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THE SENSITIVENESS OF THE EYE TO LIGHT AND COLOR

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*The Sensitiveness of the Eye to Light and Color.**

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Presented by PROF. FRANK ALLEN, Ph. D., F.R.S.C.

(Read May 28, 1913)

In an extensive paper on "The Luminous Equivalent of Radiation," Nutting† thus summarizes some of the more important characteristics of the visual response to radiation:

I. "Sensibility to Slight Differences in Wave Length, has two pronounced maxima, one in the yellow and one in the green; and two slight maxima in the extreme blue and red. These maxima vary considerably with the individual and probably also with the intensity of the radiation used."

II. "Sensibility to Radiation of Varying Intensity:

Sensibility falls off steadily with increasing intensity. It is approximately inversely proportional to the intensity over a wide range. The ratio of optical intensity to intensity of radiation increases more rapidly for red than for blue and green (*Purkinje phenomenon*)."

III. "Sensibility to small Differences in Intensity":

The least perceptible increment measured as a fraction of the whole is approximately:

(1) Independent of Intensity (*Fechner's Law*). It is about ·016 for moderate and high intensities and greater for very low and extremely high intensities.

(2) Independent of Wave Length (*König's Law*) at constant luminosity, extremes again excepted.

(3) Independent of the Individual."

With the exception of number I the above characteristics of the visual response to radiation have been verified by several observers. But concerning the sensibility of the retina to slight changes in the wave-length of the light no investigation of a very exhaustive nature appears to have been conducted. Perhaps the best recent data are those due to Dr. Olga Steindler;‡ but even there a spectrum of only one intensity appears to have been considered. With a view then, in the first place of verifying results already obtained by Steindler and others,

*To be printed also in the Physical Review.

†P. G. Nutting: Bulletin of the Bureau of Standards, 1908, Vol. 5, No. 2, page 265.

‡Wien Sitz.: IIa, 115, pp. 1-24, 1906.

and in the second place, of investigating the behaviour of these maxima with changes in the intensity of the radiation used, the following observations were made.

When it was found that all of the chief characteristics might be investigated by the apparatus to be used in the case of number I, and in view of the fact that it appeared to be an original method, it was decided to investigate all three cases. It was thought that the accuracy with which the well known phenomena were established might indicate the reliability of the results obtained in the first case.

Historical *

Mandelstamm † appears to have been the first to investigate the color sensitiveness of the human eye for slight differences in the wave length of light observed. By shifting the plates of an ophthalmometer so as to give a just perceptible color difference, he obtained a means of observing sensibility in different parts and obtained in this way maxima of sensibility in the region of the D and F lines. Dobrowolosky ‡ came to the same conclusion by similar means. Peirce[4] investigated the sensitiveness of the eye to slight differences of color by having two identical spectral bands one immediately above the other, the upper one of which might be shifted. The object of the experiment was to see how small a displacement could be infallibly *detected* and *named in direction* by the observer in different parts of the spectrum. He found maxima situated similarly to those found by other investigators. More accurate results were first obtained by König and Dieterici.[5] Uhthoff[6] also investigated the differences in wave length for just observable color differences. Brodhun,[7] himself color blind, gives measurements after the method of König for mean intensities. Exner[8] also gives a single service of measurements on a widely dispersed spectrum. In this connection also Steindler[9] reports observations upon twelve subjects. A spectrum with a dispersion of about 85 cm. at the point observed was obtained by means of an arc light and a concave grating. The light from the spectrum at this point fell upon two totally reflecting prisms placed vertically one above the other and after reflections from a second

*Historical references taken from Dr. Steindler's paper.

†Grafe's Archiv: Bd. 13, p. 399.

‡Ebenda, Bd. 18, p. 99.

[4] American Journal of Science, Vol. 26 (1883), p. 299.

[5] Annalen der Physik und Chemie, Bd. 22, p. 579.

[6] Grafe's Archives, Bd. 34, 4, p. 1.

[7] Zeitschr. für Psych. und Phys. Bd. 34 (1892), p. 89.

[8] L. c. p. 875.

[9] Wien Sitz.: IIa, 115, pp. 1-24, 1906.

larger prism two adjacent fields appeared in the eye-piece. Since the upper of the two smaller prisms was moveable along a horizontal scale light of the same or slightly different wave length might be viewed in the field. In this way the amount of change necessary to produce a just perceptible but distinct color difference was measured. Values thus obtained were mean values for at least ten observations made by the same person on the same portion of the spectrum.

Description of Apparatus.

One arrangement of apparatus was found sufficient for all investigations and is essentially that used by Allen* to measure the luminosity of the spectrum. The general arrangement is shown in Fig. 1. The light from an acetylene flame A after concentration by lenses B and B[1], passed through the opening C from a light proof chamber M; then through two nicol prisms (E and F) arranged with their principal

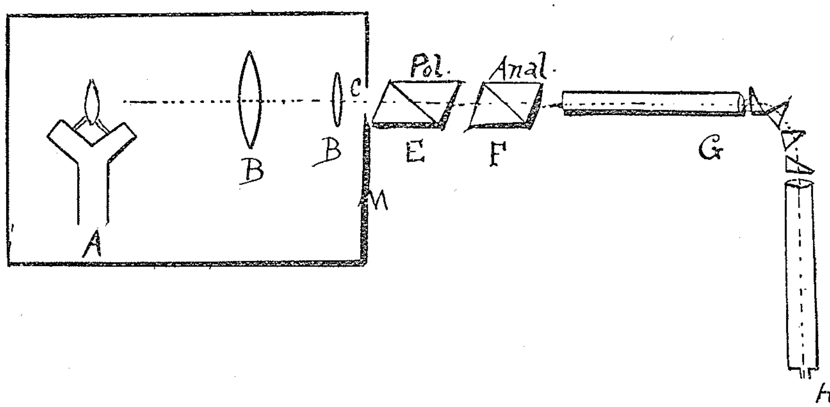

FIG. 1.

sections horizontal; thence through the spectrometer G and was finally viewed in a Hilger eye-piece H in which all the light of the spectrum except a narrow central band was cut off by means of adjustable shutters. The observer was therefore able to subject his eye to a light stimulus of any desired wave-length. A constant gas pressure was taken to indicate a constant luminosity in the source of light. The intensity of the spectrum was controlled by rotating the polariser. The principal plane of the analyser remained horizontal for *all* investigations in order that the displacement of the light waves as they met

*Frank Allen: Phil. Mag. 1911, Vol. 21, No. 125, page 604.

the prism might be perpendicular to its face. In this way it was thought to avoid a variation in intensity of light through reflection from the surfaces of the prisms.

The spectrometer used was of the Hilger Automatic type with four prisms equivalent to three sixty degree prisms and giving a dispersion slightly in excess of twelve degrees.

I.

Each characteristic of the visual response to radiation as outlined in the commencement of this paper, was treated as a separate investigation. Observations were made to show:

1. Whether sensibility to slight difference in wave length has two pronounced maxima, one in the yellow and one in the green, and two light maxima in the extreme blue and red.

2. Whether these maxima vary with the intensity of the radiation used.

1. In the investigation concerning spectra of varying intensity, other conditions remaining constant, spectra of six different intensities were examined. Only observations made with my own eye (the right) which seems quite normal as to color sensations, are considered in this paper. Upon the spectrum of maximum intensity in all parts, which for present purposes, has been represented by unity, and upon a spectrum of intensity ·25, observations were made to determine the minimum *decrease* in intensity by which the initial intensity was changed in effecting a change in sensation that was just perceptible. In the case of spectra at all other intensities considered, the minimum *increase* in the original intensity necessary to produce a just perceptible change in the sensation was determined.

Method of procedure.

The principal sections of the nicols were horizontal and so gave the brightest spectrum obtainable with the chosen arrangement of apparatus. Observations were made upon thirty-two portions of this spectrum; these are given in Table I.

Light of wave length $\cdot 42\mu$ was first observed. This was allowed to act upon the retina for four seconds. Then the polariser was rotated at a uniformly rapid rate; an effort was made to keep the rate at which the polariser was rotated, approximately constant for all observations. As soon as a change in the sensation of light was perceived the rotation was stopped. The angle through which rotation had taken place was noted. This was denoted by α; since unity had been taken to represent the amount of light passing through the prisms before the polariser was rotated, the difference between 1 and $\text{Cos}^2\ \alpha$ i.e., $\text{Sin}^2\ \alpha$ will represent the amount by which the original intensity of the light was *diminished* in effecting a change of sensation that was just perceptible. The reciprocal of this value ($\sin^2 \alpha$) taken to represent the sensibility of the eye has been plotted as a function of the corresponding wave length. This is shown in Curve 1, Fig. 2. The other chosen portions of the spectrum were observed in rapid succession in a similar way.

A dimmer spectrum was next considered. This was obtained by setting the polariser before each observation so that its principal plane made an angle of 51° with the horizontal, i.e. with the plane of the analyser. This initial angle was denoted by β. Since $\text{Cos}^2\beta = \text{Cos}^2 51°$ is equal to ·395, it was considered that each portion of the spectrum chosen for investigation was 39·5% of the intensity of corresponding portions in the spectrum at maximum intensity. With this initial arrangement light as observed in the eye-piece was, as before, allowed to act upon the retina for four seconds. Then the polariser was rotated so as to *increase* the intensity of the light. As soon as the intensity had just noticeably increased the rotation was stopped. The angle between the planes of the nicols was read and denoted by α. Since $\text{Cos}^2 51°$ represents the intensity of the stimulus at the begining and $\text{Cos}^2\ \alpha$ the intensity of the light affecting the eye at the moment of perceptible change, $\text{Cos}^2\alpha - \text{Cos}^2 51°$ will represent the amount by which the original intensity (which for each observation is 39·5% of the maximum intensity of the spectrum at the chosen point) was *increased* in affecting a change of sensation that was just perceptible. As before, the reciprocal of this value was taken to represent the sensibility of the eye and has been plotted as a function of the corresponding wave length. The results are shown graphically in Curve II, Fig. 2. The portions of the spectrum observed and the observations made upon each are given in Table II.

Upon the spectrum of intensity ·25 observations were made to determine the least perceptible *decrease* in the initial stimulus necessary to produce a noticeable change in the sensation of light. This was done by following the method described for the spectrum at maximum intensity. The results are given in Table III and are shown graphically in Curve III, Fig. 2. The spectrum of intensity ·25 was obtained by setting the plane of the polariser at an angle of 60° with the plane of the analyser.

In the case of other spectra investigated observations were made for the least perceptible *increase* in the initial stimulus, following the method used upon the spectrum second in order of brightness, as described above. These spectra were of relative intensities ·060; ·025 and ·0054 respectively,—the spectrum of maximum intensity being

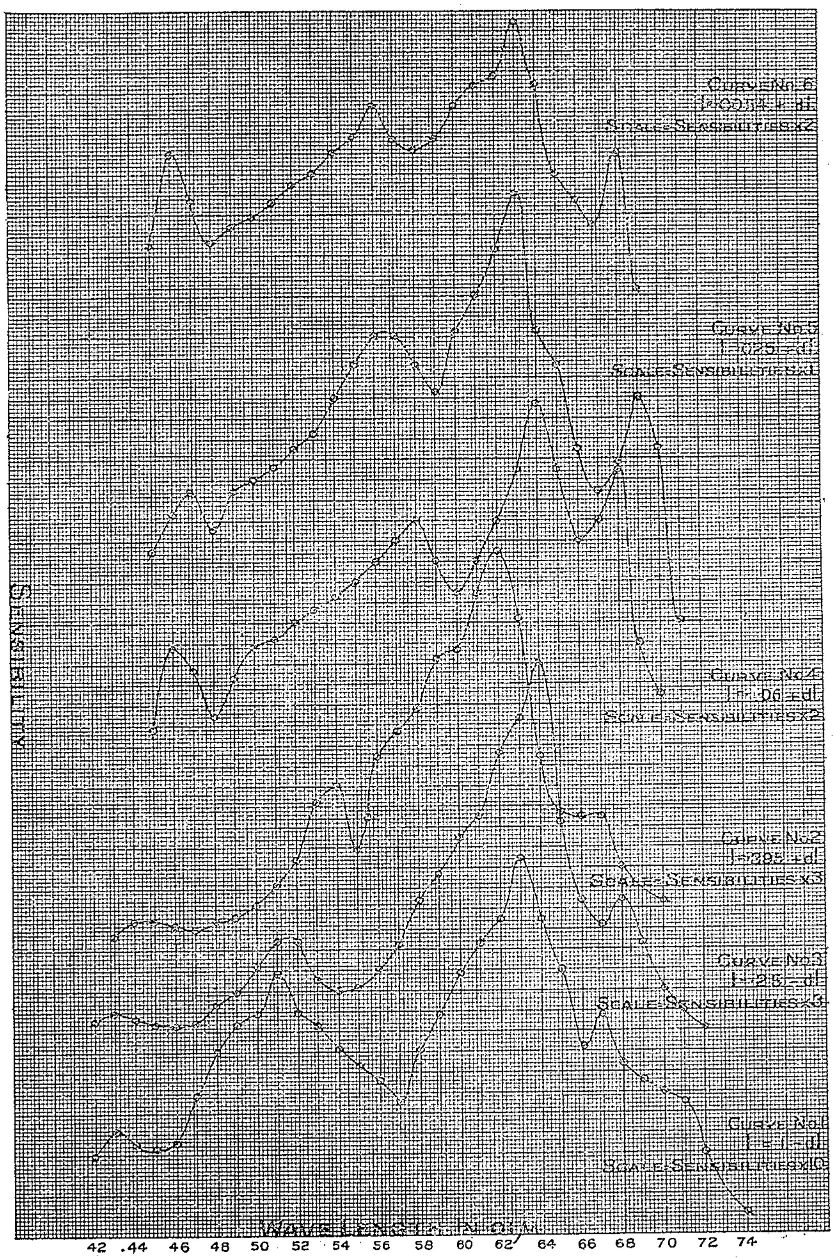
SENSIBILITY
CURVE NO. 6
SCALE-SENSIBILITIES x2
CURVE NO. 5
CURVE NO. 4
SCALE-SENSIBILITIES x2
CURVE NO. 2
SCALE-SENSIBILITIES x3
CURVE NO. 3
SCALE-SENSIBILITIES x3
CURVE NO. 1
SCALE-SENSIBILITIES x10
WAVE LENGTH IN
42 .44 46 48 50 52 54 56 58 60 62 64 66 68 70 72 74

Figure 2

considered as equal to unity in all parts. The initial conditions to give spectra of such intensities for each observation were the inclination of the principal planes of the nicols to each other at angles of 75° 45′; 80° 45′ and 85° 45′ respectively. The results are given in Tables IV, V and VI and shown graphically in Curves IV, V and VI, fig. 2.

Tables of data.

In Tables I to VI λ indicates the wave length of light observed. The column headed "Scale Readings" gives the observed reading on the attached scale, after each rotation. With each new intensity the zero of the scale was observed and taken into account to find the angle between the principle sections of the nicols. These values will be found in the column headed (α). Since β denotes the initial angle between the principal sections, in the column under $\text{Cos}^2\beta$ will be found the intensity of the spectrum considered in each case. In the columns under $\text{Sin}^2\alpha$ Table I and under $\text{Cos}^2\beta - \text{Cos}^2\alpha$ Table III will be found the amount by which the initial intensity was *diminished* to effect the minimum noticeable change of sensation. In Tables II and IV to VI, under $\text{Cos}^2\alpha - \text{Cos}^2\beta$ are given the amounts by which the original intensity (as shown under $\text{Cos}^2\beta$) had to be increased to effect a change of sensation just perceptible. Under "Sensibility" are the values of the reciprocals of the amounts of minimum perceptible change in intensity. In order that the various curves might be plotted in one figure these reciprocals, which are the ordinates for the different curves, have been multiplied by arbitrary constants as indicated in the figure and then plotted to the scale shown.

2. *A Spectrum of Equal Luminosity in all parts.*

When measuring the luminosity of the spectrum Allen* took observations upon nineteen portions of it, determining in each case the angle which the principal plane of the polariser should make with the horizontal, i.e., with the plane of the analyser in order that each portion as viewed in the eye-piece should have equal luminosity. Therefore since the apparatus used in the present investigation was also used by Allen in approximately the same arrangement with the same gas jet, gas pressure and slit width, it was considered that, by observing these same portions and setting the plane of the polariser at the corresponding angle for each, equality of luminosity in the parts observed would be obtained. This would it was thought be equivalent to observing a spectrum of equal luminosity in all parts. The next problem considered, then, was that of determining the sensibility of the eye to slight

*Phil. Mag., 1911. Vol. 21., p 604.

differences in intensity when the light stimulus was of equal intensity at each point of observation. For this purpose observations similar in principle to those already described, were made upon this "Equivalent of a Spectrum of Equal Luminosity."

Method of procedure.

The method of procedure was first to adjust the telescope of the spectrometer for light as near the ultra-violet as it was possible to make an observation. This was of wave length $\cdot 460\mu$. The plane of the polariser was adjusted to make an angle of 35° with the plane of the analyser. This gave to the light as viewed in the eye-piece a luminosity equal to the luminosity of the spectrum where the light waves were $\cdot 414\mu$—the principal sections of the nicols then being parallel. This latter luminosity was taken as the standard of brightness since all portions observed were first reduced to this luminosity. After the manner of former observations the light as viewed in the eye-piece was allowed to act upon the retina, the polariser rotated and the angle after rotation noted. This angle was denoted by α. If β denote the initial angle (35°) between the planes of the nicols, then since α was taken to denote the angle between the planes after rotation, $(\mathrm{Cos}^2\alpha - \mathrm{Cos}^2\beta)$ will be proportional to the amount of light by which the initial intensity was *increased* to effect a noticeable change in the sensation. *This amount of light* $(Cos^2\alpha - Cos^2\beta)$, *will be a fraction of the total intensity at the point considered.* But since the luminosity of each part was reduced to that of the standard, it is reasonable to expect that, at those points where the spectrum is brighter and therefore more intense, a smaller portion of the total intensity will be required to produce a noticeable change in the sensation than will be required where the total brightness is much less. That is: the brighter the spectrum at the point of observation the smaller the fraction of total intensity necessary to effect a noticeable change in the sensation. Hence we may say that the luminosity of each part of the spectrum is inversely proportional to the portion of total intensity required to effect a noticeable change in the sensation—the sensation in each case being the result of a light stimulus varying only in wave length. Therefore, plotting the reciprocals of the various values of $(\mathrm{Cos}^2\alpha - \mathrm{Cos}^2\beta)$ as functions of the corresponding wave lengths a luminosity curve for the spectrum should be obtained. This is shown in Fig. 3.

In the above case the reciprocals of the various portions of total light added cannot represent sensibility because in one instance we may take the reciprocal of a large quantity of light at low intensity and

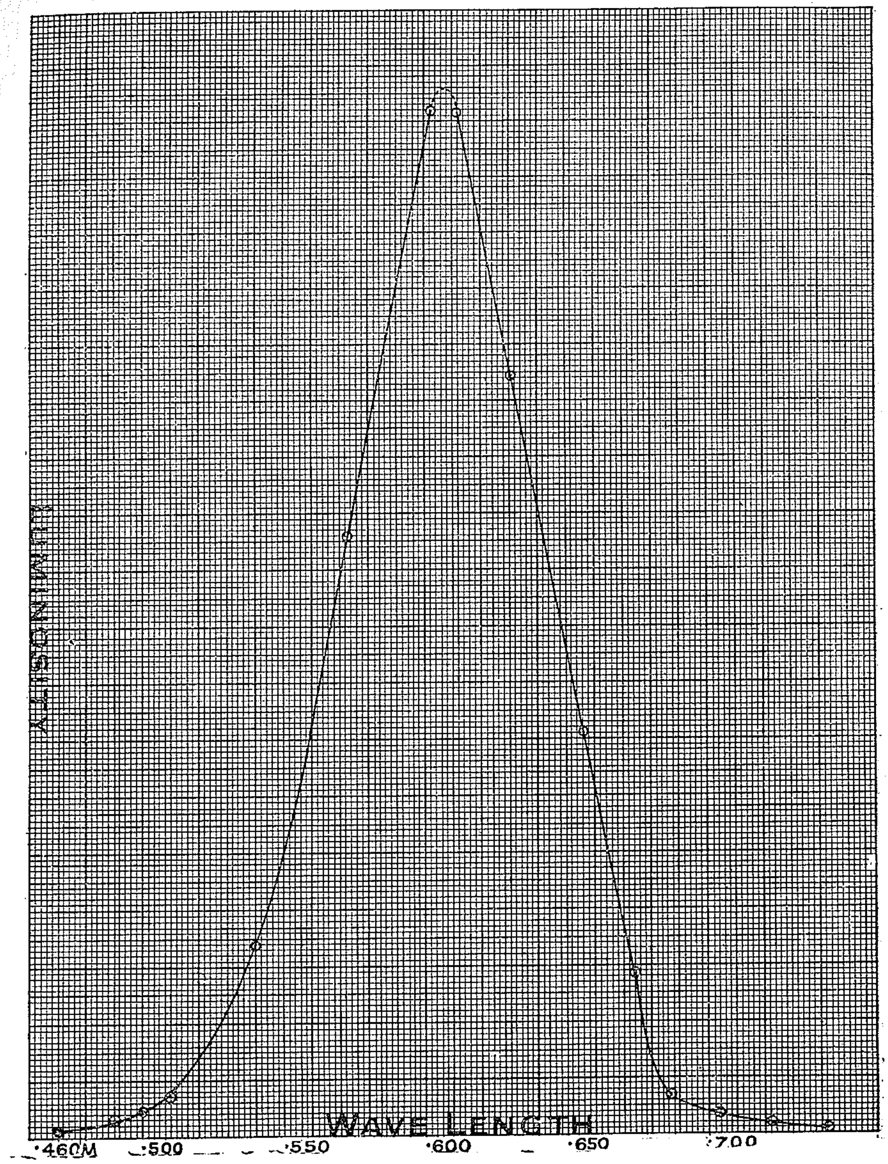

Figure 3

in another the reciprocal of a small quantity at high intensity. And while the reciprocals differ vastly in value the actual illuminating power of the two quantities may be indentical. In the one the increment of light was added at low intensity and therefore more of it was required; in the other it was added at high intensity and less of it was required. Hence before calculating reciprocals for the purpose of showing sensibility, the various increments of light were first reduced to a common standard of intensity. This was the intensity of the standard of brightness, viz., the intensity of light of wave length $\cdot 414\mu$ as observed in the eye-piece while the principal sections of the nicols were parallel.

Method of Reducing to a Common Standard of Intensity.

We may assume that the luminosity of any portion of the spectrum is proportional to its intensity. But the luminosity is inversely proportional to the fraction of total intensity, at the point considered, that will give a luminosity equal to the luminosity of the standard of brightness. But this fraction of total intensity is proportional to the square of the cosine of the angle between the principal sections of the nicols—the light going through giving a luminosity equal to the luminosity of our standard of brightness. That is: the luminosity of any portion of the spectrum is inversely proportional to the square of the cosine of the angle between the principal sections of the nicols when the light going through from that portion gives a luminosity equal to that of the standard. Therefore, taking the luminosity of the standard as unity, the luminosity of the spectrum at any point considered in terms of this standard is given by $1/\cos^2\beta$ where β is the angle between the principal sections of the nicols when just sufficient of the light considered is going through to give a luminosity equal to the luminosity to the standard. And since intensity is assumed proportional to luminosity, the intensity of light at any point in the spectrum in terms of the intensity of the standard is given by $1/\mathrm{Cos}^2\beta$.

Again, α denotes the angle between the principal sections of the nicols after the rotation already explained. Therefore, the fraction of total intensity by which the initial intensity i.e., that represented by $\mathrm{Cos}^2\beta$, was increased to give the least perceptible change in the sensation, may be represented by $(\mathrm{Cos}^2\alpha - \mathrm{Cos}^2\beta)$. The intensity of any given portion of light depends upon its position in the spectrum. Therefore, the fraction of total intensity will have an intensity depending upon the position of the point in the spectrum at which it was added, i.e. depending upon the value of the angle β. But it has been shown that the intensity of light from any portion of the spectrum may be expressed in terms of the standard of brightness by multiplying by $1/\mathrm{Cos.}^2\beta$. Henceit was thought that by multiplying $(\mathrm{Cos}^2\alpha - \mathrm{Cos}^2\beta)$ by $1/\mathrm{Cos}^2\beta$ the various increments of light added to produce a perceptible change in the sensation in each case would be of equal intensity i.e., they would have an intensity equal to the intensity of the light in the standard. The reciprocals of this product it was thought would represent sensibility. The portions of the

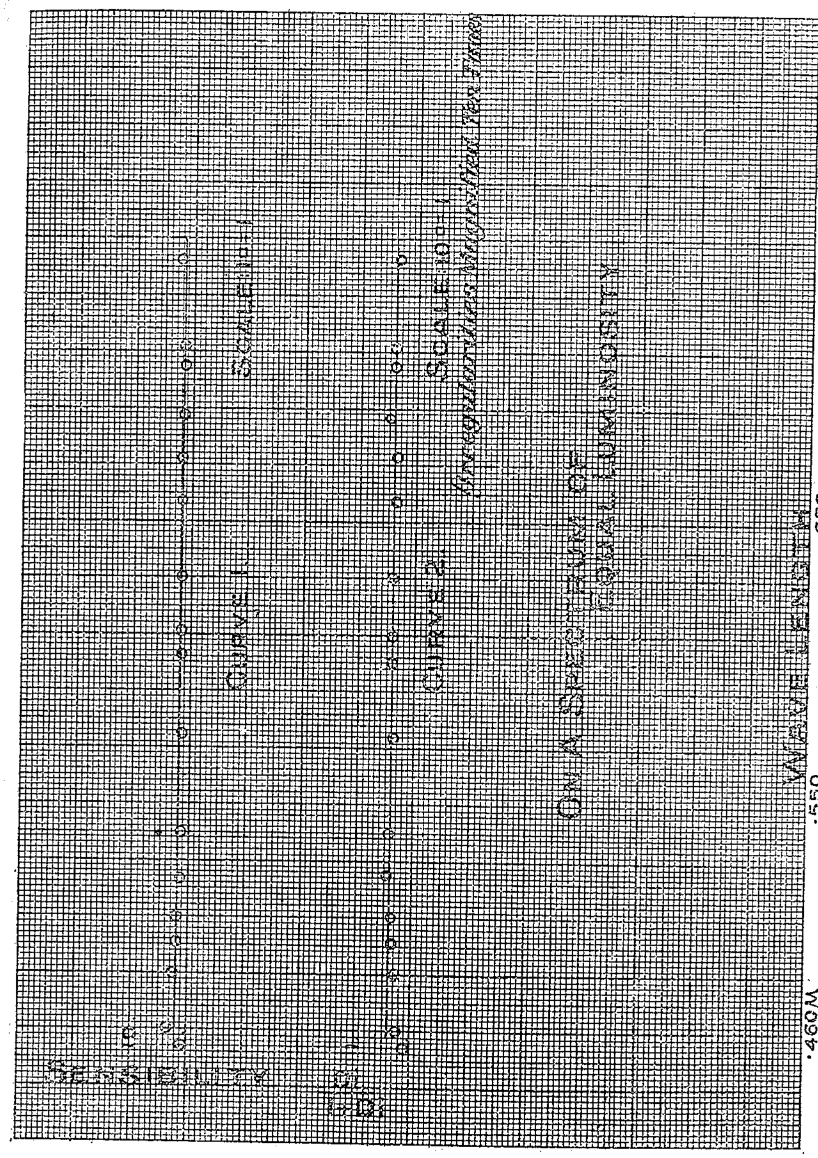

Figure 4

spectrum observed with the corresponding data and calculations are given in Table XV. The sensibility in each case was plotted as a function of the corresponding wave length as shown in Curve 1, Fig. 4, thus showing sensibility to slight differences in intensity using the equivalent of a spectrum of uniform luminosity.

Table of data.

In Table XV, λ indicates the portion of the spectrum upon which observations was made. "β" denotes the inclination of the plane of the polariser to the plane of the analyser. In the column under "α" will be found the inclination of the plane of the polariser to the plane of the analyser *after rotation has taken place.* $\text{Cos}^2\beta$ will represent the amount of light going through the nicols before the rotation of the polariser, and $\text{Cos}^2\alpha$ the amount of light going through when this rotation was completed. In the column under "Least Perceptible Increments" will be found the value of $(\text{Cos}^2\alpha - \text{Cos}^2\beta)$ which will represent the fraction of total intensity which had to be added to the initial intensity $(\text{Cos}^2\beta)$ to effect a change of sensation that was just perceptible. The reciprocals of those values plotted as a function of the corresponding wave length gave the luminosity curve found in Fig. 3. As explained above those increments were reduced to light of uniform intensity by dividing by $\text{Cos}^2\beta$ in each case. The reciprocals of these latter results were taken to represent sensibility and will be found under "Sensibility." Under $\text{Di}/(\text{I}+\text{Di})$ will be found the value of the least perceptible increment divided by the total intensity (after rotation) which was used to get curve 2, Fig. 4 to demonstrate König's Law.

II.

Sensibility to Radiation of Varying Intensity.

As a demonstration of the steady falling off of sensibility with increasing intensity and to show that it is approximately inversely proportional to the intensity over a wide range, observations were made as follows:

The same apparatus as used in the preceding was used in this; and the same arrangement. First, the telescope of the spectrometer was adjusted for light of a given wave length, for example ·420 μ (see Table VII), and rigidly fixed. The nicols were then crossed so as to exclude all light from the eye-piece. The angle indicated on the attached scale was read and this reading was taken to indicate an inclination of 90° between the principal sections of the nicols. The polariser was then rotated and at the first sensation of light experienced through the eye-piece, rotation was stopped. The angle indicated on the attached scale was again read. Let $\alpha_{,1}$ denote this angle which is necessarily less than 90°. Now $\text{Cos}^2 90=0$ and $\text{Cos}^2 \alpha_1$ which is slightly greater than 0 will, as explained in the fore part of this paper, represent the amount of stimulus necessary to produce a sensation of light. This amount of light ($\text{Cos}^2 \alpha_1$), as an initial stimulus was allowed to fall upon the retina and the polariser *further* rotated. As soon as the sensation of light as experienced from the initial stimulus was perceived to change rotation was again stopped, and the angle indicated on the scale read. If α_2 denote this angle, $\text{Cos}^2 \alpha_2$ will represent the amount of light going through the nicols *when rotation was stopped.* But $\text{Cos}^2 \alpha_1$, represents the amount of light going through the nicols *before the last rotation began.* Therefore ($\text{Cos}^2 \alpha_2 - \text{Cos}^2 \alpha_1$) will represent the amount of light by which the initial stimulus ($\text{Cos}^2 \alpha_1$) was increased to produce a change in the sensation that was just perceptible. Observing again the amount of light now in the eye-piece ($\text{Cos}^2 \alpha_2$) and further rotating the polariser until as before another change of sensation was experienced we obtained the angle α_3. Since in this case $\text{Cos}^2 \alpha_2$ represents the initial stimulus ($\text{Cos}^2 \alpha_3 - \text{Cos}^2 \alpha_2$) will represent the increment of light necessary to produce a change of sensation that was just perceptible. In this way the light going through the nicols at the end of each observation was the light upon which each subsequent observation was made until full brightness was reached. That is: the intensity of light used for any observation is represented by $\text{Cos}^2 \alpha_n$ and the increment added by ($\text{Cos}^2 \alpha_{n+1} - \text{Cos}^2 \alpha_n$) where n represents the number of observations made from the crossing of the prisms. In this way several portions of the spectrum were examined. The radiation observed in each portion varied only in intensity and hence the sensibility to radiation of varying intensity was obtained. The reciprocal of ($\text{Cos}^2 \alpha_{n+1} - \text{Cos}^2 \alpha_n$), taken to represent sensibility, was plotted as a function of the corresponding initial intensity, $\text{Cos}^2 \alpha_n$. The results are shown in Fig. 5. The radiation used in obtaining the different curves is indicated in the figure. The portions of the spectrum observed with the data obtained and the calculations made from them are given in tables VII, VIII, X, XII, XIII and XIV.

Starting with the spectrum at maximum brightness and observing the least perceptible decrease in intensity at each step till complete darkness was reached, data were obtained from which the curves indicated in Fig. 5 as "curves of decrements," have been plotted. These data are to be found in Tables IX and XI. These curves seem to indicate that the *increase* in sensibility with *decreasing* intensity follows the law for *decreasing* sensibility with *increasing* intensity.

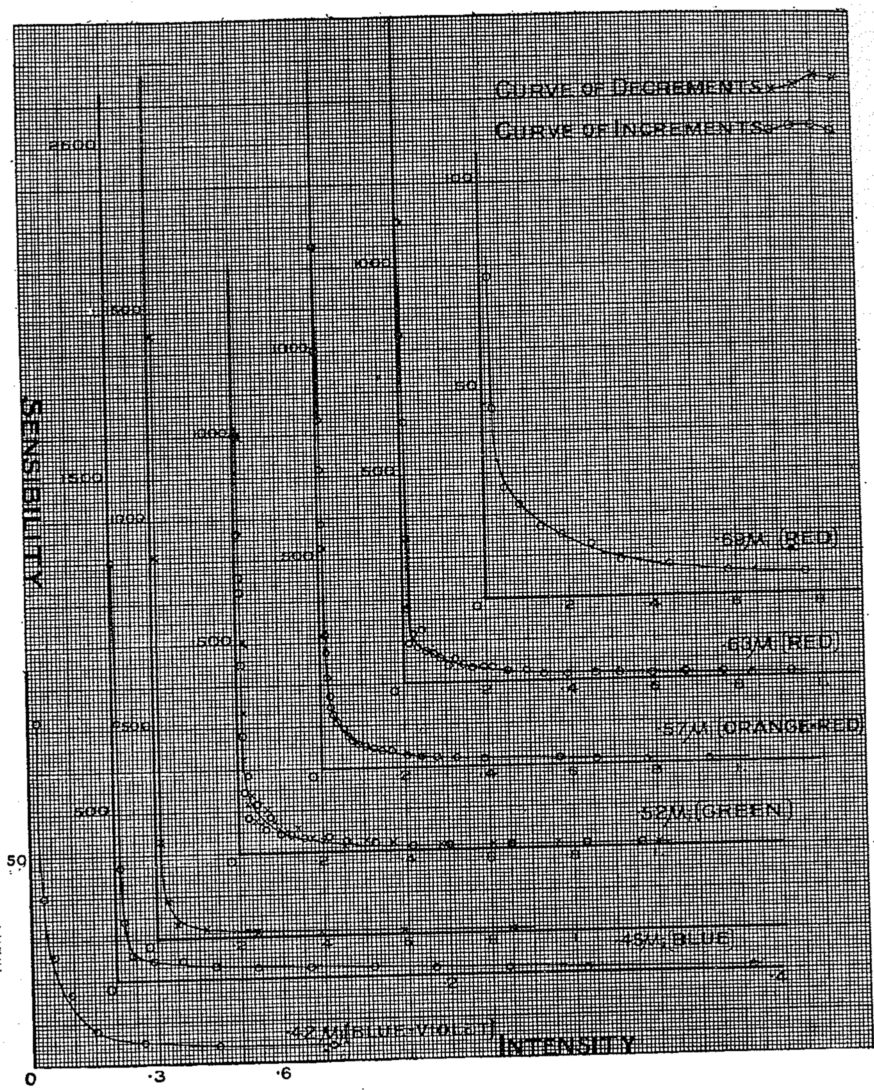

Figure 5

Tables of Data.

In Tables VII to XIV "*a*" indicates the angle between the planes of the nicols for each observation; "Cos²*a*" the intensity of the light upon which the observation was made. In Tables IX and XI "Least perceptible decrement" is the amount by which the intensity of the light "Cos²*a*" was *diminished* in effecting a change in the sensation that was just perceptible. In the other tables of this part "Least perceptible increment" is the amount by which the intensity of the light was *increased* in effecting a change of sensation that was just perceptible. "Sensibility" is the value of the reciprocal of the least perceptible increment, or decrement, as the case may be.

III.

Sensibility to small differences in Intensity.

1. Without further observation it may be shown from the data in Tables VII to XIV that the least perceptible increment or decrement measured as a fraction of the whole is approximately independent of intensity.

It has already been shown (II) that, with any constant wave length, the intensity of the light used for any observation say, the nth, may be represented by $\mathrm{Cos}^2\alpha_n$ and the least perceptible increment by $(\mathrm{Cos}^2\alpha_{n+1} - \mathrm{Cos}^2\alpha_n)$. Let

$$\frac{\mathrm{Cos}^2\alpha_{n+1} - \mathrm{Cos}^2\alpha_n}{\mathrm{Cos}^2\alpha_{n+1}} = k \text{ or } \mathrm{Cos}^2\alpha_{n+1} - \mathrm{Cos}^2\alpha_n = k\,\mathrm{Cos}^2\alpha_{n+1}$$

There appears to be a difference of opinion as to whether the denominator here should be $\mathrm{Cos}^2\alpha_n$ or $\mathrm{Cos}^2\alpha_{n+1}$. Substituting the proper values for α_n, curves have been plotted as shown in Fig. 6. It was impossible in some cases to mark all points on the curve because of their coming so close when the intensity was low and so representative points are shown.

2. Again, from the data given in Table XV it may be shown whether "the least perceptible increment measured as a fraction of the whole is approximately independent of the wave length (König's Law) at constant luminosity." For since the data given in this table were obtained by making observations upon different portions of what was thought to be the equivalent of a spectrum of uniform luminosity, we may infer that while the wave length varied with each observation, the intensity of the light as observed in the eye--piece of the spectrometer remained constant for all observations. Take for example, the data furnished by the third observation in the table referred to above. It is shown there that the amount of light going through the nicols at the beginning of the observation is represented by ·1170, while the amount of light going through at the end of the observation is represented by ·1849 showing that ·0679 of the total intensity of the spectrum where the wave length equaled $\cdot 491\mu$, had to be added to

·1170 of total intensity to produce a change in the sensation that was just perceptible. This portion of light reduced to the intensity of the standard of brightness at the polariser was found to be represented

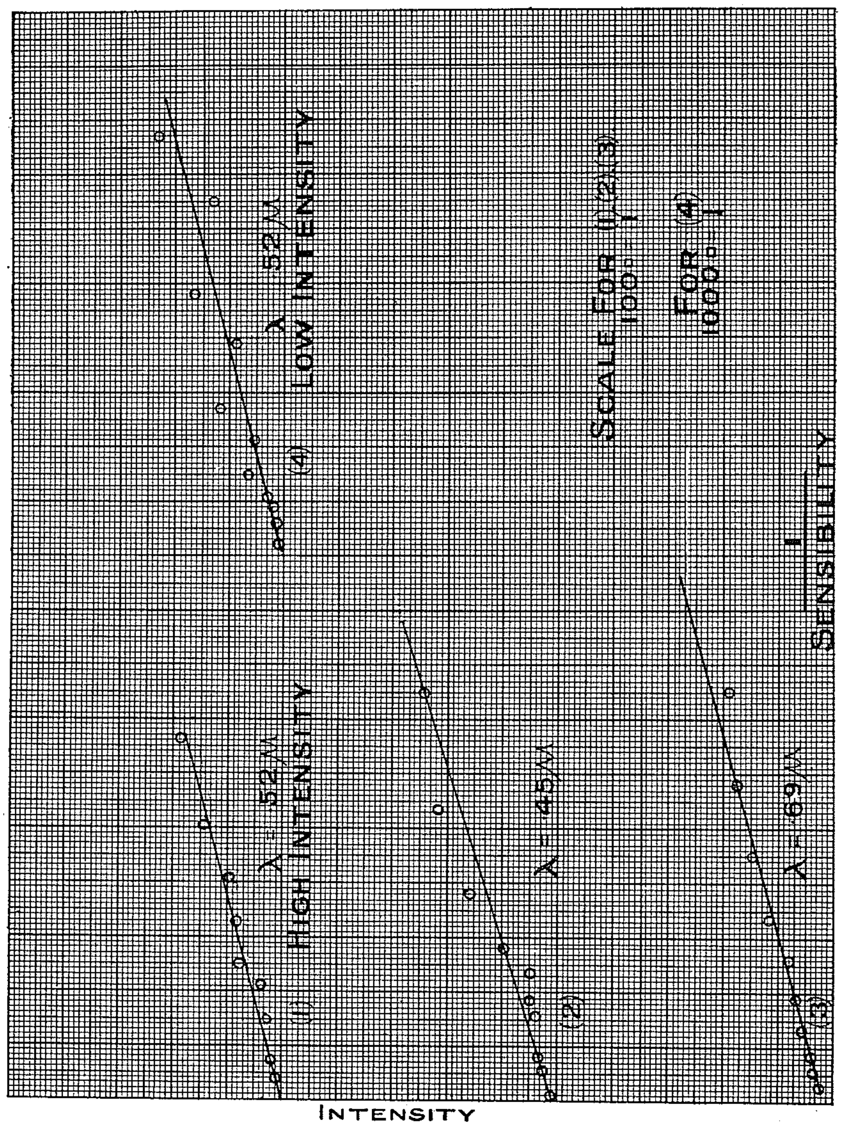

Figure 6

by ·58 as shown in the table, which gives us the least perceptible increment with an intensity at the polariser equal to the intensity of the standard. Now, as already stated, the intensity of the light under observation is represented by ·1170 of the total intensity of the spectrum in portion observed. But this fraction of total intensity gives a luminosity in the eye-piece equal to the standard of brightness which in this paper is represented by unity, and all portions of the spectrum, as observed, were of equal brightness. Therefore, we may represent the luminosity of each of those portions by unity, and hence also their intensities as observed in the eye-piece by unity. Now König's Law stated mathematically is of the form $Di/(I+Di)=k$ where Di represents the least perceptible increment and I a constant intensity in the source of light as first observed. Hence making the proper substitutions in the above formula from data in Table XV and plotting, Curve 2, Fig. 4 was obtained.

Discussion of Results.

In general, we may conclude that the experiments described in part I of this paper strongly support the view that, except in the case of spectra from light at very low intensity, the general character of the sensibility curve does not vary with varying intensity of the source. If, however, the spectrum is one of very low intensity the two pronounced maxima, one in the yellow and one in the green tend to diminish while the two slight maxima in the blue and red apparently maintain their prominence. When the spectrum is of uniform intensity in all parts these maxima appear either to disappear entirely, leaving a uniform curve parallel to the horizontal axis, or to become scarcely more than noticeable. Referring to Table XV it will be noticed that in the regions of $\mu=\cdot 564$ and $\mu=\cdot 648$ slight maxima still persist. I do not know whether these indications have a real physical significance or are due to inaccurate observations. I am inclined to adopt the latter reason for their appearance. This could only be settled absolutely by the work of several observers upon uniform spectra at different intensities. If, indeed, these maxima really exist in however slight a degree, we are forced to conclude that König's Law as interpreted in this paper is not *absolutely* correct, for these maxima still persist as is shown under $Di/(I+Di)$, Table XV. Conclusions reached regarding the steady fall in sensibility with increasing intensity, and the application of Fechner's Law, are quite in accord with accepted results.

I desire to acknowledge my indebtedness to the kindness and valuable advice of Professor Frank Allen, director of the Department of Physics, University of Manitoba, at whose suggestion the investigations described in this paper were undertaken; also to Dr. R. K.

McClung of the same department for valuable discussions and kind interest shown. The method employed is also due to Professor Allen.

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NOTE.—A most comprehensive study conducted by Dr. H. E. Ives and several assistants, has lately appeared in a series of five papers (Phil. Mag., Vol. 24, 1912, pp. 149-188; 352-370; 744-751; 845-863) under the general title "Studies in the Photometry of Lights of Different Colors." Because of the fact that the writing of the present paper was completed before the publication of Dr. Ives papers no reference has been made to them. I should like to point out however, that, though in obtaining the Luminosity Curve for the Spectrum, I was dependent upon other means to get a spectrum of uniform luminosity, the human eye when undisturbed by the presence of a second color is capable of distinguishing changes in luminosity to a very fine and uniform degree of sensitiveness as is shown by the smoothness of his luminosity curve.

TABLE I.

λ	Scale Reading	a	$\text{Sin}^2 a$	$\text{Cos}^2 \beta$	Sensibility
·42μ	34° ′	24° ′	·1648	1·	6·04
·43	33	23	·1521		6·55
·44	33 30	23 30	·1584		6·31
·45	35	25	·1780		5·64
·46	33 30	23 30	·1584		6·31
·47	31 30	21 30	·1339		7·44
·48	30	20	·1169		8·55
·49	29 30	19 30	·1108		9·17
·50	29	19	·1056		9·46
·51	28 30	18 30	·1004		9·94
·52	29	19	·1056		9·46
·53	29 30	19 30	·1108		9·17
·54	30	20	·1169		8·55
·55	30 30	20 30	·1225		8·16
·56	31	21	·1281		7·8
·57	32	22	·1398		7·25
·58	30	20	·1169		8·55
·59	29	19	·1056		9·46
·60	28	18	·0954		10·48
·61	27 30	17 30	·0900		11·11
·62	27	17	·0852		11·73
·63	26	16	·0756		13·33
·64	27	17	·0852		11·73
·65	28	18	·0954		10·48
·66	30	20	·1169		8·55
·67	29	19	·1056		9·46
·68	30 30	20 30	·1225		8·16
·69	31	21	·1281		7·80
·70	31 30	21 30	·1339		7·44
·71	32	22	·1398		7·25
·72	34	24	·1648		6·04
·73	38	28	·2199		4·54

TABLE II.

λ	Scale Reading	a	Cos^2a	$Cos^2\beta$	$Cos^2a-Cos^2\beta$	Sensibility
·43μ	47° 30′	38° 30′	·611	·395	·216	4·6
·44	50	41	·568		·173	5·7
·45	50	41	·568		·173	5·7
·46	49	40	·586		·191	5·2
·47	48 30	39 30	·594		·199	5·
·48	49 30	40 30	·577		·182	5·4
·49	50 30	41 30	·559		·164	6·
·50	51 30	42 30	·543		·148	6·7
·51	53 15	44 15	·512		·117	8·5
·52	54 30	45 30	·490		·095	10·5
·53	56 10	47 10	·461		·066	15·1
·54	56 30	47 30	·455		·060	16·6
·55	55	46	·481		·086	11·6
	56	47	·465		·070	14·2
·56	57	48	·447		·052	19·2
·57	57 15	48 15	·442		·047	21·2
·58	57 30	48 30	·438		·043	23·2
·59	57 50	48 50	·432		·037	27·
·60	57 55	48 55	·431		·036	27·7
·61	58 10	49 10	·426		·031	32·2
·62	58 20	49 20	·423		·028	35·7
·63	57 30	48 30	·438		·033	30·3
·64	57	48	·447		·052	19·2
·65	56	47	·465		·070	14·2
·66	56	47	·465		·070	14·2
·67	56	47	·465		·070	14·2
·68	54	45	·499		·104	9·6
·69	52	43	·534		·139	7·1

TABLE III.

λ	Scale Reading	a	Cos^2a	$Cos^2\beta$	$Cos^2a-Cos^2\beta$	Sensibility
·42μ	70° ′	70° ′	·1169	·25	·1331	7·5
·43	69	69	·1281		·1219	8·2
·44	69 30	69 30	·1225		·1275	7·8
·45	70	70	·1169		·1331	7·5
·46	70 30	70 30	·1108		·1392	7·1
·47	70 10	70 10	·1149		·1351	7·4
·48	68 15	68 15	·1369		·1131	8·8
·49	67 30	67 30	·1459		·1041	9·6
·50	66	66	·1648		·0852	11·76
·51	65	65	·1780		·0720	13·8
·52	65	65	·1780		·0720	13·8
·53	66 30	66 30	·1584		·0916	10·9
·54	67 15	67 15	·1489		·1011	9·89
·55	67	67	·1521		·0979	10·21
·56	66	66	·1648		·0852	11·73
·57	65	65	·1780		·0720	13·88

TABLE III.—*Continued.*

λ	Scale Reading	α	Cos²α	Cos²β	Cos²α – Cos²β	Sensibility
·58	64	64	·1918		·0582	17·18
·59	63 30	63 30	·1989		·0511	19·56
·60	63	63	·2052		·0448	22·32
·61	62 45	62 45	·2088		·0412	24·27
·62	62 15	62 15	·2162		·0338	29·58
·63	62 5	62 5	·2190		·0310	32·25
·64	61 50	61 50	·2227		·0273	36·63
·65	62 50	62 50	·2079		·0421	23·75
·66	64	64	·1918		·0582	17·18
·67	64 35	64 35	·1840		·0660	15·15
·68	64	64	·1918		·0582	17·18
·69	65	65	·1780		·0720	13·88
·70	67	67	·1521		·0979	10·21
·71	69	69	·1281		·1219	8·2
·72	71	71	·1056		·1444	6·9

TABLE IV.

λ	Scale Reading	α	Cos²α	Cos²β	Cos²α– Cos²β	Sensibility
·45	81° ′	71° 45′	·097	·060	·037	27
·46	82	72 45	·087		·027	37
·47	81 50	72 35	·089		·029	34·4
·48	81 15	72	·095		·035	28·5
·49	81 45	72 30	·090		·030	33·3
·50	82	72 45	·087		·027	37·
·51	82 10	72 55	·086		·026	38·
·52	82 15	73	·085		·025	40·
·53	82 20	73 5	·084		·024	41·6
·54	82 25	73 10	·083		·023	43·4
·55	82 30	73 15	·082		·022	44·9
·56	82 35	73 20	·081		·021	47·6
·57	82 45	73 30	·080		·020	50·
·58	82 50	73 35	·079		·019	52·6
·59	82 40	73 25	·081		·021	47·6
·60	82 30	73 15	·083		·023	43·4
·61	82 40	73 25	·081		·021	47·6
·62	82 50	73 35	·079		·019	52·6
·63	83	73 45	·077		·017	58·8
·64	83 20	74 5	·075		·015	66·6
·65	83	73 45	·077		·017	58·8
·66	82 45	73 30	·080		·020	50·
·67	82 50	73 35	·079		·019	52·6
·68	83	73	·077		·017	58·8
·69	82	72 45	·087		·027	37
·70	81 30	72 15	·092		·032	1·2 3

TABLE V.

λ	Scale Reading	a	$Cos^2 a$	$Cos^2 \beta$	$Cos^2 a - Cos^2 \beta$	Sensibility
·45μ	86° ′	76° 45′	·052	·025	·027	37·
·46	86 45	77 30	·046		·021	47
·47	87	77 45	·044		·019	52
·48	86 30	77 15	·048		·023	43
·49	87	77 45	·044		·019	52
·50	87 15	78	·043		·018	55
·51	87 25	78 10	·042		·017	58
·52	87 30	78 15	·041		·016	62
·53	87 40	78 25	·040		·015	66
·54	87 55	78 40	·038		·013	76
·55	88	78 45	·038		·012	83
·56	88 10	78 55	·036		·011	90
·57	88 5	78 50	·037		·011	90
·58	88	78 45	·038		·012	83
·59	87 55	78 40	·038		·013	76
·60	88 5	78 50	·037		·011	91
·61	88 15	79	·036		·010	100
·62	88 25	79 10	·035		·009	111
·63	88 30	79 15	·034		·008	125
·64	88 10	78 55	·136		·011	91
·65	87 50	78 35	·038		·012	83
·66	87 30	78 15	·041		·016	62
·67	87 5	77 50	·044		·019	52
·68	87 25	78 10	·042		·017	58
·69	88	78 45	·038		·013	76
·70	87 30	78 15	·041		·016	62
·71	83	73 45	·077		·052	19

TABLE VI.

λ	Scale Reading	a	$Cos^2 a$	$Cos^2 \beta$	$Cos^2 a - Cos^2 \beta$	Sensibility
	° ′	° ′				
·45	93	83 45	·0116	·0054	·0062	16·1
·46	93 45	84 30	·0090		·0036	27·7
·47	93 30	84 15	·0100		·0046	21·7
·48	93	83 45	·0116		·0062	16·1
·49	93 15	84	·0108		·0054	18·5
·50	93 20	84 5	·0106		·0052	19·2
·51	93 30	84 15	·0100		·0046	21·7
·52	93 35	84 20	·0096		·0042	23·8
·53	93 40	84 25	·0094		·0040	25·
·54	93 45	84 30	·0090		·0036	27·7
·55	93 50	84 35	·0088		·0034	29·4
·56	93 55	84 40	·0084		·0030	33·3
·57	93 50	84 35	·0088		·0034	29·4
·58	93 45	84 30	·0090		·0036	27·7
·59	93 50	84 35	·0088		·0034	29·4
·60	93 55	84 40	·0084		·0030	33·3
·61	94	84 45	·0082		·0028	35·7
·62	94 5	84 50	·0081		·0027	37·0
·63	94 10	84 55	·0077		·0023	43·4
·64	94	84 45	·0082		·0028	35·7
·65	93 40	84 25	·0094		·0040	25·
·66	93 30	84 15	·0100		·0045	21·7
·67	93 15	84	·0108		·0054	18·5
·68	93 45	84 30	·0090		·0036	27·7
·69	92	82 45	·0158		·0104	9·6
·70	89	79 45	·0313		·0259	3·8

TABLE VII.

Least perceptible increment as observed upon light of varying intensity and of wave length=·42μ.

Scale Reading		a		Cos^2a	Least percep'ble Increment	Sensibility
°	′	°	′			
99		90			·0210	47
90	30	81	30	·0210	·0121	82
88	30	79	30	·0331	·0249	40
85		76		·0580	·0374	26
81		72		·0954	·0567	17
76		67		·1521	·1131	8
68		59		·2652	·1823	5
57		48		·4475	·2716	3
41		32		·7191	·2809	3
9		0		1·0000		

TABLE VIII.

Least perceptible increment as observed upon light of varying intensity and of wave length=45μ.

Scale Reading		a		Cos^2a	Least percep'ble Increment	Sensibility
°	′	°	′			
99		90		·0	·0004	2500
97	45	88	45	·0004	·0008	1250
97		88		·0012	·0013	769
96	5	87	5	·0025	·0029	344
94	45	85	45	·0054	·0055	181
93		84		·0109	·0122	81·9
90	15	81	15	·0231	·0166	60·2
87	30	78	30	·0397	·0188	53·1
85		76		·0585	·0260	38·4
82	5	73	5	·0845	·0324	30·8
79		70		·1169	·0369	27·
75	55	66	55	·1538	·0364	27·
73	5	64	5	·1902	·0448	22·3
70		61		·2350	·0457	21·8
67		58		·2807	·0983	10·1
61		52		·3790	·1559	6·4
52	15	43		·5349	·2150	4·6
39		30		·7499	·2391	4·1
15		6		·9890		

TABLE IX.

Least perceptible decrement as observed upon light of varying intensity and of wave length=·45μ

Scale Reading °	′	a °	′	Cos^2a	Least percep'ble Decrement	Sensibility
0		0		1·	·1530	
23		23		·8470	·2603	3·8
40		40		·5867	·1992	5·
51	30	51	30	·3875	·1562	6·4
61	15	61	15	·2313	·1144	8·5
70		70		·1169	·0663	15·
77		77		·0506	·0313	31·
82		82		·0193	·0117	85·
85		85		·0076	·0043	232·
86	40	86	40	·0033	·0011	909·
87	15	87	15	·0022	·0007	1428·
87	45	87	45	·0015	·0005	2000·
88	10	88	10	·0010	·0005	2000·
88	40	88	40	·0005	·0003	3333·
89	10	89	10	·0002	·0002	5000·
90		90		·0		

TABLE X.

Least perceptible increment as observed upon light of varying intensity and of wave length—.52 μ.

Scale Reading °	′	a °	′	Cos^2a	Least percep'ble Increment	Sensibility
90		90		·0	·0001	10000·
89	30	89	30	·0001	·0003	3333·
89		89		·0004	·0004	2500
88	30	88	30	·0008	·0004	2500
88		88		·0012	·0008	1250
87	30	87	30	·0020	·0008	1250
87		87		·0028	·0010	1000
86	30	86	30	·0038	·0010	1000
86		86		·0048	·0013	769
85	30	85	30	·0061	·0015	666
85		85		·0076	·0016	625
84	30	84	30	·0092	·0022	454
83	45	83	45	·0114	·0036	277
83		83		·0150	·0069	145
81	45	81	45	·0219	·0054	185
80	30	80	30	·0273	·0123	81
78	30	78	30	·0396	·0085	117
77	15	77	15	·0481	·0169	59
75	10	75	10	·0650	·0125	80
73	45	73	45	·0775	.0233	42
71	30	71	30	·1008	·0237	42
69	15	69	15	·1245	·0282	36
67		67		·1527	·0327	30
64	30	64	30	·1854	·0300	33
62	15	62	15	·2154	·0347	28
60		60		·2501	·0790	12·6
55		55		·3291	·0842	11·9
50		50		·4133	·0955	10·4
44	30	44	30	·5088	·1457	6·8
36		36		·6545	·1796	5·6
24		24		·8341	·1358	7·3
10		10		·9699		

TABLE XI.

Least perceptible decrement as observed upon light of varying intensity and of wave length $= \cdot 52\mu$.

Scale Reading °	′	a °	′	$\text{Cos}^2 a$	Least perceptible decrement	Sensibility
39		29		·763	·160	6·25
49		39		·603	·122	8·2
56		46		·481	·112	8·9
62	30	52	30	·369	·074	13·5
67		57		·295	·053	18·8
70	30	60	30	·242	·043	23·2
73	30	63	30	·198	·038	26·3
76	25	66	25	·160	·026	38·4
78	28	68	28	·134	·018	55·5
80	8	70		·116	·021	47·6
82		72		·095	·015	66·6
83	30	73	30	·080	·013	76·9
84	55	74	55	·067	·010	100
86	5	76	5	·057	·015	66·6
88		78		·042	·012	83·3
89	55	79	55	·030	·007	142·8
91	15	81	5	·023	·008	125·
92	45	82	45	·015	·003	333·
93	30	83	30	·012	·002	500·
94	10	84	10	·010	·004	225·
95	30	85	30	·006	·003	333·
96	50	86	50	·003	·001	1000·
97	15	87	15	·002		

TABLE XII.

Least perceptible increment as observed upon light of varying intensity and of wave length $= \cdot 57\mu$.

Scale Reading °	′	a °	′	$\text{Cos}^2 a$	Least perc'ble Increment	Sensibility
90		90				
97	45	88	45	·0004		
97	29	88	29	·0007	·0004	2500·
97	2	88	2	·0011	·0006	1666·6
96	35	87	35	·0017	·0008	1250·
96	5	87	5	·0025	·0011	909
95	33	86	33	·0036	·0012	833
95		86		·0048	·0015	714
94	26	85	26	·0063	·0017	588
93	51	84	51	·0080	·0020	526
93	13	84	13	·0100	·0032	312
92	23	83	23	·0132	·0037	270
91	30	82	30	·0169	·0047	213
90	32	81	32	·0216	·0056	178
89	30	80	30	·0272	·0070	142
88	20	78	20	·0342	·0082	122
87	6	78	6	·0424	·0091	109
85	50	76	50	·0515	·0110	90

TALBE XII.—*Continued.*

Least perceptible increment as observed upon light of varying intensity and of wave length $=\cdot 57\mu$.

Scale Reading		a		$\text{Cos}^2 a$	Least percep'ble Increment	Sensibility
84	30	75	30	·0625	·0125	80
83	5	74	5	·0750	·0150	66
81	32	72	32	·0900	·0162	61
79	56	70	56	·1062	·0191	52
78	15	69	15	·1256	·0229	43·6
76	20	67	20	·1482	·0265	37·7
74	15	65	15	·1747	·0305	32·7
72		63		·2052	·0358	27·9
69	35	60	35	·2410	·0420	23·8
66	50	57	50	·2830	·0499	20
63	45	54	45	·3329	·0665	15
59	45	50	45	·3994	·0698	14·3
55	45	46	45	·4692	·0903	11
50	30	41	30	·5595	·1277	7·8
43		34		·6872	·1463	6·8
33		24		·8335	·1446	6·9
17	30	8	30	·9781		

TABLE XIII.

Least perceptible increment as observed upon light of varying intensity and of wave length $=\cdot 63\mu$.

Scale Reading		a		$\text{Cos}^2 a$	Least Percep'ble Increment	Sensibility
°	′	°	′			
97	15	88	15	·0009	·0006	1666·
96	45	87	45	·0015	·0012	833
96		87		·0027	·0016	625
95	10	86	10	·0043	·0029	344
94	5	85	5	·0072	·0057	175
92	25	83	25	·0129	·0058	172
91	5	82	5	·0187	·0108	92
89	5	80	5	·0295	·0089	112
87	40	78	40	·0384	·0113	88
86	5	77	5	·0497	·0083	120
85		76		·0580	·0132	75
83	30	74		·0712	·0164	60
81	45	72	45	·0876	·0173	57
80	5	71	5	·1049	·0225	44
78	5	69	5	·1274	·0154	64
76	45	67	45	·1428	·0220	45
75		66		·1648	·0270	37
73		64		·1918	·0281	35
71		62		·2199	·0301	33
69		60		·2500	·0459	21
66		57		·2959	·0405	24
63	30	54	30	·3364	·0504	19
60	30	61	30	·3868	·0742	13
56	10	47	10	·4610	·0559	17
53		44		·5169	·0775	12
48	30	39	30	·5944	·0845	11
45	30	34	30	·6789	·0814	12
38	15	29	15	·7603	·0732	13
33		24		·8335	·0900	11
25		16		·9235		

TABLE XIV.

Least perceptible increment as observed upon light of varying intensity and of wave length$=\cdot 69\mu$.

Scale Reading °	′	a °	′	Cos²a	Least percept'ble Increment	Sensibility
99		90		·0	·0169	59·1
91	30	82	30	·0169	·0130	76·9
89		80		·0299	·0225	44·4
85	45	76	45	·0524	·0376	26·5
81	30	72	30	·0900	·0439	22·7
77	30	68	30	·1339	·0579	17·2
73		64		·1918	·0652	15·3
68	30	59	30	·2570	·0794	12·5
63	30	54	30	·3364	·1111	9
57		48		·4475	·1392	7·2
48		40		·5867	·1771	5·6
38		29		·7638	·1848	5·4
22		13		·9486		

TABLE XV.

λ	β		α		Cos²α	Cos²β	Least Percep'le Increment	Luminosity	Inc. Cos² β	Sensibility	Di / I+Di
	°	′	°	′							
·460μ	35		22		·8593	·6711	·1882	5·3	·28	3·56	·22
·480	63		57		·2959	·2060	·0899	11·1	·43	2·29	·30
·491	70		64	30	·1849	·1170	·0679	14·7	·58	1·72	·36
·500	76	45	73	18	·0826	·0525	·0301	33·2	·57	1·75	·36
·515	83		80	30	·0272	·0148	·0124	80·6	·83	1·20	·45
·530	84	54	83	15	·0138	·0079	·0059	169·5	·74	1·35	·42
·564	87		85	45	·0052	·0033	·0019	526·3	·58	1·72	·36
·593	87	40	87		·0027	·0016	·0011	909·0	·68	1·47	·40
·601	87	40	87		·0027	·0016	·0011	909·0	·68	1·47	·40
·621	87	15	86	25	·0038	·0023	·0015	666·6	·65	1·54	·39
·648	86		85		·0076	·0048	·0028	357·1	·58	1·72	·36
·663	84		82	15	·0179	·0109	·0070	142·8	·64	1·56	·39
·677	79		75		·0665	·0364	·0301	33·2	·82	1·22	·45
·695	73	24	68	30	·1339	·0816	·0523	19·1	·64	1·56	·39
·712	55	24	45	30	·4900	·3224	·1676	5·9	·52	1·92	·39
·733	37	30	17		·9139	·6295	·2844	3·5	·45	2·22	·31