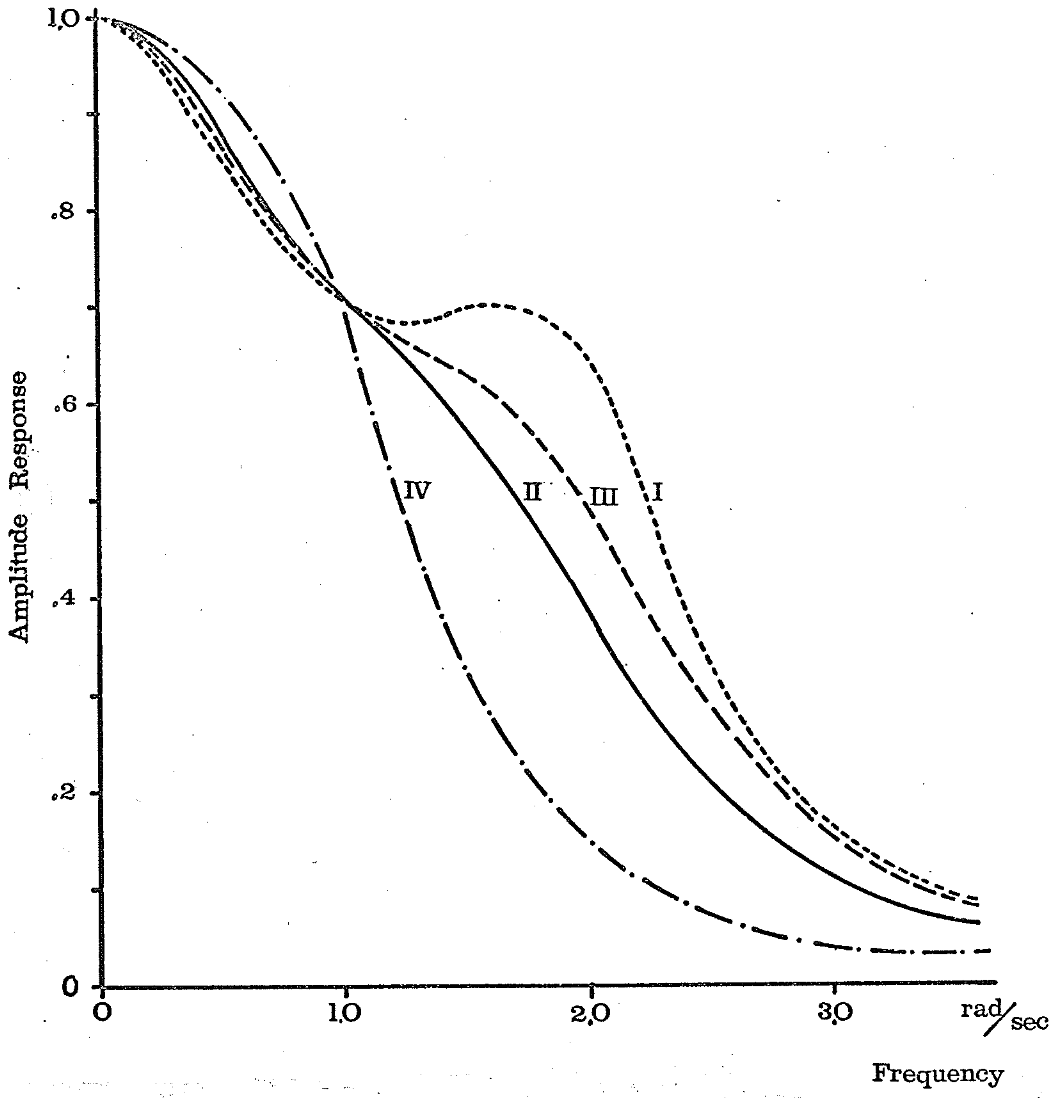


Fig. IV.5 Frequency response for the fourth order filters.

CHAPTER V

CONCLUSION

This thesis presents methods of improving the transient responses of low-pass filters. The first method is the pole-shifting method. The radius vectors of the poles are allowed to move by varying the angles between the radius vectors and the negative real axis, resulting in substantial improvements. The rise time, delay time and the overshoot versus angles are plotted in graphs so that one can find the optimal points corresponding to the figures of merit of the transient response. It is found, however, that the angles should not be greater than 70 degrees to avoid undesirable frequency responses.

Another approach to the problem is presented in chapter IV. Some error criteria of the actual and ideal responses are defined in terms of some workable parameters. The weighted sum of the integrals of the errors is then minimized by the pattern search routine. Remarkable results are obtained and are tabulated in Table IV.2. The time and frequency responses are also plotted for references. The small rise time (1.761 sec.) and delay time (1.395 sec.) are much superior to those of the

Butterworth filter which gives a rise time and delay time of 2.291 and 2.135 seconds respectively. The overshoot for all the results obtained are either zero or negligible. The 5th or higher order cases can be obtained easily by the same procedures.

The pole-shifting method, though not as efficient as the extremum method, gives much more insight into the relationship of the pole pattern and transient response. However, the numerical method gives a much better result because of the higher degrees of freedom.

The results obtained in the work are very useful in the design of filters for pulse networks. Since the transmission zeros are all at infinity, the filters considered can be realized as ladder networks by standard procedures.

It is the author's feeling that more research should be carried on in both the analytical approach and the numerical approach as the two methods complement each other.

APPENDIX A

COMPUTER SUBPROGRAMS

Several computer subprograms have been written for the IBM 360/65 to calculate both the frequency and time responses. The language used is WATFIV, a modified version of standard FORTRAN.

The flow diagrams are given in the following pages. Only the 4th order case is presented. The 3rd and 5th order cases are similar. The numeric 4 is included in the names of the subroutines to indicate a 4th order case.

The following is a brief description of the subprograms.

ROOT4 To locate the pole position $p_i = \sigma_i + j\omega_i$ with a given equation of the locus and an associated angle θ_i.

NORMW4 To calculate the frequency response and to interpolate the bandwidth (FINW) by comparing H(s) with 0.7071. The step size is reduced successively until the error is smaller than a given bound.

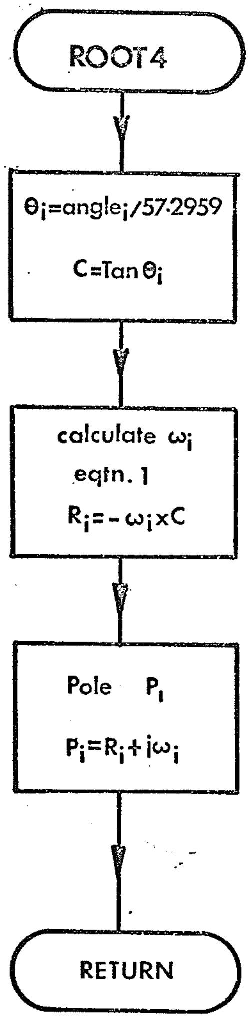
PLOC4 To check whether poles are simple conjugate, double or located on the negative real axis. A value of -1,0 or 1 is assigned to an indicater 'NCASE'. The value thus assigned to NCASE will guide the subsequent subprograms to choose different equations for different types of poles.

COPF4 To calcuate the residues of the poles according to the equations given.

FUNCTION H - Subroutines TRTD4 and OSHT4 will call this external function to calculate the value of the unit-step response f(t) at a given point t according to the type of poles.

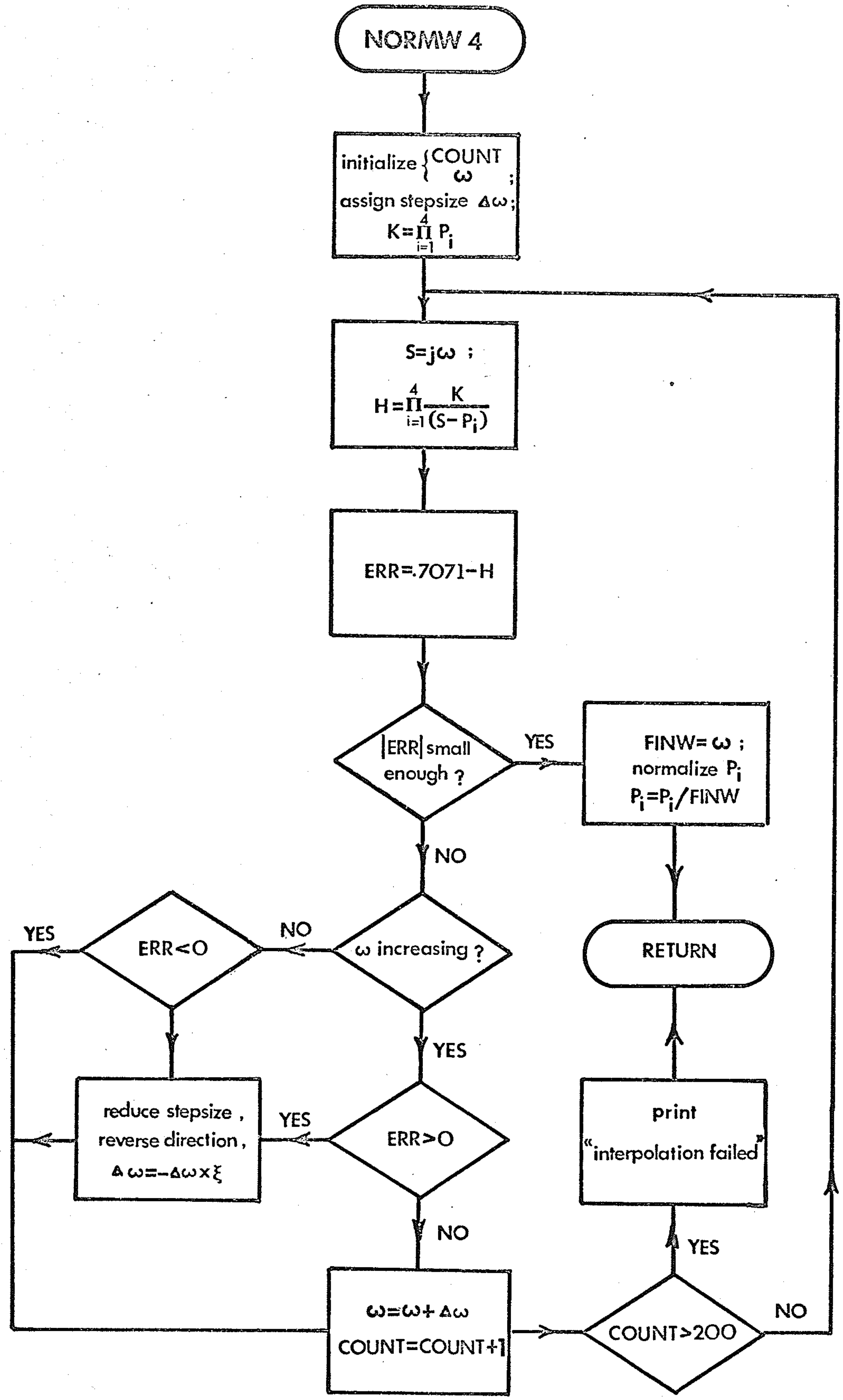
TRTD4 To interpolate the value of t when f(t) equals to 0.1, 0.5 and 0.9. The procedure is similar to NORMW4. TD is the value of t when f(t) differs 0.5 by less than the error bound. TR equals to $t_{.9}-t_{.1}$.

OSHT4 To calculate the overshoot of the time response; n points are calculated between t_i and t_f. The maximum point $f(t_m)$ is obtained by a sort program. The interval is reduced by re-assigning t_i and t_f a distance $\Delta t/2$ to the left and to the right of the maximum point. The procedure is repeated until the interval (t_f-t_i) is small enough. Overshoot is $(f(t_m)-1)\times 100\%$.

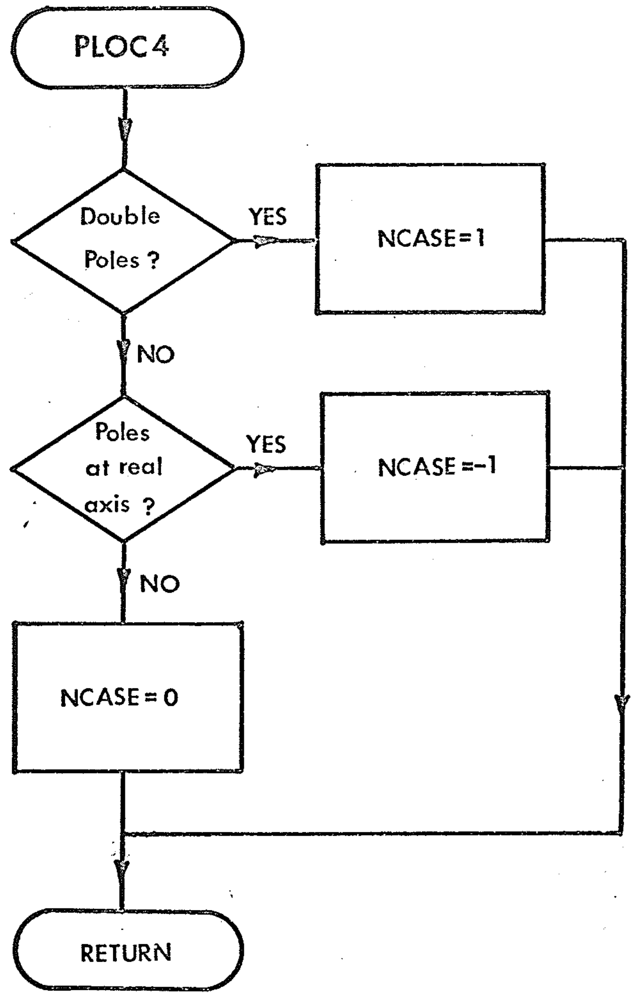


$$\omega_i = \mathrm{SQRT}(4(b+1)^2 C^2 + 1) - 2(b+1)C$$

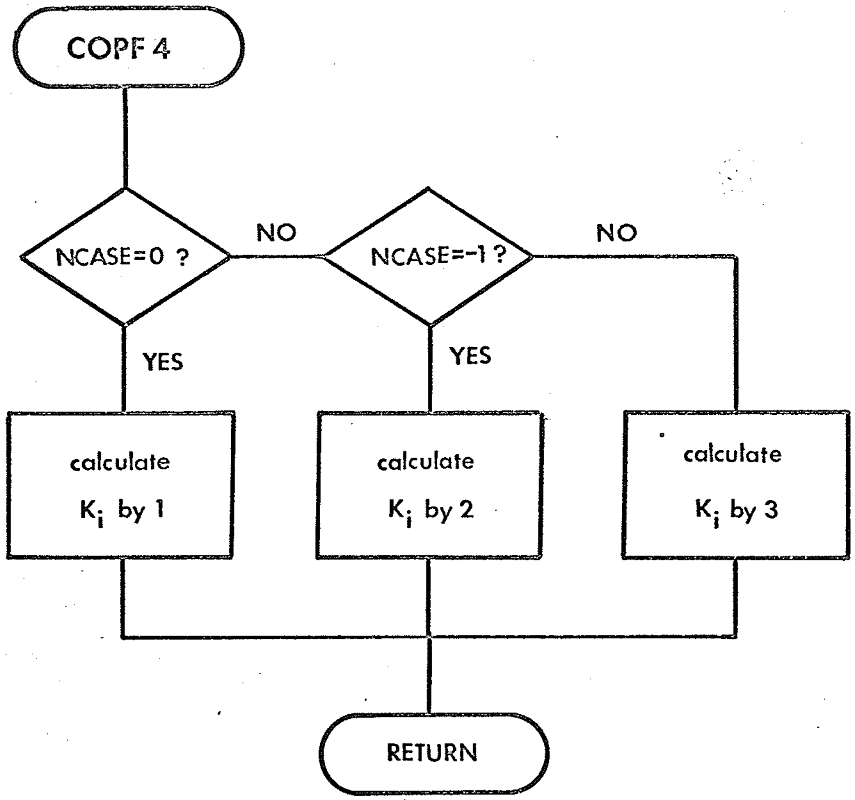
Flow diagram for ROOT4 - to locate pole position from the given equation.



Flow diagram for NORMW4 - to find bandwidth and new pole positions after normalization.



Flow diagram for PLOC4 - to check types of poles and assign value to NCASE.



1 $K_i = C/\left(P_i \prod_{j \neq i}(P_i - P_j)\right)$

2 $K_1 = C/\left(P_1 \prod_{j=2}^{4}(P_1 - P_j)\right)$

$K_2 = C/(P_2(P_2 - P_1)(P_2 - P_4))$

$K_3 = CMPLX(1,0) - K_1 - K_4$

$K_4 = CONJG(K_1)$

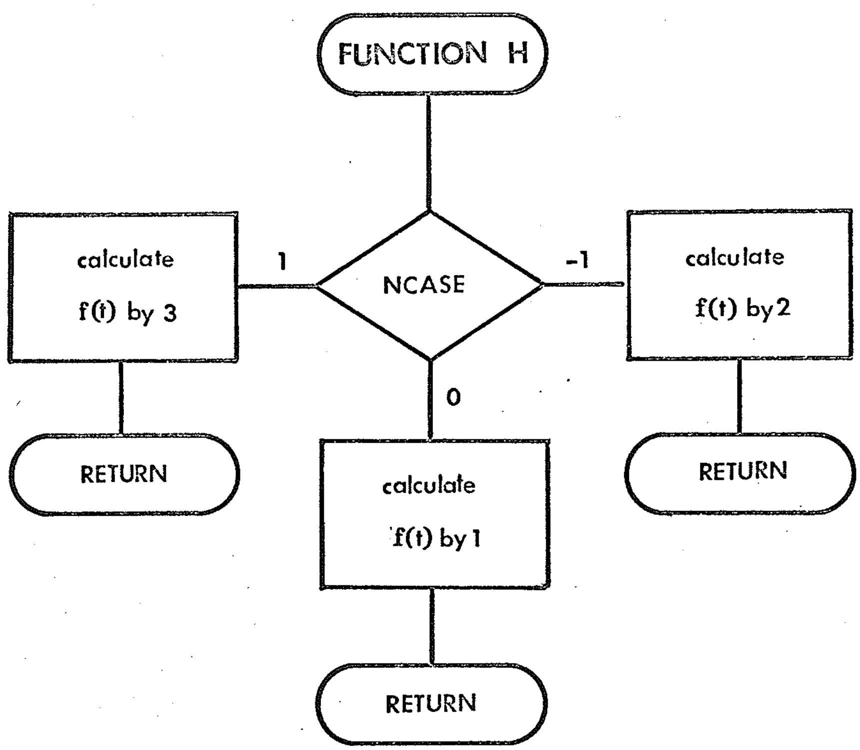
3 $K_1 = C/(A(A - P_3)^2)$

$K_2 = -C(3A - P_3)/(A^2(A - P_3)^3)$

$K_3 = CONJG(K_2)$

$K_4 = CONJG(K_1)$

Flow diagram for COPF4 - to calculate the residues of the poles.

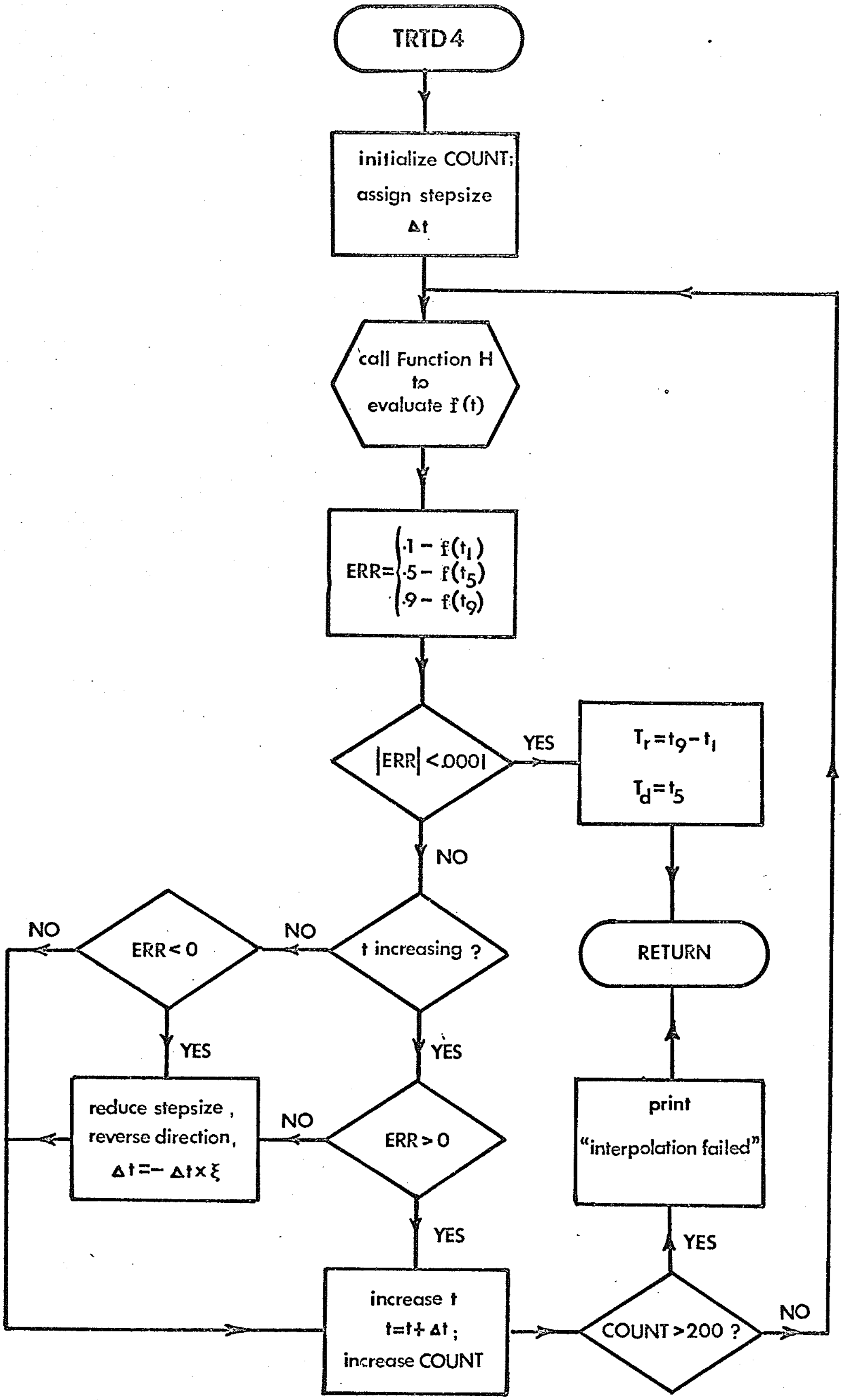


1 $$f(t) = 1 + \sum_{i=1}^{4} K_i e^{p_i t}$$

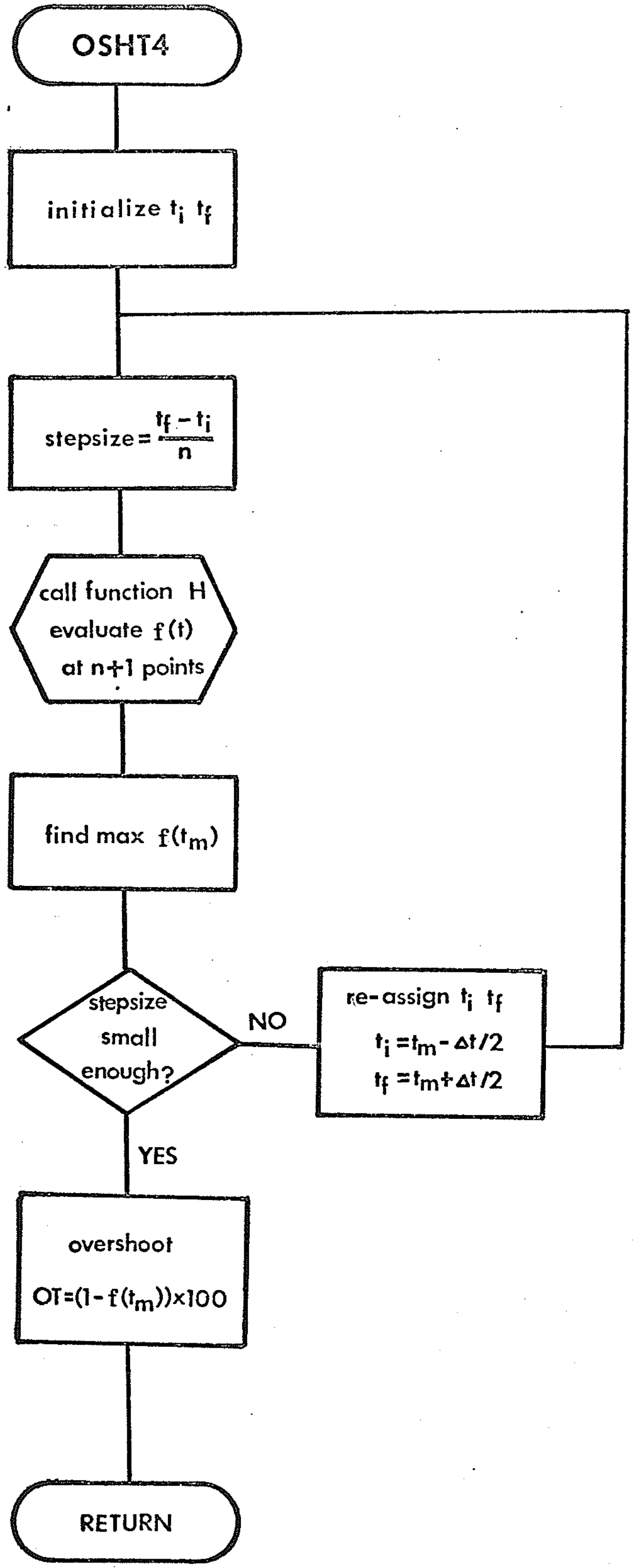
2 $$f(t) = 1 + K_1 e^{p_1 t} + (K_2 t + K_3 e^{p_2 t}) + K_4 e^{p_4 t}$$

3 $$f(t) = 1 + (K_1 t + K_2) e^{p_1 t} + (K_3 + K_4 t) e^{p_4 t}$$

Flow diagram for FUNCTION H - to calcuate the unit-step response f(t).



Flow diagram for TRTD4 - to interpolate the values of t_r and t_d.



Flow diagram for OSHT4 - to calculate the overshoot.

APPENDIX B

PATTERN SEARCH

Pattern search [17] is a direct search routine for minimizing an objective function $E(\underline{W})$ of several variables, where $\underline{W}^T=(W_1...W_k)$. The arguement W_i is varied until the minimum $E(\underline{W})$ is obtained. Figure B.1 shows a two dimensional example of the pattern search.

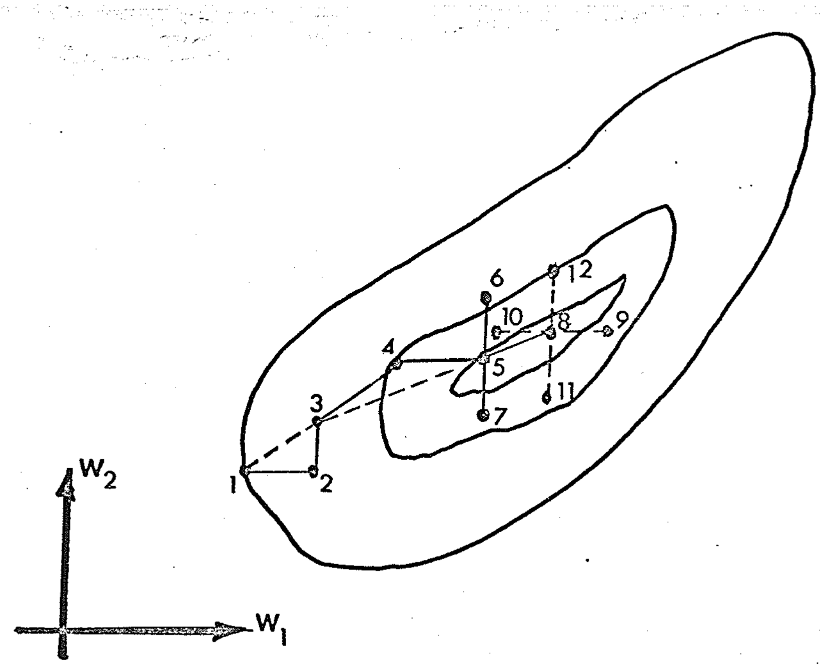


Fig. B.1 Example of a 2-dimensional pattern search.

The pattern search is successfully applied to the work of this thesis in two cases. In the first case, the arguements w_i are the angles between the pole-vectors and the negative real axis. These poles are located on the parabolic contour. The objective function $E(\underline{W})$ is simply the rise time.

In the second case, the objective function is the performance index $\mathcal{J}$ and the arguements are 'a' and 'b' or 'A','B' and 'D' for the third or fourth order case.

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