A COMPARISON OF METHODS FOR COMPUTING VALUE AT RISK IN FINANCIAL MARKETS

By Jonathon Driedger

A Thesis
Submitted to the Faculty of Graduate Studies
in Partial Fulfillment of the Requirements
for the Degree of

Master of Science

Department of Agribusiness and Agricultural Economics
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ABSTRACT

Financial crises in the late 1990's in areas such as Asia and Latin America, as well as the collapse of trading organizations such as Long-Term Capital Management, have led to increased emphasis on measuring and controlling risk. One of the most common methods for measuring market risk is Value at Risk (VaR). VaR can be defined as the "...maximum amount of money that may be lost on a portfolio over a given period of time, with a given level of confidence" (Best 1998). This study examines several different methods for computing VaR. The first method is the parametric method, with VaR being computed using the normal distribution and the t-distribution across twenty-eight different futures markets. The second method is the historical simulation method, which is examined across the same twenty-eight futures markets. The historical simulation method is then compared with the parametric method for computing VaR. The third section examines VaR from the perspective of an investment manager with a longer holding period, such as one month or longer. Monthly VaR is computed and then scaled using $\sqrt{t}$ to compute VaR for longer holding periods.

Results show that the parametric method is an acceptable method for computing VaR, and better captures risk at the 99 percent confidence level when using a t-distribution instead of using the traditional normal distribution. In addition, results show that the historical simulation method is also an acceptable method for computing VaR, particularly when a long parameter of 1000 days is used. VaR can also be accurately computed for portfolios with holding periods of one month. However, scaling one-month VaR to longer holding periods using the $\sqrt{t}$ rule tends to overestimate VaR.

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CHAPTER 1

INTRODUCTION

Considerable research has been conducted in the area of financial risk management in recent years, as financial crises in the late 1990's in areas such as Asia and Latin America, as well as the collapse of trading organizations such as Long-Term Capital Management, have led to an increased emphasis on measuring and controlling risk. Accurate assessment of risk by management is important for both investors and regulators. There are several different categories of financial risk, including market risk, credit risk, liquidity risk and operational risk (Jorion 1997). This study focuses on market risk. One challenge for risk managers and regulators is finding a measure of risk that captures the firm's market risk, and describes this risk in terms that can be understood by most managers and compared across firms.

One of the most widely used measures of market risk is Value at Risk (VaR) (Smithson 1998; Hull 1999), which attempts to measure the maximum downside market risk at a given confidence level. Therefore, the objective of this study is to examine the performance of various VaR methods for measuring risk. VaR can be defined as the "...maximum amount of money that may be lost on a portfolio over a given period of time, with a given level of confidence" (Best, 1998), and makes the statement "...we are X percent certain that we will not lose V dollars in the next N days" where V is the Value at Risk, N is the time horizon and X is the confidence level (Hull and White 1998). Essentially, VaR attempts to measure the likelihood of losses at the left-hand tail of the distribution of asset returns.

Numerous researchers have studied the reliability of Value at Risk models, including Beder (1995), Hendricks (1996) and Best (1998). Beder examined portfolios of US Treasury strips, the S&P 500 index and S&P 500 index options, and concluded that VaR results are quite sensitive to the methodology used and associated assumptions. Hendricks examined foreign exchange rate portfolios using a variety of methods for computing VaR, and found that the VaR measures examined slightly underestimated risk at the 99 percent confidence level. Best (1998) examined fifteen different assets using eight different measures for computing VaR, and concluded that the only models tested that performed well at the 99 percent confidence level were the Exponentially Weighted Moving Average (EWMA) with a weight of 0.97 and the GARCH (1,1) model.

Often a main assumption made when computing VaR is that asset returns are normally distributed (Best 1998, Alexander 2001). However, many financial asset price series contain 'outliers' with large price increases and decreases that cause the distribution of returns to have more kurtosis than is assumed by a normal distribution (Duffie and Pan 1997). This kurtosis has the effect of increasing volatility leading to an underestimation of VaR.

This study examines the performance of two common methods for computing VaR, the parametric method and the historical simulation method. Chapter Two examines the parametric method for computing daily VaR on a number of futures markets comparing the t-distribution to the commonly used normal distribution at the 99 percent confidence level. Use of a t-distribution maintains the ease of use and computation of the parametric method while allowing for a higher frequency of returns at the extreme ends of the return distribution.

Chapter Three examines the performance of the historical simulation method for computing daily VaR on a number of futures markets at the 99 percent confidence level. This method is easy to understand and compute, uses actual price data, and makes no statistical assumptions regarding the distribution of asset returns. VaR is computed using parameters of 100, 250 and 1000 days. These results are then compared to the performance of the parametric method for computing VaR on the same futures markets.

Chapter Four examines the performance of VaR for measuring risk in institutional investment portfolios with a holding period of one month or longer. The parametric method is used to compute monthly VaR at the 99 percent confidence level for portfolios of stocks and bonds. The monthly VaR is then scaled up using the $\sqrt{t}$ rule to compute 3 month, 6 month, 9 month and 12 month VaR.

Chapter Five is a summary of chapters two, three and four. It compares the results of the different VaR methods of computation, and describes how VaR can be a useful risk management tool in a variety of situations.

CHAPTER TWO

IMPROVED PARAMETRIC METHODS FOR COMPUTING VALUE AT RISK

Introduction

Increased market volatility in recent years and financial crises in the 1990's in areas such as Asia and Latin America has led to an increased emphasis on measuring and controlling market risk. The Bank of International Settlements defines market risk as "the risk that the value of on- or off-balance-sheet positions will be adversely affected by movements in equity and interest rate markets, currency exchange rates and commodity prices." (Alexander 2001). One of the most widely used measures of market risk is Value at Risk (Smithson 1998; Hull 1999), and VaR attempts to measure the maximum downside loss under normal market conditions given a predetermined confidence level. VaR is most commonly used to measure market risk for applications such as banks, financial firms and investment portfolios, as well as trading portfolios such as cash and futures.

This chapter uses the parametric method for computing VaR, as it was the first widely used method and is still the most common method for computing VaR (Smithson 1998, Best 1998). This method is also used in the popular JP Morgan VaR program *RiskMetrics*. The information in this study should be useful to managers in financial and trading firms, as it is one of the first studies to examine the usefulness of the t-distribution for computing parametric VaR over a broad range of markets, and may be a worthwhile improvement over the traditional normal distribution which has difficulty with the large price changes and fat distribution tails often found in financial markets.

Theory

Numerous researchers have studied the reliability of Value at Risk models, including Beder (1995), Hendricks (1996) and Best (1998). Beder examined portfolios of US Treasury strips, the S&P 500 index and S&P 500 index options, and concluded that VaR results are quite sensitive to the methodology used and associated assumptions. Hendricks examined foreign exchange rate portfolios using a variety of methods for computing VaR, including the equally weighted and Exponentially Weighted Moving Average measures for computing the standard deviation in VaR. Hendricks found that these particular VaR measures slightly underestimated risk at the 99 percent level.

Best (1998) examined fifteen different assets consisting of interest rates, commodities, equity indices and exchange rates. Using eight different measures for computing VaR, Best concluded that the only models tested that perform well at the 99 percent confidence level are the Exponentially Weighted Moving Average (EWMA) with a weight of 0.97 and the GARCH (1,1) model. Since both of these models performed about equally well, Best suggests that the extra computation time and estimation difficulty required by the GARCH model may not be justified for common VaR applications.

VaR began to gain wide application in the banking industry in the mid-1990's as banks began to adjust their capital for risk in financial markets as required by regulators. For example, the Basle Accord from the Bank of International Settlements was implemented using bank capital requirements as a multiple of VaR for banks in its member countries (Butler 1999). VaR measures the maximum downside market risk at a given confidence level. VaR can be defined as the "...maximum amount of money that may be lost on a portfolio over a given period of time, with a given level of confidence" (Best, 1998), and makes the statement "...we are X percent certain that we will not lose V dollars in the next N days" where V is the Value at Risk, N is the time horizon and X is the confidence level (Hull and White 1998). Essentially, VaR attempts to measure the likelihood of losses at the left-hand tail of the distribution of asset returns. But, VaR does not measure the size of potential losses beyond the chosen confidence level. There is no way to estimate an absolute worst possible loss for a portfolio, because price distributions are often assumed to be approximated by the continuous probability distribution with unlimited tails (Jorion 1997). VaR also does not consider potential losses in a specific portfolio from non-market factors such as political risk, liquidity risk, operational risk, fraud, etc.

VaR is most commonly computed at the 95 percent or 99 percent confidence levels. The Basle Accord from the Bank of International Settlements used by bank regulators uses a 99 percent VaR when setting regulatory capital requirements for commercial banks (Butler 1999, Alexander 2001). A one-day VaR, e.g. the maximum amount of losses expected over one day, is usually calculated for in-house use at most financial institutions, since most financial assets such as futures portfolios traded on organized exchanges can often be liquidated within twenty-four hours, and the profits and losses of most trading firms are also computed daily. However, a longer holding period would be used if computing VaR for assets that are difficult to liquidate in one day. As well, firms need to be careful not to make the mistake of claiming a 99 percent confidence level VaR as a *certainty*, rather than an expectation that the firm will not lose greater than that amount only one percent of the time. Accuracy of a given VaR computation can change considerably with changes in the assumptions and inputs entered into the model. One of the keys to proper implementation of VaR is understanding the parameters and inputs of the chosen model.

There are several reasons why it is important to accurately compute a firm's VaR. First, underestimating VaR mistakenly exposes a firm to more risk than it wishes, and this leads to potentially larger losses than expected. Secondly, overestimating VaR means that a firm sets aside more capital than is necessary and this is an inefficient use of capital. A third reason for the importance of accurate VaR computation is that VaR is being increasingly used by regulatory authorities for determining the capital requirements for commercial banks, and so inaccurate VaR models may lead to inadequate capital reserves being held (Best 1998).

Parametric VaR theory and assumptions

Market risk may be described as the potential profit or loss for a portfolio that is left unchanged over a period of *h* days. The formula for the change in portfolio value is

$$\Delta_h P_t = P_{t+h} - P_t \tag{2.1}$$

where P is the price of the portfolio on day *t* (Alexander 2001). A 100(1-α)% h-period value at risk measure is the nominal amount C such that

$$\text{Prob}(\Delta P < -C) = \alpha \tag{2.2}$$

where ΔP denotes the change in portfolio value over a prespecified holding period h, and α is a sufficiently small probability (Alexander and Leigh, 1997). VaR is largely determined by the volatility of the portfolio returns over the holding period since volatility is measured by standard deviation. The standard deviation is multiplied by the Z-value or t-value associated with the chosen statistical distribution and confidence level and the initial portfolio dollar amount to arrive at a dollar value. This dollar value is the VaR of the portfolio for the chosen confidence level.

Often a main assumption made in the parametric method for computing VaR is that asset returns are normally distributed (Best 1998, Alexander 2001). If equation (2.1) is the *h*-day portfolio return, it is assumed that

$$\Delta_h P_t \sim N(\mu_t, \sigma^2_t) \tag{2.3}$$

The 100α% h-period VaR is that number $VaR_{\alpha,h}$ such that $Prob([\Delta_h P_t - \mu_t]/\sigma_t) = \alpha$. Since $[\Delta_h P_t - \mu_t]/\sigma_t \sim N(0,1)$ and denoting $[\Delta_h P_t - \mu_t]/\sigma_t$ by the standard normal variate Z_t, $Prob(Z_t<[-VaR_{\alpha,h}- \mu_t]/\sigma_t = \alpha$. But for a standard normal variate Z_t, $Prob(Z_t<Z_\alpha) = \alpha$ where Z_α is the 100αth percentile of the standard normal density. Therefore $[-VaR_{\alpha,h}- \mu_t]/\sigma_t = -Z_\alpha$. Written another way, the formula for parametric VaR is,

$$VaR_{\alpha,h} = Z_\alpha \sigma_t - \mu_t. \tag{2.4}$$

Sometimes researchers assume that μ=0 when calculating daily returns, but this assumption has very little effect on a one-day holding period VaR computation because the expected return in one day is very small when compared with the standard deviation of daily returns (Hull 1999). Since Z_α is the constant given by the choice of statistical distribution, it becomes σ_t that determines the VaR (Alexander 2001).

One of the appealing properties of the parametric method for computing VaR is the ease of use and computation. However, the assumption of normal distribution of returns is often criticized since many financial asset price series contain 'outliers' with large price increases and decreases that cause the distribution of returns to have more kurtosis than is assumed by a normal distribution (Duffie and Pan 1997). In other words, there are more observations both at the center and the tails of the distribution than is assumed by a normal distribution. This kurtosis, or 'fat tails', has the effect of increasing volatility, or σ_t, leading to an underestimation of VaR. This is particularly evident at the 99 percent confidence level where the distribution of daily returns for many financial assets is significantly heavier than the tails of the normal distribution (Best 1998). As a result there will be more days when losses exceed the VaR estimate than a firm would anticipate based on the normal distribution. Use of the t-distribution for VaR maintains ease of computation while still capturing the greater risk at the tails of the distribution better than the normal distribution (Jorion 1996).

Data and Procedure

The data used for this study is the continuous nearby futures price for twenty-eight futures markets from January 1, 1988 to December 31, 2000. This includes a broad range of futures markets, including commodities, currencies, bonds and the S&P 500. Nearby futures prices are used because these markets are sufficiently liquid and futures markets are often held in the portfolios of trading firms and financial firms. Past data before 1988 is also used in order to allow results to be computed beginning on January 1, 1988. Data are obtained from the Institute for Financial Markets.

Measures and parameters for parametric VaR

Two alternative measures are used to compute daily standard deviation, with two alternative parameters used for each measure. The two measures used are: 1) the equally weighted standard deviation and 2) the Exponentially Weighted Moving Average (EWMA) standard deviation. The standard deviations from these measures are multiplied by the Z-values from the normal distribution, and alternatively the t-distribution with 10, 6, and 4 degrees of freedom, to compute daily VaRs. T-distributions of 6 and 4 degrees of freedom are chosen because these are the parameters used by Jorion's study of the collapse of the Long-Term Capital Management hedge fund (Jorion 2000). A t-distribution with 10 degrees of freedom is also chosen because it provides an additional standard deviation multiplier between the normal distribution and the t-distribution with 6 degrees of freedom used by Jorion. Four steps are used in the VaR computation procedure.

Step One: Compute daily volatility

The formula for equally weighted standard deviation is as follows:

$$\sigma = \sqrt{\frac{\sum (X-\mu)^2}{(n-1)}} \tag{2.5}$$

where X is the actual daily return (percent price change), n is the length of the observation period and μ is the expected return. This equally weighted measure for computing standard deviation is chosen because it is a commonly used measure for computing standard deviation that places equal weight on each day's price. A 50-day parameter is chosen because this is a commonly used parameter for estimating shorter-term volatility and should capture short-term fluctuations in volatility better than longer parameters. A 250-day parameter is chosen because it alternatively captures longer-term volatility. As well, there are approximately 250 trading days in a calendar year and the Basle Accord suggests using at least one year of data history when computing VaR (Alexander 2001).

Standard deviation (volatility) is computed using a walk forward procedure. That is, each day the oldest observation from the previous day's computation is dropped and the latest observation is added. For example, the 50-day standard deviation calculation on January 3, 1998 uses the 50 trading days prior to January 3. For the computation on January 4, the oldest trading day is dropped and the latest (January 3) is added.

The second measure for computing standard deviation is the Exponentially Weighted Moving Average (EWMA), with the formula:

$$\sigma_t = \sqrt{\lambda\sigma_{t-1}^2 + (1-\lambda)X_{t-1}^2} \qquad (2.6)$$

where λ is the weight given to the previous day's volatility estimate and $(1-\lambda)$ is the weight given to the previous day's return (Hendricks 1996). The aim of this measure is to better capture recent changes in volatility than the equally weighted standard deviation as more weight is put on the most recent days' return (Hendricks 1996). No specific number of observations is given when using the EWMA. Theoretically the observation period goes back to infinity, but in reality the observation period is more limited because after a given number of days the daily weightings become relatively insignificant. The observation period is therefore implied by the weighting scheme.

The EWMA measure of computation is included for comparison because it is commonly used, and also used by the JP Morgan *RiskMetrics* system, which is among the most widely used VaR programs available. The 0.94 weight is chosen because this is the chosen weight used by the JP Morgan RiskMetrics system for daily VaR calculations. The 0.97 weighting is also chosen because it uses a longer observation period to estimate the next period's standard deviation, which theoretically should give more accurate estimates nearer to the end of the distribution tail (Best 1998). The different weights used, 0.94 and 0.97, imply observations periods of approximately 30 and 100 days respectively (Best 1998). Interaction of λ and $(1-\lambda)$ means the lower the decay factor λ, the faster the decay in the influence of a given decay period (Hendricks 1996).

The exponentially weighted moving average measure is equivalent to the IGARCH (1,1) conditional volatility model (Alexander 2001). The GARCH family of volatility measures uses the equation

$$\sigma^2 = \gamma V + \alpha\mu_{n-1}^2 + \beta\sigma_{n-1}^2 \qquad (2.7)$$

and $\gamma + \alpha + \beta$ must sum to one. The EWMA measure is the GARCH (1,1) model where $\gamma = 0$, $\alpha = 1 - \lambda$ and $\beta = \lambda$. Best (1998) examines eight different measures for computing VaR in his study of fifteen different assets, including the exponentially weighted moving average with a weight of 0.97 and the GARCH (1,1) model. Since both of these models performed about equally well at the 99 percent confidence level, Best suggests that the extra computation time and estimation difficulty required by the GARCH model may not be justified for common VaR applications.

Step Two: Compute Value at Risk

For comparing the VaR computations, a \$1,000,000 long position is assumed for each futures market. The \$1,000,000 portfolio is multiplied by the computed daily standard deviation to get a standard deviation in a dollar amount. This daily dollar standard deviation is then multiplied by the appropriate one-tail Z-value or t-value from the selected statistical distribution to get the 99 percent confidence level daily VaR. In this study the daily asset standard deviations are multiplied by 2.33 to get the 99 percent confidence level for the normal distribution, 2.764 for the t-distribution with 10 degrees of freedom, 3.143 for the t-distribution with 6 degrees of freedom, and 3.747 for the t-distribution with 4 degrees of freedom, based on statistical tables.

For example, the S&P 500 futures on January 3, 1995 has a computed daily standard deviation of 0.653 percent using the 0.94 EWMA. This translates into a standard deviation of $6,530 for the $1,000,000 position. To get a 99 percent confidence VaR, $6,530 is multiplied by 2.33 to get a VaR of $15,214.90 using the normal distribution. Using a t-distribution with 10 degrees of freedom, $6,530 is multiplied by 2.764 for a VaR of $18,048.09. For t-distributions with 6 degrees of freedom and 4 degrees of freedom, $6,530 is multiplied by 3.143 and 3.747 to arrive at VaRs of $20,522.85 and $24,466.79, respectively. If the actual loss on January 3, 1995 is more than $18,048.09, then January 3, 1995 would count as an exception day using the normal distribution, as the VaR is exceeded. The same approach applies to the VaR computations for the t-distributions with 10, 6 and 4 degrees of freedom.

<u>Step Three: Compare Value at Risk with actual risk</u>

The computed VaR is compared daily with the actual one day return on the $1,000,000 portfolio. If the actual loss exceeds the computed VaR, that day is counted as an exception day. It is expected at the 99 percent confidence level that actual losses exceed the VaR one percent of the time, or one day out of 100 days. Since there are about 250 trading days in a year, it is expected that there will be roughly 2.5 exceptions per year if the VaR model is accurate. The number of times that actual daily losses exceed the VaR is summed over the entire period. Since there are approximately 3100 daily observations, it is expected that there will be about 31 exception days when actual losses exceed the daily VaR.

For example, Figure 2.1 shows the daily returns on the $1,000,000 portfolio for the British Pound from 1990 to 1994. Daily returns are arrived at by multiplying the daily percentage return by the $1,000,000 portfolio. Below the daily returns are lines representing daily VaR using the normal distribution and the t-distribution with 10 degrees of freedom. The daily VaR is computed by multiplying the portfolio amount by the daily standard deviation, and the daily dollar standard deviation by the Z-value or t-value for the associated confidence level, as explained in step 2. In this case, for the 99 percent confidence level, the Z-value for the normal distribution is 2.33 standard deviations and t-value for 10 degrees of freedom is 2.764. The standard deviation is computed using the 0.94 EWMA. An exception day is counted whenever the actual return crosses the VaR line. The actual return line should not cross the VaR line more than one percent of the time, or 2.5 exception days per year (due to approximately 250 trading days in one year), if the VaR model is accurate. Since daily standard deviation is multiplied by a larger amount when a t-distribution with 10 degrees of freedom is used, the line representing this VaR is below the line representing VaR using the normal distribution. As a result, the line representing the t-distribution will likely be crossed less frequently than the line representing the normal distribution.

Step 4: Determine if the number of Value at Risk exceptions lie within an acceptable range at the 99 percent level

The total number of exception days when actual losses exceed the daily VaR can be considered a random variable that has a binomial distribution, where the probability of an exceptional loss, or exception day, is one percent for a 99 percent confidence VaR (Alexander 2001). A Type I error check is done to determine the accuracy of the VaR models. A Type I error is made when a valid model has been erroneously rejected. This method is chosen because it is the method specified in the Basle Accord to be sure that regulators are not rejecting a valid model (Best 1998). A confidence level is calculated to determine the range of acceptable number of observations exceeding the daily VaR computation. While regulators are only concerned that a bank does not underestimate its VaR computation, the banks themselves will also want to be careful not to overestimate their VaR, which would result in carrying excess capital. As a result a two-tailed test is used. This use of a Type I error check has a tendency to give the benefit of the doubt to validating the VaR model (Best 1999). The confidence interval is calculated using a Z-score for a binomial distribution:

$$Z = \frac{X - Np}{\sqrt{Npq}} \tag{2.8}$$

where X is the number of exception days where losses exceed the VaR, N is the number of days over which the VaR model is tested, p is the required confidence level expressed as a fraction and q=1-p. A 95 percent confidence range for evaluating the validity of a model is commonly used (Best 1999, Alexander 2001).

A Z-score of 1.96 provides a 95 percent confidence level for the two-tail test to validate the VaR model. A binomial distribution is approximately normal when N is large and p is small. For example, if crude oil has 3103 daily observations for which the daily VaR is computed. The 95 percent confidence range for a 99 percent level VaR model would be calculated as follows: $X = 3103*0.01 \pm 1.96*\sqrt{(3103)(0.01)(0.99)}$ which gives a confidence range between 20 and 42 exceptions. This confidence range is converted into a 95 percent confidence range around the expected value of one percent. For the expected value of one percent, this range goes from 0.66 percent at the low end to 1.34 percent at the high end. The VaR models that have the percentage of exceptions fall within the confidence range are considered acceptable models.

Results

Descriptive statistic results

Table 2.1 shows the descriptive statistics of daily returns over the period from 1988 through 2000 for the twenty-eight futures markets used in this study. One statistic of relevance to this study is kurtosis. A value greater than 0 indicates a positive level of kurtosis when compared to the normal distribution. A positive level of kurtosis means a greater peak and fatter tails than the normal distribution, indicating there are more large price moves than if the returns are normally distributed. A value of less than 0 indicates no peak and thinner tails than the normal distribution. Nearly all futures markets showed positive kurtosis. Positive kurtosis is likely one of the reasons that a number of the futures markets are shown to be not normally distributed at the 99 percent confidence level, as the Kolmogorov-Smirnov Statistic rejected the normality of returns for all markets. This may provide evidence that a number of futures markets could be better represented by a t-distribution.

T-distribution with 10 degrees of freedom VaR results

The right hand side of Table 2.2 shows the results for computing VaR when using the t-distribution with 10 degrees of freedom. Results show that of the four distributions in this study, the t-distribution with 10 degrees of freedom provides the most accurate distribution from which to compute VaR. For all four of the parameters used to compute VaR for each market, most of the futures markets fall within the acceptable range for percentage of exception days when using 10 degrees of freedom. The VaR computations with the highest number of markets in the acceptable range are the 0.94 EWMA and the 250 day equally weighted moving average, as can be seen in Table 2.2 in the second and third columns from the right. Both of these parameters have twenty-two of the twenty-eight markets in the acceptable range. Markets where the percentage of exception days falls within the acceptable range are indicated by an asterisk in the table. The 50 day equally weighted parameter has twenty-one markets in the acceptable range and the 0.97 EWMA parameter has eighteen markets in the acceptable range.

For example, the US 10-year note futures market VaR using the 50 day equally weighted parameter and the t-distribution with 10 degrees of freedom has 28 exception days when actual losses exceed the computed VaR over the observation period (3221 days). This translates into 0.87 percent of days being exception days and falls into the acceptable range as computed in Step Four. This can be seen in the fifth column in Table 2.2. Similarly, the 250 day equally weighted parameter has 24 days where the actual losses exceed the VaR for a percentage of exception days of 0.75 percent. The 0.94 EWMA and the 0.97 EWMA parameters have 30 days and 24 days where losses exceed VaR which compute to 0.93 percent and 0.75 percent exception days. As a result, the US 10-year note futures market falls in the acceptable range when using the t-distribution for all four of the VaR computations.

Normal distribution VaR results

VaR computations appear to be considerably less accurate when using the normal distribution than when using the t-distribution with 10 degrees of freedom, as indicated by relatively few asterisks. Results on the left side of Table 2.2 show that the normal distribution underestimates VaR at the 99 percent confidence level. Of the four different parameters for computing VaR, the highest number of markets that fall within the acceptable range for percentage of exception days is seven markets for the 250 day equally weighted parameter. These markets are the Japanese Yen, pork bellies, cotton, lumber, US T-bills, wheat and soybean meal. The other parameters have two markets (pork bellies and lumber), three markets (pork bellies, lumber and wheat) and six markets (Japanese Yen, cotton, lumber, heating oil and wheat) in the acceptable range for the 50 day equally weighted, 0.94 EWMA and 0.97 EWMA, respectively.

Again using the example of the US 10-year note futures market, the 50 day equally weighted parameter using the normal distribution has 58 days when losses exceed the computed VaR. This is 1.80 percent of total observations which is above the upper limit of the acceptable range. The 250 day equally weighted parameter has 49 days when losses exceed VaR, or 1.52 percent, while the 0.94 EWMA has 57 days for 1.77 percent and the 0.97 EWMA has 50 days for 1.55 percent. All of the different normal distribution VaR computations give a percentage of exception days that exceed the 1.34 percent upper limit of the acceptable range. Therefore the normal distribution is a less acceptable model for computing VaR for the US 10-year note futures market for any of the VaR methods used in this study.

Of all the computations that fall outside of the acceptable range, all of the markets have a percentage of exception days that exceeds the upper limit of the acceptable range (with the exception of pork bellies using the 0.94 EWMA which has 0.64 percent exceptions, and is below the lower limit of the acceptable range). This means that for every model that is not acceptable, there are more days when losses exceed the VaR than would be expected at the 99 percent confidence level. Clearly the t-distribution with 10 degrees of freedom more accurately captures the heavy tails of the actual return distribution than does the normal distribution when computing VaR.

T-distribution with 6 degrees of freedom and 4 degrees of freedom VaR results

VaR computed using the t-distribution with 6 degrees of freedom and 4 degrees of freedom overestimate VaR by a large margin. The left side of Table 2.3 shows the results when using the t-distribution with 6 degrees of freedom. The 250 day equally weighted parameter has eight out of twenty-eight markets in the acceptable range, the most of any of the parameters using the t-distribution with 6 degrees of freedom. The 50 day equally weighted parameter and the 0.94 EWMA parameter both have six markets with a percentage of exception days in the acceptable range while the 0.97 EWMA has only three markets in the acceptable range. All of the markets not in the acceptable range for percentage of exception days are below the lower limit of the acceptable range. This shows that using the t-distribution with 6 degrees of freedom consistently overestimates the likelihood of large losses at the 99 percent confidence level.

The results for the t-distribution with 4 degrees of freedom are shown on the right side of Table 2.3. All of the VaR computations using the four parameters for all of the twenty-eight futures markets have a percentage of exception days that is below the lower limit of the acceptable range. Most of the percentage of exception days is less than half the lower limit of 0.66 percent. This shows that a t-distribution with 4 degrees of freedom overestimates VaR by an even larger margin than the t-distribution with 6 degrees of freedom.

The results in Table 2.2 and 2.3 are consistent with expectations, as the distributions of asset returns show more kurtosis than is assumed with the normal distribution. Results are similar when grouping the individual futures markets into equally weighted portfolios (see Table 2.4 and Table 2.5). These results are consistent with expectations that VaR computations using the normal distribution at the 99 percent confidence level would underestimate the percentage of days when actual losses exceeded the VaR, and that the fatter tails of the t-distribution would better capture the likelihood of extreme market moves. The results of this study using a broader range of markets than earlier studies confirms this, and are also consistent with the study done by Hendricks (1996) which found that all of the parametric VaR computations using the normal distribution slightly underestimated risk at the 99 percent confidence level. These results are also consistent with the study performed by Best (1998) who found that the only measures that accurately computed VaR at the 99 percent confidence level were the 0.97 EWMA and the GARCH (1,1) model.

Summary

Value at Risk (VaR) is defined as the maximum amount of funds that a firm is likely to lose under normal market conditions over a specified period of time given a predetermined confidence level. As a result of increased emphasis on risk management in financial markets and regulatory policy, VaR has become a more widely used measure of market risk by banks, financial institutions, trading firms and others. One method for computing VaR is the parametric method while using the normal distribution. However, many financial assets have return distributions that show more kurtosis, or "fatter tails", than the normal distribution, and so can cause firms to underestimate their risk exposure.

The results of the study over twenty-eight futures markets show that the commonly used normal distribution underestimates VaR, with more days showing greater losses than the VaR than would be expected at the 99 percent confidence level. The t-distribution with 10 degrees of freedom proposed as an alternative shows greater accuracy in computing VaR at the 99 percent confidence level, with most futures markets having a number of exception days that fall within the acceptable range for four different VaR models. VaR computations using the t-distribution with 6 degrees of freedom and 4 degrees of freedom overestimated VaR at the 99 percent confidence level, as too few days show actual losses that exceed the VaR.

Since VaR computations using the normal distribution underestimate risk at the 99 percent confidence level, a number of firms may wish to consider the t-distribution as an alternative to the normal distribution for VaR computations. A t-distribution with 10 degrees of freedom was found to capture VaR more accurately than the normal distribution for the individual markets examined in this study, and firms may wish to consider using it if they face similar markets, given its accuracy, ease of use, and understanding.

Table 2.1 Descriptive Statistics of Daily Percentage Returns for Individual Futures Markets and Portfolios of Futures Markets for the Period 1988 - 2000

Commodity	Variance	Standard Deviation	Skewness	Kurtosis	Kolmogorov-Smirnov Normality Test
British Pound	0.00004282	0.00654395	-0.1342462	3.45750088	0.068136*
Japanese Yen	0.00005818	0.00762787	0.82353527	8.65025517	0.067360*
Canadian Dollar	0.00000960	0.00309884	-0.0346065	3.2130829	0.046460*
Swiss Franc	0.00005861	0.00765564	0.15548861	1.98141034	0.050607*
Currency Group	0.00002128	0.00461335	0.30275211	3.57179494	0.055912*
S&P 500	0.00010328	0.01016283	-0.5022593	7.2749361	0.069782*
Live Cattle	0.00005954	0.00771648	0.00617629	0.7371848	0.034727*
Feeder Cattle	0.00005053	0.00710832	-0.0356092	1.21269853	0.041830*
Live/Lean Hogs	0.0001935	0.01391028	0.01265993	1.8675009	0.046033*
Pork Bellies	0.00052233	0.02285461	0.08878612	-0.0259246	0.023304*
Livestock Group	0.00009869	0.00993416	0.05092065	0.57457213	0.027659*
Cotton	0.00016266	0.01275387	0.04200413	0.43530844	0.035610*
Sugar #11	0.00041437	0.02035605	-0.0223915	3.66533686	0.060859*
Coffee	0.00065498	0.02559264	0.84309496	11.6460241	0.077845*
Cocoa	0.00033398	0.01827503	0.49092748	3.08961403	0.040049*
Orange Juice	0.00040864	0.02021485	1.71710397	24.4945034	0.087097*
Lumber	0.00024374	0.01561233	0.09549735	0.07925092	0.041775*
Soybean Oil	0.0001794	0.01339413	0.20526056	1.79276058	0.043720*
Soft Group	0.00005968	0.00772544	0.2127128	1.93605306	0.031118*
Gold	0.00006598	0.00812273	0.09724441	16.0584117	0.087971*
Silver	0.00020505	0.0143197	0.05957934	4.53698998	0.073651*
Metals Group	0.00010395	0.0109549	-0.0150118	5.98830742	0.067618*
Heating Oil	0.00037054	0.01924938	-0.9767473	20.1377422	0.055060*
Crude Oil	0.00042591	0.02063771	-1.1825536	21.875497	0.062246*
Energy Group	0.00037677	0.01941069	-1.1648563	22.9329677	0.063360*
US T-bills	0.00003152	0.00561455	-0.0686337	1.61580697	0.037087*
US 10-year Notes	0.000014	0.00374126	-0.0850085	1.998643	0.042411*
F.I. Group	0.00002141	0.00462747	-0.0777897	1.76129033	0.036683*
Corn	0.00016289	0.0127627	0.15769813	3.32271395	0.054440*
Wheat	0.00017738	0.01331823	0.09554215	1.56618109	0.028496*
Oats	0.00030573	0.01748525	0.07826203	1.64140125	0.054765*
Soybeans	0.00016587	0.01287903	-0.0283290	4.88386318	0.055187*
Soybean Meal	0.00018708	0.01367786	0.15808852	3.73084226	0.054710*
Canola	0.00012762	0.01129676	0.08408700	1.2871044	0.044384*
Grains Group	0.00011641	0.01078923	0.12869911	2.59269578	0.041565*

*null hypothesis of normality is rejected at the 99 percent confidence level.

Table 2.2 Normal Distribution and t-distribution with 10 degrees of freedom: Percentage of Exception Days Where Actual Losses Exceed VaR at the 99 Percent Confidence Level for Individual Futures Markets for the Period 1988 - 2000

	Normal Distribution				t- distribution with 10 d.o.f			
Commodity	50 day equal	250 day equal	0.94 EWMA	0.97 EWMA	50 day equal	250 day equal	0.94 EWMA	0.97 EWMA
British Pound	2.14[a]	2.17	2.48	1.98	1.24*	1.18*	1.33*	1.15*
Japanese Yen	1.55	1.27*	1.43	1.30*	0.77*	0.81*	0.77*	0.71*
Canadian Dollar	2.17	2.26	1.98	1.89	1.39	1.24*	1.15*	1.24*
Swiss Franc	1.64	1.52	1.64	1.49	0.90*	0.71*	0.77*	0.74*
Live Cattle	1.98	1.47	1.67	1.63	0.86*	0.54	0.74*	0.64
Feeder Cattle	2.17	1.69	2.10	1.85	1.02*	0.92*	0.86*	0.76*
Live/Lean Hogs	2.01	1.82	2.01	1.82	0.61	0.67*	0.67*	0.57
Pork Bellies	0.86*	0.99*	0.86*	0.64	0.19	0.06	0.22	0.06
Cotton	1.36	1.20*	1.36	1.30*	0.49	0.26	0.62	0.36
Sugar #11	1.66	2.01	1.79	1.75	1.01*	1.04*	0.91*	0.84*
Coffee	2.21	1.91	1.85	1.75	1.30*	1.36	0.91*	0.84*
Cocoa	1.56	1.59	1.49	1.33*	0.52	0.78*	0.45	0.55
Orange Juice	1.75	1.62	1.75	1.66	1.01*	1.07*	1.01*	0.91*
Lumber	1.07*	0.71*	1.01*	0.84*	0.39	0.36	0.32	0.29
Soybean Oil	1.56	1.46	1.53	1.46	0.75*	0.88*	0.84*	0.75*
Gold	1.85	2.07	2.27	1.88	1.28*	1.34*	1.15*	1.18*
Silver	1.82	1.63	1.72	1.60	0.99*	1.02*	1.08*	0.96*
Heating Oil	1.48	1.51	1.42	1.16*	0.87*	0.84*	0.71*	0.68*
Crude Oil	1.84	1.80	1.84	1.58	0.84*	1.06*	0.71*	0.61
S&P 500	1.92	1.58	2.04	1.73	1.24*	1.05*	1.18*	1.05*
US T-bills	1.61	1.27*	1.83	1.49	0.93*	0.59	0.84*	0.78*
US 10-year Notes	1.80	1.52	1.77	1.55	0.87*	0.75*	0.93*	0.75*
Corn	1.74	2.10	1.68	1.65	1.13*	1.13*	1.03*	0.94*
Wheat	1.45	1.32*	1.32*	1.20*	0.65	0.84*	0.65	0.52
Oats	2.00	1.87	2.00	1.71	1.20*	0.90*	0.87*	0.90*
Soybeans	1.97	1.84	1.97	1.81	0.81*	1.20*	0.81*	0.81*
Soybean Meal	1.58	1.29*	1.52	1.36	0.74*	0.90*	0.68*	0.65
Canola	1.71	1.74	1.61	1.58	0.74*	0.68*	0.58	0.55

[a]Note: Given the 99% confidence level, actual losses would be expected to exceed the VaR computation one percent of the time; for example of 3100 trading days, 31 days would be expected to have losses exceeding the VaR, or one percent.

* Indicates percentage of exception days that fall *within* acceptable expected range, from Step Four.

Table 2.3 T-distribution with 6 and 4 degrees of freedom: Percentage of Exception Days Where Actual Losses Exceed VaR at the 99 Percent Confidence Level For Individual Futures Markets for the Period 1988 - 2000

	t-distribution with 6 d.o.f				t-distribution with 4 d.o.f.			
Commodity	50 day equal	250 day equal	0.94 EWMA	0.97 EWMA	50 day equal	250 day equal	0.94 EWMA	0.97 EWMA
British Pound	0.74*[a]	0.62	0.68*	0.59	0.34	0.31	0.28	0.28
Japanese Yen	0.46	0.43	0.40	0.43	0.34	0.25	0.25	0.25
Canadian Dollar	0.81*	0.71*	0.77*	0.71*	0.53	0.46	0.43	0.37
Swiss Franc	0.37	0.40	0.40	0.37	0.12	0.12	0.09	0.09
Live Cattle	0.25	0.22	0.19	0.16	0.06	0.00	0.06	0.03
Feeder Cattle	0.41	0.51	0.35	0.35	0.22	0.13	0.19	0.13
Live/Lean Hogs	0.22	0.35	0.19	0.13	0.03	0.03	0.03	0.00
Pork Bellies	0.00	0.00	0.06	0.00	0.00	0.00	0.00	0.00
Cotton	0.29	0.16	0.23	0.16	0.13	0.10	0.10	0.13
Sugar #11	0.52	0.55	0.65	0.55	0.36	0.26	0.32	0.36
Coffee	0.71*	0.84*	0.52	0.55	0.26	0.42	0.26	0.23
Cocoa	0.23	0.26	0.32	0.16	0.13	0.13	0.03	0.10
Orange Juice	0.65	0.68*	0.71*	0.65	0.42	0.19	0.36	0.29
Lumber	0.19	0.23	0.16	0.16	0.10	0.16	0.06	0.06
Soybean Oil	0.49	0.39	0.39	0.36	0.23	0.03	0.10	0.10
Gold	0.89*	0.80*	0.83*	0.73*	0.48	0.57	0.51	0.38
Silver	0.61	0.67*	0.70*	0.54	0.38	0.29	0.35	0.32
Heating Oil	0.52	0.52	0.42	0.45	0.32	0.23	0.23	0.19
Crude Oil	0.42	0.61	0.39	0.42	0.32	0.39	0.32	0.32
S&P 500	0.90*	0.74*	0.84*	0.74*	0.40	0.37	0.40	0.37
US T-bills	0.56	0.37	0.56	0.50	0.22	0.16	0.25	0.19
US 10-year Notes	0.53	0.37	0.56	0.37	0.31	0.22	0.25	0.19
Corn	0.68*	0.61	0.48	0.52	0.19	0.23	0.16	0.06
Wheat	0.23	0.55	0.32	0.26	0.06	0.16	0.16	0.10
Oats	0.65	0.45	0.48	0.58	0.19	0.13	0.16	0.16
Soybeans	0.58	0.78*	0.48	0.45	0.26	0.29	0.13	0.19
Soybean Meal	0.42	0.68*	0.39	0.39	0.19	0.26	0.19	0.16
Canola	0.52	0.29	0.32	0.32	0.13	0.06	0.10	0.10

[a]Note: Given the 99 percent confidence level, actual losses would be expected to exceed the VaR computation one percent of the time; for example of 3100 trading days, 31 days would be expected to have losses exceeding the VaR, or one percent.
* Indicates percentage of exception days that fall *within* acceptable expected range, from Step Four.

Table 2.4 Normal Distribution and t-distribution with 10 degrees of freedom: Percentage of Exception Days Where Actual Losses Exceed VaR at the 99 Percent Confidence Level for Portfolios of Futures Markets for the Period 1988 - 2000

	Normal distribution				t-distribution with 10 d.o.f.			
Portfolio	50 day equal	250 day equal	0.94 EWMA	0.97 EWMA	50 day equal	250 day equal	0.94 EWMA	0.97 EWMA
Grains	2.00[a]	1.87	1.65	1.68	0.84*	0.78*	0.84*	0.71*
Currencies	1.61	1.39	1.77	1.61	0.93*	0.68*	0.87*	0.71*
Livestock	1.37	1.08*	1.24*	1.12*	0.45	0.29	0.35	0.41
Soft	1.36	1.04*	1.33*	0.97*	0.45	0.32	0.45	0.39
Fixed Income	2.08	1.77	2.14	1.92	1.06*	0.75*	1.06*	0.87*
Gold / Silver	1.82	1.44	1.88	1.53	0.99*	0.77*	1.02*	0.99*
Energy	1.77	1.74	1.68	1.35	0.81*	1.10*	0.71*	0.71*

[a]Note: Given the 99% confidence level, actual losses would be expected to exceed the VaR estimate one percent of the time; for example of 3100 trading days, 31 days would be expected to have losses exceeding the VaR, or one percent.

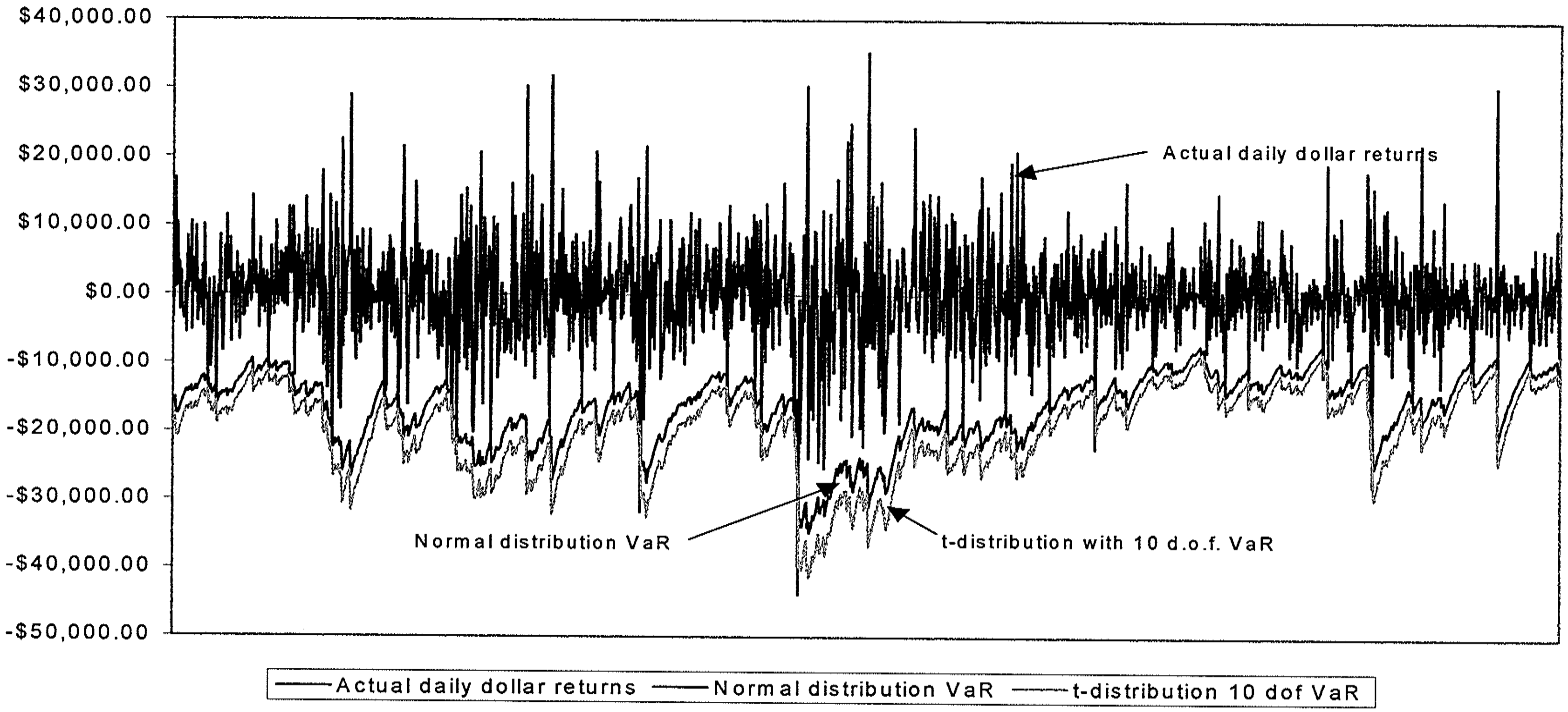
* Indicates percentage of exception days that fall *within* acceptable expected range, from Step Four.

Table 2.5 T-distribution with 6 and 4 degrees of freedom: Percentage of Exception Days Where Actual Losses Exceed VaR at the 99 Percent Confidence Level For Portfolios of Futures Markets for the Period 1988 - 2000

Portfolio	t-distribution with 6 d.o.f.				t-distribution with 4 d.o.f.			
	50 day equal	250 day equal	0.94 EWMA	0.97 EWMA	50 day equal	250 day equal	0.94 EWMA	0.97 EWMA
Grains	0.48[a]	0.52	0.36	0.45	0.13	0.22	0.06	0.10
Currencies	0.50	0.37	0.50	0.31	0.15	0.22	0.12	0.12
Livestock	0.22	0.16	0.13	0.16	0.00	0.03	0.06	0.00
Soft	0.25	0.23	0.26	0.23	0.13	0.06	0.16	0.10
Fixed Income	0.62	0.34	0.53	0.50	0.28	0.16	0.31	0.25
Gold / Silver	0.77*	0.64	0.70*	0.61	0.41	0.41	0.45	0.41
Energy	0.32	0.52	0.39	0.35	0.29	0.32	0.23	0.26

[a]Note: Given the 99% confidence level, actual losses would be expected to exceed the VaR estimate one percent of the time; for example of 3100 trading days, 31 days would be expected to have losses exceeding the VaR, or one percent.
* Indicates percentage of exception days that fall *within* acceptable expected range, from Step Four.

Figure 2.1. British Pound Futures Daily Dollar Returns for a $1,000,000 Portfolio compared to 99 percent confidence VaR estimation: using a Normal Distribution and a t-distribution with 10 dof, and a 0.94 EWMA, 1990 - 1994



Note: An exception day is counted whenever the actual return crosses the VaR line. The actual return line should not cross the VaR line more than one percent of the time, or 2.5 exception days per year.

CHAPTER THREE

A TEST OF THE HISTORICAL SIMULATION METHOD FOR COMPUTING VALUE AT RISK

Introduction

The historical simulation is another common method for computing Value at Risk in financial markets. This method is used by many large financial institutions (Pritsker 2001) and is an accepted method for computing VaR for the Basle Accord (Butler 1999). The advantages of this method are that it is easy to understand and compute, uses actual price data, and makes no statistical assumptions regarding the distribution of asset returns.

This evaluation of the historical simulation method for computing VaR provides important information on the performance and reliability of this method and compares it with the commonly used parametric method. This study is unique because it tests historical simulation across a wider range of financial instruments and over a longer time period than past studies. This is accomplished by testing twenty-eight futures markets over thirteen years of daily prices. The historical method is then compared with another popular method for computing VaR, parametric VaR, using the same futures markets over the same time horizon. This information should be useful to academics and managers in financial and trading firms as it comprehensively examines the performance of the well known historical simulation method for computing VaR over a broader range of markets and a wider variety of parameters than earlier studies.

Theory

Numerous researchers have studied the reliability of Value at Risk models, including Beder (1995), Hendricks (1996) and Best (1998). Beder examined portfolios of US Treasury strips, the S&P 500 index and S&P 500 index options, and concluded that VaR results are quite sensitive to the methodology used and associated assumptions. For example, Beder's study shows that historical simulation VaR results can be twice as high if the VaR computation period is changed from 100 trading days to 250 trading days.

Hendricks (1996) examined foreign exchange rate portfolios using a variety of methods for computing VaR. Results showed that the historical simulation method using parameters of 50, 100, 250 and 500 trading days all underestimate VaR at the 99 percent confidence level. However, the parameter of 1,250 trading days estimates VaR at the 99 percent confidence interval fairly accurately. Best (1998) examined fifteen different assets consisting of interest rates, commodities, equity indices and exchange rates using eight different methods for computing VaR. Best found that the historical simulation method with a 50 day parameter underestimates VaR at the 99 percent confidence level, and therefore suggests that a longer parameter should be used.

VaR measures the maximum downside market risk at a given a certain confidence level. VaR can be defined as the "…maximum amount of money that may be lost on a portfolio over a given period of time, with a given level of confidence" (Best, 1998) and makes the statement "…we are X percent certain that we will not lose V dollars in the next N days" where V is the Value at Risk, N is the holding period and X is the confidence level (Hull and White 1998). Essentially, VaR attempts to measure the likelihood of losses at the left-hand tail of the distribution of asset returns.

Market risk may be described as the potential profit or loss for a portfolio that is left unchanged over a period of *h* days. The formula for the change in portfolio value is

$$\Delta_h P_t = P_{t+h} - P_t \qquad (3.1)$$

where P is the price of the portfolio on day *t* (Alexander 2000). A 100(1-α)% h-period VaR measure is the nominal amount C such that

$$\text{Prob}(\Delta P < -C) = \alpha \qquad (3.2)$$

where ΔP denotes the change in portfolio value over a prespecified holding period h, and α is a sufficiently small probability (Alexander and Leigh, 1997).

The historical simulation method may have a number of advantages over the parametric method because it uses less restrictive distributional assumptions. Often a main assumption made in parametric methods for computing VaR is that asset returns are normally distributed (Best 1998; Alexander 2001). If equation (1) is the *h*-day portfolio return, it is assumed that

$$\Delta_h P_t \sim N(\mu_t, \sigma^2_t) \qquad (3.3)$$

or the changes in the value of the portfolio are normally distributed with a mean μ and variance σ^2. This assumption of normal distribution of returns is often criticized since many financial asset price series contain 'outliers' with large price increases and decreases that cause the distribution of returns to have more kurtosis than is assumed by the normal distribution (Duffie and Pan 1997). Since parametric VaR is computed based on portfolio standard deviation, this kurtosis, or 'fat tails', has the effect of increasing volatility, or σ_t, often leading to an underestimation of parametric VaR. As a result there may be more days when losses exceed the VaR computation than a firm would anticipate based on the normal distribution. However, the historical simulation overcomes this problem by using the actual price histories of the assets, and therefore no assumption regarding the distribution of asset returns is required.

Historical Simulation

The basic idea of the historical simulation method for computing VaR is fairly straightforward. For example, if a 1000 day time horizon is chosen, the daily percentage returns are computed from the past 1000 days of profits and losses and ordered from the largest loss to the largest gain. The 99 percent VaR is the 990^{th} lowest calculated portfolio value so that 10 days, or one percent of losses, exceeds this amount. This use of actual historical price changes is the distinguishing feature of historical simulation (Linsmeier and Pearson 2000).

In a linear portfolio, the h-day portfolio return $\Delta_h P_t/P_t$ is represented as the weighted sum of the returns R_i to assets,

$$\Delta_h P_t/P_t = w_i R_{it} + \ldots + w_k R_{kt} \tag{3.4}$$

where w_i are the portfolio weights and they sum to one. The historical data are obtained on each R_i, and then the portfolio price changes over h days are simulated as

$$\Delta_h P_t = \Sigma(w_i P_t) R_{i,t} = \Sigma p_i R_{i,t} \tag{3.5}$$

where p_i are the actual amounts invested in each asset. This allows h-day theoretical portfolio returns to be calculated. The empirical h-day portfolio return is obtained by building a histogram of the h-day differences $\Delta P_t = P_{t+h} - P_t$ for all t. The historical $VaR_{\alpha,h}$ is the lower 100αth percentile of this distribution (Alexander 2001).

For example, Figure 3.1 shows the histogram that would be used for the gold futures market on February 17, 1994. The 1000 daily percentage returns for the gold nearby futures contract prior to February 17, 1994 are compiled as a histogram and used to determine the VaR. Looking at the left-hand tail of the distribution, it can be seen that the 99 percent confidence return is –2.55 percent, or the eleventh lowest return, while the 95 percent confidence VaR is –1.25 percent, or the 51st lowest return.

<u>Advantages of the Historical Simulation method</u>

Advantages of the historical simulation method for computing VaR are that this method is relatively simple to understand, easy to compute, and relatively accurate. Another advantage of the historical method is that no statistical assumptions need to be made regarding the distribution of returns. Other methods of computing VaR often incorrectly assume a normal distribution, but the fat tails that exist in the real distribution of returns for most assets may cause the VaR to be underestimated. By using actual price data, the historical method accounts for fat tails and greater frequency of large price movements. A summary of some of the advantages of the historical simulation method of computing VaR can be found in Table 3.1.

Data and Procedure

The data used for this study is the continuous nearby futures price for twenty-eight futures markets from January 1, 1988 to December 31, 2000. This includes a broad range of markets, including commodities, currencies, bonds and the S&P 500. Nearby futures prices are used because these markets are sufficiently liquid and futures are often held in portfolios of trading firms and financial firms. Past data before 1988 is additionally used in order to allow results to be computed beginning on January 1, 1988. Data is from the Institute for Financial Markets.

Parameters for the Historical Simulation method

A major decision to be made when modeling the historical simulation method is the length of observation period, or parameter. Shorter observation periods may better capture short-term movements in the underlying risk, with the assumption being that the future is adequately represented by recent history. However, short observation periods may not always be a very representative example of market behavior and therefore may not provide an accurate picture of the future. Longer observation periods may be more robust and more likely to include more extreme price moves and all the characteristics of the distribution of the portfolio. In the end, a balance must be struck regarding the length of observation period.

The alternative parameters used in this study are the previous 100, 250 and 1000 trading days prior to the date of the VaR computation. The 100 day parameter is chosen because it is a commonly used parameter in historical simulation and is designed to capture recent market moves. The 250 day parameter is chosen because there are approximately 250 trading days in a year and the Basle Accord recommends using at least one year of data in computing VaR. The 1000 day parameter is chosen because it goes back approximately four years and therefore captures a longer history of market moves in its computation, and is therefore more likely to include extreme moves in the observation period. Five steps are used here to compute VaR.

Step One: Compute the 99 percent lowest return from the chosen observation period

In computing the VaR for each of the twenty-eight futures markets, the previous 100, 250 and 1000 previous days' percentage returns are computed. These returns are ordered from the lowest return to the highest return. The return that gives exactly one percent of returns below it is used to compute the 99 percent confidence level VaR. For example, computing the VaR for the 1000 day parameter, the previous 1000 daily returns are ordered from the worst to the best. The eleventh lowest return is chosen so that exactly ten observations, or one percent, are worse. Appendix A provides an example for the computation for crude oil on January 1, 1995 using the 100 day parameter.

Step Two: Compute VaR

Multiply the 99 percent lowest return from Step One by an assumed $1,000,000 long position in the futures market. The resulting dollar value is the VaR for that particular day.

Step Three: Compare VaR with Actual Risk

The computed VaR is compared daily with the actual one day return on the $1,000,000 portfolio. If the actual loss exceeds the VaR, that day is counted as an exception day. It is expected that at the 99 percent confidence level that actual losses exceed the VaR one percent of the time, or one day out of 100 days. Since there are about 250 trading days in a year, it is expected that there will be roughly 2.5 exceptions per year if the VaR model is accurate. The number of times that actual daily losses exceed the VaR are summed over the entire period. Since there are approximately 3100 daily observations, it is expected that there will be about 31 exception days when the actual losses exceed the daily VaR.

For example, Figure 3.2 shows the daily returns on the $1,000,000 portfolio for the corn futures market from 1990 to 1995. Daily returns are arrived at by multiplying the daily percentage return by the $1,000,000 portfolio. Below the daily returns are lines representing daily VaR computed using parameters of 100 days, 250 days and 1000 days. Daily VaR is arrived at by taking the 99 percent lowest return from each parameter and multiplying this return by the $1,000,000 portfolio. An exception day is counted whenever the actual return line crosses the VaR line. If the VaR model is accurate, the actual return line should not cross the VaR line more than one percent of the time, or 2.5 exception days per year (due to approximately 250 trading days in one year). Each of the parameters reacts to different levels of volatility of returns. The 100 day VaR is much more responsive to periods of high or low volatility. The 250 day parameter is somewhat less responsive to volatility changes, and the 1000 day parameter shows little change to short-term changes in volatility.

<u>Step Four: Determine if the number of VaR exceptions lie within an acceptable range at the 99 percent level</u>

As in Chapter Two, the total number of exception days when actual losses exceed the daily VaR can be considered a random variable that has a binomial distribution, where the probability of an exception day is one percent for a 99 percent confidence VaR (Alexander 2001). A Type I error check is done to determine the accuracy of the VaR models. A Type I error is made when a valid model has been erroneously rejected. A confidence range is calculated to determine the range of acceptable number of exception days when the VaR computation is exceeded. As in Chapter Two, a two-tailed test is used. This use of a Type I error check has a tendency to give the benefit of the doubt to validating the VaR model (Best 1998). The confidence interval is calculated using a Z-score for a binomial distribution:

$$Z = \frac{X - Np}{\sqrt{Npq}} \qquad (3.6)$$

where X is the number of exceptions or values exceeding the VaR estimate, N is the number of days over which the VaR model is tested, p is the required confidence level expressed as a fraction and q = 1 - p. A 95 percent confidence range for evaluating the validity of a model is commonly used (Best 1998, Alexander 2001). A Z-score of 1.96 provides a 95 percent confidence interval for the two-tailed test to validate the VaR model. This confidence range is converted into a 95 percent confidence range around the expected value of one percent. For the expected value of one percent, this range goes form 0.66 percent at the lower limit to 1.34 percent at the upper limit. The VaR models that have the number of exceptions fall within the confidence range are considered acceptable models.

Step Five: Compare historical simulation VaR with the parametric VaR

The historical simulation models are compared with different parametric measures for computing VaR using the same futures markets over the same period. For comparing the various VaR computations, again a $1,000,000 long position in each futures market is assumed. Since parametric VaR is computed based on standard deviation, a daily computation for standard deviation is made using two alternative measures for computing standard deviation with two alternative parameters for each measure. This provides four different computations for daily standard deviation. The $1,000,000 portfolio is multiplied by the daily standard deviation to get a daily standard deviation in a dollar amount. This daily dollar volatility is then multiplied by the appropriate Z-value or t-value to get the 99 percent confidence level daily VaR estimate. In this study the four daily standard deviations are multiplied by 2.33 to get the 99 percent confidence level for the normal distribution and by 2.764 to get the estimate for the t-distribution with 10 degrees of freedom. Therefore, there are four different VaR computations for each of the twenty-eight futures markets for both of the statistical distributions used in this study. A detailed description of the procedure for computing parametric VaR can be found in Chapter Two.

Results

<u>1000 day historical simulation</u>

Results for the 1000 day parameter for each of the twenty-eight futures markets can be seen in the third column in Table 3.2. Twenty-two of the futures markets have a percentage of exception days that fall within the acceptable range, as indicated by asterisks and as computed in Step Four. This indicates that the 1000 day parameter is a reasonably accurate measure for computing historical simulation VaR.

For example, the Swiss Franc futures market using the 1000 day parameter has 33 days over the entire period when actual losses exceed VaR. Since there are 3227 observations in total for the Swiss Franc, this means that 1.02 percent of days are exception days. Since 1.02 percent is almost exactly the expected one percent exception days, and easily within the acceptable range of between 0.66 percent and 1.34 percent, the 1000 day parameter model is an acceptable model for the Swiss Franc.

250 day historical simulation

The 250 day parameter underestimates VaR in all but six of the twenty-eight markets tested. The second column in Table 3.2 shows that only the futures markets for the S&P 500, live cattle, silver, US T-bills, US 10-year notes and soybean meal had a percentage of exception days that fall within the acceptable range. All of the other futures markets have a percentage of exception days that exceeds the upper limit of the acceptable range. This shows that the 250 day parameter is not a long enough observation period to capture the likelihood of large negative price movements. For example, the Swiss Franc futures market has 55 days over the observation period when actual losses exceed the computed daily VaR. This comes out to 1.70 percent of the 3227 observations and is above the upper limit of the acceptable range.

100 day historical simulation

The results in Table 3.2 show that none of the futures markets has a percentage of exception days that falls within the acceptable range for the 100 day parameter. Every market has a percentage of exception days that is above the upper limit of the acceptable range. As well, every market has a percentage of exception days that is in excess of the percentage of exception days when using the 250 day parameter. It is apparent that the 100 day parameter underestimates VaR at the 99 percent confidence level by a wide margin, as there is a greater percentage of days with losses exceeding the VaR than expected.

These results are consistent with expectations as it was expected that the longer parameter would more effectively measure VaR at the 99 percent confidence level. However, there are relatively large differences in accuracy between the different parameters. Results are similar when grouping the individual futures markets into equally weighted portfolios (see Table 3.3). These results are consistent with previous studies of the historical simulation method. Best (1998) examined only the 50 day parameter and finds that this observation period provides an inaccurate model of VaR at the 99 percent confidence level. Hendricks (1996) examined parameters of 50, 100, 250, 500 and 1,250 days and finds that only the parameter of 1,250 days does not underestimate VaR at the 99 percent confidence level.

Parametric VaR

Table 3.4 shows the results for computing VaR using the parametric method over the same time period. The t-distribution with 10 degrees of freedom has significantly more futures markets with a percentage of exception days in the acceptable range than does the normal distribution for each of the VaRs computed using the two standard deviation measures with two parameters for each. The number of markets in the acceptable range when using the t-distribution with 10 degrees of freedom ranges from eighteen markets out of twenty-eight for the 0.97 EWMA, to twenty-two markets for the 0.94 EWMA and the 250 day equally weighted parameters. The normal distribution, in contrast, has a range of only two markets out of twenty-eight in the acceptable range for the 50 day equally weighted parameter to six markets in the acceptable range for the 250 day equally weighted parameter. The 0.94 EWMA has only three markets in the acceptable range while the 0.97 EWMA has six markets in the acceptable range.

Historical simulation VaR versus parametric VaR

The historical simulation method for a linear portfolio using a parameter of 1000 days was found to be a more accurate method for computing VaR. The historical simulation method showed twenty-two of twenty-eight markets having a percentage of exception days in the acceptable range. The parametric method with the assumption of normal distribution of returns showed only eight markets or fewer in the acceptable range for the four different parameters used. This may be due to the long period used in the historical simulation being better able to capture the likelihood of large negative price moves, while the normal distribution and its thinner tails underestimates the frequency of large losses.

However, the historical simulation method with the 1000 day parameter performed with similar accuracy to the parametric method using a t-distribution with 10 degrees of freedom, as both show approximately the same number of futures markets having a percentage of exception days in the acceptable range. The historical simulation method using parameters of 100 days and 250 days had very few markets in the acceptable range, much fewer than the parametric method using a t-distribution with 10 degrees of freedom.

Summary

Value at Risk (VaR) is defined as the maximum amount of funds that a firm is likely to lose under normal market conditions over a specified period of time given a predetermined level of confidence. As a result of increased emphasis on risk management in financial markets and regulatory policy, VaR has become a widely used measure of market risk. One method for computing VaR is the historical simulation method. This method has a number of advantages over other methods of computing VaR, including its relative ease of computation and understanding, no statistical assumptions are made regarding the distribution of asset returns, and the fact that the data used represents actual market behavior.

The results of the study over twenty-eight futures markets show that a long parameter of 1000 days more accurately computes VaR at the 99 percent confidence level than shorter parameters of 100 days and 250 days, both of which underestimate VaR. The 1000 day parameter also more accurately computed VaR than the well known parametric method under the common assumption of normal distribution of asset returns. This indicates that the historical simulation method is an acceptable method for computing daily VaR in a linear portfolio over a wide range of futures markets, particularly when using a large sample parameter of 1000 days. Given its accuracy, ease of use, and no need for statistical assumptions, the historical simulation method is an acceptable method for computing VaR, especially if the parametric method is to be used with the normal distribution, which may underestimate Value at Risk.

Table 3.1 Advantages of the Historical Simulation Method for Computing Value at Risk

Advantages
– No statistical distribution assumptions necessary
– Uses actual market prices and volatility
– Outliers are included in the data
– Relatively easy to calculate and understand

Table 3.2 Historical Simulation VaR: Percentage of Exception Days Where Actual Losses Exceed VaR at the 99 Percent Confidence Level for Individual Futures Markets for the Period 1988 - 2000

Commodity	100 Previous Trading Days	250 Previous Trading Days	1000 Previous Trading Days
British Pound	1.92[a]	1.46	0.99*
Japanese Yen	1.92	1.36	1.12*
Canadian Dollar	2.23	1.52	1.12*
Swiss Franc	2.23	1.70	1.02*
S&P 500	2.29	1.30*	1.27*
Live Cattle	2.20	1.18*	0.83*
Feeder Cattle	1.75	1.40	0.73*
Live/lean Hogs	2.33	1.63	1.31*
Pork Bellies	3.09	2.10	2.20
Cotton	2.27	1.36	1.14*
Sugar #11	2.11	1.49	1.04*
Coffee	1.98	1.56	1.30*
Cocoa	2.04	1.56	1.49
Orange Juice	2.17	1.40	1.53
Lumber	3.02	2.17	1.59
Soybean Oil	2.43	1.36	1.10*
Gold	2.04	1.50	1.05*
Silver	2.04	1.20*	0.83*
Heating Oil	2.03	1.58	1.48
Crude Oil	2.32	1.61	1.42
US T-bills	1.74	1.21*	0.90*
US 10-year Notes	2.05	1.27*	0.75*
Corn	2.58	1.61	1.07*
Wheat	1.84	1.42	1.20*
Oats	2.42	1.45	1.13*
Soybeans	2.29	1.42	1.20*
Soybean Meal	2.03	1.29*	1.23*
Canola	2.78	1.49	1.13*

[a]Note: Given the 99% confidence level, actual losses would be expected to exceed the VaR computation one percent of the time; for example, of 3100 trading days, 31 days would be expected to have losses exceeding the VaR, or one percent.
*Indicates percentage of exception days that fall *within* acceptable range, from Step Five.

Table 3.3 Historical Simulation VaR: Percentage of Exception Days Where Actual Losses Exceed VaR at the 99 Percent Confidence Level for Portfolios of Futures Markets for the Period 1988 - 2000

Portfolio	100-day	250-day	1000-day
Currencies Group	1.92[a]	1.46	0.74*
Livestock Group	2.01	1.34*	1.24*
Soft Group	2.14	1.49	1.23*
Gold/Silver Group	1.91	1.15*	0.77*
Energy	1.32*	1.06*	1.03*
Fixed Income Group	1.86	1.21*	0.87*
Grains Group	2.33	1.45	1.23*

[a]Note: Given the 99 percent confidence level, actual losses would be expected to exceed the VaR computation one percent of the time; for example, of 3100 trading days, 31 days would be expected to have losses exceeding the VaR, or one percent.
*Indicates percentage of exception days that fall *within* acceptable range, from Step Five.

Table 3.4. Parametric VaR: Percentage of Exception Days Where Actual Losses Exceed the VaR at the 99 Percent Confidence Level for Individual Futures Markets for the Period 1988 - 2000

	50 day equally weighted		250 day equally weighted		0.94 EWMA		0.97 EWMA	
Commodity	Normal Dist.	t-distribution 10 dof	Normal Dist.	t-distribution 10 dof	Normal Dist.	t-distribution 10 dof	Normal Dist.	t-distribution 10 dof
British Pound	2.14[a]	1.24*	2.17	1.18*	2.48	1.33*	1.98	1.15*
Japanese Yen	1.55	0.77*	1.27*	0.81*	1.43	0.77*	1.30*	0.71*
Canadian dollar	2.17	1.39	2.26	1.24*	1.98	1.15*	1.89	1.24*
Swiss Franc	1.64	0.90*	1.52	0.71*	1.64	0.77*	1.49	0.74*
Live cattle	1.98	0.86*	1.47	0.54	1.67	0.74*	1.63	0.64
Feeder cattle	2.17	1.02*	1.69	0.92*	2.10	0.86*	1.85	0.76*
Live/lean hogs	2.01	0.61	1.82	0.67*	2.01	0.67*	1.82	0.57
Pork bellies	0.86*	0.19	0.99*	0.06	0.86*	0.22	0.64	0.06
Cotton	1.36	0.49	1.20*	0.26	1.36	0.62	1.30*	0.36
Sugar #11	1.66	1.01*	2.01	1.04*	1.79	0.91*	1.75	0.84*
Coffee	2.21	1.30*	1.91	1.36	1.85	0.91*	1.75	0.84*
Cocoa	1.56	0.52	1.59	0.78*	1.49	0.45	1.33*	0.55
Orange Juice	1.75	1.01*	1.62	1.07*	1.75	1.01*	1.66	0.91*
Lumber	1.07*	0.39	0.71*	0.36	1.01*	0.32	0.84*	0.29
Soybean Oil	1.56	0.75*	1.46	0.88*	1.53	0.84*	1.46	0.75*
Gold	1.85	1.28*	2.07	1.34*	2.27	1.15*	1.88	1.18*
Silver	1.82	0.99*	1.63	1.02*	1.72	1.08*	1.60	0.96*
Heating Oil	1.48	0.87*	1.51	0.84*	1.42	0.71*	1.16*	0.68*
Crude Oil	1.84	0.84*	1.80	1.06*	1.84	0.71*	1.58	0.61
S&P 500	1.92	1.24*	1.58	1.05*	2.04	1.18*	1.73	1.05*
US T-bills	1.61	0.93*	1.27*	0.59	1.83	0.84*	1.49	0.78*
US 10-year Notes	1.80	0.87*	1.52	0.75*	1.77	0.93*	1.55	0.75*
Corn	1.74	1.13*	2.10	1.13*	1.68	1.03*	1.65	0.94*
Wheat	1.45	0.65	1.32*	0.84*	1.32*	0.65	1.20*	0.52
Oats	2.00	1.20*	1.87	0.90*	2.00	0.87*	1.71	0.90*
Soybeans	1.97	0.81*	1.84	1.20*	1.97	0.81*	1.81	0.81*
Soybean meal	1.58	0.74*	1.29*	0.90*	1.52	0.68*	1.36	0.65
Canola	1.71	0.74*	1.74	0.68*	1.61	0.58	1.58	0.55

[a]Note: Given the 99 percent confidence level, actual losses would be expected to exceed the VaR computation one percent of the time; for example, of 3100 trading days, 31 days would be expected to have losses exceeding the VaR, or one percent.
*indicates percentage of exception days that fall *within* acceptable expected range, from Step Five.

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Figure 3.1. Historgram Example of Gold Futures Market: 1000 Daily Percentage Returns Prior to February 17, 1994

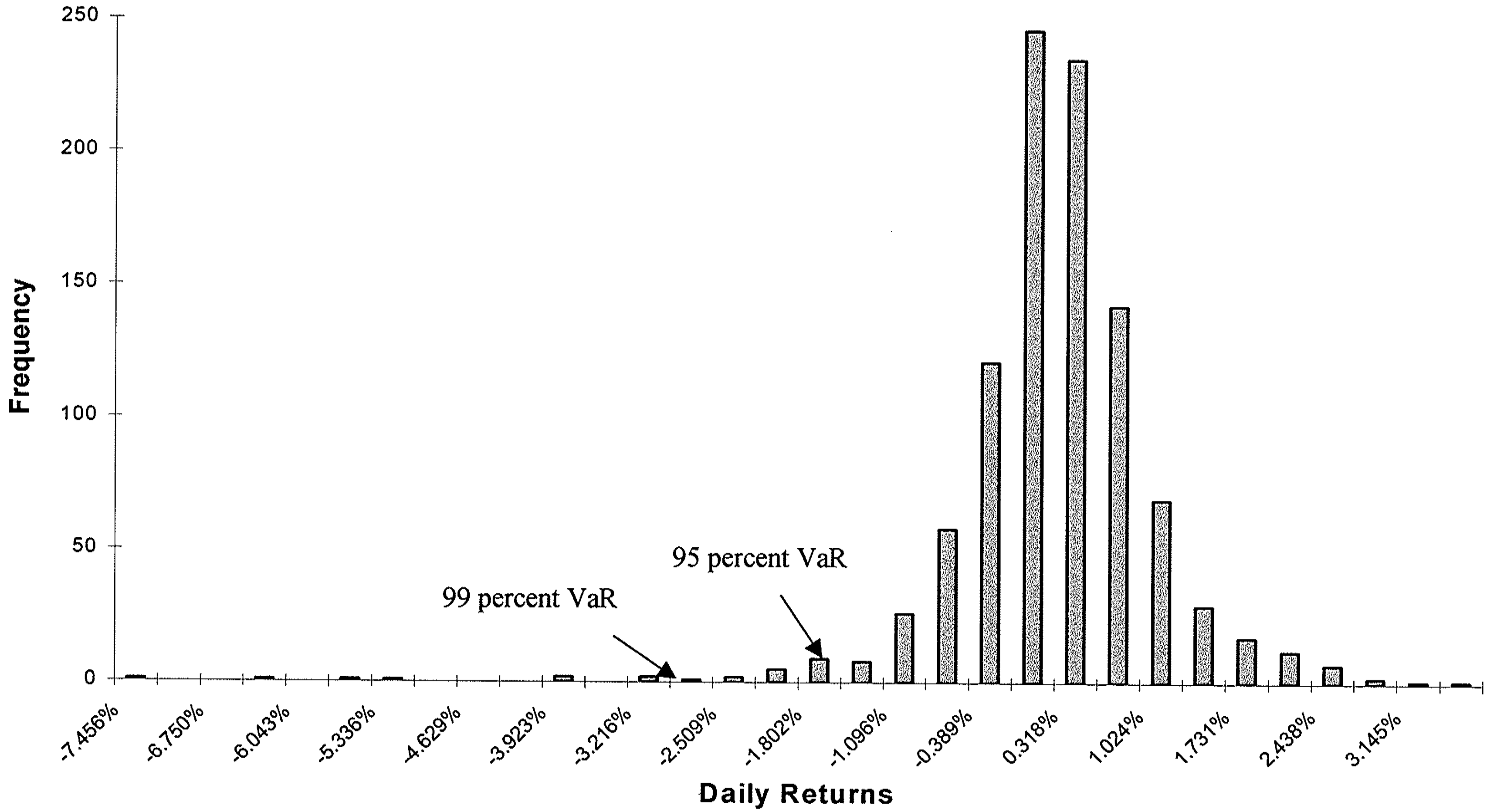
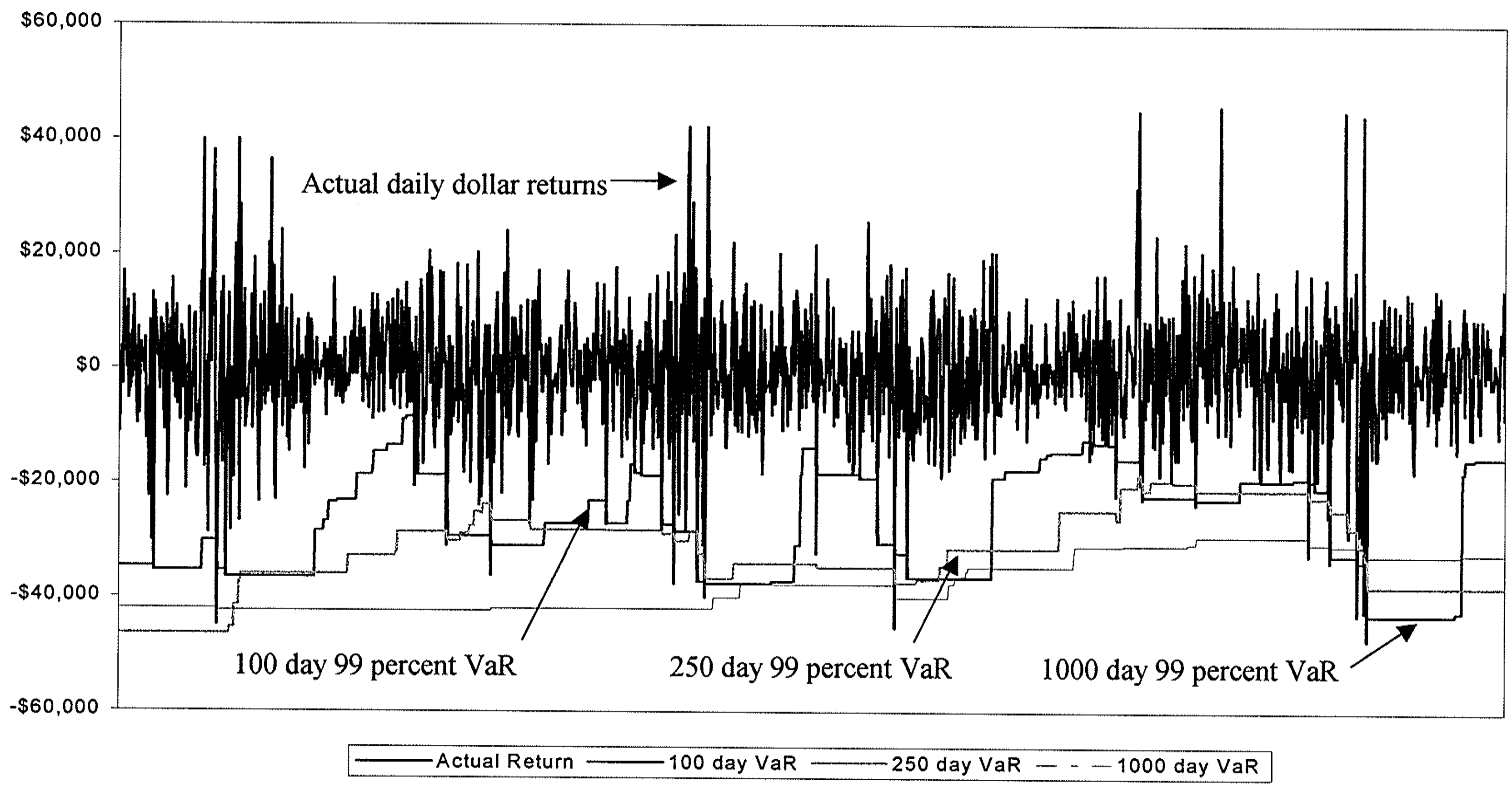


Figure 3.2. Corn Futures Daily Dollar Returns for a $1,000,000 Portfolio and 99 percent confidence VaR Computation with Parameters of 100, 250 and 1000 days for 1990 - 1995



Note: An exception day is counted whenever the actual return line crosses the VaR line. The actual return line should not cross the VaR line more than one percent of the time.

CHAPTER FOUR

VALUE AT RISK METHODS FOR INSTITUTIONAL INVESTMENT PORTFOLIOS

Introduction

The purpose of this study is to examine the performance of Value at Risk (VaR) for measuring risk in investment portfolios. One of the most widely used measures of market risk is VaR (Smithson 1998; Hull 1999). VaR attempts to measure the maximum downside loss under normal market conditions given a predetermined confidence level, and is typically used to manage risk for portfolios with short holding periods such as one day or one week. However, VaR is beginning to be used by investment firms with longer holding periods, such as one month or longer.

This study uses the parametric method for computing VaR. The parametric method is still the most common method used for computing VaR (Smithson 1998, Best 1998), and is also used in the popular JP Morgan program *RiskMetrics*, as well as being an acceptable method for computing VaR for the Basle Accord (Butler 1999). In this study, monthly VaR is computed for a number of portfolios consisting of stocks and bonds to determine the accuracy of computing VaR at the 99 percent confidence level using monthly returns. The monthly VaR is then 'scaled' to compute three month, six month, nine month and twelve month VaR. Scaling is done by the square-root-of-time rule, or the $\sqrt{t}$ rule, with t being the holding period for the portfolio. The study starts with a brief overview of VaR. This is followed by a discussion on VaR for longer holding periods and problems with the implied assumptions made when scaling by $\sqrt{t}$. A description of the methods and data for testing the VaR models succeeds this, followed by the results of the study and a summary.

The information in this study is unique because it examines VaR from the perspective of a longer monthly investment horizon, whereas most previous studies examine daily VaR. This study also examines the square-root-of-time rule using monthly data. This information should be useful to managers of mutual funds, pension funds, endowments and institutions with long term investment portfolios who are seeking to better understand the performance of Value at Risk measures.

Theory

VaR measures the maximum downside market risk at a predetermined confidence level. VaR can be defined as the "…maximum amount of money that may be lost on a portfolio over a given period of time, with a given level of confidence" (Best, 1998), and makes the statement "…we are X percent certain that we will not lose V dollars in the next N days" where V is the Value at Risk, N is the time horizon and X is the confidence level (Hull and White 1998). A one-day VaR, e.g. the amount of losses expected over one day, is usually calculated at most financial institutions where portfolios are often traded daily. However, a longer holding period would be used for institutions that hold financial assets for longer periods of time for investment.

The profit or loss for a portfolio that is left unchanged over a period of *h* days is

$$\Delta_h P_t = P_{t+h} - P_t \tag{4.1}$$

where P is the price of the portfolio on day *t* (Alexander 2001). A 100(1-α)% h-period Value at Risk measure is the nominal amount C such that

$$\text{Prob}(\Delta P < -C) = \alpha \tag{4.2}$$

where ΔP denotes the change in portfolio value over a prespecified holding period h, and α is a sufficiently small probability (Alexander and Leigh, 1997). For example, a firm reporting a 99 percent one month VaR of $10 million on a $200 million portfolio would expect to lose $10 million on the portfolio only 1 percent of the time, or one month out of one hundred, due to movement in asset prices.

Parametric VaR

The parametric method for computing VaR first requires computing standard deviation, or volatility, for the portfolio of assets. Once this portfolio standard deviation is computed, this figure is multiplied by the appropriate Z-value to get the VaR estimate for the desired confidence level (i.e. portfolio standard deviation is multiplied by 2.33 to get a 99 percent confidence level VaR).

Often a main assumption made in the parametric method of computing VaR is that asset returns are normally distributed (Best 1998, Alexander 2001). If equation (1) is the *h*-day portfolio return, it is assumed that

$$\Delta_h P_t \sim N(\mu_t, \sigma^2_t) \qquad (4.3)$$

The 100α% h-period VaR is that number $VaR_{\alpha,h}$ such that $Prob([\Delta_h P_t - \mu_t]/\sigma_t) = \alpha$. Since $[\Delta_h P_t - \mu_t]/\sigma_t \sim N(0,1)$ and denoting $[\Delta_h P_t - \mu_t]/\sigma_t$ by the standard normal variate Z_t, $Prob(Z_t<[-VaR_{\alpha,h}- \mu_t]/\sigma_t = \alpha$. But for a standard normal variate Z_t, $Prob(Z_t<Z_\alpha) = \alpha$ where Z_α is the 100αth percentile of the standard normal density. Therefore $[-VaR_{\alpha,h}- \mu_t]/\sigma_t = -Z_\alpha$. Written another way, the formula for parametric VaR is

$$VaR_{\alpha,h} = Z_\alpha \sigma_t - \mu_t. \qquad (4.4)$$

Often the distribution of financial price series show more kurtosis, or 'fat tails', than is assumed by the normal distribution. Use of the t-distribution allows the incorporation of fatter tails in the distribution while maintaining the ease of use of standard deviation.

Value at Risk for longer holding periods

Most studies on VaR have focused on shorter holding periods. Studies by Beder (1995), Hendricks (1996) and Best (1998) have all focused on one-day VaR. This is a common holding period for organizations that frequently turn over their portfolios, such as trading firms. But VaR can also be applied to situations where one-day changes in value are of a lesser significance than the value of the portfolio over a longer period of time, such as a month, or longer.

When a longer holding period VaR is required, often the standard practice is to scale up the shorter period VaR to arrive at the longer holding period VaR. Scaling is where the daily VaR is multiplied by the square root of time, or $\sqrt{t}$, to arrive at a VaR for the desired holding period. In other words,

$$VaR_{(t)} = VaR_{(1)} \times \sqrt{t} \qquad (4.5)$$

where $VaR_{(1)}$ and $VaR_{(t)}$ denote one-period VaR and the t-period VaR respectively (Odening and Hinrichs 2002). For example, a one month VaR of $1,000 is multiplied by $\sqrt{9}$ to arrive at a 9 month VaR of $3,000. The $\sqrt{t}$ scaling factor is used because parametric VaR is a multiple of the portfolio volatility, or standard deviation, and volatility is commonly multiplied by $\sqrt{t}$ to compute multiple day volatility from one-day volatility.

The use of scaling by $\sqrt{t}$ is not only common practice in industry, but is also widely accepted by regulators, including the Basle Committee from the Bank of International Settlements (Diebold, Hickman, Inoue and Schuermann 1997). Iacono and Skeie (1996) describe use of the $\sqrt{t}$ rule as a reasonable approximation under certain conditions. The $\sqrt{t}$ translation is exact only under the following circumstances:

- The underlying factors are joint normally distributed. Any linear combination of normally distributed random variables is itself normally distributed, with a fixed relationship between standard deviations and tail probabilities. As long as the distribution of asset returns per unit of time is independent and identically distributed, the standard deviation is proportional to $\sqrt{t}$.
- There is constant volatility. In order for asset returns per unit of time to be independently and identically distributed, the variance must be constant over time.
- No drift of underlying factors. As the mean of a normal distribution changes, all its quantiles must change by the same amount.
- No optionality (non-linearity) in the portfolio. Specifically, the portfolio must have zero gamma, zero time decay and a delta that is not sensitive to time.

If any of the above criteria are not met, the $\sqrt{t}$ translation is only an approximation of the true VaR (Iacono and Skeie 1996). Iacono and Skeie suggest that while in reality these conditions rarely hold, for many portfolios they will "nearly" hold thereby allowing $\sqrt{t}$ to be used as a reasonable shortcut.

It is important to realize that long-term and short-term risk measurement are two different problems. Many of the simplifying assumptions used in short-term forecasting are not applicable to longer horizons. Three of these common short-term assumptions are:

- The mean of returns can be safely assumed to be zero. This assumption is basically ignoring any upward and downward trend in the asset price, as well as growth due to dividends and the time value of money.
- Successive 1-day returns have the same volatility and are independent of one another. This implies that the variance of returns is a linear function of time.
- Returns follow a random walk. Broadly speaking, this means that returns wander away from any starting point with no particular direction and have a normal distribution at each point in time.

Generally, these assumptions do not have a great impact on risk calculations for the short horizon. Any upward or downward drift in prices is not likely to affect the mean returns in a short period of time. Similarly, scaling up daily volatility to obtain 10-day volatility does not introduce a large bias in the calculations. Such approximations, however, can create problems for long-term horizon forecasting. The mean of the returns will not be zero in the long term, and scaling up daily volatilities to a long-term horizon would introduce a considerable error in the estimate. Moreover, the random walk model does not always provide the best explanation for the dynamics of financial returns over long horizons (Kim, Maltz and Mina 1999).

Diebold, et al (1997) warn that scaling may produce volatilities that are correct on average, but magnify the volatility fluctuations, whereas in reality they should in fact be damped (Diebold, Hickman, Inoue and Schuermann 1997). In other words, use of scaling means that volatility continues to increase in magnitude with time. But this may often not be the case for many financial price series. Rather than the volatility of financial asset prices increasing indefinitely with time, volatility tends to show a certain amount of mean-reversion (Simmons 2000; Kim, Maltz and Mina 1999). For any assets that show mean reversion, scaling by $\sqrt{t}$ far into the future will overestimate volatility and therefore VaR. However, scaling for shorter time periods is more likely to be reasonably accurate.

Other problems with computing long-term VaR include the fact that short-term estimates of volatility may not be valid over long time horizons, such as during times of structural shifts in the market or changes in fiscal or monetary policy. As well, there are complications when one considers the opportunities provided by using derivatives to hedge the portfolio, and a VaR estimate for a given time-interval assumes that one will not trade out of the position if losses get too excessive, which can cause an overestimation of VaR (Simmons 2000). Therefore, one purpose of this study is to examine whether the $\sqrt{t}$ rule is a useful approximation to compute future VaR from the current monthly VaR for investment returns.

Data and Procedure

The data for this study is the total monthly returns for large company US stocks, US long-term government bonds, US Treasury Bills and US long-term corporate bonds. Total returns include both capital gains and income (dividend and coupon payments). The period of the data is from January, 1936 to December, 2000. Past data before 1936 is also used in order to allow results to be computed beginning on January, 1936. Data is obtained from Ibbotson Associates.

Four portfolios of varying asset mixes are used for the analysis in order to replicate asset allocations that might be found in various institutional and other investment portfolios with long-term investment horizons. The first portfolio consists of 60 percent large company stocks and 40 percent long-term government bonds. The second portfolio is comprised of 60 percent stocks, 25 percent long-term government bonds, 10 percent corporate bonds and 5 percent T-bills. The third portfolio consists entirely of large company stocks. The fourth portfolio contains 60 percent long-term government bonds, 30 percent long-term corporate bonds and 10 percent T-bills.

Step One - Compute parametric VaR

This study uses two different measures for computing standard deviation with two alternative parameters used for each measure. The first measure is the equally weighted standard deviation using parameters of 12 months and 120 months. The second measure for computing standard deviation is the exponentially weighted moving average (EWMA), with parameters of 0.94 and 0.97.

The standard deviation computations are multiplied by the Z-value and t-value from the normal distribution and the t-distribution with 10 degrees of freedom and 6 degrees of freedom to arrive at the appropriate VaR. A more detailed description of the parametric method for computing VaR is provided in Chapter Two. For comparing the Value at Risk computations, a $1,000,000 long position is assumed for each of the portfolios. Total monthly returns for each portfolio are computed to get a monthly profit or loss. Monthly standard deviation is computed using the two measures with two parameters each. The $1,000,000 portfolio is multiplied by the monthly standard deviation computations to get a monthly standard deviation in dollar amount. This monthly standard deviation is then multiplied by 2.33 to get the 99 percent confidence level for the normal distribution, and 2.764 to get the estimate for the t-distribution with 10 degrees of freedom, and 3.143 for the t-distribution with 6 degrees of freedom.

Step Two - Compare VaR with actual risk

The computed monthly VaR is compared with the actual monthly returns on the $1,000,000 portfolio. If the actual loss exceeds the VaR, that month is counted as an exception month. It is expected at the 99 percent confidence level that actual losses exceed the VaR one percent of the time, or one month out of 100. Since there are 779 monthly observations for each portfolio, it is expected that there will be about eight exception months when actual losses exceed the monthly VaR, or one percent.

For example, total monthly return for large company stocks was -6.56 percent and for long term US government bond returns was -8.41 percent for the month of October 1979. The total returns for the portfolio of 60 percent stocks and 40 percent long term government bonds was -7.30 percent (computed by 0.6*-6.56 + 0.4*-8.41). This results in a loss of $73,000 on the $1,000,000 portfolio. The monthly standard deviation calculation for October 1979 using the 0.97 EWMA is 2.942 percent. The 99 percent confidence level VaR for the $1,000,000 portfolio of 60 percent stocks and 40 percent long-term government bonds using the normal distribution is therefore $68,538.95 (computed by $1,000,000*2.9146*2.33). Therefore, October 1979 is counted as an exception month for the normal distribution since the actual losses exceeded the VaR. When using a t-distribution with 10 degrees of freedom, the monthly standard deviation is multiplied by 2.764 to get a VaR of $81,305.43 (computed by $1,000,000*2.9146*2.764). Since the actual losses are less than the VaR, October 1979 is not counted as an exception month when computing VaR using the t-distribution with 10 degrees of freedom.

Step Three - Scale up monthly VaR estimates for 3, 6, 9 and 12 months

The monthly VaRs are multiplied by the square root of the holding period to arrive at a VaR for future periods. For example, the six month VaR is computed by multiplying the monthly VaR by $\sqrt{6}$. Scaling is used to compute VaR for three month, six month, nine month and one year periods ahead. The VaRs are then compared for accuracy with the actual returns for the previous three months, six months, nine months and one year.

Results

Monthly VaR

Results in Table 4.1 show that monthly VaR can be computed relatively accurately using the parametric method. Each of the portfolios using each of the four parametric measures have a percentage of exception months that is near the expected one percent. The normal distribution is the more accurate distribution of the three distributions used in the study for the 120 month equally weighted, 0.94 EWMA and 0.97 EWMA parameters. The 0.94 EWMA has the narrowest range of percentage of exception days between the four portfolios, ranging from 1.03 percent exception days to 1.28 percent. The t-distribution is more accurate for the 12 month equally weighted moving average parameter, as the normal distribution has a percentage of exception days in excess of 2 percent for each of the portfolios, twice the expected amount. Therefore, results show that the parametric method accurately computes monthly VaR at the 99 percent confidence level for portfolios of stocks and bonds.

Figure 4.1 shows the results for the bond portfolio using the 0.94 EWMA VaR with the normal distribution. The upper line shows the actual monthly returns on the bond portfolio, arrived at by multiplying the percentage monthly return by the assumed $1,000,000 position in the portfolio. Monthly VaR is computed by multiplying the portfolio amount by the monthly standard deviation, and then by 2.33 to arrive at the 99 percent confidence level using the normal distribution. Standard deviation is computed using the 0.94 EWMA. An exception day is counted whenever the actual return crosses the VaR line. The actual return line should not cross the VaR line more than one percent of the time, or one month out of 100 months, if the VaR model is accurate.

Scaling monthly VaR to compute 3, 6, 9 and 12 month VaR

Results show that scaling monthly VaR by $\sqrt{t}$ may not be an accurate method for computing VaR for longer holding periods. Table 4.2 shows the results for scaling monthly VaR by $\sqrt{3}$ to compute 3 month VaR. All of the parametric measures overestimate VaR for portfolios one, two and three for both the normal distribution and the t-distribution. This is shown by the fact that there are less than one percent of exception days for each of these three portfolios using any of the four measures, with the single exception of the stock portfolio using the 120 month equally weighted moving average measure. The rest of the percentage of exception days are 0.77 percent or less. However, the bond portfolio results are different. All four of the parametric VaR computations for the bond portfolio using the normal distribution underestimate VaR. In other words, there is a higher percentage of exception days than the one percent. The percentage of exception days ranges from 1.41 percent for the 0.94 EWMA parameter to 2.05 percent for the 120 month equally weighted moving average. Results for the bond portfolio using the t-distribution with 10 degrees of freedom are mixed, with a range of only 0.26 percent exception days for the 0.94 EWMA parameter to the near-expected 1.16 percent exceptions for the 120 month equally weighted moving average.

Scaling monthly VaR by $\sqrt{6}$ to compute 6 month VaR overestimates VaR at the 99 percent confidence level. Table 4.3 shows that each of the parameters for each of the portfolios have a lower percentage of exception days than the expected one percent. The highest percentage of exception days is 0.64 percent, less than two-thirds the expected one percent. The one exception is the bond portfolio using the normal distribution and the 120 month equally weighted parameter which has 1.54 exception days. The results indicate that computing 6 month VaR by $\sqrt{6}$ overestimates VaR at the 99 percent confidence level.

Figure 4.2 and Figure 4.3 show the results for the one month VaR and the 6 month VaR, respectively, for the portfolio consisting of 60 percent stocks and 40 percent long-term government bonds. The results in Figure 4.2 are computed the same was as in the results for Figure 4.1. The results in Figure 4.3 are computed the same as the results in Figure 1, except that the 6 month return is computed and the VaR is computed by multiplying the one month VaR computation by $\sqrt{6}$. In Figure 4.2 the actual return line crosses the VaR line eight times, which is 1.03 percent of the 779 observations, precisely our expected amount. In Figure 4.3, the actual 6 month return line crosses the 6 month VaR line only twice, or 0.26 percent of the time, much lower than the expected one percent. It can be seen by looking at Figure 4.3 that scaling the monthly VaR by $\sqrt{6}$ overestimates the 6 month VaR.

Scaling monthly VaR by $\sqrt{9}$ to compute 9 month VaR and by $\sqrt{12}$ to compute 12 month VaR both overestimate VaR at the 99 percent confidence level. Table 4.4 shows that the nine month VaR computations give between 0 percent and 0.51 percent exception months, which is half or less of the expected one percent. Table 4.5 shows that the twelve month VaR gives between 0 percent and 0.26 percent exception months, which is one-quarter or less of the expected amount. The results show that scaling overestimates VaR by a considerable margin for monthly data, and that the estimate errs by a greater amount the further into the future that is scaled.

Summary

Value at Risk (VaR), a widely used measure of market risk, is defined as the maximum amount of funds that a firm is likely to lose under normal market conditions over a specified period of time give a predetermined confidence level. VaR has traditionally been used for computing daily risk for firms such as banks and trading firms, but other institutions with investment portfolios with longer holding periods have started to use VaR to measure longer term risk.

This study computes monthly VaR using total monthly returns for four different portfolios consisting of stocks and bonds. The monthly VaRs are then scaled up by $\sqrt{t}$ to determine if a firm can accurately compute longer holding period VaR from the monthly VaR estimate. The results of the study show that monthly VaR can often be measured reasonably accurately using the parametric method for computing VaR under the assumption of normal distribution of returns, with the exception of the 12 month equally weighted VaR, which underestimated risk with the normal distribution. VaR was underestimate at the 99 percent confidence level using the parametric method for the bond portfolio, although less so for the exponentially weighted moving average VaR.

The results for scaling monthly VaR by $\sqrt{t}$ to arrive at longer holding period VaRs show that scaling overestimated VaR for most portfolios for 3 months, and for all of the portfolios for 6, 9 and 12 months. The level of overestimation also increases as the holding period increases. The results are consistent with expectations, since many of the simplifying assumptions regarding the behavior of asset returns when scaling by $\sqrt{t}$ may not be applicable to longer time horizons.

This study indicates that parametric methods for computing VaR for total monthly returns are fairly accurate and therefore may be a useful tool for portfolio managers with an investment horizon of approximately one month. However, computing VaR for longer holding periods by scaling monthly VaR using the $\sqrt{t}$ rule overestimates VaR at the 99 percent confidence level, and therefore risk managers should be cautious when computing VaR for holding periods of three months or longer using a scaling procedure.

Table 4.1 One Month Parametric VaR: Percentage of Extreme Months Where Actual Losses Exceeded the VaR Estimate at the 99 Percent Confidence Level for Investment Portfolios for the Period 1936 - 2000

Portfolio	12 month equally weighted STD			120 month equally weighted STD			0.94 EWMA STD			0.97 EWMA STD		
	Norm	10 dof	6 dof	Norm	10 dof	6 dof	Norm	10 dof	6 dof	Norm	10 dof	6 dof
60% stocks, 40% gov. bonds	2.05	0.77	0.64	0.90	0.39	0.00	1.03	0.00	0.00	0.90	0.00	0.00
60% stocks, 25% gov. bonds, 10% corporate bonds, 5% T-bills	2.05	0.90	0.64	1.16	0.51	0.13	1.03	0.00	0.00	1.03	0.00	0.00
100% stocks	2.18	1.41	0.64	0.90	0.51	0.39	1.16	0.00	0.00	1.16	0.00	0.00
60% gov. bonds, 30% corp. bond, 10% T-bills	2.05	1.03	0.77	1.80	0.77	0.39	1.28	0.00	0.00	1.41	0.00	0.00

Note: Given the 99 percent confidence level, actual losses would be expected to exceed the VaR computation one percent of the time; for example, of 1000 trading months, 10 months would be expected to have losses exceeding the VaR, or one percent.
STD is standard deviation
Norm is normal distribution
dof is degrees of freedom
EWMA is Exponentially Weighted Moving Average

Table 4.2 3 Month Parametric and VaR: Percentage of Extreme Months Where Actual Losses Exceeded the Estimated 3 Month 99 Percent Confidence VaR When Scaling the One Month VaR by $\sqrt{3}$ for Investment Portfolios for the Period 1936 – 2000

Portfolio	12 month equally weighted STD			120 month equally weighted STD			0.94 EWMA STD			0.97 EWMA STD		
	Norm	10 dof	6 dof	Norm	10 dof	6 dof	Norm	10 dof	6 dof	Norm	10 dof	6 dof
60% stocks, 40% gov. bonds	0.64	0.26	0.00	0.77	0.51	0.26	0.64	0.26	0.13	0.51	0.26	0.26
60% stocks, 25% gov. bonds, 10% corporate bonds, 5% T-bills	0.77	0.26	0.00	0.77	0.51	0.26	0.64	0.26	0.13	0.64	0.26	0.26
100% stocks	0.77	0.26	0.13	1.03	0.51	0.39	0.77	0.26	0.13	0.90	0.39	0.26
60% gov. bonds, 30% corp. bond, 10% T-bills	1.67	0.77	0.39	2.05	1.16	0.39	1.41	0.26	0.26	1.93	0.39	0.00

Note: Given the 99 percent confidence level, actual losses would be expected to exceed the VaR computation one percent of the time; for example, of 1000 trading months, 10 months would be expected to have losses exceeding the VaR, or one percent.
STD is standard deviation
Norm is normal distribution
dof is degrees of freedom
EWMA is Exponentially Weighted Moving Average

Table 4.3 6 Month Parametric VaR: Percentage of Extreme Months Where Actual Losses Exceeded the Estimated 6 Month 99 Percent Confidence VaR When Scaling the One Month VaR by $\sqrt{6}$ for Investment Portfolios for the Period 1936 – 2000

Portfolio	12 month equally weighted STD			120 month equally weighted STD			0.94 EWMA STD			0.97 EWMA STD		
	Norm	10 dof	6 dof	Norm	10 dof	6 dof	Norm	10 dof	6 dof	Norm	10 dof	6 dof
60% stocks, 40% gov. bonds	0.39	0.00	0.00	0.39	0.13	0.00	0.26	0.00	0.00	0.326	0.13	0.00
60% stocks, 25% gov. bonds, 10% corporate bonds, 5% T-bills	0.39	0.00	0.00	0.39	0.13	0.13	0.26	0.13	0.00	0.26	0.13	0.00
100% stocks	0.26	0.13	0.00	0.39	0.13	0.13	0.39	0.13	0.00	0.39	0.13	0.00
60% gov. bonds, 30% corp. bond, 10% T-bills	0.26	0.26	0.13	1.54	0.51	0.13	0.64	0.13	0.00	0.64	0.26	0.00

Note: Given the 99 percent confidence level, actual losses would be expected to exceed the VaR computation one percent of the time; for example, of 1000 trading months, 10 months would be expected to have losses exceeding the VaR, or one percent.
STD is standard deviation
Norm is normal distribution
dof is degrees of freedom
EWMA is Exponentially Weighted Moving Average

Table 4.4 9 Month Parametric VaR: Percentage of Extreme Months Where Actual Losses Exceeded the Estimated 9 Month 99 Percent Confidence VaR When Scaling the One Month VaR by $\sqrt{9}$ for Investment Portfolios for the Period 1936 – 2000

Portfolio	12 month equally weighted STD			120 month equally weighted STD			0.94 EWMA STD			0.97 EWMA STD		
	Norm	10 dof	6 dof	Norm	10 dof	6 dof	Norm	10 dof	6 dof	Norm	10 dof	6 dof
60% stocks, 40% gov. bonds	0.13	0.00	0.00	0.13	0.13	0.00	0.26	0.00	0.00	0.13	0.00	0.00
60% stocks, 25% gov. bonds, 10% corporate bonds, 5% T-bills	0.13	0.00	0.00	0.26	0.13	0.00	0.26	0.00	0.00	0.26	0.00	0.00
100% stocks	0.13	0.00	0.00	0.13	0.13	0.00	0.13	0.00	0.00	0.13	0.00	0.00
60% gov. bonds, 30% corp. bond, 10% T-bills	0.13	0.00	0.00	0.39	0.13	0.00	0.00	0.00	0.00	0.13	0.00	0.00

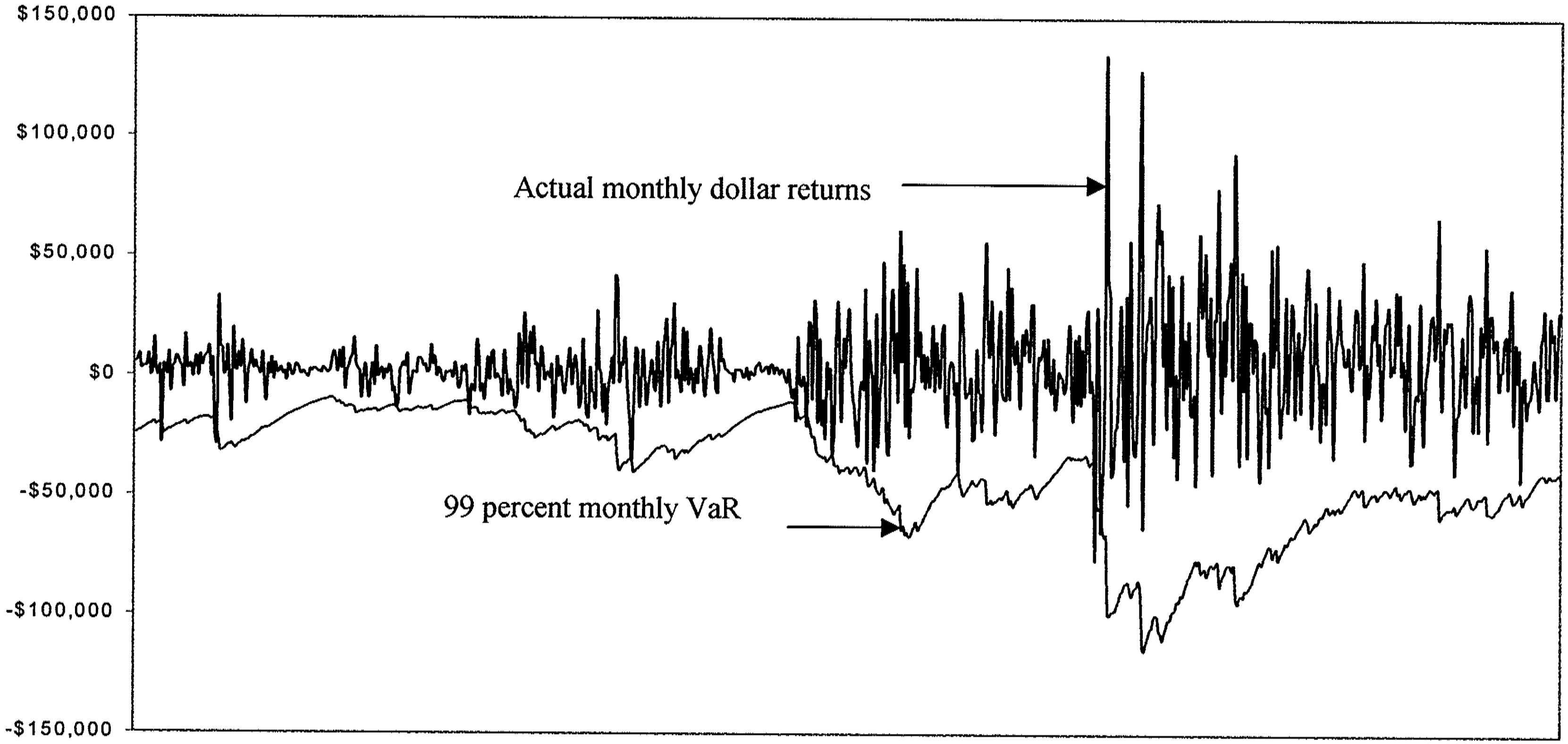
Note: Given the 99 percent confidence level, actual losses would be expected to exceed the VaR computation one percent of the time; for example, of 1000 trading months, 10 months would be expected to have losses exceeding the VaR, or one percent.
STD is standard deviation
Norm is normal distribution
dof is degrees of freedom
EWMA is Exponentially Weighted Moving Average

Table 4.5 12 Month Parametric VaR: Percentage of Extreme Months Where Actual Losses Exceeded the Estimated 12 Month 99 Percent Confidence VaR When Scaling the One Month VaR by $\sqrt{12}$ for Investment Portfolios for the Period 1936 – 2000

Portfolio	12 month equally weighted STD			120 month equally weighted STD			0.94 EWMA STD			0.97 EWMA STD		
	Norm	10 dof	6 dof	Norm	10 dof	6 dof	Norm	10 dof	6 dof	Norm	10 dof	6 dof
60% stocks, 40% gov. bonds	0.13	0.00	0.00	0.13	0.13	0.00	0.13	0.00	0.00	0.13	0.00	0.00
60% stocks, 25% gov. bonds, 10% corporate bonds, 5% T-bills	0.13	0.00	0.00	0.13	0.13	0.00	0.13	0.00	0.00	0.13	0.13	0.00
100% stocks	0.13	0.00	0.00	0.13	0.13	0.00	0.13	0.00	0.00	0.13	0.13	0.00
60% gov. bonds, 30% corp. bond, 10% T-bills	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

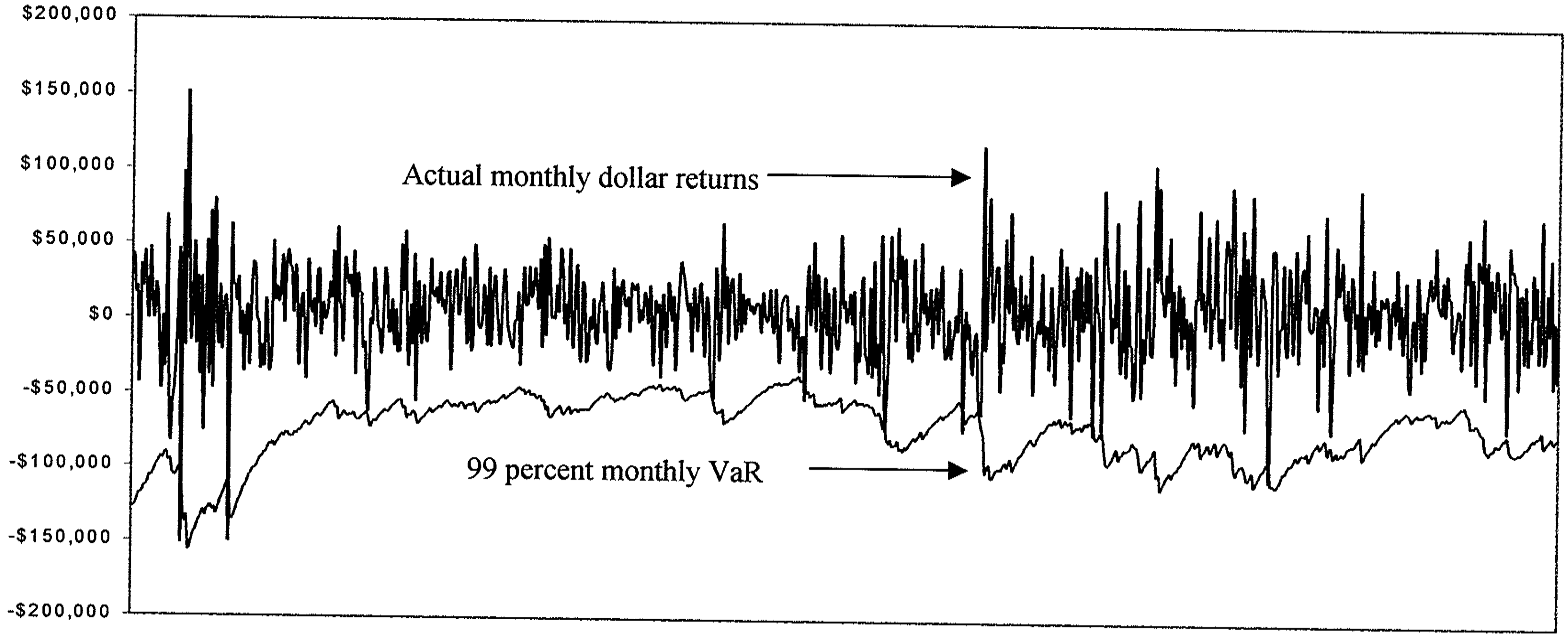
Note: Given the 99 percent confidence level, actual losses would be expected to exceed the VaR computation one percent of the time; for example, of 1000 trading months, 10 months would be expected to have losses exceeding the VaR, or one percent.
STD is standard deviation
Norm is normal distribution
dof is degrees of freedom
EWMA is Exponentially Weighted Moving Average

Figure 4.1. Bond Portfolio Total Monthly Returns and the 99 Percent Confidence VaR Using the 0.94 EWMA and the Normal Distribution for 1936 - 2000



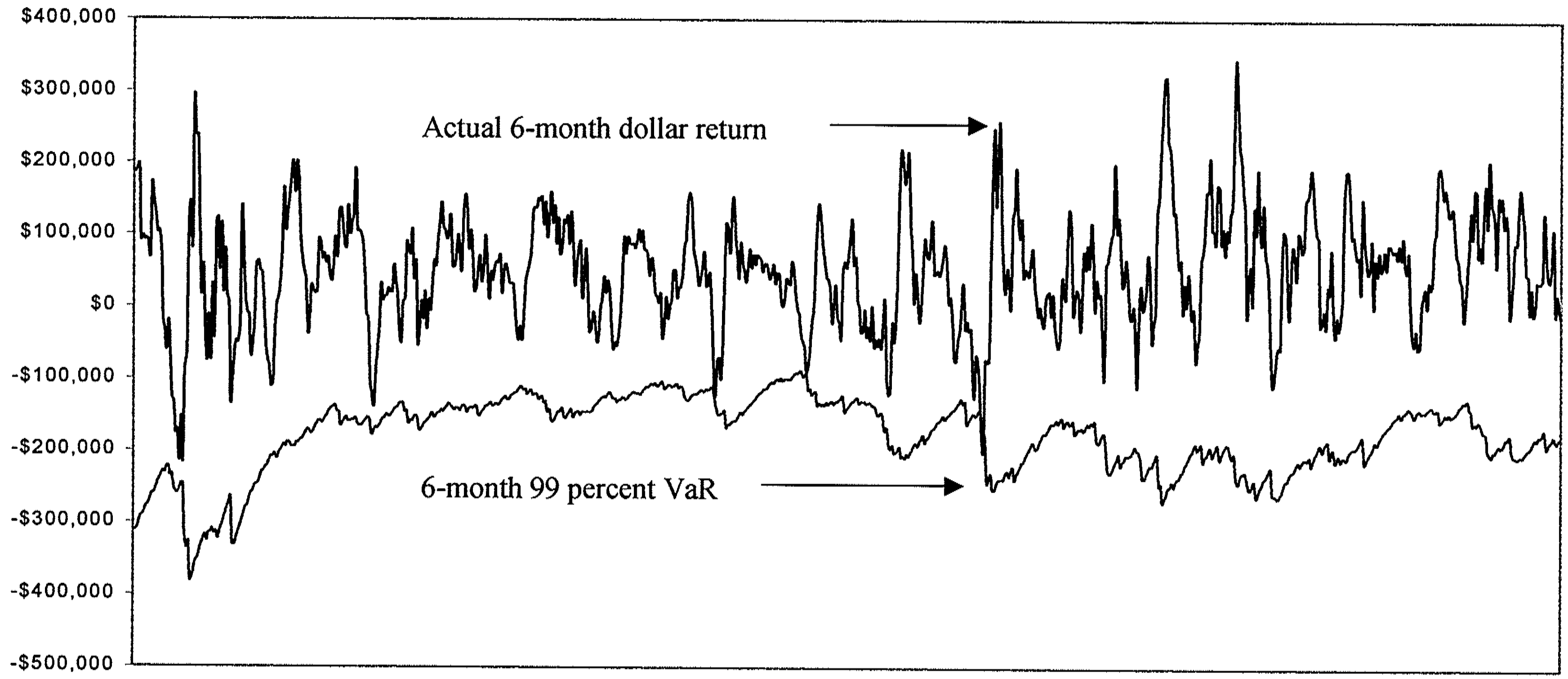
Note: An exception month is counted whenever the actual return line crosses the VaR line. The actual return line should not cross the VaR line more than one percent of the time, or one month out of 100.

Figure 4.2. 60 percent Stocks and 40 percent Long-term Government Bonds Portfolio Actual Monthly Total Returns and the 99 Percent Confidence VaR using the 0.94 EWMA and the Normal Distribution for 1936 - 2000



Note: An exception month is counted whenever the actual return line crosses the VaR line. The actual return line should not cross the VaR line more than one percent of the time, or one month out of 100.

Figure 4.3. 6 Month Ahead Scaling: 60 percent Stocks and 40 percent Long-term Government Bonds Portfolio, 99 Percent Confidence VaR Using the 0.94 EWMA and the Normal Distribution for 1936 - 2000



Note: An exception month is counted whenever the actual return line crosses the VaR line. The actual return line should not cross the VaR line more than one percent of the time, or one month out of 100.

CHAPTER 5

SUMMARY

Recent years have seen an increased emphasis on measuring and controlling risk, partially due to market events such as the financial crises in the late 1990's in areas such as Asia and Latin America, as well as the collapse of trading organizations such as Long-Term Capital Management. One of the most widely used measures of market risk is Value at Risk (VaR) (Smithson 1998; Hull 1999), which attempts to measure the maximum downside market risk at a given confidence level. VaR can be defined as the "...maximum amount of money that may be lost on a portfolio over a given period of time, with a given level of confidence" (Best 1999). The objective of this study is to examine the performance of various VaR methods for measuring risk in financial markets. VaR methods are tested by calculating how many times actual portfolio losses exceed the computed VaR over a period of past price history for daily data. Any day where actual losses exceed the computed VaR is counted as an exception day. If the percentage of exception days is within the significant range, then the VaR model is considered acceptable. Chapter Two and Chapter Three examine daily VaR in a number of futures markets. Chapter Two examines the parametric method for computing VaR comparing the traditional normal distribution with the t-distribution. Chapter Three examines the historical simulation method for computing VaR, and then compares this method with the parametric method. Chapter Four examines the parametric method for computing monthly VaR using total returns for stocks and bonds. The monthly VaR is then scaled by $\sqrt{t}$ to compute 3, 6, 9 and 12 month VaR.

Improved Parametric Methods for Computing Value at Risk

The objective of Chapter Two is to examine the performance of the parametric method for computing VaR in twenty-eight futures markets. The normal distribution is commonly used for computing parametric VaR. However, asset returns often show more kurtosis, or fat tails, than is assumed by the normal distribution. The t-distribution may be a useful alternative because it maintains the ease of use of standard deviation while allowing for a higher frequency of returns at the extreme ends of the distribution.

Results indicate daily VaR is more accurately computed at the 99 percent confidence level when using the t-distribution with 10 degrees of freedom. Computing VaR using the normal distribution tended to underestimate risk, or had more days when actual losses exceeded the VaR than would be expected at the 99 percent confidence level. Computing VaR using the t-distribution with 6 degrees of freedom and 4 degrees of freedom tended to overestimate VaR, or as there were too few days when actual losses exceeded the daily VaR.

These results may have implications for managers in financial and trading firms. Since the t-distribution with 10 degrees of freedom showed more accuracy at the 99 percent confidence level than the commonly used normal distribution, firms may wish to consider using the t-distribution given its accuracy, ease of use, and understanding.

A Test of the Historical Simulation Method for Computing Value at Risk

The objective of Chapter Three was to examine the historical simulation method for computing VaR, and then compare these results with the parametric method for computing VaR. A significant advantage of the historical simulation method, aside from its accuracy and ease of use, is the fact it is a nonparametric method and therefore no statistical assumptions need to be made regarding the distribution of asset returns. By using actual price data the historical simulation method accounts for fat tails and greater frequency of large price movements than is assumed when using the traditional normal distribution in the parametric method of computation.

Results show that the historical simulation method is relatively accurate for computing 99 percent confidence one-day VaR when using a longer parameter of 1000 days. The 1000 day parameter also computed VaR more accurately than the parametric method using the common assumption of normal distribution of returns, as the parametric method using the normal distribution tended to underestimate VaR. The historical simulation method using shorter parameters of 100 days and 250 days tend to underestimate VaR.

The implications of these results are that the historical simulation method is confirmed as being an acceptable method for computing daily VaR for linear portfolios, particularly when using a large sample parameter of 1000 days. Given the other advantages of this method, firms may wish to consider the historical simulation method as a valid method for VaR computations.

Value at Risk Methods For Institutional Investment Portfolios

The purpose of Chapter Four was to examine VaR from the perspective of an investor with a holding period of one month or longer, as opposed to the more common one-day holding period. Total returns for large company US stocks, US corporate and long-term government bonds, and US T-bills are used to comprise several portfolios that are representative of portfolios held by institutional investors such as mutual funds or pension funds. The parametric method is used to compute monthly VaR for each of the portfolios. The monthly VaR is then scaled by $\sqrt{t}$ to compute VaR for 3, 6, 9 and 12 months. While the assumptions surrounding the use of scaling by $\sqrt{t}$ do not exactly represent asset price behavior, they are a good approximation and therefore it is common to assume that these assumptions are close enough to allow the use of $\sqrt{t}$ to be reasonably accurate.

Results show that monthly VaR computations are relatively accurate at the 99 percent confidence level using the parametric method. However, scaling by $\sqrt{t}$ tended to overestimate risk, or there were fewer months when the actual losses exceeded the VaR computation. Also, the VaR overestimation increased the further into the future that the monthly VaR was scaled.

The implications of this study are that VaR measures risk relatively accurately at the 99 percent confidence level for monthly returns, and therefore investment managers may wish to consider using VaR to manage risk for longer holding periods. However, managers should be cautious when scaling monthly VaR by $\sqrt{t}$ to compute VaR for longer holding periods as this practice tended to overestimate VaR.

Conclusions

The results of the three studies show that VaR can be a useful tool for managing market risk in a variety of financial markets. However, the accuracy of a VaR method is dependant on the assumptions that are made in a given model and therefore managers should use caution when computing VaR. Given the assumptions surrounding the various

VaR methods and the fact that VaR makes no attempt to measure losses on days when the VaR is exceeded, VaR may be most effective when used in conjunction with other risk measurement techniques and methods.

REFERENCES

Alexander, C.O. and C.T. Leigh. "On the Covariance Matrices Used in Value at Risk Models." *J. Derivatives* 4(1997):50-62.

Alexander, C.O. *Market Models: A Guide to Financial Data Analysis*: New York: Wiley, 2001.

Beder, T.S. "VAR: Seductive but Dangerous." *Finan. Analysts J.* 51(1995):12-24.

Bernstein, P. *Capital Ideas:* New York: Free Press, 1992.

Best, Philip. *Implementing Value at Risk*: New York: John Wiley & Sons, 1998.

Boudoukh, J., M. Richardson and R.F. Whitelaw. "Investigation of a Class of Volatility Estimators." *J. Derivatives,* 4(1997):63-71.

Butler, Cormac. *Mastering Value at Risk:* London: Prentice Hall, 1999.

Christofferson, P., F. X. Diebold, and T. Schuermann. "Horizon Problems and Extreme Events in Financial Risk Management." *Fed. Res. Bank N.Y. Econ. Policy Rev.,* (October 1998):109-118. 1998.

Cootner, P. ed. *The Random Character of Stock Prices.* Cambridge, MA: MIT Press, 1964.

Culp, C., R. Mensink, and A. Neves. "Value at Risk For Asset Managers." *Derivatives Quarterly*, 5(1999).

Diebold, F. X., A. Hickman, A. Inoue, and T. Schuermann. "Converting 1-Day Volatility to h-Day Volatility: Scaling by $\sqrt{h}$ is Worse Than You Think." *Wharton Financial Institutions Center Working Paper* no. 97-34. 1997.

Duffie, D. and J. Pan. "An Overview of Value at Risk." *J. Derivatives,* 4(1997):7-49.

Hendricks, D. "Evaluation of Value-at-Risk Models Using Historical Data." *Fed. Res. Bank N.Y. Econ. Policy Rev.*, 2(April 1996):39-69.

Hull, J. and A. White. "Value at Risk When Daily Changes in Market Variables Are Not Normally Distributed." *J. Derivatives,* 5(1998):9-19.

Hull, John C. *Options, Futures, and other Derivative Securities, 4th Edition.* New Jersey: Prentice Hall, 1999.

Ibbotson Associates. *Stocks, Bonds, Bills & Inflation 2001 (1926-2000).* J.W. Wiley & Sons, c2001.

Jorion, P. "Risk²: Measuring the Risk in Value at Risk." *Finan. Analysts J.*, 52(1996):47-56.

------. *Value at Risk: A New Benchmark for Measuring Derivatives Risk.* Irwin, 1997.

------. "Risk Management Lessons from Long-Term Capital Management." *European Financial Management*, 6(2000):277-300.

Kendall, M. "The Analysis of Time Series, Part I: Prices." *J. of the Royal Statistical Society.* 96(1953):11-25. 1953.

Kim, J., A. Malz, and J. Mina. *LongRun Technical Document*, 1st ed., New York: Morgan Guaranty Trust Company, 1999.

Leuthold, M and M.R. Manfredo. "Value-at-Risk Analysis: A Review and the Potential for Agricultural Applications." *Review of Agricultural Economics,* 21(1999):99-111.

Linsmeier, T. and N. Pearson. "Value at Risk." *Finan. Analysts J.,* March/April (2000):47-67.

Odening, M. and J. Hinrichs. "Assessment of Market Risk in Hog Production using Value-at-Risk and Extreme Value Theory." Paper prepared for *the Annual Meeting of the American Agricultural Economics Association* 2002 in Long Beach, CA. 2002.

Siegel, J. *Stocks for the Long Run: The Definitive Gude to Finanical Market Returns and Long-Term Investment Strategies, 2nd Edition.* New York: McGraw-Hillm, 1998.

Simons, K. "The Use of Value at Risk by Institutional Investors." *New England Economic Review,* November/December 2000:21-30.

Smithson, C. *Managing Financial Risk, 3rd Edition.* New York: McGraw-Hill, 1999.

Zangari, P., J. Longerstaey. *RiskMetrics Technical Document.* 4th Ed. New York: Morgan Guaranty Trust Company, 1996.

APPENDIX A

100-Day Historical Simulation Example of Returns for Crude Oil For January 1, 1995

Appendix A – 100 Day Historical Simulation Example of Returns for Crude Oil For January 1, 1995.

Step 1 – Collect the daily percentage returns from the previous 100 days

Day	Date	Price	Percentage Return
-100	August 3, 1994	19.69	-0.203%
-99	August 4, 1994	19.71	0.102%
-98	August 5, 1994	18.97	-3.754%
…	…	…	…
-3	December 28, 1994	17.79	0.850%
-2	December 29, 1994	17.72	-0.393%
-1	December 30, 1994	17.76	0.226%

Step 2 – Order the daily percentage returns from lowest to highest

Rank	Percentage Return
100	-3.754%
99	-3.649%
98	-3.066%
97	-2.993%
…	…
2	2.694%
1	2.859%

Step 3 – Multiply Percentage Changes by Current $1,000,000 Portfolio Value

Rank	Dollar Return
100	-$37,540
99	-$36,490
98	-$30,660
97	-$29,930
…	…
2	$26,940
1	$28,590

Step 4 – Select the Desired VaR Confidence Level

If a 99% confidence level is desired, choose the value where loss only is exceeded one percent of the time. In this example the figure would be -$36,490. Therefore the 99% Value at Risk on January 1, 1995 for crude oil using the 100 day historical simulation method would be $36,490.