Approaches to the Consideration of Reliability in Water Distribution Systems

A thesis
presented to the University of Manitoba
in partial fulfillment of the
requirements for the degree of
Master of Science
in
Civil Engineering

by

A. V. Coals

Winnipeg, Manitoba

APPROACHES TO THE CONSIDERATION OF RELIABILITY IN WATER DISTRIBUTION SYSTEMS

BY

ALISON V. COALS

A thesis submitted to the Faculty of Graduate Studies of the University of Manitoba in partial fulfillment of the requirements of the degree of

MASTER OF SCIENCE

I hereby declare that I am the sole author of this thesis.

I authorize the University of Manitoba to lend this thesis to other institutions or individuals for the purpose of scholarly research.

A. V. Coals

I further authorize the University of Manitoba to reproduce this thesis by photocopying or by other means, in total or in part, at the request of other institutions or individuals for the purpose of scholarly research.

A. V. Coals

ABSTRACT

In the past, water distribution systems have been designed with the aim of keeping costs to a minimum, with reliability being of secondary importance. An attempt to balance the 'least cost' aspect with improved reliability is made, through use of optimization techniques. Three different iterative methods are developed, each incorporating a measure of reliability into a system. These approaches use linear programming in a general form, and also goal programming, a particular type of linear programming.

ACKNOWLEDGEMENTS

I would like to thank Professor I.C. Goulter for his advice and guidance given throughout the preparation of this thesis. I would also like to thank Tony Kettler, whose suggestions and insights were of great assistance.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

Chapter I

INTRODUCTION

The deterioration of various supply services in urban areas has become a source of major concern in recent years. These services include water distribution systems. The demands on these systems are constantly increasing with population and related industrial expansion. As costs of replacing or upgrading these networks are also increasing, ways of designing more reliable systems which can deal with these greater demands, while keeping costs to a minimum, are being sought.

Previous research into the design of least cost water distribution systems has used optimization, a decision-making process involving mathemtical modelling, to obtain 'optimal' layouts at minimum cost subject to hydraulic constraints. The optimization models used in this study retain this single-objective approach, either minimizing cost subject to reliability constraints (Approaches 1 and 3), or maximizing reliability, in an indirect way, subject to cost constraining (Approach 2).

Optimization techniques have long been used to deal with reliability in structural design, where the structure is considered in its entirety (Khachaturian, in Cohn, ed., 1969). Given the relative success of these techniques in the structural field, attempts to extend their use into other fields of civil engineering have been made. One such attempt is the use of optimization in the design of water distribution networks. A number of models exist to find optimal least cost solutions (Karmeli et al.,1968, Alperovits and Shamir, 1977, Quindry et al, 1981), but few have explicitly incorporated the aspect of reliability into their systems.

The probability of the pipe network being unable to provide the design flow at a given node, and the implicit significance of individual pipe failure, is very similar to the definition of a structure failing, given the failure of at least one of its members. This definition has been used in the development of a least cost design model, in which the probability of failure of a 'primary' path to supply a node is considered (Kettler and Goulter, 1983). The primary path is that route from the source to the node which carries the greatest portion of the demand at that node. The sum of the probabilities of failure of each link in the path becomes the probability of failure of the path. The capabilities of other pipes supplying that node in other paths are not considered in this approach; thus a true measure of reliability is not achieved.

In order to improve upon the basic approach of Kettler and Goulter (1983), three different iterative approaches have been developed. As a first approach, the idea of simultaneous failure of all paths to a node is considered. This is achieved by determining the probability of all links immediately connected to a given node failing in the same time period. A one-year time period has been used in this study, simply to keep numbers from becoming so small that changes would not easily be observed. It is assumed that adequate supply to the node is maintained if at least one link is capable of meeting the demand; thus if all links supplying the node fail, the system fails.

This introduces the idea of redundancy in a system. In previous work redundancy has been assumed to exist as long as there are two links, or paths, supplying a node. If, however, 'true' redundancy is to exist in the network, each demand point shoud be connected to the source by at least two distinct paths, each of which could supply the design flow to the node if necessary. Each path then, should have the capacity to supply the node to which it is connected. This implies that each of those paths should have similar capacities. A second approach attempts to achieve 'true' redundancy, ensuring adequate flow capacity, by imposing a predetermined 'acceptable' reliability at each node, and then using optimization to minimize the difference in reliabilities of each of the links connected to a node.

The third approach returns to the original concept of path failure, but incorporates the idea of simultaneous failure of all links, as developed in the first approach. The three methods are described in greater detail in Chapters IV, V, and VI respectively.

These approaches to the consideration of reliability are compared (Chapter VII), and their effects on the least cost design of a water distribution network, obtained through optimization procedures, analysed.

Chapter II

LITERATURE REVIEW

2.1 DEVELOPMENT OF RELIABILITY IN OPTIMIZATION MODELS OF WATER DISTRIBUTION NETWORKS

A number of optimization models have been developed in the search for optimum (least cost) design of both branched and looped water distribution systems. Recently, most have been based on the linear programming model formulated by Karmeli et al. (1968), where a branched water distribution network is optimized, by means of computer, to obtain a minimum cost solution. The flows in each section of pipe are predetermined, and the total friction loss in each are determined by the product of the head gradient and the length of pipe. The decision variable in the formulation represents the length of a specific diameter pipe, given a number of different diameter pipes to chose from. The system is constrained by both pressure and length restrictions. A minimum allowable pressure, which must be maintained, is set at each node. This is achieved by ensuring that the sum of the pressure losses in the section along a path is equal to, or less than, the allowable pressure difference. The length restriction is met by setting the sum of the lengths of all the different diameter pipes in a section equal to the total length of the section.

Alperovits and Shamir (1977) develop this model to include looped systems. To ensure hydraulic consistency, a further constraint is added, whereby the summation of head losses in each loop is set equal to zero. 'Optimality' is achieved through an iterative procedure, in which the dual variables associated with the loop constraints are used in a gradient search for adjusting flows. Quindry et al. (1979) correct the original formulation, and include the dual variables associated with the path constraints in the gradient technique. This results in a cheaper solution, but causes the system to revert to a branched network, using the paths selected for the headloss constraints. By 'removing' the remaining links in the loops, the system loses all measure of redundancy.

Schaake and Lai (1969) use linear programming to optimize looped water distribution system design, but employ a different decision variable. Rather than selecting lengths of different diameter pipes in each section, their decision variable is a diameter-based value derived from the Hazen-Williams formula, and is proportional to the flow in each section of pipe. In order to maintain hydraulic consistency, initial heads at each node are assumed, and a continuity constraint is applied so that the summation of flows at each node is zero.

Quindry et al. (1981) add a gradient search to the model of Schaake and Lai, using the dual solution to adjust the head at each nodel. An iterative procedure is then applied, in which optimal flows are found and pressures modified until no further improvement occurs.

Templeman (1982), in his discussion of this model, observes that

> Optimization tends to remove redundancy, and any spare capacity which is not immediately required by the design demand pattern is optimized out. Thus all flexibility is removed. The optimization process is not at fault here, it merely extrapolates the design process to its logical limits. Such faults are inherent in the design process itself which does not directly incorporate resilience, flexibility,and reliability into the design process.[1]

Few attempts to overcome this tendency towards a branched network, and to provide redundancy in a looped system, have been made. Rowell and Barnes (1982) and Morgan and Goulter (1982) have both developed models in which some degree of reliability has been ensured, by providing at least two independent and adequate paths to each node from the source(s). Goulter and Morgan (1984) show that Rowell and Barnes achieve reliability at the expense of hydraulic consistency, whereas Morgan and Goulter (1982) maintain hydraulic consistency while approaching the reliability issue. It should be noted that this latter model can be solved by satisfying the reliability constraints without providing true redunancy. A more recent study by Goulter and Morgan

---

[1] Templeman, A.B. 'Discussion of Looped Water Distibution Systems', Journal of the Environmental Engineering Division, ASCE, p.599, June 1982.

(1985a) attempts to provide a measure of redundancy by adding an additional constraint set. Further work by Morgan and Goulter (1985b) produces a model which can consider broken pipes and multiple demand patterns simultaneously.

Kettler and Goulter (1983) approach the reliability question directly, by considering the probability of pipe failure within the system. As most of the previous models have used a decision variable based on pipe diameter, an attempt to relate pipe diameter with the number of failures/km/year has been made. A statistical analysis shows a strong linear relationship between these parameters, where larger diameter, more expensive, pipes have lower failure rates. Using these failure rates, the probability of failure of each link in the network is determined using the Poisson probability distribution, and from these the probability of failure of each supply path is found.

The results of the statistical analysis performed by Kettler and Goulter are supported by a number of other studies. O'Day (1980, 1982, 1983) proposes an analytical approach as necessary for determining pipe replacements, based on the Philadelphia water supply system. Ciottini (1983), in another study on the Philadelphia system, supports the conclusion reached by O'Day that age is not as important as previously thought, and that pipe diameter plays a significant role in failure rates. Sullivan (1982), in a study conducted in Boston, identifies the same phenomenon of increased failure rate for smaller diameter pipes. Brcic (1983) uses pipe breakage data from St.Catharines to illustrate the tendency of smaller diameter pipes to have a higher failure rate than those having larger diameters. These results agree with an earlier study by Fitzgerald (1968), who shows that a larger pipe with a larger wall thickness has a lower incidence of failure. Clark et al. (1982) develop an economic replacement analysis using two equations, where the number of years between installation and the first failure/repair is shown to be a linear function of a number of different variables, one of which is pipe diameter. It should be noted that these studies all indicate that rate of breakage is dependent on pipe diameter. This suggests that models using diameter as a variable should be capable of considering tradeoffs between the larger, more expensive pipes, and the smaller, less expensive, but less reliable (in terms of failure rate) pipes.

2.2 RELIABILITY CONSIDERATION IN STRUCTURAL ANALYSIS

Khachaturian, Schmit, Fox, Moses, Lind, Cornell, and Prager (Cohn, ed., 1969) all investigate the issue of reliability in structural design using optimization techniques. Moses describes the 'weakest-link' structure, which fails if any single component fails. The probability of failure is defined as a function of the joint probability of failure of all members in the structure. Khachaturian also considers the reliability of an entire structure, in which probability of failure is determined from the probability density function of the load and resistance of each member. Optimization techniques are then used to minimize the weight of the structure, using decision variables representing the area of each member. The probability of failure (constraints) are set equal to a predetermined value.

The studies reviewed above support the use of both optimization techniques and probabilities of failure in developing a technique which will maximize overall system reliability at least cost. This technique is used in both Approaches 1 and 3 (see Chapters IV and VI).

Chapter III

GENERAL FORMULATION OF THE OPTIMIZATION MODEL

Optimization is a decision-making process, which uses a mathematical model to represent the problem or system under consideration. Once formulated, the model is solved algebraically, in such a way that the 'best', or optimal, result is selected from a number of possible alternatives.

The general objective is to design a looped water distribution system at least cost, subject to reliability constraints. A number of linear programming models have been developed, where minimizing cost has been the aim, subject to hydraulic and other physical constraints (Karmeli et al. 1968, Alperovits and Shamir 1977, Quindry et al. 1981). All functions in linear programming formulations must be expressed in linear form, in order to solve algebraically. The formulation used in this work is based on these models, but incorporates reliability into the constraint set.

The decision variable, which is obtained from the solution of the model, is the length of pipe of a certain diameter. The total number of decision variables is the product of the number of links in the system and the number of different pipe diameters in each link.

The objective function is developed using unit costs of all the different pipe diameters. The total cost of pipe in a given link is calculated by summing the products of the lengths of each size of pipe and the corresponding unit costs. The objective function is determined by summing the costs of all links in the system.

The sum of the lengths of all different diameter pipes in any link must equal the specified length of that link. This is the first constraint on the system. There will be a length constraint to correspond with each link in the network.

Hydraulic consistency must be ensured throughout the system. To achieve this, the pressure at each node must be kept above the minimum permissible level. As the constraint must be in linear form, in the linear programming formulation, loss of energy in any given pipe is expressed as

$$J = 1.13 \times 10^{12}(Q/C)^{1.852}D^{-4.87} \qquad (1)$$

where J = friction loss (m/km)

Q = flow (m^3/hour)

C = friction factor

D = pipe diameter (mm)

from the Hazen Williams formula. By multiplying the friction loss by the length of that particular pipe, the pressure head loss is obtained. The summation of the pressure losses over all links is set equal to (in the loops) or less than or equal to (in the paths) the minimum permissible head loss. There will be a pressure head loss constraint for each path and loop in the system.

The basic reliability constraint was developed by Kettler and Goulter (1983). It has been shown that pipe breakage rates can be used in a Poisson probability distribution to determine probability of failure of an individual link j,

$$P(1+)_j = \sum_{x=1}^{\infty} \frac{e^{-\lambda_j} \lambda_j^x}{x!} \qquad (2)$$

where $P(1+)_j$ = probability of one or more failures in a given year for link j

λ_j = expected number of breaks/yr in link j

$$= \sum_{k=1}^{ND} r_{jk} x_{jk} \quad \forall j \qquad (3)$$

where r_{jk} = average number of breaks in pipe diameter k in link j

For each link in the system there will be a corresponding reliability constraint.

The model can be expressed mathematically as follows:

<u>Objective Function</u>:

$$\text{minimize} \quad \sum_{j=1}^{NL} \sum_{k=1}^{n(j)} c_{jk} x_{jk} \qquad (4)$$

subject to

Constraints:

Length: the length of pipe in each link must be equal to the length of the link

$$\sum_{k=1}^{n(j)} x_{jk} = L_j \quad \forall \text{ links } j \qquad (5)$$

Head loss: Minimum permissible head at each node must be satisfied

$$H_o - \sum_{j \epsilon p(n)} \sum_{k=1}^{n(j)} J_{jk} \, x_{jk} \geq H_{n_{min}} \; \forall \text{ nodes} \qquad (6)$$

Path: Head loss along any path between two nodes must equal the difference in head between those two nodes

$$\sum_{j \epsilon p'(b)} \sum_{k=1}^{n(j)} J_{jk} \, x_{jk} = B_p \qquad (7)$$

Loop: For a looped system, the total head loss around a loop must equal zero, i.e. $B_p = 0$.

Reliability: The expected total number of breaks of all pipe sizes in each link must not exceed exceed the maximum number of failures specified in link j.

$$\sum_{k=1}^{n(j)} r_{jk} \, x_{jk} \leq R_j \quad \forall \text{ links } j \qquad (8)$$

Non-negativity: $x_{jk} \geq 0 \quad \forall \; j,k$ (9)

where:

x_{jk} = length of pipe of diameter k in link j (km)

c_{jk} = cost of pipe of diameter k in link j (\$/km)

L_j = length of pipe required in link j (km)

$H_{n_{min}}$ = minimum allowable head at node n (m)

B_p = net head loss along path p in the network (m)

J_{jk} = hydraulic gradient for pipe diameter k in link j (m/km)

n(j) = number of different pipe diameters in link j

H_o = original head at source (m)

p(n) = links in the path from the source to node n

p'(b)= links in the path associated with net head loss B_p

r_{jk} = expected number of breaks/km/year for diameter k in link j

R_{jk} = maximum allowable number of failures per year in link j

This is the basic formulation used in the three approaches discussed in the following chapters. The model varies slightly in Approach 2, details of which can be found in Chapter V.

Chapter IV

APPROACH 1: NODE ISOLATION

A system is said to have failed if the demand of any node in that system cannot be met. The probability of failure to supply any node is determined by considering the probability of simultaneous failure of all links directly connected to that node. It is assumed that adequate supply to a node is maintained as long as at least one link is capable of meeting the demand. Thus, if all links supplying that node fail, the node is isolated, and the system fails.

4.1 THEORY

The reliability of a system has been defined as a function of all paths leading to a node failing simultaneously (Kettler and Goulter 1983). This has already been shown to apply in the field of structural design, where the structure is considered as a whole. This allows the incorporation of restrictions such as safety limits, stress and strain yield points, etc., resulting in optimal designs. The reliability of such structures is based on probability theory.

Based on the definition that a structure fails when at least one of its members fails, the probability of failure of the structure is given by

$$p_F = \int_{-\infty}^{\infty} [\ 1 - \prod_{k=1}^{n} \int_{x_k}^{\infty} f_{y_k}(y)\, dy\ ]\ f_x(x)\, dx \qquad (10)$$

where n = number of members,

x,y = various stresses induced by loading and resistance

This can be approximated by

$$p_F = 1 - \prod_{k=1}^{n} (\ 1 - p_k\ ) \approx \sum_{k=1}^{n} p_k \qquad (11)$$

i.e. the probability of failure of the structure is equal to the sum of the probability of failure of its members, when p is small. (Khachaturian, 1969)

This probability theory can also be applied to a water distribution system. System failure has already been defined as the failure to supply any one node within the system. Thus, the probability of node isolation can be expressed using equation (11), where n equals the number of links supplying that node.

The objective is to maximize improvement in reliability while minimizing cost. In this approach, this is achieved by improving the reliability of the node most likely to fail.

The probability of a node being completely isolated, that is, the probability of no supply, is defined as the joint probability of all links supplying the node failing simultaneously. This can be written mathematically as

$$P_N = p_1 . p_2 . \ldots . p_n \qquad (12)$$

where P_N = probability of node N being isolated

p_n = probability of failure of link n, a link connected to node N

= f(pipes in the link)

Failure in one link is assumed to be independent of failure in other links. Thus, the probability of all links connected to node N failing simultaneously is given by

$$P_N = k_1 x_1 . k_2 x_2 . \ldots \; k_j x_j \qquad j \epsilon c(n) \qquad (13)$$

where P_N = probability of isolation of node N

x_j = length of link j

k_j = probability of one or more failures per unit length of link j

c(n)= set of links supplying node N

In order to maximize improvement in reliability and minimize the cost of the system, it is necessary to devise some means of representing these two aims. It has been shown in Equation 2 that probability of link failure can be determined from the Poisson distribution. By differentiating this distribution with respect to the expected number of breaks per year in each of the links supplying the node, values representing the improvement in node reliability are obtained. The rate of change in the probability of failure to supply a node with unit change in the probability of failure of each link can be expressed as follows.

$$\frac{\delta P_N}{\delta P_j} = \frac{\delta P_j}{\delta \lambda_j} \prod_{l \epsilon c'(n)} P_l$$

$$= \left[\sum_{x=1}^{\infty} \frac{e^{-\lambda_j}(x\lambda_j^{x-1} - \lambda_j^{x})}{x!} \right] \left[\prod_{l \epsilon c'(n)} \sum_{x=1}^{\infty} \frac{(e^{-\lambda_l} \lambda_l^{x})}{x!} \right] \quad \forall\, j \epsilon c(n) \qquad (14)$$

where P_N = probability of node N being isolated

P_j = probability of one or more failures in link j

λ_j, λ_l = number of breaks/km/yr in links j,l

c'(n) = set of links supplying node N, excluding link j

c(n) = complete set of links supplying node N

The least cost component of the approach represents the change in cost to the system with changes in reliability. This is expressed as $\delta C/\delta R_j$, where δR_j is the change in maximum allowable breaks/km/yr in link j. This value is obtained from the linear programming (LP) solution, described in the next section. Each of the constraints in the LP formulation previously discussed has associated with it a dual variable. For the reliability constraints, these dual variables represent the change in cost to the system with unit reduction in the right-hand-side value (the maximum allowable breaks/km/yr). Mathematically, this can be written as $\delta C/\delta R_j$.

The ratio of cost and reliability, expressed as

$$\frac{\delta C/\delta R_j}{\delta P_N/\delta P_j} \qquad j\epsilon c(n) \qquad (15)$$

can be used to find the maximum possible improvement in reliability which can be achieved at least cost to the system by reducing the reliability of link j.

4.2 METHOD

The linear program formulated in the previous section is applied to the network illustrated in Figure 1. The amount of input required is minimal - identification of demand points (nodes), and the associated demands; flows carried by the links; and costs and breakage rates of different diameter pipes. The data for this layout can be found in Tables 1-3. The costs and breakage rates used are those determined by Kettler and Goulter (1983), and are based on City of Winnipeg data. A uniform pressure head distribution is assumed, with the maximum permissible head loss from the source to any node being 25m.

The linear program was solved using LINDO (Linear INteractive Discrete Optimiser), a computer package, and an initial starting solution obtained. The solution gives optimum lengths for each pipe diameter in each link.

Using these lengths, the probability of one or more failures in each link (P_j) is calculated using the Poisson dis-

Figure 1: Distribution Network

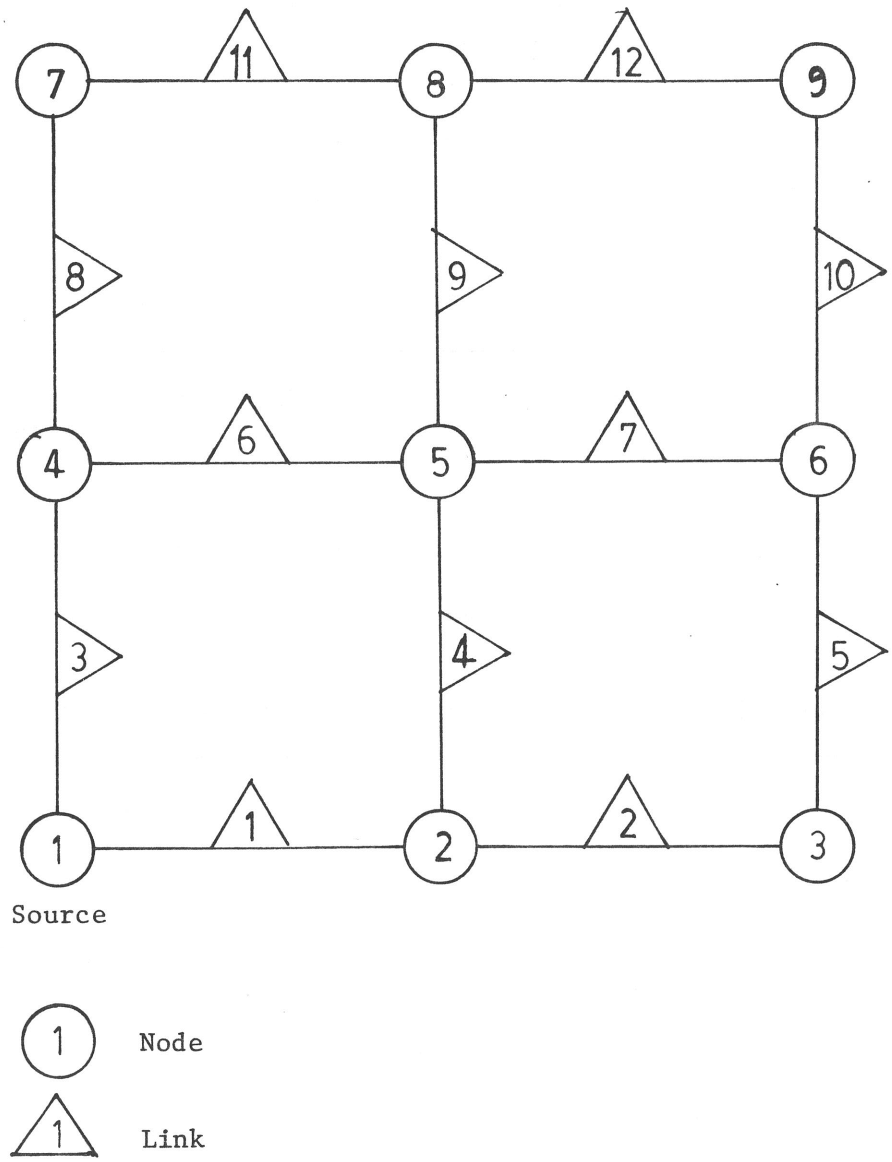

TABLE 1

Demand at Nodes in the Network

Node	Demand (m^3/hour)
1	-750 (source)
2	75
3	75
4	75
5	75
6	75
7	75
8	75
9	225

Note: Maximum permissible head loss from source to any node = 25m.

TABLE 2

Link Data for Network

Link	U/S Node	D/S Node	Length (m)	Assumed Flow (m^3/hour)
1	1	2	1000	450
2	2	3	1000	125
3	1	4	1000	300
4	2	5	1000	250
5	3	6	1000	50
6	4	5	1000	125
7	5	6	1000	175
8	4	7	1000	100
9	5	8	1000	125
10	6	9	1000	150
11	7	8	1000	25
12	8	9	1000	75

tribution. The Poisson coefficient λ_j is found by determining the actual number of expected breaks/km/year. Once all the P_j 's have been found, the probability of failure to supply each node can be calculated.

TABLE 3

Cost and Breakage Data for Network

Pipe size (mm)	Cost (\$/km)	Expected Number of Breaks/km/year
350	98,600	0.05
300	69,200	0.07
250	43,200	0.39
200	24,100	0.71
150	16,900	1.04
100	14,300	1.36

* Source: Kettler and Goulter (1983)

The node with the highest probability of isolation is identified. For each of the links supplying that node, the dual variable associated with the reliability constraint is found from the LP solution. This is the value $\delta C/\delta R_j$, the change in cost with unit reduction of maximum allowable number of breaks/km/year in link j.

The rate of improvement in the reliability of this 'worst' node ($\delta P_N/\delta P_j$) is then calculated, through differentiation of the Poisson distribution with respect to change in the expected number of breaks/km/year for each of the links (Equation 14). The ratio of least cost and node reliability improvement is determined for each of the links. Links composed of larger diameter pipes are more reliable than those having smaller diameters, but are more expensive. Thus there will be a tradeoff between cost and reliability. To obtain maximum reliability and minimum cost, the smaller(est) ratio is chosen. The link corresponding to this ratio becomes the candidate for improvement. The right-hand-side (RHS) of its reliability constraint will be reduced,[2] and the model run again. The process is repeated until an acceptable probability of isolation of all nodes in the system is achieved.

In order to obtain dual variables for the reliability constraints in the LP solution $(\delta C/\delta R_j)$, these constraints must be binding, i.e. the number of expected breaks/km/year (λ_j) must be at the upper bound, as set by the RHS of the constraint (R_j). If the maximum allowable number of breaks, R_j, is not equal to the expected number of breaks/km/year, the constraint is not binding, and the dual variable will equal zero. A number of preliminary trials are necessary to obtain binding constraints, although it is impossible to have ALL reliability constraints binding at once (due to the sensitivity of the simplex algorithm used to solve the LP). In this approach, the RHS's are gradually reduced until as many reliability constraints as possible are binding. This requires a certain amount of manipulation prior to solving the LP.

---

[2] The higher the value of the link reliability (i.e. expected number of breaks/year), the more unreliable it is. In order to improve the reliability, this value must be reduced. Thus, to maximize $\delta P_N/\delta P_j$, the minimum value is chosen.

A further complication arising from the LP solution involves the choice of pipe diameters. The solution provided by the LP sometimes results in the splitting of a link into two (or more) different and not necessarily adjacent (size-wise) diameters. This selection of non-adjacent pipe diameter constitutes a significant difference from the solution provided by other LP models which do not consider reliability explicitly. The selection of non-adjacent pipes is also contrary to current engineering practice. In the model, this problem is overcome by reducing the choice of pipes available, so that the larger, more reliable, pipe diameters remain.

4.3 RESULTS

A summary of the results obtained by this approach is given in Table 4.

The initial least cost design (Iteration 1) produces probabilities of node isolation ranging from 0.0036 (node 2) to 0.2666 (node 7). Node 7 is identified as the 'worst' node, and the ratios of $(\delta C/\delta R_j)/(\delta P_N/\delta P_j)$ are calculated for each of the connecting links - links 8 and 11 (see Fig.1). Link 11 is found to have the lower ratio (230.17, as compared to 298.01 for link 8), and so it becomes the candidate for reliability improvement. The difference between the probabilities of isolation for Node 7 and Node 3 (the next 'worst') determines the size of the increment, found to be 0.01. The reliability of link 11, which had been 0.70 expected breaks per km per year, is increased by 0.01 to 0.69 for the next trial.

Each of the next 5 runs indicate that Node 7 has the highest probability of being isolated, and thus link 11 is improved each time. In each of these runs, the reliability of link 11 is increased by 0.01. The ratio for link 11, which still remains the minimum, decreases by (-)2.3 in each of the 5 runs. Thus the average change in probability of failure to supply Node 7 per Δ0.01 link reliability is 0.00265. However, run 7, with R(11) = 0.65, gives probability of one or more failures at Node 7 as 0.2531, and at node 3, 0.2554. The difference between these is divided by the average change of 0.00265, giving the size of the reliability increment as 0.01. Link 5, indicated by the minimum ratio criterion, is improved by 0.01, from 0.71 to 0.70 maximum permissible breaks.

This process was repeated until all nodes reached a certain acceptable level of reliability (or probability of isolation). In this example, a node isolation probability of 0.2500 was set as acceptable, and this improvement was achieved at a cost of $5,152.

A more detailed breakdown of the results of the initial and final iterations using this approach is given in Tables 7 and 8.

TABLE 4

Summary of Node Isolation Approach

Iteration	'Worst' Node	Probability of Isolation	'Worst' Link	Reliability of Link	Total Cost ($ x 10^3)
1	7	0.2666	11	0.700	470.211
2	7	0.2639	11	0.690	470.816
3	7	0.2613	11	0.680	471.421
4	7	0.2586	11	0.670	472.027
5	7	0.2558	11	0.660	472.632
6	3	0.2554	5	0.710	473.237
7	7	0.2531	11	0.650	473.830
8	3	0.2529	2	0.698	474.435
9	3	0.2527	2	0.697	474.492
10	3	0.2524	2	0.696	474.550
11	3	0.2522	5	0.700	474.657
12	7	0.2503	11	0.640	475.363

Chapter V

APPROACH 2: GOAL PROGRAMMING

5.1 THEORY

In the previous approach, the probability of all links to a node failing simultaneously was considered. Failure to supply a node implies failure of the network. Such an approach does not consider all the issues of redundancy, a major contributor to the reliability of any system. For example, a node may become isolated if two out of three supplying links fail, if the remaining link does not have the capacity to meet the demand of the node. This approach assumes that each link connected to a node will be capable of supplying the entire demand of that node; thus if all links but one fail at the same time, the system itself does not fail. It follows that if all links are to have this capability, they will also have similar reliabilities. Due to hydraulic restrictions this is not feasible, but if a 'uniform' reliability is chosen for all links supplying a node, deviations from this standard could be minimized, using goal programming.

Goal programming is an optimization technique incorporating a number of objectives, rather than one single objective, as in linear programming. The concept behind this approach involves defining a goal(numeric) representing each objective, and then solving in such a way that the sum of the deviations from the goals is minimized. Both under- and over-achievement of each goal is permitted.

Goals can be expressed mathematically by

$$\sum_{i=1}^{N} a_{ik} x_{ik} + d_k^- - d_k^+ = G_k \qquad k = 1,2,\ldots..K \qquad (16)$$

where K = number of objectives

N = number of decision variables

x_{ik} = decision variable (assumed to be linear)

a_{ik} = coefficient

d_k^- = underachievement of goal k

d_k^+ = overachievement of goal k

G_k = goal for objective k

In addition, these deviations can be weighted, to indicate the relative importance of each goal. In general, the objective function can be written

$$\min \sum_{k=1}^{K} w_k (d_k^+ + d_k^-) \qquad (17)$$

where w_k is the weight associated with goal k, and K is the number of objectives or goals.

The numeric values of the goals to be used in this approach are determined from the node reliabilities calculated in Approach 1.[3] Given these node reliabilities, and knowing the number of links connected to each node, a 'uniform' link reliability for the links can be calculated, as follows.

$$UR_j = R_N^{1/NL_N} \qquad (18)$$

where UR_j = 'uniform' reliability for link j

R_N = reliability of node N

NL_N = number of links supplying node N

can be calculated. This will represent the reliability required of each link connected to a given node. As each link in the system is connected to a node at both ends, there will be two goals per link, derived from the two different node reliabilities. This will result in an infeasible solution, given two different equalities to satisfy. As the greater reliability is being sought (i.e. minimum number of failures), the lower goal is kept, and the other, redundant goal is discarded.

The 'uniform' reliability can be converted into the corresponding pipe diameter. This pipe diameter, however, may not constitute a true or discrete diameter - instead it may represent a combination of several pipe sizes comprising the

---

[3] It should be noted that the node reliabilities obtained in Approach 1 do not have to be used. Nodes may be assigned other reliabilities by the decision-maker, who may have prior knowledge about the relative importance of the various demand points in the system.

link.

The constraint set used to maintain physical feasibility in the GP formulation is similar to that used for the LP in Approach 1. This is discussed in more detail in the following section.

5.2 METHOD

Using the initial starting solution of Approach 1, the goal program can be formulated. Node reliabilites, calculated in the previous approach, are used to determine goals, which take the form of 'uniform' link reliabilities, one for all links supplying any particular node. These 'uniform' link reliabilities correspond to certain pipe diameters.

In the example network (Figure 1), all links are of equal length. It can be assumed that, in this particular case, links having similar capacities will have not only similar reliabilities, but also similar diameters. The 'uniform' (or 'average') diameters corresponding to the 'uniform' link reliabilities therefore become implicit goals.

The objective is to minimize the deviations from the uniform reliabilities at each node. For example, Node 1 has a reliability of 0.02730, and is supplied by two links. Therefore, each link would have a uniform reliability of 0.16523 (R_1^{1/NL_1}, where NL_1 = number of links supplying node 1 = 2).

By interpolation, this link reliability would correspond to a uniform pipe diameter of about 285mm, or 11". There is of course no such standard pipe size in reality. A combination of 300mm and 250mm diameters is required. The objective will be to minimize the deviations from this 'uniform' reliability.

This is achieved by setting up goals as follows.

$$\sum_{i=1}^{ND} r_{ij} x_{ij} + d_j^- - d_j^+ = UR_j \quad \forall \; j \tag{19}$$

where x_{ij} = length of pipe of diameter i in link j

r_{ij} = expected number of breaks/km/year for diameter i in link j

d_j^- = underachievement of goal

d_j^+ = overachievement of goal

UR_j = 'uniform' pipe reliability

$= R_N^{1/NL_N}$

ND = number of diameters in link j

The objective function is expressed as

$$\sum_{j=1}^{NL} w_j \, (d_j^+ + d_j^-) \tag{20}$$

where w_j = weight associated with link j

NL = number of links

The length and pressure head loss constraints remain the same as in the previous LP formulation (equations 5-7, 9). An additional constraint incorporating cost is required. The objective function from the LP forms the constraint, and the least cost obtained from its solution becomes the RHS value. The cost constraint is written

$$\sum_{j=1}^{NL} \sum_{i=1}^{ND} c_{ij} \, x_{ij} \leq C \qquad (21)$$

where C = least cost from corresponding LP solution (i.e. value of C will increase with each iteration)

c_{ij} = cost associated with diameter i in link j

The GP is then solved by LINDO. It should be noted that this approach is used in conjunction with Approach 1, since the node reliabilities calculated in the latter have been used in determining the goals.

Initially, all weights are set to 1.0, in effect a non-weighted model. This does not reflect the significance of each link within the system as a whole. The entire process was repeated, with non-unit weights. These non-unit weights are determined by the ratio of flow entering node n from link j to the total flow from all links,

$$w_j = \frac{Q_j}{\sum_{l \in c(n)} Q_l} \qquad \forall j \qquad (22)$$

(based on Morgan and Goulter, 1985b), and substituted into the objective function of the GP.

5.3 RESULTS

Results are summarized and compared to those of the least cost LP model in Tables 7,8,10, and 11, and are discussed in Chapter VII.

Chapter VI

APPROACH 3: PATH FAILURE

6.1 THEORY

This approach is based on the ideas developed by Kettler and Goulter (1983), where the probability of failure of individual paths supplying a node is assessed. Probabilities of no failure for all paths leading to a node are calculated using the Poisson distribution, as follows.

$$P(0)_p = 1 - \sum_{x=1}^{\infty} \frac{e^{-\lambda_p} \lambda_p^{x}}{x!} \qquad \forall \text{ paths } p \qquad (23)$$

where $P(0)_p$ = probability of zero failures in path p

λ_p = the expected number of breaks in path p

$= \sum_{j \epsilon c(p)} \lambda_j$

where c(p) = all links in path p

The path with the lowest 'probability of no failure' is identified as the 'worst' path. The reliability of each link in that path is determined, and those with a high expected number of breaks become possible candidates for improvement. The decision as to which of these links will have the RHS value of its associated reliability constraint reduced is based on the dual variable $(\delta C/\delta R_j)$. The link having the lowest dual variable is chosen for reliability improvement. This is not a true measure of reliability, as failure of all paths connected to the node are not considered at any one time.

In this approach, an attempt is made to apply the ratio developed in Chapter IV, as an alternative method of identifying unsatisfactory links in the system, considering both least cost and overall system reliability simultaneously. The probability of one or more failures of each path in the system is determined, and the 'worst' path identified - i.e. that path with the highest probability of one or more failures. As before, the dual variable associated with the reliability constraints $(\delta C/\delta R_j)$ is used. The other component of the ratio becomes $(\delta P_p/\delta P_j)$, the improvement in path reliability with unit improvement in the reliability of link j.

This can be expressed as

$$\frac{\delta P_p}{\delta P_j} = \left[\sum_{x=1}^{\infty} \frac{e^{-\lambda_j}(x\lambda_j^{x-1} - \lambda_j^{x})}{x!} \right] \left[\prod_{l \epsilon c'(p)} \sum_{x=1}^{\infty} \frac{(e^{-\lambda_l}\lambda_l^{x})}{x!} \right] \quad \forall\ j \epsilon c(p) \qquad (24)$$

where P_p = probability of one or more failures of path p

P_j = probability of one or more failures in link j

λ_j, λ_l = number of breaks/km/yr in links j,l

$c'(p)$ = set of links comprising path p, excluding link j c(p) = complete set of links comprising path p

The link having the lowest ratio,

$$\frac{\delta C/\delta R_j}{\delta P_p/\delta P_j} \qquad j\epsilon c(p) \qquad (25)$$

is chosen as the most suitable for reliability improvement.

It should be noted that this approach does not recognize the ability of other 'secondary' paths to provide adequate flow when the 'worst' path fails. It merely requires that 'at least one' path be capable of supply. However, the method does demonstrate an alternative way of using pipe breakage probability in the explicit consideration of reliability.

6.2 METHOD

The same LP formulation and starting solution used in Approach 1 are employed. Using this solution, optimum lengths for each pipe diameter are obtained. These lengths are used to determine the expected number of breaks (λ_j) in each link j. From these values, the expected number of breaks, or reliability, of each path p (λ_p) is computed by summing the reliabilities of the links (λ_j) comprising that path. The value (λ_p) is used in the Poisson distribution to calculate the probability of one or more failures in each path, as in Kettler and Goulter (1983). The 'worst' path - that path with the highest probability of one or more failures is identified. For each link in that path, the associated dual variable ($\delta C/\delta R_j$) is obtained from the LP solution, and ($\delta P_p/\delta P_j$), the maximum improvement component, is calculated.

Ratios of $(\delta C/\delta R_j)/(\delta P_p/\delta P_j)$ for each link in the 'worst' path are determined, and the link with the smallest ratio is chosen. The RHS of its associated reliability constraint is reduced by an amount determined by a sensitivity analysis provided by LINDO, and partially by the judgement of the decision-maker. The process is then repeated until a satisfactory level of path reliability has been reached.

6.3 RESULTS

A summary of the iterations involved in this approach are shown in Table 5. It was necessary to alter the starting solution in order to have binding constraints for all links in the 'worst' path. This was achieved by reducing the allowable number of expected breaks in link 3 to 0.38 (from 0.39). This explains the difference in cost of $3,279. (The amount by which the RHS of the reliability constraint can be reduced before it affects the optimal solution is obtained from the sensitivity analysis).

In iterations 6 and 9, the problem of non-binding constraints arises. (The impossibility of having all link reliability constraints binding simultaneously has been discussed previously). When there is a change in 'worst' path, as happens here (from path 6 to 5), an adjustment must be made to the maximum number of acceptable breaks of the the non-binding link in the 'worst' path. The process can then continue as before.

Changes in the probability of path failure with successive iterations are shown in Table 6. A value of 0.8600 has been selected as the allowable probability of one or more simultaneous failures on the paths. This may seem high, but it must be remembered that all reliabilities are in terms of a one year time period. In fact, breakages generally would be of a few days duration at most. The one year time span has been used merely to keep numbers from becoming so small that the changes would not be easily discernible.

Pipe lengths and diameters chosen in the initial and final iterations are shown in Table 9.

TABLE 5

Summary of Iterations for Approach 3

Iteration	'Worst' Path	Prob.of 1+ fail.	'Worst' Link	Link Rel.	Cost ($\$x10^3$)
1	6	0.9198	12	0.700	473.49
2	6	0.9198	11	0.700	473.49
3	6	0.9157	11	0.650	476.52
4	6	0.8907	11	0.390	492.22
5	6	0.8896	11	0.380	493.04
6	5	0.8647	6 NB	0.390	509.75
7	6	0.8645	11	0.175	510.78
8	5	0.8633	6	0.380	511.56
9	6	0.8630	8 NB	0.754	518.59
10	6	0.8618	11	0.165	519.23
11	5	0.8534	6	0.310	526.97
	(6	0.8480 - all path reliabilities < 0.8600)			

* NB = non-binding

TABLE 6

Initial and Final Probabilities of Path Failure

Path	Probability of 1+ Failure	
	Initial	Final
1	0.8453	0.8447
2	0.5933	0.5906
3	0.7591	0.7567
4	0.7737	0.7532
5	0.8660	0.8534
6	0.9206	0.8480

Chapter VII

DISCUSSION

It can be seen from Tables 7-9 that limiting the maximum number of breaks in a link can result in the selection of a number of different pipe diameters. A maximum of two different diamters per link was imposed in previous work (Kettler and Goulter, 1983), but this has not been implemented in this work. By allowing a greater freedom of choice in diameters, the problem of non-binding reliability constraints described in the previous chapters was alleviated to some extent.

The ratio used in these approaches can be described as minimizing cost and maximizing reliability. This can be expressed as

$$\frac{\min \dfrac{\delta C}{\delta R_j}}{\max \dfrac{\delta P_{P/N}}{\delta P_j}} \quad \rightarrow \quad \text{min ratio} \qquad (26)$$

It should be noted that reliability has been defined as the expected number of breaks per km per year per link. In order to improve the reliability then, the RHS of the constraint is reduced, rather than physically increased. Thus, the denominator of the ratio can be seen as a minimization, although strictly speaking this is not so.

TABLE 7

Results of Initial Iterations for Approaches 1 and 2

	Approach 1		Approach 2 (Uniform weights)		(Non-uniform)		Assoc. Weight
Link	Length (m)	Diam. (mm)	Length (m)	Diam. (mm)	Length (m)	Diam. (mm)	
1	61	350	6	350	150	350	0.68
	939	300	994	300	850	300	
2	38	250	-	-	-	-	0.15
	962	200	1000	200	1000	200	
3	1000	250	1000	250	1000	250	0.32
4	-	-	-	-	368	350	
	857	300	917	300	275	300	0.53
	143	250	83	250	357	250	
5	-	-	817	250	-	-	0.15
	1000	200	183	200	1000	200	
6	-	-	-	-	70	300	0.26
	1000	250	1000	250	930	250	
7	200	300	286	300	286	300	0.52
	800	250	714	250	714	250	
8	3	250	303	250	-	-	0.06
	862	200	319	200	698	200	
	136	150	378	150	302	150	
9	532	250	-	-	-	-	0.27
	468	200	1000	200	1000	200	
10	-	-	342	300	110	300	
	1000	250	133	250	597	250	0.67
	-	-	525	200	243	200	
11	31	250	213	250	-	-	0.06
	969	200	787	200	1000	200	
12	31	250	-	-	586	250	0.33
	969	200	1000	200	414	200	
Σ deviations			= 1.8758		= 0.4100		

TABLE 8

Results of Final Iterations for Approaches 1 and 2

	Approach 1		Approach 2 (Uniform weights)		(Non-uniform)		Assoc. Weight
Link	Length (m)	Diam. (mm)	Length (m)	Diam. (mm)	Length (m)	Diam. (mm)	
1	50 950	350 300	8 992	350 300	194 806	350 300	0.68
2	47 953	250 200	- 1000	- 200	- 1000	- 200	0.15
3	1000	250	1000	250	1000	250	0.32
4	- 869 131	- 300 250	- 915 85	- 300 250	335 315 350	350 300 250	0.53
5	75 912 13	250 200 150	830 170 -	250 200 -	- 1000 -	- 200 -	0.15
6	- 1000	- 250	- 1000	- 250	229 771	300 250	0.26
7	200 800	300 250	294 706	300 250	294 706	300 250	0.52
8	7 853 140	250 200 150	498 81 421	250 200 150	- 714 286	- 200 150	0.06
9	532 468	250 200	- 1000	- 200	- 1000	- 200	0.27
10	- 1000 -	- 250 -	365 100 535	300 250 200	133 564 303	300 250 200	0.67
11	219 781	250 200	272 728	250 200	- 1000	- 200	0.06
12	31 969	250 200	- 1000	- 200	586 414	250 200	0.33
Σ deviations			= 1.8324		= 0.4005		

TABLE 9

Results of Initial and Final Iterations - Approach 3

	Initial			Final		
Link	Length (m)	Diam. (mm)	Expected no.breaks	Length (m)	Diam. (mm)	Expected no.breaks
1	101 899	350 300	0.068	168 832	350 300	0.067
2	38 962	250 200	0.698	45 955	250 200	0.696
3	31 969	300 250	0.380	31 969	300 250	0.380
4	857 143	300 250	0.116	853 147	300 250	0.117
5	1000	200	0.710	1000	200	0.710
6	- 1000	- 250	0.390	250 750	300 250	0.310
7	200 800	300 250	0.326	221 779	300 250	0.326
8	3 862 136	250 200 150	0.754	38 821 141	250 200 150	0.744
9	532 468	250 200	0.540	531 469	250 200	0.540
10	1000	250	0.390	1000	250	0.390
11	- 31 969	- 250 200	0.700	1000 - -	300 - -	0.070
12	31 969	250 200	0.700	63 938	250 200	0.690

TABLE 10

Summary of Initial and Final Link Reliabilities

	Initial			Final		* **
Link	Appr.1	Appr.2(UW)	Appr.2(W)	Appr.1	Appr.2(UW)	Appr.2(W)
1	0.069	0.070	0.067	0.069	0.070	0.066
2	0.698	0.710	0.710	0.695	0.710	0.710
3	0.390	0.390	0.390	0.390	0.390	0.390
4	0.116	0.097	0.177	0.112	0.097	0.175
5	0.710	0.449	0.710	0.690	0.444	0.710
6	0.390	0.390	0.368	0.390	0.390	0.317
7	0.326	0.298	0.298	0.326	0.296	0.296
8	0.754	0.738	0.810	0.754	0.690	0.804
9	0.540	0.710	0.710	0.540	0.710	0.710
10	0.390	0.449	0.449	0.390	0.444	0.444
11	0.700	0.642	0.710	0.640	0.623	0.710
12	0.700	0.710	0.522	0.700	0.710	0.522

* Reliabilities expressed as expected number of breaks/km/year
** UW = uniform, unit weight; W = non-uniform, non-unit weight

For purposes of comparison, a measure of 'system reliability' has been devised. It represents the average, or mean, node reliability in the system in that particular iteration, and can be expressed as

$$\frac{\sum_{n=1}^{N} R_n}{N} \qquad (27)$$

where ΣR_n = the sum of all node reliabilities and N = the number of nodes. The comparison is shown in Table 11. The node isolation (first) approach shows greatest improvement in system reliability, and the third, weighted GP shows the least. It should be remembered that the LP method (Approach 1) is an 'extreme' approach, in that it assumes adequate supply to a node as long as there is one link in operation - flow/supply capacities of other links are not considered, nor is redundancy. The GP method, where all links supplying a node are taken into account, attempts to satisfy the redundancy requirement, while recognizing the ability of the system to adjust to failures of individual links in the system.

It can be seen that the non-unit weighted GP has the highest value of system reliability throughout the analysis (i.e. it is the most unreliable). This, and the fact that it produces the least overall improvement in the reliability of the system, is believed to be a result of the weights themselves. In the first GP formulation, all weights were set uniformly to 1.0, which in effect implies no weighting at all. In applying weights, the significance of the various links is altered. In the previous approaches, links 8/11 and 2/5 appear critical, and consequently have their diameters increased. When weights reflecting the relatively small amount of flow carried are applied, their importance decreases, as does the size required.

In the node isolation approach (Approach 1), the reliability of links 2,5, and 11 have been altered, as indicated by the minimum ratio criterion. This causes a variation in

TABLE 11

Summary of System Reliability Measure

Iteration	Approach 1	Approach 2	
		Uniform weights	Non-uniform weights
1	0.20068	0.18305	0.20403
5	0.19565	0.17917	0.20333
6	0.19440	0.17825	0.20261
7	0.19346	0.17813	0.20215
8	0.19220	0.17651	0.20182
10	0.19203	0.17648	0.20152
11	0.19195	0.17630	0.20136
12	0.19104	0.17531	0.20091
Overall improvement =	0.00964	0.00774	0.00312

lengths of different pipe diameters in those links, and also in links 1 and 4. As a result, the reliabilities of all nodes connected to those links change. It can be seen from Table 12 that the reliabilities of nodes 4 and 9 do not change throughout the analysis. This is to be expected, as the pipes in links 3,6,8 (node 4) and 10,12 (node 9) do not vary in lengths, and hence, reliabilities.

In the uniform-weighted (or 'non-weighted') GP approach, the reliabilities of nodes 1 and 2 remain constant. The associated goals are very restrictive. Large diameter pipes must be chosen in order to carry the larger flows, and to meet the reliability conditions inherent in the goals. As a result, there can be virtually no variation in pipe sizes,

TABLE 12

Summary of Node Reliabilities

Node	Initial			Final		
	Appr.1	Appr.2(UW)	Appr.2(W)	Appr.1	Appr.2(UW)	Appr.2(W)
1	0.02683	0.02730	0.02613	0.02691	0.02730	0.02574
2	0.00555	0.00482	0.00842	0.00537	0.00482	0.00820
3	0.49558	0.31879	0.50410	0.47955	0.31524	0.50410
4	0.11468	0.11225	0.11625	0.11468	0.10495	0.09940
5	0.00794	0.00800	0.01378	0.00768	0.00795	0.01166
6	0.09027	0.06008	0.09500	0.08773	0.05835	0.09331
7	0.52780	0.47380	0.57510	0.48256	0.42987	0.57084
8	0.26451	0.32363	0.26314	0.24184	0.31405	0.26314
9	0.27300	0.31879	0.23438	0.27300	0.31524	0.23177

UW = uniform, unit weight
W = non-uniform, non-unit weight

so node reliability is not affected. Further away from the source, the flows are smaller, and the goals are less restrictive.

When non-uniform, non-unit weights are applied, the significance of each link comes into effect. Here, the reliabilities of nodes 3 and 8 remain unchanged. The links supplying these nodes have relatively low weightings (2,5 -> 0.15, 9 -> 0.27, 11 -> 0.06, 12 -> 0.33), and so do not contribute as much to the system. It is those links having greater significance (higher weightings) that change, choosing larger diameter pipes, and thus increasing the reliability of the system.

The path failure approach uses the same criteria for identification, i.e. the ratio of cost with reliability improvement, as the node isolation approach. It also assumes adequate supply to a node if at least one link/path is capable of meeting the demand. If the node reliabilities in the final iteration of Approach 3 are calculated, the system is found to have a final reliability of 0.11740, at a cost of $526,970. From Table 5 it can be seen that Iteration 3 of Approach 3 has a cost similar to the cost of the final iteration of Approaches 1 and 2. The system reliability for that iteration is found to be 0.19272, at a cost of $476,520. From Table 13 it can be seen that this approach is comparable to Approach 1.

TABLE 13

Comparison of System Reliability Measure and Cost

Approach	'System Reliability'	Cost
1	0.19104	$475,363
2 (UW)	0.17531	475,363
(W)	0.20091	475,363
3	0.19272	476,520

The method used in Approach 3 is very cumbersome to use, as it requires a great deal of ongoing manipulation of the formulation in order to overcome the difficulty of non-binding constraints. In addition, it gives no direct measure of 'system reliability', and as it makes no allowance for redundancy, should not be considered a viable approach.

Plots of system reliability against least cost can be found in Appendix B. Results of a linear regression give R^2 values of 0.9964, 0.9898, and 0.9377 for Approaches 1, 2(uniform weights), and 2(non-uniform, non-unit weights) respectively. These results confirm the expected relationship between increased cost and improvement in system reliability.

Chapter VIII

CONCLUSIONS

Previous research in the field of pipe network reliability have suggested that true reliability is a function of the probability of simultaneous failure of all paths to a node. By utilizing and adapting the methodology used in such work, a measure of system reliability in water distribution networks has been developed, which can be used in conjunction with the least cost design objective.

By introducing the idea of joint probability it has been possible to consider maximum node reliability at least cost to the system as a whole (Approach 1). The problem with redundancy in the original path system, as proposed by Kettler and Goulter (1983), can be overcome to some extent through the goal programming approach (Approach 2). This approach seems most suitable for further work in this area. The use of non-uniform non-unit weights seems preferable, as it incorporates the relative importance of each link in the entire system. This prevents larger-than-necessary pipe diameters being chosen, thus contributing to the objective of minimizing cost. It does this at the expense of maximizing reliability, as a higher value of overall system reliability (i.e. less reliable) is obtained. This illustrates the tradeoff between two objectives, represented by the two components of the ratio.

The node reliabilities calculated in Approach 1 have been used to determine the 'uniform' link reliabilities for Approach 2. However, the decision-maker may assign other predetermined node reliabilities in this second approach, allowing for more reliable supply service to certain areas - hospitals, for example. This would eliminate the need for using the Node Isolation approach (1) in conjuction with the GP process. A maximum system cost must also be selected by the decision-maker, for use in the GP constraint set.

The third approach, in which the least cost/reliability ratio is applied to the path failure method developed by Kettler and Goulter (1983), is considered to be of least value. Although the ratio aspect of the approach can be used to identify unsatisfactory components in the system, the 'system reliability', as defined in this thesis, is not determined directly. Like Approach 1, this method requires only that 'at least one' path be capable of supplying a node. Thus a 'true' measure of reliability still cannot be provided by considering supply paths in a system in this way.

REFERENCES

Alperovits, E., and U. Shamir, 'Design of Optimal Water Distribution Systems', Water Resources Research, Vol.13, No.6, pp 885-900, Dec. 1977.

Brcic, C., 'Criteria for Replacement of Watermains', Paper presented at the 1983 AWWA/OMWA Joint Annual Conference, Toronto, Ontario, Canada, 1983

Ciottoni, A.S., 'Computerized Data Management in Determining Causes of Water Main Breaks', Proceedings of the 1983 International Symposium on Urban Hydrology, Hydraulics, and Sediment Control, University of Kentucky, Lexington, Kentucky, pp 323-329, 1983.

Clark, Robert M., Cheryl L. Stafford, and James A. Goodrich, 'Water Distribution Systems: A Spatial and Cost Evaluation', Journal of the Water Resources Planning and Management Division, ASCE, Vol.108, No.WR3, pp 243-256, Oct. 1972.

Cohn, M.Z. (ed.), An Introduction to Structural Optimization', University of Waterloo, pp 179, 1969.

Fitzgerald, J.H., 'Corrosion as a Primary Cause of Cast Iron Main Breaks', Journal of the AWWA, Vol.60, No.8, pp 882-897, 1968.

Goulter, I.C., and D.R. Morgan, 'Discussion of 'Obtaining the Layout of Water Distribution Systems' by Rowell et al.', Journal of the Hydraulic Division, ASCE, Vol.110, No.1, pp 67-8, 1984.

Goulter, I.C., and D.R. Morgan, 'An Integrated Approach to the Layout and Design of Water Distribution Networks', Civil Engineering Systems, Vol.2, No.2, pp 104-113, June 1985 (a).

Hillier, Frederick S., and Gerald J. Lieberman, Introduction to Operations Research, Third Edition, Holden-Day Inc., Oakland, pp 829, 1980.

Karmeli, David, Y.Gadish, and S. Mayers, 'Design of Optimal Water Distribution Networks', Journal of the Pipeline Division, ASCE, Vol.94, No.PL1, pp 1-9, October 1968.

Kettler, A.J., and I.C. Goulter, 'Reliability Consideration in the Least Cost Design of Looped Water Distribution Systems', Proceedings of the Tenth International Symposium on Urban Hydrology,Hydraulics, and Sediment Control, University of Kentucky, Lexington, Kentucky, pp 305-312, 1983.

Kettler, A.J., and I.C. Goulter, 'An Analysis of Pipe Breakage in Urban Water Distribution Networks', Canadian Journal of Civil Engineering, Vol. 12, No.2, pp 286-293, June 1985.

Morgan, D.R., and I.C. Goulter, 'Least Cost Layout and Design of Looped Water Distribution Systems', Proceedings of the Ninth International Symposium on Urban Hydrology,Hydraulics, and Sediment Control, University of Kentucky, Lexington, Kentucky, pp 65-72, 1982.

Morgan, D.R., and I.C. Goulter, 'Optimal Urban Water Distribution Design', Water Resources Research, Vol.21, No.5, pp 642-652, May 1985 (b).

O'Day, D.K., C.M. Fox, and G.M. Juguet, 'Aging Urban Water Systems: A Computerized Case Study', Public Works, Vol.3, No.8, pp 61-111, 1980.

O'Day, D.K., 'Organizing and Analysing Break Data for Making Water Main Replacement Decisions', Journal of the AWWA, Vol.74, No.11, pp 589-596, 1982.

O'Day, D.K., 'Analysing Infrastructure Condition - a Practical Approach', Civil Engineering, ASCE, Vol.53, No.4, pp 39-42, 1983.

Quindry, G.E., E.D. Brill, Jr., and J.C. Liebman, 'Water Distribution Systems Design Criteria', Final Report NSF Grant 7619120, Department of Civil Engineering, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801, March 1979.

Quindry, Gerald E., E. Downey Brill, Jr., and Jon C. Liebman, 'Optimization of Looped Water Distribution Systems', Journal of the Environmental Engineering Division, ASCE, Vol.104, No.EE4, pp 665-679, August 1981.

Rowell, William F., and J. Wesley Barnes, 'Obtaining Layout of Water Distribution Systems', Journal of the Hydraulics Division, ASCE, Vol.108, No.HY1, pp 137-148, January 1982.

Schaake, J.C., and D. Lai, 'Linear Programming and Dynamic Programming Application to Water Distribution Network Design 3. Engineering System Analysis of the Primary Water Distribution Network of New York City', 111 p., Dept. of Civil Engineering, MIT, Cambridge, Mass., 1969

Sullivan, J.P., 'Maintaining Aging Systems - Boston's Approach', _Journal of the AWWA_, Vol.74, No.11, pp 554-559, 1982.

Templeman, Andrew B., 'Discussion of 'Optimization of Looped Water Distribution Systems' by Quindry et al.', _Journal of the Environmental Engineering Division, ASCE_, Vol.108, No.EE3, pp 599-602, June 1982.

Appendix A

COMPUTER PROGRAMS AND MODEL FORMULATIONS

The following program is used in Approach 1. The data-sets required are found at the end of this Appendix.

```
C
C   THIS PROGRAM TAKES RESULTS FROM LINDO, AND OTHER DATASETS,
C   AND CALCULATES FAILURE RATES OF PIPES IN A NETWORK.  A RATIO
C   OF MINIMUM COST OF IMPROVEMENT TO MAXIMUM IMPROVEMENT IN
C   RELIABILITY OF SUPPLY TO A NODE IS CALCULATED
C
C
C   VARIABLE LIST:
C
C         A(I,K)          :   COEFFICIENT OF CONSTRAINT I, KTH ELEM
C         AHL(I)          :   ACTUAL HEAD LOSS FOR PATH I
C         AJ(J,K)         :   J VALUE FOR KTH DIAMETER OF JTH LINK
C         ALEN(J,K)       :   MATRIX OF LENGTHS OF PIPE DIAMETERS K
C                             JTH LINK
C         B(I)            :   R.H.S. OF CONSTRAINT I
C         C(J,K)          :   COST OF PIPE SIZE K IN LINK J
C         CHOICE(I)       :   NUMBER OF PIPES CONNECTED TO NODE I
C         COST(J)         :   COST ASSOCIATED WITH PIPE SIZE J
C         CV              :   HAZEN-WILLIAMS COEFFICIENT
C         D(J,K)          :   DIAMETER OF KTH CHOICE OF JTH LINK
C         DEMAND(I)       :   DEMAND AT NODE I
C         DIAM(J)         :   DIAMETER OF PIPE SIZE J
C         DR(I,LL)        :   DERIVATIVE OF RELIABILITY
C                             OF LINK LL WRT NODE I
C         DRL(J)          :   DERIVATIVE OF PROBABILITY OF FAILURE
C                             OF LINK J
C         DRN(J)          :   DERIVATIVE OF PROBABILITY OF FAILURE
C                             OF LINK J WRT CONNECTING NODE
C         DSOL(J)         :   DUAL VARIABLE OF RELIABILITY CONSTRAI
C                             ASSOCIATED WITH LINK J
C         DUAL(I)         :   DUAL VARIABLE ASSOCIATED WITH CONSTRA
C         F(J)            :   FACTOR USED TO REPRESENT PROBABILITIE
C                             IN CALCULATING FINAL RATIO
C         IBUG            :   0 IF CONSTRAINT COEFFICIENTS ARE NOT
C                               TO BE WRITTEN,
C                             1 IF CONSTRAINT COEFFICIENTS ARE TO B
C                               WRITTEN
C         ICON(I,J)       :   LINK NUMBER J CONNECTED TO NODE I
C         LEN(J)          :   LENGTH OF LINK J
C         LN(J)           :   LINK NUMBER J
C         ME              :   TOTAL NUMBER OF EQUALITY CONSTRAINTS
C         MI              :   TOTAL NUMBER OF INEQUALITY CONSTRAINT
C         MLE             :   NUMBER OF LENGTH CONSTRAINTS
C         MLO             :   NUMBER OF LOOP CONSTRAINTS
C         MP              :   NUMBER OF PATH CONSTRAINTS
C         MR              :   NUMBER OF RELIABILITY CONSTRAINTS
C         MT              :   TOTAL NUMBER OF CONSTRAINTS
C         N               :   TOTAL NUMBER OF CANDIDATE DIAMETERS F
C                             LINKS IN THE NETWORK
C         NC              :   NUMBER OF CLASSES OF PIPE DIAMETERS
C                             AVAILABLE
```

```
C        NL                   :   NUMBER OF LINKS IN THE NETWORK
C        NN                   :   NUMBER OF NODES IN THE NETWORK
C        NODE(J,1)            :   SOURCE NODE FOR LINK 1
C        NODE(J,2)            :
C        NP                   :   NUMBER OF PIPE DIAMETERS PER LINK
C        NPL(I,K)             :   LINK NUMBER K IN ITH PATH
C        NPLO(I,K)            :   LINK NUMBER K IN ITH LOOP
C        NWORST               :   NODE WITH HIGHEST PROBABILTY OF FAILU
C                                 TO SUPPLY
C        PHL(J)               :   HEAD LOSS IN LINK J
C        PFL(J)               :   PROBABILITY OF FAILURE OF LINK J
C        PFSN(I)              :   PROBABILITY OF FAILURE TO SUPPLY NODE
C        RATIO(J)             :   RATIO OF MINIMUM COST OF IMPROVING
C                                 RELIABILITY OF LINK J AND MAXIMUM
C                                 IMPROVEMENT OF LINK J WRT NODE UNDER
C                             :   CONSIDERATION
C        Q(J)                 :   FLOW THROUGH LINK J
C        R(J)                 :   ACTUAL RELIABILTY OF LINK J
C        REL(K)               :   RELIABILITY ASSOCIATED WITH DIAMETER
C        SELECT(I,K)          :   LINK NUMBER OF PIPE K CONNECTED TO NO
C        SOURCE(I)            :   NODE NUMBER OF SOURCE NODE
C        TRN(I)               :   RELIABILITY OF NODE I
C        WPROB                :   PROBABILITY OF FAILURE OF WORST NODE
C        X(J)                 :   VECTOR OF LENGTHS OF ALL PIPE DIAMETE
C                                 J IN THE NETWORK
C
C
C
C            SYSTEM LAYOUT DATA
C
      REAL LEN(120)
      DIMENSION Q(60),A(60,120),B(120),C(120,10),D(120,10),DIAM(1
     $    COST(10),REL(10),LN(40),NODE(40,2),SELECT(30,120),
     $    CHOICE(30),ICON(30,120),DEMAND(60),SOURCE(10),NPL(10,6)
     $    NPLO(10,6),X(120),DUAL(120),DSOL(60),AJ(60,10),AHL(60),
     $    PHL(60),R(60),TRN(120),ALEN(60,10),PFL(40),
     $    PFSN(40),DRL(40),DR(40,10),F(40),DRN(40),RATIO(40)
      REWIND 9
      READ(9,100)NL,NN,IBUG
  100 FORMAT(3I4)
C     WRITE(6,105)NL,NN
C 105 FORMAT(//,' NUMBER OF LINKS=',I4,2X,'NUMBER OF NODES=',I4,/
C     WRITE(6,110)
C 110 FORMAT(1H ,'LINK',5X,'FROM',5X,'TO',5X,'LENGTH (M)',2X,'FLO
C    $/H)')
      READ(9,140)N,NP,NC
  140 FORMAT(3I4)
      DO 130 J=1,NL
         READ(9,120)LN(J),NODE(J,1),NODE(J,2),LEN(J),Q(J)
  120    FORMAT(3I3,2F8.1)
C        WRITE(6,125)LN(J),NODE(J,1),NODE(J,2),LEN(J),Q(J)
C 125    FORMAT(1H ,I3,6X,I3,5X,I3,2(4X,F8.1))
  130 CONTINUE
C     WRITE(6,150)N,NP
C 150 FORMAT(//,' CANDIDATE DIAMETERS: TOTAL=',I4,2X,'PER LINK=',
```

```
C     WRITE(6,155)
C 155 FORMAT(///,' DIAMETER (MM)',5X,'COST ($10**3/KM)',5X,'RELIA
C    $  (BREAKS/KM/YR)',4X,'CLASS')
C
C          READ IN PIPE DIAMETERS  (MM)
C
      DO 200 J=1,NC
          READ(9,160)DIAM(J),COST(J),REL(J)
  160     FORMAT(F6.1,F6.2,F6.3)
C         WRITE(6,170)DIAM(J),COST(J),REL(J),J
C 170     FORMAT(1H ,4X,F6.1,15X,F6.2,20X,F5.3,15X,I3)
  200 CONTINUE
C
C             READ IN C VALUES
C
      READ(9,205)CV
  205 FORMAT(F4.1)
C
C     READ IN DEMAND AT EACH NODE  (M**3/H)
C
C     WRITE(6,210)
C 210 FORMAT(///,' NODE',5X,'DEMAND (M**3/H)',/)
      DO 250 I=1,NN
           READ(9,220)DEMAND(I)
  220      FORMAT(F5.1)
C          WRITE(6,230)I,DEMAND(I)
C 230      FORMAT(1H ,I3,7X,F6.1)
  250 CONTINUE
C
C     DIAMETERS FOR EACH LINK
C
      DO 202 J=1,NL
         L=1
         DO 201 K=1,NP
            IF(K.LT.5) GOTO 203
            IF(Q(J).GT.100.0) GOTO 203
            L=K+1
            GOTO 103
  203       L=K
  103       D(J,K)=DIAM(L)
  201    CONTINUE
  202 CONTINUE
C
C     'J' VALUE FOR EACH DIAMETER K IN LINK J
C
      DO 270 J=1,NL
         DO 260 K=1,NP
            AJ(J,K)=(1.13*10E11)*((Q(J)/CV)**1.852)/(D(J,K)**4.87
  260    CONTINUE
  270 CONTINUE
C
C     READ IN NUMBER OF CONSTRAINTS
C
      READ(9,299)MP,MR,MLO,MLE
  299 FORMAT(4I4)
```

```
      MI=MP+MR
      ME=MLO+MLE
      MT=MI+ME
      M1=MP+1
      M2=MI+1
      M3=MLO+MI
      M4=M3+1
C
C     IDENTIFY LINKS IN EACH PATH AND LOOP
C
      DO 426 I=1,MP
         READ(9,425)(NPL(I,K),K=1,4)
 425     FORMAT(4I4)
 426  CONTINUE
      DO 428 I=1,MLO
         READ(9,427)(NPLO(I,K),K=1,4)
 427     FORMAT(4I4)
 428  CONTINUE
C
C     READ IN RHS OF CONSTRAINTS
C
      READ(9,429)(B(I),I=1,MT)
 429  FORMAT(6(2X,F5.2),/,12(1X,F5.3),/,4(2X,F5.3),/,12(1X,F5.3))
C
C     CALCULATE THE NUMBER OF POSSIBLE LINKS CONNECTED TO EACH NO
C
      DO 1100 I=1,NN
            SOURCE(I)=0.0
            IF(DEMAND(I).LT.0) SOURCE(I)=1
            L=0
            DO 1000 J=1,NL
                IF(NODE(J,1).NE.I .AND. NODE(J,2).NE.I) GOTO 1000
                L=L+1
                SELECT(I,L)=J
1000        CONTINUE
            CHOICE(I)=L
1100  CONTINUE
C
C    READ IN COST COEFFICIENTS AND COEFFICIENTS OF CONSTRAINT MAT
C
      DO 445 J=1,NL
         READ(10,390)(C(J,L),L=1,NP)
 390     FORMAT(4X,4(F5.2,3X),F5.2)
 445  CONTINUE
      DO 310 I=1,MP
         READ(10,300)(A(I,K),K=1,N)
 300     FORMAT(5(4X,F7.3))
 310  CONTINUE
      DO 991 I=M1,MI
         READ(10,993)(A(I,K),K=1,N)
 993     FORMAT(5(F4.2,3X))
 991  CONTINUE
      DO 820 I=M2,M3
         READ(10,780)(A(I,K),K=1,N)
 780     FORMAT(5(F7.3,3X))
```

```
  820 CONTINUE
      DO 900 I=M4,MT
         READ(10,860)(A(I,K),K=1,N)
  860    FORMAT(5(F3.1,3X))
  900 CONTINUE
C
C     READ VALUE OF OBJECTIVE FUNCTION - TOTAL COST
C
      READ(11,600)OBJ
  600 FORMAT(////,12X,F11.6)
      WRITE(6,601)OBJ
  601 FORMAT(//,' OBJECTIVE FUNCTION: TOTAL LEAST COST=',F10.5)
C
C     READ IN RESULTS (LENGTHS OF PIPE DIAMETER) FROM LINDO
C
      READ(11,90)(X(J),J=1,N)
   90 FORMAT(//,60(18X,F9.7,/))
      WRITE(6,68)
   68 FORMAT(//,' LENGTH OF PIPE - DIAMETER:',/,7X,'(1)',7X,'(2)'
     $7X,'(3)',7X,'(4)',7X,'(5)',//)
      WRITE(6,87)(X(J),J=1,N)
   87 FORMAT(1H ,5F10.3,/)
C
      WRITE(6,92)
   92 FORMAT(//,7X,'CONSTRAINT',11X,'CHANGE IN COST')
      READ(11,93)(DUAL(I),I=1,MT)
   93 FORMAT(//,34(31X,F14.6,/))
      DO 95 I=1,MT
              WRITE(6,94)I,DUAL(I)
   94         FORMAT(/,10X,I3,12X,F14.2)
   95 CONTINUE
C
C     CALCULATE HEAD LOSS IN EACH LINK
C
      K=1
      KNP=NP
      DO 1870 J=1,NL
         LL=1
         PHL(J)=0.0
         DO 1860 L=K,KNP
            ALEN(J,LL)=X(L)
            PHL(J)=PHL(J)+AJ(J,LL)*ALEN(J,LL)
            LL=LL+1
 1860    CONTINUE
         K=K+NP
         IF(KNP.EQ.N) GOTO 1870
         KNP=KNP+NP
 1870 CONTINUE
C
      DO 337 J=1,NL
         WRITE(6,333)J,PHL(J)
  333    FORMAT(/,'HEAD LOSS IN LINK',I3,' =',F6.3)
  337 CONTINUE
C
C     CALCULATE HEAD LOSS ALONG PATHS
```

```
C
      DO 222 I=1,MP
         AHL(1)=0.0
         DO 221 K=1,N
            AHL(I)=AHL(I)+A(I,K)*X(K)
  221    CONTINUE
         WRITE(6,223)I,AHL(I)
  223    FORMAT(/,3X,'HEAD LOSS FOR PATH',I2,' =',F6.2)
  222 CONTINUE
C
C           CALCULATE LINK RELIABILITIES
C
      DO 550 J=1,NL
            I=J+MP
            R(J)=0.0
                 DO 510 K=1,N
                      R(J)=R(J)+A(I,K)*X(K)
  510            CONTINUE
  550 CONTINUE
      WRITE(6,560)
  560 FORMAT(//,7X,'LINK',11X,'RELIABILITY (BREAKS/KM/YR)',4X,'RH
     $'DC/DR')
      DO 590 J=1,NL
         I=J+MP
         DSOL(J)=DUAL(I)
         WRITE(6,570)J,R(J),B(I),DSOL(J)
  570    FORMAT(/,7X,I3,15X,F8.3,15X,F8.3,5X,F8.2)
  590 CONTINUE
C
C
C
      DO 1210 I=1,NN
           DO 1200 J=1,NL
                ICON(I,J)=0
 1200      CONTINUE
 1210 CONTINUE
      DO 1230 I=1,NN
           L=CHOICE(I)
           DO 1220 K=1,L
               J=SELECT(I,K)
               ICON(I,J)=J
 1220      CONTINUE
 1230 CONTINUE
      DO 1245 I=1,NN
           TRN(I)=1
           DO 1243 J=1,NL
               IF(ICON(I,J).EQ.0) GOTO 1243
               TRN(I)=TRN(I)*R(J)
 1243      CONTINUE
 1245 CONTINUE
      WRITE(6,1250)(J,J=1,NL)
 1250 FORMAT(///,' NODE',5X,'LINKS',54X,'NODE RELIABILITY',/,9X,1
     $I3,2X))
      DO 1270 I=1,NN
             WRITE(6,1260)I,(ICON(I,J),J=1,NL),TRN(I)
```

```
 1260        FORMAT(//,I3,4X,12(2X,I3),5X,F8.5)
 1270 CONTINUE
C
C     CALCULATE PROBABILITY OF 1+ FAILURE OF EACH LINK
C
      DO 1880 J=1,NL
         PFL(J)=0.0
         DRL(J)=0.0
         LX=1
         LXFACT=1
 1800    PFL(J)=PFL(J)+((1.0/EXP(R(J)))*(R(J)**LX)/LXFACT)
         TEMP=(1.0/EXP(R(J)))*(LX*R(J)**(LX-1)-R(J)**LX)/LXFACT
         DRL(J)=DRL(J)+TEMP
         LX=LX+1
         LXFACT=LXFACT*LX
         IF(LX.LE.10) GOTO 1800
         WRITE(6,1900)J,PFL(J),DRL(J)
 1900    FORMAT(//,5X,'PROBABILITY OF 1+ FAILURE OF LINK',I3,2X,'
     $F5.3,' ; DP(LINK)/DLAMBDA =',F10.6)
 1880 CONTINUE
C
C      CALCULATE PROBABILITY OF NO SUPPLY AT EACH NODE
C
      DO 1890 I=1,NN
         PFSN(I)=1.0
         DO 1885 J=1,NL
            IF(ICON(I,J).EQ.0.0) GOTO 1885
            PFSN(I)=PFSN(I)*PFL(J)
 1885    CONTINUE
         WRITE(6,1883)I,PFSN(I)
 1883    FORMAT(//,2X,'PROBABILITY OF SIM. FAILURE TO SUPPLY NODE
     $I3,2X,'=',F6.4)
 1890 CONTINUE
C
C     PICK WORST NODE AND CALCULATE RATIO OF CHANGE IN COST WRT
C         RELIABILITY AND CHANGE IN RELIABILITY OF LINKS TO NODE
C
      WRITE(6,1999)
 1999 FORMAT(///,10X,'***********************************',//)
      NWORST=1
      WPROB=PFSN(1)
      DO 2000 I=2,NN
         IF(PFSN(I).LT.WPROB) GOTO 2000
         NWORST=I
         WPROB=PFSN(I)
 2000 CONTINUE
      WRITE(6,2100)NWORST,WPROB
 2100 FORMAT(//,2X,'NODE WITH HIGHEST PROBABILITY OF 1+ FAILURE I
     $I3,2X,'PROBABILITY OF SIM. FAILURE =',F6.4)
C
C
      I=NWORST
      L=CHOICE(I)
      DO 2115 LL=1,L
         DR(I,LL)=1.0
```

```
         DO 2110 J=1,NL
            F(J)=PFL(J)
            IF(ICON(I,J).EQ.0.0) F(J)=1.0
            IF(ICON(I,J).EQ.SELECT(I,LL)) F(J)=DRL(J)
            DR(I,LL)=DR(I,LL)*F(J)
 2110    CONTINUE
         J=SELECT(I,LL)
         DRN(J)=DR(I,LL)
 2115 CONTINUE
      WRITE(6,2120)
 2120 FORMAT(//,2X,'LINKS',5X,'DC/DR',6X,'DRN/DRL',6X,'RATIO',//)
      I=NWORST
      DMIN=10000000.0
      DO 2200 J=1,NL
         IF(ICON(I,J).EQ.0.0) GOTO 2200
         RATIO(J)=DSOL(J)/DRN(J)
         WRITE(6,2150)ICON(I,J),DSOL(J),DRN(J),RATIO(J)
 2150    FORMAT(/,4X,I2,3X,F8.2,5X,F8.5,2X,F10.4)
         IF(RATIO(J).GE.DMIN) GOTO 2200
         DMIN=RATIO(J)
 2200 CONTINUE
      WRITE(6,2300)DMIN
 2300 FORMAT(//,2X,'MIN RATIO =',F10.4)
      STOP
      END
//GO.FT09F001 DD DSN=COALS.LAYOUT.DATA,
//   DISP=SHR
//GO.FT10F001 DD DSN=COALS.PIPES,
//   DISP=SHR
//GO.FT11F001 DD DSN=COALS.PIPANS,
//   DISP=SHR
```

This program is used in Approach 3, the path failure approach. It uses the same datasets as the previous program, which are found at the end of this listing.

```
C
C   THIS PROGRAM TAKES RESULTS FROM LINDO, AND OTHER DATASETS,
C   AND CALCULATES FAILURE RATES OF PIPES IN A NETWORK.
C   PROBABLILITY OF FAILURE OF PATHS IN THE SYSTEM IS CALCULATED.
C
C
C   VARIABLE LIST:
C
C       A(I,K)          :   COEFFICIENT OF CONSTRAINT I, KTH ELEM
C       AHL(I)          :   ACTUAL HEAD LOSS FOR PATH I
C       AJ(J,K)         :   J VALUE FOR KTH DIAMETER OF JTH LINK
C       ALEN(J,K)       :   MATRIX OF LENGTHS OF PIPE DIAMETERS K
C                           JTH LINK
C       B(I)            :   R.H.S. OF CONSTRAINT I
C       C(J,K)          :   COST OF PIPE SIZE K IN LINK J
C       CHOICE(I)       :   NUMBER OF PIPES CONNECTED TO NODE I
C       COST(J)         :   COST ASSOCIATED WITH PIPE SIZE J
C       CV              :   HAZEN-WILLIAMS COEFFICIENT
C       D(J,K)          :   DIAMETER OF KTH CHOICE OF JTH LINK
C       DEMAND(I)       :   DEMAND AT NODE I
C       DIAM(J)         :   DIAMETER OF PIPE SIZE J
C       DRL(J)          :   DERIVATIVE OF PROBABILITY OF FAILURE
C                           OF LINK J
C       DRP(J)          :   DERIVATIVE OF PROBABILITY OF FAILURE
C                           OF LINK J WRT CONNECTING PATH
C       DSOL(J)         :   DUAL VARIABLE OF RELIABILITY CONSTRAI
C                           ASSOCIATED WITH LINK J
C       DUAL(I)         :   DUAL VARIABLE ASSOCIATED WITH CONSTRA
C       EB(I)           :   EXPECTED NUMBER OF BREAKS IN PATH I
C       F(J)            :   FACTOR USED TO REPRESENT PROBABILITIE
C                           IN CALCULATING FINAL RATIO
C       IBUG            :   0 IF CONSTRAINT COEFFICIENTS ARE NOT
C                             TO BE WRITTEN,
C                           1 IF CONSTRAINT COEFFICIENTS ARE TO B
C                             WRITTEN
C       ICON(I,J)       :   LINK NUMBER J CONNECTED TO NODE I
C       LEN(J)          :   LENGTH OF LINK J
C       LN(J)           :   LINK NUMBER J
C       ME              :   TOTAL NUMBER OF EQUALITY CONSTRAINTS
C       MI              :   TOTAL NUMBER OF INEQUALITY CONSTRAINT
C       MLE             :   NUMBER OF LENGTH CONSTRAINTS
C       MLO             :   NUMBER OF LOOP CONSTRAINTS
C       MP              :   NUMBER OF PATH CONSTRAINTS
C       MR              :   NUMBER OF RELIABILITY CONSTRAINTS
C       MT              :   TOTAL NUMBER OF CONSTRAINTS
C       N               :   TOTAL NUMBER OF CANDIDATE DIAMETERS F
C                           LINKS IN THE NETWORK
C       NC              :   NUMBER OF CLASSES OF PIPE DIAMETERS
C                           AVAILABLE
```

```
C        NL                 :    NUMBER OF LINKS IN THE NETWORK
C        NN                 :    NUMBER OF NODES IN THE NETWORK
C        NODE(J,1)          :    SOURCE NODE FOR LINK 1
C        NODE(J,2)          :
C        NP                 :    NUMBER OF PIPE DIAMETERS PER LINK
C        NPL(I,K)           :    LINK NUMBER K IN ITH PATH
C        NPLO(I,K)          :    LINK NUMBER K IN ITH LOOP
C        NWORST             :    PATH WITH HIGHEST PROBABILTY OF FAILU
C                                TO SUPPLY
C        PHL(J)             :    HEAD LOSS IN LINK J
C        PFL(J)             :    PROBABILITY OF FAILURE OF LINK J
C        PFP (I)            :    PROBABILITY OF FAILURE TO SUPPLY PATH
C        Q(J)               :    FLOW THROUGH LINK J
C        R(J)               :    ACTUAL RELIABILTY OF LINK J
C        REL(K)             :    RELIABILITY ASSOCIATED WITH DIAMETER
C        SELECT(I,K)        :    LINK NUMBER OF PIPE K CONNECTED TO NO
C        SOURCE(I)          :    NODE NUMBER OF SOURCE NODE
C        WPROB              :    PROBABILITY OF FAILURE OF WORST PATH
C        X(J)               :    VECTOR OF LENGTHS OF ALL PIPE DIAMETE
C                                J IN THE NETWORK
C
C
C
C           SYSTEM LAYOUT DATA
C
      REAL LEN(120)
      DIMENSION Q(60),A(60,120),B(120),C(120,10),D(120,10),DIAM(1
     $    COST(10),REL(10),LN(40),NODE(40,2),SELECT(30,120),
     $    CHOICE(30),ICON(30,120),DEMAND(60),SOURCE(10),NPL(10,6)
     $    NPLO(10,6),X(120),DUAL(120),DSOL(60),AJ(60,10),AHL(60),
     $    PHL(60),R(60),ALEN(60,10),PFL(40),EB(10),
     $    DRL(40),F(40),DRP(40),PFP(40),DR(40,10),RATIO(40,10)
      REWIND 9
      READ(9,100)NL,NN,IBUG
  100 FORMAT(3I4)
C     WRITE(6,105)NL,NN
C 105 FORMAT(//,' NUMBER OF LINKS=',I4,2X,'NUMBER OF NODES=',I4,/
C     WRITE(6,110)
C 110 FORMAT(1H ,'LINK',5X,'FROM',5X,'TO',5X,'LENGTH (M)',2X,'FLO
C    $/H)')
      READ(9,140)N,NP,NC
  140 FORMAT(3I4)
      DO 130 J=1,NL
         READ(9,120)LN(J),NODE(J,1),NODE(J,2),LEN(J),Q(J)
  120    FORMAT(3I3,2F8.1)
C        WRITE(6,125)LN(J),NODE(J,1),NODE(J,2),LEN(J),Q(J)
C 125    FORMAT(1H ,I3,6X,I3,5X,I3,2(4X,F8.1))
  130 CONTINUE
C     WRITE(6,150)N,NP
C 150 FORMAT(//,' CANDIDATE DIAMETERS: TOTAL=',I4,2X,'PER LINK=',
C     WRITE(6,155)
C 155 FORMAT(///,' DIAMETER (MM)',5X,'COST ($10**3/KM)',5X,'RELIA
C    $  (BREAKS/KM/YR)',4X,'CLASS')
C
C           READ IN PIPE DIAMETERS  (MM)
```

```
C
      DO 200 J=1,NC
          READ(9,160)DIAM(J),COST(J),REL(J)
  160     FORMAT(F6.1,F6.2,F6.3)
C         WRITE(6,170)DIAM(J),COST(J),REL(J),J
C 170     FORMAT(1H ,4X,F6.1,15X,F6.2,20X,F5.3,15X,I3)
  200 CONTINUE
C
C               READ IN C VALUES
C
      READ(9,205)CV
  205 FORMAT(F4.1)
C
C     READ IN DEMAND AT EACH NODE  (M**3/H)
C
C     WRITE(6,210)
C 210 FORMAT(///,' NODE',5X,'DEMAND (M**3/H)',/)
      DO 250 I=1,NN
           READ(9,220)DEMAND(I)
  220      FORMAT(F5.1)
C          WRITE(6,230)I,DEMAND(I)
C 230      FORMAT(1H ,I3,7X,F6.1)
  250 CONTINUE
C
C     DIAMETERS FOR EACH LINK
C
      DO 202 J=1,NL
         L=1
         DO 201 K=1,NP
            IF(Q(J).GT.100.0) GOTO 203
            L=K
            GOTO 103
  203       L=L+1
  103       D(J,K)=DIAM(L)
  201    CONTINUE
  202 CONTINUE
C
C     'J' VALUE FOR EACH DIAMETER K IN LINK J
C
      DO 270 J=1,NL
         DO 260 K=1,NP
            AJ(J,K)=(1.13*10E11)*((Q(J)/CV)**1.852)/(D(J,K)**4.87
  260    CONTINUE
  270 CONTINUE
C
C     READ IN NUMBER OF CONSTRAINTS
C
      READ(9,299)MP,MR,MLO,MLE
  299 FORMAT(4I4)
      MI=MP+MR
      ME=MLO+MLE
      MT=MI+ME
      M1=MP+1
      M2=MI+1
      M3=MLO+MI
```

```
      M4=M3+1
C
C     IDENTIFY LINKS IN EACH PATH AND LOOP
C
      DO 426 I=1,MP
         READ(9,425)(NPL(I,K),K=1,4)
  425    FORMAT(4I4)
  426 CONTINUE
      DO 428 I=1,MLO
         READ(9,427)(NPLO(I,K),K=1,4)
  427    FORMAT(4I4)
  428 CONTINUE
C
C     READ IN RHS OF CONSTRAINTS
C
      READ(9,429)(B(I),I=1,MT)
  429 FORMAT(6(2X,F5.2),/,12(1X,F5.3),/,4(2X,F5.3),/,12(1X,F5.3))
C
C     CALCULATE THE NUMBER OF POSSIBLE LINKS CONNECTED TO EACH NO
C
      DO 1100 I=1,NN
           SOURCE(I)=0.0
           IF(DEMAND(I).LT.0) SOURCE(I)=1
           L=0
           DO 1000 J=1,NL
               IF(NODE(J,1).NE.I .AND. NODE(J,2).NE.I) GOTO 1000
               L=L+1
               SELECT(I,L)=J
 1000      CONTINUE
           CHOICE(I)=L
 1100 CONTINUE
C
C    READ IN COST COEFFICIENTS AND COEFFICIENTS OF CONSTRAINT MAT
C
      DO 445 J=1,NL
         READ(10,390)(C(J,L),L=1,NP)
  390    FORMAT(4X,4(F5.2,3X),F5.2)
  445 CONTINUE
      DO 310 I=1,MP
         READ(10,300)(A(I,K),K=1,N)
  300    FORMAT(5(4X,F7.3))
  310 CONTINUE
      DO 991 I=M1,MI
         READ(10,993)(A(I,K),K=1,N)
  993    FORMAT(5(F4.2,3X))
  991 CONTINUE
      DO 820 I=M2,M3
         READ(10,780)(A(I,K),K=1,N)
  780    FORMAT(5(F7.3,3X))
  820 CONTINUE
      DO 900 I=M4,MT
         READ(10,860)(A(I,K),K=1,N)
  860    FORMAT(5(F3.1,3X))
  900 CONTINUE
C
```

```
C     READ IN RESULTS (LENGTHS OF PIPE DIAMETER) FROM LINDO
C
      READ(11,90)(X(J),J=1,N)
   90 FORMAT(///////,60(18X,F9.7,/))
      WRITE(6,68)
   68 FORMAT(//,' LENGTH OF PIPE - DIAMETER:',/,7X,'(1)',7X,'(2)'
     $7X,'(3)',7X,'(4)',7X,'(5)',//)
      WRITE(6,87)(X(J),J=1,N)
   87 FORMAT(1H ,5F10.3,/)
C
      WRITE(6,92)
   92 FORMAT(//,7X,'CONSTRAINT',11X,'CHANGE IN COST')
      READ(11,93)(DUAL(I),I=1,MT)
   93 FORMAT(//,34(31X,F14.6,/))
      DO 95 I=1,MT
              WRITE(6,94)I,DUAL(I)
   94         FORMAT(/,10X,I3,12X,F14.2)
   95 CONTINUE
C
C     CALCULATE HEAD LOSS IN EACH LINK
C
      K=1
      KNP=NP
      DO 1870 J=1,NL
         LL=1
         PHL(J)=0.0
         DO 1860 L=K,KNP
            ALEN(J,LL)=X(L)
            PHL(J)=PHL(J)+A(J,LL)*ALEN(J,LL)
            LL=LL+1
 1860    CONTINUE
         K=K+NP
         IF(KNP.EQ.N) GOTO 1870
         KNP=KNP+NP
 1870 CONTINUE
C
      DO 337 J=1,NL
C        WRITE(6,333)J,PHL(J)
C 333    FORMAT(/,'HEAD LOSS IN LINK',I3,' =',F6.3)
  337 CONTINUE
C
C     CALCULATE HEAD LOSS ALONG PATHS
C
      DO 222 I=1,MP
         AHL(1)=0.0
         DO 221 K=1,N
            AHL(I)=AHL(I)+A(I,K)*X(K)
  221    CONTINUE
         WRITE(6,223)I,AHL(I)
  223    FORMAT(/,3X,'HEAD LOSS FOR PATH',I2,' =',F6.2)
  222 CONTINUE
C
C            CALCULATE LINK RELIABILITIES
C
      DO 550 J=1,NL
```

```
         I=J+MP
         R(J)=0.0
              DO 510 K=1,N
                   R(J)=R(J)+A(I,K)*X(K)
  510         CONTINUE
  550 CONTINUE
      WRITE(6,560)
  560 FORMAT(//,7X,'LINK',11X,'RELIABILITY (BREAKS/KM/YR)',4X,'RH
     $'DC/DR')
      DO 590 J=1,NL
         I=J+MP
         DSOL(J)=DUAL(I)
         WRITE(6,570)J,R(J),B(I),DSOL(J)
  570    FORMAT(/,7X,I3,15X,F8.3,15X,F8.3,5X,F8.2)
  590 CONTINUE
C
C     CALCULATE PROBABILITY OF 1+ FAILURE OF EACH LINK
C
      DO 1880 J=1,NL
         PFL(J)=0.0
         DRL(J)=0.0
         LX=1
         LXFACT=1
 1800    PFL(J)=PFL(J)+((1.0/EXP(R(J)))*(R(J)**LX)/LXFACT)
         TEMP=(1.0/EXP(R(J)))*(LX*R(J)**(LX-1)-R(J)**LX)/LXFACT
         DRL(J)=DRL(J)+TEMP
         LX=LX+1
         LXFACT=LXFACT*LX
         IF(LX.LE.10) GOTO 1800
         WRITE(6,1900)J,PFL(J),DRL(J)
 1900    FORMAT(//,5X,'PROBABILITY OF 1+ FAILURE OF LINK',I3,2X,'
     $F5.3,' ; DP(LINK)/DLAMBDA =',F10.6)
 1880 CONTINUE
C
C      CALCULATE REL. & PROB. OF FAILURE (1+) OF EACH PATH
C
      DO 1890 I=1,MP
         EB(I)=0.0
         DO 1887 K=1,4
         DO 1885 J=1,NL
            IF(NPL(I,K).NE.J) GOTO 1885
            EB(I)=EB(I)+R(J)
 1885    CONTINUE
 1887    CONTINUE
 1890 CONTINUE
         PFP(I)=0.0
         DO 1790 I=1,MP
             LX=1
             LXFACT=1
 1700       PFP(I)=PFP(I)+((1.0/EXP(EB(I)))*(EB(I)**LX)/LXFACT)
            LX=LX+1
            LXFACT=LXFACT*LX
            IF(LX.LE.10) GOTO 1700
         WRITE(6,1883)I,EB(I),PFP(I)
 1883    FORMAT(//,2X,'PATH',I3,2X,'EXPECTED BREAKS=',F6.4,5X,'PR
```

```
     $OF 1+ FAILURE =',F6.4)
 1790    CONTINUE
C
C     PICK WORST PATH AND CALCULATE RATIO OF CHANGE IN COST WRT
C          RELIABILITY AND CHANGE IN RELIABILITY OF LINKS TO NODE
C
      WRITE(6,1999)
 1999 FORMAT(///,10X,'**********************************',//)
      NWORST=1
      WPROB=PFP(1)
      DO 2000 I=2,MP
         IF(PFP(I).LT.WPROB) GOTO 2000
         NWORST=I
         WPROB=PFP(I)
 2000 CONTINUE
      WRITE(6,2100)NWORST,WPROB
 2100 FORMAT(//,2X,'PATH WITH HIGHEST PROBABILITY OF 1+ FAILURE I
     $I3,2X,'PROBABILITY OF  FAILURE =',F6.4)
C
C
      I=NWORST
      DO 2115 L=1,4
         DR(I,L)=1.0
         DO 2110 K=1,4
            J=NPL(I,K)
            F(J)=PFL(J)
            IF(NPL(I,L).EQ.J) F(J)=DRL(J)
            DR(I,L)=DR(I,L)*F(J)
 2110    CONTINUE
          J=NPL(I,L)
         RATIO(I,L)=DSOL(J)/DR(I,L)
 2115 CONTINUE
      WRITE(6,2120)
 2120 FORMAT(//,2X,'LINKS',6X,'PROB.FAILURE',5X,'DRL',8X,'DC/DR',
     $'DRP/DRL',5X,'RATIO',//)
      I=NWORST
      DO 2300 K=1,4
      DO 2200 J=1,NL
         IF(NPL(I,K).NE.J) GOTO 2200
         WRITE(6,2150)NPL(I,K),PFL(J),DRL(J),DSOL(J),DR(I,K),RATI
 2150    FORMAT(/,4X,I2,8X,F8.3,5X,F8.6,5X,F8.2,5X,F8.3,5X,F8.2)
 2200 CONTINUE
 2300 CONTINUE
      STOP
      END
//GO.FT09F001 DD DSN=COALS.LAYOUT.DATA,
//  DISP=SHR
//GO.FT10F001 DD DSN=COALS.PIPES,
//  DISP=SHR
//GO.FT11F001 DD DSN=COALS.PIPANS,
//  DISP=SHR
```

Data for the predetermined layout used in these programs is found in a data set, in the following format.

```
  12    9   0                    - No. of links, nodes (0=IBUG)
  60    5   6                    - No. of candidate diameters
   1   1   2  1000.0    450.0   - Link no., corresponding nodes,
   2   2   3  1000.0    125.0       link length, and flow.
   3   1   4  1000.0    300.0
   4   2   5  1000.0    250.0
   5   3   6  1000.0     50.0
   6   4   5  1000.0    125.0
   7   5   6  1000.0    175.0
   8   4   7  1000.0    100.0
   9   5   8  1000.0    125.0
  10   6   9  1000.0    150.0
  11   7   8  1000.0     25.0
  12   8   9  1000.0     75.0
 300.0 69.20 0.070               - Pipe diameter, cost/km, and
 250.0 43.20 0.390                  expected no. breaks/km/yr
 200.0 24.10 0.710
 150.0 16.90 1.040
 350.0 98.60 0.050
 100.0 14.30 1.360
130.0                            - C value, Hazen-Williams eq'n
-750.0                           - Demands at nodes
  75.0
  75.0
  75.0
  75.0
  75.0
  75.0
  75.0
 225.0
   6  12   4  12                 - No. of eq'ns for each type of
                                   constraint (path, reliability,
                                   loop, length)
   1   2   5  10                 - Links in each path
   1   4   7  10
   1   4   9  12
   3   6   7  10
   3   6   9  12
   3   8  11  12
   1   4   6   3                 - Links in each loop
   2   5   7   4
   6   9  11   8
   7  10  12   9
  25.00  25.00  25.00  25.00  25.00  25.00    - RHS values (path)
 0.069 0.698 0.380 0.117 0.710 0.310     - RHS values (reliabilit
 0.326 0.744 0.540 0.390 0.070 0.690
  0.000  0.000  0.000  0.000             - RHS values (loop)
 1.000 1.000 1.000 1.000 1.000 1.000     - RHS values (length)
 1.000 1.000 1.000 1.000 1.000 1.000
```

The linear programming formulation used in Approach 1 and 3 is found in the following dataset. It is set up in such a way that it can be used by LINDO, to find an optimum solution.

```
MIN 69.20AV+43.20AW+24.10AX+16.90AY+98.60AZ
    +69.20BV+43.20BW+24.10BX+16.90BY+98.60BZ
    +69.20CV+43.20CW+24.10CX+16.90CY+98.60CZ
    +69.20DV+43.20DW+24.10DX+16.90DY+98.60DZ
    +69.20EV+43.20EW+24.10EX+16.90EY+14.30EZ
    +69.20FV+43.20FW+24.10FX+16.90FY+98.60FZ
    +69.20GV+43.20GW+24.10GX+16.90GY+98.60GZ
    +69.20HV+43.20HW+24.10HX+16.90HY+14.30HZ
    +69.20IV+43.20IW+24.10IX+16.90IY+98.60IZ
    +69.20JV+43.20JW+24.10JX+16.90JY+98.60JZ
    +69.20KV+43.20KW+24.10KX+16.90KY+14.30KZ
    +69.20LV+43.20LW+24.10LX+16.90LY+14.30LZ
S.T.  9.733AV + 23.650AW +  0.000AX +  0.000AY +  4.594AZ
     +0.000BV +  2.206BW +  6.539BX +  0.000BY +  0.000BZ
     +0.000CV +  0.000CW +  0.000CX +  0.000CY +  0.000CZ
     +0.000DV +  0.000DW +  0.000DX +  0.000DY +  0.000DZ
     +0.000EV +  0.404EW +  1.198EX +  4.864EY +  0.000EZ
     +0.000FV +  0.000FW +  0.000FX +  0.000FY +  0.000FZ
     +0.000GV +  0.000GW +  0.000GX +  0.000GY +  0.000GZ
     +0.000HV +  0.000HW +  0.000HX +  0.000HY +  0.000HZ
     +0.000IV +  0.000IW +  0.000IX +  0.000IY +  0.000IZ
     +0.000JV +  3.092JW +  0.000JX +  0.000JY +  0.000JZ
     +0.000KV +  0.000KW +  0.000KX +  0.000KY +  0.000KZ
     +0.000LV +  0.000LW +  0.000LX +  0.000LY +  0.000LZ< 25.00
      9.733AV + 23.650AW +  0.000AX +  0.000AY +  4.594AZ
     +0.000BV +  0.000BW +  0.000BX +  0.000BY +  0.000BZ
     +0.000CV +  0.000CW +  0.000CX +  0.000CY +  0.000CZ
     +3.277DV +  7.963DW +  0.000DX +  0.000DY +  0.000DZ
     +0.000EV +  0.000EW +  0.000EX +  0.000EY +  0.000EZ
     +0.000FV +  0.000FW +  0.000FX +  0.000FY +  0.000FZ
     +1.693GV +  4.113GW +  0.000GX +  0.000GY +  0.000GZ
     +0.000HV +  0.000HW +  0.000HX +  0.000HY +  0.000HZ
     +0.000IV +  0.000IW +  0.000IX +  0.000IY +  0.000IZ
     +0.000JV +  3.092JW +  0.000JX +  0.000JY +  0.000JZ
     +0.000KV +  0.000KW +  0.000KX +  0.000KY +  0.000KZ
     +0.000LV +  0.000LW +  0.000LX +  0.000LY +  0.000LZ< 25.00
      9.733AV + 23.650AW +  0.000AX +  0.000AY +  4.594AZ
     +0.000BV +  0.000BW +  0.000BX +  0.000BY +  0.000BZ
     +0.000CV +  0.000CW +  0.000CX +  0.000CY +  0.000CZ
     +3.277DV +  7.963DW +  0.000DX +  0.000DY +  0.000DZ
     +0.000EV +  0.000EW +  0.000EX +  0.000EY +  0.000EZ
     +0.000FV +  0.000FW +  0.000FX +  0.000FY +  0.000FZ
     +0.000GV +  0.000GW +  0.000GX +  0.000GY +  0.000GZ
     +0.000HV +  0.000HW +  0.000HX +  0.000HY +  0.000HZ
     +0.000IV +  2.206IW +  6.539IX +  0.000IY +  0.000IZ
     +0.000JV +  0.000JW +  0.000JX +  0.000JY +  0.000JZ
     +0.000KV +  0.000KW +  0.000KX +  0.000KY +  0.000KZ
```

```
+0.000LV +  0.856LW +  2.539LX +  0.000LY +  0.000LZ< 25.00
 0.000AV +  0.000AW +  0.000AX +  0.000AY +  0.000AZ
+0.000BV +  0.000BW +  0.000BX +  0.000BY +  0.000BZ
+4.593CV + 11.161CW +  0.000CX +  0.000CY +  0.000CZ
+0.000DV +  0.000DW +  0.000DX +  0.000DY +  0.000DZ
+0.000EV +  0.000EW +  0.000EX +  0.000EY +  0.000EZ
+0.908FV +  2.206FW +  0.000FX +  0.000FY +  0.000FZ
+1.693GV +  4.113GW +  0.000GX +  0.000GY +  0.000GZ
+0.000HV +  0.000HW +  0.000HX +  0.000HY +  0.000HZ
+0.000IV +  0.000IW +  0.000IX +  0.000IY +  0.000IZ
+0.000JV +  3.092JW +  0.000JX +  0.000JY +  0.000JZ
+0.000KV +  0.000KW +  0.000KX +  0.000KY +  0.000KZ
+0.000LV +  0.000LW +  0.000LX +  0.000LY +  0.000LZ< 25.00
 0.000AV +  0.000AW +  0.000AX +  0.000AY +  0.000AZ
+0.000BV +  0.000BW +  0.000BX +  0.000BY +  0.000BZ
+4.593CV + 11.161CW +  0.000CX +  0.000CY +  0.000CZ
+0.000DV +  0.000DW +  0.000DX +  0.000DY +  0.000DZ
+0.000EV +  0.000EW +  0.000EX +  0.000EY +  0.000EZ
+0.908FV +  2.206FW +  0.000FX +  0.000FY +  0.000FZ
+0.000GV +  0.000GW +  0.000GX +  0.000GY +  0.000GZ
+0.000HV +  0.000HW +  0.000HX +  0.000HY +  0.000HZ
+0.000IV +  2.206IW +  6.539IX +  0.000IY +  0.000IZ
+0.000JV +  0.000JW +  0.000JX +  0.000JY +  0.000JZ
+0.000KV +  0.000KW +  0.000KX +  0.000KY +  0.000KZ
+0.000LV +  0.856LW +  2.539LX +  0.000LY +  0.000LZ< 25.00
 0.000AV +  0.000AW +  0.000AX +  0.000AY +  0.000AZ
+0.000BV +  0.000BW +  0.000BX +  0.000BY +  0.000BZ
+4.593CV + 11.161CW +  0.000CX +  0.000CY +  0.000CZ
+0.000DV +  0.000DW +  0.000DX +  0.000DY +  0.000DZ
+0.000EV +  0.000EW +  0.000EX +  0.000EY +  0.000EZ
+0.000FV +  0.000FW +  0.000FX +  0.000FY +  0.000FZ
+0.000GV +  0.000GW +  0.000GX +  0.000GY +  0.000GZ
+0.000HV +  1.459HW +  4.326HX + 17.559HY +  0.000HZ
+0.000IV +  0.000IW +  0.000IX +  0.000IY +  0.000IZ
+0.000JV +  0.000JW +  0.000JX +  0.000JY +  0.000JZ
+0.046KV +  0.112KW +  0.332KX +  0.000KY +  0.000KZ
+0.000LV +  0.856LW +  2.539LX +  0.000LY +  0.000LZ< 25.00
 0.07AV+0.39AW+0.00AX+0.00AY+0.05AZ
+0.00BV+0.00BW+0.00BX+0.00BY+0.00BZ
+0.00CV+0.00CW+0.00CX+0.00CY+0.00CZ
+0.00DV+0.00DW+0.00DX+0.00DY+0.00DZ
+0.00EV+0.00EW+0.00EX+0.00EY+0.00EZ
+0.00FV+0.00FW+0.00FX+0.00FY+0.00FZ
+0.00GV+0.00GW+0.00GX+0.00GY+0.00GZ
+0.00HV+0.00HW+0.00HX+0.00HY+0.00HZ
+0.00IV+0.00IW+0.00IX+0.00IY+0.00IZ
+0.00JV+0.00JW+0.00JX+0.00JY+0.00JZ
+0.00KV+0.00KW+0.00KX+0.00KY+0.00KZ
+0.00LV+0.00LW+0.00LX+0.00LY+0.00LZ<0.069
 0.00AV+0.00AW+0.00AX+0.00AY+0.00AZ
+0.07BV+0.39BW+0.71BX+1.04BY+0.05BZ
+0.00CV+0.00CW+0.00CX+0.00CY+0.00CZ
+0.00DV+0.00DW+0.00DX+0.00DY+0.00DZ
+0.00EV+0.00EW+0.00EX+0.00EY+0.00EZ
+0.00FV+0.00FW+0.00FX+0.00FY+0.00FZ
```

```
+0.00GV+0.00GW+0.00GX+0.00GY+0.00GZ
+0.00HV+0.00HW+0.00HX+0.00HY+0.00HZ
+0.00IV+0.00IW+0.00IX+0.00IY+0.00IZ
+0.00JV+0.00JW+0.00JX+0.00JY+0.00JZ
+0.00KV+0.00KW+0.00KX+0.00KY+0.00KZ
+0.00LV+0.00LW+0.00LX+0.00LY+0.00LZ<0.698
 0.00AV+0.00AW+0.00AX+0.00AY+0.00AZ
+0.00BV+0.00BW+0.00BX+0.00BY+0.00BZ
+0.07CV+0.39CW+0.00CX+0.00CY+0.00CZ
+0.00DV+0.00DW+0.00DX+0.00DY+0.00DZ
+0.00EV+0.00EW+0.00EX+0.00EY+0.00EZ
+0.00FV+0.00FW+0.00FX+0.00FY+0.00FZ
+0.00GV+0.00GW+0.00GX+0.00GY+0.00GZ
+0.00HV+0.00HW+0.00HX+0.00HY+0.00HZ
+0.00IV+0.00IW+0.00IX+0.00IY+0.00IZ
+0.00JV+0.00JW+0.00JX+0.00JY+0.00JZ
+0.00KV+0.00KW+0.00KX+0.00KY+0.00KZ
+0.00LV+0.00LW+0.00LX+0.00LY+0.00LZ<0.380
 0.00AV+0.00AW+0.00AX+0.00AY+0.00AZ
+0.00BV+0.00BW+0.00BX+0.00BY+0.00BZ
+0.00CV+0.00CW+0.00CX+0.00CY+0.00CZ
+0.07DV+0.39DW+0.00DX+0.00DY+0.00DZ
+0.00EV+0.00EW+0.00EX+0.00EY+0.00EZ
+0.00FV+0.00FW+0.00FX+0.00FY+0.00FZ
+0.00GV+0.00GW+0.00GX+0.00GY+0.00GZ
+0.00HV+0.00HW+0.00HX+0.00HY+0.00HZ
+0.00IV+0.00IW+0.00IX+0.00IY+0.00IZ
+0.00JV+0.00JW+0.00JX+0.00JY+0.00JZ
+0.00KV+0.00KW+0.00KX+0.00KY+0.00KZ
+0.00LV+0.00LW+0.00LX+0.00LY+0.00LZ<0.117
 0.00AV+0.00AW+0.00AX+0.00AY+0.00AZ
+0.00BV+0.00BW+0.00BX+0.00BY+0.00BZ
+0.00CV+0.00CW+0.00CX+0.00CY+0.00CZ
+0.00DV+0.00DW+0.00DX+0.00DY+0.00DZ
+0.00EV+0.39EW+0.71EX+1.04EY+0.00EZ
+0.00FV+0.00FW+0.00FX+0.00FY+0.00FZ
+0.00GV+0.00GW+0.00GX+0.00GY+0.00GZ
+0.00HV+0.00HW+0.00HX+0.00HY+0.00HZ
+0.00IV+0.00IW+0.00IX+0.00IY+0.00IZ
+0.00JV+0.00JW+0.00JX+0.00JY+0.00JZ
+0.00KV+0.00KW+0.00KX+0.00KY+0.00KZ
+0.00LV+0.00LW+0.00LX+0.00LY+0.00LZ<0.710
 0.00AV+0.00AW+0.00AX+0.00AY+0.00AZ
+0.00BV+0.00BW+0.00BX+0.00BY+0.00BZ
+0.00CV+0.00CW+0.00CX+0.00CY+0.00CZ
+0.00DV+0.00DW+0.00DX+0.00DY+0.00DZ
+0.00EV+0.00EW+0.00EX+0.00EY+0.00EZ
+0.07FV+0.39FW+0.00FX+0.00FY+0.00FZ
+0.00GV+0.00GW+0.00GX+0.00GY+0.00GZ
+0.00HV+0.00HW+0.00HX+0.00HY+0.00HZ
+0.00IV+0.00IW+0.00IX+0.00IY+0.00IZ
+0.00JV+0.00JW+0.00JX+0.00JY+0.00JZ
+0.00KV+0.00KW+0.00KX+0.00KY+0.00KZ
+0.00LV+0.00LW+0.00LX+0.00LY+0.00LZ<0.310
 0.00AV+0.00AW+0.00AX+0.00AY+0.00AZ
```

+0.00BV+0.00BW+0.00BX+0.00BY+0.00BZ
+0.00CV+0.00CW+0.00CX+0.00CY+0.00CZ
+0.00DV+0.00DW+0.00DX+0.00DY+0.00DZ
+0.00EV+0.00EW+0.00EX+0.00EY+0.00EZ
+0.00FV+0.00FW+0.00FX+0.00FY+0.00FZ
+0.07GV+0.39GW+0.00GX+0.00GY+0.00GZ
+0.00HV+0.00HW+0.00HX+0.00HY+0.00HZ
+0.00IV+0.00IW+0.00IX+0.00IY+0.00IZ
+0.00JV+0.00JW+0.00JX+0.00JY+0.00JZ
+0.00KV+0.00KW+0.00KX+0.00KY+0.00KZ
+0.00LV+0.00LW+0.00LX+0.00LY+0.00LZ<0.326
0.00AV+0.00AW+0.00AX+0.00AY+0.00AZ
+0.00BV+0.00BW+0.00BX+0.00BY+0.00BZ
+0.00CV+0.00CW+0.00CX+0.00CY+0.00CZ
+0.00DV+0.00DW+0.00DX+0.00DY+0.00DZ
+0.00EV+0.00EW+0.00EX+0.00EY+0.00EZ
+0.00FV+0.00FW+0.00FX+0.00FY+0.00FZ
+0.00GV+0.00GW+0.00GX+0.00GY+0.00GZ
+0.00HV+0.39HW+0.71HX+1.04HY+0.00HZ
+0.00IV+0.00IW+0.00IX+0.00IY+0.00IZ
+0.00JV+0.00JW+0.00JX+0.00JY+0.00JZ
+0.00KV+0.00KW+0.00KX+0.00KY+0.00KZ
+0.00LV+0.00LW+0.00LX+0.00LY+0.00LZ<0.744
0.00AV+0.00AW+0.00AX+0.00AY+0.00AZ
+0.00BV+0.00BW+0.00BX+0.00BY+0.00BZ
+0.00CV+0.00CW+0.00CX+0.00CY+0.00CZ
+0.00DV+0.00DW+0.00DX+0.00DY+0.00DZ
+0.00EV+0.00EW+0.00EX+0.00EY+0.00EZ
+0.00FV+0.00FW+0.00FX+0.00FY+0.00FZ
+0.00GV+0.00GW+0.00GX+0.00GY+0.00GZ
+0.00HV+0.00HW+0.00HX+0.00HY+0.00HZ
+0.00IV+0.39IW+0.71IX+0.00IY+0.00IZ
+0.00JV+0.00JW+0.00JX+0.00JY+0.00JZ
+0.00KV+0.00KW+0.00KX+0.00KY+0.00KZ
+0.00LV+0.00LW+0.00LX+0.00LY+0.00LZ<0.540
0.00AV+0.00AW+0.00AX+0.00AY+0.00AZ
+0.00BV+0.00BW+0.00BX+0.00BY+0.00BZ
+0.00CV+0.00CW+0.00CX+0.00CY+0.00CZ
+0.00DV+0.00DW+0.00DX+0.00DY+0.00DZ
+0.00EV+0.00EW+0.00EX+0.00EY+0.00EZ
+0.00FV+0.00FW+0.00FX+0.00FY+0.00FZ
+0.00GV+0.00GW+0.00GX+0.00GY+0.00GZ
+0.00HV+0.00HW+0.00HX+0.00HY+0.00HZ
+0.00IV+0.00IW+0.00IX+0.00IY+0.00IZ
+0.00JV+0.39JW+0.00JX+0.00JY+0.00JZ
+0.00KV+0.00KW+0.00KX+0.00KY+0.00KZ
+0.00LV+0.00LW+0.00LX+0.00LY+0.00LZ<0.390
0.00AV+0.00AW+0.00AX+0.00AY+0.00AZ
+0.00BV+0.00BW+0.00BX+0.00BY+0.00BZ
+0.00CV+0.00CW+0.00CX+0.00CY+0.00CZ
+0.00DV+0.00DW+0.00DX+0.00DY+0.00DZ
+0.00EV+0.00EW+0.00EX+0.00EY+0.00EZ
+0.00FV+0.00FW+0.00FX+0.00FY+0.00FZ
+0.00GV+0.00GW+0.00GX+0.00GY+0.00GZ
+0.00HV+0.00HW+0.00HX+0.00HY+0.00HZ

```
+0.00IV+0.00IW+0.00IX+0.00IY+0.00IZ
+0.00JV+0.00JW+0.00JX+0.00JY+0.00JZ
+0.07KV+0.39KW+0.71KX+0.00KY+0.00KZ
+0.00LV+0.00LW+0.00LX+0.00LY+0.00LZ<0.070
 0.00AV+0.00AW+0.00AX+0.00AY+0.00AZ
+0.00BV+0.00BW+0.00BX+0.00BY+0.00BZ
+0.00CV+0.00CW+0.00CX+0.00CY+0.00CZ
+0.00DV+0.00DW+0.00DX+0.00DY+0.00DZ
+0.00EV+0.00EW+0.00EX+0.00EY+0.00EZ
+0.00FV+0.00FW+0.00FX+0.00FY+0.00FZ
+0.00GV+0.00GW+0.00GX+0.00GY+0.00GZ
+0.00HV+0.00HW+0.00HX+0.00HY+0.00HZ
+0.00IV+0.00IW+0.00IX+0.00IY+0.00IZ
+0.00JV+0.00JW+0.00JX+0.00JY+0.00JZ
+0.00KV+0.00KW+0.00KX+0.00KY+0.00KZ
+0.00LV+0.39LW+0.71LX+0.00LY+0.00LZ<0.690
  9.733AV+ 23.650AW+  0.000AX+  0.000AY+  4.594AZ
 -0.000BV+  0.000BW+  0.000BX+  0.000BY+  0.000BZ
 -4.593CV- 11.161CW-  0.000CX-  0.000CY-  0.000CZ
 +3.277DV+  7.963DW+  0.000DX+  0.000DY+  0.000DZ
 +0.000EV+  0.000EW+  0.000EX+  0.000EY+  0.000EZ
 -0.908FV-  2.206FW-  0.000FX-  0.000FY-  0.000FZ
 +0.000GV+  0.000GW+  0.000GX+  0.000GY+  0.000GZ
 +0.000HV+  0.000HW+  0.000HX+  0.000HY+  0.000HZ
 +0.000IV+  0.000IW+  0.000IX+  0.000IY+  0.000IZ
 +0.000JV+  0.000JW+  0.000JX+  0.000JY+  0.000JZ
 +0.000KV+  0.000KW+  0.000KX+  0.000KY+  0.000KZ
 +0.000LV+  0.000LW+  0.000LX+  0.000LY+  0.000LZ=0.000
  0.000AV+  0.000AW+  0.000AX+  0.000AY+  0.000AZ
 +0.000BV+  2.206BW+  6.539BX+  0.000BY+  0.000BZ
 +0.000CV+  0.000CW+  0.000CX+  0.000CY+  0.000CZ
 -3.277DV-  7.963DW-  0.000DX-  0.000DY-  0.000DZ
 +0.000EV+  0.404EW+  1.198EX+  4.864EY+  0.000EZ
 +0.000FV+  0.000FW+  0.000FX+  0.000FY+  0.000FZ
 -1.693GV-  4.113GW-  0.000GX-  0.000GY-  0.000GZ
 +0.000HV+  0.000HW+  0.000HX+  0.000HY+  0.000HZ
 +0.000IV+  0.000IW+  0.000IX+  0.000IY+  0.000IZ
 +0.000JV+  0.000JW+  0.000JX+  0.000JY+  0.000JZ
 +0.000KV+  0.000KW+  0.000KX+  0.000KY+  0.000KZ
 +0.000LV+  0.000LW+  0.000LX+  0.000LY+  0.000LZ=0.000
  0.000AV+  0.000AW+  0.000AX+  0.000AY+  0.000AZ
 +0.000BV+  0.000BW+  0.000BX+  0.000BY+  0.000BZ
 +0.000CV+  0.000CW+  0.000CX+  0.000CY+  0.000CZ
 +0.000DV+  0.000DW+  0.000DX+  0.000DY+  0.000DZ
 +0.000EV+  0.000EW+  0.000EX+  0.000EY+  0.000EZ
 +0.908FV+  2.206FW+  0.000FX+  0.000FY+  0.000FZ
  0.000GV+  0.000GW+  0.000GX+  0.000GY+  0.000GZ
 -0.000HV-  1.459HW-  4.326HX- 17.559HY-  0.000HZ
 +0.000IV+  2.206IW+  6.539IX+  0.000IY+  0.000IZ
+ 0.000JV+  0.000JW+  0.000JX+  0.000JY+  0.000JZ
 -0.046KV-  0.112KW-  0.332KX-  0.000KY-  0.000KZ
 +0.000LV+  0.000LW+  0.000LX+  0.000LY+  0.000LZ=0.000
  0.000AV+  0.000AW+  0.000AX+  0.000AY+  0.000AZ
 +0.000BV+  0.000BW+  0.000BX+  0.000BY+  0.000BZ
 +0.000CV+  0.000CW+  0.000CX+  0.000CY+  0.000CZ
```

```
 +0.000DV+  0.000DW+  0.000DX+  0.000DY+  0.000DZ
 +0.000EV+  0.000EW+  0.000EX+  0.000EY+  0.000EZ
 +0.000FV+  0.000FW+  0.000FX+  0.000FY+  0.000FZ
 +1.693GV+  4.113GW+  0.000GX+  0.000GY+  0.000GZ
 +0.000HV+  0.000HW+  0.000HX+  0.000HY+  0.000HZ
 -0.000IV-  2.206IW-  6.539IX-  0.000IY-  0.000IZ
 +0.000JV+  3.092JW+  0.000JX+  0.000JY+  0.000JZ
 +0.000KV+  0.000KW+  0.000KX+  0.000KY+  0.000KZ
 -0.000LV  -0.856LW-  2.539LX-  0.000LY-  0.000LZ=0.000
 1.0AV+1.0AW+0.0AX+0.0AY+1.0AZ
+0.0BV+0.0BW+0.0BX+0.0BY+0.0BZ
+0.0CV+0.0CW+0.0CX+0.0CY+0.0CZ
+0.0DV+0.0DW+0.0DX+0.0DY+0.0DZ
+0.0EV+0.0EW+0.0EX+0.0EY+0.0EZ
+0.0FV+0.0FW+0.0FX+0.0FY+0.0FZ
+0.0GV+0.0GW+0.0GX+0.0GY+0.0GZ
+0.0HV+0.0HW+0.0HX+0.0HY+0.0HZ
+0.0IV+0.0IW+0.0IX+0.0IY+0.0IZ
+0.0JV+0.0JW+0.0JX+0.0JY+0.0JZ
+0.0KV+0.0KW+0.0KX+0.0KY+0.0KZ
+0.0LV+0.0LW+0.0LX+0.0LY+0.0LZ=1.000
 0.0AV+0.0AW+0.0AX+0.0AY+0.0AZ
+0.0BV+1.0BW+1.0BX+0.0BY+0.0BZ
+0.0CV+0.0CW+0.0CX+0.0CY+0.0CZ
+0.0DV+0.0DW+0.0DX+0.0DY+0.0DZ
+0.0EV+0.0EW+0.0EX+0.0EY+0.0EZ
+0.0FV+0.0FW+0.0FX+0.0FY+0.0FZ
+0.0GV+0.0GW+0.0GX+0.0GY+0.0GZ
+0.0HV+0.0HW+0.0HX+0.0HY+0.0HZ
+0.0IV+0.0IW+0.0IX+0.0IY+0.0IZ
+0.0JV+0.0JW+0.0JX+0.0JY+0.0JZ
+0.0KV+0.0KW+0.0KX+0.0KY+0.0KZ
+0.0LV+0.0LW+0.0LX+0.0LY+0.0LZ=1.000
 0.0AV+0.0AW+0.0AX+0.0AY+0.0AZ
+0.0BV+0.0BW+0.0BX+0.0BY+0.0BZ
+1.0CV+1.0CW+0.0CX+0.0CY+0.0CZ
+0.0DV+0.0DW+0.0DX+0.0DY+0.0DZ
+0.0EV+0.0EW+0.0EX+0.0EY+0.0EZ
+0.0FV+0.0FW+0.0FX+0.0FY+0.0FZ
+0.0GV+0.0GW+0.0GX+0.0GY+0.0GZ
+0.0HV+0.0HW+0.0HX+0.0HY+0.0HZ
+0.0IV+0.0IW+0.0IX+0.0IY+0.0IZ
+0.0JV+0.0JW+0.0JX+0.0JY+0.0JZ
+0.0KV+0.0KW+0.0KX+0.0KY+0.0KZ
+0.0LV+0.0LW+0.0LX+0.0LY+0.0LZ=1.000
 0.0AV+0.0AW+0.0AX+0.0AY+0.0AZ
+0.0BV+0.0BW+0.0BX+0.0BY+0.0BZ
+0.0CV+0.0CW+0.0CX+0.0CY+0.0CZ
+1.0DV+1.0DW+0.0DX+0.0DY+0.0DZ
+0.0EV+0.0EW+0.0EX+0.0EY+0.0EZ
+0.0FV+0.0FW+0.0FX+0.0FY+0.0FZ
+0.0GV+0.0GW+0.0GX+0.0GY+0.0GZ
+0.0HV+0.0HW+0.0HX+0.0HY+0.0HZ
+0.0IV+0.0IW+0.0IX+0.0IY+0.0IZ
+0.0JV+0.0JW+0.0JX+0.0JY+0.0JZ
```

```
+0.0KV+0.0KW+0.0KX+0.0KY+0.0KZ
+0.0LV+0.0LW+0.0LX+0.0LY+0.0LZ=1.000
 0.0AV+0.0AW+0.0AX+0.0AY+0.0AZ
+0.0BV+0.0BW+0.0BX+0.0BY+0.0BZ
+0.0CV+0.0CW+0.0CX+0.0CY+0.0CZ
+0.0DV+0.0DW+0.0DX+0.0DY+0.0DZ
+0.0EV+1.0EW+1.0EX+1.0EY+0.0EZ
+0.0FV+0.0FW+0.0FX+0.0FY+0.0FZ
+0.0GV+0.0GW+0.0GX+0.0GY+0.0GZ
+0.0HV+0.0HW+0.0HX+0.0HY+0.0HZ
+0.0IV+0.0IW+0.0IX+0.0IY+0.0IZ
+0.0JV+0.0JW+0.0JX+0.0JY+0.0JZ
+0.0KV+0.0KW+0.0KX+0.0KY+0.0KZ
+0.0LV+0.0LW+0.0LX+0.0LY+0.0LZ=1.000
 0.0AV+0.0AW+0.0AX+0.0AY+0.0AZ
+0.0BV+0.0BW+0.0BX+0.0BY+0.0BZ
+0.0CV+0.0CW+0.0CX+0.0CY+0.0CZ
+0.0DV+0.0DW+0.0DX+0.0DY+0.0DZ
+0.0EV+0.0EW+0.0EX+0.0EY+0.0EZ
+1.0FV+1.0FW+0.0FX+0.0FY+0.0FZ
+0.0GV+0.0GW+0.0GX+0.0GY+0.0GZ
+0.0HV+0.0HW+0.0HX+0.0HY+0.0HZ
+0.0IV+0.0IW+0.0IX+0.0IY+0.0IZ
+0.0JV+0.0JW+0.0JX+0.0JY+0.0JZ
+0.0KV+0.0KW+0.0KX+0.0KY+0.0KZ
+0.0LV+0.0LW+0.0LX+0.0LY+0.0LZ=1.000
 0.0AV+0.0AW+0.0AX+0.0AY+0.0AZ
+0.0BV+0.0BW+0.0BX+0.0BY+0.0BZ
+0.0CV+0.0CW+0.0CX+0.0CY+0.0CZ
+0.0DV+0.0DW+0.0DX+0.0DY+0.0DZ
+0.0EV+0.0EW+0.0EX+0.0EY+0.0EZ
+0.0FV+0.0FW+0.0FX+0.0FY+0.0FZ
+1.0GV+1.0GW+0.0GX+0.0GY+0.0GZ
+0.0HV+0.0HW+0.0HX+0.0HY+0.0HZ
+0.0IV+0.0IW+0.0IX+0.0IY+0.0IZ
+0.0JV+0.0JW+0.0JX+0.0JY+0.0JZ
+0.0KV+0.0KW+0.0KX+0.0KY+0.0KZ
+0.0LV+0.0LW+0.0LX+0.0LY+0.0LZ=1.000
 0.0AV+0.0AW+0.0AX+0.0AY+0.0AZ
+0.0BV+0.0BW+0.0BX+0.0BY+0.0BZ
+0.0CV+0.0CW+0.0CX+0.0CY+0.0CZ
+0.0DV+0.0DW+0.0DX+0.0DY+0.0DZ
+0.0EV+0.0EW+0.0EX+0.0EY+0.0EZ
+0.0FV+0.0FW+0.0FX+0.0FY+0.0FZ
+0.0GV+0.0GW+0.0GX+0.0GY+0.0GZ
+0.0HV+1.0HW+1.0HX+1.0HY+0.0HZ
+0.0IV+0.0IW+0.0IX+0.0IY+0.0IZ
+0.0JV+0.0JW+0.0JX+0.0JY+0.0JZ
+0.0KV+0.0KW+0.0KX+0.0KY+0.0KZ
+0.0LV+0.0LW+0.0LX+0.0LY+0.0LZ=1.000
 0.0AV+0.0AW+0.0AX+0.0AY+0.0AZ
+0.0BV+0.0BW+0.0BX+0.0BY+0.0BZ
+0.0CV+0.0CW+0.0CX+0.0CY+0.0CZ
+0.0DV+0.0DW+0.0DX+0.0DY+0.0DZ
+0.0EV+0.0EW+0.0EX+0.0EY+0.0EZ
```

```
+0.0FV+0.0FW+0.0FX+0.0FY+0.0FZ
+0.0GV+0.0GW+0.0GX+0.0GY+0.0GZ
+0.0HV+0.0HW+0.0HX+0.0HY+0.0HZ
+0.0IV+1.0IW+1.0IX+0.0IY+0.0IZ
+0.0JV+0.0JW+0.0JX+0.0JY+0.0JZ
+0.0KV+0.0KW+0.0KX+0.0KY+0.0KZ
+0.0LV+0.0LW+0.0LX+0.0LY+0.0LZ=1.000
 0.0AV+0.0AW+0.0AX+0.0AY+0.0AZ
+0.0BV+0.0BW+0.0BX+0.0BY+0.0BZ
+0.0CV+0.0CW+0.0CX+0.0CY+0.0CZ
+0.0DV+0.0DW+0.0DX+0.0DY+0.0DZ
+0.0EV+0.0EW+0.0EX+0.0EY+0.0EZ
+0.0FV+0.0FW+0.0FX+0.0FY+0.0FZ
+0.0GV+0.0GW+0.0GX+0.0GY+0.0GZ
+0.0HV+0.0HW+0.0HX+0.0HY+0.0HZ
+0.0IV+0.0IW+0.0IX+0.0IY+0.0IZ
+0.0JV+1.0JW+0.0JX+0.0JY+0.0JZ
+0.0KV+0.0KW+0.0KX+0.0KY+0.0KZ
+0.0LV+0.0LW+0.0LX+0.0LY+0.0LZ=1.000
 0.0AV+0.0AW+0.0AX+0.0AY+0.0AZ
+0.0BV+0.0BW+0.0BX+0.0BY+0.0BZ
+0.0CV+0.0CW+0.0CX+0.0CY+0.0CZ
+0.0DV+0.0DW+0.0DX+0.0DY+0.0DZ
+0.0EV+0.0EW+0.0EX+0.0EY+0.0EZ
+0.0FV+0.0FW+0.0FX+0.0FY+0.0FZ
+0.0GV+0.0GW+0.0GX+0.0GY+0.0GZ
+0.0HV+0.0HW+0.0HX+0.0HY+0.0HZ
+0.0IV+0.0IW+0.0IX+0.0IY+0.0IZ
+0.0JV+0.0JW+0.0JX+0.0JY+0.0JZ
+1.0KV+1.0KW+1.0KX+0.0KY+0.0KZ
+0.0LV+0.0LW+0.0LX+0.0LY+0.0LZ=1.000
 0.0AV+0.0AW+0.0AX+0.0AY+0.0AZ
+0.0BV+0.0BW+0.0BX+0.0BY+0.0BZ
+0.0CV+0.0CW+0.0CX+0.0CY+0.0CZ
+0.0DV+0.0DW+0.0DX+0.0DY+0.0DZ
+0.0EV+0.0EW+0.0EX+0.0EY+0.0EZ
+0.0FV+0.0FW+0.0FX+0.0FY+0.0FZ
+0.0GV+0.0GW+0.0GX+0.0GY+0.0GZ
+0.0HV+0.0HW+0.0HX+0.0HY+0.0HZ
+0.0IV+0.0IW+0.0IX+0.0IY+0.0IZ
+0.0JV+0.0JW+0.0JX+0.0JY+0.0JZ
+0.0KV+0.0KW+0.0KX+0.0KY+0.0KZ
+0.0LV+1.0LW+1.0LX+0.0LY+0.0LZ=1.000
END
LEAVE
```

The following formulations are used in Approach 2. It can be seen from the objective functions that the first model presented here is the uniform weighted model, and that the second is the non-uniform, non-unit weighted model. As before, they have been set up for use by LINDO.

DM represents under-achievement of the goal, and DP the over- achievement.

```
MIN 1.0DM1 +1.0DP1 +1.0DM2 +1.0DP2 +1.0DM3 +1.0DP3
   +1.0DM4 +1.0DP4 +1.0DM5 +1.0DP5 +1.0DM6 +1.0DP6
   +1.0DM7 +1.0DP7 +1.0DM8 +1.0DP8 +1.0DM9 +1.0DP9
   +1.0DM10+1.0DP10+1.0DM11+1.0DP11+1.0DM12+1.0DP12
S.T.  0.07AV+0.39AW+0.00AX+0.00AY+0.05AZ +DM1 -DP1 =0.16404
      0.07BV+0.39BW+0.71BX+0.00BY+0.05BZ +DM2 -DP2 =0.17512
      0.07CV+0.39CW+0.00CX+0.00CY+0.05CZ +DM3 -DP3 =0.16404
      0.07DV+0.39DW+0.00DX+0.00DY+0.05DZ +DM4 -DP4 =0.17512
      0.07EV+0.39EW+0.71EX+0.00EY+0.00EZ +DM5 -DP5 =0.44647
      0.07FV+0.39FW+0.00FX+0.00FY+0.05FZ +DM6 -DP6 =0.29603
      0.07GV+0.39GW+0.00GX+0.00GY+0.05GZ +DM7 -DP7 =0.29603
      0.07HV+0.39HW+0.71HX+1.04HY+0.00HZ +DM8 -DP8 =0.48584
      0.07IV+0.39IW+0.71IX+0.00IY+0.05IZ +DM9 -DP9 =0.29603
      0.07JV+0.39JW+0.71JX+0.00JY+0.05JZ +DM10-DP10=0.44647
      0.07KV+0.39KW+0.71KX+0.00KY+0.00KZ +DM11-DP11=0.62303
      0.07LV+0.39LW+0.71LX+0.00LY+0.00LZ +DM12-DP12=0.52249
      9.733AV + 23.650AW +  0.000AX +  0.000AY +  4.594AZ
     +0.908BV +  2.206BW +  6.539BX +  0.000BY +  0.428BZ
     +0.000CV +  0.000CW +  0.000CX +  0.000CY +  0.000CZ
     +0.000DV +  0.000DW +  0.000DX +  0.000DY +  0.000DZ
     +0.166EV +  0.404EW +  1.198EX +  0.000EY +  0.000EZ
     +0.000FV +  0.000FW +  0.000FX +  0.000FY +  0.000FZ
     +0.000GV +  0.000GW +  0.000GX +  0.000GY +  0.000GZ
     +0.000HV +  0.000HW +  0.000HX +  0.000HY +  0.000HZ
     +0.000IV +  0.000IW +  0.000IX +  0.000IY +  0.000IZ
     +1.270JV +  3.092JW +  9.166JX +  0.000JY +  0.601JZ
     +0.000KV +  0.000KW +  0.000KX +  0.000KY +  0.000KZ
     +0.000LV +  0.000LW +  0.000LX +  0.000LY +  0.000LZ< 25.00
      9.733AV + 23.650AW +  0.000AX +  0.000AY +  4.594AZ
     +0.000BV +  0.000BW +  0.000BX +  0.000BY +  0.000BZ
     +0.000CV +  0.000CW +  0.000CX +  0.000CY +  0.000CZ
     +3.277DV +  7.963DW +  0.000DX +  0.000DY +  1.547DZ
     +0.000EV +  0.000EW +  0.000EX +  0.000EY +  0.000EZ
     +0.000FV +  0.000FW +  0.000FX +  0.000FY +  0.000FZ
     +1.693GV +  4.113GW +  0.000GX +  0.000GY +  0.800GZ
     +0.000HV +  0.000HW +  0.000HX +  0.000HY +  0.000HZ
     +0.000IV +  0.000IW +  0.000IX +  0.000IY +  0.000IZ
     +1.270JV +  3.092JW +  9.166JX +  0.000JY +  0.601JZ
     +0.000KV +  0.000KW +  0.000KX +  0.000KY +  0.000KZ
     +0.000LV +  0.000LW +  0.000LX +  0.000LY +  0.000LZ< 25.00
```

```
 9.733AV + 23.650AW +  0.000AX +  0.000AY +  4.594AZ
+0.000BV +  0.000BW +  0.000BX +  0.000BY +  0.000BZ
+0.000CV +  0.000CW +  0.000CX +  0.000CY +  0.000CZ
+3.277DV +  7.963DW +  0.000DX +  0.000DY +  1.547DZ
+0.000EV +  0.000EW +  0.000EX +  0.000EY +  0.000EZ
+0.000FV +  0.000FW +  0.000FX +  0.000FY +  0.000FZ
+0.000GV +  0.000GW +  0.000GX +  0.000GY +  0.000GZ
+0.000HV +  0.000HW +  0.000HX +  0.000HY +  0.000HZ
+0.908IV +  2.206IW +  6.539IX +  0.000IY +  0.428IZ
+0.000JV +  0.000JW +  0.000JX +  0.000JY +  0.000JZ
+0.000KV +  0.000KW +  0.000KX +  0.000KY +  0.000KZ
+0.352LV +  0.856LW +  2.539LX +  0.000LY +  0.000LZ< 25.00
 0.000AV +  0.000AW +  0.000AX +  0.000AY +  0.000AZ
+0.000BV +  0.000BW +  0.000BX +  0.000BY +  0.000BZ
+4.594CV + 11.161CW +  0.000CX +  0.000CY +  2.168CZ
+0.000DV +  0.000DW +  0.000DX +  0.000DY +  0.000DZ
+0.000EV +  0.000EW +  0.000EX +  0.000EY +  0.000EZ
+0.908FV +  2.206FW +  0.000FX +  0.000FY +  0.428FZ
+1.693GV +  4.113GW +  0.000GX +  0.000GY +  0.800GZ
+0.000HV +  0.000HW +  0.000HX +  0.000HY +  0.000HZ
+0.000IV +  0.000IW +  0.000IX +  0.000IY +  0.000IZ
+1.270JV +  3.092JW +  9.166JX +  0.000JY +  0.601JZ
+0.000KV +  0.000KW +  0.000KX +  0.000KY +  0.000KZ
+0.000LV +  0.000LW +  0.000LX +  0.000LY +  0.000LZ< 25.00
 0.000AV +  0.000AW +  0.000AX +  0.000AY +  0.000AZ
+0.000BV +  0.000BW +  0.000BX +  0.000BY +  0.000BZ
+4.594CV + 11.161CW +  0.000CX +  0.000CY +  2.168CZ
+0.000DV +  0.000DW +  0.000DX +  0.000DY +  0.000DZ
+0.000EV +  0.000EW +  0.000EX +  0.000EY +  0.000EZ
+0.908FV +  2.206FW +  0.000FX +  0.000FY +  0.428FZ
+0.000GV +  0.000GW +  0.000GX +  0.000GY +  0.000GZ
+0.000HV +  0.000HW +  0.000HX +  0.000HY +  0.000HZ
+0.908IV +  2.206IW +  6.539IX +  0.000IY +  0.428IZ
+0.000JV +  0.000JW +  0.000JX +  0.000JY +  0.000JZ
+0.000KV +  0.000KW +  0.000KX +  0.000KY +  0.000KZ
+0.352LV +  0.856LW +  2.539LX +  0.000LY +  0.000LZ< 25.00
 0.000AV +  0.000AW +  0.000AX +  0.000AY +  0.000AZ
+0.000BV +  0.000BW +  0.000BX +  0.000BY +  0.000BZ
+4.594CV + 11.161CW +  0.000CX +  0.000CY +  2.168CZ
+0.000DV +  0.000DW +  0.000DX +  0.000DY +  0.000DZ
+0.000EV +  0.000EW +  0.000EX +  0.000EY +  0.000EZ
+0.000FV +  0.000FW +  0.000FX +  0.000FY +  0.000FZ
+0.000GV +  0.000GW +  0.000GX +  0.000GY +  0.000GZ
+0.600HV +  1.459HW +  4.326HX + 17.559HY +  0.000HZ
+0.000IV +  0.000IW +  0.000IX +  0.000IY +  0.000IZ
+0.000JV +  0.000JW +  0.000JX +  0.000JY +  0.000JZ
+0.046KV +  0.112KW +  0.332KX +  0.000KY +  0.000KZ
+0.352LV +  0.856LW +  2.539LX +  0.000LY +  0.000LZ< 25.00
 9.733AV+ 23.650AW+  0.000AX+  0.000AY+  4.594AZ
+0.000BV+  0.000BW+  0.000BX+  0.000BY+  0.000BZ
-4.594CV- 11.161CW-  0.000CX-  0.000CY-  2.168CZ
+3.277DV+  7.963DW+  0.000DX+  0.000DY+  1.547DZ
+0.000EV+  0.000EW+  0.000EX+  0.000EY+  0.000EZ
-0.908FV-  2.206FW-  0.000FX-  0.000FY-  0.428FZ
+0.000GV+  0.000GW+  0.000GX+  0.000GY+  0.000GZ
```

```
+0.000HV+  0.000HW+  0.000HX+  0.000HY+  0.000HZ
+0.000IV+  0.000IW+  0.000IX+  0.000IY+  0.000IZ
+0.000JV+  0.000JW+  0.000JX+  0.000JY+  0.000JZ
+0.000KV+  0.000KW+  0.000KX+  0.000KY+  0.000KZ
+0.000LV+  0.000LW+  0.000LX+  0.000LY+  0.000LZ=0.000
 0.000AV+  0.000AW+  0.000AX+  0.000AY+  0.000AZ
+0.908BV+  2.206BW+  6.539BX+  0.000BY+  0.428BZ
+0.000CV+  0.000CW+  0.000CX+  0.000CY+  0.000CZ
-3.277DV-  7.963DW-  0.000DX-  0.000DY-  1.547DZ
+0.166EV+  0.404EW+  1.198EX+  0.000EY+  0.000EZ
+0.000FV+  0.000FW+  0.000FX+  0.000FY+  0.000FZ
-1.693GV-  4.113GW-  0.000GX-  0.000GY-  0.800GZ
+0.000HV+  0.000HW+  0.000HX+  0.000HY+  0.000HZ
+0.000IV+  0.000IW+  0.000IX+  0.000IY+  0.000IZ
+0.000JV+  0.000JW+  0.000JX+  0.000JY+  0.000JZ
+0.000KV+  0.000KW+  0.000KX+  0.000KY+  0.000KZ
+0.000LV+  0.000LW+  0.000LX+  0.000LY+  0.000LZ=0.000
 0.000AV+  0.000AW+  0.000AX+  0.000AY+  0.000AZ
+0.000BV+  0.000BW+  0.000BX+  0.000BY+  0.000BZ
+0.000CV+  0.000CW+  0.000CX+  0.000CY+  0.000CZ
+0.000DV+  0.000DW+  0.000DX+  0.000DY+  0.000DZ
+0.000EV+  0.000EW+  0.000EX+  0.000EY+  0.000EZ
+0.908FV+  2.206FW+  0.000FX+  0.000FY+  0.428FZ
+0.000GV+  0.000GW+  0.000GX+  0.000GY+  0.000GZ
-0.600HV-  1.459HW-  4.326HX- 17.559HY-  0.000HZ
+0.908IV+  2.206IW+  6.539IX+  0.000IY+  0.428IZ
+0.000JV+  0.000JW+  0.000JX+  0.000JY+  0.000JZ
-0.046KV-  0.112KW-  0.332KX-  0.000KY-  0.000KZ
+0.000LV+  0.000LW+  0.000LX+  0.000LY+  0.000LZ=0.000
 0.000AV+  0.000AW+  0.000AX+  0.000AY+  0.000AZ
+0.000BV+  0.000BW+  0.000BX+  0.000BY+  0.000BZ
+0.000CV+  0.000CW+  0.000CX+  0.000CY+  0.000CZ
+0.000DV+  0.000DW+  0.000DX+  0.000DY+  0.000DZ
+0.000EV+  0.000EW+  0.000EX+  0.000EY+  0.000EZ
+0.000FV+  0.000FW+  0.000FX+  0.000FY+  0.000FZ
+1.693GV+  4.113GW+  0.000GX+  0.000GY+  0.800GZ
+0.000HV+  0.000HW+  0.000HX+  0.000HY+  0.000HZ
-0.908IV-  2.206IW-  6.539IX-  0.000IY-  0.428IZ
+1.270JV+  3.092JW+  9.166JX+  0.000JY+  0.601JZ
+0.000KV+  0.000KW+  0.000KX+  0.000KY+  0.000KZ
-0.352LV  -0.856LW-  2.539LX-  0.000LY-  0.000LZ=0.000
 1.0AV+1.0AW+0.0AX+0.0AY+1.0AZ
+0.0BV+0.0BW+0.0BX+0.0BY+0.0BZ
+0.0CV+0.0CW+0.0CX+0.0CY+0.0CZ
+0.0DV+0.0DW+0.0DX+0.0DY+0.0DZ
+0.0EV+0.0EW+0.0EX+0.0EY+0.0EZ
+0.0FV+0.0FW+0.0FX+0.0FY+0.0FZ
+0.0GV+0.0GW+0.0GX+0.0GY+0.0GZ
+0.0HV+0.0HW+0.0HX+0.0HY+0.0HZ
+0.0IV+0.0IW+0.0IX+0.0IY+0.0IZ
+0.0JV+0.0JW+0.0JX+0.0JY+0.0JZ
+0.0KV+0.0KW+0.0KX+0.0KY+0.0KZ
+0.0LV+0.0LW+0.0LX+0.0LY+0.0LZ=1.000
 0.0AV+0.0AW+0.0AX+0.0AY+0.0AZ
+1.0BV+1.0BW+1.0BX+0.0BY+1.0BZ
```

```
+0.0CV+0.0CW+0.0CX+0.0CY+0.0CZ
+0.0DV+0.0DW+0.0DX+0.0DY+0.0DZ
+0.0EV+0.0EW+0.0EX+0.0EY+0.0EZ
+0.0FV+0.0FW+0.0FX+0.0FY+0.0FZ
+0.0GV+0.0GW+0.0GX+0.0GY+0.0GZ
+0.0HV+0.0HW+0.0HX+0.0HY+0.0HZ
+0.0IV+0.0IW+0.0IX+0.0IY+0.0IZ
+0.0JV+0.0JW+0.0JX+0.0JY+0.0JZ
+0.0KV+0.0KW+0.0KX+0.0KY+0.0KZ
+0.0LV+0.0LW+0.0LX+0.0LY+0.0LZ=1.000
 0.0AV+0.0AW+0.0AX+0.0AY+0.0AZ
+0.0BV+0.0BW+0.0BX+0.0BY+0.0BZ
+1.0CV+1.0CW+0.0CX+0.0CY+1.0CZ
+0.0DV+0.0DW+0.0DX+0.0DY+0.0DZ
+0.0EV+0.0EW+0.0EX+0.0EY+0.0EZ
+0.0FV+0.0FW+0.0FX+0.0FY+0.0FZ
+0.0GV+0.0GW+0.0GX+0.0GY+0.0GZ
+0.0HV+0.0HW+0.0HX+0.0HY+0.0HZ
+0.0IV+0.0IW+0.0IX+0.0IY+0.0IZ
+0.0JV+0.0JW+0.0JX+0.0JY+0.0JZ
+0.0KV+0.0KW+0.0KX+0.0KY+0.0KZ
+0.0LV+0.0LW+0.0LX+0.0LY+0.0LZ=1.000
 0.0AV+0.0AW+0.0AX+0.0AY+0.0AZ
+0.0BV+0.0BW+0.0BX+0.0BY+0.0BZ
+0.0CV+0.0CW+0.0CX+0.0CY+0.0CZ
+1.0DV+1.0DW+0.0DX+0.0DY+1.0DZ
+0.0EV+0.0EW+0.0EX+0.0EY+0.0EZ
+0.0FV+0.0FW+0.0FX+0.0FY+0.0FZ
+0.0GV+0.0GW+0.0GX+0.0GY+0.0GZ
+0.0HV+0.0HW+0.0HX+0.0HY+0.0HZ
+0.0IV+0.0IW+0.0IX+0.0IY+0.0IZ
+0.0JV+0.0JW+0.0JX+0.0JY+0.0JZ
+0.0KV+0.0KW+0.0KX+0.0KY+0.0KZ
+0.0LV+0.0LW+0.0LX+0.0LY+0.0LZ=1.000
 0.0AV+0.0AW+0.0AX+0.0AY+0.0AZ
+0.0BV+0.0BW+0.0BX+0.0BY+0.0BZ
+0.0CV+0.0CW+0.0CX+0.0CY+0.0CZ
+0.0DV+0.0DW+0.0DX+0.0DY+0.0DZ
+1.0EV+1.0EW+1.0EX+0.0EY+0.0EZ
+0.0FV+0.0FW+0.0FX+0.0FY+0.0FZ
+0.0GV+0.0GW+0.0GX+0.0GY+0.0GZ
+0.0HV+0.0HW+0.0HX+0.0HY+0.0HZ
+0.0IV+0.0IW+0.0IX+0.0IY+0.0IZ
+0.0JV+0.0JW+0.0JX+0.0JY+0.0JZ
+0.0KV+0.0KW+0.0KX+0.0KY+0.0KZ
+0.0LV+0.0LW+0.0LX+0.0LY+0.0LZ=1.000
 0.0AV+0.0AW+0.0AX+0.0AY+0.0AZ
+0.0BV+0.0BW+0.0BX+0.0BY+0.0BZ
+0.0CV+0.0CW+0.0CX+0.0CY+0.0CZ
+0.0DV+0.0DW+0.0DX+0.0DY+0.0DZ
+0.0EV+0.0EW+0.0EX+0.0EY+0.0EZ
+1.0FV+1.0FW+0.0FX+0.0FY+1.0FZ
+0.0GV+0.0GW+0.0GX+0.0GY+0.0GZ
+0.0HV+0.0HW+0.0HX+0.0HY+0.0HZ
+0.0IV+0.0IW+0.0IX+0.0IY+0.0IZ
```

```
+0.0JV+0.0JW+0.0JX+0.0JY+0.0JZ
+0.0KV+0.0KW+0.0KX+0.0KY+0.0KZ
+0.0LV+0.0LW+0.0LX+0.0LY+0.0LZ=1.000
 0.0AV+0.0AW+0.0AX+0.0AY+0.0AZ
+0.0BV+0.0BW+0.0BX+0.0BY+0.0BZ
+0.0CV+0.0CW+0.0CX+0.0CY+0.0CZ
+0.0DV+0.0DW+0.0DX+0.0DY+0.0DZ
+0.0EV+0.0EW+0.0EX+0.0EY+0.0EZ
+0.0FV+0.0FW+0.0FX+0.0FY+0.0FZ
+1.0GV+1.0GW+0.0GX+0.0GY+1.0GZ
+0.0HV+0.0HW+0.0HX+0.0HY+0.0HZ
+0.0IV+0.0IW+0.0IX+0.0IY+0.0IZ
+0.0JV+0.0JW+0.0JX+0.0JY+0.0JZ
+0.0KV+0.0KW+0.0KX+0.0KY+0.0KZ
+0.0LV+0.0LW+0.0LX+0.0LY+0.0LZ=1.000
 0.0AV+0.0AW+0.0AX+0.0AY+0.0AZ
+0.0BV+0.0BW+0.0BX+0.0BY+0.0BZ
+0.0CV+0.0CW+0.0CX+0.0CY+0.0CZ
+0.0DV+0.0DW+0.0DX+0.0DY+0.0DZ
+0.0EV+0.0EW+0.0EX+0.0EY+0.0EZ
+0.0FV+0.0FW+0.0FX+0.0FY+0.0FZ
+0.0GV+0.0GW+0.0GX+0.0GY+0.0GZ
+1.0HV+1.0HW+1.0HX+1.0HY+0.0HZ
+0.0IV+0.0IW+0.0IX+0.0IY+0.0IZ
+0.0JV+0.0JW+0.0JX+0.0JY+0.0JZ
+0.0KV+0.0KW+0.0KX+0.0KY+0.0KZ
+0.0LV+0.0LW+0.0LX+0.0LY+0.0LZ=1.000
 0.0AV+0.0AW+0.0AX+0.0AY+0.0AZ
+0.0BV+0.0BW+0.0BX+0.0BY+0.0BZ
+0.0CV+0.0CW+0.0CX+0.0CY+0.0CZ
+0.0DV+0.0DW+0.0DX+0.0DY+0.0DZ
+0.0EV+0.0EW+0.0EX+0.0EY+0.0EZ
+0.0FV+0.0FW+0.0FX+0.0FY+0.0FZ
+0.0GV+0.0GW+0.0GX+0.0GY+0.0GZ
+0.0HV+0.0HW+0.0HX+0.0HY+0.0HZ
+1.0IV+1.0IW+1.0IX+0.0IY+1.0IZ
+0.0JV+0.0JW+0.0JX+0.0JY+0.0JZ
+0.0KV+0.0KW+0.0KX+0.0KY+0.0KZ
+0.0LV+0.0LW+0.0LX+0.0LY+0.0LZ=1.000
 0.0AV+0.0AW+0.0AX+0.0AY+0.0AZ
+0.0BV+0.0BW+0.0BX+0.0BY+0.0BZ
+0.0CV+0.0CW+0.0CX+0.0CY+0.0CZ
+0.0DV+0.0DW+0.0DX+0.0DY+0.0DZ
+0.0EV+0.0EW+0.0EX+0.0EY+0.0EZ
+0.0FV+0.0FW+0.0FX+0.0FY+0.0FZ
+0.0GV+0.0GW+0.0GX+0.0GY+0.0GZ
+0.0HV+0.0HW+0.0HX+0.0HY+0.0HZ
+0.0IV+0.0IW+0.0IX+0.0IY+0.0IZ
+1.0JV+1.0JW+1.0JX+0.0JY+1.0JZ
+0.0KV+0.0KW+0.0KX+0.0KY+0.0KZ
+0.0LV+0.0LW+0.0LX+0.0LY+0.0LZ=1.000
 0.0AV+0.0AW+0.0AX+0.0AY+0.0AZ
+0.0BV+0.0BW+0.0BX+0.0BY+0.0BZ
+0.0CV+0.0CW+0.0CX+0.0CY+0.0CZ
+0.0DV+0.0DW+0.0DX+0.0DY+0.0DZ
```

```
+0.0EV+0.0EW+0.0EX+0.0EY+0.0EZ
+0.0FV+0.0FW+0.0FX+0.0FY+0.0FZ
+0.0GV+0.0GW+0.0GX+0.0GY+0.0GZ
+0.0HV+0.0HW+0.0HX+0.0HY+0.0HZ
+0.0IV+0.0IW+0.0IX+0.0IY+0.0IZ
+0.0JV+0.0JW+0.0JX+0.0JY+0.0JZ
+1.0KV+1.0KW+1.0KX+0.0KY+0.0KZ
+0.0LV+0.0LW+0.0LX+0.0LY+0.0LZ=1.000
 0.0AV+0.0AW+0.0AX+0.0AY+0.0AZ
+0.0BV+0.0BW+0.0BX+0.0BY+0.0BZ
+0.0CV+0.0CW+0.0CX+0.0CY+0.0CZ
+0.0DV+0.0DW+0.0DX+0.0DY+0.0DZ
+0.0EV+0.0EW+0.0EX+0.0EY+0.0EZ
+0.0FV+0.0FW+0.0FX+0.0FY+0.0FZ
+0.0GV+0.0GW+0.0GX+0.0GY+0.0GZ
+0.0HV+0.0HW+0.0HX+0.0HY+0.0HZ
+0.0IV+0.0IW+0.0IX+0.0IY+0.0IZ
+0.0JV+0.0JW+0.0JX+0.0JY+0.0JZ
+0.0KV+0.0KW+0.0KX+0.0KY+0.0KZ
+1.0LV+1.0LW+1.0LX+0.0LY+0.0LZ=1.000
 69.20AV+43.20AW+ 0.00AX+ 0.00AY+98.60AZ
+69.20BV+43.20BW+24.10BX+ 0.00BY+98.60BZ
+69.20CV+43.20CW+ 0.00CX+ 0.00CY+98.60CZ
+69.20DV+43.20DW+ 0.00DX+ 0.00DY+98.60DZ
+69.20EV+43.20EW+24.10EX+ 0.00EY+ 0.00EZ
+69.20FV+43.20FW+ 0.00FX+ 0.00FY+98.60FZ
+69.20GV+43.20GW+ 0.00GX+ 0.00GY+98.60GZ
+69.20HV+43.20HW+24.10HX+16.90HY+ 0.00HZ
+69.20IV+43.20IW+24.10IX+ 0.00IY+98.60IZ
+69.20JV+43.20JW+ 0.00JX+ 0.00JY+98.60JZ
+69.20KV+43.20KW+24.10KX+ 0.00KY+ 0.00KZ
+69.20LV+43.20LW+24.10LX+ 0.00LY+ 0.00LZ<=474.657
END
LEAVE
```

```
MIN 0.68DM1 +0.68DP1+0.15DM2+0.15DP2+0.32DM3+0.32DP3
   +0.53DM4+0.53DP4+0.15DM5+0.15DP5+0.26DM6+0.26DP6
   +0.52DM7+0.52DP7+0.06DM8+0.06DP8+0.27DM9+0.27DP9
   +0.67DM10+0.67DP10+0.06DM11+0.06DP11+0.33DM12+0.33DP12
S.T.  0.07AV+0.00AW+0.00AX+0.00AY+0.05AZ +DM1-DP1=0.16380
      0.07BV+0.39BW+0.71BX+0.00BY+0.05BZ +DM2-DP2=0.17705
      0.07CV+0.39CW+0.00CX+0.00CY+0.05CZ +DM3-DP3=0.16380
      0.07DV+0.39DW+0.00DX+0.00DY+0.05DZ +DM4-DP4=0.17705
      0.07EV+0.39EW+0.71EX+0.00EY+0.00EZ +DM5-DP5=0.44859
      0.07FV+0.39FW+0.00FX+0.00FY+0.05FZ +DM6-DP6=0.29851
      0.07GV+0.39GW+0.00GX+0.00GY+0.05GZ +DM7-DP7=0.29851
      0.00HV+0.39HW+0.71HX+1.04HY+1.00HZ +DM8-DP8=0.48584
      0.00IV+0.39IW+0.71IX+0.00IY+0.00IZ +DM9-DP9=0.29851
      0.07JV+0.39JW+0.71JX+0.00JY+0.00JZ +DM10-DP10=0.44859
      0.07KV+0.39KW+0.71KX+0.00KY+0.00KZ +DM11-DP11=0.64192
      0.07LV+0.39LW+0.71LX+0.00LY+0.00LZ +DM12-DP12=0.52249
      9.733AV +  0.000AW +  0.000AX +  0.000AY +  4.594AZ
     +0.908BV +  2.206BW +  6.539BX +  0.000BY +  0.428BZ
     +0.000CV +  0.000CW +  0.000CX +  0.000CY +  0.000CZ
     +0.000DV +  0.000DW +  0.000DX +  0.000DY +  0.000DZ
     +0.166EV +  0.404EW +  1.198EX +  0.000EY +  0.000EZ
     +0.000FV +  0.000FW +  0.000FX +  0.000FY +  0.000FZ
     +0.000GV +  0.000GW +  0.000GX +  0.000GY +  0.000GZ
     +0.000HV +  0.000HW +  0.000HX +  0.000HY +  0.000HZ
     +0.000IV +  0.000IW +  0.000IX +  0.000IY +  0.000IZ
     +1.270JV +  3.092JW +  9.166JX +  0.000JY +  0.000JZ
     +0.000KV +  0.000KW +  0.000KX +  0.000KY +  0.000KZ
     +0.000LV +  0.000LW +  0.000LX +  0.000LY +  0.000LZ< 25.00
      9.733AV +  0.000AW +  0.000AX +  0.000AY +  4.594AZ
     +0.000BV +  0.000BW +  0.000BX +  0.000BY +  0.000BZ
     +0.000CV +  0.000CW +  0.000CX +  0.000CY +  0.000CZ
     +3.277DV +  7.963DW +  0.000DX +  0.000DY +  1.547DZ
     +0.000EV +  0.000EW +  0.000EX +  0.000EY +  0.000EZ
     +0.000FV +  0.000FW +  0.000FX +  0.000FY +  0.000FZ
     +1.693GV +  4.113GW +  0.000GX +  0.000GY +  0.800GZ
     +0.000HV +  0.000HW +  0.000HX +  0.000HY +  0.000HZ
     +0.000IV +  0.000IW +  0.000IX +  0.000IY +  0.000IZ
     +1.270JV +  3.092JW +  9.166JX +  0.000JY +  0.000JZ
     +0.000KV +  0.000KW +  0.000KX +  0.000KY +  0.000KZ
     +0.000LV +  0.000LW +  0.000LX +  0.000LY +  0.000LZ< 25.00
      9.733AV +  0.000AW +  0.000AX +  0.000AY +  4.594AZ
     +0.000BV +  0.000BW +  0.000BX +  0.000BY +  0.000BZ
     +0.000CV +  0.000CW +  0.000CX +  0.000CY +  0.000CZ
     +3.277DV +  7.963DW +  0.000DX +  0.000DY +  1.547DZ
     +0.000EV +  0.000EW +  0.000EX +  0.000EY +  0.000EZ
     +0.000FV +  0.000FW +  0.000FX +  0.000FY +  0.000FZ
     +0.000GV +  0.000GW +  0.000GX +  0.000GY +  0.000GZ
     +0.000HV +  0.000HW +  0.000HX +  0.000HY +  0.000HZ
     +0.000IV +  2.206IW +  6.539IX +  0.000IY +  0.000IZ
     +0.000JV +  0.000JW +  0.000JX +  0.000JY +  0.000JZ
     +0.000KV +  0.000KW +  0.000KX +  0.000KY +  0.000KZ
     +0.352LV +  0.856LW +  2.539LX +  0.000LY +  0.000LZ< 25.00
      0.000AV +  0.000AW +  0.000AX +  0.000AY +  0.000AZ
     +0.000BV +  0.000BW +  0.000BX +  0.000BY +  0.000BZ
     +4.593CV + 11.161CW +  0.000CX +  0.000CY +  2.168CZ
```

+0.000DV + 0.000DW + 0.000DX + 0.000DY + 0.000DZ
+0.000EV + 0.000EW + 0.000EX + 0.000EY + 0.000EZ
+0.908FV + 2.206FW + 0.000FX + 0.000FY + 0.428FZ
+1.693GV + 4.113GW + 0.000GX + 0.000GY + 0.800GZ
+0.000HV + 0.000HW + 0.000HX + 0.000HY + 0.000HZ
+0.000IV + 0.000IW + 0.000IX + 0.000IY + 0.000IZ
+1.270JV + 3.092JW + 9.166JX + 0.000JY + 0.000JZ
+0.000KV + 0.000KW + 0.000KX + 0.000KY + 0.000KZ
+0.000LV + 0.000LW + 0.000LX + 0.000LY + 0.000LZ< 25.00
0.000AV + 0.000AW + 0.000AX + 0.000AY + 0.000AZ
+0.000BV + 0.000BW + 0.000BX + 0.000BY + 0.000BZ
+4.593CV + 11.161CW + 0.000CX + 0.000CY + 2.168CZ
+0.000DV + 0.000DW + 0.000DX + 0.000DY + 0.000DZ
+0.000EV + 0.000EW + 0.000EX + 0.000EY + 0.000EZ
+0.908FV + 2.206FW + 0.000FX + 0.000FY + 0.428FZ
+0.000GV + 0.000GW + 0.000GX + 0.000GY + 0.000GZ
+0.000HV + 0.000HW + 0.000HX + 0.000HY + 0.000HZ
+0.000IV + 2.206IW + 6.539IX + 0.000IY + 0.000IZ
+0.000JV + 0.000JW + 0.000JX + 0.000JY + 0.000JZ
+0.000KV + 0.000KW + 0.000KX + 0.000KY + 0.000KZ
+0.352LV + 0.856LW + 2.539LX + 0.000LY + 0.000LZ< 25.00
0.000AV + 0.000AW + 0.000AX + 0.000AY + 0.000AZ
+0.000BV + 0.000BW + 0.000BX + 0.000BY + 0.000BZ
+4.593CV + 11.161CW + 0.000CX + 0.000CY + 2.168CZ
+0.000DV + 0.000DW + 0.000DX + 0.000DY + 0.000DZ
+0.000EV + 0.000EW + 0.000EX + 0.000EY + 0.000EZ
+0.000FV + 0.000FW + 0.000FX + 0.000FY + 0.000FZ
+0.000GV + 0.000GW + 0.000GX + 0.000GY + 0.000GZ
+0.000HV + 1.459HW + 4.326HX + 17.559HY + 0.000HZ
+0.000IV + 0.000IW + 0.000IX + 0.000IY + 0.000IZ
+0.000JV + 0.000JW + 0.000JX + 0.000JY + 0.000JZ
+0.046KV + 0.112KW + 0.332KX + 0.000KY + 0.000KZ
+0.352LV + 0.856LW + 2.539LX + 0.000LY + 0.000LZ< 25.00
9.733AV+ 0.000AW+ 0.000AX+ 0.000AY+ 4.594AZ
+0.000BV+ 0.000BW+ 0.000BX+ 0.000BY+ 0.000BZ
-4.593CV- 11.161CW- 0.000CX- 0.000CY- 2.168CZ
+3.277DV+ 7.963DW+ 0.000DX+ 0.000DY+ 1.547DZ
+0.000EV+ 0.000EW+ 0.000EX+ 0.000EY+ 0.000EZ
-0.908FV- 2.206FW- 0.000FX- 0.000FY- 0.428FZ
+0.000GV+ 0.000GW+ 0.000GX+ 0.000GY+ 0.000GZ
+0.000HV+ 0.000HW+ 0.000HX+ 0.000HY+ 0.000HZ
+0.000IV+ 0.000IW+ 0.000IX+ 0.000IY+ 0.000IZ
+0.000JV+ 0.000JW+ 0.000JX+ 0.000JY+ 0.000JZ
+0.000KV+ 0.000KW+ 0.000KX+ 0.000KY+ 0.000KZ
+0.000LV+ 0.000LW+ 0.000LX+ 0.000LY+ 0.000LZ=0.000
0.000AV+ 0.000AW+ 0.000AX+ 0.000AY+ 0.000AZ
+0.908BV+ 2.206BW+ 6.539BX+ 0.000BY+ 0.428BZ
+0.000CV+ 0.000CW+ 0.000CX+ 0.000CY+ 0.000CZ
-3.277DV- 7.963DW- 0.000DX- 0.000DY- 1.547DZ
+0.166EV+ 0.404EW+ 1.198EX+ 0.000EY+ 0.000EZ
+0.000FV+ 0.000FW+ 0.000FX+ 0.000FY+ 0.000FZ
-1.693GV- 4.113GW- 0.000GX- 0.000GY- 0.800GZ
+0.000HV+ 0.000HW+ 0.000HX+ 0.000HY+ 0.000HZ
+0.000IV+ 0.000IW+ 0.000IX+ 0.000IY+ 0.000IZ
+0.000JV+ 0.000JW+ 0.000JX+ 0.000JY+ 0.000JZ

```
+0.000KV+  0.000KW+  0.000KX+  0.000KY+  0.000KZ
+0.000LV+  0.000LW+  0.000LX+  0.000LY+  0.000LZ=0.000
 0.000AV+  0.000AW+  0.000AX+  0.000AY+  0.000AZ
+0.000BV+  0.000BW+  0.000BX+  0.000BY+  0.000BZ
+0.000CV+  0.000CW+  0.000CX+  0.000CY+  0.000CZ
+0.000DV+  0.000DW+  0.000DX+  0.000DY+  0.000DZ
+0.000EV+  0.000EW+  0.000EX+  0.000EY+  0.000EZ
+0.908FV+  2.206FW+  0.000FX+  0.000FY+  0.428FZ
+0.000GV+  0.000GW+  0.000GX+  0.000GY+  0.000GZ
-0.000HV-  1.459HW-  4.326HX- 17.559HY-  0.000HZ
+0.000IV+  2.206IW+  6.539IX+  0.000IY+  0.000IZ
+0.000JV+  0.000JW+  0.000JX+  0.000JY+  0.000JZ
-0.046KV-  0.112KW-  0.332KX-  0.000KY-  0.000KZ
+0.000LV+  0.000LW+  0.000LX+  0.000LY+  0.000LZ=0.000
 0.000AV+  0.000AW+  0.000AX+  0.000AY+  0.000AZ
+0.000BV+  0.000BW+  0.000BX+  0.000BY+  0.000BZ
+0.000CV+  0.000CW+  0.000CX+  0.000CY+  0.000CZ
+0.000DV+  0.000DW+  0.000DX+  0.000DY+  0.000DZ
+0.000EV+  0.000EW+  0.000EX+  0.000EY+  0.000EZ
+0.000FV+  0.000FW+  0.000FX+  0.000FY+  0.000FZ
+1.693GV+  4.113GW+  0.000GX+  0.000GY+  0.800GZ
+0.000HV+  0.000HW+  0.000HX+  0.000HY+  0.000HZ
-0.000IV-  2.206IW-  6.539IX-  0.000IY-  0.000IZ
+1.270JV+  3.092JW+  9.166JX+  0.000JY+  0.000JZ
+0.000KV+  0.000KW+  0.000KX+  0.000KY+  0.000KZ
-0.352LV  -0.856LW-  2.539LX-  0.000LY-  0.000LZ=0.000
 1.0AV+0.0AW+0.0AX+0.0AY+1.0AZ
+0.0BV+0.0BW+0.0BX+0.0BY+0.0BZ
+0.0CV+0.0CW+0.0CX+0.0CY+0.0CZ
+0.0DV+0.0DW+0.0DX+0.0DY+0.0DZ
+0.0EV+0.0EW+0.0EX+0.0EY+0.0EZ
+0.0FV+0.0FW+0.0FX+0.0FY+0.0FZ
+0.0GV+0.0GW+0.0GX+0.0GY+0.0GZ
+0.0HV+0.0HW+0.0HX+0.0HY+0.0HZ
+0.0IV+0.0IW+0.0IX+0.0IY+0.0IZ
+0.0JV+0.0JW+0.0JX+0.0JY+0.0JZ
+0.0KV+0.0KW+0.0KX+0.0KY+0.0KZ
+0.0LV+0.0LW+0.0LX+0.0LY+0.0LZ=1.000
 0.0AV+0.0AW+0.0AX+0.0AY+0.0AZ
+1.0BV+1.0BW+1.0BX+0.0BY+1.0BZ
+0.0CV+0.0CW+0.0CX+0.0CY+0.0CZ
+0.0DV+0.0DW+0.0DX+0.0DY+0.0DZ
+0.0EV+0.0EW+0.0EX+0.0EY+0.0EZ
+0.0FV+0.0FW+0.0FX+0.0FY+0.0FZ
+0.0GV+0.0GW+0.0GX+0.0GY+0.0GZ
+0.0HV+0.0HW+0.0HX+0.0HY+0.0HZ
+0.0IV+0.0IW+0.0IX+0.0IY+0.0IZ
+0.0JV+0.0JW+0.0JX+0.0JY+0.0JZ
+0.0KV+0.0KW+0.0KX+0.0KY+0.0KZ
+0.0LV+0.0LW+0.0LX+0.0LY+0.0LZ=1.000
 0.0AV+0.0AW+0.0AX+0.0AY+0.0AZ
+0.0BV+0.0BW+0.0BX+0.0BY+0.0BZ
+1.0CV+1.0CW+0.0CX+0.0CY+1.0CZ
+0.0DV+0.0DW+0.0DX+0.0DY+0.0DZ
+0.0EV+0.0EW+0.0EX+0.0EY+0.0EZ
```

+0.0FV+0.0FW+0.0FX+0.0FY+0.0FZ
+0.0GV+0.0GW+0.0GX+0.0GY+0.0GZ
+0.0HV+0.0HW+0.0HX+0.0HY+0.0HZ
+0.0IV+0.0IW+0.0IX+0.0IY+0.0IZ
+0.0JV+0.0JW+0.0JX+0.0JY+0.0JZ
+0.0KV+0.0KW+0.0KX+0.0KY+0.0KZ
+0.0LV+0.0LW+0.0LX+0.0LY+0.0LZ=1.000
0.0AV+0.0AW+0.0AX+0.0AY+0.0AZ
+0.0BV+0.0BW+0.0BX+0.0BY+0.0BZ
+0.0CV+0.0CW+0.0CX+0.0CY+0.0CZ
+1.0DV+1.0DW+0.0DX+0.0DY+1.0DZ
+0.0EV+0.0EW+0.0EX+0.0EY+0.0EZ
+0.0FV+0.0FW+0.0FX+0.0FY+0.0FZ
+0.0GV+0.0GW+0.0GX+0.0GY+0.0GZ
+0.0HV+0.0HW+0.0HX+0.0HY+0.0HZ
+0.0IV+0.0IW+0.0IX+0.0IY+0.0IZ
+0.0JV+0.0JW+0.0JX+0.0JY+0.0JZ
+0.0KV+0.0KW+0.0KX+0.0KY+0.0KZ
+0.0LV+0.0LW+0.0LX+0.0LY+0.0LZ=1.000
0.0AV+0.0AW+0.0AX+0.0AY+0.0AZ
+0.0BV+0.0BW+0.0BX+0.0BY+0.0BZ
+0.0CV+0.0CW+0.0CX+0.0CY+0.0CZ
+0.0DV+0.0DW+0.0DX+0.0DY+0.0DZ
+1.0EV+1.0EW+1.0EX+0.0EY+0.0EZ
+0.0FV+0.0FW+0.0FX+0.0FY+0.0FZ
+0.0GV+0.0GW+0.0GX+0.0GY+0.0GZ
+0.0HV+0.0HW+0.0HX+0.0HY+0.0HZ
+0.0IV+0.0IW+0.0IX+0.0IY+0.0IZ
+0.0JV+0.0JW+0.0JX+0.0JY+0.0JZ
+0.0KV+0.0KW+0.0KX+0.0KY+0.0KZ
+0.0LV+0.0LW+0.0LX+0.0LY+0.0LZ=1.000
0.0AV+0.0AW+0.0AX+0.0AY+0.0AZ
+0.0BV+0.0BW+0.0BX+0.0BY+0.0BZ
+0.0CV+0.0CW+0.0CX+0.0CY+0.0CZ
+0.0DV+0.0DW+0.0DX+0.0DY+0.0DZ
+0.0EV+0.0EW+0.0EX+0.0EY+0.0EZ
+1.0FV+1.0FW+0.0FX+0.0FY+1.0FZ
+0.0GV+0.0GW+0.0GX+0.0GY+0.0GZ
+0.0HV+0.0HW+0.0HX+0.0HY+0.0HZ
+0.0IV+0.0IW+0.0IX+0.0IY+0.0IZ
+0.0JV+0.0JW+0.0JX+0.0JY+0.0JZ
+0.0KV+0.0KW+0.0KX+0.0KY+0.0KZ
+0.0LV+0.0LW+0.0LX+0.0LY+0.0LZ=1.000
0.0AV+0.0AW+0.0AX+0.0AY+0.0AZ
+0.0BV+0.0BW+0.0BX+0.0BY+0.0BZ
+0.0CV+0.0CW+0.0CX+0.0CY+0.0CZ
+0.0DV+0.0DW+0.0DX+0.0DY+0.0DZ
+0.0EV+0.0EW+0.0EX+0.0EY+0.0EZ
+0.0FV+0.0FW+0.0FX+0.0FY+0.0FZ
+1.0GV+1.0GW+0.0GX+0.0GY+1.0GZ
+0.0HV+0.0HW+0.0HX+0.0HY+0.0HZ
+0.0IV+0.0IW+0.0IX+0.0IY+0.0IZ
+0.0JV+0.0JW+0.0JX+0.0JY+0.0JZ
+0.0KV+0.0KW+0.0KX+0.0KY+0.0KZ
+0.0LV+0.0LW+0.0LX+0.0LY+0.0LZ=1.000

```
 0.0AV+0.0AW+0.0AX+0.0AY+0.0AZ
+0.0BV+0.0BW+0.0BX+0.0BY+0.0BZ
+0.0CV+0.0CW+0.0CX+0.0CY+0.0CZ
+0.0DV+0.0DW+0.0DX+0.0DY+0.0DZ
+0.0EV+0.0EW+0.0EX+0.0EY+0.0EZ
+0.0FV+0.0FW+0.0FX+0.0FY+0.0FZ
+0.0GV+0.0GW+0.0GX+0.0GY+0.0GZ
+0.0HV+1.0HW+1.0HX+1.0HY+0.0HZ
+0.0IV+0.0IW+0.0IX+0.0IY+0.0IZ
+0.0JV+0.0JW+0.0JX+0.0JY+0.0JZ
+0.0KV+0.0KW+0.0KX+0.0KY+0.0KZ
+0.0LV+0.0LW+0.0LX+0.0LY+0.0LZ=1.000
 0.0AV+0.0AW+0.0AX+0.0AY+0.0AZ
+0.0BV+0.0BW+0.0BX+0.0BY+0.0BZ
+0.0CV+0.0CW+0.0CX+0.0CY+0.0CZ
+0.0DV+0.0DW+0.0DX+0.0DY+0.0DZ
+0.0EV+0.0EW+0.0EX+0.0EY+0.0EZ
+0.0FV+0.0FW+0.0FX+0.0FY+0.0FZ
+0.0GV+0.0GW+0.0GX+0.0GY+0.0GZ
+0.0HV+0.0HW+0.0HX+0.0HY+0.0HZ
+0.0IV+1.0IW+1.0IX+0.0IY+0.0IZ
+0.0JV+0.0JW+0.0JX+0.0JY+0.0JZ
+0.0KV+0.0KW+0.0KX+0.0KY+0.0KZ
+0.0LV+0.0LW+0.0LX+0.0LY+0.0LZ=1.000
 0.0AV+0.0AW+0.0AX+0.0AY+0.0AZ
+0.0BV+0.0BW+0.0BX+0.0BY+0.0BZ
+0.0CV+0.0CW+0.0CX+0.0CY+0.0CZ
+0.0DV+0.0DW+0.0DX+0.0DY+0.0DZ
+0.0EV+0.0EW+0.0EX+0.0EY+0.0EZ
+0.0FV+0.0FW+0.0FX+0.0FY+0.0FZ
+0.0GV+0.0GW+0.0GX+0.0GY+0.0GZ
+0.0HV+0.0HW+0.0HX+0.0HY+0.0HZ
+0.0IV+0.0IW+0.0IX+0.0IY+0.0IZ
+1.0JV+1.0JW+1.0JX+0.0JY+0.0JZ
+0.0KV+0.0KW+0.0KX+0.0KY+0.0KZ
+0.0LV+0.0LW+0.0LX+0.0LY+0.0LZ=1.000
 0.0AV+0.0AW+0.0AX+0.0AY+0.0AZ
+0.0BV+0.0BW+0.0BX+0.0BY+0.0BZ
+0.0CV+0.0CW+0.0CX+0.0CY+0.0CZ
+0.0DV+0.0DW+0.0DX+0.0DY+0.0DZ
+0.0EV+0.0EW+0.0EX+0.0EY+0.0EZ
+0.0FV+0.0FW+0.0FX+0.0FY+0.0FZ
+0.0GV+0.0GW+0.0GX+0.0GY+0.0GZ
+0.0HV+0.0HW+0.0HX+0.0HY+0.0HZ
+0.0IV+0.0IW+0.0IX+0.0IY+0.0IZ
+0.0JV+0.0JW+0.0JX+0.0JY+0.0JZ
+1.0KV+1.0KW+1.0KX+0.0KY+0.0KZ
+0.0LV+0.0LW+0.0LX+0.0LY+0.0LZ=1.000
 0.0AV+0.0AW+0.0AX+0.0AY+0.0AZ
+0.0BV+0.0BW+0.0BX+0.0BY+0.0BZ
+0.0CV+0.0CW+0.0CX+0.0CY+0.0CZ
+0.0DV+0.0DW+0.0DX+0.0DY+0.0DZ
+0.0EV+0.0EW+0.0EX+0.0EY+0.0EZ
+0.0FV+0.0FW+0.0FX+0.0FY+0.0FZ
+0.0GV+0.0GW+0.0GX+0.0GY+0.0GZ
```

```
+0.0HV+0.0HW+0.0HX+0.0HY+0.0HZ
+0.0IV+0.0IW+0.0IX+0.0IY+0.0IZ
+0.0JV+0.0JW+0.0JX+0.0JY+0.0JZ
+0.0KV+0.0KW+0.0KX+0.0KY+0.0KZ
+1.0LV+1.0LW+1.0LX+0.0LY+0.0LZ=1.000
 69.20AV+ 0.00AW+ 0.00AX+ 0.00AY+98.60AZ
+69.20BV+43.20BW+24.10BX+ 0.00BY+98.60BZ
+69.20CV+43.20CW+ 0.00CX+ 0.00CY+98.60CZ
+69.20DV+43.20DW+ 0.00DX+ 0.00DY+98.60DZ
+69.20EV+43.20EW+24.10EX+ 0.00EY+ 0.00EZ
+69.20FV+43.20FW+ 0.00FX+ 0.00FY+98.60FZ
+69.20GV+43.20GW+ 0.00GX+ 0.00GY+98.60GZ
+ 0.00HV+43.20HW+24.10HX+16.90HY+ 0.00HZ
+ 0.00IV+43.20IW+24.10IX+ 0.00IY+ 0.00IZ
+69.20JV+43.20JW+24.10JX+ 0.00JY+ 0.00JZ
+69.20KV+43.20KW+24.10KX+ 0.00KY+ 0.00KZ
+69.20LV+43.20LW+24.10LX+ 0.00LY+ 0.00LZ<=470.211
END
LEAVE
```

Appendix B

SYSTEM RELIABILITY AND LEAST COST PLOTS

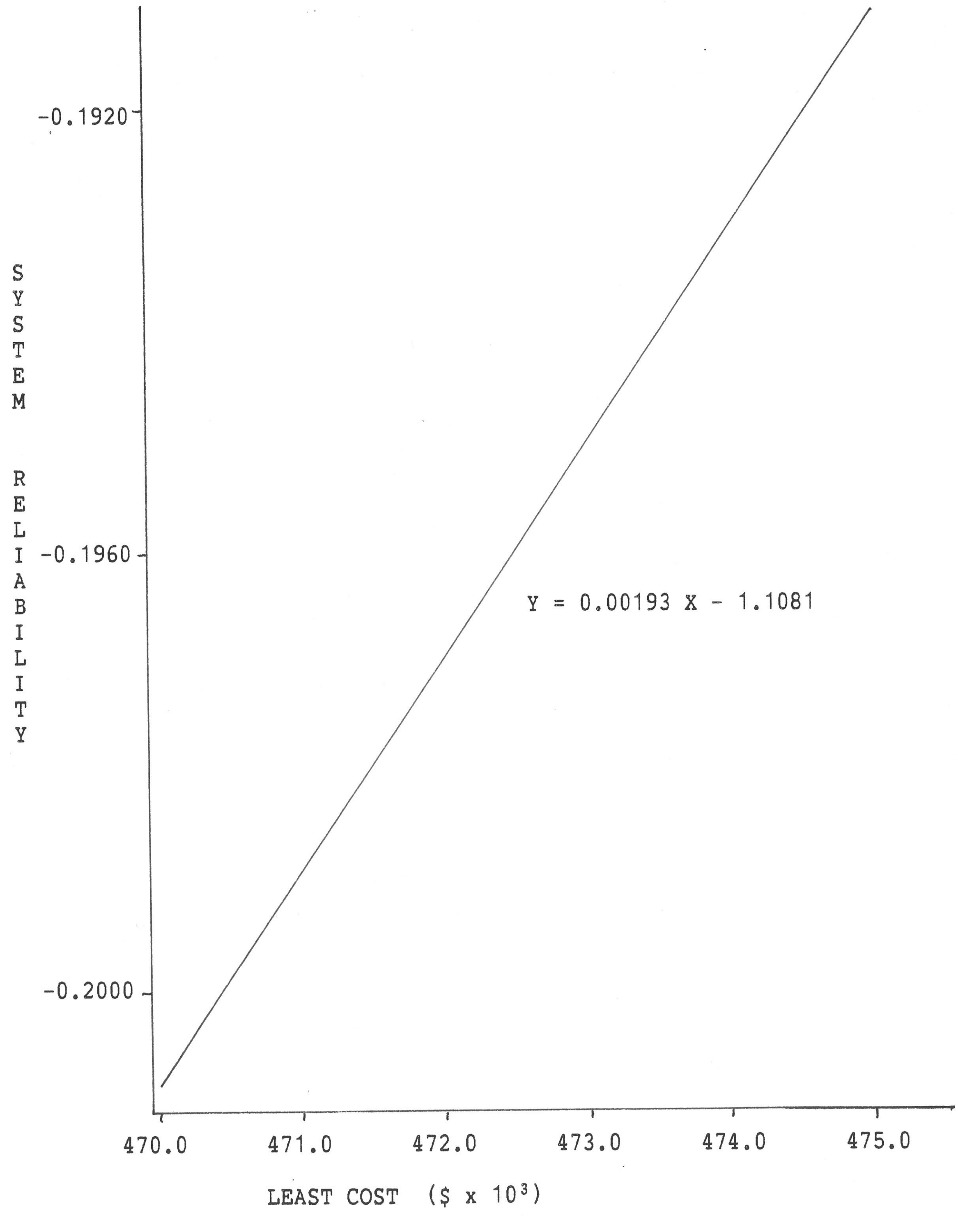

Figure 2: Plot of Least Cost and System Reliability: Approach 1

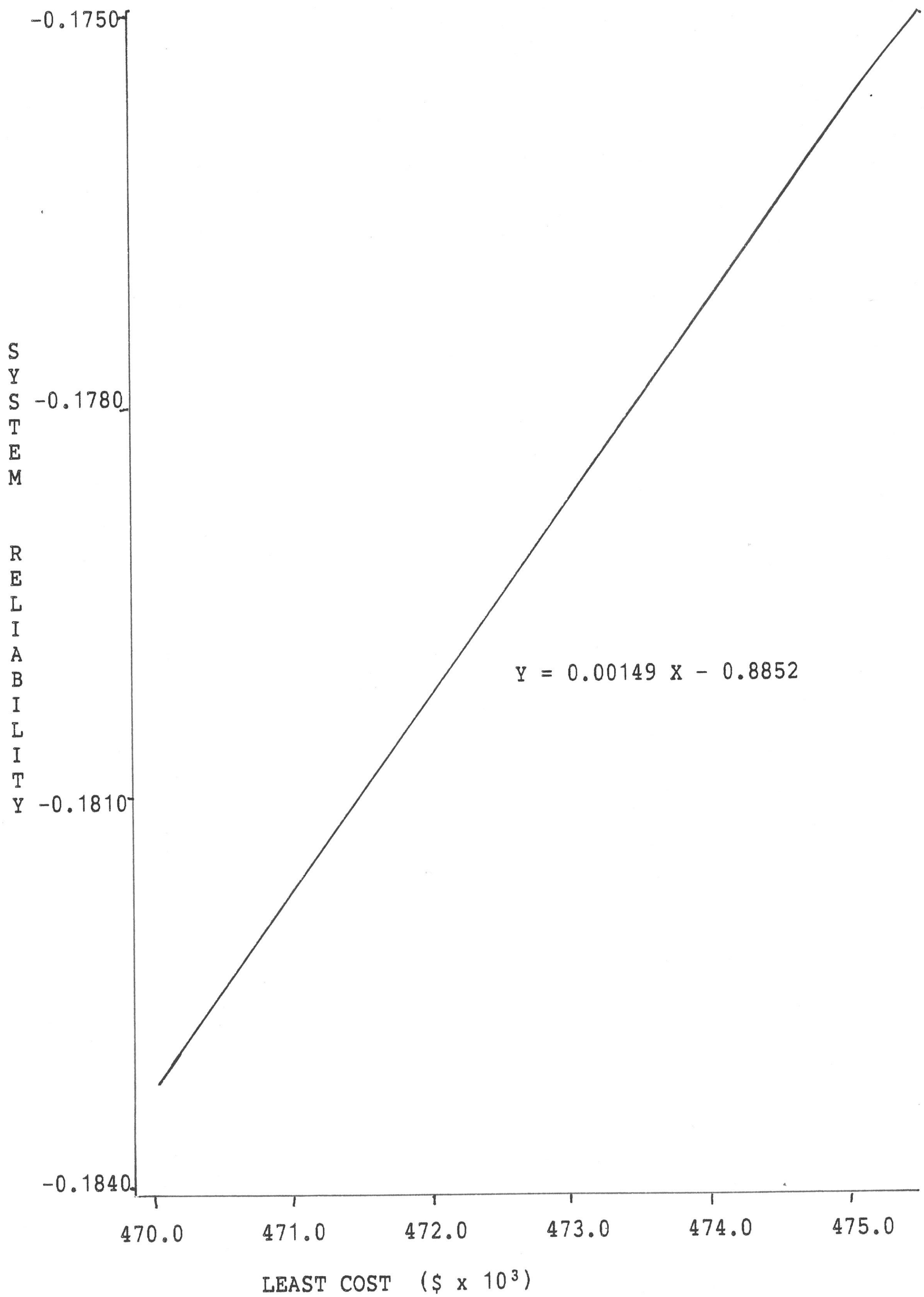

Figure 3: Plot of Least Cost and System Reliability: Approach 2 (Uniform Weights)

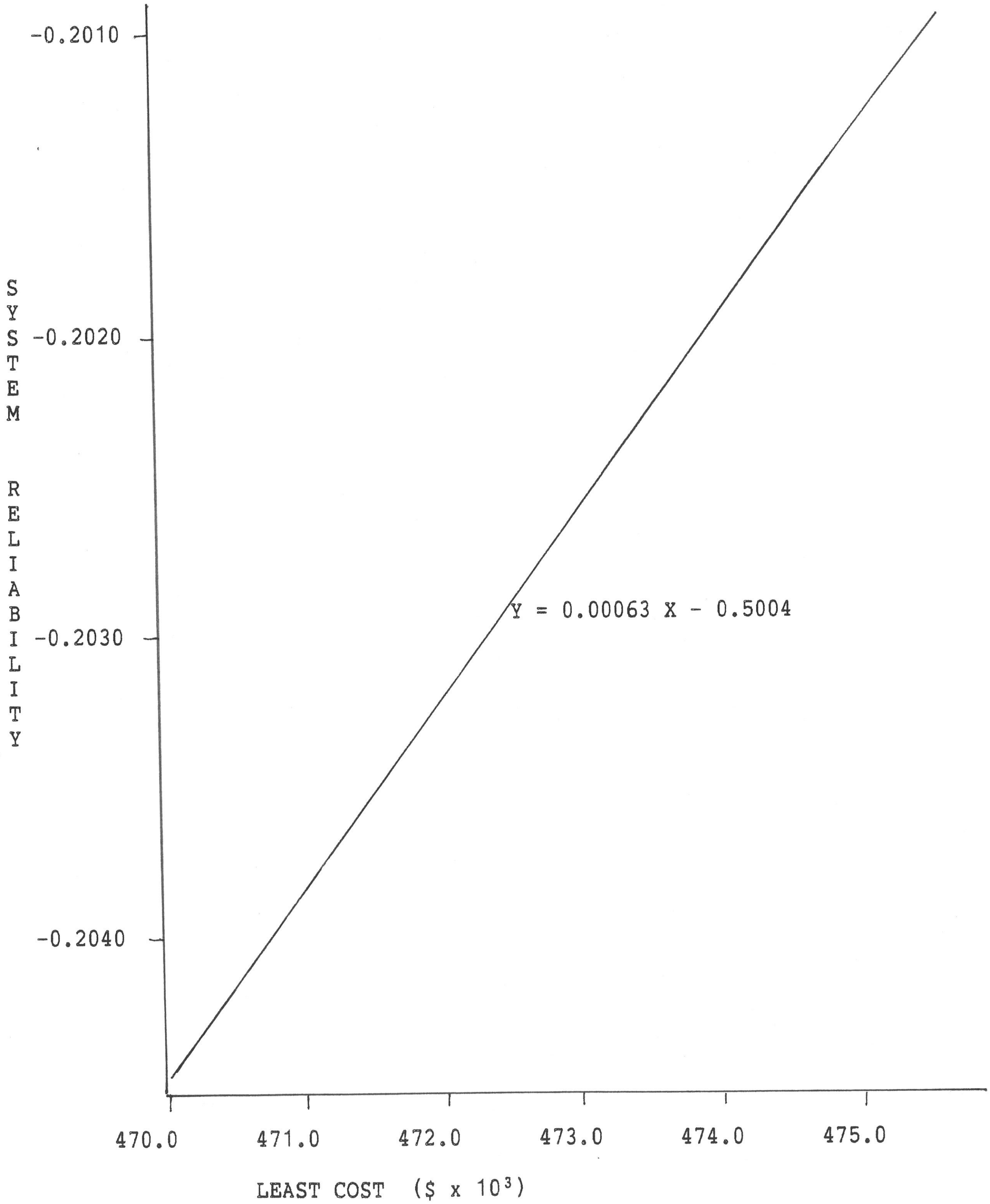

Figure 4: Plot of Least Cost and System Reliability: Approach 2 (Non-uniform Weights)