

TWO-FREQUENCY POWER TRANSMISSION  
for  
EXTRA-LONG DISTANCE LINES

by  
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To My Mother

who moved mountains

to let me go to school.

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## ABSTRACT

The two-frequency transmission of power cannot be regarded as an entirely new concept. Examples of the tying in of two power systems of different frequencies are not hard to find. Until comparatively recent times, North America had a variety of frequencies, of which 60 and 25 cycles were the most common; and for certain operations these frequencies were tied together. In Europe such ties exist between 50 cycles system and the 16-2/3 cycles of the various electric railway systems. These systems are nothing but a special case of the two-frequency transmission of power.

The two-frequency method is considered here as a possibility in extra-long distance power transmission, and as an alternative to the half-wave length mode of power transmission. In theory, it utilizes the concepts of half-wave length power transmission which itself can be extended to a full wave length. But, because of the conversion devices that must accompany this arrangement, there is the added advantage of better control over voltage and current profiles of the line.

The obvious implication of this method is the utilization of one frequency different from the standard consumer frequency. What effect, if any, this other frequency has on commonly known system parameters such as reactance, eddy currents, and hysteresis losses on power system devices is examined, and where possible, an attempt is made to relate observed isolated effects to reveal the interdependence of

some of them. In cases where such interdependence does not exist, the influence of frequency on any particular parameter is presented for a fairly wide range of frequencies so that the choice of any frequency within the range can be dictated by sound engineering practice and economics.

In the presentation of this thesis a brief review is made of the conventional long distance transmission of power and the comparatively recent half-wave length method. The theory and mode of the steady state operation of the two-frequency line is described also. The effect of increasing frequency is presented, and theoretical calculations for a hypothetical two-frequency line are carried out. In addition, a survey is made of existing and probable methods of frequency conversion.

The author feels that this method of power transmission offers reliability, and a greater degree of flexibility than is known at present in the extra-long distance transmission and distribution of energy.

CHAPTER ONE  
ASPECTS OF POWER TRANSMISSION

In the effort to harness water resources for power, attention was mainly directed to water resources that are nearer to centers of population. As the demand for power increased, and these nearby water resources became depleted, attention was naturally focused on more remote potential power sites. Problems which hitherto were absent or only slightly present, now manifested themselves to a high degree, and new ways had to be found either to eliminate or circumvent them. What these problems are, and how they have been overcome, is the purpose of this and the subsequent chapters.

1.1 Fundamental Problems of Extra-long Distance Power Transmission

As far as the power engineer is concerned, an equation<sup>1</sup> of interest for a power system as shown in figure 1 is:

$$P_r = \frac{|E_s||E_r|}{|B|} \cos(b-d) - \frac{|A||E_r|^2}{|B|} \cos(b-a) \dots 1$$

where  $P_r$  is the power received,

$E_s$  is the sending end voltage,

$E_r$  is the receiving end voltage,

$d$  is the angle between  $E_s$  and  $E_r$ ,

$A$  and  $B$  refer to the ABCD constants of a four terminal network, with  $a$  and  $b$  as their respective phase angles.

For a lossless line - and most power lines are made

Nearly lossless - equation 1 is given approximately as

$$P_r = \frac{E_r E_s \sin d}{B} \dots \dots \dots 2$$

and the maximum transferable power is:

$$P_r = \frac{E_r E_s}{B} \dots \dots \dots 3$$

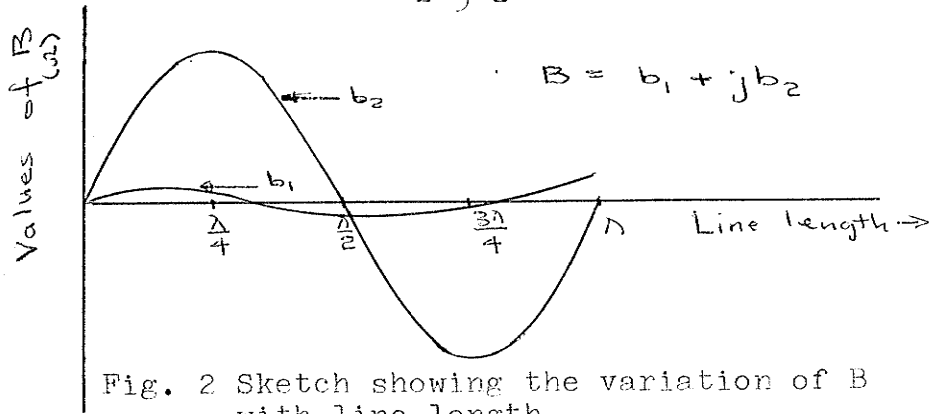
To a certain degree, the angle  $d$  is within the control of the engineer, while  $B$  is a quantity which for a given frequency, is determined by the combined influence of the line length and all non-zero impedance equipment installed on the line. Obviously  $B$  is a limiting factor on the maximum transferable power. The point of interest then is how does  $B$  vary?



Fig.1 One line diagram of an electric system

A sketch of the variation of  $B$  with line length is provided in figure 2. It has real and imaginary components, the latter part of which varies almost sinusoidally with line length. If equation 2 is now re-examined in the light of this fact, it will then be seen that for any line less than





or equal to one-quarter wavelength the maximum transferable power decreases as the line length increases. For the standard 60 cycle frequency, a quarter wavelength is about 750 miles so that for any line from 350 to 750 miles, the engineer inevitably has to grapple with this increased value of  $B$ .

An equally fundamental and yet undesirable characteristic is the charging reactive power which must be supplied to the open circuited line. Figure 3 displays a plot<sup>2</sup> of charging reactive power versus line length.

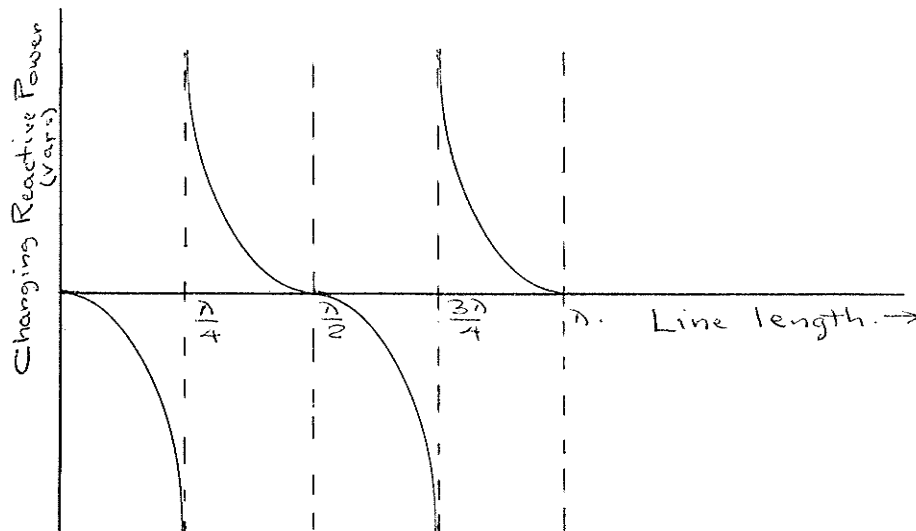


Fig. 3 Sketch showing Reactive power versus line length for an open circuited line.

In figure 4 the ratio of receiving end voltage for the open circuited line to sending end voltage is plotted for lines less than, or equal to, one full wave length..

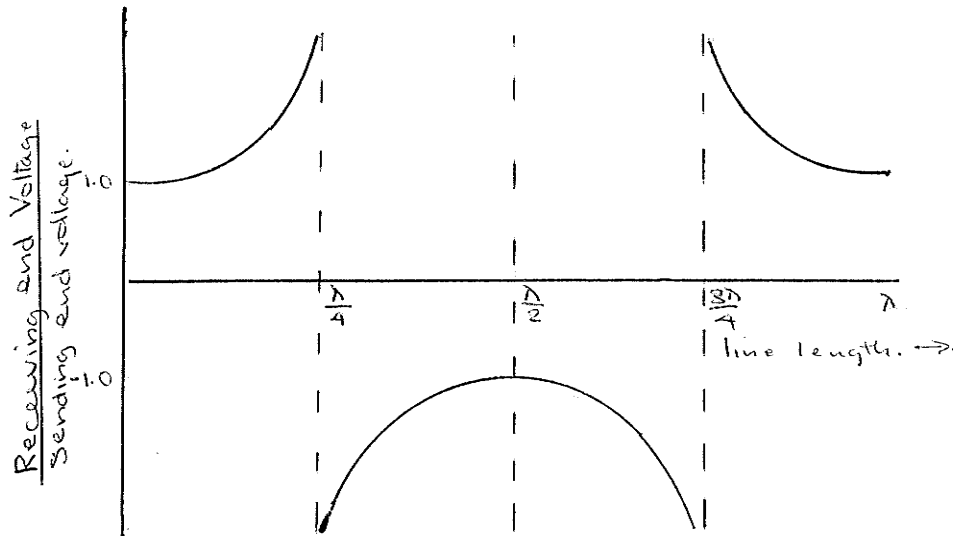


Fig. 4 Sketch: Ratio of receiving end voltage as a function of line length.

A study of figures 3 and 4 reveals increasing values of voltage and charging vars as the line length increases within the first quarter-wave length and at points near the quarter-wave length position, these tend to become very large, and theoretically infinite for a lossless line. Beyond this position, both these quantities decrease from opposite values of infinity, reaching a minimum at half-wave length position, and thereafter increasing again. These are obviously undesirable characteristics for which solutions must be found.

Another problem which must be considered too, is the one of voltage rise in the mid-point of the line if the magnitudes of the receiving and sending end voltages are held constant. The voltage rise increases as the line length increases. Figure 5 sketches the voltage profiles of three transmission lines of different lengths.<sup>2</sup>

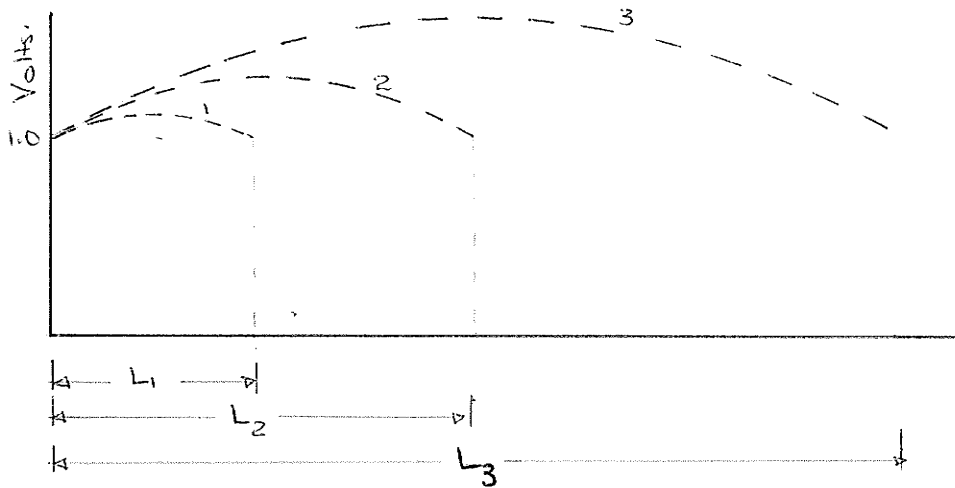


Fig. 5 Sketches of internal line voltages.

## CHAPTER TWO

### SOLUTIONS TO EXTRA-LONG DISTANCE POWER TRANSMISSION PROBLEMS

To combat all the aforementioned effects, one method which has been devised has been to sectionalize the transmission line.<sup>2</sup> (See figure 6.) This solves the problem of

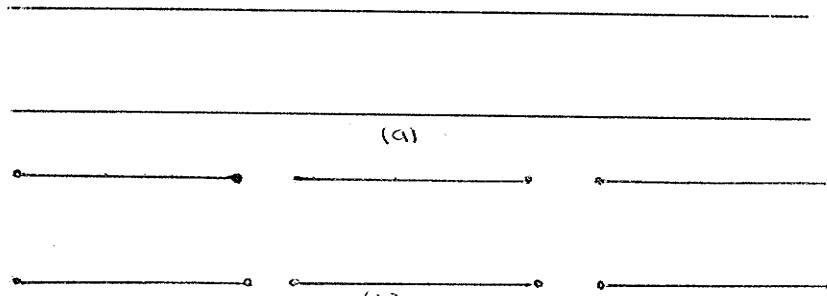


Fig. 6a Long line. 6b Sections of long line.

high voltage ratios between receiving and sending ends of each section and consequently for the whole line. To avoid the high charging currents, each section is shunt compensated. (See figure 7.) To obtain greater maximum transferable power, each section is series compensated; the effect being to decrease the value of  $B$ . In other words, an electrically short line is being made out of a physically long line. With the line so compensated there

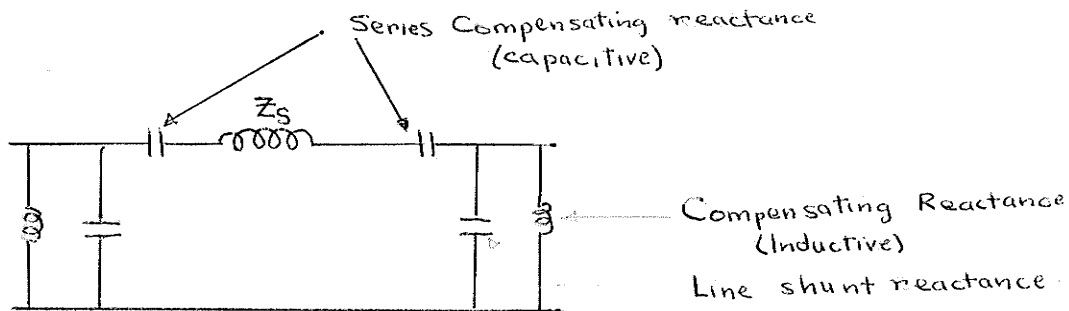


Fig. 7 Compensated section of a line.  
 $Z_s$  is the line series impedance.

come the risks of serious and sustained overvoltages and these may become the limiting factor on the maximum transferable power.

## 2.1 Direct Current Transmission

Another possibility for long distance transmission of power is the dc method. Basically this method involves the rectification of alternating current power, which is then transmitted and inverted into alternating current power again. By this method, the problems of stability, transient or otherwise, are sidestepped as much as possible. It suffers from a high initial cost. A diagrammatic representation of such a line is given in figure 8.

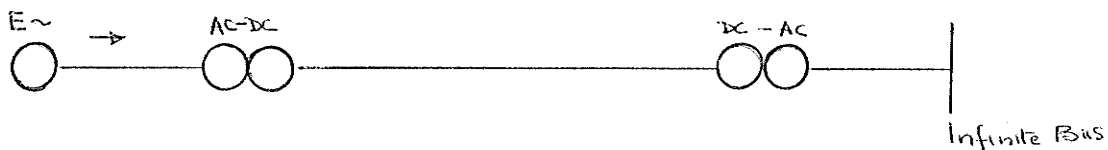


Fig. 8 One line diagram of a d.c. line.

## 2.2 Half-wave Length Power Transmission<sup>5</sup>

In the half-wave length method of power transmission, the undesirable inhibitory influence of  $B$  on maximum transferable power is very greatly minimized by utilizing the semi-cyclic variation of the imaginary part of  $B$ . (In a lossless case there will be no real part of  $B$ .) Thus a line which is between 180 and 270 electrical degrees in length behaves in almost every respect as a line which is between

0 and 90 electrical degrees. In other words, charging currents and terminal voltage magnitudes are the same. Fault currents at the line terminals are no larger than those of conventional lines. But there is a great diminution of fault currents as fault points occur progressively farther from the terminals. It might just be well to point out at this stage that from stability considerations, line lengths between 90 and 180 electrical degrees while having small magnitudes of  $B$ , are not applicable.<sup>4</sup>

However, in actual practice no power lines are up to 180 electrical degrees on the standard 60 cycle frequency, so the technique used is to lengthen the line. A number of tee sections, each of the same characteristic impedance as the line, is placed in series anywhere along the line. The total number added must be such as to stretch the line to between 180 and 270 degrees.

Perhaps one great advantage inherent in this system is the incorporation into the tee sections of all transformer reactances on the line. However, a serious shortcoming of this type of line is the great fluctuations of the line internal voltages as the line load is varied. (See figure 9.) Such wide voltage fluctuations<sup>3</sup> make it difficult to adjust for lightning protection. Also there is a problem of intermediate tapping of power for which solutions are still being sought.

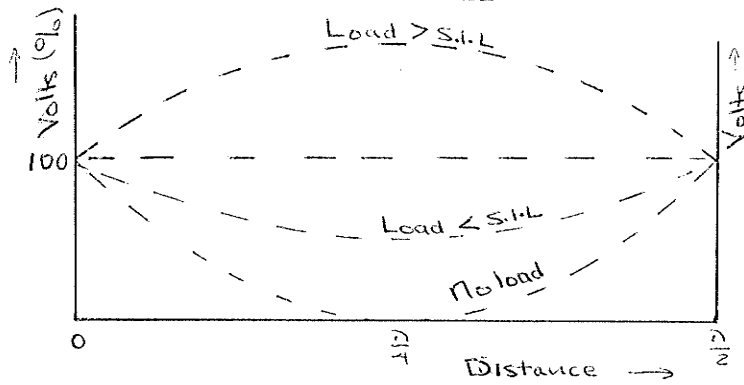


Fig. 9 Voltage profiles for a half-wave length line.

Data and design<sup>5</sup> of such a transmission line are presented below as a matter of interest.

Line length = 900 miles

Series impedance =  $.0123 + j0.249$  pu per 100 miles

Characteristic impedance =  $1.22 - j.0303$  ohms pu

Propagation constant  $\alpha + jB = .005 + j0.204$  ohms per 100 miles

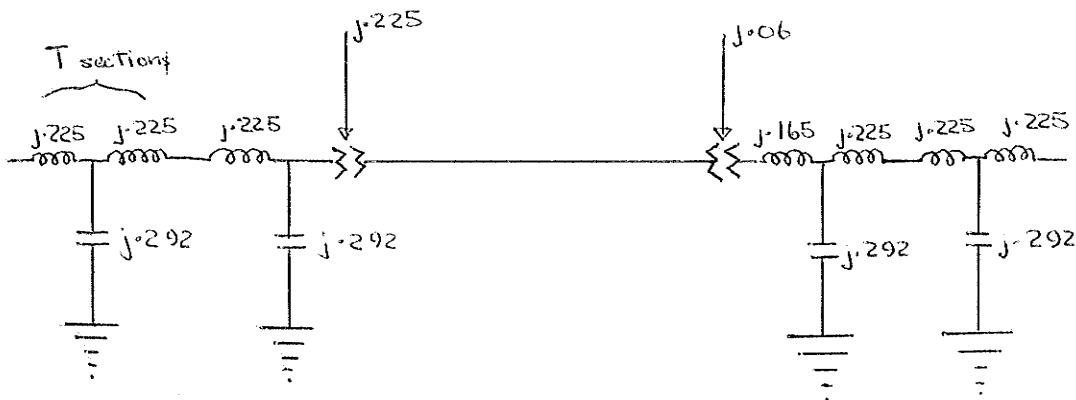


Fig. 10 One line diagram of a hypothetical half-wave length line.

## CHAPTER THREE

### TWO-FREQUENCY METHOD

Before introducing the discussion of the two-frequency method, the point must be established that consideration is given only to steady state operation.

#### 3.1 Basic Principle

As has been indicated earlier in this paper, the two-frequency method is based on the half-wave length principle but differs from it in one fundamental respect. This is the adoption of an alternative procedure of making a given physical length correspond to a desired electrical length by choice of frequency. (The electrical line length is inversely proportional to frequency.) For instance, a 900 mile transmission line operating at 60 cps is less than a half-wave length, but such a line operating at 120 cps would be more than a half-wave length.

The two-frequency method realizes a desired electrical length by the simultaneous use of two frequencies and such a line would be as shown in figure 11.

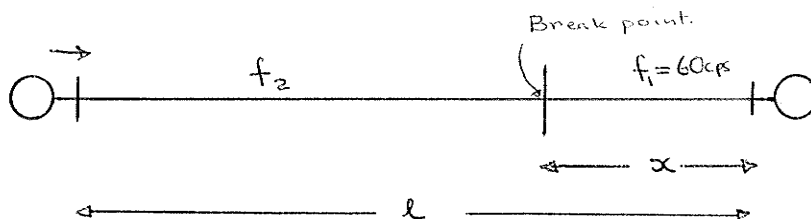


Fig. 11 One line diagram of a two-frequency line.



If the distances  $l$  and  $x$  are fixed, and the line impedance is purely reactive, and if  $f_1$  is held at 60 cycles, then an equation for computing  $f_2$  such that the line is 180 electrical degrees is\*

$$f_2 = \frac{(180 - x\phi_1)f_1}{(l - x)\phi_1}$$

where  $x$  is the distance from the receiving end

and  $\phi$  is the phase shift per unit distance at  $f_1$ .

For the purpose of this thesis,  $f_2$  will be referred to as the primary frequency and the point of frequency discontinuity, so to say, will be referred to as the break-point. Break-point distances will be measured from the receiving end of the transmission line. Frequency versus break-point curves for three different lengths of transmission line are shown in figure 12. Though break-points can be placed anywhere along the line, not all of them are realizable. Break-points near a quarter-wave-length involve higher frequencies but apart from this, the drastic voltage fluctuations likely to occur at such points, particularly during open circuits, would seriously affect the operation of any equipments installed at those points.

---

\*Suppose phase shift at  $f_1$  is  $\phi_1$  degrees per unit distance. Then at  $f_2$ , the phase shift is  $f_2 \phi_1$  degrees per unit distance. Thus

for a combined phase shift of 180 degrees we will have the relation

$$x\phi_1 + (l - x)\frac{f_2}{f_1}\phi_1 = 180^\circ$$

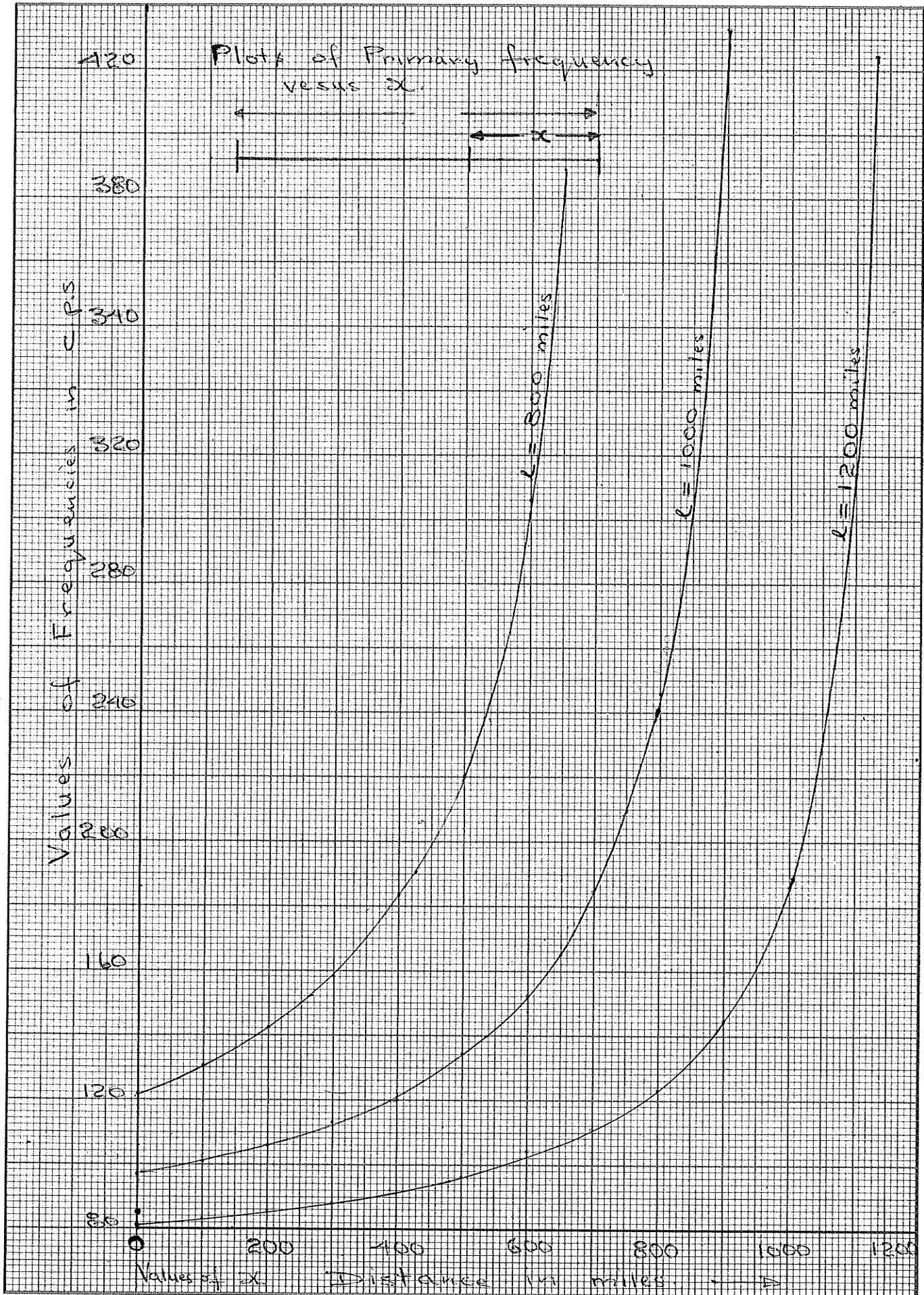


Fig. 12 Plots of primary frequency versus breakpoint.

The question may be asked why it is necessary to have a break-point other than zero. The answer is that only a non-zero break-point operation of the line is likely to offer the fullest use of the resources of the system. This will be better understood and appreciated when the mode of operation of a typical two-frequency line is fully explained.

In practice, most transmission lines are less than a half-wave length on the standard consumer frequency in which case higher frequencies would be needed to bring them to a half-wave length electrically. But power must be available to the customer at consumer frequency so that the need for frequency converters becomes evident.

The converting devices contemplated have components that can supply reactive power to the transmission line. This reactive power may be employed for stub-matching<sup>17</sup> purposes and thus help keep line voltage changes as small as possible. In this connection, attention should be directed again to the sketches of voltage profiles for the half-wave length line. (See figure 9, page 11.)

For stub-matching purposes the reactive power would have to be introduced at some distance away from the load. It will be necessary to wait until the development of the section dealing with tuned transmission lines for this point to be clarified.

In the light of the foregoing, it will be seen that the two-frequency method eliminates the necessity for tee sections. (An illustration of the use of tee sections is shown in figure 10.) Furthermore, the complications incident to the use of capacitors in a high voltage line are avoided. Also, the method offers prospects of the control of the internal line voltages which, in turn, means better lightning adjustments.<sup>3</sup>

## CHAPTER FOUR

### FREQUENCY EFFECTS

Power transmission involves the use of equipment such as transformers and generators, all of which have components that are affected by frequency. Also, some transmission line parameters, inextricably bound up with other variables that respond to frequency, are invariably affected. The effects on line parameters and system devices are separately considered.

#### 4.1 Effect on System Stability

As has been mentioned earlier under "Fundamental Problems," the crux of both the half wavelength and the two-frequency methods is the lengthening of a line in order to obtain a desired value of  $B$ , (see page 5,) for purposes of greater transfer of power. Thus from equation 2,

$$P_r = \frac{E_s E_r \sin d}{B}$$

so that all systems having equal values of  $B$  would have the same steady state power capabilities, and would be unaffected by their generation and transmission frequencies.

##### 4.1.1 Transient Stability

Synchronous generators and motors play an important part in the transient stability of a transmission system.

The angular momenta of synchronous machines are usually temporarily adjusted to meet various fault conditions as a means of improving system transient stability.

From classical mechanics the momentum of a rotating mass is given as

$$M = I\omega$$

where  $M$  is in joule-sec/rad,

$I$  is the moment of inertia of the object,

and  $\omega$  is the angular velocity of the object.

In general for a constant speed of rotation and a fixed rotating mass, the angular momentum becomes constant.

Hence, as will be shown later under "Generators", generators of the same capacity, the same magnetic core flux density, and mechanical speed have the same momentum.

The angular momentum can also be expressed in a derived set of units. The resulting mathematical expression for a single machine is then

$$M = \frac{GH}{180f} = \frac{K}{f}$$

where  $M$  is in megajoule-seconds per electrical degree,

$G$  is the rating of the machine in megavolt-ampere,

$H$  is the stored energy in megajoules/machine rating  
in megavolt-amperes.

Thus the angular momentum has a reciprocal relation with frequency.

The angular momentum appears in the swing equation in the following way:

$$M \frac{d^2\phi}{dt^2} = dP \dots\dots\dots 5$$

where dP is the accelerating power,

and  $\phi$  is the angular position of the rotor of a synchronous machine.

Thus the indirect effect of frequency on the acceleration of the rotor angle is seen by substituting for M in equation 5.

Hence,

$$\frac{K}{f} \times \frac{d^2\phi}{dt^2} = dP$$
$$\text{or } \frac{d^2\phi}{dt^2} = \frac{dP}{K} \times f$$

Therefore, the rotor acceleration is directly proportional to frequency. This proportionality in turn implies that the critical clearing time for a given fault condition on a power system becomes smaller and smaller as the frequency increases.

#### 4.2 Frequency Effects on Dielectrics

The variation of breakdown strength of dielectrics with frequency<sup>6</sup> is given by an empirical relation

$$R_f = \frac{K}{f^n}$$

where  $R_f$  = ratio of dielectric strength at frequency  $f$   
to 60 cycle strength,

$f$  = frequency in cycles per second,

$n$  = constant = 0.137,

$K$  = constant depending on the relative strength  
of the material under consideration.

This relation is presented graphically in figure 14 for  $K = 1.75$ . The curve points to the necessity of using more insulating material as the frequency increases. But the rate at which the ratio decreases is considerably slower at the higher frequencies than at the lower ones.

#### 4.3 Effect on Magnetic Circuits

Core losses are due to the variation of flux in a magnetic circuit; and fall into two classes: hysteresis and eddy current losses. The eddy current losses are resistance losses as produced when current flows through any material. Expressions for these losses have been arrived

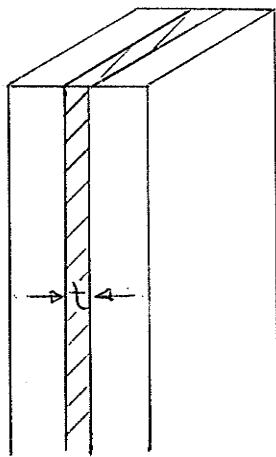


Fig.13 Magnetic core  
showing a lamina  
of thickness "t"

at for laminated materials with alternating excitation, and they could be the basis on which frequency effects could be examined. The losses<sup>7</sup> for the magnetic core shown are:

Hysteresis loss  $P_h = K_h f B_m^x$  watts  
per unit volume or weight,



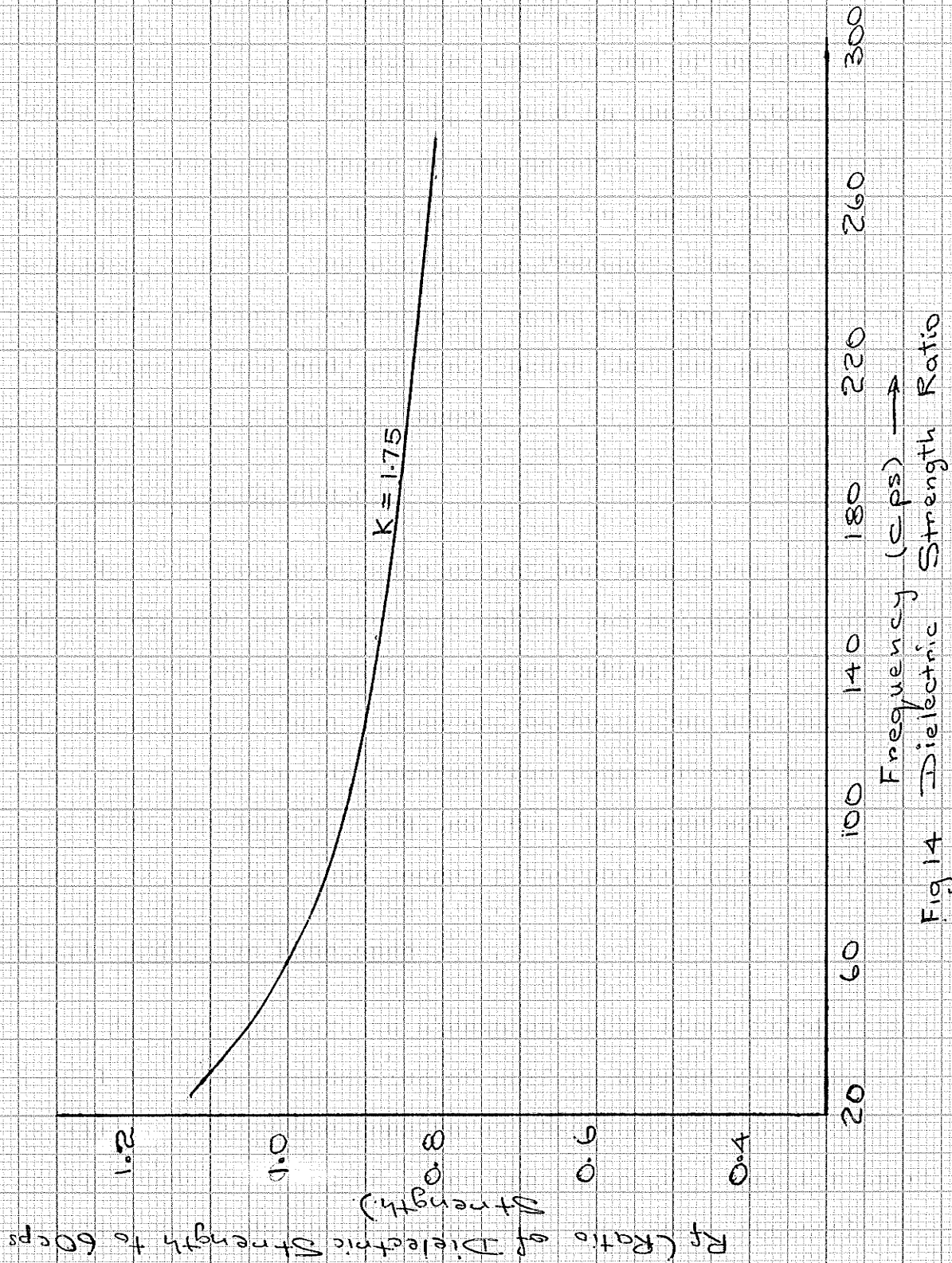


Fig 14 Dielectric Strength Ratio

$$\text{Eddy current loss } P_e = K_e t^2 f^2 B^2$$

watts per unit volume or weight

where the constants  $K_h$  and  $K_e$  depend on the characteristic and electrical resistance of the material. The exponent  $x$  is a constant between 0.5 and 2.3,  $t$  is the thickness of a lamina, and  $f$  is the excitation frequency. The equation obviously indicates hysteresis losses that are proportional to frequency, and eddy current losses that are proportional to the square of the frequency.

#### 4.4 Effect on Conductors

The phenomenon of skin effect has been known for a long time, and there are many tables and calculations for a goodly number of conductor shapes taking it into consideration. But these tables and calculations were made for frequencies so far removed from power frequencies, that they can hardly be relied upon for power work.<sup>8</sup> Some work has been done recently to rectify this situation.

##### 4.4.1 Inductive Reactance

Most of the work in Aluminium Cable Steel Reinforced (ACSR) is on the assumption of uniform current distribution. Proximity effects are considered absent since conductors of overhead transmission lines are invariably very far apart relative to conductor diameter. For the larger stranded conductors, both tests and calculations show skin effect

on inductive reactance to be negligible. Under these conditions the inductive reactance of ACSR conductors is directly proportional to frequency.

#### 4.4.2 Skin Effect Resistance

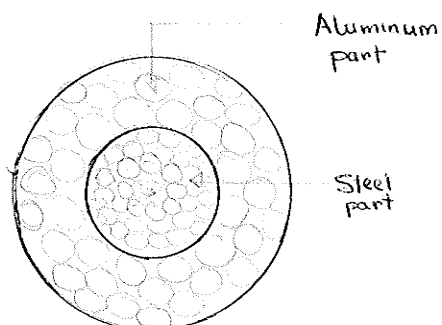


Fig. 15a ACSR conductor.

If the thickness of the aluminum layer is large, skin effect causes an increase of resistance over the direct current value. Work done indicates that the degree of skin effect on stranded ACSR

conductors is nearly the same as that of a uniform conductor of the same inside and outside dimensions, and the same resistance to direct current as the aluminum part. Thus ACSR conductors having two or more layers could be analyzed on

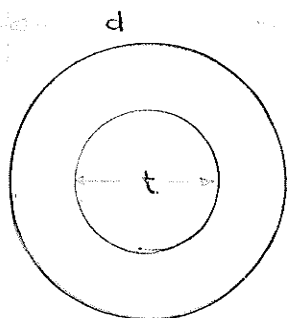


Fig. 15b Hollow tubular conductor.

the same pattern as for hollow tubular conductors as shown in figure 15b. Plots of ac-dc resistance ratio as a function of  $\sqrt{\frac{f}{\gamma_{ac}}}$  for different values of the ratio  $\frac{t}{d}$  are available.<sup>8</sup>

The dc resistance ratio was calculated for ACSR 1,780,000 cmils twin conductors. Data for ACSR 1,780,000 was not available during the investigation, and as such the conductor was assumed to be 54/19 and the outside and inside diameters, and the dc resistance of the aluminum layers were extrapolated

from plots made of all 54/19 ACSR conductors available in standard engineering handbooks.<sup>9</sup> The results obtained from the extrapolations are:

Outside diameter = 1.60 inches

Inside diameter = .535 inches

DC resistance per mile  
per conductor = .059 ohm

From figure 15b the ratio  $\frac{t}{d}$  is given by the ratio of inside diameter to outside diameter. Thus the ratio of ac resistance to dc resistance was obtained for a variety of frequencies from reference 8. A plot of this ratio for different frequencies is given in figure 16.

#### 4.5 Effect on Line Parameters

The work done so far in this section has been to examine the separate effects of frequency on certain system parameters without relating them to other system parameters that may be affected one way or the other by frequency. Thus line constants and other system variables that are composites of frequency influenced parameters are re-examined; where possible, attempt will be made to present the overall effects of these components.

In travelling wave engineering the parameters of interest are:

$Z_0$  = the characteristic impedance

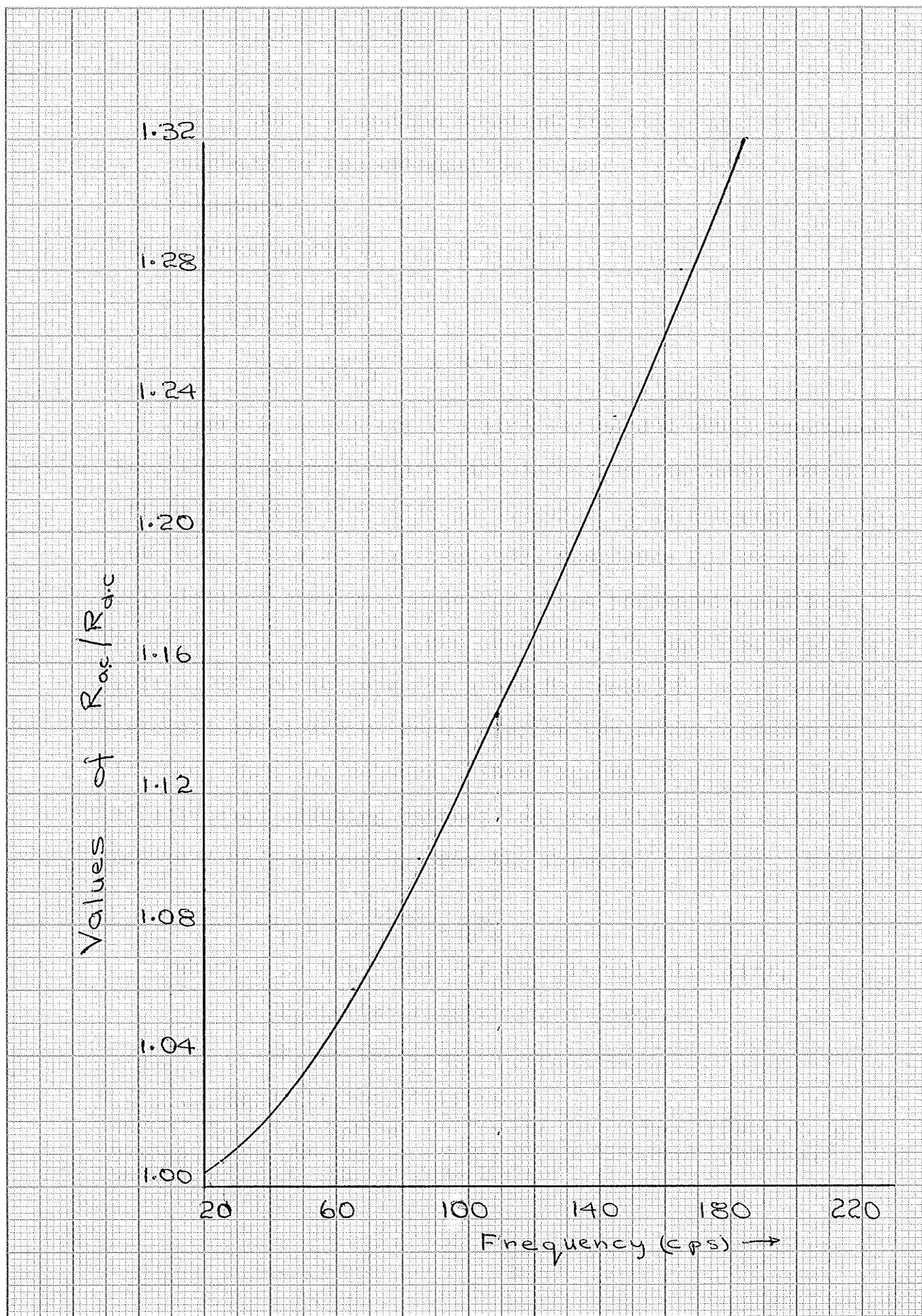


Fig 16 AC/DC resistance ratio versus frequency plot

$\gamma$  = the propagation constant and its components  $\alpha$  and  $\beta$  where  $\alpha$  is the attenuation constant and  $\beta$  the phase constant.

The expression for  $Z_0$  and  $\gamma$  are:<sup>17</sup>

$$Z_0 = \sqrt{\frac{Z_s}{Y}}$$

$$\gamma = \sqrt{Z_s Y} = \alpha + j\beta$$

where  $Z_s$  and  $Y$  are the line series and shunt reactance. From the sections dealing with inductive reactance and skin effect resistance of ACSR conductors, it is now possible to obtain  $Z_s$  and  $Y$  at any frequency, and hence  $Z_0$ ,  $\alpha$  and  $\beta$ . Calculations\* have been made for these parameters and are presented in graphical form in figures 17a and 17b. Plots are made at 25 and 165 cps for A and B, ( $\cosh \gamma l$  and  $Z_0 \sinh \gamma l$  respectively,) where these refer to the ABCD constants of the four terminal network. (See figures 18 and 19.) Line efficiencies assuming matched load conditions are calculated for varying frequencies from the formula<sup>16</sup>

$$P_r = P_s e^{-2\alpha d} = \eta P_s$$

where  $d$  is the length of the transmission line,

$P_r$  is the receiving end power,

$P_s$  is the sending end power,

and  $\eta$  is the efficiency.

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\*Note: These calculations are for the Twin Chukar, 1,780,000 cmil conductor<sup>2</sup> and the results are given on a 250 $\Omega$  base impedance, where applicable. (See figures 17a and 19.)



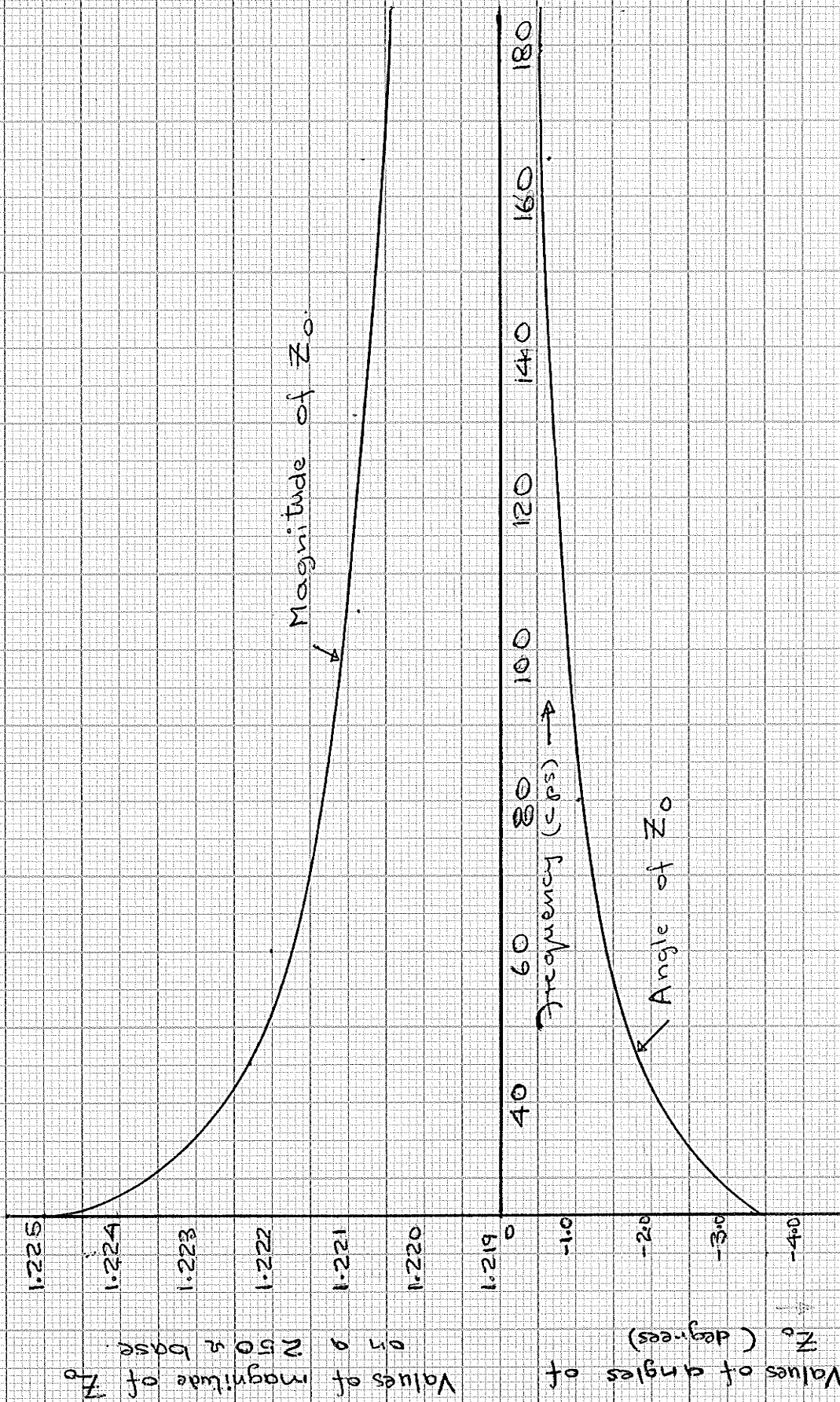


Fig 17a. Magnitude and Phase Angle as a function of frequency.

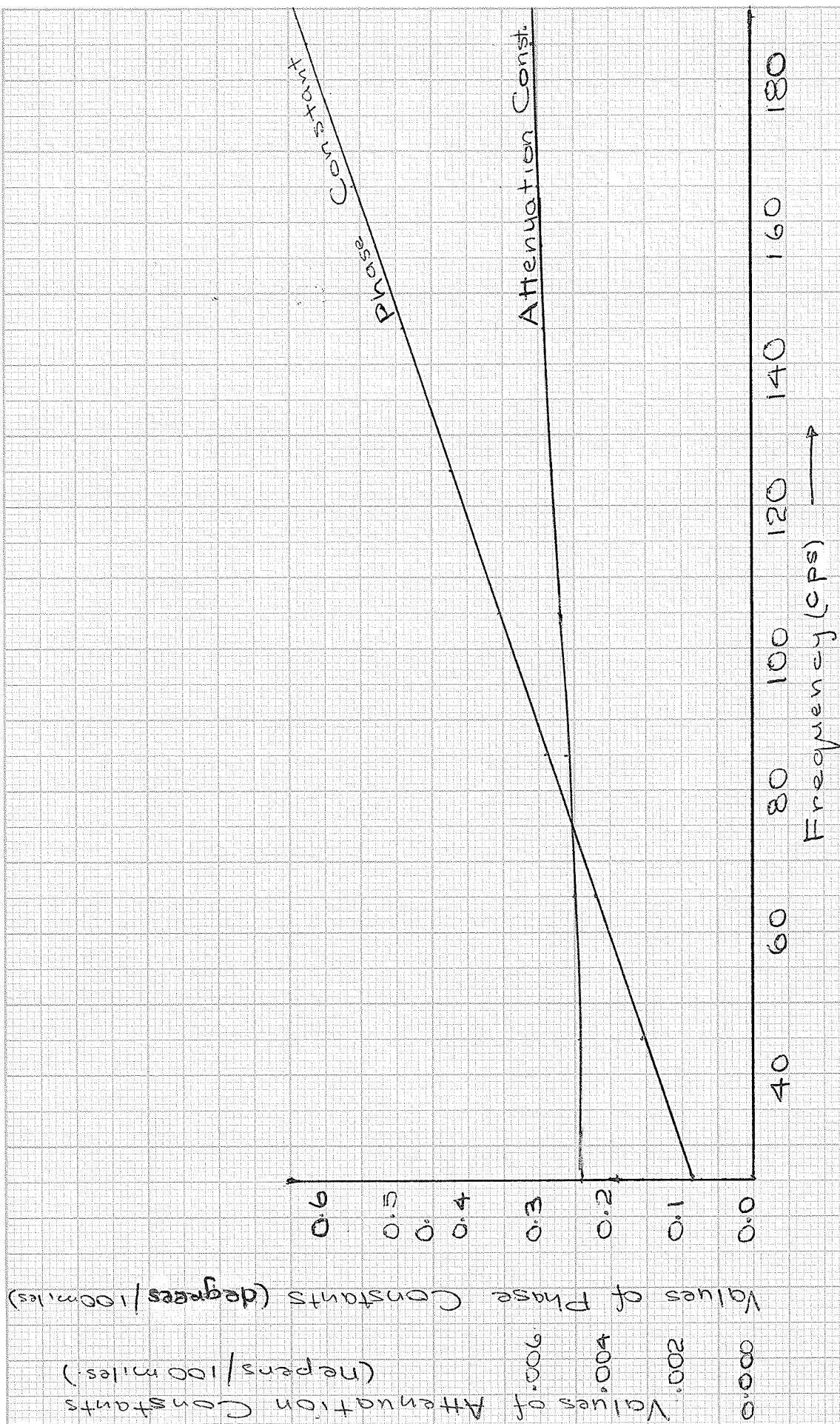


Fig 17b Attenuation and phase constant as a function of frequency.



Line length in hundreds of miles	E F F I C I E N C I E S		
	25 cycles	60 cycles	165 cycles
0	1.000	1.000	1.000
1	.990	.990	.987
2	.980	.980	.976
3	.971	.970	.964
4	.962	.960	.952
5	.952	.951	.941
6	.943	.941	.929
7	.934	.932	.918
8	.925	.923	.907
9	.916	.913	.896

Table 1.

A few discernible facts from the curves, (figures 17a and b,) are:

- (a) relatively rapid decreases in both the magnitudes and phase angles of the characteristic impedance of the line at the lower frequencies, but these slow down at the higher frequencies.
- (b) the phase and attenuation constants both increase with frequency, but what is of particular interest is the degree to which line efficiency is affected. (See table 1 above.)
- (c) a study of figure 18 will show that the effect of higher frequency operation has been to suppress

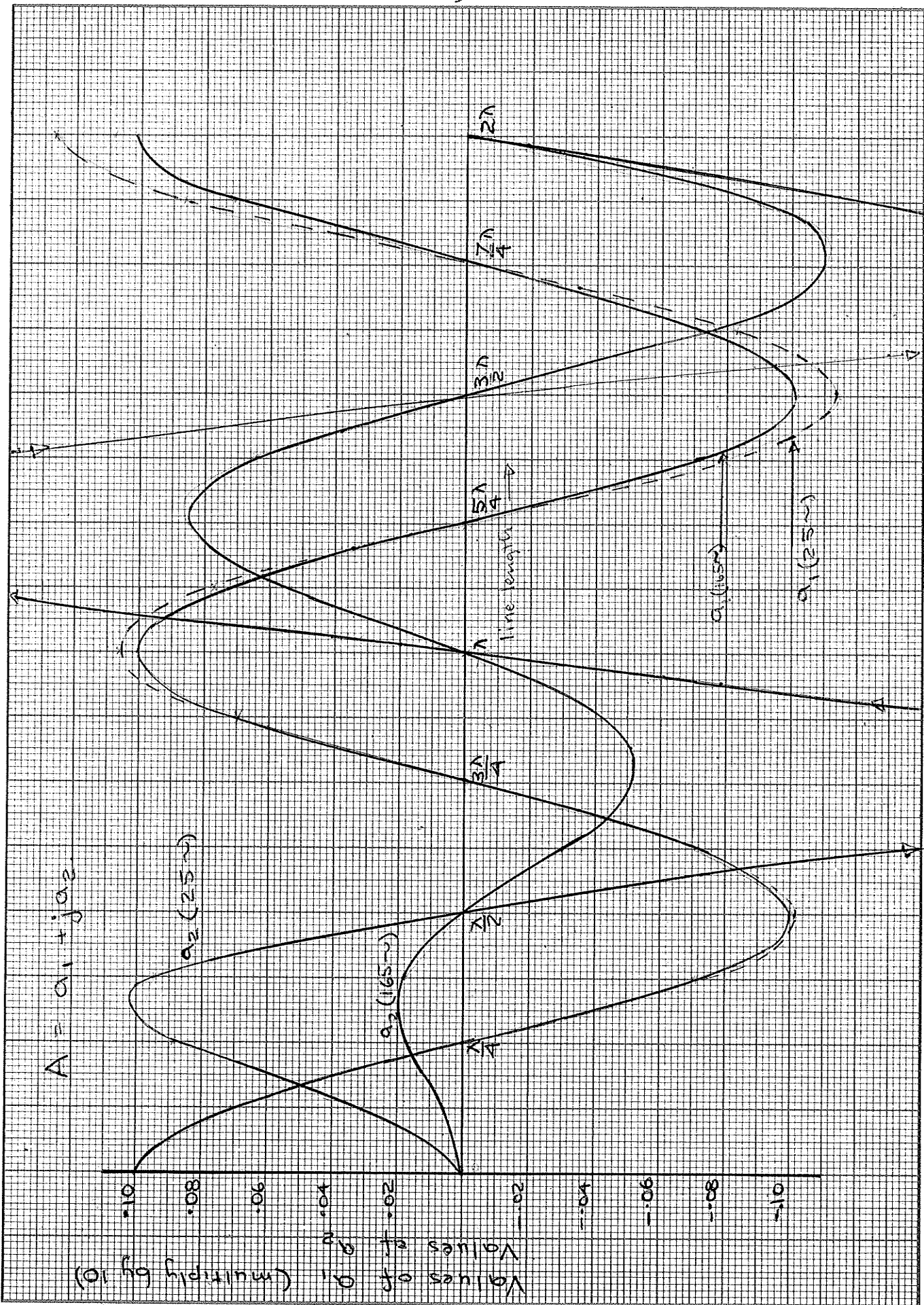


Fig 18 Plot of A as a function of line length.

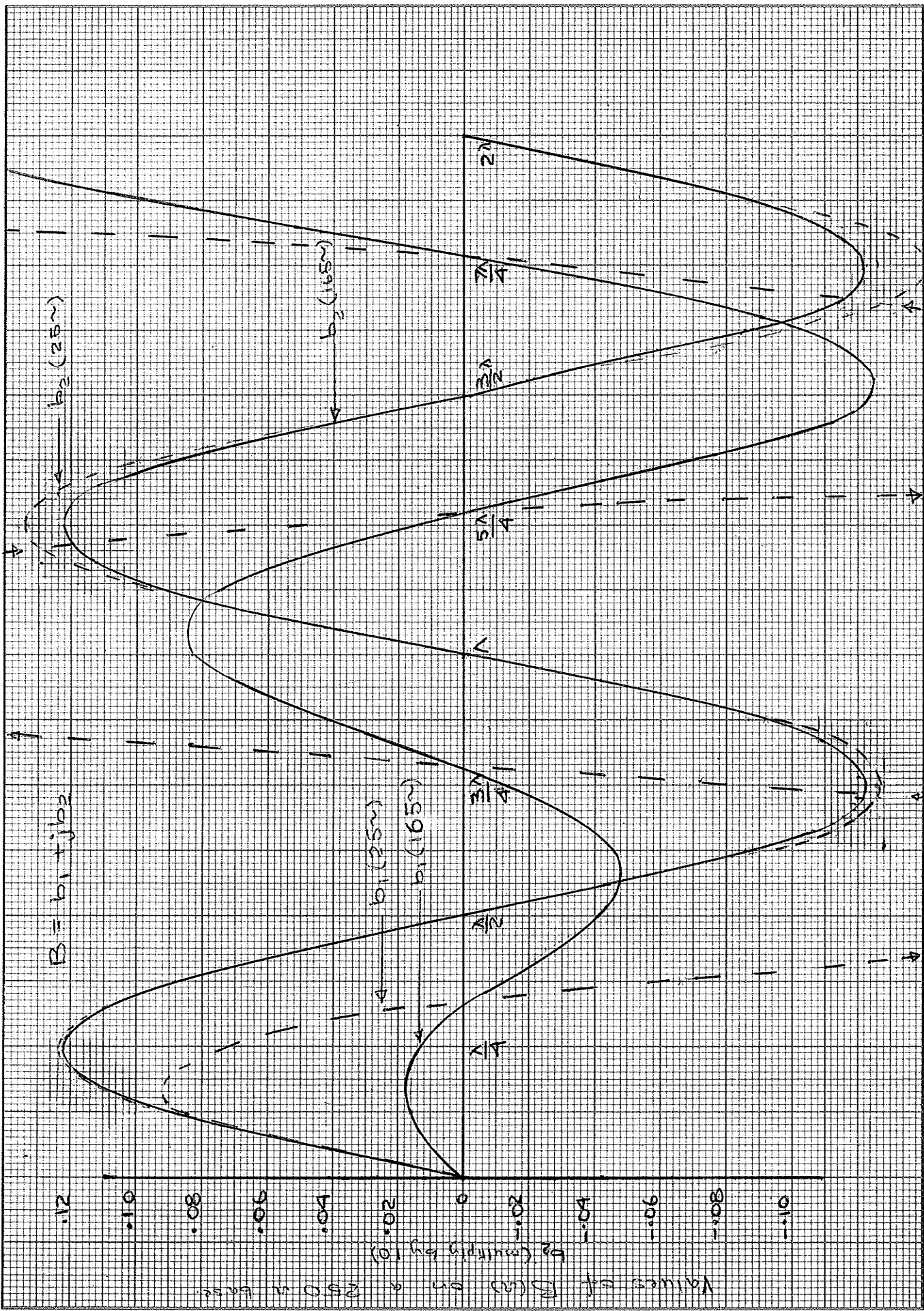


Fig 19. Plot of B as a function of line length.

the imaginary part of A while figure 19 will show the suppression of the real part of B. These effects are due to the more rapid increase of line reactance over resistance. The overall effect of higher frequencies has been to produce A and B constants that closely approximate those of a lossless line. However, this should not be construed as meaning that the line is less lossy.

#### 4.6 Overall Effect on Transformer

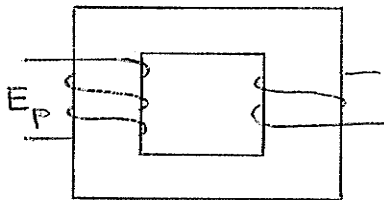


Fig. 20 Simplified diagram of a transformer.

A simplified diagram of a transformer is given in figure 20.

From transformer theory<sup>10</sup> the voltage on the primary windings is given as

$$E_p = 4.44fN_pAB_{\max} \dots \dots 6$$

Where  $f$  = frequency of excitation,

$B$  = flux density,

$A$  = the cross sectional area of the core,

$N_p$  = number of turns in the primary.

For constant values of  $E_p$ ,  $N_p$  and  $B_{\max}$ , it can be seen from equation 6 that  $A$  and  $f$  are inversely proportional. Therefore, for power transmission at a fixed voltage level, the cross-sectional area of the magnetic circuit decreases as the frequency increases. For most magnetic core devices however,  $N_p$  and  $B_{\max}$  will depend on stray loss consideration.

Another result of this decrease in cross-sectional area is a saving in copper and a consequent reduction of the transformer resistive losses. A mathematical analysis for either a circular or square cross-sectional area of core will reveal that each turn length is reduced by the square root of the ratio of frequencies.

#### 4.6.1 Transformer Core Losses

The magnetic core losses were given earlier in this section. Suppose it is desired to compare the core losses

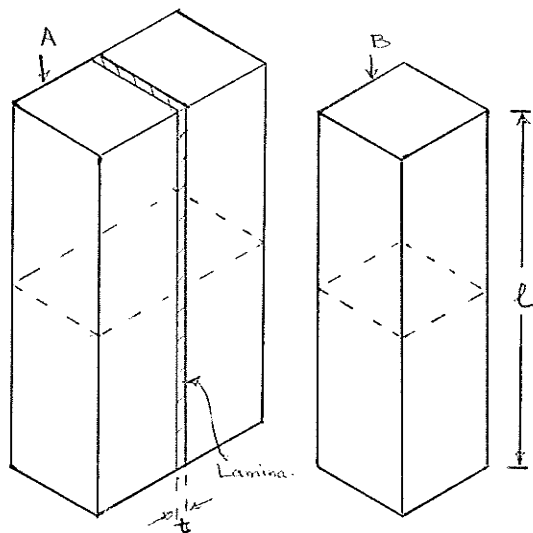


Fig. 21 Arms of transformers A and B

of two transformers A and B, having equal voltage ratings. To take advantage of a simple frequency ratio assume that the frequency  $f_A$  of A is half the frequency  $f_B$  of B. Assume also that the flux densities of A and B are the same. Then the cross-sectional area of A is twice that of B. For the

configuration shown in figure 21, the core thickness of A is twice that of B.

In general the magnetic core is a pile of thin sheets - laminae - of the magnetic material. If the lamination thicknesses are equal the number of laminae  $N_A$  of A is

equal to the number of laminae  $N_B$  of B. From the core loss formula for hysteresis, the loss per core is

$$P_{hA} = K_h f_A B_m l t N_A \dots\dots\dots 7$$

$$\text{and } P_{hB} = K_h f_B B_m l t N_B \dots\dots\dots 8$$

where  $l$  is the length of the magnetic material.

$$\text{But } f_B = 2f_A$$

$$\text{Also } N_B = 1/2N_A$$

Therefore substituting into equation 8,

$$\begin{aligned} P_{hB} &= K_h 2f_A B_m l / 2N_A l \\ &= K_h f_A B_m N_A l \end{aligned}$$

$$\therefore P_{hB} = P_{hA}$$

To calculate the eddy current losses, if the same assumptions are made as for hysteresis loss calculation, then the total eddy current loss for A is

$$P_{eA} = K_e t^2 f_A^2 B_m^2 N_A l \dots\dots\dots 9$$

and for B,

$$P_{eB} = K_e t^2 f_B^2 B_m^2 N_B l \dots\dots\dots 10$$

Substituting into 10 for  $f_B$  and  $N_B$ ,

$$P_{eB} = K_e t^2 / f_A^2 l / 2N_A l$$

$$\text{Hence } P_{eB} = 2P_{eA}$$

What has been established is that for the same output voltage, and flux density of the magnetic material, the hysteresis losses are independent of frequency, while

the eddy current losses are directly proportional to frequency. It may be necessary to add here that the heat dissipation may become more difficult as a result of the greatly reduced surface area.

#### 4.6.2 Stray Load Losses

Stray load losses are those miscellaneous losses which occur when a machine is loaded. They are caused mainly by leakage flux and by the distortion of the main exciting flux. These losses are impossible to calculate<sup>14</sup> and certainly not easy to measure even in a completed machine. For this reason, no definite statement will be made with regard to frequency effects.

#### 4.6.3 Leakage Reactance of Transformers

The leakage reactance is functionally dependent upon the flux linkages per ampere turn and hence is directly proportional to the number of turns of the winding. In actuality, however, not all fluxes of a transformer secondary or primary link the primary or the secondary respectively. Under certain simplifying assumptions an approximate formula<sup>11</sup> has been derived for the leakage reactance of the core type of transformer, and this formula could serve as the basis for the examination of frequency effects on armature leakage reactance. For the core type transformer of figure 22, the leakage reactance is given as

$$X_e = Kfm(d_3 + \frac{d_1}{3} + \frac{d_2}{3}) \dots \dots \dots 11$$

where  $K =$  constant,

$m =$  the mean length of a turn ( $2\pi r_f$ ),

and  $f =$  frequency.

The reactance is related to the physical dimensions of the transformer component parts. The mean radius is made up of

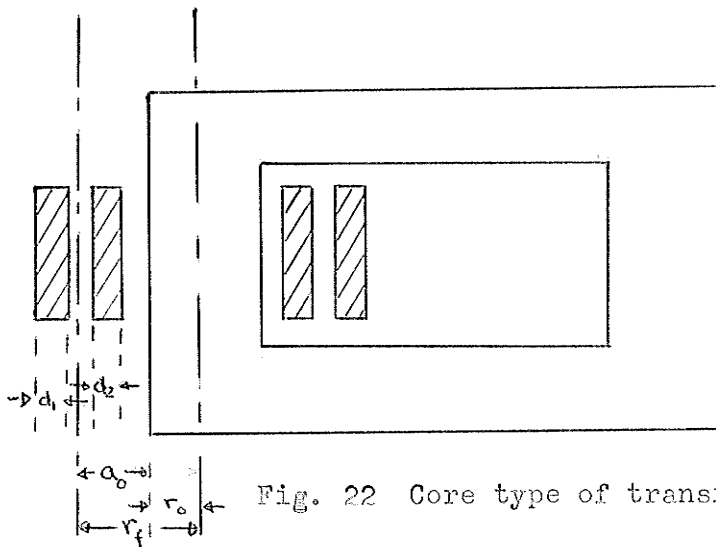


Fig. 22 Core type of transformer.

a layer of magnetic material, an insulating layer between the secondary winding and the magnetic core, the layer of the secondary windings, and half the insulating layer between the secondary and the primary windings. Of all these, only the magnetic layer is likely to become appreciably smaller while the insulating materials may tend to increase with frequency, though this is of negligible effect. Consequently the mean radius at any frequency can at best be expressed as



$$r_f = a_o + r_o \sqrt{\frac{f_o}{f}}^*$$

where  $a_o$  and  $r_o$  are as shown in figure 22. (Note  $r_o$  is a dimension within the magnetic circuit.) Substituting into equation 11, it will be seen that the leakage reactance expression consists of a constant component and another which decreases with frequency. At relatively higher frequencies it can be correctly assumed that the leakage reactance is directly proportional to frequency.

In general, however, leakage reactances could be made as small as possible with perhaps short circuit considerations as the limiting factor. Thus for the purposes of high frequency transmission the nominal value of leakage reactance as is now commonly used, should be applicable.

#### 4.7 Generators

The evaluation of high frequency generators will be on the basis of equal generated voltages and equal mechanical speed of rotation inasmuch as most low-head hydro generators have slow moving rotors. The basic expression<sup>10</sup> for the generated voltage in conductors subjected to a varying magnetic field is:

$$E = Kf\phi_m N \quad \text{or} \quad KfB_m AN \dots\dots\dots 12$$

\*Note: If two circular cross-sectional areas are  $A_o$  and  $A_f$  at frequencies  $f_o$  and  $f$ , then from inverse proportionality

$$\frac{A_o}{A} = \frac{f}{f_o}, \text{ i.e., } \frac{\pi r_o^2}{\pi r_f^2} = \frac{f}{f_o}, \text{ therefore, } r_f = r_o \sqrt{\frac{f_o}{f}}$$

The frequency is related to the mechanical speed by

$$f = \frac{nP}{120}$$

where  $n$  is the number of revolutions of the machine field,  $P$  is the number of poles, and  $A$  is the cross-sectional area of the pole. If  $f$  is substituted into equation 12, the result is:

$$E = K_n P B_m A N / 120 \dots 13$$

If  $n$ , and  $N$  are held constant,

$$E = K_1 p B_m A$$

If yet the same maximum flux density is desired

$$E = K_2 p A \text{ -- an inverse proportionality.}$$

In other words, for the same generated voltage the number of poles would have to be increased even though the size of each pole will be proportionately decreased. For comparatively very high frequencies a greatly increased number of poles may be needed and this may perhaps pose some constructional difficulties that not only make the cost of such machines fairly expensive, but also add more weight to an already bulky machine.

However, if it were possible to operate at an increased speed, then holding  $p$ ,  $N$  and  $B_m$  constant in equation 13,

$$E = K_3 n A$$

Under the condition of greater speed, the power per weight of the machine is obviously increased. To obtain this greater speed, which alone can give this advantage of

greater power per weight, direct connections of generators to water wheels must be avoided, or else radically new methods of generator designs will be called for.

#### 4.7.1 Armature Reaction

When a synchronous machine is delivering power, the current in its windings creates a magnetomotive force. This magnetomotive force can be divided into two components, one of which the armature reaction passes through the main magnetic circuit where it causes a distortion of the main excitation field, and the other, armature leakage reactance which has a fairly localized influence.

##### 4.7.1.1 Armature Leakage Reactance

For purposes of calculation,<sup>11</sup> the armature leakage flux may be further subdivided into four parts:

- 1) slot leakage flux
- 2) end-connection leakage flux
- 3) zig-zag leakage flux, and
- 4) belt leakage flux.

The leakage reactance is then the product of the interlinkages between the armature leakage flux per unit of current, and the time rate of change of the flux. Reactance formulae for each of these components are known<sup>11</sup> but are not presented in this paper. But it can be safely said that there is a direct proportionality between them

and frequency, since the other variables that make up the expressions are related entirely to the physical dimensions in the armature and these in themselves are virtually unaffected by frequency.

#### 4.7.1.2 Armature Reaction

The effect of the armature reaction<sup>11</sup> on the field is dependent upon the magnitudes of the armature current, the arrangement of the windings, the magnetic circuits, and the power factor.

#### 4.8 Corona Losses

Typical line designs take the influence of corona into consideration. The phenomenon of corona has not yielded itself to exact mathematical analysis and as such most work on it has been empirical. It is known that corona is induced when the voltage gradient at the surface of a conductor exceeds a certain value, the Visual Corona Onset Voltage Gradient. An empirical expression<sup>12</sup> for this voltage is:

$$E_o = 30m(1 + 0.301/\sqrt{r}) \quad \text{KV/cm}$$

where  $m$  = an empirical constant determined on the basis of surface condition of the conductor, and

$r$  = the radius of the conductor.

The expression obviously shows no frequency influence in inducing the Visual Corona Onset Voltage so that if a line is designed below this value, corona will not be a factor

to be reckoned with in choosing a frequency for power transmission. However, once the onset voltage gradient is exceeded, corona, with its accompanying losses, occurs. For such a case, using Nigol and Cassan's formula<sup>12</sup> the corona loss in kilowatt per conductor per mile is:

$$P = Kfr^2 \frac{dP}{2\pi} E_e^2 \ln\left(\frac{E_e}{E_o}\right)$$

where  $f$  = frequency

$r$  = conductor radius in cm

$K$  is an empirical constant dependent on weather and conductor surface condition,  $E_o$  is the Visual Corona Onset Voltage Gradient, and  $E_e$  is a voltage dependent on the combined<sup>12</sup> influence of the line voltage and mechanical composition and disposition of the conductor. Also  $dP$  is a value calculated from the line dimension and voltage.

Thus, once corona sets in, the loss per mile per conductor increases proportionately with frequency -- a rather gloomy picture, particularly if it is noted that contemplated frequencies may range from 80 to 250 cycles per second. But corona losses are also affected by the level of voltage under which the conductor operates. The corona power loss formula has a voltage squared factor which shows clearly the greatly increased losses associated with higher operating voltages.

Two important points are worth recalling at this stage. Firstly, the two-frequency method offers a possibility



of transmitting power that is greater than the line characteristic power without risking the excessively higher voltages that would otherwise result in an unmatched half-wave length line. (It might be useful to take a look at the voltage profiles of the three cases given under "Tuning Possibilities of the Two-frequency Line.") Secondly, the anticipated tuning devices of the two-frequency method are integral parts of the frequency converting devices and would be of very little extra cost should it be necessary to obtain specially designed converters to provide for yet greater tuning capacities. The first point becomes more significant when it is considered that the lower voltage level of transmission line as is made possible in the two-frequency method, might make the difference between a transmission line being or not being stressed to visual corona onset voltage gradient; it might very well be the factor responsible for obtaining a design that may be corona free for small overloads on the line.

## CHAPTER FIVE

### FREQUENCY CONVERTERS

The operation of the two-frequency mode of power transmission necessitates the use of frequency converting devices, and a survey of existing and possible means of such converting devices will now be presented.

It will be pointed out again that the frequencies contemplated must of necessity be above the standard consumer frequencies, otherwise most of the few advantages that go with the method are likely to be lost. As has been shown in the section under "Frequency Effects," any power transmission device with magnetic circuit will become bulky and unwieldy at lower frequencies. Therefore, the range of frequencies contemplated for extra-long distance lines will be above 60 cycles.

#### 5.1 Classification of Frequency Converters

Frequency changing devices fall within two categories: synchronous and non-synchronous, and a table classifying various devices that are in present-day use is given below. The classification is important because the tie chosen will determine the ease of expanding the system to cope with additional capacity, and the manner in which increased load demands on the systems are met. For a synchronous tie, in the event of a sudden increase in the load of one system,

Synchronous Ties

Synchronous-Synchronous

Induction-Synchronous Frequency Converter

Non-Synchronous Ties

Induction-Synchronous Speed Regulating

Static Electronic Frequency Changer

Induction Motor Set

the demand is met collectively and the proportion to which the separate systems meet this demand is determined by their relative capacities and their respective speed regulations. Thus two systems of the same capacities and speed regulations will share loads equally. In addition, synchronous ties can be easily expanded to meet increased demand.<sup>16</sup> These are the distinguishing features between the synchronous and non-synchronous ties.

Another important distinction is made on the basis of frequency coupling. The fixed frequency ratio type of converters does not allow any departure from its design ratio, and any prolonged deviations would break the tie between the connected systems. In general, it is designed to withstand small deviations of short duration only.

Flexible ratio types of converters are designed to accommodate frequency deviations of a more serious and



durable nature. In actuality, no converter is inherently flexible with respect to frequency. The flexibility such as is intended here is one attained through the use of a series of regulating devices. As such, flexible type of converters are economical only in large power projects.<sup>13</sup>

## 5.2 Types of Converters<sup>13, 16</sup>

As shown on the foregoing page, there are many types of frequency changers, but each has some definite characteristics that make it best suited for certain special requirements of system performance. These devices with their special characteristics will now be described.

### 5.2.1 Synchronous-Synchronous

A synchronous-synchronous converter is simply a synchronous motor driving a synchronous generator, with each of the units having separate d-c excitation. It is the set most commonly found. It is of a fixed frequency ratio type in which case any tendency of the systems connected by it to depart from this ratio may pull it out of synchronism.

### 5.2.2 Induction Motor Set

Both the induction and the synchronous sets are similar in operation except that the induction motor slip responds to load changes and consequently brings about frequency deviations as the load is varied.

### 5.2.3 Induction-Synchronous Speed-regulating

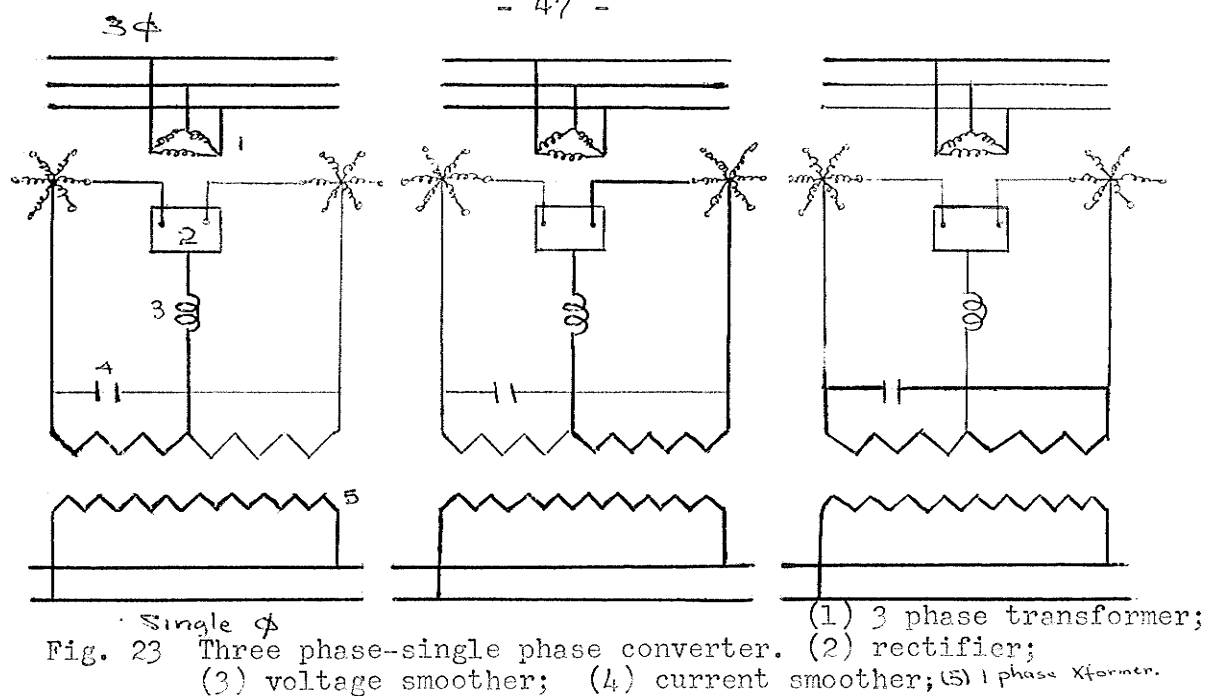
In essence this is a Scherbius<sup>13</sup> type of speed control for operating induction motors above and below synchronism. It is mostly used as a converter in cases where frequencies may vary within wide limits.

### 5.2.4 Electronic Static Frequency Changer

The electronic static frequency changer is a static equipment which in its basic form will convert power of a given frequency to direct current power. This conversion is by means of grid controlled rectifiers, the operation of which is now common knowledge. The conversion of one frequency to another involves a process of AC-DC and a reverse process for DC-AC, but what is considered here is a converter that performs the entire AC-AC operation in one unit.

#### 5.2.4.1 Basic Principles of the Static AC-AC Converter<sup>18a</sup>

The operating principles can be illustrated by means of a three phase to single phase power conversion. These principles apply to a three phase to three phase network since the same considerations hold for each phase. The diagram essential for an understanding of the basic operation is shown in figure 23. These diagrams show how single phase current is obtained from the main three phase line.



The cathode current of the grid controlled rectifier flows through the single phase transformer in one direction at first and then the other. The single phase current and voltage are far from sinusoidal and to overcome this shortcoming the inductance "3" and the condenser "4" are introduced into the network to smooth out the voltage and current respectively. The voltage and current smoothers may not be as simplified as indicated in the diagram, but in the main they are no more than a combination of capacitors and inductances of varying capacities to meet design specifications. The smoothing condenser has a fairly large capacity so that apart from its current smoothing function, it can supply a good deal of reactive current into the single phase network.

In figure 23 the heavy lines show the flow of reactive power both in the conversion circuit and the single phase

network. The significance of this as far as the two-frequency line is concerned is that only active power flows in the transmission line from the sending end up to the converter, while reactive power, certainly a by-product, can be supplied to the load.

Again it will be emphasized that the entire process is one of AC rectification and DC inversion, but these two actions have only been performed in what looks like a single stage. By this means the cost duplication is considerably reduced. The diagram for the three phase AC-AC conversion is given in figure 24.<sup>18b</sup>

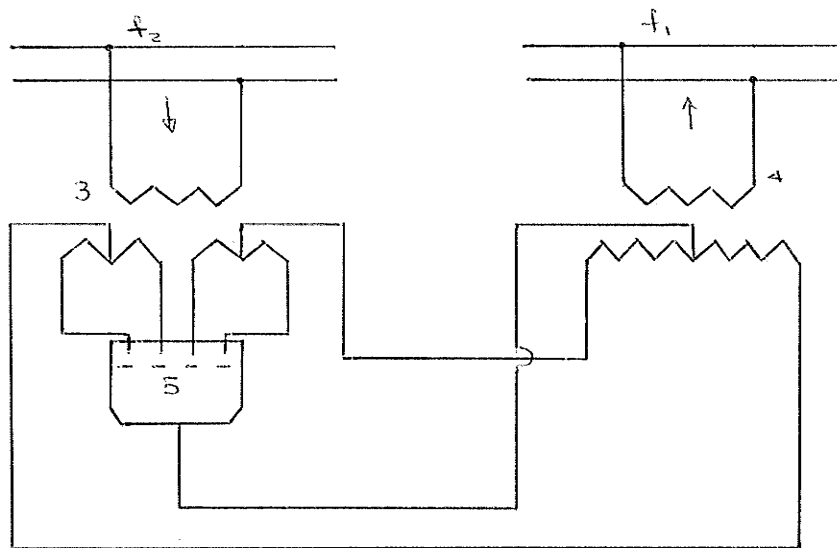


Fig. 24 One phase only of a 3 phase-3 phase AC-AC converter.  
3 and 4 - transformers; 5 - converter.

To take care of the frequency deviations that are likely to occur in either of the networks, this device can be equipped with auxiliaries which enable it to have variable frequency ratios.

### 5.2.5 Induction-Synchronous Frequency Converter

Neither of the devices discussed in the foregoing provides an electromagnetic continuity such as would make the entire system amenable to one continuous analysis. What is desirable is a device that provides a sort of cable tie. One such device is the induction-synchronous frequency converter.<sup>13</sup>

It is known from Induction Machine theory that if the stator of the machine is excited from a source of a constant frequency, a revolving magnetic field is set up and if the rotor were held fixed, i.e., zero slip, a voltage of the same frequency as the stator would appear in the rotor terminals. However, if the rotor were allowed to move it would try to keep abreast of the flux. But if a load is attached to the rotor, it would then slip behind the rotating field of the stator, the extent of this slip being governed by the load. The rotor frequency at a given slip  $S$  is then

$$f_r = S f_s$$

where  $f_s$  is the stator frequency.

A diagrammatic representation of the machine is shown in figure 25.

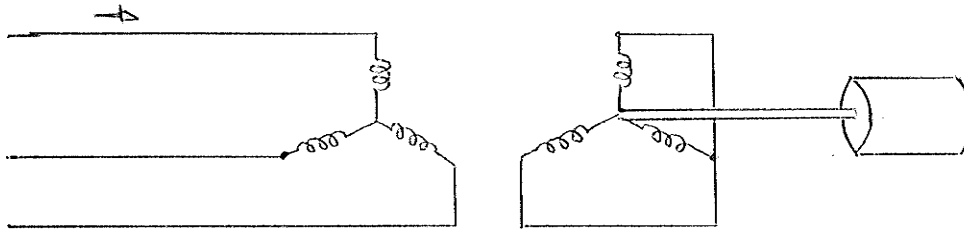


Fig 25 Induction motor diagram.

The power flow for such a machine is governed also by the slip. The power expended in the rotor is<sup>10</sup>

$$P_r = sP_s$$

where  $P_s$  is the input power at the stator and the power transferred to the load is  $(1-s)P_s$  under normal motoring operation. This rotor power is virtually lost and is the cause of the low efficiencies associated with the induction machines.

For frequency conversion purposes no power is actually lost except the usual losses associated with rotating machines. In this case the power transmitted across the air gap is available for consumption either through the shaft of the rotor which could be used to drive a synchronous generator, or through the rotor which derives its energy through normal electromagnetic processes as with the ordinary transformer.<sup>10</sup> The two powers may or may not be of the same frequency, but either of them could be made to assume any predetermined frequencies, made equal or otherwise.

Assuming, therefore, that both these secondary frequencies are equal, an arrangement for frequency conversion and transmission, along the same line is as shown in figure 26. The diagram is a graphic illustration of the strong tie between the two A.C. systems. They are connected by a common magnetic field, and to all intents and purposes behave as an electrical cable tie, save, of course, for the much larger impedance drop than would normally be encountered in a cable. But this difference, as will be shown later, poses no problem for the two-frequency line. The synchronous generator could supply an entirely different line, so long

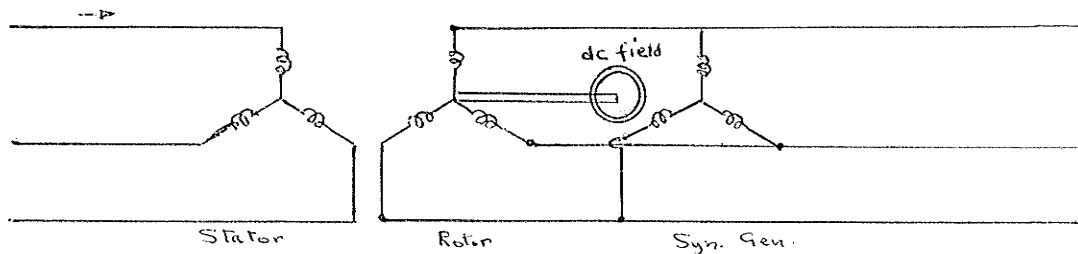


Fig. 26 Induction Synchronous Frequency Converter.

as it is an infinite bus. The only limitations imposed on the utilization of the generated energy of the synchronous generators are that the feeder line be such that would ensure system stability, and no greater demand than is provided for by the transformation ratio of the set can be made of it. But if the tuning advantages are not to be lost, both the rotor and the synchronous machine

should feed the same line.

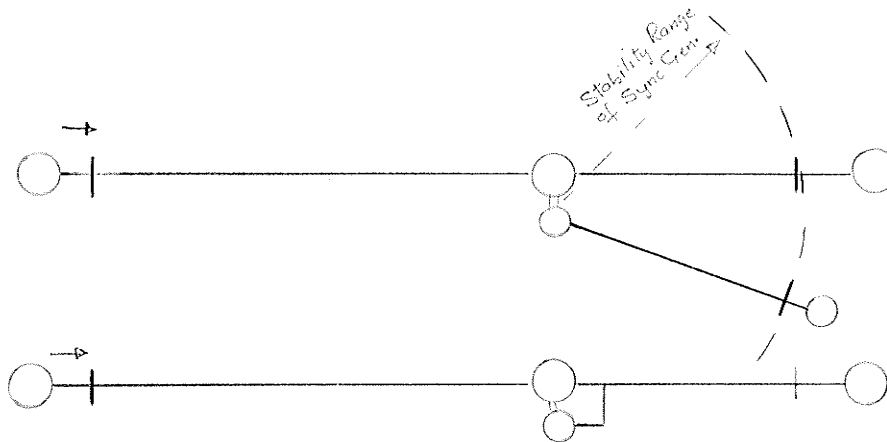


Fig. 27 Connections for two-frequency line.

The possibilities expressed above are illustrated in figure 27. The amount of power available at each bus, as explained earlier, depends entirely on the frequency transformation ratio. Thus by a judicious choice of primary frequency, estimated power demands of an intermediate city could be met while the remaining continues on an onward, perhaps much further, journey. This way, the hitherto unsolved problem of intermediate tapping of half-wave length power is remedied even though the problem is only sidestepped.\*

A schematic diagram for a possible double line system is shown in figure 28. If bus 5 is within the stability range of the synchronous generator, some of its power could be diverted to the main line. By regulating the excitation of a synchronous machine on an infinite bus, the machine can be made to supply lagging or leading Kvars.

It is in the light of this that the section "Tuning

\*The interpretation of this statement is subject to the limitations as explained on page 13.



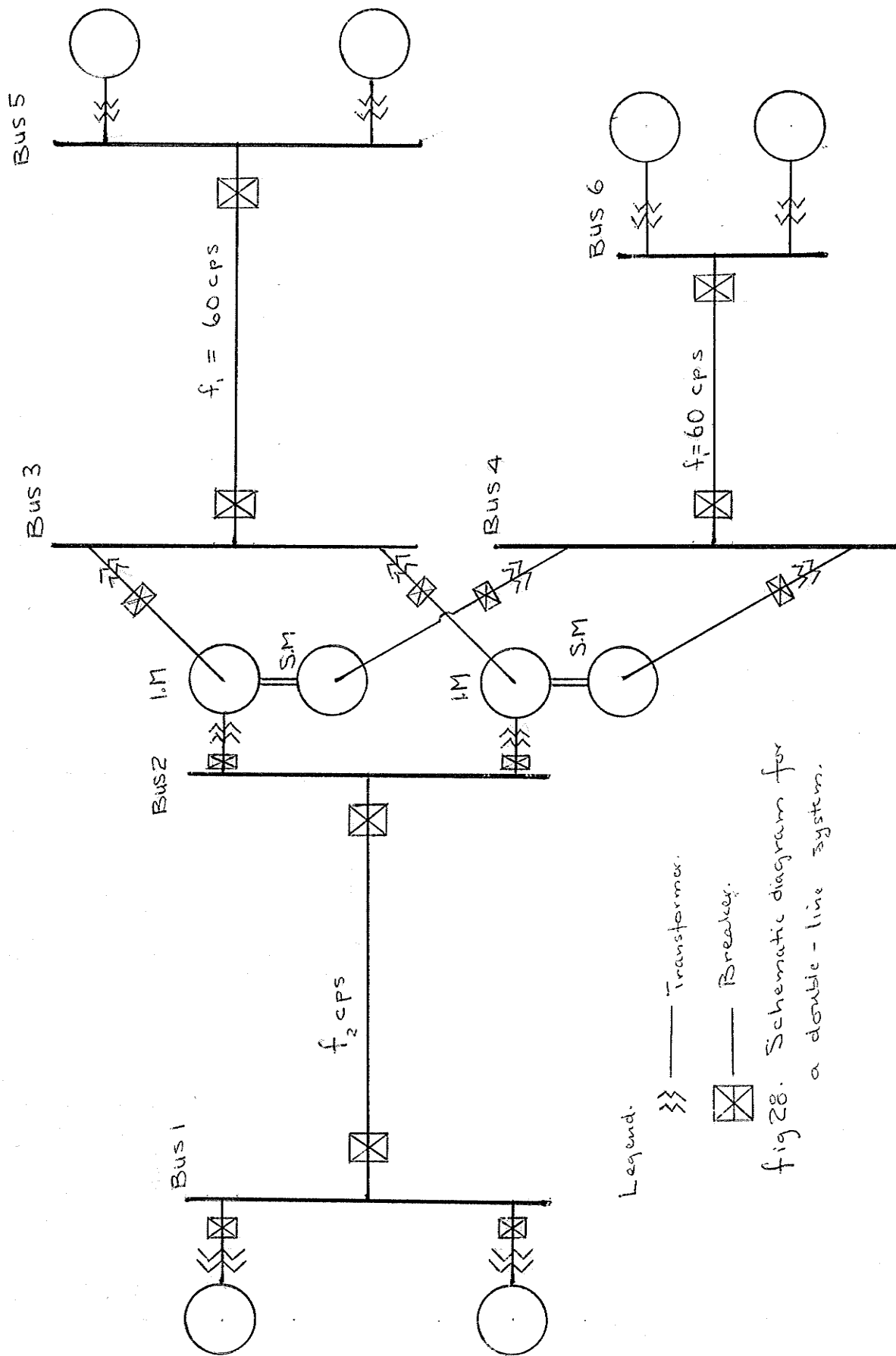


fig 28. Schematic diagram for a double-line system.

Possibilities of the Two-Frequency Line," should be closely examined. Taking advantage of the tuning capacity of the synchronous generator, line voltages for overload conditions, (i.e., loads greater than surge impedance loading,) could be held down to a certain degree, at least much below what they would otherwise have been.

Induction-synchronous frequency converters can theoretically be built to any size.<sup>13</sup> One of the largest built so far was in 1934 and had a 35MW capacity.<sup>19</sup> It seemed to have been at a disadvantage because existing synchronous-synchronous converter sets under the same circumstances proved to be more efficient. Induction machines envisaged under frequency operations need not have disproportionately bulkier rotors as do those of the 60/25 cycle machines, and this could improve their performance.

#### 5.2.6 Static Frequency Doubler<sup>20</sup>

Another device which has some exciting possibilities for higher frequency work is the static frequency doubler.

In its simplest form the doubler is a magnetic core device, the basic structure of which is shown in figure 29. The core magnetic characteristic is shown (idealized) in figure 29b. The two outer windings are series connected but are so wound that the voltages induced by an exciting source at  $P_1$  and  $P_3$  are not only equal, but also add

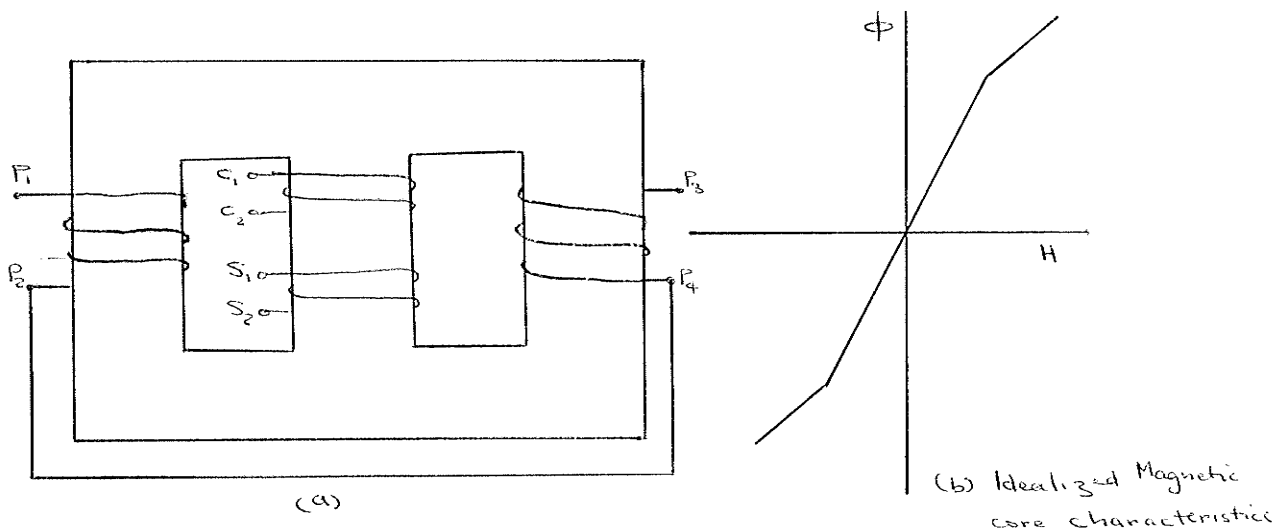


Fig. 29 Frequency doubler and idealized BH curve.

together. This condition also means that the fluxes set up by an exciting current flowing through the windings aid each other everywhere except at the middle leg, where they are in opposition and tend to cancel each other out.

If now a direct current source is applied at terminals  $C_1$ - $C_2$  and is of sufficiently high voltage to push the outer magnetic paths very close to saturation, then if a current now flows through  $P_1$  and  $P_2$ , one of the fluxes will push the nearly-saturated leg further into saturation, while the other pushes it out of saturation. But because of the non-linearity of the magnetic circuit, these two effects, while opposite, are not equal. The contribution of each leg to the centre leg is sketched on figure 30. The flux which actually flows through the centre leg is the resultant

of these two. It is seen to be at twice the frequency of the original exciting source. Therefore, a voltage of twice the frequency appears at the terminals  $C_1-C_2$ .

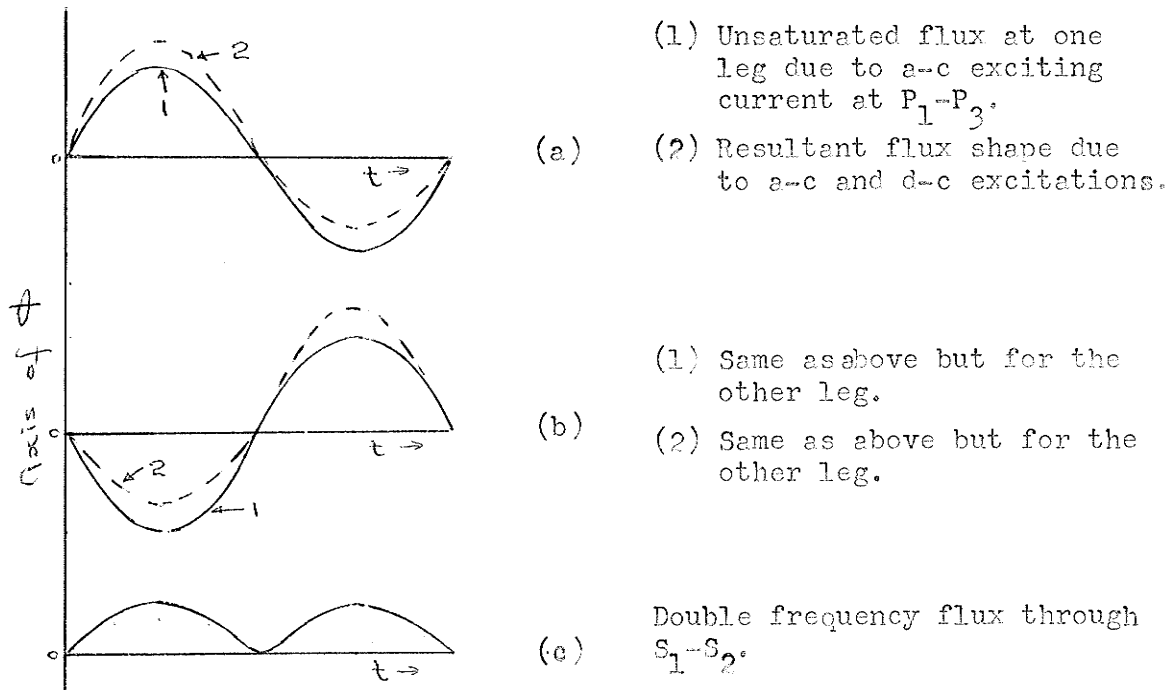


Fig. 30 Sketch illustrating how doubling is obtained.

With a doubler installed at the sending end terminal of a transmission line, generators need not have the large number of poles required if they were operated at relatively higher frequencies.

The doubler was conceived as a means of obtaining high speeds from induction motors. An experimental model having a 1.5kw capacity and approximately 65% efficiency, has been built. It is still in its research stage but there is no doubt that it holds out very high promise of higher frequency application.

## CHAPTER SIX

### TUNING POSSIBILITIES OF THE TWO-FREQUENCY LINE

Both the induction frequency and the electronic frequency converters have possibilities for tuned transmission of power. There is a number of reasons why a tuned line is desirable. It permits a greater flow of power to the load and where lines are lossy, losses are least when the load is matched to the line. Above all, it permits the transfer of powers greater than the characteristic line load, while keeping the line voltages well below what they would have been for the unmatched loads.

Tuning can be obtained either by single or double stub-matching.<sup>17</sup> In single stub-matching, reactance is placed in shunt at a certain distance from the load. (See figure 31a) The value of the reactance and the distance are such as to present a matched load to the source. The resulting voltage profile is flat up to the stub position and thereafter assumes a shape dependent on the load condition. (See figure 31b.)

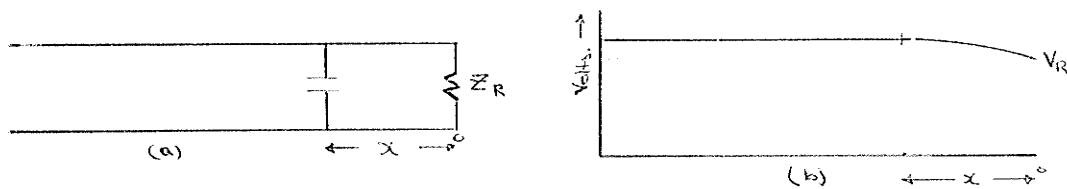


Fig. 31 Single stub line diagram and possible voltage profile.

If the values are compromised, the line is no longer matched and the flat portion of the voltage profile will no longer be flat. But the overall profile will be better than that of the unmatched line.

The method of evaluating the distance, the reactance, and the voltage are available in any standard text on Electromagnetic Wave theory.

The appropriate shunting reactance may be capacitive or inductive. Static condensers and reactors can be employed as shunts in power work. For this purpose, a synchronous condenser or reactor could be employed just as well. Synchronous equipments provide very fine control and have the added advantage of being able to supply and absorb reactive power.<sup>22</sup>

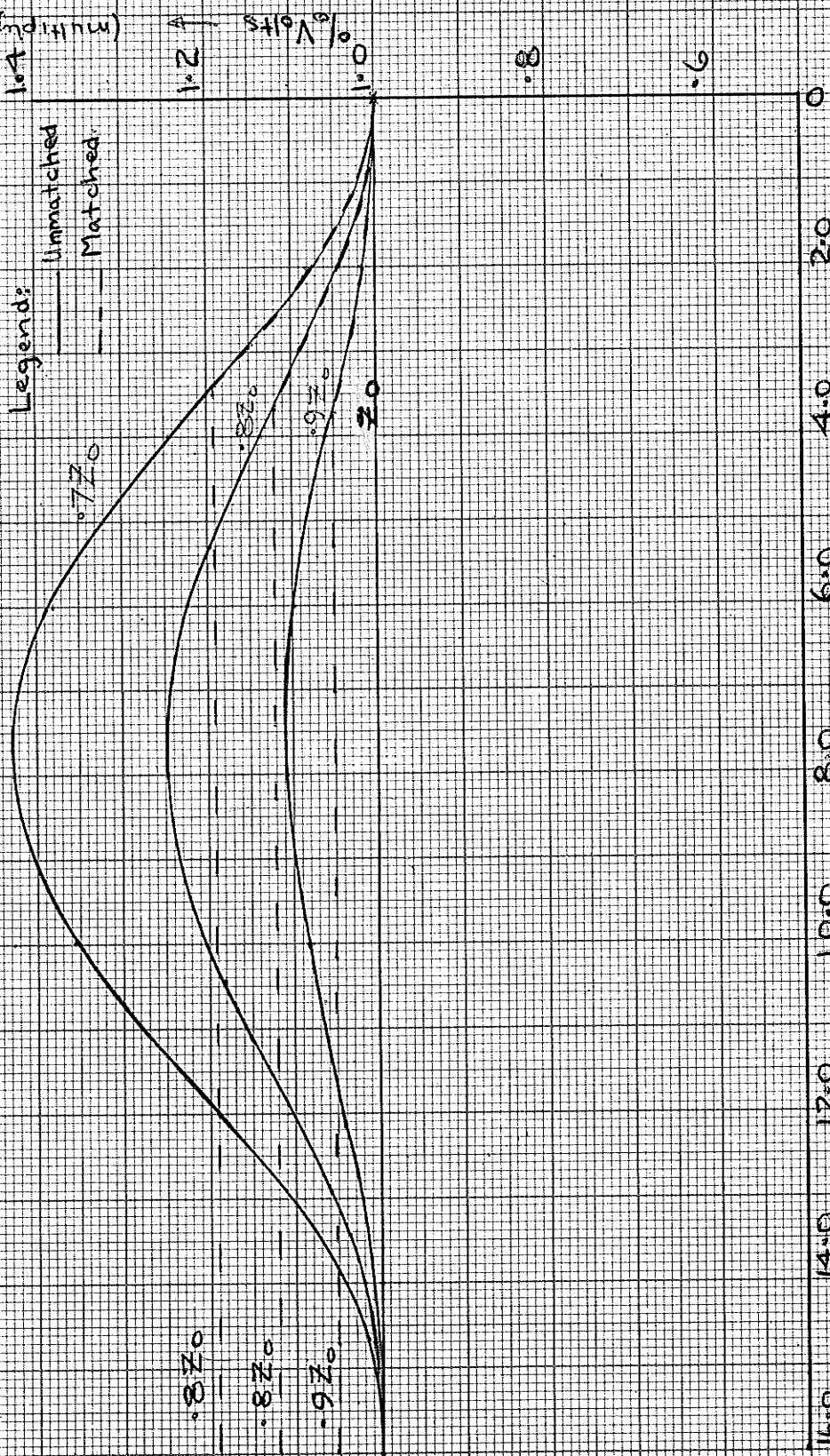
Results are presented for the voltage profile corresponding to certain degrees of mismatch. (See figure 32.) The following plots are all made in figure 33.

- a) Maximum expected line voltage versus load, when the line is single stub-matched.
- b) The required shunting susceptance versus load.

It can be seen that if loads greater than the line characteristic load are to be supplied, tuning will keep voltages within not too disastrous levels. Besides, for long lines there has always been the problem<sup>15</sup> of maintaining voltages at all points within  $\pm 10$  per cent of design values so that for underload or overload conditions tuning could help keep the voltages within these limits.

Voltage Profiles for  
various receiving  
end loads  
(multiply by 100)

Legend:  
— Unmatched  
--- Matched



Distance in hundreds of miles from Load.

Fig 32

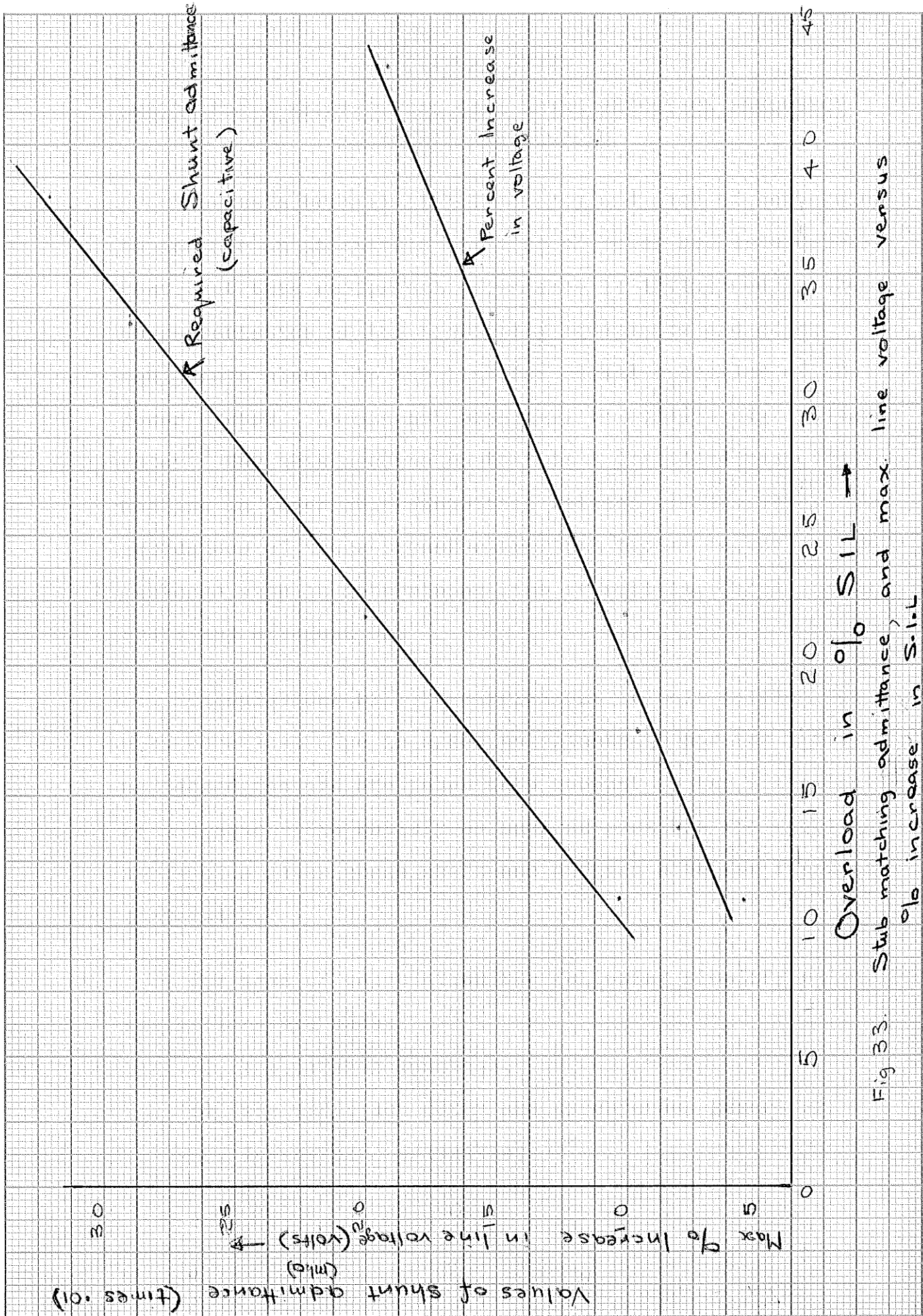


Fig 33. Stub matching admittance, and max. line voltage versus % increase in S.I.L.



## CHAPTER SEVEN

### DESIGN FOR A HYPOTHETICAL LINE USING INDUCTION FREQUENCY CONVERTER SYNCHRONOUS

The main transmission frequency and site of conversion to achieve an acceptable transfer of power are interdependent, but this reciprocal dependence is influenced by the characteristics of the converting equipments and other equipments placed in the line for normal operation of the power system. It has not been easy to build a high voltage rotor<sup>13</sup> for this machine so that in a practical case the machine is invariably sandwiched between transformers. The impedances of these transformers then become factors to be reckoned with in determining line overall impedance, phase shift and attenuation.

Calculations will be carried out in this section for a hypothetical line in order to illustrate how the above-mentioned factors are taken into account in the determination of a frequency to satisfy as much as possible the requirements of a given phase shift and a fixed site for frequency conversion.

For ease of calculation the following simplifying assumptions are made:

- a) the line is lossless,
- b) the induction motor and synchronous generator losses are negligible,

- c) irrespective of frequency, the transformers have an impedance of .1 pu since in general, transformer impedances are designed for the purpose of limiting short circuit currents to reasonable values.

NOTE: For want of data on induction converters as would seem practical for large power transmission, the reactances used for the stator and the rotor of the converter are those of the 35MW capacity mentioned on page 54. For the purpose of this chapter, exact values of reactances are unnecessary.

The reactance of the synchronous generator is assumed equal to that of the induction machine rotor. The stator and rotor reactances are 6.6 and 5.4 per cent respectively.

#### LINE DATA

Line length	= 900 miles
Distance of conversion site from load = 100 miles	
Phase shift of transmission line at 60 cps	= .204 rads/100 miles or 11.7°/100 miles
Characteristic Impedance	= 1.22 per cent

#### PROBLEM

Determine the operating frequency such that the phase difference between the terminals of the line is approximately 190°.

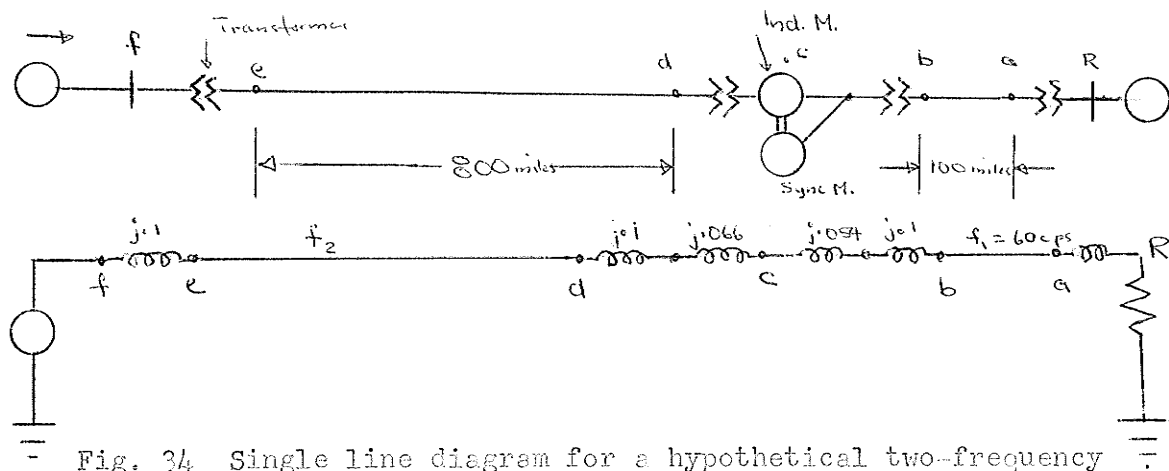


Fig. 34 Single line diagram for a hypothetical two-frequency line using an induction synchronous frequency converter.

SOLUTION

Suppose receiving end voltage  $V_r = 1.0/0^\circ$

$$\therefore V_a = V_R \times \left( \frac{Z_R + Z_{aR}}{Z_R} \right)$$

Substituting,

$$\begin{aligned} V_a &= 1.0/0^\circ \frac{(1.22 + j.1)}{1.22} \\ &= 1/4^\circ 41 = 1 + j.082 \end{aligned}$$

$$\text{But, } V_a = V_a^+ (1 + k_a)$$

where  $k_a$  is reflection coeff at "a" and  $V_a^+$  is the incident voltage at "a".

$$\begin{aligned} k_a &= \frac{Z_a - Z_o}{Z_a + Z_o} = \frac{1.22 + j.1 - 1.22}{1.22 + j.1 + 1.22} \\ &= .041/87^\circ 39'' \end{aligned}$$

$$\begin{aligned} \therefore V_a^+ &= \frac{1/4^\circ 41''}{1 + .041/87^\circ 39''} \\ &= 1/2^\circ 20'' \end{aligned}$$

$$\text{But } V_b^+ = V_a^+ e^{\beta d}; \text{ and } ab = 100 \text{ miles}$$

$$\begin{aligned} \text{Hence } V_b^+ &= (1.0/2^\circ 20'')(1/11.7^\circ) \\ &= 1.0/14.03^\circ \end{aligned}$$

$$\therefore V_b = (1 + k_b) V_b^+$$

$$\begin{aligned} \text{But } k_b &= k_a e^{-j2\beta d} = (.041/87^\circ 39'')(\angle -23.4) \\ &= .041/64^\circ 15'' \end{aligned}$$

$$\begin{aligned} \text{Hence } V_b &= (1.0/14.03^\circ)(1 + .041/64^\circ 15'') \\ &= 1.0177/16.1^\circ \end{aligned}$$

$$\begin{aligned} \text{Now } Z_b &= Z_o \frac{(1 + k_b)}{1 - k_b} \\ &= 1.22 \frac{(1 + .0178 + j.037)}{1 - .0178 - j.037} \end{aligned}$$

$$= 1.245 + j.092 = 1.245 \angle 4.14^\circ$$

$$\text{Now } V_b = \frac{Z_b}{Z_b + Z_{bd}} V_d$$

$$\begin{aligned} \therefore V_d &= \frac{Z_b + Z_{bd}}{Z_b} V_b \\ &= \frac{1.245 + j.092 + j.319}{1.245 \angle 4.14^\circ} \\ &= 1.051 \angle 14.14^\circ \end{aligned}$$

$$Z_d = 1.245 + j.4112$$

$$\begin{aligned} \therefore k_d &= \frac{Z_d - Z_o}{Z_d + Z_o} = \frac{1.245 + j.412 - j1.22}{1.245 + j.412 + j1.22} \\ &= .1645 \angle 77.01^\circ \end{aligned}$$

$$\text{Now } V_d = V_d^+ (1 + k_d)$$

$$\begin{aligned} \therefore V_d^+ &= \frac{1.051 \angle 14.14^\circ}{1 + .1645 \angle 77.01^\circ} \\ &= \frac{1.051 \angle 14.14^\circ}{1.048 \angle 8.8^\circ} \\ &= 1.005 \angle 5.16^\circ \end{aligned}$$

The point to remember at this stage is:

between  $V_s$  and  $V_r$  the phase difference is approximately  $190^\circ$ .

Hence, the permissible phase shift through the transmission line and the reactance  $Z_{ef}$  will be approximately  $190^\circ - 5^\circ$ .

Hence, from here on, the solution will be by trial and error.

$\therefore$  assume the phase angle of  $V_e^+$  that would eventually lead to an overall phase shift of  $190^\circ$  at 'e' is  $185^\circ$ .

$$\text{i. e., } V_e^+ = V_d^+ \angle \theta^\circ = |V_d^+| \angle 185^\circ$$

$$\begin{aligned}\text{But } V_d^+ &= 1.005/5^\circ 16'' \\ \phi &= 185 - 5^\circ 16'' = 179^\circ 44''\end{aligned}$$

In other words, this should be the total phase shift when the voltage traverses the 800 miles. Hence, the phase shift per 100 miles

$$= \frac{179^\circ 44''}{8} = 22.4^\circ \text{ approx.}$$

Hence, the operating frequency  $f_2$  is

$$f_2 = 22.4 \frac{60.0}{11.4} = 118 \text{ cps.}$$

For this phase shift of  $179^\circ 44''$

$$\begin{aligned}k_e &= k_d e^{-j2(179^\circ 44'')} \\ &= (.1645/77^\circ 1'')(1/-359^\circ 28'') \\ &= .1645/-282^\circ 27''\end{aligned}$$

If solved

$$V_e = 1.05/193.85^\circ$$

Phase angle is already too large.

$$\text{Try } V_e^+ = V_d^+ / \phi^\circ = V_d^+ / 175^\circ$$

$$\text{Since } V_d^+ = 1.005/5^\circ 16''$$

$$\phi = 175 - 5^\circ 16'' = 169^\circ 44''$$

∴ required phase shift per 100 miles

$$= \frac{169^\circ 44''}{8} = 21.3^\circ$$

Hence required frequency  $f_2$  is

$$f_2 = \frac{21.3 \times 60}{11.7}$$

$$= 109 \text{ cps.}$$

$$\begin{aligned}\therefore k_e &= k_d e^{-j2(169^\circ 44'')} \\ &= (.1645/77^\circ 1'')(1/-339^\circ 28'')\end{aligned}$$

$$= .1645 \angle -262^{\circ} 27''$$

$$\begin{aligned} \therefore V_e &= V_e^+(1 + k_e) \\ &= (1.005 \angle 175^{\circ})(1 + .1645 \angle -262^{\circ} 27'') \\ &= .997 \angle 184.45^{\circ} \end{aligned}$$

$$\begin{aligned} \text{Now, } Z_e &= 1.22 \frac{(1 + k_e)}{1 - k_e} = 1.22 \frac{(.997 + j.163)}{1.0216 - j.163} \\ &= 1.17 \text{ ohms} \end{aligned}$$

$$\therefore \frac{V_e}{V_f} = \frac{Z_e}{Z_e + Z_{fe}} = \frac{1.17}{1.17 + j.1}$$

$$\begin{aligned} \therefore V_f &= \frac{(1.17 + j.1)}{1.17} V_e \\ &= \frac{(1.17 + j.1)}{1.17} (.997 \angle 184.45^{\circ}) \\ &= (1 \angle 4.87^{\circ})(.997 \angle 184.45^{\circ}) \\ &= .997 \angle 189.52^{\circ} \end{aligned}$$

This is nearly  $190^{\circ}$  and the frequency satisfying the conditions is about 110 cps.

## CHAPTER EIGHT

### SUMMARY AND CONCLUSION

The concept of two-frequency transmission as intended here involves the use of frequencies higher than the standard consumer frequencies. These higher frequencies may bring about appreciable savings on the magnetic materials of transformers and other magnetic core devices. But, it is quite likely that heat dissipation may become more difficult as the magnetic material becomes greatly reduced.

Quite rightly the power engineer is more concerned with losses than with anything else. Magnetic core losses on the whole increase rapidly with frequency, but are less rapid when considered for assembled units, like generators and transformers, all having the same capacity. The increased losses on a reduced surface area will further worsen the problems of heat dissipation.

On dielectrics there is generally a decrease in strength as frequency increases. Dielectric core losses of insulation will increase rapidly as frequency increases, and these losses may bring about considerable heating and deterioration of the insulation. Transmission line losses may be negligible.

Increase in line and equipment reactances are of no concern also, since these reactance values are all combined

in arriving at an operating frequency. However, the leakage reactance of the armature may be some source of trouble in voltage regulation.

A rise in frequency will cause a directly proportional rise in corona losses of transmission lines. But since corona is not induced by frequency, the only solution would be to design lines that induce minimal corona losses or lines which are entirely corona free.

The other losses arise as a result of the effect of skin effect on resistance. Although the resistance, and hence the losses, can always be reduced by the use of bundled phase conductors, in relation to those losses occurring at 60 or 50 cycles, higher frequency losses will always be greater. However, where such losses become significant, advantage might be taken of the tuning capabilities of the converter system. This might decrease the losses.

On converting devices there is a fairly wide range from which to make a choice. This variety makes it possible to obtain the most practical and economical set for any given situation. Theoretically at least, the induction machine seems to be the ideal for extra-long distance work. This machine has generally not met with favour<sup>19</sup> in the opinions of engineers, but it is hoped that the unique possibilities it offers will draw immediate attention to it.



It is not the purpose of this thesis to delve into an economic consideration of the two-frequency mode of power transmission. But it can be said that, in the entire analysis, there is no single device or suggestion that is so obviously uneconomical and hence undeserving of consideration. The idea of higher frequencies for power transmission is not new. In fact, "up to about 1890, the most common frequency was 133 cycles per second."<sup>22</sup>

Pioneer projects invariably cost more and this has always militated against new ideas which are not very obviously cheaper than existing ones. The two-frequency method is certainly not presented as a universal and final solution to the problems of extra-long distance transmission. It has its problems, too, for which solutions must be found. It is only one way out of a difficult situation and this thesis is merely trying to serve as a pointer to that way.

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