## THE UNIVERSITY OF MANITOBA

# COMMUTATION REACTANCE IN HVDC TRANSMISSION SYSTEMS

ВΥ

WALTER PYL

#### A Thesis

Submitted to the Faculty of Graduate Studies

in Partial Fulfillment of the Requirements for the

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# "COMMUTATION REACTANCE IN HVDC TRANSMISSION SYSTEMS"

by

#### WALTER PYL

A dissertation submitted to the Faculty of Graduate Studies of the University of Manitoba in partial fulfillment of the requirements of the degree of

MASTER OF SCIENCE

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#### **ABSTRACT**

Commutation reactance influences many aspects of HVDC systems. This thesis is primarily concerned with showing how the optimum range of commutation reactance can be determined based on reliable valve operation and low capital equipment and operating costs and how the resulting reactance in the commutating circuit affects the HVDC system.

Arcback requirements of mercury are valve rectifiers and dc fault currents for thyristor valves are shown to determine the minimum commutation reactance allowable. Both are limited by reactance in the commutating circuit. Reactive power and regulation requirements as well as higher equipment ratings are shown to limit the maximum commutation reactance.

Once the commutation reactance for a particular scheme is determined, its effect on system operation is considered. Generating uncharacteristic harmonics due to unbalanced reactances in the commutating circuit is discussed as well as the need for compensating commutation reactance common to several inverters.

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# NOTATION

E' <sub>L</sub>	rms line-to-line transformer secondary voltage
E <sub>m</sub>	phase-to-ground peak voltage
E <sub>mF.L.</sub>	phase-to-ground peak voltage corresponding to
	full load
$E_1, E_2, E_S, E_T$	phase-to-ground rms voltages
e <sub>a</sub> ,e <sub>b</sub> ,e <sub>c</sub>	instantaneous phase-to-ground voltages
IL	total rms transformer secondary current
I d	direct current
I <sub>dF.L.</sub>	full load (rated) direct current
I <sub>a</sub> ,I <sub>b</sub> ,I <sub>c</sub>	rms phase currents
Iab	rms value of the arcback current
i <sub>1</sub> ,i <sub>2</sub> ,i <sub>3</sub> ,i <sub>4</sub> ,i <sub>5</sub>	, i 6 instantaneous valve currents
i <sub>a</sub> ,i <sub>b</sub> ,i <sub>c</sub>	transformer secondary phase currents
$I(n)_{o}$	rms value of nth harmonic current with no overlap
ν <sub>ο</sub>	average dc no load output voltage
VoF.L.	average dc no load output voltage corresponding
·	to full load
$v_d$	average dc output voltage.
V (n)	rms value of nth harmonic voltage
X <sub>s</sub>	per unit system reactance on converter transformer
	base
X <sub>T</sub>	per unit converter transformer leakage reactance on
	own base
X <sub>C</sub> .	per unit commutation reactance on converter

transformer base

•	$oldsymbol{\cdot}$
X <sub>M</sub> .	per unit mutual reactance on converter transformer
	base
Xab	per unit arcback reactance on converter transformer
	base
<b>L</b> :	phase inductance .
Ls	phase inductance of the ac system
L <sub>T</sub>	phase inductance of the converter transformer
S	system short circuit capacity
p .	pulse number of the converter
n	harmonic number
μ	commutation overlap angle
α	delay angle of a valve measured with respect to the
	natural commutation point
$\gamma_{o}$	minimum de-ionization angle of an inverter valve
γ	de-ionization angle of an inverter valve ( $\gamma > \gamma_{_{\scriptsize O}}$ )
β	advance angle of firing an inverter valve
λ ,	power factor
ω	radian frequency

## Chapter I

## INTRODUCTION.

HVDC technology has developed considerably in recent decades and today HVDC transmission schemes are in operation as viable alternatives to AC transmission. Much work has been done to solve the many technical problems associated with this relatively new technology.

Rectification, inversion, reactive power requirements, harmonic generation, apparatus and converter control are some of the numerous areas which have required analysis in developing successful HVDC system designs.

An important aspect which is common to all these areas of study of HVDC systems is commutation reactance. Many technical papers have been written on various aspects related to HVDC design and operation; however, none deal directly with the many effects of commutation reactance on HVDC systems.

This thesis attempts to correlate and present a unified treatment of commutation reactance to show its varied and complex influence on direct current system technology. This thesis and the attached bibliography point out areas where the consideration of commutation reactance is important.

assumptions which simplify the derivation of these equations. Chapter II introduces the basic converter equations based on these assumptions. The effect of reactance in the commutating circuit is included by defining the commutation overlap angle and showing its relationship to the commutation reactance. In conclusion, there is a discussion of the assumptions used to derive the equations and their validity.

Since both mercury arc and thyristor valves are temperature sensitive, the current which they may be required to conduct transiently must be limited. This duty is performed by the reactance in the commutating circuit. It is shown that a finite commutation reactance is necessary and the criteria for determining the lower commutation reactance limit is explained based on thyristor and mercury arc valve thermal requirements.

Commutation reactance common to a number of inverter bridges operating in series on the dc side causes interaction between the valve groups. The need to compensate and exclude any common reactance from the respective commutating circuits is discussed in Chapter IV. The effects of operating under this condition and possible compensation schemes are illustrated.

Generation of harmonics is considered in Chapter V.

The effect of commutation reactance on characteristic current harmonic magnitudes is shown. Conditions necessary for uncharacteristic current harmonic generation and in particular, the effect of unbalanced phase reactance on uncharacteristic current harmonics is examined.

Regulation and the relationship between the commutating reactance and the converter transformer rating, reactive power requirement and tap range is considered in Chapter VI. A sample calculation is done to demonstrate how these parameters can be determined and why a minimum converter transformer leakage reactance compatible with valve requirements should be chosen.

The final chapter examines the role of major system components in the successful operation of HVDC converters and their contribution to the commutation reactance.

As well, Appendix I proposes two methods for the measurement and calculation of the commutation reactance of operating HVDC converters. Both methods are based on the use of oscillograms to determine parameter values which are then substituted into mathematical expressions to yield the required results.

In recognition of the increasing use of solid state thyristor valves, comparisons are made throughout this thesis between thyristor and mercury arc valves.

### Chapter II

### BASIC CONVERTER EQUATIONS

To attempt a study of an HVDC system, it is necessary to be acquainted with the basic equations governing converter operation. Equations have been derived by Adamson and  $\operatorname{Hingorani}^1$ ,  $\operatorname{Cory}^2$ , and  $\operatorname{Kimbark}^3$  to simplify the understanding of the operation of an HVDC system.

In this chapter, the equations which represent the effect of reactance in the commutating circuit of the converter will be considered.

References 1, 2, and 3 make the following assumptions in deriving the basic converter equations and these apply to the equations given in this chapter:

- i) constant dc outputi.e., infinite inductance ( smoothing reactor )on the dc side,
- ii) impedance at the ac bus supplying the converter transformer and the dc converter is zero,
- iii) the rectifying element is an ideal short circuit when conducting and an ideal open circuit when not conducting,
- iv) the magnetization admittance and series resistance

of the converter transformer is neglected,

v) sinusoidal and balanced three phase wave

voltages at the supply point to the dc converter.

Further, two practical considerations must also be included. The mercury arc valves or thyristors used in practical HVDC installations can be controlled up to the point where each begins to conduct. This is accomplished by the application of a negative bias signal to the grid of a mercury arc valve or to the gate of a thyristor. When conduction does occur, due to the transformer reactance, it is not possible to initiate an instantaneous rate of change of current.

The assumptions provide the basis for the derivation of HVDC converter equations.

# II.1 Rectifier Equations

Figure 2-1 shows a wye connected transformer secondary, six valves comprising the rectifier, the valve currents and the output voltage for a rectifier operating with a finite firing angle and finite transformer leakage reactance.

The equation for the dc output current of this rectifier is:

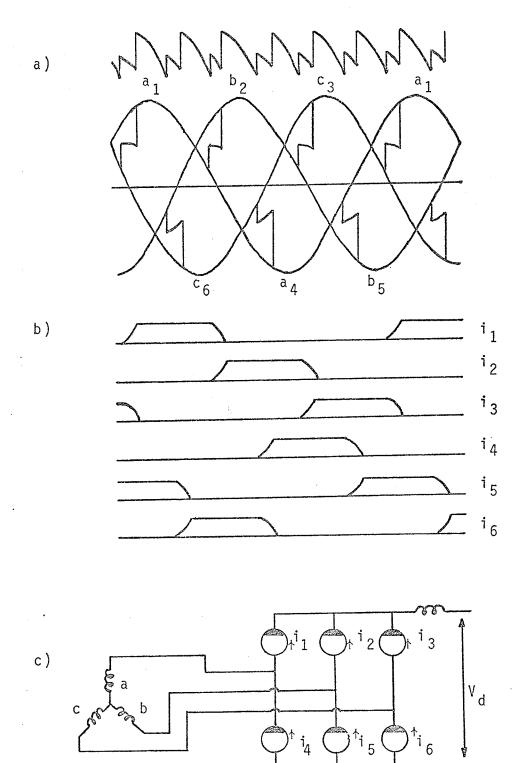


Figure 2-1: a) Output voltage, b) valve currents, and c) rectifier for condition of finite delay and commutation overlap angle

$$I_{d} = \frac{\sqrt{3} E_{m}}{2m!} \left[\cos \alpha - \cos (\alpha + \mu)\right]$$
 (2.1)

where

 $\sqrt{3}E_{m}$  - phase to phase peak voltage supplying the rectifier

ωL - leakage reactance of the converter transformer

α - firing (delay) angle

μ - commutation overlap angle

The dc output current is related to the total ac rms phase current by

$$I_L = I_d \sqrt{2/3 (1 - 3\Psi (\alpha, \mu))}$$
 (2.2)

where

Ψ (α,μ) = 
$$\frac{1}{2\pi}$$
  $\left[\frac{\sin \mu (2+\cos (2\alpha+\mu)) - \mu (1 + 2\cos \alpha \cos (\alpha+\mu))}{(\cos \alpha - \cos (\alpha+\mu))^2}\right]$ 

which is applicable when finite commutation reactance is being considered.

The factor  $\sqrt{1-3\Psi}$   $(\alpha,\mu)$  can be evaluated for a particular  $\alpha$  and  $\mu$  from figure 3.19 on page 46 of reference 1.

Equation (2.2) reduces to

$$I_{L} = \sqrt{2/3} \quad I_{d}$$
 (2.3)

for  $\alpha = \mu = 0^{\circ}$ .

The average dc output voltage is given by

$$V_{d} = \frac{V_{o}}{2} \left[ \cos \alpha + \cos (\alpha + \mu) \right]$$
 (2.4)

where

 $\mbox{\ensuremath{V_{0}}}$  - average dc no load voltage and is related to the ac supply voltage by

$$V_{o} = \frac{3\sqrt{3}E_{m}}{\pi} \tag{2.5}$$

The drop in the dc power from  ${}^V{}_{0}{}^{I}{}_{d}$  to  ${}^V{}_{d}{}^{I}{}_{d}$  is the result of a lagging power factor on the ac side due to the effects of the delay and commutation overlap angles.

The power factor can be evaluated by equating real power on the ac and dc sides resulting in

$$\lambda = \cos \phi = \sqrt{2} \frac{V_d I_d}{3E_m I_L}$$
 (2.6)

Substituting (2.2), (2.4), and (2.5) into (2.6) yields

$$\lambda = \cos \phi = \frac{3}{2\pi} \frac{\cos \alpha + \cos(\alpha + \mu)}{\sqrt{1 - 3\Psi(\alpha, \mu)}}$$
 (2.7)

The converter transformer rating is based on the maximum ac voltage and current which the converter can handle. This situation corresponds to operation at  $\alpha=\mu=\sigma^0$  since the voltage and current are the highest as seen from equations (2.3) and (2.4).

The corresponding rating is found from (2.3) and (2.5) and is

$$MVA_{T} = \sqrt{3} E_{L}I_{L} = \frac{3E_{m}}{\sqrt{2}} I_{L}$$
From equations (2.2) and (2.4), at full load
$$MVA_{T} = \frac{\pi}{3} V_{OF.L.}I_{dF.L.} \qquad (2.8)$$

Knowing the transformer rating, the commutation reactance in per unit and its relationship to the other parameters can be found.

On the converter transformer base

$$X_c = \frac{\omega L}{Z_b}$$

where  $Z_b$  - base impedance =  $\frac{E_L^2}{MVA_T}$ 

$$\therefore X_{c} = \frac{\omega L \sqrt{3}E_{L}I_{L}}{E_{L}^{2}} = \frac{2\omega LI_{d}F.L.}{\sqrt{3}E_{m}F.L.}$$
 (2.9)

Using this relation, rearranging and substituting into equation (2.1) yields

$$\chi_{c} = \cos \alpha - \cos(\alpha + \mu) \tag{2.10}$$

From equation (2.10), the commutation overlap angle in terms of the commutation reactance is known for a given value

of  $\alpha$ . However, it should be noted that equations (2.9) and (2.10) are exact only when rated quantities are considered.

## II.2 Inverter Equations

Variation of the delay angle causes the dc output voltage to vary over the range  $V_0$  to zero. For a rectifier, the delay angles considered vary from  $0^0$  to  $90^0$ .

For delay angles beyond  $90^{\circ}$ , the dc output voltage becomes negative. However, the direct current cannot change direction because of the uni-directional property of the inversion elements. Therefore, for values of  $\alpha$  greater than  $90^{\circ}$ , the dc power,  $V_d^{\rm I}_d$ , becomes negative and rather than supplying power to the dc system, the inverter removes power from the dc system and supplies the ac system. This is the basic principle governing the operation of a dc inverter.

Since inversion results from the delay of firing the inversion element (valve or thyristor) beyond  $90^{\circ}$ , the requirement for an ac supply is evident. Inversion is not possible without an ac supply to energize the converter transformer.

The equation for the dc current of the inverter is

$$I_{d} = \frac{\sqrt{3} E_{m}}{2\omega L} \quad (\cos \gamma - \cos \beta)$$
 (2.11)

where

 $\gamma$  is the de-ionization angle necessary to allow the valve which is commutating out to de-ionize,

 $\beta$  is the advance angle of firing which includes  $\mu$  and  $\gamma$  .

Figure 2-2 shows the inverter bridge, phase voltages and output current. As well it defines the angles  $\mu$ ,  $\gamma$  and  $\beta$ .

The dc output voltage of the inverter is given by 
$$V_{d} = \frac{V_{o}}{2} \left[\cos \beta + \cos \gamma\right]$$
 (2.12)

# II.3 Validity of Assumptions

The preceding equations were based on the five assumptions listed on page 5. In actual HVDC systems, these assumptions may not prove to be completely accurate.

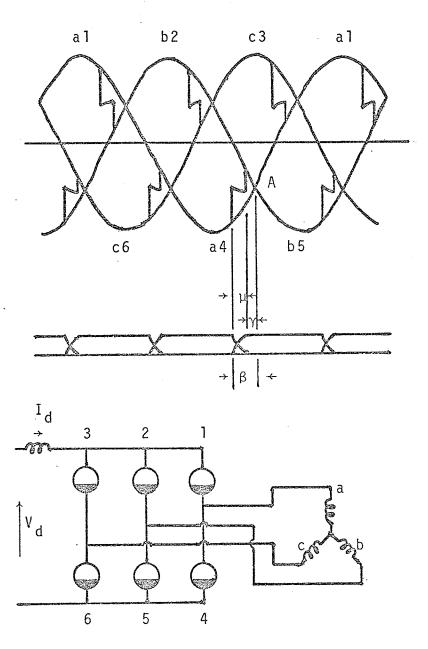


Figure 2-2: Inverter phase voltages and output current.

The following sections discuss the validity of these assumptions and their effect on the accuracy of the equations.

i) constant dc outputi.e. infinite inductance (smoothing reactance)on the dc side.

The dc voltage on the converter side of the line reactor has some ripple as shown in figure 2-1. A six pulse rectifier produces harmonic voltages of order 6, 12, 18, 24, etc. The magnitude of these harmonics at zero commutation and firing angle, as a percentage of the average dc voltage, are 4.04, 0.99, 0.44, and 0.25 (reference 1) respectively for the harmonics listed above. It should be noted that these harmonics are of small magnitude.

To see the effect of the smoothing reactor and dc filters on these harmonics consider the circuit shown in figure 2-3. To simplify the explanation, assume that the input voltage is a combination of a dc component and a sixth harmonic component, and can be expressed as A + B cos  $6\omega t$ . As well, the dc filter shown is tuned for the sixth

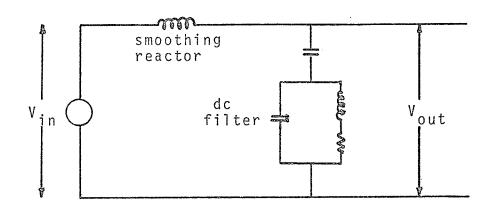


Figure 2-3: Circuit used to show effect of dc smoothing reactor and dc filters on dc output.

harmonic.

The output voltage (actual dc line voltage) can be written as

$$V_{out} = \frac{Z_2}{Z_1 + Z_2} V_{in}$$

$$= \frac{Z_2}{Z_1 + Z_2} A + \frac{Z_2}{Z_1 + Z_2} B \cos 6\omega t \qquad (2.13)$$

where

 $Z_1$  - impedance of smoothing reactor

Z<sub>2</sub> - 6th harmonic filter impedance.

At zero frequency (for direct current ) the smoothing reactor impedance ( $Z_1$ ) is zero and the filter impedance is infinite due to the series capacitor. The term  $\frac{Z_2}{Z_1+Z_2}$  can be

rearranged to  $\frac{1}{1 + \frac{Z_1}{Z_2}}$  and is unity for direct

current.

At the sixth harmonic frequency, the smoothing reactor impedance is large while the filter impedance (Z<sub>2</sub>) approaches zero. The term  $\frac{Z_2}{Z_1+Z_2}$ 

therefore approaches zero.

The smoothing reactor impedance is very large and if the filter is not ideal, forces  $\frac{Z_2}{Z_1+Z_2}$ 

to approach zero.

As a result,

 $V_{\text{out}} = (1)A + (0)B \cos 6\omega t$ = A

and the dc line voltage consists only of a dc component.

Since the dc voltage is smooth and the transmission line elements are linear, the dc current is also smooth.

The preceding development can be extended to the other dc voltage harmonics. However, dc filters are generally used to filter the 6th and 12th harmonics which are of the highest magnitude. The higher order harmonics for which filters are not provided are passed through into the dc line. However, due to their small magnitudes, their

presence is insignificant.

ii) impedance at the ac bus supplying the dc converter is zeroi.e. ac system has infinite capacity

The effect of assuming an infinite bus at the supply point to the dc converter is that the ac system does not affect the converter operation.

The commutation reactance is simply the converter transformer reactance only.

This situation is approached in actual HVDC systems when harmonic self-tuned filters are used. The source of sinusoidal voltage becomes the ac filter bus and the ac system reactance does not affect the commutation reactance.

iii) rectifier is an ideal short circuit when conducting and an ideal open circuit when not conducting.

Normal HVDC mercury arc valves operating at full load have a forward voltage drop of approximately 50 volts. This drop is less than 1% of a typical

high voltage converter bridge voltage rating and neglecting it does not alter the results. In high voltage valves, arc drop is practically constant for all loads and may be taken into account as a load dependent resistance in series with the valve if a more detailed representation is required.

The assumption of the converter element being an ideal open circuit results in the neglect of the reverse leakage current. However, a typical leakage current is in the order of milliamperes and does not contribute significantly to full load current.

The figures for thyristors are generally of the same order although the power loss in a thyristor bridge is slightly greater than that for a mercury arc bridge.

iv) magnetization and resistance of the converter
transformer is neglected

The exciting current in a large power transformer is usually less than 5% of the full load current.

This small percentage does not affect the voltage regulation of the ac system. Use of the ideal transformer turns ratio is not affected.

The winding resistance of large power transformers is of the order of 1% - 2% of the transformer leakage reactance. It can be considered in the calculation of the dc parameters but its inclusion also has an insignificant effect.

v) sinusoidal and balanced  $3\phi$  voltages at the dc supply point

The effect of assuming a balanced  $3\phi$  sinusoidal supply voltage is to provide equal conduction periods for each valve.

In actual systems, slight unbalance and distortion of the supply voltage is expected and the controls must be capable of adjusting to these conditions.

### Chapter III

## COMMUTATION REACTANCE

A typical HVDC system with its sending and receiving end systems is shown in figure 3-1. Although there are many variations of this basic scheme in use or under consideration, the majority of HVDC schemes are of this basic type.

valve begins to conduct the current increases from zero to the operating value but due to the reactance in the commutating circuit, the increase cannot be instantaneous. Commutation reactance is that reactance directly taking part in the operation of the valve group and is defined as "the reactance between the valve group and the source of fundamental ac voltage",

In figure 3-1, referring to the sending end system, the filter bus is considered the fundamental voltage source for the dc system, since the low impedance of the filters to harmonics results in an essentially sinusoidal voltage on the filter bus, particularly when self-tuned filters are used. In this case, the commutation reactance excludes all system reactances beyond the filters and consists only of the converter transformer leakage reactance and the anode reactors if these are present.

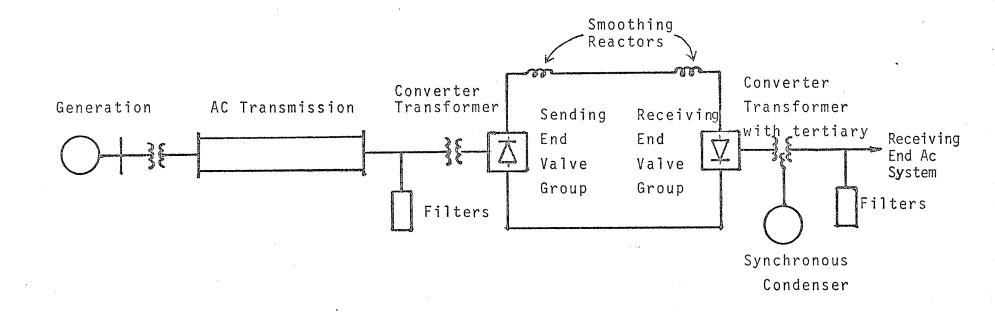


Figure 3-1: Typical HVDC transmission system showing the sending and receiving end ac systems.

A similar analysis of the receiving end system can also be made although in the case of figure 3-1, the simple result is complicated by the use of the synchronous condenser on the tertiary of the converter transformer.

In the case where ac filters are not included, the commutation reactance includes the converter transformer reactance, the ac transmission line reactance and the subtransient reactance ( X"d ) of the generators. If synchronous condensers are used in the ac system for var support, their subtransient reactance must also be included.

Once the required dc current and voltage magnitudes are determined for the dc scheme, the minimum commutation reactance must be determined. For mercury arc rectifiers, this commutation reactance must be sufficient to limit the rate of change of current at current zero to prescribed limits and also limit possible arcback current magnitudes. For solid state thyristor converters, the commutation reactance must be sufficient to limit dc side fault currents. These aspects are considered in the following sections.

# III.1 Limiting the Current Derivative and Arcback

Figure 3.2 shows the current in one valve during the conduction period. At the end of the conduction period, that is, at point A, the current becomes zero. At this point, the rate of change of current must be within an allowable limit prescribed by the valve designer. The limiting parameter is the commutation reactance. The higher the commutation reactance, the lower  $\frac{di}{dt}$ . A consequence of too great a current derivative is valve failure due to arcback.

The rate of change of current can be calculated from  $\frac{di}{dt} = \frac{\omega I}{X_C} dF \cdot L \cdot \sin(\alpha + \mu)$  (3.1)

where

 $X_{c}$  = commutation reactance in p.u.  $\alpha + \mu$  = end of commutation

The derivation of equation (3.1) is given in Appendix III.

When arcback occurs, a valve which has completed commutation and has negative voltage on the anode conducts in the reverse direction ( cathode to anode ). This type of fault

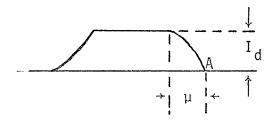


Figure 3-2: Valve current showing point at which current becomes zero at the end of the commutation period (rectifier).

is prevalent in rectifiers which have negative voltage on the anode for  $240^{\circ}$ .

Figure 3-3 shows the circuit conditions during an arcback. Assuming valve 1 has completed commutation with valve 2 and has arced-back, it is seen that a short circuit exists on transformer phases a and b. Since the voltage on rectifier valves remains negative for 240°, short circuit currents may attain values greater than 10 times the normal full-load current and rectifier blocking is essential.

A large short circuit current causes a large temperature rise in the valve and may result in permanent damage. The limiting parameter is the transformer winding leakage reactance. If this were originally designed for a low value, serious consequences may result should an arcback occur.

A further effect of this short circuit current is the resulting force on the transformer windings. The large currents involved cause axial forces on the transformer windings and care must be taken to ensure adequate construction and bracing.

Otherwise, the windings may shift in the tank with respect to the core position and affect the normal operation of the unit.

Several types of arcback may occur, depending on the number of valves involved and how the arcback is terminated. These are:

- i) successful, where only two valves are involved.

  These are the arcback valve and the valve conducting in the forward direction ( valves 1 and 2 respectively in figure 3-3). The arcback is terminated when the forward conducting valve is blocked.
- ii) partially successful, where the two forward conducting valves are involved for one period of conduction each, but are blocked as their currents go to zero.
- iii) unsuccessful blocking, where the two forward conducting valves are unable to block.

The type of arcback generally designed for is the condition known as "partially successful" .

Figure 3-3 shows the rectifier, phase voltages and valve currents during the arcback. At  $\omega t = \alpha + \mu$ , valve 1 has commutated out. If at this time, since it now has negative voltage on the anode with respect to the cathode, it arcs back, a short circuit exists on transformer phases a and b . The driving voltage is the difference between  $e_a$  and  $e_b$ .

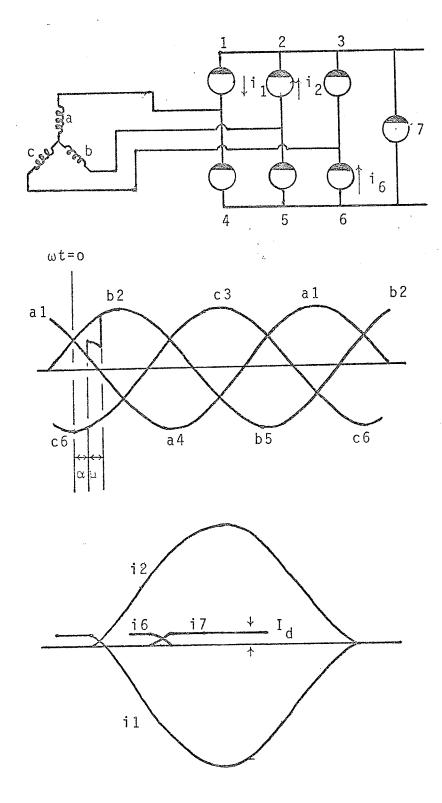


Figure 3-3: Circuit conditions, phase voltages and arcback current for arcback involving valves 1 and 2

When the arcback occurs, the non-conducting valves are blocked and the bypass valve de-blocked. At  $\omega t=90^{\circ}$ , the voltages across valve 6 and the bypass valve are equal and commutation between the two begins. At the end of this commutation period, valve 6 is blocked and only valves 1, 2 and 7 conduct.

The arcback current is determined by the driving potential (  ${\rm e}_{\rm b}$  -  ${\rm e}_{\rm a}$  ) and the reactances in phases a and b. The current through valve 2 is given by

$$\frac{di_2}{dt} = \frac{1}{2L} (e_b - e_a)$$

and after integration

$$i_2 = \frac{-\sqrt{3} E_m}{2 \omega L} \cos \omega t + c$$

where (  $e_b - e_a$  ) =  $\sqrt{3} E_m \sin \omega t$ 

 $\omega L$  = reactance in each phase supplying the arcback.

Substituting the boundary condition that at

$$\omega t = \alpha + \mu$$
,  $i_2 = I_d$ 

the constant of integration is

$$c = \frac{\sqrt{3} E_{m}}{2\omega L} \cos (\alpha + \mu) + Id$$

and hence,

$$i_2 = \frac{\sqrt{3} E_m}{2\omega L} \qquad \left[ \cos (\alpha + \mu) - \cos \omega t \right] + I_d$$

Differentiating  $i_2$  with respect to  $\omega t$  and equating the resulting expression to zero, indicates that the maximum value of  $i_2$  occurs at  $\omega t = 180^{\circ}$ . At this point, the arcback current magnitude begins to decrease and reaches zero at  $\omega t = 360^{\circ} - \alpha$ .

The maximum arcback current is given by the expression

$$i_2 = \frac{\sqrt{3} E_m}{2\omega l} \left[ \cos (\alpha + \mu) + 1 \right] + I_d$$

However, 
$$X_c = \frac{2\omega L I_d F.L.}{\sqrt{3} E_m F.L.}$$
 (2.9)

and, substituting, this expression yields

$$i_2 = \frac{I_d F.L}{X_c} \cdot \left[ \cos (\alpha + \mu) + 1 \right] + I_d$$

which at full load becomes

$$i_2 = \frac{I_d F \cdot L}{X_c} \left[ 1 + \cos (\alpha + \mu) + X_c \right]$$

Let 
$$K = \begin{bmatrix} 1 + \cos(\alpha + \mu) + X_c \end{bmatrix}$$

$$\therefore i_2 = \frac{K I_d F.L.}{X_c}$$

For typical values of  $\alpha$ ,  $\mu$  and  $X_c$ , K is of the order 1.9 . However, in the calculation, the commutation reactance was assumed equal to the arcback reactance. In actual cases the peak arcback current  $^{20}$  is given more closely by

$$I_{ab} (peak) = \frac{K I_d}{X_{ab}} F.L. \qquad (3.2)$$

where

 $I_{dF.L.}$  = rated valve group current if there is more than one valve group per pole

 $X_{ab}$  = rated reactance between the voltage source (generation) and the valve group (per unit)

K = constant depending on the arcback condition, circuit connection and X/R ratio of the arcback impedance and is typically equal to 1.94 for partially successful blocking.

There are 5 cases to be considered when choosing the correct commutation reactance to limit arcback currents. These can be grouped according to the type of ac system to which they

are connected.

- Strong AC System
  - i) with filters
  - ii) without filters
- 2. Weak AC System
  - i) with filters
  - ii) without filters low reactance case
  - iii) without filters high reactance case

In this context a strong ac system is defined <sup>17</sup> as one where the system short circuit capacity at the point of connection to the converter is very much larger than the converter MVA rating. A weak ac system is defined as one where the system short circuit capacity at the point of connection to the converter is comparable to the converter station MVA rating.

Case 1-i) can be represented by the circuit in figure 3-4. The arcback reactance is given by

$$X_{ab} = X_s + X_c \tag{3.3}$$

where the per unit system reactance on the transformer base is

$$X_s = \frac{\pi}{3} \quad V_o \quad I_d / S$$

S = short circuit capacity of the system in MVA

$$\frac{\pi}{3}$$
 V<sub>o</sub> I<sub>d</sub> = MVA<sub>b</sub> = transformer MVA rating

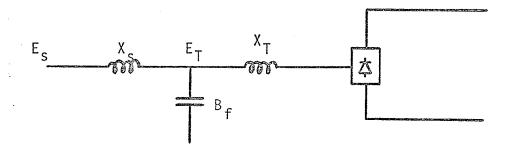


Figure 3-4: Equivalent circuit of ac system with filters (  $X_c = X_T$  )

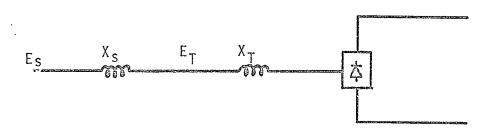


Figure 3-5: Equivalent circuit of ac system without filters ( $X_c = X_s + X_T$ )

Substituting equation (3.2) into (3.3) and solving for the commutation reactance  $X_{\rm C}$  yields

$$X_c = \frac{K I_d F.L. - X_s}{I_{ab}}$$

which gives the minimum commutation reactance necessary to limit the arcback current.

Case 1-ii) is shown in Figure 3-5. Since there are no filters in this case, the point  $E_{\mathsf{T}}$  is not the source of fundamental ac voltage. As a result, the commutation reactance is given by

$$X_c = X_s + X_T$$

where

$$X_s = \frac{MVA_b}{s}$$

The arcback reactance is then

$$X_{ab} = X_c = X_s + X_T$$

Case 2-i) can be represented by Figure 3-4. In this case, the commutation reactance  $X_{\rm c}$ , is chosen to satisfy the limitation on the current derivative. Since the system reactance is quite high, the minimum arcback reactance is inevitably surpassed and only the arcback current need be

calculated. The arcback reactance is given by

$$X_{ab} = X_s + X_c$$

However, because the system reactance can be quite high,  $E_s$  can be considerably higher than  $E_T$  and a load flow should be done to determine its value. Knowing the voltage behind reactance and the arcback reactance, the arcback current can be determined.

Case 2-ii) is a difficult case to design for, since the commutation reactance which is comprised of the system reactance plus the converter transformer reactance is equal to the arcback reactance.

The commutation reactance must be small enough that double commutation does not occur, yet the arcback reactance must be large enough that the arcback current limit is not exceeded. This condition may be difficult to satisfy simultaneously in an economic manner.

In the context of case 2-iii), high reactance indicates a reactance large enough to cause double commutation. This case has not yet been applied to a dc transmission scheme and is very difficult to study analytically.

## III.2 Limiting D.C. Side Fault Current

Thyristors are more temperature sensitive than mercury arc valves. If subjected to excessive temperatures, they will be destroyed.

For this reason, the maximum current a thyristor may be required to conduct under a fault condition must be known. Several papers <sup>18, 19</sup> have been written regarding the fault currents in thyristor valves. Since thyristors are destroyed if current is conducted in the reverse direction, no phenomenon equivalent to arcback in mercury arc valves need be considered with respect to thyristors. However, faults on the dc side can result in high through-valve currents. With respect to thyristor rectifiers, the two worst fault locations are shown in figure 3-6.

These fault conditions are:

- i) across the bridge terminals at the commencement of commutation between two valves.
- ii) across a valve which has completed commutation.

Faults beyond the smoothing reactor are much lower in magnitude due to the large smoothing reactance.

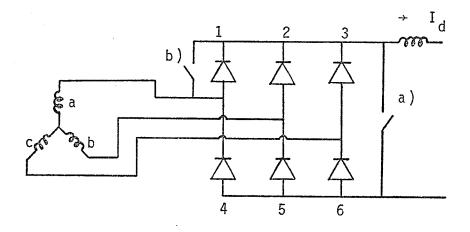


Figure 3-6: Rectifier bridge showing location of two worst dc faults a) across the bridge, b) across a valve

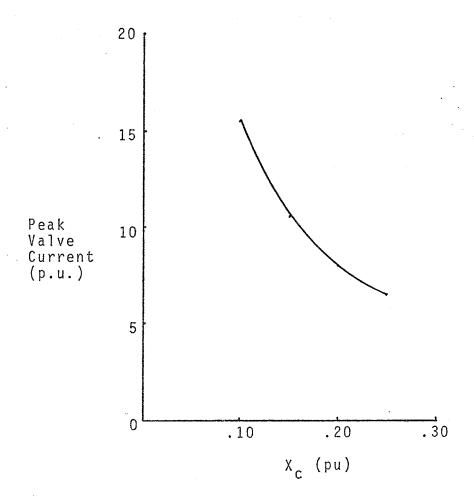


Figure 3-7: Peak valve current due to a fault across the bridge at the beginning of commutation (rectifier)

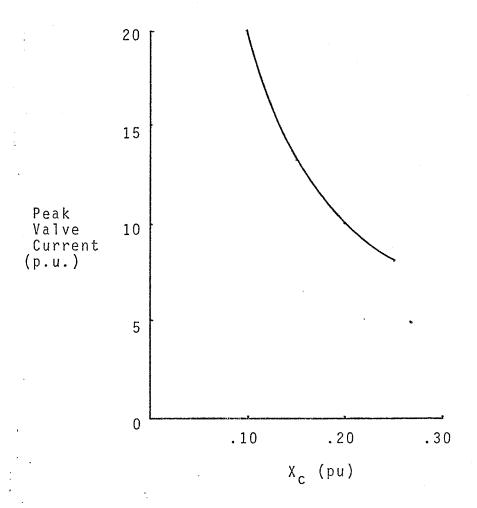


Figure 3-8: Peak valve current due to a fault across a valve at the end of commutation (rectifier)

If immediate blocking of non-conducting valves is assumed, the fault current is a single uni-directional pulse with a peak magnitude in the order of 10 p.u.

As is the case with arcback, the limiting parameter is the circuit reactance.

Figures 3-7 and 3-8 show the fault current magnitude as functions of the commutation reactance for the case where a system of infinite short circuit capacity is supplying the rectifier. Appendix II gives the derivation of the applicable equations used in determining these curves.

Although the fault of case ii) results in the higher thyristor fault current, as compared to case i), under certain system conditions, the reverse may be true.

Case i) involves a commutation during the fault interval since the fault is applied as commutation commences. As a result, the limiting reactance is only the commutation reactance. The system reactance is excluded if self-tuned filters are used.

Case ii) does not involve a commutation and can be compared to the arcback cases discussed previously. As a result,

the system reactance must be included in the fault current calculation. As a result, if the system reactance is significant, the thyristor current during this fault may be less than the corresponding current for case i).

Hence both conditions should be studied when evaluating the highest thyristor fault current magnitude.

## Chapter IV

#### EFFECT OF REACTANCE ON INVERTER OPERATION

mentioned that after completion of conduction by a valve, a period of time during which a negative anode-cathode voltage exists across the valve is necessary for de-ionization of the valve. Otherwise, if the anode to cathode voltage becomes positive before complete de-ionization occurs and grid control takes effect, the valve will conduct.

To reduce inverter var requirement, the de-ionization angle is normally as small as reliable inverter operation will allow. However, under certain conditions, when a number of inverters are operating in series and part of the commutation reactance is common to all inverters, it may be difficult to maintain an adequate de-ionization angle.

This chapter examines the circuit conditions which may cause this difficulty, the effect on the operation of the inverters and discusses a number of methods of eliminating this condition.

Figure 4-1 shows the three phase voltages existing on the ac side of an inverter and the voltage across valve 6. As can be

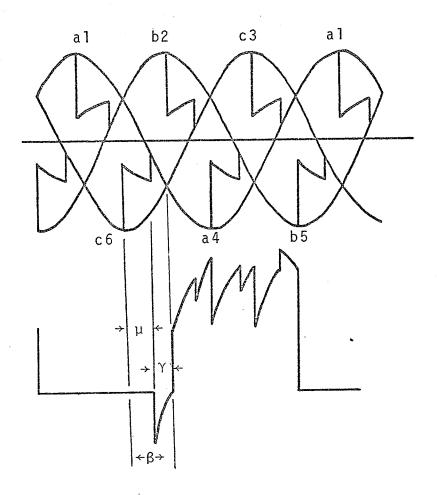


Figure 4-1: Phase voltages and voltage across valve 6 for an inverter.

seen, the period of time corresponding to negative voltage corresponds to the de-ionization period. Also, in the case shown,  $\mu + \gamma = 60^{\circ} = \beta$  where  $60^{\circ}$  is the angle between successive commutations. For larger commutation angles, the period  $\gamma = 60^{\circ} - \mu$  will be decreased and may reach a point where adequate de-ionization time is unavailable. This condition exists for large values of direct current or for a large commutation reactance.

To provide for both the minimum de-ionization time and the possibility of a large commutation overlap angle, the advance angle of firing ( $\beta$ ) can be increased. This leads to larger reactive power consumption and higher voltage stresses on the valves. For this reason, the inverter should be designed such that the commutation overlap angle does not increase beyond the practical range of approximately  $20^{\circ}-30^{\circ}$ .

Consider the operation of two 6-pulse inverters operating in series on the dc side and in parallel on the ac side as a 12-pulse group. In certain applications where filtering of the harmonics is not required, the commutating reactance for each group is made up of the corresponding converter transformer reactance and the system reactance. In such a case, the system reactance is common to both groups. As a result, a commutation occurring in one valve group affects the operation of the second valve group.

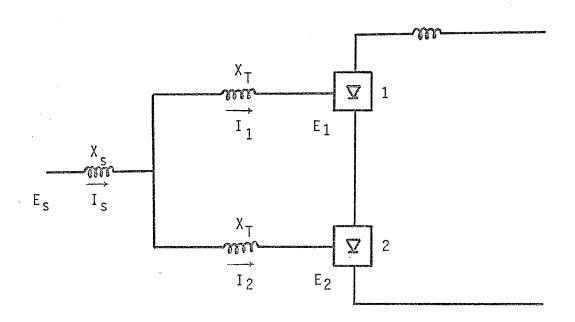


Figure 4-2: Single phase equivalent circuit showing the common system reactance

Figure 4-2 shows a single phase circuit which illustrates this, where  ${\rm E_s}$ ,  ${\rm E_1}$  and  ${\rm E_2}$  are measured with respect to a common reference.

Should E  $_1$  change due to a commutation within bridge 1, the current I  $_1$  will change by some amount  $\Delta I\,.$ 

Initially,

$$E_1 = E_s - L_s = \frac{dI_s}{dt} - L_T = \frac{dI_1}{dt}$$

and

$$E_2 = E_s - L_s = \frac{dI_s}{dt} - L_T = \frac{dI_2}{dt}$$

where

 $I_1 + I_2 = I_s$  and inductances  $L_s$  and  $L_T$  correspond to the reactances  $X_s$  and  $X_T$ 

After the change in  $I_1$  has occurred, the equations

become

$$E_{11} = E_s - L_s \frac{dI_s}{dt} - L_T \frac{d(I_1 + \Delta I)}{dt}$$

where  $I_s$  is now

$$I_1 + \Delta I + I_2$$

and

$$E_{21} = E_s - L_s \qquad \frac{dI_s}{dt} - L_T \qquad \frac{dI_2}{dt}$$

$$= E_s - L_s \qquad \frac{d(I_1 + \Delta I + I_2)}{dt} \qquad - L_T \qquad \frac{dI_2}{dt}$$

Now, the change in  $\mathbf{E}_2$  due to a change in  $\mathbf{E}_1$  is given by

$$\frac{E_{21} - E_{2}}{E_{11} - E_{1}} = \frac{-\frac{L_{s}}{\frac{d(\Delta I)}{dt}}}{-\frac{L_{s}}{\frac{d(\Delta I)}{dt}} - \frac{d(\Delta I)}{dt}}$$
$$= \frac{\frac{L_{s}}{L_{s} + L_{T}}}$$

From this it can be seen that the mutual interaction effect can be negated by making  $\mathsf{L}_\mathsf{S}$  equal to zero. That is, to remove the mutual effect of the system inductance.

Figure 4-3 shows the voltage across a typical valve in a 12-pulse group where mutual commutation reactance is a factor.

The dents a and b are due to commutations in the second valve group. Of particular importance is the dent a. From figure 4-3, the total angle between the end of one commutation and the end of the next successive commutation is  $30^{\circ}$ . If the commutation overlap angle is greater than that shown in the

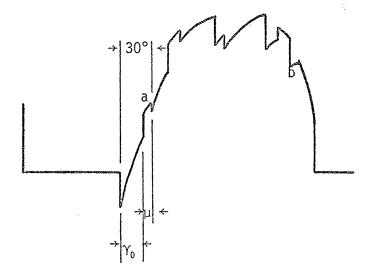


Figure 4-3: Voltage across a typical valve in a 12-pulse group with common reactance.

diagram (  $\mu > 30^{\circ} - \gamma_{\circ}$  ), the de-ionization period would be decreased and the effects would be the same as those described earlier in this chapter. As mentioned, a practical value of  $\mu$  is  $20^{\circ}-30^{\circ}$ . This would require the de-ionization angle to be approximately  $10^{\circ}$  which is considerably less than the practical value of  $18^{\circ}$ .

For this reason, it is necessary to eliminate the mutual effects described.

#### IV.1 Methods of Compensation

In HVDC transmission schemes for which self-tuned harmonic filters are required, the ac system reactance does not take part in the commutation process. The filters which effectively short circuit the high frequency current components to earth, prevent their entering into the ac network and result in an essentially sinusoidal voltage on their terminals. For this reason the commutation reactance is only the reactance between the filters and the individual valve groups of which no portion is common to both groups. Hence, the use of self-tuned filters eliminates the mutual interaction completely.

A second method which may be used when filters are not required is shown in Figure 4-4. In this case, which has been

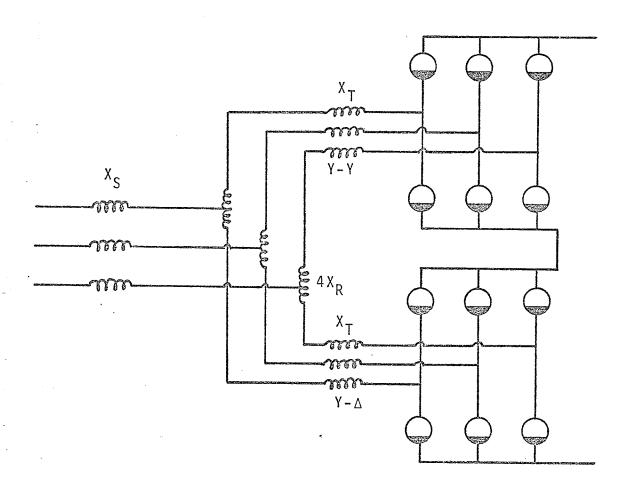


Figure 4-4: Compensation of common reactance by use of centre-tapped reactors.

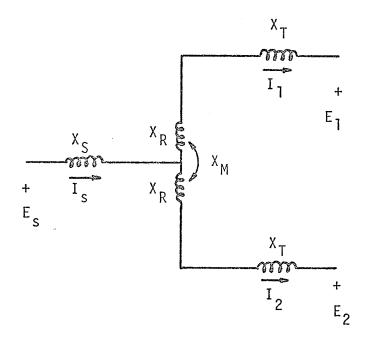


Figure 4-5: Single phase equivalent of centre-tapped reactor compensation scheme.

described in the literature  $^{10}$ ,  $^{11}$ , a centre-tapped reactor is placed in each phase between the ac system and each valve group as shown.

The required reactance of these reactors can be determined in the following manner. Figure 4-5 shows a single phase equivalent of the scheme.

In this scheme, an increase in current  $I_1$  due to a commutation in group 1, increases the voltage drop across the ac system reactance resulting in a change in  $E_2$ . To compensate, such that group 2 is unaffected by this occurrence, the mutual coupling between the two halves of the centre-tapped reactor must be such that the increase in  $I_1$  results in a corresponding voltage rise across the half of the reactor  $X_R$  associated with group 2.

Therefore, writing the equations

$$E_1 = E_s - X_s I_s - X_R I_1 + X_M I_2 - X_T I_1$$

$$E_2 = E_s - X_s I_s - X_R I_2 + X_M I_1 - X_T I_2$$

and substituting

$$I_s = I_1 + I_2$$

results in

$$E_1 = E_s - X_s I_1 - X_s I_2 - X_R I_1 + X_M I_2 - X_T I_1$$
  
 $E_2 = E_s - X_s I_1 - X_s I_2 - X_R I_2 + X_M I_1 - X_T I_2$ 

To eliminate the effect of  $\mathbf{I}_2$  on  $\mathbf{E}_1$  and  $\mathbf{I}_1$  on  $\mathbf{E}_2$ 

$$- X_{s}I_{2} + X_{M}I_{2} = 0$$

and

$$- X_{S}I_{1} + X_{M}I_{1} = 0$$

This condition is satisfied for  $X_s = X_M$ 

The commutation reactance for each valve group is  $X_S + X_R + X_T$ . To minimize  $X_R$  and still satisfy the relationship  $X_S = X_M$  a reactor with a coupling coefficient of approximately 1.0 is required. As a result each half of the reactor is equal to  $X_S$ , hence  $X_R = X_S$ .

The resulting commutation reactance becomes  $X_T + 2X_S$  which is higher than the original  $X_S + X_T$ . However, the transformer can be designed for a lower leakage reactance. The effect of the compensating reactor is to make each valve group appear to be supplied by a separate ac system.

The total reactance of the centre-tapper reactor can be found from consideration of Figure 4-6. Assuming a supply between terminals 1 and 2, the following loop equation can be written

$$E_{12} = X_T I + X_R I + X_R I + X_M I + X_T I$$

Combining terms

$$E_{12} = [2X_T + 2(X_R + X_M)] I$$

The voltage drop due to the centre-tapped reactor is  $2(X_R + X_M)I$  which results in an equivalent reactance of  $2(X_R + X_M) = 4X_R = 4X_S$ .

Since the system reactance  $(X_s)$  tends to vary depending on the ac system configuration,  $X_R$  should be chosen to correspond to the largest value of  $X_s$ . When  $X_R$  is greater than  $X_s$ , overcompensation occurs and the dents caused by commutation in an adjacent valve group become negative. No serious difficulty arises from this condition.

The preceding discussion applies to both mercury arc and thyristor valves. With thyristors, however, the required minimum de-ionization angle is slightly less than that for mercury arc, and, as a result, compensation may not be required in some cases where it would be if mercury arc valves were used.

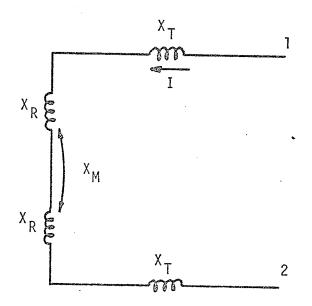


Figure 4-6: Circuit considered to evaluate total reactance of the centre-tapped reactors.

## Chapter V

#### HARMONICS

The non-linear operation of conversion devices results in the generation of higher order harmonics of the fundamental frequency. These harmonics are found in the ac current supplying the converter and in the direct voltage generated by the converter.

This chapter examines the effect of commutation reactance on the harmonics generated by an HVDC converter. The effect on current and voltage harmonic magnitude is pointed out, as well as the influence on the generation of characteristic and uncharacteristic current harmonics.

A simplifying assumption relating to harmonic studies in HVDC systems is that the direct current is perfectly smooth with no ripple (infinite inductance on the dc side of the converter). This assumption simplifies the mathematical analysis of current harmonics and, since in practice the dc line reactors are of the order of 1 henry, the resulting error is not of sufficient magnitude to warrant the further complication resulting from the consideration of ripple.

# V.1 Characteristic Harmonics

To appreciate better the influence of reactance on characteristic harmonics, they will be considered both with and without reactance in the commutating circuit.

#### Neglecting Commutation Reactance

The assumption of no commutation reactance results in a rectangular ac line current waveshape. The lack of inductance between the source of sinusoidal voltage and the converter allows for an infinite rate of rise of current through the valves when commutation occurs from the nonconducting to the conducting states.

Assumption of a finite firing angle only results in a shift of the current waveforms with respect to the commutating voltage waveforms and does not affect the harmonic magnitude except to alter the magnitude of the direct current.

Figures 5-1 and 5-2 show the ac line currents feeding converters through Y-Y and Y- $\Delta$  connected converter transformers respectively.

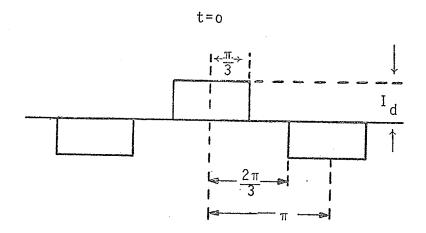


Figure 5-1: Y-Y transformer secondary phase current.

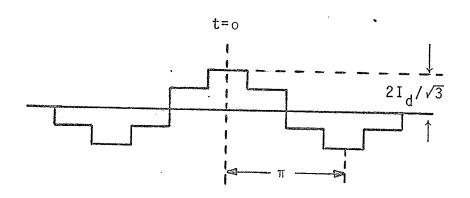


Figure 5-2:  $Y-\Delta$  transformer secondary phase current.

In the literature<sup>1</sup>, it has been shown that the resulting Fourier analysis of these waveforms gives the solutions

$$i_{\gamma-\gamma} = \frac{2\sqrt{3}}{\pi} I_d \left[\cos \omega t - \frac{1}{5}\cos 5 \omega t + \frac{1}{7}\cos 7 \omega t - \frac{1}{11}\cos 11 \omega t + \frac{1}{13}\cos 13 \omega t + \ldots\right]$$

$$i_{\gamma-\Delta} = \frac{2\sqrt{3}}{\pi} I_d \left[\cos \omega t + \frac{1}{5}\cos 5 \omega t - \frac{1}{7}\cos 7 \omega t - \frac{1}{11}\cos 11 \omega t + \frac{1}{13}\cos 13 \omega t + \ldots\right]$$

The harmonics can be seen to be of the order  $6k\pm1$  where k is any integer.

The rms value of any harmonic component can be evaluated by standard procedures and is

$$I_{(n)_0} = \frac{1}{\sqrt{2}} \left( \frac{2\sqrt{3}}{n\pi} I_d \right) = \frac{\sqrt{6}}{n\pi} I_d$$

which for the fundamental results in

$$I_{(1)_0} = \frac{\sqrt{6}}{\pi} I_d$$

Previously it had been shown that the total rms current evaluated for a rectangular waveform is

$$I_{d} = \sqrt{\frac{2}{3}} I_{d}$$
 (2.3)

Of particular practical importance in the study of HVDC converter generated harmonics is that the summation of  $i_{Y-Y}$  and  $i_{Y-\Delta}$  results in the cancellation of harmonics of order 5, 7, 17, 19, .... and the doubling of harmonics of order 11, 13, 23, 25, .... In practice, this is accomplished by paralleling Y-Y and Y- $\Delta$  converter transformers on the ac side. Under this condition, the 5, 7, 17, 19, .... order harmonics circulate between the transformers and do not enter the ac system except under conditions of valve group unbalance.

Therefore, filters are necessary only for the harmonics of order  $n = kp \pm 1$  where

p = 6 for 1 bridge

p = 12 for 2 bridges phase displaced by  $30^{\circ}$ 

In theory, through the use of phase shifting, p can be increased to 24 for four bridges and result in additional harmonic cancellations but this practice is uneconomical due to the additional complexity of the transformers.

# Finite Commutation Reactance

Consideration of finite commutation reactance results in a different current waveform than that illustrated in figures 5-1 and 5-2. Figure 5-3 shows the voltage and current waveforms

resulting from consideration of finite delay and commutation angles.

The resulting current waveform can be analyzed by dividing it into three parts. Taking t=o as shown in figure 5-3, these are

1) 
$$i_p = I_d \left[ \frac{\cos \alpha - \cos \omega t}{\cos \alpha - \cos (\alpha + \mu)} \right]$$
 for  $\alpha < \omega t < \alpha + \mu$ 

2) 
$$i_q = I_d$$
 for  $\alpha + \mu < \omega t < \alpha + \frac{2\pi}{3}$ 

3) 
$$i_r = I_d \left[ 1 - \frac{\cos \alpha - \cos (\omega t - \frac{2\pi}{3})}{\cos \alpha - \cos (\alpha + \mu)} \right]$$

$$for \alpha + \frac{2\pi}{3} < \omega t < \alpha + \mu + \frac{2\pi}{3}$$
(5.1)

Fourier analysis has been completed on this waveform and results have been illustrated in the literature  $^{1}$  as plots of harmonic current as a percentage of the fundamental versus varying commutation angles for different angles of delay.

These curves indicate that

- i) harmonic magnitude as a percentage of the fundamental decreases with increasing angle
- ii) higher order harmonics tend to decrease more rapidly than lower orders. Each reach a minimum

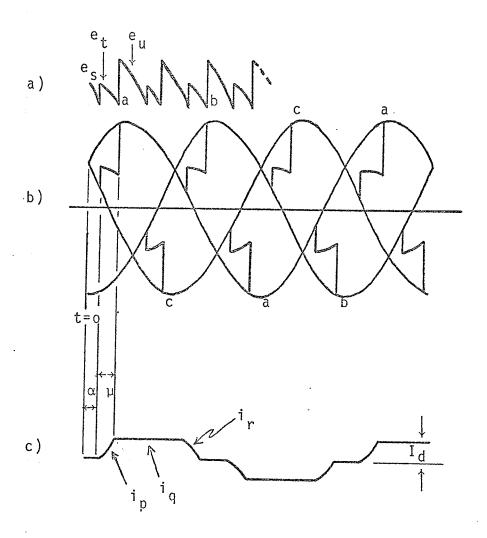


Figure 5-3: Current and voltage waveforms considering finite delay and commutation angles.

- a) output dc voltage
- b) phase voltages
- c) transformer secondary phase current

at an angle  $\mu$  =  $\frac{2\pi}{n}$  and increase slightly thereafter.

- iii) for a constant angle  $\mu$ , changes in the magnitude of the harmonics with changes in angle  $\alpha$ , are slight.
- iv) as  $\mu$  decreases, the magnitude of the harmonics increase and reach a maximum not exceeding

$$I_{(n)_0} = \frac{I_{(1)_0}}{n}$$

A further useful relationship which has been developed is the fundamental in terms of  $I_{(1)}$  and  $I_{(1)}$ , its active and reactive components, as a function of  $\mu$ .

Assuming a sinusoidal ac voltage supplying the converter, harmonic current components can be neglected since harmonic voltages do not exist at the supply point. As a result the exact ac power feeding the converter is

$$P_{ac} = \sqrt{3} E_{L} I(1)_{a}$$

Neglecting losses and equating the ac and dc power,  $I\left(1\right)_{a} \quad \text{can be found in terms of the dc parameters } \alpha \text{ and } \mu \text{ and a constant}$ 

$$K_{(1)_a} = \frac{\cos \alpha + \cos (\alpha + \mu)}{2}$$

can be defined to relate  $I_{(1)}$  and  $I_{(1)}$ .

The ac side reactive power resulting from phase shift and commutation angle can be found from the equation

$$P_{r} = \frac{1}{2\pi} \int_{0}^{\pi} e_{q} i d\omega t$$

where e is the phase-to-phase voltage across the converter bridge and can be written

$$e = \sqrt{3} E_m \cos (\omega t - \pi/6)$$

considering the point of voltage crossover ( zero commutating voltage ) as reference. Therefore, the quadrature component of e is

$$e_q = \sqrt{3} E_m \sin (\omega t - \pi/6)$$
.

The current I, using the same reference, must be divided into the three parts  $i_p$ ,  $i_q$ ,  $i_r$  discussed previously.

Substitution results in a new constant relating  $^{\rm I}$ (1) $_{\rm r}$  and  $^{\rm I}$ (1) $_{\rm o}$ 

$$K_{(1)_r} = \frac{2\mu + \sin 2\alpha - \sin 2(\alpha + \mu)}{4 \left[\cos \alpha - \cos(\alpha + \mu)\right]}$$

The fundamental component of current is found from

$$I_{(1)} = \sqrt{I_{(1)}_{a}^{2} + I_{(1)}_{r}^{2}} = I_{(1)}_{0} \sqrt{K_{(1)}_{a}^{2} + K_{(1)}_{r}^{2}}$$

The use of the factors  $K_{\left(1\right)_a}$  and  $K_{\left(1\right)_r}$  gives the relationship between the parameters  $\alpha$  and  $\mu$  and the exact converter power factor. The exact power factor can be evaluated from

$$\sqrt{3}$$
 E<sub>L</sub> I<sub>(1)</sub> cos  $\theta$  = V<sub>d</sub> I<sub>d</sub>

Substituting  $I_{(1)} = I_{(1)} = I$ 

$$\cos \theta = \frac{\cos \alpha + \cos (\alpha + \mu)}{2 \int_{0}^{K(1)} \frac{2}{a} + K(1)_{r}^{2}}$$

and is readily identified as the approximate power factor modified by the term

$$\frac{1}{\sqrt{K_{(1)_a}^2 + K_{(1)_r}^2}}$$

This term also relates  $I_{(1)}$  and  $I_{(1)}$  as

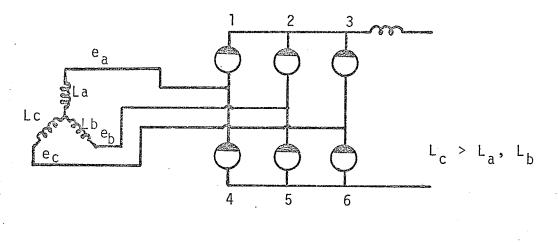
$$\frac{I(1)_{0}}{I(1)} = \frac{1}{\sqrt{K(1)_{a}^{2} + K(1)_{r}^{2}}}$$

## V.2 Uncharacteristic Harmonics

Up to this point, only characteristic harmonics have been discussed; that is, harmonics of order  $kp\pm1$ . Factors responsible for producing uncharacteristic harmonics are summarized by Kauferle, Mey and Rogowsky based on references 7, 8 and others. These factors are:

- i) control angle dissymmetryi.e. firing pulses are not equally spacedor of equal duration.
- iii) dissimilar commutating reactances.

When one or more of the three conditions listed above are found to exist, harmonics of orders other than those termed "characteristic" can occur. It can be concluded that all three factors result in valve conduction periods which are not equal.



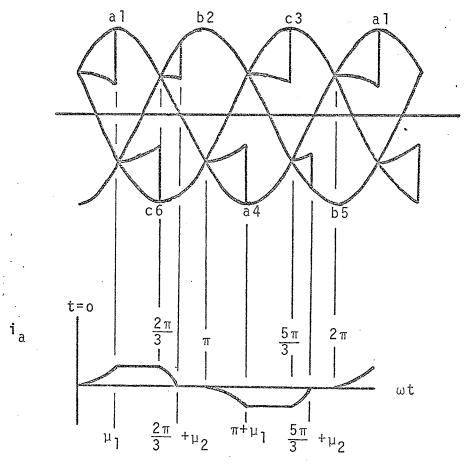


Figure 5-4: Phase voltages and phase a current corresponding to unbalanced commutation reactance.

With respect to this thesis, let us now consider the case where the commutation reactance in one of the phases supplying the rectifier is not equal to that in the other phases.

Figure 5-4 shows the supply transformer secondary, the phase voltages and the phase a current corresponding to such a condition where  $L_{\rm c}$  is greater than  $L_{\rm a}$  or  $L_{\rm b}$ .

As a result of the unbalanced phase reactances, the overlap angles at the beginning and at the end of conduction are not equal (  $\mu_1 \neq \mu_2$  ) for phase a or b .

The resulting current waveform can be analyzed for harmonic content in the same manner as shown previously in this chapter. For analysis purposes, the waveform must be divided into eight intervals.

i) 
$$i(\omega t) = I_d \qquad \left[ \frac{1 - \cos \omega t}{1 - \cos \mu_1} \right] \quad \text{for } o < \omega t < \mu_1$$

ii) 
$$i(\omega t) = I_d$$
 for  $\mu_1 < \omega t < 120^0$ 

iii) 
$$i(\omega t) = I_d \left[ 1 - \frac{1 - \cos(\omega t - 60^\circ)}{1 - \cos\mu_2} \right]$$
  
for  $120^\circ < \omega t < (120^\circ + \mu_2)$ 

iv) 
$$i(\omega t) = 0$$
 for  $(120^{\circ} + \mu_2) < \omega t < 180^{\circ}$ 

v)  $i(\omega t) = -I_d \left[ \frac{1 - \cos(\omega t - 180^{\circ})}{1 - \cos\mu_1} \right]$ 

for  $180^{\circ} < \omega t < (180^{\circ} + \mu_1)$ 

vi)  $i(\omega t) = -I_d$  for  $(180^{\circ} + \mu_1) < \omega t < 300^{\circ}$ 

vii)  $i(\omega t) = -I_d \left[ 1 - \frac{1 - \cos(\omega t - 300^{\circ})}{1 - \cos\mu_2} \right]$ 

viii) 
$$i(\omega t) = 0$$
 for  $(300^{\circ} + \mu_2) < \omega t < 360^{\circ}$ 

for  $300^{\circ} < \mu_1 < (300^{\circ} + \mu_2)$ 

From Fourier Series Theory, the harmonic components can be found by substituting into the following formula

$$i(\omega t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega t + b_n \sin n\omega t$$

where  $a_0 = 0$  since, over one period, the waveform has an average value of zero and

$$a_n = \frac{1}{\pi} \int_0^{2^{\pi}} i(\omega t) \cos n\omega t d\omega t$$

$$b_n = \frac{1}{\pi} \int_0^{2\pi} i(\omega t) \sin n\omega t \, d\omega t$$

A computer program was written to calculate the  ${\bf a}_n$  and  ${\bf b}_n$  terms and the magnitude of the resulting harmonics for varying values of  $\mu_1$  and  $\mu_2$  .

Figures 5-5 to 5-8 show the results as plots of the magnitude of the harmonic current as a percentage of the fundamental, versus  $\mu_1$  for varying  $\mu_2.$ 

Figures 5-5 and 5-6 show the third and ninth harmonics which are not characteristic. As can be seen from these graphs, their magnitude becomes zero for the condition of equal phase reactances as shown by equal  $\mu_1$  and  $\mu_2$ . For unbalances, the harmonic magnitude increases.

As in the case with characteristic harmonics, the harmonic magnitudes decrease for increasing harmonic number.

Figures 5-7 and 5-8 show the characteristic fifth and seventh harmonics. The graphs indicate that the magnitudes of these harmonics have alternate maxima and minima as  $\mu_1$  differs from  $\mu_2$ . At locations where  $\mu_1 = \mu_2$ , the corresponding magnitudes are the same as those given in the literature  $^1$  for zero delay angle.

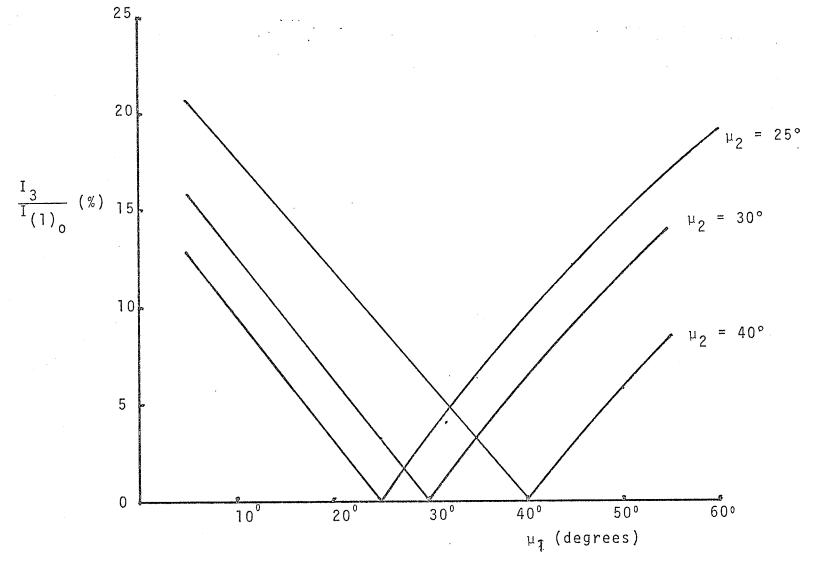


Figure 5-5: Variation of the third harmoic magnitude with varying  $\mu_1$  and  $\mu_2$ .

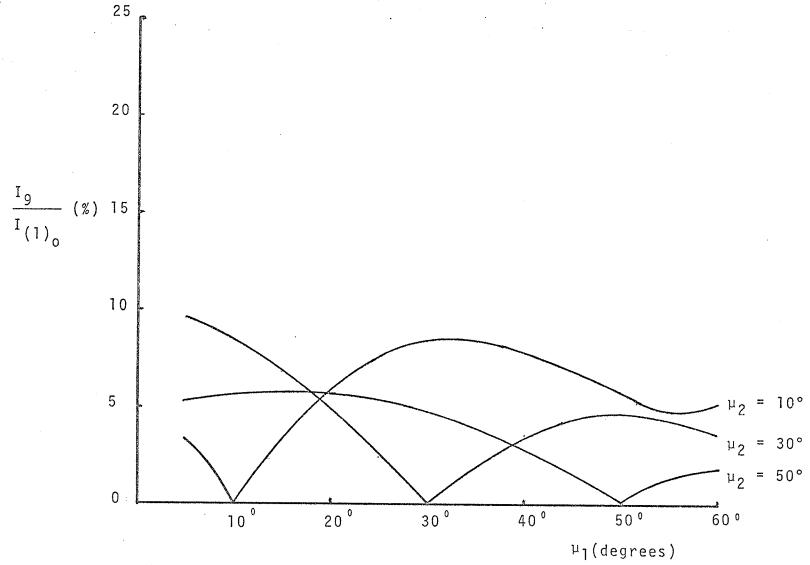


Figure 5-6: Variation of the ninth harmonic magnitude with varying  $\mu_1$  and  $\mu_2$ .

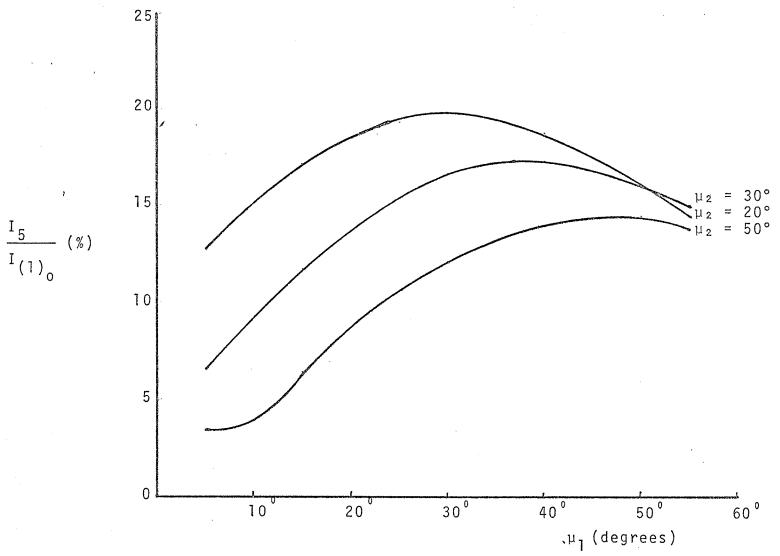


Figure 5-7: Variation of the fifth harmonic magnitude with varying  $\mu_1$  and  $\mu_2$ .

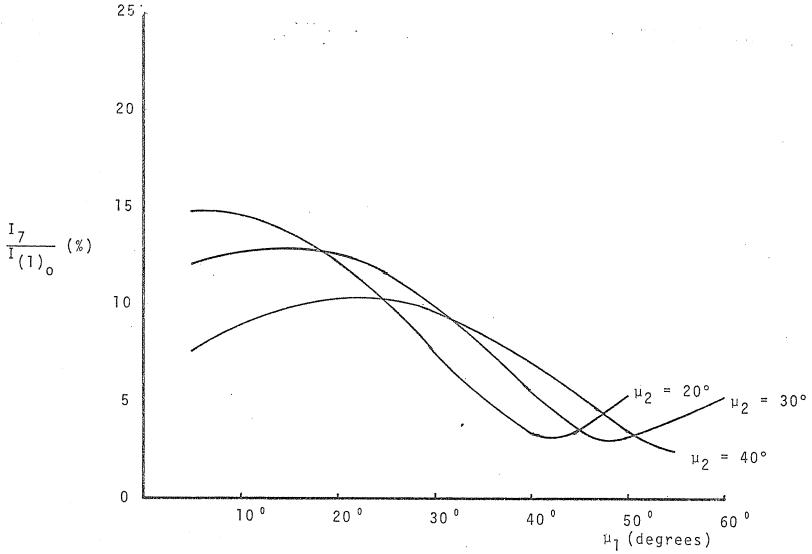


Figure 5-8: Variation of the seventh harmonic magnitude with varying  $\mu_1$  and  $\mu_2$ .

Under the condition of unbalanced phase reactances, even harmonics are not generated but odd triple harmonics are ( that is 3rd, 9th, 15th, 21st, .... ).

In the cases studied, the delay angle was assumed to be zero (natural commutation). The consideration of finite delay angles would not affect the generation of the odd triple harmonics although the magnitudes would be altered. The increased complexity in evaluating the Fourier components did not warrant the consideration of finite delay.

# V.3 Direct Voltage Harmonics

The output voltage of an HVDC rectifier is dependent on the number of phases, the delay angle, and the commutation overlap angle. Figure 5-3 shows a typical output waveform for a 6 pulse converter.

To evaluate the harmonics present in the waveform, it can be divided into three parts

$$e_s = \sqrt{3} E_m \cos (\omega t + \pi/6)$$
 for  $0 < \omega t < \alpha$ 

$$e_t = \frac{1}{2} \left[ \sqrt{3} E_m \cos (\omega t - \pi/6) + \sqrt{3} E_m \cos (\omega t + \pi/6) \right]$$

$$=\frac{3}{2} E_{m} \cos \omega t \qquad \qquad \text{for } \alpha < \omega t < (\alpha + \mu)$$

$$e_{u} = \sqrt{3} E_{m} \cos (\omega t - \pi/6) \qquad \qquad \text{for } (\alpha + \mu) < \omega t < \pi/3$$

The results of the Fourier analysis of this waveform are found in the literature  $^{1}$ . The rms value of the nth harmonic is

$$V_{(n)} = \frac{V_0}{\sqrt{2(n^2-1)}} \int \frac{(n-1)^2 \cos^2([n+1] \mu/2) + (n+1)^2 \cos^2([n-1] \mu/2) - (n-1)(n+1) \cos([n+1] \mu/2) \cos([n-1] \mu/2) \cos(2\alpha + \mu)}{2(n-1)(n+1) \cos([n+1] \mu/2) \cos([n-1] \mu/2) \cos(2\alpha + \mu)}$$

This expression reduces to  $V_{(n)_0} = V_0$   $\frac{\sqrt{2}}{(n^2-1)}$  for the ideal case of  $\alpha = \mu = 0^0$  and where

n = mp

with

m = integer

p = phase number

Curves of  $\frac{V\left(n\right)}{V_{0}}$  plotted against  $_{\mu}$  for different values of  $\alpha$  are available in  $^{0}$  reference 1.

These curves indicate:

- i) unlike current harmonic magnitudes which decrease with increasing values of  $\alpha$  for constant  $\mu$ , direct voltage harmonic magnitudes increase with increasing  $\alpha$  for constant  $\mu$ .
- harmonic magnitudes for given values of  $\alpha$  decrease and reach a minimum at a value of  $\mu = \frac{\pi}{n}, \text{ increase again to a maximum at}$   $\mu \approx \frac{2\pi}{n}, \text{ then decrease to a second minimum at}$   $\mu \simeq \frac{3\pi}{n}. \text{ Harmonics corresponding to small values}$  of  $\alpha(<10^{0})$  tend to increase over the practical range of commutation overlap angle.
- iii) at  $\mu$  =  $\frac{\pi}{n+1}$  and  $\frac{\pi}{n-1}$  , the harmonic magnitudes are equal for all values of  $\alpha.$

## V.4 Effects of Harmonics on the System

As has been shown, current and voltage harmonics are a byproduct of the conversion process. Current harmonics, both characteristic and uncharacteristic, can cause difficulties in the ac system supplying the converter as well as in the converter itself. These harmonics, and in particular, the lower order harmonics which are of higher magnitude can be responsible for any of the following:

- in ac machines without damper windings or with inadequate damper windings, the presence of harmonic currents causes additional heating in the rotors.
  - ii) in capacitors, harmonic voltages increase the dielectric loss and increase the possibility of dielectric breakdown.
  - iii) ac systems which are not resonant at the supply frequency may be resonant at one of the harmonic frequencies. Overvoltages on the system due to resonance can cause equipment failures.
  - iv) harmonics causing distortion of the converter supply voltage may shift the natural commutating point. The result is unequal firing angles in rectifiers and unequally timed advance angle of firing in inverters. The result is generation of uncharacteristic harmonics which may add to the distortion already existing and cause further inaccuracies in firing.
  - v) generation of lower order harmonics which fall in the audio range may cause interference with voice frequency communication systems if these are in the vicinity of the ac transmission lines.

Fortunately, these harmonics can be reduced in magnitude in a number of ways. The use of harmonic filters to bypass the current harmonics to ground is one of the most effective for characteristic harmonics. Through the use of the inherent 30° phase shift of wye-delta connected converter transformers and harmonic filters, the effect of characteristic harmonics can be virtually eliminated economically. Filters could also be used for uncharacteristic harmonics; however, this is usually an uneconomic approach. The best solution to non-characteristic harmonic generation is proper design of the converter controls, adequate filtering of characteristic harmonics and ensuring equal commutation reactances in each phase.

### Chapter VI

## REGULATION AND REACTIVE POWER REQUIREMENTS

The commutation reactance as seen by the converter plays an integral part in determining equipment ratings, reactive power requirements, voltage regulation and various other system parameters. This chapter examines the effect of commutation reactance in determining system requirements.

Consider the rectifier and ac system shown in figure 6-1. The components making up this system include the dc rectifier, a converter transformer which is represented as an ideal transformer with a turns ratio of n:1 and a constant reactance  $X_{\mathsf{T}}$  representing the leakage reactance, a full set of self-tuned filters on the high voltage bus and an ac system which supplies the real and reactive power to the rectifier.

Since a set of self-tuned filters is present, the commutation reactance is equal to the converter transformer leakage reactance. In the initial design of the system, an economic valve group rating is chosen. Subsequent to this, the minimum commutation reactance, to satisfy arcback or thyristor fault current requirements, is determined in the manner explained in Chapter III. This sets a minimum commutation reactance limit.

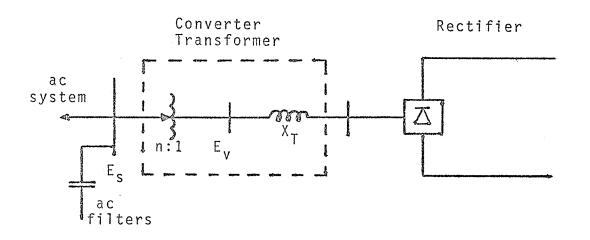


Figure 6-1: Circuit for determining converter transformer tap range and leakage reactance.

### VI.1 Regulation

At this stage, the converter transformer tap range, leakage reactance and MVA rating is determined. Economics will generally determine the leakage reactance provided the minimum reactance is exceeded. Standard reactances are available for transformers and higher or lower reactances generally involve additional design and hence are more costly.

The maximum loading requirement of the dc system then determines the MVA rating of the transformer as well as the required regulation. To see the basic effect of commutation reactance on the choice of converter transformer rating and tap range, consider the rectifier of figure 6-1 under the following conditions.

Assume that the valve group is rated for 1000 A dc at 200 KV dc with a firing angle of 10°. The filter bus is held at 230 KV under all loading conditions (ac system has infinite capacity). To determine the optimum converter transformer leakage reactance, rating and tap range consider practical transformer reactances in the range 15% to 30% on the transformer base.

The commutation overlap angle for a transformer reactance of 15% and a rectifier loading as above can be calculated from

$$\mu = \cos^{-1} \left[ \cos \alpha - X_c \right] - \alpha \qquad (2.10)$$

Substituting  $\alpha$  =  $10^{\,0}$  and  $\rm X_{\,C}$  =  $\rm X_{\,T}$  = 0.15 p.u., the resulting overlap angle is 23.4  $^{\,0}$  .

At rated load, the power factor is given by 
$$\lambda = \cos \phi = \frac{3}{2\pi} \frac{\cos \alpha + \cos (\alpha + \mu)}{\sqrt{1 - 3 \Psi (\alpha, \mu)}}$$
 (2.7)

where the term (  $\sqrt{1-3~\Psi~(\alpha,\mu)}$  ) evaluated for  $\alpha$  =  $10^{0}$ ,  $\mu$  = 23.4 is 0.970 .

Substituting the above values, the power factor is 0.896 and  $\phi$  = 26.4°. Since the overlap angle is included, this calculated power factor is that seen at the primary of the converter transformer and includes the var loss in the transformer.

The average dc no load voltage from equation (2.4) is

$$V_0 = \frac{2 V_d}{\cos \alpha + \cos (\alpha + \mu)} = \frac{400 \text{ KV}}{1.8196} = 219.8 \text{ KV}$$

and the corresponding ac r.m.s. line-to-line commutating voltage is

$$E_{V} = \frac{\pi V_{O}}{3\sqrt{2}} = 162.8 \text{ KV}$$

The required transformer rating is then

$$MVA_{T} = 1.047 V_{oF.L.}I_{dF.L.}$$

$$= (1.047) (219.8 KV) (1000 A)$$

$$= 23 0. 1 MVA$$

where the factor 1.047 is used to allow margin for harmonics in the ac current.

The transformer reactance, in ohms, on the valve winding base is

$$\omega L = \frac{E_v^2 X_c}{MVA_T} = 17.28 \Omega$$

The valve Winding voltage,  $\boldsymbol{E}_{\boldsymbol{v}},$  is related to the primary bus voltage by

$$\frac{E_s}{E_v} = \frac{n}{1}$$

and since  $\mathbf{E}_{s}$  is held at 230 KV, the required turns ratio is

$$n = \frac{E_s}{E_V} = \frac{230 \text{ KV}}{162.8 \text{ KV}} = 1.413$$

This gives the required turns ratio at maximum load ( rated load ).

If the minimum load transmitted by this dc system is 10% of maximum capability, the maximum transformer turns ratio is determined in the following manner.

The direct current at 200 KV dc is 100 A. If it is assumed that operation at  $\alpha$  =  $10^0$  is preferred, the transformer tap changer will operate to satisfy

$$V_{0} = \frac{V_{d} + \frac{3\omega L I_{d}}{\pi}}{\cos \alpha}$$

where

$$V_d = 200 \text{ KV}$$

$$\omega L = 17.28\Omega$$

$$I_d = 100A$$

$$\alpha = 10^{\circ}$$

The result is

$$V_0 = 204.8 \text{ KV}$$

and the corresponding ac line-to-line commutating voltage is

$$E_V = \frac{\pi V_0}{3\sqrt{2}} = 151.6 \text{ KV}$$

which corresponds to a turns ratio of

$$n = \frac{230 \text{ KV}}{151.7 \text{ KV}} = 1.517$$

The tap range, defined as maximum turns ratio required divided by the turns ratio at maximum load minus one is

$$(\frac{1.517}{1.413} - 1.0)$$
 100% =  $7.36\%$ 

These calculations were done for transformer reactances in the range 15% to 30% and the resulting tap ranges under the same maximum and minimum loading conditions, are shown in figure 6-2.

The conclusion drawn from figure 6-2 is that as the transformer reactance increases, the required tap range also increases.

The actual choice of tap range depends on other factors in addition to the loading. Some factors which could affect the tap range are

- i) variation in the voltage on the ac bus. In the example, this was kept constant at 230 KV.
- ii) operation for extended periods of time with a firing angle greater than 10° results in a larger reactive power demand.

## VI.2 Reactive Power Requirement

Once a commutation reactance is chosen to satisfy arcback and fault limiting requirements, the reactive power requirement of the converter must be considered.

If an HVDC converter could be operated with no delay angle and had no reactance in the commutating circuit, the ac voltage and current would be in-phase. The rectifier would appear as a unity power factor load to the ac system and the only reactive power required from the ac system would be that necessary to supply the  $I^2X$  losses in the ac transmission.

In practical systems, several factors influence the reactive power consumption of a converter. With the introduction of a delay angle, the voltage and fundamental current waveforms are no longer in phase. A lagging power factor is introduced and reactive power is required. Similarly, consideration of reactance in the ac circuit introduces the concept of commutation overlap. A shift in phase occurs and a lagging power factor is again introduced into the circuit. During commutation, a phase-to-phase short circuit exists on the secondary of the converter transformer for the duration of the commutation overlap period. This is essentially a zero power factor load if circuit resistance is neglected and requires generator vars.

To illustrate the converter var demand due to overlap and delay angles and particularly to show the variation with commutation reactance, the calculations of the previous section were extended. Consider figure 6-3 which shows the variation of reactive to real power for different values of commutation reactance for the rectifier of figure 6-1.

For these curves, the reactive to real power ratio increases with a slope of approximately 1:1 with the per unit commutating reactance if a linear relationship is assumed. For a rectifier rated 1000 MW with a firing angle of zero, an increase in commutating reactance from 0.2 to 0.25 p.u. results in a reactive power requirement increase from approximately 500 MVAR to 550 MVAR.

Since an increasing var demand in the ac system generally means an increase in var generating capability (addition of synchronous condensers, capacitor banks or an increase in generator var capability), it is economically appealing to keep the commutation reactance as small as possible consistent with thyristor or mercury arc valve requirements.

The supply of extra reactive power increases the cost of the dc scheme.

As well, a higher var demand by the rectifier affects the converter transformer rating and hence, the cost. Figure 6-4 shows the variation in transformer MVA rating with commutation reactance and figure 6-5 shows the variation of the valve winding voltage with commutation reactance. Both factors increase as commutation reactance increases, thereby increasing the cost.

At the receiving end, the reactive power requirement is a function of the de-ionization angle  $\gamma$  and inverter commutation overlap angle. The magnitude is comparable to the rectifier reactive power requirement.

The foregoing discussion indicates clearly that it is advantageous to choose as small a transformer reactance as is practical. The result is a more economic system based on reduced ratings, var and regulation requirements.

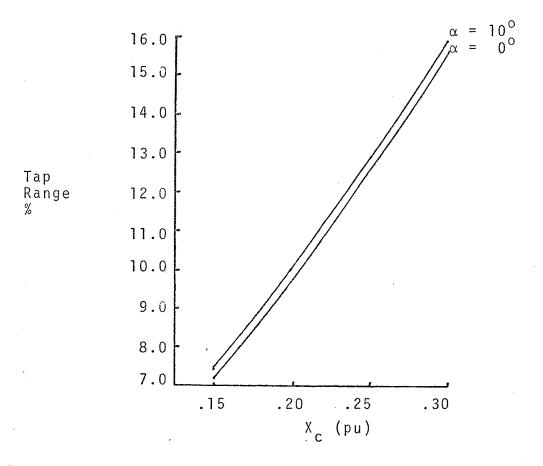


Figure 6-2: Variation of the tap range as a function of per unit commutation reactance for two values of  $\boldsymbol{\alpha}$ 

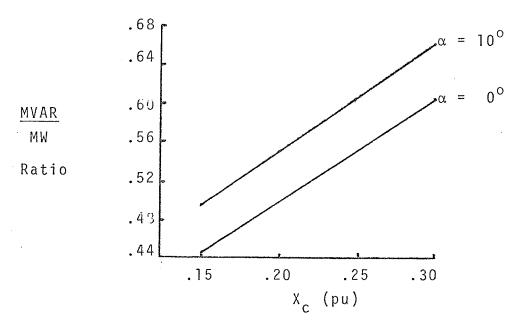


Figure 6-3: Variation of the MVAR/MW ratio as a function of per unit commutation reactance for two values of  $\alpha$  (at full load).

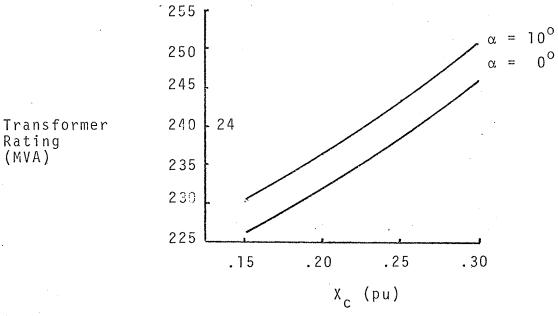


Figure 6-4: Variation of transformer rating as a function of per unit commutation reactance for two values of  $\alpha.$ 

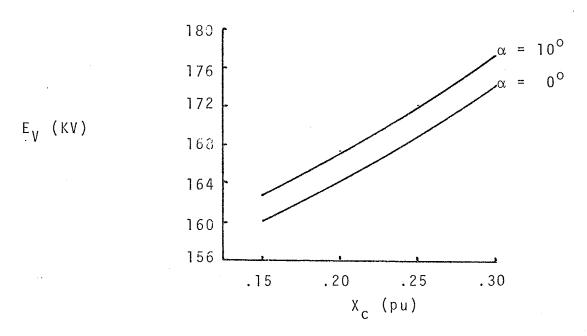


Figure 6-5: Variation of the commutating voltage as a function of per unit commutation reactance for two values of  $\alpha.$ 

#### Chapter VII

#### EFFECT OF APPARATUS ON COMMUTATION REACTANCE

In HVDC schemes, the ac systems at the receiving and sending ends effect the commutation reactance as seen by the converters. In all cases, each or a number of the following pieces of equipment are required:

- 1) converter transformers
- 2) anode reactors
- 3) synchronous condensers
- 4) harmonic filters
- 5) generators, transformers and transmission lines.

In the following sections, each of the items listed will be discussed as to its effect on the dc system but with particular emphasis on its contribution to the commutation reactance.

## VII.1 Converter Transformers

Converter transformers are required primarily to step down the transmission voltage to the level required for the efficient operation of the converter. On load tap changers with a wide tap range are used to maintain this level for widely varying transmission line loadings, although their response time

is too slow ( in the order of 1 second per tap change ) to benefit the system during a transient condition.

If auxilliary equipment is required, such as synchronous condensers or harmonic filters, it may be most economic to include a low voltage tertiary winding in the transformer. A synchronous condenser or filter on the tertiary does not require insulation rated for the high primary voltage.

There are three basic converter transformer winding configurations applicable to HVDC schemes. These are

- a) two winding
- b) two winding with a low voltage tertiary
- c) primary winding with two valve windings.

In each case, the leakage reactances are applicable in the calculation of the commutation reactance. These circuits are shown in figure 7-1. For scheme c), since there is a common primary winding for two valve windings, there may be an interaction between valve groups during commutation. For this reason, compensation may be required.

The reactances contributing to the commutation reactance in each of the three cases are:

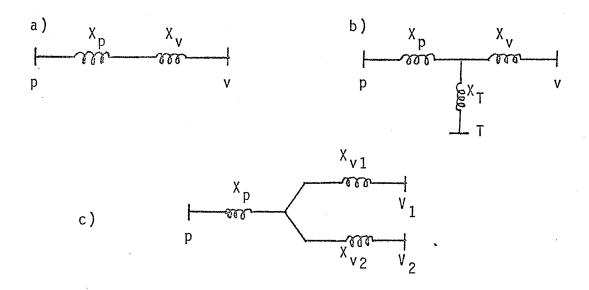


Figure 7-1: Basic converter transformer equivalent circuits:

- a) two windingb) two winding with low voltage tertiaryc) primary winding with two valve windings.

- a)  $X_c = X_p + X_v + \text{system reactance}$
- b)  $X_c = X_v + ((X_p + system reactance) in parallel with (X_T + tertiary to ground reactance))$
- c)  $X_c = X_{v_1} + (X_p + system reactance)$

The practical values of the winding reactances listed above can be varied by the designer depending on the system requirements. In practical applications where a tertiary winding is required, it is located between the primary and valve windings to insure a small tertiary reactance magnitude.

#### VII.2 Anode Reactors

As the delay angle of the converter valves is increased, the anode-cathode voltage prior to conduction increases. Various stray capacitances between the anode and cathode discharge when conduction occurs resulting in a high frequency transient which affects the current through the valve. "Ringing" occurs as the current builds up from zero to  $I_d$ . Several difficulties can result from this transient, the most important of which are radio frequency interference and arc-quenching of the valve.

The frequency of the oscillations is often in the radio frequency range and if not suppressed can affect programming

reception in the vicinity of the converter station. The second difficulty results from the amplitude of the oscillations. The amplitude may be large enough to cause the valve current to pass through a zero. The result is arc-quenching and corresponding high overvoltage.

A high frequency choke in series with the anode can alter the frequency and amplitude of the oscillations, such that these difficulties can be controlled.

The anode reactor adds in series to the valve winding reactance of the converter transformer. To decrease the combined reactance, the transformer reactance can be designed for a lower value and the overall effect on the commutation reactance will be unchanged.

### VII.3 Synchronous Condensers

HVDC converters require a large amount of reactive power due to the phase-to-phase short circuit which occurs during commutation and which essentially is a zero power factor load on the system for the overlap period and also due to the commutation delay angle which causes a phase displacement between the ac voltage and current.

For typical systems, the reactive requirement may amount to 60% - 70% of the real power transfer corresponding to a converter power factor of 82% - 86%.

The ac systems at each end of the dc transmission system may not have adequate capacity to supply the var requirement. Several alternative sources are readily available. Synchronous condensers, filters or capacitor banks can be used for this purpose. Synchronous condensers operated in the overexcited state can supply vars to the system.

Due to the advanced controls available, synchronous condensers have the added advantage of giving excellent transient support to aid system stability. Their presence also increases the system short circuit capacity which increases the effectiveness of the converter controls.

Synchronous condensers are normally located on the tertiary bus of the converter transformer. Several advantages are gained from this location, primarily, a decrease in converter transformer MVA rating and a lowering of the insulation requirements.

In the determination of commutation reactance, the subtransient reactance of the synchronous condenser is used.

Such is the case since commutation is essentially a phase-tophase short circuit of the transformer secondary.

#### VII.4 Harmonic Filters

The presence of harmonics in the ac system is not compatible with good system operation. At the present time, the most successful method of eliminating harmonics is through the use of harmonic filters.

These filters are generally tuned R, L, C series circuits in which a separate arm is provided for each harmonic current expected in the system. The presence of harmonic currents can be found analytically through the methods described in chapter V and during operation of the dc scheme, these results can be checked by actual system measurements.

The majority of schemes incorporate filters for the 5th, 7th, 11th and 13th harmonics as well as a high pass filter for higher harmonics. Each arm is tuned to appear as a low resistance path compared to the ac system for the respective harmonic current.

Theoretically there are three possible locations for the filters. These are:

a) on the system side (primary) of the converter

transformer,

- b) on the tertiary of the converter transformer, and
- c) on the valve side of the converter transformer.

In actual practice, c) is unacceptable in that it results in a very low commutation reactance by excluding the converter transformer reactance from participating in the commutation process. As a result, currents due to arcback are dangerously high and the time derivative of the current at  $I_d$  =0 is higher than the valves can withstand.

Location b) results in a saving in the filter costs due to the lower insulation. Ideally the tertiary winding reactance should be zero, to ensure that no characteristic harmonics enter the ac system. This is not practically possible.

The most favoured location is at the primary bus where guarantees of harmonic voltage can be made. However, in the case where synchronous condensers are not used to supply reactive power, these vars are supplied by the system and the filters and result in a larger converter transformer MVA rating.

A further consideration is the use of self-tuned filters. As the filter goes off-tune due to a frequency variation in the

system or a parameter change due to a change in temperature, either the inductor or capacitor in the filter arm is varied to bring the filter back in tune. The best method for self-tuning is the use of a control system which monitors VI sin  $\phi$  where

V = harmonic voltage across the filter

I = harmonic current through the filter

 $\phi$  = angle between V and I.

Under tuned conditions, V and I are in phase and the impedance of the filter is the arm resistance R. Under this condition, the quantity VI sin  $\phi$  is zero and no change is required in the tuning. When VI sin  $\phi$  is not zero, the simplest tuning arrangement is to change the arm inductance.

Self-tuning filters are particularly advantageous where the possibility of de-tuning is great. For large, stiff systems where frequency variations are very small, the advantages of self-tuning are lost.

When connected in parallel with the ac system, these filters provide a low impedance path to ground for the harmonics. As a result, the harmonic voltage at the primary bus is small due to the small  $I_n$  drop across the filter. For this reason, the filter bus is assumed to be the source of fundamental frequency voltage and reactances on the ac system side do not take part in

commutation.

## VII.5 Generators, Tranformers, and Transmission Lines

In HVDC systems where self-tuned filters are used, ac transmission reactance has no effect on the commutation reactance.

Sending end systems, however, can be designed with no filtering. This is possible where the rectifier is in an isolated location and electrically near the generation. In such a case, the generators are designed with sufficient capability to absorb the harmonic currents generated by the converter.

Under this condition, the transmission line impedance, generator transformer impedance and subtransient reactance of the generators must be included in the calculation of the commutation reactance. To cover the entire range of possible commutation reactances, care must be taken to consider cases with different line and generator outages. Also, if several generating stations are combined to supply an HVDC rectifier station, the sharing of harmonics among the different generators must be known.

## Chapter VIII

#### CONCLUSIONS

The aim of this thesis was to show the importance of commutation reactance on the design and operation of an HVDC system. Since commutation reactance affects many aspects of the overall scheme, it was necessary to consider it with respect to the converters as well as the ac sending and receiving end systems.

It was shown that reactance in the commutating circuit is necessary to limit transient current through the thyristor or mercury arc valves. Based on this requirement, for a particular type and rating of valve, there is a minimum commutation reactance which must be sufficient to prevent the possibility of a major valve failure. Decreasing this limit, decreases the cost of the scheme, at the expense of reliability.

As well, economic considerations limit increasing the commutation reactance beyond that necessary to meet the valve requirements. Any increase in commutation reactance increases the steady-state reactive power requirement of the converter, the converter transformer rating and the tap range as well as increasing the voltage stresses on the valve. All of these factors result in higher equipment costs.

Consequently, reliability and economics must be considered simultaneously in deciding on the final value of commutation reactance.

With respect to the operation of several phase displaced inverters in series, it was shown that any reactance common to the commutating circuits of several inverters causes reduction of the de-ionization period available to the valves. When the de-ionization period is reduced beyond that necessary for successful valve de-ionization, it is necessary to eliminate the common reactance. Filters on the primary of the converter transformers exclude the common system reactance by establishing the commutating voltage at this location. Alternatively, compensating reactors are used which compensate the voltage drop across the common reactance seen by a valve group when a valve in an adjacent bridge begins to conduct. This is an important consideration which must also be considered.

It was also shown how commutation reactance alters the converter transformer phase current waveshape resulting in a change in the harmonic content. This only influences the magnitudes of characteristic harmonics.

Of particular significance was the generation of odd triple harmonic uncharacteristic current harmonics due to unbalanced phase reactances. These harmonics are not filtered

and hence enter the ac system.

A summary of equipment necessary for a dc transmission scheme with particular emphasis on its effect on commutation reactance was also included. Anyone involved in a dc scheme must be aware of the functions each serves and how they influence the reactance in the commutating circuit. As well, two methods are given for the measurement of the commutation reactance of an existing system.

In conclusion, it is hoped that this thesis provides an insight into the importance of understanding the role which commutation reactance plays in a dc scheme. Its effects on various aspects of the system, both technical and economic, cannot be taken lightly.

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## Appendix A1

#### MEASUREMENT AND CALCULATION OF COMMUTATION REACTANCE

In practical HVDC converter systems, it may be desirable to measure the commutation reactance as seen by the converter.

To simplify this measurement, it should be possible to make it accurately by obtaining the necessary dc parameters from an oscillogram and substituting into a mathematical expression.

Two methods described in the literature  $^{12}$  which allow this to be done are:

- i) commutation overlap method
- ii) overlap-area loss method.

To mathematically express the commutation reactance in terms of the dc parameters, the following assumptions are made:

- the phases have equal commutation reactance and the resistance is negligible as compared to the reactance.
- ii) the ac supply is balanced and sinusoidal.
- iii) external capacitances and the effect of capacitances on commutation are negligible.

#### A1.1 Commutation Overlap Method

Derivation of the mathematical expression is based on the period during which commutation occurs. If during this period, the commutation voltage between two phases is  $\sqrt{3}~E_m~\sin~\omega t,~then~integration~and~solution~for~i~in~the~loop~equation~yields$ 

$$i = \frac{\sqrt{3} E_{\rm m}}{2\omega L} \cos \omega t + C$$

Applying the boundary conditions that at t =  $\frac{\alpha}{\omega}$ , i = 0, and at t =  $\frac{(\alpha + \mu)}{\omega}$ , i = I d yields

$$I_{d} = \frac{\sqrt{3} E_{m}}{2\omega L} \left[ \cos \alpha - \cos (\alpha + \mu) \right]$$

Solving for  $\omega L$  yields

$$\omega L = \frac{\sqrt{3} E_{m}}{2 I_{d}} \left[ \cos \alpha - \cos (\alpha + \mu) \right]$$
 (A1.1)

The parameters  $E_m$ ,  $I_d$ ,  $\alpha$  and  $\mu$  can be measured from an oscillogram and substituted in (A1.1). The resulting value for  $\omega L$  is that reactance as seen by the commutating valves and can be compared to the theoretical value calculated from design parameters.

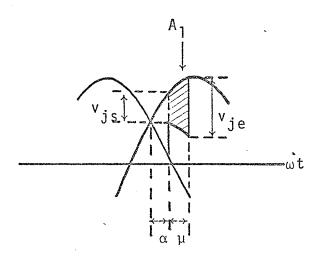


Figure A1-1:Parameters required to calculate  $\;\omega L\;$  by the  $\;$  Overlap-Area Loss Method.

# A1.2 Overlap-Area Loss Method

During the commutation of two valves, there is a drop in the dc output voltage.

 $\label{lem:commutation} \mbox{Figure A1-1 shows the voltage during commutation.} \\ \mbox{During this period}$ 

$$\int_{\alpha}^{\alpha_{+}\mu} e_{c} d\omega t = 2\omega L \int_{\alpha}^{I} dd_{c}$$

where

e<sub>c</sub> = commutation voltage
i<sub>c</sub> = commutation current

$$\int_{\alpha}^{\alpha+\mu} e_{c} d\omega t = 2\omega L I_{d}$$

which also equals two times the overlap-area loss. That is

$$2\omega L I_d = 2A_1$$

If the area  $A_1$  in figure A1-1 is considered to be a trapezoid with parallel sides  $v_{js}$  and  $v_{je}$  where  $v_{js} = \text{voltage jump across the valves at } \omega t = \alpha \text{, and}$ 

 $v_{je}$  = voltage jump across the valves at  $\omega t$  =  $\alpha$  +  $\mu$  and a width  $\mu$  equal to the overlap angle, then  $A_1 = \frac{1}{2} \; (v_{js} \; + \; v_{je}) \mu$  =  $\omega L \; I_d$ 

Therefore, measurement of  $v_{\mbox{\scriptsize js}},\ v_{\mbox{\scriptsize je}},$  and  $I_{\mbox{\scriptsize d}}$  yields  $\omega L,$  the commutation reactance.

# Appendix II

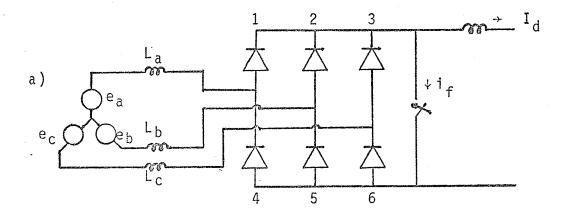
# CALCULATION OF THYRISTOR CURRENTS DUE TO D.C. SIDE FAULTS

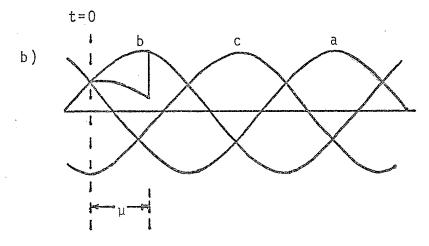
The two dc side fault conditions resulting in the highest thyristor currents, as stated in Chapter II, are

- fault across the bridge terminals at the commencement of commutation between two valves.
- fault across a valve which has completed commutation.

The equations for the thyristor current in terms of the commutating reactance will be determined for these two conditions under the following assumptions:

- a) ac system has infinite capacity and therefore system reactance has no effect.
- b) no change in  $I_d$  is assumed to take place in the interval considered.
- c) all phase inductances are equal.
- d) at the time of fault application, all nonconducting valves are blocked.





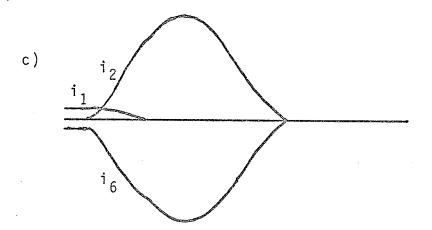


Figure A2-1: a) Circuit condition, b) phase voltages, and c) thyristor currents for a fault across the valve bridge terminals.

# AII.1 Fault Across the Bridge Terminals

Consider figure A2-1 which shows the circuit arrangement, ac voltages and thyristor currents.

The three ac phase voltages are defined as

$$e_a = E_m \cos (\omega t + 60^{\circ})$$

$$e_b = E_m \cos (\omega t - 60^0)$$

$$e_c = E_m \cos (\omega t - 180^0)$$

The worst condition will be considered, that is, a fault applied at  $\omega t$  = 0 when valves 1 and 2 begin to commutate.

The analysis is best approached by considering two intervals

i) 
$$0 < \omega t < \mu$$

# Interval i) $0 < \omega t < \mu$

During this interval, the following equation can be written:

$$e_a - L_a \frac{di_1}{dt} = e_b - L_b \frac{di_2}{dt} = e_c + L_c \frac{di_6}{dt}$$
 (A2.1)

where 
$$i_6 = i_f + I_d$$
  
and  $i_1 + i_2 = i_f + I_d$ 

From equation (A2.1),

$$e_a - e_b = L_a \frac{di_1}{dt} - L_b \frac{di_2}{dt}$$

however,

$$i_1 = i_6 - i_2$$

$$\therefore e_a - e_b = L \left[ \frac{di_6}{dt} - 2 \frac{di_2}{dt} \right]$$

and rearranging,

$$\frac{di_6}{dt} - 2 \frac{di_2}{dt} = \frac{e_a - e_b}{L} = \frac{-\sqrt{3} E_m \sin \omega t}{L}$$

Integration yields

$$i_6 - 2i_2 = \frac{\sqrt{3} E_m \cos \omega t}{\omega L} + C$$
 (A2.2)

and substituting the boundary condition that at  $\omega t$  = 0,  $i_2$  = 0 and  $i_6$  =  $I_d$  yields

$$C = I_d - \frac{\sqrt{3} E_m}{\omega}$$

Rearranging equation (A2.2) and substituting for the constant of integration C yields

$$i_6 - 2i_2 = \frac{\sqrt{3} E_m}{\omega L} \left[ \cos \omega t - 1 \right] + I_d$$

At the end of this interval, that is, at  $\omega t$  =  $\mu$  , i  $_6$  = i  $_2$  since commutation has been completed.

$$\therefore i_2 = \frac{\sqrt{3} E_m}{\omega L} \left[ 1 - \cos \mu \right] - I_d \qquad (A2.3)$$

However, to find the value of the overlap angle  $\mu,$  an equation for  $i_6$  is required. From equation (A2.1),

$$e_a - L_a \frac{di_1}{dt} = e_c + L_c \frac{di_6}{dt}$$

and after rearranging and integrating

$$i_1 + i_6 = \frac{\sqrt{3} E_m}{2\omega L} \left[ \sqrt{3} \sin \omega t + \cos \omega t \right] + C \qquad (A2.4)$$

At  $\omega t = 0$ ,  $i_1 = i_6 = I_d$ 

$$\therefore C = 2I_d - \frac{\sqrt{3} E_m}{2\omega l}$$

and equation (A2.4) becomes

$$i_1 + i_6 = \frac{\sqrt{3} E_m}{2\omega L}$$
  $\left[\sqrt{3} \sin \omega t + \cos \omega t - 1\right] + 2 I_d$ 

At the end of commutation,  $i_1 = 0$  and therefore

$$i_6 = \frac{\sqrt{3} E_m}{2\omega L} \left[ \sqrt{3} \sin \mu + \cos \mu - 1 \right] + 2I_d$$
 (A2.5)

To determine the overlap angle  $\mu$ , since at  $\omega t = \mu$ ,  $i_2 = i_6$ , equate equations (A2.3) and (A2.5) which yield

$$-\frac{3}{2}\cos \mu - \frac{\sqrt{3}}{2}\sin \mu = \frac{-3}{2} + \frac{3\omega L I_d}{\sqrt{3} E_m}$$
 (A2.6)

However,

$$X_{c} = \frac{2\omega L I_{dF}.L.}{\sqrt{3} E_{mF}.L.}$$
 (2.9)

and at full load, equation (A2.6) becomes

$$-3\cos\mu - \sqrt{3}\sin\mu = -3 + 3X_{c}$$
 (A2.7)

Equation (A2.7) can then be solved for  $\mu$  by successive substitutions.

# Interval ii) ωt > μ

Valves 2 and 6 are conducting and the following equation can be written

$$e_b - L_b \frac{di_2}{dt} = e_c + L_c \frac{di_6}{dt}$$

but  $i_2 = i_6$ 

$$\therefore e_b - e_c = 2L \frac{di_2}{dt}$$

Rearranging and integrating yields

and since, at  $\omega t = \mu$ 

$$i_2 = \frac{\sqrt{3} E_m}{\omega L} \left[ 1 - \cos \mu \right] - I_d$$
 (A2.3)

solving for C and substituting into equation (A2.8) yields

$$i_2 = \frac{\sqrt{3}E_m}{\omega L} \left[ \frac{\sqrt{3}}{4} \sin\omega t - \frac{\cos\omega t}{4} + 1 - \frac{3\cos\mu}{4} - \frac{\sqrt{3}}{4} \sin\mu \right] - I_d$$
(A2.9)

The maximum value of  $i_2$  can be found by differentiating equation (A2.9) with respect to t and setting it to zero. This yields

$$\frac{di_2}{dt} = 0 = \frac{\sqrt{3} E_m}{L} \left[ \frac{\sqrt{3}}{4} \cos \omega t + \frac{\sin \omega t}{4} \right]$$
 (A2.10)

and  $0 = \sqrt{3} \cos \omega t + \sin \omega t$ 

for which the solution is  $\omega t = 120^{\circ}$ . This corresponds to the point where the ac driving voltage,  $e_b - e_c$ , is zero.

Substituting 
$$\omega t = 120^{\circ}$$
,  $X_c = \frac{2\omega L I_{dF}.L.}{\sqrt{3} E_{mF}.L.}$  and

rearranging equation (A2.9) yields the maximum value of  $i_2$  corresponding to a full load condition.

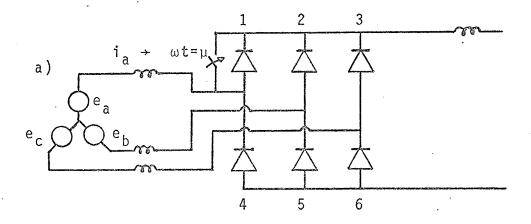
$$\frac{i_2}{I_{dF,L}} = \frac{\sqrt{3}}{\chi_c} \left[ \sqrt{3} \left( 1 - \frac{\cos \mu}{2} \right) - \frac{\sin \mu}{2} \right] - 1$$

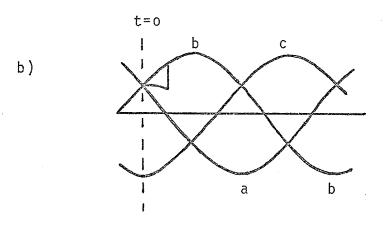
where  $\mu$  is as found from equation (A2.7).

The point where i  $_2$  becomes zero is found by setting i  $_2\text{=0}$  in equation (A2.9) and solving for  $\omega t,$  which yields

$$\sqrt{3}$$
 sin  $\omega t$  -  $\cos \omega t$  =  $2X_c$  - 4 + 3  $\cos \mu$  +  $\sqrt{3}$   $\sin \mu$ 

This equation can be solved for  $\omega t$  by successive substitution.





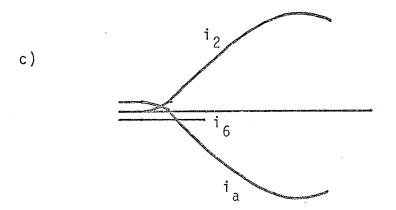


Figure A2-2: a) Circuit condition, b) phase voltages, and c) thyristor currents for a fault across a valve.

# AII.2 Fault Across a Valve

Consider figure A2-2 which shows the circuit arrangement, ac voltages and thyristor currents. The phase voltages are as defined previously.

The worst condition arises for a fault across valve 1 at  $\omega t$  =  $\mu$ , that is, when valve 1 has completed commutation.

When the fault is applied, the following equation can be written

$$e_a + L_a \frac{di_a}{dt} = e_b - L_b \frac{di_2}{dt}$$
 (A2.11)

It should be noted that phase a current is no longer equal to valve 1 current.

Rearranging equation (A2.11) and substituting  $i_a = i_2 - I_d$  results in

$$\frac{\text{di}_2}{\text{dt}} = \frac{e_b - e_a}{2L} = \frac{\sqrt{3} E_m \sin \omega t}{2L}$$

and integration yields

$$i_2 = \frac{-\sqrt{3} E_m}{2\omega L} \cos \omega t + C \qquad (A2.12)$$

At  $\omega t = \mu$ ,  $i_2 = I_d$ , therefore the constant of integration is

$$C = \frac{\sqrt{3} E_{m}}{2\omega I} \cos \mu + I_{d}$$

Substituting this into equation (A2.12) and rearranging yields

$$i_2 = \frac{\sqrt{3} E_m}{2\omega L} \left[ \cos \mu - \cos \omega t \right] + I_d \qquad (A2.13)$$

Differentiating equation (A2.13), setting  $\frac{di_2}{dt} = 0$ , and solving for  $\omega t$  yields  $\omega t = 180^{\circ}$ .

That is, the maximum value of  $i_2$  occurs at  $\omega t$  =  $180^{\circ}$  which corresponds to the ac driving voltage  $(e_b-e_a)$  being equal to zero.

Substituting  $\omega t$  =  $180^{\rm O}$  into equation (A2.13) yields the maximum value of  $i_2$ 

$$\frac{i_2}{I_{dF.L.}} = 1 + \left[ \frac{1 + \cos \mu}{X_{c}} \right]$$

#### APPENDIX III

#### CURRENT DERIVATIVE AT CURRENT ZERO

Referring to figure 5-3 which shows a typical transformer phase current waveform, the current derivative at the end of commutation can be evaluated from the portion of the current waveform referred to as  $i_r$ . From the text, the equation describing  $i_r$  is

$$i_r = I_d \left[ 1 - \frac{\cos \alpha - \cos (\omega t - \frac{2\pi}{3})}{\cos \alpha - \cos (\alpha + \mu)} \right]$$
 (5.1)

and  $i_r$  becomes zero at  $\omega t = \alpha + \mu + \frac{2\pi}{3}$ 

Differentiation with respect to time yields

$$\frac{di_r}{dt} = \frac{I_d}{\cos \alpha - \cos (\alpha + \mu)} \left[ -\omega \sin (\omega t - \frac{2\pi}{3}) \right] \quad (A3.1)$$

Substituting equation  $_{(2.10)}$  and evaluating (A3.1) at  $\omega t = \alpha + \mu + \frac{2\pi}{3} \ , \ \text{yields the magnitude of the current derivative}$  at current zero. The result is

$$\frac{di_r}{dt} = \frac{\omega I_{dF} \cdot L}{\chi_c} \sin (\alpha + \mu)$$

$$\omega t = \alpha + \mu + \frac{2\pi}{3}$$
(A3.2)