A Portfolio Optimization Model Combining Pooling and Group Buying of Reinsurance Under an Asset Liability Management Approach

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ABSTRACT

Some insurance firms are faced with the unique challenge of managing risks that are large, infrequent, and potentially highly correlated within geographic regions and/or across product lines. An example of this is crop insurance, which includes weather risk, and leads to a portfolio of risks with high variance. A solution to this problem is undertaken in this study, through using a combination of pooling and private reinsurance in a portfolio approach. This approach takes advantage of offsetting risks across regions, in order to reduce risk in a cost effective manner.

An asset liability management (ALM) approach is used to examine the entire crop insurance sector for Canada. This is the first study to focus on pooling for an entire insurance sector in a country, and it uses all major crops from 1978-2009, across 10 regions (provinces). Chapter two develops an innovative insurance portfolio under a full premium pool, combining a self managed insurance pool and private reinsurance using the coefficient of variation (CV) of the loss coverage ratio (LCR), Model 3. Results show that this portfolio approach reduces risk across regions.

Chapter three, in contrast to chapter two, uses a reinsurance premium pool, where regions contribute only a portion of their risk to a reinsurance pool. An improved insurance portfolio model is developed in chapter three, using combinatorial optimization with a genetic algorithm to combine a self managed reinsurance pool and private reinsurance, Model C. Results show that this reinsurance portfolio model efficiently reduces risk.

Chapter four uses a similar approach to chapter three, except that it allows for dependence (correlation) across regions. Results for this model (Model CC) are consistent with those of chapter three, indicating the effectiveness of the portfolio approach when correlation is present across regions. Overall, the portfolio models developed in each of the three chapters (Model 3, Model C, and Model CC), produce acceptable surplus, survival probability, and deficit at ruin, indicating that the portfolio approach using pooling is efficient for reducing risk. Beyond crop insurance, the portfolio models can be applied to other large natural disaster and weather related insurance, and other portfolio applications.

Keywords: risk management, asset liability management, portfolio optimization, genetic algorithm, combinatorics, pooling, reinsurance, crop insurance.

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CHAPTER 1

INTRODUCTION

Risk management is an integral part of organizational processes, and there is a potential for many insurance organizations to realize meaningful gains by optimizing their current risk management policies. Most insurance organizations are often faced with risks that are small, frequent, and uncorrelated, for which the variance of aggregate risks in the portfolio is low. But conversely, some firms are faced with a unique challenge of managing risks that are large, infrequent, and potentially highly correlated across geographic regions and/or product lines. This leads to a portfolio of aggregate risks with high variance.

A firm may reduce the risk associated with a portfolio of high variance risks by purchasing private reinsurance. This option can be costly though, as reinsurers may in some cases charge relatively large brokerage fees and high premiums for this service. As an alternative option that eliminates the higher cost associated with private reinsurance, a firm may manage risk internally by pooling (combining) its insurance business across a number of geographic regions and/or product lines. In practice, however, this approach is seldom fully implemented, as a self managed pool is often insufficiently diversified because risks may be large and correlated. As a result, private reinsurance is often needed in addition to pooling, if pooling is to be successful.

Three distinct approaches are explored in this study, and each successive approach is presented as a separate chapter (Table 1.1). Throughout this study, an asset liability management (ALM) approach is used to evaluate the three alternative models presented in each chapter. Risk measures including surplus, survival probability, and deficit at ruin are compared using a simulation model. This study examines the complete crop insurance sector for Canada, and considers the ten provincial non-profit crop insurance corporations. This is the first study to focus on pooling for an entire insurance sector in a country, and utilizes a comprehensive data set that includes 32 years of actual premiums and liabilities (from 1978-2009), across 10 regions (provinces) for 279 crop types.

Crop insurance is a particularly suitable and comprehensive example to consider for pooling and private reinsurance. One reason is that risks can be very large. A second reason is that crop insurance claims may have high variability from year to year, and weather liabilities are often correlated within regions yet are less correlated across regions (Wang and Zang, 2003). These two factors create a potential opportunity for pooling within this sector. For example, the provincial crop insurance firms in Canada do not currently diversify geographically across regions (provinces), but rather only diversify across products (crops) within a region (province).

Developing a Full Premium Insurance Pooling Model to Reduce Risk

Chapter two models a full premium pool, where all of the risks from each of the ten provinces are pooled together into one large countrywide pool (i.e. 100% of the crop insurance premiums are transferred from each region to the shared pool). This approach represents the maximum diversification that can be achieved within an insurance sector in an entire country. The objective of this chapter is to analyze the diversification that can be achieved through pooling insurance business across a number of geographic regions (provinces) and products (crops), as a possible solution to better manage a portfolio of aggregate risks with high variance. Figure 1.1 shows the three models that are developed under a full premium pool, including Model 1 (self managed insurance pool), Model 2 (self managed insurance pool that also purchases private reinsurance as a group), and Model 3 (a portfolio approach to combine a self managed insurance pool and private reinsurance using CV of the LCR). An eight step methodology is presented for the innovative insurance portfolio model, Model 3.

Reinsurance Premium Pool: Combinatorial Optimization with a Genetic Algorithm to Combine Pooling and Private Reinsurance to Reduce Risk

Chapter three, in contrast to chapter two, uses a countrywide reinsurance premium pool, where regions (provinces) contribute only a portion of their risk to the pool (i.e. only a portion of the insurance premium, reinsurance premium, from each region is transferred to the shared pool). This is different than chapter two, where a full premium pool was assumed. One of the objectives of chapter three is to model a reinsurance premium pool to address one of the potential limitations of chapter two, which is the possible reluctance of some provinces in transferring control of their region to cooperate in the pool. A reinsurance premium pool is an incremental approach to pooling which allows regions to continue operating independently, and pool only a portion of the risk. The second objective of chapter three is to develop an improved insurance portfolio model that combines a self managed reinsurance pool and private reinsurance, using combinatorial optimization with a genetic algorithm, Model C, compared to chapter two that uses CV of the LCR, Model 3.

Figure 1.2 shows the three models that are developed in chapter three under the assumption of a reinsurance premium pool. This includes Model A (self managed reinsurance pool), Model B (group buying of private reinsurance), and Model C (portfolio approach to combine Model A and B using combinatorial optimization with a genetic algorithm). Model C is based on the hypothesis that a portfolio of aggregate risks with high variance can be optimally managed by segregating it into two groups, and then combining them. The first group of risks within the portfolio is uncorrelated, and diversified enough to be managed internally within a pool. This group of uncorrelated risks within the pool naturally offsets, which lowers the variance of the aggregate risks within this set. The second group of risks within the portfolio is correlated, and therefore the risks do not sufficiently offset, which leads to a high variance portfolio of risks. This second group of risks is ceded to private reinsurers who are better diversified for risks that are large and correlated.

Similar to chapter two, an eight step methodology is also presented for Model C, however, the focus in chapter three is on step 4 and step 7. Step 4 covers how to use combinatorial optimization with a genetic algorithm to segregate the portfolio, and step 7 shows how to calculate surplus using an ALM surplus approach under a reinsurance premium pool.

Dependence across Regions Under a Reinsurance Premium Pool: Combinatorial Optimization with a Genetic Algorithm to Combine Pooling and Private Reinsurance to Reduce Risk

Chapter four uses a similar approach to chapter three, except that it allows for dependence (correlation) across regions (provinces). In chapter three, a portfolio approach was used to combine a self managed reinsurance pool and private reinsurance using optimization, Model C. In chapter four, however, the assumption of dependence across regions is incorporated into the optimized portfolio model, Model CC. The objective of this chapter is to analyze the effectiveness of the portfolio optimization model under dependence across regions, compared to chapter three that assumed independence across regions.

The intent of this chapter is to analyze the consistency of results between the portfolio models in chapter three and chapter four. Analyzing the portfolio optimization model under dependence across regions helps to ensure that the group of aggregate risks within the portfolio that is retained within the self managed reinsurance pool remains sufficiently diversified in the presence of correlated LCR's. Ignoring dependencies among regions may lead to inaccurate estimates of the risk, and this may alter the diversification that can be achieved through pooling. For example, pooling may be less effective when correlation is allowed in the model, while private reinsurance may be more effective in this case. Figure 1.3 shows the three models that are developed under a reinsurance premium pool with the added assumption of dependence across regions. Model AA, is a self managed reinsurance pool, Model BB, is group buying of private reinsurance, and Model CC, is a portfolio approach to combine a self managed reinsurance pool and private reinsurance using optimization.

Table 1.1: Summary of Nine Alternative Insurance Models: Corresponding Chapter, Description, and Characteristics

Overview of Models				**Model Characteristics**	
Models	Chapter	Initial Surplus	Model Description	Premium Contributed to Pool	Objective
Model 1	2	1 Times Premium $(1*E(X)*L_i)$ $872,486,863	Self managed insurance pool	Full premium pool (i.e. 100% of risk is transferred to the shared pool)	Maximum diversification; Combined approach uses CV
Model 2			Self managed insurance pool that also purchases private reinsurance as a group		
Model 3			**Portfolio approach** to combine a self managed insurance pool and private reinsurance using CV of the LCR		
Model A	3	0 Times Premium $(0*E(X)*L_i)$ $0	Self managed reinsurance pool	Reinsurance premium pool (i.e. ~10% of risk is transferred to the shared pool)	Incremental pooling overcomes potential cross subsidization; Combined approach uses combinatorial optimization with a genetic algorithm
Model B			Group buying of private reinsurance		
Model C			**Portfolio approach** to combine Model A and B using combinatorial optimization with a genetic algorithm		
Model AA	4	0 Times Premium $(0*E(X)*L_i)$ $0	Self managed reinsurance pool under correlation across regions	Reinsurance premium pool (i.e. ~10% of risk is transferred to the shared pool)	Allows for dependence (correlation) across regions (provinces); Combined approach uses combinatorial optimization with a genetic algorithm
Model BB			Group buying of private reinsurance under correlation across regions		
Model CC			**Portfolio approach** to combine Model AA and BB using combinatorial optimization with a genetic algorithm under correlation across regions		

Notes: Models 1, 2, and 3, correspond to **chapter 2**, and represent a full premium pool. A full premium pool corresponds to 100% of the risk from each region (province) allocated to a shared pool. The three models considered in this chapter have an initial surplus equivalent to 1 times the premium loading ($872 million

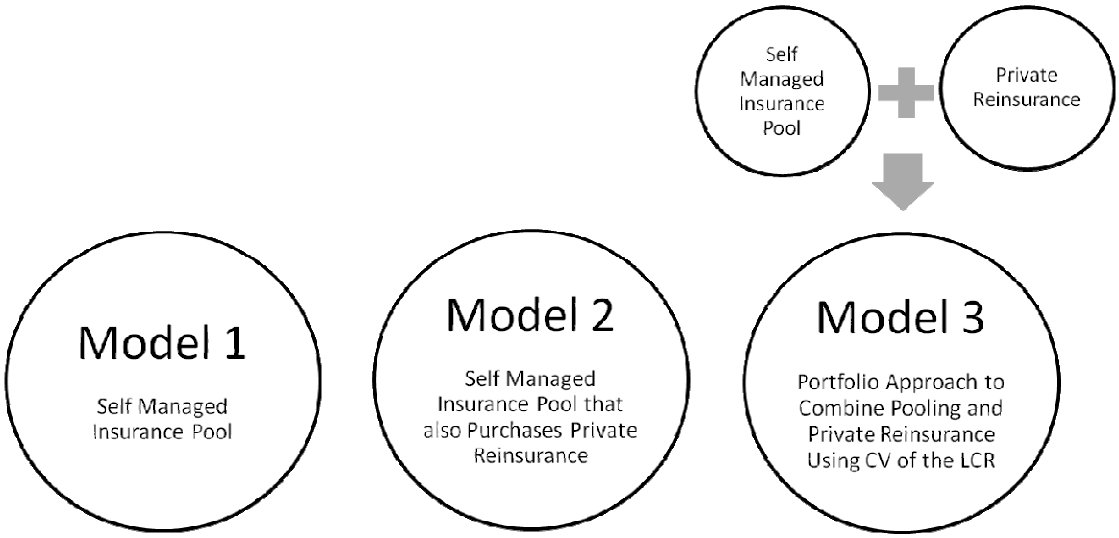
Table 1.1 (Continued)

shared across 10 provinces). The objective of this chapter is to consider the maximum diversification that can be achieved across products and/or geographic regions.

Chapter 3 develops three models, Models A, B, and C, under an incremental pooling approach. Instead of a full premium pool as in chapter two, chapter three considers a reinsurance premium pool, where approximately 10% of the risk is transferred to the shared pool. The three models considered in this chapter assume an initial surplus of $0, in order to save upfront capital expenditure. The objective of chapter three is to consider an incremental pooling approach in order to overcome any potential cross-subsidization of premiums across regions. The second objective of this chapter is to develop a more accurate portfolio approach using combinatorial optimization and a genetic algorithm to combine reinsurance pooling and private reinsurance.

Similar to chapter three, **chapter four** develops three models, Models AA, BB, and CC, under the assumption of an incremental pooling approach where the initial surplus is also zero. The main difference of chapter four is the consideration of dependence (correlation) across regions. The objective of chapter four is to analyze the consistency of results between the portfolio model in chapter three (Model C), and the one in chapter four (Model CC).

Figure 1.1 Description of Models 1, 2, and 3, Used in Chapter Two: Full Premium Insurance Pooling Model to Reduce Risk

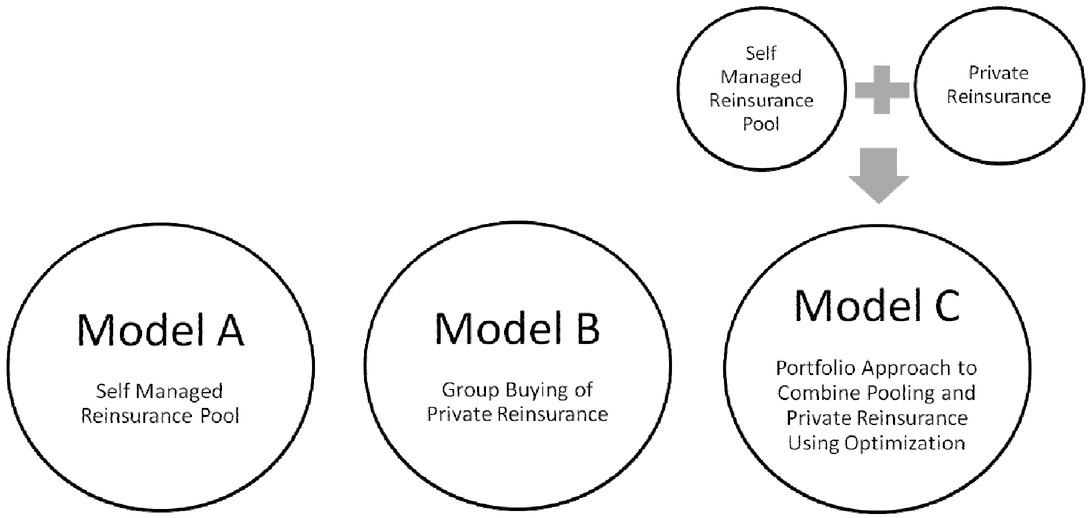


Notes: Chapter two models a full premium pool, where regions contribute 100% of their insurance premium to the pool, representing the maximum diversification that can be achieved within an insurance sector in an entire country. **Model 1,** is a self managed insurance pool that combines all risks from each region (province) into its own insurance company. This approach takes advantage of the natural offsetting of risks in the portfolio that are uncorrelated across regions (provinces) and/or products (crops), and therefore the portfolio risk is reduced for the insurance company. The advantage of this model is that costly reinsurance brokerage fees are eliminated. The disadvantage of this approach is that the pool may not be sufficiently diversified to offset extreme losses that are widespread, such as a drought that destroys the crops in a large region of the country.

A second alternative, **Model 2,** is proposed where instead of the firm relying exclusively on the natural offsetting of uncorrelated risks within the pool to reduce the variance of the portfolio, the self managed insurance pool also purchases private reinsurance as a group. The advantage of this model is that the private reinsurance firm is assumed to be better diversified than the self managed insurance pool in Model 1, as it holds a well diversified portfolio of risks that are uncorrelated with other international liabilities in the portfolio (e.g. from other countries outside of Canada, and other sectors outside of agriculture). The disadvantage of this approach is that private reinsurance is more expensive.

A third alternative, **Model 3,** is proposed to blend the benefits of Model 1 and Model 2. Model 3, a portfolio approach, uses the coefficient of variation (CV) of the loss coverage ratio (LCR) to segregate the portfolio into one group of risks with low CV that are appropriate for retaining in a self managed insurance pool. The second group of risks within the portfolio with high CV is ceded externally to private reinsurers. The assumption is that this will overcome the Model 1 problem of insufficient diversification due to extreme events that are widespread (e.g. all regions facing the same large risk in the same year). At the same time, Model 3 should benefit from the risks that have low CV and are retained within the pool (Model 1) as this is a lower cost option than private reinsurance.

Figure 1.2 Description of Models A, B, and C, Used in Chapter Three: Reinsurance Premium Pool Using Combinatorial Optimization with a Genetic Algorithm to Combine Pooling and Private Reinsurance to Reduce Risk



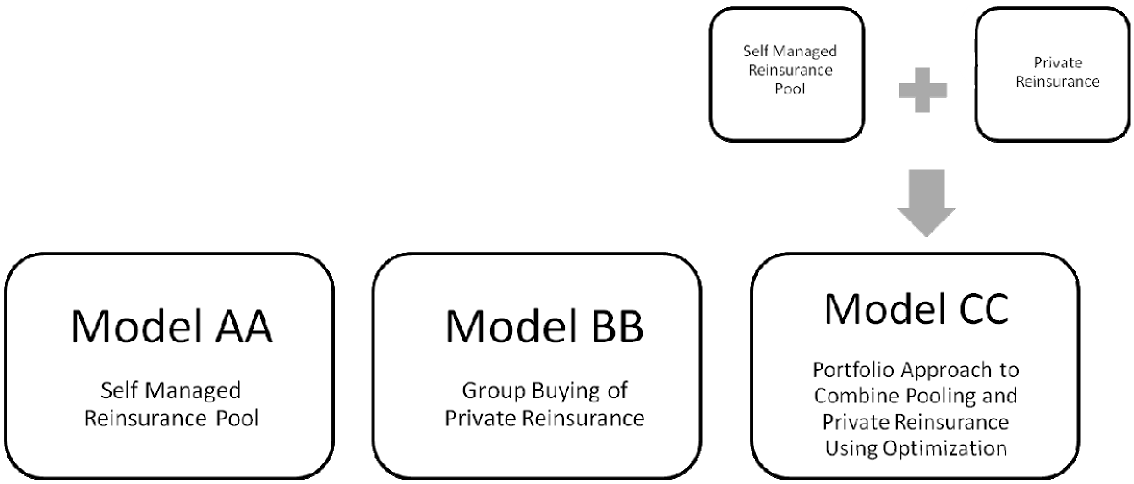
Notes: Chapter three models a reinsurance premium pool, where regions (provinces) contribute only a portion of their risks to the pool, as compared to chapter two, where a full premium pool was assumed. This addresses one of the potential limitations of chapter two, which is the reluctance that some provinces may have in transferring control of their region to cooperate in the pool. A reinsurance premium pool is an incremental approach to pooling which allows regions to continue operating independently, where only a portion of risks are pooled (10%). The remaining 90% of risks are retained within each provincial crop insurance company.

The first model, **Model A**, is a self managed reinsurance pool that combines reinsurance premium from different regions into its own internal reinsurance company. This model takes advantage of risks that naturally offset in the portfolio and are uncorrelated across regions and/or products in order to reduce risk to the reinsurance company. The advantage of this model is that it saves brokerage fees normally paid to private reinsurers. However, the disadvantage of this approach is that the pool may not be sufficiently diversified for extreme events that are widespread, such as a drought that destroys the crops across a many regions in a country.

A second alternative, **Model B,** is proposed where instead of the firm providing its own reinsurance as in Model A, it purchases reinsurance with other firms as a group from the private reinsurance market. The advantage of private reinsurance is that it is better diversified than the self reinsurance firm in Model A, due to the well diversified portfolio of risks that are uncorrelated with other international liabilities in its portfolio. The disadvantage of this approach is that private reinsurance is more expensive.

A third alternative, **Model C,** is proposed to blend the benefits of Model A and Model B. Using combinatorial optimization with a genetic algorithm, Model C takes one group of risks from within the portfolio that is relatively uncorrelated and naturally offsetting (and therefore has reduced variance in the portfolio), and retains it within a self managed reinsurance pool. The second group of risks within the portfolio that is correlated and does not sufficiently offset (and therefore has high variance in the portfolio), is ceded externally to private reinsurers. The assumption is that private reinsurance from Model B will overcome the Model A problem of insufficient diversification from extreme events that are widespread (e.g. all regions facing the same large risks in the same year). Also, Model C should benefit from the risks that are uncorrelated with a lower portfolio variance that are retained within the pool (Model A), as this is a lower cost option than private reinsurance (Model B).

Figure 1.3 Description of Models AA, BB, and CC, Used in Chapter Four: Dependence Across Regions (Provinces) Under a Reinsurance Premium Pool Using Combinatorial Optimization with a Genetic Algorithm to Combine Pooling and Private Reinsurance to Reduce Risk



Notes: Chapter four develops three reinsurance pool models under the assumption of a reinsurance premium pool, which are very similar to Models A, B, and C developed in chapter three, but with the added assumption of *dependence across regions*. This includes Model AA, a self managed reinsurance pool model, Model BB, a private reinsurance model, and Model CC, a portfolio model to combine pooling and private reinsurance using combinatorial optimization with a genetic algorithm.

CHAPTER 2
DEVELOPING A FULL PREMIUM INSURANCE POOLING MODEL TO REDUCE RISK

Introduction

When firms are faced with managing a portfolio of aggregate risks with high variance, pooling insurance business within a small geographic region or across insufficient product lines often results in inadequate diversification. However, pooling insurance business across many regions in a country or across a large number of product lines, however, may improve the amount of diversification that can be achieved. Chapter two models a full premium pool, where all of the risk from each of the ten provinces is pooled together into one large countrywide pool, representing the maximum diversification that can be achieved within an insurance sector in an entire country. The objective of this chapter is to analyze the diversification that can be achieved through pooling insurance business across a number of geographic regions (provinces) and products (crops), as a possible solution to better manage a portfolio of aggregate risks with high variance. In this solution, an eight step methodology is developed for an innovative insurance portfolio model that combines a self managed insurance pool and private reinsurance using the coefficient of variation (CV) of the loss coverage ratio (LCR), Model 3.

Three alternative pooling and reinsurance models are developed under a full premium pool, which are evaluated under an asset liability management (ALM) approach. Using simulation, risk measures including surplus, survival probability, and deficit at ruin are considered. This study focuses on the complete crop insurance sector for Canada, considering ten provincial non-profit crop insurance corporations to evaluate the models. A comprehensive data set is considered, including 32 years of actual premiums and liabilities (from 1978-2009), across 10 regions for 279 crop types.

The first model, **Model 1,** is a self managed insurance pool that combines all of the risks from each region into its own joint insurance company. This approach benefits from risks in the portfolio that naturally offset and are uncorrelated across regions and/or products, and therefore the portfolio risk is reduced for the insurance company. The advantage of this model is that costly reinsurance brokerage fees are eliminated. The disadvantage of this approach is that the pool may not be sufficiently diversified to offset extreme losses that are widespread, such as a drought that destroys the crops in a large region of the country.

A second alternative, **Model 2,** is proposed where instead of the firm relying exclusively on the natural offsetting of uncorrelated risks within the pool to reduce the variance of the portfolio, the self managed insurance pool also purchases private reinsurance as a group. The advantage of this model is that the private reinsurance firm is assumed to be better diversified than the self managed insurance pool in Model 1, as it holds a well diversified portfolio of risks that are uncorrelated with other international risks in the portfolio (e.g. from other countries outside of Canada, and other sectors outside of agriculture). The disadvantage of this approach is that private reinsurance is more expensive.

A third alternative, **Model 3,** is proposed to blend the benefits of Model 1 and Model 2. Model 3 uses a portfolio approach to combine a self managed insurance pool and group buying of private reinsurance using the coefficient of variation (CV) of the loss coverage ratio (LCR). The portfolio is segregated into one group of risks with low CV that are appropriate for retaining in a self managed insurance pool. The second group of risks within the portfolio with high CV is ceded externally to private reinsurers. The assumption is that this will overcome the Model 1 problem of insufficient diversification due to extreme events that are widespread (e.g. all regions facing the same large risk in the same year). At the same time, Model 3 should benefit from the risks that have low CV and are retained within the pool as this is a lower cost option than private reinsurance.

The remainder of this chapter is organized as follows. The next section discusses past literature and background, focusing on possible alternatives for reducing the variance in a portfolio of correlated risks. Following this is a section on data, and then an eight step methodology is presented for the innovative risk management portfolio approach to combine a self managed insurance pool and group buying of private reinsurance using the CV of the LCR, Model 3. Results of the asset liability management (ALM) model for the three alternative insurance models are compared and contrasted under a full premium pool, and a summary section is presented.

Literature

<u>Private Reinsurance</u>

One solution to reducing the risk associated with a high variance portfolio of risks is for an insurance company to purchase private reinsurance. Reinsurance is a means of risk management where an insurer purchases insurance from a reinsurer. Transferring risk to a private reinsurer helps to stabilize premium rates from year to year, and maintain an adequate reserve to keep premium rates low. When risks are small and uncorrelated across regions, reinsurance premiums include relatively low brokerage fees and therefore premiums are priced closer to the actuarially fair rate of the expected loss of the risk transferred. However, when risks are large and correlated across regions, private reinsurance premiums are often comprised of substantially higher brokerage fees to account for the uncertainty of large and variable losses from year to year, which results in reinsurance premiums which are costly.

Reinsurance premiums have also been reported to be increasing in recent years (Holzheu and Lechner, 2007). Some reasons for reinsurance premium increases may include agency problems, solvency problems due to large natural catastrophes, low investment returns by reinsurance companies, and changes in risk profiles such as larger losses or increased variability of losses (AON, 2009; Woodward et al., 2010). Increasing reinsurance premiums may pose a significant concern to government, corporations, and end users (e.g. agricultural producers), because it leads to higher costs which may decrease competitiveness or program participation (Cox and Lin, 2007). Further, if insurance firms cannot afford to purchase private reinsurance due to high premium rates, they may be faced with additional risk exposure. This could impact reserve levels, and possibly put the insurance company at an increased risk of default.

In crop insurance, the level of risk associated with a portfolio of risks has been measured to be roughly ten times larger than the portfolio risk faced in conventional insurance with independent risk exposures (Miranda and Glauber, 1997). Therefore, crop premiums are often comprised of substantial loading fees making reinsurance premiums expensive relative to the expected loss of the risk transferred (Froot, 2001). Private informed estimates state that private reinsurance brokerage fees for crop insurance in Canada are estimated to be 35% or more of total reinsurance premium payments.

Pooling as a Possible Solution to a Portfolio of Aggregate Risks with High Variance

A focus of this thesis is risk management approaches for a portfolio of aggregate risks with high variance, making it different than the portfolio risk faced by other insurance firms with portfolios of low variance (Skees and Barnett, 1999; Miranda and Glauber, 1997; Quiggin, 1994). When firms are faced with the challenge of managing a portfolio of aggregate risks with high variance, private reinsurance is traditionally the risk management approach that is implemented. However, the problem with this approach is that it is costly.

When private reinsurance is too expensive, retaining risks internally within a firm's pool to diversify losses may be a suitable alternative (Mahul and Stutley, 2010). In a self managed insurance pool, diversification is achieved by combining a number of individual risks from geographic regions and/or product lines, in order to offset risks and reduce the variance of the portfolio. The advantage of a self managed insurance pool is that it eliminates the high brokerage fees associated with private reinsurance. However, the disadvantage of this approach is that the self managed insurance pool may not be sufficiently diversified for extreme events that are widespread.

It is sometimes believed that pooling insurance business within regions does not substantially reduce the variance of the portfolio comprised of risks that are large and correlated such as in crop insurance (Miranda and Glauber, 1997). But contrary to this statement, crop areas located in different states in the U.S. that are far apart across regions are often found to have low or negative correlation (Wang and Zhang, 2003). Similarly, it is found that as the geographic area of the insurance pool increases, the degree of correlation is reduced, suggesting that correlation may be geographically related (Skees et al., 2005; Goodwin, 2001).

Self insured pools can be observed in some catastrophic reinsurance funds throughout the world. For example, 18 countries in the Caribbean have pooled their premiums in a self managed reinsurance fund in the event of hurricanes or earthquakes. It is estimated that they will save approximately 40% in brokerage fees relative to private reinsurance (Worldbank Reinsurance Focus, 2007). Similarly, some U.S. States have responded to private reinsurance premium increases (upwards of 15% per year) by creating self managed reinsurance funds for hurricanes and other catastrophic insurance protection (AON, 2009).

Group Buying of Reinsurance as a Possible Solution to a Portfolio of Aggregate Risks with High Variance

The group buying of private reinsurance refers to all regions coming together in a pool as one entity to purchase reinsurance as a group (e.g. ten provinces), compared to each region purchasing private reinsurance on their own. This group approach may lead to increased buying power, allowing the pool to negotiate a reduced rate. This could be because private reinsurers are willing to decrease their profit margins in exchange for a larger volume of business. While there are no direct examples of group buying of reinsurance in agriculture, this approach can be observed in some health insurance markets where many small companies join together to gain collective market power to negotiate reduced rates.

Insurance purchasing pools have been successfully utilized in the healthcare sector for lowering the costs of rapidly rising prescription drug costs (Health and Human Services, 2004; NGA, 2004; Warn, 2005; Globe and Mail, 2010; National Post, 2010). By purchasing prescription drugs from pharmaceutical manufacturers as a large group, increased market power is often achieved. Furthermore, by promising to purchase a larger volume the seller may be motivated to reduce rates (e.g. brokerage rates) in exchange for a larger contract that could bring economies of scale in administration and underwriting costs.

For U.S. prescription drug purchasing pools, savings are estimated to be 5 to 15 percent of total prescription drug costs (Warn, 2005). For example, Georgia's intra-state pooling program saved the state approximately $60 million in 2001 and 2002 (Health and Human Services, 2004). Similarly, a multi-state pooling program between Delaware, Missouri, New Mexico, and West Virginia was able to negotiate substantial manufacturer discounts for prescription drugs for state employees. In West Virginia alone, savings of $25 million were realized in 2004 (NGA, 2004). More recently, in Canada there has been a movement to create a national agency for all 13 provinces and territories to purchase prescription drugs, medical supplies, and equipment in an effort to lower health care costs (Globe and Mail, 2010; National Post, 2010). These examples suggest that group buying may be capable of reducing costs by increasing market power, a strategy that could be useful in negotiating lower private reinsurance premiums in crop insurance as well as other insurance sectors and large organizations.

Asset Liability Management (ALM) Modeling

In an effort to manage an insurance portfolio and help improve the longevity of an insurance fund, the asset liability management (ALM) approach has often become a preferred model. ALM is "the act of planning, acquiring, and directing the flow of funds through a financial organization to generate adequate and stable earnings, maintain adequate liquidity, and steadily build capital, while taking reasonable and measured business risks" (Cole and Featherstone, 1997). Therefore, an ALM approach is an important tool for managing the significant impact that liabilities, such as indemnity payments and interest rates, can have on the performance and survival of an insurance fund.

Crop Insurance Problems

Crop insurance has generally been successful in a number of countries when supported by government, but without a subsidy it has often been considered unsuccessful (Miranda and Glauber, 1997). The traditional explanation for crop insurance market difficulties has been asymmetric information, including adverse selection and moral hazard (Skees and Reed, 1986; Holstrom, 1979; Chambers, 1989; Quiggin, 1994; Nelson and Loehman, 1997; Cohen and Siegelman, 2010).

Moral hazard occurs when the insured acts less carefully because of the protection received by the insurer, compared to if it were fully exposed. This has an effect on the probability of losses, although these actions are hidden from the insurer.

Adverse selection on the other hand occurs when the insurer does not have access to the same information as the insured, and the insured is able to hide the fact that its demand for insurance is positively correlated to its risk level.

Crop Insurance Operation in Canada

In Canada, each provincial crop insurance company operates its own crop insurance program independently of the other nine provinces. The crop insurance program is voluntary for farmers, and costs are subsidized by government. The federal government pays 36 percent of premiums, the provincial government pays 24 percent of premiums, and producers (farmers) are responsible for 40 percent of costs. The crop insurance program in the United States operates very similarly to Canadian crop insurance, however in the U.S., crop insurance is sold by private firms.

Each province's crop insurance company also has the option of participating in a reinsurance arrangement offered by the federal government under the Farm Income Protection Act (FIPA). For the Federal reinsurance program, each participating province contributes premiums to the federal reinsurance account based on their assessed risk of a payout. At such time when crop insurance payments surpass the provinces accumulated premium reserves, and a deductible of 2.5% of the province's crop insurance liabilities, a payment from the federal reinsurance account is issued. Residual indemnities are then distributed 75/25, with the federal reinsurance being responsible for the 75% share.

If funds in the federal reinsurance account fall short of the required reinsurance payment, the Minister of Finance is responsible for advancing the funds to the reinsurance account. The outstanding advances are repaid from future reinsurance premiums. In 2010, five of the ten Canadian provinces participated in the federal-provincial reinsurance arrangement, including Alberta, Saskatchewan, Manitoba, New Brunswick, and Nova Scotia. In addition to having the option of participating in the federal reinsurance program, provincial crop insurance companies can also purchase reinsurance from the private reinsurance market. In 2010, five of the ten Canadian provinces, British Columbia, Alberta, Manitoba, Ontario, and PEI, purchased private reinsurance.

As an example of the crop insurance program in Manitoba, the following outlines the indemnity payment to farmers (Boyd et al., 2011). It is assumed that the average wheat yield for a farmer has been 2 tonnes per hectare over the past number of years, and that the farmer purchases crop insurance to insure 1 hectare at 80% coverage of a $300 per ton value. In the example, it is also assumed that the farmer experiences crop loss, and harvests only 60% of the expected 2 ton value. Therefore, the indemnity payment to the farmer would be (80% - 60%) x $300 x 2 tons = $120 per hectare. Generally, farmers prefer to insure their crop yield for approximately 60% to 80% of their historical yield levels.

Data

The entire crop insurance sector for Canada is empirically analyzed using historical crop loss data from 1978 through 2009, obtained from Agriculture and Agri-Food Canada's Production Insurance National Statistical System (PINSS). This comprehensive data set represents an entire insurance sector in a country, including 32 years of historical indemnities and liabilities (from which the loss coverage ratio, LCR is calculated), across 10 regions (provinces) for 279 crops. The ten regions considered include British Columbia, Manitoba, New Brunswick, Newfoundland, Nova Scotia, Ontario, Prince Edward Island, Quebec and Saskatchewan. In addition, two groupings of provinces are considered, including PROV5 (AB, BC, MB, ON and PEI) and PROV10 (AB, BC, MB, ON, PEI, SK, NFLD, NB, NS and QC).

Methodology

An asset liability management (ALM) approach is used to develop an innovative risk management portfolio approach to combine a self managed insurance pool and group buying of private reinsurance. There are eight basic steps that are necessary for a portfolio approach to combine the benefits of pooling and reinsurance using coefficient of variation (CV) of the loss coverage ratio (LCR) under the assumption of a full premium pool (Table 2.1). These steps include: developing the ALM surplus approach; development and description of three alternative pooling and reinsurance models (Models 1, 2, and 3), calculation of reinsurance layers using the loss coverage ratio (LCR) and a loss elimination ratio (LER), using CV of the LCR to segregate the portfolio of risks; simulating claims and calculation of the expected insurance premium and reinsurance premium, and forecasting of liabilities using a least squares regression model. From these variables, surplus is calculated over each forecasting period (e.g. 30 years) under a full premium pool, using an ALM process. In addition, risk measures such as survival probability and deficit at ruin are computed in order to evaluate the most effective model.

Step 1: Developing the ALM Surplus Approach

The first step for creating the combined pooling and private reinsurance model includes developing an asset liability management (ALM) approach. The ALM approach responsibly matches assets and liabilities, and considers the potential result of mismatching (Gerstner et al., 2008). Using this approach, surplus U_t, which is the balance in the insurance company's account at the end of the t_{th} period, is calculated for each forecasting period (e.g. 30 years) as shown in Equation [2.1] (Klugman et al., 2008; Bowers et al., 1997). The surplus process is computed over successive time periods, followed by the calculation of other risk measures such as survival probability and deficit at ruin. Together, these risk measures can be used to evaluate the model, as explained in the seven steps that follow.

$$U_t = U_{t-1} + P_t + C_t - S_t \qquad [2.1]$$

where

(U_t) Surplus at the beginning of the t_{th} period.

(P_t) Premiums at the beginning of the t_{th} period.

(C_t) Interest on retained cash at the end of the t_{th} period.

(S_t) Claims at the end of the t_{th} period.

Step 2: Development and Description of Three Alternative Pooling and Reinsurance Models (Models 1, 2, and 3)

Step two involves developing and defining three alternative pooling and reinsurance models that are later evaluated in order to find the best model. Each model examines two groupings of regions, including one group comprised of five provinces (PROV5: AB, BC, MB, ON and PEI), and one group comprised of all 10 provinces (PROV10: AB, BC, MB, ON, PEI, SK, NFLD, NB, NS and QC). The first grouping of regions, PROV5, is selected based on the provinces that participated in private reinsurance arrangements in 2010. The second grouping of regions, PROV10, is based on possible future participation by all ten provinces.

Each model assumes that regions operate collectively with the other participating regions, and all risks are allocated to a proposed self insurance pool. This is an alternative to current procedures where each region (provinces) manages its own corporation independently. This approach represents the maximum possible diversification within a sector, where all farmer premiums from all regions (provinces) within a country are pooled together.

The first alternative approach is **Model 1,** where a smaller insurance organization (province) along with others, forms their own joint reinsurance company. Each small organization from different geographic regions pools all of their risk and contributes all of their insurance premiums into their jointly owned insurance company that is self managed. This approach assumes that large losses across regions (provinces) will be uncorrelated and offsetting, and therefore risk will be reduced for the company. The advantage is that the purchase of higher cost private reinsurance can be avoided and brokerage fees are eliminated, providing more efficiency. The disadvantage is that in a bad year when losses across regions do not adequately offset, the joint insurance company could experience ruin because it is not sufficiently diversified to the same extent as a well diversified international reinsurer (e.g. risks from outside of Canada and outside of agriculture).

The second alternative approach is **Model 2,** where instead of the firm only providing its own insurance as in Model 1, it also cooperates with the other smaller insurance organizations and purchases private reinsurance as a group from the international reinsurance market. Private reinsurance is purchased to cover approximately 10% of the risks in the pool, and the private reinsurer pays a claim to the pool if the losses fall within the specified range of coverage, which is explained in more detail in step three. This model assumes that the private reinsurance firm is better diversified than the self managed pool in Model 1 and is much less likely to fail. The reduced risk of failure is because the private reinsurer holds a well diversified portfolio of risks that offset with international risks from multiple countries (e.g. outside of Canada) across different sectors and products (e.g. outside of agriculture). The disadvantage of this approach is that private reinsurance is more expensive, however, some savings are realized through group buying. It is assumed that group buying allows the pool to gain market power, and negotiate a reduced brokerage rate which is conservatively assumed to be 25%, compared to the individual rate it would otherwise pay, estimated to be 35%.

Model 3 is a portfolio approach that combines a self managed insurance pool and group buying of private reinsurance. It uses CV of the LCR, a normalized measure of dispersion, to segregate the portfolio of risks into two groups before combining them. The first group contains risks that have low CV of the LCR, which are assumed to be less risky to the pool are therefore retained internally within a self insurance pool. A second group of risks from within the portfolio with high CV of the LCR, are assumed to be more risky to the pool and are ceded externally to private reinsurers. For this second group of risks, private reinsurance is purchased for a layer that covers approximately 10% of liabilities. The reinsurer pays a claim to the pool if losses occur within the specified range of coverage for the select group of risks that have high CV, which is explained in more detail in step three.

The private reinsurance purchased by the pool should help overcome the Model 1 problem of insufficient diversification for extreme events that are widespread (e.g. if all regions face the same large risk in the same year and the joint reinsurance company fails). Model 3 should benefit from the group of risks with low CV that are retained within the pool, as this is a lower cost option than private reinsurance. Rather than paying reinsurance brokerage fees on the entire portfolio, brokerage is only incurred on the group of risks with high CV that may benefit the most from the improved diversification of private reinsurance.

For the group of risks that are ceded to private reinsurers, it is assumed that group buying will bring a slight reduction in brokerage fees. In this model, brokerage fees are assumed to be 30%, compared to current individual rates that are estimated to be 35%. While group buying of reinsurance may help to negotiate a reduced rate as in Model 2 (where brokerage is assumed to be 25%), the selective nature of purchasing private reinsurance for only the group of high CV risks in Model 3 may limit the amount of savings that can be negotiated. Private reinsurers have indicated that the group of "higher risks" in the portfolio, may not necessarily be high risk to the reinsurer, and so it would be acceptable for the reinsurer to sell the reinsurance. As long as risks are not correlated with the reinsurer's portfolio, then it is not "high risk" to the reinsurer. Therefore, private reinsurers may be more effective at offsetting high variance risks.

Step 3: Calculation of Reinsurance Layers Using the Loss Coverage Ratio (LCR) and a Loss Elimination Ratio (LER)

This step involves setting the reinsurance coverage layer for each individual region and the self managed reinsurance pool, using the expected loss coverage ratio (LCR) and the loss elimination ratio (LER). The reinsurance coverage layer corresponds to the percentage of risks for which the insurance firm seeks coverage. For example, in this study a reinsurance layer that equates to a loss elimination ratio (LER) of 90% is used to set coverage levels for approximately 10% of risks which is explained next. When losses occur within the specified range of the LCR, the private reinsurer pays a claim to the pool, up to a maximum of 10% of the LCR.

The LCR is a measure of risk that reflects the annual loss, and is calculated as the ratio of total indemnities to total liabilities, multiplied by 100 (Woodward et al., 2010). Indemnity refers to the amount of claims paid out to customers if there is a loss, and coverage (liability) is the total amount of insurance carried. In general, LCR is often referred to as the "loss-cost ratio." The general rule of thumb is that in any given year, a LCR greater than 20% is considered a disaster. In this study, however, since the denominator (total liability) represents the total coverage, LCR is referred to as the "loss-coverage ratio." Further, actuaries use a similar measure to establish actuarially sound premiums, pure premium, which is represented by indemnities over exposure.

The LER (Promislow, 2006) measures the fraction of expected losses that is eliminated from the expected liability due to an imposed deductible. Utilizing a LER helps to take into consideration the varying risk levels that face each region, which therefore aids in preventing the problem of cross subsidization of premiums (e.g. regions facing lower variance portfolios of risks subsidize regions facing higher variance portfolios of risks). In this study, it is assumed that the LER is equal to 90%, which equates to coverage on approximately 10% of liabilities. The lower bound reinsurance layer d, and the upper bound reinsurance layer u=d+0.10, is manipulated to satisfy the equation below.

$$LER = 90\% = \frac{E(X_j) - E(Y_j)}{E(X_j)} \Rightarrow E(Y_j) = 0.1 * E(X_j) \qquad [2.2]$$

where

- E(Xj) is the expected LCR or insurance premium for region j, and
- E(Yj) is the reinsurance premium for region j.

Given the reinsurance coverage layer for each region, the corresponding reinsurance premium can be calculated based on the following equation.

$$y = \begin{cases} 0 \Rightarrow 0 \le X \le d \\ x - d \Rightarrow d \le X \le u \\ u - d \Rightarrow X \ge u \end{cases} \qquad [2.3]$$

As an example, Figure 2.1 shows historical LCR data for the province of Manitoba, and depicts a reinsurance coverage layer between d=15% and u=25%. All of the losses that fall below the lower bound reinsurance coverage of d=15%, are retained by MASC. The losses (LCR) that exceed 15%, up to 10%, are transferred to the private reinsurer.

Step 4: Using Coefficient of Variation (CV) to Segregate the Portfolio

Step four covers using the coefficient of variation (CV) of the loss coverage ratio (LCR) to segregate the portfolio into two groups which are then combined. The first group of risks within the portfolio with low CV is retained internally within the self managed insurance pool. The second group of risks within the portfolio with high CV is ceded to private reinsurers. CV is a calculation of risk that measures the dispersion of a probability distribution, which is calculated as the ratio of standard deviation σ to the mean μ. This statistic is useful for comparing the degree of variation from one risk to another, even if the means are substantially different from each other.

In this study, the historical LCR for each crop is aggregated and the CV is calculated. Using CV the crops are sorted from highest to lowest, and the crops with the highest CV that comprise approximately 20% of the entire portfolio are selected for ceding to private reinsurers. The remaining second group of risks with low CV that comprise approximately 80% of the portfolio, is retained internally within the firms self managed insurance pool.

Step 5: Simulating Claims and Calculation of the Expected Insurance Premium and Reinsurance Premium

Step 5 involves the simulation of claims as represented by the loss coverage ratio (LCR), as well as the calculation of the expected insurance premium E(X) and reinsurance premium E(Y). Simulation of the LCR is necessary for each region as well as for the two groups of risks in the portfolio for Model 3, as described in the previous step using CV of the LCR to segregate the portfolio. Historical LCR data over 32 years is aggregated for each region, as well as for the group of high CV risks and low CV risks for Model 3. Palisade's @Risk software is used to determine the best statistical distribution for each of the data sets, based on the Kolmogorov-Smirnov test (K-S).

The K-S statistic is a fit statistic that is used to measure how good the distribution fits the input data and how confident you can be that the data was produced by the distribution function.

$$D_n = \sup[|F_n(x) - \hat{F}(x)|] \qquad [2.4]$$

where

n = total number of data points

$\hat{F}(x)| =$ the fitted cumulative distribution function

$F_n(x) = \frac{N(x)}{n}$

$N(x)$ = the number of X_i's less than x

For the K-S statistic, the smaller the value, the better the fit. Distributions including loglogistic, gamma, and weibull are found to be the best statistical fit for the various data sets, with significance levels ranging from 0.043 to 0.093. Once the distribution was chosen, parameter values were generated from @Risk, and then used to simulate LCR 5,000 times, 30 years in the future.

Besides using the parameter values from each distribution to simulate the LCR, these values are also used to calculate the expected insurance payment E(X) and the reinsurance payment E(Y). For the loglogistic distribution, the expected insurance payment E(X) is described by Equation [2.5], and the reinsurance payment E(Y) is described by Equation [2.6].

$$E(X) = \theta * \Gamma(1+1/\gamma) * \Gamma(1-1/\gamma) \quad [2.5]$$

$$\begin{aligned} E(Y) = &\left[\theta * \Gamma(1+1/\gamma)\Gamma(1-1/\gamma)\beta(1+1/\gamma, 1-1/\gamma; \mu) + u(1-\mu)\right] - \\ &\left[\theta * \Gamma(1+1/\gamma)\Gamma(1-1/\gamma)\beta(1+1/\gamma, 1-1/\gamma; \mu) + d(1-\mu)\right] \end{aligned} \quad [2.6]$$

where X is the annual loss per 100 of exposure, and Y is the reinsurance premium according to the coverage layer X^x, and has a loglogistic distribution with parameters γ and θ (Klugman et al., 2008).

For the gamma distribution, the expected insurance payment E(X) is described by Equation [2.7], and the expected reinsurance payment E(Y) is described by Equation [2.8].

$$E(X) = \theta * \alpha \quad [2.7]$$

$$E(Y) = \left[\alpha\theta\,\Gamma\left(\alpha+1;\frac{u}{\theta}\right) + u\left(1-\Gamma\left(\alpha;\frac{u}{\theta}\right)\right)\right] - \left[\alpha\theta\,\Gamma\left(\alpha+1;\frac{d}{\theta}\right) + d\left(1-\Gamma\left(\alpha;\frac{d}{\theta}\right)\right)\right] \qquad [2.8]$$

where X is the annual loss per 100 of exposure, and Y is the reinsurance premium according to the coverage layer X^x, and has a gamma distribution with parameters α and θ (Klugman et al., 2008).

For the weibull distribution, the expected insurance payment E(X) is described by Equation [2.9], and the expected reinsurance payment E(Y) is described by Equation [2.10].

$$E(X) = \theta\Gamma\left(1+\frac{1}{\tau}\right) \qquad [2.9]$$

$$E(Y) = \left[\theta\Gamma\left(1+\frac{1}{\tau}\right)\Gamma\left[1+\frac{1}{\tau};\left(\frac{u}{\theta}\right)^{\tau}\right] + ue^{-\left(\frac{u}{\theta}\right)^{\tau}}\right] - \left[\theta\Gamma\left(1+\frac{1}{\tau}\right)\Gamma\left[1+\frac{1}{\tau};\left(\frac{d}{\theta}\right)^{\tau}\right] + de^{-\left(\frac{d}{\theta}\right)^{\tau}}\right] \qquad [2.10]$$

where X is the annual loss per 100 of exposure, and Y is the reinsurance premium according to the coverage layer X^x, and has a weibull distribution with parameters θ and τ (Klugman et al., 2008).

Once E(X) and E(Y) are calculated, the 10% layer of reinsurance coverage is set according to the process described in step 3. Table 2.2 lists the statistical distribution, reinsurance coverage layer, and expected reinsurance premium for each region.

Step 6: Forecasting of Liabilities Using a Least Squares Regression Model

Following the simulation of claims, liabilities (L_j) are forecasted for each of the data sets 30 years into the future. Assuming a linear trend, a least squares regression model is developed from 32 years of historical liabilities from 1978 through 2009.

Step 7: Calculating Surplus Using an ALM Surplus Approach Under a Full Premium Pool

Next, each of the variables described in the proceeding steps are then used to calculate surplus at the end of each forecasting period (e.g. 30 years) under a full premium pool. As previously described in Equation [2.1], surplus is comprised of four inputs including the balance in the fund at the beginning of the t_{th} period (U_t), the total premium collected during the time period (P_t), the interest earned on cash retained in the fund (C_t), and the total claims paid during the period (S_t). The calculation for each of the four inputs is explained next.

U_t is the balance in the fund at the beginning of the t_{th}, where $U_0 = u$ is the initial surplus. In this chapter, initial surplus is the equivalent to one times the expected loss $\sum(1 * E(X) * L_j)$. This equates to all five regions (provinces) contributing a combined total of \$499,172,745, and all ten regions (provinces) contributing a combined total of \$872,486,863.

P_t is the total premium collected during the time period, and the formula varies according to the assumptions in each model. **Model 1** has one component related to premium (P^1), as shown in Equation [2.11]. This includes the insurance premium contributed by each region j to the self insurance pool as represented by P^1.

Model 2 has two components related to premium (P^2), as shown in Equation [2.12]. This includes the insurance premium contributed by each region (province) to the pool as in Model 1 above (P^1). In addition, the pool pays premiums to the private reinsurer (P^2) to cover 10% of the risks in the portfolio (p). A brokerage fee θ_1 equal to 25% is applied to account for the reduced reinsurance brokerage that can be negotiated due to group buying, due to the current 35% individual brokerage rate.

Model 3 also has two components related to premium (P^3), as shown in Equation [2.13]. This includes the insurance premium contributed by each region to the pool as in Model 1 above (P^1). In addition, the pool pays premiums to the private reinsurer (P^3) to cover 10% of the second group of risks in the portfolio with high CV of the LCR (r). Further, a brokerage fee θ_2 equal to 30% is applied. This rate accounts for the reduced reinsurance brokerage that can be negotiated as a result of group buying (as in Model 2 where brokerage is assumed to be 25%), yet also takes into consideration the potentially higher reinsurance brokerage rate due to selectively reinsuring only the second group of risks within the portfolio with high CV of the LCR.

Model 1 (self managed insurance pool): $P_t^{(1)} = (\sum_{j=1}^{10} E(X_j) * L_j)$ [2.11]

Model 2 (group buying of private reinsurance): $P_t^{(2)} = P_t^{(1)} - ((1+\theta_1) * E(Y_p) * L_p)$ [2.12]

Model 3 (combination): $P_t^{(3)} = P_t^{(1)} - ((1+\theta_2) * E(Y_r) * L_r)$ [2.13]

where

- $E(X_j)$ is the insurance premium rate per dollar of coverage for region j, based on coverage for all of the risk in the pool.

- $E(Y_p)$ is the reinsurance premium rate per dollar of coverage ceding for reinsurance to cover all risks in the portfolio; $E(Y_r)$ is the reinsurance premium rate per dollar of coverage for the pool, based on coverage for only the second group of risks within the portfolio with high CV of the LCR.
- L_j refers to the corresponding total liability in region j; L_p refers to the total liability in the portfolio based on all risks allocated to the pool from all regions; L_r refers to the total liability in the portfolio associated with only the second group of risks within the portfolio with high CV of the LCR.

C_t is the interest earned on the surplus retained during the period, $u_{t-1} * i$. The rate of interest i, is assumed to be an average return of 3% which is based on the assumption that in order to meet liquidity requirements, a large portion of surplus is held in reserve earning little or no return, while the remainder of surplus is split among various short term holdings.

S_t is the total claims paid out during the period. The formula varies according to the assumptions in the model. **Model 1** has one component pertaining to claims (S^1), as shown in Equation [2.14]. This includes claims that the self managed insurance pool pays to each region when losses occur within the specified reinsurance coverage layer.

Model 2 has two components related to claims (S^2), as shown in Equation [2.15]. This includes claims that the self managed insurance pool pays to each region as in Model 1 above. In addition, the private reinsurer pays a claim to the self insurance pool when losses occur within the specified reinsurance coverage layer, based on all risks in the portfolio (p).

Model 3 also has two components related to a claim (S^3), as shown in Equation [2.16]. This includes claims that the self insurance pool pays to each region as in Model 1 above. In addition, the private reinsurer pays a claim to the self insurance pool when losses occur within the specified reinsurance coverage layer, but, only the second group of risks within the portfolio that have high CV of the LCR (r).

Model 1 (self managed insurance pool): $S_t^{(1)} = \sum_{j=1}^{10} (X_{t,j} * L_j)$ [2.14]

Model 2 (group buying of private reinsurance): $S_t^{(2)} = S_t^{(1)} - (Y_{t,p} * L_p)$ [2.15]

Model 3 (combination): $S_t^{(3)} = S_t^{(1)} - (Y_{t,r} * L_r)$ [2.16]

where

- $X_{t,j}$ is the loss coverage ratio (LCR) in period t, in region j.
- L_j refers to the total liability in each region j; L_p is the total liability in the portfolio contributed by all regions for coverage of all risks in the pool; L_r is the total liability in the portfolio for coverage of only the second group of risks within the portfolio with high CV of the LCR.
- $Y_{t,j}$ is the reinsurance premium in time t, in region j; $Y_{t,p}$ is the reinsurance premium at time t, based on all risks in the portfolio (p); $Y_{t,r}$ is the reinsurance payment at time t, based on coverage for only the second group of risks within the portfolio with high CV of the LCR (r).

Step 8: Determining Survival Probability, and Deficit at Ruin, to Evaluate the Most Effective Model

The last step is determining risk measures of survival probability, and deficit at ruin for each of the forecasting periods (e.g. 30 years) is based on the surplus calculation performed in the previous step. Together these three risk measures help to evaluate the models and determine the most effective approach. To be acceptable, a model must have a reasonable level of surplus, survival probability, and low deficit at ruin. Models that are substantially inferior in any of these three risk measures put the insurance firm at serious risk of default.

The objective of the ALM approach is to optimally maintain solvency, by ensuring that assets (incoming cash flows) exceed liabilities (outgoing cash flows) (Huaung, 2010). One of the most important questions to answer in insurance is the probability that the fund will survive, or conversely, the probability that the fund will ruin. Survival probability is an extension of surplus that measures the point at which capital is exhausted. Simply put, if the surplus in a given time period is positive then the insurance firm is said to survive, while if the surplus is negative the insurance firm is said to experience ruin. Given N=5,000 simulations where L represents the number of iterations that produce a negative surplus, the probability of ultimate ruin can be estimated at each discrete time interval as L/N, where the survival probability is 1-(L/N).

A high probability of ruin indicates instability, and measures such as purchasing private reinsurance, or raising premiums should be considered (Dickson, 2005). Despite the many advantages and popularity of using survival probability, the main drawback is that it is essentially a binary measure of solvent, or insolvent (Gerber et al., 1987; Kaas et al., 2008; Rolski et al., 1999; Mango, 2006). In other words, this measure does not indicate the extent of shortfall, only that the balance becomes negative.

Deficit at ruin refers to the distribution of losses that result when surplus becomes negative (Gerber and Shiu, 1997), and the amount of shortfall is recorded for each simulation if ruin occurs within the forecasting period, t (e.g. 30 years). The year in which the balance becomes negative is identified, and the corresponding negative surplus at this time period is discounted to t=0 (where i is assumed to be 3%). From this information, a distribution for the deficit at ruin is then constructed. The advantage of this risk measure is that it goes a step beyond the probability of ruin, and provides information on the severity of losses. Therefore, this measure provides a better indicator of safety, measuring the whole tail of the distribution rather than a single percentile (Mango, 2006).

Results

To evaluate the three alternative insurance models under the assumption of a full premium pool, an asset liability management (ALM) approach using simulation is used to compare surplus, survival probability, and deficit at ruin. Model 1 is a self managed insurance pool model, and Model 2 is a self managed insurance pool and also includes group buying of private reinsurance. Model 3 is a portfolio approach to combine a self managed insurance pool and private reinsurance using CV of the LCR to segregate the portfolio and combine the two approaches.

Surplus

Surplus is a measure of risk that reflects the balance in the account at the end of time t. Figures 2.2, 2.3, and 2.4 show the average surplus, 95th percentile of surplus, and 5th percentile of surplus, respectively, for Models 1, 2, and 3. The average surplus over the 30 year forecasting period is compared in each of the three models in Figure 2.2. The self managed insurance pool model, Model 1, achieves the highest surplus by year 30 ($2.2 billion). This can be attributed to the elimination of costly brokerage fees, which helps the fund accumulate wealth at an increased rate compared to the other two models.

Model 2, a self managed insurance pool that also purchases private reinsurance as a group, produces the smallest average surplus ($807 million). This inferior result corresponds to the expensive reinsurance brokerage fees that are incurred for all of the risks in the portfolio. This demonstrates the long term effect of private reinsurance, which causes the fund balance to deplete each successive year.

Lastly, the portfolio approach that combines a self managed insurance pool and group buying of private reinsurance using the CV of the LCR, Model 3, achieves lower surplus than Model 1 but higher surplus than Model 2 ($1.7 billion). This middle result corresponds to the blended approach where costly reinsurance brokerage fees are incurred for only the second group of risks within the portfolio with high CV of the LCR.

The 95th percentile of surplus, which corresponds to some of the best case scenarios of the simulations, is compared for the three models in Figure 2.3. The 95th

percentile of surplus means that 95% of simulation values are below this value. In other words, this value represents the 250th best case scenario of the 5,000 surplus values determined through simulation. Figure 2.3 demonstrates the considerable effect that expensive reinsurance brokerage fees can have on depleting surplus over the long-term, particularly when losses are not extreme (as represented by the 95th percentile). For example, the surplus values of Model 1 (8.7 billion) and Model 3 ($8.0 billion) are approximately 181% and 167% higher than Model 2 surplus ($4.8 billion), respectively.

The 5th percentile of surplus, which corresponds to some of the worst case scenarios of the simulations, is compared for the three models in Figure 2.4. The 5th percentile of surplus means that 95% of simulation values are above this value. In other words, this value represents the 4,750th worst case scenario of the 5,000 surplus values determined through simulation. This figure demonstrates the major benefit offered by private reinsurers, which is a reduction in the risk that transpires for large losses that can be widespread (as represented by the 5th percentile). For example, the surplus values of Model 1 (-$6.5 billion) and Model 3 (-$6.4 billion) are approximately 144% and 142% higher than Model 2 surplus (-$4.5 billion), respectively.

Survival Probability

Table 2.3 lists summary results for Models 1, 2, and 3, including survival probability at year 30, for each of the ten regions (provinces), in addition to two groupings of regions (PROV5 and PROV10). Overall, survival probability is improved for the three pooled models compared to the ten individual regions. Further, survival probability is improved for the grouping of 10 regions, compared to the grouping of 5 regions. Figure 2.5 compares the survival probability of Models 1, 2, and 3, under the scenario of all ten regions participating in the pool (PROV10). At the end of the 30 year projection period, Model 2 has the highest survival probability of 62.8%, Model 1 has survival of 56.8%, and Model 3 has survival of 53.6%.

Figure 2.5 shows that survival probability decreases over time for all three models, due to the conservative definition of ruin used here, ultimate ruin. In this study, once the surplus in a particular simulation becomes negative (e.g. is ruined), the simulation is not allowed to continue and possibly become nonnegative again. For example, given 5000 simulations, if 100 simulations are ruined in year 1, then survival probability for year 1 is calculated as (5000-100)/(5000) = 98%. In the second year, only the 4900 'surviving' simulations are allowed to continue. In year 2, 120 simulations ruin, and survival probability for year 2 is calculated as (4900-120)/(5000) = 95.6%. Therefore it is possible to have an increasing surplus function, yet a decreasing survival function. In general, a high and stable survival probability is imperative for the long term success of an insurance company.

The high survival probability of Model 2 reflects the improved diversification of the private reinsurer compared to the self managed insurance pool in Model 1. Private reinsurers hold a well diversified portfolio of international risks from various countries (e.g. outside of Canada) and sectors (e.g. outside of agriculture). Therefore, private reinsurers may be more efficient at offsetting risks that are correlated. Comparatively, Model 1 produces survival probability that is lower than Model 2, which shows that a self managed insurance pool is capable of less diversification through the natural offsetting claims compared to a private reinsurer. Finally, contrary to expectation, the portfolio approach to combine pooling and private reinsurance using CV of the LCR, Model 3, does not produce survival probability in the middle of Model 1 and Model 2. Instead, survival probability for this model is the lowest.

Coefficient of Variation (CV) of the Loss Coverage Ratio (LCR), and Risk Reduction

The coefficient of variation (CV) of the loss coverage ratio (LCR) represents an additional risk measure of normalized dispersion. This risk measure demonstrates that diversification (risk reduction) is achieved through geographically pooling of risks across regions, compared to pooling of risks within each individual region. Table 2.3 shows that the CV of the LCR is lower for pooled models, Models 1, 2, and 3, compared to individual regions that have higher CV of the LCR. Results show that PROV5 has a lower CV of the LCR of 44.11% compared to PROV10 which is 51.88%, which is contrary to expectation that as more provinces are added to the pool CV of the LCR should be reduced to reflect improved diversification. This opposing result is likely due to the inclusion of the two provinces with the highest CV within the pool, SK (136.27%) and NB (173.81%), comprising PROV10 but not PROV5.

Deficit at Ruin

Deficit at ruin is a very important measure of risk that looks at the size of the loss. While survival probability indicates whether the fund “survived”, deficit at ruin is needed to provide insight as to the severity of the loss. For example, a $10 loss has much different implications than a $10 million loss. In this example, survival probability would indicate that ruin occurred, however, it would not be capable of distinguishing between the sizes of losses, as measured by deficit at ruin.

Figure 2.6 shows that the deficit at ruin is found to be the smallest (least severe) for Model 3, a portfolio approach to combine a self managed insurance pool and group buying of private reinsurance (-$554 million). This is likely due to the combined approach that saves costly reinsurance brokerage fees by selectively reinsuring only the second group of risks within the portfolio with high CV of the LCR, while retaining the first group of risks within the portfolio with low CV of the LCR. Conversely, the deficit at ruin is largest (most severe) for the self managed insurance pool model, Model 1 (-$687 million). This is because the self managed insurance pool is not as well diversified as the private reinsurer.

Model 2, a self managed insurance pool that also purchases private reinsurance, produces deficit at ruin that is slightly more severe than Model 3, but less severe than Model 1 (-$579 million). This result shows that based on the empirical data, there is no additional advantage to reinsuring all risks in the portfolio, compared to selectively reinsuring only the second group of risks within the portfolio with high CV of the LCR. Therefore, although Model 3 experiences ruin slightly more often than Model 1 and Model 2 (as measured by lower survival probability), the amount by which the account falls short is the smallest.

<u>Overview of Results</u>

Results confirm that diversification is improved as regions (provinces) combine their insurance business across multiple geographic regions and products. Table 2.3

shows that survival probability is higher for the pooled models, Model 1, 2, and 3, compared to individual regions. In addition, survival probability is higher for the group consisting of all 10 regions, PROV10, compared to the group of only 5 regions, PROV5. It is also observed that CV of the LCR is lower for the pooled models compared to individual regions.

As expected, results from the ALM evaluation shows that Model 3, a portfolio approach to combine a self managed insurance pool and group buying of private reinsurance using CV of the LCR, is a promising innovative risk management approach that provides adequate surplus and survival probability, and improves deficit at ruin. The self managed insurance pool model, Model 1, produces high surplus and survival probability, however, the deficit of ruin is the most severe. Conversely, the group buying of private reinsurance model, Model 2, produces high survival probability and adequate deficit at ruin, however, the surplus is very low. Table 2.3 shows a summary of the three alternative pooling and reinsurance models, and Table 2.4 shows the ranking of the three alternative pooling and reinsurance models based on surplus, survival probability, and deficit at ruin.

Summary

This chapter modeled a full premium pool, where all of the risks from each of the ten provinces were pooled together into one large countrywide pool, which represented the maximum diversification that could be achieved within an insurance sector in an entire country. Three alternative pooling and reinsurance models were developed under a full premium pool, and were evaluated under an asset liability management (ALM) approach. Model 1 was a self managed insurance pool, Model 2 was a self managed insurance pool that also purchased private reinsurance as a group, and Model 3 used a portfolio approach to combine a self managed insurance pool and group buying of private reinsurance using the CV of the LCR. Using simulation, risk measures including surplus, survival probability, and deficit at ruin were considered. The eight step methodology was presented for the new portfolio approach that combined pooling and private reinsurance using CV of the LCR, Model 3. The models were evaluated using the complete crop insurance sector for Canada, which included 32 years of actual premiums and liabilities (1978-2009), across 10 provinces for 279 crop types.

This chapter analyzed the diversification that could be achieved through pooling insurance business across multiple geographic regions (provinces) and products (crops), as a possible solution to help reduce the variance of aggregate risks in a portfolio with high variance. An innovative insurance portfolio model, Model 3, was developed in order to combine the benefits of both pooling and private reinsurance, using CV of the LCR to segregate a portfolio of aggregate risks with high variance into two groups, and then combine them. The first group of risks within the portfolio with low CV of the LCR was retained internally within the insurance firms self managed insurance pool. The second group of risks within the portfolio with high CV of the LCR was ceded to private reinsurers.

Results showed that diversification was improved through geographic pooling of risks across regions compared to the pooling of risks within individual regions (as represented by lower CV of the LCR and high survival probability for Models 1, 2, and 3, compared to individual regions). While the self managed insurance pool model, Model 1, produced high surplus and adequate survival probability, in the event that ruin occurred, the deficit at ruin was the most severe. Conversely, Model 2, a self managed insurance pool that also purchased private reinsurance, successfully achieved high survival probability and an adequate deficit at ruin, however, surplus was very low.

A portfolio approach to combine a self managed insurance pool and group buying of private reinsurance, Model 3, allows regions to take advantage of natural hedging across geographical regions and products. Despite the large geographic region that Canada occupies, however, a self managed insurance pool still required the help of some additional private reinsurance to stabilize losses when risks were extreme and widespread. Results showed that Model 3 reduced the amount of private reinsurance that needed to be purchased by 50%, without incurring additional risk (as measured by surplus, survival probability, and deficit at ruin). This constitutes a promising innovative risk management strategy.

Table 2.1 Summary of Methodology using CV of the LCR Under a Full Premium Pool

Step 1	Developing the ALM Approach
Step 2	Development and Description of Three Alternative Pooling and Reinsurance Models (Models 1, 2, and 3)
Step 3	Calculation of Reinsurance Layers Using the Loss Coverage Ratio (LCR) and a Loss Elimination Ratio (LER)
Step 4	Using Coefficient of Variation to Segregate the Portfolio
Step 5	Simulating Claims and Calculation of the Expected Insurance Premium and Reinsurance Premium
Step 6	Forecasting of Liabilities Using a Least Squares Regression Model
Step 7	Calculating Surplus Using and ALM Surplus Process Under a Full Premium Pool
Step 8	Determining Survival Probability, and Deficit at Ruin to Evaluate the Most Effective Model

Notes: This table provides a summary of the methodology employed in this chapter to combine pooling and private reinsurance using the coefficient of variation (CV) of the loss coverage ratio (LCR). This table highlights the eight basic steps that are necessary to produce an ALM surplus model, where the portfolio is segregated into two groups, which are then combined. The first group of risks within the portfolio with low CV of the LCR is retained internally within a self managed reinsurance pool. The second group of risks from within the portfolio with high CV of the LCR is ceded externally to private reinsurers.

The eight basis steps include the developing the ALM approach, development and description of three alternative pooling and reinsurance models, calculation of reinsurance layers using the loss coverage ratio (LCR) and a loss elimination ratio (LER), using coefficient of variation (CV) to segregate the portfolio, simulating claims and calculation of the expected insurance and reinsurance premium, and the forecasting of liabilities. From these variables, surplus can be calculated over each of the forecasting periods (e.g. 30 years) using an ALM process. Finally, the surplus calculation can be used to calculate the risk measures of survival probability, and deficit at ruin to evaluate the most effective model.

Table 2.2 Statistical Distribution, Reinsurance Premium, and Reinsurance Coverage Layers by Region

Province	Distribution	Reinsurance Premium: $E(Y_j)$	Reinsurance Layer 1: "u"	Reinsurance Layer 2: "d"
Alberta (AB)	Loglogistic	1.1504%	14.5%	24.5%
British Columbia (BC)	Gamma	0.8362%	12.7%	22.7%
Manitoba (MB)	Loglogistic	0.8142%	11.7%	21.7%
Ontario (ON)	Weibull	0.6737%	10.1%	20.1%
Prince Edward Island (PEI)	Gamma	0.8805%	11.8%	21.8%
Saskatchewan (SK)	Loglogistic	1.1157%	15.4%	25.4%
New Brunswick (NB)	Loglogistic	1.5760%	18.8%	28.8%
Newfoundland (NFLD)	Weibull	1.3839%	19.2%	29.2%
Nova Scotia (NS)	Weibull	0.6845%	8.6%	18.6%
Quebec (QC)	Gamma	0.7039%	9.8%	19.8%

Notes: This table summarizes the best statistical distribution, expected reinsurance premium, and the lower and upper bound reinsurance coverage layer for each of the 10 geographic regions (provinces). The historical loss coverage ratio (LCR) from 1978 through 2009 is examined for each of 10 regions, and Palisade's @Risk is used to choose the best statistical distribution to be used in simulation. The LCR is simulated 5,000 times, 30 years in the future. Loglogistic, Gamma, and Weibull distributions are found to be the best fit for the data, using the Kolmogorov-Smirnov fit statistic.

The reinsurance payment $E(Y_j)$ for region j is calculated using Equations [2.4] through [2.9] that correspond to the selected distribution. The upper and lower bound reinsurance coverage layer, "u" and "d", are manipulated in order to yield a loss elimination ratio (LER) of 90% to help ensure that premiums are allocated equitably across regions.

Table 2.3 Summary Results for 10 Regions and 3 Alternative Pooling and Reinsurance Models

Province	Initial Surplus	Surv. Prob. (Yr 30)	Surplus (Yr 30)	CV of the LCR
AB	$226,809,302	42.9%	$591,533,544	76.06%
BC	$37,190,012	37.8%	$96,387,166	68.73%
MB	$109,843,767	41.0%	$256,641,817	99.22%
ON	$115,138,008	36.4%	$302,817,431	64.75%
PEI	$10,191,655	40.4%	$23,679,117	55.10%
SK	$291,727,361	41.4%	$711,677,117	136.27%
NB	$8,609,738	41.6%	$19,284,266	173.81%
NFLD	$141,161	36.6%	$372,149	65.25%
QC	$71,622,817	38.9%	$177,455,671	57.36%
NS	$1,213,041	44.4%	$3,160,155	47.92%
Model	**Initial Surplus**	**Surv. Prob. (Yr 30)**	**Surplus (Yr 30)**	**CV of the LCR**
M1, PROV5	$499,172,745	53.3%	$ 1,271,059,509	44.11%
M2, PROV5	$499,172,745	56.2%	$ 493,243,640	44.11%
M3, PROV5	$499,172,745	49.4%	$1,008,801,665	44.11%
M1, PROV10	$872,486,863	56.8%	$ 2,183,008,868	51.88%
M2, PROV10	$872,486,863	62.8%	$ 807,521,333	51.88%
M3, PROV10	$872,486,863	53.6%	$1,682,889,013	51.88%

Notes: This table shows that diversification is improved for geographic pooling of risks across regions (provinces), compared to pooling within regions. For example, survival probability and coefficient of variation (CV) is lower for each individual region compared to pooled models, Models 1, 2, and 3. Furthermore, diversification, as measured by survival probability, is improved for a pooled model of 10 regions (PROV10) compared to 5 regions (PROV5). While the CV was also expected to be lower for PROV10 compared to PROV5, the opposing results are likely due to the high risk region of SK and NB that comprise PROV10 but not PROV5. In comparing the three alternative pooling and reinsurance models, Model 3 is found to be a promising innovative risk management approach that lowers reinsurance premium costs, and produces adequate surplus, survival probability, and the lowest deficit at ruin. M1 is Model 1, M2 is Model 2, and M3 is Model 3.

The three alternative pooling and group buying of private reinsurance models are described next. The first model, Model 1 is a self insurance pool that does not purchase private reinsurance. Model 2 is

Table 2.3 (Continued)

similar to Model 1, however, in addition the pool purchases private reinsurance as a group for all risks in the portfolio. Model 3 combines Model 1 and Model 2 using CV of the LCR to identify one group of risks within the portfolio with low CV of the LCR to be retained in the a self managed insurance pool as in Model 1, and a second group of risks within the portfolio with high CV of the LCR to be ceded to private reinsurers as in Model 2.

The initial surplus, the amount of surplus required in the fund at t=0, is assumed to be equal to one times the premium loading, which is 1*E(X)*Li. Survival probability is a measure of fund stability that represents the percentage of iterations that have a positive fund balance. Surplus represents the account balance to be allocated to an ALM approach. Coefficient of variation (CV) is a normalized measure of dispersion of the loss coverage ratio (LCR), where crops that have higher CV of the LCR are considered to have more risk than crops with low CV of the LCR.

Table 2.4 Ranking of Models 1, 2, and 3 for Surplus, Survival Probability, and Deficit at Ruin

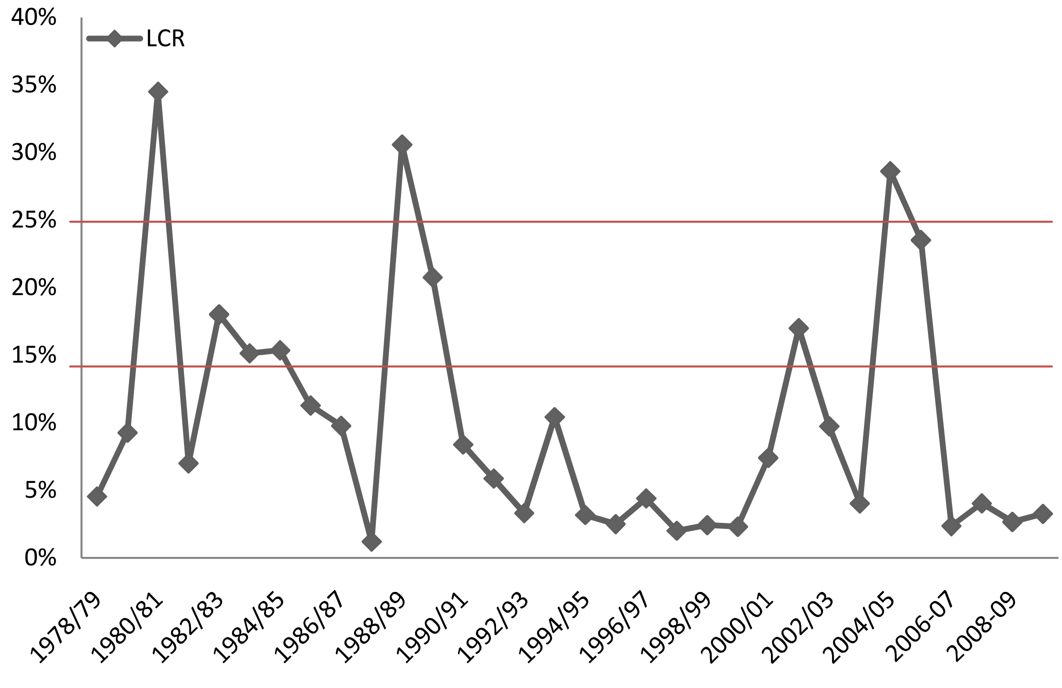
Risk Measure	Best Ranking	Middle Rank	Worst Ranking
Surplus	Model 1	Model 3	Model 2
Survival Probability	Model 2	Model 1	Model 3
Deficit at Ruin	Model 3	Model 2	Model 1

Notes: This table compares survival probability, surplus, and deficit at ruin, for the three pooling and reinsurance models under a full premium pool for the grouping of 10 regions (PROV10). The results show that Model 3, which combines a self managed insurance pool (Model 1) and group buying of private reinsurance (Model 2), is a promising innovative risk management approach that reduces the amount of private reinsurance that needs to be purchased by 50%, and achieves adequate surplus, survival probability, and the lowest deficit at ruin.

The three alternative pooling and group buying of private reinsurance models are described next. The first model, Model 1 is a self insurance pool that does not purchase private reinsurance. Model 2 is similar to Model 1, however, in addition the pool purchases private reinsurance as a group for all risks in the portfolio. Model 3 combines Model 1 and Model 2 using CV of the LCR to identify one group of risks within the portfolio with low CV of the LCR to be retained in the a self managed insurance pool as in Model 1, and a second group of risks within the portfolio with high CV of the LCR to be ceded to private reinsurers as in Model 2.

Survival probability is a measure of fund stability that represents the percentage of iterations that have a positive fund balance. Surplus represents the account balance to be allocated to an ALM approach. Deficit at ruin measures the severity of shortfall when the balance becomes negative.

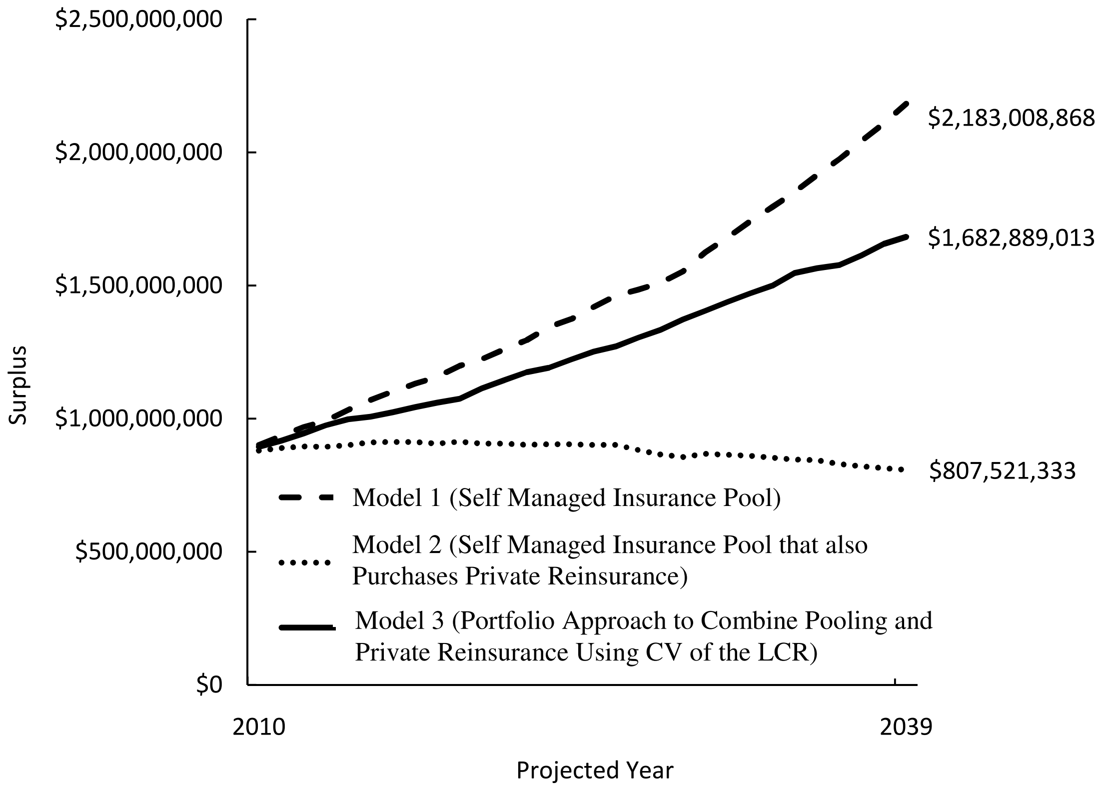
Figure 2.1 Loss Coverage Ratio (LCR) for the Province of Manitoba, and Reinsurance Coverage for Layers 15% to 25%



Notes: This figure shows historical LCR from 1978 through 2009 for the region (province) of Manitoba. The upper and lower bound reinsurance coverage layer of 15% to 25% is also shown. In this example, the provincial crop insurance company is responsible for losses, as measured by LCR, below the first layer of reinsurance, 15%. The losses (LCR) that exceed 15%, up to 10%, are transferred to the private reinsurer.

The loss coverage ratio, LCR, is the ratio used to establish actuarially sound premiums and is a measure of risk. This ratio is also used to show the annual loss for each province and is calculated by dividing total indemnities by total liabilities (total coverage), multiplied by 100.

Figure 2.2 Average Surplus for Models 1, 2, and 3



Notes: This figure shows the average surplus for Models 1, 2, and 3, for all 10 regions, provinces (PROV10) under a full premium pool. With an initial surplus equal to 1 times the premium loading (e.g. 1*E(X)*Li), results show that the self managed insurance pool model, Model 1, achieves the largest surplus. This is because of the costly brokerage fees that are eliminated which helps surplus accumulate at a faster rate. The combined pooling and private reinsurance model, Model 3, achieves surplus that is lower than Model 1. The group buying of private reinsurance model, Model 2, achieves the lowest surplus.

Surplus, the balance in the account during each time period, is compared over 30 years in the future. Surplus is calculated by the following formula: $U_t = U_{t-1} + P_t + C_t - S_t$, where ($U_t$) is the surplus in the account at the beginning of the t_{th} period, (P_t) is the premiums collected during the t_{th} period, (C_t) is the interest earned on the surplus the t_{th} period, and (S_t) is any claims paid out during the t_{th} period.

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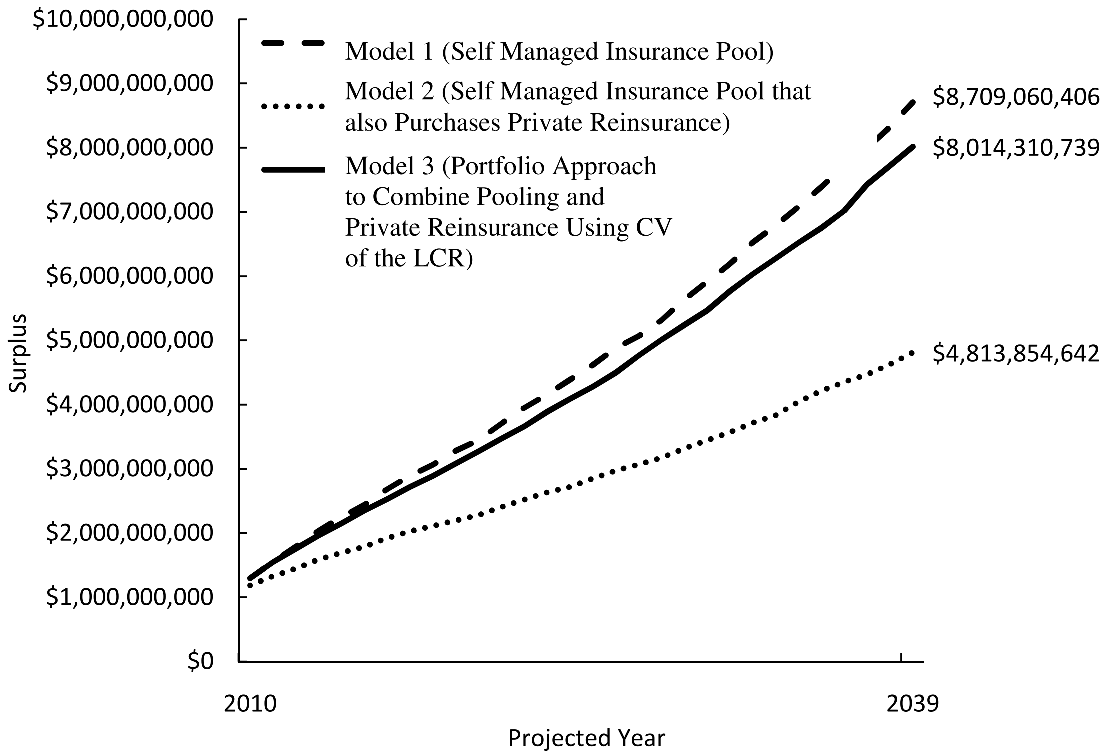
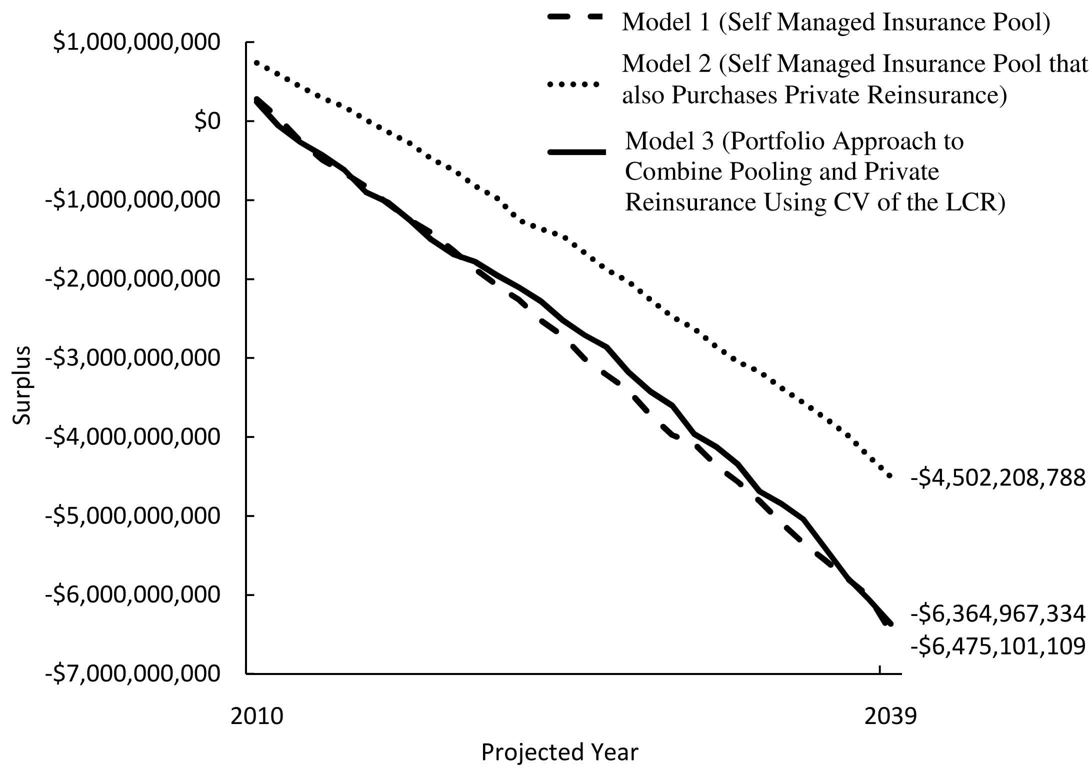
Figure 2.3 95th Percentile of Surplus for Models 1, 2, and 3 (Best Case Scenarios)



Notes: This figure shows the 95th percentile of surplus for Models 1, 2, and 3, for all 10 regions, provinces (PROV10) under a full premium pool. The 95th percentile of surplus means that 95% of simulation values are below this value. In other words, this value represents the 250th best case scenario of the 5,000 surplus values determined through simulation. With an initial surplus equal to 1 times the premium loading (e.g. 1*E(X)*Li), results show the self managed insurance pool model, Model 1, produces the largest surplus under the best case scenarios when losses are least severe and uncorrelated. The combined pooling and private reinsurance model, Model 3, produces surplus that is slightly smaller than Model 1. The group buying of private reinsurance model, Model 2, produces surplus that is substantially lower than the other two models. This demonstrates the considerable effect that expensive reinsurance brokerage fees can have on depleting surplus over the long-term, particularly when losses are not extreme.

Surplus, the balance in the account during each time period, is compared over 30 years in the future. Surplus is calculated by the following formula: $U_t = U_{t-1} + P_t + C_t - S_t$, where ($U_t$) is the surplus in the account at the beginning of the t_{th} period, (P_t) is the premiums collected during the t_{th} period, (C_t) is the interest earned on the surplus the t_{th} period, and (S_t) is any claims paid out during the t_{th} period.

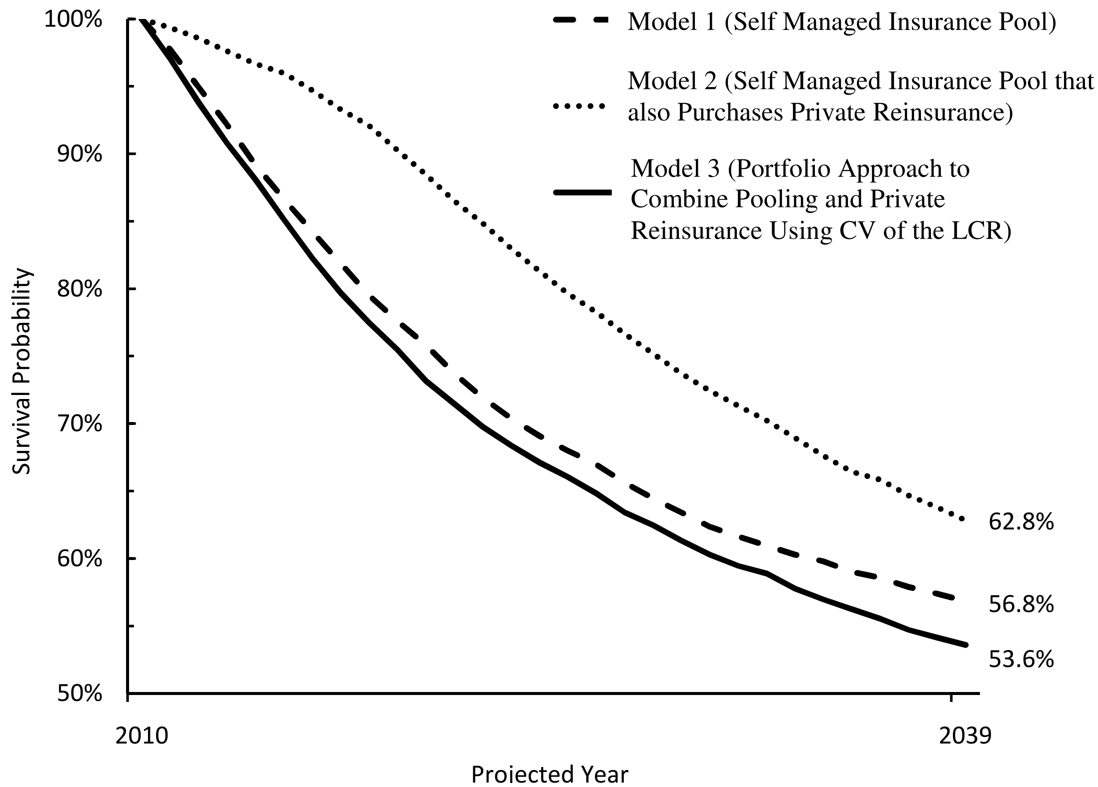
Figure 2.4 5th Percentile of Surplus for Models 1, 2, and 3 (Worst Case Scenarios)



Notes: This figure shows the 5th percentile of surplus for Models 1, 2, and 3, for all 10 regions, provinces (PROV10) under a full premium pool. The 5th percentile of surplus means that 95% of simulation values are above this value. In other words, this value represents the 4,750th worst case scenario of the 5,000 surplus values determined through simulation. With an initial surplus equal to 1 times the premium loading (e.g. 1*E(X)*Li), results show the group buying of private reinsurance model, Model 2, produces the largest surplus. This is because private reinsurers are better diversified than a self managed insurance pool when losses are large and correlated. The self managed insurance pool model, Model 1, as well as the combined pooling and private reinsurance model, Model 3, achieve similar results of low surplus.

Surplus, the balance in the account during each time period, is compared over 30 years in the future. Surplus is calculated by the following formula: $U_t = U_{t-1} + P_t + C_t - S_t$, where ($U_t$) is the surplus in the account at the beginning of the t_{th} period, (P_t) is the premiums collected during the t_{th} period, (C_t) is the interest earned on the surplus the t_{th} period, and (S_t) is any claims paid out during the t_{th} period.

Figure 2.5 Survival Probability for Models 1, 2, and 3

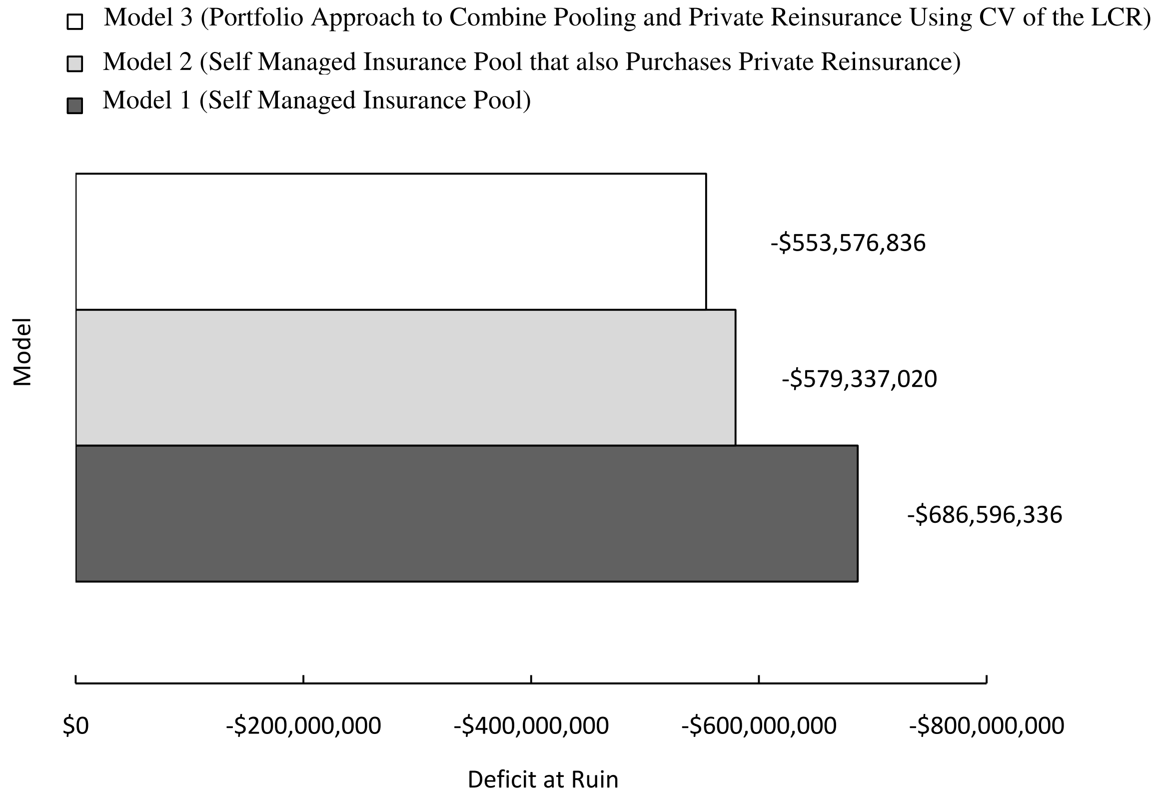


Notes: This figure shows survival probability for Models 1, 2, and 3, for all 10 regions, provinces (PROV10) under a full premium pool. With an initial surplus equal to 1 times the premium loading (e.g. 1*E(X)*Li), results show that Model 2, the group buying of private reinsurance model, has the highest survival probability. This is because private reinsurers are better diversified and 'ruin' less frequently, compared to a self managed insurance pool that ruins more frequently The self managed reinsurance pool model, Model 1, and the combined pooling and private reinsurance model, Model 3, produce the lowest survival probability, respectively.

The three alternative pooling and group buying of private reinsurance models are described next. The first model, Model 1 is a self insurance pool that does not purchase private reinsurance. Model 2 is similar to Model 1, however, in addition the pool purchases private reinsurance as a group for all risks in the portfolio. Model 3 combines Model 1 and Model 2 using CV of the LCR to identify one group of risks within the portfolio with low CV of the LCR to be retained in the a self managed insurance pool as in Model 1, and a second group of risks within the portfolio with high CV of the LCR to be ceded to private reinsurers as in Model 2.

Survival probability is compared over 30 years in the future and refers to the probability that the fund will 'survive' without ruin (e.g. have enough reserves to pay claims). This study applies a conservative definition of ruin, ultimate ruin, where once surplus becomes negative it is not possible for it to continue. Therefore, it is possible to have increasing surplus, yet decreasing survival probability.

Figure 2.6 Average Deficit at Ruin for Models 1, 2, and 3



Notes: This figure shows the average deficit at ruin for Models 1, 2, and 3, for all 10 regions, provinces (PROV10) under a full premium pool. With an initial surplus equal to 1 times the premium loading (e.g. 1*E(X)*Li), results show that self managed insurance pool model, Model 1, produces the most severe deficit at ruin. This is because a self managed insurance pool is insufficiently diversified for losses that are large and correlated. The combined pooling and private reinsurance model, Model 3, has the lowest deficit at ruin. This is because the first group of risks in the portfolio that have low CV of the LCR are retained in the self managed insurance pool, and therefore avoids costly reinsurance brokerage fees. However, the second group of risks within the portfolio that has high CV of the LCR is ceded to private reinsurers who are more efficient at diversifying this group of risks. The group buying of private reinsurance model, Model 2, has a deficit that is marginally higher (more severe) than Model 3. Therefore, although Model 3 ruins slightly more frequently than Model 1 and 2, the shortfall is less.

The three alternative pooling and group buying of private reinsurance models are described next. The first model, Model 1 is a self insurance pool that does not purchase private reinsurance. Model 2 is similar to Model 1, however, in addition the pool purchases private reinsurance as a group for all risks in the portfolio. Model 3 combines Model 1 and Model 2 using CV of the LCR to identify one group of risks within the portfolio with low CV of the LCR to be retained in the a self managed insurance pool as in Model 1, and a second group of risks within the portfolio with high CV of the LCR to be ceded to private reinsurers as in Model 2.

Deficit at ruin represents the severity of ruin, and demonstrates the degree to which the funds liabilities exceed its assets. To calculate deficit at ruin, a distribution of simulated surplus values just prior to ruin is created. In addition, the surplus value prior to ruin is discounted according to the period in which ruin occurred to t=0 at an assumed rate of 3%.

CHAPTER 3

REINSURANCE PREMIUM POOL: COMBINATORIAL OPTIMIZATION WITH A GENETIC ALGORITHM TO COMBINE POOLING AND PRIVATE REINSURANCE TO REDUCE RISK

Introduction

Chapter three, in contrast to chapter two, uses a reinsurance premium pool, where regions (provinces) contribute only a portion of their risk to the pool, and therefore allocate only a portion of the insurance premium, reinsurance premium, to the pool. This is different than chapter two, where a full premium pool was assumed. One of the objectives of chapter three is to model a reinsurance premium pool to address one of the potential limitations of chapter two, which is the reluctance that some provinces may have in transferring control of their region to cooperate in the pool. A reinsurance premium pool is an incremental approach to pooling which allows regions to continue operating independently, where only a portion of risks, approximately 10%, are pooled. The remaining 90% of risks are retained within each provincial crop insurance company.

The second objective of chapter three is to develop a more efficient insurance portfolio model, using a more precise method than the CV of the LCR in chapter two, to segregate the portfolio into two parts and then combine them. In chapter three, combinatorial optimization with a genetic algorithm is used to optimally manage a portfolio of aggregate risks with high variance. The first group of risks within the portfolio is uncorrelated, and diversified enough to be managed internally within a pool. This group of uncorrelated risks within the pool naturally offsets, which lowers the variance of the aggregate risks within this set. The second group of risks within the portfolio is correlated, and therefore the risks do not sufficiently offset, which leads to a high variance portfolio of risks. This second group of risks is ceded to private reinsurers who are better diversified for risks that are large and correlated.

Under the assumption of a reinsurance premium pool here, this chapter develops three alternative pooling and reinsurance models which are evaluated under an ALM approach. Using simulation, risk measures including surplus, survival probability, and deficit at ruin are considered. As in the previous chapter, this chapter focuses on the complete crop insurance sector for Canada, considering ten provincial non-profit crop insurance corporations to evaluate the models. A comprehensive data set is considered, including 32 years of actual premiums and liabilities (from 1978-2009), across 10 regions for 279 crops.

The first model, **Model A**, is a self managed reinsurance pool that combines only a portion of the risk from each geographic region (province) into its own internal reinsurance company. This model benefits from risks that naturally offset and are uncorrelated across regions and/or products in order to reduce risk to the reinsurance company. The advantage of this model is that it saves brokerage fees normally paid to private reinsurers. However, the disadvantage of this approach is that the pool may not be sufficiently diversified for extreme events that are widespread, such as a drought that destroys the crops across a many regions in a country.

A second alternative, **Model B,** is proposed where instead of the firm providing its own reinsurance as in Model A, it purchases reinsurance with other firms as a group from the private reinsurance market. The advantage of private reinsurance is that it is better diversified than the self reinsurance firm in Model A, due to the well diversified portfolio of risks that are uncorrelated with other international risks in its portfolio. The disadvantage of this approach is that private reinsurance is more expensive.

A third alternative, **Model C,** is proposed to blend the benefits of Model A and Model B using a portfolio approach. Using combinatorial optimization with a genetic algorithm, Model C takes one group of risks from within the portfolio that is relatively uncorrelated and naturally offsetting (and therefore has reduced variance in the portfolio), and retains it within a self managed reinsurance pool. The second group of risks within the portfolio that is correlated and does not sufficiently offset (and therefore has high variance in the portfolio), is ceded externally to private reinsurers. The benefit of this approach is that private reinsurance from Model B will overcome the Model A problem of insufficient diversification from extreme events that are widespread (e.g. all regions facing the same large risks in the same year). Also, Model C should benefit from the risks that are uncorrelated with a lower portfolio variance that are retained within the pool (Model A), as this is a lower cost option than private reinsurance (Model B).

The remainder of this chapter is organized as follows. The next section discusses past literature and background. Following this, the data and then methodology are discussed where a genetic algorithm is used to optimize and combine a self managed reinsurance pool and group buying of private reinsurance model (Model C). Results of the asset liability management (ALM) model for the three models under the assumption of a reinsurance premium pool are compared and contrasted, and a summary section is presented.

Literature

While the concept of pooling regarding crop insurance has historically faced a number of constraints, in recent years pooling has become more feasible. The first concept of crop insurance in Canada dates back to 1939, when the federal government implemented the Prairie Farm Assistance Act (PFAA). This income protection program resembled a crop insurance program, and protection was provided for a block or township basis. Premiums were pooled between the provinces of Manitoba, Saskatchewan, and Alberta (Hueth and Furtan, 1994). However, this pooling became a problem, as farmers in some provinces were charged too low a premium, while others were charged too high a premium, relative to their risk level.

Around the mid 1950's, farmers began to question the PFAA, not convinced that they were receiving adequate protection. One of the major criticisms of the pooling program that had developed was that premiums were not differentiated between regions. There was perceived inequity in risk levels between provinces, and it was felt that regions that had high yields with low variability were subsidizing the farmers in other regions that had low yields with much higher risk. This feeling of inequity was supported by a comparison of premiums collected and claims paid to each province between 1939 and 1955 (House of Commons, 1955). Manitoba collected approximately \$13.2 million in premiums, compared to \$24.6 million in Alberta, and \$47.9 million in Saskatchewan. In Manitoba \$8.2 million in claims were paid, in Alberta it was \$42.5 million, and in Saskatchewan it was \$126.4 million. The ratio of claims to premiums was 62% for

Manitoba, 173% for Alberta, and 264% for Saskatchewan. It appeared that significant cross-subsidization was occurring.

Further criticism cited that farmers had different needs across regions, and that in order for the program to be successful the details had to be individualized (Hueth & Furtan, 1994). In 1955, Manitoba cited that the P.F.A.A. was primarily designed for drought, the primary risk in Saskatchewan for example, but not Manitoba's main crop hazard (House of Commons, 1955). Therefore, there was a strong push to move to a more independent approach where each province was responsible for its own crop insurance program and reinsurance needs. In 1959, the Federal Crop Insurance Act was passed, and each province was given authority over its own crop insurance program. However, the different types and levels of risk between provinces and the associated different premium levels are now much better understood, now that reliable yield history is available. Therefore, the possibility for pooling of reinsurance across provinces is now much more feasible, compared to earlier times. However, some regions will continue to have reservations of transferring control of their region in order to cooperate in the pool. As a result, a more gradual incremental approach to a full premium pool, such as a reinsurance pool proposed here, may be a more realistic first step to making changes to the crop insurance program Canada.

Data

This chapter uses the same comprehensive data set from chapter two, which represents an entire insurance sector in a country. Historical crop losses from 1978 through 2009 are obtained from Agriculture and Agri-Food Canada's Production Insurance National Statistical System (PINSS). This includes 32 years of actual premium rates and indemnities (from which the loss coverage ratio, LCR is calculated) across all ten Canadian provinces for all 279 crops. The ten Canadian provinces include Alberta, British Columbia, Manitoba, New Brunswick, Newfoundland, Nova Scotia, Ontario, Prince Edward Island, Quebec and Saskatchewan.

Methodology

An asset liability management (ALM) approach is used to develop an optimal insurance portfolio approach to combine a self managed reinsurance pool and group buying of private reinsurance. In this chapter, there are eight steps that are necessary to optimize the benefits of combined pooling and private reinsurance using combinatorial optimization with a genetic algorithm, under a reinsurance premium pool (Table 3.1). The focus of the methodology is on step 4 and step 7, as these two steps differ from the eight step methodology presented in chapter two for the portfolio approach to combine pooling and private reinsurance using the CV of the LCR. Step 4 outlines the process to using combinatorial optimization with a genetic algorithm to segregate a portfolio, and step 7 shows how surplus is calculated using an ALM surplus approach under a reinsurance premium pool.

The eight steps to the methodology developed in this chapter, include developing the ALM surplus approach, development and description of three alternative pooling and reinsurance models (Models A, B, and C), using combinatorial optimization with a genetic algorithm to segregate the portfolio, simulating claims and calculation of the expected insurance and reinsurance premium, and the forecasting of liabilities using a least squares regression model. From these variables, surplus can be calculated over each of the 30 forecasting periods considered in this study under a reinsurance premium pool, using an ALM process. In addition, survival probability, and deficit at ruin can be determined in order to evaluate the most effective model.

Step 1: Developing the ALM Surplus Approach

The first step includes developing an asset liability management (ALM) approach, which follows the surplus process shown in Equation [3.1]. This step is explained in detail in chapter two.

$$U_t = U_{t-1} + P_t + C_t - S_t \qquad [3.1]$$

where

(U_t) Surplus at the beginning of the t_{th} period.

(P_t) Premiums at the beginning of the t_{th} period.

(C_t) interest on retained cash at the end of the t_{th} period.

(S_t) Claims at the end of the t_{th} period.

Step 2: Development and Description of Three Alternative Pooling and Reinsurance Models (Models A, B, and C)

Step two involves developing and defining three alternative pooling and reinsurance models under a reinsurance premium pool, that are later evaluated in order to find the best model. Each model assumes that each region continues to operate individually, but in addition, a portion of the risks from each region are contributed to the self managed reinsurance pool. The corresponding reinsurance premium that each region allocates to the pool is based on a 10% layer of reinsurance coverage that is specific to the region, yielding a loss elimination ratio (LER) of 90%. This variable reinsurance layer, which determines the level of loss that must be incurred in order to trigger a reinsurance payment, helps to ensure that reinsurance premiums are equitable among contributing regions.

The first alternative approach is **Model A**, where a smaller insurance organization along with others, forms their own joint reinsurance company. Each smaller organization from different geographic regions (provinces) contributes a portion of their risk and associated reinsurance premium into their jointly owned reinsurance company that is self managed. This approach is based on the assumption that large losses across regions will be uncorrelated and offsetting, and therefore risk will be reduced for the insurance company. The advantage of forming a joint reinsurance company is that the purchase of higher cost private reinsurance can be avoided, providing more efficiency. The disadvantage is that in a bad year when losses across regions do not adequately offset, the reinsurance company could experience ruin because it is not sufficiently diversified to the same extent as a well diversified international reinsurer. It is also assumed that in exchange for reinsurance coverage, a reinsurance premium rate equivalent to the actuarially fair rate, plus a loading fee of 35% (which is equivalent to the current estimate of private reinsurance brokerage fees) is charged to each region, and allocated to the pool to help build reserves in the insurance fund.

A second alternative approach is **Model B,** where instead of the firm (e.g. province) providing its own reinsurance as above, it cooperates with the other smaller insurance organizations (provinces) and purchases private reinsurance as a group from the international reinsurance market. The advantage to this approach is that the private reinsurance firm is better diversified than the self managed pool in Model A, and is much less likely to fail because it holds a well diversified portfolio comprised of international liabilities from multiple countries (e.g. outside of Canada) across different insurance sectors (e.g. outside of agriculture). The disadvantage of this approach is that private reinsurance is more expensive, however, some savings are realized through the group buying of private reinsurance. Group buying of private reinsurance brings market power and allows the self managed reinsurance pool to negotiate a reduced brokerage rate, which is conservatively assumed to be 25% compared to the individual rate it would otherwise pay (estimated to be 35%).

Model C is a portfolio approach to combine Model A and Model B, and captures the benefits of both reinsurance pooling and private reinsurance. This model uses combinatorial optimization with a genetic algorithm to segregate a portfolio into two groups, and then combines them. The first group contains risks that are relatively uncorrelated across the smaller insurance organizations, and therefore the variance of the portfolio is reduced which is appropriate for pooling internally across the smaller firms (provinces) (Model A). A second group of high variance risks are then ceded to private reinsurers (Model B). The private reinsurance should overcome the Model A problem of insufficient diversification from extreme events that are widespread (e.g. if all regions face the same large risk in the same year and the joint reinsurance company fails). In addition, Model C should benefit from the low variance risks that are uncorrelated and retained within the pool (Model A) as this is a lower cost option than private reinsurance. This option is lower cost because rather than paying reinsurance brokerage fees on the entire portfolio, reinsurance brokerage is only incurred on the second group of high variance risks in the portfolio that may benefit the most from improved diversification of private reinsurance.

For the group of high variance risks that is ceded to private reinsurers, it is assumed that group buying will bring a slight reduction in the brokerage rate to 30%. While group reinsurance buying may help to negotiate a reduced rate as in Model B (where brokerage fees are assumed to be 25%), the selective nature of Model C reinsuring only 'high variance' risks may reduce the amount of savings that can be negotiated. Private reinsurers have indicated that aggregate risks in the portfolio with high variance may not necessarily be high risk to the reinsurer, and so it would be acceptable for the reinsurance to sell the reinsurance. As long as risks are not correlated with the reinsurer's portfolio, then it is not "high risk" to the reinsurer. Therefore, private reinsurers may be more effective at offsetting these high variance risks.

Step 3: Calculation of Reinsurance Layers using the Loss Coverage Ratio (LCR) and a Loss Elimination Ratio (LER)

This step covers the calculation of reinsurance coverage layers using the loss coverage ratio (LCR) and a loss elimination ratio (LER). This step is explained in detail in chapter two.

Step 4: Using Combinatorial Optimization with a Genetic Algorithm to Segregate the Portfolio

Step four includes construction of an optimization model to identify the group of low variance risks in the portfolio to be managed internally in the firms self managed reinsurance pool, as well as the group of high variance risks for ceding to private reinsurers. Optimization is an area of applied mathematics, and is a methodology for selecting an optimal solution based on three components, including a set of decision variables, an objective function, and a set of constraints. Optimization problems can be categorized based on the characteristics of each component. For example, linear, quadratic, convex, second-order cone, and integer and mixed integer programming are some of the most common classifications of optimization problems (Pachamanova & Fabozzi, 2010) .

Quadratic Programming

When the objective function of a model is quadratic, and the constraints are in linear form, then the optimization problem is called quadratic (Cornueljols and Tutuncu, 2007). Quadratic programming (QP) is a special form of nonlinear programming, and is predominately used in finance for trading models and asset allocation models, such as mean-variance portfolio optimization (MVO) which was developed by Markowitz in 1952. The typical MVO approach solves for the weight of each asset, and determines the composition of a portfolio of assets that minimizes risk while achieving a predetermined level of expected return.

The portfolio optimization approach developed in this study is a quadratic programming problem, and uses the basic framework set forth by Markowitz. However, rather than solving for the weight of each asset, this study solves for the combination of discrete assets (crops), each with a fixed portfolio weight (based on liability) that is always known. This optimization problem is quite large as it considers 279 crop types across 10 regions (provinces). The goal of the optimization problem is to minimize the variance of the loss coverage ratio (LCR) in the pool for the whole country (Model C). The portfolio is optimally divided into one group of risks with low variance that is retained in the self managed reinsurance pool (Model A), and another group of risks with high variance that is ceded to private reinsurers (Model B).

Combinatorial Optimization using a Genetic Algorithm

In this study, the classical MVO problem is modified to include a constraint for the discrete grouping of risks (crops). As a result, this problem belongs to a subset of optimization problems called combinatorial optimization. The discrete combination of risks is analogous to a "switch", where assets are turned either "on" or "off" to reflect risks that are either retained in the self managed reinsurance pool, or ceded to private reinsurers, respectively. Further, the optimization model accounts for risk levels that vary depending on whether the model indicator states the risk should be ceded to private reinsurers, or retained within the self managed reinsurance pool. For example, if the risk is considered to be turned "on", which means that it is retained in the self managed reinsurance pool, then the risk to the pool is the LCR. However, if the risk is considered to be turned "off", which means that it is ceded to private reinsurers, then the risk to the pool is the difference between the LCR and the reinsurance coverage from the private reinsurer.

A closed form solution does not exist to solve this optimization problem. Given that there are 279 crop types considered in the portfolio, iterating through every possible combination quickly becomes computationally exhaustive, and unfeasible. For example, to solve this problem for 279 crop types, there are 279! (factorial) possible combinations. For a group of only 20 risks, there are 20! (factorial) possible combinations, which equates to 2,432,902,008,176,640,000 possibilities. Therefore, an optimization algorithm must be used to find the local optimal solution. A local optimum is not necessarily the best solution, but it is the best solution in a neighborhood of feasible solutions (Pachamanova and Fabozzi, 2010), which satisfies the given constraints on the system. This is in contrast to finding the global optimum solution, which is the best solution among the group of local optimal solutions, but can be computationally exhaustive for large combinations as in this study.

Depending on the classification of optimization problem, different algorithms are used. A useful algorithm that works well for discrete and combinatorial optimizations problems is the branch and bound technique. While this algorithm is not used in this study, it works by dividing the search space and generating a sequence of sub problems

(Clausen, 1999). The search space is represented by a tree, and each node in the tree is identified with a sub problem, derived from previous sub problems, on the path leading to the root of the tree (SAS, 2010). The disadvantage to this approach is that it may never reach the global optimum because it typically stops when the first local optimum is found (Pachamanova and Fabozzi, 2010).

Randomized search algorithms, such as the genetic algorithm, try to improve upon the solution that is found by an algorithm such as a branch and bound technique, by including a degree of randomness as part of its logic. Therefore, a genetic algorithm is used in this study to solve the combinatorial optimization problem. Genetic algorithms are useful in cases where the objective function is nonlinear, and are particularly efficient when some of the decision variables are integer numbers or binary, which is relevant in this study since we are solving for a discrete combination of risks (crops) (Pachamanova and Fabozzi, 2010).

A genetic algorithm is based on the evolution of a complex biological system (Greer and Ruhe, 2004). At the beginning of the algorithm, the initial population of chromosomes is generated using an encoded string of integers or binary bits (Pachamanova & Fabozzi, 2010). The combinations are evaluated using a fitness score, and the combinations that are considered most fit are crossed over to create better solutions. The program continues to evolve until the fitness function is maximized, and the best solution is obtained (Palisade Corporation, 2009).

In this study, a combinatorial optimization model for an insurance portfolio is solved using a genetic algorithm to determine the optimal group of risks within the portfolio with low variance to be retained internally in the self managed reinsurance pool, without private reinsurance. The remaining second group of risks from within the portfolio with high variance is ceded externally to private reinsurers. Therefore, Z is the optimal combination of crops to retain in the self managed reinsurance pool such that the variance of the LCR to the pool is minimized, and is described by Equation [3.2] as follows,

$$Z = \sum_{i=1}^{I} \alpha_i L_i W_i \qquad [3.2]$$

where

α_i is the indicator that corresponds to whether a particular crop is retained in the pool = 1, or alternatively ceded to private reinsurers = 0.

L_i is the weight of the specific crop liability as a proportion of the entire portfolio, which is always known.

W_i is the measured risk to the pool according to the loss coverage ratio (LCR), and dependent on the indicator α_i. When α_i is 1, meaning that the claim is retained in the self managed reinsurance pool, W_i is simply the LCR of the crop. When α_i is 0, meaning that the risk is ceded to private reinsurers, W_i is equal to the LCR of the crop less the specified reinsurance coverage provided by the private reinsurer. For example, where the LCR of a particular crop is 20%, when α_i is 1, W_i is 20%. However, when α_i is 0 (and where the reinsurance coverage layer is assumed to be 15% to 25%), W_i is 15% (20% - 5%).

The optimization problem is subject to the discrete combination of crops shown as follows.

$$\alpha_i = 0,1$$

In addition, the optimization problem is constrained by a percentage of the portfolio weight. The portfolio weight is given a value of 50% in this study, in order to examine the benefits of pooling (Model A) and private reinsurance (Model B) under equal weights. This provides equal weight of 50% for Model A and 50% for Model B, for construction of Model C.

$$\sum \alpha_i L_i = 0.50$$

To solve this system of linear equations, the function is written in terms of a vector as shown in Equation [3.3].

$$Z = A' LW \qquad [3.3]$$

where

$$A = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \dots \\ \alpha_I \end{bmatrix} \qquad L = \begin{bmatrix} L_1 & 0 & 0 & 0 \\ 0 & L_2 & 0 & 0 \\ 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & L_i \end{bmatrix} \qquad W = \begin{bmatrix} W_1 \\ W_2 \\ \dots \\ W_I \end{bmatrix}$$

The target function is minimized as shown in Equation [3.4].

$$\text{VAR(Z)} = \text{VAR(A'LW)} = (A'L)\Omega(L'A) \qquad [3.4]$$

where

$$VAR\ (W\ ) = \Omega$$

The goal is to find the combination of α_i's that minimize [3.4], s.t. the constraint $\sum \alpha_i L_i = 0.50$.

Step 5: Simulating Claims and Calculation of Expected Insurance Premium and Reinsurance Premium

In Step 5, the simulating of claims, and the calculation of the expected insurance premium E(X) and reinsurance premium E(Y) is outlined. Based on the combinatorial optimization model described in the previous section, the optimal group of low variance risks to retain for pooling, and the group of high variance risks to cede for private reinsurance are identified. These groupings are used to aggregate 32 years of historical LCR data into two data sets for each of 10 regions, including one data set for high variance risks, and one data set for low variance risks. In this study, 10 Canadian provinces are considered, therefore this equates to 20 data sets comprised of historical LCR's.

Palisade's @Risk software is used to analyze each data set, and determine the best statistical distribution, and corresponding parameter values. Loglogistic and gamma distributions are found to be the distributions best suited for the data, based on the Kolmogorov-Smirnov test with critical values calculated for 5% significance (see chapter two for more detail). The parameter values are then used to simulate claims (LCR) 5,000 times, 30 years in the future, for each data set. In addition, the simulation output has a random catastrophic event added, with LCR=25%, with an occurrence rate of once per 30 years that occurs at the same time in all 20 data sets. This additional risk event helps to ensure that the simulation includes sufficient risk in the tail of the distribution, and represents the fact that severe losses tend to be widespread and correlated across regions.

Besides using the parameter values from each distribution to simulate LCR, parameter values are also used to calculate the expected insurance payment, E(X), and the reinsurance payment E(Y). For the loglogistic distribution, the expected insurance payment E(X) is described by Equation [3.5], and the reinsurance payment E(Y) is described by Equation [3.6].

$$\theta * \Gamma(1 + 1/\gamma) * \Gamma(1 - 1/\gamma) \quad [3.5]$$

$$\begin{aligned}&[\theta * \Gamma(1+1/\gamma)\Gamma(1-1/\gamma)\beta(1+1/\gamma, 1-1/\gamma; \mu) + u(1-\mu)] - \\ &[\theta * \Gamma(1+1/\gamma)\Gamma(1-1/\gamma)\beta(1+1/\gamma, 1-1/\gamma; \mu) + d(1-\mu)]\end{aligned} \quad [3.6]$$

where X is the annual loss per 100 of exposure, and Y is the reinsurance premium according to the coverage layer X^x, and has a loglogistic distribution with parameters γ and θ (Klugman et al., 2008).

For the gamma distribution, the expected insurance payment E(X) is described by Equation [3.7], and the expected reinsurance payment E(Y) is described by Equation [3.8].

$$\theta * \Gamma(1 + 1/\gamma) * \Gamma(1 - 1/\gamma) \quad [3.7]$$

$$\begin{aligned}&[\theta * \Gamma(1+1/\gamma)\Gamma(1-1/\gamma)\beta(1+1/\gamma, 1-1/\gamma;\mu) + u(1-\mu)] - \\ &[\theta * \Gamma(1+1/\gamma)\Gamma(1-1/\gamma)\beta(1+1/\gamma, 1-1/\gamma;\mu) + d(1-\mu)]\end{aligned} \quad [3.8]$$

Where X is the annual loss per 100 of exposure, and Y is the reinsurance premium according to the coverage layer X^x, and has a gamma distribution with parameters α and θ(Klugman et al., 2008).

To calculate the 10% layer of reinsurance coverage for each region, the expected insurance payment E(X), and the reinsurance payment E(Y) are used. To address the potential for cross subsidization of premiums, where regions facing lower variance portfolios subsidize regions facing higher variance portfolio, a loss elimination ratio (LER) of 90% can be applied to help ensure equitable reinsurance premium contributions.

Step 6: Forecasting of Liabilities using a Least Squares Regression Model

Next, liabilities (Li) are forecasted in each of the 20 data sets, for a 30 year projection horizon. As described in chapter two, a linear relationship is assumed and a least squares regression model is developed using 32 years of historical liabilities.

Step 7: Calculating Surplus using an ALM Approach Under a Reinsurance Premium Pool

In Step 7, each of the variables described in the proceeding steps can then be used to calculate surplus at the end of each forecasting period (e.g. 30 years) under a reinsurance premium pool. As described in Equation [1], surplus is comprised of four inputs, including the surplus balance in the fund at the beginning of the t_{th} period (U_t), the total premium collected during the t_{th} period (P_t), the interest earned on the surplus in the t_{th} period (C_t), and the total claims paid during the t_{th} period (S_t). The calculation for each of the four inputs is explained next.

U_t is the surplus balance in the fund in the t_{th} period, where $U_0 = u$ is the initial surplus. In this study, a conservative approach is taken and initial surplus is assumed to be \$0. This conservative approach reflects the fact that provinces continue to operate independently, and therefore do not have sufficient capital to allocate to a reinsurance premium pool upfront. Instead, regions (provinces) must allocate their capital to individual reserve accounts to meet future demands in each province. Depending on the risk preference and ALM model of the insurance firm, however, initial surplus can be increased to provide additional stability to the fund.

P_t is the total premium collected during the time period. The formula varies according to the model. **Model A** has one component related to premium, as shown in Equation [3.9]. This includes the premium contributed from each region j, to the self managed reinsurance pool as represented by P^1. In addition, a loading fee θ of 35% is applied.

Model B has two components related to premium, as shown in Equation [3.10]. This includes the reinsurance premium contributed by each region to the pool as in Model A above (P^1). In addition, the pool pays premiums to the private reinsurer (P^2) based on all risks in the portfolio (p). Further, a brokerage fee η_1 equal to 25% is applied, which accounts for the reduction due to group buying (compared to current individual province reinsurance brokerage fees estimated to be 35%).

Model C also has two components related to premium, as shown in Equation [3.11]. This includes the reinsurance premium contributed by each region to the pool as in Model A above (P^1). In addition, the pool pays premiums to the private reinsurer (P^3) based on only the second group of high variance risks within the portfolio (r). Further, a brokerage fee η_2, equal to 30% is applied, which accounts for the reduction in fees due to group buying as in Model 2 (where brokerage is 25%). Using a high brokerage fees also considers that because only 'high variance' risks are ceded to private reinsurers, less savings may be able to be negotiated.

Model A (self managed reinsurance pool)
$$P_t^{(1)} = (\sum_{j=1}^{10} (E(Y_j) * L_j) * (1+\theta)) \quad [3.9]$$

Model B (group buying of private reinsurance)
$$P_t^{(2)} = P_t^{(1)} - (E(Y_p) * L_p * (1+\eta_1)) \quad [3.10]$$

Model C (combination)
$$P_t^{(3)} = P_t^{(1)} - (E(Y_r) * L_r * (1+\eta_2)) \quad [3.11]$$

where

- E(Yj) is the reinsurance premium contributed by each region j to the pool to cover a portion of the region's risk; E(Yp) is the reinsurance premium paid to the private reinsurer for coverage on all of the risk in the pool; E(Yr) is the reinsurance premium paid to the private reinsurer for coverage for only the second group of high variance risks within the portfolio.
- L_j refers to the corresponding total liability in region j; L_p refers to the total liability in the portfolio based on all risks allocated to the pool from all regions; L_r refers to the total liability in the portfolio associated with only the second group of risks within the portfolio with high CV of the LCR.

C_t is the interest earned on the surplus during the t_{th} period, $u_{t-1} * i$. The rate of interest, i, is assumed to be an average return of 3% which is based on the assumption that in order to meet liquidity requirements, a large portion of surplus is held in reserve earning little or no return while the remainder of surplus is split between various short term holdings.

S_t is the total claims paid during the period. The formula varies according to the model. **Model A** has one component pertaining to claims, as shown in Equation [3.12]. This includes claims that the self managed reinsurance pool pays to each region (S^1) when losses occur within the regions reinsurance coverage layer. **Model B**, as shown in Equation [3.13], has two components related to a claim. This includes claims that the self managed reinsurance pool pays to each region as in Model A above (S^1). In addition, the private reinsurer pays a claim to the self managed reinsurance pool (S^2) when losses occur within the reinsurance coverage layer for all risks in the portfolio (p). **Model C**, as shown in Equation [3.14], has two components related to a claim. This includes claims that the self managed reinsurance pool pays to each region as in Model A above (S^1). In addition, the private reinsurer pays a claim to the self managed reinsurance pool (S^3) when losses occur within the reinsurance coverage layer for only the second group of high variance risks in the portfolio (r).

Model A (self managed reinsurance pool) $$S_t^{(1)} = -(\sum_{j=1}^{10}(X_{t,j} - Y_{t,j}) * L_j) \qquad [3.12]$$

Model B (group buying of private reinsurance) $$S_t^{(2)} = ((X_{tp} - Y_{tp}) * L_p) - S_t^{(1)} \qquad [3.13]$$

Model C (combination) $$S_t^{(3)} = ((X_{tr} - Y_{tr}) * L_r) - S_t^{(1)} \qquad [3.14]$$

where

- $X_{t,j}$ is the loss coverage ratio (LCR) at period t, in region j.
- L_j refers to the total liability in each region j; L_p is the total liability in the portfolio contributed by all regions for coverage of all risks in the pool; L_r is the total liability in the portfolio for coverage of only the second group of risks within the portfolio with high CV of the LCR.
- $Y_{t,j}$ is the reinsurance premium in time t, in region j; $Y_{t,p}$ is the reinsurance premium at time t, based on all risks in the portfolio (p); $Y_{t,r}$ is the reinsurance payment at time t, based on coverage for only the second group of risks within the portfolio with high CV of the LCR (r).

Step 8: Determination of Survival Probability, and Deficit at Ruin, to Evaluate the Most Effective Model

In order to evaluate the most effective model, the final step involves calculating survival probability and deficit at ruin for each of the forecasting periods (e.g. 30 years) based on the surplus calculation determined in step 7.

Results

To evaluate the three alternative insurance models under the assumption of a reinsurance premium pool, an asset liability management (ALM) approach using simulation is used to compare surplus, survival probability, and deficit at ruin. Model A is a self managed reinsurance pool model, Model B is a group buying of private reinsurance model, and Model C is a portfolio approach to combine pooling (Model A) and private reinsurance (Model B) using combinatorial optimization with a genetic algorithm.

Surplus

Figures 3.1, 3.2, and 3.3 show the average surplus, 95th percentile of surplus, and 5th percentile of surplus, respectively, for Models A, B, and C. The average surplus shown in Figure 3.1, is largely a reflection of the reinsurance brokerage fees that are incurred in each of the three models. The self managed reinsurance pool model, Model A, does not purchase private reinsurance and therefore eliminates all reinsurance brokerage fees. This helps surplus to grow at an increased rate compared to the other two models, achieving surplus of $1.9 billion by year 30.

The group buying of private reinsurance model, Model B, incurs reinsurance brokerage fees for all risks in the portfolio. As a result, surplus is lowest in this model because the costly reinsurance brokerage fees deplete the fund balance at an increased rate, resulting in surplus of $620 million by year 30.

Model C, the portfolio approach to combine pooling and private reinsurance using optimization, achieves surplus in the middle of Model A and Model B. This result is in keeping with purchasing private reinsurance on a select group of risks in the pool that account for 50% of the risks in the portfolio. This corresponds to average surplus of $1.1 billion by year 30, which is roughly 50% lower than Model A, and 50% higher than Model B.

The *95th percentile of surplus* is presented in Figure 3.2, and corresponds to 95% of simulated surplus values that fall below this value. In other words, this value represents the 250th best case scenario of the 5,000 surplus values determined through simulation. As expected, Model A is superior for the best case simulations when losses are not extreme and widespread, achieving surplus of $3.4 billion by year 30. This is related to the elimination of costly brokerage fees which allows surplus to grow at an increased rate, without the negative impact of severe and highly correlated losses.

The group buying of private reinsurance model, Model B, achieves the lowest surplus of $2.2 billion by year 30. When losses are less severe, as represented by the 95th percentile of surplus, this model does not substantially benefit from the added diversification provided by private reinsurers, and instead is hindered by the costly reinsurance that is purchased.

The portfolio approach to combine pooling and private reinsurance using optimization, Model C, offers a slight improvement over Model B and achieves surplus of $2.5 billion by year 30. This is a reflection of the savings that are realized from incurring costly brokerage fees only a select group of risks in the portfolio.

The *5th percentile of surplus* is presented in Figure 3.3, and corresponds to 5% of simulated values that fall below this value. In other words, this value represents the

4,750th worst case scenario of the 5,000 surplus values determined through simulation. In the short term (first 8 years), the group buying of private reinsurance model, Model B, achieves the highest surplus at the 5th percentile. However, over the long term this model quickly becomes inferior as the reinsurance brokerage fees deplete the fund balance (-$717 million). Conversely, the self managed reinsurance pool model, Model A, performs the worst initially, however, around the 8th year this model quickly becomes the best performer (-$357 million). This is due to the elimination of costly reinsurance brokerage fees each year, which allows surplus to accumulate and stabilize the fund. The portfolio approach to combine Model A and Model B using optimization, Model C, achieves fairly consistent surplus over the duration of the 30 year forecasting period (-$336 million). This is an important finding that highlights the consistency of this model in both the short and long term.

Survival Probability

The survival probability of Models A, B, and C are compared in Figure 3.4. This figure shows that survival probability decreases over time for all three models, which is due to the conservative definition of ruin used here, ultimate ruin. As explained in chapter two, in this study once a simulation is ruined, the process is not allowed to continue. Therefore it is possible to have an increasing surplus function, yet a decreasing survival function.

Results show that Model A, the self managed reinsurance pool that does not purchase private reinsurance, has the highest survival probability (40.7%). This is because pooling of insurance business across multiple geographic regions and products, allows the natural offsetting of risks. Therefore, the variance of aggregate risks in the portfolio is reduced without the need of higher cost private reinsurance. The reinsurance brokerage fees that are saved in this model allows surplus to grow at an increased rate, providing extra surplus to stabilize the fund and help the pool survive when risks are large and correlated. Model C, which combines Model A and Model B using optimization, achieves adequate surplus that is lower than Model A, yet higher than Model B (33.2%). The group buying of private reinsurance model, Model B, produces the lowest survival probability (18.8%). Table 3.2 lists summary results including survival probability at year 30, for Models A, B, and C. Therefore, when only survival probability is considered, Model A is the most effective. However, since Model A showed too high a deficit at ruin, Model C was considered to be overall superior to Model A.

Deficit at Ruin

Figure 3.5 shows the average deficit at ruin for Models A, B, and C, where the initial surplus is assumed to be $0. The group buying of private reinsurance model, Model B, has the smallest average deficit at ruin of-$37.4 million. This highlights the main benefit of private reinsurance, which is improved diversification due to a well diversified portfolio of international risks that offset risks across multiple countries and insurance sectors. The portfolio approach to combine pooling and private reinsurance using optimization, Model C, produces deficit at ruin of -$68.4 million, which is slightly larger than Model B. The self managed reinsurance pool model, Model A, produces the most severe deficit at ruin of -$127.8 million. If only deficit at ruin is considered, then

Model B is most effective. However, since Model B shows too low a surplus and survival probability, Model C is considered to be overall superior to Model B.

Overview of Results

To be acceptable, a model must have a reasonable level of each of the three risk measures considered in this study, including surplus, survival probability, and deficit at ruin. As expected, empirical results from the ALM evaluation show that Model C is superior overall. It uses a portfolio approach to optimally combine pooling (Model A) and private reinsurance (Model B), in an innovative risk management approach that is more efficient than a risk management approach that uses exclusively only reinsurance pooling or private reinsurance. Model C provides acceptable risk measures across all three measures, including adequate surplus, survival probability, and deficit at ruin (Table 3.2). The self managed reinsurance pool, Model A, achieves the highest surplus and survival probability, however, the deficit of ruin is the most severe of the three models considered. Conversely, a group buying of private reinsurance model, Model B, produces the lowest surplus and survival probability. However, the deficit at ruin is the least severe. A summary result of the three models is shown in Table 3.2, and the ranking of the models based on surplus, survival probability, and deficit at ruin is shown in Table 3.3.

Summary

Chapter three modeled a reinsurance premium pool where regions contribute only a portion of their risk to the pool. This addressed one of the potential limitations of chapter two, which is the potential reluctance that some regions may have in transferring control of their region to cooperate in the pool. A reinsurance premium pool provided an incremental approach to pooling which allowed regions to continue operating independently and contribute only a portion of risks, approximately 10%, to the pool. The remaining 90% of risks were retained within each provincial crop insurance company.

In addition, chapter three developed a more efficient reinsurance portfolio model, using a more precise method than the CV of the LCR used in chapter two, to segregate the portfolio into two parts and then combine them. In chapter three, combinatorial optimization with a genetic algorithm was used to optimally manage a portfolio of aggregate risks with high variance (Model C). The first group of risks within the portfolio was uncorrelated, and diversified enough to be managed internally within a pool. This group of uncorrelated risks within the pool naturally offset, which lowered the variance of the aggregate risks within this set. The second group of risks within the portfolio was correlated, and therefore the risks did not sufficiently offset, which led to a high variance portfolio of risks. Therefore, in order to decrease the variance of the aggregate risks in the portfolio, the second group of risks was ceded to private reinsurers who are better diversified for risks that are large and correlated.

Three alternative pooling and reinsurance models were developed under a reinsurance premium pool, which were evaluated under an asset liability management (ALM) approach. Model A was a self managed reinsurance pool, Model B was a group buying of private reinsurance model, and Model C was a portfolio approach to combine pooling (Model A) and private reinsurance (Model B) using optimization. Using simulation, risk measures including surplus, survival probability, and deficit at ruin were considered. To evaluate the models, the complete crop insurance sector for Canada was analyzed, which included 32 years of actual premiums and liabilities, across 10 provinces for 279 crop types.

In this chapter, eight steps were presented to optimize the benefits of combined pooling and private reinsurance using combinatorial optimization with a genetic algorithm, under the assumption a reinsurance premium pool. The methodology focused on step 4 and step 7, as these two steps differed from the eight step methodology presented in chapter two for the portfolio approach to combine pooling and private reinsurance using the CV of the LCR. Step 4 outlined the process to using combinatorial optimization with a genetic algorithm to segregate a portfolio, and step 7 showed how surplus was calculated using an ALM surplus approach under a reinsurance premium pool.

To be acceptable, a model must achieve a reasonable level of surplus, survival probability, and deficit at ruin. A model that is inferior in any of these areas would not be considered efficient, as the model would be faced with high risk of default. Results showed that the portfolio approach to combine pooling and private reinsurance, Model C, was overall superior. It overcame the problem of insufficient diversification for large and correlated risks associated with pooling, as reported by the most severe deficit at ruin in Model A. In addition, the Model C problem overcame the costly brokerage fees associated with private reinsurance, as reported by very low surplus and survival probability in Model B. Model C used a portfolio approach and successfully combined the two risk management approaches into a more efficient model than either pooling or private reinsurance on its own. Model C produced acceptable measures of risk in all three categories, as reported by high surplus, survival probability, and low deficit at ruin.

Table 3.1 Summary of Methodology Steps using Combinatorial Optimization with a Genetic Algorithm Under a Reinsurance Premium Pool

Step 1	Developing the ALM Surplus Approach
Step 2	Development and Description of Alternative Pooling and Reinsurance Models (Models A, B, and C)
Step 3	Calculation of Reinsurance Coverage Layers Using the Loss Coverage Ratio (LCR) and the Loss Elimination Ratio (LER)
***Step 4**	Using Combinatorial Optimization With a Genetic Algorithm to Segregate the Portfolio
Step 5	Simulating Claims and Calculation of the Expected Insurance Premium and Reinsurance Premium
Step 6	Forecasting of Liabilities Using a Least Squares Regression Model
***Step 7**	Calculating Surplus Using an ALM Surplus Process Under a Reinsurance Premium Pool
Step 8	Determination of Survival Probability, and Deficit at Ruin to Evaluate the Most Effective Model

*This step varies from chapter 2.

Notes: This table provides a summary of the methodology steps that are necessary to optimally combine the benefits of a self managed reinsurance pool (Model A), and group buying of private reinsurance (Model B), in order to create Model C. This table highlights the eight basic steps that are necessary to produce an ALM surplus model that segregates a portfolio into two groups, which are then combined. The first group of risks is uncorrelated, and therefore the variance of the aggregate risks within this set is reduced. The second group of risks within the portfolio is correlated, and therefore in order to reduce the variance of the aggregate risks within this set, private reinsurance is purchased for this select group.

The eight basic steps to this approach, include developing the ALM surplus approach, the development and description of three alternative pooling and private reinsurance models (Models A, B, and C), calculation of reinsurance coverage layers using the LCR and the LER, using combinatorial optimization with a genetic algorithm to segregate the portfolio, simulating claims and calculation of the expected insurance and reinsurance premium, and the forecasting of liabilities using a least squares regression model. From these variables, surplus can be calculated over each of the forecasting periods under a reinsurance premium pool, using an ALM process. Finally, survival probability and deficit at ruin can be determined based on surplus, to evaluate the most effective model.

Table 3.2 Overview of Results for Model A, B, and C

	Model A Self managed reinsurance pool	**Model B** Group Buying of Private Reinsurance	**Model C** Model A and B Combined using Optimization
Initial Surplus	$0	$0	$0
Survival Probability (Yr 30)	40.7%	18.8%	33.2%
Surplus (Yr 30): Average	$1,937,319,125	$620,177,959	$1,129,239,639
Surplus (Yr 30): 5th Percentile	$356,723,008	-$717,070,845	-$335,799,527
Surplus (Yr 30): 95th Percentile	$3,350,427,897	$2,199,933,250	$2,494,623,079
Deficit at Ruin (Yr 30): Average	-$127,810,178	-$37,444,275	-$68,361,518

Notes: This table compares survival probability, surplus, and deficit at ruin, for the three pooling and reinsurance models. The results show that survival probability and surplus is the highest under Model A, a self managed reinsurance pool, but the deficit at ruin is the most severe. The group buying of private reinsurance model, Model B, has the lowest survival probability and surplus, however, the deficit of ruin is the smallest (roughly 1/3 the severity of Model A). Model C, which combines Model A and B using optimization, demonstrates stable results across all three risk measures. Therefore, Model C is the most effective model overall.

Three alternative pooling and group buying of private reinsurance models, consisting of ten Canadian provinces (AB, BC, MB, ON, PEI, SK, NL, NB, NS and QC), are compared. The expected reinsurance premium from each province is allocated to the shared pool, based on a 10% layer of reinsurance coverage that is specific to each province. Model A is a self managed reinsurance pool that does not purchase private reinsurance. Model B is similar to Model A, however, in addition the pool purchases private reinsurance as a group for all liabilities in the portfolio, for a 10% layer of reinsurance coverage. Model C combines Model A and B using optimization, and segregates the portfolio into one group of risks that are uncorrelated to retain in the self reinsurance pool, and a second group of risks that are correlated to cede to private reinsurers.

The initial surplus, the amount of surplus required in the fund at t=0, is assumed to be equal to $0. Survival probability is a measure of fund stability that represents the percentage of iterations that have a positive fund balance. Surplus represents the account balance to be allocated to an ALM approach. Deficit at ruin measures the severity of shortfall when the balance becomes negative.

Table 3.3 Ranking of Models A, B, and C for Surplus, Survival Probability, and Deficit at Ruin

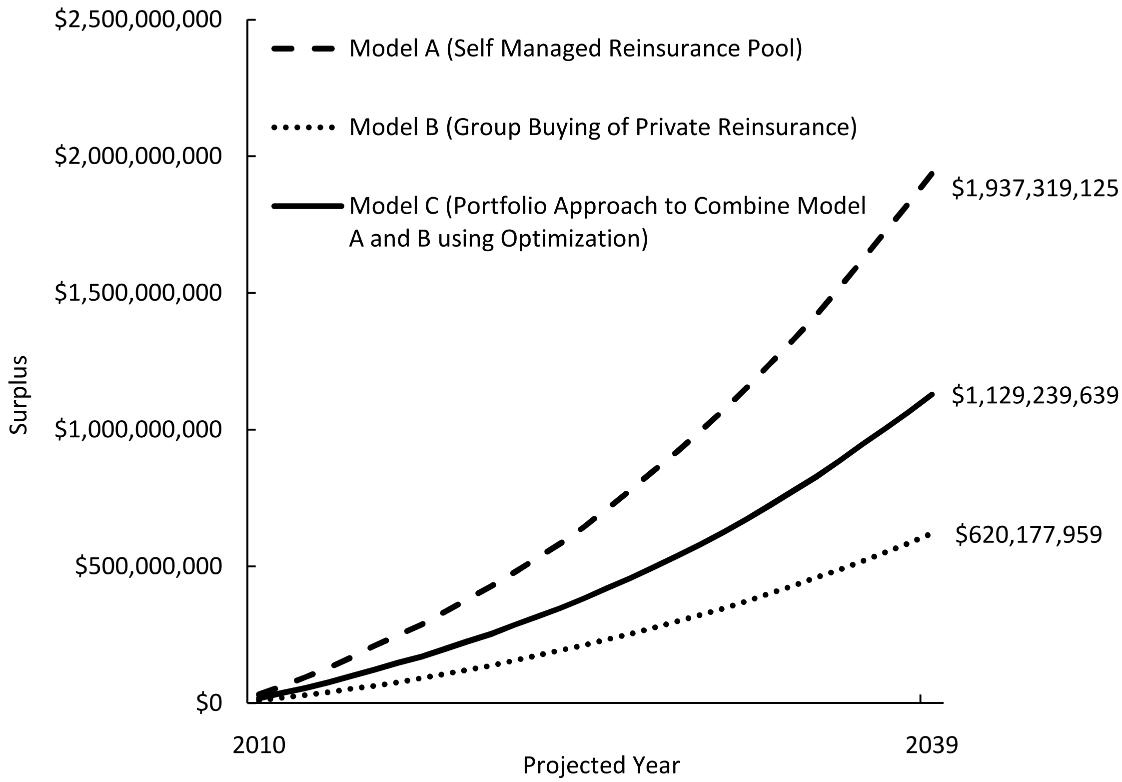
Risk Measure	Best Ranking	Middle Rank	Worst Ranking
Surplus	Model A	Model C	Model B
Survival Probability	Model A	Model C	Model B
Deficit at Ruin	Model B	Model C	Model A

Notes: This table compares survival probability, surplus, and deficit at ruin, for the three pooling and reinsurance models. The results show that Model C, which combines a self managed reinsurance pool (Model A) and group buying of private reinsurance (Model B) achieves stable results in the middle for all three risk measures. This is in comparison to Model A which ranks best for surplus and survival probability, however, ranks worst for deficit at ruin. Conversely, Model B ranks best for deficit at ruin, however, it ranks worst for surplus and survival probability.

Three alternative pooling and group buying of private reinsurance models, consisting of ten Canadian provinces (AB, BC, MB, ON, PEI, SK, NL, NB, NS and QC), are compared. The expected reinsurance premium from each province is allocated to the shared pool, based on a 10% layer of reinsurance coverage that is specific to each province. Model A is a self managed reinsurance pool that does not purchase private reinsurance. Model B is similar to Model A, however, in addition the pool purchases private reinsurance as a group for all liabilities in the portfolio, for a 10% layer of reinsurance coverage. Model C combines Model A and B using optimization, and segregates the portfolio into one group of risks that are uncorrelated to retain in the self reinsurance pool, and a second group of risks that are correlated to cede to private reinsurers.

Survival probability is a measure of fund stability that represents the percentage of iterations that have a positive fund balance. Surplus represents the account balance to be allocated to an ALM approach. Deficit at ruin measures the severity of shortfall when the balance becomes negative.

Figure 3.1 Average Surplus for Models A, B, and C

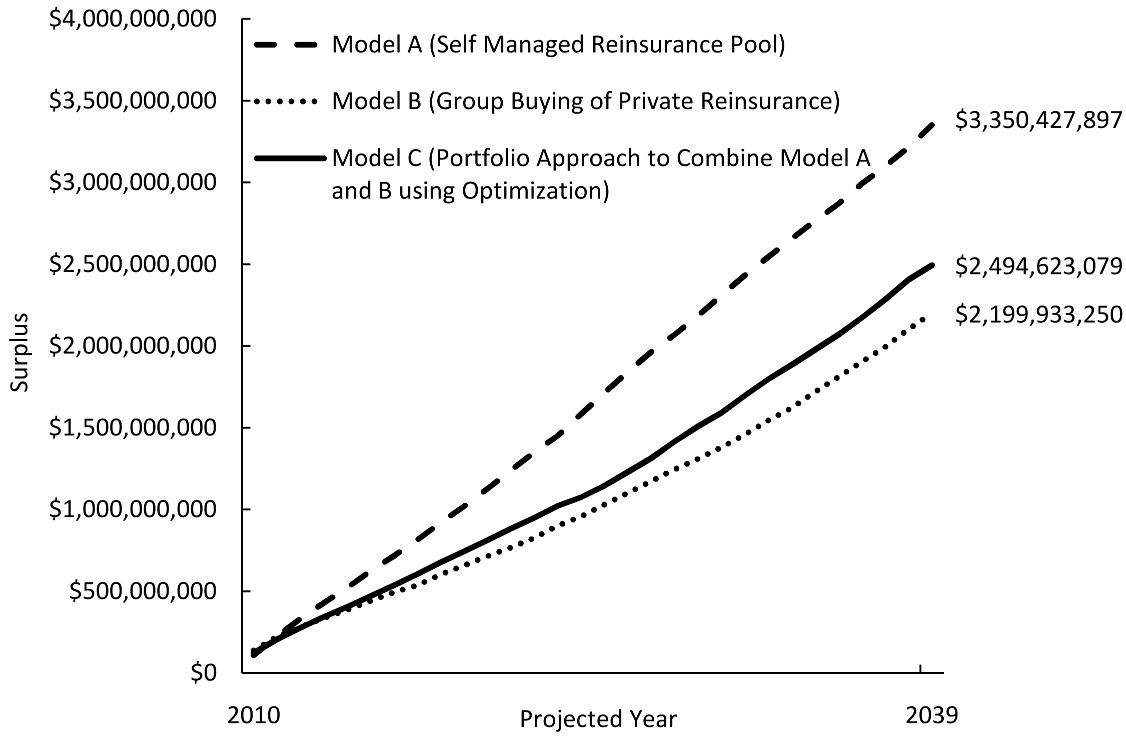


Notes: This figure shows the average surplus for Models A, B, and C at $0 initial surplus. The results show that the self managed reinsurance pool, Model A, achieves the highest surplus over the 30 year projection period. The group buying of private reinsurance model, Model B, achieves the lowest surplus. Model C, which optimally combines Model A and Model B, produces surplus that is in between.

Three alternative pooling and group buying of private reinsurance models, consisting of ten Canadian provinces (AB, BC, MB, ON, PEI, SK, NL, NB, NS and QC), are compared. The expected reinsurance premium from each province is allocated to the shared pool, based on a 10% layer of reinsurance coverage that is specific to each province. Model A is a self managed reinsurance pool that does not purchase private reinsurance. Model B is similar to Model A, however, in addition the pool purchases private reinsurance as a group for all liabilities in the portfolio, for a 10% layer of reinsurance coverage. Model C combines Model A and B using optimization, and segregates the portfolio into one group of risks that are uncorrelated to retain in the self reinsurance pool, and a second group of risks that are correlated to cede to private reinsurers.

Surplus, the balance in the account during each time period, is compared over 30 years in the future. Surplus is calculated by the following formula: $U_t = U_{t-1} + P_t + C_t - S_t$, where ($U_t$) is the surplus in the account at the beginning of the t_{th} period, (P_t) is the premiums collected during the t_{th} period, (C_t) is the interest earned on the surplus the t_{th} period, and (S_t) is any claims paid out during the t_{th} period.

Figure 3.2 95th Percentile of Surplus for Models A, B, and C (Best Case Scenarios)

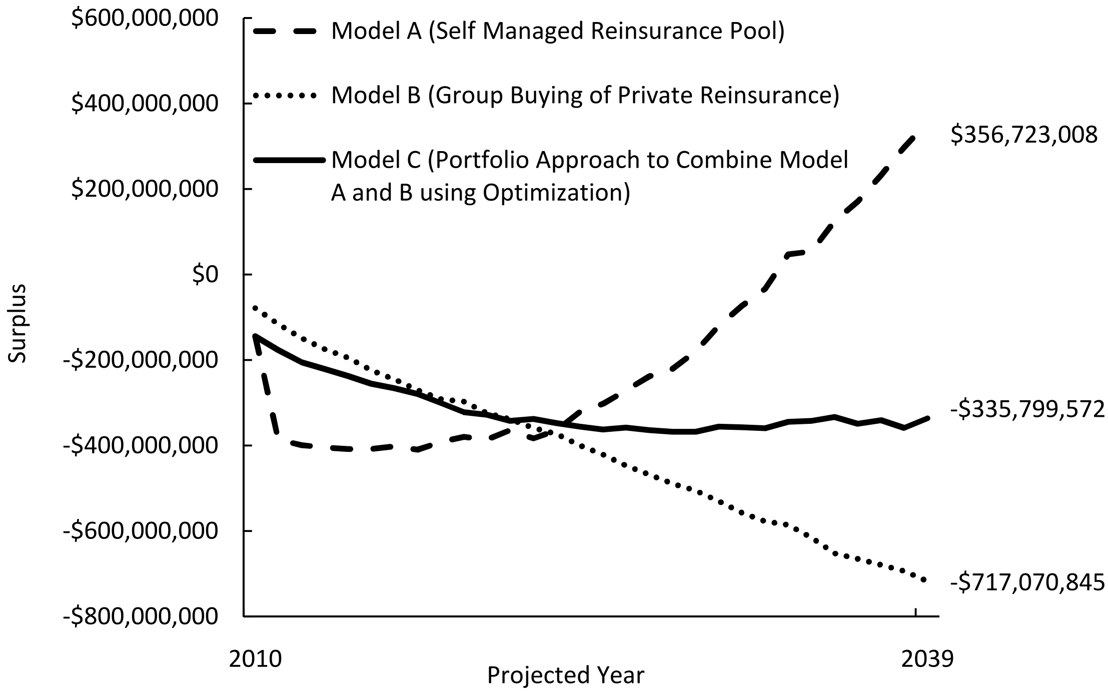


Notes: This figure shows the 95th percentile of surplus for Models A, B, and C at $0 initial surplus. The 95th percentile corresponds to the value where 95% of simulated surplus values fall below. In other words, this value represents close to the best case scenario of simulated surplus. The results show that the self managed pooling model, Model A, performs very well when losses are less severe (as represented by the 95th percentile). This is related to the elimination of costly brokerage fees that allow surplus to grow at an increased rate. The group buying of private reinsurance model, Model B, achieves the lowest surplus. When losses are less severe, this model does not substantially benefit from the costly private reinsurance that is purchased. The optimally combined pooling and private reinsurance model, Model C, offers a slight improvement over Model B. This is a reflection of the savings that are realized from paying costly brokerage fees only a select group of risks in the portfolio.

Three alternative pooling and group buying of private reinsurance models, consisting of ten Canadian provinces (AB, BC, MB, ON, PEI, SK, NL, NB, NS and QC), are compared. The expected reinsurance premium from each province is allocated to the shared pool, based on a 10% layer of reinsurance coverage that is specific to each province. Model A is a self managed reinsurance pool that does not purchase private reinsurance. Model B is similar to Model A, however, in addition the pool purchases private reinsurance as a group for all liabilities in the portfolio, for a 10% layer of reinsurance coverage. Model C combines Model A and B using optimization, and segregates the portfolio into one group of risks that are uncorrelated to retain in the self reinsurance pool, and a second group of risks that are correlated to cede to private reinsurers.

Surplus, the balance in the account during each time period, is compared over 30 years in the future. Surplus is calculated by the following formula: $U_t = U_{t-1} + P_t + C_t - S_t$, where ($U_t$) is the surplus in the account at the beginning of the t_{th} period, (P_t) is the premiums collected during the t_{th} period, (C_t) is the interest earned on the surplus the t_{th} period, and (S_t) is any claims paid out during the t_{th} period.

Figure 3.3 5^{th} Percentile of Surplus for Models A, B, and C (Worst Case Scenarios)

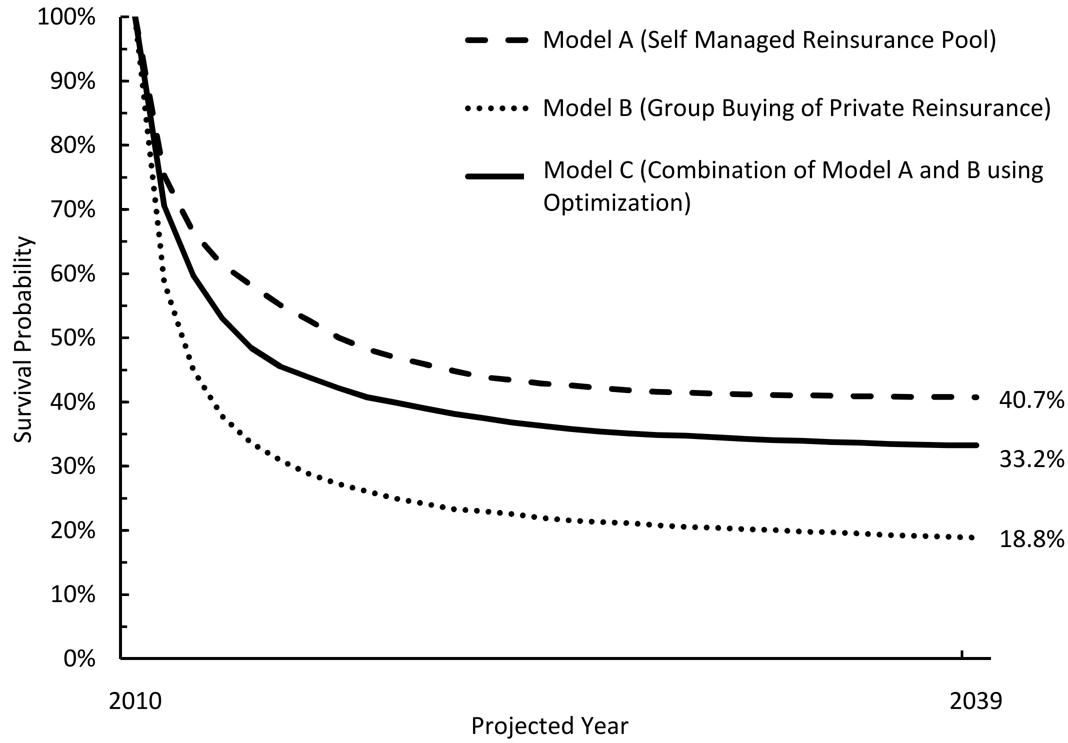


Notes: This figure shows the 5^{th} percentile of surplus for Models A, B, and C at $0 initial surplus. The 5^{th} percentile corresponds to 95% of simulated surplus values that fall above this value. In other words, this value represents close to the worst case scenario of simulated surplus. The results show that the group buying of private reinsurance model, Model B, is superior initially. However, in the long term the costly reinsurance brokerage fees deplete the fund balance which causes surplus to fall substantially. The self managed reinsurance pool model, Model A, presents contrasting results. While this model performs worst in the first 8 years, over time this model substantially improves and achieves the highest surplus. This is due to costly brokerage fees that are eliminated each year which contribute to growth in surplus and allows the fund to stabilize. The combined pooling and private reinsurance model, Model C, is fairly consistent over time. This is a reflection of the combined approach which benefits from the additional diversification provided by private reinsurers, particularly in the early years as the fund builds stability. In addition, the savings that are realized from paying costly brokerage fees only on a select group of risks in the portfolio helps to improve the surplus over the long term.

Three alternative pooling and group buying of private reinsurance models, consisting of ten Canadian provinces (AB, BC, MB, ON, PEI, SK, NL, NB, NS and QC), are compared. The expected reinsurance premium from each province is allocated to the shared pool, based on a 10% layer of reinsurance coverage that is specific to each province. Model A is a self managed reinsurance pool that does not purchase private reinsurance. Model B is similar to Model A, however, in addition the pool purchases private reinsurance as a group for all liabilities in the portfolio, for a 10% layer of reinsurance coverage. Model C combines Model A and B using optimization, and segregates the portfolio into one group of risks that are uncorrelated to retain in the self reinsurance pool, and a second group of risks that are correlated to cede to private reinsurers.

Surplus, the balance in the account during each time period, is compared over 30 years in the future. Surplus is calculated by the following formula: $U_t = U_{t-1} + P_t + C_t - S_t$, where ($U_t$) is the surplus in the account at the beginning of the t_{th} period, (P_t) is the premiums collected during the t_{th} period, (C_t) is the interest earned on the surplus the t_{th} period, and (S_t) is any claims paid out during the t_{th} period.

Figure 3.4 Survival Probability for Models A, B, and C

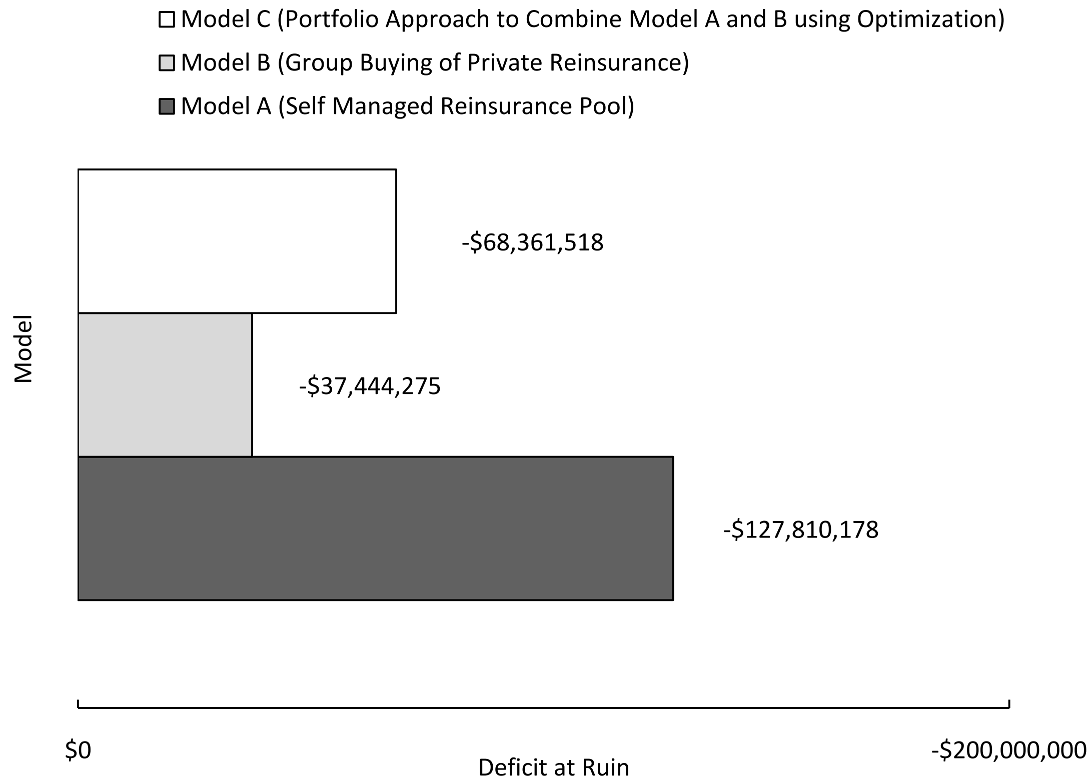


Notes: This figure compares survival probability for the three models at an initial surplus of $0. The results show that Model A, the self managed reinsurance pool that does not purchase private reinsurance, has the highest survival probability. Model A is closely followed by Model C, which is a combination of Model A and Model B using optimization. Model B, the group buying of private reinsurance model, produces the lowest survival probability. Therefore, Model A offers the greatest fund stability, and the smallest probability of default. Therefore, when only survival probability is considered, Model A is the most effective.

Three alternative pooling and group buying of private reinsurance models, consisting of ten Canadian provinces (AB, BC, MB, ON, PEI, SK, NL, NB, NS and QC), are compared. The expected reinsurance premium from each province is allocated to the shared pool, based on a 10% layer of reinsurance coverage that is specific to each province. Model A is a self managed reinsurance pool that does not purchase private reinsurance. Model B is similar to Model A, however, in addition the pool purchases private reinsurance as a group for all liabilities in the portfolio, for a 10% layer of reinsurance coverage. Model C combines Model A and B using optimization, and segregates the portfolio into one group of risks that are uncorrelated to retain in the self reinsurance pool, and a second group of risks that are correlated to cede to private reinsurers.

Survival probability is compared over 30 years in the future and refers to the probability that the fund will 'survive' without ruin (e.g. have enough reserves to pay claims). This study applies a conservative definition of ruin, ultimate ruin, where once the account ruins it is not possible for it to become nonnegative. This means that it is possible for surplus to be increasing over time, while survival probability decreases over time.

Figure 3.5 Average Deficit at Ruin for Models A, B, and C



Notes: This figure shows the average deficit at ruin, for the three models at an initial surplus of $0. Model A produces the most severe deficit at ruin, which highlights the disadvantage of a pooling model when losses are extreme and lead to ruin. Conversely, Model B has the lowest deficit at ruin, demonstrating the advantage of the additional diversification private reinsurers can offer when losses are large and widespread. Model C has a deficit that is slightly larger than Model B, however, it significantly improves on Model A (roughly ½ the severity of Model A). Selectively ceding only the second group of correlated risks within the portfolio to private reinsurers helps to reduce the risk exposure that is present in the self managed pooling approach. If only deficit at ruin is considered, then Model B is most effective.

Three alternative pooling and group buying of private reinsurance models, consisting of ten Canadian provinces (AB, BC, MB, ON, PEI, SK, NL, NB, NS and QC), are compared. The expected reinsurance premium from each province is allocated to the shared pool, based on a 10% layer of reinsurance coverage that is specific to each province. Model A is a self managed reinsurance pool that does not purchase private reinsurance. Model B is similar to Model A, however in addition the pool purchases private reinsurance as a group for all liabilities in the portfolio, for a 10% layer of reinsurance coverage. Model C combines Model A and B using optimization, and segregates the portfolio into one group of risks that are uncorrelated to retain in the self reinsurance pool, and a second group of risks that are correlated to cede to private reinsurers.

Deficit at ruin represents the severity of ruin, and demonstrates the degree to which the funds claims exceed its assets. To calculate deficit at ruin, a distribution of simulated surplus values just prior to ruin is created. In addition, the surplus value prior to ruin is discounted according to the period in which ruin occurred to t=0 at an assumed rate of 3%.

CHAPTER 4

DEPENDENCE ACROSS REGIONS UNDER A REINSURANCE PREMIUM POOL: COMBINATORIAL OPTIMIZATION USING A GENETIC ALGORITHM TO COMBINE POOLING AND PRIVATE REINSURANCE TO REDUCE RISK

Introduction

Chapter four uses a similar approach to chapter three, except that it allows for dependence (correlation) across regions (provinces), under the assumption of a multivariate normal distribution. In chapter three, a portfolio approach was used to combine a self managed reinsurance pool and private reinsurance using optimization, Model C. In chapter four, however, the assumption of dependence across regions is incorporated into the optimized portfolio model, Model CC. The objective of this chapter is to analyze the effectiveness of the portfolio optimization model under dependence across regions, compared to chapter three that assumed independence across regions. If results of the portfolio optimization models are consistent between the two chapters, then the portfolio models will be considered most effective.

The intent is to ensure that the group of aggregate risks within the portfolio that is retained within the self managed reinsurance pool remains sufficiently diversified in the presence of correlated LCR's across regions. Ignoring dependencies among regions may lead to inaccurate estimates of the risk, and this may alter the diversification that can be achieved through pooling. For example, pooling may be less effective when correlation is allowed in the model, while private reinsurance may be more effective in this case.

In chapter four, three alternative insurance models are developed under a reinsurance premium pool, which are very similar to Models A, B, and C developed in chapter three, but with the added assumption of dependence across regions. These are Models AA, a self managed reinsurance pool model, Model BB, a group buying of private reinsurance model, and Model CC, a portfolio approach to combine Model AA and Model BB using optimization. As in the previous two chapters, risk measures including surplus, survival probability, and deficit at ruin were considered using simulation, and the three models are evaluated analyzing the entire crop insurance sector for Canada.

The remainder of this chapter is organized as follows. The next section discusses data, followed by methodology which focuses on the fifth step of an eight step methodology to optimally combine pooling and private reinsurance. Step five focuses on simulating claims under dependence across regions. Results of the asset liability management (ALM) model for the three models are compared and contrasted, and a summary section is presented.

Data

This chapter utilizes the same comprehensive data presented in the previous two chapters. This includes 32 years of historical crop losses (from 1978 through 2009), obtained from Agriculture and Agri-Food Canada's Production Insurance National Statistical System (PINSS). The data set covers premium rates and indemnities (from which the loss coverage ratio, LCR is calculated) across ten Canadian provinces (Alberta, British Columbia, Manitoba, New Brunswick, Newfoundland, Nova Scotia, Ontario, Prince Edward Island, Quebec and Saskatchewan) for 279 crops.

Methodology

An asset liability management (ALM) approach is used to develop an innovative portfolio risk management approach that combines reinsurance pooling and group buying of private reinsurance. Earlier in chapter two, eight basic steps were presented in order to combine the benefits of pooling and private reinsurance under a full premium pool, using the coefficient of variation (CV) of the loss coverage ratio (LCR) (Table 2.1). In chapter three, eight similar steps were presented in order to more efficiently combine the benefits of reinsurance pooling and private reinsurance using combinatorial optimization with a genetic algorithm (Table 3.1).

Here in chapter four, the same eight basic steps that were presented in chapter three to optimize the benefits of combined reinsurance pooling and private reinsurance, are used in this chapter. However, the fifth step is modified to include dependence across regions for simulated risks (Table 4.1). The focus of the methodology is on step 5, which is simulating claims under dependence across regions.

Here in chapter four, the fifth step outlines how dependence across regions is incorporated into the simulation of claims across regions, represented by the loss coverage ratio (LCR). Using the output from the optimization model in step 4 in chapter three, 32 years of historical LCR data is grouped into two data sets for each of the ten regions considered. For each region, the historical LCR's are determined for the first group of relatively uncorrelated risks within the portfolio that are retained in the self managed reinsurance pool, as well as for the second group of correlated risks within the portfolio that are ceded to private reinsurers. Therefore, 20 data sets in total are considered.

To help normalize the data, a logit transformation is applied to the LCR's using Equation [4.1].

$$Logit\,(LCR) = \log \frac{LCR}{1 - LCR} \qquad [4.1]$$

Using the normalized data, Spearman's rank correlation is calculated for a 20 by 20 correlation matrix, which reflects the dependence of the LCR across the two groups within the portfolio for each region. To calculate Spearman's correlation coefficient (ρ), the data is first ranked, followed by the calculation of the sum of the squares of the differences of the ranks. Finally, Equation [4.2] is used to calculate the correlation coefficient.

$$\rho = 1 - \frac{6\sum d_i^2}{n(n^2 - 1)} \qquad [4.2]$$

Where

$d_i^2 = x_i - y_i$, the differences between the ranks of each observation on the two variables.

Statistical software such as Palisade's StatTools can be used to obtain the correlation matrix (Palisade Corporation, 2009). Spearman's rank correlation is a non-parametric measure of statistical dependence between variables. The sign of the coefficient indicates the direction of association. If, as one variable increases, the other variable increases (decreases), the rank correlation will be positive (negative). Table 4.2 shows the correlation matrix for the group of uncorrelated risks within the portfolio that are retained in the self managed reinsurance pool (LR) and the group of correlated risks within the portfolio that are ceded to private reinsurers (HR), for each of the 10 regions. The output shows non zero correlation coefficients, which reflects the varying levels of association across regions. The historical correlation among regions indicates that allowing for dependence (correlation) across regions in the model, is therefore a realistic assumption.

In chapter three, a random catastrophic event was added to the simulation, with LCR=25%. This catastrophic event had an occurrence rate of once per 30 years that occurred at the same time in all 20 data sets. This additional risk event was included in chapter three to help ensure that the simulation included sufficient risk in the tail of the distribution, and represented the fact that severe losses tend to be widespread and correlated across regions.

However, in chapter four, a more precise measure of correlation is considered in the simulation. Dependence of LCR across regions is factored into all aspects of the simulation, using a multivariate normal distribution, and the RiskCorrmat function in @Risk (Palisade Corporation, 2009). This function has the form (CorrMat, Index), where CorrMat is a matrix of correlations, and Index is an index of the variables being correlated. The RiskCorrmat function is entered as the last argument of a random @RISK function. Using the correlation structure, LCR is simulated 5,000 times for each of the 20 datasets, for 30 years in the future.

Results

To evaluate the three alternative insurance models developed under dependence across regions, an asset liability management (ALM) approach using simulation is used to compare surplus, survival probability, and deficit at ruin. Model AA is a self managed reinsurance pool model, Model BB is a group buying of private reinsurance model, and Model CC is a portfolio approach to combine pooling (Model AA) and private reinsurance (Model BB) using optimization.

<u>Surplus</u>

The average surplus, 95th percentile of surplus, and 5th percentile of surplus is shown in Figures 4.1, 4.2, and 4.3, respectively. Assuming a multivariate normal distribution and dependence across regions, Figure 4.1 shows the average surplus for Models AA, BB, and CC, where the initial surplus is conservatively assumed to be $0. The average surplus is highest for Model AA, the self managed reinsurance pool model, at $1.7 billion by year 30, due to eliminating costly reinsurance brokerage fees which allows surplus to grow at an increased rate. This is followed by Model CC, a portfolio approach to combine pooling (Model AA) and private reinsurance (Model BB) using optimization, and achieves surplus of $1.2 billion by year 30. The group buying of private reinsurance model, Model BB, produces the lowest average surplus of $931 million by year 30.

The *95th percentile of surplus* is presented in Figure 4.2 for Models AA, BB, and CC, where the initial surplus is conservatively assumed to be $0. The 95th percentile corresponds to 95% of simulated surplus values that fall below this value. In other words, this value represents the 250th best case scenario of the 5,000 surplus values determined through simulation. As expected, Model AA achieves the highest surplus of $3.6 billion by year 30, for the best case scenarios that represent risks that are not extreme and widespread. The success of Model AA is a reflection of the diversification that is achieved through the sufficient offsetting of relatively small and uncorrelated risks in the portfolio. In addition, this model does not incur costly reinsurance brokerage fees, and therefore surplus grows at a much faster rate than the other two models that purchase private reinsurance. It was expected that Model BB would produce the lowest surplus at the 95th percentile due to costly reinsurance brokerage fees that are incurred on all risks in the portfolio. It was also expected that Model CC would produce surplus somewhere in the middle of Model AA and Model BB, since only 50% of the risks in the portfolio purchased private reinsurance. However, the results showed that Model CC produced the lowest surplus ($2.7 billion), and Model BB offered a marginal improvement ($2.8 billion).

The 5th percentile of surplus is presented in Figure 4.3 for Models AA, BB, and CC, where the initial surplus is conservatively assumed to be $0. The 5th percentile corresponds to 5% of simulated values that fall below this value. In other words, this value represents the 4,750th worst case scenario of the 5,000 surplus values determined through simulation. While the self managed reinsurance pool model, Model AA, achieves the lowest surplus over the first 17 years of the simulation, by the end of the projection horizon, year 30, Model AA has the highest surplus of -$436 million by year 30. This reflects the instability of this model in the short term for worst case scenarios when risks are large and correlated. However, over time Model AA stabilizes as reserves build, due to savings from eliminating the high cost associated with private reinsurance.

The group buying of private reinsurance model, Model BB, is the superior model in the short term for worst case scenarios, however, over time this model steadily decreases to a surplus of -$689 million by year 30. Model CC, which optimally combines pooling (Model AA) and private reinsurance (Model CC), performs very well initially with results just slightly reduced from Model BB. However, unlike Model BB which steadily declines over time, Model CC stabilizes and achieves surplus that is only marginally lower than Model AA at -$499 million by year 30. Therefore, for the worst case scenarios that represent losses that are large and correlated, Model CC is superior overall, providing stable results over both the short term and long term.

Survival Probability

The survival probability of Models AA, BB, and CC under dependence across regions is compared in Figure 4.4. The survival probability results correspond to the results of average surplus identified above. Due to the elimination of costly reinsurance brokerage fees, the self managed reinsurance pool model, Model AA, achieves the highest survival probability of 40.0% by year 30. This can be attributed to the fact that Model AA has the highest surplus, and therefore there is a smaller probability of ruin (surplus becoming negative), even in years when losses are large and widespread.

Model CC, a portfolio approach to combine pooling (Model AA) and private reinsurance (Model BB) using optimization, achieves survival probability that is slightly lower than Model AA at 34.9% by year 30. This finding corresponds to the blended approach of the model, where the advantage of eliminating costly private reinsurance premiums on the group of risks retained in the self managed reinsurance pool are balanced against the better diversification provided by private reinsurers for the group of risks ceded to private reinsurers. The group buying of private reinsurance model, Model BB, produces the lowest survival probability of the three models of 23.9% by year 30. This corresponds to the fact that Model BB produces the lowest surplus, and therefore there is a larger probability that surplus will become negative.

Deficit at Ruin

While surplus and survival probability show that Model AA is superior, followed by Model CC, and lastly Model BB, the average deficit at ruin identifies a substantial weakness of Model AA. Deficit at ruin indicates the severity of losses, and specifies the dollar amount by which the surplus falls short. Figure 4.5 shows the average deficit at ruin for the three models under dependence across regions, where initial surplus is $0. Model BB has the smallest average deficit at ruin of -$49.5 million, which can be attributed to the purchase of private reinsurance. Private reinsurance helps to dampen the effect of severe and widespread losses since a portion of risk is transferred to the reinsurer. Model CC has an average deficit at ruin of -$67.3 million, that is slightly more severe than Model BB. Model BB has the most severe deficit at ruin at -$97.7 million.

Comparison of average deficit at ruin results under dependence across regions (provinces) in chapter four, to the chapter three assumption of independence of risks across regions, reveals interesting results. Private reinsurance is often considered more effective than pooling at diversifying large and highly correlated risks. Therefore, it would be expected that the self managed reinsurance pool model, Model AA, would have a more severe deficit at ruin under dependence across regions compared to Model A, where independence is assumed. Instead, Model AA has a smaller deficit at ruin of -$97.7 million, compared to Model A which has a deficit of ruin of -$127.8 million. For the group buying of private reinsurance model, Model BB, the deficit of ruin is larger under the assumption of dependence across regions at -$49.5 million, compared to the Model B assumption of independence of -$37.4 million. The combined pooling and private reinsurance model that uses optimization, reports consistent results for the deficit at ruin under both the Model CC assumption of dependence across regions of -$67.3 million, and the Model C assumption of independence of -$68.4 million.

Overview of Results

To be acceptable, a model must have a reasonable level of surplus, survival probability, and deficit at ruin. Results from the ALM evaluation under the assumption of dependence across regions shows that Model CC is superior overall. Model CC optimally combines pooling (Model AA), and private reinsurance (Model BB) in an innovative risk management approach that is more efficient than a risk management approach that uses exclusively only reinsurance pooling or private reinsurance. Model CC overcomes the problem of severe deficit of ruin due to lack of diversification for large and correlated losses in Model AA. Further, Model CC overcomes the problem of low surplus and survival probability due to costly reinsurance brokerage fees in Model BB. As a result, Model CC achieves acceptable results across all three risk measures considered, including high surplus, survival probability, and low deficit at ruin (Table 4.2).

The self managed reinsurance pool model, Model AA, achieves the highest surplus and survival probability, however, the deficit of ruin is the most severe of the three models considered. Conversely a group buying of private reinsurance model, Model BB, produces the lowest surplus and survival probability, however, the deficit at ruin is the least severe. The ranking of the three models based on surplus, survival probability, and deficit at ruin under dependence across regions, are shown in Table 4.3. These results are consistent with results obtained in chapter three, which showed that Model C was superior under the assumption of independence across regions (Table 3.2). This is an important result, showing that a self managed reinsurance pool remains effective at diversifying risk for the group of aggregate risks within the portfolio that are uncorrelated and retained within the firm. The second group of risks from within the portfolio that are correlated is managed efficiently through the purchase of private reinsurance.

Summary

Chapter four used a similar approach to the optimization model presented in chapter three, Model C, however, in addition the assumption of dependence across regions was incorporated into the model, resulting in Model CC. The objective of this chapter was to ensure that the portfolio optimization model remained effective in the presence of dependent risks across regions. Specifically, the intent of chapter four was to make certain that the first group of aggregate risks within the portfolio that was retained within the self managed reinsurance pool, remained sufficiently diversified in the presence of correlated simulations of LCR across regions (i.e. the results of Model C in chapter three were consistent with the results of Model CC in chapter four).

In chapter four, three alternative pooling and reinsurance models were developed, which corresponds to Models A, B, and C developed in chapter three, but with the additional assumption of dependence across regions when simulating LCR. As in chapter three, a reinsurance premium pool was also assumed, and the three models were evaluated under an asset liability management (ALM) approach. Model AA was a self managed reinsurance pool model, Model BB was a group buying of private reinsurance model, and Model CC combined pooling (Model AA) and private reinsurance (Model BB) using optimization. Simulation was used to compute surplus, survival probability, and deficit at ruin. The three models were evaluated using the complete crop insurance sector for Canada, which included 32 years of actual premiums and liabilities, across 10 provinces for 279 crop types.

In this chapter, there are eight steps that are necessary to optimize the benefits of combined pooling and private reinsurance using combinatorial optimization with a genetic algorithm, under the assumption a reinsurance premium pool (Table 3.1). The focus of the methodology is on step 4 and step 7, as these two steps differ from the eight step methodology presented in chapter 2 for the portfolio approach to combine pooling and private reinsurance using the CV of the LCR. Step 4 outlines the process to using combinatorial optimization with a genetic algorithm to segregate a portfolio, and step 7 how surplus is calculated using an ALM surplus approach under a reinsurance premium pool.

Results showed that under dependence across regions, Model CC, which combined reinsurance pooling (Model AA) and private reinsurance (Model BB) using optimization, was superior overall. This result is also consistent with results in chapter three which showed that under independence across regions, Model C was superior. Therefore, an optimized approach that combined pooling and private reinsurance remained the most efficient insurance portfolio model, compared to either reinsurance pooling, or private reinsurance used on its own. This optimized approach overcame the problem associated with pooling, which is the lack of diversification as reported by the most severe deficit at ruin in Model AA. Further, this model overcame the problem of costly reinsurance brokerage fees associated with private reinsurance as reported by the lowest surplus and survival probability in Model BB. Model CC, combined the benefits of both risk management approaches, and achieved stable results across all three measures, including high surplus, high survival probability, and low deficit at ruin.

Table 4.1 Summary of Methodology Steps Under Dependence Across Regions

Step 1	Developing the ALM Surplus Approach
Step 2	Development and Description of Alternative Pooling and Reinsurance Models (Models AA, BB, and CC)
Step 3	Calculation of Reinsurance Coverage Layers Using the Loss Coverage Ratio (LCR) and the Loss Elimination Ratio (LER)
Step 4	Using Combinatorial Optimization with a Genetic Algorithm to Segregate the Portfolio
***Step 5**	Simulating Claims Under *Dependence across Regions* and Calculation of the Expected Insurance and Reinsurance Premium
Step 6	Forecasting of Liabilities Using a Least Squares Regression Model
Step 7	Calculating Surplus Using and ALM Surplus Process Under a Reinsurance Premium Pool
Step 8	Determination of Survival Probability, and Deficit at Ruin, to Evaluate the Most Effective Model

*This step varies from chapter three, because of the assumption of dependence across regions.

Notes: This table provides a summary of the methodology steps that are necessary to optimally combine the benefits of a self managed reinsurance pool (Model AA), and group buying of private reinsurance (Model BB) under the assumption of dependence across regions, in order to create Model CC. This table highlights the eight basic steps that are necessary to produce an ALM surplus model that segregates a portfolio into two groups to be combined. The first group of risks is uncorrelated, and therefore the variance of the aggregate risks within this set is reduced. The second group of risks within the portfolio is correlated, and therefore in order to reduce the variance of the aggregate risks within this set, private reinsurance is purchased for this select group.

The eight basic steps to this approach, include developing the ALM surplus approach, the development and description of three alternative pooling and private reinsurance models, calculation of reinsurance coverage layers using the LCR and the LER, using combinatorial optimization with a genetic algorithm to segregate the portfolio, simulating claims under *dependence across regions* and calculation of the expected insurance and reinsurance premium, and the forecasting of liabilities using a least squares regression model. From these variables, surplus can be calculated over each of the forecasting periods using an ALM process. Finally survival probability and deficit at ruin can be determined based on surplus to evaluate the most effective model.

Table 4.2 Spearman's Correlation Coefficient Matrix for Aggregate Risks in Each Data Set

	AB-HR	AB-LR	MB-HR	MB-LR	ON-HR	ON-LR	BC-HR	BC-LR	NB-HR	NB-LR	NS-HR	NS-LR	PEI-HR	PEI-LR	QC-HR	QC-LR	SK-HR	SK-LR	NFLD-HR	NFLD-LR
AB-HR	1.0																			
AB-LR	0.8	1.0																		
MB-HR	0.1	0.0	1.0																	
MB-LR	0.2	0.3	0.7	1.0																
ON-HR	0.3	0.4	0.3	0.3	1.0															
ON-LR	0.3	0.3	0.1	0.2	0.7	1.0														
BC-HR	-0.1	0.0	-0.1	0.1	-0.2	-0.2	1.0													
BC-LR	0.1	0.3	-0.2	0.2	0.0	-0.1	0.7	1.0												
NB-HR	-0.2	-0.3	-0.2	-0.2	-0.1	-0.1	0.2	0.1	1.0											
NB-LR	-0.3	-0.4	-0.3	-0.4	-0.5	-0.1	0.3	-0.1	0.4	1.0										
NS-HR	-0.2	-0.2	0.0	0.0	-0.2	-0.1	0.2	0.0	0.1	0.4	1.0									
NS-LR	-0.2	-0.2	-0.3	-0.3	-0.4	-0.4	-0.2	0.0	-0.1	0.5	0.0	1.0								
PEI-HR	0.2	0.1	-0.1	0.2	-0.2	0.0	0.1	0.1	0.0	0.1	-0.1	0.3	1.0							
PEI-LR	0.2	0.3	0.5	0.5	0.4	0.4	0.0	-0.1	-0.1	-0.1	-0.3	-0.5	-0.2	1.0						
QC-HR	0.2	0.3	0.1	0.2	-0.2	-0.2	0.2	0.3	0.0	0.2	0.2	0.2	0.1	-0.3	1.0					
QC-LR	-0.1	0.2	0.3	0.3	0.3	0.2	0.3	0.3	0.0	-0.5	0.2	-0.3	-0.1	0.3	-0.1	1.0				
SK-HR	0.5	0.5	0.5	0.5	0.3	0.2	0.1	0.1	-0.2	-0.3	-0.2	-0.5	0.0	0.9	0.2	0.2	1.0			
SK-LR	0.4	0.6	0.2	0.4	0.3	0.3	0.3	0.3	0.0	-0.2	-0.2	-0.5	-0.1	0.7	0.2	0.3	0.7	1.0		
NFLD-HR	0.2	0.3	0.2	0.2	0.3	0.3	0.2	0.0	0.0	-0.4	-0.1	-0.2	-0.1	0.4	0.4	0.4	0.3	0.5	1.0	
NFLD-LR	-0.2	-0.2	-0.2	-0.4	0.1	0.1	-0.1	-0.2	0.3	0.6	-0.2	0.2	-0.2	0.2	-0.5	0.0	-0.2	0.0	0.1	1.0

Note: This table shows that the correlation coefficient is non zero, meaning that dependence (correlations) across regions (provinces) is a realistic assumption for the model. Spearman's correlation coefficient is calculated for the loss coverage ratios (LCR's) for the first group of risks from within the portfolio that are uncorrelated and are retained in the self managed reinsurance pool (LR), as well as the second group of risks from within the portfolio that are correlated and do not sufficiently offset and are therefore ceded to private reinsurers (HR), for each of 10 Canadian Provinces. LCR is a measure of risk that reflects the annual loss calculated as the ratio of total indemnities to total liabilities, multiplied by 100. Spearman's rank correlation is a non-parametric measure of statistical dependence between variables. The sign of the coefficient indicates the direction of association. If, as one variable increases, the other variable increases (decreases), the rank correlation will be positive (negative).

Table 4.3 Overview of Results Under Dependence Across Regions for Models AA, BB, and CC

	Model AA Self managed reinsurance pool	Model BB Group Buying of Private Reinsurance	Model CC Combination Optimization
Initial Surplus	$0	$0	$0
Survival Probability (Yr 30)	40.0%	23.9%	34.9%
Balance (Yr 30): Avg.	$1,703,772,411	$931,325,269	$1,153,662,898
Balance (Yr 30): 5th Perc.	-$436,877,318	-$688,814,356	-$499,038,084
Balance (Yr 30): 95th Perc.	$3,601,932,961	$2,830,293,042	$2,706,780,845
Deficit at Ruin (Yr 30): Avg.	-$97,733,347	-$49,514,507	-$67,318,775

Notes: Rank correlations of the loss coverage ratio (LCR) are used to consider dependence across regions for three alternative pooling and group buying of private reinsurance models. This table shows that Model AA achieves the highest surplus and survival probability, however, the deficit at ruin is the most severe. Model BB produces the lowest surplus and survival probability, however, the deficit at ruin is the least severe. Model CC is a portfolio approach to combine the benefits of both Model AA and Model BB, and achieves acceptable results for all three risk measures considered. This includes high surplus, survival probability, and low deficit at ruin.

Each model considers a pool of ten Canadian provinces including, AB, BC, MB, ON, PEI, SK, NL, NB, NS and QC. The expected reinsurance premium from each province is allocated to the shared pool, based on a 10% reinsurance coverage layer. Three models are developed under a reinsurance premium pool, under the assumption of *dependence across regions*. Model AA is a self managed reinsurance pool that does not purchase private reinsurance. Model BB is similar to Model AA, however, in addition the pool purchases private reinsurance as a group for all liabilities. Model CC is an optimization model that combines Model AA (self managed reinsurance pool), and Model BB (group buying of private reinsurance). The optimization model segregates the portfolio and determines the first group of uncorrelated risks within the portfolio to retain internally in the self managed reinsurance pool, and the second group of correlated risks from within the portfolio to cede externally to private reinsurers.

The initial surplus, the amount of surplus required in the fund at t=0, is assumed to be equal to $0. Survival probability is a measure of fund stability that represents the percentage of iterations that have a positive fund balance. Surplus represents the account balance to be allocated to an ALM approach. Deficit at ruin measures the severity of shortfall when the balance becomes negative.

Table 4.4 Ranking of Models AA, BB, and CC Under Dependence Across Regions, for Surplus, Survival Probability, and Deficit at Ruin

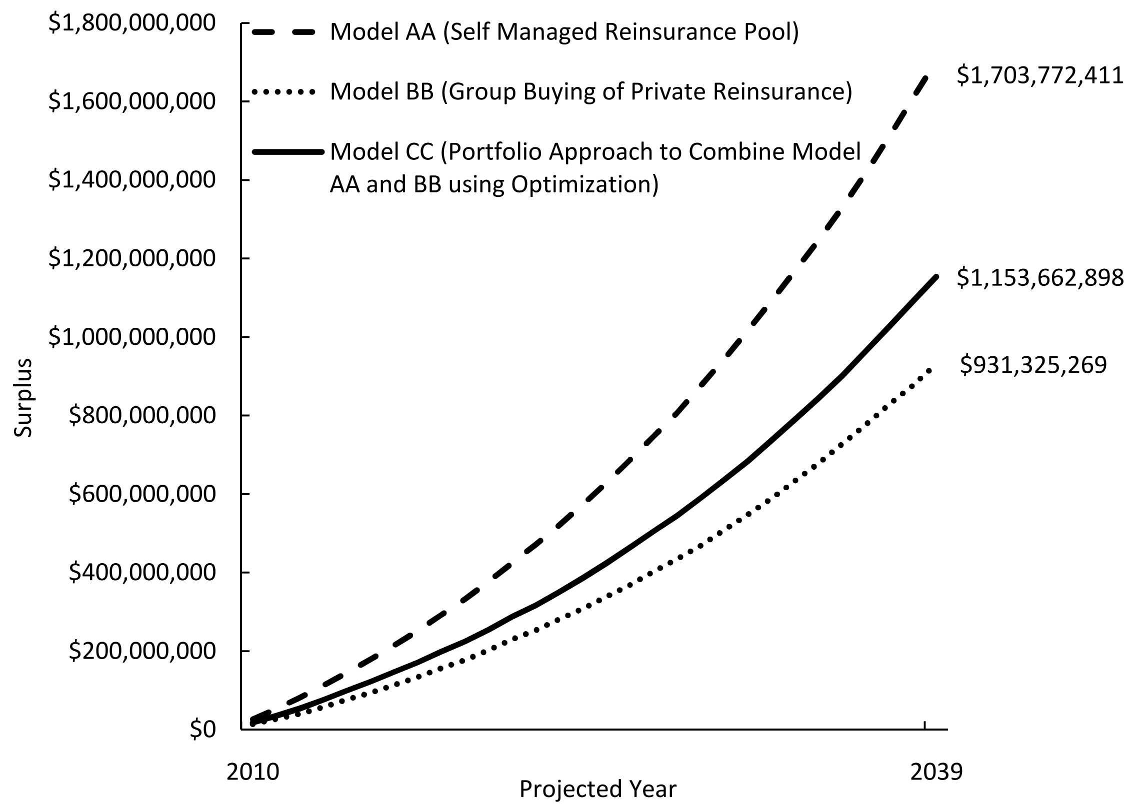
Risk Measure	Best Ranking	Middle Rank	Worst Ranking
Surplus	Model AA	Model CC	Model BB
Survival Probability	Model AA	Model CC	Model BB
Deficit at Ruin	Model BB	Model CC	Model AA

Notes: This table compares survival probability, surplus, and deficit at ruin, for the three pooling and reinsurance models considered. Results show that Model CC, which combines pooling (Model AA) and private reinsurance (Model BB), achieves stable results across all three risk measures. The self reinsurance pool model, Model AA, achieves high surplus and survival probability, however, the deficit at ruin is the most severe. The group buying of private reinsurance model, Model BB, achieves the lowest surplus and survival probability, however, the deficit at ruin is the least severe.

Each model considers a pool of ten Canadian provinces including, AB, BC, MB, ON, PEI, SK, NL, NB, NS and QC. The expected reinsurance premium from each province is allocated to the shared pool, based on a 10% reinsurance coverage layer. Three models are developed under a reinsurance premium pool, under the assumption of *dependence across regions*. Model AA is a self managed reinsurance pool that does not purchase private reinsurance. Model BB is similar to Model AA, however, in addition the pool purchases private reinsurance as a group for all liabilities. Model CC is an optimization model that combines Model AA (self managed reinsurance pool), and Model BB (group buying of private reinsurance). The optimization model segregates the portfolio and determines the first group of uncorrelated risks within the portfolio to retain internally in the self managed reinsurance pool, and the second group of correlated risks from within the portfolio to cede externally to private reinsurers.

Survival probability is a measure of fund stability that represents the percentage of iterations that have a positive fund balance. Surplus represents the account balance to be allocated to an ALM approach. Deficit at ruin measures the severity of shortfall when the balance becomes negative.

Figure 4.1 Average Surplus Under Dependence Across Regions for Models AA, BB, and CC

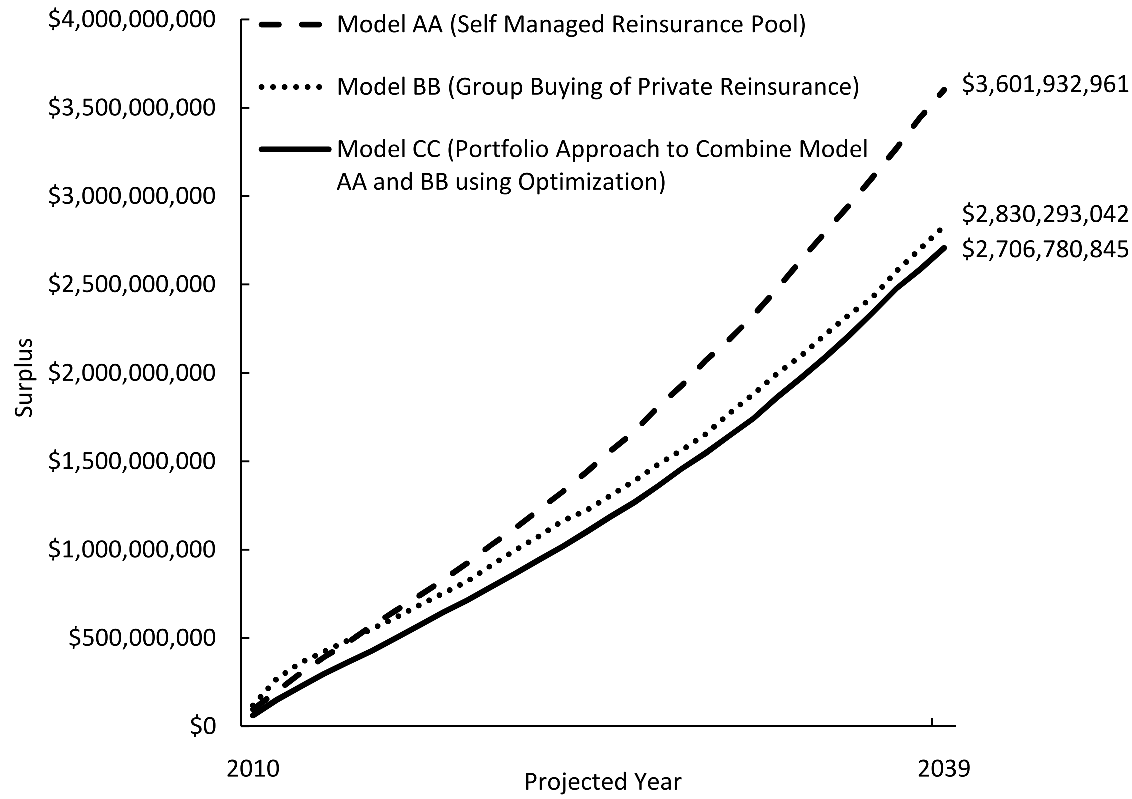


Notes: This figure shows the average surplus for Models AA, BB, and CC, under dependence across regions with an initial surplus of $0. Results show that self managed reinsurance premium pool, Model AA, achieves the highest average surplus of $1.7 billion. This is because costly reinsurance brokerage fees are eliminated in this model, which allows surplus to grow at an increased rate. Following this, the combined reinsurance pooling and private reinsurance model that uses optimization, Model CC, achieves surplus of $1.2 billion. Model BB, the group buying of private reinsurance model, produces the lowest average surplus of $931 million.

Each model considers a pool of ten Canadian provinces including, AB, BC, MB, ON, PEI, SK, NL, NB, NS and QC. The expected reinsurance premium from each province is allocated to the shared pool, based on a 10% reinsurance coverage layer. Three models are developed under a reinsurance premium pool, under the assumption of dependence across regions. Model AA is a self managed reinsurance pool that does not purchase private reinsurance. Model BB is similar to Model AA, however in addition the pool purchases private reinsurance as a group for all liabilities. Model CC is an optimization model that combines Model AA (self managed reinsurance pool), and Model BB (group buying of private reinsurance). The optimization model segregates the portfolio and determines the first group of uncorrelated risks within the portfolio to retain internally in the self managed reinsurance pool, and the second group of correlated risks from within the portfolio to cede externally to private reinsurers.

Surplus, the balance in the account during each time period, is compared over 30 years in the future. Surplus is calculated by the following formula: $U_t = U_{t-1} + P_t + C_t - S_t$, where ($U_{t-1}$) is the previous time period balance in the fund, (P_t) is the premiums collected during the time period, (C_t) is the interest earned on cash retained during the period, and (S_t) is any claims paid out during the period.

Figure 4.2 95th Percentile of Surplus Under Dependence Across Regions for Models AA, BB, and CC (Best Case Scenarios)

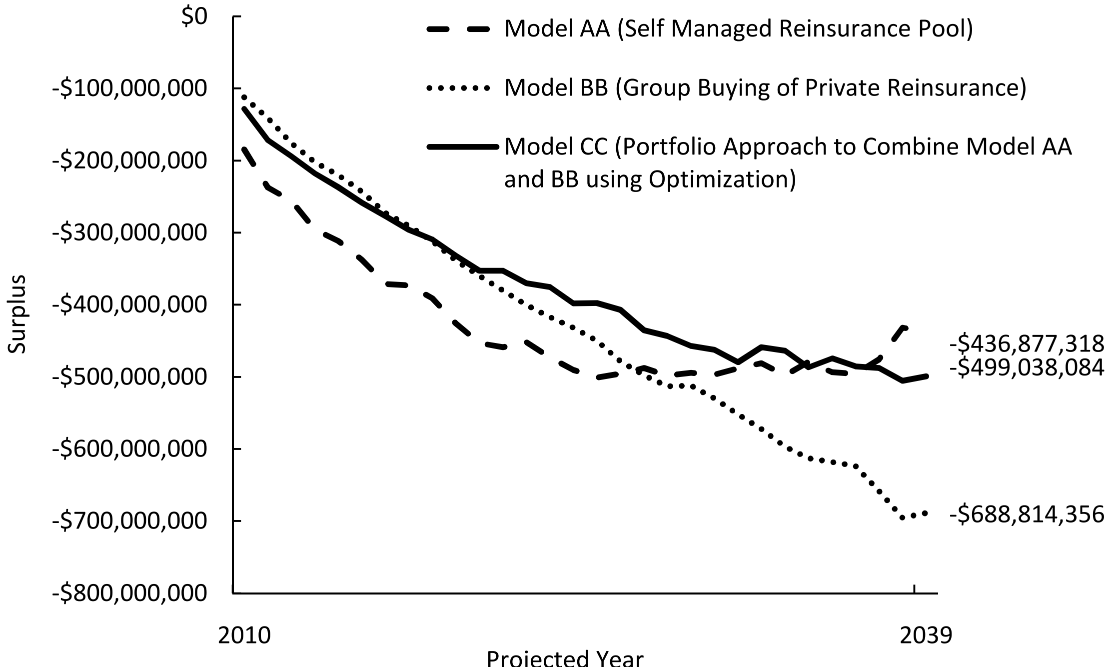


Notes: This figure shows the 95th percentile of surplus for Models AA, BB, and CC, under dependence across regions at an initial surplus of $0. The 95th percentile corresponds to the value where 95% of simulated surplus values fall below. In other words, this value represents close to the best case scenario of simulated surplus. The results show when losses are less severe (as represented by the 95th percentile), the self managed pooling model, Model AA, achieves surplus that is substantially larger than the other two competing models that purchase private reinsurance, Model BB and Model CC, respectively.

Each model considers a pool of ten Canadian provinces including, AB, BC, MB, ON, PEI, SK, NL, NB, NS and QC. The expected reinsurance premium from each province is allocated to the shared pool, based on a 10% reinsurance coverage layer. Three models are developed under a reinsurance premium pool, under the assumption of dependence across regions. Model AA is a self managed reinsurance pool that does not purchase private reinsurance. Model BB is similar to Model AA, however in addition the pool purchases private reinsurance as a group for all liabilities. Model CC is an optimization model that combines Model AA (self managed reinsurance pool), and Model BB (group buying of private reinsurance). The optimization model segregates the portfolio and determines the first group of uncorrelated risks within the portfolio to retain internally in the self managed reinsurance pool, and the second group of correlated risks from within the portfolio to cede externally to private reinsurers.

Surplus, the balance in the account during each time period, is compared over 30 years in the future. Surplus is calculated by the following formula: $U_t = U_{t-1} + P_t + C_t - S_t$, where ($U_{t-1}$) is the previous time period balance in the fund, (P_t) is the premiums collected during the time period, (C_t) is the interest earned on cash retained during the period, and (S_t) is any claims paid out during the period.

Figure 4.3 5th Percentile of Surplus Under Dependence Across Regions for Models AA, BB, and CC (Worst Case Scenarios)

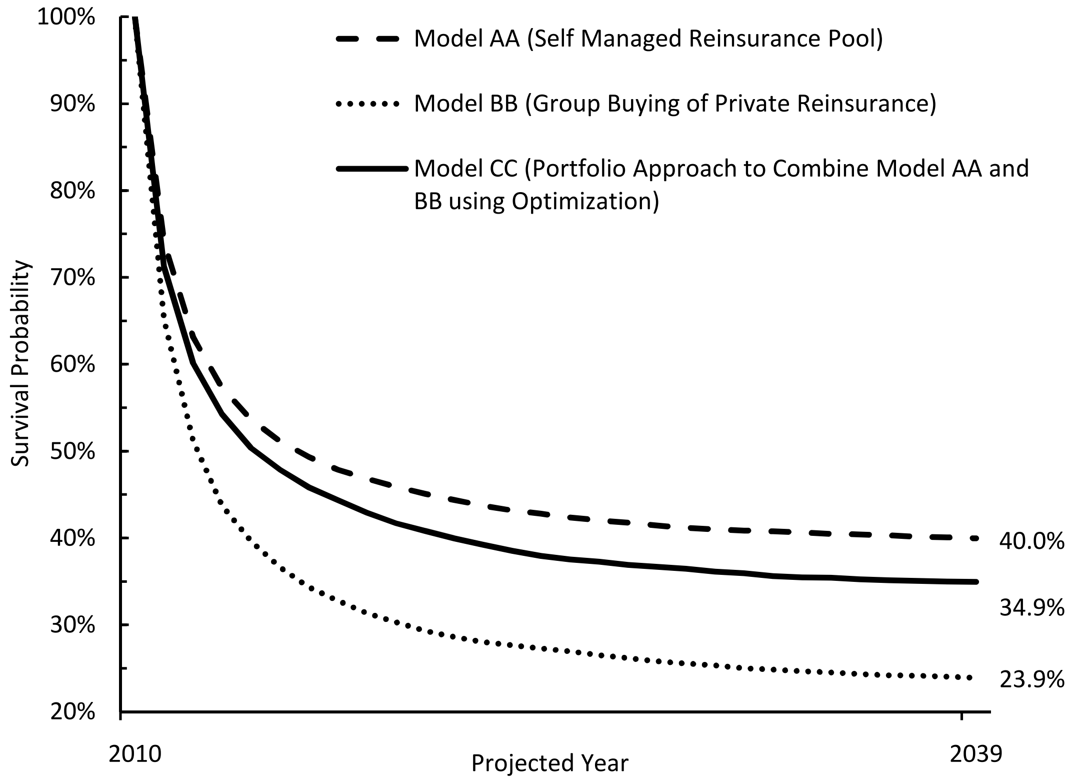


This figure shows the 5th percentile of surplus for Models AA, BB, and CC, under dependence across regions at an initial surplus of $0. The 5th percentile corresponds to 95% of simulated surplus values that fall above this value. In other words, this value represents close to the worst case scenario of simulated surplus. The results show that the group buying of private reinsurance model, Model BB, is superior initially. In the long term, however, the costly reinsurance brokerage fees deplete the fund balance which causes surplus to fall consistently over the 30 year projection horizon. The self managed reinsurance pool model, Model AA, achieves the lowest surplus initially. However, over time the surplus improves and is superior by year 30. The combined reinsurance pooling and private reinsurance pool model that uses optimization, Model CC, consistently performs well over the duration of the 30 years. At year 30, surplus is marginally lower than Model AA.

Each model considers a pool of ten Canadian provinces including, AB, BC, MB, ON, PEI, SK, NL, NB, NS and QC. The expected reinsurance premium from each province is allocated to the shared pool, based on a 10% reinsurance coverage layer. Three models are developed under a reinsurance premium pool, under the assumption of *dependence across regions*. Model AA is a self managed reinsurance pool that does not purchase private reinsurance. Model BB is similar to Model AA, however, in addition the pool purchases private reinsurance as a group for all liabilities. Model CC is an optimization model that combines Model AA (self managed reinsurance pool), and Model BB (group buying of private reinsurance). The optimization model segregates the portfolio and determines the first group of uncorrelated risks within the portfolio to retain internally in the self managed reinsurance pool, and the second group of correlated risks from within the portfolio to cede externally to private reinsurers.

Surplus, the balance in the account during each time period, is compared over 30 years in the future. Surplus is calculated by the following formula: $U_t = U_{t-1} + P_t + C_t - S_t$, where ($U_{t-1}$) is the previous time period balance in the fund, (P_t) is the premiums collected during the time period, (C_t) is the interest earned on cash retained during the period, and (S_t) is any claims paid out during the period.

Figure 4.4 Survival Probability Under Dependence Across Regions for Models AA, BB, and CC

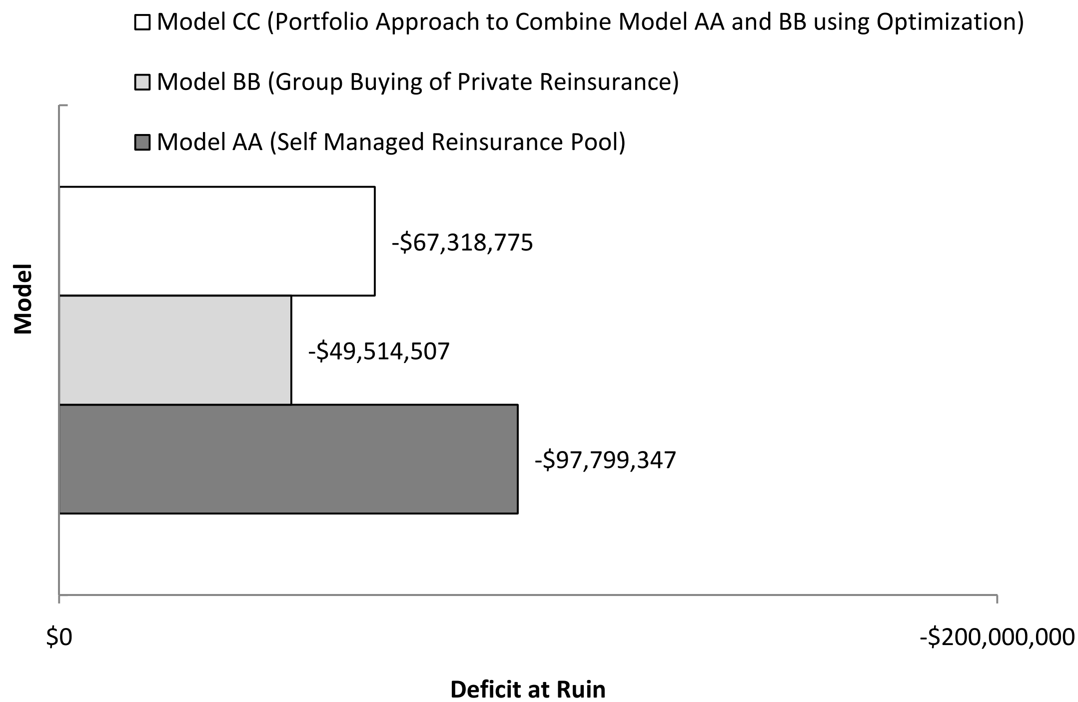


Notes: This figure compares survival probability for Models AA, BB, and CC, under dependence across regions at an initial surplus of $0. Results show that the self managed reinsurance pool model, Model AA, achieves the highest survival probability. This is likely a result of eliminating costly reinsurance brokerage fees, which helps the insurance fund grow surplus at an increased rate, providing stability. Model CC, which combines reinsurance pooling and private reinsurance using optimization, achieves survival probability that is slightly lower than Model AA. Model BB, the group buying of private reinsurance model, achieves the lowest survival probability.

Each model considers a pool of ten Canadian provinces including, AB, BC, MB, ON, PEI, SK, NL, NB, NS and QC. The expected reinsurance premium from each province is allocated to the shared pool, based on a 10% reinsurance coverage layer. Three models are developed under a reinsurance premium pool, under the assumption of *dependence across regions*. Model AA is a self managed reinsurance pool that does not purchase private reinsurance. Model BB is similar to Model AA, however, in addition the pool purchases private reinsurance as a group for all liabilities. Model CC is an optimization model that combines Model AA (self managed reinsurance pool), and Model BB (group buying of private reinsurance). The optimization model segregates the portfolio and determines the first group of uncorrelated risks within the portfolio to retain internally in the self managed reinsurance pool, and the second group of correlated risks from within the portfolio to cede externally to private reinsurers.

Survival probability is compared over 30 years in the future and refers to the probability that the fund will 'survive' without ruin (e.g. have enough reserves to pay claims). This study applies the definition of ultimate ruin, where once the account ruins it is not possible for it to become nonnegative (e.g. decreasing function).

Figure 4.5 Average Deficit at Ruin Under Dependence Across Regions for Models AA, BB, and CC



Notes: This figure shows the average deficit at ruin at year 30, for Models AA, BB, and CC, under dependence across regions at an initial surplus of $0. The self reinsurance pool model, Model AA, produces the most severe deficit at ruin, which highlights the disadvantage of a pooling model when losses are extreme and lead to ruin. The group buying of private reinsurance model, Model BB, has the lowest deficit at ruin, demonstrating the advantage of the additional diversification private reinsurers provide when losses are large and widespread. The portfolio approach to combine pooling and private reinsurance using optimization, Model CC, has a deficit at ruing that is slightly larger than Model BB, however, it is substantially improved compared to Model AA, which is roughly twice the severity.

Each model considers a pool of ten Canadian provinces including, AB, BC, MB, ON, PEI, SK, NL, NB, NS and QC. The expected reinsurance premium from each province is allocated to the shared pool, based on a 10% reinsurance coverage layer. Three models are developed under a reinsurance premium pool, under the assumption of dependence across regions. Model AA is a self managed reinsurance pool that does not purchase private reinsurance. Model BB is similar to Model AA, however in addition the pool purchases private reinsurance as a group for all liabilities. Model CC is an optimization model that combines Model AA (self managed reinsurance pool), and Model BB (group buying of private reinsurance). The optimization model segregates the portfolio and determines the first group of uncorrelated risks within the portfolio to retain internally in the self managed reinsurance pool, and the second group of correlated risks from within the portfolio to cede externally to private reinsurers.

Deficit at ruin represents the severity of ruin, and demonstrates the degree to which the funds claims exceed its assets. To calculate deficit at ruin, a distribution of simulated surplus values just prior to ruin is created. In addition, the surplus value prior to ruin is discounted according to the period in which ruin occurred to t=0 at an assumed rate of 3%.

CHAPTER 5

SUMMARY

Some insurance firms are faced with the unique challenge of managing a portfolio of aggregate risks with high variance. This is because risks in the portfolio may be large, infrequent, and potentially highly correlated across geographic regions and/or product lines. In order to manage the high variance portfolio, insurance firms often rely on private reinsurance. The main problem with private reinsurance, however, is that reinsurance brokerage fees are often costly, and these fees appear to be increasing in recent years. An alternative risk management approach that eliminates the cost associated with private reinsurance is pooling. Pooling insurance business across a number of geographic regions and/or products may help to reduce the portfolio variance of aggregate risks, however, this approach may be insufficiently diversified to handle risks that are large and correlated.

Therefore, the objective of this study was to develop a more efficient insurance portfolio model to help overcome the problem of costly reinsurance brokerage fees associated with private reinsurance, and the problem of insufficient diversification for large and correlated risks associated with pooling. An asset liability management (ALM) approach was used to empirically evaluate the alternative insurance models presented in each chapter. The focus of this study was the entire crop insurance sector for Canada, which included 32 years (from 1978-2009) of historical indemnities and liabilities (from which the loss coverage ratio, LCR was calculated), across 10 provinces for 279 crops. Risk measures including surplus, survival probability, and deficit at ruin were considered. The innovative risk management portfolio approach to combine pooling and private reinsurance using combinatorial optimization with a genetic algorithm, was found to be superior overall (Model CC).

Developing a Full Premium Insurance Pooling Model to Reduce Risk

Chapter two modeled a full premium pool, where crop insurance risks from each of the ten provinces are pooled together into one large countrywide pool, representing the maximum diversification that could be achieved within an insurance sector in an entire country. The objective of this chapter was to analyze the diversification that could be achieved through pooling insurance business across a number of geographic regions (provinces) and products (crops), as a possible solution to better manage a portfolio of aggregate risks with high variance. Three alternative insurance models were developed under a full premium pool, including Model 1 (self managed insurance pool), Model 2 (self managed insurance pool that also purchases private reinsurance as a group), and Model 3 (a portfolio approach to combine pooling and private reinsurance using the CV of the LCR). An eight step methodology was developed for the innovative insurance portfolio model, Model 3, which used the CV of the LCR to segregate a portfolio into two groups, and then combine them. The three models were evaluated under an asset liability management (ALM) surplus model using simulation.

Results showed that diversification was improved through geographic pooling of risks across regions compared to the pooling of risks within regions. In addition, Model 3, a portfolio approach to combine pooling and private reinsurance using the

CV of the LCR, was found to reduce risk through pooling, resulting in a more efficient insurance portfolio. Model 3 showed that it offered the lowest (least severe) deficit at ruin, and adequate surplus, however, the survival probability was the lowest. With further refinement to the portfolio segregation method, the surplus of Model 3 could possibly be improved.

Reinsurance Premium Pool: Combinatorial Optimization with a Genetic Algorithm to Combine Pooling and Private Reinsurance to Reduce Risk

Chapter three, in contrast to chapter two, used a countrywide reinsurance premium pool, where regions (provinces) contributed only a portion of their risk to the pool. This was different than chapter two, where a full premium pool was assumed, and regions contributed all of their risk to the pool. One of the objectives of chapter three was to model a reinsurance premium pool to address one of the potential limitations of chapter two, which was the possible reluctance of some provinces in transferring control of their region to cooperate in the pool. A reinsurance premium pool is an incremental approach to pooling which allows regions to continue operating independently, where only a portion of risks are pooled. The second objective of chapter three was to develop a more efficient insurance portfolio model that combined a self managed reinsurance pool and private reinsurance, using combinatorial optimization with a genetic algorithm, Model C. This is in contrast to chapter two that used CV of the LCR, Model 3.

Three alternative insurance models were developed under a reinsurance premium pool, including Model A (self managed reinsurance pool), Model B (group buying of private reinsurance), and Model C (a portfolio approach to combine Model

A and Model B using optimization). To optimally manage a portfolio of aggregate risks with high variance using Model C, the portfolio was segregated into two groups, and then combined. The first group of risks within the portfolio was uncorrelated, and diversified enough to be managed internally within a pool. This group of uncorrelated risks within the pool naturally offset, which lowered the variance of the aggregate risks within this set. The second group of risks within the portfolio was correlated, and therefore the risks did not sufficiently offset, which led to a high variance portfolio of risks. This second group of risks was ceded to private reinsurers who were better diversified for risks that were large and correlated. To evaluate the three models, an asset liability management (ALM) surplus model was developed.

Results revealed the weakness of the pooling and private reinsurance models, Models A and Model B, respectively. Although Model A achieved the highest surplus and survival probability, in the event of ruin, the deficit of ruin was the most severe. This was due to a self managed reinsurance pool being insufficiently diversified for risks that were large and correlated. Model B successfully achieved the lowest deficit at ruin, however, the expensive reinsurance brokerage fees that were incurred resulted in the lowest surplus and survival probability. Model C, the portfolio approach that combined Model A and Model B, achieved acceptable results in all three risk areas. This included adequately high surplus and survival probability, and adequately low deficit at ruin.

Dependence across Regions Under a Reinsurance Premium Pool: Combinatorial Optimization with a Genetic Algorithm to Combine Pooling and Private Reinsurance to Reduce Risk

Chapter four used a similar approach to chapter three, except that it allowed for dependence (correlation) across regions (provinces). In chapter three, a portfolio approach was used to combine a self managed reinsurance pool and private reinsurance using combinatorial optimization with a genetic algorithm, Model C. In chapter four, however, the assumption of dependence across regions was incorporated into the portfolio approach that used combinatorial optimization model with a genetic algorithm, Model CC. The objective of this chapter was to analyze the effectiveness of the portfolio optimization model under dependence across regions, compared to chapter three that assumed independence across regions.

The intent of this chapter was to ensure that the group of aggregate risks within the portfolio that was retained within the self managed reinsurance pool remained sufficiently diversified in the presence of correlated LCR's across regions (e.g. ensure consistency of results between Model C in chapter three, and Model CC in chapter four). This additional assumption of dependence was important because ignoring dependencies caused by factors such as similar weather, could lead to inaccurate estimates of the risk. This could alter the diversification that could be achieved through pooling.

In chapter four, three alternative reinsurance models were developed under a reinsurance premium pool, which were very similar to Models A, B, and C developed in chapter three, but with the added assumption of dependence across regions. This included Model AA, a self managed reinsurance pool model, Model BB, a group buying of private reinsurance model, and Model CC, a portfolio approach to combine Model AA and Model BB using combinatorial optimization with a genetic algorithm. Rank correlations of the LCR's across regions were used to achieve the desired cross correlation structure, and an asset liability management (ALM) surplus model was developed to evaluate the three models using simulation.

Results confirmed that under dependence across regions, the optimization model that combined pooling and private reinsurance, Model CC, was overall superior to models BB and CC. Results for Model CC are similar to those of Model C in chapter three, indicating consistency between the two models. Therefore, the chapter three and chapter four analysis showed how the problem of insufficient diversification for large correlated risks associated with pooling could be overcome using a combined pooling and reinsurance model. This lower cost self managed reinsurance pool could be used, with the addition of private reinsurance, to form a more efficient reinsurance portfolio model (Modes C and CC).

Limitations and Constraints

While there are a number of benefits associated with combining pooling and reinsurance, there are a few constraints to reinsurance pooling. First, insurance companies with business units organized by geographic regions or product lines may be reluctant to cooperate with each other in a self managed reinsurance pool. This may be because they compete internally with each other within the parent company. Secondly, control may be an issue because cooperation among business units may cause individual business units to lose some control of their operations. Thirdly, a number of non profit insurance firms cooperating together in a self managed reinsurance pool may have disagreements regarding reinsurance premium levels. However, using a loss elimination ratio (LER) to determine reinsurance coverage levels as undertaken in this study will help overcome the issue of premium level disagreements (e.g. the premium level disagreement issue can arise when one firm or business unit may feel that it is lower risk and should pay lower premiums, in order to avoid subsidizing other firms or business units in the reinsurance pool who feel they have larger risks and are not paying sufficiently high premium levels).

Additional Applications and Future Research

Firms most likely to benefit from the combined pooling and private reinsurance approach developed in this study, are those who face portfolios with high variance portfolios of risks due to large but infrequent risks, which are correlated within geographic regions, but are less correlated across geographic regions. These types of firms include those facing large natural disasters and weather events, such as earthquakes, floods, hurricanes, monsoons, and excessive heat or cold, etc. Further, the portfolio models developed in this study could also be applied within large organizations where there is potential to utilize natural hedging across business units that operate in various geographic regions, or across different product lines. The portfolio models represent an innovative approach for industry to reduce insurance costs without incurring additional risk.

APPENDIX GLOSSARY OF TERMS

Asset liability management (ALM) - the act of managing liabilities that arise due to the mismatching of assets (incoming cash flows) and liabilities (outgoing cash flows). This requires careful planning to balance the generation of adequate and stable earnings, maintaining adequate liquidity, and steadily building capital.

Coefficient of variation (CV) - CV is a calculation of risk that measures the dispersion of a probability distribution, which is calculated as the ratio of standard deviation σ to the mean μ, and often multiplied by 100. This statistic is useful for comparing the degree of variation from one risk to another, even if the means are substantially different from each other.

Combinatorial optimization – a branch of applied mathematics, in which an optimization problem can be reduced to a discrete set of feasible solutions, and which the goal is to find the best solution.

Deficit at ruin - deficit at ruin refers to the distribution of losses that result when surplus becomes negative. The advantage of this risk measure is that it goes a step beyond the probability of ruin, and provides information on the severity of losses.

Genetic algorithm – is a search heuristic that follows the process of natural evolution. A genetic algorithm is often used to find solutions to optimization problems, and uses techniques inspired by natural evolution, such as inheritance, mutation, selection, and crossover.

Indemnity – indemnity is the amount of claims paid out to customers if there is a loss.

Insurance premium – the insurance premium E(X), is the expected loss coverage ratio (LCR) which is used to determine the insurance premium contributed by each region to the pool.

Liability (coverage) – liability (L_i), is the total amount of insurance carried.

Loss coverage ratio (LCR) – LCR is a measure of risk that reflects the annual loss. LCR is calculated as the ratio of total indemnities to total liabilities (total coverage), multiplied by 100. As well, the LCR is sometimes referred to as the loss cost ratio. Further, this ratio is very similar to the pure premium, except that instead of total liabilities in the denominator, exposure is in the denominator.

Loss elimination ratio (LER) – LER is the percentage of the loss which the insurer does not pay due to the deductible. The LER can be calculated as $\frac{E(X_j) - E(Y_j)}{E(X_j)}$

Model 1 – Model 1, used in chapter two, assumes that an organization forms a single "insurance company" for the whole country (or large organization), rather than 10 individual companies (or business units). A full premium pool is assumed where each region contributes all of their insurance premiums into a self managed insurance pool that is managed internally by the firm. The advantage of Model 1 is that the purchase of high cost private reinsurance can be avoided and brokerage fees are eliminated, providing more efficiency. The disadvantage is that in a bad year when losses across regions do not adequately offset, the reinsurance company experiences ruin because it is not sufficiently diversified to the same extent as a well diversified international reinsurer.

Model 2 – Model 2, used in chapter two, assumes that the self managed insurance pool that is managed internally within the firm as in Model 1, also purchases private reinsurance for a 10% layer of reinsurance coverage. The advantage of Model 2 is that purchasing private reinsurance helps to stabilize the self managed insurance pool, and lessen the severity of the deficit at ruin. This is because a private reinsurance firm is better diversified than the self managed pool in Model 1, because it holds a well diversified portfolio of risks that offset with international liabilities from multiple countries across different sectors and products. The disadvantage is that private reinsurance is more expensive.

Model 3 – Model 3, used in chapter two, assumes that the benefits of both pooling (Model 1) and private reinsurance (Model 2) can be combined. Coefficient of variation (CV), a normalized measure of dispersion, is used to segregate a portfolio into a first group of liabilities with low CV that are appropriate for retaining internally within a self insurance pool (Model 1). A second group of liabilities with high CV, are then ceded to private reinsurers (Model 2). Private reinsurance is purchased for the select group of high CV liabilities, for a 10% layer of reinsurance coverage. This model assumes that the additional private reinsurance purchased by the pool will overcome the Model 1 problem of insufficient diversification for extreme events that are widespread (e.g. if all regions face the same large risk in the same year). At the same time, Model 3 should benefit from the liabilities with low CV that are retained within the pool, as this is a lower cost option than private reinsurance. Rather than paying brokerage fees on the entire portfolio, brokerage is only incurred on the group of liabilities with high CV that may benefit the most from the improved diversification.

Model A – Model A, used in chapter three, is very similar to Model 1, however instead of assuming a full premium pool, a more gradual incremental approach where each region contributes only a portion of their insurance premium, reinsurance premium, to the self managed reinsurance pool. The advantage is that the purchase of high cost private reinsurance can be avoided, providing more efficiency. The disadvantage is that in a bad year when losses across regions do not adequately offset, the reinsurance company experiences ruin because it is not sufficiently diversified to the same extent as a well diversified international reinsurer.

Model B – Model B, used in chapter three, is very similar to Model 2, however instead of assuming a full premium pool, a more gradual incremental approach where each region contributes only a portion of their insurance premium, reinsurance premium, to the self managed reinsurance pool. Instead of Model B providing its own reinsurance as in Model A, the pool also purchases private reinsurance as a group from the international reinsurance market. The advantage to this approach is that the private reinsurance firm is better diversified than the self managed pool in Model A, because it holds a well diversified portfolio comprised of international liabilities from multiple countries, across different sectors and products. The disadvantage of this approach is that private reinsurance is more expensive.

Model C – Model C, used in chapter three, combines Model A and Model B, and is very similar to Model 3, however, instead of assuming a full premium pool, a more gradual incremental approach where each region contributes only a portion of their insurance premium, reinsurance premium, to the self managed reinsurance pool. In addition, Model C further refines the process of identifying the group of liabilities that are best suited for retaining internally in the self reinsurance pool, and the group of liabilities that are best suited for ceding to private reinsurers, using combinatorial optimization and a genetic algorithm. The assumption is that private reinsurance should overcome the Model A problem of insufficient diversification from extreme events that are widespread (e.g. if all regions face the same large risk in the same year). At the same time, Model C should benefit from the low variance liabilities that are uncorrelated and retained within the pool as this is a lower cost option than private reinsurance. Rather than paying brokerage fees on the entire portfolio, brokerage is only incurred on the select group of high variance liabilities that may benefit the most from improved diversification.

Model AA – Model AA, used in chapter four, is very similar to Model A, however, in this models assumes dependence of the LCR's across regions.

Model BB – Model BB, used in chapter four, is very similar to Model B, however, in this models assumes dependence of the LCR's across regions.

Model CC – Model CC, used in chapter four, is very similar to Model C, however, in this models assumes dependence of the LCR's across regions.

Pooling – risk pooling is fundamental to insurance, whereby similar liabilities are pooled together in order to offset losses and decrease the variance of the portfolio. This risk management approach tends to be most effective for a low variance portfolio of liabilities, where losses are small and frequent, and uncorrelated.

Premium – premium is the amount paid for insurance coverage for a specific period of time.

Private reinsurance – private reinsurance is one solution to reducing a portfolio of high variance liabilities, where losses are large but infrequent, and correlated. Reinsurance is a means of risk management where an insurer purchases insurance from a reinsurer, helping to stabilize premium rates from year to year and maintain an adequate reserve to keep premium rates low.

Reinsurance premium – the reinsurance premium E(Y) is the portion of insurance premium E(X) that is paid to the private reinsurer in return for coverage on a portion of liabilities in the portfolio.

Surplus - surplus U_t, which is the balance in the insurance company's account at the end of the t_{th} period, surplus can be calculated for each forecasting period using the following formula: $U_t = U_{t-1} + P_t + C_t - S_t$, where ($U_{t-1}$) is the surplus in the prior period; (P_t) refers to premiums; (C_t) is the interest retained on cash; (S_t) refers to claims.

Survival probability - survival probability is an extension of surplus that measures the point at which capital is exhausted. If the surplus in a given time period is positive then the insurance firm is said to survive, while if the surplus is negative the insurance firm is said to experience ruin. Given N=5,000 simulations, and where L represents the number of iterations that produce a negative surplus, the probability of ultimate ruin can be estimated at each discrete time interval as L/N (where the survival probability is 1-(L/N)). A high probability of ruin indicates instability, and measures such as purchasing private reinsurance, or raising premiums should be considered

Ultimate ruin – ultimate ruin is a conservative measure of the traditional measure of probability of ruin. For ultimate ruin, once the surplus in a particular simulation becomes negative (e.g. is ruined), the simulation is not allowed to continue and possibly become nonnegative again. For example, given 5000 simulations, 100 simulations are ruined in year 1, and survival probability for year 1 is calculated as (5000-100)/(5000) = 98%. In the second year, only the 4900 'surviving' simulations are continued. In year 2, 120 simulations ruin, and survival probability for year 2 is calculated as (4900-120)/(5000) = 95.6%. Therefore it is possible to have an increasing surplus function, yet a decreasing survival function.

Self insurance – self insurance refers to a risk management method that is carried out internally within a firm. Rather than purchasing private reinsurance for example, a firm can set aside the actuarially calculated premiums to cover the future uncertain loss.

Simulation - simulation is an attempt to model a hypothetical situation or process. Simulating generally entails changing variables to represent certain key characteristics or behaviors in order to make a prediction about the behavior of the system. In this study, simulation is performed by computer software, Palisade's @Risk.

Pure premium – pure premium refers to the expected loss per unit exposure, without taking into consideration fixed expenses, variable expenses, and a provision for profit and contingencies.

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