SOME ECONOMIC CONSIDERATIONS INVOLVED IN FERTILIZER USE

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ABSTRACT

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There were two major objectives in this study. Several production functions were analyzed to determine which function provided the best estimate of the fertilizer-yield response relationship. A linear programming analysis based on an existing farm in central Manitoba was employed to investigate the ability of fertilizer to compete with other production inputs for initial operating capital and total operating capital in the farm business.

The data for the production function analysis were obtained from experiments conducted at the Wabowden Substation for the years 1956 to 1958 inclusive. The 1956 data were previously analyzed by Gilson and Bjarnarson. The 1957 and 1958 data were analyzed in this study. The purpose was to provide verification or rejection of the conclusions of the Gilson and Bjarnarson study. An existing farm operation was used on which to base the linear programming analysis.

The significant findings of the above investigation

were as follows.

1. The quadratic function with squared terms provides the best estimate of the fertilizer-yield response relationship. Gilson and Bjarnarson also selected the quadratic function as the best estimating equation.

2. Fertilizer is highly competitive for limited initial capital and total capital with other production increasing inputs in the farm business. With original yields decreases in level of fertilizer use occured only when total capital was very limiting. Levels of fertilizer use were not significantly affected by decreases in initial capital or increases in fertilizer cost. A reduction in yields decreased the marginal productivity of fertilizer in programs computed with limited total capital.

ACKNOWLEDGMENTS

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Appreciation is expressed to the member of the Carman District Farm Business Association who allowed his farm to be used as the basis for the linear programming analysis.

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CHAPTER I

INTRODUCTION

Agriculture in Canada is presently in a critical phase of its development. Historically, industry has progressed from the stage in which primitive methods were used to produce a limited output to the present stage where skilled technical and administrative abilities are employed to produce an output restricted only by the inherent conditions of maximum profit. Agriculture has passed the primitive subsistence stage and is now developing into a highly commercial, competitive business. It has as yet not reached a stage of development comparable to that of other industries. In the past, most attention has been given to the physical and technical aspects of production. In the future, more attention must be centred on the economics of production. In order for the farm operator to compete, the various facets of production must be in balance.

The success or failure of many farmers as they enter this commercial phase of agricultural production will depend on their ability to adapt to change. Many will remain stagnant because they lack the knowledge, financial resources, or managerial ability to accept new ideas and adapt to technological changes. Others will progress because they possess these qualities. The successful farmer must have knowledge of the economic as well as physical aspects of production.

One of the most important factors determining the extent of progress is the use of resources in the production of farm products. One potentially important resource is commercial fertilizer. The purpose of this thesis is to investigate several of the economic aspects of fertilizer as a production input in Manitoba.

With the diversified agriculture in Canada one could assume there would be significant variations in the use of fertilizer among different regions. Western Canada, excluding British Columbia, contains over two-thirds of the cropland in Canada. The production of grain is dominant with smaller areas of livestock and general mixed farming. With the emphasis on grain production, and few livestock to provide manure for fertilization, one would expect commercial fertilizer to be a prominent item in the prairie farmers' operating expense account. However, such is not the case. Preliminary investigations have shown that Manitoba farmers are consistently applying fertilizer at less than the recommended rates.[1] Farming is extensive rather than intensive. Many reasons have been advanced to explain why farmers do not use more

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[1] J.C. Gilson, M.H. Yeh, and G. A. Matheson. "Economic Considerations Involved in Fertilizer Use" (Unpublished progress report, Department of Agricultural Economics and Farm Management, University of Manitoba, 1961).

fertilizer. It is possible that variability in yields and weather, summer fallowing practices, marketing quotas, cash income, and many other factors have a definite influence on the farmers' decision whether or not to use fertilizer.

In 1960 Manitoba farmers were operating approximately eleven per cent of the total improved land in Canada.[2] Their expenditure on fertilizer was only three per cent of the national total of 72.5 million dollars.[3] Within the Province of Manitoba the use of fertilizer gradually increased to a maximum expenditure of $3,377,000 in 1953.[4] The expenditure dropped sharply in 1954 and 1955, remained fairly constant to 1958, and showed a strong upward rise in 1959 to slightly over 2 million dollars.[5] Indications are that farmers may be starting a new upward trend in the use of fertilizer.

Many farmers in Manitoba are using fertilizer at the present time. However, to use fertilizer is not enough. Some attempt must be made to determine the levels of application which will provide the maximum return for the investment made in fertilizer. The purpose of this thesis is to investigate some of the considerations involved in the profit-

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[2]Total improved land includes field, garden, nursery and orchard cropland; summerfallow; pasture; and any other improved land.

[3]Dominion Bureau of Statistics, Quarterly Bulletin of Agricultural Statistics. Ottawa, Canada. January-March, 1961, Page 11.

[4]*Ibid*. January-March 1954. Page 12.

[5]*Ibid*. January-March 1961. Page 17.

able application of fertilizer to cropland in Manitoba.

Objectives

There is no one primary objective in this thesis. Rather, several objectives of equal importance will be considered.

The first objective is the evaluation of several production functions as methods of predicting fertilizer-yield relationships under agricultural conditions existing in Manitoba. The types of equations to be included in the evaluation are the polynomial quadratic, the square root quadratic, and the exponential or Cobb-Douglas equations. The data used in the determination of the best prediction equation is also employed in investigating several other aspects of economic fertilizer application. The determination of the most profitable combinations of nitrogen and phosphorus; the least cost combination of nitrogen and phosphorus; and the marginal rate of substitution between nitrogen and phosphorus will be pursued. Emphasis is placed on the methodological procedure and the determination of the best prediction equation.

A second objective of this study is an investigation of the ability of fertilizer to compete with other production inputs for the available operating capital in the farm business. Several situations will be considered in order to determine the effect of varying levels of yield and available capital on the use of fertilizer. The price of fertilizer will be varied to determine the effect on the use of fertilizer, and other farm enterprises. Linear programming, a relatively new research technique, is employed for these purposes.

Scope

The data for the empirical analysis of the fertilizer-yield relationship are obtained from experiments conducted at the Wabowden Substation, Wabowden, Manitoba, during the years 1956 to 1958 inclusive.[6] In the experiment, varying levels of nitrogen and phosphorus were applied to plots of Parkland barley. The results provide adequate data for plotting a complete production surface.

The case study method is used with linear programming to investigate the economic considerations in fertilizer use.

A farm located on Osborne clay soil in Central Manitoba was chosen as a base for the linear programming analysis. Osborne clay is a heavy clay with imperfect drainage. The cooperator on whose farm the study is based is a member of the Carman District Farm Business Association. As far as was

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[6] The Wabowden Experimental Substation, Canada Department of Agriculture, is located in a 4 million acre clay belt of potentially arable land, approximately 400 miles north of Winnipeg.

practicable the level of resources was not changed from that available on the farm. The levels of capital available, yields and the price of fertilizer were varied in accordance with the inquiry into the competitive ability of fertilizer.

Chapter 2 provides a brief theoretical and historical background for the analysis of the production functions. A non-mathematical exposition of linear programming is also presented to provide a basic understanding of this research technique.

Chapter 3 will present the results of the production function analysis.

Chapter 4 will present the results of the linear programming analysis.

In Chapter 5, consideration will be given to conclusions and recommendations for future study.

CHAPTER II

HISTORICAL AND THEORETICAL BACKGROUND

PART I - THE PRODUCTION FUNCTIONS

Historical Review

The theory of production functions is a relatively new concept in the comprehensive discipline of general economic theory. The significant contributions have all been made since 1900. However, several concepts which are important in present day analyses of resource-product relationships were advanced before any serious consideration was given to the actual application of such relationships to agriculture.

The concept of diminishing returns, so important in the theory of production, is credited to Thomas Robert Malthus. He applied the principle to population expansion and the relative contraction of the food supply. He hypothesized that the world population would increase at a geometric rate while the supply of food available increased at an arithmetic rate. Malthus extended his theories in an article published anonymously in 1798.[1]

The Law of Diminishing Returns is defined as: "the

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[1]Thomas Robert Malthus. An Essay on the Principle of Population As It Affects the Future Improvement of Society (London: T. Bensley, 1803, 2nd ed.)

addition of a variable input to fixed inputs results first in total returns which increase at an increasing rate, second in total returns which increase at a decreasing rate, and third in total returns which decrease with increases in the variable inputs".[2]

The principle of the Law of Diminishing Returns was questioned by Justus von Liebig.[3] As an alternative he proposed the Law of the Minimum. The Law stated that plant growth is determined by any essential nutrient which is present at a limited level. When maximum growth is reached, as dictated by the critical factor, further increases in growth are impossible unless the input of the critical factor is increased. While von Liebig did not present any algebraic equations to represent his concept of the crop production function, he implied constant marginal returns. Several later scientists including Bondorff, Plessing, and Boresch have advanced various algebraic interpretations of Liebigs' Law of the Minimum.[4]

Mitscherlich and Spillman, working independently,

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[2]Lawrence A. Bradford and Glenn L. Johnson, Farm Management Analysis (New York: John Wiley and Sons, Inc., 1953), Page 113.

[3]Justus von Liebig, The Natural Laws of Husbandry, (London: Walton and Maberly, 1863).

[4]Earl O. Heady and John L. Dillon, Agricultural Production Functions (Ames, Iowa: The Iowa State University Press, 1961) Page 10.

developed similar exponential yield equations. Mitscherlich is generally credited with being the first person to attempt definition of the algebraic nature of the fertilizer-yield response relationship. He developed an equation of the logarithmic form, $\log A - \log (A-y) = CX$, where A is the maximum yield obtainable by the addition of the input X; and C is a proportionality constant defining the rate at which marginal yields decrease. The constant C is defined for all crops as being devoid of influence by the type of crop, climate, or other environmental factors. The equation proposed by Mitscherlich met with objections because it did not allow for decreasing yields. Decreasing marginal products were permitted but not negative marginal products. Mitscherlich consequently developed a new equation which permitted decreasing total product.[5]

The equation developed by W. J. Spillman[6] is of the form, $Y = M - AR^X$, where X is the quantity of variable input; Y is the quantity of total output; M is the theoretical maximum output attainable by increasing X; A is the output which can be added to total product by increasing X; and R is the coefficient defining the ratio by which marginal productivity

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[5] Ibid, Page 11.

[6] W. J. Spillman, "Exponential Yield Curves in Fertilizer Experiments". U.S.D.A. Technical Bulletin 348, Washington, D.C. 1933.

of X declines. The total output without the addition of the variable input X is M-A. Spillman assumed that constants in the equation were affected by environmental factors.

The Spillman function does not allow a diminishing total product; this inadequacy is also a characteristic of the Mitscherlich equation. The Spillman equation permits either increasing or decreasing marginal productivity, but they cannot be present in the same function. As a result the equation may give misleading results especially at higher input levels. It also assumes that the ratio of marginal products is constant over all ranges of output.

RELEVANT FUNCTIONS

In this thesis three forms of equations are employed to investigate the fertilizer-yield response relationship. The primary objective is to determine the equation which is the best predictor of this relationship in the environmental conditions of Manitoba. In addition, data complementary to the above investigation will be presented. The three functions to be considered are the Cobb-Douglas[7] or power function, the polynomial quadratic equation, and the quadratic square root equation.

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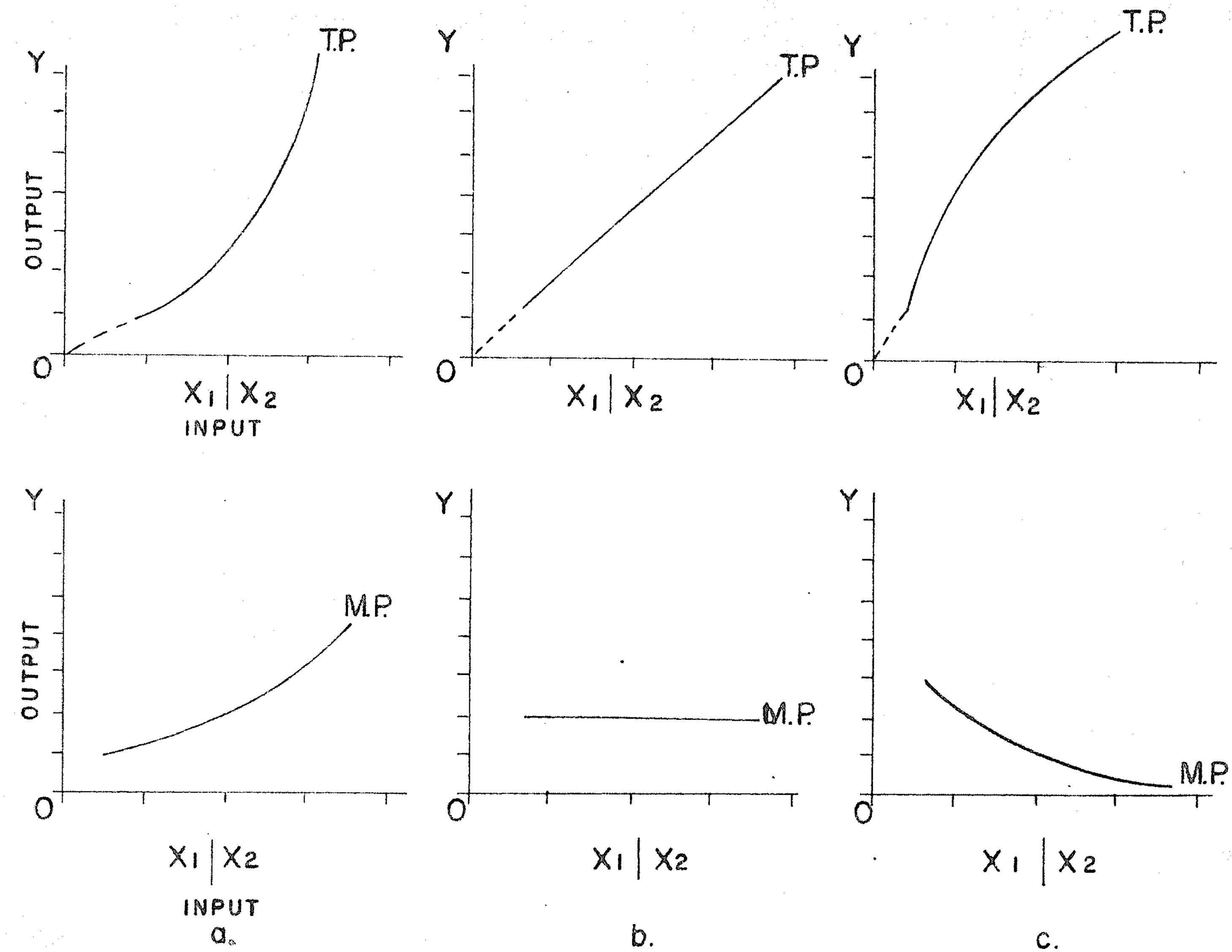
[7] C.S. Cobb and Paul H. Douglas, "A Theory of Production", The American Economic Review, Volume XVIII, Supplement, March, 1928.

<u>Cobb-Douglas Function</u>

The Cobb-Douglas equation is of the form $Y = aX_1^{b_1} X_2^{b_2}$ -------$X_n^{b_n}$. In logarithmic notation, $\log Y = \log a + b_1 \log X_1 + b_2 \log X_2 +$ ------- $+ b_n \log X_n$, the equation is linear. Y is the dependent variable, yield, and X_1 and X_2 are independent variable inputs, representing in this case, nitrogen and phosphorus, respectively. The regression coefficients b_1, b_2 -------b_n are also elasticities of production with respect to X_1, X_2 -------X_n. For the three functions being investigated, the regression coefficients are a measurement of changes which occur in the dependent variable as the inputs are varied. The coefficient "a" is a constant which indicates yield with no application of the variable inputs.

The production function of the Cobb-Douglas form permits increasing, constant, or decreasing marginal productivity. Such characteristics of marginal productivity are, of course, dependent on the characteristics of the total product curve. A disadvantage of the Cobb-Douglas function is that it does not allow decreasing total product. As with the Spillman function this may lead to irrational estimates at higher input levels. Decreasing or increasing marginal productivity is not possible in the same function.

If all relevant inputs are included in the equation, the sum of the b_i values indicate the returns to scale. If Σ



<u>Figure 2.1</u>: Input-Output Relationships for Cobb-Douglas Function showing a. increasing b. constant c. decreasing marginal productivity.

b_i's = 1, the returns to scale are constant. If $\sum b_i$'s > 1, the marginal product is increasing, while marginal product is decreasing if $\sum b_i$'s < 1.

The characteristics of the Cobb-Douglas function are illustrated in Figure 2.1.

Quadratic Function with Squared Terms

A quadratic form of a fertilizer-yield response function has several advantages over the Cobb-Douglas function. For this study the equation is of the form: $Y = a + b_1X_1 + b_2X_2 + b_3X_1^2 + b_4X_2^2 + b_5X_1X_2$. Y is the dependent variable; X_1 and X_2 are variable inputs; b_1 ----- b_5 are regression coefficients, and a is a constant indicating Y with no application of X_1 and X_2.

The quadratic equation allows both decreasing and negative marginal productivity, but not increasing and decreasing marginal products, in the same equation. The quadratic function with squared terms assumes that marginal products decline at a constant rate.

The total product and marginal product curves of Figure 2.2 illustrate the characteristics of the quadratic equation with the squared terms.

Quadratic Function with Square Root Terms

The square root form of the quadratic equation is Y =

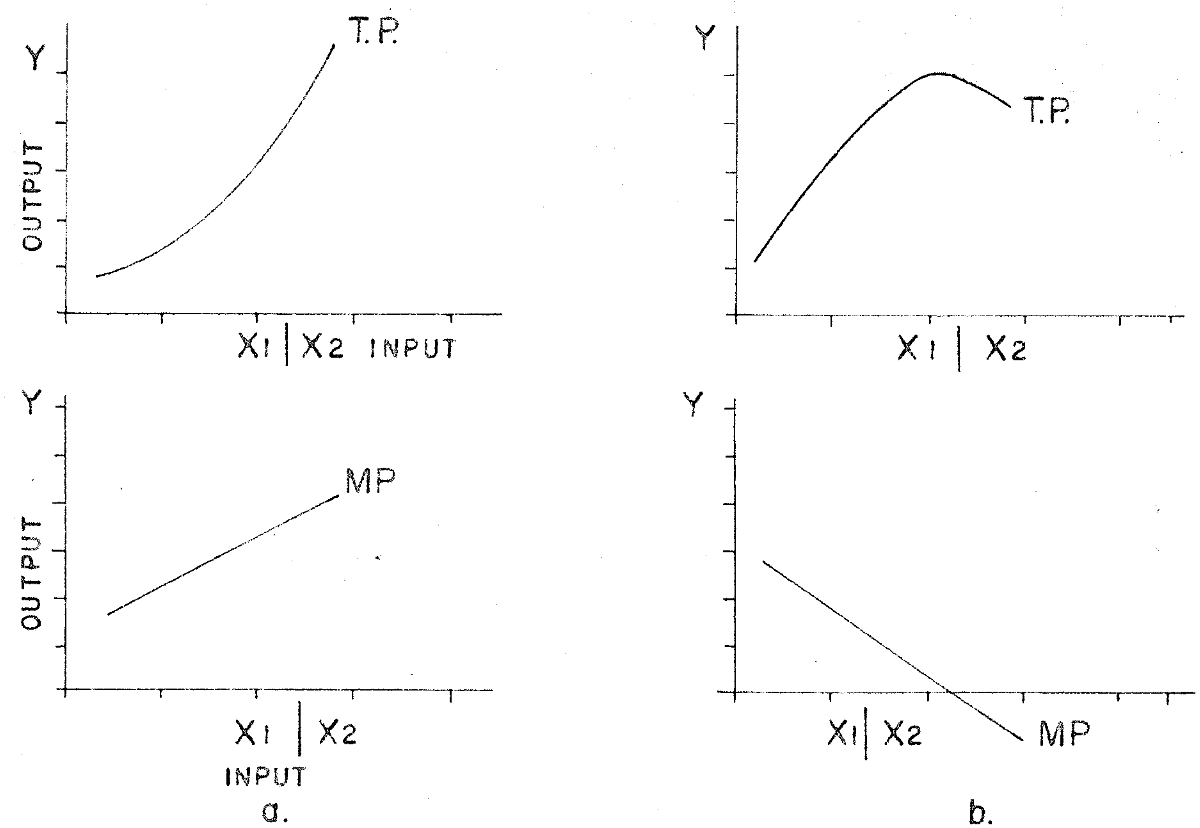


Figure 2.2: Input-Output Relationships for Quadratic Function with Squared Terms showing a. increasing b. decreasing and negative marginal productivity.

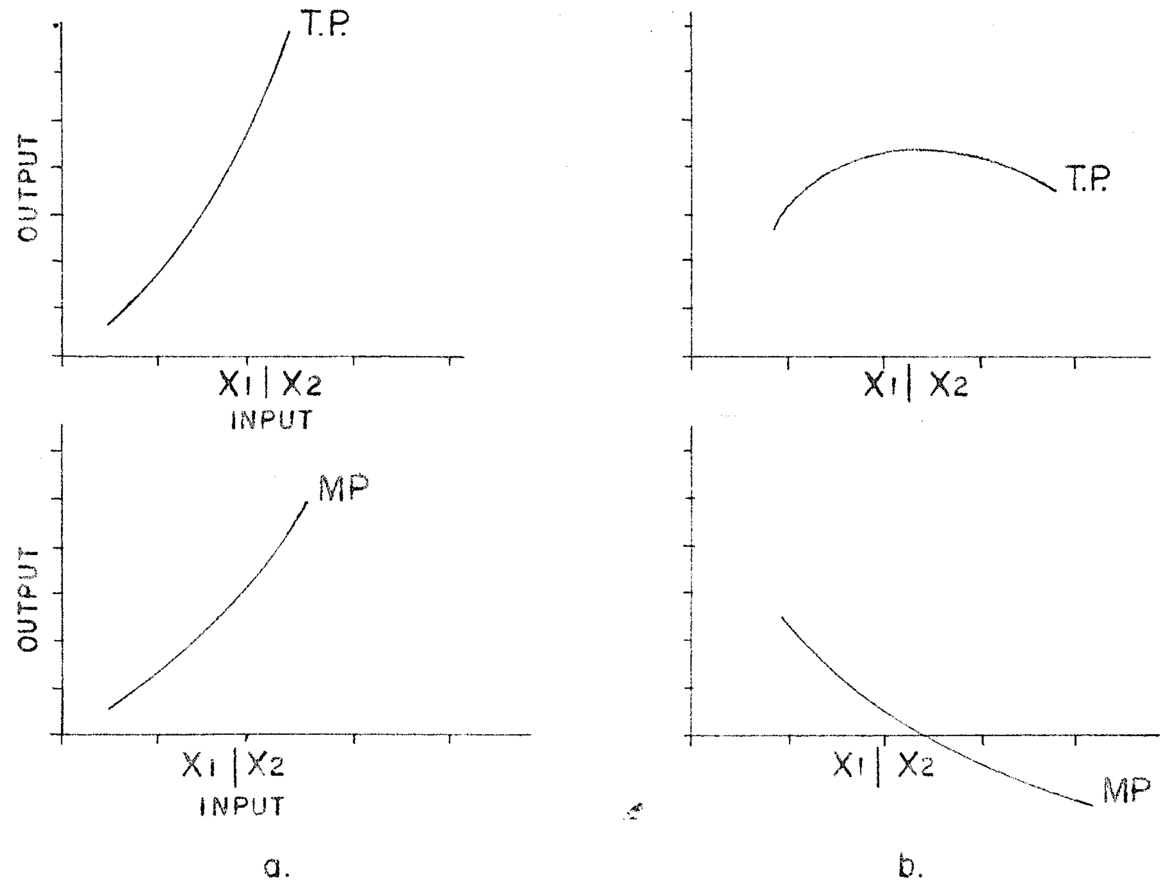


Figure 2.3: Input-Output Relationships for Quadratic Function with Square Root Terms showing a. increasing b. decreasing and negative marginal productivity.

$a + b_1X_1 + b_2X_2 + b_3\sqrt{X_1} + b_4\sqrt{X_2} + b_5\sqrt{X_1X_2}$. This function exhibits the same characteristics as the quadratic equation. In addition, it is similar to the Cobb-Douglas function in that the marginal products decline at a diminishing rate.

The general form of the quadratic square root equation is shown in Figure 2.3. The function allows large marginal productivity over small inputs (Figure 2.3a) followed by small and fairly constant marginal products. As indicated in Figure 2.3b this results in a relatively flat total product curve. Further inputs may result in a diminishing total product curve and negative marginal productivity, (Figure 2.3b).

In addition to the characteristics outlined above, the quadratic equations allow the variable inputs to be substitutes for small outputs; substitutes or complements at higher output levels; and technical complements only at maximum output.

PART II - LINEAR PROGRAMMING

Introduction to Linear Programming

Linear programming may be defined as the maximization or minimization of a linear function subject to certain linear restrictions. In this thesis concern is with the maximization of profit subject to restrictions on the available quantities of land, labor, building space, feed, and capital.

In this thesis, the basic concepts necessary for an understanding of programming will be presented. There are many excellent works to which the reader may make reference in order to expand his knowledge of this research technique.[8]

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[8] A. Charnes, W.W. Cooper, and W. Henderson, An Introduction to Linear Programming (New York: John Wiley and Sons, Inc., 1953).

Robert Dorfman, Application of Linear Programming To Theory of the Firm (Berkley and Los Angeles: The University of California Press, 1951).

R.K. Eyvindson, "Economic Aspects of Farm Organization on Red River Clay", (Unpublished Master's thesis, University of Manitoba, Winnipeg, 1961).

J.C. Gilson, "An Application of Linear Programming to Farm Planning", Technical Bulletin No. 2, Department of Agricultural Economics, University of Manitoba, March, 1960.

________________, "Economic Aspects of Alternative Crop Rotations and Beef Production Systems", (An Application of Linear Programming), Technical Bulletin No. 3, Department of Agricultural Economics, University of Manitoba, May, 1960.

Earl O. Heady and Wilfred Candler, Linear Programming Methods, (Ames, Iowa: The Iowa State College Press, 1958).

Within the framework of farm management, linear programming may be thought of as a method of handling a very large, complex farm budget. In traditional budgeting procedures the number of alternative enterprises that can be considered is limited. In addition, the enterprises cannot be considered simultaneously. The value of any budget is determined by the skill and ability of the analyst. To be of any value, budgeting must be done by a person with knowledge of agricultural production, economics, and practical, common-sense farm management. There is no advantage, in linear programming, when a partial budget on one enterprise is being determined. It then becomes a matter of allocating resources to derive maximum profit from one enterprise rather than allocating resources among several enterprises to obtain maximum profit.

The most important step in the linear programming procedure is the calculation of resource requirement coefficients, prices, and expenses. This step is similar to that of budgeting in that the analyst must have knowledge of production, economics, and management. The remaining steps can be completed by a clerk with little knowledge of the theory of programming.

Assumptions of Linear Programming

There are four major assumptions in linear programming.

First, it is assumed that resources used are additive. The sum of the resources used by individual processes or enterprises must be equal to the sum of the resources used by several enterprises considered simultaneously. This assumption excludes any interaction or complementary effect between two processes.

Second, a linear resource-product relationship holds for linear programming.

Third, resources and products are assumed to be continuous. Resources can be used, and products produced, in fractional quantities. Hence, a process may require the use of 1.34 hours of labor. The most profitable plan may specify the production of 145.97 market hogs. For practical application this can be taken as 146 market hogs without destroying the validity of the conclusions. In fact, most resources and many products may be specified in fractional quantity terms.

Fourth, there is assumed to be a limit to the number of alternative processes which can be considered in determining which combination of processes will return the maximum profit. Unless the number of alternative processes is finite, no final most profitable solution can be found.

One important specific assumption must be made with respect to this study. With three exceptions, prices, yields, resource-product coefficients, costs, and levels of available resources are assumed to be constant. The level of operating capital is varied to determine the effect of varying levels of capital on the use of fertilizer. The price of fertilizer is varied to determine what effect such a change has upon the use of fertilizer and the intensity of use of other processes. Yields are varied, holding fertilizer applications constant to determine what effect this has on the level of use of fertilizer.

METHOD OF EXPOSITION

There are two basic approaches which may be taken in attempting an explanation of linear programming. The exposition may be mathematical or non-mathematical. The latter course is chosen in this thesis. The following discussion relies heavily on an article by Robert Dorfman, as a reference.[9]

The Concept of a Process

The idea of a process or activity is basic to the study of linear programming. A process may be defined as a specific method for producing a particular output. The produnction of barley using a specific analysis of fertilizer is one process, another plan with a different analysis represents an entirely different process. There would be an infinite number of processes by which barley could be produced since

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[9] Robert Dorfman, 'Mathematical or "Linear" Programming: A Non-Mathematical Exposition', *The American Economic Review*, Volume 43, December, 1953. pp. 797-825. The diagrams used in the exposition were taken directly from this article and adopted to an agricultural problem.

the slightest change in the ratio of the nutrients creates a new process. However, in order to comply with the assumption of finiteness, the number of processes is assumed to be limited. In fact, many products can be produced by only a few processes. In such cases, resources cannot be substituted for each other except by changing the levels at which entire technical processes are used since each process uses resources in fixed proportions.

The concept of a process may be illustrated diagrammatically, as in Figure 2.4.

In Figure 2.4, OM is a process vector representing the production of barley, using nitrogen and phosphorus. All other resource requirements necessary for the production of barley by process M are assumed to be present at the required levels. Since OM is a straight line a linear resource-product relationship is implied. For the production of one acre of barley, OD pounds of nitrogen and OA pounds of phosphorus are required. Similarly, the requirements for two and three acres of production can be determined. The proportions in which nitrogen and phosphorus can be used in process M are fixed for all levels of production. Since OQ = QH = HK, constant returns to scale are indicated.

Two Processes

The process concept can be expanded to include more

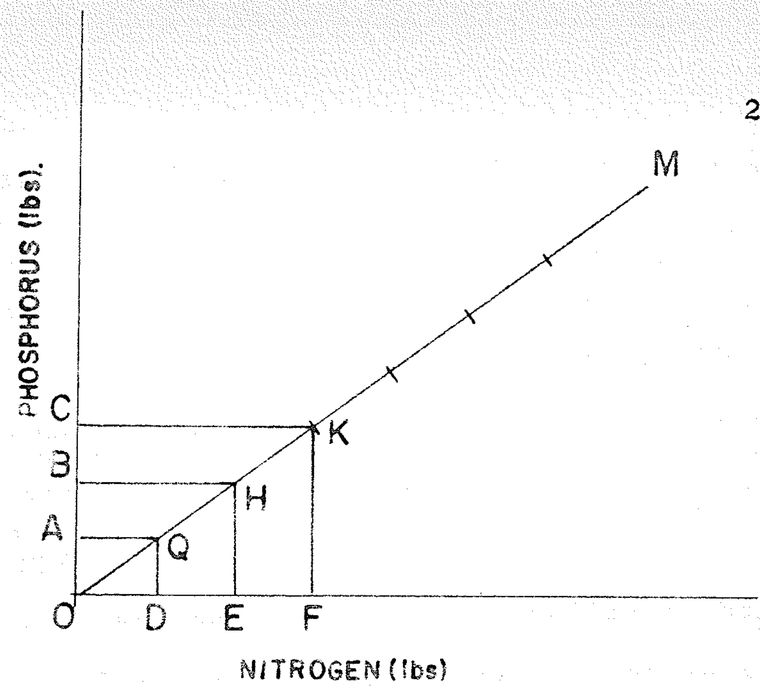


Figure 2.4: Illustration of a Process by means of a Resource-Resource Relationship.

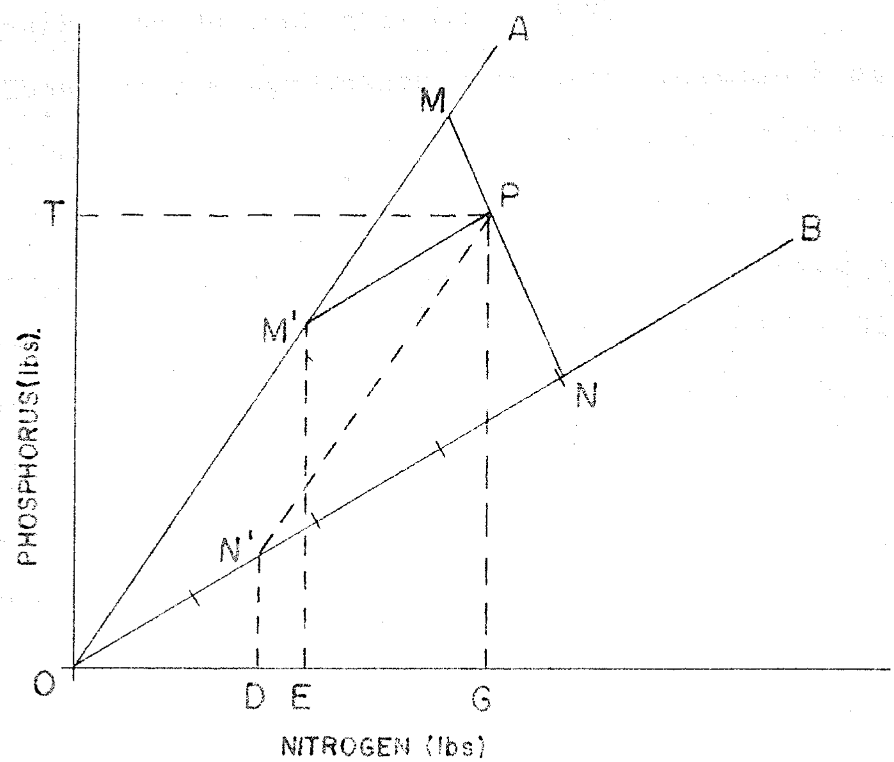


Figure 2.5: Two Processes and Two Resources

than one process. Figure 2.5 illustrates the case in which two different processes are used in the production of barley. This case illustrates additivity of resources.

The two processes are represented by vectors OA and OB. The scales on the two process vectors are different. This indicates that the productivity of the two processes differ with respect to the use of nitrogen and phosphorus.

Let M and N represent the same production of barley by processes A and B respectively. Then the line MN is an isoquant and any point on MN represents a combination of processes A and B producing the same output as either process A or B alone. This can be proven as follows. Any point P is taken on line MN and the line M'P drawn parallel to OB. ON' is marked on OB such that ON' = M'P.

Consider the production plan using process A at the level of OM' units and process B at the level of ON' units. This plan uses OE units of nitrogen for process A and OD units of nitrogen for process B. Utilizing the parallelogram law it is seen that OD = EG. Therefore, OD + OE = EG + OE = OG units of nitrogen. The quantity of nitrogen used by a combination of processes, A and B is, therefore, OG units. This follows from the assumption of additivity. The phosphorus requirements for point P can be found in a similar manner to be OT units.

To prove that MN is an isoquant it must be shown that output OM' plus output ON' equals the output OM which equals the output ON.

$$\frac{\text{Output }(M')}{\text{Output }(M)} = \frac{OM'}{OM} \qquad \frac{\text{Output }(N')}{\text{Output }(N)} = \frac{ON'}{ON}$$

But Output (M) = Output (N)

Therefore, $\frac{\text{Output }(M')}{\text{Output }(N)} = \frac{OM'}{OM}$

If P lies on the isoquant MN then:

$$\frac{\text{Output }(M') + \text{Output }(N')}{\text{Output }(M)} = 1$$

$$\frac{\text{Output }(M') + \text{Output }(N')}{\text{Output }(M)} = \frac{OM'}{OM} + \frac{ON'}{ON} = \frac{OM'}{OM} + \frac{M'P}{ON} =$$

$$= \frac{OM'}{OM} + \frac{M'M}{OM} = \frac{OM}{OM} = 1$$

Therefore, point P lies on the line MN which must be an isoquant. If there are two processes, and two points of equi-output are chosen on them, the line joining the points is an isoquant.

The Resource-Resource Relationship and the Discontinuous Isoquant

By expanding the diagram for processes, the discontinuous isoquant which is implicit in linear programming can be illustrated. In Figure 2.6, four processes for the production of barley are shown. Figure 2.7 shows the isoquant of the traditional marginal analysis. As the number of processes is increased to infinity the isoquants take the form of smooth

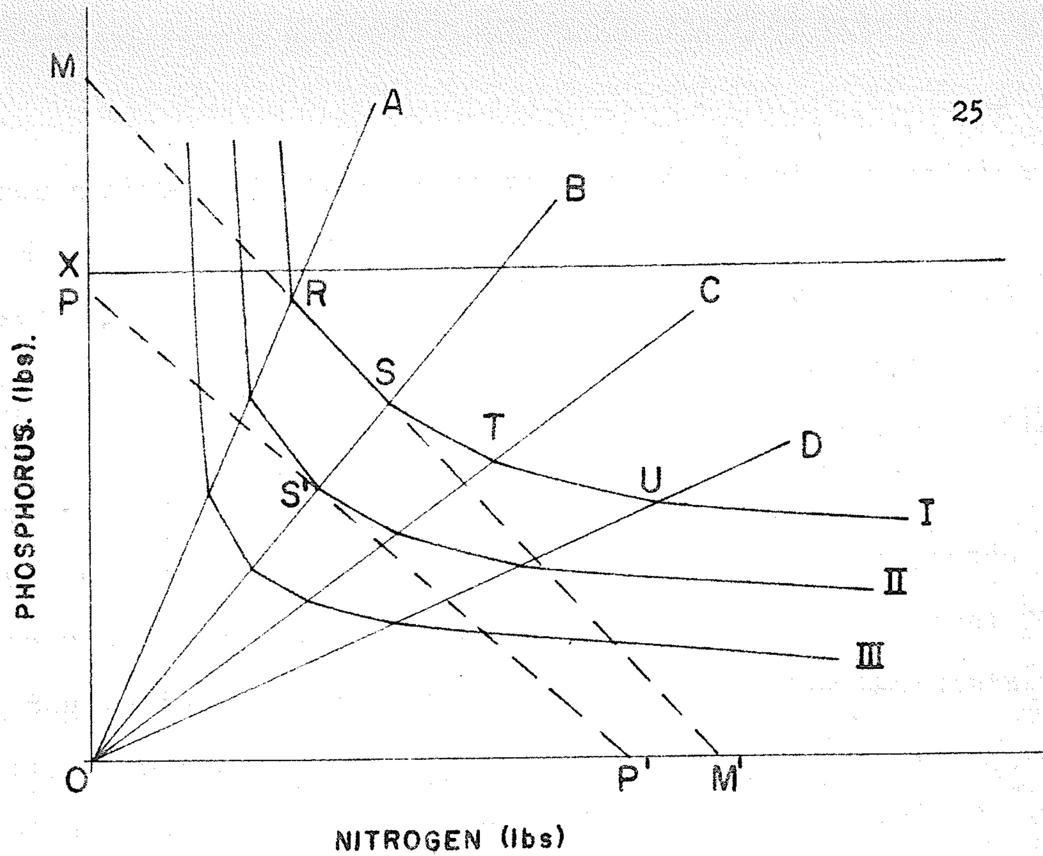


Figure 2.6: Resource-Resource Relationship: Discontinuous Isoquants

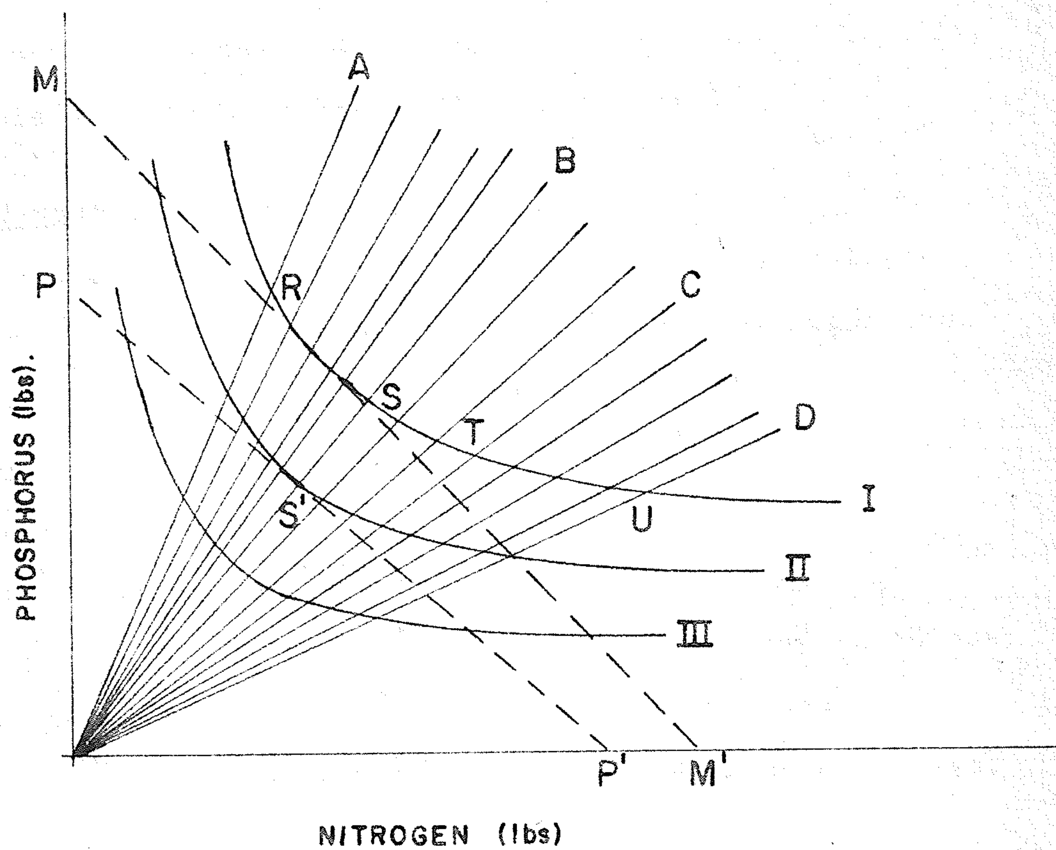


Figure 2.7: Resource-Resource Relationship: Continuous Isoquants

continuous curves. However, a baisc assumption of programming is that the number of processes available for the production of any output are finite. The discontinuous isoquant is then justified.

Point R on isoquant I in Figure 2.6 indicates a specific output obtained by using process A. Similarly, points S, T, and U represent identical outputs obtained by using processes B, C, and D, respectively. By joining these points of equal output, the isoquant I is obtained. Similarly, for isoquants II and III. Following from the discussion of Figure 2.5 and considering isoquant I, it is seen that an identical output can be obtained in several ways. Processes A, B, C or D; or combinations of processes A and B, B and C, or C and D, will all produce an output represented by isoquant I.

Optimum Output

Two price lines can be introduced into the analysis at this point. The price relationship between nitrogen and phosphorus as indicated by the price line PP' shows that maximum output is obtained by using process B and the level of OS' units.

If the price relationship illustrated by price line MM' exists, optimum output is indicated by isoquant I. Since the price line MM' coincides with segment RS of isoquant I, maximum output can theoretically be obtained in several ways.

Process A, process B or some combination of the two can be used to obtain optimum output.

The price line MM' points out a fact which is basic to linear programming. Given a number of processes for producing the same output, the one most profitable process will be found. In this case, the final most profitable plan may include process A or process B but not a combination of the two processes. Linear programming does not permit two processes to combine in obtaining maximum production of a specific output. Only two resources, nitrogen and phosphorus, in the processes A and B are being considered here. Many resources are, of course, required to form either process A or B. Even though processes A and B are capable of providing the same output, other resources required in their formation will dictate whether process A or process B will provide the method of production in the final plan.

The Resource-Product Relationship

The relevant production function in linear programming is also a discontinuous function with a finite number of points rather than an infinite number of points which produce a continuous function. The discontinuous function can be derived from the system of isoquants that has been discussed.

Suppose that the quantity of phosphorus available is limited to the amount OX as measured on the vertical axis of

Figure 2.6. The production function of Figure 2.8 can be derived from Figure 2.6 when the amount of phosphorus is constant and the quantity of nitrogen is variable. As is seen from the diagram the production function is discontinuous with discrete points indicating the requirements of nitrogen for the production of barley, when phosphorus is constant at amount OX.

An implicit assumption of the preceding explanation has been the motive of maximum output. Maximum output, however, is not equivalent to maximum profit. Dependent on subjective restrictions set by the farm operator, such as the number of hours he is willing to work per day, the explicit goal of any farm enterprise is to provide maximum return to the fixed and variable factors of production. This can be done quite simply by converting physical products to value products. In the diagrams which have been used to explain the basics of linear programming no major changes need to be made. The scale on the processes are changed to measure output in monetary rather than physical terms.

The Product-Product Relationship

The discussion thus far has demonstrated the resource-resource and resource-product relationships as they apply to linear programming. The third concept of the traditional marginal analysis to be assessed for validity with respect

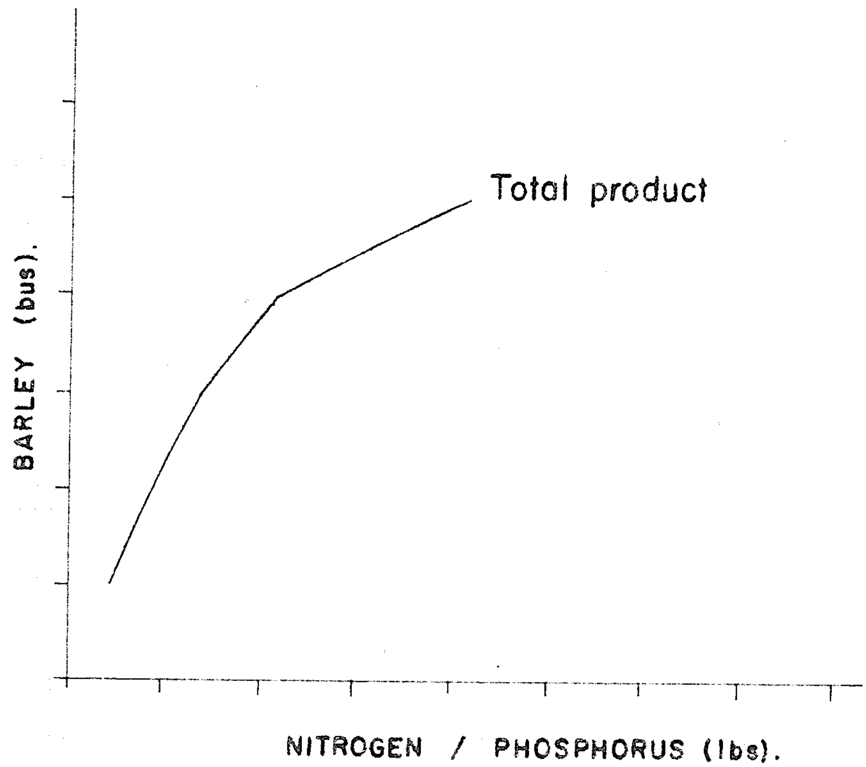


Figure 2.8: Resource-Product Relationship: Discontinuous Production Function (Derived from Figure 2.6).

to programming is the product-product relationship. The objective here is to determine the combination of products to return maximum profit.

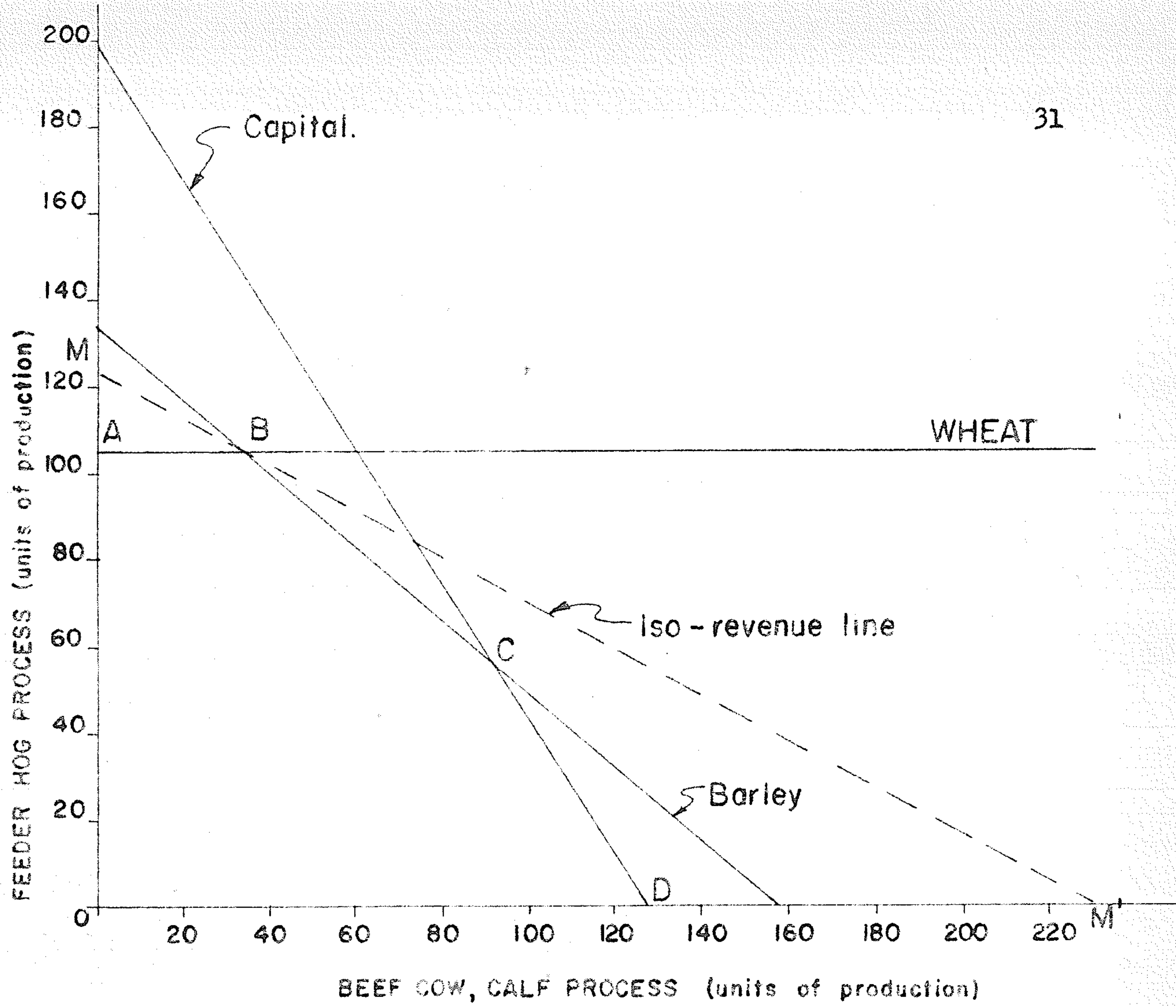
The product-product principle for linear programming is represented by a discontinuous transformation curve with discrete points, as shown in Figure 2.9. The analogous curve for the traditional analysis is shown in Figure 2.10.

In determining the discontinuous curve three resources, used in the feeder hog and beef cow calf processes, are considered. Table 2.1 gives the level of resources available and the resource requirements of each process per unit of production. All other resources are assumed to be non-limiting.

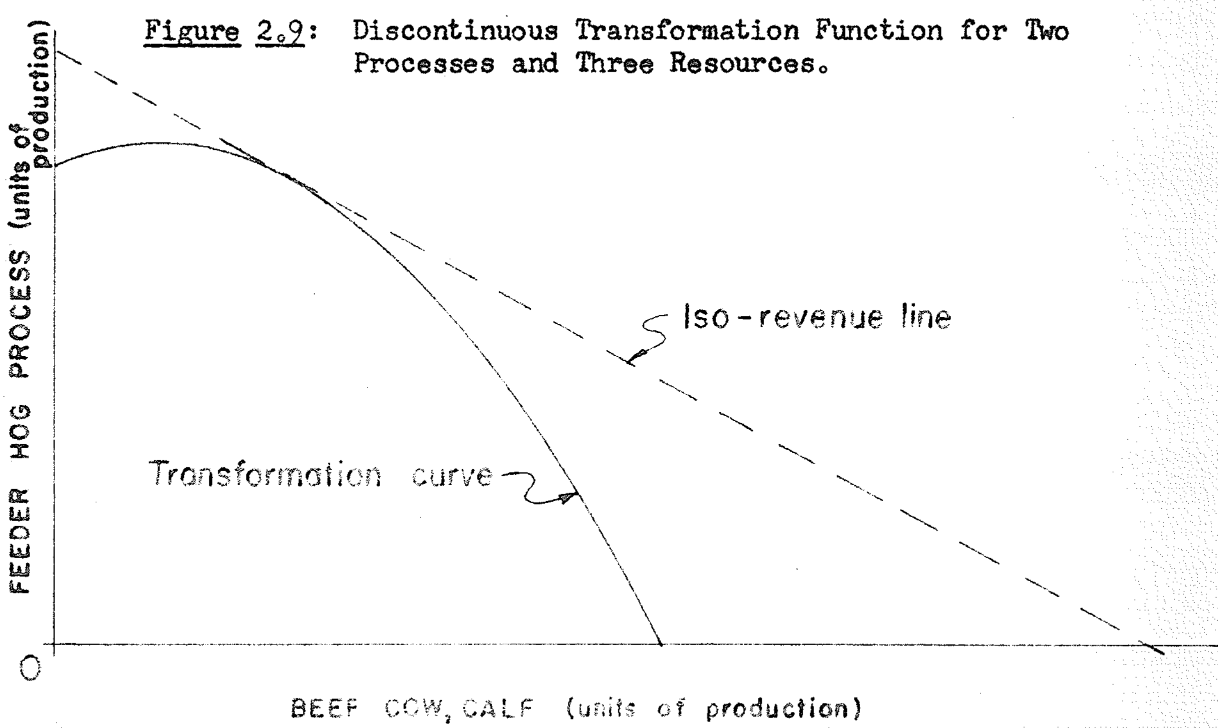
Table 2.1. Use of Limited Resources in Feeder Hog and Beef Cow Calf Processes.

Resource	Amount Available	Beef Cow Calf: Required Per Unit of Production	Beef Cow Calf: Maximum Production (Units)	Feeder Hogs: Required Per Unit of Production	Feeder Hogs: Maximum Production (Units)
Wheat	3000 lbs.	--	--	28. lbs.	107.
Barley	40,000 "	252. lbs.	159.	303. "	132.
Capital	4000 $	31. $	129.	20. $	200.

Of the three resources considered, wheat is the most limiting in the production of feeder hogs. This is indicated by point A in Figure 2.9. The level of the feeder hog



<u>Figure 2.9</u>: Discontinuous Transformation Function for Two Processes and Three Resources.



<u>Figure 2.10</u>: Continuous Transformation Function for Two Processes.

process is limited to 107 units. The cow calf process does not require wheat, therefore its production is not affected by the availability of this resource. With 40,000 pounds of barley available 159 units of the cow calf process or 132 units of the feeder hog process can be produced. Four thousand dollars of capital limits cow calf production to 129 units, point D, on Figure 2.9, while feeder hog production could be expanded to 200 units. Therefore, the most limiting resources for beef cow calf and feeder hog production are capital and wheat, respectively.

The relevant area on which production decisions must be based is bounded by the discontinuous transformation curve ABCD. Any point on segment AB indicates maximum production of feeder hogs if no units of the cow-calf process are produced. Point D indicates maximum production of the beef cow calf process if no hogs are produced. Given the limitations on wheat, barley, and capital, BCD represents combinations of the hog and beef processes. The combination that will return maximum profit is determined by the iso-revenue line. The iso-revenue line indicates the ratio of prices of the competing products and defines all possible combinations of hogs and beef which will return the same profit. In Figure 2.9, the iso-revenue line MM' indicates that the combination of hogs and cow-calf production necessary for maximum profit is at point B on the transformation curve. Thus production for maximum profit requires 107 units of the feeder hog process and 33 units of the beef cow calf process. It is significant to note the price ratio can change considerably before any adjustment will be made in the production plan. In the traditional analysis the slightest change in the price ratio requires a change in the production plan if the objective of maximum profit is to be maintained.

The exposition of the product-product principle has been limited to two processes because of the difficulty of representing more than two processes graphically. The mechanism of the linear program allows the number of alternative processes to be very large as long as it is finite. The various possible processes are considered simultaneously in the procedure of finding that combination which will yield maximum profit.

CHAPTER III

THE PRODUCTION FUNCTIONS: ANALYSIS AND RESULTS

The experimental data on which the analysis of this chapter is based were obtained from experiments conducted at the Wabowden Substation.[1] In the experiments, varying levels of nitrogen and phosphorus were applied to plots of Parkland barley and the yield response measured. For an explanation of the experimental design see the study by Gilson and Bjarnarson.[2] The data were collected for a period of three years, 1956-58 inclusive. The 1956 data were analyzed and the results reported by Gilson and Bjarnarson.[3] This chapter is primarily an extension of the analysis, by Gilson and Bjarnarson, for the years 1957 and 1958 and the combined composite data for the three years. Where necessary reference will be made to the aforementioned study in order to maintain a note of cohesion for the three years. Pertinent information from the Gilson and Bjarnarson report is included in Appendix I.

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[1] See footnote 2, Chapter 1, Page 5.

[2] J.C. Gilson and V.W. Bjarnarson, "Effects of Fertilizer Use on Barley in Northern Manitoba", Journal of Farm Economics, Volume XL, No. 4, November, 1958, pp. 932-941. It is suggested that this report be read in conjunction with this chapter.

[3] Ibid.

The major objective of the analysis is the evaluation of three possible estimating equations, in order to select the equation which provides the best estimate of actual barley yields. Economic aspects of fertilizer use which are complementary to the evaluation of the equations are also considered. Isoquants and rates of substitution between nitrogen and phosphorus are determined. Predicted yield response surfaces are developed. Least cost combinations and most profitable applications of nitrogen and phosphorus are derived from the basic equations.

The Evaluation of the Equations

The three equations to be evaluated are the quadratic with squared terms; the quadratic with square root terms; and the Cobb-Douglas function. For purposes of brevity the first two equations will hereafter be referred to as the quadratic equation and the square root equation. The characteristics and general form of the three equations were discussed in Chapter II.

The problems associated with the determination of the best estimating or predicting equation are set out by Heady:

> "Direct" tests are not available for choosing between such "widely different" functions as the Spillman, Cobb-Douglas, cross-product or square-root equations. We may select, however, in terms of such statistics as the multiple coefficient of correlation and the magnitude of residual variance quantities.

Also, plotting the derived quantities against the sample observations may aid in choice and selection.[4]

The technique of multiple regression analysis was used to obtain coefficients of determination and regression coefficients for each equation for each year and the composite data. The equations derived from the 1956 data by Gilson and Bjarnarson are included.

1956:

1. Quadratic: $\hat{Y}=8.2714+.2715N+.3106P-.0020N^2-.0014P^2+.0006NP$

2. Square Root: $\hat{Y}=8.3100-.1319N-.0350P+1.6590\sqrt{N}+1.8404\sqrt{P}+.1020\sqrt{NP}$

3. Cobb-Douglas: $\hat{Y}=7.5509N^{.0971}P^{.2444}$

1957:

4. Quadratic: $\hat{Y}=21.4012+.4539N+.5234P-.0029N^2-.0036P^2+.0037NP$

5. Square Root: $\hat{Y}=28.2051-.0002N-.1277P+.3913\sqrt{N}+1.6977\sqrt{P}+.5103\sqrt{NP}$

6. Cobb-Douglas: $\hat{Y}=16.2370N^{.1685}P^{.1756}$

1958:

7. Quadratic: $\hat{Y}=41.9530+.2279N+1.2162P-.0022N^2-.0081P^2+.0022NP$

8. Square Root: $\hat{Y}=40.2534-.1475N-.6368P+.8408\sqrt{N}+9.9821\sqrt{P}+.2981\sqrt{NP}$

9. Cobb-Douglas: $\hat{Y}=32.7990N^{.0327}P^{.2158}$

Composite:

10. Quadratic: $\hat{Y}=22.0391+.3976N+.7374P-.0031N^2-.0049P^2+.0024NP$

11. Square Root: $\hat{Y}=25.3981-.0946N-.2963P+.9946\sqrt{N}+4.5556\sqrt{P}+.3011\sqrt{NP}$

12. Cobb-Douglas: $\hat{Y}=15.6300N^{.1009}P^{.2156}$

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[4] Earl O. Heady, Glenn L. Johnson and Lowell S. Hardin (eds), Resource Productivity, Returns to Scale, and Farm Size (Ames, Iowa: The Iowa State College Press, 1956) pp. 13-14.

The coefficients of determination (R^2) and the values of the t-tests for the regression coefficients for the 12 equations are shown in Table 3.1. The F-test was used to test the significance of the coefficients of determination.

Table 3.1. Coefficients of Determination and the t-Tests for the Twelve Equations.

Equation	Value R^2	t-test values for the regression coefficients of the following terms				
		N	P	N^2 or $\sqrt{N}$	P^2 or $\sqrt{P}$	NP or $\sqrt{NP}$
1	.8422*	4.62**	5.28**	4.64**	3.23**	1.45^{+++}
2	.8667*	2.80**	.74	2.74*	3.03**	2.13^{+}
3	.8819*	1.83^{++}	4.59**	--	--	--
4	.9495*	6.25**	7.20**	-5.37**	-6.72**	7.25**
5	.9629*	-.004	-2.34^{+}	.56	2.41^{+}	9.22**
6	.9019*	12.06**	12.57**	--	--	--
7	.8431*	2.02^{++}	10.94**	-2.52*	-9.90**	2.42^{+}
8	.9208*	-1.69^{+++}	-7.30**	.75	8.89**	3.38**
9	.8694*	2.22^{+}	14.66**	--	--	--
10	$.3212^{++}$	2.05^{+}	3.80**	-2.19^{+}	-3.45**	1.60^{+++}
11	$.2925^{++}$	-.50	-1.44^{+++}	.41	1.88^{++}	1.59^{+++}
12	$.3179^{++}$	2.96**	6.34**	--	--	--

$^{+++}$Significant at the 10 per cent level
$^{++}$Significant at the 5 per cent level
$^{+}$Significant at the 2.5 per cent level
*Significant at the 1.0 per cent level
**Significant at the .5 per cent level

The F-test for coefficients of determination tested significant at the one per cent level for equations 1 to 9 inclusive and at the five per cent level for equations 10 to 12 inclusive. The coefficient of determination measures the proportion of variance in the dependent variable, yield, attributable to the independent variables, nitrogen and phosphorus. Values for equations 1, 2, and 3 were taken from the Gilson and Bjarnarson study.[5]

Predicted yields were derived from the 12 equations. The actual and predicted yields for each year are shown in Tables 3.2 and 3.5 inclusive. The yield response surfaces diagrammed in Figures 3.1 to 3.16 inclusive, provide a three dimensional view of the data contained in Tables 3.2 to 3.5. The response surface represents the yield response curves for all possible combinations of nitrogen and phosphorus within the limits of the experimental applications of these nutrients. If nitrogen is held constant at 20 pounds, while phosphorus applications are varied from 0 to 120 pounds, the response curve or total product curve AB of Figure 3.2 is obtained. Similarly, if phosphorus is held constant at 20 pounds while nitrogen is varied the total product curve CD is obtained.

In addition to statistical measures, visual comparisons of actual and predicted yield values were made to aid in the selection of the best estimating equation. An examination of

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[5] J.C. Gilson and V.W. Bjarnarson, op. cit.

Table 3.2. Actual and Predicted Yields of Barley, Estimated by the Three Equations, for Varying Levels of Nitrogen and Phosphorus - 1956.

LBS P_2O_5		LBS N 0	20	40	60	80	120
0	Actual	7.8	9.6	13.2	16.0	12.2	9.0
	Quadratic	8.27	12.80	15.91	17.32	17.13	11.91
	Square-Root	8.31	13.09	13.53	13.25	12.60	10.65
	Cobb-Douglas	7.17	9.72	10.43	10.87	11.19	11.66
20	Actual	20.2	22.1	27.2	26.2	23.4	23.1
	Quadratic	13.92	18.79	22.04	23.69	23.74	19.00
	Square-Root	15.84	22.66	23.94	24.31	24.21	23.18
	Cobb-Douglas	15.41	20.90	22.42	23.37	24.06	25.07
40	Actual	18.8	26.0	29.0	38.0	26.4	29.3
	Quadratic	18.45	23.56	27.05	28.94	29.23	24.97
	Square-Root	18.54	26.22	27.85	28.48	28.61	27.96
	Cobb-Douglas	18.40	24.95	26.77	27.89	28.72	29.93
60	Actual	15.3	22.1	26.9	28.9	28.7	32.1
	Quadratic	21.87	27.22	30.95	33.08	33.61	29.83
	Square-Root	20.46	28.78	30.68	31.52	31.82	31.46
	Cobb-Douglas	20.41	27.67	29.69	30.94	31.86	33.20
80	Actual	23.2	27.9	33.0	37.9	37.0	31.7
	Quadratic	24.16	29.75	33.72	36.09	36.86	33.56
	Square-Root	21.97	30.83	32.96	33.97	34.42	34.31
	Cobb-Douglas	21.97	29.78	31.95	33.30	34.28	35.73
120	Actual	27.5	31.9	37.0	40.2	44.0	33.6
	Quadratic	25.38	31.45	35.90	38.75	40.00	37.66
	Square-Root	24.27	34.05	36.55	37.86	38.55	38.85
	Cobb-Douglas	24.36	33.03	35.44	36.93	38.03	39.63

Table 3.3. Actual and Predicted Yields of Barley, Estimated by the Three Equations, for Varying Levels of Nitrogen and Phosphorus - 1957.

LBS P_2O_5		LBS N					
		0	20	40	60	80	120
0	Actual	25.4	30.3	30.2	35.2	31.1	32.8
	Quadratic	21.40	29.33	34.96	38.30	39.34	34.54
	Square-Root	28.20	29.95	30.67	31.22	31.69	32.46
	Cobb-Douglas	16.24	26.90	30.23	32.37	33.98	36.38
20	Actual	36.5	44.6	48.7	54.2	56.9	59.6
	Quadratic	30.43	39.86	46.98	51.81	54.34	52.52
	Square-Root	33.24	45.19	50.14	53.94	57.14	62.50
	Cobb-Douglas	27.48	45.52	51.16	54.76	57.50	61.56
40	Actual	31.6	52.2	58.7	53.1	65.3	74.3
	Quadratic	36.59	47.51	56.12	62.45	66.47	67.64
	Square-Root	33.83	50.01	56.71	61.85	66.18	73.45
	Cobb-Douglas	31.03	51.41	57.78	61.86	64.94	69.53
60	Actual	33.9	47.6	59.3	71.1	72.3	81.5
	Quadratic	39.88	52.29	62.40	70.21	75.73	79.88
	Square-Root	33.69	53.12	61.16	67.33	72.53	81.25
	Cobb-Douglas	33.32	55.20	62.04	66.43	69.73	74.66
80	Actual	38.6	54.3	66.3	80.6	86.5	83.3
	Quadratic	40.30	54.20	65.80	75.10	82.11	89.24
	Square-Root	33.17	55.33	64.50	71.54	77.48	87.43
	Cobb-Douglas	35.05	58.06	65.26	69.87	73.34	78.53
120	Actual	29.9	53.9	63.8	79.0	90.1	93.0
	Quadratic	32.51	49.40	53.98	76.27	86.26	99.36
	Square-Root	31.48	58.22	69.30	77.80	84.96	96.97
	Cobb-Douglas	37.64	62.35	70.07	75.03	78.76	84.32

Table 3.4. Actual and Predicted Yields of Barley, Estimated by the Three Equations, for Varying Levels of Nitrogen and Phosphorus - 1958.

LBS P_2O_5		LBS N					
		0	20	40	60	80	120
0	Actual	44.5	33.1	40.3	44.5	31.7	34.6
	Quadratic	41.95	45.63	47.55	47.71	46.10	37.62
	Square-Root	40.25	41.06	39.67	37.92	35.97	31.76
	Cobb-Douglas	32.80	36.18	37.00	37.50	37.85	38.36
20	Actual	64.1	73.2	79.8	82.1	86.7	71.7
	Quadratic	63.04	67.60	70.39	71.43	70.71	63.98
	Square-Root	72.16	78.93	80.01	80.15	79.80	78.27
	Cobb-Douglas	62.61	69.05	70.63	71.58	72.25	73.22
40	Actual	81.9	87.1	87.1	90.2	82.4	89.1
	Quadratic	77.64	83.08	86.76	88.68	88.83	83.87
	Square-Root	77.91	87.16	89.26	90.18	90.50	90.08
	Cobb-Douglas	72.71	80.19	82.03	83.12	83.91	85.03
60	Actual	76.8	93.6	99.5	99.9	102.5	87.2
	Quadratic	85.76	92.08	96.64	99.44	100.48	97.27
	Square-Root	79.37	90.50	93.39	94.92	95.74	96.17
	Cobb-Douglas	79.36	87.52	89.53	90.73	91.58	92.81
80	Actual	77.6	95.9	104.9	90.8	107.4	105.6
	Quadratic	87.41	94.61	100.04	103.72	105.64	104.20
	Square-Root	78.59	91.32	94.87	96.90	98.16	99.31
	Cobb-Douglas	84.44	93.13	95.26	96.54	97.45	98.75
120	Actual	75.2	89.6	85.1	82.4	100.7	100.8
	Quadratic	71.26	80.22	87.41	92.85	96.53	98.60
	Square-Root	73.18	88.59	93.25	96.14	98.11	100.46
	Cobb-Douglas	92.16	101.74	103.98	105.36	106.36	107.78

Table 3.5. Actual and Predicted Yields of Barley, Estimated by the Three Equations, for Varying Levels of Nitrogen and Phosphorus - Composite Data.

LBS P_2O_5		LBS N					
		0	20	40	60	80	120
0	Actual	25.90	24.33	27.90	31.90	25.00	25.47
	Quadratic	22.04	28.74	32.95	34.66	33.88	24.82
	Square-Root	25.40	27.95	27.90	27.43	26.73	24.94
	Cobb-Douglas	15.63	21.15	22.68	23.62	24.32	25.34
20	Actual	40.27	46.63	51.90	54.17	55.67	51.47
	Quadratic	34.82	42.47	47.63	50.29	50.45	43.29
	Square-Root	40.38	48.96	51.41	52.84	53.76	54.68
	Cobb-Douglas	29.82	40.34	43.26	45.07	46.40	48.33
40	Actual	44.10	55.10	58.27	60.43	58.03	64.23
	Quadratic	43.66	52.26	58.37	61.98	63.09	57.82
	Square-Root	43.44	54.51	57.98	60.22	61.80	63.84
	Cobb-Douglas	34.62	46.84	50.24	52.33	53.88	56.12
60	Actual	42.00	54.43	61.90	66.63	67.83	66.93
	Quadratic	48.57	58.12	65.17	69.73	71.78	68.42
	Square-Root	44.53	57.51	61.78	64.62	66.72	69.62
	Cobb-Douglas	37.79	51.12	54.82	57.11	58.80	61.25
80	Actual	46.47	59.37	68.07	69.77	76.97	73.53
	Quadratic	49.54	60.04	68.04	73.54	76.55	75.08
	Square-Root	44.60	59.20	64.14	67.49	70.02	73.64
	Cobb-Douglas	40.20	54.39	58.33	60.77	62.56	65.17
120	Actual	44.20	58.47	61.97	67.20	78.27	75.80
	Quadratic	39.68	52.07	61.97	69.37	74.27	76.59
	Square-Root	42.98	60.29	66.35	70.56	73.81	78.66
	Cobb-Douglas	43.88	59.36	63.66	66.32	68.27	71.12

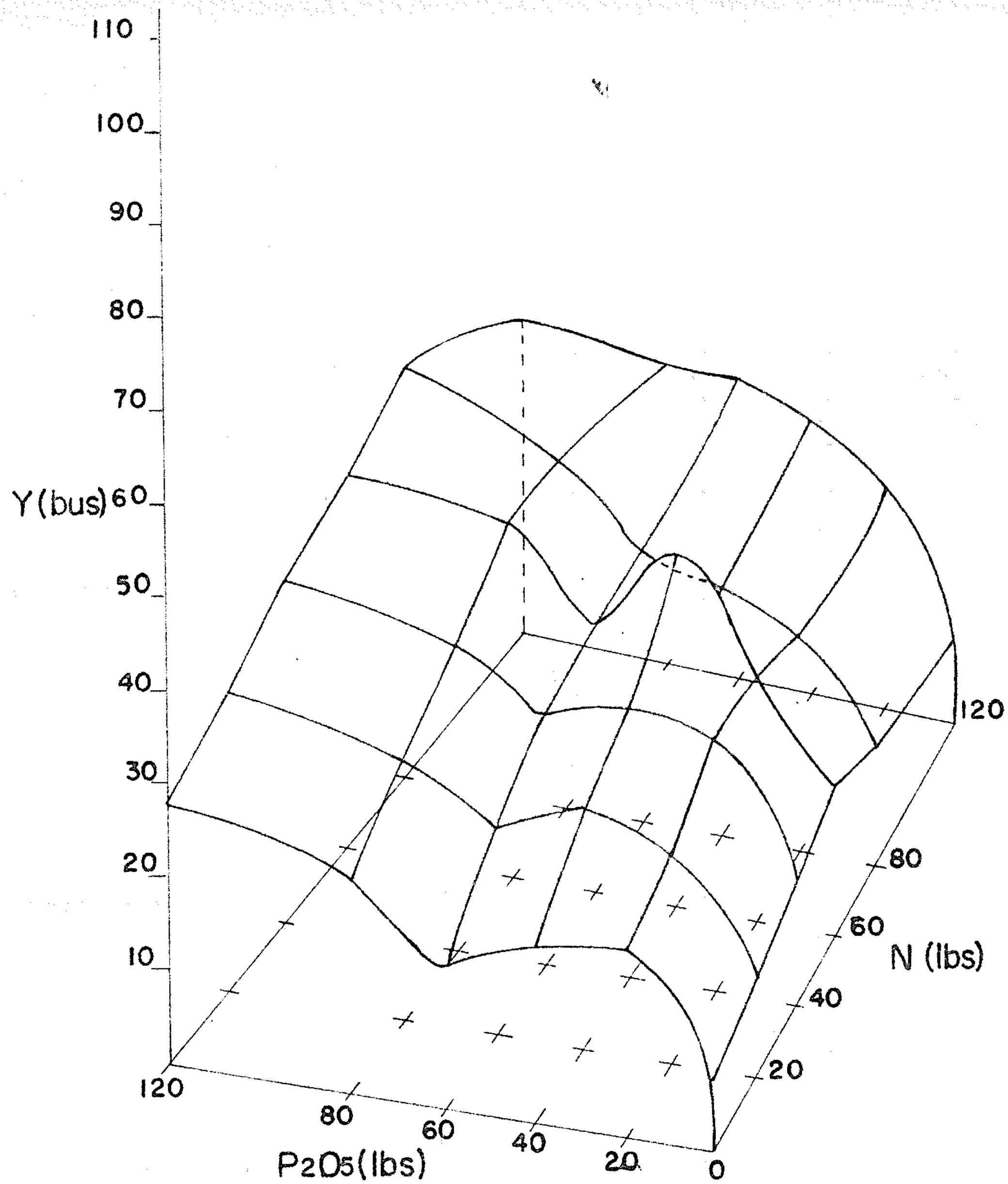


Figure 3:1: Actual Yield Response Surface for Barley - 1956.

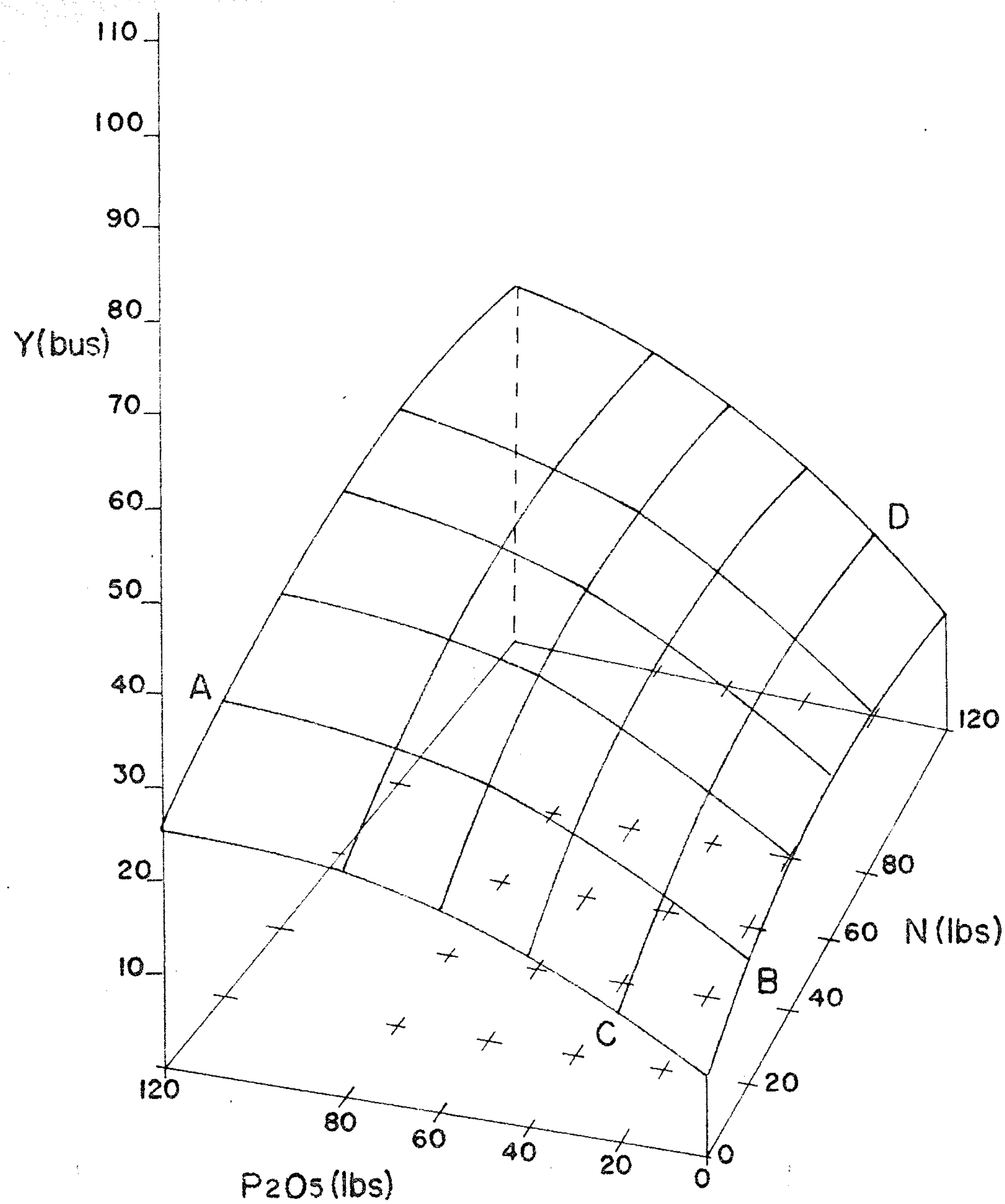
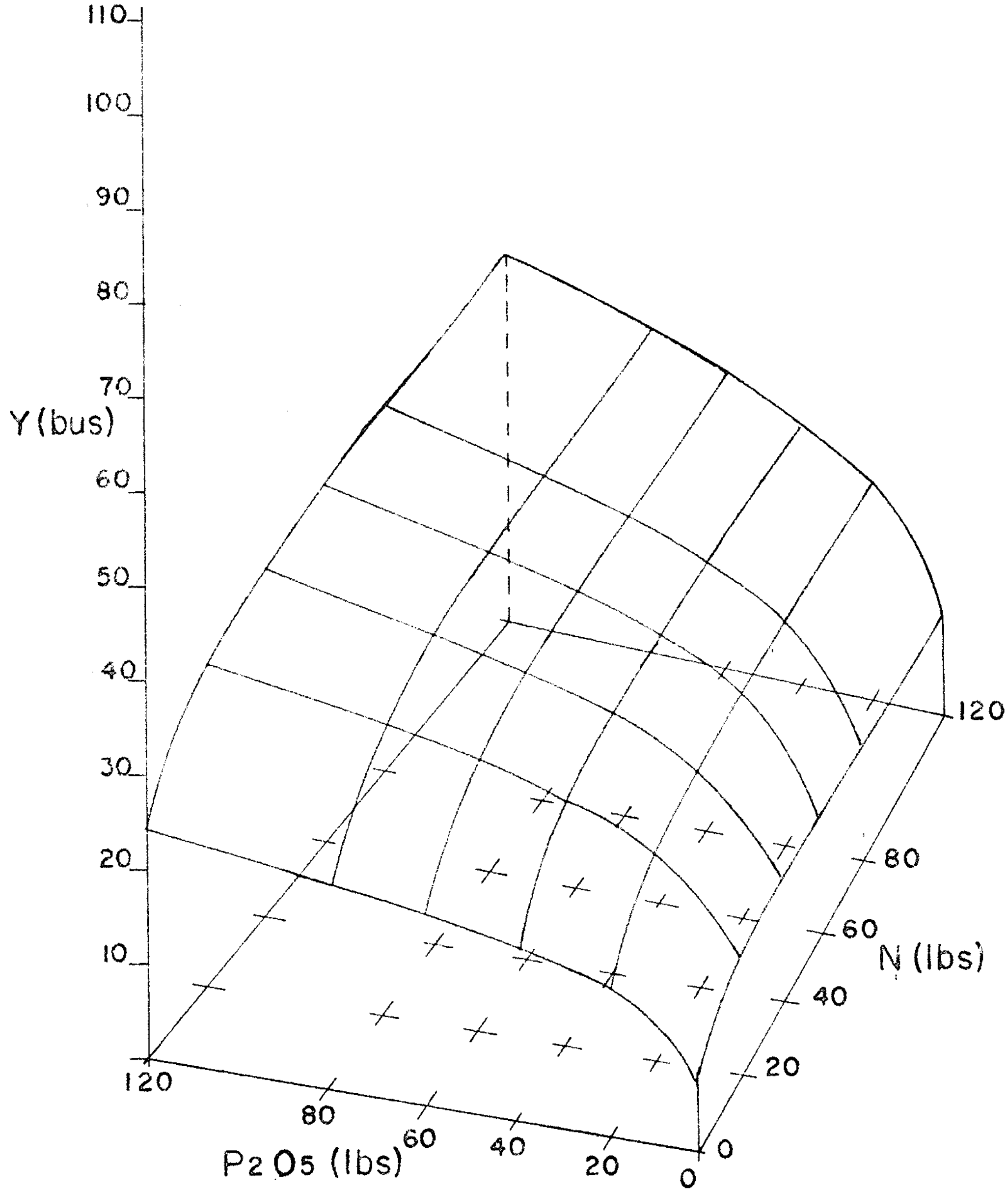


Figure 3.2: Predicted Yield Response Surface for Barley, Derived by the Quadratic Function - 1956.



<u>Figure 3.3</u>: Predicted Yield Response Surface for Barley, Derived by the Square Root Function - 1956

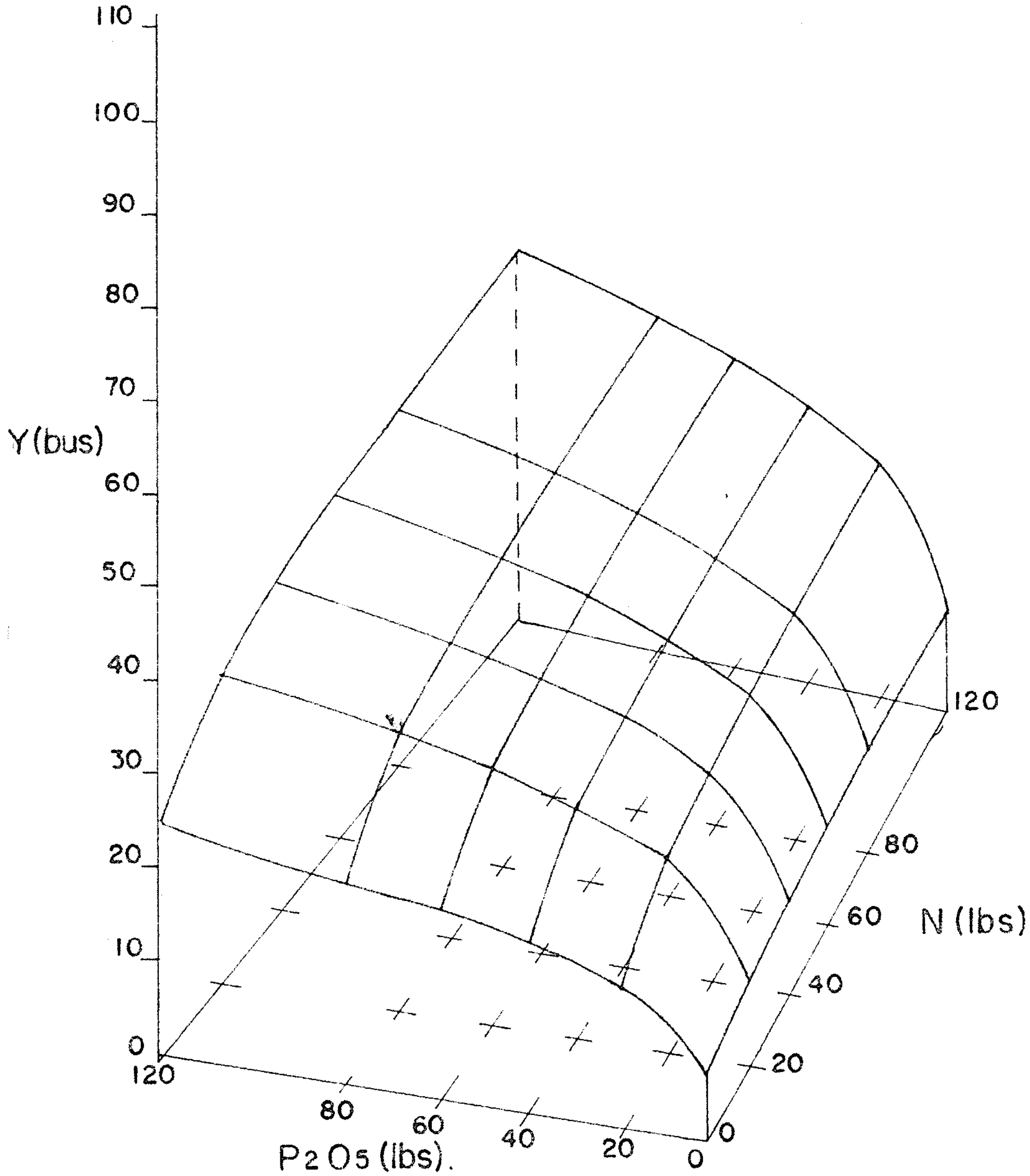


Figure 3.4: Predicted Yield Response Surface for Barley, Derived by the Cobb-Douglas Function - 1956.

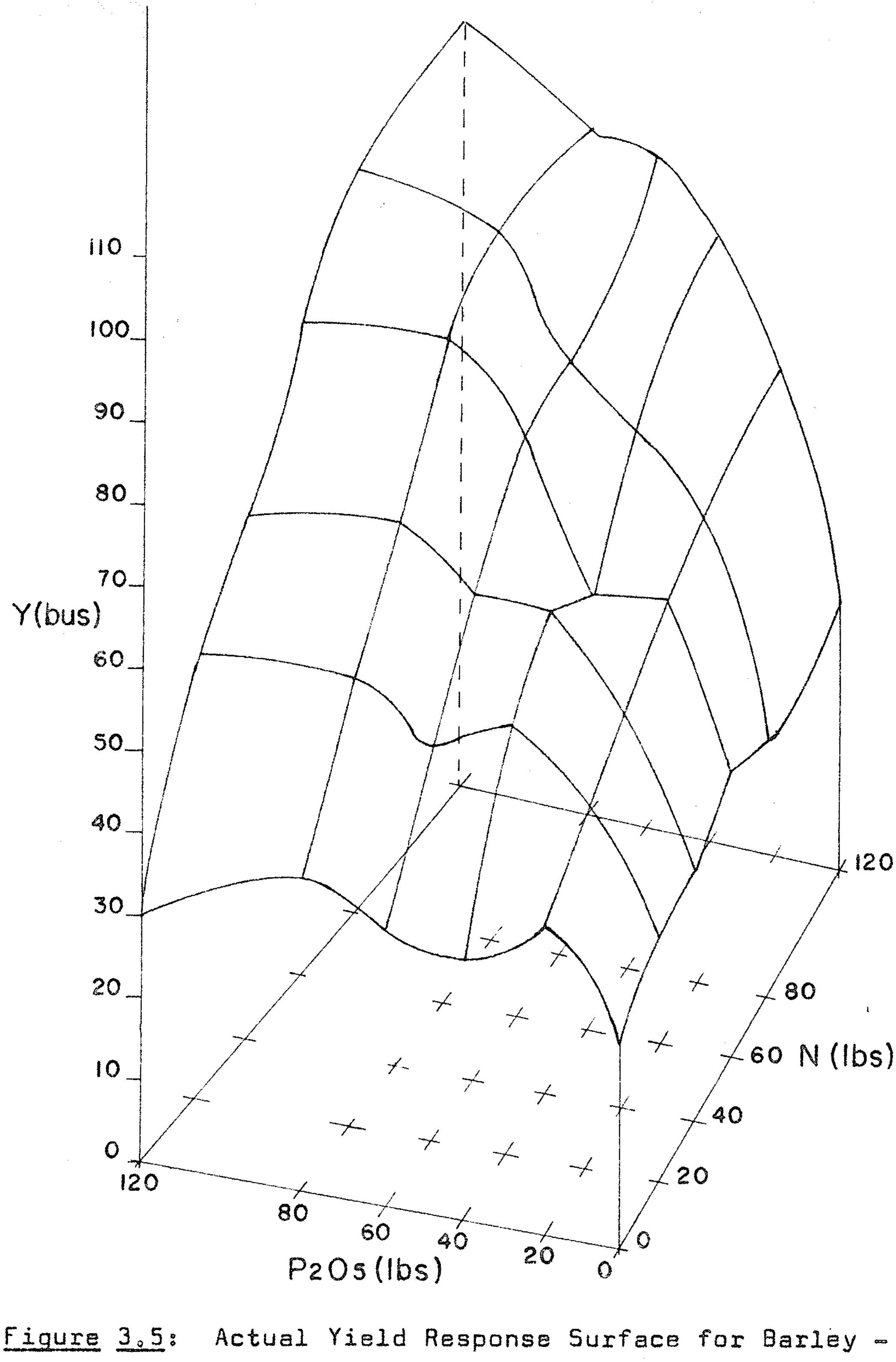


Figure 3.5: Actual Yield Response Surface for Barley - 1957.

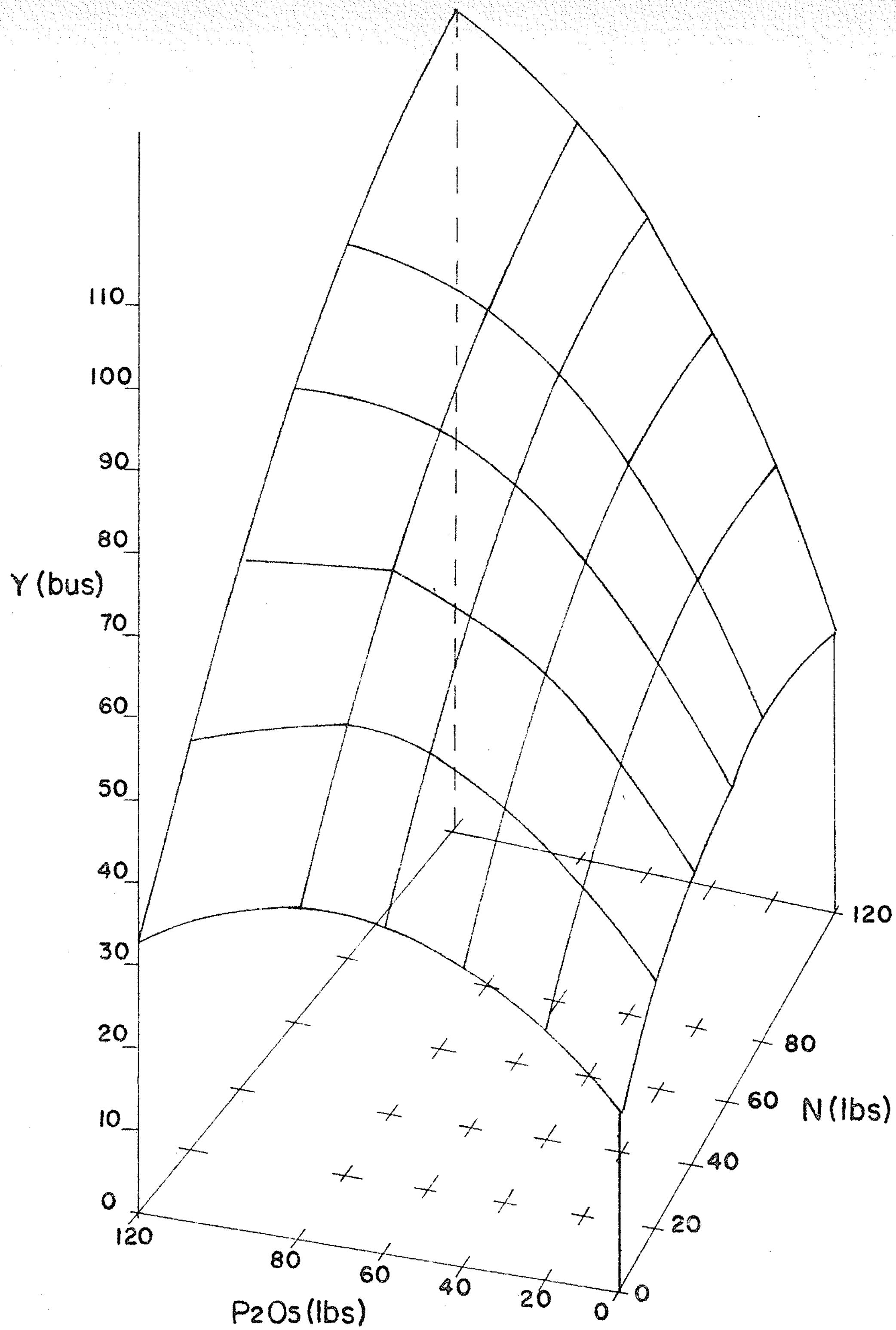


Figure 3.6: Predicted Yield Response Surface for Barley, Derived by the Quadratic Function - 1957.

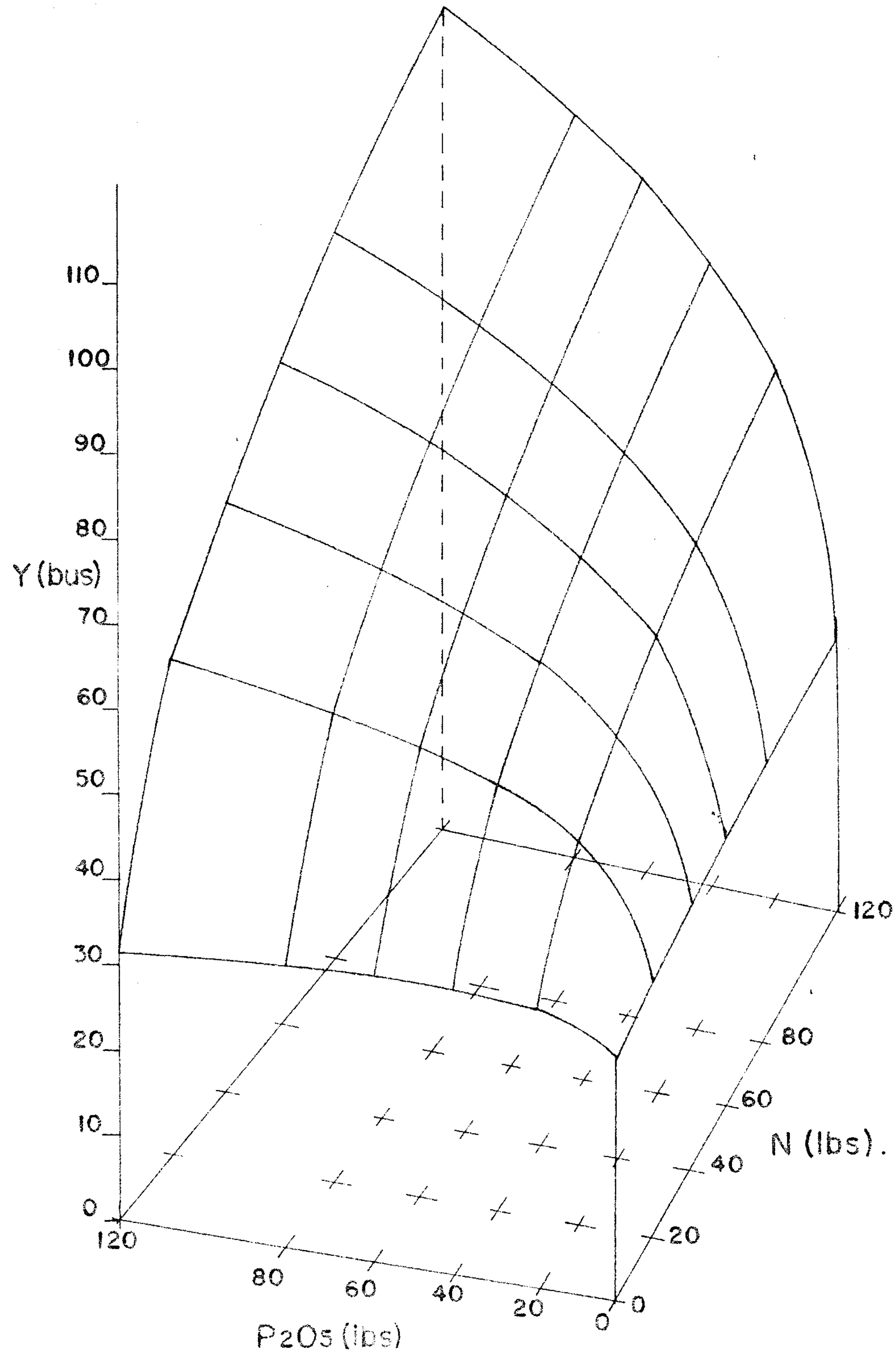


Figure 3.7: Predicted Yield Response Surface for Barley, Derived by the Square Root Function - 1957.

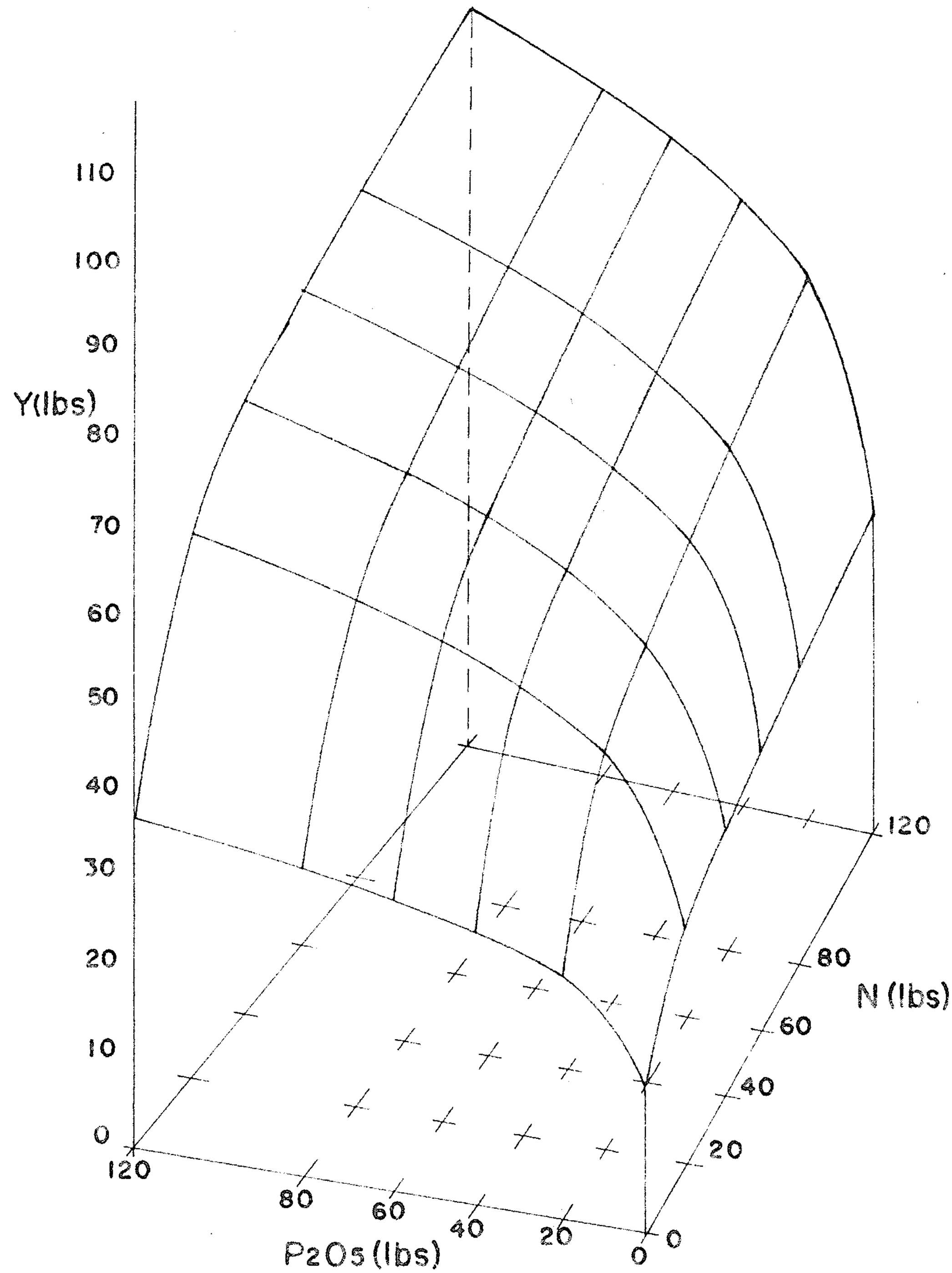


Figure 3.8: Predicted Yield Response Surface for Barley, Derived by the Cobb-Douglas Function - 1957.

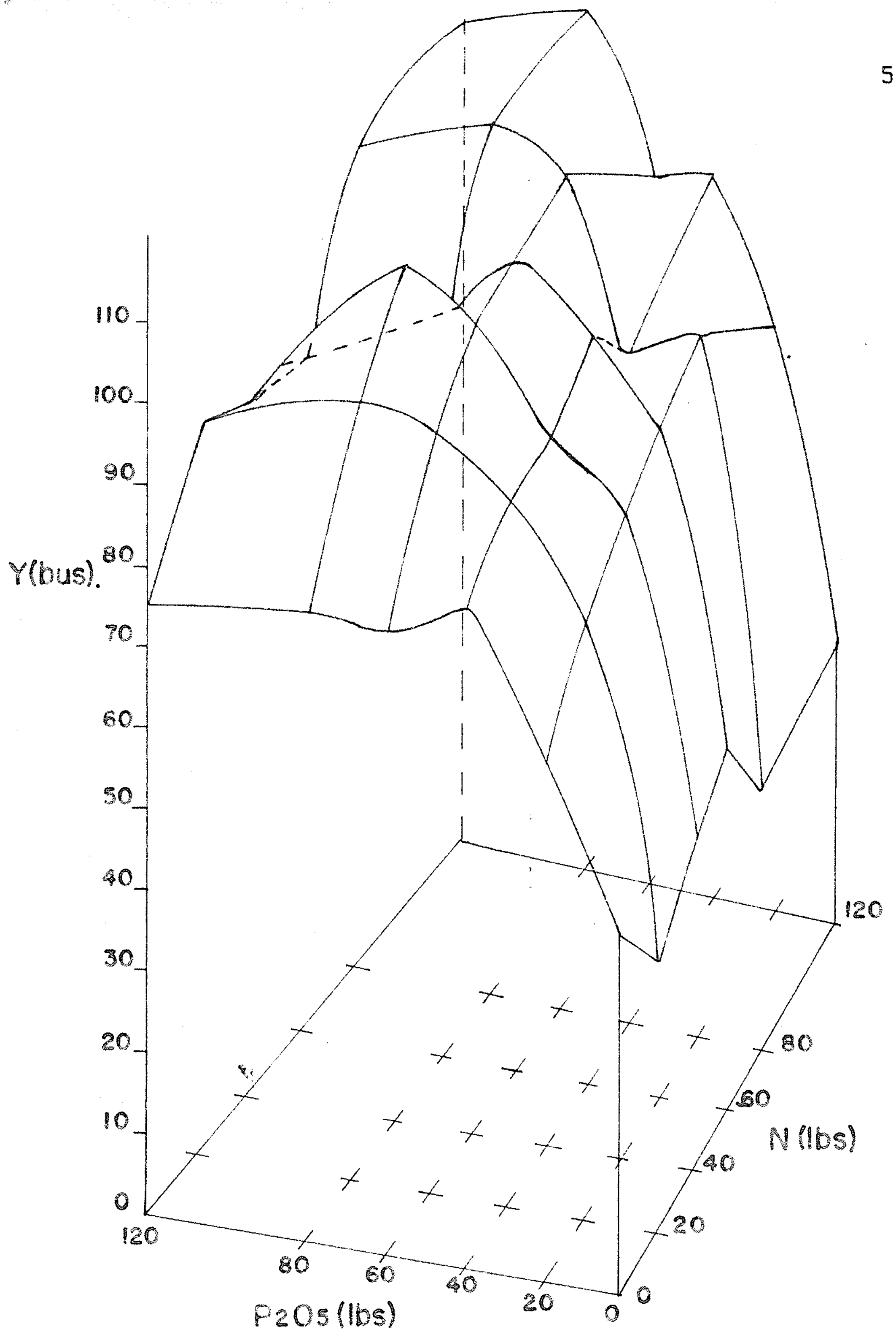


Figure 3.9: Actual Yield Response Surface for Barley - 1958.

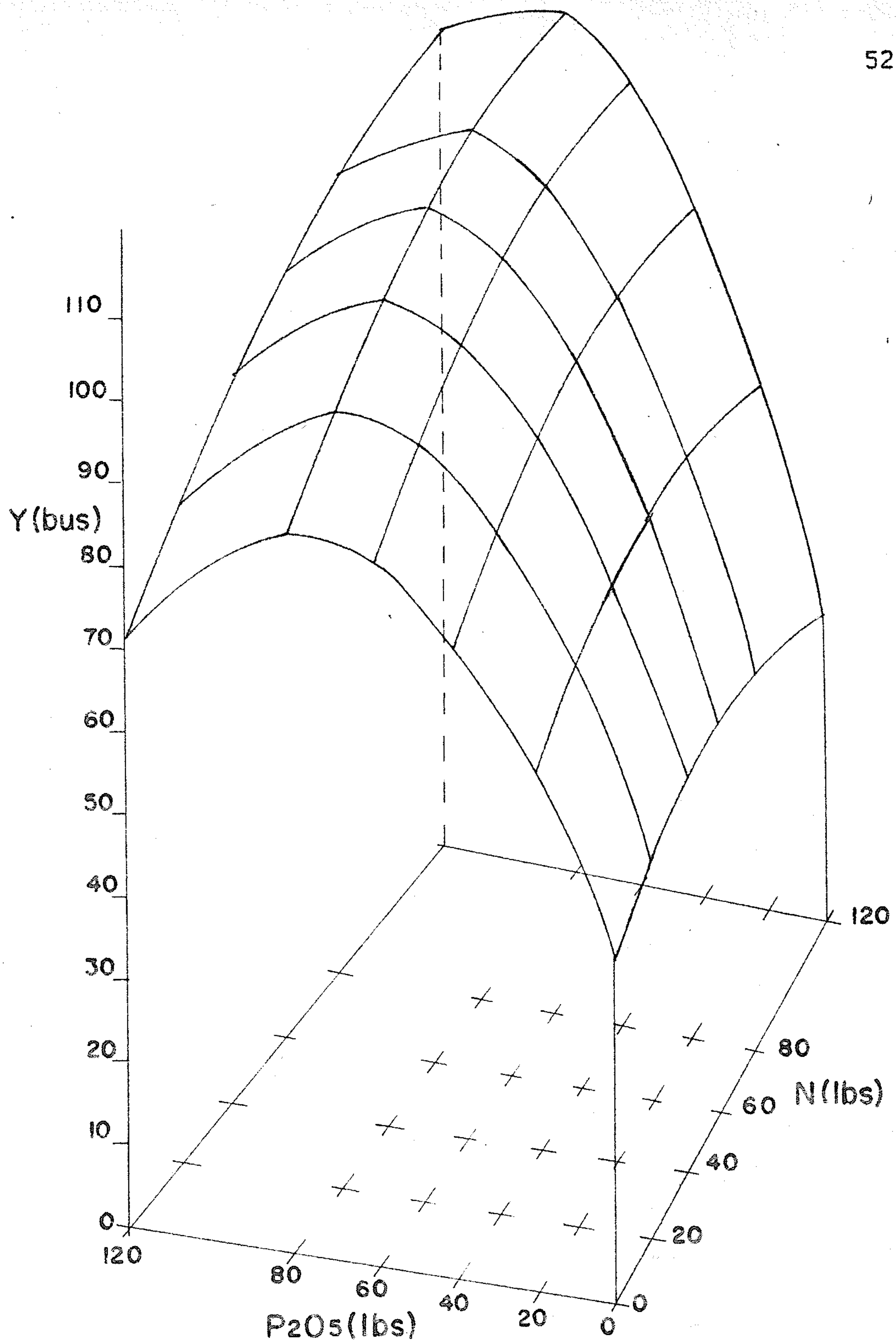


Figure 3.10: Predicted Yield Response Surface for Barley, Derived by the Quadratic Function - 1958.

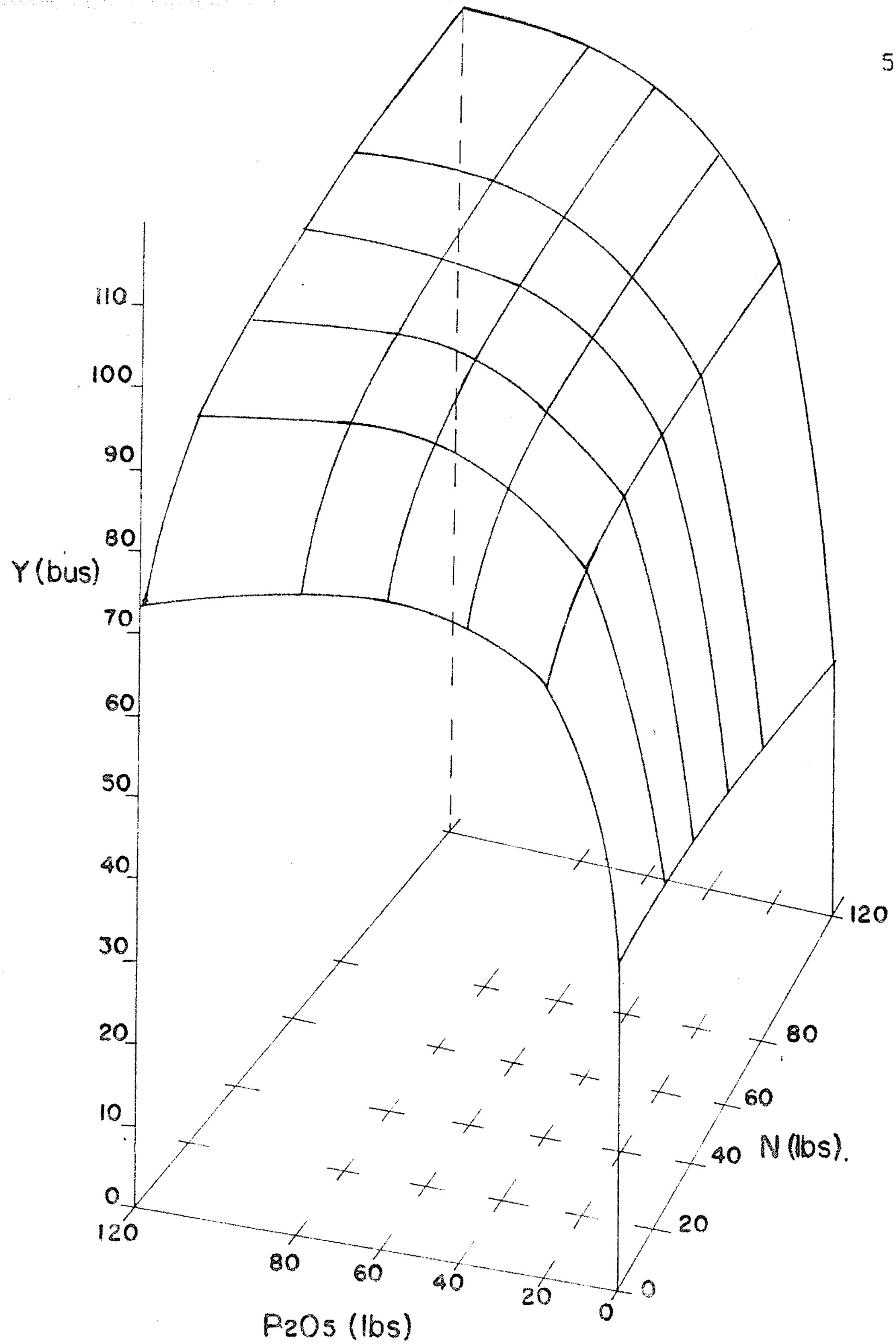


Figure 3.11: Predicted Yield Response Surface for Barley, Derived by the Square Root Function - 1958.

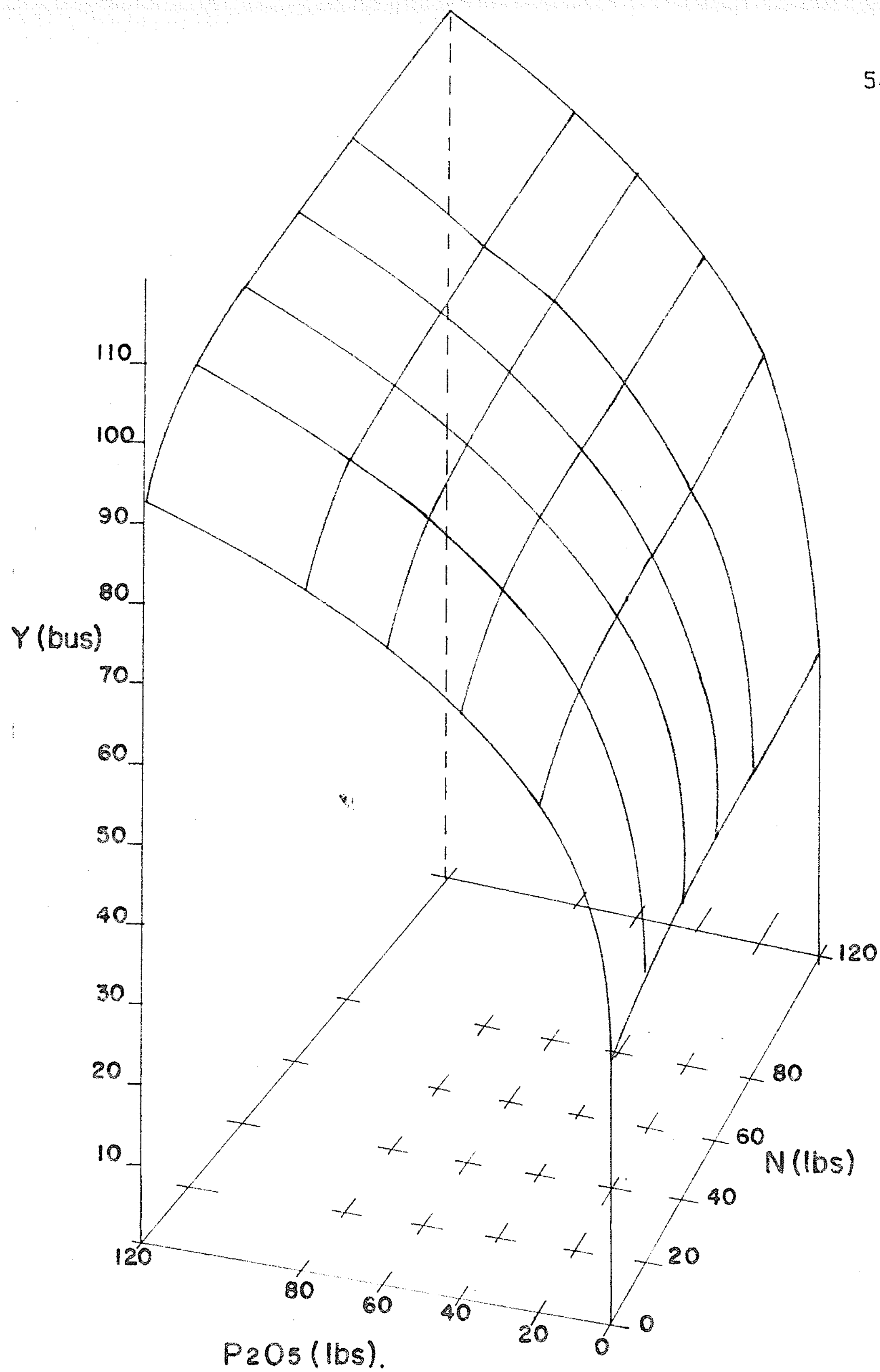


Figure 3.12: Predicted Yield Response Surface for Barley, Derived by the Cobb-Douglas Function - 1958.

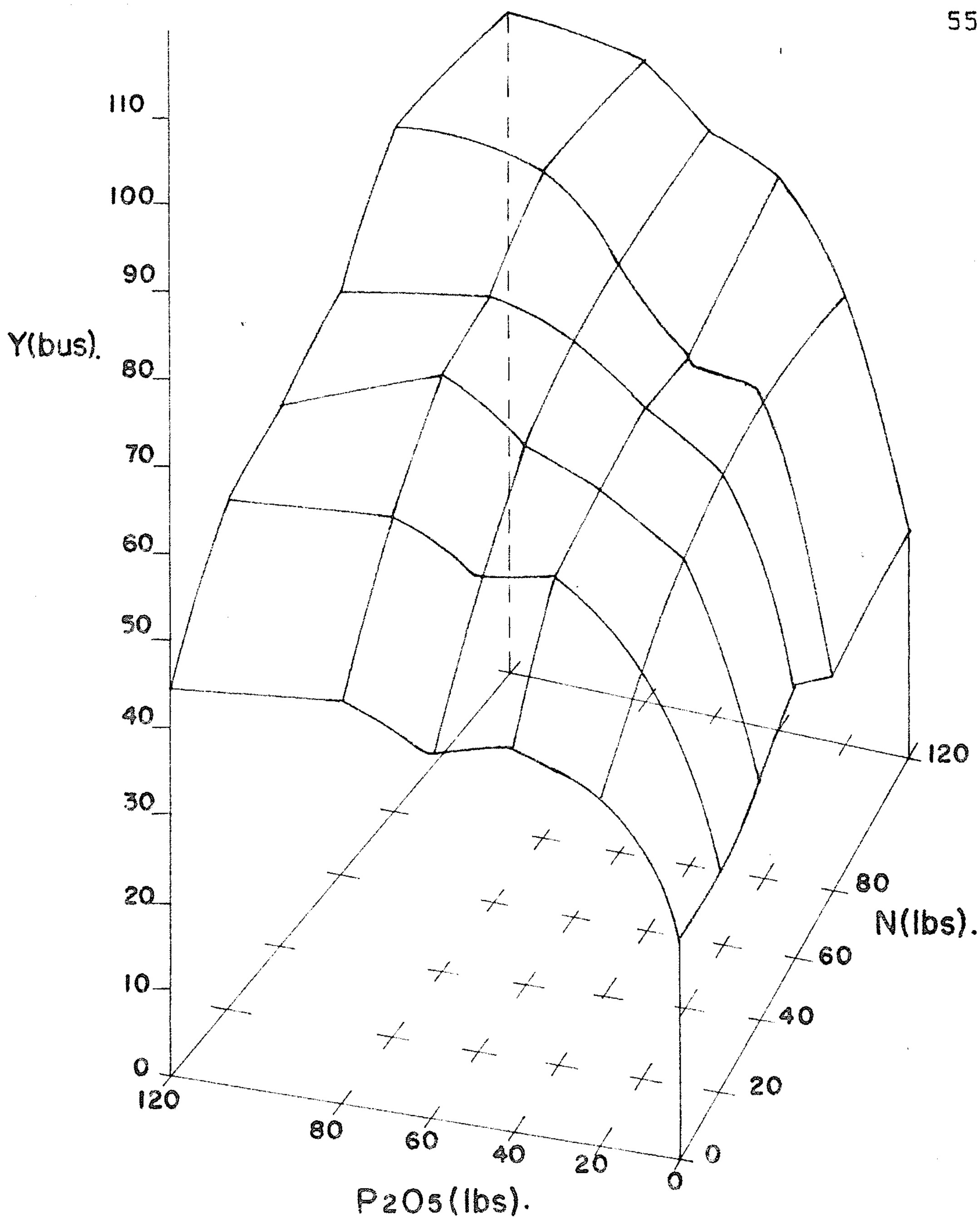


Figure 3.13: Actual Yield Response Surface for Barley - Composite Data.

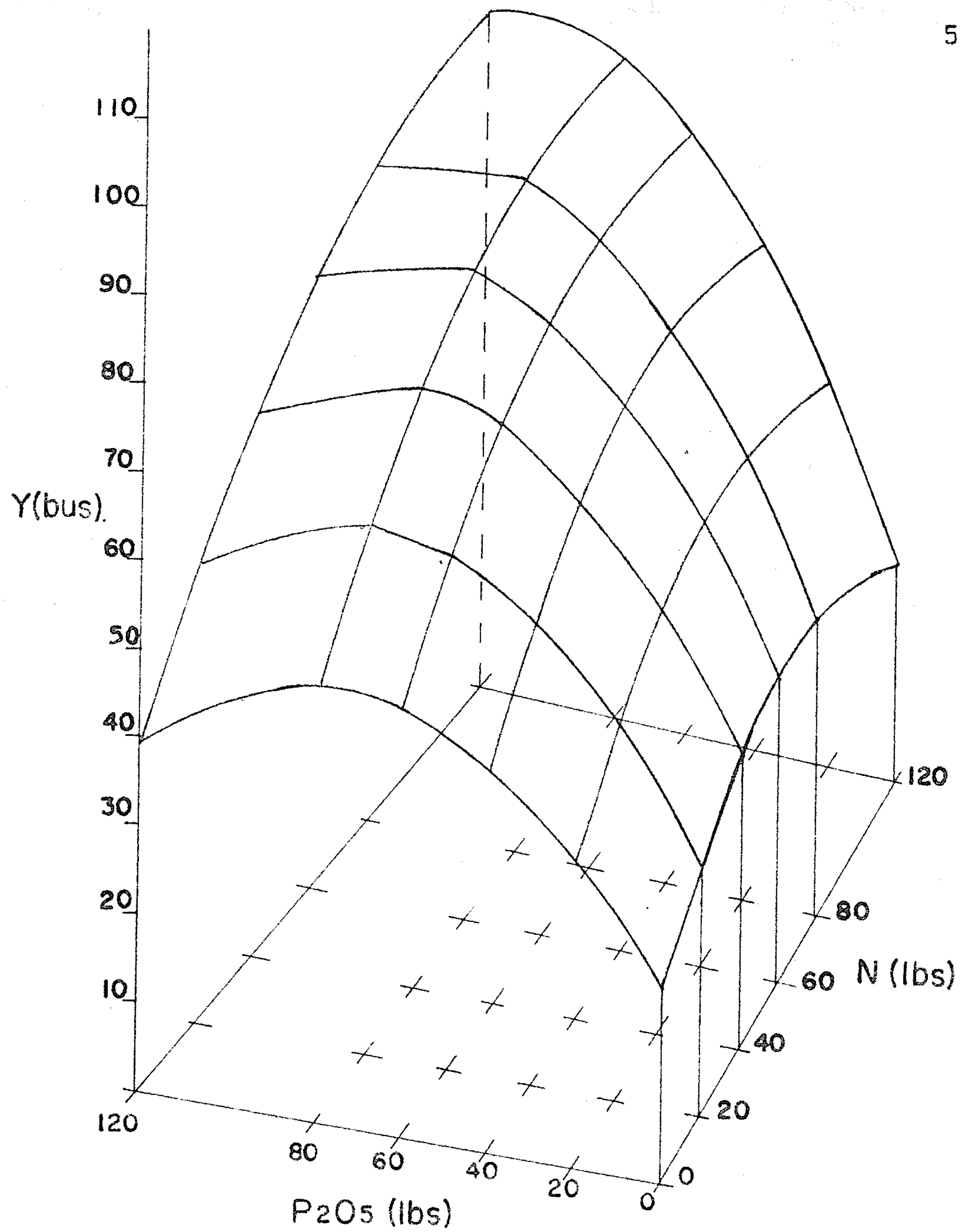


Figure 3.14: Predicted Yield Response Surface for Barley, Derived by the Quadratic Function - Composite Data.

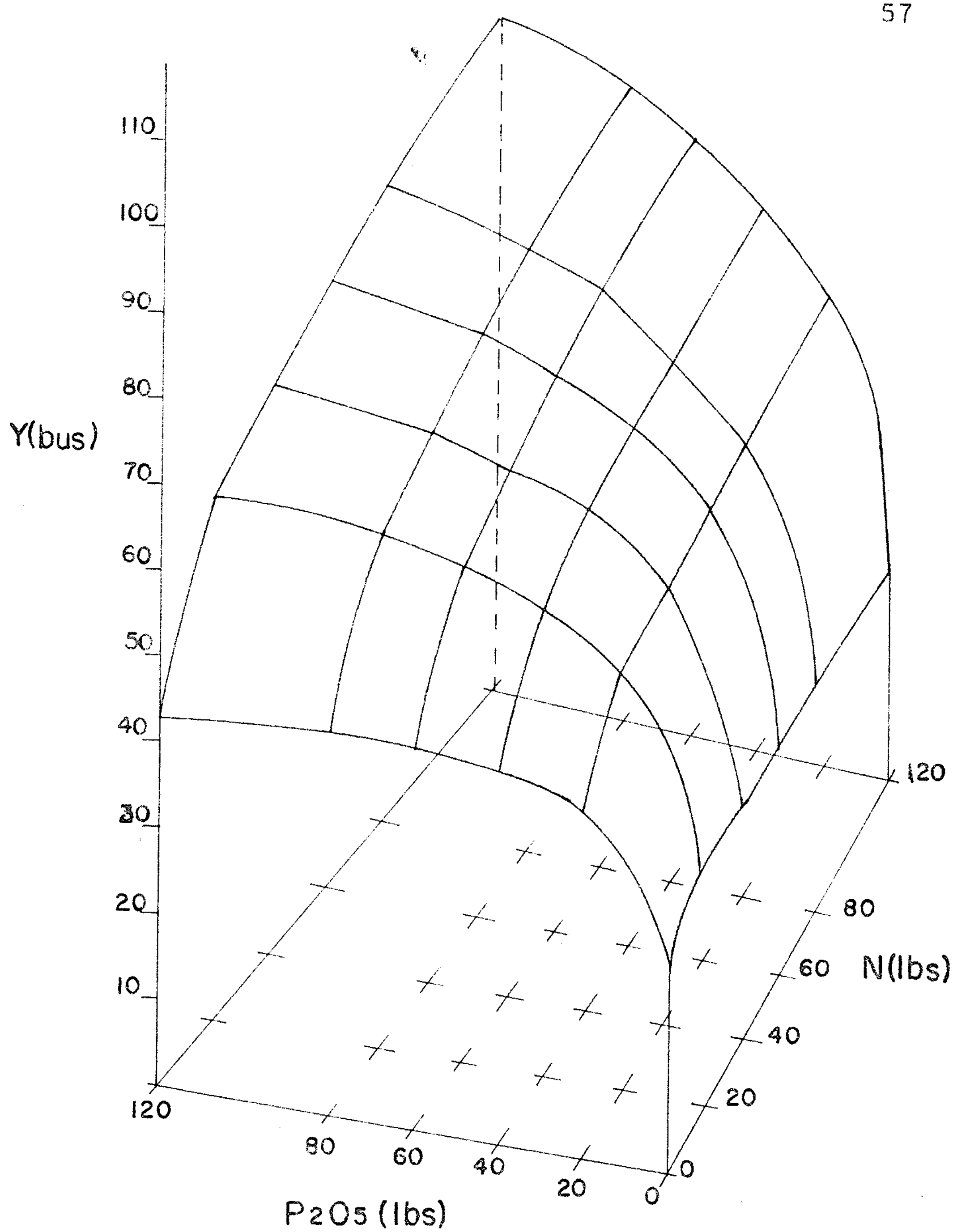


Figure 3.15: Predicted Yield Response Surface for Barley, Derived by the Square Root Function - Composite Data.

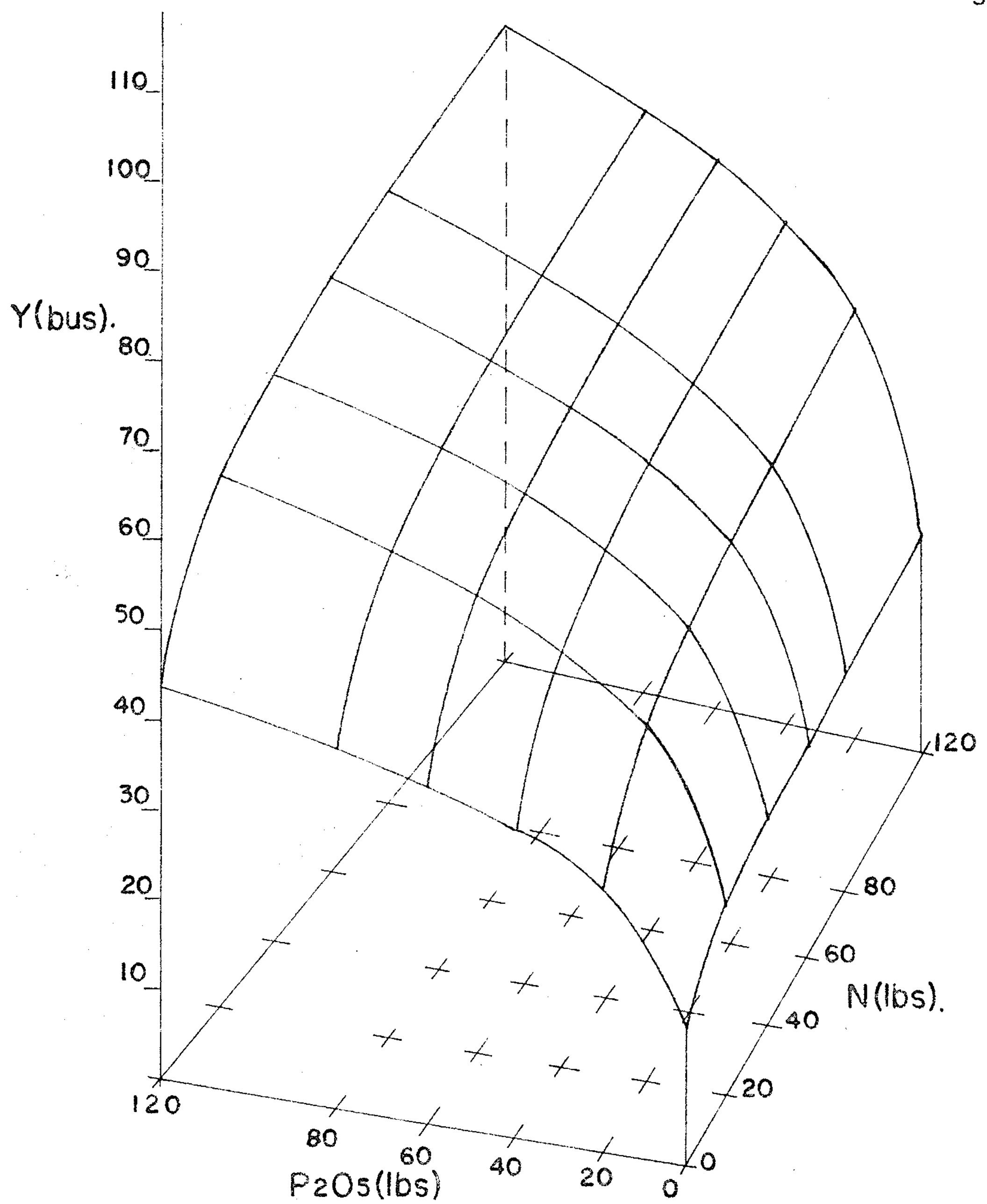


Figure 3.16: Predicted Yield Response Surface for Barley, Derived by the Cobb-Douglas Function - Composite Data.

the actual data indicated that a diminishing total product was present for higher levels of nitrogen and phosphorus. For this reason, the Cobb-Douglas function was discarded as a possible estimating equation. This function does not allow a diminishing total product and may produce erroneous estimates at high application levels of nitrogen and phosphorus. However, for purposes of comparison, complete results from the Cobb-Douglas function are included in the analysis.

As noted by Gilson and Bjarnarson, the comparative advantage of either the quadratic or the square root function as an estimating equation is very slight. With the exception of the composite data, the coefficients of determination for the square root equations indicate a slightly higher degree of correlation between yield and fertilizer nutrients than do the quadratic equations. The t-test values indicate the quadratic function to be the better predicting equation. The regression coefficients of the quadratic equation consistently tested at a higher or as high a level of significance as did the coefficients of the square root equation.

Actual yields and yields predicted by the quadratic and square root functions (Tables 3.2 to 3.5) were compared for each combination of nitrogen and phosphorus. Two general conclusions were drawn from this comparison. The square root equation provided the best yield estimates for rates of phosphorus up to, and including 60 pounds of P_2O_5; for levels in excess of 60 pounds the quadratic equation provided the best estimates. For applications of nitrogen neither equation showed a distinct advantage, although the square root equation provided slightly better estimates at extremely low and high rates of nitrogen application. This comparison indicates that the selection of a prediction equation could be influenced by the levels of application of the nutrients under investigation. For the range of applic- ations in this study the visual comparison does not permit selection of either equation as the best estimator of yield. This conclusion was verified by a comparison of the actual and predicted yield response surfaces.

The foregoing discussion indicates the difficulty in selecting either the quadratic or square root function as the best estimating equation. A decision to select one of these two equations must be based not only on statistical measures but also a common-sense examination of actual and predicted yields. The selection of the quadratic equation for predicting yields is based on three facts. The coefficients of determination, although better for the square root equation were very close. The t-tests were significantly better for the quadratic equation. The visual comparison of actual and predicted yields for the two equations did not show a distinct advantage for either equation.

The choice of the quadratic equation is made with one important qualification. It is evident that more research is needed in the analysis of experimental fertilizer-yield response data. This need is apparent when yield data for the three years are examined. Figure 3.17 shows that the marginal productivity for small incremental inputs of phosphorus increased sharply from 1956 to 1957 and 1958. Maximum attainable yields have also greatly increased. These observations invite further research. It is suggested that the following may be pertinent to such an investigation:

1. Climatic factors including precipitation, number of frost-free days, length of growing season, and temperature.
2. Date of seeding.
3. Preparation of the seedbed.

Such research would require the co-operation of soil and plant scientists, and agricultural economists.

Rates of Substitution between Nitrogen and Phosphorus In Producing Specific Yields

Combinations of nitrogen and phosphorus to produce specified yields are shown in Tables 3.6 to 3.16 inclusive.[6] The isoquants which were plotted from the data in these Tables

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[6] Equations for determining isoquants are presented in Appendix II.

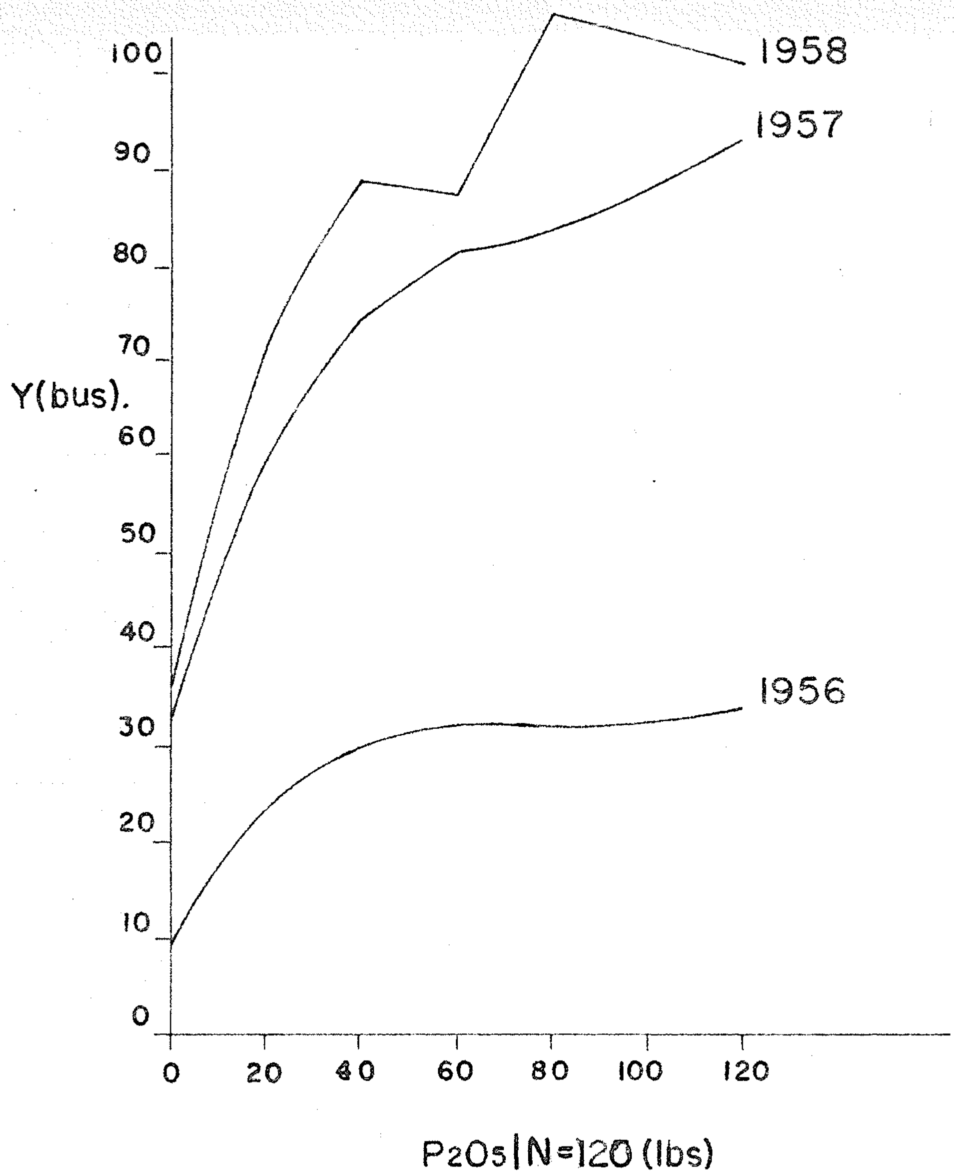

Figure 3.17: Total Product Curves for Barley, with Nitrogen Constant at 120 Pounds and Phosphorus Varied for the Years 1956, 1957, and 1958.

are illustrated in Figures 3.18 to 3.28 inclusive. The table and figure for the 1956 quadratic equation, as derived by Gilson and Bjarnarson, are included in Appendix I.

Complete results for all equations and years have been calculated. Discussion will be based on the quadratic equation. Where necessary, reference will also be made to the other equations.

Combinations of nitrogen and phosphorus in producing specified yields have not been calculated for all rates of nitrogen. Only relevant combinations which occur within the boundaries of the ridgelines have been determined. Within the ridgelines, represented by dashed lines on the figures, substitution between nitrogen and phosphorus is possible. Outside the ridgelines application of both nutrients must be increased in order to maintain production at a specified yield. The area outside the ridgelines is, therefore, an irrational area of application. Accordingly, marginal rates of substitution have been calculated only for combinations of nitrogen and phosphorus within the ridgelines.[7]

Columns 4 and 5 of Table 3.6 indicate that 60 bushels of barley can be produced by several combinations of nitrogen and phosphorus. This is also illustrated in Figure 3.18.

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[7]Equations for determining marginal rates of substitution are presented in Appendix II.

Forty pounds of nitrogen and 41.47 pounds of phosphorus produce a yield of 60 bushels. If nitrogen is increased to 60 pounds, phosphorus can be reduced to 20.92 pounds and yield maintained at 60 bushels. A further increase in nitrogen to 80 pounds reduces the phosphorus requirements for 60 bushels of barley to 8.80 pounds.

Columns 1 and 2 of Table 3.7 illustrate irrational applications of nitrogen and phosphorus. No nitrogen and 18.36 pounds of phosphorus will produce 60 bushels of barley. Sixty bushels may also be produced by combining 40 pounds of nitrogen and 11.43 pounds of phosphorus. If the nitrogen application is increased to 60 pounds, an increase in phosphorus is also necessary for yield to be maintained at 60 bushels. On the 60 bushel isoquant shown in Figure 3.19 the combination of 60 pounds of nitrogen and 12.38 pounds of phosphorus lies outside the boundaries of the upper ridge-line. Such an application is therefore irrational.

Marginal rates of substitution of phosphorus for nitrogen are given in columns 3, 6, and 9 of Tables 3.6 to 3.16. The decrease in phosphorus application as nitrogen levels are increased indicates a decreasing marginal rate of substitution of phosphorus for nitrogen. Column 3 of Table 3.6 shows that 1 pound of nitrogen will replace 8.74 pounds of phosphorus when no nitrogen and 61.85 pounds of phosphorus are combined to produce 40 bushels of barley. If 20 pounds of nitrogen and 20.38 pounds of phosphorus are combined, one pound of nitrogen will replace 0.92 pounds of phosphorus. One pound of nitrogen will replace only 0.35 pounds of phosphorus when 40 pounds of nitrogen and 2.5 pounds of phosphorus are combined in the production of 40 bushels of barley.

The results derived by the use of the Cobb-Douglas function require explanation. Since the ridgelines coincide with the nitrogen and phosphorus axes a broader range of combinations of nitrogen and phosphorus is obtained. This is a technical characteristic of the Cobb-Douglas function and does not give it any advantage in the analysis of yield response.

The importance of determining marginal rates of substitution is evident when least cost combinations of nitrogen and phosphorus are calculated.

Least Cost Combinations

Least cost combinations of nutrients have a practical use for the farmer making decisions relating to fertilizer use. For any given yield it is desirable to know what combination of nutrients will provide that yield at the least cost. In the ideal situation the farmer will want the com-

Table 3.6. Combinations of N and P_2O_5 to Produce Specified Yields of Barley and Rates of Substitution of N and P_2O_5, Derived by the Quadratic Equation - 1957.

40 Bushels			60 Bushels			80 Bushels		
LBS N (1)	LBS P_2O_5 (2)	MARGINAL RATE OF SUBSTITUTION OF P_2O_5 FOR N (3)	LBS N (4)	LBS P_2O_5 (5)	MARGINAL RATE OF SUBSTITUTION OF P_2O_5 FOR N (6)	LBS N (7)	LBS P_2O_5 (8)	MARGINAL RATE OF SUBSTITUTION OF P_2O_5 FOR N (9)
0	61.85	-8.74	0	--	--	0	--	--
20	20.38	- .92	20	--	--	20	--	--
40	2.5	- .35	40	41.47	-1.01	40	--	--
60	--	--	60	20.92	- .58	60	67.90	-1.39
80	--	--	80	8.80	- .03	80	39.85	- .26
120	--	--	120	--		120	15.85	

Table 3.7. Combinations of N and P_2O_5 to Produce Specified Yields of Barley and Rates of Substitution of N and P_2O_5, Derived by the Quadratic Equation - 1958.

60 Bushels			80 Bushels			100 Bushels		
LBS N (1)	LBS P_2O_5 (2)	MARGINAL RATE OF SUBSTITUTION OF P_2O_5 FOR N (3)	LBS N (4)	LBS P_2O_5 (5)	MARGINAL RATE OF SUBSTITUTION OF P_2O_5 FOR N (6)	LBS N (7)	LBS P_2O_5 (8)	MARGINAL RATE OF SUBSTITUTION OF P_2O_5 FOR N (9)
0	18.36	- .29	0	44.43	- .66	0	--	--
20	12.72	- .16	20	35.77	- .32	20	--	--
40	11.43	- .07	40	32.52	- .19	40	--	--
60	12.38	--	60	32.73	- .18	60	74.30	- .39
80	15.41	--	80	35.88		80	79.92	- .53
120	15.66	--	120	52.02		120	--	--

Table 3.8. Combinations of N and P_2O_5 to Produce Specified Yields of Barley and Rates of Substitution of N and P_2O_5, Derived by the Quadratic Equation - Composite Data.

40 Bushels			60 Bushels		
LBS N (1)	LBS P_2O_5 (2)	MARGINAL RATE OF SUBSTITUTION OF P_2O_5 FOR N (3)	LBS N (4)	LBS P_2O_5 (5)	MARGINAL RATE OF SUBSTITUTION OF P_2O_5 FOR N (6)
0	30.56	-1.08	0	--	
20	17.17	- .51	20	--	
40	13.28	- .26	40	52.36	- .86
60	15.45	- .08	60	39.39	- .24
80	22.99		80	52.84	
120	59.53		120	--	

Table 3.9. Combinations of N and P_2O_5 to Produce Specified Yields of Barley and Rates of Substitution of N and P_2O_5, Derived by the Square Root Equation - 1956.

20 Bushels			30 Bushels		
LBS N (1)	LBS P_2O_5 (2)	MARGINAL RATE OF SUBSTITUTION OF P_2O_5 FOR N (3)	LBS N (4)	LBS P_2O_5 (5)	MARGINAL RATE OF SUBSTITUTION OF P_2O_5 FOR N (6)
0	54.61	--	0	318.06	--
20	9.98	- .26	20	71.45	-2.44
40	7.33	- .05	40	54.80	- .44
60	7.08		60	49.44	- .14
80	7.78		80	48.18	- .00
120	10.80		120	51.20	

Table 3.10. Combinations of N and P_2O_5 to Produce Specified Yields of Barley and Rates of Substitution of N and P_2O_5, Derived by the Square Root Equation - 1957.

40 Bushels			60 Bushels			80 Bushels		
LBS N (1)	LBS P_2O_5 (2)	MARGINAL RATE OF SUBSTITUTION OF P_2O_5 FOR N (3)	LBS N (4)	LBS P_2O_5 (5)	MARGINAL RATE OF SUBSTITUTION OF P_2O_5 FOR N (6)	LBS N (7)	LBS P_2O_5 (8)	MARGINAL RATE OF SUBSTITUTION OF P_2O_5 FOR N (9)
0	--	--	0	--	--	0	--	--
20	7.68	- .34	20	164.91	-28.48	20	--	--
40	3.99	- .10	40	54.15	- 1.58	40	--	--
60	2.60	- .03	60	34.47	- .62	60	138.17	-3.66
80	1.86	- .03	80	25.39	- .34	80	91.83	-1.48
120	1.17	- .01	120	19.21	- .17	120	56.41	- .54

Table 3.11. Combinations of N and P_2O_5 to Produce Specified Yields of Barley and Rates of Substitution of N and P_2O_5, Derived by the Square Root Equation - 1958.

60 Bushels			80 Bushels			100 Bushels		
LBS N (1)	LBS P_2O_5 (2)	MARGINAL RATE OF SUBSTITUTION OF P_2O_5 FOR N (3)	LBS N (4)	LBS P_2O_5 (5)	MARGINAL RATE OF SUBSTITUTION OF P_2O_5 FOR N (6)	LBS N (7)	LBS P_2O_5 (8)	MARGINAL RATE OF SUBSTITUTION OF P_2O_5 FOR N (9)
0	3.64		0	--	--	0	--	--
20	3.50	-.004	20	21.78	- .18	20	--	--
40	3.64		40	19.99	- .04	40	--	--
60	4.02		60	19.80		60	--	--
80	4.52		80	20.25		80	--	--
120	5.81		120	22.14		120	88.00	- .03

Table 3.12. Combinations of N and P_2O_5 to Produce Specified Yields of Barley and Rates of Substitution of N and P_2O_5, Derived by the Square Root Equation - Composite Data.

40 Bushels			60 Bushels		
LBS N (1)	LBS P_2O_5 (2)	MARGINAL RATE OF SUBSTITUTION OF P_2O_5 FOR N (3)	LBS N (4)	LBS P_2O_5 (5)	MARGINAL RATE OF SUBSTITUTION OF P_2O_5 FOR N (6)
0	18.46	--	0	--	--
20	5.19	- .09	20	--	--
40	4.19	- .02	40	49.37	- .79
60	3.91	- .01	60	36.04	- .28
80	3.90		80	34.46	- .17
120	4.26		120	28.27	- .05

Table 3.13. Combinations of N and P_2O_5 to Produce Specified Yields of Barley and Rates of Substitution of N and P_2O_5, Derived by the Cobb-Douglas Equation - 1956.

20 Bushels			40 Bushels		
LBS N (1)	LBS P_2O_5 (2)	MARGINAL RATE OF SUBSTITUTION OF P_2O_5 FOR N (3)	LBS N (4)	LBS P_2O_5 (5)	MARGINAL RATE OF SUBSTITUTION OF P_2O_5 FOR N (6)
0	--	--	0	--	--
20	16.37	- .32	20	85.90	-1.71
40	12.42	- .12	40	65.31	- .65
60	10.57	- .08	60	55.59	- .37
80	9.44	- .05	80	49.55	- .25
120	8.04	- .03	120	42.17	- .14

Table 3.14. Combinations of N and P_2O_5 to Produce Specified Yields of Barley and Rates of Substitution of N and P_2O_5, Derived by the Cobb-Douglas Equation - 1957.

40 Bushels			60 Bushels			80 Bushels		
LBS N (1)	LBS P_2O_5 (2)	MARGINAL RATE OF SUBSTITUTION OF P_2O_5 FOR N (3)	LBS N (4)	LBS P_2O_5 (5)	MARGINAL RATE OF SUBSTITUTION OF P_2O_5 FOR N (6)	LBS N (7)	LBS P_2O_5 (8)	MARGINAL RATE OF SUBSTITUTION OF P_2O_5 FOR N (9)
0	--	--	0	--	--	0	--	--
20	9.46	- .45	20	92.26	-4.43	20	474.20	-22.75
40	4.87	- .12	40	47.42	-1.14	40	243.80	- 5.85
60	3.30	- .05	60	32.14	- .51	60	165.20	- 2.64
80	2.51	- .03	80	24.38	- .29	80	125.30	- 1.50
120	1.70	- .01	120	16.41	- .13	120	84.92	- .68

Table 3.15. Combinations of N and P_2O_5 to Produce Specified Yields of Barley and Rates of Substitution of N and P_2O_5, Derived by the Cobb-Douglas Equation - 1958.

60 Bushels			80 Bushels			100 Bushels		
LBS N (1)	LBS P_2O_5 (2)	MARGINAL RATE OF SUBSTITUTION OF P_2O_5 FOR N (3)	LBS N (4)	LBS P_2O_5 (5)	MARGINAL RATE OF SUBSTITUTION OF P_2O_5 FOR N (6)	LBS N (7)	LBS P_2O_5 (8)	MARGINAL RATE OF SUBSTITUTION OF P_2O_5 FOR N (9)
0	--	--	0	--	--	0	--	--
20	10.45	-.08	20	39.63	-.30	20	111.40	-.84
40	9.40	-.04	40	35.65	-.14	40	100.20	-.38
60	8.85	-.02	60	33.50	-.08	60	94.19	-.24
80	8.47	-.02	80	32.06	-.06	80	90.16	-.17
120	7.96	-.01	120	30.20	-.04	120	84.92	-.11

Table 3.16. Combinations of N and P_2O_5 to Produce Specified Yields of Barley and Rates of Substitution of N and P_2O_5, Derived by the Cobb-Douglas Equation - Composite Data.

40 Bushels			60 Bushels		
LBS N (1)	LBS P_2O_5 (2)	MARGINAL RATE OF SUBSTITUTION OF P_2O_5 FOR N (3)	LBS N (4)	LBS P_2O_5 (5)	MARGINAL RATE OF SUBSTITUTION OF P_2O_5 FOR N (6)
0	--	--	0	--	--
20	19.23	- .45	20	126.20	-2.95
40	13.90	- .16	40	91.20	-1.07
60	11.51	- .09	60	75.34	- .59
80	10.05	- .06	80	65.92	- .39
120	8.32	- .03	120	54.45	- .21

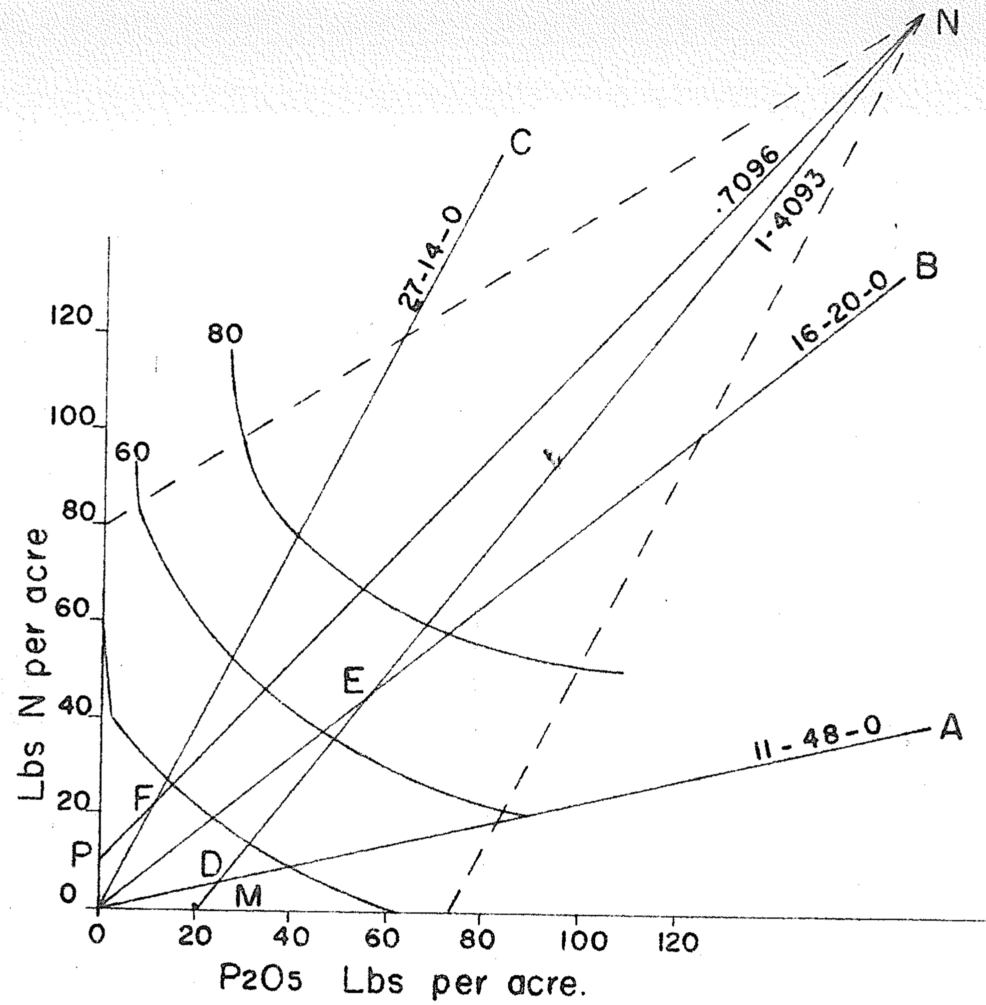


Figure 3.18: Combinations of Nitrogen and Phosphorus to Produce Specified Barley Yields, Derived by the Quadratic Equation, 1957.

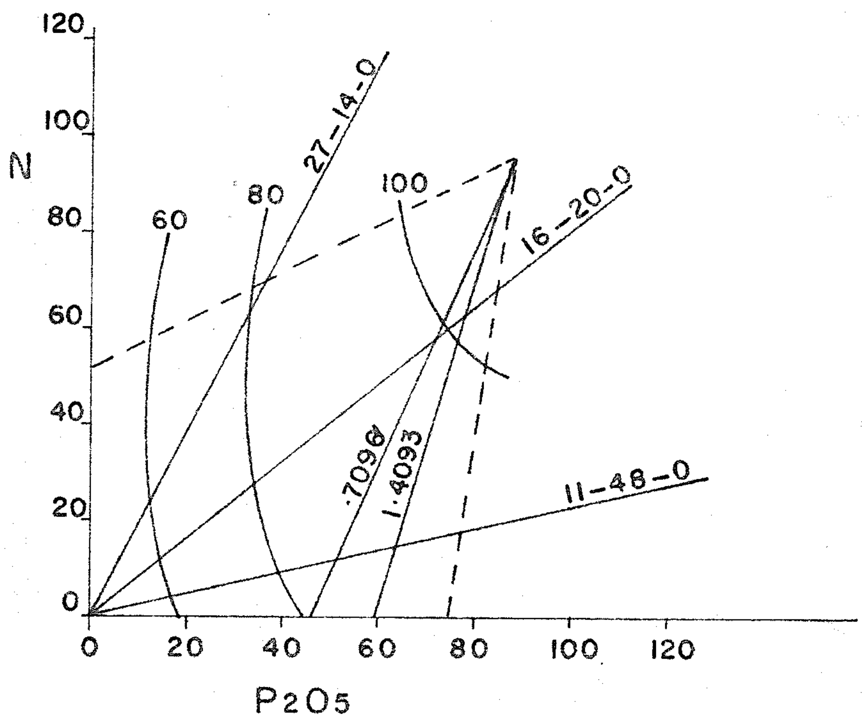


Figure 3.19: Combinations of Nitrogen and Phosphorus to Produce Specified Barley Yields, Derived by the Quadratic Equation, 1958.

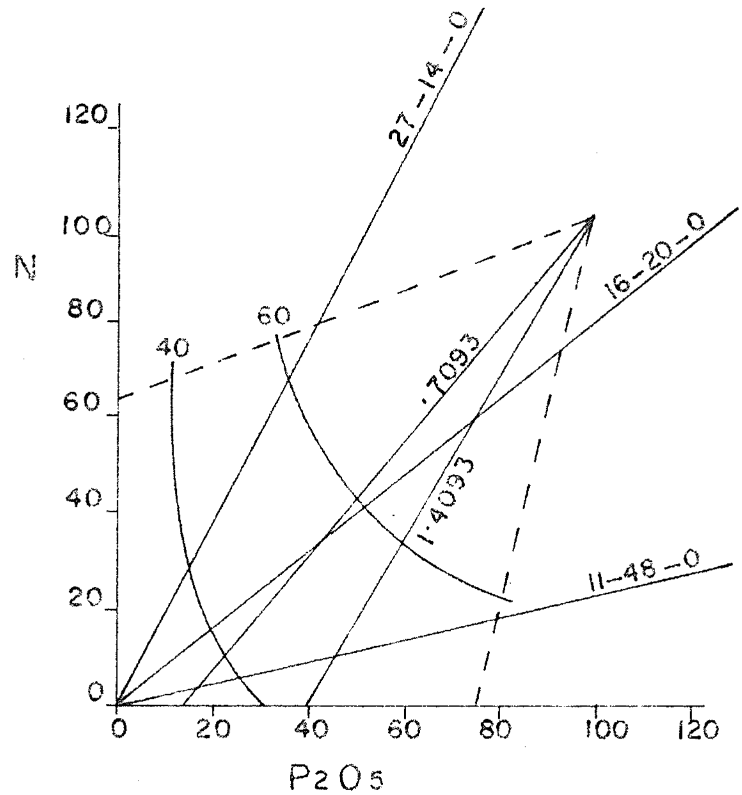


Figure 3.20: Combinations of Nitrogen and Phosphorus to Produce Specified Barley Yields, Derived by the Quadratic Equation, Composite Data.

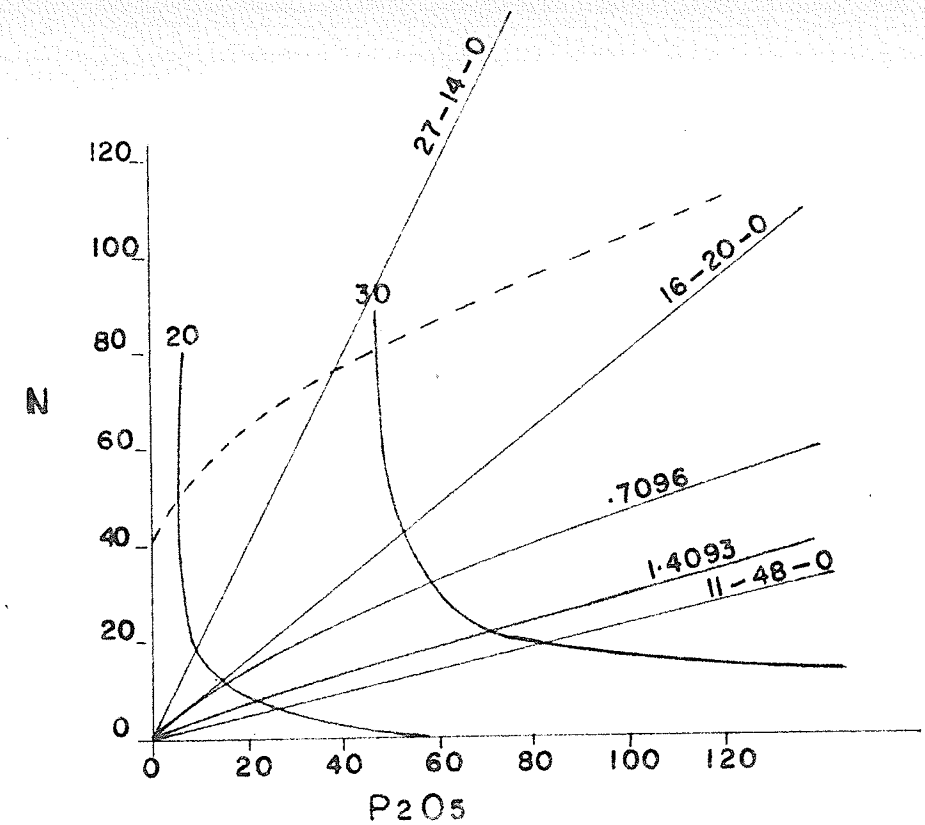


Figure 3.21: Combinations of Nitrogen and Phosphorus to Produce Specified Barley Yields, Derived by the Square Root Equation - 1956.

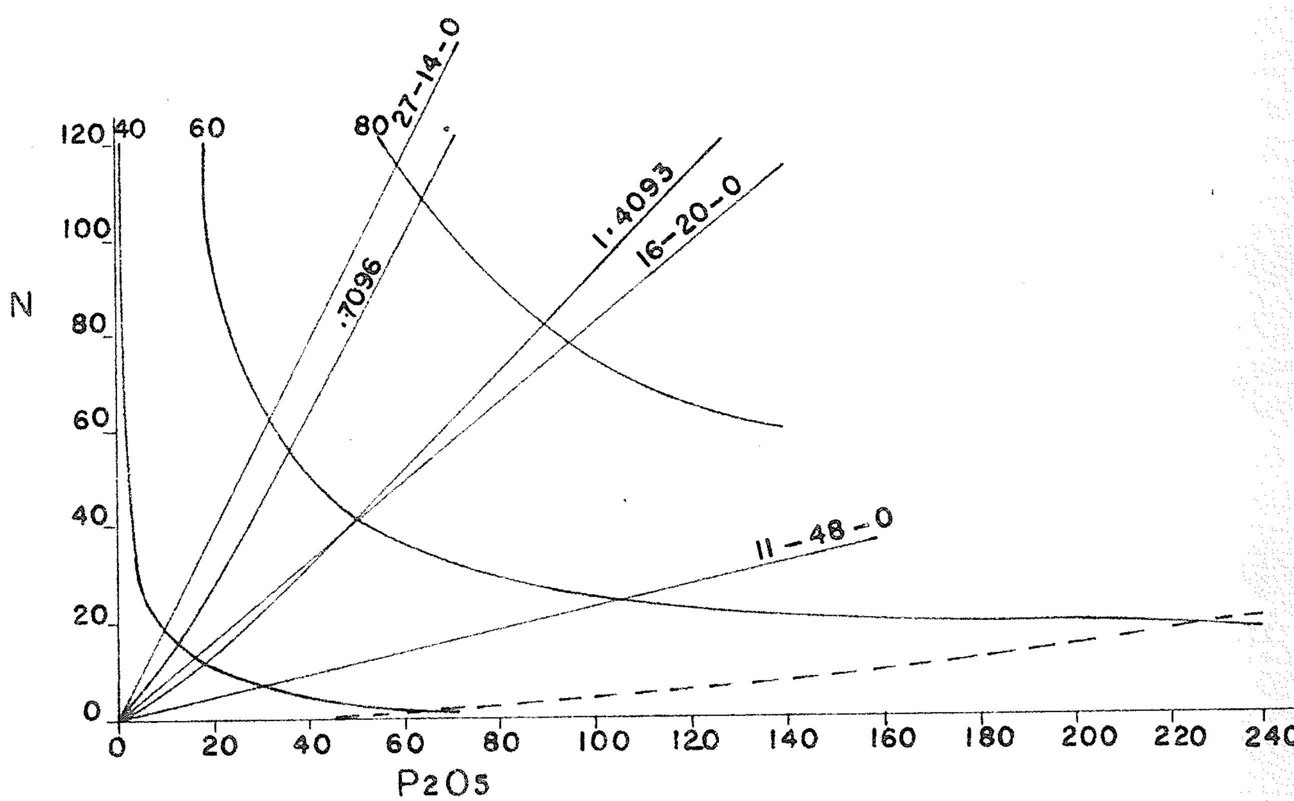
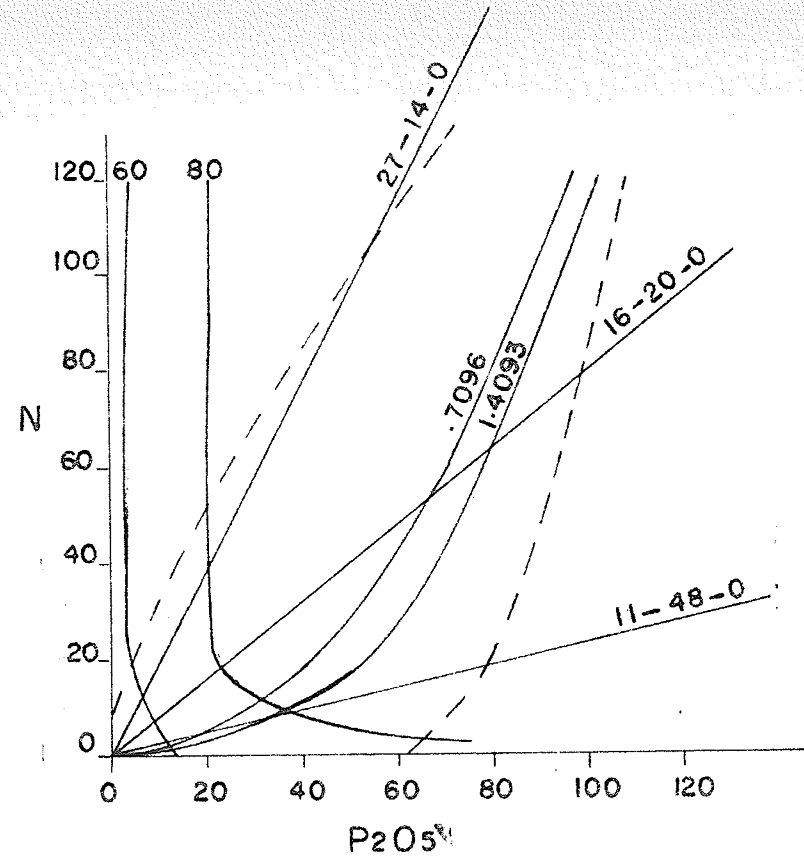
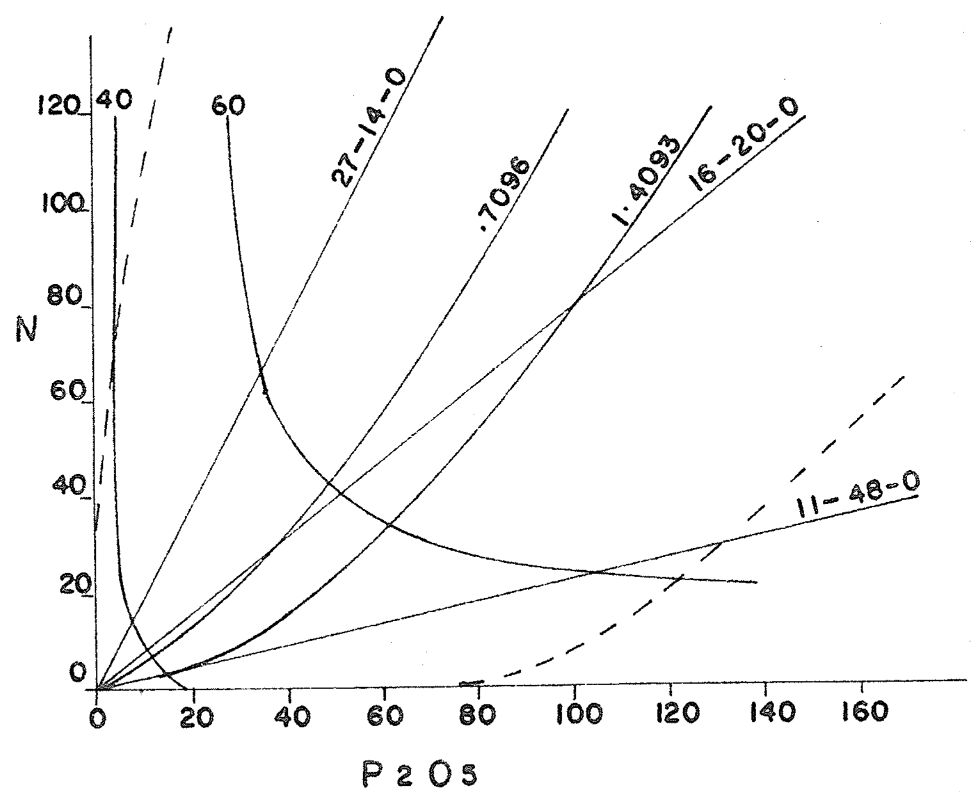


Figure 3.22: Combinations of Nitrogen and Phosphorus to Produce Specified Barley Yields, Derived by the Square Root Equation - 1957.



<u>Figure 3.23</u>: Combinations of Nitrogen and Phosphorus to Produce Specified Barley Yields, Derived by the Square Root Equation - 1958.



<u>Figure 3.24</u>: Combinations of Nitrogen and Phosphorus to Produce Specified Barley Yields, Derived by the Square Root Equation - Composite Data.

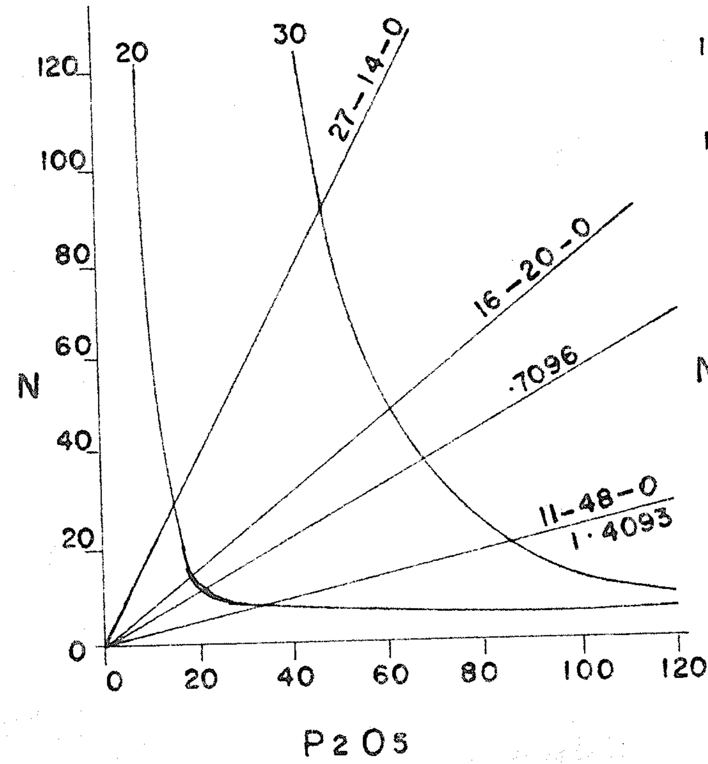


Figure 3.25: Combinations of Nitrogen and Phosphorus to Produce Specified Barley Yields, Derived by the Cobb-Douglas Equation-1956.

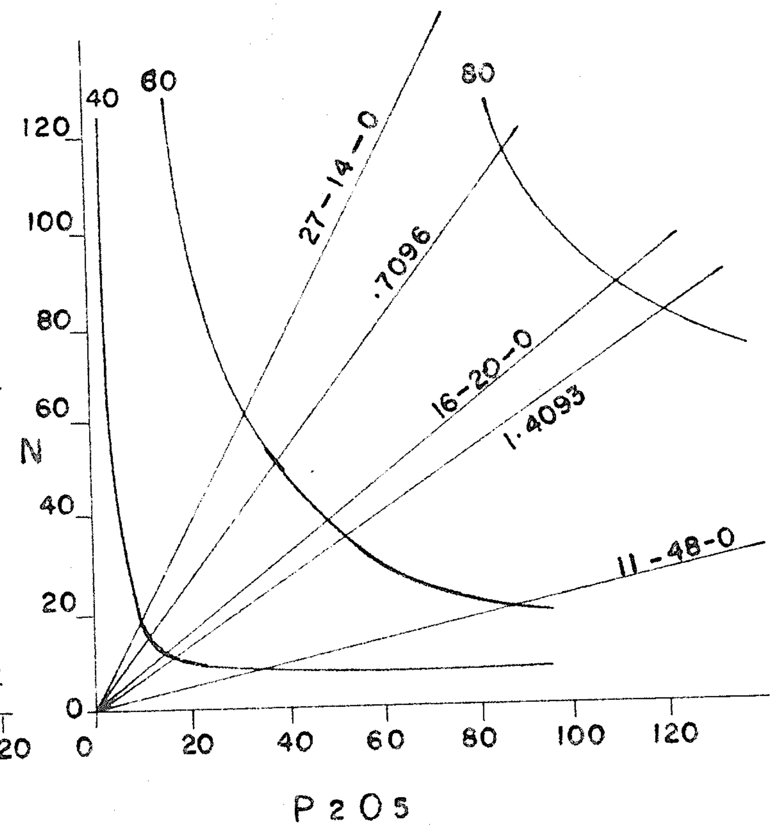


Figure 3.26: Combinations of Nitrogen and Phosphorus to Produce Specified Barley Yields, Derived by the Cobb-Douglas Equation-1957.

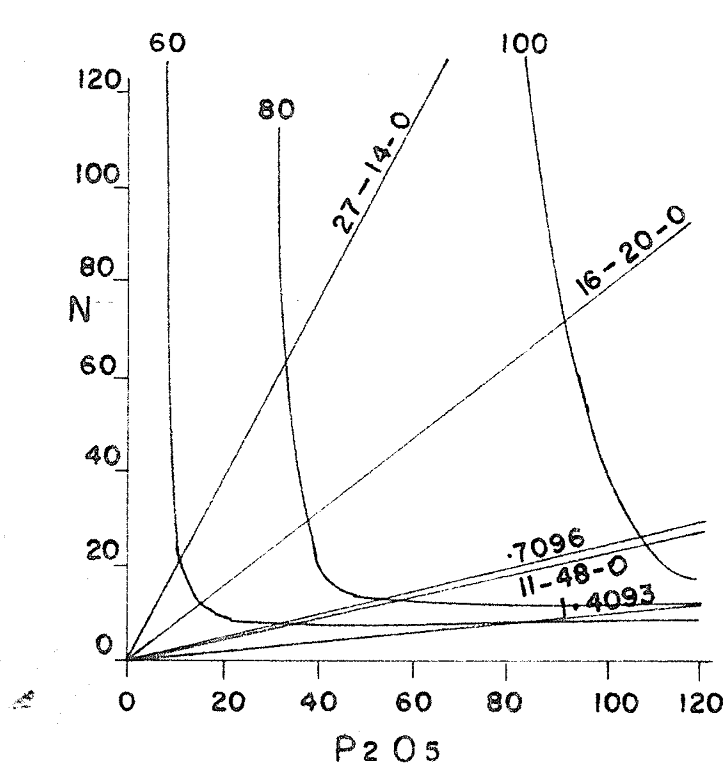


Figure 3.27: Combinations of Nitrogen and Phosphorus to Produce Specified Barley Yields, Derived by the Cobb-Douglas Equation-1958.

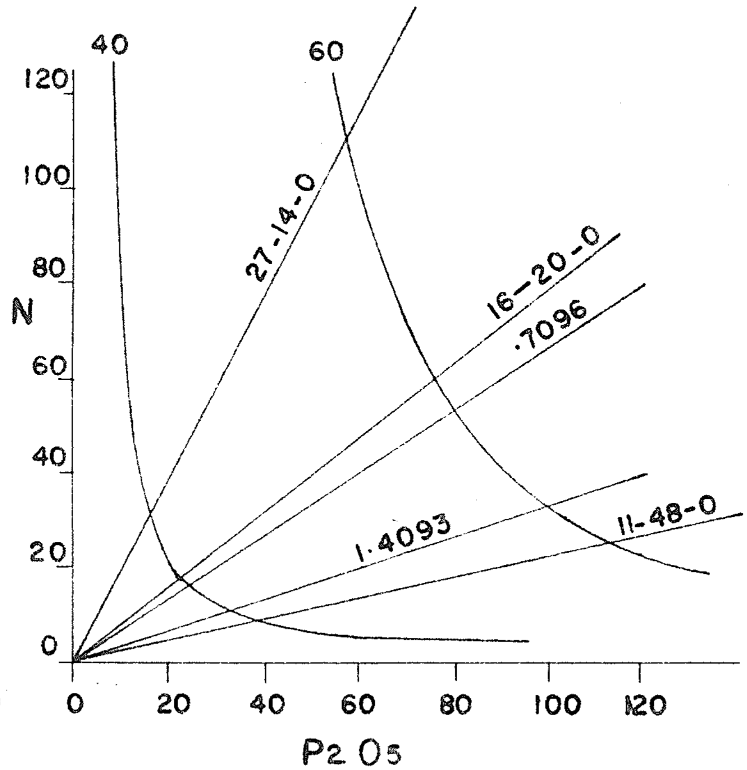


Figure 3.28: Combinations of Nitrogen and Phosphorus to Produce Specified Barley Yields, Derived by the Cobb-Douglas Equation-Composite Data.

bination of nutrients which will provide the expected yield at the lowest cost.

The least cost combination of nitrogen and phosphorus in producing a given yield is determined by equating the marginal rate of substitution of phosphorus and nitrogen with the inverse price ratio of these nutrients.[8]

The general function for expressing least cost is

$$M.R.S.\ P_2O_5 \text{ for } N = \frac{dP}{dN} = \frac{P_N}{P_P}$$

Least cost combinations were calculated for all equations for two price ratios. The two price ratios are $\frac{P_N}{P_P} = \frac{.1212}{.0860} = 1.4093$ and $\frac{P_N}{P_P} = \frac{.0860}{.1212} = .7096$.

Least cost combination isoclines derived for the two price ratios are shown for all equations in Figures 3.18 to 3.28. These figures also show the ratios in which three popular commercial fertilizers 11-48-0, 16-20-0, and 27-14-0 are available.[9] For the quadratic equation, an examination of the figures shows that the least cost lines coincide with the commercial fertilizer ratio lines only at discrete points.

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[8] Equations for calculating least cost combinations are presented in Appendix II.

[9] The commercial fertilizer formulas are interpreted as follows. Using 11-48-0 as an example, the fertilizer contains 11 per cent available nitrogen, 48 per cent available phosphorus, and no potash.

For the square root equation there are even fewer points of coincidence.

Figure 3.18 is derived from the 1957 quadratic equation. The isocline MN determines the least cost combinations of nitrogen and phosphorus when P_N = 12.12 cents per pound and P_P = 8.60 cents per pound. The isocline PN determines least cost combinations of nitrogen and phosphorus when P_N = 8.60 cents per pound and P_P = 12.12 cents per pound. The fertilizers 11-48-0, 16-20-0, and 27-14-0, are represented by vectors OA, OB, and OC, respectively. Examination of Figure 3.18 shows that 11-48-0 provides least cost combination of nitrogen and phosphorus at only one point, D, when P_N is 12.12 cents per pound. The 16-20-0 fertilizer provides least cost at point E under the assumed price ratio. The 27-14-0 fertilizer approaches least cost at the point F when P_N is 8.60 cents per pound.

Since the actual prices of nitrogen and phosphorus approach 12.12 and 8.60 cents, respectively, the isocline MN is most acceptable for practical considerations.

The interpretation of Figure 3.18 may be applied to the succeeding Figures 3.19 to 3.28.

Most Profitable Application of Nitrogen and Phosphorus

If the prices of nitrogen, phosphorus, and barley are assumed, the most profitable levels of application of the nutrients can be determined for each equation for each year. The following prices are assumed: P_N = 12.12 cents per pound; P_P = 8.60 cents per pound, and barley = 93 cents per bushel.

In order to determine most profitable applications the predicting equation is differentiated with respect to nitrogen and phosphorus. Each equation is equated with the corresponding ratio of the price of the nutrient divided by the price of barley.[10]

Table 3.17 shows the most profitable applications of nitrogen and phosphorus for the 3 years and the composite data, derived by the quadratic equation. Predicted yields from these applications are also included in Table 3.17.

Table 3.17. Most Profitable Applications of Nitrogen and Phosphorus and Predicted Yields, derived by the Quadratic Equation.

	N (lbs)	P_2O_5 (lbs)	$\hat{Y}$ (bus.)
1956	48.3	88.2	35.8
1957	122.0	122.7	100.1
1958	61.0	77.6	103.7
Composite	63.0	77.5	73.9

No specific conclusions can be drawn from the data in Table

[10] Equations for determining most profitable applications are presented in Appendix II.

3.17. The optimum rates of application are dependent on the prices of the nutrients and barley. Changes in prices would change the most profitable application of nitrogen and phosphorus. Ideally, the relationship of nutrient cost and product price is an important consideration in determining rates of application. Since fertilizer prices remain relatively constant, consideration could conceivably be given to such a relationship in practical fertilizer recommendations, based on varying product prices.

Table 3.18. Most Profitable Applications of Nitrogen and Phosphorus and Predicted Yields, derived by the Square-Root Equation.

	N (lbs)	P_2O_5 (lbs)	$\hat{Y}$ (bus.)
1956	24.6	84.6	31.9
1957	50.9	19.5	31.5
1958	32.5	56.1	93.2
Composite	79.0	99.7	72.1

Table 3.19. Most Profitable Applications of Nitrogen and Phosphorus and Predicted Yields, derived by the Cobb-Douglas Equation.

	N (lbs)	P_2O_5(lbs)	$\hat{Y}$ (bus.)
1956	24.0	85.2	30.8
1957	114.5	168.1	88.8
1958	31.4	291.4	124.9
Composite	54.4	163.6	70.2

In 1957, a yield of 100.1 bushels was obtained for applications of 122.0 pounds of nitrogen and 122.7 pounds of phosphorus. In 1958 61.0 pounds of nitrogen and 77.6 pounds of phosphorus produced a yield of 103.7 bushels.

Tables 3.18 and 3.19 show results obtained with the square root and Cobb-Douglas functions respectively. Estimates obtained by the square root equation are generally lower than estimates obtained by the quadratic equation. The erratic estimates of the Cobb-Douglas function do not permit any conclusions to be drawn for this equation.

CHAPTER IV

THE LINEAR PROGRAMMING ANALYSIS

The objective of the linear programming analysis is the evaluation of commercial fertilizer as a competitive input in the farm business. The chapter is considered in three parts. Part I presents the framework of the analysis. In Part II the results of the analysis with initial operating capital are presented. Part III presents the results of the analysis with total operating capital. These terms are fully explained in Part I.

PART I - PROCEDURES

There are several distinct steps necessary in the development of the framework for the linear programming analysis. The procedures are outlined as follows:

1. Selection of the base farm.
2. Description of the base farm.
3. Determination of available resources.
4. Selection of processes or enterprises to be considered in the linear programming analysis.
5. Calculation of input-output coefficients.
6. Description of the programs.

Selection of the Base Farm

The choice of the case study method necessitated the selection of a farm operation on which to base the analysis. There are two accepted methods of determining the base farm. The first is the development of a hypothetical farm model, and the second is the selection of an existing farm unit on which to base the case study.[1] An existing farm unit was selected for the linear programming analysis.

Description of the Base Farm

The base farm selected is located on Osborne clay soil in Central Manitoba.[2] The farm was chosen for several reasons. Osborne clay soil has been considered one of the problem soils of Manitoba. The recent trend in thought is that its productive capacity has been underestimated. Very few studies of an economic nature have been done on the Osborne clay soil. The land on the base farm is classed as IIIdl land. This flat land is subject to water-logging. The surface run-off and internal percolation of water is slow. The land is fairly productive with good management practices.

The farm consists of 480 acres of land of which 470 acres are arable. The farm operator owns 320 acres and rents 160 acres on a long term lease. At the present time no

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[1]See R.K. Eyvindson, "Economic Aspects of Farm Organization on Red River Clay". (Unpublished Master's thesis, The University of Manitoba, Winnipeg, 1961), pp. 40-41, for a discussion of the selection of a base farm.

[2]The owner of the base farm is a member of the Carman District Farm Business Association.

definite rotation is followed. The use of fertilizer has been gradually increased over the past several years on cereal grain crops. Present cropping plans include the production of wheat, oats, flax, and forage. Barley is not grown on this farm.

The operator is establishing a purebred herd of Brown Swiss dairy cattle. At the present time cream is sold. The herd size is not large enough to justify a fluid milk contract. For programming purposes the assumption is made that a contract for fluid milk could be obtained. The farm operation includes a swine enterprise in which market hogs are raised from birth. Poultry are kept for on farm consumption. A small number of steers and young calves are fattened out and sold each year.

Building space for livestock is limited. If the dairy cattle enterprise continues to expand, construction of a dairy barn will be necessary.

The present farm organization includes a complete complement of machinery. If the dairy fluid milk or beef cow calf enterprise is included in the optimum plan, the purchase of forage harvesting machinery will be necessary. This purchase is necessary since zero grazing would be used for providing pasture for the two cattle enterprises. If these two enterprises do not enter the optimum plan the purchase of additional equipment will not be necessary.

At the present time very little labor is hired by the farm operator. The operator does not wish to expand his business to the point where hired labor is required on a continuous basis.

Determination of Resource Restrictions

When the base farm is selected it is then necessary to determine the supply of resources available for production. Where practicable, resources are maintained at present farm levels for this study.

The acreage of arable land is maintained at 470 acres for linear programming.

For linear programming all labor is provided by the farm operator. Labor is divided into spring, summer, fall, and winter labor in divisions determined by the farm operator. The division is as follows:

Spring Labor	May 1st - June 15th
Summer Labor	June 16th - August 15th
Fall Labor	August 16th - October 31st
Winter Labor	November 1st - April 30th.

There is no definite restriction placed on dairy cattle building space. Capital requirements for the dairy cattle enterprise include the variable costs of the construction of a dairy barn. Space for beef and swine enterprises is divided into rest of year space and winter space.

There are two methods of treating operating capital in linear programming. In the first method the level of capital defined as initial available operating capital is determined at the start of the year. As processes produce output, cash surpluses may be re-invested in the farm business. Total capital expenditure in any one year may then exceed the level of initial operating capital.

In the second method initial operating capital and total capital expenditure are assumed to be identical. At the beginning of the year, the total amount of operating capital available for expenditure throughout the year is determined. In this case, receipts from farm production are re-invested in the farm business only to the limit of the available capital determined at the beginning of the year.

The amount of initial operating capital is varied from $2500 to $10,000, in increments of $2500. Total operating capital is varied from $5000 to $20,000 in increments of $2500.

Table 4.1 shows the level at which the various resources are available.

Table 4.1: Resource Restrictions for Linear Programming.

Resource	Restriction
Land	470 acres
Spring Labor	480 hours
Summer Labor	676 hours

Table 4.1: Resource Restrictions for Linear Programming. (cont'd).

Resource	Restriction
Fall Labor	792 hours
Winter Labor	1267 hours
Rest of Year Space	1400 square feet
Winter Space	1400 square feet
Initial Capital	10,000 dollars maximum
Total Capital	20,000 dollars maximum

Selection of Enterprises

There are three types of enterprises in this study. They are, a. crop rotations, b. livestock production, and c. buying and selling activities.

Crop Rotations. Nine basic crop rotations were developed.[3] Each rotation is fertilized at a zero, low, medium, and high rate of application.[4] Therefore, there are 36 possible rotation processes for consideration in the linear programming analysis. The distinguishing characteristics of each rotation are described briefly.[5]

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[3]The crop rotations, rates of fertilizer application, and expected yields used in this study were developed by Dr. R.A. Hedlin, Dr. R.J. Soper, and A.O. Ridley, Department of Soil Science, University of Manitoba.

[4]Low, medium, and high levels are relative terms between crops. A medium rate of application for one crop could be a high level for another crop.

[5]See Table III.1, Appendix III, for a detailed description of each rotation process. Table III.1 shows the rate and level of fertilization and the expected yields for each rotation.

Rotation 1: A 4 year rotation of flax, wheat, oats seeded, and clover fallow.

Rotation 2: A 3 year rotation of wheat, oats seeded, and clover fallow.

Rotation 3: A 10 year rotation of wheat, oats seeded, forage, forage, forage (broken), breaking fallow, flax, wheat, oats seeded, and clover fallow.

Rotation 4: A 9 year rotation of wheat, oats seeded, forage, forage, forage (broken), breaking fallow, wheat, oats seeded, and clover fallow.

Rotation 5: A 2 year rotation of wheat seeded, and clover fallow.

Rotation 6: A 10 year rotation of wheat, oats seeded, forage, forage, forage, forage (broken), breaking fallow, wheat, oats seeded, and clover fallow.

Rotation 7: A 5 year rotation of wheat, oats seeded, forage, forage (broken), and breaking fallow.

Rotation 8: A 6 year rotation of wheat, oats seeded, forage, forage, forage (broken), and breaking fallow.

Rotation 9: A 5 year rotation of wheat seeded, forage, forage, forage (broken), and breaking fallow.

Livestock Enterprises. There are two beef production enterprises developed for consideration in the analysis. These are a cow-calf enterprise and a yearling steer enterprise. Input-output coefficients for the cow-calf enterprise include feed, labor, and capital requirements for the mature cow, a replacement heifer, and the stocker calves. Stocker calves are sold at 7 months of age weighing approximately 450 pounds. Revenue in this enterprise is obtained from the marketing of stocker calves and the salvage cow. Artificial insemination is used to assure a good quality breeding stock.

The yearling steer enterprise consists of the purchase of steers at 710 pounds and sale at 1010 pounds. This enterprise requires only winter labor.

A dairy fluid milk enterprise is included in the analysis. Coefficients for this enterprise include requirements for the mature cow, a replacement heifer, and veal calves. Calves not kept for replacement purposes are marketed as vealers at 175 pounds. Revenue is obtained from the sale of fluid milk, the salvage cow, and veal calves. Artificial insemination is used for breeding.

There are two feeder market hog enterprises. They are winter hogs and rest of year hogs. Winter hogs require only winter labor; rest of year hogs require spring, summer, and fall labor. For these processes, weanling pigs are purchased at 20 pounds and fed to a market weight of 190 pounds.

Buying and Selling Enterprises. The buying and selling enterprises are included to permit the purchase of production resources or the marketing of surplus primary products. Selling enterprises are included for wheat, oats, flax, and hay. These products can be sold or fed to livestock. Buying enterprises are included for wheat, oats, barley, and hay. Since barley is not produced on the farm, the inclusion of any livestock in the optimum plans will necessitate the purchase of barley.

Input-Output Coefficients

Coefficients are required for each process or enterprise.[6] Land, labor, and capital requirements are calculated for all rotations. Output coefficients for wheat, oats, flax, and hay production are necessary. These are derived from Table III.1, Appendix III. Labor, feed, and operating capital requirements for livestock are calculated. Prices for livestock output and the grain products must also be determined. The coefficients used in this study are presented in Table III.2, Appendix III.

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[6]Department of Agricultural Economics, University of Manitoba, "Farm Management Handbook", (Unpublished). This handbook was the source of most coefficients. In addition, much assistance was given by various staff members of the Departments of Soil Science and Animal Science, University of Manitoba.

Description of Programs

Initial Operating Capital: A total of forty programs were computed at two yield levels. Table III.1 shows the original yields. Reduced yields were calculated from the original yields in the following manner. All wheat, oats, and flax yields of Table III.1 were reduced by 10 per cent. All forage yields were reduced by 20 per cent. The levels of fertilization shown in Table III.1 were held constant for both the original and reduced yields. Twenty programs were computed for each yield level. In addition, the levels of initial operating capital and fertilizer cost were varied. The cost of fertilizer was increased in increments of 20 per cent, to 180 per cent of the actual cost. In Table 4.2a the actual fertilizer cost is indicated by the index 100. The fertilizer cost which is 20 per cent greater than actual cost is indicated by the index 120. Similarly for the index numbers of 140, 160, and 180. The original yield level is represented by the index 100 in Table 4.2a. The index 80-90 indicates reduced yields in Table 4.2b.

Columns 1, 2, and 3 of Tables 4.2a and 4.2b show the identifying characteristics of each program.

Total Operating Capital: The same general procedures were followed in computing programs with total operating capital. Ten programs were computed at the two yield levels with total operating capital variable and fertilizer cost constant at the actual level. The results of the programs are shown in Table 4.5.

PART II - RESULTS - PROGRAMS COMPUTED WITH INITIAL OPERATING CAPITAL

The objective of the linear programming analysis is an attempt to evaluate the ability of commercial fertilizer to compete for initial available operating capital with other production resources in the farm business. To this end, the level of fertilizer use is examined in situations where yields, initial capital, and fertilizer cost are varied. The specific situations are outlined below.

1. The levels of initial capital varied from a non-limiting level to several limiting levels and the effect on fertilizer use determined. The hypothesis is advanced that as the level of available operating capital is lowered, the use of fertilizer will be decreased. This will provide an indication of the competitive ability of fertilizer. The effect of varying capital is considered for original and decreased yields.

2. The cost of fertilizer is increased. Cost increases are in increments of 20 per cent. It is hypothesized that level of use will be decreased as cost is increased. Consideration is given to the effect of increasing cost, for both yield levels.

The interpretation of the analysis may be clarified by an explanation of Tables 4.2a and 4.2b. These tables include all the information relevant to the discussion of the linear programming results. Programs 1 to 20 (Table 4.2a) were computed for original (100) yields with capital and fertilizer cost variable. Programs 21 to 40 (Table 4.2b) were computed for decreased (80-90) yields, with capital and fertilizer cost variable.

Columns 1, 2, and 3 of Tables 4.2a and 4.2b indicate the levels of initial capital, yields, and fertilizer cost, respectively, for each program.

Column 4 shows the profit of the optimum (most profitable) plan of each program. The profit determined in linear programming represents the return to fixed costs of operation. In order to determine "net" profit all fixed costs would have to be subtracted from the figure given in Column 4. Determination of "net" profit is not necessary for the purposes of this thesis.

Column 5 indicates the level at which rotations are fertilized in the optimum plans. The following designations apply:[7]

F_3 - a high level of fertilizer application

F_2 - a medium level of fertilizer application

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[7]Table III.1 shows the rate of fertilization, the analysis of fertilizer used, and the expected yield for each level of fertilization.

F_1 - a low level of fertilizer application

F_0 - a zero level of application

No optimum plans were computed for less than a medium level of application.

Productive processes or enterprises which appear in the optimum plans for each program are presented in Columns 6 to 15. Enterprises such as yearling steers which did not appear in any final plan are omitted from Tables 4.2a and 4.2b.

Columns 16 to 20 include all resources which were present at surplus levels in the optimum plans. Resources, such as land, which were limiting in all plans are omitted from the Tables.

The understanding of Tables 4.2a and 4.2b may be further simplified by discussing the results of a specific program. Program 8 is selected for this program. In Program 8, initial capital is limited to $5000, original yields are assumed, and expenditure for fertilizer is 140 per cent of actual cost. The profit to the fixed costs is $8048.57. Rotation 2 is fertilized at a high level and rotation 9 at a medium level. The optimum plan specifies 370.38 acres of Rotation 2 and 99.62 acres of Rotation 9. All available land has been employed in production in the final plan.

Column 8 indicates that the optimum plan specifies the production of 3.61 units of the dairy fluid milk enterprise. One of the restrictions on this process was that at least 24 units must be specified in the optimum plan. Otherwise, for practical considerations the dairy fluid milk process would be discarded if Program 8 was to be implemented on the farm.

The optimum plan specifies the production of 93.52 winter hogs and 212.12 rest of year hogs.

Columns 11, 12, and 13 indicate surpluses of wheat, oats, and hay. The optimum plan includes the sale of 4119.24 bushels of wheat, 4411.94 bushels of oats, and 125.04 tons of hay. The purchase of 2042.72 bushels of barley is necessary.

For this analysis oats and barley are not substitutable in feed rations. If livestock rations require barley it must be purchased since no barley is produced on the farm.

In the optimum plan of Program 8, all available spring and summer labor is used. The fall and winter seasons provide surplus labor. There are 213.65 hours of fall labor and 666.25 hours of winter labor idle in the optimum plan. Winter space is also available at a non-limiting level. A maximum of 212.12 winter hogs could be produced with the 1400 square feet of available winter space. Since only 94 hogs are produced, there is a surplus of 782.77 square feet of winter space.

Column 21 indicates that initial capital is a limiting resource at the $5000 level.

The results are presented in 4 sections. They are:

1. General observations.
2. The change in fertilizer use as initial capital is decreased, for the two yield levels.
3. The change in fertilizer use as fertilizer cost is increased, for two yield levels.

ns for Programs 1 to 20, Computed
al Yields.

3	4	5		6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
				Productive Processes in Optimum Plan										Surplus Resources in Optimum Plan					
rt. st	Profit	Fert. Level R_2	R_9	Rotations R_2 Acres	R_9 Acres	Dairy Fluid Milk (Units)	Winter Hogs (head)	Rest of Year Hogs (head)	Sell Wheat (Bus)	Sell Oats (Bus)	Sell Hay (Tons)	Buy Barley (Bus)	Buy Hay (Tons)	Surplus Spring Labor (Hrs)	Surplus Summer Labor (Hrs)	Surplus Fall Labor (Hrs)	Surplus Winter Labor (Hrs)	Surplus Winter Space (Sq.Feet)	Surplus Initial Capital ($)
00	8337.56	F_3	F_3	370.38	99.62	3.61	30.48	212.12	4168.51	4685.58	125.04	1644.00	-	-	-	213.65	784.45	1198.83	-
20	7947.58	F_3	F_3	370.38	99.62	3.61	19.36	212.12	4173.72	4733.77	125.04	1573.76	-	-	-	213.65	805.28	1272.19	-
40	7559.91	F_3	F_2	370.38	99.62	3.61	8.32	212.12	4158.97	4781.67	125.04	1503.94	-	-	-	213.65	826.00	1345.10	-
60	7315.36	F_2	F_2	370.38	99.62	3.61	1.35	212.12	4038.79	3577.29	125.04	1459.90	-	-	-	213.65	839.07	1391.10	-
80	7068.78	F_2	F_2	384.55	85.45	3.61	-	212.12	4093.52	3797.34	106.38	1438.44	-	-	31.86	215.45	870.84	1400.00	-
00	8827.22	F_3	F_3	370.38	99.62	3.61	115.68	212.12	4128.78	4315.86	125.04	2182.80	-	-	-	213.65	624.69	636.50	-
20	8436.99	F_3	F_3	370.38	99.62	3.61	104.69	212.12	4133.74	4364.27	125.04	2112.72	-	-	-	213.65	645.86	709.03	-
0	8049.57	F_3	F_2	370.38	99.62	3.61	93.52	212.12	4119.24	4411.94	125.04	2042.72	-	-	-	213.65	666.25	782.77	-
0	7805.00	F_2	F_2	370.38	99.62	3.61	86.68	212.12	3998.79	3208.15	125.04	1998.88	-	-	-	213.65	679.64	827.91	-
0	7567.68	F_2	F_2	370.38	99.62	3.61	79.78	212.12	4002.21	3237.24	125.04	1955.90	-	-	-	213.65	692.00	873.43	-
0	9317.32	F_3	F_3	370.38	99.62	3.61	200.90	212.12	4088.64	3946.68	125.04	2721.00	-	-	-	213.65	464.91	74.08	-
0	8927.34	F_3	F_3	370.38	99.62	3.61	189.78	212.12	4093.85	3994.86	125.04	2650.76	-	-	-	213.65	485.75	147.43	-
0	8539.67	F_3	F_2	370.38	99.62	3.61	178.73	212.12	4079.10	4042.77	125.04	2580.94	-	-	-	213.65	506.47	220.35	-
0	8295.12	F_2	F_2	370.38	99.62	3.61	171.76	212.12	3958.92	2838.38	125.04	2536.89	-	-	-	213.65	519.54	266.35	-
0	8057.45	F_2	F_2	370.38	99.62	3.61	164.99	212.12	3962.31	2867.83	125.04	2494.48	-	-	-	213.65	532.24	311.05	-
0	9381.86	F_3	F_3	370.38	99.62	3.61	212.12	212.12	4083.38	3898.01	125.04	2791.94	-	-	-	213.65	443.87	-	2170.68
0	9055.58	F_3	F_3	370.38	99.62	3.61	212.12	212.12	4083.62	3897.77	125.04	2791.94	-	-	-	213.65	443.87	-	1844.40
0	8731.42	F_3	F_2	370.38	99.62	3.61	212.12	212.12	4063.70	3897.77	125.04	2791.94	-	-	-	213.65	443.87	-	1520.25
0	8527.16	F_2	F_2	370.38	99.62	3.61	212.12	212.12	3940.01	2663.40	125.04	2791.94	-	-	-	213.65	443.87	-	1315.95
0	8328.40	F_2	F_2	370.38	99.62	3.61	212.12	212.12	3940.01	2663.40	125.04	2791.94	-	-	-	213.65	443.87	-	1117.22

Table 4.2a: Optimum Plans for Programs 1 to 20, Computed With Original Yields.

Program Characteristics							Productive Processes in Optimum Plan										Surplus Resources in Optimum Plan			
	1	2	3	4	5		6	7	8	9	10	11	12	13	14	15	16	17	18	19
Program No.	Initial Capital	Yield Level	Fert. Cost	Profit	Fert. Level R_2	R_9	Rotations R_2 Acres	R_9 Acres	Dairy Fluid Milk (Units)	Winter Hogs (head)	Rest of Year Hogs (head)	Sell Wheat (Bus)	Sell Oats (Bus)	Sell Hay (Tons)	Buy Barley (Bus)	Buy Hay (Tons)	Surplus Spring Labor (Hrs)	Surplus Summer Labor (Hrs)	Surplus Fall Labor (Hrs)	Surplus Winter Labor (Hrs)
1	2500	100	100	8337.56	F_3	F_3	370.38	99.62	3.61	30.48	212.12	4168.51	4685.58	125.04	1644.00	-	-	-	213.65	784.45
2	"	"	120	7947.58	F_3	F_3	370.38	99.62	3.61	19.36	212.12	4173.72	4733.77	125.04	1573.76	-	-	-	213.65	805.28
3	"	"	140	7559.91	F_3	F_2	370.38	99.62	3.61	8.32	212.12	4158.97	4781.67	125.04	1503.94	-	-	-	213.65	826.00
4	"	"	160	7315.36	F_2	F_2	370.38	99.62	3.61	1.35	212.12	4038.79	3577.29	125.04	1459.90	-	-	-	213.65	839.07
5	"	"	180	7068.78	F_2	F_2	384.55	85.45	3.61	-	212.12	4093.52	3797.34	106.38	1438.44	-	-	31.86	215.45	870.84
6	5000	100	100	8827.22	F_3	F_3	370.38	99.62	3.61	115.68	212.12	4128.78	4315.86	125.04	2182.80	-	-	-	213.65	624.69
7	"	"	120	8436.99	F_3	F_3	370.38	99.62	3.61	104.69	212.12	4133.74	4364.27	125.04	2112.72	-	-	-	213.65	645.86
8	"	"	140	8049.57	F_3	F_2	370.38	99.62	3.61	93.52	212.12	4119.24	4411.94	125.04	2042.72	-	-	-	213.65	666.25
9	"	"	160	7805.00	F_2	F_2	370.38	99.62	3.61	86.68	212.12	3998.79	3208.15	125.04	1998.88	-	-	-	213.65	679.64
10	"	"	180	7567.68	F_2	F_2	370.38	99.62	3.61	79.78	212.12	4002.21	3237.24	125.04	1955.90	-	-	-	213.65	692.00
11	7500	100	100	9317.32	F_3	F_3	370.38	99.62	3.61	200.90	212.12	4088.64	3946.68	125.04	2721.00	-	-	-	213.65	464.91
12	"	"	120	8927.34	F_3	F_3	370.38	99.62	3.61	189.78	212.12	4093.85	3994.86	125.04	2650.76	-	-	-	213.65	485.75
13	"	"	140	8539.67	F_3	F_2	370.38	99.62	3.61	178.73	212.12	4079.10	4042.77	125.04	2580.94	-	-	-	213.65	506.47
14	"	"	160	8295.12	F_2	F_2	370.38	99.62	3.61	171.76	212.12	3958.92	2838.38	125.04	2536.89	-	-	-	213.65	519.54
15	"	"	180	8057.45	F_2	F_2	370.38	99.62	3.61	164.99	212.12	3962.31	2867.83	125.04	2494.48	-	-	-	213.65	532.24
16	10,000	100	100	9381.86	F_3	F_3	370.38	99.62	3.61	212.12	212.12	4083.38	3898.01	125.04	2791.94	-	-	-	213.65	443.87
17	"	"	120	9055.58	F_3	F_3	370.38	99.62	3.61	212.12	212.12	4083.62	3897.77	125.04	2791.94	-	-	-	213.65	443.87
18	"	"	140	8731.42	F_3	F_2	370.38	99.62	3.61	212.12	212.12	4063.70	3897.77	125.04	2791.94	-	-	-	213.65	443.87
19	"	"	160	8527.16	F_2	F_2	370.38	99.62	3.61	212.12	212.12	3940.01	2663.40	125.04	2791.94	-	-	-	213.65	443.87
20	"	"	180	8328.40	F_2	F_2	370.38	99.62	3.61	212.12	212.12	3940.01	2663.40	125.04	2791.94	-	-	-	213.65	443.87

ns for Programs 21 to 40, Computed
l Yields.

3	4	5		6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
				Productive Processes in Optimum Plan										Surplus Resources in Optimum Plan					
ct. st	Profit	Fert. Level R_2	R_9	Rotations R_2 Acres	R_9 Acres	Dairy Fluid Milk (Units)	Winter Hogs (head)	Rest of Year Hogs (head)	Sell Wheat (Bus)	Sell Oats (Bus)	Sell Hay (Tons)	Buy Barley (Bus)	Buy Hay (Tons)	Surplus Spring Labor (Hrs)	Surplus Summer Labor (Hrs)	Surplus Fall Labor (Hrs)	Surplus Winter Labor (Hrs)	Surplus Winter Space (Sq.Feet)	Surplus Initial Capital ($)
00	6676.11	F_3	-	470.00	-	.65	23.06	212.12	4080.12	5773.15	-	1506.42	6.15	-	224.05	226.25	1003.98	1247.77	-
20	6300.06	F_3	-	470.00	-	.65	12.35	212.12	4085.14	5819.61	-	1438.68	6.15	-	224.05	226.25	1024.08	1318.50	-
40	6061.23	F_2	-	470.00	-	.65	5.54	212.12	3947.42	4439.12	-	1395.66	6.15	-	224.05	226.25	1036.84	1363.42	-
60	5872.36	F_2	-	470.00	-	.65	.16	212.12	3949.84	4462.45	-	1361.64	6.15	-	224.05	226.25	1046.93	1398.95	-
80	5668.25	F_2	-	470.00	-	.23	-	212.12	3952.83	4494.64	-	1347.74	2.16	5.98	231.33	234.32	1077.93	1400.00	-
00	7165.98	F_3	-	470.00	-	.65	108.27	212.12	4040.21	5403.74	-	2045.01	6.15	-	224.05	226.25	844.22	685.39	-
20	6789.93	F_3	-	470.00	-	.65	97.56	212.12	4045.23	5450.20	-	1977.27	6.15	-	224.05	226.25	864.31	756.12	-
40	6551.10	F_2	-	470.00	-	.65	90.75	212.12	3907.42	4069.71	-	1934.25	6.15	-	224.05	226.25	877.08	801.05	-
60	6396.85	F_2	-	470.00	-	.65	90.33	212.12	3907.62	4106.58	-	1931.38	6.15	-	224.05	226.25	877.87	803.82	-
80	6172.81	F_2	-	470.00	-	.65	79.97	212.12	3912.47	4116.45	-	1866.10	6.15	-	224.05	226.25	897.29	872.20	-
00	7655.85	F_3	-	470.00	-	.65	193.48	212.12	4000.30	5034.34	-	2583.60	6.15	-	224.05	226.25	684.45	123.02	-
20	7279.76	F_3	-	470.00	-	.65	182.79	212.12	4005.29	5080.87	-	2515.89	6.15	-	224.05	226.30	704.61	193.60	-
40	7040.97	F_2	-	470.00	-	.65	175.96	212.12	3867.51	3700.31	-	2472.83	6.15	-	224.05	226.25	717.31	238.67	-
60	6852.11	F_2	-	470.00	-	.65	170.58	212.12	3870.03	3723.64	-	2438.81	6.15	-	224.05	226.25	727.40	274.20	-
80	6662.68	F_2	-	470.00	-	.65	165.18	212.12	3872.56	3747.05	-	2404.69	6.15	-	224.05	226.25	737.52	309.83	-
00	7937.03	F_3	F_3	370.38	99.62	3.61	212.12	212.12	3637.13	3281.18	93.16	2791.94	-	-	-	213.65	443.87	-	725.84
20	7610.56	F_3	F_2	370.38	99.62	3.61	212.12	212.12	3619.02	3281.97	93.16	2791.94	-	-	-	213.65	443.87	-	403.97
40	7380.25	F_2	F_2	370.38	99.62	3.61	212.12	212.12	3508.09	2170.03	93.16	2791.94	-	-	-	213.65	443.87	-	169.07
60	7179.76	F_2	F_2	370.38	99.62	3.61	212.12	212.12	3515.77	2200.83	93.16	2791.94	-	-	5.02	213.65	443.87	-	-
80	6974.53	F_2	F_2	370.38	99.62	3.61	212.12	212.12	3565.93	2401.99	93.16	2791.94	-	-	37.84	215.78	443.87	-	-

Table 4.2b: Optimum Plans for Programs 21 to 40, Computed with Reduced Yields.

Program Characteristics | Productive Processes in Optimum Plan | Surplus Resources in Optimum Plan

Pro-gram No.	1 Initial Capital	2 Yield Level	3 Fert. Cost	4 Profit	5 Fert. Level R_2	5 Fert. Level R_9	6 Rotations R_2 Acres	7 Rotations R_9 Acres	8 Dairy Fluid Milk (Units)	9 Winter Hogs (head)	10 Rest of Year Hogs (head)	11 Sell Wheat (Bus)	12 Sell Oats (Bus)	13 Sell Hay (Tons)	14 Buy Barley (Bus)	15 Buy Hay (Tons)	16 Surplus Spring Labor (Hrs)	17 Surplus Summer Labor (Hrs)	18 Surplus Fall Labor (Hrs)	19 Surplus Winter Labor (Hrs)	S W S (
21	2500	80-90	100	6676.11	F_3	-	470.00	-	.65	23.06	212.12	4080.12	5773.15	-	1506.42	6.15	-	224.05	226.25	1003.98	1
22	"	"	120	6300.06	F_3	-	470.00	-	.65	12.35	212.12	4085.14	5819.61	-	1438.68	6.15	-	224.05	226.25	1024.08	1
23	"	"	140	6061.23	F_2	-	470.00	-	.65	5.54	212.12	3947.42	4439.12	-	1395.66	6.15	-	224.05	226.25	1036.84	1
24	"	"	160	5872.36	F_2	-	470.00	-	.65	.16	212.12	3949.84	4462.45	-	1361.64	6.15	-	224.05	226.25	1046.93	1
25	"	"	180	5668.25	F_2	-	470.00	-	.23	-	212.12	3952.83	4494.64	-	1347.74	2.16	5.98	231.33	234.32	1077.93	1
26	5000	80-90	100	7165.98	F_3	-	470.00	-	.65	108.27	212.12	4040.21	5403.74	-	2045.01	6.15	-	224.05	226.25	844.22	
27	"	"	120	6789.93	F_3	-	470.00	-	.65	97.56	212.12	4045.23	5450.20	-	1977.27	6.15	-	224.05	226.25	864.31	
28	"	"	140	6551.10	F_2	-	470.00	-	.65	90.75	212.12	3907.42	4069.71	-	1934.25	6.15	-	224.05	226.25	877.08	
29	"	"	160	6396.85	F_2	-	470.00	-	.65	90.33	212.12	3907.62	4106.58	-	1931.38	6.15	-	224.05	226.25	877.87	
30	"	"	180	6172.81	F_2	-	470.00	-	.65	79.97	212.12	3912.47	4116.45	-	1866.10	6.15	-	224.05	226.25	897.29	
31	7500	80-90	100	7655.85	F_3	-	470.00	-	.65	193.48	212.12	4000.30	5034.34	-	2583.60	6.15	-	224.05	226.25	684.45	
32	"	"	120	7279.76	F_3	-	470.00	-	.65	182.79	212.12	4005.29	5080.87	-	2515.89	6.15	-	224.05	226.30	704.61	
33	"	"	140	7040.97	F_2	-	470.00	-	.65	175.96	212.12	3867.51	3700.31	-	2472.83	6.15	-	224.05	226.25	717.31	
34	"	"	160	6852.11	F_2	-	470.00	-	.65	170.58	212.12	3870.03	3723.64	-	2438.81	6.15	-	224.05	226.25	727.40	
35	"	"	180	6662.68	F_2	-	470.00	-	.65	165.18	212.12	3872.56	3747.05	-	2404.69	6.15	-	224.05	226.25	737.52	
36	10,000	80-90	100	7937.03	F_3	F_3	370.38	99.62	3.61	212.12	212.12	3637.13	3281.18	93.16	2791.94	-	-	-	213.65	443.87	
37	"	"	120	7610.56	F_3	F_2	370.38	99.62	3.61	212.12	212.12	3619.02	3281.97	93.16	2791.94	-	-	-	213.65	443.87	
38	"	"	140	7380.25	F_2	F_2	370.38	99.62	3.61	212.12	212.12	3508.09	2170.03	93.16	2791.94	-	-	-	213.65	443.87	
39	"	"	160	7179.76	F_2	F_2	370.38	99.62	3.61	212.12	212.12	3515.77	2200.83	93.16	2791.94	-	-	5.02	213.65	443.87	
40	"	"	180	6974.53	F_2	F_2	370.38	99.62	3.61	212.12	212.12	3565.93	2401.99	93.16	2791.94	-	-	37.84	215.78	443.87	

4. An examination of the relationship between the extent of fertilizer use and the levels at which specific enterprises are present in the optimum plans.

Section 1 - General Observations

Only two of the nine basic rotations entered the optimum plans. Rotation 2, a 3 year sequence of wheat, oats seeded, and clover fallow, was the dominant rotation. It appeared in all optimum plans. Rotation 9, a 5 year rotation of wheat seeded, forage, forage, forage (broken), and breaking fallow was present in all programs except Programs 21 to 35 inclusive.[8]

Both swine production enterprises were specified in the optimum plans. Rest of year hog production was at the maximum level for all plans. The dairy fluid milk enterprise appeared in all optimum plans at a very low level. In practical applications this process would be discarded.

Resources which limited further expansion in Programs 1 to 20 were initial capital, land, spring labor, summer labor, and rest of year hog space. Capital, land, spring labor, and rest of year space were limiting resources in the programs with reduced yields.

Section 2 - The Effect of Decreasing Initial Capital

Table 4.3 shows the level of fertilizer use, for

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[8]Under different assumptions, different rotations could have appeared in the optimum plans. The fact that only rotations 2 and 9 were significant must be considered only within the limitations of the input-output coefficients of this analysis. Changes in input-output coefficients and assumptions could alter the type of rotations which would appear in the optimum plans.

Table 4.3: Use of Fertilizer Related to Level of Initial Capital and Varying Fertilizer Cost, with Original Yields.

$ Capital	Fertilizer Cost (Per Cent of Actual) 100+ R_2*	R_9**	120 R_2	R_9	140 R_2	R_9	160 R_2	R_9	180 R_2	R_9
10,000	High	High	High	High	High	Med.	Med.	Med.	Med.	Med.
7,500	"	"	"	"	"	"	"	"	"	"
5,000	"	"	"	"	"	"	"	"	"	"
2,500	"	"	"	"	"	"	"	"	"	"

*R_2 denotes rotation 2

**R_9 denotes rotation 9

+100 denotes actual fertilizer cost

Table 4.4: Use of Fertilizer Related to Level of Initial Capital and Varying Fertilizer Cost, with Reduced Yields.

$ Capital	Fertilizer Cost (Per Cent of Actual) 100 R_2	R_9	120 R_2	R_9	140 R_2	R_9	160 R_2	R_9	180 R_2	R_9
10,000	High	High	High	Med.	Med.	Med.	Med.	Med.	Med.	Med.
7,500	"	–	"	–	"	–	"	–	"	–
5,000	"	–	"	–	"	–	"	–	"	–
2,500	"	–	"	–	"	–	"	–	"	–

original yields, when initial capital is decreased. An examination of Table 4.3 shows that decreases in initial capital had no effect on the level of fertilizer use. For any specific fertilizer cost and either rotation, the level of use remains constant as initial capital is decreased from $10,000 to $2,500. Only if the rate of application changed within any one column would a response to diminishing levels of initial capital be indicated.

Table 4.4 shows the level of fertilizer use, for decreased yields, when capital is decreased. The same conclusion is reached as for the original yields.

The following direct and indirect conclusions are made.

1. Within the range of initial capital considered in this study, the level of fertilizer application is not affected by decreases in the level of initial available operating capital.

2. The following inference is drawn from the above conclusion. Since the level of use is not affected by initial capital changes, fertilizer exhibits a highly competitive ability to compete for capital with other inputs for the initial operating capital.

Prudence must be exercised in attempting any comprehensive generalizations from the above conclusions. In the strictest sense the conclusions can be applied only to the base farm. Since the base farm is assumed typical of the area it is permissible to make general statements about the effect of varying capital on fertilizer use.

Section 3 - The Effect of Increasing Fertilizer Cost

The data presented in Tables 4.3 and 4.4 is used in evaluating the effect of an increase in fertilizer cost on the level of fertilizer applications.

With original yields, the level of fertilizer use shows no response to increased fertilizer cost until the fertilizer cost has been increased to 140 per cent of actual. A fertilizer cost of 140 per cent of actual reduces the quantity used on Rotation 9 to a medium level. With a further increase in fertilizer cost, both Rotations 2 and 9 are fertilized at the medium level. No further decreases in use occur, within the cost range considered in the study.

The same general pattern is followed with reduced yields. The fertilizer cost level at which application is reduced is decreased from the 140 per cent level to the 120 per cent level. At actual costs, both rotations are fertilized at the high level. At the 120 per cent cost level, Rotation 2 receives a high application while the application on Rotation 9 is reduced to the medium level. Application remains at the medium level for Rotation 2 for all succeeding increases in fertilizer cost. Rotation 9 does not appear in the optimum plan for levels of $2,500, $5,000, and $7,500 of capital.

The conclusions of the above analysis are as follows.

1. Fertilizer use is affected only when relatively great increases take place in the cost of fertilizer. Fert-

ilizer prices have generally been lowered over the past few years. Large increases in prices are very unlikely. Therefore, for practical considerations, the change in fertilizer costs will not affect the level of use of fertilizer.

2. A reduction in yields increases the sensitivity of fertilizer use to increases in fertilizer cost. If yields were severely depressed, a slight increase in cost may reduce fertilizer use. The rate of application remained constant while yields were reduced. This is equivalent to a diminishing response to fertilizer. Therefore, the lower the response of yield to fertilizer application, the greater will be the sensitivity of fertilizer use to fertilizer cost.

Section 4 - Fertilizer Use and Processes in the Optimum Plans

A comparison of fertilizer levels and specific enterprises in the various optimum plans provide further estimates of the competitive ability of fertilizer. The comparison to be considered is as follows. The changes in the levels of enterprises are related to the level of fertilizer. The fertilizer cost is varied and initial capital and yield levels held constant. The winter hog and rest of year hog processes are selected for comparison with the level of fertilizer use.

The optimum plans of Programs 6 to 10 (Table 4.2a) are used for this comparison. Initial capital is fixed at

$5000 and yields are constant at the original (100) level. As fertilizer cost is increased, the amount of capital available for expenditure on processes is decreased if the fertilizer use remains unchanged. In Program 6, fertilization is at the high level for both rotations. Production of winter hogs is 115.68 head. When fertilizer cost is increased to the 120 level, fertilization remains unchanged and production of winter hogs is reduced to 104.69 head. Further increases in fertilizer cost are accompanied by corresponding decreases in the production of winter hogs. This is true also at the 180 cost level where fertilization on both rotations is reduced to the medium (F_2) level. The conclusion is drawn that the marginal return on initial capital is greater for fertilizer than the winter hog process. Only when initial capital is non-restrictive (Programs 16 to 21) are winter hogs produced at the maximum level of 212.12 hogs.

A comparison of fertilizer use and the rest of year hogs process shows a different relationship. At the limiting level of $5000 of initial capital, rest of year hog production remains at the maximum level of production as fertilizer cost is increased. This indicates that the marginal productivity of capital is greater for the rest of year hogs process than for fertilizer.

Section 2 considered the effect of diminishing initial capital on the use of fertilizer. A decrease in initial capital produces an effect similar to an increase in fertilizer cost. An initial capital level of $10,000 permits the maximum production of winter hogs at 212.12. With fertilizer cost constant at the actual cost level, a reduction in initial capital to $2500, decreases winter hog production to 30.48 head.

The results of the analysis show that fertilizer is highly competitive with other inputs for initial capital in the farm business. Decreases in initial capital did not affect fertilizer use. Extremely great increases in fertilizer cost reduced the level of use of fertilizer. For practical purposes, this reduction is not significant. A comparison of fertilizer use and process levels in the optimum plans indicated, (1) the marginal return is greater to fertilizer than to winter hogs, (2) the marginal return is less to fertilizer than to rest of year hogs, when initial capital is limited.

PART III - RESULTS - PROGRAMS COMPUTED WITH TOTAL OPERATING CAPITAL

In Part III, the level of fertilizer use is related to changes in yields and total available operating capital. Eight programs were computed for purposes of studying this relationship. Two programs were computed for extremely limited and non-limited total capital. The optimum plans for the programs are shown in Table 4.5.

The results of Programs 41 to 50 are presented in three sections.

1. Programs 41 and 50.

2. The change in fertilizer use as total capital is decreased, for the two yield levels.

3. An examination of the relationship between the extent of fertilizer use and the levels at which specific enterprises are present in the optimum plans.

Section 1 - Programs 41 and 50.

Program 41 was computed with original yields and $5000 total capital. The optimum plan of Program 41 specifies 453.14 acres of Rotation 2 fertilized at the medium level. Therefore, 16.85 acres of land are left idle. Under the assumptions of this program fertilizer shows a marginal productivity greater than the dairy fluid milk, rest of year hog, and winter hog enterprises. This is indicated by the absence of these enterprises at the medium level of application. For practical applications, the most profitable plan with very limited total capital, is a straight grain farm operation.

The optimum plan of Program 50, computed with non-limited total capital, is the same as the optimum plan of Program 16. Both programs were computed with original yields,

for Programs 41 to 50, Computed
Total Operating Capital and
ds.

Productive Processes in Optimum Plan — Surplus Resources in Optimum Plan

t. t	4 Profit	5 Fert. Level R_2 R_9	6 Rotation R_2 Acres	7 R_9 Acres	8 Dairy Fluid Milk (Units)	9 Winter Hogs (head)	10 Rest of Year Hogs (head)	11 Sell Wheat (Bus)	12 Sell Oats (Bus)	13 Sell Hay (Tons)	14 Buy Barley (Bus)	15 Surplus Land (acres)	16 Surplus Spring Labor (Hrs)	17 Surplus Summer Labor (Hrs)	18 Surplus Fall Labor (Hrs)	19 Surplus Winter Labor (Hrs)	20 Surplus Rest of Year Space (Sq.Feet)	21 Surplus Winter Space (Sq.Feet)	22 Surplus Capital ($)
0	5353.96	F_2 -	453.14	-	-	-	-	4361.51	5840.40	-	-	16.85	122.01	378.28	419.06	1100.69	1400.	1400.	-
0	6740.75	F_2 F_2	386.11	83.88	10.11	-	-	4150.46	4216.99	37.90	310.71	-	-	46.21	248.07	363.67	1400.	1400.	-
0	5437.47	F_2 F_2	399.26	70.73	9.51	-	6.55	3755.99	3871.26	-	333.48	-	-	75.45	248.73	406.57	1356.76	1400.	-
0	7502.54	F_3 F_3	367.42	102.57	7.98	-	81.80	4202.41	5006.43	88.12	761.13	-	-	-	233.20	521.92	860.09	1400.	-
0	6121.53	F_2 F_2	447.20	22.79	3.06	-	159.20	3884.18	4247.30	-	1100.23	-	-	175.49	231.46	872.85	349.26	1400.	-
0	8160.65	F_3 F_3	370.38	99.62	3.61	-	211.78	4182.84	4818.22	124.94	1449.61	-	-	-	213.70	840.77	2.19	1400.	-
0	6721.25	F_2 F_2	463.82	6.17	.82	63.06	212.12	3899.06	4104.20	-	1764.61	-	-	210.71	225.47	916.24	-	983.75	-
0	8715.88	F_3 F_3	370.38	99.62	3.61	96.28	212.12	4137.66	4400.26	125.03	2059.86	-	-	-	213.65	661.06	-	764.52	-
0	7276.23	F_2 F_2	463.82	6.17	.82	159.59	212.12	3853.82	3685.66	-	2374.67	-	-	210.71	225.47	735.24	-	346.65	-
0	9381.86	F_3 F_3	370.38	99.61	3.61	212.12	212.12	4083.37	3898.00	125.03	2791.93	-	-	-	213.65	443.86	-	-	1999.90

Table 4.5: Optimum Plans for Programs 41 to 50, Computed with Variable Total Operating Capital and Variable Yields.

Program Characteristics | Productive Processes in Optimum Plan | Surplus Resources in Optimum Plan

Program No.	1 Total Capital	2 Yield Level	3 Fert. Cost	4 Profit	5 Fert. Level R_2 R_9	6 Rotation R_2 Acres	7 Rotation R_9 Acres	8 Dairy Fluid Milk (Units)	9 Winter Hogs (head)	10 Rest of Year Hogs (head)	11 Sell Wheat (Bus)	12 Sell Oats (Bus)	13 Sell Hay (Tons)	14 Buy Barley (Bus)	15 Surplus Land (acres)	16 Surplus Spring Labor (Hrs)	17 Surplus Summer Labor (Hrs)	18 Surplus Fall Labor (Hrs)	19 Surplus Winter Labor (Hrs)	S R Y (
41	5000	100	100	5353.96	F_2 -	453.14	-	-	-	-	4361.51	5840.40	-	-	16.85	122.01	378.28	419.06	1100.69	1
42	7500	100	100	6740.75	F_2 F_2	386.11	83.88	10.11	-	-	4150.46	4216.99	37.90	310.71	-	-	46.21	248.07	363.67	1
43	7500	80-90	100	5437.47	F_2 F_2	399.26	70.73	9.51	-	6.55	3755.99	3871.26	-	333.48	-	-	75.45	248.73	406.57	1
44	10,000	100	100	7502.54	F_3 F_3	367.42	102.57	7.98	-	81.80	4202.41	5006.43	88.12	761.13	-	-	-	233.20	521.92	
45	10,000	80-90	100	6121.53	F_2 F_2	447.20	22.79	3.06	-	159.20	3884.18	4247.30	-	1100.23	-	-	175.49	231.46	872.85	
46	12,500	100	100	8160.65	F_3 F_3	370.38	99.62	3.61	-	211.78	4182.84	4818.22	124.94	1449.61	-	-	-	213.70	840.77	
47	12,500	80-90	100	6721.25	F_2 F_2	463.82	6.17	.82	63.06	212.12	3899.06	4104.20	-	1764.61	-	-	210.71	225.47	916.24	
48	15,000	100	100	8715.88	F_3 F_3	370.38	99.62	3.61	96.28	212.12	4137.66	4400.26	125.03	2059.86	-	-	-	213.65	661.06	
49	15,000	80-90	100	7276.23	F_2 F_2	463.82	6.17	.82	159.59	212.12	3853.82	3685.66	-	2374.67	-	-	210.71	225.47	735.24	
50	20,000	100	100	9381.86	F_3 F_3	370.38	99.61	3.61	212.12	212.12	4083.37	3898.00	125.03	2791.93	-	-	-	213.65	443.86	

actual fertilizer cost and non-limited operating capital.

Section 2 - The Effect of Decreasing Total Capital

As previously stated, Program 50 was computed with a non-limited capital level of $20,000. Rotations 2 and 9 were fertilized at the high level. As capital was decreased to $15,000, a limiting level, with original yields no change in fertilizer application occured. Successive decreases in available total capital did not effect fertilizer use. With original yields, capital must be reduced to the $7500 level before the level of fertilizer is reduced to the medium level for both rotations.

For reduced yields, the level of fertilizer application remains constant at the medium level of application for all levels of total capital. When yields are reduced a shift to Rotation 2 also occured. Rotation 2 is a non-forage rotation, Rotation 9 is a forage rotation. The shift to Rotation 2, when yields are reduced, is probably due to the fact that forage yields were reduced by a greater percentage than were the grain yields.

Total capital effects the level of fertilizer use only when capital is at very limiting levels. At any level of capital greater than $10,000 fertilizer is applied at the high level. Increases in total capital permit increased production of both hog enterprises, as indicated in Table 4.5.

Section 3 - Fertilizer Use and Processes in the Optimum Plan.

Programs 44 and 45 are used to study the relationship between fertilizer use and processes in the optimum plans. In Program 44 with $10,000 of total capital and original yields, 81.80 rest of year market hogs are specified in the optimum plan. Fertilization is at the high level. When yields are reduced (Program 45) fertilization is decreased to a medium level while output of rest of year hogs is increased to 159.20 head. The conclusion is drawn that when yields are reduced, the marginal productivity of rest of year hogs exceeds the marginal productivity of fertilizer. Therefore, capital is invested in the rest of year hog enterprise and fertilizer use reduced. The rest of the programs may be interpreted in a similar manner.

The results of Programs 41 to 50 can be summarized as follows.

1. The use of fertilizer is affected by very limiting levels of total available operating capital.

2. Under extremely limiting ($5000) total capital the marginal return to fertilizer is greater than to the livestock enterprises.

3. When yields are reduced the marginal return to fertilizer is reduced. This indicates that in the practical situation, the ability of fertilizer to compete with other productive inputs for total capital, is at least partially determined by the yield response to fertilizer applications.

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

Two major conclusions have resulted from the consideration of several of the economic aspects of fertilizer use.

The quadratic function with squared terms was selected as the best estimator of the fertilizer yield response relationship. The study by Gilson and Bjarnarson also resulted in the selection of the quadratic function as the best estimating equation. Substitution rates, least cost combinations, and the most profitable applications of nitrogen and phosphorus were considered on the basis of the quadratic equation.

The linear programming results indicated a low sensitivity of fertilizer use to level of initial capital and increases in fertilizer cost. Decreases in yields slightly increased the sensitivity of fertilizer use to initial capital changes and increases in fertilizer cost. Decreases in level of initial capital did not change the level of use of fertilizer. Large increases in fertilizer cost reduced fertilizer use from high to medium levels. Such a decrease is not significant in view of practical experience. In the past few years, fertilizer prices have decreased rather than increased. The winter hog process indicated a lower marginal return to initial capital than did fertilizer. Fertilizer showed a lower marginal return to initial capital than did the rest of year hogs process. The over-all conclusion of the above results was that fertilizer is highly competitive with other production inputs for available initial capital in the farm business.

Extreme reductions in total operating capital were required before fertilizer use was reduced. With limited total capital and reduced yields, the marginal productivity of fertilizer is reduced in comparison to the marginal productivity of the rest of year hog process.

Recommendations

The data used for determining the best estimating equation was obtained from experiments conducted at the Wabowden Substation in northern Manitoba. Comparable data are not available for the southern regions of the province. Similar analysis based on data from the southern regions would be very valuable.

The functions employed in determining the best estimating equation included only two of the many possible variables which may influence yield and response to fertilizer application. Future studies should give consideration to other variables such as rainfall, temperature, frost-free period, and the length of the growing season. It is hypothesized that such independent variables could account for the change in the basic shape of the production function from 1956 to 1958 (Figure 3.17).

The linear programming analysis of this thesis has provided a preliminary evaluation of the ability of fertilizer to compete for available initial operating capital with other productive inputs in the farm business. An objective of any similar future research should be the refinement and expansion of specific procedures for analysing fertilizer use. Changes in procedures would permit specific comparisons to be made between fertilizer and other inputs. Only indirect comparisons of this nature are possible in this thesis.

The validity of any linear programming analysis is dependent on the degree of reliability of the input-output coefficients. Such coefficients must be subject to constant revision. The goal of the researcher should be to obtain the best coefficients available. This goal requires the close cooperation of the agricultural economist, physical scientist, and biological scientist.

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APPENDIX I

Table I.1. Combinations of N and P_2O_5 to Produce Two Given Yields of Barley and Marginal Replacement Rates of N and P_2O_5.

20 Bus. Barley			30 Bus. Barley		
Lbs. N	Lbs. P_2O_5	Rate of Substitution of P_2O_5 for N (Lbs. P_2O_5 replaced by 1 lb. N) $\frac{dP}{dN}$	Lbs. N	Lbs. P_2O_5	Rate of Substitution of P_2O_5 for N (Lbs. P_2O_5 replaced by 1 lb. N) $\frac{dP}{dN}$
(1)	(2)	(3)	(4)	(5)	(6)
0	50	-1.77	20	83.0	-2.67
10	34	-1.14	25	71.9	-1.73
20	24	- .81	30	64.0	-1.27
30	17	- .58	40	54.0	- .78
40	16	- .42	50	48.0	- .49
			55	46.0	- .37
			60	44.0	- .26
			70	42.5	- .07

This table is taken from:

J.C. Gilson and V.W. Bjarnarson, "Effects of Fertilizer Use on Barley in Northern Manitoba", Journal of Farm Economics, Volume XL, No. 4, November, 1958, Page 936.

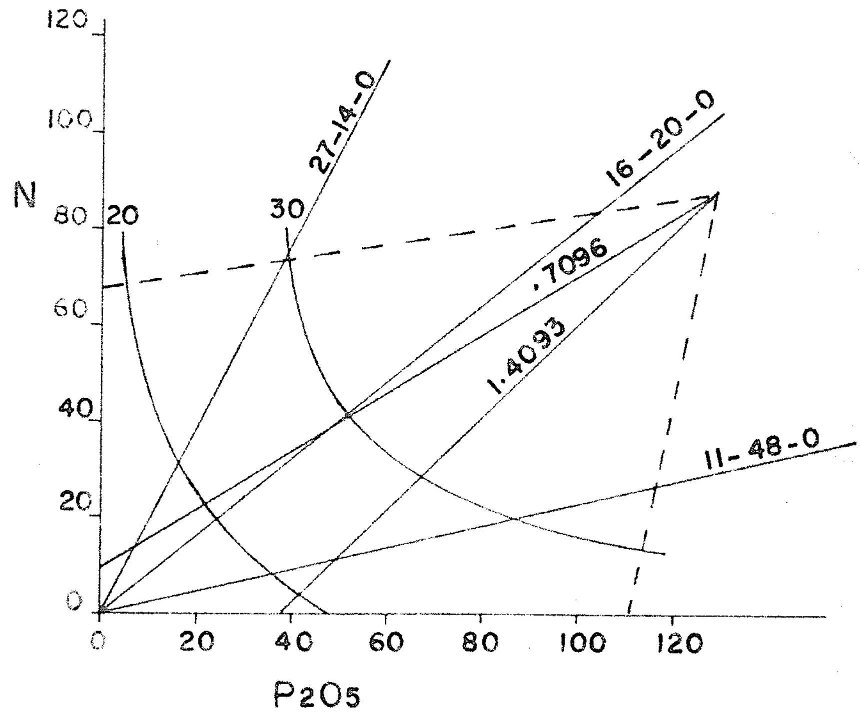

Figure I.1: **Combinations of Nitrogen and Phosphorus to produce Specified Barley Yields, Derived by the Quadratic Equation, 1956.**

NOTE: This figure is derived from: J.C. Gilson and V.W. Bjarnarson. "Effects of Fertilizer Use on Barley in Northern Manitoba", Journal of Farm Economics, Volume XL, No. 4, November, 1958, Page 937.

APPENDIX II

EQUATIONS FOR DETERMINING ISOQUANTS

The Quadratic Function

Given the general equation,

$$\hat{Y} = a + b_1N + b_2P + b_3N^2 + b_4P^2 + b_5NP,$$

With N varied from 0 to 120 pounds:

$$P = \frac{-(b_2 + b_5N) \pm \sqrt{(b_2 + b_5N)^2 - 4b_4\ (b_1N + b_3N^2 + a - \hat{Y})}}{2b_4}$$

The Square Root Function

Given the general equation,

$$\hat{Y} = a + b_1N + b_2P + b_3\sqrt{N} + b_4\sqrt{P} + b_5\sqrt{NP},$$

With N varied from 0 to 120 pounds:

$$P = \left[\frac{-(b_4 + b_5\sqrt{N}) \pm \sqrt{(b_4 + b_5\sqrt{N})^2 - 4b_2[b_1N + b_3\sqrt{N} + a - \hat{Y}]}}{2b_2}\right]^2$$

The Cobb-Douglas Function

Given the general equation, $\hat{Y} = aN^{b_n} P^{b_p}$,

With N varied from 0 to 120 pounds:

$$P = \left[\frac{\hat{Y}}{aN^{b_n}}\right]^{\frac{1}{b_p}}$$

EQUATIONS FOR MARGINAL RATES OF SUBSTITUTION

The Quadratic Function

Given the general equation,

$$\hat{Y} = a + b_1N + b_2P + b_3N^2 + b_4P^2 + b_5NP,$$

from which combinations of N and P_2O_5 to produce specific yields are obtained; the M.R.S. P_2O_5 for N, for each specific yield is calculated by the following equation.

$$\text{M.R.S. } P_2O_5 \text{ for } N = -\frac{b_1 + 2b_3N + b_5P}{b_2 + 2b_4P + b_5N}$$

The Square Root Function

Given the general equation,

$$\hat{Y} = a + b_1N + b_2P + b_3\sqrt{N} + b_4\sqrt{P} + b_5\sqrt{NP},$$

from which combinations of N and P_2O_5 to produce specific yields are obtained; the M.R.S. P_2O_5 for N, for each specific yield is calculated by the following equation.

$$\text{M.R.S. } P_2O_5 \text{ for } N = -\frac{\sqrt{P}\,(2b_1\sqrt{N} + b_3 + b_5\sqrt{P})}{\sqrt{N}\,(2b_2\sqrt{P} + b_4 + b_5\sqrt{N})}$$

The Cobb-Douglas Function

Given the general equation, $\hat{Y} = aN^{b_n} P^{b_p}$, from which combinations of N and P_2O_5 to produce specific yields are obtained; the M.R.S. P_2O_5 for N for each specific yield is calculated by the following equation.

$$\text{M.R.S. } P_2O_5 \text{ for } N = -\frac{b_nP}{b_pN}$$

EQUATIONS FOR LEAST COST COMBINATIONS

The Quadratic Function

Given the general equation,

$$\hat{Y} = a + b_1N + b_2P + b_3N^2 + b_4P^2 + b_5NP,$$

with N varied from 0 to 120 pounds and C equal the ratio of the prices of nitrogen and phosphorus $\left(\frac{P_N}{P_P}\right)$:

$$P = \frac{b_1 - Cb_2 + N\,(2b_3 - Cb_5)}{2Cb_4 - b_5}$$

The Square Root Function

Given the general equation,

$$\hat{Y} = a + b_1N + b_2P + b_3\sqrt{N} + b_4\sqrt{P} + b_5\sqrt{NP},$$

with N varied from 0 to 120 pounds and C equal the ratio of the prices of nitrogen and phosphorus $\left(\frac{P_N}{P_P}\right)$:

$$P = \left[\frac{-[b_3 + \sqrt{N}\,(2b_1 - 2Cb_2)] \pm \sqrt{[b_3 + \sqrt{N}\,(2b_1 - 2Cb_2)]^2 - 4b_5(-Cb_4\sqrt{N} - Cb_5N)}}{2b_5}\right]^2$$

The Cobb-Douglas Function

Given the general equation, $\hat{Y} = aN^{b_n}\,P^{b_p}$,

with N varied from 0 to 120 pounds:

$$P = \frac{b_p\;P_N}{b_n\;P_P}\;.N$$

EQUATIONS FOR MOST PROFITABLE APPLICATIONS

The Quadratic Function

Given the general equation,

$$\hat{Y} = a + b_1N + b_2P + b_3N^2 + b_4P^2 + b_5NP,$$

the most profitable application is found by a simultaneous solution of the following equations:

$$\frac{dY}{dN} = b_1 + 2b_3N + b_5P = \frac{P_N}{P_B}$$

$$\frac{dY}{dP} = b_2 + 2b_4P + b_5N = \frac{P_P}{P_B}$$

where P_B equals the price of barley.

The Square Root Function

Given the general equation,

$$\hat{Y} = a + b_1N + b_2P + b_3\sqrt{N} + b_4\sqrt{P} + b_5\sqrt{NP},$$

the most profitable application is found by a simultaneous solution of the following equations:

$$\frac{dY}{dN} = b_1 + \frac{b_3}{2\sqrt{N}} + \frac{b_5\sqrt{P}}{2\sqrt{N}} = \frac{P_N}{P_B}$$

$$\frac{dY}{dN} = b_2 + \frac{b_4}{2\sqrt{P}} + \frac{b_5\sqrt{N}}{2\sqrt{P}} = \frac{P_P}{P_B}$$

The Cobb-Douglas Function

Given the general equation, $\hat{Y} = aN^{b_n} P^{b_p}$,

the most profitable application is found by a simultaneous solution of the following equations:

$$\frac{dY}{dN} = a\ b_n N^{b_n - 1} P^{b_p} = \frac{P_N}{P_B}$$

$$\frac{dY}{dN} = a\ b_p N^{b_n} P^{b_p - 1} = \frac{P_P}{P_B}$$

APPENDIX III

Table III.1. Alternative Rotations Considered in Linear Programming Analysis.

No. 1 4 Years		Check	Low Level Fertilization	Medium Level Fertilization	High Level Fertilization
Flax	Fertilizer		20 11-48-0	40 11-48-0	40 11-48-0
	Yield	8	9	10	10
Wheat	Fertilizer		45 16-20-0	70 16-20-0	140 27-14-0
	Yield	17	19	22	25
Oats Seeded	Fertilizer		45 16-20-0	70 16-20-0	140 27-14-0
	Yield	30	34	40	50
Clover Fallow		-	-	-	-

No. 2 3 Years		Check	Low Level Fertilization	Medium Level Fertilization	High Level Fertilization
Wheat	Fertilizer		20 11-48-0	40 11-48-0	60 11-48-0
	Yield	24	27	30	31
Oats Seeded	Fertilizer		45 16-20-0	70 16-20-0	140 27-14-0
	Yield	30	34	40	50
Clover Fallow		-	-	-	-

No. 3 10 Years		Check	Low Level Fertilization	Medium Level Fertilization	High Level Fertilization
Wheat	Fertilizer		20 11-48-0	40 11-48-0	60 11-48-0
	Yield	24	27	30	31
Oats Seeded	Fertilizer		45 16-20-0	70 16-20-0	140 27-14-0
	Yield	30	34	40	50
Forage	Fertilizer		50 11-48-0	100 11-48-0	100 11-48-0
	Yield	1.0	1.5	2.0	2.0
Forage	Fertilizer		50 11-48-0	100 11-48-0	100 11-48-0
	Yield	1.5	2.0	3.0	3.0
Forage	Fertilizer		50 11-48-0	100 11-48-0	100 11-48-0
(Broken in Fall)	Yield	1.5	2.0	3.0	3.0
Breaking Fallow		-	-	-	-
Flax	Fertilizer		20 11-48-0	40 11-48-0	40 11-48-0
	Yield	10	11	12	12
Wheat	Fertilizer		45 16-20-0	70 16-20-0	140 27-14-0
	Yield	17	19	22	27
Oats Seeded	Fertilizer		45 16-20-0	70 16-20-0	140 27-14-0
	Yield	30	34	40	50
Clover Fallow		-	-	-	-

No. 4 9 Years		Check	Low Level Fertilization	Medium Level Fertilization	High Level Fertilization
Wheat	Fertilizer Yield	24	20 11-48-0 27	40 11-48-0 30	60 11-48-0 31
Oats Seeded	Fertilizer Yield	30	45 16-20-0 34	70 16-20-0 40	140 27-14-0 50
Forage	Fertilizer Yield	1.0	50 11-48-0 1.5	100 11-48-0 2.0	100 11-48-0 2.0
Forage	Fertilizer Yield	1.5	50 11-48-0 2.0	100 11-48-0 3.0	100 11-48-0 3.0
Forage (Broken in Fall)	Fertilizer Yield	1.5	50 11-48-0 2.0	100 11-48-0 3.0	100 11-48-0 3.0
Breaking Fallow		-	-	-	-
Wheat	Fertilizer Yield	24	20 11-48-0 27	40 11-48-0 30	60 11-48-0 31
Oats Seeded	Fertilizer Yield	30	45 16-20-0 34	70 16-20-0 40	140 27-14-0 50
Clover Fallow		-	-	-	-

No. 5 2 Years					
Wheat Seeded	Fertilizer Yield	18	20 11-48-0 20	40 11-48-0 23	60 11-48-0 24
Clover Fallow		-	-	-	-

No. 6 10 Years		Check	Low Level Fertilization		Medium Level Fertilization		High Level Fertilization	
Wheat	Fertilizer		20	11-48-0	40	11-48-0	60	11-48-0
	Yield	24		27		30		31
Oats Seeded	Fertilizer		45	16-20-0	70	16-20-0	140	27-14-0
	Yield	30		34		40		50
Forage	Fertilizer		50	11-48-0	100	11-48-0	100	11-48-0
	Yield	1.0		1.5		2.0		2.0
Forage	Fertilizer		50	11-48-0	100	11-48-0	100	11-48-0
	Yield	1.5		2.0		3.0		3.0
Forage	Fertilizer		50	11-48-0	100	11-48-0	100	11-48-0
	Yield	1.5		2.0		3.0		3.0
Forage	Fertilizer		50	11-48-0	100	11-48-0	100	11-48-0
(Broken in Fall)	Yield	1.0		1.5		2.0		2.0
Breaking Fallow		-		-		-		-
Wheat	Fertilizer		20	11-48-0	40	11-48-0	60	11-48-0
	Yield	24		27		30		31
Oats Seeded	Fertilizer		45	16-20-0	70	16-20-0	140	27-14-0
	Yield	30		34		40		50
Clover Fallow		-		-		-		-

No. 7 5 Years		Check	Low Level Fertilization		Medium Level Fertilization		High Level Fertilization	
Wheat	Fertilizer		20	11-48-0	40	11-48-0	60	11-48-0
	Yield	25	28		31		32	
Oats Seeded	Fertilizer		45	16-20-0	70	16-20-0	140	27-14-0
	Yield	30	34		40		50	
Forage	Fertilizer		50	11-48-0	100	11-48-0	100	11-48-0
	Yield	1.0	1.5		2.0		2.0	
Forage (Broken in Fall)	Fertilizer		50	11-48-0	100	11-48-0	100	11-48-0
	Yield	1.5	2.0		3.0		3.0	
Breaking Fallow		-	-		-		-	

No. 8 6 Years		Check	Low Level Fertilization		Medium Level Fertilization		High Level Fertilization	
Wheat	Fertilizer		20	11-48-0	40	11-48-0	60	11-48-0
	Yield	25	28		31		32	
Oats Seeded	Fertilizer		45	16-20-0	70	16-20-0	140	27-14-0
	Yield	30	34		40		50	
Forage	Fertilizer		50	11-48-0	100	11-48-0	100	11-48-0
	Yield	1.0	1.5		2.0		2.0	
Forage	Fertilizer		50	11-48-0	100	11-48-0	100	11-48-0
	Yield	1.5	2.0		3.0		3.0	
Forage (Broken in Fall)	Fertilizer		50	11-48-0	100	11-48-0	100	11-48-0
	Yield	1.5	2.0		3.0		3.0	
Breaking Fallow		-	-		-		-	

No. 9 5 Years		Check	Low Level Fertilization	Medium Level Fertilization	High Level Fertilization
Wheat Seeded	Fertilizer		20 11-48-0	40 11-48-0	60 11-48-0
	Yield	25	28	31	32
Forage	Fertilizer		50 11-48-0	100 11-48-0	100 11-48-0
	Yield	1.0	1.5	2.0	2.0
Forage	Fertilizer		50 11-48-0	100 11-48-0	100 11-48-0
	Yield	1.5	2.0	3.0	3.0
Forage (Broken in Fall)	Fertilizer		50 11-48-0	100 11-48-0	100 11-48-0
	Yield	1.5	2.0	3.0	3.0
Breaking Fallow		-	-	-	-

NOTE: Fertilization rates are presented on the basis of pounds per acre. Yields are presented on the basis of bushels per acre for grain crops and tons per acre for forage crops.

Table III.2. Input-Output Coefficients for Linear Programming.

Resource		Net Prices	-9.857	-11.207	-12.209	-14.402	-8.851
		Resource Supply Level	R_1F_0	R_1F_1	R_1F_2	R_1F_3	R_2F_0
Land	Acres	470.	1.000	1.000	1.000	1.000	1.000
Spring Labor	Hours	480.	.925	.925	.925	.925	.790
Summer Labor	Hours	676.	.592	.592	.592	.592	.657
Fall Labor	Hours	792.	1.000	1.000	1.000	1.000	.823
Winter Labor	Hours	1267.	.385	.385	.385	.385	.367
Rest of Year Space	Sq. Feet	1400.	-	-	-	-	-
Winter Space	Sq. Feet	1400.	-	-	-	-	-
Wheat	Pounds	0	-238.250	-268.250	-313.250	-358.250	-457.500
Oats	Pounds	0	-243.661	-277.661	-328.661	-413.661	-324.881
Flax	Pounds	0	-105.330	-119.330	-133.330	-133.330	-
Hay Equivalent	Pounds	0	-	-	-	-	-
Barley	Pounds	0	-	-	-	-	-
Capital	Dollars	10,000	9.857	11.207	12.209	14.402	8.851

-10.093	-11.034	-12.957	-10.474	-12.184	-13.681	-15.272	-10.169
R_2F_1	R_2F_2	R_2F_3	R_3F_0	R_3F_1	R_3F_2	R_3F_3	R_4F_0
1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.790	.790	.790	.625	.694	.694	.694	.547
.657	.657	.657	.960	1.161	1.494	1.494	.999
.823	.823	.823	.743	.743	.743	.743	.655
.367	.367	.367	.327	.327	.327	.327	.314
-	-	-	-	-	-	-	-
-	-	-	-	-	-	-	-
-517.500	-577.500	-597.498	-232.500	-262.500	-298.500	-334.500	-304.992
-370.213	-438.213	-551.545	-194.928	-222.128	-262.928	-330.928	-216.584
-	-	-	- 53.332	- 58.932	- 64.532	- 64.532	-
-	-	-	-800.000	-1100.000	-1600.000	-1600.000	-888.888
-	-	-	-	-	-	-	-
10.093	11.034	12.957	10.474	12.184	13.681	15.272	10.169

-11.853	-13.395	-14.680	-8.396	-9.045	-9.746	-10.344	-10.407
R_4F_1	R_4F_2	R_4F_3	R_5F_0	R_5F_1	R_5F_2	R_5F_3	R_6F_0
1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.603	.603	.603	.570	.570	.570	.570	.498
1.222	1.592	1.592	.780	.780	.780	.780	1.032
.655	.655	.655	.810	.810	.810	.810	.599
.314	.314	.314	.325	.325	.325	.325	.304
-	-	-	-	-	-	-	-
-	-	-	-	-	-	-	-
-345.000	-384.996	-398.328	-511.875	-661.875	-661.875	-691.875	-274.500
-246.804	-292.060	-367.694	-	-	-	-	-193.928
-	-	-	-	-	-	-	-
-1222.221	-1777.778	-1777.778	-	-	-	-	-1000.000
-	-	-	-	-	-	-	-
11.853	13.395	14.680	8.396	9.045	9.746	10.344	10.407

-12.201	-13.851	-15.013	-10.602	-12.395	-14.053	-15.205	-10.811
R_6F_1	R_6F_2	R_6F_3	R_7F_0	R_7F_1	R_7F_2	R_7F_3	R_8F_0
1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.498	.498	.498	.498	.566	.566	.566	.425
1.300	1.700	1.700	1.004	1.272	1.672	1.672	1.170
.599	.599	.599	.668	.668	.668	.668	.572
.304	.304	.304	.304	.304	.304	.304	.288
-	-	-	-	-	-	-	-
-	-	-	-	-	-	-	-
-310.500	-346.500	-358.500	-286.500	-322.500	-358.500	-370.500	-238.746
-222.128	-262.928	-330.928	-194.929	-222.129	-262.929	-330.929	-162.441
-	-	-	-	-	-	-	-
-1400.000	-2000.000	-2000.000	-1000.000	-1400.000	-2000.000	-2000.000	-1333.000
-	-	-	-	-	-	-	-
12.201	13.851	15.015	10.602	12.395	14.053	15.205	10.811

-12.760	-14.596	-15.557	-11.007	-12.924	-14.843	-15.082	66.918
R_8F_1	R_8F_2	R_8F_3	R_9F_0	R_9F_1	R_9F_2	R_9F_3	Beef Cow Calf
1.000	1.000	1.000	1.000	1.000	1.000	1.000	-
.510	.510	.510	.264	.366	.366	.366	1.050
1.505	2.060	2.060	1.322	1.724	2.390	2.390	.535
.572	.572	.572	.378	.378	.378	.378	1.400
.288	.288	.288	.256	.256	.256	.256	14.525
-	-	-	-	-	-	-	42.763
-	-	-	-	-	-	-	42.763
-268.746	-298.746	-308.748	-288.750	-324.750	-360.750	-372.750	-
-185.105	-219.105	-275.773	-	-	-	-	357.603
-	-	-	-	-	-	-	-
-1833.334	-2666.667	-2666.667	-1600.000	-2200.000	-3200.000	-3200.000	9275.269
-	-	-	-	-	-	-	252.373
12.760	14.596	15.557	11.007	12.924	14.843	15.082	31.385

65.293	304.522	17.981	15.068	1.533	.628	2.806	11.700
Year-ling Steers	Dairy Fluid Milk	Rest of Year Hogs	Winter Hogs	Sell Wheat	Sell Oats	Sell Flax	Sell Hay
-	-	-	-	-	-	-	-
-	14.260	.469	-	-	-	-	-
-	17.360	.622	-	-	-	-	-
-	19.220	.785	-	-	-	-	-
9.447	73.160	-	1.875	-	-	-	-
-	-	6.600	-	-	-	-	-
38.000	-	-	6.600	-	-	-	-
-	416.640	28.120	28.120	60.000	-	-	-
495.000	2552.512	147.420	147.420	-	34.000	-	-
-	-	-	-	-	-	56.000	-
1890.000	19,039.782	-	-	-	-	-	2000.000
1072.000	1474.196	303.350	303.350	-	-	-	-
149.837	168.584	20.021	20.021	-1.533	-.628	-2.806	-11.700

-1.533	-.628	-.930	-11.700
Buy Wheat	Buy Oats	Buy Barley	Buy Hay
-	-	-	-
-	-	-	-
-	-	-	-
-	-	-	-
-	-	-	-
-	-	-	-
-	-	-	-
-60.000	-	-	-
-	-34.000	-	-
-	-	-	-
-	-	-	-2000.000
-	-	-48.000	-
1.533	.628	.930	11.700

APPENDIX IV

A NOTE ON ILL-CONDITIONED DATA IN LINEAR PROGRAMMING

A situation known as ill-conditioned data may arise in linear programming. If this situation occurs irrational results are obtained. The input-output coefficients of Table III.2 may be used to illustrate this ill-conditioning. The input-output coefficients range in size from 0.256 hours of winter labor for Rotation 9 to 19,039.782 pounds of hay equivalent for the dairy fluid milk enterprise. Under certain conditions the computer is unable to handle coefficients of such a wide range. In order to avoid problems associated with ill-conditioning it is advisable to scale all input-output coefficients before programming is commenced. For best computer operation all coefficients should be less than 20.0. Coefficients of less than 10.0 are more preferable. Row scaling and column scaling are used. In row scaling all coefficients in a single row of Table III.2 are divided by a constant. The constant should be large enough to reduce the largest coefficient in the row to less than 20. Column scaling requires that all coefficients in one column be divided by a constant. When column scaling is used, the results must be re-scaled to obtain the proper levels of processes.