MODELS FOR CALL ACCEPTANCE IN A CELLULAR NETWORK BASED ON HANDOFF GUARANTEES

by

Md Mostafizur Rahman

A Thesis submitted to the Faculty of Graduate Studies of

The University of Manitoba

in partial fulfilment of the requirements of the degree of

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Abstract

Call admission control (CAC) is important for cellular wireless networks in order to provide quality of service (QoS) requirements to users. There are different static and adaptive call admission control schemes. The adaptive schemes are usually better than static ones, partly because the static schemes make an unrealistic assumption about the distribution of the handoff call arrival process. On the other hand, the adaptive schemes make a different unrealistic, but less severe, assumption about the distribution of the number of users in a cell. In a static scheme where new calls arrive according to the Poisson process and both call holding and cell residence times are exponentially distributed it is usually assumed that handoff arrivals follow Poisson process. However, this assumption of Poisson distributed handoff arrival process is not justified for a wireless network consisting of two or more cells. Conversely, the probability distribution of the number of calls in a cell is approximated as Gaussian or Poisson distribution in distributed schemes. In general, we find that the handoff process is better captured by a two-dimensional Markov chain for single class of calls. We use two-dimensional and four-dimensional Markov chains to capture the handoff arrival process exactly in two-cell wireless network for single and two classes of users, respectively in this thesis. This thesis develops adaptive call admission control schemes based on handoff guarantees without making any assumption about the distribution of number of users in a cell. Moreover, an efficient algorithm for analyzing the guard channel scheme, which is a static scheme, is developed in this thesis.

We propose a novel adaptive call admission control scheme for the two-cell system which accepts a new call if it can guarantee, with a certain probability, that a user's call will be maintained irrespective of its (his/her) movement in the system. Our scheme is

compared with the guard channel scheme which is a static scheme. Our adaptive scheme is found to have lower new call blocking probability than the guard channel scheme for maintaining the handoff failure probability below some threshold value. Then this adaptive scheme is extended to the case of cellular networks with multiple cells. We find that it is not always possible for our adaptive scheme to keep the handoff failure probability at QoS value at higher new call arrival rate to minimize the new call blocking probability. Therefore, another variant of this adaptive scheme is developed which we call fractional adaptive scheme. Fractional adaptive scheme can minimize the new call blocking probability as much as possible in higher new call arrival rates by keeping the handoff failure probability at the target QoS probability. Both the adaptive and fractional adaptive schemes are found to outperform the guard channel scheme in controlling the handoff failure probability in cellular wireless network consisting of more than two cells. After developing adaptive scheme for single class of users we develop adaptive scheme for two classes of users.

In the final part of this thesis we address the problem of efficient analysis of guard channel scheme with general cell residence time and general call holding duration by keeping track of the number of users in different channel holding phases.

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Chapter 1

Introduction

Call admission control (CAC) is a provisioning strategy to provide high quality of service (QoS) to users by limiting the number of call connections into a communication network, to a profitable level, while reducing the network congestion, depending on the availability of resources. In cellular wireless networks, another factor comes into play, the possibility of dropping a connected call due to the mobility of users. It is more irritating for a user's call not to be completed due to handoff failure than the call to be blocked during the new call attempt. Therefore, handoff calls are prioritized over new calls. It has been found that prioritizing handoff calls over new calls results in the decrease of the number of handoff failures and the call dropping probabilities. However, prioritizing handoff calls over new calls results in the increase of the new call blocking probability. A good CAC scheme for a wireless cellular network has to balance the call blocking and the call dropping probabilities in order to provide the desired QoS requirements. The design of CAC algorithms for mobile cellular wireless networks is especially challenging given the limited and highly variable resources, and the mobility of users encountered in such networks.

Call admission control schemes for cellular wireless networks have been extensively studied in the literature. They can be broadly classified into two categories — static schemes and distributed schemes. In the literature, static call admission control schemes, which are mainly cutoff priority schemes, are generally studied for a single cell [3, 45, 55, 90] to approximate the whole cellular wireless network. On the other hand, in distributed call admission control schemes [36, 56, 76, 79, 104], which are also called adaptive call admission control schemes, a number of neighboring cells are considered for admitting calls in a cell. In static call admission control schemes, a cell is studied to approximate the whole cellular network. In static schemes, a new call is admitted in the target cell depending only on the number of calls in that cell. On the other hand, in distributed call admission control schemes, the number of calls in the target cell and the number of ongoing calls in the neighboring cells are considered in the process of accepting a new call in the target cell. In this thesis, we first study and develop a model for a cellular system which consists of two cells. We study the two-cell wireless network to understand how the two-cell system works with the goal in mind to use the results in approximating a cellular network consisting of more than two cells.

In static call admission control schemes, exponentially distributed handoff inter-arrival time is assumed for exponentially distributed call holding time and cell residence time. In this thesis, it is shown that the assumption of exponentially distributed handoff call inter-arrival time is unrealistic for the case when inter-arrival time for new calls, call holding duration and cell residence time all follow exponential distribution. We demonstrate that handoff call inter-arrival time does not follow exponential or hyper-exponential distribution even if new call inter-arrival time, call holding duration and cell residence time are exponentially distributed. It is found that the handoff arrival to a cell is Markovian and depends on the number of ongoing calls in the neighboring cell(s). In general we notice that the handoff arrival process which is Markovian is better captured by a two-dimensional Markov chain. We consider how the number of ongoing calls in the neighboring cell affects a call in a two-cell system which is not considered for guard channel scheme in the literature. In our call admission control scheme, handoff calls are accepted whenever a handoff call arrives and there is at least one free channel. On the other hand, when a new call arrives we obtain the probability of dropping of the call in the neighboring cell if we predict that it needs to handoff based on a table which is computed off-line using transient analysis. A new call is admitted in our scheme if this probability is not more than some threshold value. In other words, our scheme accepts a new call in a cell if a guarantee can be provided with some probability that the call will be successfully handed off if it moves to the neighboring cell. Transient analysis is carried out for our proposed two-cell system using matrix geometric method [80] to compute the probability of dropping the candidate call in the neighboring cell if the candidate call needs to handoff in the neighboring cell. These dropping probabilities are computed for different numbers of ongoing calls in the current cell and the neighboring cell. These computed values of handoff drop probabilities for candidate calls in the neighboring cell are stored in a table. Using simulation (we use the term "simulation" for Monte-Carlo simulation in this thesis) study we determine the thresholds for accepting a new call for exponentially distributed new call inter-arrival time, call holding duration and cell residence time to provide the required QoS of handoff failure probabilities. Our method is designed for off-line computation since our schemes are mainly suitable for designing call admission controller whereas other distributed schemes can be considered as operational schemes as they use online computation. Our call admission control algorithm needs to look up a table for the handoff dropping probability of the candidate call being dropped for unavailability of a free channel in the neighboring cell if the candidate call is accepted. After obtaining the dropping probability of the candidate call and comparing this probability with the threshold value the candidate call is accepted or rejected in our adaptive call admission control scheme. It is found that our adaptive call admission control scheme for the two-cell wireless network better controls the admission of new calls by keeping lower the new call blocking probability and better channel utilization than the guard channel scheme while maintaining the handoff call dropping probability below some threshold value.

We extend our adaptive call admission control scheme for two-cell wireless network to a cellular wireless network consisting of multiple cells. The neighbors of the target cell are approximated with one cell and the target cell as another cell for the two-cell system. It is found that this approximation better controls the handoff call dropping probability than the guard channel scheme. Another variant of this adaptive scheme is developed which we call fractional adaptive scheme for cellular wireless network consisting of more than two cells. Our schemes for multiple-cell wireless network are found to outperform guard channel scheme in controlling handoff failure probability. Fractional adaptive scheme is found to better control the handoff failure probability than the adaptive scheme.

Our adaptive call admission control scheme for two-cell network for a single class of call is extended to two classes of calls — class 1 and class 2 calls. Class 1 call has upper bounds for handoff failure probability and new call blocking probability. Class 2 call has upper bound on handoff failure probability. We develop a four-dimensional Markov chain for two classes of calls for two-cell system. Transient analysis is carried out to get the probability of dropping of a new call in the neighboring cell if it is accepted in the target cell and needs to handoff. The values of the dropping probabilities of the two classes of calls are stored in two separate tables for different numbers of class 1 and class 2 calls in both cells. Simulation study is used to develop two separate thresholds for class 1 and class 2 calls for providing QoS to class 1 and class 2 calls. Then this scheme for two classes of calls for two-cell network is extended to multiple-cell network.

We also develop an efficient analytical technique for studying guard channel schemes when cell residence time and call holding duration are not restricted to exponential, hyper-exponential, Erlang or hyper-Erlang distributions. There are some models for guard channel schemes which capture general distributions of call holding duration and cell residence time by phase type distributions but are computationally cumbersome to implement. The state-spaces of the Markov chains for those models make the computation intractable. A tractable computational model is developed to analyze guard channel scheme with general cell residence time and call holding duration captured by phase type distributions. Our mathematical model is made computationally tractable by keeping track of the number of calls in different phases of the channel holding time instead of the phase of the channel holding time of individual calls.

The thesis is organized as follows. Some of the most important call admission control schemes are discussed in Chapter 2. The steady state analyses of existing non-priority handoff and cutoff priority system for one-cell and the two-cell system are shown in Chapter 3. Our handoff acceptance based call admission model for the two-cell network and its extension for multiple-cell wireless networks are also presented in Chapter 3. Then handoff acceptance based call admission model for two classes of users for the two-cell network and its extension for multiple-cell wireless networks are developed in Chapter 4. Our proposed model for call admission controller with general cell residence time and general call holding duration are presented in Chapter 5. The thesis is concluded with summary of our work and our future goals in Chapter 6.

Chapter 2

Related Work

Call admission control algorithms have been extensively studied in the literature. Ahmed [1] and Ghaderi and Boutaba [35] provided comprehensive surveys on static and adaptive call admission control schemes. Different handoff priority and nonpriority-based schemes have been proposed in the literature. These schemes can be broadly classified as two groups—guard channel (GC) schemes and predictive schemes. We describe these two types of schemes in Sections 2.1 and 2.2, respectively.

2.1 Guard Channel (GC) Schemes

In guard channel schemes, a number of channels are reserved for handoff calls. There are different guard channel schemes such as cutoff priority schemes, fractional guard channel schemes, rigid division-based schemes, new call bounding schemes and queueing priority schemes. These are described briefly in what follows.

2.1.1 Cutoff Priority Scheme

In this scheme, new calls are accepted when the number of ongoing calls remain below a certain threshold value. When the number of ongoing calls reaches the threshold value new calls are rejected and only handoff calls are accepted. In other words, a number of channels is reserved for handoff calls in the cutoff priority scheme. When a call is completed the channel occupied by that call is returned to the common pool of channels.

In this scheme, if there are c number of channels in a cell then g $(0 < g < c)$ number of channels are reserved for handoff calls. A handoff call is accepted if there is any free channel. On the other hand, a new call is accepted in a cell if the number of calls in that cell is less than $c - g$. Hong and Rappaport [45] first systematically studied cutoff priority scheme which was subsequently followed by many studies. Some of the cutoff priority schemes are only for voice calls [3, 4, 45, 62, 63, 70, 71, 92, 99]. On the other hand, the rest of the cutoff priority schemes are for integration of voice and data calls or integration of multiple types of calls [8, 9, 17, 57, 58, 59, 74, 82, 91, 106, 109].

In cutoff priority scheme, exponential distributions are generally assumed for cell residence time, call holding duration and channel holding time [8, 9, 17, 63, 59, 57, 58, 70, 71, 74, 92, 99, 106, 109]. Different exponential distributions with different mean values for call holding time of new and handoff calls were assumed in [32]. We also use exponential distributions for cell residence time, call holding duration and channel holding time in Chapters 3 and 4 of this thesis. On the other hand, the distributions for call holding duration and cell residence time are not restricted to exponential distribution in some call admission control schemes [3, 4, 45, 62, 82, 91]. Hong and Rappaport [45] considered uniformly distributed velocity for mobile users which translated into non-exponential distribution for cell residence time. They assumed exponentially distributed call holding time and approximated the channel holding time with an equivalent exponential distribution which is not quite realistic. Rappaport [91] analyzed cutoff priority scheme for integration of multiple classes of calls for sum of exponentially distributed cell residence time and exponentially distributed call holding duration. Orlik and Rappaport [82] extends the model [91] to analyze the cutoff priority scheme for integration of different classes of calls for sum of hyper-exponentially distributed cell residence time and call holding duration. Alfa and Li [3, 4] considered general distribution for cell residence time and call holding duration captured by phase type distribution. In another paper, Li and Alfa [62] used general distribution for cell residence time and call holding duration. We also use general distribution for cell residence time and call holding duration captured by phase type distribution for analyzing guard channel scheme in Chapter 5. However, our method requires a substantially less state-space than [3, 4, 62].

Exponential distributions are generally assumed for inter-arrival times of new and handoff calls or independent Poisson arrival processes are assumed for new and handoff calls [8, 9, 17, 63, 57, 58, 59, 62, 70, 71, 74, 92, 99, 106, 109]. Poisson distribution is also used for new call arrival process in this thesis. Conversely, handoff arrival processes from a micro-cell is exactly captured whereas handoff arrivals from a macro-cell is approximated with Poisson Process in the cutoff priority scheme of Rappaport and Hu [92]. In contrast, Markovian arrival process (MAP) is considered for correlated arrival process in Alfa and Li [3, 4]. On the other hand, Markovian handoff arrival process is accurately captured by two-dimensional Markov chain for two-cell wireless networks in Chapters 3 and 4 of this thesis. Alternatively, handoff arrival process is approximated with Poisson distribution in Chapter 5.

Handoff arrival process is usually expressed in terms of new call blocking and handoff failure probabilities, probability of handoff and new call arrival process. However, dependence of handoff arrival process on these aforementioned quantities are not considered in [3, 4, 59].

New call blocking and handoff failure probabilities are the two user-perceived performance measurements which are studied in all the call admission control schemes. We study new call blocking and handoff failure probabilities as well as channel utilization which is service provider-perceived measurement. Some of the cutoff priority schemes in the literature have other performance measurements such as distributions of mean and actual call holding time and new call and handoff call blocking periods. The other performance measurements besides new call blocking and handoff failure probabilities of some cutoff priority schemes are concisely described below. The description is followed by discussion on tools and techniques used for analyzing the schemes.

Alfa and Li [3] obtained distributions and expectations of new call and handoff call blocking periods. In another paper [4], they computed distributions of busy period in orbit of new calls. An orbit is an imaginary space where all retrying calls go into while waiting to make retrials.

Li and Alfa [62] analyzed the distributions and the expectations of actual call holding time of a new call and related conditional distributions and the expectations, as well as the boundary of the mean of the actual call holding time of an incomplete call and a complete call. Later Chao and Li [9] and Li and Chao [63] derived expressions for average actual call connection time for both complete call and incomplete calls, as well as the probability for a call to be incomplete or complete.

Li and Alfa [62] developed modified offered load (MOL) approximations of the blocking probability of new and handoff calls. Mitchell and Sohraby [74] used fixed point approximation (FPA) model of loss networks [50] to derive approximate formulas for call completion probabilities of different classes of calls.

Alfa and Li [4] considered retrial of new calls when the call attempts fail. New calls which arrive to find the number of busy channels over a threshold value go into a space which is called orbit. The retrial time is considered to be of phase type distribution and the retrial continues until the new call gets a channel or leaves the orbit.

A blocked call in a cell is either disconnected from the network or is deemed as a handoff call in a neighboring cell in the scheme of Li and Chao [63]. It was proved that the stationary distribution of this cellular mobile network has a product form for this mechanism. Chao and Li [9] extended the cutoff priority scheme of Li and Chao [63] for multiple classes of calls where a certain number of channels in a cell are reserved for handoff calls of each class.

Ma et al [70] carried out availability analysis of channels in a cell where each channel in a cell can fail and each failed channel can be repaired. Trivedi et al [99] extended the work [70] by developing fast recursive formulas for the loss probabilities of new calls and handoff calls. Algorithms were developed for determining the optimal number of guard channels and the optimal number of channels.

Madan et al [71] developed mobile assisted handoff protocols for IS-136 standard.

Bandwidth adaptations were considered by Chou and Shin [17], Li et al [58], Li and Chao [63] and Zheng et al [109]. Chou and Shin [17] developed a new performance measure, upgrade/degrade frequency (UDF), to analyze the perception of user-perceived QoS.

There are different techniques and tools to solve multi-dimensional Markov chain for analyzing cutoff priority scheme. Li et al [59] used Stochastic Petri Net (SPN) for analyzing their hybrid cutoff priority scheme. Yin et al [106] used the recursive solution technique of Herzog et al [44] to solve the two-dimensional Markov chain. Alfa and Li [3, 4] and Li and Alfa [62] used the technique of Gaver et al [33]. We also use the technique of Gaver et al [33] in this thesis. Transform based methods were used in some analysis such as [58].

2.1.2 Fractional Guard Channel Scheme

In fractional guard channel scheme (FGCS), new calls are accepted with some probabilities. Ramjee et al [90] introduced FGCS for cellular wireless networks supporting only voice calls. Later Cruz-Pérez et al [18] proposed a fractional channel reservation (FCR) scheme which was followed by other schemes [19, 26, 43, 54, 55].

In the FGCS of Ramjee et al [90], a new call is accepted with a probability which depends on the number of busy channels in the cell. It was assumed that each cell has total c channels and a new call is accepted with probability α_i $(0 \leq \alpha_i \leq 1)$ when there are i $(0 \leq i < c)$ busy channels in the cell. Cutoff priority scheme is a special case of FGCS. In cutoff priority scheme, $\alpha_i = 1$ for $0 \leq i \leq c - g - 1$ and $\alpha_i = 0$ for $c - g \leq i \leq c$. Ramjee et al [90] developed another scheme called limited fractional guard channel policy (LFGCP) which accepts new calls when the number of calls is below some threshold $c - g - 1$ and accepts a new call with some probability α $(0 < \alpha \leq 1)$ when the number of ongoing calls is $c - g$ and blocks new calls when the number of ongoing calls is more than $c - g$. Cruz-Pérez et al [18] proposed FCR scheme for mobile communication systems which is the LFGCP of Ramjee et al [90]. In the first thinning scheme of Fang [26], admission control for multiple classes of calls was proposed which used the idea of fractional guard channel policy of Ramjee et al [90]. Similarly, Heredia-Ureta et al [43] developed multiple fractional channel reservation (MFCR) scheme by extending the FCR scheme of Cruz-Pérez et al [18] for multi-service mobile wireless networks to differentiate service and to provide the QoS constraints of different amount of bandwidths, new call blocking probabilities and forced termination probabilities of different types of calls. Cruz-Pérez and Ortigoza-Guerrero [19] proposed a scheme which combines regulated call dropping (RCD) with FCR approach for slow and fast moving users in a cellular wireless network to increase fairness between slow and fast mobile users. Chau et al [10] and Leong et al [54, 55] extended LFGCP for integration of voice/data integration.

Leong et al [54] approximated three-dimensional Markov chain to two-dimensional Markov chain for performance analysis. Chau et al [10] used simulated annealing to obtain optimal control parameters. Heredia-Ureta et al [43] developed heuristic algorithm to determine the optimum numbers of reserved channels to achieve maximum system capacity for MFCR scheme.

Ramjee et al [90], Cruz-Pérez et al [18], Fang [26] and Heredia-Ureta et al [43] did not consider in their analytical models the dependence of handoff call arrival rate on new call arrival rate, cell residence time, call holding duration, number of guard channels and total number of channels in a cell. In this thesis, we use the idea of FGCS [90] to develop fractional adaptive scheme in Chapter 3.

2.1.3 Rigid Division-Based Schemes

In the rigid division based scheme, the channels of a cell are divided into two groups and one group is accessible for both handoff and new calls while the other group is only accessible to handoff calls. In rigid division based schemes a number of channels always remain unavailable to the new calls. On the other hand, in cutoff priority scheme any channel unaccessible to new calls in a cell due to the number of channels occupied in the cell is not below a threshold can become available to new calls when the number of occupied channels in the cell goes below that threshold. Kulavaratharasah and Aghvami [51] introduced three different rigid division-based schemes which differ in the order of access of shared channel pool and handoff-only channels by handoff calls. No detailed analytical model for performance evaluation was provided in [51].

The schemes of Kulavaratharasah and Aghvami [51] was followed by the schemes of Luo et al [69] which is known as measurement-based pre-assignment (MPr) technique. In this scheme, the number of channels reserved for only handoff calls change with periodic measurement. Token buffer mechanism is used for channel allocation to handoff calls. The MPr technique [69] was extended for one type of calls to multiple types of calls where multiple handoff-only channel pools are kept reserved for different types of handoff calls and a common channel pool for different types of new and handoff calls. Expressions were derived for approximate computation of the number of reserved channels for different types of handoff calls for different handoff call acceptance guarantee for different types of calls. The problem with the scheme of Luo et al [69] is that the token buffer system assumes constant token freeing rate which is not justified as channel releasing rate depends on the number of channels occupied by ongoing calls.

2.1.4 New Call Bounding Scheme

In new call bounding scheme or threshold priority policy for channel assignment in cellular networks, new calls are accepted in a cell if there is any free channel and the number of ongoing new calls is below a threshold number. Gavish and Sridhar [34] and Fang and Zhang [32] carried out research on this scheme. Fang [26] later extended this scheme for multiple classes of calls in his second thinning scheme.

The channel holding times of new and handoff calls were assumed to follow exponential distribution with different mean values in the new call bounding scheme of Fang and Zhang [32]. A two dimensional Markov chain was used to analyze the performance of this scheme. Fractional new call bounding scheme was also developed in [32]. In fractional new call bounding scheme, a new call is accepted with probability β_i $(0 \leq \beta_i \leq 1)$ when there is any free channel and the number of ongoing call is i. A method was developed for approximate solution of cutoff priority and fractional guard channel scheme. Fang [26] proposed his second thinning scheme for multiple classes of calls which is an extension of fractional new call bounding scheme [32].

Gavish and Sridhar [34] and Fang and Zhang [32] compared threshold priority policy with cutoff priority policy and non-prioritized handoff scheme with respect to trade-off between new call blocking probability and the forced termination probability. It was found that threshold priority policy is the best among these three policies with respect to the call completion probability.

The schemes of Fang and Zhang [32] and Fang [26] did not consider the dependence of handoff arrival rate on the new call arrival rate, cell residence time, call holding duration and threshold number for new call acceptance in their analysis and simulation.

2.1.5 Queueing Priority (QP) Schemes

In these schemes, when calls cannot be allocated free channels they are queued which reduce the new call blocking or handoff call dropping probabilities. There are different queueing priority schemes for single class of call such as [6, 7, 16, 20, 38, 45, 53, 61, 65, 66, 67, 73, 83, 98, 107]. On the other hand, there are queueing priority schemes for voice/data integration or multiple classes of calls such as [11, 40, 84, 97, 103]. These schemes differ in the queueing of new or handoff or both types of calls.

Guerin [38], Lau and Maric [53], Daigle and Jain [20]and Panoutsopoulos et al [83] used queues for only new calls. Hong and Rappaport [45], Lin et al [66, 67], Li and Alfa [61], Li et al [65] and Tekinay and Jabbari [98] proposed buffers for only handoff calls. Chang et al [6, 7], McMillan [73], Choi and Sohraby [16] and Yoon and Un [107] considered queueing for both new and handoff calls.

First in First out (FIFO) queueing discipline was generally considered for queueing calls. Last in First out (LIFO) queueing mechanism was used for new calls in [107] and same queue was used for queueing new and handoff calls. Handoff calls were queued in FIFO manner with preemptive priority over queued new calls in [107]. Tekinay and Jabbari [98] introduced measurement based priority scheme (MBPS) for queueing handoff calls by prioritizing queued handoff calls depending on the received power level from the base station. It was shown that MBPS is better than FIFO discipline for queueing handoff calls in terms of call completion probability in [66, 98].

Lau and Maric [53] analyzed a model for queueing of new calls like Guerin [38]. However, Lau and Maric [53] considered the effects of the mobility of queued call requests in their model. In this scheme [53], queued new calls can handoff to neighboring cells which are considered as new calls in the neighboring cell.

Lin et al [68] and Li and Alfa [61] used splitting of channels for handoff calls to decrease handoff dropping probability. MAP arrival process was considered for new and handoff calls in [61]. Different exponentially distributed channel holding times for original channel and splitted channel and exponentially distributed degradation time during handoff process were assumed. New call blocking probability, forced termination probability of a handoff call, and the mean dwell time of a handoff call in the buffer were analyzed. The expressions for the busy time period of the cell, the first time to split the c-th channel and the total carried traffic were derived. Li et al [65] extended the scheme of Li and Alfa [61] by using a threshold for rejecting new calls.

Hysteresis control mechanism was used in [16, 73]. Reneging of only queued handoff calls were considered in [73] and queued both new and handoff calls were considered in [16].

Chang et al [6] analyzed a hierarchical cellular system where a number of micro-cells are placed inside a macro-cell in high-traffic areas to enhance system capacity. On the other hand, macro-cells cover all of the territory to provide service in low-traffic areas and to provide channels for calls overflowing from the overlaid micro-cells. Reneging of queued calls were used in [6, 7].

Preemptive priority was used in [11, 97]. The scheme of Chen et al [11] is a hybrid of complete sharing and complete partitioning scheme whereas the scheme of Tang and Li [97] is a complete sharing scheme. Bandwidth adaptation was used for data calls in [97].

Finite buffer were proposed for handoff data calls in [84] and for both new and handoff data calls in [103]. The scheme of Haung et al [40] can be considered as an extension of the movable boundary scheme for voice/data integration of Wieselthier and Ephremides [100] or as an extension of hybrid of complete sharing (CS) and complete partitioning (CP) schemes studied by Epstein and Schwartz [22].

Dependence of handoff arrival process on new call arrival process was not considered in [20, 38, 40, 61, 65, 97].

2.2 Predictive Schemes

In predictive schemes, predictions are made to set thresholds on the number of users that can be admitted to a cell so that the current calls handing off in the near future will be ensured of having channels available to them. Naghshineh and Schwartz [79] developed a distributed call admission control (DCAC) scheme which is the first notable predictive scheme. This scheme was followed by many other predictive schemes. Some of these predictive schemes are for single class of users such as DCAC [79], generalized distributed call admission control (gDCAC) and mobility-aware distributed call admission control (mDCAC) of Jiang et al [49], probability index scheme and the hybrid control scheme of Peha and Sutivong [85], event based resource estimation technique [76], predictive channel reservation (PCR) schemes of Chiu and Bassiouni [12], stable dynamic call admission (SDCA) of Wu et al [104] and one step prediction (OSPRED) algorithm of Epstein [21]. The rest of the predictive algorithms are for integration of voice/data or for multiple classes of calls [2, 5, 14, 15, 21, 23, 46, 77, 75, 52, 56, 81, 108]. These schemes also differ in the computation of dropping probabilities of accepted calls.

Time is divided into control periods in DCAC and all schemes based on DCAC such as [23, 36, 49, 52]. Wu et al [104] also divided time into control periods. At the start of every control period each neighboring cell exchanges information about the total number of calls. In [23, 36, 49, 52, 79], a new call is accepted in a cell after computing overload (the incident of handoff or new call arrivals when all the channels of a cell are occupied) probabilities of current and neighboring cells in some time ahead. If the overload probabilities are below certain threshold value the new call is accepted. The overload probabilities of cells determine the probabilities of dropping handoff calls or blocking of new calls in a time period. The overload probability of a cell is computed by approximating binomially distributed number of calls in a cell by normal distribution.

gDCAC [49] generalizes the arrival process of new calls of DCAC. mDCAC [49] considers the difference of handoff support between low and high mobility calls in making the call admission control decision in order to improve channel utilization. How low and high mobility calls are identified for mDCAC scheme is not clear. gDCAC and mDCAC compute the cell overload probability in the same manner as the cell overload probability of the DCAC scheme [79] but differ in the computation of the probability of the presence in the current cell and handoff to the neighboring cell of an ongoing call.

OSPRED, multimedia one step prediction (MMOSPRED), independent multi-class one step prediction complete sharing variant (IMOSP-CS) and independent multi-class one step prediction reservation variant (IMOSP-RES) algorithms [23] were developed based on the DCAC scheme [79]. In IMOSP-CS algorithm handoff users completely share the available bandwidth while in the IMOSP-RES algorithm the handoff reservation partitions are dynamically changed on predictive demand to ensure that the dropping probabilities of each class are met independently.

Kwon et al [52] extended the DCAC scheme of Naghshineh and Schwartz for adaptive bandwidth allocation and call admission control of wireless networks. The problem with the scheme of Naghshineh and Schwartz [79] and all predictive schemes based on this scheme is that the approximation of binomial distribution by normal distribution may cause inaccuracy as normal distribution has negative tail.

In the SDCA scheme of Wu et al [104], at the start of each control period, the number of calls and the number of calls admitted in the previous control period are exchanged among immediate neighbors, second nearest neighbors and third nearest neighbors. The admission policies in SDCA and extended fractional guard channel (EFGC) scheme of Ghaderi and Baoutaba [36] are stochastic which spreads new call arrivals evenly over a control period. Call accepted in a control period with some probability can result in the under utilization of the channels in a cell. EFGC was developed for voice/data whereas SDCA was developed for single class of calls. In SDCA, the expression for the time-dependent call dropping probability was computed by the diffusion approximation of the channel occupancy to get the value of acceptance ratio in the cellular wireless network. Li et al [60] extended the SDCA scheme [104] by estimating dynamically the changing traffic parameters such as new call arrival rate, handoff arrival rate and the channel occupancy in each cell through local on-line estimation.

The probability index scheme of Peha and Sutivong [85] computed a probability index using transient analysis of the number of ongoing calls in a cell and the cell transition matrix. We also use transient analysis for our adaptive schemes in this thesis but our transient analysis is carried out on the number of ongoing calls in two cells.

Epstein [21] developed multi-media dynamic reservation (MMDR) algorithm. The MMDR algorithm is a completely distributed measurement-based algorithm which does not require any information from the neighboring cells. In this algorithm, the bandwidth is divided into different partitions for different classes new and handoff calls. The partitions are adjusted as a result of comparing measured new call blocking blocking and handoff call dropping probabilities to the QoS requirements specified at the outset.

Mišić et al [76] proposed an adaptive bandwidth reservation technique by modeling the number of probable handoff arrivals from the surrounding cells for the lifetime of a call. The binomially distributed bandwidth requirement for handoff calls from neighboring cells was approximated with Poisson distribution. The bandwidth requirement was computed by collecting information from the base stations in the surrounding rings with update at handoff and call termination events. These computations involve simple arithmetic operations only. Mišić et al extended their scheme for single class of calls [76] to multiple classes of calls [77]. Mišić and Bun [75] analyzed the performance of the scheme of Mišić et al [76] in presence of nonuniform new call arrival rate for single and multiple cells. Mišić and Tam [78] analyzed the performance of the scheme of Mišić et al [76] in presence of nonuniform mobility of users and nonuniform new call arrival rate. The approximation of the binomial distribution by Poisson distribution used in these schemes is unrealistic when the mean and variance of the binomial distribution differ more than ten percent. This inaccurate approximation may cause inaccurate prediction of bandwidth requirements for the schemes of Mišić et al [76, 77], Mišić and Bun [75] and Mišić and Tam [78].

Levine et al [56] argued that every mobile terminal with an active wireless connection exerts an influence upon the cells and their base stations in the vicinity of its current location and along its direction of travel and the area of influence moves with the movement of the active mobile terminal. The base stations and their cells currently being influenced by a mobile terminal are said to form a shadow cluster, because the region of influence follows the movements of the active mobile terminal like a shadow. The shadow clusters of active mobile terminals were used to estimate future resource requirements and to perform call admission decisions in wireless networks based on QoS requirements and local traffic conditions in [56]. Islam and Murshed [46] also developed a call admission control scheme based on tracking the position, direction, and speed of a mobile unit together with the duration of a particular call to accurately estimate the cell visiting probability in order to identify a shadow cluster of cells the unit is most likely to visit. Aljadhai and Znati [5] introduced call admission control based on mobility prediction. They developed most likely cluster of cells a user can visit like shadow cluster concept based on prediction. Yu and Leung [108] statistically predicted user mobility based on the mobility history of users for call admission control. Akyildiz and Wang [2] introduced user mobility profile (UMP) which is a combination of historic records and predictive patterns of the movement of mobile terminals. UMP helps to keep track of the mobile terminals and UMP and shadow cluster concept can be used for call admission control in wireless networks. The problem with the schemes of Levine et al [56], Islam and Murshed [46], Aljadhai and Znati [5], Yu and Leung [108] and Akyildiz and Wang [2] is that the accurate tracking of the trajectory of mobile users is complex.

Oliveira et al [81] and Choi and Shin [14, 15] proposed predictive admission control schemes for accepting and allocating bandwidth to multiple classes of calls. Their schemes allocate bandwidth to a call in the cell where the call request originates and reserves bandwidth in all neighboring cells. The amount of bandwidth to reserve is dynamically adjusted, reflecting the current network conditions.

The schemes of Choi and Shin [15] and Soh and Kim [94] proposed to utilize the path/location information available from the car navigation system or global positioning system (GPS). In another paper, Soh and Kim [95], incorporated road topology information into the prediction technique to yield better prediction accuracy for mobile terminals that are carried in vehicles. However, the techniques proposed in [94, 95] are too complex to implement in wireless networks.

In the PCR schemes of Chiu and Bassiouni [12] and the predictive schemes of Xu et al [105], reservation requests are sent to neighboring cells based on extrapolating the motion of mobile telephones. Use of reservation pooling, queuing of reservation requests, hybrid approach for integration of guard channels and a threshold distance to control the timing of reservation requests were their suggested enhancements to minimize the effect of false reservations and to improve the throughput of the cellular system. The schemes of Chiu and Bassiouni [12] and Xu et al [105] suffer from the complexity of tracking and extrapolating the motion of mobile telephones.

We also develop distributed schemes which we call adaptive schemes in Chapters 3 and 4 for a two-cell wireless network and then extend those schemes for a multiple-cell network. The handoff arrival processes are exactly captured in our adaptive schemes for two-cell network which result in better performance.

Chapter 3

Model for Call Acceptance Based on Handoff Guarantees

In this chapter, we study and develop models for cellular wireless network consisting of two cells to understand how the two-cell system works with the goal in mind to extend this two-cell system to approximate a cellular network that has more than two cells. However, in the literature, a cell is generally studied to approximate the whole network for cutoff priority scheme as the state-space of the Markov chain needed to model the whole cellular wireless network is exponentially large. Moreover, it is generally assumed that the inter-arrival time of both the new calls and handoff calls to a cell are exponentially distributed. However, Chlebus and Ludwin [13], Rajaratnam and Takawira [88], Sohraby [96], Sidi and Starobinski [93], Hegde [41] and Hegde and Sohraby [42] showed non-Poisson handoff arrival process for Poisson new call arrival process and exponentially distributed cell residence time and call holding duration due to blocking. Hegde and Sohraby [42] analyzed blocking probability using single and multi-dimensional Markov chains for single and multiple cells. Their emphasis was on one and two cells as the computations become complex for larger number of cells. Their model, which captured handoff rate to a cell for multiple cells by cell residence time and the number of ongoing calls in the neighboring cells, was used to analyze blocking probability for bursty traffic. We also use a two-cell model like Hegde and Sohraby [42] but our focus is to develop models for call acceptance based on handoff guarantees. Part of the work presented in this chapter is also reported in [87].

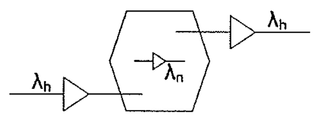

Figure 3.1: Relation between new and handoff call arrival rate in a cell for one-cell system

Before explaining the two-cell system in Section 3.2, for completeness, we visit the one-cell system for non-prioritized handoff and cutoff handoff priority channel schemes in Section 3.1. The steady-state distributions for different number of customers, mean handoff arrival rate, new call blocking probability and handoff call failure probability for the one-cell and the two cell systems are shown in Sections 3.1 and 3.2, respectively. After developing the analytical model for steady-state distribution of the two-cell system, a model is developed for call acceptance based on handoff guarantees in Section 3.3 using transient analysis. Our model for call acceptance based on handoff guarantees is extended from two cells to multiple cells in Section 3.3.2. Some numerical examples are provided in Section 3.4 at the end of this chapter.

In this chapter, the new call arrivals to a cell are assumed to follow Poisson distribution with mean λ_n for both the one-cell and the two-cell systems and handoff arrivals are assumed to follow Poisson distribution with mean λ_h for the one-cell system. Both call holding time and cell residence time are assumed to follow exponential distributions with means $\frac{1}{\mu}$ and $\frac{1}{\eta}$, respectively for both the one cell and the two cell systems and with this assumption it can be shown that the channel holding time is exponentially distributed with mean $\frac{1}{\mu_c} = \frac{1}{\mu+\eta}$ and handoff probability is $P_h = \frac{\eta}{\mu+\eta}$.

3.1 One-Cell System

In this section, the one-cell system is studied where the analysis of a homogenous cellular wireless network is approximated with the analysis of a single cell of that network. In Figure 3.1, the new call and handoff call arrivals to a cell and handoff call generation from a cell to its neighboring cells are shown. The analysis done by Hong and Rappaport [45] for non-prioritized handoff scheme and the cutoff priority guard channel scheme for the one cell system are shown in Sections 3.1.1 and 3.1.2, respectively.

3.1.1 Non-prioritized Handoff Scheme

In the non-prioritized handoff scheme, no channels are reserved for handoff calls. New and handoff calls are treated in the same manner in this scheme. It can be easily shown that for a cell with C channels, the probability of having j $(0 \leq j \leq C)$ calls, based on the M/M/C/C results [37], is

$$P_j = \frac{\frac{(\frac{\lambda}{\mu_c})^j}{j!}}{\sum_{k=0}^{C} \frac{(\frac{\lambda}{\mu_c})^k}{k!}}, \tag{3.1}$$

where $\lambda = \lambda_n + \lambda_h$. When all the C channels of a cell are occupied, any new call is blocked and any incoming handoff call is dropped. Therefore, the new call blocking probability P_b and the handoff call failure probability P_{hf} can be expressed as

$$P_b = P_{hf} = P_C = \frac{\frac{(\frac{\lambda}{\mu_c})^C}{C!}}{\sum_{k=0}^{C} \frac{(\frac{\lambda}{\mu_c})^k}{k!}}. \tag{3.2}$$

Handoff call arrival rate to a cell is λ_h and the handoff call generated from a cell for its neighboring cells is $(\lambda_n(1 - P_b) + \lambda_h(1 - P_{hf}))P_h$. In the steady state, the rate of arrival of handoff calls to a cell and the rate of leaving handoff calls from that cell to the neighboring cells are same. Equating the net rate of calls entering and leaving a cell, we get $\lambda_h = (\lambda_n(1 - P_b) + \lambda_h(1 - P_{hf}))P_h$. Therefore, the handoff call arrival rate λ_h can be expressed as

$$\lambda_h = \frac{\lambda_n P_h(1 - P_C)}{1 - P_h(1 - P_C)}. \tag{3.3}$$

Handoff arrival rate λ_h, probability distribution P_j $(0 \leq j \leq C)$ of j number of calls in a cell, new call blocking probability P_b and handoff failure probability P_{hf} can all be determined by iteratively solving Equations (3.1), (3.2) and (3.3). We can find the mean number of call in a cell for the non-prioritized handoff scheme. The mean number of calls in a cell is $\overline{N} = \sum_{i=1}^{C} iP_i$ which can be computed after getting the probability distribution of the number of ongoing calls in a cell. Then the expected channel utilization can be computed which is the ratio of the mean number of calls in a cell to the total number of channels in a cell. Therefore, the expected channel utilization for two cell system with non-prioritized handoff scheme is $\frac{\overline{N}}{C}$.

3.1.2 Cutoff Priority Guard Channel Handoff Scheme

In this scheme, handoff calls are prioritized over new calls and g channels among total C channels of a cell are reserved for handoff calls in such a way that the new call blocking probability is minimized while keeping the handoff call dropping probability below a threshold value P_{th}. As handoff calls are prioritized over new calls handoff calls are accepted if there is any free channel in the cell and new calls are accepted in this scheme only when the total number of ongoing calls in the cell is less than $C - g = K$. In this scheme, the probability of having j $(0 \leq j \leq C)$ calls in a cell can be expressed as

$$P_j = \begin{cases} \dfrac{\frac{(\frac{\lambda}{\mu_c})^j}{j!}}{\sum_{k=0}^{K} \frac{(\frac{\lambda}{\mu_c})^k}{k!} + \sum_{k=K+1}^{C} \frac{(\lambda)^K (\lambda_h)^{k-K}}{\frac{(\mu_c)^k}{k!}}}, 0 \leq j \leq K \\ \dfrac{\frac{(\lambda)^K (\lambda_h)^{j-K}}{\frac{(\mu_c)^j}{j!}}}{\sum_{k=0}^{K} \frac{(\frac{\lambda}{\mu_c})^k}{k!} + \sum_{k=K+1}^{C} \frac{(\lambda)^K (\lambda_h)^{k-K}}{\frac{(\mu_c)^k}{k!}}}, K < j \leq C \end{cases} \tag{3.4}$$

where $\lambda = \lambda_n + \lambda_h$. In this scheme, a new call is blocked if the number of occupied channels is more than $C - g - 1 = K - 1$ channels. Therefore, the new call blocking probability P_b in this scheme can be stated as

$$P_b = \sum_{k=K}^{C} P_k. \tag{3.5}$$

In this scheme, a handoff call is dropped when all the C channels of a cell are occupied. Therefore, the handoff failure probability P_{hf} can be expressed as

$$P_{hf} = P_C. \tag{3.6}$$

Handoff call arrives to a cell with rate λ_h and handoff call is generated in a cell for its neighbors with rate $(\lambda_n(1 - P_b) + \lambda_h(1 - P_{hf}))P_h$. Equating the handoff call coming to a cell and the outgoing handoff rate from that cell we get $\lambda_h = (\lambda_n(1-P_b)+\lambda_h(1-P_{hf}))P_h$. Therefore, the handoff arrival rate λ_h to a cell can be stated as

$$\lambda_h = \frac{\lambda_n P_h(1 - P_b)}{1 - P_h(1 - P_{hf})}. \tag{3.7}$$

We can find the mean handoff call arrival rate λ_h, new call blocking probability P_b, handoff call dropping probability P_{hf} by solving Equations (3.4), (3.5), (3.6) and (3.7) iteratively for given values of mean new call arrival rate λ_n, mean cell residence time $\frac{1}{\eta}$ and mean call holding time $\frac{1}{\mu}$. Now, the mathematical expression for the cutoff priority guard channel scheme can be written as

$$\begin{gathered}\text{compute } g \text{ which}\\ \text{minimizes } P_b\\ \text{subject to } P_{hf} \leq P_{th}.\end{gathered}$$

The optimum value of g can be found using enumeration to satisfy the above objective function using Equations (3.4), (3.5), (3.6) and (3.7).

The mean number of calls in a cell for cutoff priority guard channel handoff scheme can be computed. The mean number of calls in a cell is $\overline{N} = \sum_{i=1}^{C} iP_i$ and the expected channel utilization for two cell system with non-prioritized handoff scheme is $\frac{\overline{N}}{C}$.

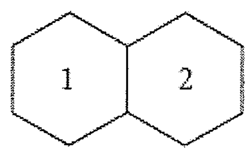

Figure 3.2: The Two-Cell System

3.2 Two-Cell System

In this section, we study a wireless network consisting of two identical cells. We denote the two cells as cells 1 and 2 as shown in Figure 3.2. New calls are generated in cells 1 and 2 and handoff calls can arrive from cell 1 to cell 2 and from cell 2 to cell 1. We assume that there are no external handoff calls from outside of these two cells and calls from these two cells do not handoff to outside. In other words, new and handoff calls are generated, accepted, blocked, dropped and completed within these two cells. We assume that a user with an ongoing call can remain in his current cell or move to the neighboring cell where his call can continue if there is any free channel in the neighboring cell. The new call generation in both cells are assumed to follow Poisson processes with the same mean λ_n, the call duration is exponentially distributed with mean $\frac{1}{\mu}$ and the cell residence times of a call in both cells are exponentially distributed with the same mean $\frac{1}{\eta}$. With this assumption it can be observed that a cell with k ongoing calls can have a call completion (the transition of state for the cell from k ongoing calls to $k-1$ ongoing calls for completion of a call) with rate $k\,\mu$ and handoff to the neighboring cell with rate $k\eta$ and the number of ongoing call k in a cell varies which makes the assumption for Poisson process with fixed mean for the handoff process inaccurate. In the two-cell systems, we show that the handoff arrival does not follow Poisson process or handoff inter-arrival time is not exponentially distributed for exponentially distributed new call inter-arrival times, cell residence time and call holding duration. The handoff arrival process, distribution of the number of ongoing calls in a cell, new call blocking probability, handoff failure probability and channel utilization are analyzed for non-priority handoff scheme and cutoff priority guard channel scheme in the following sections 3.2.1 and 3.2.2, respectively.

3.2.1 Non-prioritized Handoff Scheme for Two-Cell System

In this scheme, a new call or a handoff call is accepted in a cell when there is any free channel. We consider a continuous time Markov chain for this two-cell system based on exponential distribution assumptions for new call inter-arrival time, cell residence time. The state space of this Markov chain is $\mathcal{S} = \{(i,j) : 0 \le i \le C;\ 0 \le j \le C\}$ where i and j represent the total number of ongoing calls in cells 1 and 2, respectively. In this continuous time Markov chain increase or decrease in the number of calls in a cell can be at most one. This Markov chain can have the following types of transitions:

- Increase in the total number of calls in cell 1 due to arrival of a new call to cell 1, i.e., $(i,j) \to (i+1,j)$ with rate λ_n for $0 \le i < C$ and $0 \le j \le C$.
- Increase in the total number of calls in cell 2 due to arrival of a new call to cell 2, i.e., $(i,j) \to (i,j+1)$ with rate λ_n for $0 \le i \le C$ and $0 \le j < C$.
- Increase in the total number of calls in cell 1 and decrease in the total number of calls in cell 2 due to arrival of a handoff call from cell 2 to cell 1, i.e., $(i,j) \to (i+1,j-1)$ with rate $j\eta$ for $0 \le i < C$ and $0 < j \le C$.
- Increase in the total number of calls in cell 2 and decrease in the total number of calls in cell 1 due to arrival of a handoff call from cell 1 to cell 2, i.e., $(i,j) \to (i-1,j+1)$ with rate $i\eta$ for $0 < i \le C$ and $0 \le j < C$.

- Decrease in the total number of calls in cell 1 due to completion of a call in cell 1, i.e., $(i,j) \rightarrow (i-1,j)$ with rate $i\,\mu$ for $0 < i \leq C$ and $0 \leq j < C$.

- Decrease in the total number of calls in cell 2 due to completion of a call in cell 2, i.e., $(i,j) \rightarrow (i,j-1)$ with rate $j\,\mu$ for $0 \leq i < C$ and $0 < j \leq C$.

- Decrease in the total number of calls in cell 1 due to completion of a call in cell 1 or handoff failure for a call from cell 1 to cell 2 as a user with ongoing call moves from cell 1 to cell 2 but all the channels in cell 2 are occupied, i.e., $(i, C) \rightarrow (i-1, C)$ with rate $i\,\mu_c$ for $0 < i \leq C$.

- Decrease in the total number of calls in cell 2 due to completion of a call in cell 2 or handoff failure for a call from cell 2 to cell 1 as a user with ongoing call moves from cell 2 to cell 1 but all the channels in cell 1 are occupied, i.e., $(C,j) \rightarrow (C,j-1)$ with rate $j\,\mu_c$ for $0 < j \leq C$.

An example of a Markov chain for the non-prioritized handoff scheme for a two-cell system for $C = 3$ can be found in Figure 2 of Sidi and Starobinski [93]. In this figure, two-tuple (i,j) of each node represents the state of the system (i.e., the numbers of calls in cells 1 and 2, respectively) and the label on each arrow between any two nodes represents the rate of change from one state to another in the direction of the arrow. The means of new call arrival rate and cell residence time are denoted as λ and $\frac{1}{\gamma}$, respectively in that figure. On the other hand, λ_n and $\frac{1}{\eta}$ are used in this chapter for means of new call arrival rate and cell residence time, respectively.

The generator matrix of a Markov chain for non-prioritized handoff scheme can be represented in the following way

$$Q = \begin{bmatrix} B_0 & U_0 & & & & \\ D_1 & B_1 & U_1 & & & \\ & D_2 & B_2 & U_2 & & \\ & & \ddots & \ddots & \ddots & \\ & & & D_{C-1} & B_{C-1} & U_{C-1} \\ & & & & D_C & B_C \end{bmatrix}. \tag{3.8}$$

Here, B_i, U_i and D_i represent respectively no change, increase by one and decrease by one in the number of ongoing calls in cell 1 when the number of ongoing calls in cell

1 is i. B_i $(0 \leq i \leq C)$, U_i $(0 \leq i \leq C-1)$ and D_i $(1 \leq i \leq C)$ are square matrices of order $C+1$. B_i has nonzero elements in the diagonal, subdiagonal and superdiagonal. The diagonal elements of B_i represent the rate of no change in the number of ongoing calls in both cells 1 and 2. The superdiagonal elements of B_i represent the rate of a new call arrival and acceptance in cell 2. The subdiagonal elements of B_i show the rate of completion of an ongoing call in cell 2. U_i has nonzero elements in the diagonal and subdiagonal. The diagonal of U_i indicate the rate of a new call arrival and acceptance in cell 1. The subdiagonal elements of U_i represent the rate of a successful handoff call from cell 2 to cell 1. D_i has nonzero elements in the diagonal and superdiagonal. The first C diagonal elements of D_i show the rate of a call completion in cell 1 and the last diagonal element of D_i show the sum of the rates of a call completion in cell 1 and a handoff failure from cell 1 to cell 2. The superdiagonal elements of D_i represent the rate of a successful handoff call from cell 1 to cell 2.

The structure of B_i $(0 \leq i \leq C)$ is shown in Table 3.1.

D_i $(1 \leq i \leq C)$ has the following structure

$$D_i = \begin{bmatrix} i\,\mu & i\eta & & & & \\ & i\,\mu & i\eta & & & \\ & & i\,\mu & i\eta & & \\ & & & \ddots & \ddots & \\ & & & & i\,\mu & i\eta \\ & & & & & i\,\mu_c \end{bmatrix}. \tag{3.10}$$

U_i $(0 \leq i \leq C-1)$ has the following structure

$$U_i = \begin{bmatrix} \lambda_n & & & & & \\ \eta & \lambda_n & & & & \\ & 2\eta & \lambda_n & & & \\ & & \ddots & \ddots & & \\ & & & (C-1)\eta & \lambda_n & \\ & & & & C\eta & \lambda_n \end{bmatrix}. \tag{3.11}$$

The generator matrix Q of Equation (3.8) is irreducible for the state-space $\mathcal{S}$. Let us assume that $x = [x_0, x_1, \ldots, x_C]$ is the steady state probability vector for the continuous

Table 3.1: Structure of B_i in Non-prioritized Handoff Scheme for Two Cell System

B_i $(0 \leq i \leq C)$ has the following structure

$$
B_i = \begin{bmatrix}
B_i(0,0) & B_i(0,1) & & & & & \\
B_i(1,0) & B_i(1,1) & B_i(1,2) & & & & \\
& B_i(2,1) & B_i(2,2) & B_i(2,3) & & & \\
& \ddots & \ddots & \ddots & & & \\
& & B_i(k-1,k-2) & B_i(k-1,k-1) & B_i(k-1,K) & & \\
& & & B_i(k,k-1) & B_i(k,k) & B_i(k,k+1) & \\
& & & & \ddots & \ddots & \ddots \\
& & & & B_i(C-2,C-1) & B_i(C-1,C-1) & B_i(C-1,C) \\
& & & & & B_i(C-1,C) & B_i(C,C)
\end{bmatrix}, \tag{3.9}
$$

where $B_i(k,k) = -(2\,\lambda_n + (i+k)\,\mu_c)$ for $(0 \leq i,k \leq C-1)$, $B_i(C,C) = -(\lambda_n + (i+C)\,\mu_c)$ for $(0 \leq i \leq C-1)$, $B_C(k,k) = -(\lambda_n + (k+C)\,\mu_c)$ for $(0 \leq k \leq C-1)$, $B_C(C,C) = -2C\,\mu_c$, $B_i(k,k+1) = \lambda_n$ for $(0 \leq i \leq C, 0 \leq k \leq C-1)$, $B_i(k,k-1) = k\,\mu$ for $(0 \leq i \leq C-1, 1 \leq k \leq C)$ and $B_C(k,k-1) = k\,\mu_c$ for $(1 \leq k \leq C)$.

time Markov chain Q in that equation which satisfies $xQ = \mathbf{0}^T$ and $x\mathbf{1} = 1$ where $\mathbf{0}$ and $\mathbf{1}$ are respectively column vectors of zeros and ones of appropriate dimension. Here, x_i $(0 \leq i \leq C)$ is a row vector having $C+1$ scalar elements and $x_i = [x_{i,0}, x_{i,1}, \ldots, x_{i,C}]$. $x_{i,j}$ represents the steady state probability of having i ongoing calls in cell 1 and j ongoing calls in cell 2. If we let $\prod_{i=1}^{n} \beta_i = \beta_1\beta_2 \ldots \beta_n$ for matrices $\beta_1, \beta_2, \ldots, \beta_n$ we can find the steady-state probability vector x_i $(1 \leq i \leq C)$ using the technique of Gaver et al [33] in the following way

$$x_i = x_0 \prod_{k=1}^{i} (U_{k-1}(-E_k)^{-1}) \quad (1 \leq i \leq C) \tag{3.12}$$

where x_0 satisfies $x_0 E_0 = \mathbf{0}^T$ and

$$x_0(I + \sum_{i=1}^{C} \prod_{k=1}^{i} (U_{k-1}(-E_k)^{-1}))\mathbf{1} = 1. \tag{3.13}$$

Here, E_k $(0 \leq k \leq C)$ is recursively determined by $E_C = B_C$ and

$$E_k = B_k + U_k(-E_{k+1})^{-1} D_{k+1} \quad (0 \leq k \leq C-1). \tag{3.14}$$

When there are i $(0 \leq i \leq C)$ ongoing calls in cell 1 it generates handoff traffic for cell 2 with rate $i\eta$. The probability of having i ongoing calls in cell 1 is $x_i\mathbf{1}$. The handoff rate due to an individual user from a cell to the neighboring cell is η and for l $(1 \leq l \leq C)$ users in a cell the handoff rate to the neighboring cell is $l\eta$. At any time the handoff rate from cell 1 to cell 2 changes with the number of ongoing calls in cell 1 and the handoff rate from cell 2 to cell 1 depends on the number of ongoing calls in cell 2. For exponential distributed inter-arrival time or Poisson distributed arrival process the transition to another state occurs with a fixed rate. However, the handoff arrival process from and to a cell in our two cell system does not remain fixed. Handoff to a cell depends on the number of ongoing calls in the neighboring cell. On the other hand, for hyper-exponential distribution the change from one state to another state can occur with any rate of the fixed number of exponential distributions that constitute the hyper-exponential distribution. For handoff process the number of users that constitute the handoff process does not remain fixed. Moreover, the change between handoff generation rates are correlated. The handoff generation rate can switch from $l\eta$ to $(l-1)\eta$ but not to the rate $(l-2)\eta$ (for $l > 2$). Similarly, the handoff generation rate cannot change directly from rate $l\eta$ to $(l+2)\eta$ (for $l+1 \leq C$). However, hyper-exponentially distributed variables can change from one rate to another of the rates which constitute the hyper-exponential random variable. Therefore, the handoff inter-arrival time from and to a cell in the two-cell system is neither exponentially nor hyper-exponentially distributed. In other words the conclusion is that the handoff call arrival process does not follow a Poisson process with a constant rate when the new call inter-arrival time is exponentially distributed and both the call holding duration and the cell residence time are exponentially distributed for a two-cell system with non-prioritized handoff scheme.

The mean arrival rate $\bar{\lambda}_{h_1}$ from cell 2 to cell 1 can be computed as

$$\bar{\lambda}_{h_1} = \sum_{i=0}^{C} \sum_{j=1}^{C} j\eta x_{i,j}. \tag{3.15}$$

On the other hand, the mean arrival rate $\bar{\lambda}_{h_2}$ for handoff process from cell 1 to cell 2 can be expressed as

$$\bar{\lambda}_{h_2} = \sum_{i=1}^{C} \sum_{j=0}^{C} i\eta x_{i,j}. \tag{3.16}$$

However, the mean arrival rate $\bar{\lambda}_{h_1}$ for handoff process from cell 2 to cell 1 and the mean arrival rate $\bar{\lambda}_{h_1}$ from cell 1 to cell 2 are same as both the cells are identical. Therefore, the mean arrival rate of handoff call to any cell in the two-cell system is $\bar{\lambda}_h = \bar{\lambda}_{h_1} = \bar{\lambda}_{h_2}$.

In this scheme, the handoff failure and the new call blocking probabilities in a cell are not same like the case with non-priority handoff scheme with Poisson handoff arrival process in a cell with a constant rate parameter. In this scheme, a call is handed off from cell 1 to cell 2 if cell 1 has ongoing calls and handoff from cell 1 to cell 2 fails if all the channels of cell 2 are occupied. When there is no ongoing calls in cell 1 there is no possibility of handoff of a call from cell 1 to cell 2. We cannot compute the handoff failure probability in cells 1 and 2 by direct use of Poisson Arrivals See Time Averages (PASTA) [102] as PASTA is used for computation of new call blocking and handoff failure probabilities of a cell in one cell system. We can use the approach of PASTA to compute handoff failure probability of a cell in our two-cell system which is described below.

An outside observer sees in time T the average number of handoff traffic generated from cell 1 to cell 2 as $\sum_{i=1}^{C} i\eta x_i \mathbf{1} T = \sum_{i=1}^{C} \sum_{j=0}^{C} i\eta x_{i,j} T$ and the amount of handoff traffic from cell 2 to cell 1 as $\sum_{i=0}^{C}\sum_{j=1}^{C} j\eta x_{i,j} T$. On the other hand, the outside observer sees in time T the average number of handoff traffic dropped in cell 1 as $\sum_{j=1}^{C} j\eta x_{C,j} T$ and the amount of handoff traffic dropped in cell 2 as $\sum_{i=1}^{C} i\eta x_{i,C} T$. Hence, the handoff failure probability P_{hf_1} in cell 1 can be expressed as

$$P_{hf_1} = \frac{\sum_{j=1}^{C} j x_{C,j}}{\sum_{i=0}^{C}\sum_{j=1}^{C} j x_{i,j}}. \tag{3.17}$$

Similarly, the handoff failure probability P_{hf_2} in cell 2 can be stated as

$$P_{hf_2} = \frac{\sum_{i=1}^{C} i x_{i,C}}{\sum_{i=1}^{C}\sum_{j=0}^{C} i x_{i,j}}. \tag{3.18}$$

A new call is blocked in cell 1 if all the channels of cell 1 are occupied. Therefore, the new call blocking probability P_{b_1} in cell 1 can be expressed as

$$P_{b_1} = x_C \mathbf{1} = \sum_{j=0}^{C} x_{C,j}. \tag{3.19}$$

In the same way the new call blocking probability P_{b_2} in cell 2 can be computed which is

$$P_{b_2} = \sum_{i=0}^{C} x_{i,C}. \tag{3.20}$$

As both cells have same number of channels, same new call generation process and same cell residence time and all the calls follow same call holding time distribution, the handoff failure and the new call blocking probabilities in both cells are same, i.e., $P_{b_1} = P_{b_2} = P_b$ and $P_{hf_1} = P_{hf_2} = P_{hf}$.

We can find the mean number of calls in a cell for the two-cell system with non-prioritized handoff scheme. The mean number of calls in cell 1 is $\overline{N_1} = \sum_{i=1}^{C} i x_i \mathbf{1} = \sum_{i=1}^{C}\sum_{j=0}^{C} i x_{i,j}$ and the mean number of calls in cell 2 is $\overline{N_2} = \sum_{i=0}^{C}\sum_{j=1}^{C} j x_{i,j}$. As both the cells are identical the mean number of calls in both cells are same which is $\overline{N} = \overline{N_1} = \overline{N_2}$. Therefore, the expected channel utilization for the two-cell system with non-prioritized handoff scheme is $\frac{\overline{N}}{C}$.

3.2.2 Cutoff Priority Guard Channel Handoff Scheme for Two-Cell System

In the last section, the non-prioritized handoff scheme for the two-cell system is analyzed. The problem with the non-prioritized handoff scheme is that the call admission controller does not differentiate in the allocation of channels for new and handoff calls when there is any free channel which results in quite similar handoff failure and new call blocking probabilities. A perception of poor QoS is more imminent when a caller's ongoing call is dropped for handoff failure than failure to get a connection when he/she attempts to make a new call. Therefore, handoff calls need to be prioritized over new calls to keep the handoff failure probability significantly lower than the new call blocking probability which can be performed by reserving some channels as guard channels for handoff calls. In this section, we develop and analyze the cutoff priority guard channel handoff scheme for the two cell system. It is assumed that there are two cells both having C channels each. In each cell, g number of channels are reserved as guard channels for only handoff calls. A handoff call is accepted in a cell when there is any free channel in the cell. On the other hand, a new call is accepted in a cell if the number of ongoing calls in that cell is less than $K = C - g$.

Consider a continuous time Markov chain for this two-cell system. The state space of this Markov chain is $\mathbb{S} = \{(i,j) : 0 \leq i \leq C;\ 0 \leq j \leq C\}$ where i and j represent respectively the total number of ongoing calls in cells 1 and 2. The summation of the ongoing calls in both cells cannot be more than $2C - g = C + K$ (i.e. $0 \leq i + j \leq 2C - g$) as there are g number of guard channels in both cells 1 and 2. In this continuous time Markov chain the increase or the decrease in the number of ongoing calls in a cell can be at most one and the following types of transitions can occur

- Increase in the total number of calls in cell 1 due to arrival of a new call to cell 1, i.e., $(i,j) \to (i+1,j)$ with rate λ_n for $0 \leq i < C - g$ and $0 \leq j \leq C$.
- Increase in the total number of calls in cell 2 due to arrival of a new call to cell 2, i.e., $(i,j) \to (i,j+1)$ with rate λ_n for $0 \leq i \leq C$ and $0 \leq j < C - g$.
- Increase in the total number of calls in cell 1 and decrease in the total number of calls in cell 2 due to arrival of a handoff call from cell 2 to cell 1, i.e., $(i,j) \to (i+1,j-1)$

with rate $j\eta$ for $0 \leq i < C$ and $0 \leq j \leq \min(C, 2C - g - i)$.

- Increase in the total number of calls in cell 2 and decrease in the total number of calls in cell 1 due to arrival of a handoff call from cell 1 to cell 2, i.e., $(i,j) \rightarrow (i-1, j+1)$ with rate $i\eta$ for $0 < i \leq C$ and $0 \leq j \leq \min(C-1, 2C-g-i)$.

- Decrease in the total number of calls in cell 1 due to completion of a call in cell 1, i.e., $(i,j) \rightarrow (i-1,j)$ with rate $i\,\mu$ for $0 < i \leq C$ and $0 \leq j \leq \min(C-1, 2C-g-i)$.

- Decrease in the total number of calls in cell 2 due to completion of a call in cell 2, i.e., $(i,j) \rightarrow (i, j-1)$ with rate $j\,\mu$ for $0 \leq i < C$ and $0 < j \leq \min(C, 2C-g-i)$.

- Decrease in the total number of calls in cell 1 due to completion of a call in cell 1 or a handoff failure for a call from cell 1 to cell 2 as a user with ongoing call moves from cell 1 to cell 2 but all the channels in cell 2 are occupied, i.e., $(i, C) \rightarrow (i-1, C)$ with rate $i\,\mu_c$ for $0 < i \leq C - g$.

- Decrease in the total number of calls in cell 2 due to completion of a call in cell 2 or a handoff failure for a call from cell 2 to cell 1 as a user with ongoing call moves from cell 2 to cell 1 but all the channels in cell 1 are occupied, i.e., $(C,j) \rightarrow (C, j-1)$ with rate $j\,\mu_c$ for $0 < j \leq C - g$.

An example of a Markov chain for the guard channel call admission control scheme for a two-cell system is shown in Figure 3.3 where $C = 3$ and $g = 1$. In this figure, two-tuple (i,j) of each node represents the state of the system (i.e., the numbers of calls in cells 1 and 2, respectively) and the label on each arrow between any two nodes represents the rate of change from one state to another in the direction of the arrow. We can represent such Markov chain using the following generator matrix

$$Q = \begin{bmatrix} B_0 & U_0 & & & & \\ D_1 & B_1 & U_1 & & & \\ & D_2 & B_2 & U_2 & & \\ & & \ddots & \ddots & \ddots & \\ & & & D_{C-1} & B_{C-1} & U_{C-1} \\ & & & & D_C & B_C \end{bmatrix}. \tag{3.21}$$

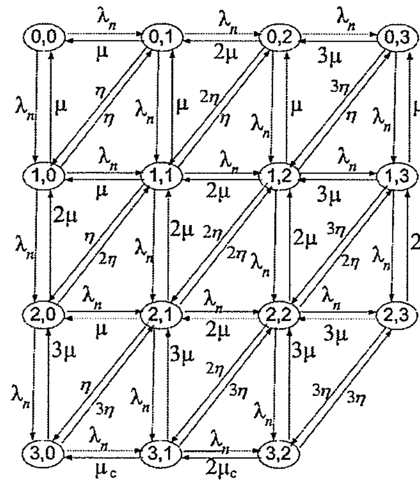

Figure 3.3: An example of a Markov chain for the guard channel call admission control scheme for a two-cell system

Here, B_i $(0 \le i \le C)$ are square matrices of order L_i where $L_i = C+1$ for $0 \le i \le K$ and $L_i = 2C - i - g + 1 = C + K - i + 1$ for $K+1 \le i \le C$. D_i $(1 \le i \le C)$ are $L_i \times L_{i-1}$ matrices. U_i $(0 \le i \le C-1)$ are $L_i \times L_{i+1}$ matrices.

The structure of B_i $(0 \le i \le K-1)$ and B_i $(K \le i \le C)$ are shown in Tables 3.2 and 3.3, respectively.

D_i $(1 \le i \le K)$ has the following structure

$$D_i = \begin{bmatrix} i\,\mu & i\eta & & & & \\ & i\,\mu & i\eta & & & \\ & & i\,\mu & i\eta & & \\ & & & \ddots & \ddots & \\ & & & & i\,\mu & i\eta \\ & & & & & i\,\mu_c \end{bmatrix}. \tag{3.24}$$

Table 3.2: Structure of B_i $(0 \le i \le K-1)$ in Cutoff Priority Guard Channel Handoff Scheme for Two Cell System

B_i $(0 \le i \le K-1)$ has the following structure

$$B_i = \begin{bmatrix} B_i(0,0) & B_i(0,1) & & & & & & \\ B_i(1,0) & B_i(1,1) & B_i(1,2) & & & & & \\ & B_i(2,1) & B_i(2,2) & B_i(2,3) & & & & \\ & \ddots & \ddots & \ddots & & & & \\ & & B_i(K-1,K-2) & B_i(K-1,K-1) & B_i(K-1,K) & & & \\ & & & B_i(K,K-1) & B_i(K,K) & & & \\ & & & \ddots & \ddots & & & \\ & & & & B_i(C-1,C-2) & B_i(C-1,C-1) & & \\ & & & & & B_i(C,C-1) & B_i(C,C) & \end{bmatrix}, \tag{3.22}$$

where $B_i(j,j) = -(2\,\lambda_n + (i+j)\,\mu_c)$ for $(0 \le j \le K-1)$, $B_i(j,j) = -(\lambda_n + (i+j)\,\mu_c)$ for $(K \le j \le C)$, $B_i(j,j+1) = \lambda_n$ for $(0 \le j \le K-1)$ and $B_i(j,j-1) = j\,\mu$ for $(1 \le j \le C)$.

Table 3.3: Structure of B_i $(K \leq i \leq C)$ in Cutoff Priority Guard Channel Handoff Scheme for Two Cell System

B_i $(K \leq i \leq C)$ has the following structure

$$B_i = \begin{bmatrix} B_i(0,0) & B_i(0,1) & & & & & & \\ B_i(1,0) & B_i(1,1) & B_i(1,2) & & & & & \\ & B_i(2,1) & B_i(2,2) & B_i(2,3) & & & & \\ & \ddots & \ddots & \ddots & & & & \\ & & B_i(K-1,K-2) & B_i(K-1,K-1) & B_i(K-1,K) & & & \\ & & & B_i(K,K-1) & B_i(K,K) & & & \\ & & & \ddots & \ddots & & & \\ & & & & B_i(L_i-2,L_i-3) & B_i(L_i-2,L_i-2) & & \\ & & & & & B_i(L_i-1,L_i-2) & B_i(L_i-1,L_i-1) \end{bmatrix} \tag{3.23}$$

where $B_i(j,j) = -(\lambda_n + (i+j)\,\mu_c)$ for $(0 \leq j \leq K-1)$, $B_i(j,j) = -(i+j)\,\mu_c$ for $(K \leq j < L_i)$, $B_i(j,j+1) = \lambda_n$ for $(0 \leq j \leq K-1)$, $B_i(j,j-1) = j\,\mu$ for $(K \leq i \leq C-1, 1 \leq j < L_i)$ and $B_C(j,j-1) = j\,\mu_c$ for $(1 \leq j < L_C)$.

D_i $(K+1 \le i \le C)$ has the following structure

$$D_i = \begin{bmatrix} i\,\mu & i\eta & & & & & \\ & i\,\mu & i\eta & & & & \\ & & i\,\mu & i\eta & & & \\ & & & \ddots & \ddots & & \\ & & & & i\,\mu & i\eta & \\ & & & & & i\,\mu & i\eta \end{bmatrix}. \tag{3.25}$$

U_i $(0 \le i \le K-1)$ has the following structure

$$U_i = \begin{bmatrix} \lambda_n & & & & & \\ \eta & \lambda_n & & & & \\ & 2\eta & \lambda_n & & & \\ & & \ddots & \ddots & & \\ & & & (C-1)\eta & \lambda_n & \\ & & & & C\eta & \lambda_n \end{bmatrix}. \tag{3.26}$$

U_i $(K \le i \le C-1)$ has the following structure

$$U_i = \begin{bmatrix} & & & & \\ \eta & & & & \\ & 2\eta & & & \\ & & \ddots & & \\ & & & (L_i-2)\eta & \\ & & & & (L_i-1)\eta \end{bmatrix}. \tag{3.27}$$

The generator matrix Q of Equation (3.21) is irreducible. Let us assume that $x = [x_0, x_1, \ldots, x_C]$ is the steady state probability vector for the continuous time Markov chain Q in that equation which satisfies $xQ = \mathbf{0}^T$ and $x\mathbf{1} = 1$. Here, x_i $(0 \le i \le C)$ is a row vector having L_i scalar elements. $x_i = [x_{i,0}, x_{i,1}, \ldots, x_{i,L_i-1}]$ where $x_{i,j}$ represents the steady state probability of having i ongoing calls in cell 1 and j ongoing calls in cell 2. If we let $\prod_{i=1}^{n} \beta_i = \beta_1\beta_2\ldots\beta_n$ for matrices $\beta_1, \beta_2, \ldots, \beta_n$ then the steady-state probability vector x_i $(1 \le i \le C)$ can be determined by

$$x_i = x_0 \prod_{k=1}^{i} (U_{k-1}(-E_k)^{-1}) \quad (1 \le i \le C) \tag{3.28}$$

where x_0 satisfies $x_0E_0 = \mathbf{0}^T$ and

$$x_0(I + \sum_{i=1}^{C} \prod_{k=1}^{i} (U_{k-1}(-E_k)^{-1}))\mathbf{1} = 1. \tag{3.29}$$

E_k $(0 \leq k \leq C)$ is recursively determined by $E_C = B_C$ and

$$E_k = B_k + U_k(-E_{k+1})^{-1}D_{k+1} \quad (0 \leq k \leq C-1). \tag{3.30}$$

The handoff inter-arrival times from cell 1 to cell 2 and from cell 2 to cell 1 are neither exponentially distributed nor hyper-exponentially distributed like two-cell system with non-prioritized handoff scheme. The handoff generation process from cell 1 to cell 2 occurs with rate $i\eta$ $(0 \leq i \leq C)$ and the handoff generation rate from cell 2 to cell 1 occurs with rate $j\eta$ $(0 \leq j \leq C)$ when cells 1 and 2 have i and j ongoing calls respectively. As both the cells are identical the mean handoff arrival rate $\bar{\lambda}_{h_1}$ from cell 2 to cell 1 and the mean handoff arrival rate $\bar{\lambda}_{h_2}$ from cell 1 to cell 2 are same. In other words, the mean handoff arrival rate to a cell is $\bar{\lambda}_h = \bar{\lambda}_{h_1} = \bar{\lambda}_{h_2}$. Here, the mean handoff arrival rate from cell 2 to cell 1 can be written as

$$\bar{\lambda}_{h_1} = \sum_{j=1}^{C} \sum_{i=0}^{L_j-1} j\eta x_{i,j}. \tag{3.31}$$

On the other hand, the mean handoff arrival rate from cell 1 to cell 2 can be expressed as

$$\bar{\lambda}_{h_2} = \sum_{i=1}^{C} \sum_{j=0}^{L_i} i\eta x_{i,j}. \tag{3.32}$$

The handoff failure probabilities in cells 1 and 2 can be computed using the approach we used for computing handoff failure probabilities in Section 3.2.1. For the duration of time T an outside observer sees average number of handoff calls from cell 1 to cell 2 as $\sum_{i=1}^{C} i\eta x_1\mathbf{1}T = \sum_{i=1}^{C}\sum_{j=0}^{L_i-1} i\eta x_{i,j}T$ and average number of handoff calls from cell 2 to cell 1 as $\sum_{j=1}^{C}\sum_{i=0}^{L_j-1} j\eta x_{i,j}T$. The outside observer also notices for the duration of time T the average number of handoff calls dropped in cell 1 as $\sum_{j=1}^{L_C-1} j\eta x_{C,j}T$ and the average number of handoff calls dropped in cell 2 as $\sum_{i=1}^{L_C-1} i\eta x_{i,C}T$. Therefore, the handoff failure probability P_{hf_1} in cell 1 can be expressed as

$$P_{hf_1} = \frac{\sum_{j=1}^{L_C-1} jx_{C,j}}{\sum_{j=1}^{C}\sum_{i=0}^{L_j-1} jx_{i,j}}. \tag{3.33}$$

In the same way the handoff failure probability in cell 2 can be stated as

$$P_{hf_2} = \frac{\sum_{i=1}^{L_C-1} ix_{i,C}}{\sum_{i=1}^{C}\sum_{j=0}^{L_i-1} ix_{i,j}}. \tag{3.34}$$

Similarly, the new call blocking probabilities P_{b_1} of cell 1 and P_{b_2} of cell 2 can be expressed as

$$P_{b_1} = \sum_{i=K}^{C}\sum_{j=0}^{L_i-1} x_{i,j}. \tag{3.35}$$

$$P_{b_2} = \sum_{j=K}^{C}\sum_{i=0}^{L_j-1} x_{i,j} \tag{3.36}$$

As the two cells are identical with respect to total number of channels, number of guard channels, new call arrival rate, cell residence time and call holding duration, the two cells have the same handoff failure probabilities and the same new call blocking probabilities which we can write as $P_{hf_1} = P_{hf_2} = P_{hf}$ and $P_{b_1} = P_{b_2} = P_b$.

After getting the expressions for new call blocking and handoff failure probabilities, we can compute the optimum number of guard channels g that we require to keep the new call blocking probability minimum and the handoff call failure probability within a threshold. This optimization problem can be written as

$$\begin{gathered}\text{compute } g \text{ which}\\ \text{minimizes } P_b\\ \text{subject to } P_{hf} \leq P_{th}.\end{gathered}$$

Enumeration technique can be used to solve the above optimization problem to compute the optimum value of g.

The mean number of calls in a cell can be obtained for the two-cell system with cutoff priority guard channel handoff scheme. The mean number of calls in cell 1 is $\overline{N_1} = \sum_{i=1}^{C}\sum_{j=0}^{L_i-1} ix_{i,j}$ and the mean number of calls in cell 2 is $\overline{N_2} = \sum_{j=1}^{C}\sum_{i=0}^{L_j-1} jx_{i,j}$. As both the cells are identical the mean number of calls in both the cells are same which we can write as $\overline{N} = \overline{N_1} = \overline{N_2}$. On the other hand, the expected channel utilization for two-cell system with cutoff priority guard channel handoff scheme is $\frac{\overline{N}}{C}$.

3.3 An Adaptive Call Admission Control Scheme for Two-Cell and Multiple-Cell System

In this section, an adaptive call admission control scheme is developed for the two-cell system. In our adaptive scheme we try to ensure, with some probability, that if a call is considered for acceptance in a cell then the call gets a channel in the neighboring cell if it needs to handoff to the neighboring cell. Moreover, we guarantee, with some probability, that the acceptance of the new call in the target cell will not have adverse effect on the handoff call from the neighboring cell to the target cell of the candidate call being dropped below some threshold value. A set of constraints are proposed for acceptance of calls in the two cells of our system which implicitly prioritizes handoff calls over new calls. Let i and j represent the total numbers of calls in cells 1 and 2, respectively. A new call is accepted in cell 1 with this set of rules when the following constraints are satisfied

- There is at least one free channel in cell 1 for accepting the new call, i.e., $i < C$.
- We can guarantee with at least probability $1 - P_E$ that the call will get a free channel in cell 2 if it needs to handoff later.

We can compute the average probability of a call getting a free channel in the neighboring cell if it needs to handoff. Transient analysis can be carried out to compute the average probability $\bar{P}_E(T)$ of dropping a candidate call in cell 2 in T time ahead when it comes to cell 1 for acceptance. If the candidate call is accepted in cell 1 then we have to consider the remaining $C - 1$ channels of cell 1 and all the channels of cell 2. We can consider a continuous time Markov chain for both cells as in Section 3.2.1. However, the state-space of our Markov chain for this adaptive scheme has $C(C+1)$ elements which is different from the $(C+1)^2$-element state-space for the Markov chain of Section 3.2.1. The state space of this Markov chain for our adaptive scheme can be represented as $\{(\hat{i},\hat{j}) : 0 \leq \hat{i} \leq C-1, 0 \leq \hat{j} \leq C\}$ where $\hat{i}$ represents total number of calls in the $C-1$ channels of cell 1 and $\hat{j}$ represent total number of calls in C channels of cell 2.

In this continuous time Markov chain the transitions are similar to the transitions in Section 3.2.2 except that there is no guard channel. The following transitions can occur in this Markov chain

- Increase in the total number of calls in cell 1 due to arrival of a new call to cell 1, i.e., $(\hat{\imath},\hat{\jmath}) \rightarrow (\hat{\imath}+1,\hat{\jmath})$ with rate λ_n for $0 \leq \hat{\imath} < C-1$ and $0 \leq j \leq C$.

- Increase in the total number of calls in cell 2 due to arrival of a new call to cell 2, i.e., $(\hat{\imath},\hat{\jmath}) \rightarrow (\hat{\imath},\hat{\jmath}+1)$ with rate λ_n for $0 \leq \hat{\imath} \leq C-1$ and $0 \leq \hat{\jmath} < C$.

- Increase in the total number of calls in cell 1 and decrease in the total number of calls in cell 2 due to arrival of a handoff call from cell 2 to cell 1, i.e., $(\hat{\imath},\hat{\jmath}) \rightarrow (\hat{\imath}+1,\hat{\jmath}-1)$ with rate $\hat{\jmath}\eta$ for $0 \leq \hat{\imath} < C-1$ and $0 < \hat{\jmath} \leq C$.

- Increase in the total number of calls in cell 2 and decrease in the total number of calls in cell 1 due to arrival of a handoff call from cell 1 to cell 2, i.e., $(\hat{\imath},\hat{\jmath}) \rightarrow (\hat{\imath}-1,\hat{\jmath}+1)$ with rate $\hat{\imath}\eta$ for $0 < \hat{\imath} \leq C-1$ and $0 \leq \hat{\jmath} < C$.

- Decrease in the total number of calls in cell 1 due to completion of a call in cell 1, i.e., $(\hat{\imath},\hat{\jmath}) \rightarrow (\hat{\imath}-1,\hat{\jmath})$ with rate $\hat{\imath}\,\mu$ for $0 < \hat{\imath} \leq C-1$ and $0 \leq \hat{\jmath} < C$.

- Decrease in the total number of calls in cell 2 due to completion of a call in cell 2, i.e., $(\hat{\imath},\hat{\jmath}) \rightarrow (\hat{\imath},\hat{\jmath}-1)$ with rate $\hat{\jmath}\,\mu$ for $0 \leq \hat{\imath} < C-1$ and $0 < \hat{\jmath} \leq C$.

- Decrease in the total number of calls in cell 1 due to completion of a call in cell 1 or handoff failure for a call from cell 1 to cell 2 as a user with ongoing call moves from cell 1 to cell 2 but all the channels in cell 2 are occupied, i.e., $(\hat{\imath}, C) \rightarrow (\hat{\imath}-1, C)$ with rate $\hat{\imath}\,\mu_c$ for $0 < \hat{\imath} \leq C-1$.

- Decrease in the total number of calls in cell 2 due to completion of a call in cell 2 or handoff failure for a call from cell 2 to cell 1 as a user with ongoing call moves from cell 2 to cell 1 but all the channels in cell 1 are occupied, i.e., $(C-1,\hat{\jmath}) \rightarrow (C-1,\hat{\jmath}-1)$ with rate $\hat{\jmath}\,\mu_c$ for $0 < \hat{\jmath} \leq C$.

An example of a Markov chain for admission control scheme for a two-cell system is shown in Figure 3.4 with $C = 3$ when a user attempts a new call in the target cell having at least one channel free. Two-tuple in each node represents the state of the Markov chain where the first number of the tuple represents the number of calls in the two channels of the target cell excluding the free channel which can be allocated to the candidate call. The second number of the two-tuple represents the number of calls in the

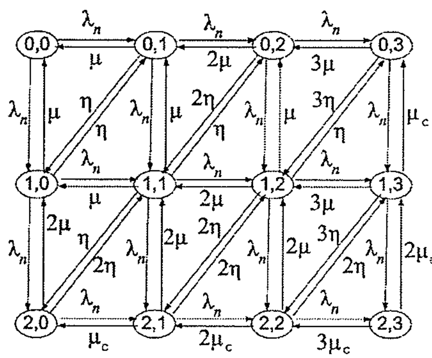

Figure 3.4: An example of a Markov chain for adaptive call admission control scheme for a two-cell system

three channels of the neighboring cell. Each arrow associated with two nodes represents the change from one state to another state of the Markov chain in the direction of the arrow. We can represent such continuous time Markov chain using the following matrix

$$\tilde{Q} = \begin{bmatrix} \tilde{B}_0 & \tilde{U}_0 & & & & \\ \tilde{D}_1 & \tilde{B}_1 & \tilde{U}_1 & & & \\ & \tilde{D}_2 & \tilde{B}_2 & \tilde{U}_2 & & \\ & & \ddots & \ddots & \ddots & \\ & & & \tilde{D}_{C-2} & \tilde{B}_{C-2} & \tilde{U}_{C-2} \\ & & & & \tilde{D}_{C-1} & \tilde{B}_{C-1} \end{bmatrix}. \tag{3.37}$$

Here, $\tilde{B}_{\hat{\imath}}$, $\tilde{U}_{\hat{\imath}}$ and $\tilde{D}_{\hat{\imath}}$ represent respectively no change, increase by one and decrease by one in the number of ongoing calls in $(C-1)$ channels of cell 1 when the number of ongoing calls in $(C-1)$ channels of cell 1 is $\hat{\imath}$. $\tilde{B}_{\hat{\imath}}$ $(0 \leq \hat{\imath} \leq C-1)$, $\tilde{U}_{\hat{\imath}}$ $(0 \leq \hat{\imath} \leq C-2)$ and $\tilde{D}_{\hat{\imath}}$ $(1 \leq \hat{\imath} \leq C-1)$ are square matrices of order $C+1$. $\tilde{B}_{\hat{\imath}}$ has nonzero elements in the diagonal, subdiagonal and superdiagonal. The diagonal elements of $\tilde{B}_i$ represent the rate of no change in the number of ongoing calls in both cells 1 and 2. The superdiagonal elements of $\tilde{B}_{\hat{\imath}}$ represent the rate of a new call connection in cell 2. The subdiagonal elements of $\tilde{B}_{\hat{\imath}}$ show the rate of completion of an ongoing call in cell 2. $\tilde{U}_{\hat{\imath}}$ has nonzero elements in the diagonal and subdiagonal. The diagonal of $\tilde{U}_{\hat{\imath}}$ indicate the rate of a new call connection in cell 1. The subdiagonal elements of $\tilde{U}_{\hat{\imath}}$ represent the rate of a successful handoff call from cell 2 to cell 1. $\tilde{D}_{\hat{\imath}}$ has nonzero elements in the diagonal and superdiagonal. The diagonal elements of $\tilde{D}_{\hat{i}}$ show the rate of a call completion in cell 1. The superdiagonal elements of $\tilde{D}_{\hat{i}}$ represent the rate of a successful handoff call from cell 1 to cell 2.

The structure of $\tilde{B}_{\hat{i}}$ $(0 \leq \hat{i} \leq C-1)$ is shown in Table 3.4.

$\tilde{D}_{\hat{i}}$ $(1 \leq \hat{i} \leq C-1)$ has the following structure

$$\tilde{D}_{\hat{i}} = \begin{bmatrix} \hat{i}\,\mu & \hat{i}\eta & & & & \\ & \hat{i}\,\mu & \hat{i}\eta & & & \\ & & \hat{i}\,\mu & \hat{i}\eta & & \\ & & & \ddots & \ddots & \\ & & & & \hat{i}\,\mu & \hat{i}\eta \\ & & & & & \hat{i}\,\mu_c \end{bmatrix}. \tag{3.39}$$

$\tilde{U}_{\hat{i}}$ $(0 \leq \hat{i} \leq C-2)$ has the following structure

$$\tilde{U}_{\hat{i}} = \begin{bmatrix} \lambda_n & & & & & \\ \eta & \lambda_n & & & & \\ & 2\eta & \lambda_n & & & \\ & & \ddots & \ddots & & \\ & & & (C-1)\eta & \lambda_n & \\ & & & & C\eta & \lambda_n \end{bmatrix}. \tag{3.40}$$

Let $\tilde{x}(0)$ and $\tilde{x}(t)$ be $C(C+1)$-element row-vectors representing the state of the two-cell system at time 0 and t, respectively for $(C-1)$ channels in cell 1 and C channels in cell 2. In the model for admission control, we can express $\tilde{x}_{<r_0 \rightarrow r>}(0,t) = Pr\{\tilde{x}(t) = r \mid \tilde{x}(0) = r_0\}$ as the probability that the system is in state r at time t given the initial state r_0 at time zero. If we consider the system at time t, we have $\tilde{x}_{<r>}(t) = Pr\{\tilde{x}(t) = r\}$ which is conditioned on the initial state. These transient probabilities are the elements of $\tilde{x}(t) = [\tilde{x}_0(t), \tilde{x}_1(t), \ldots, \tilde{x}_{C-1}(t)]$ where $\tilde{x}_{\hat{i}}(t)$ $(0 \leq \hat{i} \leq C-1)$ is a $(C+1)$-element row vector representing the probability vector for total $\hat{i}$ calls in the $C-1$ channels of cell 1 at time t. $\tilde{x}_{\hat{i}}(t)$ can be represented as $\tilde{x}_{\hat{i}}(t) = [\tilde{x}_{\hat{i},0}(t), \tilde{x}_{\hat{i},1}(t), \tilde{x}_{\hat{i},2}(t), \ldots, \tilde{x}_{\hat{i},C}(t)]$ where $\tilde{x}_{\hat{i},\hat{j}}(t)$ $(0 \leq \hat{i} \leq C-1, 0 \leq \hat{j} \leq C)$ represents $\hat{i}$ calls in $C-1$ channels of cell 1 and $\hat{j}$ calls in C channels of cell 2 at time t. To calculate $\tilde{x}(t)$, we have to solve the Kolmogorov-forward equations which can be expressed in the matrix form as follows:

$$\frac{\mathrm{d}\tilde{x}(t)}{\mathrm{d}t} = \tilde{x}(t)\tilde{Q}. \tag{3.41}$$

Table 3.4: Structure of $\tilde{B}_{\hat{\imath}}$ in Adaptive Call Admission Control Scheme for Two Cell System

$\tilde{B}_{\hat{\imath}}$ $(0 \leq \hat{\imath} \leq C-1)$ has the following structure

$$\tilde{B}_{\hat{\imath}} = \begin{bmatrix} \tilde{B}_{\hat{\imath}}(0,0) & \tilde{B}_{\hat{\imath}}(0,1) & & & & & & \\ \tilde{B}_{\hat{\imath}}(1,0) & \tilde{B}_{\hat{\imath}}(1,1) & \tilde{B}_{\hat{\imath}}(1,2) & & & & & \\ & \tilde{B}_{\hat{\imath}}(2,1) & \tilde{B}_{\hat{\imath}}(2,2) & \tilde{B}_{\hat{\imath}}(2,3) & & & & \\ & \ddots & \ddots & \ddots & & & & \\ & & \tilde{B}_{\hat{\imath}}(\hat{\jmath}-1,\hat{\jmath}-2) & \tilde{B}_{\hat{\imath}}(\hat{\jmath}-1,\hat{\jmath}-1) & \tilde{B}_{\hat{\imath}}(\hat{\jmath}-1,\hat{\jmath}) & & & \\ & & & \tilde{B}_{\hat{\imath}}(\hat{\jmath},\hat{\jmath}-1) & \tilde{B}_{\hat{\imath}}(\hat{\jmath},\hat{\jmath}) & \tilde{B}_{\hat{\imath}}(\hat{\jmath},\hat{\jmath}+1) & & \\ & & & & \ddots & \ddots & \ddots & \\ & & & & \tilde{B}_{\hat{\imath}}(C-1,C-2) & \tilde{B}_{\hat{\imath}}(C-1,C-1) & \tilde{B}_{\hat{\imath}}(C-1,C) \\ & & & & & \tilde{B}_{\hat{\imath}}(C,C-1) & \tilde{B}_{\hat{\imath}}(C,C) \end{bmatrix}, \tag{3.38}$$

where $\tilde{B}_{\hat{\imath}}(\hat{\jmath},\hat{\jmath}) = -(2\,\lambda_n + (\hat{\imath}+\hat{\jmath})\,\mu_c)$ for $(0 \leq \hat{\imath} \leq C-2)$ and $(0 \leq \hat{\jmath} \leq C-1)$, $\tilde{B}_{\hat{\imath}}(C,C) = -(\lambda_n + (\hat{\imath}+C)\,\mu_c)$ for $(0 \leq \hat{\imath} \leq C-2)$, $\tilde{B}_{C-1}(\hat{\jmath},\hat{\jmath}) = -(\lambda_n + (\hat{\jmath}+C-1)\,\mu_c)$ for $(0 \leq \hat{\jmath} \leq C-1)$, $\tilde{B}_{C-1}(C,C) = -(2C-1)\,\mu_c$, $\tilde{B}_{\hat{\imath}}(\hat{\jmath},\hat{\jmath}+1) = \lambda_n$ for $(0 \leq \hat{\imath} \leq C-1, 0 \leq \hat{\jmath} \leq C-1)$, $\tilde{B}_{\hat{\imath}}(\hat{\jmath},\hat{\jmath}-1) = \hat{\jmath}\,\mu$ for $(0 \leq \hat{\imath} \leq C-2, 1 \leq \hat{\jmath} \leq C)$ and $\tilde{B}_{C-1}(\hat{\jmath},\hat{\jmath}-1) = \hat{\jmath}\,\mu_c$ for $(1 \leq \hat{\jmath} \leq C)$.

If we solve Equation (3.41) we get

$$\tilde{x}(t) = \tilde{x}(0)e^{\tilde{Q}t}. \tag{3.42}$$

If the candidate call at time 0 sees $\hat{\imath}$ and $\hat{\jmath}$ calls in cells 1 and 2, respectively then $(\hat{\imath}(C+1)+\hat{\jmath})$-th element (with the assumption of counting begins from the starting position as 0-th position) of $\tilde{x}(0)$ is one and all other elements of $\tilde{x}(0)$ are zero. Now, exploiting this special structure of $\tilde{x}(0)$ we can determine $\tilde{x}(t)$ which is the $(\hat{\imath}(C+1)+\hat{\jmath})$-th row of $e^{\tilde{Q}t}$. The term $e^{\tilde{Q}t}$ is not computed using formula of exponential function which is $e^{\tilde{Q}t} = I + \sum_{k=1}^{\infty} \frac{(\tilde{Q}t)^k}{k!}$ because $\tilde{Q}$ has negative elements in the diagonal which makes the computation of $e^{\tilde{Q}t}$ unstable as this computation involves repeated multiplication and addition of negative and positive quantities. Therefore, uniformization technique of Jensen [48] is used for computing $e^{\tilde{Q}t}$ which is a general and efficient way to obtain the solution from the above Equation (3.42) as it does not involve repeated multiplication and addition of negative and positive quantities. In this technique, the new transition probability matrix P is defined as

$$P = \frac{\tilde{Q}}{\Lambda} + I, \tag{3.43}$$

where Λ is not less than the maximum of the absolute values of the diagonal elements of $\tilde{Q}$ matrix. If the arrival rate of new call is greater than the inverse of mean channel holding time (i.e., $\lambda_n \geq \mu_c$) then we use $\Lambda = 2\lambda_n + (2C-3)\mu_c$. On the other hand, if the arrival rate of new call is less than the inverse of mean channel holding time (i.e., $\lambda_n < \mu_c$) then we use $\Lambda = (2C-1)\mu_c$. Now, using Equations (3.42) and (3.43) we get

$$\begin{aligned} \tilde{x}(t) &= \tilde{x}(0)e^{(P-I)\Lambda t} \\ &= \tilde{x}(0)e^{P\Lambda t}e^{-\Lambda t} \\ &= \tilde{x}(0)\textstyle\sum_{n=0}^{\infty} P^n \frac{(\Lambda t)^n}{n!} e^{-\Lambda t}. \end{aligned} \tag{3.44}$$

The infinite summation in Equation (3.44) is truncated at some value $n = M$ such that the truncation error remains below ε which can be expressed mathematically as

$$\tilde{x}(t) = \tilde{x}(0)\sum_{n=0}^{M} P^n \frac{(\Lambda t)^n}{n!} e^{-\Lambda t}, \tag{3.45}$$

where $\sum_{n=M+1}^{\infty} e^{-\Lambda t}\frac{(\Lambda t)^n}{n!} < \varepsilon$.

The probability that an accepted user in cell 1 needs handoff at time t is $(1 - e^{-\eta t})e^{-\mu t}$. Now, the probability of all the channels in cell 2 being occupied at time t is $\sum_{\hat{i}=1}^{C-1} \tilde{x}_{\hat{i},C}(t)$. Therefore, the candidate call accepted at cell 1 will be dropped at time t after its acceptance if, at that moment the call needs to handoff to cell 2, it finds all the channels occupied. This dropping of candidate call accepted at cell 1 occurs with probability $p_e(t) = e^{-\mu t}(1 - e^{-\eta t}) \sum_{\hat{i}=0}^{C-1} \tilde{x}_{\hat{i},C}(t)$. Now, the average probability $\bar{P}_E(T)$ over period T of an accepted call in cell 1 of not finding any empty channel in cell 2 if it requires handoff is

$$\bar{P}_E(T) = \frac{1}{T} \int_0^T p_e(t)\mathrm{d}t. \tag{3.46}$$

Equation (3.46) is the expression for average dropping probability of a candidate call in the neighboring cell. The implementation of this integral expression and its implementation in the two-cell system are described in Section 3.3.1. The extension of this adaptive scheme for multiple-cell wireless network and its variant fractional adaptive scheme are explained in Section 3.3.2.

3.3.1 Implementation of Adaptive Call Admission Control Scheme for Two-Cell System

The integration of Equation (3.46) is computed numerically using Simpson's method of numerical integration. In this method, the range $[0, T]$ is divided into r subintervals of equal length $l = \frac{T}{r}$ where r is an even number. The numerical integration can be expressed as

$$\begin{aligned} \bar{P}_E(T) &= \tfrac{l}{3T}(f_0 + 4f_1 + 2f_2 + 4f_3 + 2f_4 \\ &\quad + \cdots + 2f_{r-2} + 4f_{r-1} + f_r) \\ &= \tfrac{1}{3r}(f_0 + 4f_1 + 2f_2 + 4f_3 + 2f_4 \\ &\quad + \cdots + 2f_{r-2} + 4f_{r-1} + f_r), \end{aligned} \tag{3.47}$$

where $f_k = p_e(\frac{kT}{r})$, $0 \le k \le r$.

There are $C(C+1)$ different values that $\tilde{x}(0)$ can have. For different $C(C+1)$ values of $\tilde{x}(0)$ we can compute values of $\bar{P}_E(T)$. The threshold P_E is one of the $C(C+1)$ values for different initial states of $\tilde{x}(0)$. We can determine this threshold value of P_E for accepting a new call by running simulation experiments with the goal of minimizing the new call

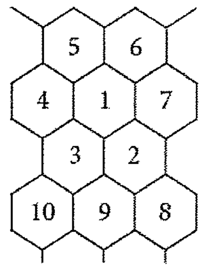

Figure 3.5: Hexagonal structure of cellular wireless network

blocking probability while keeping the handoff failure probability below a prespecified value. The admission controller can keep the table of $\bar{P}_E(T)$ and the threshold values of P_E in memory. Whenever a new call comes to cell 1 then the admission controller needs to check whether the $(\hat{i}(C+1)+\hat{j})$-th entry of the table storing values of $\bar{P}_E(T)$ is not more than P_E. If this condition is satisfied the call is accepted in cell 1. On the other hand, when a call comes to cell 2 it needs to check the $(\hat{j}(C+1)+\hat{i})$-th entry of the table storing values of $\bar{P}_E(T)$ is not more than P_E. If this condition is satisfied the call is accepted in cell 2. We conjecture that the acceptance of a new call in our model guarantees a successful handoff for the accepted call with some probability. Guard channel scheme fails to guarantee this handoff in a cell for different number of calls in the neighboring cells.

The advantage of our model is that we can compute $\bar{P}_E(T)$ offline for different values of $\tilde{x}(0)$ and keep these values in a table. Whenever a new call attempt is made in a cell the call admission control algorithm needs to get the value of $\bar{P}_E(T)$ for decision about acceptance or rejection of the call.

3.3.2 Extension to Multiple-Cell Wireless Network

We can extend our adaptive call admission control scheme for two-cell system to multiple-cell wireless network. Here, each cell contains C channels, new call inter-arrival time to each cell is exponentially distributed with mean λ_n, cell residence time in each cell is exponentially distributed with mean $\frac{1}{\eta}$ and call holding duration is exponentially distributed with mean $\frac{1}{\mu}$. The target cell is approximated with cell 1 of the two-cell system and the number of calls in the target cell is assumed as the number of calls in cell 1 of the two-cell system. On the other hand, the immediate neighboring cells are approximated with cell 2 of the two-cell system and the ceiling of the average number of calls in the immediate neighboring cells is assumed as the number of calls in cell 2 of the two-cell system. In this approximation, let us assume that our target cell F_t has i calls in C channels and the target cell has J immediate neighbors $F_{n_1}, F_{n_2}, \ldots, F_{n_J}$ having $l_{n_1}, l_{n_2}, \ldots, l_{n_J}$ calls respectively. A new call is blocked in the target cell if $i = C$. On the other hand, we can use the values in the table for accepting a call in the target cell for $i < C$ and $j = \lceil \frac{\sum_{k=1}^{J} l_{n_k}}{J} \rceil$. The table contains values for probability of dropping a candidate call in the target cell if it needs to handoff to the neighboring cell. We need to determine the threshold value P_E by carrying out simulation to keep the handoff failure probability below that value. When a call comes to the target cell with at least one free channel we use $\hat{i} = i$ and $\hat{j} = j$ of Section 3.3.1. The admission controller needs to check whether the $(i(C+1)+j)$-th entry of the table storing values of $\bar{P}_E(T)$ is not more than P_E. In lower load the handoff failure probability is very low and needs no prioritization of handoff calls over new calls. As a result, the value of P_E is the highest value of the values of handoff failure probabilities of candidate call in the neighboring cell stored in the table. If this condition is satisfied the call is accepted in the target cell. Otherwise, the new call is blocked. An example of hexagonal cell structure of cellular wireless network is shown in Figure 3.5. If a new call comes to cell 1 then cell 1 is the target cell. If there is at least one free channel in cell 1 then i is the value of the number of calls in cell 1 and the value of j is computed as the ceiling of the average number of calls in the immediate neighboring cells 2, 3, 4, 5, 6 and 7. We show the adaptive call admission control algorithm for target cell in the multiple-cell system using the following pseudo-code.

```
Adaptive Algorithm
if(handoff call)
    if(i < C)
        accept the handoff call;
    else /*if(i = C)*/
        reject the call;
else /* if(new call) */
```

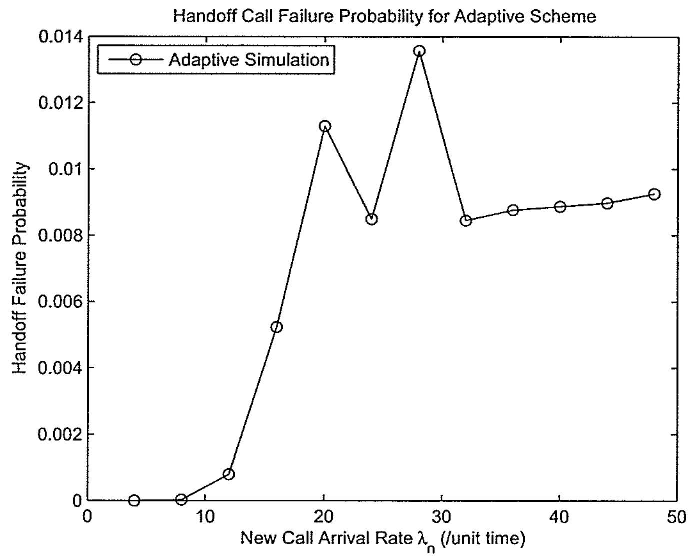

Figure 3.6: Simulation results of handoff call failure probability for Adaptive scheme in nineteen-cell wrap around structure

if($i = C$)
  reject the call
else /* if($i < C$) */
  sum$\leftarrow$ 0;
  for $k \leftarrow 1$ to J do
    sum $\leftarrow$ sum + l_{n_k};
  $j \leftarrow \lceil(\frac{\text{sum}}{J})\rceil$;
  Get $(i(C+1)+j)$-th entry
  $\bar{P}_E(T)$ from table;
  if($\bar{P}_E(T) \leq P_E$)
    accept the call;
  else /* if ($\bar{P}_E(T) > P_E$) */
    reject the call;

End of Adaptive Algorithm

Handoff failure probability remains below upper bound of QoS handoff failure probability for lower new call arrival rates. However, at higher loads the QoS handoff failure probability can be maintained by increasing the new call blocking probability with lower channel utilization. A good admission control scheme tries to satisfy the QoS bound for handoff calls while minimizing new call blocking probability and maximizing channel utilization. An example of adaptive algorithm's not providing the exact desired handoff call failure probability of 0.015 in higher load is shown in Figure 3.6 for $\frac{1}{\mu} = 0.25$ unit time, $\frac{1}{\eta} = 1.0$ unit time and $\lambda_n = 4.0$ to 48.0 calls per unit time. The new call blocking probability could be minimized and channel utilization could be increased if the adaptive algorithm could maintain the handoff failure probability at the desired QoS upper bound of 0.015. Therefore, we develop another variant of this adaptive algorithm which we call fractional adaptive algorithm to maintain the handoff failure probability at the desired QoS upper bound of handoff failure probability at higher new call arrival rates. The approach similar to fractional guard channel scheme [90] is used for developing fractional adaptive algorithm. In this algorithm, we select the threshold value P_E which is the immediate higher value than the threshold value used in adaptive scheme from the table of the adaptive scheme. In the fractional adaptive scheme, a new call is always accepted in a target cell if the $(i(C+1)+j)$-th entry of the table storing values of $\bar{P}_E(T)$ is less than P_E. On the other hand, a new call is accepted with some probability P_{th} when the $(i(C+1)+j)$-th entry of the table storing values of $\bar{P}_E(T)$ is equal to P_E. We can choose the value of P_{th} by simulation in such a way that the handoff failure probability is approximately equal to the desired handoff failure probability value. Adaptive scheme can be considered as a special case of fractional adaptive scheme where $P_{th} = 1$ and P_E is chosen in such a way that handoff failure probability is not more than the desired handoff failure probability. We show the fractional adaptive call admission control algorithm for target cell in the multiple-cell system using the following pseudo-code.

```
Fractional Adaptive Algorithm
if(handoff call)
    if(i < C)
        accept the handoff call;
    else /*if(i = C)*/
        reject the call;
else /* if(new call) */
```

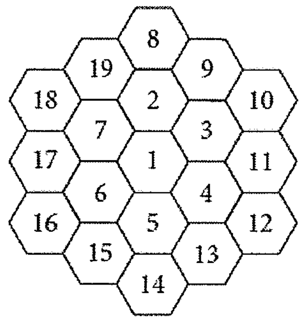

Figure 3.7: Cellular network consisting of nineteen cells with wrap-around structure

if($i = C$)
  reject the call
else /* if($i < C$) */
  sum$\leftarrow$ 0;
  for $k \leftarrow 1$ to J do
    sum $\leftarrow$ sum + l_{n_k};
  $j \leftarrow \lceil(\frac{\text{sum}}{J})\rceil$;
  Get $(i(C+1)+j)$-th entry
  $\bar{P}_E(T)$ from table;
  if($\bar{P}_E(T) < P_E$)
    accept the call;
  else if ($\bar{P}_E(T) = P_E$)
    $u \leftarrow$ rand(0, 1);
    if ($u \leq P_{th}$)
      accept the call;
    else /* if ($u > P_{th}$) */
      reject the call;
  else /* if ($\bar{P}_E(T) > P_E$) */
    reject the call;

End of Fractional Adaptive Algorithm

3.4 Numerical Examples

We carry out analytical and discrete event simulation studies on the two-cell system of Figure 3.2. Simulation studies are also performed on the nineteen-cell (multiple-cell) wireless network of Figure 3.7 with wrap-around structure where each cell has six neighbors (e.g., cell 12 has neighboring cells 4, 11, 13, 17, 18 and 19). In wrap-around cellular networks opposite sides of boundary cells wrap around to eliminate the finite size effect. In the absence of available data in the literature to use for assessing this model, hypothetical values were selected. We assume that there are ten channels in each cell. The cell residence time and call holding duration are assumed to be exponentially distributed with means 1.0 unit time and 0.25 unit time, respectively. New call arrival process is assumed to follow Poisson distribution. Mean call arrival rate is varied from 4.0 calls per unit time to 48.0 calls per unit time. Our goal is to minimize the new call blocking probability while keeping handoff call failure probability below 0.015. We use $T = 5.0$ unit time for our adaptive admission control scheme in numerical examples. 100000.0 unit time is chosen as warm-up time for the simulation. Ten independent replications of length 50000.0 unit time are performed in the simulation studies. We discuss numerical examples for two-cell and multiple-cell wireless networks in Sections 3.4.1 and 3.4.2, respectively which are followed by sensitivity analysis of the results on the value of T in Section 3.4.3. In the figures, we denote analytical and simulation results for non-priority handoff scheme as "Guard0 Analytical" and "Guard0 Simulation", respectively. We denote analytical results for guard channel schemes with one and two guard channels as "Guard1 Analytical" and "Guard2 Analytical", respectively. Similarly, we denote simulation results for guard channel schemes with one and two guard channels as "Guard1 Simulation" and "Guard2 Simulation", respectively. The simulation results for adaptive and fractional adaptive scheme are represented as "Adaptive Simulation" and "Fractional Adaptive Simulation", respectively.

3.4.1 Numerical Result for Two-Cell System

Analytical studies are carried out on non-priority handoff scheme and guard channel scheme with one and two guard channels. Simulation studies are performed on non-priority handoff scheme, guard channel scheme with one and two guard channels and our

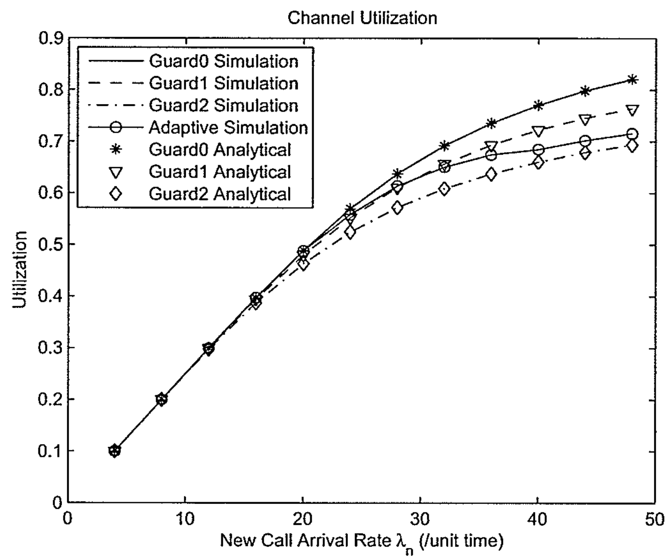

Figure 3.8: Analytical and simulation results of channel utilization for the two-cell system with guard channel scheme

adaptive CAC scheme.

Analytical and simulation results of channel utilization, new call blocking probability and handoff call failure probability are shown in Figures 3.8, 3.9 and 3.10, respectively. We can see that our analytical and simulation results agree well for non-priority handoff scheme and guard channel scheme with one and two guard channels from these figures.

Figure 3.10 shows that non-priority handoff scheme and guard channel scheme with one guard channel fail to keep the handoff failure probability below 0.015 for higher new call arrival rate (28.0 to 48.0 calls per unit time) while they have higher channel utilization (Figure 3.8) and lower new call blocking probability (Figure 3.9). On the other hand, our adaptive CAC scheme and guard channel scheme with two guard channels succeed to keep the handoff failure probability below the required value 0.015. Moreover, our adaptive call admission control scheme has higher channel utilization and lower new call blocking probability than the guard channel scheme with two guard channels.

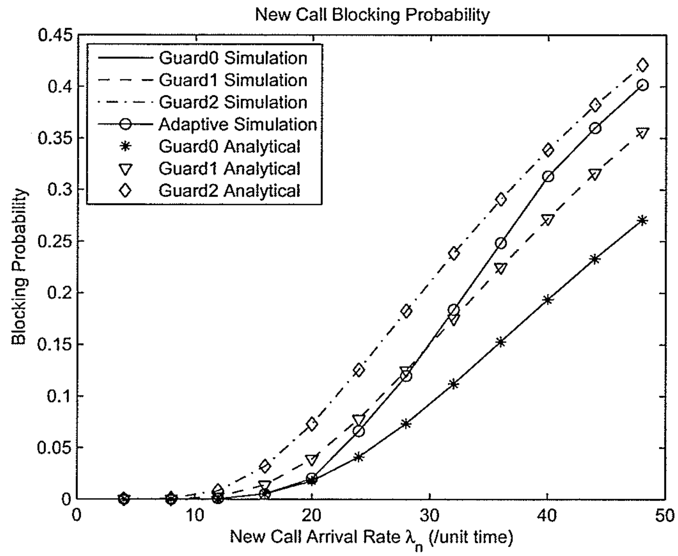

Figure 3.9: Analytical and simulation results of new call blocking probability for the two-cell system with guard channel scheme

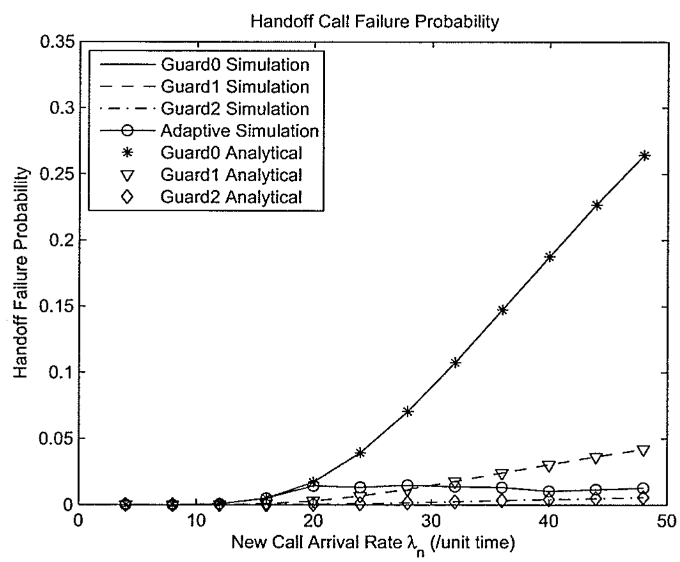

Figure 3.10: Analytical and simulation results of handoff failure probability for the two-cell system with guard channel scheme

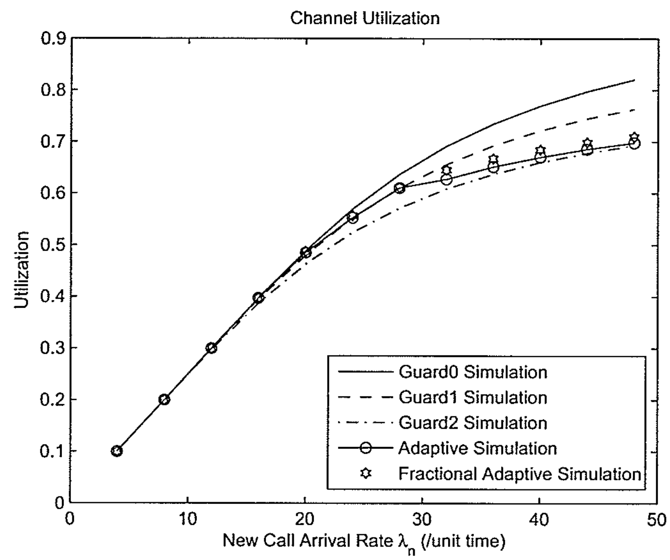

Figure 3.11: Simulation results of channel utilization for nineteen-cell wrap around structure

3.4.2 Numerical Result for Multiple-Cell System

Channel utilization, new call blocking probability and handoff failure probability are studied using simulation for guard channel scheme. The guard channel schemes are with zero, one and two guard channels. Simulation is also carried out for adaptive and fractional adaptive CAC schemes for the wrapped around nineteen-cell wireless network of Figure 3.7. Results of simulation studies for channel utilization, new call blocking and handoff failure probabilities are presented in Figures 3.11, 3.12 and 3.13, respectively.

Figures 3.11 and 3.12 show that for lower new call arrival rates (4.0 to 24.0 calls per unit time), channel utilization and new call blocking probabilities are almost the same for the different schemes. At higher new call arrival rates channel utilization and new call blocking probabilities are largest for non-priority handoff scheme. With respect to channel utilization and new call blocking probabilities the remaining schemes can be ordered in decreasing order as guard channel scheme with one guard channel, fractional adaptive scheme, adaptive scheme and guard channel scheme with two guard channels. Figure 3.13 shows that handoff failure probability is lower than 0.015 for all the schemes in lower loads. However, at higher loads only adaptive, fractional adaptive and guard

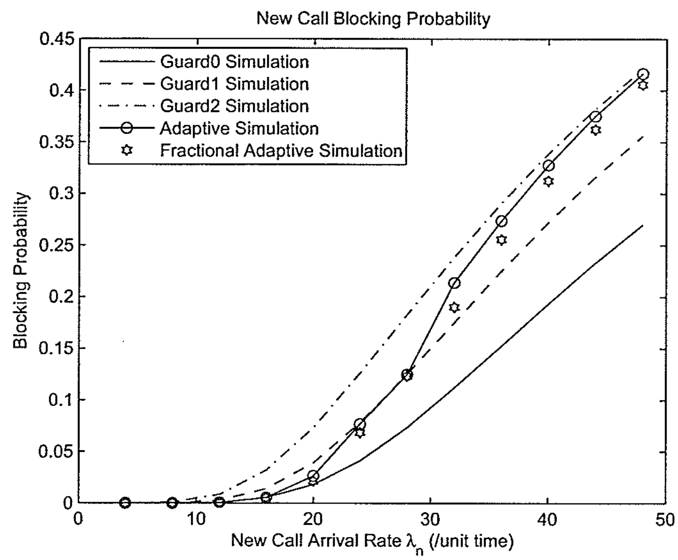

Figure 3.12: Simulation results of new call blocking probability for nineteen-cell wrap around structure

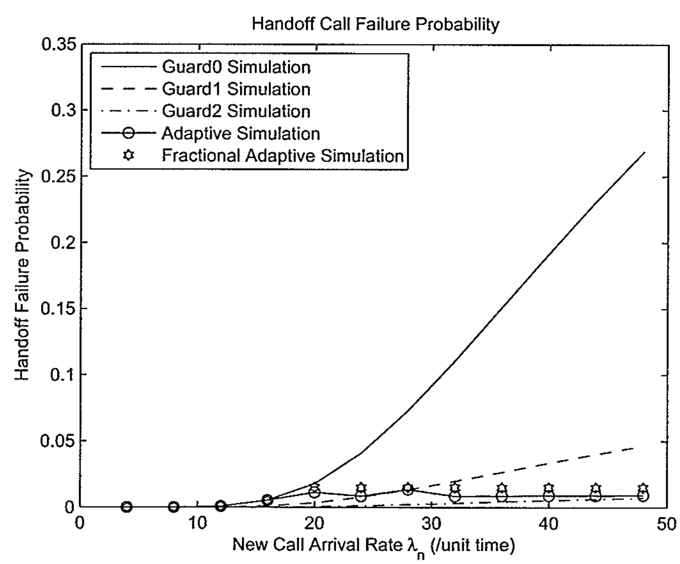

Figure 3.13: Simulation results of handoff call failure probability for nineteen-cell wrap around structure

Table 3.5: Summary of results for nineteen-cell wrap around structure

Schemes	Handoff Failure Probability	New Call Blocking Probability	Channel Utilization	Overall
"Guard0"	"Not Satisfy"	1st	1st	–
"Guard1"	"Not Satisfy"	2nd	2nd	–
"Guard2"	"Satisfy"	5th	5th	3rd
"Adaptive"	"Satisfy"	4th	4th	2nd
"Fractional" "Adaptive"	"Satisfy"	3rd	3rd	1st

channel scheme with two guard channels can keep the handoff failure probability below 0.015. We can see from Figure 3.13 that the handoff failure probability can be kept quite close to 0.015 by adaptive scheme at higher loads. On the other hand, fractional adaptive scheme keeps the handoff failure probability almost equal to 0.015. It is clear that at higher loads adaptive and fractional adaptive schemes better control handoff failure probability than the guard channel scheme, and fractional adaptive scheme is better than adaptive scheme in controlling the handoff failure probability to a threshold value.

The results for higher new call arrival rates (in the range of 28.0 to 48.00 calls per unit time) are summarized in Table 3.5. "Satisfy" and "Not Satisfy" are used to represent the objective view whether a scheme satisfies the QoS handoff failure probability. Ranking of the schemes are done depending on the measurements of two performance metrics of each scheme — new call blocking probability and channel utilization. Lower new call blocking probability and higher channel utilization are perceived better. An overall ranking is shown in the last column of Table 3.5 depending on the satisfaction of QoS handoff failure probability, lower new call blocking probability and higher channel utilization.

3.4.3 Sensitivity of Numerical Examples on the Values of T

$T = \frac{5}{\eta}$ is used in the numerical examples presented in Sections 3.4.1 and 3.4.2. In this section, the sensitivity of performance metrics with respect the value of T for adaptive

Table 3.6: Sensitivity Analysis for twenty-four calls per unit time new call arrival rate

$T\eta$	P_b	P_{hf}	Utilization
1.0	0.066265	0.0133	0.558371
1.5	0.066265	0.0133	0.558371
2.0	0.066265	0.0133	0.558371
2.5	0.066265	0.0133	0.558371
3.0	0.066265	0.0133	0.558371
3.5	0.066265	0.0133	0.558371
4.0	0.066265	0.0133	0.558371
4.5	0.066265	0.0133	0.558371
5.0	0.066265	0.0133	0.558371
5.5	0.066265	0.0133	0.558371
6.0	0.066265	0.0133	0.558371
6.5	0.066265	0.0133	0.558371
7.0	0.066265	0.0133	0.558371

scheme in the two-cell system. The time period T is varied from $\frac{1}{\eta}$ to $\frac{7}{\eta}$ in step of $\frac{1}{2\eta}$ for $\frac{1}{\eta} = 1.0$ unit time, $\frac{1}{\mu} = 4.0$ unit time and $C = 10$. We use the value of a particular index of offline tables for new call acceptance. Two sets of simulation experiments are carried out for two different values of mean new call arrival rates — $\lambda_n = 24.0$ calls per unit time and $\lambda_n = 32.0$ calls per unit time. The threshold value for $\lambda_n = 24.0$ calls per unit time is chosen as the dropping probability in the offline table at index one hundred and three whereas he threshold value for $\lambda_n = 32.0$ calls per unit time is chosen as the dropping probability in the offline table at index ninety-six. The results of these two sets of experiments are presented in Tables 3.6 and 3.7. In these tables, $T\eta$ denotes the ratio of time period T to mean cell residence time $\frac{1}{\eta}$, P_b denotes new call blocking probability, P_{hf} denotes handoff call failure probability and "Utilization" denotes channel utilization in a cell. Table 3.6 shows no change in the results for the values of T from $\frac{1}{\eta}$ to $\frac{7}{\eta}$. On the other hand, Table 3.7 shows no change in the results for the values of T from $\frac{1}{\eta}$ to $\frac{5.5}{\eta}$. However, Table 3.7 presents change in performance for the value of T from $\frac{6.0}{\eta}$ to $\frac{7.0}{\eta}$. The

Table 3.7: Sensitivity Analysis for thirty-two calls per unit time new call arrival rate

$T\eta_1$	P_b	P_{hf}	Utilization
1.0	0.184203	0.013797	0.650461
1.5	0.184203	0.013797	0.650461
2.0	0.184203	0.013797	0.650461
2.5	0.184203	0.013797	0.650461
3.0	0.184203	0.013797	0.650461
3.5	0.184203	0.013797	0.650461
4.0	0.184203	0.013797	0.650461
4.5	0.184203	0.013797	0.650461
5.0	0.18416	0.013782	0.650543
5.5	0.18416	0.013782	0.650543
6.0	0.183882	0.013877	0.650659
6.5	0.183882	0.013877	0.650659
7.0	0.182871	0.014531	0.651425

change in result can be explained from the computation of tables of average dropping probabilities. In these tables, same weight is given to dropping probabilities of different time durations. However, the instantaneous probability of handoff $(1 - e^{-\eta t})e^{-\mu t}$ at time t to neighboring cell varies on time t. Moreover, the term $(1 - e^{-\eta t})e^{-\mu t}$ becomes zero when t is zero or infinity. On the other hand, the term $(1 - e^{-\eta t})e^{-\mu t}$ has positive value for $0 < t < \infty$. This variation of instantaneous handoff dropping probability can impact the computation of the average handoff dropping probability over period T using Equation (3.46) for different values of T. Another factor that impacts the accuracy of handoff dropping probabilities is the value of Λ. Higher values of T and Λ can result in severe roundoff errors. At lower values of Λ and T the elements in an offline table for dropping probabilities has a particular ordering of values stored in different indices which changes at higher values of Λ and T. Therefore, Λ should be chosen as the maximum of the absolute values of the diagonal elements of the generator matrix of Equation (3.37) and choice of the value of T should be carefully performed.

Chapter 4

Call Admission Control Scheme for Two Classes of Users

Data and multi-media services are now supported along with voice in existing 2G and emerging 3G and 4G cellular wireless networks like wired networks. However, bandwidth and the number of channels in a cellular wireless network are limited whereas bandwidth is relatively cheap in wired networks and it is relatively cheap to overprovision in wired networks for operation in low congestion region. Therefore, supporting different classes of calls such as voice, data and multimedia in cellular wireless networks is challenging as the limited number of channels in wireless link need to be allocated properly to satisfy quality of service (QoS) requirements of each class of traffic while the channels are utilized as efficiently as possible. Handoff of ongoing calls for mobility of users makes the channel allocation even more complex than wired networks. Therefore, the design of a call admission control scheme for multi-service wireless networks becomes even more challenging for satisfying different QoS requirements for different classes of calls with limited number of channels. In this chapter, an adaptive call admission control scheme is developed for two classes of users. This scheme can be considered as an extension of the adaptive call admission control scheme for single class of call of Chapter 3.

There are different call admission control schemes for multiple classes of users which can be broadly classified as static and predictive schemes. In static schemes [9, 10, 11, 19, 22, 26, 40, 43, 54, 55, 64, 57, 58, 59, 69, 74, 97, 100, 103, 106, 109], the numbers of different classes of calls in a cell are considered for call acceptance in that cell. On the other hand, in the predictive schemes [2, 5, 21, 23, 36, 46, 77, 49, 56, 81, 108], calls are accepted based on the numbers of different classes of calls in the target cell and the neighboring cells. In static schemes, a cell is studied to approximate the whole cellular network and these schemes suffer from the approximation of handoff arrival process as Poisson process. Conversely in predictive schemes, the number of calls in the target cell and in the neighboring cells are considered for call acceptance. Predictive schemes suffer either from approximation of number of handoff arrivals in a cell as normal [21, 23, 36] or Poisson [77] distribution or from complex tracking of the mobility of users [2, 46, 56]. In this chapter, we first study a cellular wireless network consisting of two cells for two classes of users. The two-cell wireless network is studied to understand how the two-cell system works with the goal in mind to use the results in approximating a cellular network consisting of more than two cells. We propose an adaptive call admission control scheme for two classes of calls with different QoS requirements.

The two classes of calls considered in this chapter are termed as class 1 and class 2 calls. Class 1 calls have upper bounds for handoff failure probability and new call blocking probability and class 2 calls only have upper bound on handoff failure probability. At first non-prioritized and dual threshold reservation (DTR) schemes are analyzed in this chapter for the two-cell system for class 1 and class 2 calls. Then an adaptive call admission control scheme is developed using the modeling approach for non-prioritized scheme to support the two types of calls with above mentioned QoS requirements. In this adaptive scheme, both class 1 and class 2 handoff calls are accepted whenever a handoff call arrives and there is at least one free channel. On the other hand, when a new class 1 or class 2 call arrives we get the probability of dropping of the call in the neighboring cell if we predict that it needs to handoff based on two tables which are computed off-line using transient analysis. A new class 1 or class 2 call is admitted in our scheme if this probability is not more than some threshold value. In other words, our scheme accepts a new call in a cell if a guarantee can be provided with some probability that the call will be successfully handed off if it moves to the neighboring cell. Transient analysis is carried out for our proposed two-cell system using matrix geometric method [80] to compute the probability of dropping the candidate call in the neighboring cell if the candidate call needs to handoff in the neighboring cell. These dropping probabilities are computed and stored in tables for different number of two classes of calls in the current cell and the neighboring cell. Appropriate two different threshold drop probabilities are determined using simulation for two classes of calls depending on these stored values. The call admission controller scheme is extended for multiple-cell wireless networks after designing the scheme for two-cell system. The suitability of our scheme in comparison to dual threshold reservation (DTR) scheme [106] and non-priority scheme is demonstrated through numerical examples. Our adaptive call admission control scheme is found to maintain the QoS requirements of both classes of calls while providing as much channel utilization as possible.

The rest of the chapter is organized as follows. In Sections 4.1 and 4.2, we show the steady state analysis of the non-prioritized scheme and the DTR scheme, respectively for the two-cell system. Our proposed model for two-cell system and its approximation for multiple-cell system are explained in Section 4.3. Section 4.4 presents our analytical and simulation results.

4.1 Non-prioritized Scheme for Two-Cell System

In this section, we discuss non-prioritized scheme for the two-cell system for two classes of users. The two-cell system is shown in Figure 3.2. We assume that the two-cell wireless system consists of two identical cells where the cells are identified as cells 1 and 2, respectively. Users can only move from one cell to another and each cell contains C channels. There are two classes of calls — class 1 and class 2 calls. We assume that cell residence time, call holding duration and new call inter-arrival times are exponentially distributed. We assume that the means of cell residence time, call holding duration and new call inter-arrival time of class 1 calls are $\frac{1}{\eta_1}$, $\frac{1}{\mu_1}$ and $\frac{1}{\lambda_1}$, respectively. Similarly, the means of cell residence time, call holding duration and new call inter-arrival time of class 2 calls are $\frac{1}{\eta_2}$, $\frac{1}{\mu_2}$ and $\frac{1}{\lambda_2}$, respectively. Each call of both classes only require one channel. The channel holding time in a cell for class 1 and class 2 calls are exponentially distributed with means $\frac{1}{\mu_{c_1}} = \frac{1}{\mu_1+\eta_1}$ and $\frac{1}{\mu_{c_2}} = \frac{1}{\mu_2+\eta_2}$, respectively. We follow the assumptions about number of channels, call arrival process, cell residence time and call holding duration throughout the chapter.

The non-prioritized scheme for the two-cell system can be represented by a continuous time Markov chain. The state space of this Markov chain is $\{(i, j, k, l) : 0 \leq i, j, k, l \leq$

C; $0 \leq i+j \leq C$; $0 \leq k+l \leq C\}$ where i and j represent the total number of class 1 and class 2 ongoing calls, respectively in cell 1 and k and l represent the total number of class 1 and class 2 ongoing calls, respectively in cell 2. In this continuous time Markov chain increase or decrease in the number of calls in a cell can be at most one. This Markov chain can have the following types of transitions

- Increase in the total number of class 1 calls in cell 1 due to arrival of a new class 1 call to cell 1, i.e., $(i,j,k,l) \rightarrow (i+1,j,k,l)$ with rate λ_1 for $0 \leq i+j < C$.

- Increase in the total number of class 2 calls in cell 1 due to arrival of a new class 2 call to cell 1, i.e., $(i,j,k,l) \rightarrow (i,j+1,k,l)$ with rate λ_2 for $0 \leq i+j < C$.

- Increase in the total number of class 1 calls in cell 2 due to arrival of a new class 1 call to cell 2, i.e., $(i,j,k,l) \rightarrow (i,j,k+1,l)$ with rate λ_1 for $0 \leq k+l < C$.

- Increase in the total number of class 2 calls in cell 2 due to arrival of a new class 2 call to cell 2, i.e., $(i,j,k,l) \rightarrow (i,j,k,l+1)$ with rate λ_2 for $0 \leq k+l < C$.

- Decrease in the total number of class 1 calls in cell 1 due to completion of a class 1 call in cell 1, i.e., $(i,j,k,l) \rightarrow (i-1,j,k,l)$ with rate $i\,\mu_1$ for $0 < i \leq C$ and $0 \leq k+l < C$.

- Decrease in the total number of class 2 calls in cell 1 due to completion of a class 2 call in cell 1, i.e., $(i,j,k,l) \rightarrow (i,j-1,k,l)$ with rate $j\,\mu_2$ for $0 < j \leq C$ and $0 \leq k+l < C$.

- Decrease in the total number of class 1 calls in cell 2 due to completion of a class 1 call in cell 2, i.e., $(i,j,k,l) \rightarrow (i,j,k-1,l)$ with rate $k\,\mu_1$ for $0 \leq i+j < C$ and $0 < k \leq C$.

- Decrease in the total number of class 2 calls in cell 2 due to completion of a class 2 call in cell 2, i.e., $(i,j,k,l) \rightarrow (i,j,k,l-1)$ with rate $l\,\mu_2$ for $0 \leq i+j < C$ and $0 < l \leq C$.

- Increase in the total number of class 1 calls in cell 1 and decrease in the total number of class 1 calls in cell 2 due to arrival of a class 1 handoff call from cell 2 to cell 1, i.e., $(i, j, k, l) \rightarrow (i+1, j, k-1, l)$ with rate $k\eta_1$ for $0 \leq i+j < C$ and $0 < k \leq C$.

- Increase in the total number of class 2 calls in cell 1 and decrease in the total number of class 2 calls in cell 2 due to arrival of a class 2 handoff call from cell 2 to cell 1, i.e., $(i, j, k, l) \rightarrow (i, j+1, k, l-1)$ with rate $l\eta_2$ for $0 \leq i+j < C$ and $0 < l \leq C$.

- Increase in the total number of class 1 calls in cell 2 and decrease in the total number of class 1 calls in cell 1 due to arrival of a class 1 handoff call from cell 1 to cell 2, i.e., $(i, j, k, l) \rightarrow (i-1, j, k+1, l)$ with rate $i\eta_1$ for $0 < i \leq C$ and $0 \leq k+l < C$.

- Increase in the total number of class 2 calls in cell 2 and decrease in the total number of class 2 calls in cell 1 due to arrival of a class 2 handoff call from cell 1 to cell 2, i.e., $(i, j, k, l) \rightarrow (i, j-1, k, l+1)$ with rate $j\eta_2$ for $0 < j \leq C$ and $0 \leq k+l < C$.

- Decrease in the total number of class 1 calls in cell 1 due to completion of a class 1 call in cell 1 or handoff failure for a class 1 call from cell 1 to cell 2 as a user with ongoing class 1 call moves from cell 1 to cell 2 but all the channels in cell 2 are occupied, i.e., $(i, j, k, l) \rightarrow (i-1, j, k, l)$ with rate $i\,\mu_{c_1}$ for $0 < i \leq C$ and $k+l = C$.

- Decrease in the total number of class 2 calls in cell 1 due to completion of a class 2 call in cell 1 or handoff failure for a class 2 call from cell 1 to cell 2 as a user with ongoing class 2 call moves from cell 1 to cell 2 but all the channels in cell 2 are occupied, i.e., $(i, j, k, l) \rightarrow (i, j-1, k, l)$ with rate $j\,\mu_{c_2}$ for $0 < j \leq C$ and $k+l = C$.

- Decrease in the total number of class 1 calls in cell 2 due to completion of a class 1 call in cell 2 or handoff failure for a class 1 call from cell 2 to cell 1 as a user with ongoing class 1 call moves from cell 2 to cell 1 but all the channels in cell 2 are occupied, i.e., $(i, j, k, l) \rightarrow (i, j, k-1, l)$ with rate $k\,\mu_{c_1}$ for $i+j = C$ and $0 < k \leq C$.

- Decrease in the total number of class 2 calls in cell 2 due to completion of a class 2 call in cell 2 or handoff failure for a class 2 call from cell 2 to cell 1 as a user with ongoing class 2 call moves from cell 2 to cell 1 but all the channels in cell 2 are occupied, i.e., $(i, j, k, l) \rightarrow (i, j, k, l-1)$ with rate $l\,\mu_{c_2}$ for $i + j = C$ and $0 < l \leq C$.

We can represent such Markov chain by the generator matrix Q. Q is a $\frac{(C+1)^2(C+2)^2}{4} \times \frac{(C+1)^2(C+2)^2}{4}$ matrix and has following structure

$$Q = \begin{bmatrix} B_0 & U_0 & & & & \\ D_1 & B_1 & U_1 & & & \\ & D_2 & B_2 & U_2 & & \\ & & \ddots & \ddots & \ddots & \\ & & & D_{C-1} & B_{C-1} & U_{C-1} \\ & & & & D_C & B_C \end{bmatrix}. \tag{4.1}$$

Here, B_i $(0 \leq i \leq C)$ is a $\frac{(C+1)(C+2)(C-i+1)}{2} \times \frac{(C+1)(C+2)(C-i+1)}{2}$ matrix, U_i $(0 \leq i \leq C-1)$ is a $\frac{(C+1)(C+2)(C-i+1)}{2} \times \frac{(C+1)(C+2)(C-i)}{2}$ matrix and D_i $(1 \leq i \leq C)$ is a $\frac{(C+1)(C+2)(C-i+1)}{2} \times \frac{(C+1)(C+2)(C-i+2)}{2}$ matrix. B_i $(0 \leq i \leq C)$, D_i $(1 \leq i \leq C)$ and U_i $(0 \leq i \leq C-1)$ represent respectively no change, decrease by one and increase by one in the number of class 1 calls in cell 1 when the number of class 1 calls in cell 1 is i. We use three tuples (j, k, l) to represent the row and column indices for block matrices B_i's, U_i's and D_i's where j, k and l are already defined. We can compute the distance between (j, k, l) from $(0, 0, 0)$ as $j\frac{(C+1)(C+2)}{2} + \sum_{n=0}^{k-1}(C - n + 1) + l$. For $B_i((j_1, k_1, l_1), (j_2, k_2, l_2))$, $U_i((j_1, k_1, l_1), (j_2, k_2, l_2))$ and $D_i((j_1, k_1, l_1), (j_2, k_2, l_2))$, the first three tuple (j_1, k_1, l_1) represents j_1 number of class 2 calls in cell 1, k_1 number of class 1 calls in cell 2 and l_1 number of class 2 calls in cell 2 before transition. On the other hand, the second three tuple (j_2, k_2, l_2) represents j_2 number of class 2 calls in cell 1, k_2 number of class 1 calls in cell 2 and l_2 number of class 2 calls in cell 2 after transition. The diagonal entries of B_i are shown in Table 4.1. The non-zero nondiagonal entries of B_i and nonzero entries of U_i and D_i are given below

$$B_i((j, k, l), (j + 1, k, l)) = \lambda_2, \; 0 \leq j < C - i. \tag{4.3}$$

Table 4.1: Diagonal entries of B_i for non-prioritized scheme

$$B_i((j,k,l),(j,k,l)) = \begin{cases} -(2(\lambda_1+\lambda_2)+(i+k)\,\mu_{c_1}+(j+l)\,\mu_{c_2}), & 0 \le i+j < C, 0 \le k+l < C \\ -((\lambda_1+\lambda_2)+(i+k)\,\mu_{c_1}+(j+l)\,\mu_{c_2}), & 0 \le i+j < C, k+l = C \\ -((\lambda_1+\lambda_2)+(i+k)\,\mu_{c_1}+(j+l)\,\mu_{c_2}), & i+j = C, 0 \le k+l < C \\ -((i+k)\,\mu_{c_1}+(j+l)\,\mu_{c_2}), & i+j = C, k+l = C \end{cases} \tag{4.2}$$

$$B_i((j,k,l),(j,k+1,l)) = \lambda_1,\ 0 \le k < C-l. \tag{4.4}$$

$$B_i((j,k,l),(j,k,l+1)) = \lambda_2,\ 0 \le l < C-k. \tag{4.5}$$

$$B_i((j,k,l),(j+1,k,l-1)) = l\eta_2,\ 0 < l \le C-k, 0 \le j < C-i. \tag{4.6}$$

$$B_i((j,k,l),(j-1,k,l+1)) = j\eta_2,\ 0 < j \le C-i, 0 \le l < C-k. \tag{4.7}$$

$$B_i((j,k,l),(j-1,k,l)) = \begin{cases} j\,\mu_2, & 0 < j < C-i, 0 \le k+l < C \\ j\,\mu_{c2}, & 0 < j < C-i, k+l = C \end{cases} \tag{4.8}$$

$$B_i((j,k,l),(j,k-1,l)) = \begin{cases} k\,\mu_1, & 0 < k \le C, 0 \le i+j < C \\ k\,\mu_{c1}, & 0 < k \le C, i+j = C \end{cases} \tag{4.9}$$

$$B_i((j,k,l),(j,k,l-1)) = \begin{cases} l\,\mu_2, & 0 < l \le C-k, 0 \le i+j < C \\ l\,\mu_{c2}, & 0 < l \le C, i+j = C \end{cases} \tag{4.10}$$

$$U_i((j,k,l),(j,k,l)) = \lambda_1,\ 0 \le j < C-i. \tag{4.11}$$

$$U_i((j,k,l),(j,k-1,l)) = k\eta_1,\ 0 < k \le C, 0 \le j < C-i. \tag{4.12}$$

$$D_i((j,k,l),(j,k,l)) = \begin{cases} i\,\mu_1, & 0 < i \le C, 0 \le k+l < C \\ i\,\mu_{c1}, & 0 < i \le C, k+l = C \end{cases} \tag{4.13}$$

$$D_i((j,k,l),(j,k+1,l)) = i\eta_1,\ 0 < i \le C, 0 < k+l \le C. \tag{4.14}$$

The generator matrix Q of Equation (4.1) is irreducible. Let us assume that $x = [x_0, x_1, \ldots, x_C]$ is the steady state probability vector for the continuous time Markov chain Q in Equation (4.1) which satisfies $xQ = \mathbf{0}^T$ and $x\mathbf{1} = 1$ where $\mathbf{0}$ and $\mathbf{1}$ are respectively column vectors of zeros and ones of appropriate dimension. Here, x_i $(0 \le i \le C)$ is a row vector having $\frac{(C+1)(C+2)(C-i+1)}{2}$ scalar elements and $x_i = [x_{i,0}, x_{i,1}, \ldots, x_{i,C-i}]$. $x_{i,j}$ $(0 \le$

$j \le C-i$) is a row vector having $\frac{(C+1)(C+2)}{2}$ elements and $x_{i,j} = [x_{i,j,0}, x_{i,j,1}, \ldots, x_{i,j,C}]$. $x_{i,j,k}$ ($0 \le k \le C$) is a row vector having ($C-k+1$) elements and $x_{i,j,k} = [x_{i,j,k,0}, x_{i,j,k,1}, \ldots, x_{i,j,k,C-k}]$. We need to compute the distance of $x_{i,j,k,l}$ from $x_{i,0,0,0}$ for finding the indices corresponding to three tuples (j_1, k_1, l_1) and (j_2, k_2, l_2) of $B_i((j_1, k_1, l_1), (j_2, k_2, l_2))$, $U_i((j_1, k_1, l_1), (j_2, k_2, l_2))$ and $D_i((j_1, k_1, l_1), (j_2, k_2, l_2))$. Let d be the distance of $x_{i,j,k,l}$ from $x_{i,0,0,0}$. We can express d as

$$d = j\frac{(C+1)(C+2)}{2} + kC - \frac{k(k+1)}{2} + l. \tag{4.15}$$

We can find the steady-state probability vector x_i ($1 \le i \le C$) using the technique of Gaver et al [33] in the following way:

$$x_i = x_0 \prod_{m=1}^{i} (U_{m-1}(-E_m)^{-1}), \quad (1 \le i \le C) \tag{4.16}$$

where x_0 satisfies $x_0 E_0 = \mathbf{0}^T$ and

$$x_0(I + \sum_{i=1}^{C} \prod_{m=1}^{i} (U_{m-1}(-E_m)^{-1}))\mathbf{1} = 1. \tag{4.17}$$

Here, E_m ($0 \le m \le C$) is recursively determined by $E_C = B_C$ and

$$E_m = B_m + U_m(-E_{m+1})^{-1} D_{m+1}, \quad (0 \le m \le C-1). \tag{4.18}$$

We can compute the mean handoff arrival rate $\bar{\lambda}_{h_1}^{(2,1)}$ of class 1 call from cell 2 to cell 1 as

$$\bar{\lambda}_{h_1}^{(2,1)} = \sum_{i=0}^{C} \sum_{j=0}^{C-i} \sum_{k=1}^{C} \sum_{l=0}^{C-k} k\eta_1 x_{i,j,k,l}. \tag{4.19}$$

On the other hand, the mean rate $\bar{\lambda}_{h_1}^{(1,2)}$ for class 1 handoff arrival process from cell 1 to cell 2 can be expressed as

$$\bar{\lambda}_{h_1}^{(1,2)} = \sum_{i=1}^{C} \sum_{j=0}^{C-i} \sum_{k=0}^{C} \sum_{l=0}^{C-k} i\eta_1 x_{i,j,k,l}. \tag{4.20}$$

In the same manner we can compute the mean handoff arrival rates of class 2 calls $\bar{\lambda}_{h_2}^{(2,1)}$ and $\bar{\lambda}_{h_2}^{(1,2)}$, respectively from cell 2 to cell 1 and from cell 1 to cell 2 as

$$\bar{\lambda}_{h_2}^{(2,1)} = \sum_{i=0}^{C} \sum_{j=0}^{C-i} \sum_{k=0}^{C-1} \sum_{l=1}^{C-k} l\eta_2 x_{i,j,k,l}. \tag{4.21}$$

$$\bar{\lambda}_{h_2}^{(1,2)} = \sum_{i=0}^{C-1}\sum_{j=1}^{C-i}\sum_{k=0}^{C}\sum_{l=0}^{C-k} j\eta_2 x_{i,j,k,l}. \tag{4.22}$$

However, the mean arrival rate $\bar{\lambda}_{h_1}^{(2,1)}$ of class 1 call from cell 2 to cell 1 and the mean handoff arrival rate $\bar{\lambda}_{h_1}^{(1,2)}$ for handoff process from cell 1 to cell 2 are same and the mean arrival rate $\bar{\lambda}_{h_2}^{(2,1)}$ of class 2 call from cell 2 to cell 1 and the mean handoff arrival rate $\bar{\lambda}_{h_2}^{(1,2)}$ for handoff process from cell 1 to cell 2 are same as both the cells are identical. Therefore, the mean arrival rate of class 1 handoff call to any cell in the two-cell system is $\bar{\lambda}_{h_1} = \bar{\lambda}_{h_1}^{(2,1)} = \bar{\lambda}_{h_1}^{(1,2)}$ and the mean arrival rate of class 2 handoff call to any cell in the two-cell system is $\bar{\lambda}_{h_2} = \bar{\lambda}_{h_2}^{(2,1)} = \bar{\lambda}_{h_2}^{(1,2)}$.

An approach similar to PASTA [102] can be used to compute class 1 and class 2 handoff failure probabilities in a cell in the two-cell system. In this approach, an outside observer sees $\sum_{i=0}^{C}\sum_{j=0}^{C-i}\sum_{k=1}^{C}\sum_{l=0}^{C-k} k\eta_1 x_{i,j,k,l}T$ and $\sum_{i=1}^{C}\sum_{j=0}^{C-i}\sum_{k=0}^{C}\sum_{l=0}^{C-k} i\eta_1 x_{i,j,k,l}T$ class 1 handoff traffics from cell 2 to cell 1 and from cell 1 to cell 2, respectively over time T. On the other hand, the outside observer sees $\sum_{i=0}^{C}\sum_{k=1}^{C}\sum_{l=0}^{C-k} k\eta_1 x_{i,C-i,k,l}T$ and $\sum_{i=1}^{C}\sum_{j=0}^{C-i}\sum_{k=0}^{C} i\eta_1 x_{i,j,k,C-k}T$ class 1 handoff traffics to be dropped in cell 1 and cell 2, respectively in time T. Moreover, handoff failure probability of a class of traffic in a cell can be expressed as the ratio of the amount of the handoff traffic of that class dropped in the cell to the amount the handoff traffic of that class comes to the cell over a period of time. Therefore, the handoff failure probabilities $P_{hf_1}^{(2,1)}$ and $P_{hf_1}^{(1,2)}$ of class 1 calls from cell 2 to cell 1 and from cell 1 to cell 2, respectively can be computed as

$$P_{hf_1}^{(2,1)} = \frac{\sum_{i=0}^{C}\sum_{k=1}^{C}\sum_{l=0}^{C-k} k\eta_1 x_{i,C-i,k,l}T}{\sum_{i=0}^{C}\sum_{j=0}^{C-i}\sum_{k=1}^{C}\sum_{l=0}^{C-k} k\eta_1 x_{i,j,k,l}T} = \frac{\sum_{i=0}^{C}\sum_{k=1}^{C}\sum_{l=0}^{C-k} kx_{i,C-i,k,l}}{\sum_{i=0}^{C}\sum_{j=0}^{C-i}\sum_{k=1}^{C}\sum_{l=0}^{C-k} kx_{i,j,k,l}}. \tag{4.23}$$

$$P_{hf_1}^{(1,2)} = \frac{\sum_{i=1}^{C}\sum_{j=0}^{C-i}\sum_{k=0}^{C} i\eta_1 x_{i,j,k,C-k}T}{\sum_{i=1}^{C}\sum_{j=0}^{C-i}\sum_{k=0}^{C}\sum_{l=0}^{C-k} i\eta_1 x_{i,j,k,l}T} = \frac{\sum_{i=1}^{C}\sum_{j=0}^{C-i}\sum_{k=0}^{C} ix_{i,j,k,C-k}}{\sum_{i=1}^{C}\sum_{j=0}^{C-i}\sum_{k=0}^{C}\sum_{l=0}^{C-k} ix_{i,j,k,l}}. \tag{4.24}$$

Similarly, the handoff failure probabilities $P_{hf_2}^{(2,1)}$ and $P_{hf_2}^{(1,2)}$ of class 2 calls from cell 2 to cell 1 and from cell 1 to cell 2, respectively can be expressed as

$$P_{hf_2}^{(2,1)} = \frac{\sum_{i=0}^{C}\sum_{k=0}^{C-1}\sum_{l=1}^{C-k} lx_{i,C-i,k,l}}{\sum_{i=0}^{C}\sum_{j=0}^{C-i}\sum_{k=0}^{C-1}\sum_{l=1}^{C-k} lx_{i,j,k,l}}. \tag{4.25}$$

$$P_{hf_2}^{(1,2)} = \frac{\sum_{i=0}^{C-1}\sum_{j=1}^{C-i}\sum_{k=0}^{C} jx_{i,j,k,C-k}}{\sum_{i=0}^{C-1}\sum_{j=1}^{C-i}\sum_{k=0}^{C}\sum_{l=0}^{C-k} jx_{i,j,k,l}}. \tag{4.26}$$

The new call blocking probabilities $P_{b_1}^{(1)}$ and $P_{b_2}^{(1)}$ of class 1 and 2 calls, respectively in cell 1 can be expressed as

$$P_{b_1}^{(1)} = P_{b_2}^{(1)} = \sum_{i=0}^{C} x_{i,C-i}\mathbf{1} = \sum_{i=0}^{C}\sum_{k=0}^{C}\sum_{l=0}^{C-k} x_{i,C-i,k,l}. \tag{4.27}$$

Similarly, we can compute the new call blocking probabilities $P_{b_1}^{(2)}$ and $P_{b_2}^{(2)}$ of class 1 and 2 calls respectively in cell 2 as

$$P_{b_1}^{(2)} = P_{b_2}^{(2)} = \sum_{i=0}^{C}\sum_{j=0}^{C-i}\sum_{k=0}^{C} x_{i,j,k,C-k}. \tag{4.28}$$

As cell 1 and cell 2 are identical, new call arrival process for class 1 and class 2 calls are same in cell 1 and cell 2, cell residence time of class 1 and class 2 calls are identically distributed in both cells, we get $P_{hf_1}^{(2,1)} = P_{hf_1}^{(1,2)} = P_{hf_1}$, $P_{hf_2}^{(2,1)} = P_{hf_2}^{(1,2)} = P_{hf_2}$ and $P_{b_1}^{(1)} = P_{b_2}^{(1)} = P_{b_1}^{(2)} = P_{b_2}^{(2)} = P_b$.

We can find the mean number of calls in a cell for the two-cell system. If we denote $\overline{N_1}$ and $\overline{N_2}$ as the mean number of calls in cell 1 and cell 2 then

$$\overline{N_1} = \sum_{i=0}^{C}\sum_{j=0}^{C-i}(i+j)x_{i,j}\mathbf{1} = \sum_{i=0}^{C}\sum_{j=0}^{C-i}\sum_{k=0}^{C}\sum_{l=0}^{C-k}(i+j)x_{i,j,k,l}. \tag{4.29}$$

and

$$\overline{N_2} = \sum_{i=0}^{C}\sum_{j=0}^{C-i}\sum_{k=0}^{C}\sum_{l=0}^{C-k}(k+l)x_{i,j,k,l}. \tag{4.30}$$

As both the cells are identical the mean number of calls in both cells are same which is $\overline{N} = \overline{N_1} = \overline{N_2}$. Therefore, the expected channel utilization for the two-cell system is $\frac{\overline{N}}{C}$.

4.2 Dual Threshold Reservation (DTR) Scheme for Two-Cell System

In dual threshold reservation (DTR) scheme, there are two thresholds K_1 and K_2 and $0 < K_2 \leq K_1 \leq C$. In the DTR scheme, a class 2 new or handoff call is accepted in a cell if the number of ongoing calls in that cell is less than K_2. A class 1 new call is accepted in a cell 1 if the number of ongoing calls in that cell is less than K_1. On the other hand, a class 1 handoff call is accepted in a cell if there is any free channel in that cell. The non-priority scheme for two classes of users is a special case of the DTR scheme. The DTR scheme for $K_1 = K_2 = C$ is the non-priority scheme for two classes of users.

The state-space for the four-dimensional Markov chain for the DTR scheme for two cells can be represented as $\{(\breve{i}, j, \breve{k}, l) : 0 \leq \breve{i}, \breve{k} \leq C;\ 0 \leq \breve{i} + \breve{k} \leq C + K_1;\ 0 \leq j, l \leq K_2;\ 0 \leq j \leq \min(\breve{i}, K_2);\ 0 \leq l \leq \min(\breve{k}, K_2)\}$. Here, $\breve{i}$ and $\breve{k}$ represent respectively the total numbers of ongoing calls in cells 1 and 2. j and l represent the numbers of class 2 ongoing calls in cells 1 and 2, respectively. Therefore, the numbers of class 1 calls in cells 1 and 2 are respectively $\breve{i} - j$ and $\breve{k} - l$. The Markov chain can have the following types of transitions:

- Increase in the total number of class 1 calls in cell 1 due to arrival of a new class 1 call to cell 1, i.e., $(\breve{i}, j, \breve{k}, l) \rightarrow (\breve{i} + 1, j, \breve{k}, l)$ with rate λ_1 for $0 \leq \breve{i} < K_1$.

- Increase in the total number of class 2 calls in cell 1 due to arrival of a new class 2 call to cell 1, i.e., $(\breve{i}, j, \breve{k}, l) \rightarrow (\breve{i} + 1, j + 1, \breve{k}, l)$ with rate λ_2 for $0 \leq \breve{i} < K_2$.

- Increase in the total number of class 1 calls in cell 2 due to arrival of a new class 1 call to cell 2, i.e., $(\breve{i}, j, \breve{k}, l) \rightarrow (\breve{i}, j, \breve{k} + 1, l)$ with rate λ_1 for $0 \leq \breve{k} < K_1$.

- Increase in the total number of class 2 calls in cell 2 due to arrival of a new class 2 call to cell 2, i.e., $(\breve{i}, j, \breve{k}, l) \rightarrow (\breve{i}, j, \breve{k} + 1, l + 1)$ with rate λ_2 for $0 \leq \breve{k} < K_2$.

- Decrease in the total number of class 1 calls in cell 1 due to completion of a class 1 call in cell 1, i.e., $(\breve{i}, j, \breve{k}, l) \rightarrow (\breve{i} - 1, j, \breve{k}, l)$ with rate $(\breve{i} - j)\,\mu_1$ for $0 < \breve{i} - j \leq C$ and $0 \leq \breve{k} \leq \min(C - 1, C + K_1 - \breve{i})$.

- Decrease in the total number of class 2 calls in cell 1 due to completion of a class 2 call in cell 1, i.e., $(\breve{i}, j, \breve{k}, l) \rightarrow (\breve{i} - 1, j - 1, \breve{k}, l)$ with rate $j\,\mu_2$ for $0 < j \leq \min(K_2, \breve{i})$ and $0 \leq \breve{k} < K_2$.

- Decrease in the total number of class 1 calls in cell 2 due to completion of a class 1 call in cell 2, i.e., $(\breve{i}, j, \breve{k}, l) \rightarrow (\breve{i}, j, \breve{k} - 1, l)$ with rate $(\breve{k} - l)\,\mu_1$ for $0 \leq \breve{i} < C$ and $0 < \breve{k} \leq \min(C, C + K_1 - \breve{i})$.

- Decrease in the total number of class 2 calls in cell 2 due to completion of a class 2 call in cell 2, i.e., $(\breve{i}, j, \breve{k}, l) \to (\breve{i}, j, \breve{k}-1, l-1)$ with rate $l\,\mu_2$ for $0 \le \breve{i} < K_2$ and $0 < l \le \min(K_2, \breve{k})$.

- Increase in the total number of class 1 calls in cell 1 and decrease in the total number of class 1 calls in cell 2 due to arrival of a class 1 handoff call from cell 2 to cell 1, i.e., $(\breve{i}, j, \breve{k}, l) \to (\breve{i}+1, j, \breve{k}-1, l)$ with rate $(\breve{k}-l)\eta_1$ for $0 \le \breve{i} < C$ and $0 < \breve{k} - l \le \min(C, C + K_1 - \breve{i})$.

- Increase in the total number of class 2 calls in cell 1 and decrease in the total number of class 2 calls in cell 2 due to arrival of a class 2 handoff call from cell 2 to cell 1, i.e., $(\breve{i}, j, \breve{k}, l) \to (\breve{i}+1, j+1, \breve{k}-1, l-1)$ with rate $l\eta_2$ for $0 \le \breve{i} < K_2$ and $0 < l \le \min(K_2, \breve{k})$.

- Increase in the total number of class 1 calls in cell 2 and decrease in the total number of class 1 calls in cell 1 due to arrival of a class 1 handoff call from cell 1 to cell 2, i.e., $(\breve{i}, j, \breve{k}, l) \to (\breve{i}-1, j, \breve{k}+1, l)$ with rate $(\breve{i}-j)\eta_1$ for $0 < \breve{i} - j \le C$ and $0 \le \breve{k} \le \min(C-1, C+K_1-\breve{i})$.

- Increase in the total number of class 2 calls in cell 2 and decrease in the total number of class 2 calls in cell 1 due to arrival of a class 2 handoff call from cell 1 to cell 2, i.e., $(\breve{i}, j, \breve{k}, l) \to (\breve{i}-1, j-1, \breve{k}+1, l+1)$ with rate $j\eta_2$ for $0 < j \le \min(\breve{i}, K_2)$ and $0 \le \breve{k} < K_2$.

- Decrease in the total number of class 1 calls in cell 1 due to completion of a class 1 call in cell 1 or handoff failure for a class 1 call from cell 1 to cell 2 as a user with ongoing class 1 call moves from cell 1 to cell 2 but all the channels in cell 2 are occupied, i.e., $(\breve{i}, j, \breve{k}, l) \to (\breve{i}-1, j, \breve{k}, l)$ with rate $(\breve{i}-j)\,\mu_{c_1}$ for $0 < \breve{i} - j \le K_1$ and $\breve{k} = C$.

- Decrease in the total number of class 2 calls in cell 1 due to completion of a class 2 call in cell 1 or handoff failure for a class 2 call from cell 1 to cell 2 as a user with ongoing class 2 call moves from cell 1 to cell 2 but at least K_2 channels in cell 2 are occupied, i.e., $(\breve{i}, j, \breve{k}, l) \to (\breve{i}-1, j-1, \breve{k}, l)$ with rate $j\,\mu_{c_2}$ for $0 < j \le \min(K_2, \breve{i})$ and $K_2 \le \breve{k} \le \min(C + K_1 - \breve{i}, C)$.

- Decrease in the total number of class 1 calls in cell 2 due to completion of a class 1 call in cell 2 or handoff failure for a class 1 call from cell 2 to cell 1 as a user with ongoing class 1 call moves from cell 2 to cell 1 but all the channels in cell 1 are occupied, i.e., $(\check{\imath}, \jmath, \check{k}, l) \rightarrow (\check{\imath}, \jmath, \check{k}-1, l)$ with rate $(\check{k}-l)\,\mu_{c_1}$ for $\check{\imath} = C$ and $0 < \check{k} - l \leq K_1$.

- Decrease in the total number of class 2 calls in cell 2 due to completion of a class 2 call in cell 2 or handoff failure for a class 2 call from cell 2 to cell 1 as a user with ongoing class 2 call moves from cell 2 to cell 1 but at least K_2 channels in cell 1 are occupied, i.e., $(\check{\imath}, \jmath, \check{k}, l) \rightarrow (\check{\imath}, \jmath, \check{k}-1, l-1)$ with rate $l\,\mu_{c_2}$ for $K_2 \leq \check{\imath} \leq C$ and $0 < l \leq \min(\check{k}, K_2)$.

We can represent such a Markov chain by the generator matrix Q. Q is a square matrix of order $\frac{(K_2+1)^2}{4}(2C^2 + (4K_1 - 4K_2 + 6)C - 2K_1{}^2 + K_2{}^2 + 2K_1 - 4K_2 + 4)$ and has following structure

$$Q = \begin{bmatrix} B_0 & U_0 & & & & \\ D_1 & B_1 & U_1 & & & \\ & D_2 & B_2 & U_2 & & \\ & & \ddots & \ddots & \ddots & \\ & & & D_{C-1} & B_{C-1} & U_{C-1} \\ & & & & D_C & B_C \end{bmatrix}. \tag{4.31}$$

Here, $B_{\check{\imath}}$ $(0 \leq \check{\imath} \leq C)$ is an $M^{(\check{\imath})} \times M^{(\check{\imath})}$ matrix where

$$M^{(\check{\imath})} = \begin{cases} \frac{(\check{\imath}+1)(K_2+1)(2C-K_2+2)}{2}, 0 \leq \check{\imath} \leq K_2 \\ \frac{(K_2+1)^2(2C-K_2+2)}{2}, K_2+1 \leq \check{\imath} \leq K_1 \\ \frac{(K_2+1)^2(2(C-\check{\imath}+K_1)-K_2+2)}{2}, K_1+1 \leq \check{\imath} \leq C \end{cases} \tag{4.32}$$

$D_{\check{\imath}}$ $(1 \leq \check{\imath} \leq C)$ is an $M^{(\check{\imath})} \times M^{(\check{\imath}-1)}$ matrix and $U_{\check{\imath}}$ $(0 \leq \check{\imath} \leq C-1)$ is an $M^{(\check{\imath})} \times M^{(\check{\imath}+1)}$ matrix. $B_{\check{\imath}}$ $(0 \leq \check{\imath} \leq C)$, $D_{\check{\imath}}$ $(1 \leq \check{\imath} \leq C)$ and $U_{\check{\imath}}$ $(0 \leq \check{\imath} \leq C-1)$ represent respectively no change, decrease by one and increase by one in the total number of ongoing calls in cell 1 when the total number of ongoing calls in cell 1 is $\check{\imath}$. For $B_{\check{\imath}}((\jmath_1, \check{k}_1, l_1), (\jmath_2, \check{k}_2, l_2))$, $U_{\check{\imath}}((\jmath_1, \check{k}_1, l_1), (\jmath_2, \check{k}_2, l_2))$ and $D_{\check{\imath}}((\jmath_1, \check{k}_1, l_1), (\jmath_2, \check{k}_2, l_2))$, the first three tuple $(\jmath_1, \check{k}_1, l_1)$ represents $\jmath_1$ number of class 2 calls in cell 1, $\check{k}_1$ number of ongoing calls in cell 2 and l_1

number of class 2 calls in cell 2 before transition. On the other hand, the second three tuple $(j_2, \breve{k}_2, l_2)$ represents j_2 number of class 2 calls in cell 1, $\breve{k}_2$ number of ongoing calls in cell 2 and l_2 number of class 2 calls in cell 2 after transition. The diagonal entries of $B_{\breve{i}}$ are shown in Table 4.2. The nonzero nondiagonal entries of B_i and nonzero entries of U_i and D_i are given below

$$B_{\breve{i}}((j,\breve{k},l),(j,\breve{k}+1,l)) = \lambda_1,\ 0 \le \breve{k} < K_1. \tag{4.34}$$

$$B_{\breve{i}}((j,\breve{k},l),(j,\breve{k}+1,l+1)) = \lambda_2,\ 0 \le \breve{k} < K_2. \tag{4.35}$$

$$B_{\breve{i}}((j,\breve{k},l),(j,\breve{k}-1,l)) = \begin{cases} (\breve{k}-l)\,\mu_1, & 0 < \breve{k}-l \le \min(C+K_1-\breve{i},\, C), 0 \le \breve{i} < C \\ (\breve{k}-l)\,\mu_{c_1}, & 0 < \breve{k}-l \le K_1, \breve{i} = C \end{cases} \tag{4.36}$$

$$B_{\breve{i}}((j,\breve{k},l),(j,\breve{k}-1,l-1)) = \begin{cases} l\,\mu_2, & 0 < l \le K_2, 0 \le \breve{i} < K_2 \\ l\,\mu_{c_2}, & 0 < l \le K_2, K_2 \le \breve{i} \le C \end{cases} \tag{4.37}$$

$$U_{\breve{i}}((j,\breve{k},l),(j,\breve{k},l)) = \lambda_1,\ 0 \le \breve{i} < K_1. \tag{4.38}$$

$$U_{\breve{i}}((j,\breve{k},l),(j+1,\breve{k},l)) = \lambda_2,\ 0 \le \breve{i} < K_2. \tag{4.39}$$

$$U_{\breve{i}}((j,\breve{k},l),(j,\breve{k}-1,l)) = (\breve{k}-l)\eta_1,\ 0 < \breve{k}-l \le \min(C+K_1-\breve{i},\, C), 0 \le \breve{i} < C. \tag{4.40}$$

$$U_{\breve{i}}((j,\breve{k},l),(j+1,\breve{k}-1,l-1)) = l\eta_2,\ 0 < l \le K_2, 0 \le \breve{i} < K_2. \tag{4.41}$$

$$D_{\breve{i}}((j,\breve{k},l),(j,\breve{k},l)) = \begin{cases} (\breve{i}-j)\,\mu_1, & 0 < \breve{i}-j \le C, 0 \le \breve{k} < \min(C, C+K_1-\breve{i}) \\ (\breve{i}-j)\,\mu_{c_1}, & 0 < \breve{i}-j \le K_1, \breve{k} = C \end{cases} \tag{4.42}$$

Table 4.2: Nonzero diagonal entries of $B_{\breve{i}}$

$$B_{\breve{i}}((j,\breve{k},l),(j,\breve{k},l)) = \begin{cases} -(2(\lambda_1+\lambda_2)+(\breve{i}-j+\breve{k}-l)\,\mu_{c_1}+(j+l)\,\mu_{c_2}), & 0 \le \breve{i},\breve{k} < K_2 \\ -(2\,\lambda_1+\lambda_2+(\breve{i}-j+\breve{k}-l)\,\mu_{c_1}+(j+l)\,\mu_{c_2}), & 0 \le \breve{i} < K_2, K_2 \le \breve{k} < K_1 \\ -(\lambda_1+\lambda_2+(\breve{i}-j+\breve{k}-l)\,\mu_{c_1}+(j+l)\,\mu_{c_2}), & 0 \le \breve{i} < K_2, K_1 \le \breve{k} \le C \\ -(2\,\lambda_1+\lambda_2+(\breve{i}-j+\breve{k}-l)\,\mu_{c_1}+(j+l)\,\mu_{c_2}), & K_2 \le \breve{i} < K_1, 0 \le \breve{k} < K_2 \\ -(2\,\lambda_1+(\breve{i}-j+\breve{k}-l)\,\mu_{c_1}+(j+l)\,\mu_{c_2}), & K_2 \le \breve{i} < K_1, K_2 \le \breve{k} < K_1 \\ -(\lambda_1+(\breve{i}-j+\breve{k}-l)\,\mu_{c_1}+(j+l)\,\mu_{c_2}), & K_2 \le \breve{i} < K_1, K_1 \le \breve{k} \le C \\ -(\lambda_1+\lambda_2+(\breve{i}-j+\breve{k}-l)\,\mu_{c_1}+(j+l)\,\mu_{c_2}), & K_1 \le \breve{i} \le C, 0 \le \breve{k} < K_2 \\ -(\lambda_1+(\breve{i}-j+\breve{k}-l)\,\mu_{c_1}+(j+l)\,\mu_{c_2}), & K_1 \le \breve{i} \le C, K_2 \le \breve{k} < K_1 \\ -((\breve{i}-j+\breve{k}-l)\,\mu_{c_1}+(j+l)\,\mu_{c_2}), & K_1 \le \breve{i} \le C, K_1 \le \breve{k} \le \min(C, C+K_1-\breve{i}) \end{cases} \quad (4.33)$$

$$D_{\check{i}}((j,\check{k},l),(j,\check{k}+1,l)) = (\check{i}-j)\eta_1,\ 0 < \check{i}-j \leq C, 0 \leq \check{k} < \min(C, C+K_1-\check{i}). \quad (4.43)$$

$$D_{\check{i}}((j,\check{k},l),(j-1,\check{k},l)) = \begin{cases} j\,\mu_2, & 0 < j \leq K_2, 0 \leq \check{k} < K_2 \\ j\,\mu_{c2}, & 0 < j \leq K_2, K_2 \leq \check{k} \leq \min(C+K_1-\check{i}, C) \end{cases} \quad (4.44)$$

$$D_{\check{i}}((j,\check{k},l),(j-1,\check{k}+1,l+1)) = j\eta_2,\ 0 < j \leq K_2, 0 \leq \check{k} < K_2. \quad (4.45)$$

The generator matrix Q of Equation (4.31) is irreducible. Let us assume that $x = [x_0, x_1, \ldots, x_C]$ is the steady state probability vector for the continuous time Markov chain Q in Equation (4.31) which satisfies $xQ = \mathbf{0}^T$ and $x\mathbf{1} = 1$ where $\mathbf{0}$ and $\mathbf{1}$ are respectively column vectors of zeros and ones of appropriate dimension. Here, $x_{\check{i}}$ $(0 \leq i \leq C)$ is a row vector having $M^{(\check{i})}$ scalar elements. $x_{\check{i}}$ can be represented as $x_{\check{i}} = [x_{\check{i},0}, x_{\check{i},1}, \ldots, x_{\check{i},J^{(\check{i})}}]$ where $J^{(\check{i})} = \min(\check{i}, K_2)$. $x_{\check{i},j}$ $(0 \leq j \leq J^{(\check{i})})$ is a row vector having $\frac{(K_2+1)}{2}(2C - K_2 + 2)$ for $0 \leq \check{i} \leq K_1$ or $\frac{(K_2+1)}{2}(2(C - \check{i} + K_1) - K_2 + 2)$ for $K_1 < \check{i} \leq C$ elements. $x_{\check{i},j}$ can be represented as $x_{i,j} = [x_{i,j,0}, x_{i,j,1}, \ldots, x_{i,j,K^{(\check{i})}}]$ where $K^{(\check{i})} = C$ for $0 \leq \check{i} \leq K_1$ and $K^{(\check{i})} = C + K_1 - \check{i}$ for $K_1 < \check{i} \leq C$. $x_{\check{i},j,\check{k}}$ is a row vector having $(L^{(\check{k})}+1)$ elements where $L^{(\check{k})} = \min(\check{k}, K_2)$. $x_{\check{i},j,\check{k}}$ can be represented as $x_{\check{i},j,\check{k}} = [x_{\check{i},j,\check{k},0}, x_{\check{i},j,\check{k},1}, \ldots, x_{\check{i},j,\check{k},L^{(\check{k})}}]$. We need to compute the distance of $x_{\check{i},j,\check{k},l}$ from $x_{\check{i},0,0,0}$ for finding the indices corresponding to three tuples $(j_1, \check{k}_1, l_1)$ and $(j_2, \check{k}_2, l_2)$ of $B_{\check{i}}((j_1, \check{k}_1, l_1), (j_2, \check{k}_2, l_2))$, $U_{\check{i}}((j_1, \check{k}_1, l_1), (j_2, \check{k}_2, l_2))$ and $D_{\check{i}}((j_1, \check{k}_1, l_1), (j_2, \check{k}_2, l_2))$. Let d be the distance of $x_{\check{i},j,\check{k},l}$ from $x_{\check{i},0,0,0}$. The expression for d is given in Table 4.3.

We can find the steady-state probability vector $x_{\check{i}}$ $(1 \leq \check{i} \leq C)$ using the technique of Gaver et al [33] in the following way

$$x_{\check{i}} = x_0 \prod_{m=1}^{\check{i}} (U_{m-1}(-E_m)^{-1}), \quad (1 \leq \check{i} \leq C) \quad (4.47)$$

where x_0 satisfies $x_0 E_0 = \mathbf{0}^T$ and

$$x_0(I + \sum_{\check{i}=1}^{C} \prod_{m=1}^{\check{i}} (U_{m-1}(-E_m)^{-1}))\mathbf{1} = 1. \quad (4.48)$$

Table 4.3: The expression for distance d between $x_{\check{i},j,\check{k},l}$ and $x_{\check{i},0,0,0}$

$$d = \begin{cases} j\frac{(K_2+1)(2C-K_2+2)}{2} + \frac{\check{k}(\check{k}+1)}{2} + l, & 0 < \check{i} \leq K_1, 0 \leq \check{k} \leq K_2 \\ j\frac{(K_2+1)(2C-K_2+2)}{2} + \frac{(K_2+1)(K_2+2)}{2} + (K_2+1)(\check{k} - K_2 - 1) + l, & 0 < \check{i} \leq K_1, K_2 < \check{k} \leq C \\ j\frac{(K_2+1)(2(C-\check{i}+K_1)-K_2+2)}{2} + \frac{\check{k}(\check{k}+1)}{2} + l, & K_1 < \check{i} \leq C, 0 \leq \check{k} \leq K_2 \\ j\frac{(K_2+1)(2(C-\check{i}+K_1)-K_2+2)}{2} + \frac{(K_2+1)(K_2+2)}{2} + (K_2+1)(\check{k} - K_2 - 1) + l, & K_1 < \check{i} \leq C, K_2 < \check{k} \leq C \end{cases} \tag{4.46}$$

Here, E_m $(0 \leq m \leq C)$ is recursively determined by $E_C = B_C$ and

$$E_m = B_m + U_m(-E_{m+1})^{-1}D_{m+1}, \quad (0 \leq m \leq C-1). \tag{4.49}$$

We can compute the mean handoff arrival rate $\bar{\lambda}_{h_1}^{(2,1)}$ of class 1 call from cell 2 to cell 1 as

$$\bar{\lambda}_{h_1}^{(2,1)} = \sum_{\breve{i}=0}^{C} \sum_{j=0}^{J^{(\breve{i})}} \sum_{\breve{k}=1}^{K^{(\breve{i})}} \sum_{l=0}^{L^{(\breve{k})}} (\breve{k} - l)\eta_1 x_{\breve{i},j,\breve{k},l}. \tag{4.50}$$

On the other hand, the mean handoff arrival rate $\bar{\lambda}_{h_1}^{(1,2)}$ for handoff process from cell 1 to cell 2 can be expressed as

$$\bar{\lambda}_{h_1}^{(1,2)} = \sum_{\breve{i}=1}^{C} \sum_{j=0}^{J^{(\breve{i})}} \sum_{\breve{k}=0}^{K^{(\breve{i})}} \sum_{l=0}^{L^{(\breve{k})}} (\breve{i} - j)\eta_1 x_{\breve{i},j,\breve{k},l}. \tag{4.51}$$

In the same manner, we can compute the mean handoff arrival rates of class 2 calls $\bar{\lambda}_{h_2}^{(2,1)}$ and $\bar{\lambda}_{h_2}^{(1,2)}$, respectively from cell 2 to cell 1 and from cell 1 to cell 2 as

$$\bar{\lambda}_{h_2}^{(2,1)} = \sum_{\breve{i}=0}^{C} \sum_{j=0}^{J^{(\breve{i})}} \sum_{\breve{k}=1}^{K^{(\breve{i})}} \sum_{l=1}^{L^{(\breve{k})}} l\eta_2 x_{\breve{i},j,\breve{k},l}. \tag{4.52}$$

$$\bar{\lambda}_{h_2}^{(1,2)} = \sum_{\breve{i}=1}^{C} \sum_{j=1}^{J^{(\breve{i})}} \sum_{\breve{k}=0}^{K^{(\breve{i})}} \sum_{l=0}^{L^{(\breve{k})}} j\eta_2 x_{\breve{i},j,\breve{k},l}. \tag{4.53}$$

However, the mean arrival rate $\bar{\lambda}_{h_1}^{(2,1)}$ of class 1 call from cell 2 to cell 1 and the mean handoff arrival rate $\bar{\lambda}_{h_1}^{(1,2)}$ for handoff process from cell 1 to cell 2 are same as both the cells are identical. Similarly, the mean arrival rate $\bar{\lambda}_{h_2}^{(2,1)}$ of class 2 call from cell 2 to cell 1 and the mean handoff arrival rate $\bar{\lambda}_{h_2}^{(1,2)}$ for handoff process from cell 1 to cell 2 are same. Therefore, the mean arrival rate of class 1 handoff call to any cell in the two-cell system is $\bar{\lambda}_{h_1} = \bar{\lambda}_{h_1}^{(2,1)} = \bar{\lambda}_{h_1}^{(1,2)}$ and the mean arrival rate of class 2 handoff call to any cell in the two-cell system is $\bar{\lambda}_{h_2} = \bar{\lambda}_{h_2}^{(2,1)} = \bar{\lambda}_{h_2}^{(1,2)}$.

We can compute the handoff failure probabilities $P_{hf_1}^{(2,1)}$ and $P_{hf_1}^{(1,2)}$ of class 1 calls from cell 2 to cell 1 and from cell 1 to cell 2, respectively as

$$P_{hf_1}^{(2,1)} = \frac{\sum_{j=0}^{J(C)} \sum_{\breve{k}=1}^{K(C)} \sum_{l=0}^{L^{(\breve{k})}} (\breve{k} - l) x_{C,j,\breve{k},l}}{\sum_{\breve{i}=0}^{C} \sum_{j=0}^{J^{(\breve{i})}} \sum_{\breve{k}=1}^{K^{(\breve{i})}} \sum_{l=0}^{L^{(\breve{k})}} (\breve{k} - l) x_{\breve{i},j,\breve{k},l}}. \tag{4.54}$$

$$P_{hf_1}^{(1,2)} = \frac{\sum_{\breve{i}=1}^{K_1}\sum_{j=0}^{J(\breve{i})}\sum_{l=0}^{L(C)}(\breve{i}-j)x_{\breve{i},j,C,l}}{\sum_{\breve{i}=1}^{C}\sum_{j=0}^{J(\breve{i})}\sum_{k=0}^{K(\breve{i})}\sum_{l=0}^{L(\breve{k})}(\breve{i}-j)x_{\breve{i},j,\breve{k},l}}. \tag{4.55}$$

Similarly, the handoff failure probabilities $P_{hf_2}^{(2,1)}$ and $P_{hf_2}^{(1,2)}$ of class 2 calls from cell 2 to cell 1 and from cell 1 to cell 2, respectively can be expressed as

$$P_{hf_2}^{(2,1)} = \frac{\sum_{\breve{i}=K_2}^{C}\sum_{j=0}^{J(\breve{i})}\sum_{k=1}^{K(\breve{i})}\sum_{l=1}^{L(\breve{k})} l x_{\breve{i},j,\breve{k},l}}{\sum_{i=0}^{C}\sum_{j=0}^{J(\breve{i})}\sum_{k=1}^{K(\breve{i})}\sum_{l=1}^{L(\breve{k})} l x_{\breve{i},j,\breve{k},l}}. \tag{4.56}$$

$$P_{hf_2}^{(1,2)} = \frac{\sum_{i=1}^{C}\sum_{j=1}^{J(\breve{i})}\sum_{k=K_2}^{K(\breve{i})}\sum_{l=0}^{L(\breve{k})} j x_{\breve{i},j,\breve{k},l}}{\sum_{i=1}^{C}\sum_{j=1}^{J(\breve{i})}\sum_{k=0}^{K(\breve{i})}\sum_{l=0}^{L(\breve{k})} j x_{\breve{i},j,\breve{k},l}}. \tag{4.57}$$

The new call blocking probabilities $P_{b_1}^{(1)}$ and $P_{b_2}^{(1)}$ of class 1 and 2 calls, respectively in cell 1 can be expressed as

$$P_{b_1}^{(1)} = \sum_{\breve{i}=K_1}^{C} x_{\breve{i}}\mathbf{1} = \sum_{\breve{i}=K_1}^{C}\sum_{j=0}^{J(\breve{i})}\sum_{\breve{k}=0}^{K(\breve{i})}\sum_{l=0}^{L(\breve{k})} x_{\breve{i},j,\breve{k},l}. \tag{4.58}$$

$$P_{b_2}^{(1)} = \sum_{\breve{i}=K_2}^{C} x_{\breve{i}}\mathbf{1} = \sum_{\breve{i}=K_2}^{C}\sum_{j=0}^{J(\breve{i})}\sum_{\breve{k}=0}^{K(\breve{i})}\sum_{l=0}^{L(\breve{k})} x_{\breve{i},j,\breve{k},l}. \tag{4.59}$$

Similarly, we can compute the new call blocking probabilities $P_{b_1}^{(2)}$ and $P_{b_2}^{(2)}$ of class 1 and 2 calls respectively in cell 2 as

$$P_{b_1}^{(2)} = \sum_{\breve{i}=0}^{C}\sum_{j=0}^{J(\breve{i})}\sum_{k=K_1}^{K(\breve{i})} x_{\breve{i},j,\breve{k}}\mathbf{1} = \sum_{\breve{i}=0}^{C}\sum_{j=0}^{J(\breve{i})}\sum_{k=K_1}^{K(\breve{i})}\sum_{l=0}^{L(\breve{k})} x_{\breve{i},j,\breve{k},l}. \tag{4.60}$$

$$P_{b_2}^{(2)} = \sum_{\breve{i}=0}^{C}\sum_{j=0}^{J(\breve{i})}\sum_{k=K_2}^{K(\breve{i})} x_{\breve{i},j,\breve{k}}\mathbf{1} = \sum_{\breve{i}=0}^{C}\sum_{j=0}^{J(\breve{i})}\sum_{k=K_2}^{K(\breve{i})}\sum_{l=0}^{L(\breve{k})} x_{\breve{i},j,\breve{k},l}. \tag{4.61}$$

As cell 1 and cell 2 are identical, new call arrival process for class 1 and class 2 calls are same in cell 1 and cell 2, cell residence time of class 1 and class 2 calls are identically distributed in both cells, we get $P_{hf_1}^{(2,1)} = P_{hf_1}^{(1,2)} = P_{hf_1}$, $P_{hf_2}^{(2,1)} = P_{hf_2}^{(1,2)} = P_{hf_2}$, $P_{b_1}^{(1)} = P_{b_1}^{(2)} = P_{b_1}$ and $P_{b_2}^{(1)} = P_{b_2}^{(2)} = P_{b_2}$.

We can find the mean number of calls in a cell for the two-cell system. If the mean number of calls in cell 1 and cell 2 are N_1 and N_2, respectively then

$$\overline{N_1} = \sum_{\breve{i}=0}^{C} \breve{i} x_{\breve{i}} \mathbf{1} = \sum_{\breve{i}=0}^{C} \sum_{j=0}^{J^{(\breve{i})}} \sum_{k=0}^{K^{(\breve{i})}} \sum_{l=0}^{L^{(\breve{k})}} \breve{i} x_{\breve{i},j,\breve{k},l}. \tag{4.62}$$

and

$$\overline{N_2} = \sum_{\breve{i}=0}^{C} \sum_{j=0}^{J^{(\breve{i})}} \sum_{k=0}^{K^{(\breve{i})}} \breve{k} x_{\breve{i},j,\breve{k}} \mathbf{1} = \sum_{\breve{i}=0}^{C} \sum_{j=0}^{J^{(\breve{i})}} \sum_{k=0}^{K^{(\breve{i})}} \sum_{l=0}^{L^{(\breve{k})}} \breve{k} x_{\breve{i},j,\breve{k},l}. \tag{4.63}$$

As both the cells are identical the mean number of calls in both cells are same which is $\overline{N} = \overline{N_1} = \overline{N_2}$. Therefore, the expected channel utilization for the two-cell system is $\frac{\overline{N}}{C}$.

4.3 An Adaptive Call Admission Control Scheme for Two-Cell and Multiple-Cell System

In this section, we develop an adaptive call admission control scheme for the two-cell system. The goal of this scheme is to achieve given quality of service (QoS) new call blocking and handoff failure probabilities for class 1 calls, QoS handoff failure probability for class 2 calls while minimizing the new call blocking probability of class 2 calls. For meeting these QoS goals in our adaptive scheme we try to ensure, with some probability, that if a call is considered for acceptance in a cell then the call gets a channel in the neighboring cell if it needs to handoff to the neighboring cell. Moreover, we guarantee, with some probability, that the acceptance of the new call in the target cell will not have adverse effect on the handoff call from the neighboring cell to the target cell of the candidate call. Two sets of constraints are proposed for acceptance of calls in the two cells of our system which implicitly prioritizes handoff calls over new calls. We assume that i and j represent the total numbers of class 1 and class 2 calls, respectively in cell 1 and k and l represent the total numbers of class 1 and class 2 calls respectively in cell 2. A new class 1 call is accepted in cell 1 with this set of rules when the following constraints are satisfied

- There is at least one free channel in cell 1 for accepting the new call, i.e., $i+j < C$.

- We can guarantee with at least probability $1 - P_{E_1}$ that the call will get a free channel in cell 2 if it needs to handoff.

Similarly, a new class 2 call is accepted in cell 1 when the following constraints are satisfied

- There is at least one free channel in cell 1 for accepting the new call, i.e., $i+j < C$.
- We can guarantee with at least probability $1 - P_{E_2}$ that the call will get a free channel in cell 2 if it needs to handoff.

We can compute the average probability of an accepted call in a cell getting a free channel in the neighboring cell if it needs to handoff. Transient analysis can be carried out to compute the average probabilities $\bar{P}_{E_1}(T)$ and $\bar{P}_{E_2}(T)$ of dropping a class 1 and class 2 candidate calls in cell 2 in T time ahead when a class 1 or class 2 call comes to cell 1 for acceptance. If the candidate call is accepted in cell 1 then we have to consider the remaining $C-1$ channels of cell 1 and all the channels of cell 2. We can consider a continuous time Markov chain for both cells like in Section 4.2. However, the cardinality of the state-space of our Markov chain for this adaptive scheme is $\frac{C(C+1)^2(C+2)}{4}$ which is different from the $\frac{(K_2+1)^2}{4}(3C^2 + (2K_1 - 4K_2 + 7)C - {K_1}^2 + {K_2}^2 + K_1 - 4K_2 + 4)$-element state space for the Markov chain of Section 4.2. The state space of this Markov chain for our adaptive scheme can be represented as $\{(\hat{\imath}, \hat{\jmath}, \hat{k}, \hat{l}) : 0 \leq \hat{\imath}, \hat{\jmath} \leq C-1, 0 \leq \hat{k}, \hat{l} \leq C\}$ where $\hat{\imath}$ and $\hat{\jmath}$ represent total number of class 1 and class 2 calls respectively in the $C-1$ channels of cell 1 and $\hat{k}$ and $\hat{l}$ represent total number of class 1 and class 2 calls respectively in C channels of cell 2.

In this continuous time Markov chain the transitions are similar to the transitions in Section 4.2 with the exception that there is no channel reservation. The following transitions can occur in this Markov chain

- Increase in the total number of class 1 calls in cell 1 due to arrival of a new class 1 call to cell 1, i.e., $(\hat{\imath}, \hat{\jmath}, \hat{k}, \hat{l}) \rightarrow (\hat{\imath}+1, \hat{\jmath}, \hat{k}, \hat{l})$ with rate λ_1 for $0 \leq \hat{\imath} + \hat{\jmath} < C-1$.
- Increase in the total number of class 2 calls in cell 1 due to arrival of a new class 2 call to cell 1, i.e., $(\hat{\imath}, \hat{\jmath}, \hat{k}, \hat{l}) \rightarrow (\hat{\imath}, \hat{\jmath}+1, \hat{k}, \hat{l})$ with rate λ_2 for $0 \leq \hat{\imath} + \hat{\jmath} < C-1$.

- Increase in the total number of class 1 calls in cell 2 due to arrival of a new class 1 call to cell 2, i.e., $(\hat{\imath}, \hat{\jmath}, \hat{k}, \hat{l}) \rightarrow (\hat{\imath}, \hat{\jmath}, \hat{k}+1, \hat{l})$ with rate λ_1 for $0 \leq \hat{k}+\hat{l} < C$.

- Increase in the total number of class 2 calls in cell 2 due to arrival of a new class 2 call to cell 2, i.e., $(\hat{\imath}, \hat{\jmath}, \hat{k}, \hat{l}) \rightarrow (\hat{\imath}, \hat{\jmath}, \hat{k}, \hat{l}+1)$ with rate λ_2 for $0 \leq \hat{k}+\hat{l} < C$.

- Decrease in the total number of class 1 calls in cell 1 due to completion of a class 1 call in cell 1, i.e., $(\hat{\imath}, \hat{\jmath}, \hat{k}, \hat{l}) \rightarrow (\hat{\imath}-1, \hat{\jmath}, \hat{k}, \hat{l})$ with rate $\hat{\imath}\,\mu_1$ for $0 < \hat{\imath} \leq C-1$ and $0 < \hat{k}+\hat{l} < C$.

- Decrease in the total number of class 2 calls in cell 1 due to completion of a class 2 call in cell 1, i.e., $(\hat{\imath}, \hat{\jmath}, \hat{k}, \hat{l}) \rightarrow (\hat{\imath}, \hat{\jmath}-1, \hat{k}, \hat{l})$ with rate $\hat{\jmath}\,\mu_2$ for $0 < \hat{\jmath} \leq C-1$ and $0 < \hat{k}+\hat{l} < C$.

- Decrease in the total number of class 1 calls in cell 2 due to completion of a class 1 call in cell 2, i.e., $(\hat{\imath}, \hat{\jmath}, \hat{k}, \hat{l}) \rightarrow (\hat{\imath}, \hat{\jmath}, \hat{k}-1, \hat{l})$ with rate $\hat{k}\,\mu_1$ for $0 \leq \hat{\imath}+\hat{\jmath} < C-1$ and $0 < \hat{k} \leq C$.

- Decrease in the total number of class 2 calls in cell 2 due to completion of a class 2 call in cell 2, i.e., $(\hat{\imath}, \hat{\jmath}, \hat{k}, \hat{l}) \rightarrow (\hat{\imath}, \hat{\jmath}, \hat{k}, \hat{l}-1)$ with rate $\hat{l}\,\mu_2$ for $0 \leq \hat{\imath}+\hat{\jmath} < C-1$ and $0 < \hat{l} \leq C$.

- Increase in the total number of class 1 calls in cell 1 and decrease in the total number of class 1 calls in cell 2 due to arrival of a class 1 handoff call from cell 2 to cell 1, i.e., $(\hat{\imath}, \hat{\jmath}, \hat{k}, \hat{l}) \rightarrow (\hat{\imath}+1, \hat{\jmath}, \hat{k}-1, \hat{l})$ with rate $\hat{k}\eta_1$ for $0 \leq \hat{\imath}+\hat{\jmath} < C-1$ and $0 < \hat{k} \leq C$.

- Increase in the total number of class 2 calls in cell 1 and decrease in the total number of class 2 calls in cell 2 due to arrival of a class 2 handoff call from cell 2 to cell 1, i.e., $(\hat{\imath}, \hat{\jmath}, \hat{k}, \hat{l}) \rightarrow (\hat{\imath}, \hat{\jmath}+1, \hat{k}, \hat{l}-1)$ with rate $l\eta_2$ for $0 \leq \hat{\imath}+\hat{\jmath} < C-1$ and $0 < \hat{l} \leq C$.

- Increase in the total number of class 1 calls in cell 2 and decrease in the total number of class 1 calls in cell 1 due to arrival of a class 1 handoff call from cell 1 to cell 2, i.e., $(\hat{\imath}, \hat{\jmath}, \hat{k}, \hat{l}) \rightarrow (\hat{\imath}-1, \hat{\jmath}, \hat{k}+1, \hat{l})$ with rate $\hat{\imath}\eta_1$ for $0 < \hat{\imath} \leq C-1$ and $0 \leq \hat{k}+\hat{l} < C$.

- Increase in the total number of class 2 calls in cell 2 and decrease in the total number of class 2 calls in cell 1 due to arrival of a class 2 handoff call from cell 1 to cell 2, i.e., $(\hat{\imath},\hat{\jmath},\hat{k},\hat{l}) \to (\hat{\imath},\hat{\jmath}-1,\hat{k},\hat{l}+1)$ with rate $\hat{\jmath}\eta_2$ for $0 < \hat{\jmath} \leq C-1$ and $0 \leq \hat{k}+\hat{l} < C$.

- Decrease in the total number of class 1 calls in cell 1 due to completion of a class 1 call in cell 1 or handoff failure for a class 1 call from cell 1 to cell 2 as a user with ongoing class 1 call moves from cell 1 to cell 2 but all the channels in cell 2 are occupied, i.e., $(\hat{\imath},\hat{\jmath},\hat{k},\hat{l}) \to (\hat{\imath}-1,\hat{\jmath},\hat{k},\hat{l})$ with rate $\hat{\imath}\,\mu_{c_1}$ for $0 < \hat{\imath} \leq C-1$ and $\hat{k}+\hat{l} = C$.

- Decrease in the total number of class 2 calls in cell 1 due to completion of a class 2 call in cell 1 or handoff failure for a class 2 call from cell 1 to cell 2 as a user with ongoing class 2 call moves from cell 1 to cell 2 but all the channels in cell 2 are occupied, i.e., $(\hat{\imath},\hat{\jmath},\hat{k},\hat{l}) \to (\hat{\imath},\hat{\jmath}-1,\hat{k},\hat{l})$ with rate $\hat{\jmath}\,\mu_{c_2}$ for $0 < \hat{\jmath} \leq C-1$ and $\hat{k}+\hat{l} = C$.

- Decrease in the total number of class 1 calls in cell 2 due to completion of a class 1 call in cell 2 or handoff failure for a class 1 call from cell 2 to cell 1 as a user with ongoing class 1 call moves from cell 2 to cell 1 but all the channels in cell 2 are occupied, i.e., $(\hat{\imath},\hat{\jmath},\hat{k},\hat{l}) \to (\hat{\imath},\hat{\jmath},\hat{k}-1,\hat{l})$ with rate $\hat{k}\,\mu_{c_1}$ for $\hat{\imath}+\hat{\jmath} = C-1$ and $0 < \hat{k} \leq C$.

- Decrease in the total number of class 2 calls in cell 2 due to completion of a class 2 call in cell 2 or handoff failure for a class 2 call from cell 2 to cell 1 as a user with ongoing class 2 call moves from cell 2 to cell 1 but all the channels in cell 2 are occupied, i.e., $(\hat{\imath},\hat{\jmath},\hat{k},\hat{l}) \to (\hat{\imath},\hat{\jmath},\hat{k},\hat{l}-1)$ with rate $\hat{l}\,\mu_{c_2}$ for $\hat{\imath}+\hat{\jmath} = C-1$ and $0 < \hat{l} \leq C$.

We can represent the above continuous time Markov chain using the following generator matrix

$$\tilde{Q} = \begin{bmatrix} \tilde{B}_0 & \tilde{U}_0 & & & & \\ \tilde{D}_1 & \tilde{B}_1 & \tilde{U}_1 & & & \\ & \tilde{D}_2 & \tilde{B}_2 & \tilde{U}_2 & & \\ & & \ddots & \ddots & \ddots & \\ & & & \tilde{D}_{C-2} & \tilde{B}_{C-2} & \tilde{U}_{C-2} \\ & & & & \tilde{D}_{C-1} & \tilde{B}_{C-1} \end{bmatrix}. \tag{4.64}$$

Here, $\tilde{B}_{\hat{\imath}}$ $(0 \le \hat{\imath} \le C-1)$ is a square matrix of order $\frac{(C+1)(C+2)(C-\hat{\imath})}{2}$, $\tilde{U}_{\hat{\imath}}$ $(0 \le \hat{\imath} \le C-2)$ is a $\frac{(C+1)(C+2)(C-\hat{\imath})}{2} \times \frac{(C+1)(C+2)(C-\hat{\imath}-1)}{2}$ matrix and $\tilde{D}_{\hat{\imath}}$ $(1 \le \hat{\imath} \le C-1)$ is a $\frac{(C+1)(C+2)(C-\hat{\imath})}{2} \times \frac{(C+1)(C+2)(C-\hat{\imath}+1)}{2}$ matrix. We use three tuples $(\hat{\jmath}, \hat{k}, \hat{l})$ to represent the row and column indices for block matrices $\tilde{B}_{\hat{\imath}}$'s, $\tilde{U}_{\hat{\imath}}$'s and $\tilde{D}_{\hat{\imath}}$'s where $\hat{\jmath}$, $\hat{k}$ and l are already defined. We can compute the distance between $(\hat{\jmath}, \hat{k}, \hat{l})$ from $(0,0,0)$ as $\hat{\jmath}\frac{(C+1)(C+2)}{2} + \sum_{n=0}^{\hat{k}-1}(C-n+1) + \hat{l}$. $\tilde{B}_{\hat{\imath}}((\hat{\jmath}_1, \hat{k}_1, \hat{l}_1), (\hat{\jmath}_2, \hat{k}_2, \hat{l}_2))$ denotes no change in the number of class 1 calls in cell 1 when the number of class 1 calls in cell 1 is $\hat{\imath}$; before transition the number of class 2 calls in cell 1 is $\hat{\jmath}_1$, and the numbers of class 1 and class 2 calls in cell 2 are $\hat{k}_1$ and $\hat{l}_1$, respectively whereas after transition the number of class 2 calls in cell 1 is $\hat{\jmath}_2$, and the numbers of class 1 and class 2 calls in cell 2 are $\hat{k}_2$ and $\hat{l}_2$ respectively. $\tilde{U}_{\hat{\imath}}((\hat{\jmath}_1, \hat{k}_1, \hat{l}_1), (\hat{\jmath}_2, \hat{k}_2, \hat{l}_2))$ denotes increase in the number of class 1 calls in cell 1 by one when the number of class 1 calls in cell 1 is $\hat{\imath}$; before transition the number of class 2 calls in cell 1 is $\hat{\jmath}_1$, and the numbers of class 1 and class 2 calls in cell 2 are $\hat{k}_1$ and $\hat{l}_1$, respectively whereas after transition the number of class 2 calls in cell 1 is $\hat{\jmath}_2$, and the numbers of class 1 and class 2 calls in cell 2 are $\hat{k}_2$ and $\hat{l}_2$, respectively. $\tilde{D}_{\hat{\imath}}((\hat{\jmath}_1, \hat{k}_1, \hat{l}_1), (\hat{\jmath}_2, \hat{k}_2, \hat{l}_2))$ denotes decrease in the number of class 1 calls in cell 1 by one when the number of class 1 calls in cell 1 is $\hat{\imath}$; before transition the number of class 2 calls in cell 1 is $\hat{\jmath}_1$, and the numbers of class 1 and class 2 calls in cell 2 are $\hat{k}_1$ and $\hat{l}_1$, respectively whereas after transition the number of class 2 calls in cell 1 is $\hat{\jmath}_2$, and the numbers of class 1 and class 2 calls in cell 2 are $\hat{k}_2$ and $\hat{l}_2$, respectively. The nonzero diagonal entries of $\tilde{B}_{\hat{\imath}}$ are shown in Table 4.4. The nonzero nondiagonal entries of $\tilde{B}_{\hat{\imath}}$ and nonzero entries of $\tilde{U}_{\hat{\imath}}$ and $\tilde{D}_{\hat{\imath}}$ are given below

$$B_{\hat{\imath}}((\hat{\jmath}, \hat{k}, \hat{l}), (\hat{\jmath}+1, \hat{k}, \hat{l})) = \lambda_2, \;\; 0 \le \hat{\jmath} < C - \hat{\imath} - 1. \tag{4.66}$$

Table 4.4: Nonzero diagonal entries of $B_{\hat{\imath}}$

$$B_{\hat{\imath}}((\hat{\jmath},\hat{k},\hat{l}),(\hat{\jmath},\hat{k},\hat{l})) = \begin{cases} -(2(\lambda_1+\lambda_2)+(\hat{\imath}+\hat{k})\,\mu_{c_1}+(\hat{\jmath}+\hat{l})\,\mu_{c_2}), & 0 \le \hat{\imath}+\hat{\jmath} < C-1, 0 \le \hat{k}+\hat{l} < C \\ -((\lambda_1+\lambda_2)+(\hat{\imath}+\hat{k})\,\mu_{c_1}+(\hat{\jmath}+\hat{l})\,\mu_{c_2}), & 0 \le \hat{\imath}+\hat{\jmath} < C-1, \hat{k}+\hat{l} = C \\ -((\lambda_1+\lambda_2)+(\hat{\imath}+\hat{k})\,\mu_{c_1}+(\hat{\jmath}+\hat{l})\,\mu_{c_2}), & \hat{\imath}+\hat{\jmath} = C-1, 0 \le \hat{k}+\hat{l} < C \\ -((\hat{\imath}+\hat{k})\,\mu_{c_1}+(\hat{\jmath}+\hat{l})\,\mu_{c_2}), & \hat{\imath}+\hat{\jmath} = C-1, \hat{k}+\hat{l} = C \end{cases} \tag{4.65}$$

$$B_{\hat{\imath}}((\hat{\jmath},\hat{k},\hat{l}),(\hat{\jmath},\hat{k}+1,\hat{l})) = \lambda_1,\ 0 \le \hat{k} < C - \hat{l}. \tag{4.67}$$

$$B_{\hat{\imath}}((\hat{\jmath},\hat{k},\hat{l}),(\hat{\jmath},\hat{k},\hat{l}+1)) = \lambda_2,\ 0 \le \hat{l} < C - \hat{k}. \tag{4.68}$$

$$B_{\hat{\imath}}((\hat{\jmath},\hat{k},\hat{l}),(\hat{\jmath}+1,\hat{k},\hat{l}-1)) = l\eta_2,\ 0 < \hat{l} \le C - \hat{k}, 0 \le \hat{\jmath} < C - \hat{\imath} - 1. \tag{4.69}$$

$$B_{\hat{\imath}}((\hat{\jmath},\hat{k},\hat{l}),(\hat{\jmath}-1,\hat{k},\hat{l}+1)) = \hat{\jmath}\eta_2,\ 0 < \hat{\jmath} \le C - \hat{\imath} - 1, 0 \le \hat{l} < C - \hat{k}. \tag{4.70}$$

$$B_{\hat{\imath}}((\hat{\jmath},\hat{k},\hat{l}),(\hat{\jmath}-1,\hat{k},\hat{l})) = \begin{cases} \hat{\jmath}\,\mu_2, & 0 < \hat{\jmath} < C - \hat{\imath} - 1, 0 \le \hat{k}+\hat{l} < C \\ \hat{\jmath}\,\mu_{c2}, & 0 < \hat{\jmath} < C - \hat{\imath} - 1, \hat{k}+\hat{l} = C \end{cases} \tag{4.71}$$

$$B_{\hat{\imath}}((\hat{\jmath},\hat{k},\hat{l}),(\hat{\jmath},\hat{k}-1,\hat{l})) = \begin{cases} \hat{k}\,\mu_1, & 0 < \hat{k} \le C, 0 \le \hat{\imath}+\hat{\jmath} < C - 1 \\ \hat{k}\,\mu_{c1}, & 0 < \hat{k} \le C, \hat{\imath}+\hat{\jmath} = C - 1 \end{cases} \tag{4.72}$$

$$B_{\hat{\imath}}((\hat{\jmath},\hat{k},\hat{l}),(\hat{\jmath},\hat{k},\hat{l}-1)) = \begin{cases} \hat{l}\,\mu_2, & 0 < l \le C - \hat{k}, 0 \le \hat{\imath}+\hat{\jmath} < C - 1 \\ \hat{l}\,\mu_{c2}, & 0 < \hat{l} \le C, \hat{\imath}+\hat{\jmath} = C - 1 \end{cases} \tag{4.73}$$

$$U_{\hat{\imath}}((\hat{\jmath},\hat{k},\hat{l}),(\hat{\jmath},\hat{k},\hat{l})) = \lambda_1,\ 0 \le \hat{\jmath} < C - \hat{\imath} - 1. \tag{4.74}$$

$$U_{\hat{\imath}}((\hat{\jmath},\hat{k},\hat{l}),(\hat{\jmath},\hat{k}-1,\hat{l})) = \hat{k}\eta_1,\ 0 < \hat{k} \le C, 0 \le \hat{\jmath} < C - \hat{\imath} - 1. \tag{4.75}$$

$$D_{\hat{\imath}}((\hat{\jmath},\hat{k},\hat{l}),(\hat{\jmath},\hat{k},\hat{l})) = \begin{cases} \hat{\imath}\,\mu_1, & 0 < \hat{\imath} \le C - 1, 0 \le \hat{k}+\hat{l} < C \\ \hat{\imath}\,\mu_{c1}, & 0 < \hat{\imath} \le C - 1, \hat{k}+\hat{l} = C \end{cases} \tag{4.76}$$

$$D_{\hat{\imath}}((\hat{\jmath},\hat{k},\hat{l}),(\hat{\jmath},\hat{k}+1,\hat{l})) = \hat{\imath}\eta_1,\ 0 < \hat{\imath} \le C - 1, 0 < \hat{k}+\hat{l} \le C. \tag{4.77}$$

Let $\tilde{x}(0)$ and $\tilde{x}(t)$ be row-vectors containing $\frac{C(C+1)^2(C+2)}{4}$ scalar element representing the state of the two-cell system at time 0 and t, respectively for $(C-1)$ channels in cell 1 and C channels for cell 2. In the model for admission control, we can express $\tilde{x}_{<r_0 \to r>}(0,t) = Pr\{\tilde{x}(t) = r \mid \tilde{x}(0) = r_0\}$ as the probability that the system is in state r at time t given the initial state r_0 at time zero. If we consider the system at time t, we have $\tilde{x}_{<r>}(t) = Pr\{\tilde{x}(t) = r\}$ which is conditioned on the initial state. These transient probabilities are the elements of $\tilde{x}(t) = [\tilde{x}_0(t), \tilde{x}_1(t), \ldots, \tilde{x}_{C-1}(t)]$ where $\tilde{x}_{\hat{i}}(t)$ $(0 \le \hat{i} \le C-1)$ is a row vector containing $\frac{(C+1)(C+2)(C-\hat{i})}{2}$ scalar elements which represents the probability vector for total $\hat{i}$ class 1 calls in the $C-1$ channels of cell 1 at time t and $\tilde{x}_{\hat{i}}(t) = [\tilde{x}_{\hat{i},0}(t), \tilde{x}_{\hat{i},1}(t), \ldots, \tilde{x}_{\hat{i},C-\hat{i}-1}(t)]$ $(0 \le \hat{i} \le C-1)$. Here, each $\tilde{x}_{\hat{i},\hat{j}}(t)$ $(0 \le \hat{i} \le C-1, 0 \le \hat{j} \le C-\hat{i}-1)$ is a row-vector having $\frac{(C+1)(C+2)}{2}$ scalar elements which represents $\hat{i}$ and $\hat{j}$ ongoing calls in $C-1$ channels of cell 1. $\tilde{x}_{\hat{i},\hat{j}}(t)$ can be represented as $\tilde{x}_{\hat{i},\hat{j}}(t) = [\tilde{x}_{\hat{i},\hat{j},0}(t), \tilde{x}_{\hat{i},\hat{j},1}(t), \ldots, \tilde{x}_{\hat{i},\hat{j},C}(t)]$ where $\tilde{x}_{\hat{i},\hat{j},\hat{k}}(t)$ $(0 \le \hat{i} \le C-1, 0 \le \hat{j} \le C-\hat{i}-1, 0 \le \hat{k} \le C)$ is a row vector having $C+1-\hat{k}$ scalar elements which represents $\hat{i}$ class 1 and $\hat{j}$ class 2 calls in $(C-1)$ channels of cell 1 and $\hat{k}$ class 1 calls in C channels of cell 2. $\tilde{x}_{\hat{i},\hat{j},\hat{k}}(t)$ can be represented as $\tilde{x}_{\hat{i},\hat{j},\hat{k}}(t) = [\tilde{x}_{\hat{i},\hat{j},\hat{k},0}(t), \tilde{x}_{\hat{i},\hat{j},\hat{k},0}(t), \ldots, \tilde{x}_{\hat{i},\hat{j},\hat{k},C-\hat{k}}(t)]$. To calculate $\tilde{x}(t)$, we have to solve the Kolmogorov-forward equations which can be expressed in the matrix form as follows:

$$\frac{\mathrm{d}\tilde{x}(t)}{\mathrm{d}t} = \tilde{x}(t)\tilde{Q}. \tag{4.78}$$

If we solve Equation (4.78) we get

$$\tilde{x}(t) = \tilde{x}(0)e^{\tilde{Q}t}. \tag{4.79}$$

If the candidate call at time 0 sees $\hat{i}$ class 1 and $\hat{j}$ class 2 calls in cell 1 and $\hat{k}$ class 1 and $\hat{l}$ class 2 calls in cell 2 then $(\sum_{\check{i}=0}^{\hat{i}-1} \frac{(C+1)(C+2)(C-\check{i})}{2} + \hat{j}\frac{(C+1)(C+2)}{2} + \sum_{\check{k}=0}^{\hat{k}-1}(C-\check{k}+1)+\hat{l})$-th element (with the assumption of counting begins from the starting position as 0-th position) of $\tilde{x}(0)$ is one and all other elements of $\tilde{x}(0)$ are zero. Now, exploiting the special structure of $\tilde{x}(0)$ we can determine $\tilde{x}(t)$ which is the $(\sum_{\check{i}=0}^{\hat{i}-1} \frac{(C+1)(C+2)(C-\check{i})}{2} + \hat{j}\frac{(C+1)(C+2)}{2} + \sum_{\check{k}=0}^{\hat{k}-1}(C-\check{k}+1)+\hat{l})$-th row of $e^{\tilde{Q}t}$. The term $e^{\tilde{Q}t}$ is not computed using formula of exponential function which is $e^{\tilde{Q}t} = I + \sum_{k=1}^{\infty} \frac{(\tilde{Q}t)^k}{k!}$ because $\tilde{Q}$ has negative elements in the diagonal which makes the computation of $e^{\tilde{Q}t}$ unstable as this computation involves repeated multiplication and addition of negative and positive quantities. Therefore, uniformization technique of Jensen [48] is used for computing $e^{\tilde{Q}t}$. The method [48] is a general and efficient way to obtain the solution from the above Equation (4.79) as it does not involve repeated multiplication and addition of negative and positive quantities. In this technique, the new transition probability matrix P is defined as

$$P = \frac{\tilde{Q}}{\Lambda} + I, \tag{4.80}$$

where Λ is at least equal to the maximum of the absolute values of the diagonal elements of $\tilde{Q}$ matrix. We use $\Lambda = 2(\lambda_1 + \lambda_2) + (2C - 1)(\mu_{c_1} + \mu_{c_2})$ which is greater than the maximum of the absolute values of the diagonal elements of $\tilde{Q}$ matrix. Now, using Equations (4.79) and (4.80) we get

$$\begin{aligned} \tilde{x}(t) &= \tilde{x}(0)e^{(P-I)\Lambda t} \\ &= \tilde{x}(0)e^{P\Lambda t}e^{-\Lambda t} \\ &= \tilde{x}(0)\textstyle\sum_{n=0}^{\infty} P^n \frac{(\Lambda t)^n}{n!} e^{-\Lambda t}. \end{aligned} \tag{4.81}$$

The infinite summation in Equation (4.81) is truncated at some value $n = M$ such that the truncation error remains below ε which can be expressed mathematically as

$$\tilde{x}(t) = \tilde{x}(0)\sum_{n=0}^{M} P^n \frac{(\Lambda t)^n}{n!} e^{-\Lambda t}, \tag{4.82}$$

where $\sum_{n=M+1}^{\infty} e^{-\Lambda t}\frac{(\Lambda t)^n}{n!} < \varepsilon$.

The probabilities of an accepted class 1 call and class 2 call in cell 1 need handoff at time t are respectively $(1 - e^{-\eta_1 t})e^{-\mu_1 t}$ and $(1 - e^{-\eta_2 t})e^{-\mu_2 t}$. Now, the probability of all the channels in cell 2 being occupied at time t is $\sum_{\hat{i}=0}^{C-1}\sum_{\hat{j}=0}^{C-\hat{i}-1}\sum_{\hat{k}=0}^{C}\tilde{x}_{\hat{i},\hat{j},\hat{k},C-\hat{k}}(t)$. Therefore, the class 1 and class 2 candidate call accepted at cell 1 will be dropped at time t if at that moment the call needs to handoff to cell 2 and finds all the channels occupied. The dropping occurs with probabilities $p_{e_1}(t)$ and $p_{e_2}(t)$ for class 1 and class 2 candidate calls, respectively where

$$p_{e_1}(t) = e^{-\mu_1 t}(1 - e^{-\eta_1 t})\sum_{\hat{i}=0}^{C-1}\sum_{\hat{j}=0}^{C-\hat{i}-1}\sum_{\hat{k}=0}^{C}\tilde{x}_{\hat{i},\hat{j},\hat{k},C-\hat{k}}(t)$$

and

$$p_{e_2}(t) = e^{-\mu_2 t}(1 - e^{-\eta_2 t})\sum_{\hat{i}=0}^{C-1}\sum_{\hat{j}=0}^{C-\hat{i}-1}\sum_{\hat{k}=0}^{C}\tilde{x}_{\hat{i},\hat{j},\hat{k},C-\hat{k}}(t).$$

Now, the average probabilities $\bar{P}_{E_1}(T)$ of a class 1 accepted call and $\bar{P}_{E_1}(T)$ of class 2 accepted call in cell 1 of not finding any empty channel in cell 2 if the accepted call requires handoff are given by respectively

$$\bar{P}_{E_1}(T) = \frac{1}{T}\int_0^T p_{e_1}(t)\mathrm{d}t. \tag{4.83}$$

$$\bar{P}_{E_2}(T) = \frac{1}{T}\int_0^T p_{e_2}(t)\mathrm{d}t. \tag{4.84}$$

Equations (4.83) and (4.84) are the expressions for average dropping probability of a class 1 and a class 2 candidate calls, respectively in the neighboring cell. The implementation of these integral expressions and their implementation in the two-cell system are described in Section 4.3.1. The extension of this adaptive scheme for multiple-cell wireless network is explained in Section 4.3.2.

4.3.1 Implementation of Adaptive Call Admission Control Scheme for Two Classes of Calls in Two-Cell System

The integrations of Equations (4.83) and (4.84) are computed numerically using Simpson's method of numerical integration. In this method, the range [0, T] is divided into r subintervals of equal length $l = \frac{T}{r}$ where r is an even number. The numerical integrations can be expressed as

$$\begin{aligned}\bar{P}_{E_1}(T) &= \tfrac{l}{3T}(f_{1_0} + 4f_{1_1} + 2f_{1_2} + 4f_{1_3} + 2f_{1_4} \\ &\quad + \cdots + 2f_{1_{r-2}} + 4f_{1_{r-1}} + f_{1_r}) \\ &= \tfrac{1}{3r}(f_{1_0} + 4f_{1_1} + 2f_{1_2} + 4f_{1_3} + 2f_{1_4} \\ &\quad + \cdots + 2f_{1_{r-2}} + 4f_{1_{r-1}} + f_{1_r}),\end{aligned} \tag{4.85}$$

$$\begin{aligned}\bar{P}_{E_2}(T) &= \tfrac{l}{3T}(f_{2_0} + 4f_{2_1} + 2f_{2_2} + 4f_{2_3} + 2f_{2_4} \\ &\quad + \cdots + 2f_{2_{r-2}} + 4f_{2_{r-1}} + f_{2_r}) \\ &= \tfrac{1}{3r}(f_{2_0} + 4f_{2_1} + 2f_{2_2} + 4f_{2_3} + 2f_{2_4} \\ &\quad + \cdots + 2f_{2_{r-2}} + 4f_{2_{r-1}} + f_{2_r}),\end{aligned} \tag{4.86}$$

where $f_{1_k} = p_{e_1}(\frac{kT}{r})$ and $f_{2_k} = p_{e_2}(\frac{kT}{r})$ for $0 \le k \le r$.

There are $\frac{C(C+1)^2(C+2)}{4}$ different values that $\tilde{x}(0)$ can have. For different $\frac{C(C+1)^2(C+2)}{4}$ values of $\tilde{x}(0)$ we can compute values of $\bar{P}_{E_1}(T)$ and $\bar{P}_{E_2}(T)$. The threshold values P_{E_1}

and P_{E_2} are two of the values of $\frac{C(C+1)^2(C+2)}{4}$ values for different initial states of $\tilde{x}(0)$ for class 1 and class 2 candidate calls, respectively. We can determine these two threshold values of P_{E_1} and P_{E_2} for accepting a new class 1 call and a new class 2 call by running simulation experiments with the goal of achieving our QoS handoff failure probabilities for both classes of calls and QoS new call blocking probability for class 1 calls while minimizing the new call blocking probability of class 2 calls. The admission controller can keep two tables for $\bar{P}_{E_1}(T)$ and $\bar{P}_{E_2}(T)$ and the threshold values of P_{E_1} and P_{E_2} in memory. Whenever a new class 1 call comes to cell 1 then the admission controller needs to check whether the $(\sum_{\check{i}=0}^{\hat{i}-1} \frac{(C+1)(C+2)(C-\check{i})}{2} + \hat{j}\frac{(C+1)(C+2)}{2} + \sum_{\check{k}=0}^{\hat{k}-1}(C-\check{k}+1)+\hat{l})$-th entry of the table storing values of $\bar{P}_{E_1}(T)$ is not more than P_{E_1}. On the other hand, if a new class 2 call comes to cell 1 then the admission controller needs to check whether the $(\sum_{\check{i}=0}^{\hat{i}-1} \frac{(C+1)(C+2)(C-\check{i})}{2} + \hat{j}\frac{(C+1)(C+2)}{2} + \sum_{\check{k}=0}^{\hat{k}-1}(C-\check{k}+1)+\hat{l})$-th entry of the table storing values of $\bar{P}_{E_2}(T)$ is not more than P_{E_2}. If these conditions are satisfied the candidate calls are accepted in cell 1. On the other hand, when a call comes to cell 2 it needs to check the $(\sum_{\check{k}=0}^{\hat{k}-1} \frac{(C+1)(C+2)(C-\check{k})}{2} + \hat{l}\frac{(C+1)(C+2)}{2} + \sum_{\check{i}=0}^{\hat{i}-1}(C-\check{i}+1)+\hat{j})$-th entry of the table storing values of $\bar{P}_{E_1}(T)$ is not more than P_{E_1} for class 1 candidate call and $\bar{P}_{E_2}(T)$ is not more than P_{E_2} for class 2 candidate call. If these conditions are satisfied the candidate calls are accepted in cell 2. We conjecture that acceptance of new call in our model guarantees a successful handoff for the accepted call with some probability and satisfaction of QoS for both classes of calls. Static schemes fail to guarantee this handoff in a cell for different number of calls in the neighboring cells.

The advantage of our model is that we can compute $\bar{P}_{E_1}(T)$ and $\bar{P}_{E_2}(T)$ in T time ahead for different values of $\tilde{x}(0)$ and keep these values in two tables. Whenever a new call attempt is made in a cell the call admission control algorithm needs to get the value of $\bar{P}_{E_1}$ or $\bar{P}_{E_2}$ for decision about acceptance or rejection of the call.

4.3.2 Extension to Multiple-Cell Wireless Network

We can extend our adaptive call admission control scheme for two-cell system to multiple-cell wireless network. Here, each cell contains C channels, class 1 and class 2 new call inter-arrival time to each cell is exponentially distributed with means $\frac{1}{\lambda_{n_1}}$ and $\frac{1}{\lambda_{n_2}}$, respectively, cell residence times of class 1 and class 2 calls in each cell are exponen-

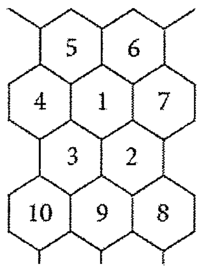

Figure 4.1: Hexagonal structure of cellular wireless network

tially distributed with means $\frac{1}{\eta_1}$ and $\frac{1}{\eta_2}$, respectively and call holding durations of class 1 and class 2 calls are exponentially distributed with means $\frac{1}{\mu_1}$ and $\frac{1}{\mu_2}$, respectively. In this approximation let us assume that our target cell F_t has i and j numbers of class 1 and class 2 calls and the target cell has $\mathfrak{N}$ immediate neighbors $F_{n_1}, F_{n_2}, \ldots, F_{n_\mathfrak{N}}$ having $r_{n_1}, r_{n_2}, \ldots, r_{n_\mathfrak{N}}$ class 1 calls, respectively. Similarly, the immediate neighbors $F_{n_1}, F_{n_2}, \ldots, F_{n_\mathfrak{N}}$ have $s_{n_1}, s_{n_2}, \ldots, s_{n_\mathfrak{N}}$ class 2 calls, respectively. A new call is blocked if $i + j = C$. On the other hand, we can use the values in the table for accepting a call in the target cell for $i + j < C$ and k is the nearest integer of $\frac{\sum_{a=1}^{\mathfrak{N}} r_{n_a}}{\mathfrak{N}}$ and l is the nearest integer of $\frac{\sum_{a=1}^{\mathfrak{N}} s_{n_a}}{\mathfrak{N}}$. The table contains values for probability of dropping a candidate call in the target cell if it needs to handoff to the neighboring cell. We need to determine the threshold value P_{E_1} and P_{E_2} by carrying out simulation to achieve the QoS handoff failure probabilities for class 1 and class 2 calls and QoS new call blocking probabilities for class 1 calls. When a call comes to the target cell with at least one free channel we use $\hat{i} = i$, $\hat{j} = j$, $\hat{k} = k$ and $\hat{l} = l$. The admission controller needs to check whether the $(\sum_{\check{i}=0}^{\hat{i}-1} \frac{(C+1)(C+2)(C-\check{i})}{2} + \hat{j}\frac{(C+1)(C+2)}{2} + \sum_{\check{k}=0}^{\hat{k}-1}(C-\check{k}+1) + \hat{l})$-th entry of the table storing values of $\bar{P}_{E_1}(T)$ is not more than P_{E_1} for class 1 new call. Similarly, for class 2 new call the admission controller needs to check whether the $(\sum_{\check{i}=0}^{\hat{i}-1} \frac{(C+1)(C+2)(C-\check{i})}{2} + \hat{j}\frac{(C+1)(C+2)}{2} + \sum_{\check{k}=0}^{\hat{k}-1}(C-\check{k}+1) + \hat{l})$-th entry of the table storing values of $\bar{P}_{E_2}(T)$ is not more than P_{E_2}. In lower class 1 and class 2 load the handoff failure probabilities are very low and no prioritization is required for both classes of handoff calls and class 1 new calls. As a result, the value of P_{E_1} is the highest value of the values of handoff failure probabilities of class 1 candidate call in the neighboring cell stored in the table of class 1 calls and the value of P_{E_2} is the highest value of the values of handoff failure probabilities of class 2 candidate call in the neighboring cell stored in the table of class 2 calls. If the condition for a call is satisfied the call is accepted in the target cell. Otherwise, the new call is blocked. An example of hexagonal cell structure of cellular wireless network is shown in Figure 4.1. If a new call comes to cell 1 then cell 1 is the target cell. If there is at least one free channel in cell 1 then i is the value of the number of class 1 calls and j is the number of class 2 calls in cell 1. The values of k and l are computed as the nearest integers of the average numbers of class 1 and class 2 calls in the immediate neighboring cells 2, 3, 4, 5, 6 and 7. We show the adaptive call admission control algorithm for target cell in the multiple-cell system using the following pseudo-code.

```
Adaptive Algorithm
if(handoff call)
    if(i + j < C)
        accept the handoff call;
    else /*if(i + j = C)*/
        reject the call;
else /* if(new call) */
    if(i + j = C)
        reject the call
    else /* if(i + j < C) */
        S_1 ← 0;
        S_2 ← 0;
        for a ← 1 to 𝔑 do
            S_1 ← S_1 + r_{n_a};
            S_2 ← S_2 + s_{n_a};
        k ← nearest integer of S_1/𝔑;
        l ← nearest integer of S_2/𝔑;
        if(class 1 new call)
            Get (∑_{ǐ=0}^{i-1} (C+1)(C+2)(C-ǐ)/2 + j(C+1)(C+2)/2 + ∑_{ǩ=0}^{k-1}(C - ǩ + 1) + ľ)-th entry
            P̄_{E_1}(T) from table;
            if(P̄_{E_1}(T) ≤ P_{E_1})
```

accept the call;

else /* if ($\bar{P}_{E_1}(T) > P_{E_1}$) */

reject the call;

else /* if(class 2 new call) */

Get ($\sum_{\check{i}=0}^{i-1} \frac{(C+1)(C+2)(C-\check{i})}{2} + j\frac{(C+1)(C+2)}{2} + \sum_{\check{k}=0}^{k-1}(C - \check{k} + 1) + \hat{l}$)-th entry

$\bar{P}_{E_2}(T)$ from table;

if($\bar{P}_{E_2}(T) \leq P_{E_2}$)

accept the call;

else /* if ($\bar{P}_{E_2}(T) > P_{E_2}$) */

reject the call;

End of Adaptive Algorithm

4.4 Numerical Examples

We carry out analytical studies for non-priority and DTR schemes and discrete event simulation studies for non-priority, DTR and adaptive call admission control schemes for two-cell system of Figure 3.2. Simulation studies are also performed for non-priority, DTR and adaptive call admission control schemes on the nineteen-cell (multiple-cell) wireless system of Figure 3.7.

In our analytical and simulation studies, we measure channel utilization of each cell and new call blocking probability, handoff failure probability and handoff arrival rate for both types of calls. We compare the performance of non-priority scheme and DTR scheme with our call admission control scheme. We assume that our goal is to minimize the new call blocking probability of class 2 calls while keeping the handoff failure probabilities of both class 1 and class 2 calls below 0.015 and the new call blocking probability of class 1 calls below 0.05.

We assume that there are four channels in each cell. The cell residence and call holding times for both class 1 and 2 calls are assumed to be exponentially distributed with means $\frac{1}{\eta_1} = \frac{1}{\eta_2} = 1.0$ unit time and $\frac{1}{\mu_1} = \frac{1}{\mu_2} = 0.25$ unit time, respectively. New call inter-arrival times for class 1 and class 2 calls are exponentially distributed. We carry out two sets of experiments for both the two-cell and nineteen-cell wireless networks. In the first set of experiments the mean call arrival rate of class 2 call is $\lambda_2 = 1.0/$ unit time. In the second set of experiments the mean call arrival rate of class 2 call is $\lambda_2 = 2.0/$ unit time. In both sets of experiments we vary the mean call arrival rate of class 1 calls from 1.0/ unit time to 5.0/ unit time. We use $T = 5.0$ unit time for numerical examples of adaptive system. For DTR scheme we use two thresholds $K_1 = 3$ and $K_2 = 2$, respectively. In the DTR scheme, a class 1 handoff call is accepted in a cell if there is at least one free channel in the cell, a new class 1 call is accepted in a cell if the number of ongoing calls in that cell is less than K_1 and a new or handoff class 2 call is accepted in a cell if the number ongoing call in that cell is less than K_2. 2000000.0 unit time is chosen as warm-up time in simulation. Ten independent replications of length 1000000.0 unit time are carried out. We discuss numerical examples for two-cell and multiple-cell wireless networks in Sections 4.4.1 and 4.4.2, respectively which are followed by sensitivity analysis of the results on the value of T in Section 4.4.3. In the figures, we denote analytical and simulation results of non-priority system by "Nonpriority Analytical" and "Nonpriority Simulation" respectively. Similarly, analytical and simulation results of the DTR scheme are denoted by "DTR Analytical" and "DTR Simulation" in the figures. In the same manner, simulation result of adaptive system is denoted as "Adaptive Simulation".

4.4.1 Numerical Result for Two-Cell System

Analytical and simulation results for class 1 new call blocking probability, class 2 new call blocking probability, class 1 handoff failure probability, class 2 handoff failure probability, channel utilization, class 1 handoff call arrival rate and class 2 handoff call arrival rate are shown in Figures 4.2, 4.3, 4.4, 4.5, 4.10, 4.11 and 4.12, respectively for first set of experiments. Similarly, analytical and simulation results for class 1 new call blocking probability, class 2 new call blocking probability, class 1 handoff failure probability, class 2 handoff failure probability, channel utilization, class 1 handoff call arrival rate and class 2 handoff call arrival rate are shown in Figures 4.6, 4.7, 4.8, 4.9, 4.13, 4.14 and 4.15, respectively for second set of experiments. We can see that our analytical and simulation results agree well for non-priority scheme from Figures 4.2-4.15. We can see from Figures 4.4 and 4.8 that our adaptive scheme and DTR scheme can maintain class 1 handoff failure probability below 0.015 for the range of class 1 new call arrival

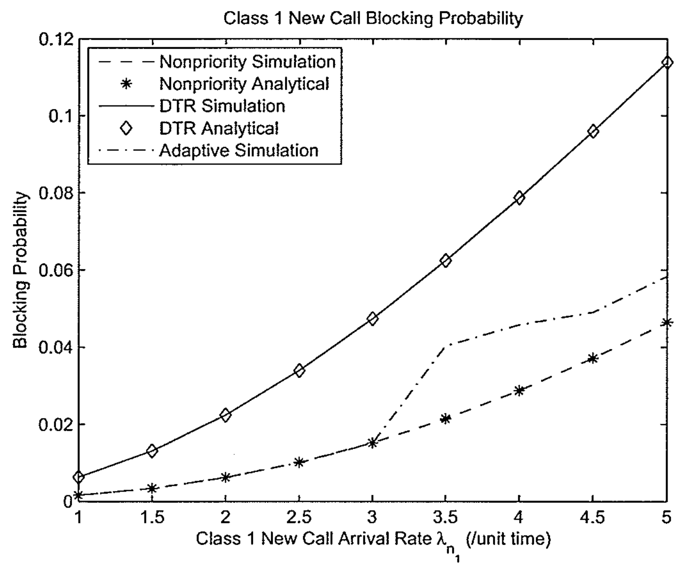

Figure 4.2: Analytical and simulation results of class 1 new call blocking probability in the two-cell system for one call per unit time for class 2 new calls

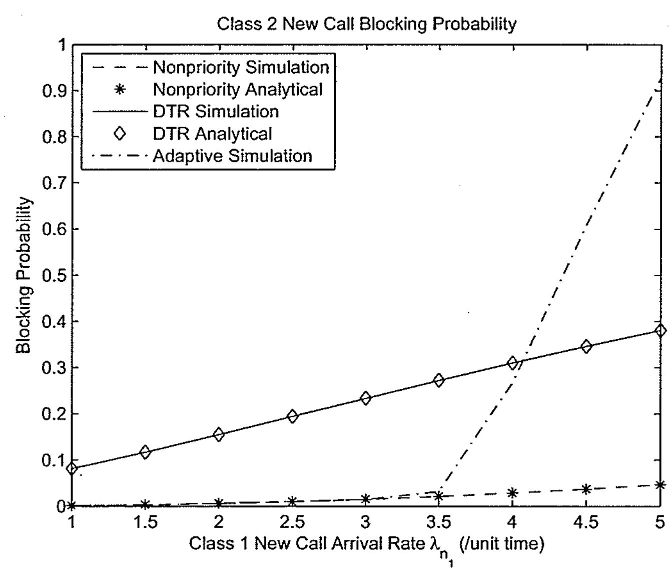

Figure 4.3: Analytical and simulation results of class 2 new call blocking probability in the two-cell system for one call per unit time for class 2 new calls

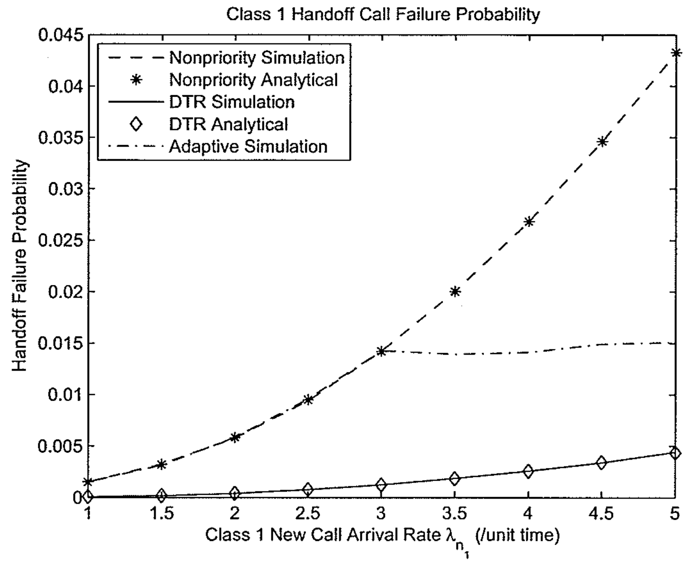

Figure 4.4: Analytical and simulation results of class 1 handoff call failure probability in the two-cell system for one call per unit time for class 2 new calls

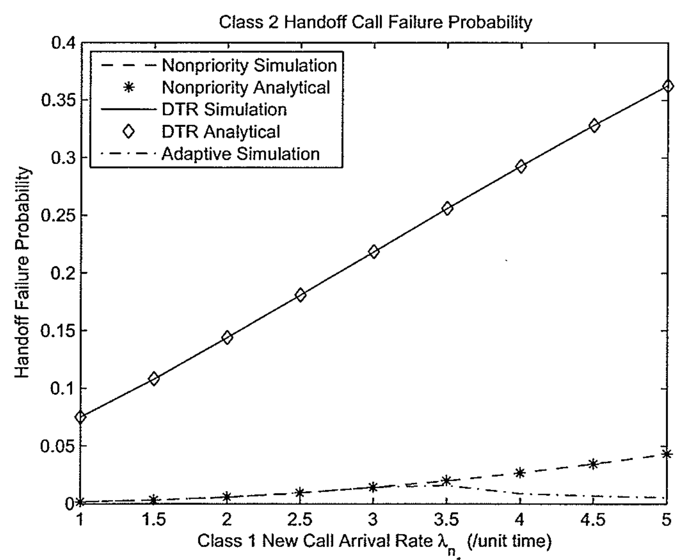

Figure 4.5: Analytical and simulation results of class 2 handoff call failure probability in the two-cell system for one call per unit time for class 2 new calls

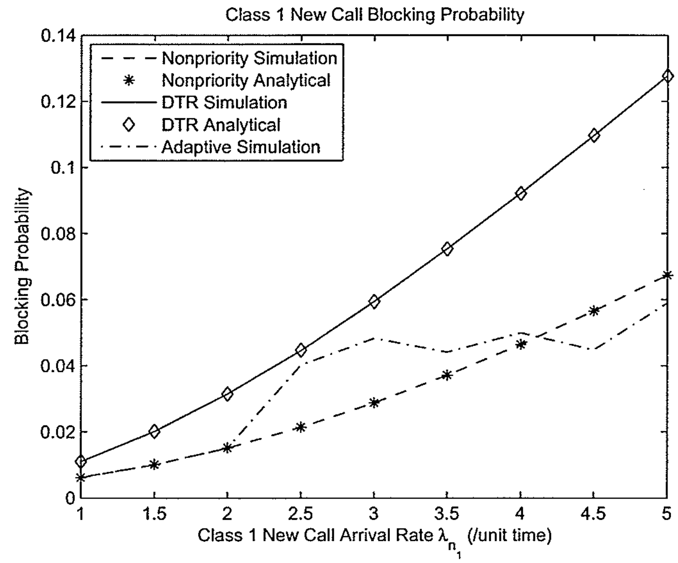

Figure 4.6: Analytical and simulation results of class 1 new call blocking probability in the two-cell system for two calls per unit time for class 2 new calls

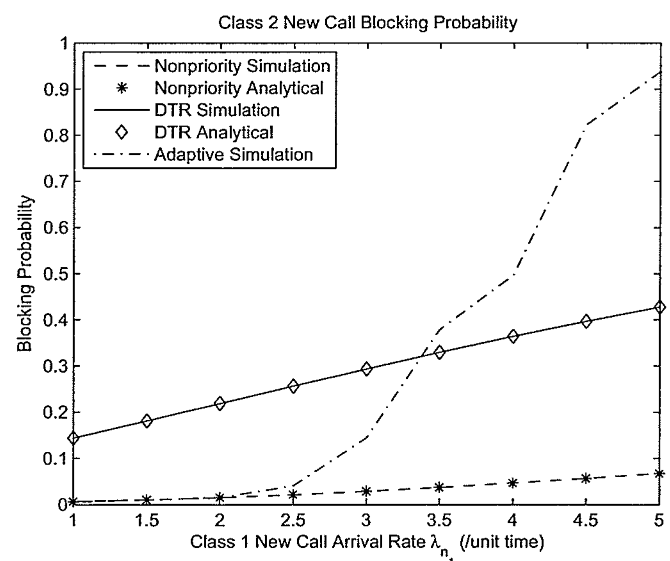

Figure 4.7: Analytical and simulation results of class 2 new call blocking probability in the two-cell system for two calls per unit time for class 2 new calls

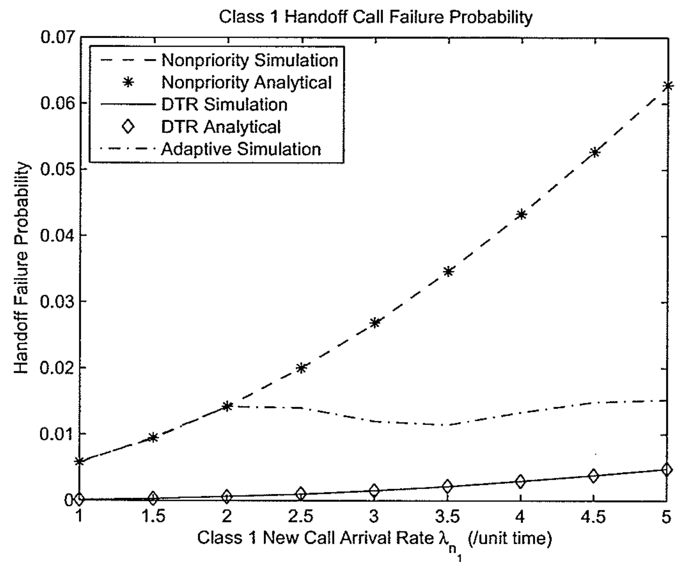

Figure 4.8: Analytical and simulation results of class 1 handoff call failure probability in the two-cell system for two calls per unit time for class 2 new calls

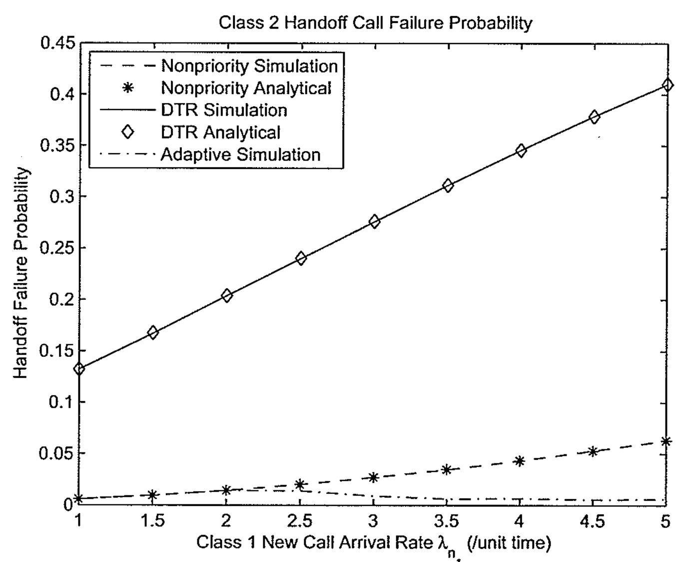

Figure 4.9: Analytical and simulation results of class 2 handoff call failure probability in the two-cell system for two calls per unit time for class 2 new calls

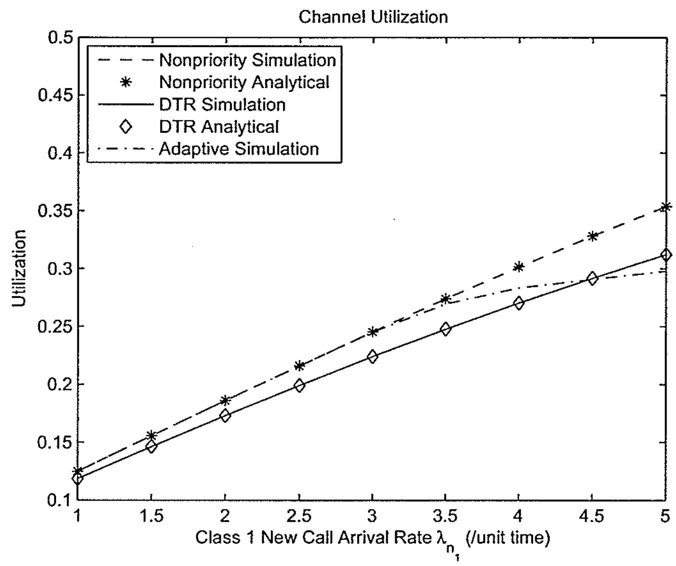

Figure 4.10: Analytical and simulation results of channel utilization in the two-cell system for one call per unit time for class 2 new calls

rates considered which the non-priority scheme fails to maintain. We can also see from Figures 4.2 and 4.6 that class 1 new call blocking probability can be maintained below 0.05 which the non-priority scheme and DTR scheme fail to maintain. However, with increasing class 1 new call arrival rate it is impossible to maintain the new call blocking probability and handoff failure probability within the given QoS.

Figures 4.2 and 4.6 show that class 1 new call blocking probability for adaptive scheme is not always less than non-priority scheme. However, class 1 new call blocking probability of our adaptive scheme can maintain the QoS limit except for 5.0/unit time arrival rate for class 1 calls. We can also see that DTR scheme cannot maintain the new call blocking probability QoS limit for class 1 calls. We can see from Figures 4.3 and 4.7 that class 2 new call blocking probability is highest for DTR scheme and our adaptive scheme, respectively at lower and higher load. New class 2 calls are blocked to achieve QoS new call blocking and handoff failure probabilities for class 1 users.

Figures 4.4 and 4.8 show that QoS requirement of class 1 handoff failure probability is not maintained by non-priority scheme while DTR scheme and adaptive scheme maintain QoS requirement of class 1 handoff failure probability. It is also found that DTR scheme

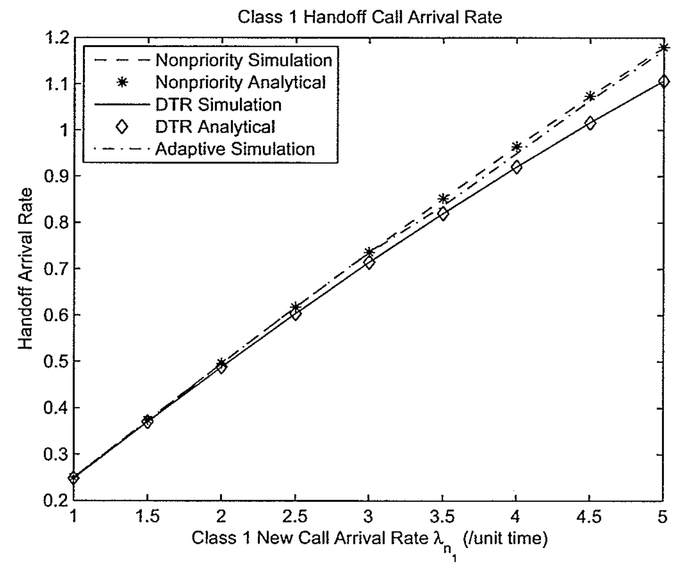

Figure 4.11: Analytical and simulation results of class 1 handoff call arrival rate in the two-cell system for one call per unit time for class 2 new calls

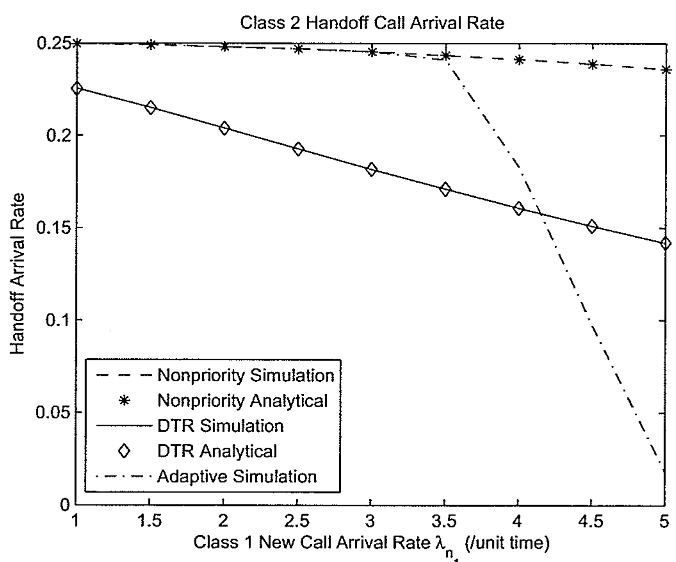

Figure 4.12: Analytical and simulation results of class 2 handoff call arrival rate in the two-cell system for one call per unit time for class 2 new calls

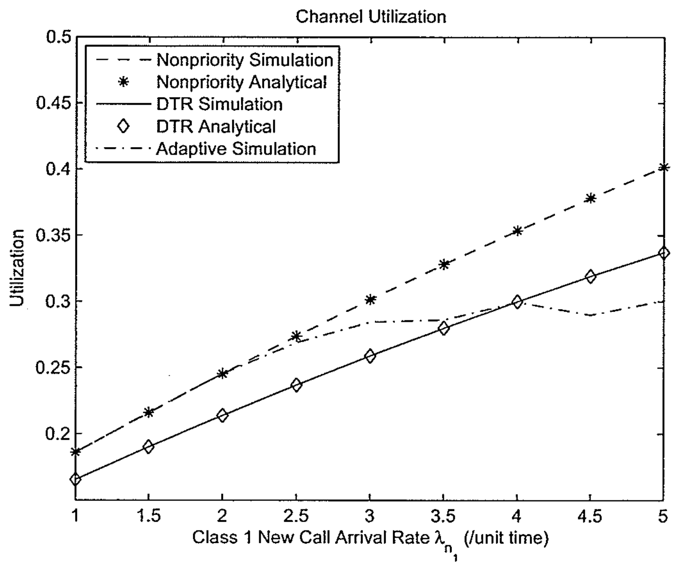

Figure 4.13: Analytical and simulation results of channel utilization in the two-cell system for two calls per unit time for class 2 new calls

has lower class 1 handoff failure probability than adaptive scheme which is achieved at the expense of QoS requirement of new call blocking probability of class 1 user and handoff failure probability of class 2 users. On the other hand, Figures 4.5 and 4.9 show that class 2 handoff failure probabilities in adaptive scheme are quite low due to low class 2 handoff arrival rate and prioritized service for class 2 handoff calls.

Non-priority scheme is found to have highest channel utilization among three schemes in Figures 4.10 and 4.13. On the other hand, adaptive scheme has higher channel utilization than DTR scheme at lower class 1 new call arrival rate. As class 1 new call arrival rate increases the channel utilization of DTR scheme becomes higher than adaptive scheme.

Figures 4.11 and 4.14 show that class 1 handoff call arrival rate is least for DTR scheme and non-priority scheme and adaptive scheme have similar class 1 handoff call arrival rate. On the other hand, class 2 handoff call arrival rate is least for DTR scheme at lower load and adaptive scheme at higher load which can be seen from Figures 4.12 and 4.15. Class 2 handoff rate is highest for non-priority scheme as non-priority scheme does not block any class 2 new or handoff call in a cell if there is any available channel in that cell.

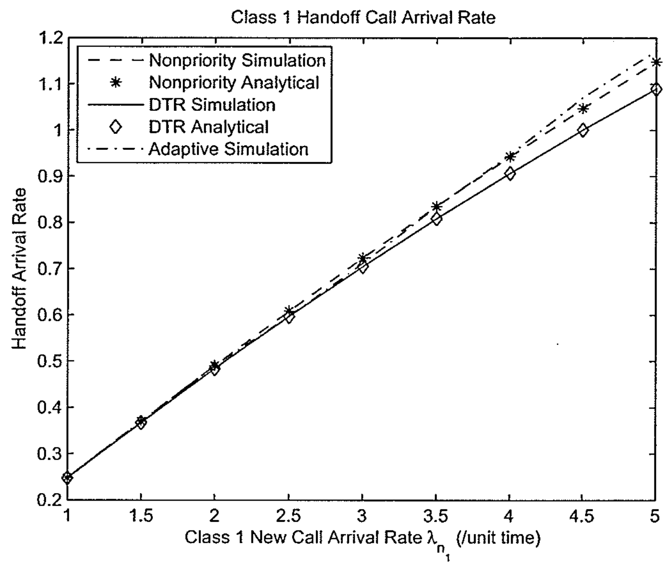

Figure 4.14: Analytical and simulation results of class 1 handoff call arrival rate in the two-cell system for two calls per unit time for class 2 new calls

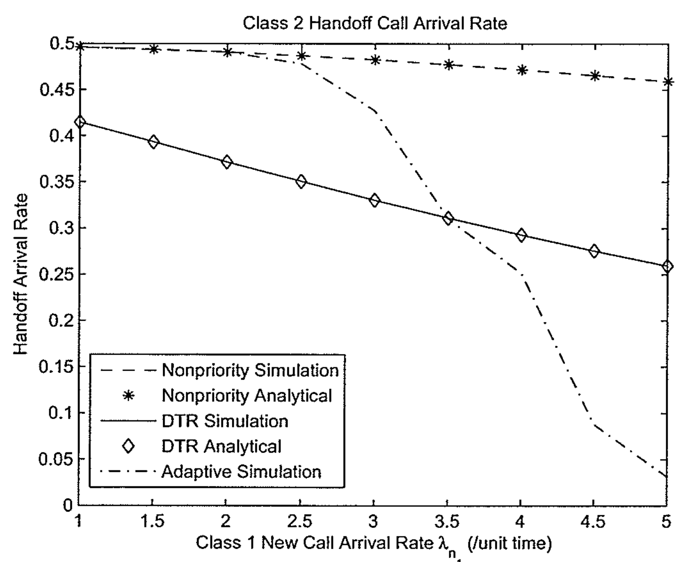

Figure 4.15: Analytical and simulation results of class 2 handoff call arrival rate in the two-cell system for two calls per unit time for class 2 new calls

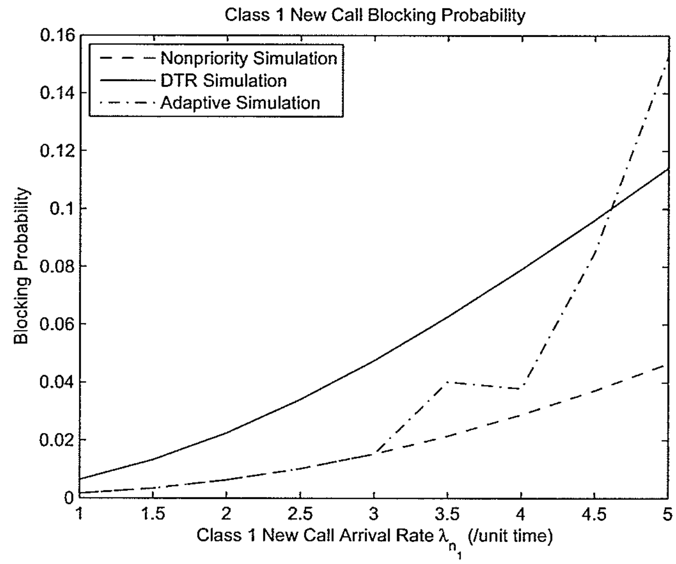

Figure 4.16: Simulation results of class 1 new call blocking probability in the nineteen-cell wrap around structure for one call per unit time for class 2 new calls

4.4.2 Numerical Result for Multiple-Cell System

Class 1 and class 2 new call blocking probabilities, class 1 and class 2 handoff call failure probabilities, channel utilization and class 1 and class 2 handoff call arrival rate are studied using simulation for non-priority, DTR and adaptive schemes for the wrapped around nineteen-cell wireless network of Figure 3.7. Results of simulation studies for class 1 new call blocking probability, class 2 new call blocking probability, class 1 handoff call failure probability and class 2 handoff call failure probability are presented in Figures 4.16, 4.17, 4.18 and 4.19, respectively for $\lambda_{n_2} = 1.0$/unit time. Similarly, simulation results for class 1 new call blocking probability, class 2 new call blocking probability, class 1 handoff call failure probability and class 2 handoff call failure probability are shown in Figures 4.20, 4.21, 4.22 and 4.23, respectively for $\lambda_{n_2} = 1.0$/unit time, .

We can see from Figures 4.16-4.23 that DTR scheme can maintain the QoS handoff failure probabilities for new call blocking and handoff failure probabilities for class 1 calls while it fails to achieve the QoS handoff failure probability for class 2 calls. We can see from these figures that adaptive scheme can maintain the QoS handoff failure probabilities for both class 1 and class 2 calls for λ_{n_1} having values from 1.0/ unit time

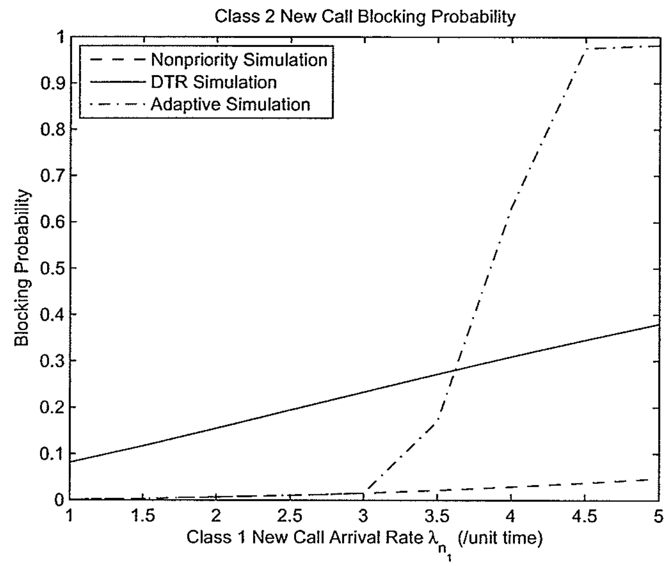

Figure 4.17: Simulation results of class 2 new call blocking probability in the nineteen-cell wrap around structure for one call per unit time for class 2 new calls

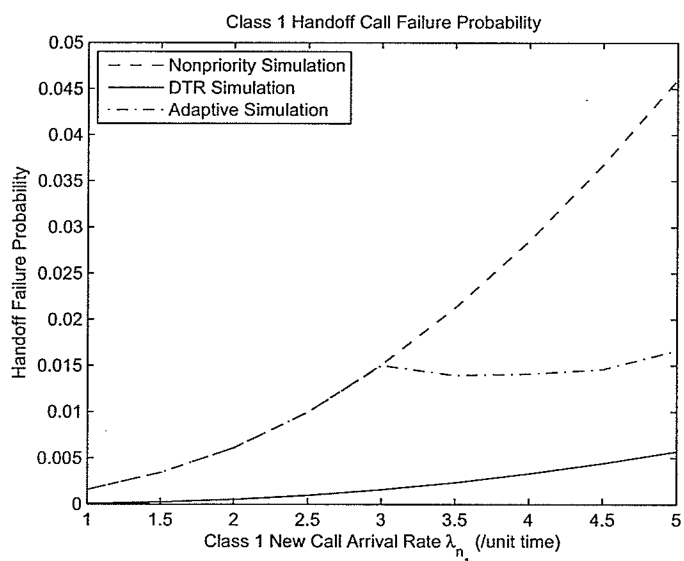

Figure 4.18: Simulation results of class 1 handoff call failure probability in the nineteen-cell wrap around structure for one call per unit time for class 2 new calls

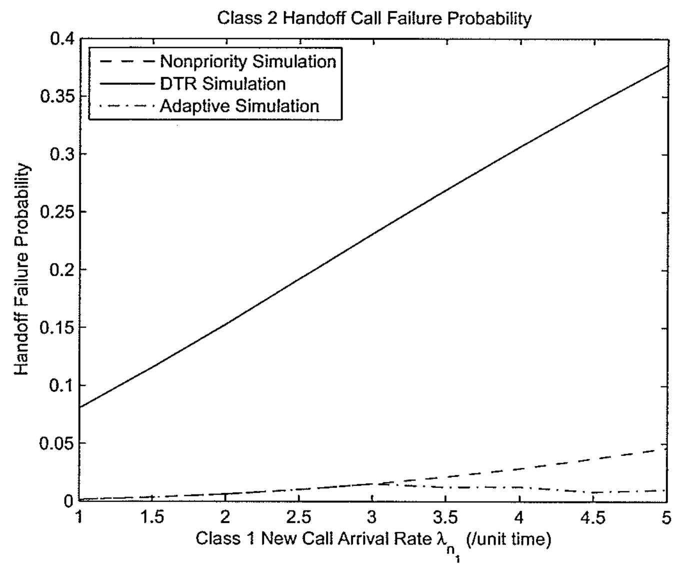

Figure 4.19: Simulation results of class 2 handoff call failure probability in the nineteen-cell wrap around structure for one call per unit time for class 2 new calls

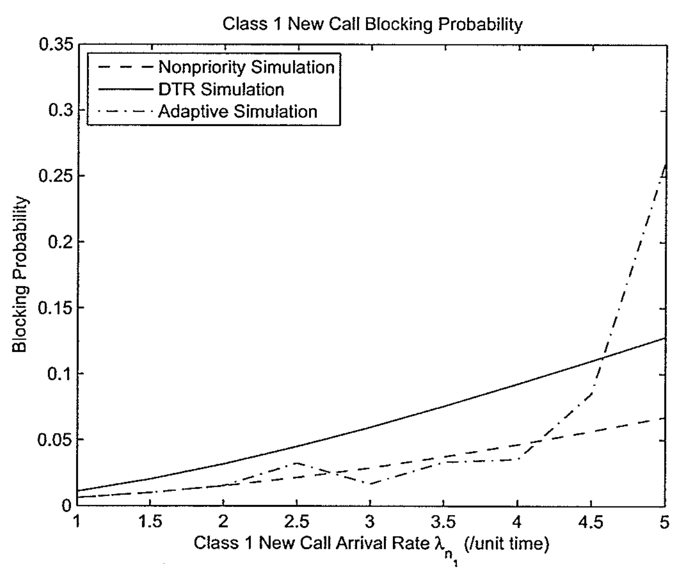

Figure 4.20: Simulation results of class 1 new call blocking probability in the nineteen-cell wrap around structure for two calls per unit time for class 2 new calls

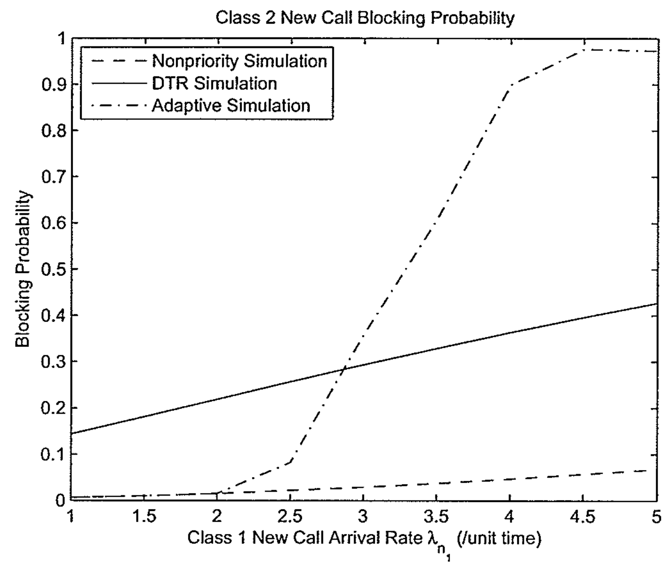

Figure 4.21: Simulation results of class 2 new call blocking probability in the nineteen-cell wrap around structure for two calls per unit time for class 2 new calls

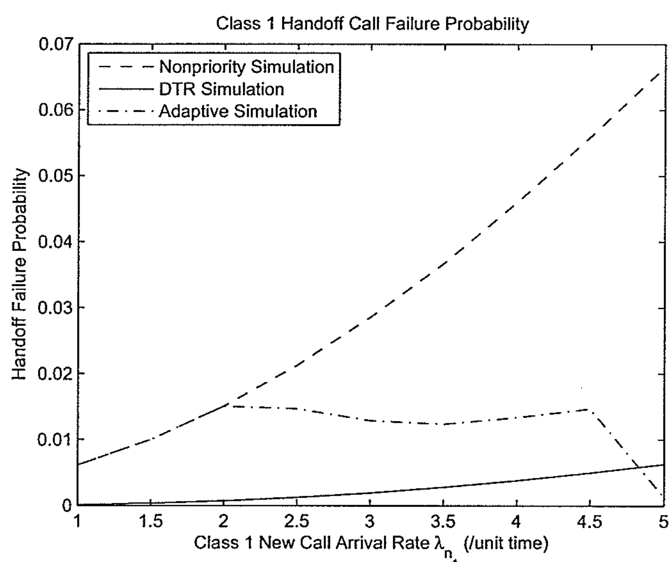

Figure 4.22: Simulation results of class 1 handoff call failure probability in the nineteen-cell wrap around structure for two calls per unit time for class 2 new calls

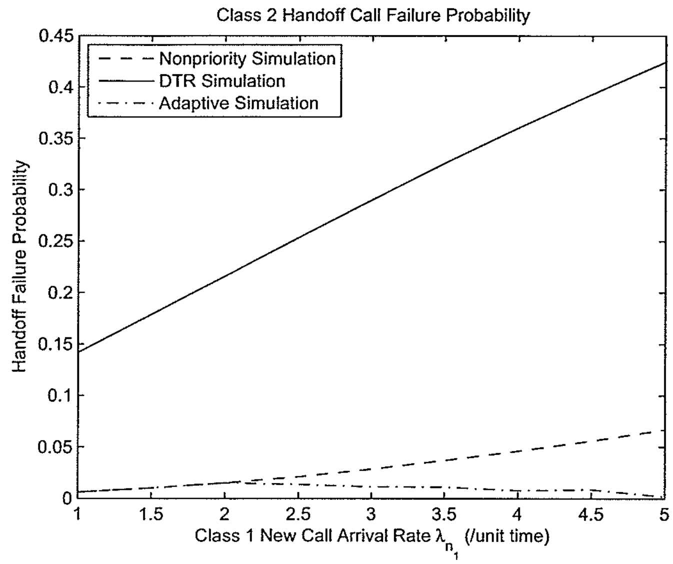

Figure 4.23: Simulation results of class 2 handoff call failure probability in the nineteen-cell wrap around structure for two calls per unit time for class 2 new calls

to 5.0/ unit time. On the other hand, adaptive scheme can achieve new call blocking probabilities for class 1 calls for λ_{n_1} having values from 1.0/ unit time to 4.0/ unit time. We can also see that at higher loads, adaptive scheme has the highest new call blocking probabilities for class 2 calls for achieving the QoS handoff failure probabilities for class 1 and class 2 calls and Qos new call blocking probability for class 1 calls. Moreover, we can see from these figures that non-priority scheme has the lowest new call blocking probability for class 2 calls while this scheme fails to achieve QoS for class 1 and class 2 calls.

We present simulation results for channel utilization and handoff arrival rate for class 1 and class 2 calls for $\lambda_{n_2} = 1.0$/ unit time in Figures 4.24, 4.25 and 4.26, respectively. Similarly, simulation results of channel utilization and handoff arrival rate for class 1 and class 2 calls are for $\lambda_{n_2} = 2.0$/ unit time in Figures 4.27, 4.28 and 4.29, respectively. We can find from Figures 4.24 and 4.27 that channel utilization is highest for non-priority which is achieved at the expense of not providing QoS for class 1 and class 2 calls. We can also see that adaptive scheme has better channel utilization than DTR scheme at lower class 1 new call arrival rate. On the other hand, DTR scheme has better channel

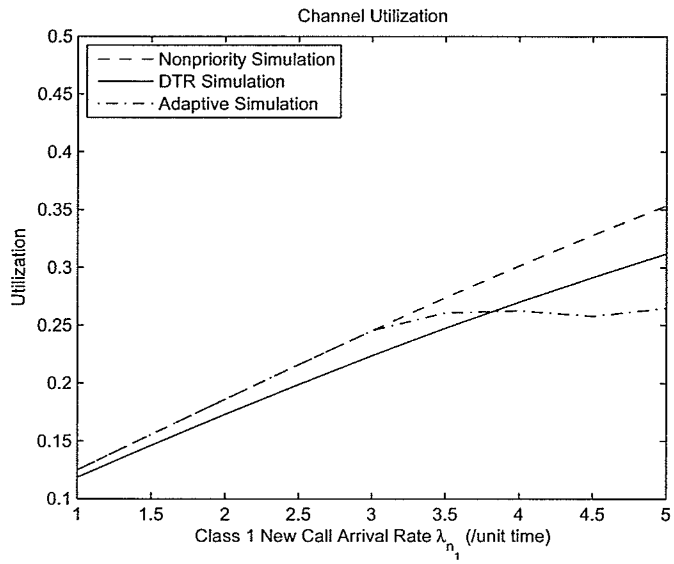

Figure 4.24: Simulation results of channel utilization in the nineteen-cell wrap around structure for one call per unit time for class 2 new calls

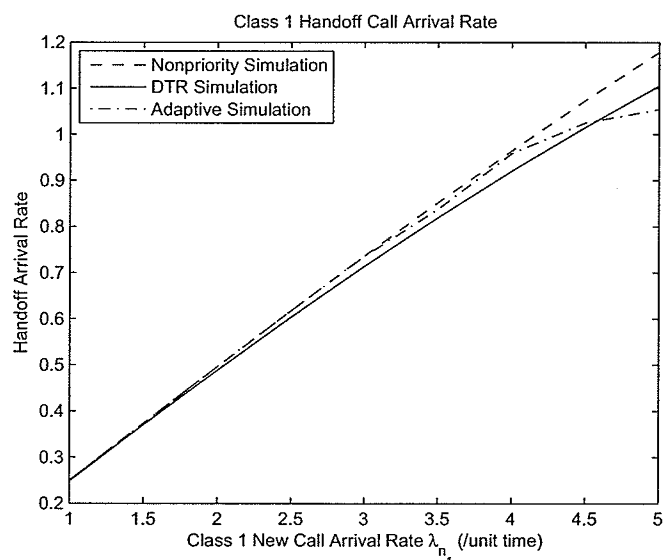

Figure 4.25: Simulation results of class 1 handoff call arrival rate in the nineteen-cell wrap around structure for one call per unit time for class 2 new calls

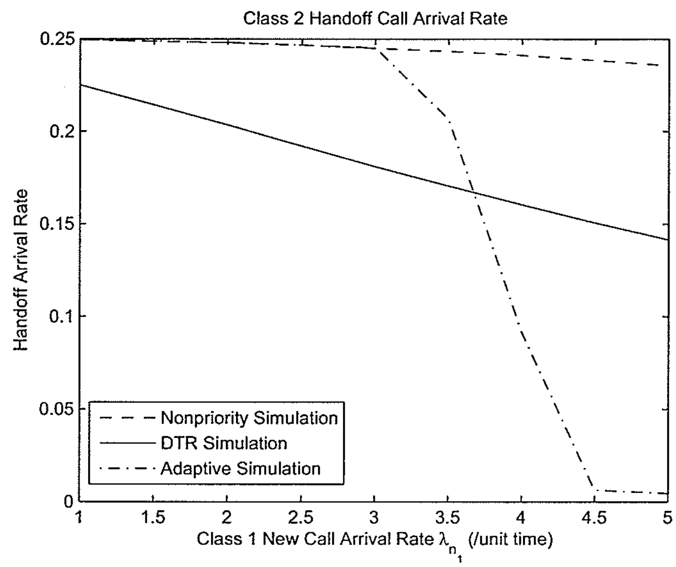

Figure 4.26: Simulation results of class 2 handoff call arrival rate in the nineteen-cell wrap around structure for one call per unit time for class 2 new calls

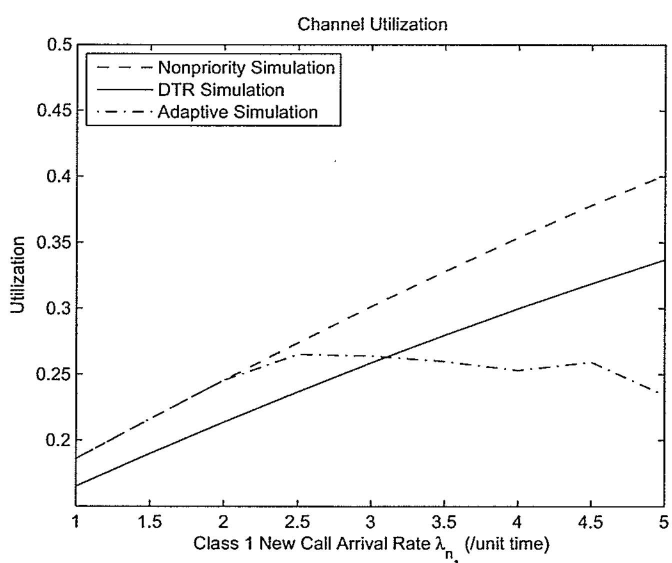

Figure 4.27: Simulation results of channel utilization in the nineteen-cell wrap around structure for two calls per unit time for class 2 new calls

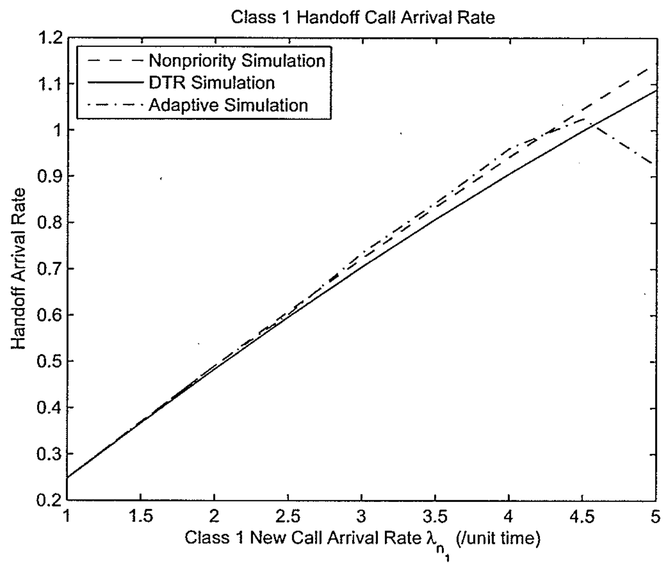

Figure 4.28: Simulation results of class 1 handoff call arrival rate in the nineteen-cell wrap around structure for two calls per unit time for class 2 new calls

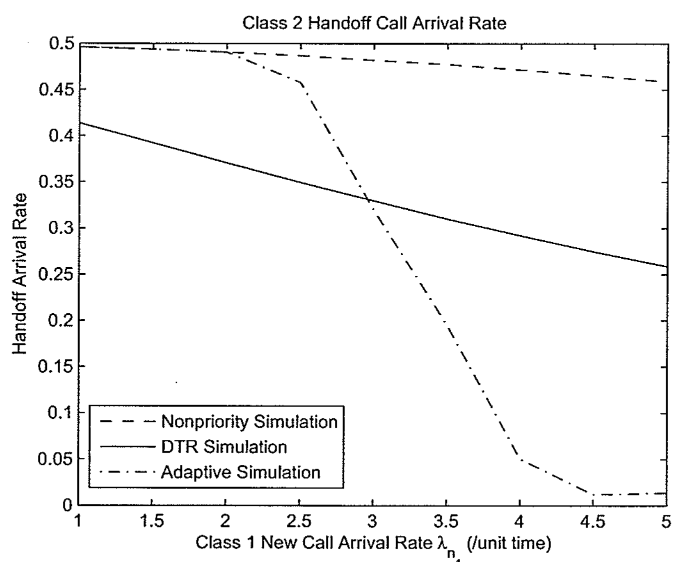

Figure 4.29: Simulation results of class 2 handoff call arrival rate in the nineteen-cell wrap around structure for two calls per unit time for class 2 new calls

utilization than adaptive scheme at higher new class 1 call arrival rate. We can find from Figures 4.25, 4.26, 4.28 and 4.29 that class 1 and class 2 handoff call arrival rate is the highest for non-priority scheme. We can also see that adaptive scheme has higher class 1 and class 2 call handoff arrival rates at lower class 1 new call arrival rate. On the other hand, DTR scheme has higher class 1 and class 2 handoff call arrival rates than adaptive scheme at higher new class 1 call arrival rate.

The numerical examples presented in Sections 4.4.1 and 4.4.2 show that our adaptive call admission control scheme can satisfy QoS new call blocking and handoff failure probabilities of class 1 users and QoS handoff failure probability of class 2 users to a certain call arrival rate which non-prioritized and DTR schemes fail to do. It is not possible to provide QoS requirements of class 1 and class 2 users at very high new call arrival rate. On the other hand, non-prioritized scheme achieves highest channel utilization which is wireless service provider-perceived utility. In contrast, DTR scheme can neither satisfy user-perceived QoS and service provider-perceived utility. However, our adaptive scheme for two classes of users can balance the user-perceived QoS and service provider-perceived utility.

4.4.3 Sensitivity of Numerical Examples on the Values of T

$T = \frac{5}{\eta_1}$ is used in the numerical examples presented in Sections 4.4.1 and 4.4.2. In this section, the sensitivity of performance metrics with respect the value of T for adaptive scheme in the two-cell system. The time period T is varied from $\frac{1}{\eta_1}$ to $\frac{7}{\eta_1}$ in step of $\frac{1}{2\eta_1}$ for $\frac{1}{\eta_1} = \frac{1}{\eta_2} = 1.0$ unit time, $\frac{1}{\mu_1} = \frac{1}{\mu_2} = 4.0$ unit time and $C = 4$. We use the value of a particular index of offline tables for new call acceptance. Two sets of simulation experiments are carried out. In the first set of experiments, $\lambda_{n_1} = 4.5$ calls per unit time and $\lambda_{n_2} = 1.0$ calls per unit time were used whereas in the second set of experiments, $\lambda_{n_1} = 4.0$ calls per unit time and $\lambda_{n_2} = 2.0$ calls per unit time were used. The results of these two sets of experiments are presented in Tables 4.5 and 4.6. In these tables, $T\eta_1$ denotes the ratio of time period T to class 1 mean cell residence time $\frac{1}{\eta_1}$, P_{b_1} denotes class 1 new call blocking probability, P_{b_2} denotes class 2 new call blocking probability, P_{hf_1} denotes class 1 handoff call failure probability, P_{hf_2} denotes class 2 handoff call failure probability and "Util" denotes channel utilization in a cell. These two tables

Table 4.5: Sensitivity analysis for first set of parameters

$T\eta_1$	P_{b_1}	P_{b_2}	P_{hf_1}	P_{hf_2}	Util
1.0	0.048162	0.700438	0.014812	0.006438	0.285379
1.5	0.048162	0.700438	0.014812	0.006438	0.285379
2.0	0.048162	0.700438	0.014812	0.006438	0.285379
2.5	0.048162	0.700438	0.014812	0.006438	0.285379
3.0	0.048162	0.700438	0.014812	0.006438	0.285379
3.5	0.048162	0.700438	0.014812	0.006438	0.285379
4.0	0.048162	0.700438	0.014812	0.006438	0.285379
4.5	0.048162	0.700438	0.014812	0.006438	0.285379
5.0	0.048162	0.700438	0.014812	0.006438	0.285379
5.5	0.048162	0.700438	0.014812	0.006438	0.285379
6.0	0.048162	0.700438	0.014812	0.006438	0.285379
6.5	0.038435	0.704231	0.016989	0.006505	0.287697
7.0	0.038435	0.704231	0.016989	0.006505	0.287697

show no change in the results for the values of T from $\frac{1.0}{\eta_1}$ to $\frac{6.0}{\eta_1}$. These tables show changes in results for T from $\frac{6.5}{\eta_1}$ to $\frac{7.0}{\eta_1}$. The changes in result can be explained from the computation of tables of average dropping probabilities can be explained from the computation of instantaneous handoff probabilities of class 1 and class 2 candidate calls in the neighboring cell. The variation of instantaneous handoff dropping probabilities impacts the computation of the average handoff dropping probabilities over period T using Equations (4.83) and (4.84) for different values of T. The term Λ can also impact the accuracy of computing handoff dropping probabilities. Higher values of T and Λ can result in severe roundoff errors. At lower values of Λ and T the elements in an offline table for dropping probabilities has a particular ordering of values stored in different indices which changes at higher values of Λ and T. Therefor, Λ should be chosen as the maximum of the absolute values of the diagonal elements of the generator matrix of Equation (4.64) and choice of the value of T should be carefully performed.

Table 4.6: Sensitivity Analysis for second set of parameters

$T\eta_1$	P_{b_1}	P_{b_2}	P_{hf_1}	P_{hf_2}	Util
1.0	0.045955	0.609908	0.012059	0.005358	0.286447
1.5	0.045955	0.609908	0.012059	0.005358	0.286447
2.0	0.045955	0.609908	0.012059	0.005358	0.286447
2.5	0.045955	0.609908	0.012059	0.005358	0.286447
3.0	0.045955	0.609908	0.012059	0.005358	0.286447
3.5	0.045955	0.609908	0.012059	0.005358	0.286447
4.0	0.045955	0.609908	0.012059	0.005358	0.286447
4.5	0.045955	0.609908	0.012059	0.005358	0.286447
5.0	0.045955	0.609908	0.012059	0.005358	0.286447
5.5	0.045955	0.609908	0.012059	0.005358	0.286447
6.0	0.045955	0.609908	0.012059	0.005358	0.286447
6.5	0.034732	0.613693	0.014500	0.005451	0.288672
7.0	0.034732	0.613693	0.014500	0.005451	0.288672

Chapter 5

Computationally Efficient Method for Analyzing Guard Channel Schemes

Most CAC models for cellular networks assume that the cell residence time and the call holding duration have exponential distributions. These assumptions make their performance analysis computationally tractable. However, the assumptions of exponential distributions can be unrealistic (especially for the cell residence time). On the other hand, those CAC models that make realistic assumption about the distributions of the cell residence time and the call holding duration suffer from computational intractability problem. In this chapter, we develop computationally tractable model for guard channel scheme without restricting the cell residence time and the call holding duration to exponential distributions. Part of the work presented in this chapter is reported in [86].

In cellular wireless networks the call holding duration and the call inter-arrival time are generally assumed to follow exponential distribution just like in the wired telephone networks. However, in cellular wireless communication networks another factor comes into play, the mobility of users. This factor translates into the cell residence time of users which affects the distribution of a user's channel holding time in a cell. Numerical analysis of static call admission control schemes are easily tractable if the channel holding time of a call in a cell is exponentially distributed. Fang et al [31] showed that for exponentially distributed call holding times the channel holding times are exponentially distributed if and only if the cell residence times are exponentially distributed. Therefore, the cell residence time is generally assumed to follow exponential distribution in static call admission control schemes [38, 54, 55, 66, 98] for tractability. Hong and Rappaport [45] assumed uniformly distributed velocity for mobile users and approximated the channel holding time with an equivalent exponential distribution which is not quite realistic. Similarly, the cell residence time is also assumed to follow exponential distribution in many of the distributed call admission control schemes for the purpose of tractability. Fang [24], Fang and Chlamtac [29] and Fang et al [30, 31, 28, 25, 27] argued for use of general distribution for the cell residence time. They analyzed channel holding time for Erlang and hyper-Erlang distributed cell residence time and call holding duration but could not obtain new call blocking and handoff failure probabilities because it is difficult to keep track of the dimension of the Markov chain. Jayasuriya et al [47] followed the approach of Ramaswami and Lucantoni [89] of keeping track of the number of users in different phases for multi-server queue with phase type distributed service time. They analyzed the new call blocking and handoff failure probabilities for cellular networks with Poisson call arrivals and generalised Erlang distributed channel holding time. They assumed generalised Erlang distributed channel holding time for both new and handoff calls which is not justified for Erlang distributed cell residence time and do not consider the dependence of handoff arrival process on new call arrival process and other relevant parameters (we will use the term "other relevant parameters" for handoff probabilities of new and handoff calls and new call blocking and handoff failure probabilities in the rest of the chapter). Orlik and Rappaport [82] analyzed the performance of a cellular wireless network for sum of hyper-exponentially distributed cell residence time and call holding duration. Marsan et al [72] analyzed hierarchical cellular system with general distributed cell residence time and call holding duration approximated by hyper-exponential, exponential or Erlang distribution. Their work suffers from too much approximation. Alfa and Li [3] captured the distributions of the call holding duration and the cell residence time completely by use of phase type distributions which is more general than exponential, Erlang and sum of hyper-exponential distributions. However, their model is computationally demanding and actually intractable using reasonable computing resources as they keep track of the phase of the channel holding time of each call which makes the state-space large.

In this chapter, we analyze the guard channel scheme by following the approach of Alfa and Li [3] to capture the general distributions for both the cell residence time and the call holding duration with phase type distributions. The performance analysis of a cellular network is made computationally tractable in our model by using the technique of Ramaswami and Lucantoni [89] to keep track of the number of users in different phases of the channel holding time. We compute the probabilities of handoff for both new and handoff calls. We analyze the new call blocking and the handoff failure probabilities for different number of guard channels. The dependence of handoff arrival process on the new call arrival rate and other relevant parameters is considered. The main contribution of this chapter is to develop a computationally tractable algorithm to analyze new call blocking and handoff failure probabilities with general cell residence time and call holding duration.

The rest of the chapter is organized as follows. In Section 5.1 we explain our proposed model for call admission controller with general cell residence time and general call holding duration. Implementation issues for the analytical model are considered in Section 5.2. Section 5.3 presents some numerical examples.

5.1 System Model

In this section, we present our model and its analysis. We introduce phase type distribution and provide necessary assumptions and relevant definitions for the model in Section 5.1.1. These assumptions are used throughout this chapter. Analytical expressions for handoff probabilities of new and handoff calls are derived in Section 5.1.2 based on the assumptions of Section 5.1.1. Then we describe how the number of calls in different phases of channel holding time can be represented instead of representing the phase of channel holding time of individual calls in Section 5.1.3. We build an analytical model in Section 5.1.4 based on the assumptions of Section 5.1.1, the analytical expressions of the handoff probabilities of Section 5.1.2 and the concept of Section 5.1.3.

5.1.1 Assumptions and Definitions

Phase type distribution is the time t ($t \in [0, \infty)$) until absorption in a Markov chain with n ($n \geq 1$) transient states and one absorbing state. It is represented as (γ, V) of dimension n. Here, $\gamma = [\gamma_1, \gamma_2, \gamma_3, \ldots, \gamma_n]$ is the initial probability vector and γ_l is the probability that the Markov chain starts from transient state l ($1 \leq l \leq n$). $\gamma\mathbf{1} = 1$ where $\mathbf{1}$ is a column vector of ones. V is an $n \times n$ transient matrix corresponding to transitions within transient states. The generator matrix of the entire Markov chain is

$$\begin{bmatrix} V & V^0 \\ \mathbf{0}' & 0 \end{bmatrix}.$$

Here, V^0 is an $n \times 1$ column-vector and $V^0 = -V\mathbf{1}$. $\mathbf{0}'$ is a row vector of zeros. The diagonal elements of V are negative and nondiagonal elements are nonnegative. The elements of the column vector V^0 are nonnegative and at least one element is positive. The probability density function (pdf) of phase type distribution can be represented as $f(t) = \gamma e^{Vt} V^0$ where e is the base of the natural logarithm. Similarly, cumulative distribution function (cdf) of the phase type distribution can be expressed as $F(t) = 1 - \gamma e^{Vt}\mathbf{1}$. The mean of the phase type distribution is $-\gamma V^{-1}\mathbf{1}$.

We consider a cellular wireless network consisting of homogenous cells. This type of network can be studied by analyzing a cell in isolation [45]. We assume that each cell has C channels and each call needs one channel. There are g ($g < C$) number of channels reserved as guard channels for handoff calls. In this model, an incoming handoff call is accepted in a cell if there is any free channel in that cell. On the other hand, a new call is accepted if the number of ongoing calls is less than $C - g$.

We assume that the new call arrivals to a cell follow a Poisson process with mean inter-arrival time $\frac{1}{\lambda_n}$. We also assume that the handoff arrivals to the cell can be approximated by a Poisson process with mean inter-arrival time $\frac{1}{\lambda_h}$.

The cell residence time R of a user in a cell is the duration of time from when he enters the cell to the moment he leaves the cell (i.e., the time a user spends in a cell is called the cell residence time of that user in that cell). R has a general phase type distribution represented by (α, T) of dimension r. The probability distribution of R is given by $P(R \leq t) = 1 - \alpha e^{Tt}\mathbf{1}$, where $\alpha\mathbf{1} = 1$, and $T\mathbf{1} = -T^0$. Now, the residual cell residence time $\bar{R}$ of a user in a cell is related to a new call attempt in this chapter. The residual cell residence time of a user in a cell at the time of a new call attempt is the time between the new call attempt and the moment the user leaves the cell. When a user makes a new call attempt in a cell it is not known how much time the user has already spent in the cell. The phase of the cell residence time of a user during a new call attempt could be exactly determined from the transient analysis of the cell residence time if the times of the user's entering the cell and attempting the new call are kept track of. As an example, if a user enters a cell at time t_1 and attempts a new call in the cell at time t_2 ($t_2 \geq t_1$) then the phase of the cell residence time of the user at time t_2 is $\alpha e^{T(t_2-t_1)}$. However, a Markov chain only keeps track of transitions among states and does not keep track of the time of any event. Therefore, we use $\bar{\alpha} = (\alpha T^{-1}\mathbf{1})^{-1}\alpha T^{-1}$, the equilibrium distribution of phases of the cell residence time, as an approximation for the phase of the cell residence time of a user in a cell making a new call attempt which we believe is reasonable. In this chapter, we use the remaining residence time of a new call in a cell as the residual cell residence time $\bar{R}$. $\bar{R}$ also has phase type distribution represented by $(\bar{\alpha}, T)$ of dimension r where $\bar{\alpha}(T + T^0\alpha) = \mathbf{0}'$ and $\bar{\alpha}\mathbf{1} = 1$. The probability distribution of $\bar{R}$ is given by $P(\bar{R} \leq t) = 1 - \bar{\alpha}e^{Tt}\mathbf{1}$. The average cell residence time is $-\alpha T^{-1}\mathbf{1}$ and the average residual cell residence time is $-\bar{\alpha}T^{-1}\mathbf{1}$.

The call holding duration H of a call is the time from the start of the call to the completion of that call. H has a general phase type distribution represented by (β, S) of dimension h, i.e., $P(H \leq t) = 1 - \beta e^{St}\mathbf{1}$ where $\beta\mathbf{1} = 1$ and $S\mathbf{1} = -S^0$. The call holding duration is used for a new call attempt to compute the channel holding time of the new call. On the other hand, the residual call holding duration $\bar{H}$ is related to a handoff call and it is observed during a handoff attempt. When an ongoing call makes a handoff attempt then the residual call holding duration represents the time between that handoff attempt and the completion of the call. As an example, if a user starts a new call at time t_2 and attempts a handoff (first, second or any handoff) at time t_3 ($t_3 \geq t_2$) then the phase of the call holding duration of the user at time t_3 is $\beta e^{S(t_3-t_2)}$. The exact probability vector of the phase for the residual call holding duration at k-th handoff ($k \geq 1$) depends on the time difference ($t_3 - t_2$) and does not depend on the number k. As time is not kept track of in a Markov chain we do not know at the time of a handoff attempt of a call how long the call has been going on. Therefore, we use an approximation for the initial probability vector for the phase of the residual call holding duration at handoff time, like we have done for the residual cell residence time, at the time of new call attempt. The approximation for the initial probability vector for residual call holding duration is the equilibrium probability vector $\bar{\beta} = (\beta S^{-1}\mathbf{1})^{-1}\beta S^{-1}$ which is used for first, second or any handoff. The residual call holding duration $\bar{H}$ has a phase type distribution represented by $(\bar{\beta}, S)$ of dimension h where $\bar{\beta}(S+S^0\beta) = \mathbf{0}'$ and $\bar{\beta}\mathbf{1} = 1$. The probability distribution function of the residual call holding duration is given by $P(\bar{H} \leq t) = 1 - \bar{\beta}e^{St}\mathbf{1}$. The average call holding duration is $-\beta S^{-1}\mathbf{1}$ and the average residual call holding duration is $-\bar{\beta}S^{-1}\mathbf{1}$.

The channel holding time S_n of a new call is the minimum of the residual cell residence time and the call holding duration, i.e., $S_n = \min\{\bar{R}, H\}$. S_n also has a phase type distribution represented by $(\delta_n, L_n) = (\bar{\alpha} \otimes \beta, T \oplus S)$ of dimension hr and $L_n\mathbf{1} = -L_n^0$ where $A \otimes B \equiv (A_{i,j}B)$ is the Kronecker product and $A \oplus B = A \otimes I + I \otimes B$ is the Kronecker sum of the matrices A and B. The probability distribution function of S_n is given by $P(S_n \leq t) = 1 - \delta_n e^{L_n t}\mathbf{1}$. The average channel holding time of a new call is $-\delta_n L_n^{-1}\mathbf{1}$.

The channel holding time S_h of a handoff call is the minimum of the cell residence time and the residual call holding duration, i.e., $S_h = \min\{R, \bar{H}\}$. S_h also has a phase type distribution represented by $(\delta_h, L_h) = (\alpha \otimes \bar{\beta}, T \oplus S)$ of dimension hr and $L_h\mathbf{1} = -L_h^0$. The probability distribution function of S_h is given by $P(S_h \leq t) = 1 - \delta_h e^{L_h t}\mathbf{1}$. The average channel holding time of a handoff call is $-\delta_h L_h^{-1}\mathbf{1}$.

We can find that the probability distribution of the channel holding time of new and handoff calls only differ in the initial probability vector $\delta_n = \bar{\alpha} \otimes \beta$ and $\delta_h = \alpha \otimes \bar{\beta}$. The transient transition matrices and the absorption column vectors of the channel holding times of new and handoff calls are same as $L_n = L_h = L = T \oplus S$ and $L_n^0 = L_h^0 = L^0 = -L\mathbf{1}$.

5.1.2 Handoff Probabilities

We need the probability density functions of the cell residence time and the residual cell residence time for computing the handoff probabilities of new and handoff calls. If we denote the probability density functions of the cell residence time and the residual cell residence time as $f_R(t)$ and $f_{\bar{R}}(t)$, respectively then $f_R(t)$ and $f_{\bar{R}}(t)$ can be mathematically expressed as

$$f_R(t) = -\alpha e^{Tt}T\mathbf{1} = \alpha e^{Tt}T^0. \tag{5.1}$$

$$f_{\bar{R}}(t) = -\bar{\alpha} e^{Tt}T\mathbf{1} = \bar{\alpha} e^{Tt}T^0. \tag{5.2}$$

Let us denote the handoff probabilities of a new call and a handoff call as P_h^n and P_h^h, respectively. P_h^n is the probability of the call holding duration being greater than the residual cell residence time. Similarly, P_h^h is the probability of the residual call holding duration being greater than the cell residence time. We can express P_h^n as

$$\begin{aligned} P_h^n &= P(H > \bar{R}) \\ &= \int_0^\infty f_{\bar{R}}(t)(1 - P(H \leq t))\mathrm{d}t \\ &= -\int_0^\infty \bar{\alpha} e^{Tt} T\mathbf{1}\beta e^{St}\mathbf{1}\mathrm{d}t \\ &= -\int_0^\infty (\bar{\alpha} \otimes \beta) e^{(T\oplus S)t}(T \otimes I)(\mathbf{1} \otimes \mathbf{1})\mathrm{d}t \\ &= -(\bar{\alpha} \otimes \beta)\left[e^{(T\oplus S)t}\right]_0^\infty (T \oplus S)^{-1}(T \otimes I)\mathbf{1} \\ &= (\bar{\alpha} \otimes \beta)(T \oplus S)^{-1}(T \otimes I)\mathbf{1} \\ &= \delta_n L^{-1}(T \otimes I)\mathbf{1}. \end{aligned} \tag{5.3}$$

Similarly, we can express P_h^h as

$$\begin{aligned} P_h^h &= P(\bar{H} > R) \\ &= \int_0^\infty f_R(t)(1 - P(\bar{H} \leq t))\mathrm{d}t \\ &= \delta_h L^{-1}(T \otimes I)\mathbf{1}. \end{aligned} \tag{5.4}$$

5.1.3 Representing the Number of Calls in Different Phases

We see from Section 5.1.1 that the channel holding time of a call has rh phases if the cell residence time has r phases and the call holding duration has h phases. If there are i $(1 \leq i \leq C)$ ongoing calls in a cell then each of the i calls can be in any of the different rh phases of the channel holding time. Let us assume that the phases are $0, 1, 2, \ldots, (rh-1)$. If we denote p_k $(0 \leq p_k \leq rh - 1)$ as the phase of k-th $(1 \leq k \leq i)$ call then we can denote a state as $(i, (p_1, p_2, \ldots, p_k, \ldots, p_i))$ for representing i calls. Here, the i-tuple $(p_1, p_2, \ldots, p_k, \ldots, p_i)$ represents the phases of i individual calls. In this representation we need $(rh)^i$ states for representing the phase of i individual calls. If there are three calls (i.e., $i = 3$) and three phases for the channel holding time (i.e., $rh = 3$) then we need $3^3 = 27$ states to represent three calls in three different phases.

Ramaswami and Lucantoni [89] keep track of the numbers of servers in different phases of service time for analyzing $GI/PH/c$ queue which reduces the size of the state-space. We follow their approach to keep track of the number a_k $(0 \leq k \leq rh-1)$ calls in phase k of the channel holding time. We can denote a state of the model as $(i, (a_0, a_1, a_2, \ldots, a_{rh-1}))$ for i $(0 \leq i \leq C)$ calls where $\sum_{k=0}^{rh-1} a_k = i$ and $0 \leq a_k \leq i$. If we denote $N_i^{(j)}$ as the number of combinations that i number of ongoing calls can be in different j phases then $N_i^{(j)} = \begin{pmatrix} i+j-1 \\ j-1 \end{pmatrix}$. As $(rh)^i > N_i^{(rh)}$ for $i \geq 2$ and $rh \geq 2$ we can just keep track of the number of calls in a cell in different phases of the channel holding time. As an example, for $rh = 3$ and $i = 3$, $(rh)^i = 27$ and $N_i^{(rh)} = 10$. We can order different combinations of i calls in rh phases of the channel holding time by sorting the strings $(a_0, a_1, \ldots, a_{rh-1})$ in ascending order. As an example, we can sort the combinations of three calls in three different phases (i.e., for $rh = 3$) as $\{(0,0,3), (0,1,2), (0,2,1), (0,3,0), (1,0,2), (1,1,1), (1,2,0), (2,0,1), (2,1,0), (3,0,0)\}$. When we sort the $N_i^{(rh)}$ combinations of i calls in rh phases we can assign an index u $(0 \leq u \leq N_i^{(rh)}-1)$ for each combination $(a_0, a_1, a_2, \ldots, a_{rh-1})$. In this notation u is zero for $(0, 0, \ldots, i)$ and u is $(N_i^{(rh)} - 1)$ for $(i, 0, \ldots, 0)$. We call u as an auxiliary variable for i $(0 \leq i \leq C)$ calls in rh phases. We can denote the state of our model as (i, u) instead of $(i, (a_0, a_1, a_2, \ldots, a_{rh-1}))$. We follow this notation in Section 5.1.4.

5.1.4 Analytical Model

We consider a two-dimensional continuous time Markov chain for an isolated cell in our model. The state-space of the Markov chain for a cell can be represented as $\{(i, u);\ 1 \leq i \leq C;\ 0 \leq u \leq N_i^{(rh)} - 1\}$ where i is the number of ongoing calls in the cell and u is the auxiliary variable for the combination of i calls in different rh phases. The generator matrix Q of this Markov chain has following structure

$$Q = \begin{bmatrix} B_0 & U_0 & & & & & \\ D_1 & B_1 & U_1 & & & & \\ & D_2 & B_2 & U_2 & & & \\ & & \ddots & \ddots & \ddots & & \\ & & & D_{C-1} & B_{C-1} & U_{C-1} \\ & & & & D_C & B_C \end{bmatrix}. \tag{5.5}$$

Here, B_i $(0 \leq i \leq C)$ is a square matrix of order $N_i^{(rh)}$, U_i $(0 \leq i \leq C-1)$ is an $(N_i^{(rh)} \times N_{i+1}^{(rh)})$ matrix and D_i $(1 \leq i \leq C)$ is an $(N_i^{(rh)} \times N_{i-1}^{(rh)})$ matrix.

$B_i((a_0, a_1, \ldots, a_{rh-1}), (b_0, b_1, \ldots, b_{rh-1}))$ denotes the change in the numbers of calls in different phases with no change in the number of calls in a cell when the number of calls in a cell is i, starting combination of calls in different phases is $(a_0, a_1, \ldots, a_{rh-1})$, finishing combination of calls in different phases is $(b_0, b_1, \ldots, b_{rh-1})$ and $\sum_{j=0}^{rh-1} a_j = \sum_{j=0}^{rh-1} b_j = i$. $U_i((a_0, a_1, \ldots, a_{rh-1}), (b_0, b_1, \ldots, b_{rh-1}))$ denotes the increase in the number of calls in the cell by one when the number of calls in the cell is i, starting combination of calls in different phases is $(a_0, a_1, \ldots, a_{rh-1})$, finishing combination of calls in different phases is $(b_0, b_1, \ldots, b_{rh-1})$, $\sum_{j=0}^{rh-1} a_j = i$ and $\sum_{j=0}^{rh-1} b_j = i+1$. $D_i((a_0, a_1, \ldots, a_{rh-1}), (b_0, b_1, \ldots, b_{rh-1}))$ denotes the decrease in the number of calls in a cell by one when the number of calls in the cell is i, starting combination of calls in different phases is $(a_0, a_1, \ldots, a_{rh-1})$, finishing combination of calls in different phases is $(b_0, b_1, \ldots, b_{rh-1})$, $\sum_{j=0}^{rh-1} a_j = i$ and $\sum_{j=0}^{rh-1} b_j = i-1$. There can be one call completion, one call arrival or change in the phase of one call in each transition as we consider a continuous time Markov chain. The nonzero entries of B_i, U_i and D_i are shown in Equations (5.6), (5.7), (5.8), (5.9), (5.10) and (5.11). Here, $L(j,k)$ represents the element of the intersection of j-th row and k-th column $(0 \leq j, k \leq rh-1)$ of the matrix L. $L^0(j)$ represents j-th element of the column vector L^0. Similarly, $\delta_n(j)$ and $\delta_h(j)$ represent the j-th element of the row-vectors δ_n and δ_h, respectively.

Let us assume that $x = [x_0, x_1, \ldots, x_C]$ is the steady state probability vector for the continuous time Markov chain Q of Equation (5.5) which satisfies $xQ = \mathbf{0}'$ and $x\mathbf{1} = 1$. Here, x_i $(0 \leq i \leq C)$ is a row vector having $N_i^{(rh)}$ scalar elements. x_i can be represented as $x_i = [x_{i,0}, x_{i,1}, \ldots, x_{i,N_i^{(rh)}-1}]$ where $x_{i,j}$ represents the steady state probability of having

Table 5.1: Non-zero entries of B_i, U_i and D_i

$$B_0((0,0,\ldots,0),(0,0,\ldots,0)) = -(\lambda_n + \lambda_h). \tag{5.6}$$

$$B_i((a_0, a_1, \ldots, a_{rh-1}),(a_0, a_1, \ldots, a_{rh-1})) = \begin{cases} \sum_{k=0}^{rh-1} a_k L(k,k) - (\lambda_n + \lambda_h), & 1 \le i < C-g \\ \sum_{k=0}^{rh-1} a_k L(k,k) - \lambda_h, & C-g \le i < C \end{cases} \tag{5.7}$$

$$B_C((a_0, a_1, \ldots, a_{rh-1}),(a_0, a_1, \ldots, a_{rh-1})) = \sum_{k=0}^{rh-1} a_k L(k,k). \tag{5.8}$$

$$B_i((a_0, a_1, \ldots, a_j, \ldots, a_k, \ldots, a_{rh-1}),(a_0, a_1, \ldots, a_j - 1, \ldots, a_k + 1, \ldots, a_{rh-1})) = a_j L(j,k)), \; a_j > 0 \text{ and } j \ne k. \tag{5.9}$$

$$U_i((a_0, a_1, \ldots, a_j, \ldots, a_{rh-1}),(a_0, a_1, \ldots, a_j + 1, \ldots, a_{rh-1})) = \begin{cases} \delta_n(j)\,\lambda_n + \delta_h(j)\,\lambda_h, & 0 \le i < C-g \\ \delta_h(j)\,\lambda_h, & C-g \le i < C \end{cases} \tag{5.10}$$

$$D_i((a_0, a_1, \ldots, a_j, \ldots, a_{rh-1}),(a_0, a_1, \ldots, a_j - 1, \ldots, a_{rh-1})) = a_j L^0(j), \; (1 \le i \le C) \text{ and } a_j > 0. \tag{5.11}$$

i ongoing calls in a cell and j is the auxiliary variable for a particular combination of i ongoing calls in rh channel holding time phases. If we let $\prod_{i=1}^{n} \beta_i = \beta_1\beta_2 \ldots \beta_n$ for matrices $\beta_1, \beta_2, \ldots, \beta_n$ we can find the steady-state probability vector x_i $(1 \leq i \leq C)$ using the technique of Gaver et al [33] in the following way

$$x_i = x_0 \prod_{k=1}^{i} (U_{k-1}(-E_k)^{-1}) \quad (1 \leq i \leq C) \tag{5.12}$$

where x_0 satisfies $x_0 E_0 = \mathbf{0}'$ and

$$x_0(I + \sum_{i=1}^{C} \prod_{k=1}^{i} (U_{k-1}(-E_k)^{-1}))\mathbf{1} = 1. \tag{5.13}$$

E_k $(0 \leq k \leq C)$ is recursively determined by $E_C = B_C$ and

$$E_k = B_k + U_k(-E_{k+1})^{-1} D_{k+1} \quad (0 \leq k \leq C-1). \tag{5.14}$$

A new call is blocked if the number of ongoing calls is more than $C-g-1$. Therefore, the new call blocking probability P_b can be expressed as

$$P_b = \sum_{i=C-g}^{C} x_i \mathbf{1} = \sum_{i=C-g}^{C} \sum_{j=0}^{N_i^{(rh)}-1} x_{i,j}. \tag{5.15}$$

A handoff fails if the number of ongoing calls in the cell is C. Therefore, the handoff failure probability P_{hf} can be expressed as

$$P_{hf} = x_C \mathbf{1} = \sum_{j=0}^{N_C-1} x_{C,j}. \tag{5.16}$$

The rate of handoff generated from a cell to its neighboring cells is $\lambda_n P_h^n(1-P_b) + \lambda_h P_h^h(1-P_{hf})$ and the rate of handoff coming to a cell is λ_h. In the steady state the incoming and outgoing handoff arrival rates are same. Equating these two rates the mean handoff arrival rate λ_h can be expressed as

$$\lambda_h = \frac{P_h^n(1-P_b)}{1 - P_h^h(1-P_{hf})} \lambda_n. \tag{5.17}$$

We can iteratively solve the above equations for given new call arrival process, cell residence time and call holding duration to determine new call blocking and handoff

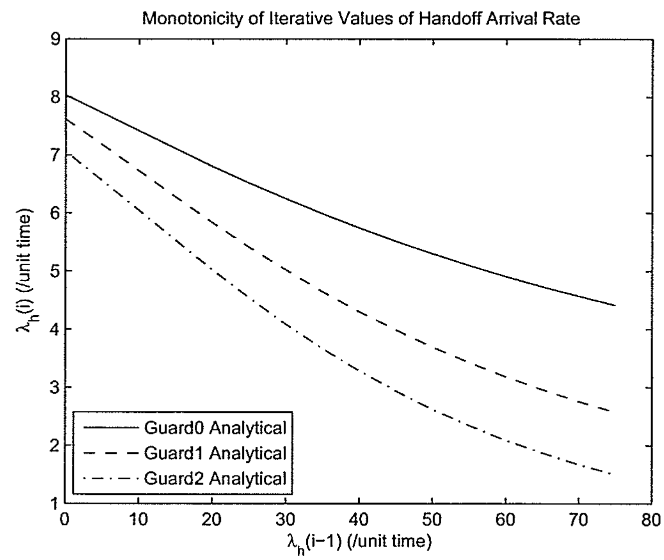

Figure 5.1: Monotonicity of the iterative values of handoff arrival rate

failure probabilities for different number of guard channels. We use initial value of handoff arrival rate as $\frac{P_h^n}{1-P_h^h}\lambda_n$ which is at least equal to the handoff arrival rate λ_h. The pseudo-code for the iterative algorithm is given below:

Solve($\alpha, \beta, T, S, \lambda_n, C, \varepsilon$)

$i \leftarrow 0$;
$\lambda_h^{(i)} \leftarrow \frac{P_h^n}{1-P_h^h}\lambda_n$;
do
  $\lambda_h \leftarrow \lambda_h^{(i)}$;
  Compute P_b and P_{hf} using Equations (5.5)-(5.16);
  $\lambda_h^{(i+1)} \leftarrow \frac{P_h^n(1-P_b)}{1-P_h^h(1-P_{hf})}\lambda_n$;
  $i \leftarrow i+1$;
while($\mid \lambda_h^{(i)} - \lambda_h^{(i-1)} \mid \geq \varepsilon$);
$\lambda_h \leftarrow \lambda_h^{(i)}$;

End of Solve Algorithm

Haring et al [39] prove analytically the monotonicity property of the handoff arrival rate for exponentially distributed handoff inter-arrival time, cell-residence time and call holding duration. They show that the resulting i-th iterative value of the handoff arrival rate $\lambda_h^{(i)}$ decreases with the increase in the previous $(i-1)$-th iterative value of the handoff arrival rate $\lambda_h^{(i-1)}$ and $\lambda_h^{(i)} = 0$ for $\lambda_h^{(i-1)} = \infty$. They show that the fixed point iteration provides a unique solution for the handoff arrival rate λ_h. We cannot prove the monotonicity property for the handoff arrival rate analytically for general distributed cell-residence time and call holding duration captured by phase type distributions. However, we find from our numerical examples reported in Figure 5.1 that the monotonicity property holds for the handoff arrival rate for general distributed cell-residence time and call holding duration. Therefore, we can conjecture that λ_h has an unique solution for our model. In Figure 5.1, the values of $\lambda_h^{(i)}$ is plotted against the values of $\lambda_h^{(i-1)}$. The figure shows the monotonicity property of the iterative values of the handoff arrival rate for $C = 10$, $\lambda_n = 48.0/\text{unit time}$, $\alpha = \begin{bmatrix} 1.0 & 0.0 \end{bmatrix}$, $T = \begin{bmatrix} -2.0 & 2.0 \\ 0.0 & -2.0 \end{bmatrix}$, $\beta = \begin{bmatrix} 1.0 & 0.0 \end{bmatrix}$ and $S = \begin{bmatrix} -10.0 & 9.0 \\ 0.0 & -10.0 \end{bmatrix}$. We use "Guard0 Analytical", "Guard1 Analytical" and "Guard2 Analytical" for $g = 0$, $g = 1$ and $g = 2$, respectively.

5.2 Implementation Issues

We need to compute the nonzero entries $B_i((a_0, a_1, \ldots, a_{rh-1}), (b_0, b_1, \ldots, b_{rh-1}))$, $U_i((a_0, a_1, \ldots, a_{rh-1}), (b_0, b_1, \ldots, b_{rh-1}))$ and $D_i((a_0, a_1, \ldots, a_{rh-1}), (b_0, b_1, \ldots, b_{rh-1}))$ of B_i, U_i and D_i matrices, respectively for i users. We need to enumerate $N_i^{(rh)}$ phase combinations $(a_0, a_1, \ldots, a_{rh-1})$ in lexicographically ascending (or descending) order for the row indices of B_i, U_i and D_i matrices. We can store $N_i^{(rh)}$ phase combinations in an $N_i^{(rh)} \times rh$ array. For each index j $(0 \le j \le N_i^{(rh)} - 1)$ we have an unique phase combination $(a_0, a_1, \ldots, a_{rh-1})$. Similarly, we can have different $(b_0, b_1, \ldots, b_{rh-1})$ phase combinations after transition for a given initial phase combination $(a_0, a_1, \ldots, a_{rh-1})$. We need to compute the column index for a phase combination $(b_0, b_1, \ldots, b_{rh-1})$ for B_i, U_i or D_i. Therefore, we need an algorithm for generating enumeration of the phase combinations in a lexicographical order and an algorithm for computing an index for a given phase combination. We generate the enumerations in lexicographically ascending order using "NexCom" algorithm which is adapted from the "NEXCOM" algorithm of Wilf [101]. Two different algorithms — "getIndex" and "getHashInd" are developed for computing the column indices of the block matrices. "getHashInd" algorithm generates index for a given combination of phases by using a hash data structure "hashTabl". "hashTabl" is computed using a simple hash generating algorithm "setHashTabl".

The enumeration of combinations of i users in $K = rh$ phases can be generated by using "NexCom" algorithm. The combination $(0, 0, \ldots, 0, i)$ is the first combination and $(i, 0, 0, \ldots, 0)$ is the last combination for i users with K different phases. Here, "IndexTable" in "NexCom" algorithm is a two-dimensional array of dimension $N_i^{(K)} \times K$ which keeps track of different $N_i^{(K)}$ combinations of i different users in K different phases. For different values of i which vary from zero to C, the total number of channels in a cell, we need to store $(C + 1)$ different "IndexTable" arrays.

```
NexCom(Integer i, K, IndexTable[][])
  Integer MTC ← 0, L, Z, G, Y[K], J;
  Y[0] ← i;
  Z ← i;
  G ← −1;
  J ← 0;
  if(K = 1)
    MTC ← 0;
    IndexTable[J][0] ← Y[0];
  else
    L ← 1;
    while(L < K)
      Y[L] ← 0;
      L ← L + 1;
  L ← 0;
  while(L < K)
    IndexTable[J][K − L − 1] ← Y[I];
    L ← L + 1;
  if(N = 0)
    MTC ← 0;
  else
```

```
        MTC ← 1;
    J ← J + 1;
    while(MTC)
        if(Z > 1)
            G ← 0;
        else
            G ← G + 1;
        Z ← Y[G];
        Y[G] ← 0;
        Y[0] ← Z − 1;
        Y[G + 1] ← Y[G + 1] + 1;
        if(Y[K − 1]! = i)
            MTC ← 1;
        else
            MTC ← 0;
        L ← 0;
        while(L < K)
            IndexTable[J][K − L − 1] ← Y[L];
            L ← L + 1
        J ← J + 1;
End of NexCom Algorithm
```

A combination table "CTabl" is computed in "genComb" algorithm using the relation $\binom{n-1}{r-1} + \binom{n-1}{r} = \binom{n}{r}$. We need the combination values to allocate memories for the block matrices B_i, U_i and D_i. We also need to compute the row and column indices of the block matrices for given combination of phases in "getIndex" algorithm and "hashTabl" data structure in "setHashTabl" algorithm.

```
genComb(Integer C1, K1, CTabl[][])
  Integer I, J, nRow, nCol;
  nRow ← C1 + 1;
  nCol ← K1 + 1;
```

```
I ← 0;
while(I < nRow)
   CTabl[I][0] ← 1;
   I ← I + 1;
I ← 0;
while(I < nCol)
   CTabl[I][I] ← 1;
   I ← I + 1;
J ← 1;
while(J < nCol)
   I ← J + 1;
   while(I < nRow)
      CTabl[I][J] ← CTabl[I − 1][J]+ CTabl[I − 1][J − 1];
      I ← I + 1;
   J ← J + 1;
```

End of genComb Algorithm

"getIndex" algorithm computes the index for a given combination $(a_0, a_1, a_2, \ldots, a_{K-1})$ of K phases of i users. For lexicographically ascending order generated combinations, the combination $(0, 0, 0, \ldots, i)$ is the zero-th combination. "getIndex" algorithm computes the distance of the combination $(a_0, a_1, a_2, \ldots, a_{K-1})$ from the combination $(0, 0, 0, \ldots, i)$ assuming the distance of the combination $(0, 0, 0, \ldots, i)$ as zero. The outer "while" loop of the algorithm iterates for $(K-1)$ phases from phase zero to phase $(K-2)$ (in "getIndex" algorithm Phase$[j] = a_j$). The inner "while" loop works on each nonzero a_j $(0 \leq j \leq K-2)$ for a_j times. The zero-th combination $(0, 0, 0, \ldots, i)$ of i users in K phases has no user for phases zero to $(K-2)$ and i users in the $(K-1)$-th phase. Suppose $a_0 \geq 1$. "getIndex" algorithm first computes the distance of $(1, 0, 0, \ldots, i-1)$ from $(0, 0, 0, \ldots, i)$. $(1, 0, 0, \ldots, i-1)$ is the next combination of $(0, i, 0, \ldots, 0)$ in the lexicographically ascending order generated enumeration. The total number of combination from $(0, 0, 0, \ldots, i)$ to $(0, i, 0, \ldots, 0)$ is $N_i^{(K-1)}$ as $(K-1)$ nonnegative numbers can be added to get i in $N_i^{(K-1)}$ ways. Therefore, the distance between $(1, 0, 0, \ldots, i-1)$ and $(0, 0, 0, \ldots, i)$ is $N_i^{(K-1)}$. Similarly, the distance from $(1, 0, 0, \ldots, i-1)$ to $(2, 0, 0, \ldots, i-2)$

is $N_{i-1}^{(K-2)}$. Hence, the distance of $(a_0, 0, 0, \ldots, i - a_0)$ from $(0, 0, 0, \ldots, i)$ is $\sum_{l=1}^{a_0} N_{i-l+1}^{(K-l)}$. After computing the distance for a_0 users in zero-th phase and $i - a_0$ users in the $(K-1)$-th phase the algorithm computes the distance of a_0 users in the zero-th phase, a_1 users in the first phase and $i - a_0 - a_1$ users in the $(K-1)$-th phase. This process continues until the outer "while" loop runs for $(K-1)$ times. "getIndex" algorithm uses "CTabl" for accessing the number $N_e^{(f)}$ from the $(e+f-1, f-1)$-th entry of "CTabl" for nonnegative integers e and f. "getIndex" algorithm accesses "CTabl" for i times for computing the distance of the phase combination for i users. For i users "getIndex" algorithm requires $O(i)$ operations.

```
Integer getIndex(Integer i, K, Phase[], CTabl[][])
  Integer Q, J, M, L;
  M ← i, L ← 0;
  J ← 0;
  while(J < K − 1)
    if(Phase[J] > 0)
      Q ← 0;
      while(Q < Phase[J])
        L ← L+ CTabl[M + K − J − 2][K − J − 2];
        M ← M − 1;
        Q ← Q + 1;
    J ← J + 1;
  return L;
End of getIndex Algorithm
```

If the number of phases K is more than two we can compute index in $O(K)$ operations by using "hashTabl", a hash data-structure which is a $(K-2) \times (C+1) \times (C+1)$ array. "getIndex" algorithm accesses "CTabl" for a_j $(0 \leq j \leq K-2)$ times for each nonzero a_j in the phase combination $(a_0, a_1, \ldots, a_{K-1})$. On the other hand, the use of "hashTabl" in "getHashInd" algorithm requires one access for each nonzero a_j $(0 \leq j \leq K-3)$. The entry "hashTabl"$[j][i - \sum_{m=0}^{j-1} a_m][a_j]$ for i users contains the distance between the phase combination $(a_0, a_1, \ldots, a_{j-1}, 0, 0, \ldots, i - \sum_{m=0}^{j-1} a_m)$ and $(a_0, a_1, \ldots, a_{j-1}, a_j, 0, \ldots, i - \sum_{m=0}^{j} a_m)$. "hashTabl" requires more state-space than

"CTabl". The entries of "hashTabl" can be filled up by the following pseudo-code of "setHashTabl" algorithm.

```
setHashTabl(Integer i, K, hashTabl[][][], CTabl[][])
 Integer i, j, k;
 l ← 0;
 while(l < K − 2)
    j ← 0;
    while(j ≤ i)
       hashTabl[l][j][0] ← 0;
       k ← 1;
       while(k ≤ j)
          hashTabl[l][j][k] ← hashTabl[l][j][k − 1]
                               + CTabl[j − k + K − l − 1][K − l − 2];
          k ← k + 1;
       j ← j + 1;
    l ← l + 1;
End of setHashTabl Algorithm
```

We can compute the index of a phase "phase" by using the hashtable using the following pseudo-code of "getHashInd" algorithm. "getHashInd" algorithm takes as input the number of ongoing calls i, number of phases K, "hashTabl" and the given combination of phases "phase". "phase" is a one-dimensional array of size K with indices 0 to $K-1$. 0 to $(K-2)$-th elements of "phase" array are used compute the index of the given combinations of i calls in K phases.

```
Integer getHashInd(Integer i, K, hashTabl[][][], phase[])
 Integer nIndex, nRem, j;
 nIndex ← 0;
 nRem ← i;
 j ← 0
 while(j < K − 2)
    nIndex ← nIndex + hashTabl[j][nRem][phase[j]];
    nRem ← nRem - phase[j];
```

$j \leftarrow j + 1$;

nIndex $\leftarrow$ nIndex + phase$[K-2]$;

return nIndex;

End of getHashInd Algorithm

5.3 Numerical Examples

We carry out analytical and discrete event simulation studies of our model of Section 5.1. We develop analytical and discrete event simulation programs for performance analysis of the guard channel scheme with general cell residence time and general call holding duration captured by phase type distributions using "C" compiler. We execute our programs for analytical and simulation studies. In this section, we first discuss the relevant parameters for analytical and simulation studies and the complexities of simulation for phase type distributed cell residence time. Next we discuss the decrease in the state-space for our analytical method followed by some analytical and simulation results.

Simulation studies are performed on a nineteen-cell wireless system of Figure 3.7 with wrap-around structure. We assume that there are ten channels in each cell, i.e., $C = 10$. Cell residence time R is represented by (α, T) where $\alpha = \begin{bmatrix} 1.0 & 0.0 \end{bmatrix}$ and $T = \begin{bmatrix} -2.0 & 2.0 \\ 0.0 & -2.0 \end{bmatrix}$. The mean of the cell residence time is 1.0 unit time and the mean of the residual cell residence time is 0.75 unit time. Similarly, call holding duration H is represented by (β, S) where $\beta = \begin{bmatrix} 1.0 & 0.0 \end{bmatrix}$ and $S = \begin{bmatrix} -10.0 & 9.0 \\ 0.0 & -10.0 \end{bmatrix}$. The mean of the call holding duration is 0.19 unit time and the mean of the residual call holding duration is 0.1474 unit time. New call arrival process is assumed to follow Poisson distribution. The mean of the new call arrival rate is varied from 4.0/unit time to 72.0/unit time. Channel holding times of new and handoff calls are given by (δ_n, L_n) and (δ_h, L_h), respectively where $\delta_n = \begin{bmatrix} 0.5 & 0.0 & 0.5 & 0.0 \end{bmatrix}$, $\delta_h = \begin{bmatrix} 0.5263 & 0.4737 & 0.0 & 0.0 \end{bmatrix}$ and

$$L_n = L_h = \begin{bmatrix} -12.0 & 9.0 & 2.0 & 0.0 \\ 0.0 & -12.0 & 0.0 & 2.0 \\ 0.0 & 0.0 & -12.0 & 9.0 \\ 0.0 & 0.0 & 0.0 & -12.0 \end{bmatrix}.$$

The mean channel holding time of a new call is 0.1632 unit time and the mean channel holding time of a handoff call is 0.1411 unit time. 10000.0 unit time is chosen as warm-up time for the simulation. Ten independent replications of length 5000.0 unit time are performed in the simulation studies.

The simulation study of cellular wireless network for phase type distributed cell residence time is more complex than the simulation study for exponentially distributed cell residence time. For exponentially distributed cell residence time, call holding duration and new call inter-arrival times, an event list is maintained for next new call arrival in a cell, call completion time and cell residence time of an ongoing call. There is no need to keep track of elapsed time of a user in a cell making new call attempt for exponentially distributed cell residence time for memoryless property of exponential distribution. In discrete event simulation for cellular wireless network new call inter-arrival times are generated for all the cells. When a user makes a successful new call attempt, the cell residence time and call holding duration are generated for the user. On the other hand, a new cell residence time is computed when a user with an ongoing call makes a successful handoff to another cell. On the other hand, the elapsed time of a user in a cell is needed to keep track of in phase type distributed cell residence time when a user makes a new call attempt. Therefore, we need to generate separate auxiliary lists for the cell residence time of users who do not have any ongoing calls in all the cells. During a successful new call attempt in a cell we need to remove one user from the auxiliary list as a random user and generate events for its call holding duration. Current call holding phase, current cell residence phase, times of leaving the current call holding duration phase and current cell residence time phase are maintained in the event list. Event list also maintains the new call inter-arrival time in all the cells. We keep the length of the auxiliary list in a cell within the range $[Lim_L, Lim_H]$. We use $Lim_L = 3C$ and $Lim_H = 5C$. In ideal situation, the length of the auxiliary list should be infinite which is not possible to implement. When a user without an ongoing call finishes cell residence time we remove the user from auxiliary list and choose one of its six neighboring cells randomly. When the number of users in auxiliary list in a cell goes below Lim_L we add a user in that auxiliary list. On the other hand, we add a user in the auxiliary list of a cell if the number of users in that auxiliary list is below Lim_H and a call completes or a failed handoff occurs.

The state-space required for our analytical model with ten channels and four different

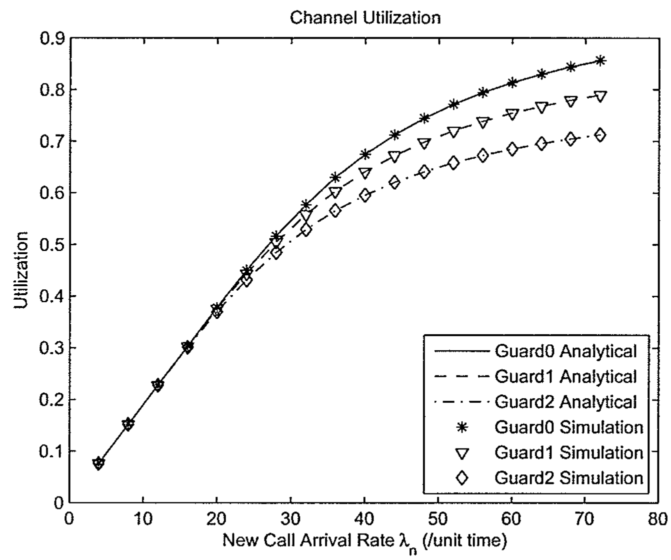

Figure 5.2: Analytical and simulation results of channel utilization

phases of channel holding time is $\sum_{i=0}^{10} \begin{pmatrix} i+3 \\ 3 \end{pmatrix} = 971$. On the other hand, if we want to keep track of the phase of the channel holding time of each call then the state-space required is $\sum_{i=0}^{10} 4^i = 1,398,101$. If the method of Alfa and Li [3] is used which keeps track of the phase of the channel holding time of each call and the number of handoff calls the state-space required will be larger than $1,398,101$. Our method substantially reduces the state-space of Markov chain and the required amount of computations.

Analytical and simulation results of channel utilization, new call blocking probability, handoff call failure probability and handoff arrival rate are shown in Figures 5.2, 5.3, 5.4 and 5.5, respectively. In these figures, we denote analytical results of non-priority handoff scheme as "Guard0 Analytical", guard channel schemes with one and two guard channels as "Guard1 Analytical" and "Guard2 Analytical", respectively. Similarly, we denote simulation results of non-priority handoff scheme as "Guard0 Simulation", guard channel schemes with one and two guard channels as "Guard1 Simulation" and "Guard2 Simulation", respectively. Figures 5.2-5.5 show close match between simulation and analytical results. From these figures we can see increase in the new call blocking probability, decrease in the handoff failure probability, decrease in the channel utilization and decrease

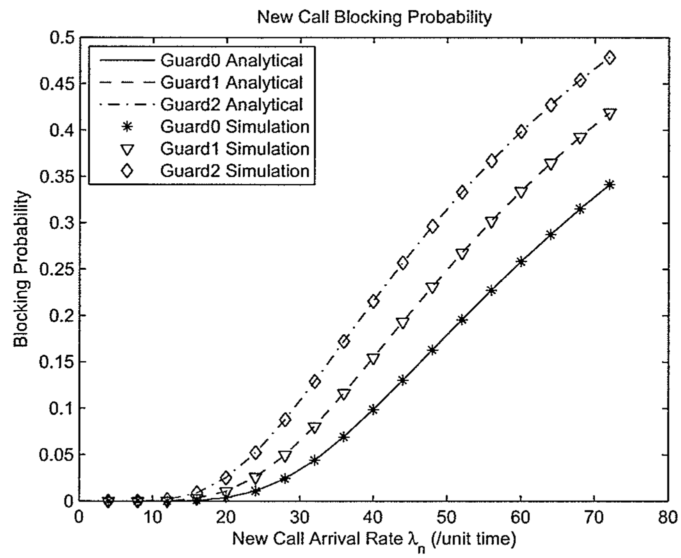

Figure 5.3: Analytical and simulation results of new call blocking probability

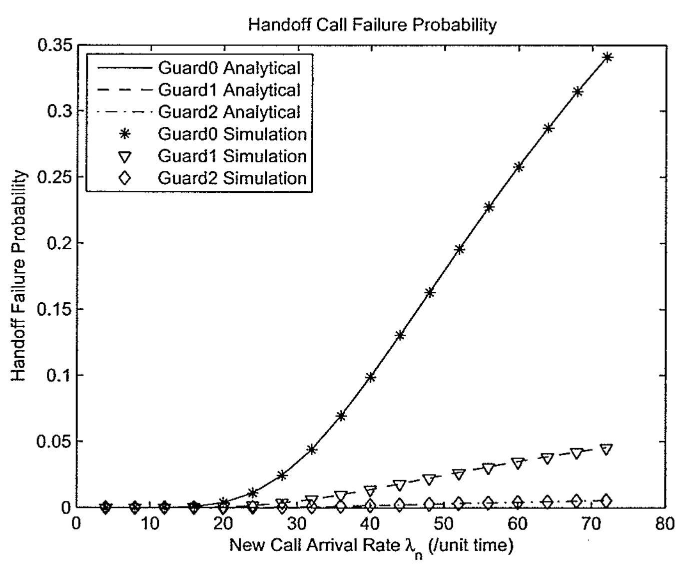

Figure 5.4: Analytical and simulation results of handoff call failure probability

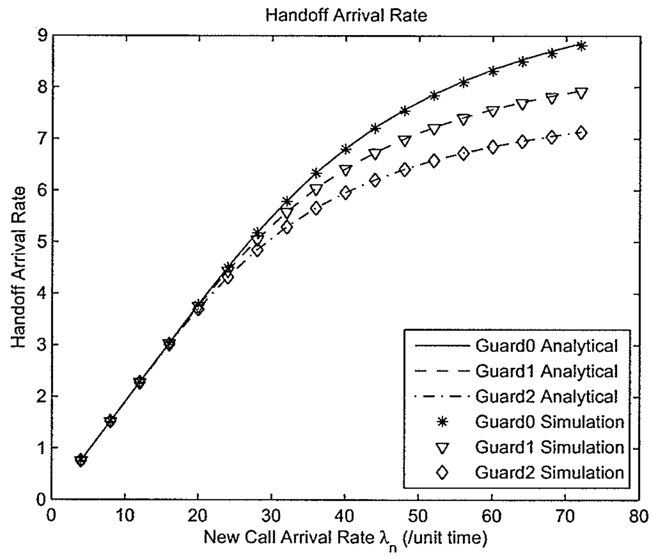

Figure 5.5: Analytical and simulation results of handoff arrival rate

in the mean handoff arrival rate with increase of guard channels from zero to two. We can achieve our desired channel utilization or handoff failure probability by changing the number of guard channels with change in the new call arrival rate. Figures 5.2 and 5.5 shows that the rate of increase of channel utilization and handoff arrival rate with increase in new call arrival rate decreases due to increase in the new call blocking and the handoff failure probabilities.

Chapter 6

Conclusion and Future Work

6.1 Conclusion

In this thesis, steady-state analysis of non-prioritized handoff and guard channel schemes are carried out for two-cell system. It is shown in those analyses that the assumption of exponentially or hyper-exponentially distributed handoff inter-arrival time is not justified. Handoff process in two-cell wireless network can be captured by a two-dimensional Markov chain. Then an adaptive call admission control scheme for the two-cell system is proposed which provides a guarantee, with some probability, that if a user is admitted the call will be completed irrespective of its movement in the two-cell system. The adaptive scheme for two cell wireless network is extended to develop the adaptive call admission control scheme for multiple cell wireless network. Then fractional adaptive scheme for multiple cell wireless system — a variant of our adaptive scheme for multiple cell system is developed. Some numerical examples are provided which show that both the adaptive and fractional adaptive schemes outperform the guard channel scheme in controlling the handoff failure probability in cellular wireless network consisting of more than two cells. The numerical examples present us some insight about the new call blocking and handoff failure probabilities with new call arrival rate, cell residence time, call holding time, number of ongoing calls in the current and the neighboring cells.

The framework presented for our adaptive schemes is very significant as it completely determines the impacts of the number of calls in the current cell and the neighboring cells. In our framework, we do not make any assumption about the handoff call arrival process or the distribution of number of calls in a cell which makes our scheme realistic. As a result, our framework provides the tool for efficient call admission controller design under realistic conditions. Our method is designed for off-line computation since our schemes are mainly suitable for designing call admission controller whereas other distributed schemes can be considered as operational schemes as they use online computation. Based on the framework for single class of traffic we extend our adaptive call admission control scheme for two classes of calls requiring different QoS.

In the first part of the thesis, we assume exponentially distributed cell residence time in our analysis as done by other researchers so that we could focus on control issues. However, cell residence time depends on position and velocity of a caller and dimension of a cell. Therefore, phase type distribution is more appropriate for cell residence time. Assumption of phase type distribution for cell residence time and exponential or phase type distributed call holding time makes channel holding time of a call phase type distributed. But it is difficult to study a multi-server system with phase type distributed server occupation time by each job for use of Kronecker product to keep track of the phase of service time in each busy server. In this thesis, an efficient computational model is developed which does not require this Kronecker product. The numbers of calls in each phase are kept track of in our model which require substantially less state-space than the model which keeps track of the phase of the channel holding time of each call. Efficient algorithms are developed for implementing the model to keep track of the number of calls in different phases. Handoff probabilities of new and handoff calls for phase type distributed cell residence time and call holding time distribution are also computed. A novel simulation technique for phase type distributed cell residence time is also proposed and implemented in this thesis.

6.2 Future Work

The following directions will be pursued for future research work.

- Transient analysis is used for our adaptive call admission controller schemes. Jensen's method [48] is applied for transient analysis. We will study alternative faster methods for transient analysis.

- Retrial of calls have not been studied. When a new call attempt fails a user usually again tries to make a successful call. On the other hand, when a handoff attempt fails a user can try to make a call again. Retrial of failed new and handoff call attempt will be considered in our future work.

- Use of absorbing Markov chain for determining the dropping probability of an accepted call will be studied. Use of absorbing Markov chain requires less computational effort than transient analysis. Moreover, use of absorbing Markov chain will enable us to get the expressions of new call blocking and handoff call failure probabilities analytically. Combination of transient analysis and simulation is used for our adaptive call admission controller schemes.

- Our model for general distributed cell residence time and call holding duration can be extended from a single class of call to multiple classes of calls.

- Relative QoS is not studied in our thesis. However, we believe that our adaptive scheme for two classes can be modified to have relative QoS among classes of calls.

- Use of multiple channels by a call is not considered in our scheme for two classes of users. We will consider multiple-channel usage by a call in our schemes for two classes of calls. Moreover, bandwidth adaptation for multiple channel usage by a call in our schemes can be considered for two classes of calls.

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List of Acronyms

CAC	Call Admission Control
cdf	Cumulative Distribution Function
CP	Complete Partitioning
CS	Complete Sharing
DCAC	Distributed Call Admission Control
DTR	Dual Threshold Reservation
EFGC	Extended Fractional Guard Channel
FCR	Fractional Channel Reservation
FGCS	Fractional Guard Channel Scheme
FIFO	First In First Out
FPA	Fixed Point Approximation
GC	Guard Channel
gDCAC	Generalized Distributed Call Admission Control
GPS	Global Positioning System
IMOSP-CS	Independent Multi-Class One Step Prediction Complete Sharing
IMOSP-RES	Independent Multi-Class One Step Prediction Reservation
LFGCP	Limited Fractional Guard Channel Policy
LIFO	Last In First Out
MAP	Markovian Arrival Process
MBPS	Measurement Based Priority Scheme
mDCAC	Mobility-Aware Distributed Call Admission Control
MFCR	Multiple Fractional Channel Reservation
MMDR	Multimedia Dynamic Reservation
MMOSPRED	Multimedia One Step Prediction

MOL	Modified Offered Load
MPr	Measurement-Based Pre-assignment
OSPRED	One Step Prediction
PCR	Predictive Channel Reservation
pdf	Probability Density Function
QoS	Quality of Service
RCD	Regulated Call Dropping
SDCA	Stable Dynamic Call Admission
SPN	Stochastic Petri Net
UDF	Upgrade/Degrade Frequency
UMP	User Mobility Profile