

TURBULENT FLOW PREDICTION IN A CONICAL DIFFUSER

by

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ABSTRACT

A study of the turbulent flow prediction near the wall of a conical diffuser is presented. The calculation technique for turbulent boundary layers, developed by Bradshaw et al.(1967) is adopted with modifications in the empirical input. A preliminary investigation of the flow field in a developing pipe flow was carried out to ascertain the accuracy of the flow prediction using this software. The predicted mean velocity and shear stress profiles, in the above study, were in excellent agreement with the experimental data.

The predicted mean velocity and shear stress distribution in the diffuser is in fairly good agreement with the experimental data. The universal law of the wall present at the diffuser inlet vanishes for certain distance and then re-emerges. Accounting for the effect of divergence in the form of extra strain rate, improves the predicted value in the initial region only. The main draw back of this technique is that it fails to predict the flow beyond one half the total length of the diffuser.

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NOMENCLATURE

C, κ	-	Constants of log-law velocity distribution
C, n	-	Constants of power law velocity distribution
C_f	-	Skin friction co-efficient
G	-	Energy-diffusion function
H	-	Shape factor (δ^*/θ)
L	-	Energy-dissipation function
R_θ	-	Reynolds number ($U_e \theta / \nu$)
R_x	-	Reynolds number ($U_e x / \nu$)
U_b	-	Pipe bulk average velocity
U, V, W	-	Mean velocity in X, Y and Z direction of flow
u, v, w	-	Fluctuating velocity in X, Y and Z direction of flow
u^*	-	Frictional velocity
U_+	-	U/u^*
Y_+	-	yu^*/ν
a_1	-	Stress-energy ratio
e	-	Extra strain rate
l	-	Mixing length
P	-	Mean static pressure or rate of turbulent energy production
p	-	Fluctuating component of the pressure field
$\overline{q}^2$	-	Turbulent stress tensor ($= \overline{u}^2 + \overline{v}^2 + \overline{w}^2$)

GREEK

μ	-	Absolute viscosity
δ	-	Boundary layer thickness
ρ	-	Density of air
δ^*	-	Displacement thickness
ν	-	Kinematic viscosity
θ	-	Momentum thickness
τ	-	Shear stress
ϵ	-	Viscous term in turbulent energy equation

SUBSCRIPT

e	-	Center line velocity
eff	-	Effective
max	-	Maximum
o	-	Initial condition
ref	-	Reference
w	-	Wall
1	-	Values at first mesh point in calculation

SUPERSCRIPT

I	-	Instantaneous flow quantities in turbulent flow
-	-	Average quantity

Chapter I

INTRODUCTION

Turbulent boundary layer flow under arbitrary flow boundary conditions, such as pressure gradient and surface roughness, is one of the most important unsolved problems of fluid mechanics. In turbulent boundary layer study very little useful theoretical analysis has been done compared to the volume of experimental analysis conducted until today. In spite of the great many experimental and theoretical studies, the behaviour of turbulent boundary layers in various flows, the nature of such flows continues to be one of the major unresolved problems of fluid flow.

The turbulent boundary layer flows can be classified into two broad categories as with or without pressure gradient. Of these, very little is known about the exact behaviour of flows with pressure gradient compared to flows without pressure gradient under various other factors affecting the flow such as geometry of the flow device, roughness of wall, viscosity of the fluid etc. Apparently we are missing a general physical model which correctly describes the turbulence in a boundary layer flow under various flow conditions.

Turbulent boundary layer flows with pressure gradient have offered a tremendous challenge both in theory and ex-

periment. Clauser(1954) tackled this problem experimentally and for the first time provided greater insight to the understanding of turbulent boundary layers in adverse pressure gradients. The paramount problem in the prediction of turbulent flows is to find an empirical expression for Reynolds stresses. Earlier hypotheses like the mixing length formula and the eddy viscosity formula related the Reynolds stresses to the mean velocity gradient. Based on the work of Townsend(1961), Bradshaw and Ferris proposed a hypothesis to convert turbulent kinetic energy equation [TKEE] into a closed equation for shear stress. Thus relating the shear stress to the turbulent kinetic energy which is governed by TKEE; a procedure which they argued as rational since it related two turbulent quantities. Using this concept, Bradshaw et al.(1967) developed a calculation procedure to predict the flow field for a two-dimensional turbulent boundary layer.

Extending this calculation procedure to other turbulent wall bounded flows, provided boundary layer approximations are valid, helps in understanding the flow in different situations. Such a study would also demonstrate the universal behaviour, if it exists, of all wall bounded flows. In practice there are several examples of wall bounded flows which are both external and internal in nature. One such flow of utmost importance belonging to the class of internal flows, is the diffuser flow.

Physically, a diffuser converts mean kinetic energy into positive pressure gradient in the direction of flow. This increases the intensity of highly energetic turbulent processes near the wall resulting in high turbulence intensity in the flow field far beyond other wall bounded flows. This high intensity of turbulence makes the diffuser research interesting and challenging, while the effect of adverse pressure gradient on the structure of turbulence is of considerable importance from the point of view of calculation.

The main aim of the present study is to predict the flow near the wall of a conical diffuser using the calculation technique developed by Bradshaw et al.(1967). In order to achieve this goal it was first necessary to test the software developed by Bradshaw & Unsworth(1974) in a well established flow field. This was accomplished by predicting the flow in a developing pipe flow. The details of the application of the theory to a pipe flow and the diffuser flow are presented in chapter II. Chapter III presents the results and discussion of the predicted flow field. A comparison with other wall bounded flows is also presented. Finally, the conclusions and suggestions for future study are presented in chapter IV.

Chapter II

THEORY AND APPLICATION

2.1 ANALYTICAL BACKGROUND

A fundamental problem of turbulent flow prediction is to obtain an empirical expression for the Reynolds stresses. These stresses appear in the time averaged momentum equation and are a reflection of the nonlinearity of the Navier Stokes equations. In general the additional force, caused due to the gradient of Reynolds stress, describes the effect of the turbulent fluctuations on the mean flow and is simply interpreted as a consequence of the mean rates of transport of momentum by the turbulent movement of the fluid. The two most popular hypotheses for the Reynolds shear stress in two dimensional incompressible boundary layer flow are the mixing length formula

$$\tau = \rho l^2 \left(\frac{\partial U}{\partial y} \right) \left| \frac{\partial U}{\partial y} \right| \quad (2.1)$$

and the eddy viscosity formula

$$\tau = \mu \frac{\partial U}{\partial y} \quad (2.2)$$

The basic assumption made in calculating the development of turbulent boundary layers using these formulae is that the shear stress profiles at a given distance from the origin of the boundary layer is uniquely related to the mean flow conditions at those stations. The other alternative is to ap-

peal to the exact transport equations for the rate of change of Reynolds stresses along a mean streamline. Since not all the terms in the above equation can be measured experimentally, such an equation if used will be a very crude approximation. Bradshaw et al.(1967) argued that the shear stress is closely related to the turbulent kinetic energy and has a much closer relationship with other parameters describing the turbulent structure. Based on the work of Townsend(1961) they proposed a hypothesis that "for any given shear stress profile $\tau(y)$ there is one and only one possible profile for any one of the other turbulent quantities (such as $\bar{q}^2$ or ϵ)". In particular, they made use of the turbulent energy equation for the rate of change $\rho\bar{q}^2/2$ along a mean stream line. For a two dimensional incompressible mean flow outside the viscous sublayer this equation can be written as (see Appendix A for details)

$$\underbrace{\frac{1}{2}\rho\left(U\frac{\partial\bar{q}^2}{\partial x} + V\frac{\partial\bar{q}^2}{\partial y}\right)}_{\text{I}} - \underbrace{\tau\frac{\partial U}{\partial y}}_{\text{II}} + \underbrace{\frac{\partial}{\partial y}(\overline{pV} + \frac{1}{2}\rho\overline{q^2V})}_{\text{III}} + \underbrace{\rho\epsilon}_{\text{IV}} = 0 \quad (2.3)$$

The terms I, II, III and IV in equation (2.3) represent advection, production, diffusion and dissipation in turbulent flow respectively. The turbulent energy equation is transformed into a closed equation for τ by defining empirical functions a_1 (stress-energy ratio), L (energy-dissipation length scale) and G (energy-diffusion function) as follows:

$$\begin{aligned}
 a_1 &= \frac{\tau}{\rho q^2} \\
 L &= \frac{(\tau/\rho)^{3/2}}{\epsilon} \\
 G &= \frac{\tau}{\rho} \frac{(\overline{p}v/ + \overline{q^2}v/2)}{\sqrt{(\tau_{\max}/\rho)}}
 \end{aligned}
 \tag{2.4}$$

Here a_1 and G are dimensionless where as L has the dimension of length. L is the most important parameter of the three because over most of the boundary layer the dissipation is much larger. a_1 , L and G depend on the shape of the shear stress profile at a given point and they are function of y/δ .

With the above definitions for a_1 , L and G equation (2.3) can be rewritten in a closed form for τ as

$$\begin{aligned}
 U \frac{\partial}{\partial x} \left(\frac{\tau}{2a_1 \rho} \right) + V \frac{\partial}{\partial y} \left(\frac{\tau}{2a_1 \rho} \right) - \frac{\tau}{\rho} \frac{\partial U}{\partial y} \\
 + \left(\frac{\tau_{\max}}{\rho} \right)^{1/2} \frac{\partial}{\partial y} (G \frac{\tau}{\rho}) + \frac{(\tau/\rho)^{3/2}}{L} = 0
 \end{aligned}
 \tag{2.5}$$

Equation (2.5) can be solved for thin turbulent shear layers (assuming a_1 , L and G are known) using the continuity equation

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0
 \tag{2.6}$$

and the boundary layer momentum equation

$$U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} = - \frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{1}{\rho} \frac{\partial \tau}{\partial y}
 \tag{2.7}$$

for two dimensional incompressible steady flow. It is readily seen that the three unknowns in the set of equations (2.5), (2.6) and (2.7) are U , V & τ .

The important approximations made in forming the above equations are

- i) the boundary layer approximation i.e., no static pressure difference across the layer, and
- ii) a_1 , L and G change much more slowly than τ and U in X direction.

A close examination of equations (2.5), (2.6) and (2.7) indicate that they are hyperbolic in nature. Solutions to the set of hyperbolic equations are obtained by the method of characteristics. The tangent of the angles γ between the three characteristics and the X -axis (figure 3) being

$$\tan \gamma = \infty, \frac{(V + \alpha \pm \beta)}{U} \quad (2.8)$$

$$\text{where } \alpha = a_1 G / \left(\frac{\tau_{\max}}{\rho} \right)$$

$$\beta = \sqrt{\left(\frac{\alpha^2 + 2a_1 \tau}{\rho} \right)}$$

Once the three characteristic directions are known, the partial differential equations reduce to ordinary differential equations containing gradients along the characteristics only. Solution of the ordinary differential equations can be obtained by finite difference methods. A detailed analysis of the above equations and the method of solution is given by Bradshaw & Unsworth (1974). This method of mod-

elling for Reynolds shear stress can be readily perceived as a refinement of mixing length theory from equation (2.5). It is evident that by neglecting advection and diffusion terms, which is true outside the viscous sublayer, equation (2.5) reduces (after dividing throughout by τ/ρ) to the mixing length formula.

Complicated wall boundary conditions are satisfied independently of the main calculation at each X step. In general, the boundary conditions at the outer edge ($U \rightarrow U_e$, $\tau \rightarrow 0$) of the boundary layer are satisfied numerically and the boundary conditions at a solid surface ($U = 0$, $V = 0$ or V_w , $\tau = \tau_w$) is satisfied by matching the solution to the logarithmic law of the wall at the first mesh point.

2.2 APPLICATION OF THE THEORY TO DEVELOPING BOUNDARY LAYER IN A PIPE

The theory developed in the previous section was used to predict the flow field in a developing turbulent boundary layer on the inside of a pipe with smooth solid wall. The flow is considered as a two dimensional turbulent boundary layer with zero pressure gradient. This translates as a constant free stream velocity throughout the flow. Since the flow is in a developing stage the effect of interaction of shear layers can be neglected. The co-ordinate system selected was X-axis along the wall (positive along the downstream direction of the flow) and Y-axis being perpendicular

to the wall. The origin of this co-ordinate system being at the trip which is situated at the beginning of the pipe. Experimental data of Kassab(1985) was used as the initial data input for calculation.

The distribution of parameters a_1 , L and G was assumed to be the same as in constant pressure two dimensional turbulent boundary layer¹. They show adequate representation over the entire pipe flow region considered for study.

The following two cases were considered for complete flow examination:

- i) using the flow integral parameters such as θ , C_f and R_θ the initial velocity profile was generated² at the starting point with Coles' law. Making use of this velocity and shear stress profiles flow field in the downstream direction was predicted.
- ii) introducing experimental velocity and shear stress profiles along with R_x and C_f at the initial point, the velocity and shear stress profiles in the downstream direction were predicted.

¹ This is demonstrated from the results obtained by Kassab(1985).

² Hereafter the generation of velocity profile using flow parameters θ , C_f and R_θ will be referred as SYNTHETIC velocity profile generation.

2.3 APPLICATION OF THEORY TO (CONICAL) DIFFUSER FLOW

It is easier to formulate a general method of predicting velocity and shear stress profiles in case of turbulent boundary layer flow or a developing pipe flow. For such flows accurate results can be established, as shown in section 3.1, by knowing the flow conditions at very few stations. When we turn our attention to the case of a turbulent flow in a conical diffuser the situation is not so simple. In diffuser flow, where the mean kinetic energy is converted into a positive pressure gradient in the direction of flow, a general method of predicting the flow will be a very difficult task. Since the main interest in the present study is the behaviour of the flow near the wall only, the existing method developed by Bradshaw et. al.(1967) is adopted with some modifications.

Basically, the flow field in a horizontal plane containing the axis of the diffuser is treated as a two dimensional turbulent boundary layer with adverse pressure gradient. The basis for such a treatment was obtained from a critical examination of the mean velocity profiles measured by Kasab(1985). It is well known that in complex turbulent flows the interaction of shear layers and extra strain rates play an important role. A realistic picture of flow prediction in conical diffusers should include the effect of these parameters on the flow field. Because of the lack of knowledge of exact interaction of various shear layers as the

flow develops in a conical diffuser, the effect of interaction was neglected in the present studies. Bradshaw & Unsworth(1974) procedure allows to account for the effect of extra strain rates on the structure of turbulence. Although this was developed for external flows, a similar procedure was adopted in the present study by making use of the diffuser geometry.

A major difference in the present study is the representation of empirical data for a_1 and L in a conical diffuser. In the case of a conical diffuser it is found that the distribution of a_1 and L as given by Bradshaw & Unsworth(1974) for a constant pressure boundary layer are no longer valid over the entire flow region. The modifications adopted to represent the distribution of a_1 and L over the entire flow field is discussed in the following sections. A similar approach to modify the distribution of energy-diffusion function (G) over the entire flow region could not be undertaken due to lack of information available on the behaviour of G .

2.3.1 Stress-Energy Ratio in Diffuser Flow

Bradshaw & Unsworth(1974) chose a constant value of a_1 ($=0.15$) in the entire boundary layer region. Although this assumption simplifies calculation procedure, it does not truly represent the behaviour of a_1 in the diffuser. The distribution of a_1 in the diffuser measured by Arora(1978) demonstrates that it is a function of both X and Y co-ordi-

nates. It can be noted that the behaviour of a_1 at a given X is essentially the same in the inner region ($y/\delta = 0.2$) of the boundary layer. In the outer region ($y/\delta > 0.2$) the difference in a_1 at a given X is more pronounced and they all fall gradually with X . A plateau of constant a_1 as seen in a turbulent boundary layer is not obvious from this study. A study of turbulent boundary layer with lateral divergence by Smits et al.(1979) also indicates a variation of a_1 as a function of both X and Y . To incorporate the above changes, a_1 was first represented as a function of Y only. A preliminary run using the above set of values improved the results considerably, specially in the inner region. When the variation in X was also included, the results obtained did not indicate a significant change.

Since the present study concentrates mainly in the inner region, a single best fit curve for the data measured by Arora(1978) was chosen. Such a representation made the calculation procedure simple and as realistic as possible. Figure 4 shows the best fit curve used in the present study along with the constant value used by Bradshaw & Unsworth(1974).

2.3.2 Dissipation Length Scale in Diffuser Flow

Dissipation length scale parameter is the most important of the three functions a_1 , L and G because over most of the boundary layer the dissipation is much larger than the ad-

vection or diffusion. The representation of this function in the flow region has a significant effect on the flow parameters such as mean velocity, shear stress etc. For a two dimensional turbulent boundary layer Bradshaw & Unsworth(1974) chose this parameter as a function of Y only. This is also true in the case of a fully developed pipe flow as shown by Lawn(1970) and Liepmann & Laufer(1974). In a diffuser treating L as a function of Y only does not represent the flow adequately over the entire region. Azad(1982) reported the distribution of L in a diffuser measured by Arora(1978). From their study it is clearly evident that L is a function of both X and Y co-ordinates. In general terms it can be said that at any given X , L varies linearly with Y initially, reaches a peak and then falls to zero on the axis. The region of linearity increases with increase in X .

After several preliminary runs, using L as a function of both X and Y co-ordinates the following procedure was adopted for calculation. The total length of the diffuser was divided into 3 equal regions (of 24 cm each). The variation of L with Y within each region was assumed to be the same but different from one region to the other. Such an assumption can also be justified from the behaviour of L as shown in Azad(1982;figure 14b). The above procedure substantially simplifies the empirical data input, keeping as much as possible the characteristic of flow in a diffuser. The varia-

tion of L in the three different regions used in the present study is shown in figure 5. It should be noted that the behaviour of L may be different depending on the specifications of the diffuser used. It was not possible to include this aspect in the present study for the lack of information available for various other diffusers.

2.3.3 Energy-Diffusion Function in Diffuser Flow

The distribution of energy diffusion function G , used by Bradshaw & Unsworth(1974), has been retained without any modification in the present study. This is due to lack of information available on the behaviour of energy diffusion function in a diffuser. It is well known that the energy diffusion function G is not a dominant function in the turbulent energy equation as compared to the dissipation length parameter L . Thus retaining the values for G used by Bradshaw & Unsworth(1974) may not have any adverse effect in the present study.

In order to study the effect of various parameters on the predicted flow field several cases were investigated. In general they can be classified as:

- i) Generating the initial velocity profile in the diffuser using the Coles' law at the inlet of the diffuser and predicting the downstream velocity and shear stress profiles under the influence of a po-

sitive stream wise pressure gradient defined using the diffuser centerline velocities.

- ii) Same as above but the effect of divergence defined using the virtual origin of the diffuser was included.
- iii) Experimental velocity and shear stress profile near the diffuser inlet as the starting profile to predict the downstream velocity and shear stress profiles, under the influence of a positive pressure gradient as defined in case (i).
- iv) Same as case (iii), but the effect of divergence defined as before was included.

The experimental³ data used was measured by Kassab(1985)⁴ in a conical diffuser of solid angle of 8° . The diffuser is fed by fully developed pipe flow from a pipe of 10.16 cm diameter. The overall length of diffuser is 71.96 cm with an inlet diameter as that of pipe and outlet diameter of 20.22 cm giving an area ratio of 4:1. The center line velocity decreases monotonically in the downstream direction due to

³ The station number referred hereafter in case of diffuser flow represents the distance in cm from the starting point of the diffuser. For experimental setup and diffuser details refer to Arora(1978) and Kassab(1985).

⁴ Kassab(1985) measured the velocity and shear stress profiles perpendicular to the axis of the diffuser. This data was compared with the profiles measured (perpendicular to the diffuser wall) by Laura(1985). The maximum variation between these two different methods of measurement was 2% in the wall region. Thus, in the present study, it was decided to use experimental data obtained by Kassab(1985) for all purposes.

the positive adverse pressure gradient created by the shape of diffuser. The X-axis was selected along the wall of the diffuser in a horizontal plane and the Y-axis being perpendicular to the wall. Figure 2 shows the details of the diffuser used along with the co-ordinate system employed.

A brief description of the software used and options available in using the software is given in Appendix B. A detailed description and data input format can be found in Bradshaw & Unsworth(1974).

Chapter III

RESULTS AND DISCUSSION

3.1 DEVELOPING BOUNDARY LAYER IN PIPE FLOW

Here turbulent boundary layer flow is considered to be two dimensional without adverse pressure gradient. The results of the two cases discussed in section 2.2 are presented for the case of a turbulent boundary layer growth in a pipe. Three stations at 2.27, 2.28 and 2.29 metres from the end of the trip⁵ were chosen for comparison of experimental and calculated quantities. In the first case mean velocity and shear stress profiles measured at station 2.27 meters from the trip was used as initial input. For the second case, at the same station, mean velocity and shear stress profiles were generated from Coles' velocity profile using flow parameters θ , C_f and R_0 . In both cases the experimental measurements of Kassab(1985) were used without any modifications.

⁵ Details of Experimental set up and equipments for turbulent boundary layer flow in pipe is given in Kassab(1985) and Azad et al.(1985)

3.1.1 Flow Parameters of Developing Boundary Layer

At first a compatibility study of the software used was carried out by comparing the boundary layer parameters of the flow generated at station 2.27 meters with that of Azad et al.(1985). The summary of this comparison using Coles' velocity profile is given in table 1. It shows that the calculated values of δ are very close (within 1-2% variation) to the experimental results presented by Azad et al.(1985). The behaviour of shape factor H in the present study contradicts their conclusion. The shape factor is found to be a function of R_0 rather than being a constant. This view is supported by the results of previous researchers. Figure 6 shows the comparison of shape factor from the present study with that of Azad et al.(1985), Coles(1956), Hama(1954), Landweber(1953), Purtell et al.(1981) and Schultz-Grunow(1940) for a turbulent boundary layer. The absolute values of shape factor in the present study is very close to that of Coles(1956) and Hama(1954).

Table 2 presents the flow parameters such as H , C_f , θ , δ and R_0 calculated for the three stations under consideration at a free stream velocity of 8.03 m/sec. These values are compared with the experimental data of Kassab(1985) at the respective stations. It is evident that the difference between the calculated and experimental quantities is about 4%. This difference is well within the experimental errors encountered in practice. The round off errors in the calculation procedure could also contribute to this difference.

Figure 7 presents the calculated flow parameters H , C_f and θ along the stream wise direction. The predicted values of θ and H , using the velocity and shear stress input at station 2.27, are slightly higher than those obtained using the Coles' velocity profile. This difference may be due to inaccuracies in the starting data as suggested by Bradshaw & Unsworth(1974).

3.1.2 The Mean Velocity Field

The mean velocity profiles, both measured and calculated at 2.27 and 2.29 metres from the trip are presented in figures 8 & 9. It is seen that at both stations the local value of the mean velocity component in X-direction obtained using Coles' velocity profile is slightly higher in the y/δ range of 0.1-0.3. The typical variation being of the order of 2-3%. A similar phenomenon is observed for the inbetween station at 2.28 metres (data not presented).

To study the velocity profiles in much more detail it is presented in terms of universal co-ordinates U_+ and Y_+ as shown in figures 10 & 11. Here simple log law of the form,

$$\frac{U}{u^*} = \frac{1}{\kappa} \log\left(\frac{u^* y}{\nu}\right) + C \quad (3.1)$$

and simple power law of the form,

$$\frac{U}{u^*} = \frac{C}{(u^* y / \nu)^n} \quad (3.2)$$

are plotted along with calculated values and experimental data of Kassab(1985). In the present case, the existence of

logarithmic region is observed in the Y_+ range of 30 to 300. This observation reinforces the idea of Purtell et al.(1981) and Azad et al.(1985) that "the logarithmic region seems to be an inherent characteristic of the turbulent boundary layer". The limits of existence of logarithmic region is discussed in Hinze(1975). According to him the upper limit of logarithmic region depends on Reynolds number. Hinze(1975) agrees with a lower limit of $Y_+ = 30$ but has suggested to consider the lower limit in the range of 50 to 60 to accommodate fully turbulent flow condition with negligible direct viscosity effect. The deviation in predicted velocity profile from that of Kassab(1985) is 2-3% in the logarithmic region and decreases substantially (gradually to 1% or less) in the outer layer.

The numerical constants in the present case for "log law" is observed to be $1/\kappa = 2.44$ and $C = 5.0$. These values are in agreement with Coles(1956) and many other published data. The constants of "Power law velocity distribution" are given as $C = 8.3$ and $n = 7$ by Hinze(1975;eq 7.57), where as Schlichting(1979;eq 20.11) obtained values of $C = 8.74$ and $n = 7$. According to several published literatures to date the value of $n = 7$ does not seem to vary. But the value suggested for C varies from 8.2 to 8.8. Assuming the value of $n = 7$, in the present study the value of C was observed to be 8.35.

It can be readily seen from the universal plots that the calculated mean velocity profile begins approximately at a Y_+ value of 30. This limitation can be attributed to the calculation procedure(ref. Bradshaw & Unsworth;1974) where the initial point in the input is taken equal to the momentum thickness θ . It is worthwhile to note that even if we are able to predict the mean velocity very close to the wall, comparison of this results with the experimental data can pose several problems. For example, it is well known that the error in measurement of accurate distance from the wall to hot wire can result in a deviation of the measured mean velocity between a linear relationship and the form $U_+ = Y_+$. Also, there is a problem of correcting the hot wire anemometer reading for heat loss due to conduction from the wire to the wall.

3.1.3 The Shear Stress Distribution

The calculated shear stress (τ) non-dimensionalized with free stream velocity (boundary layer outer parameter) is presented in figures 12 & 14. In figures 13 & 15 the same shear stress non-dimensionalized using frictional velocity⁶ (boundary layer inner parameter) is presented.

⁶ The mean velocity data were used in a cross-log plot to obtain the frictional velocity.

Since experimental shear stress profile was available only at the starting point (2.27 metres from the trip), in figures 13 & 15 (at station 2.29 metres) the shear stress profile comparison is shown for two methods of calculation only. It is observed (figure 12 & 14) that very near to the wall (within $0.1 y/\delta$) the calculated shear stress values obtained using Coles' velocity profile generation method are higher (about 5-7%) than the experimental values. But for $0.1 \leq y/\delta \leq 0.5$ this difference decreases to 1-2%. For $y/\delta > 0.5$, the behaviour is reversed and the difference between the two is about 1-2%. However, in the region $y/\delta < 0.1$ all figures(12-15) seem to exhibit a constant value of shear stress. This behaviour is in agreement with the assumption of a constant shear stress in the wall region. A similar trend in the distribution of measured turbulent shear stress was reported by Klebanoff(1954).

3.2 TURBULENT FLOW PREDICTION IN A CONICAL DIFFUSER

Here the results for the flow in a conical diffuser are presented. Again it should be noted that the main aim was to examine the behaviour of the flow near the diffuser wall. As mentioned in section 2.3 the effects of various input parameters were broadly categorized into four cases. The results obtained using the above procedure are summarized in the following sections.

3.2.1 The Mean Velocity Field

The predicted and measured non-dimensionalized mean velocity profiles are shown in figures 16-23. It is evident that in the initial stages of the flow, accounting for the extra strain rate due to the flow divergence, does not seem to greatly alter the predicted value in the absence of divergence. The difference between the two being of the order of 2-3% in the wall region. The above observation is true for synthetic as well as experimental starting profiles. The predicted values using synthetic profiles do not yield a better agreement with the experimental data, specially in the wall region, for distances upto 24 cms from the diffuser inlet. Such a difference can be expected since the synthetic starting profile using the integral flow parameters may not exactly represent the characteristics of the flow at that point. Also the shear stress profile obtained from this generated velocity profile does not agree well with the experimental measurements. A detailed discussion of shear stress will be presented in the following sections. It is surprising to note that in the outer region or towards the axis of the diffuser the predicted values are in close agreement with the measured velocity profile. It is observed from figures 16-23 that including the effect of divergence for distances beyond 24 cms from the inlet leads to predicted values being far lower than the experimental values. The variation between the two is in the range of 3-8%.

Neglecting the effect of divergence, the predicted mean velocity, using both the synthetic and experimental starting profiles, is in close agreement with the measured quantity. Thus the flow seems to behave as a two dimensional turbulent boundary layer flow with a small obstacle at the diffuser inlet (due to flow divergence) causing disturbance in the initial region of the diffuser flow. The divergence effect in the form of extra strain rate is felt in this initial region only. A study of extra strain rate reported by Azad(1982), confirms the behaviour exhibited in the present study. As the flow develops further into the diffuser the effect of divergence gradually reduces. Finally, after a certain stage (approximately one half of the total length of the diffuser) the flow attains a fully developed state and divergence has no longer any effect on the flow field.

The main drawback of this method was that in all cases flow could be predicted only upto one half of the total length of the diffuser, when the calculation was terminated due to the occurrence of separation. This results in $\tau \leq 0$ in the calculation and eventually its termination. In order to study the effect of various input parameters on separation, measured and predicted value of C_f at 30 cms from the inlet of the diffuser for all the four cases is shown in figure 24. It is readily seen that including divergence effect at this stage results in significant deviation of the calculated value from the measured. It should be noted that

any deviation in the measured and calculated values should also take into account the difficulty of obtaining C_f by preston tube method in an adverse pressure gradient. The occurrence of separation is surprising since the diffuser used was an optimum one from the performance point of view with included angle in the range of $5-10^\circ$. A close examination of the mean velocity profiles measured by Kassab(1985) reveals that the slope and magnitude of the profile decreases towards the diffuser exit. There was no indication (visual) from the shape of the profiles that the separation could have occurred within the diffuser. A more convincing and conclusive study about separation from flow visualization techniques could not be carried out in the present study.

The mean velocity profiles plotted using universal co-ordinates are presented in figures 25 to 32. At the inlet to the diffuser the fully developed pipe flow is in excellent agreement with the universal law of the wall for $Y_+ > 30$. The existence of this law is evident at approximately 1 cm from the inlet. Here the logarithmic law line (solid line in the figures 25 to 32) has been drawn keeping the slope equal to 2.44 and as a best fit to pass through the data at $Y_+ = 30$. Such a technique can be justified from the study of Huffman & Bradshaw(1972) who concluded that the slope was a universal constant even at lower Reynolds number. The Y intercept obtained from this procedure is clearly different

from the usually quoted value of 5. Further, in the region of 5 to 15 cms from the inlet the existence of a logarithmic law region is not evident as shown in figures 27 and 28. In this region, including the effect of divergence does seem to reduce the difference between the predicted and the experimental values. The disappearance of the universal logarithmic law in this initial region is not a surprise. In a study of step induced separation of turbulent boundary layers, Bradshaw & Galea(1967) revealed that the logarithmic region disappears when the pressure gradient parameter $[(\nu/\rho u^3) dp/dx]$ is in the range of 0.01 to 0.04. An examination of this parameter along the diffuser length indicates that non existence of the logarithmic law is a possibility within the first 10 cms from the inlet. A similar observation was made by Trupp et al.(1985) in a detailed analysis of the experimental data. In fact they propose a dual logarithmic law based on the conjecture that there could be two boundary layers; one on the wall of the diffuser and the other due to the pipe flow. At approximately 20 cms from the inlet the logarithmic law reappears but is limited to a very small region. A dominating wake component takes over as the flow proceeds towards the exit of the diffuser. It is seen from the experimental data (Trupp et al.;1985) that in the later half of the diffuser, the region of validity of the logarithmic law remains approximately a constant. A similar behaviour of decrease in logarithmic portion of the velocity profile was observed by Scot-

torn(1967) for a turbulent boundary layer under the influence of an adverse pressure gradient.

It should be noted that the calculated mean velocity profiles begin approximately at a higher Y_+ values (around 30) in the universal plot. The reason for it being the same as given at the end of section 3.1.2. Thus a discussion of the wall law in the viscous sublayer could not be carried out in the present study.

3.2.2 The Shear Stress Distribution

The predicted shear stress distribution for all the four cases under consideration are presented in figures 33 to 35. They represent stations 6, 18 and 30 respectively. A comparison of the experimental and predicted profiles at the above stations for all the four cases shows that they both agree qualitatively. Having the experimental velocity and shear stress profile as initial input, there is, in general, a better quantitative agreement with the experimental data at all the stations. The variation between the two being $\pm 2\%$ near the wall region. Using the synthetic profile as the initial input, the predicted shear stress profiles result in being far from agreement with the experimental specially in the initial region of the diffuser (upto nearly 10 cms from the inlet). Such a difference can be explained by examining the initial shear stress profile generated from Coles velocity profile. This distribution no longer truly represents

the turbulence characteristics at the inlet as seen in figure 33. Thus a difference in the measured and predicted value for the synthetic case is largely due to the discrepancy in the initial shear stress profile.

Accounting for the effect of divergence in the initial region (upto 10 cms) of the diffuser does not seem to greatly improve the predicted values near the wall region. However, including the effect of divergence further downstream results in predicted values being considerably higher than the measured shear stress. Again emphasizing the point that divergence effect if present is confined to the initial region only. A close examination of figures 34 and 35 show that the predicted shear stress using the synthetic data input is also in good agreement with the measured value in the wall region. A behaviour which cannot be explained at this stage of investigation.

Chapter IV

CONCLUSIONS AND SUGGESTION

4.1 CONCLUSIONS

The following conclusions can be drawn from the study of flow prediction in a developing pipe flow.

- i) The flow field in a developing pipe flow can be treated as a two-dimensional turbulent boundary layer with zero pressure gradient for prediction purposes.
- ii) The predicted mean velocity field, using the technique developed by Bradshaw et al.(1967), is in excellent agreement with that obtained experimentally. This is true for both experimental and synthetic data as the initial input. The predicted flow integral parameters match well with those obtained from experimental data. The only drawback of this prediction technique is that the predicted values do not cover the viscous sublayer region of the turbulent boundary layer.
- iii) The predicted shear stress values, using the synthetic data as the initial input, are slightly higher (5-7%) than the experimental values in the wall region ($y/\delta < 0.1$).

In the case of a conical diffuser the results obtained can be summarized as follows:

- i) Mean velocity profiles predicted upto approximately half the total length of the diffuser are in close agreement with the experimental measurements. This result is true for both experimental and synthetic data near the diffuser inlet as the initial input.
- ii) The universal logarithmic region of the mean velocity profile which is present at the inlet to the diffuser disappears in the initial region (from 2-15 cms from the inlet). In this region there seems to be a dual logarithmic law region with higher slopes and lower constants. The universal logarithmic law region reappears at a later stage but the extent to which it is valid is considerably reduced.
- iii) The predicted shear stress profiles, using the experimental data near the diffuser inlet as initial input, agree well with the measured value. Using the synthetic data as the initial input results in predicted shear stress values being far from agreement with the measured specially in the initial region.
- iv) In the initial region of the diffuser (approximately in the range of 2-15 cms from the inlet), accounting for the effect of divergence improves the

predicted value of mean velocity near the wall. The predicted shear stress values are not affected much by the inclusion of the divergence effect. However, accounting for the divergence effect in later part of the flow results in predicted velocity and shear stress profile being far from agreement with the measured values.

- v) The prediction of the flow beyond half the total length of the diffuser fails, indicating either the limitation of this technique or separation which cannot be confirmed at this stage of investigation.

4.2 SUGGESTION

- i) In the present analysis the interaction of various shear layers was neglected. Future study should include such interactions. One possibility is to use the theory available for channel flows and extend it to the case of a diffuser.
- ii) The distribution of energy diffusion function (G) available in the program was retained for lack of information available in the diffuser. A thorough experimental investigation of a_1 , L and G measured normal to the diffuser wall should help in the proper representation of these functions for calculation.

- iii) Finally, a program of flow visualization in the diffuser will greatly help in understanding the behaviour of the flow near the wall.

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Appendix A

FORMULATION OF TURBULENT ENERGY EQUATION

A.1 DERIVATION OF REYNOLDS EQUATION

The Navier-Stokes' equation for an incompressible flow is

$$\frac{\partial V_i^I}{\partial t} + V_k \frac{\partial V_i^I}{\partial x_k} = -\frac{1}{\rho} \frac{\partial P^I}{\partial x_i} + \nu \frac{\partial^2 V_i^I}{\partial x_k \partial x_k} \quad (A.1)$$

Considering steady flow and writing instantaneous velocities as the sum of mean and fluctuating quantities i.e.,

$$V_k^I = U_k + u_k ; \quad P^I = P + p$$

where U_k is mean and u_k is the fluctuating component of instantaneous velocity in turbulent flow. Then,

$$\frac{\partial U_k}{\partial x_k} = \frac{\partial u_k}{\partial x_k} = 0 ; \quad \overline{u_i} = 0 ;$$

$$\overline{U_i} = U_i ; \quad \overline{u_j \frac{\partial u_i}{\partial x_k}} \neq 0$$

Substituting the above turbulent quantities in equation (A.1) and taking average we arrive at

$$\begin{aligned} & \frac{\partial (\overline{U_i U_k})}{\partial x_k} + \frac{\partial (\overline{U_i u_k})}{\partial x_k} + \frac{\partial (\overline{U_k u_i})}{\partial x_k} + \frac{\partial (\overline{u_i u_k})}{\partial x_k} = \\ & -\frac{1}{\rho} \frac{\partial \overline{P}}{\partial x_i} - \frac{1}{\rho} \frac{\partial \overline{p}}{\partial x_i} + \nu \frac{\partial}{\partial x_k} \left(\overline{\frac{\partial U_i}{\partial x_k}} + \frac{\partial \overline{u_i}}{\partial x_k} \right) \end{aligned} \quad (A.2)$$

which reduces to

$$\rho \frac{\partial (\overline{U_i U_k})}{\partial x_k} = - \frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_k} \left(\mu \frac{\partial U_i}{\partial x_k} - \overline{\rho u_i u_k} \right) \quad (A.3)$$

Equation (A.3) is known as Reynolds' equation.

A.2 TOTAL TURBULENT ENERGY EQUATION

Rewriting equation (A.1) in terms of mean and fluctuating quantities for a steady flow (i component of flow only)

$$\frac{\partial (U_i + u_i)(U_k + u_k)}{\partial x_k} = - \frac{1}{\rho} \frac{\partial (P + p)}{\partial x_i} + \frac{\partial^2 (U_i + u_i)}{\partial x_k^2} \quad (A.4)$$

Multiplying equation (A.4) by $(U_j + u_j)$, expanding, averaging and rearranging it, we get i component of Navier-Stokes' equation as

$$\begin{aligned} U_j \frac{\partial U_j U_k}{\partial x_k} + U_j \frac{\partial \overline{u_i u_k}}{\partial x_k} + \overline{u_j \frac{\partial U_i U_k}{\partial x_k}} + \overline{u_j \frac{\partial u_i u_k}{\partial x_k}} + \overline{u_j \frac{\partial u_i u_k}{\partial x_k}} = \\ \frac{1}{\rho} U_j \frac{\partial P}{\partial x_i} - \frac{1}{\rho} \overline{u_j \frac{\partial p}{\partial x_i}} + \nu \overline{u_j \frac{\partial^2 u_i}{\partial x_k^2}} \end{aligned} \quad (A.5)$$

Similarly we can get j component of Navier-Stokes' equation by interchanging i and j in equation (A.5). Adding i and j component of Navier-Stokes' equation will yield total turbulent energy equation which can be written as

$$\begin{aligned}
& \frac{\partial (U_i U_j U_k)}{\partial x_k} + U_j \frac{\partial (\overline{u_i u_k})}{\partial x_k} + U_i \frac{\partial (\overline{u_j u_k})}{\partial x_k} + \overline{u_j u_k} \frac{\partial U_i}{\partial x_k} + \overline{u_i u_k} \frac{\partial U_j}{\partial x_k} \\
& + U_k \frac{\partial (\overline{u_i u_j})}{\partial x_k} + \frac{\partial (\overline{u_i u_j u_k})}{\partial x_k} = \frac{1}{\rho} (U_j \frac{\partial P}{\partial x_i} + U_i \frac{\partial P}{\partial x_j} + \overline{u_j \frac{\partial P}{\partial x_i}} + \overline{u_i \frac{\partial P}{\partial x_j}}) \\
& + \nu (U_j \frac{\partial^2 U_i}{\partial x_k \partial x_k} + U_i \frac{\partial^2 U_j}{\partial x_k \partial x_k} + \overline{u_j \frac{\partial^2 u_i}{\partial x_k \partial x_k}} + \overline{u_i \frac{\partial^2 u_j}{\partial x_k \partial x_k}}) \quad (A.6)
\end{aligned}$$

A.3 MEAN FLOW ENERGY EQUATION

To obtain i and j component of mean flow energy equation, Reynolds equation (A.3) is multiplied by U_j and U_i respectively. Adding i and j components and simplifying we get mean flow energy equation as

$$\begin{aligned}
\frac{\partial (U_i U_j U_k)}{\partial x_k} &= -\frac{1}{\rho} (U_j \frac{\partial P}{\partial x_i} + U_i \frac{\partial P}{\partial x_j}) + \nu (U_j \frac{\partial^2 U_i}{\partial x_k \partial x_k} + U_i \frac{\partial^2 U_j}{\partial x_k \partial x_k}) \\
&- (U_j \frac{\partial \overline{u_i u_k}}{\partial x_k} + U_i \frac{\partial \overline{u_j u_k}}{\partial x_k}) \quad (A.7)
\end{aligned}$$

A.4 TURBULENT KINETIC ENERGY EQUATION [TKEE]

(Fluctuating Turbulent Flow Energy Equation)

Turbulent kinetic energy equation is obtained by subtracting mean flow energy equation (A.7) from total turbulent energy equation. Then setting $i = j$ we get TKEE as

$$\begin{aligned}
\overline{2u_i u_k} \frac{\partial U_i}{\partial x_k} + \overline{U_k \frac{\partial \overline{u_i^2}}{\partial x_k}} + \frac{\overline{\partial u_i^2 u_k}}{\partial x_k} = \\
- \frac{1}{\rho} \left(\overline{u_i \frac{\partial p}{\partial x_i}} + \overline{u_i \frac{\partial p}{\partial x_i}} \right) + 2 \overline{v u_i \frac{\partial u_i}{\partial x_k \partial x_k}}
\end{aligned} \quad (A.8)$$

Introducing boundary layer approximations without pressure gradient i.e.,

$$\frac{1}{\rho} \frac{\partial \overline{p u}}{\partial x} = 0 ; \quad \frac{1}{2} \frac{\partial \overline{u q^2}}{\partial x} = 0 ;$$

$$\text{where } q^2 = \frac{1}{2} (u^2 + v^2 + w^2)$$

$$\text{and using } \frac{\partial V}{\partial y} = - \frac{\partial U}{\partial x}$$

Expanding equation (A.8) we get

$$\overline{u_j u_k} \frac{\partial U_i}{\partial x_k} + \frac{1}{2} \left(\overline{U_k \frac{\partial \overline{u_i^2}}{\partial x_k}} + \frac{\overline{\partial u_i^2 u_k}}{\partial x_k} \right) = - \frac{\overline{u_i}}{\rho} \frac{\partial p}{\partial x_i} + \overline{\gamma u_i \frac{\partial^2 u_i}{\partial x_k \partial x_k}} \quad (A.9)$$

$$\text{But } \overline{u_i \frac{\partial^2 u_i}{\partial x_k \partial x_k}} = \frac{1}{2} \left(\frac{\partial^2 \overline{u_i^2}}{\partial x_k \partial x_k} - \overline{\left\{ \frac{\partial u_i}{\partial x_k} \right\}^2} \right) \quad (A.10)$$

The terms $\overline{u^2} \frac{\partial U}{\partial x}$ and $\overline{v^2} \frac{\partial V}{\partial y}$ are in most cases an order of magnitude smaller than $\overline{uv} \frac{\partial U}{\partial y}$ and they can be neglected.

Since viscous diffusion is negligible, substituting for $i = 1, 2$ in equation (A.9) and adding them upon introducing

$$u_1 = u; \quad u_2 = v; \quad U_1 = U; \quad U_2 = V$$

$$x_1 = x \text{ and } x_2 = y$$

after simplification we get TKEE for two dimensional flow as:

$$\frac{\rho}{2} \left(U \frac{\partial \overline{q^2}}{\partial x} + V \frac{\partial \overline{q^2}}{\partial y} \right) + \frac{\partial}{\partial y} \left(\overline{p v} + \frac{1}{2} \rho \overline{q^2 v} \right) - \tau \frac{\partial U}{\partial y} + \rho \epsilon = 0 \quad (\text{A.11})$$

$$\text{where } \tau = - \rho \overline{u v} ; \quad \epsilon = \nu \left(\frac{\partial \overline{u}_i}{\partial x_k} \right)^2$$

Appendix B
DETAILS OF PROGRAM

B.1 OPTIONS AVAILABLE IN THE PROGRAM

In the program several options are available. They can be summarized as follows:

- i) To generate velocity and shear stress profiles at the beginning point by Coles' velocity profile family specifying θ , $U_e \theta/\nu$ and C_f (referred to as synthetic profile generation), or to experimental velocity and shear stress profiles at that point.
- ii) Option for separate dU_e/dx input or dU_e/dx from 3-point parabolic fit to U_e which is useful for free-stream velocity distribution with rapid change of gradient

$$-\frac{1}{\rho} \frac{\partial p}{\partial x} = U_e \frac{dU_e}{dx} \quad (B.1)$$

- iii) Variable surface roughness for given equivalent sand-grain roughness at given X (alternatively roughness parameter Z_0 can be input) or smooth surface. (The additive constant in the logarithmic velocity profile being evaluated from an analytical fit to the classical results of Nikuradse's sand roughness data [Schlichting(1979);Fig. 20.21]),

- iv) Distributed surface transpiration, the inner boundary condition being the "bilogarithmic" law. (With an analytic fit to a consensus of experimental data for the additive constant),
- v) Allowance for the effect of low Reynolds number on the outer layer, (by changing the length scale L in outer layer only) by an empirical function

$$\frac{\delta}{v} \sqrt{\left(\frac{\tau_{\max}}{\rho}\right)} \quad (\text{B.2})$$

to produce the observed variation of outer layer profile shape.

- vi) Allowance for the effect of longitudinal curvature and lateral divergence on the turbulence structure.
- vii) Output at selected values of X . This is achieved by modifying X step in calculations such that it is equal to the requested X step.

The option (vi) is an important factor in diffuser flows. This divergence effect is taken care of by subtracting $\tau \Sigma a e_{\text{eff}}$ term in dissipation term $(\tau^{1.5}/L)$ in the shear stress equation. a being an empirical constant and is of the order of 10 which is different for each extra strain rate e and each e_{eff} is the solution of a "lag equation" or "low-pass filter"

$$10\delta \frac{de_{\text{eff}}}{dx} = (e - e_{\text{eff}}) \quad (\text{B.3})$$

where 10δ is an empirical large-eddy "life time". The allowance for the effect of divergence on the turbulence can be suppressed in the program leaving only its effect on the continuity equation. Typical values of a for the effects of divergence and curvature can be listed as

$$\begin{aligned} a &= 10 \text{ for divergence} \\ &= 14 \text{ for convex curvature} \\ &= 9 \text{ for concave curvature} \end{aligned}$$

B.2 PROGRAM DESCRIPTION

A flow diagram of the software used is given in figure 1. In the main program minor calculations and calling of appropriate subroutines is done. ORDIN is an interpolating subroutine, which interpolates tables linearly or parabolically depending upon the sign of step (linear if positive, parabolic if negative) in calling function.

The program begins with a BLOCK DATA subprogram defining tables of various parameters such as

$$\frac{G}{\sqrt{(\tau_{\max}/U_e^2)}} \quad , \quad \frac{\delta}{\sqrt{(\tau_{\max}/U_e^2)}} \frac{dG}{dy}$$

dissipation length scale parameter L and stress-energy ratio a_1 at an interval of 0.05δ from $y = 0$ to $y = 1.45\delta$. Appropriate block data of a_1 and L are used while executing the program. The various curves of a_1 and L used here for analysis is given in figures 4 and 5 respectively.

Then the program calls START which reads all the data and different options (as mentioned in section B.1) required in particular cases. If velocity and shear stress profiles are not read in, it calls for SYNTH subroutine in which it generates velocity and shear stress profile making use of given values of θ , $U_e \theta/\nu$ and C_f , with Coles' velocity profile family

$$\frac{U}{u_*} = \frac{1}{\kappa} \log \frac{u_* y}{\nu} + C + \frac{\pi}{\kappa} (1 - \cos \frac{\pi y}{\delta}) \quad (B.4)$$

known as "log law plus cosine wake" and a mixing-length assumption for shear stress profile.

After getting velocity and shear stress profiles in SYNTH θ is converted into y step and $U_e \theta/\nu$ to $U_{ref} L/\nu$. At the end of START U and τ profiles are re-normalized by U_{ref} instead of U_e . START and SYNTH (if velocity and shear stress profiles are not read in) are used only once during a run and they are not called anywhere after this.

After START, the main program adds two extra "free stream" points to each profile and transfers the recently inputted or calculated profiles (UFUT and TFUT) into the current arrays of U and TAU . With this the new values of UFUT and TFUT are obtained by forward integration in NEWPRO. τ_{max} and $\delta_{0.5}$ are evaluated at this stage which are used in scaling G . Then INTGEQ, VANGI and CFL are called in the above order.

INTGEQ first evaluates displacement thickness δ^* and momentum thickness θ by integrating velocity profile, at current X . Then it is again calculated from the momentum-integral equation (assigned as SCHMET to differentiate THETA calculated by using velocity profile). The difference between THETA and SCHMET is a measure of accuracy of the numerical procedure as a whole. VANGL calculates V by integrating the equation along the vertical characteristic and TANCAL calculates the characteristic angles ($\tan A$ and $\tan B$) at the current value of X for later use in NEWPRO. CFL calculates the next X step where the profiles need to be calculated. If this next step would pass, or nearly reach the next value of X at which PRINT is called, the X step is reassigned so as to reach that value.

To obtain the effective value of extra strain rate an optional routine EXSTRA is called (where it solves ordinary differential lag equation (B.2)). It is assigned to the variable STRAIN the value of $\Sigma a_{eff} / U$ at the current X for use in NEWPRO and WALLBC. In next step PRINT is called if the value of X or the number of steps executed so far satisfy the required conditions.

The program now checks whether the number of steps performed or the X distance covered has reached the maximum required or whether the maximum X would be exceeded on the next step. If so, the run is stopped and the next case is begun, if any. If the roughness length input is positive then ROUGH simulates Nikuradse's sand-roughness data.

After assigning boundary values at X (i.e., X step and the values of $U_e dU_e/dx$) the program calls the wall boundary conditions routine WALLBC to calculate τ_w and U , V and τ at the first mesh point at $X + X$ step. Here modified logarithmic law equation [Bradshaw et al.(1967);Eq. 18], the ordinary differential equation along the characteristic [Bradshaw & Unsworth(1974);Eq. 7] and shear stress equation [Bradshaw & Unsworth;Eq. 11] (formula obtained by substituting a $1/5$ th power law velocity profile into the momentum equation and integrating between the wall and the first mesh point at $X + X$ step) forms three equations for τ_w , τ_1 and U_1 . The increment in U_1 , a_0 and τ_w (DU , DA and DTW) are calculated according to linearized approximation to reduce the above three equations ($F1$, $F2$ and $F3$ respectively in the program) to zero until the iteration for solid wall (or transpiration) has converged.

After achieving convergency for $F1$, $F2$ and $F3$ $V(1)$ is calculated at $X + X$ step by integrating the continuity equation from wall to the first calculation point (assuming $U/u^* = f(u^*y/\nu)$), given by

$$V(1) = V_w - \frac{U_1 y_1}{u^*} \frac{du^*}{dx} - \left(\frac{1}{U} \frac{\partial w}{\partial z} \right) \int_w^1 U dy \quad (B.5)$$

where $(1/U) \partial w / \partial z$ is independent of y and equal to $1/(x-x_0)$ in a laterally diverging flow.

In WALLBC functions PHI and F are called by the solid-wall and transpiration options respectively which are analytical functions appearing in the generalized logarithmic laws obtained by integrating the mixing-length formula with a shear stress varying in y direction. The effect of extra strain rate has been incorporated in solid-surface option by writing the turbulent energy equation in local equilibrium form with empirical allowance for extra strain rate as

$$\tau \frac{\partial U}{\partial y} = \frac{\tau^{3/2}}{L} - \left(\frac{\alpha_e}{U}\right) \tau U \quad (B.6)$$

where $\alpha_e/U = \text{STRAIN}$. If τ , or τ falls below zero at any step of the iteration the flow is deemed to have separated and $\tau_w = 0$ is returned to the main program as a signal to stop the run.

If separation has not occurred, the main program calls routine NEWPRO to calculate U and τ at X + X step (UFUT and TFUT) from second mesh point to the edge of the layer using the remaining boundary conditions. Here method of characteristic is adopted in calculating U and τ profiles from second mesh point to the edge of the layer. AL array (2 X 3) in NEWPRO calculates the co-efficients in the pair of characteristic equations along lines intersecting at the downstream mesh point [figure 3], one for in going characteristic (I1 = 1) and the other for out going characteristic (I1 = 2). Since the method of characteristic is explicit no future values of U and τ at other positions are required. A

check is made for unphysical values of y , U and τ (such as $y > y_{\max}$, $\tau < 0$ and $U > U_E$). The numerical calculation is an initial value problem because the downstream profile lies entirely in the region of influence of the upstream profile. The physical problem thus requires specification of one boundary condition at the outer edge (which is satisfied by giving velocity at outer edge of boundary layer).

The rapid stream wise changes are most likely in the inner layer as the boundary layer approximation requires the outer layer to change slowly. Thus after calculation of the downstream values of U and τ , values of V and the characteristic angles are calculated in the inner layer only. The calculations of U and τ in the inner layer is repeated with co-efficients interpolated between the upstream values and the newly calculated downstream values. The values of V and the characteristic angles corresponding to the final values of U_{FUT} and τ_{FUT} are calculated throughout the layer by the `VANGL` routine in the next step. In brief, the calculations at the first mesh point is iterated to converge in `WALLBC`. The calculation in the inner layer is iterated twice and the calculation in the outer layer is performed only once in `NEWPRO`.

Finally the main program calls `REDUCE` to reduce the number of points on the profile if it exceeds the required optional limit `I(11)`. It reduces the number of points on the profile by $2/3$ of previous number and increases y step ap-

propriately by a factor of $3/2$. To obtain velocity and shear stress profiles at new y , τ and $\log U$ are interpolated linearly. REDUCE is suppressed if $C_f < 0.0005$ and near separation, provided that the number of profile points is not about to exceed the array limit. Then the main program increments the X step count by 1 and the value of X by X step, and returns to the beginning of the main calculation loop.

B.3 EMPIRICAL INPUT PRESET IN PROGRAM

- i) Logarithmic-law constants κ is taken as 0.41 in WALLBC, C as 5.2 in the main program. Overwritten by ROUGH subroutine, if called, by linear fit to Nikuradse's sand-roughness data and again in WALLBC, if transpiration velocity is non-zero, by analytical fit to Rotta (1970) data.
- ii) The empirical constant of order 10 in the allowance for extra strain rates and time constant, 10δ , in the low-pass filter applied to the allowance set in EXSTRA.
- iii) Cosine-law wake profile and mixing-length profile in SYNTH.
- iv) The empirical function correcting L for low-Reynolds-number effects, set in the main program and used in the assignment of RL in NEWPRO.

TABLE 1

Comparison of Boundary Layer Parameters At Station 2.27 m
From The Trip

U	TYPE	δ	θ	H	R_θ
4.27	Experimental	46.00	5.46	1.346	1466
	Calculated	49.31	5.46	1.346	1467
8.03	Experimental	46.00	5.46	1.346	2756
	Calculated	46.34	5.46	1.379	2757
12.01	Experimental	46.00	5.46	1.346	4125
	Calculated	48.97	5.46	1.339	4127

TABLE 2

Turbulent Boundary Layer Parameters At Three Stations

(with free stream velocity 8.03 m/sec)

STATION CASE*		δ	θ	H	R_θ	C_f
2.27	I	46.00	5.460	1.3460	2756	3.341
	II	46.34	5.463	1.3789	2757	3.341
	III	50.17	5.535	1.4051	2796	3.341
2.28	I	45.85	5.514	1.3620	2785	3.317
	II	46.43	5.479	1.3781	2765	3.335
	III	49.66	5.533	1.4055	2794	3.059
2.29	I	45.06	5.519	1.3620	2788	3.296
	II	46.53	5.497	1.3778	2775	3.335
	III	49.83	5.563	1.4053	2810	3.007

- * CASE I - Experimental data of Kassab(1985)
CASE II - Calculated using parameters at Station 2.27 m
with Coles' velocity profile
CASE III - Calculated using data at station 2.27 m

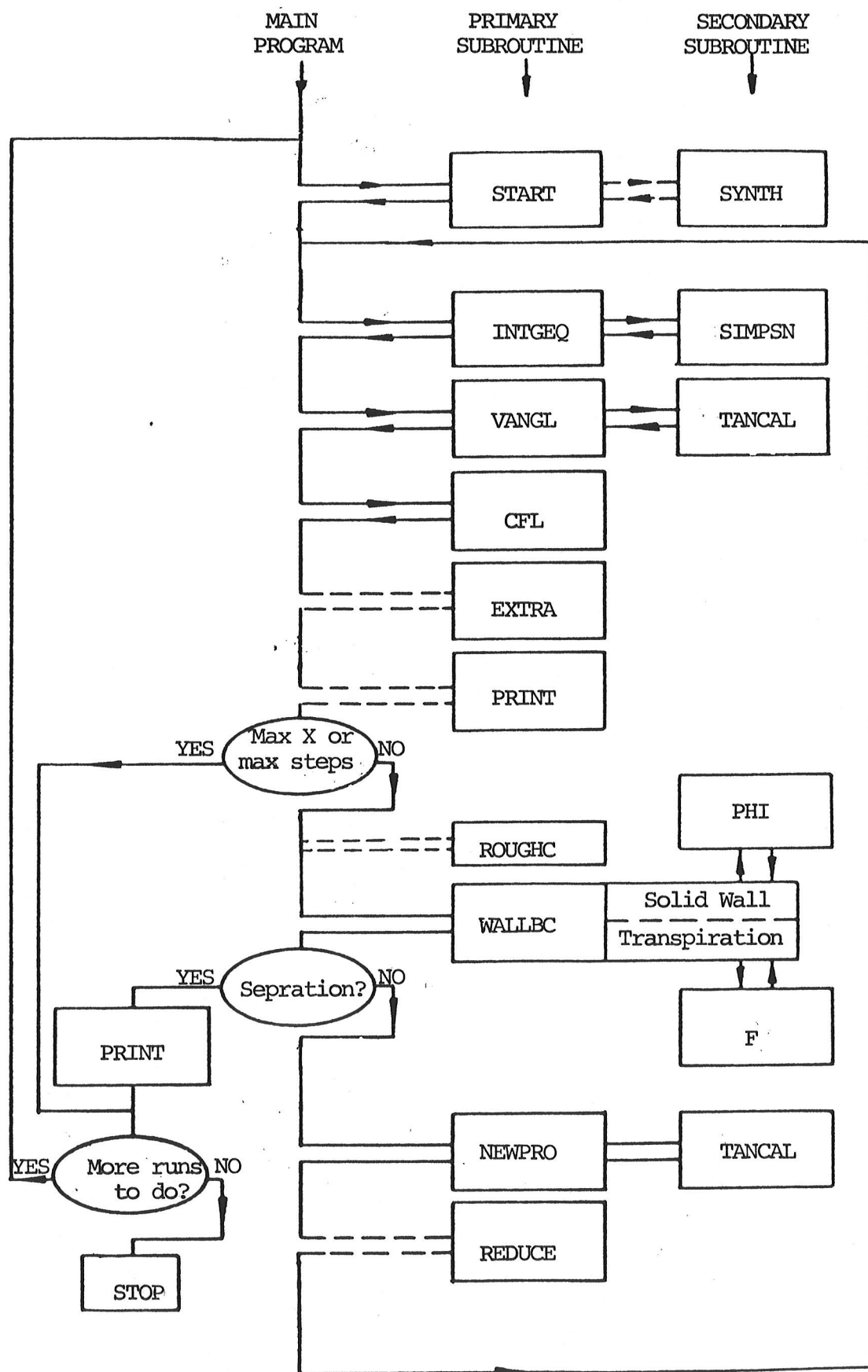


Figure 1: Flow Diagram. Dotted lines indicate optional subroutines

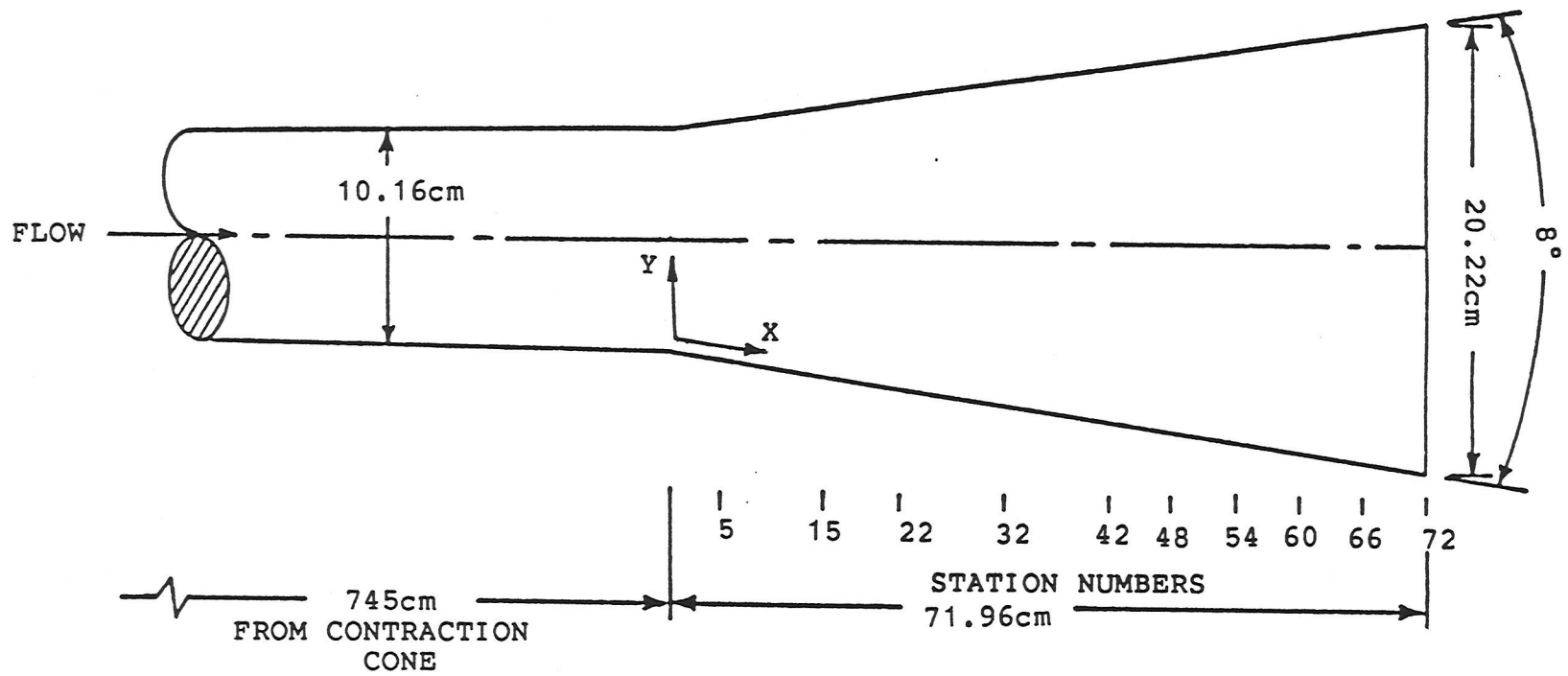


Figure 2: Diffuser Geometry

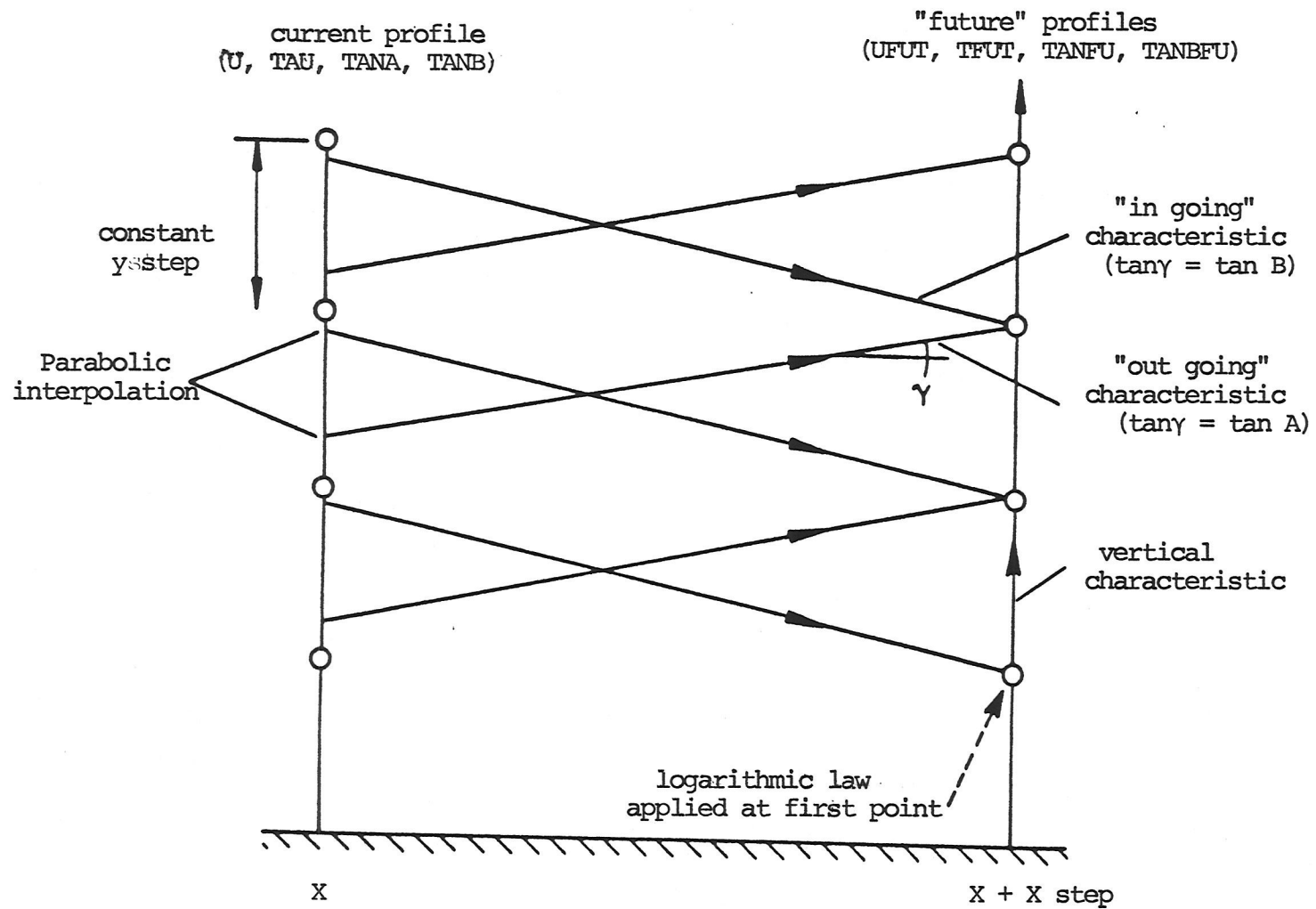


Figure 3: Characteristic MESH (Angles exaggerated)

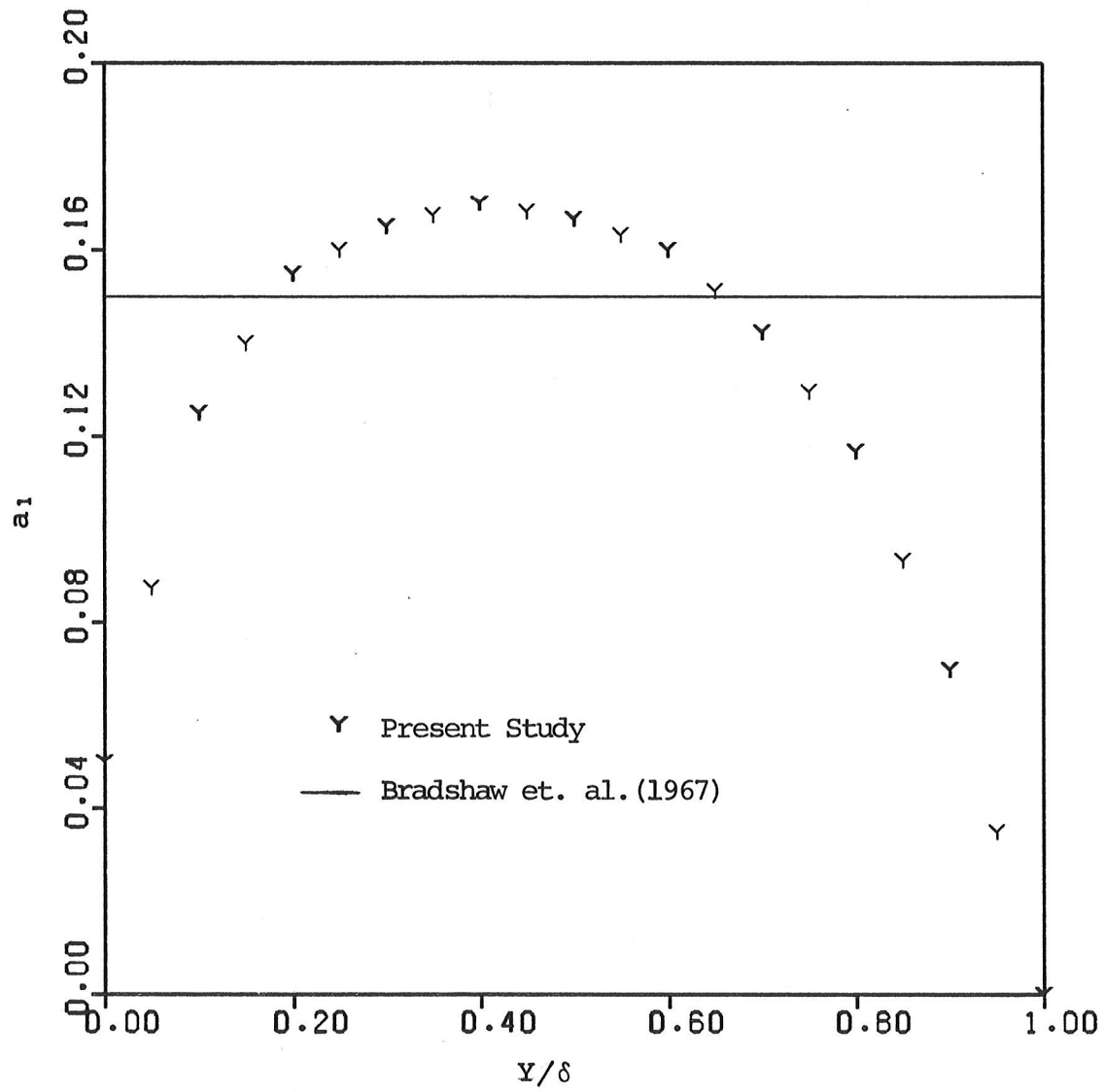


Figure 4: Variation of Stress-Energy ratio a_1 in stream wise direction

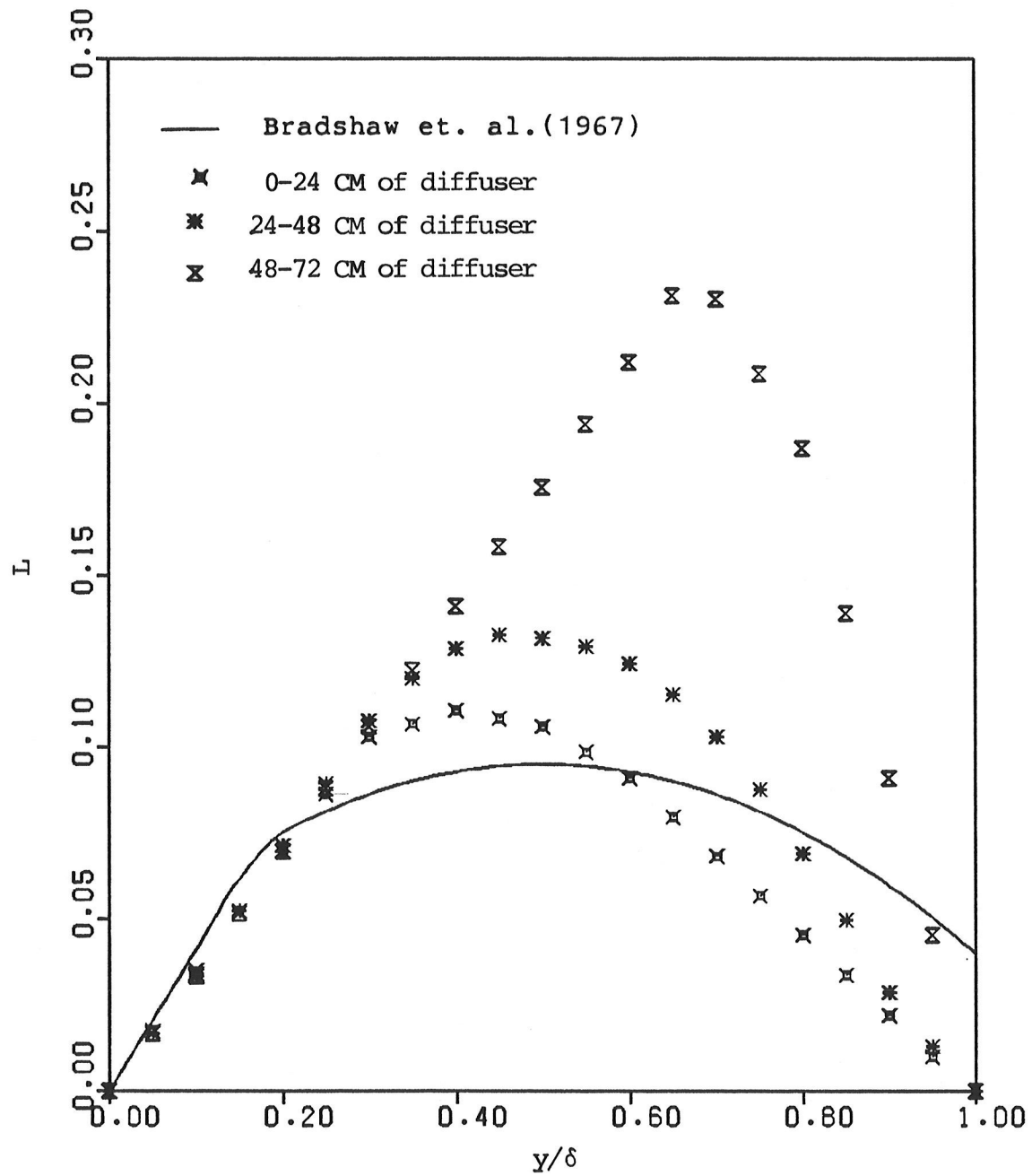


Figure 5: Variation of Dissipation Length Scale L in stream wise direction

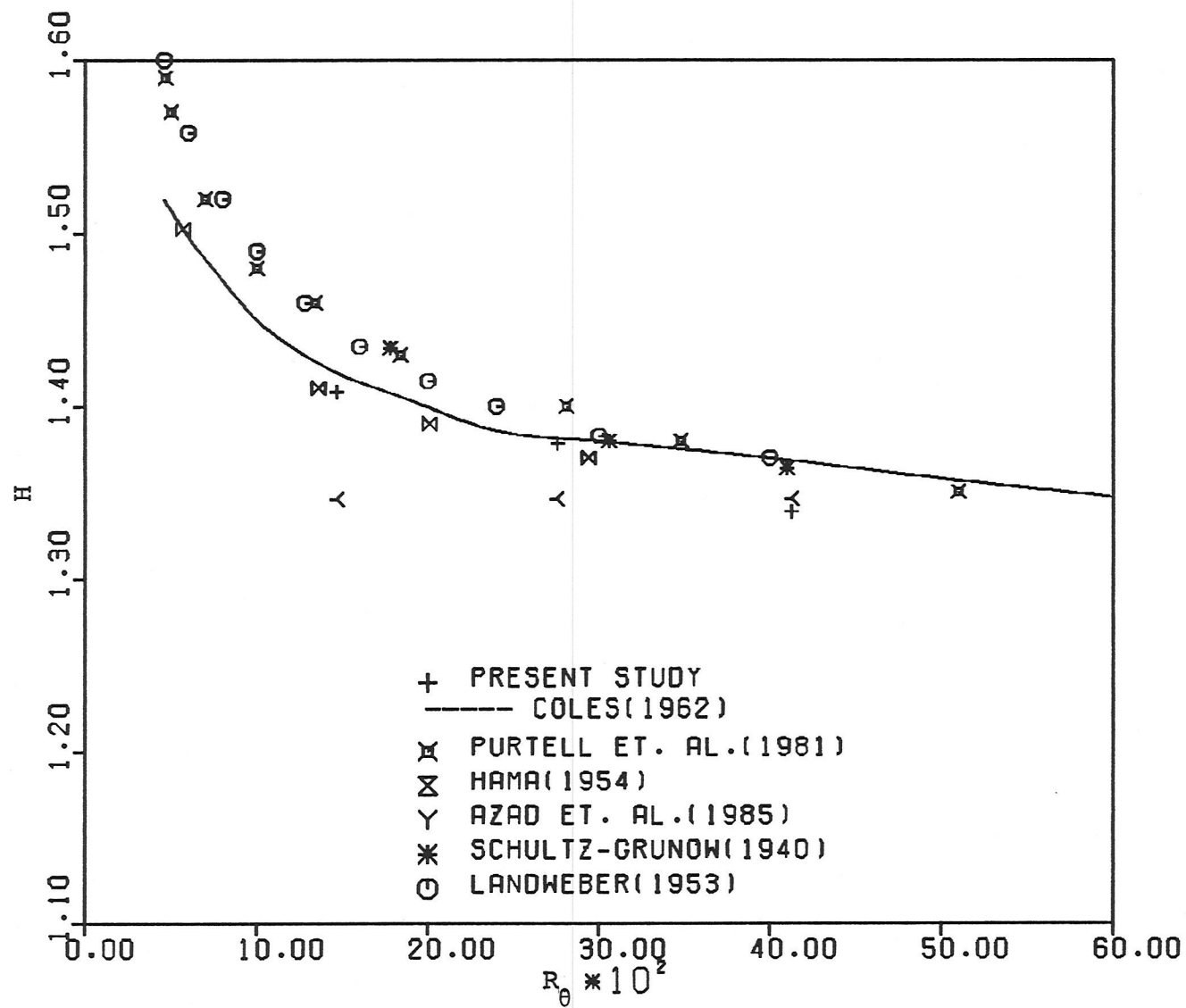
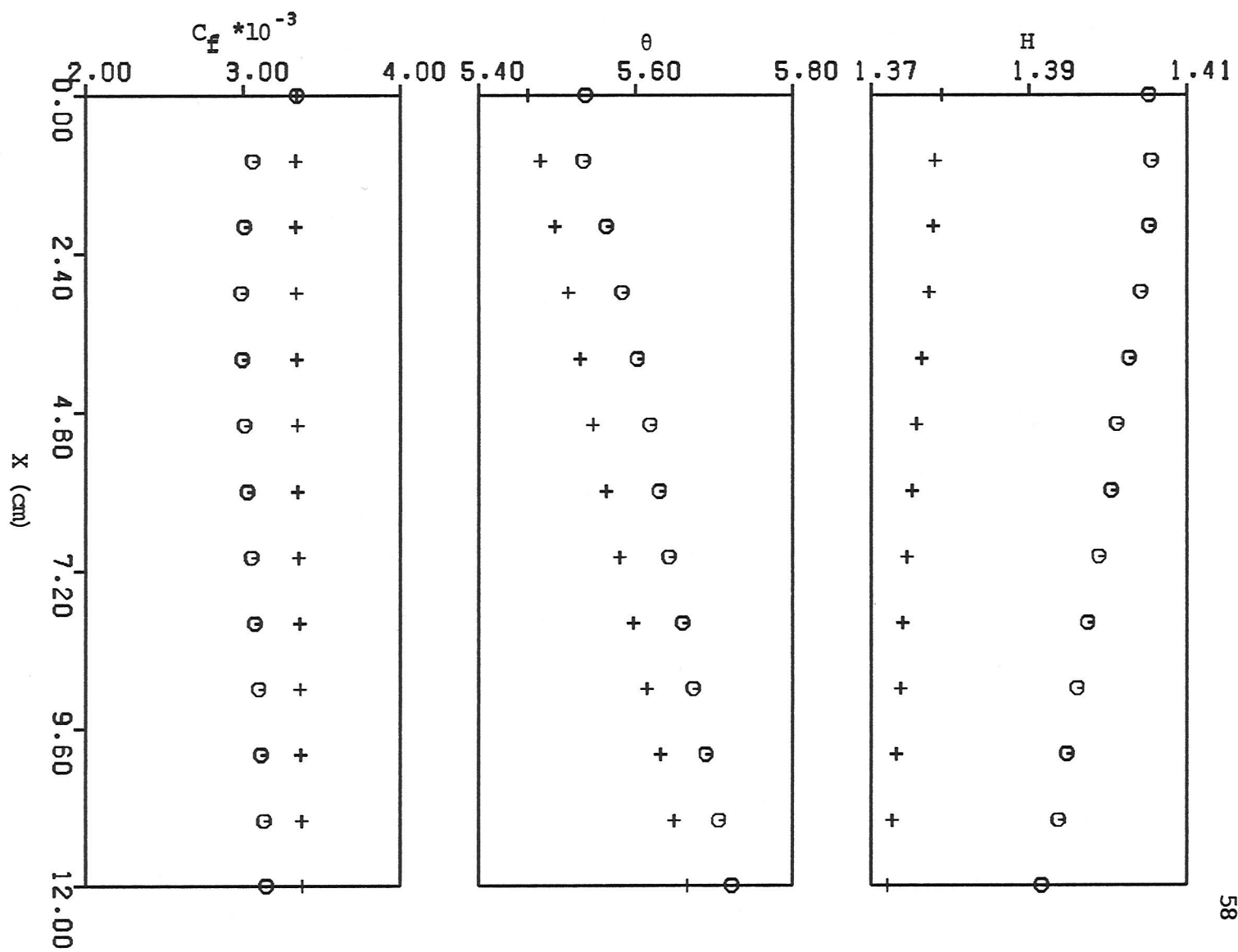


Figure 6: Variation of H with R_θ

THE FOLLOWING SYMBOLS IN FIGURES 7 TO 36 DENOTES THE FOLLOWING:

- EXPERIMENTAL
- PROFILE FED AT STATION# 6
- △ PROFILE FED AT STATION# 6 WITH DIVERGENCE
- + SYNTHETIC PROFILE GENERATION
- × SYNTHETIC PROFILE GENERATION WITH DIVERGENCE

Figure 7: Variation of C_F , H and θ in the streamwise direction



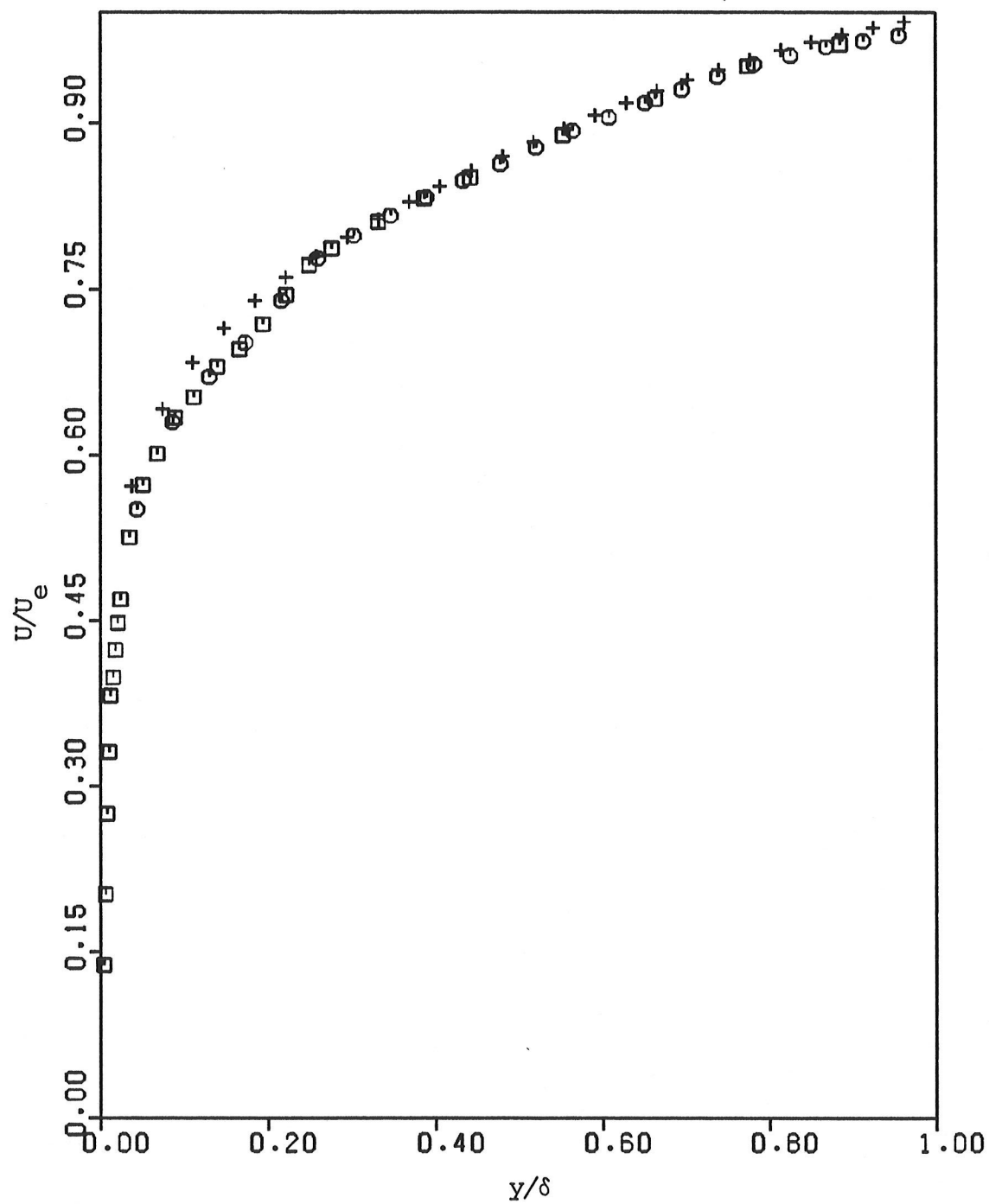


Figure 8: Velocity profile in Turbulent Boundary Layer at Station 2.27 m

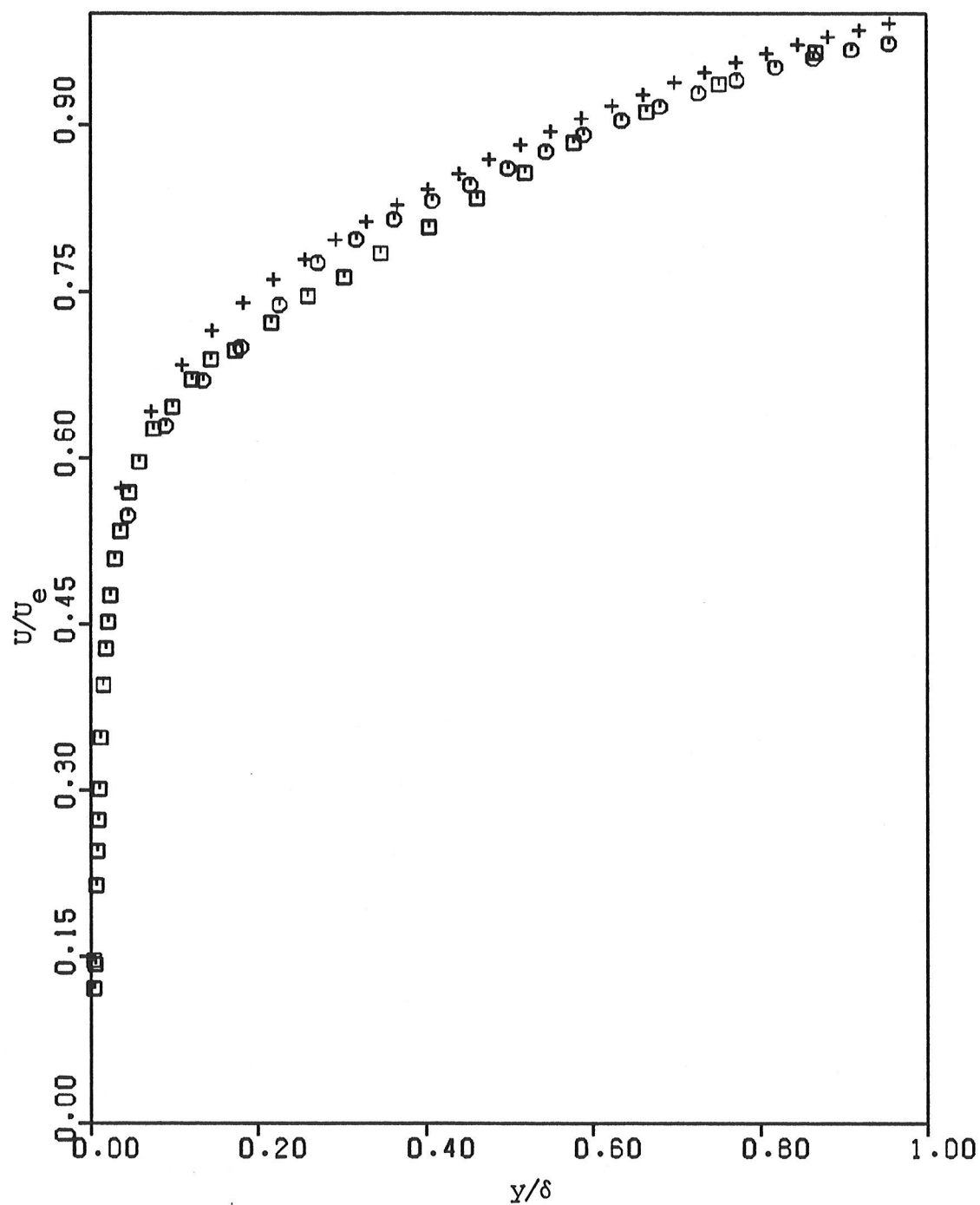


Figure 9: Velocity profile in Turbulent Boundary Layer at Station 2.29 m

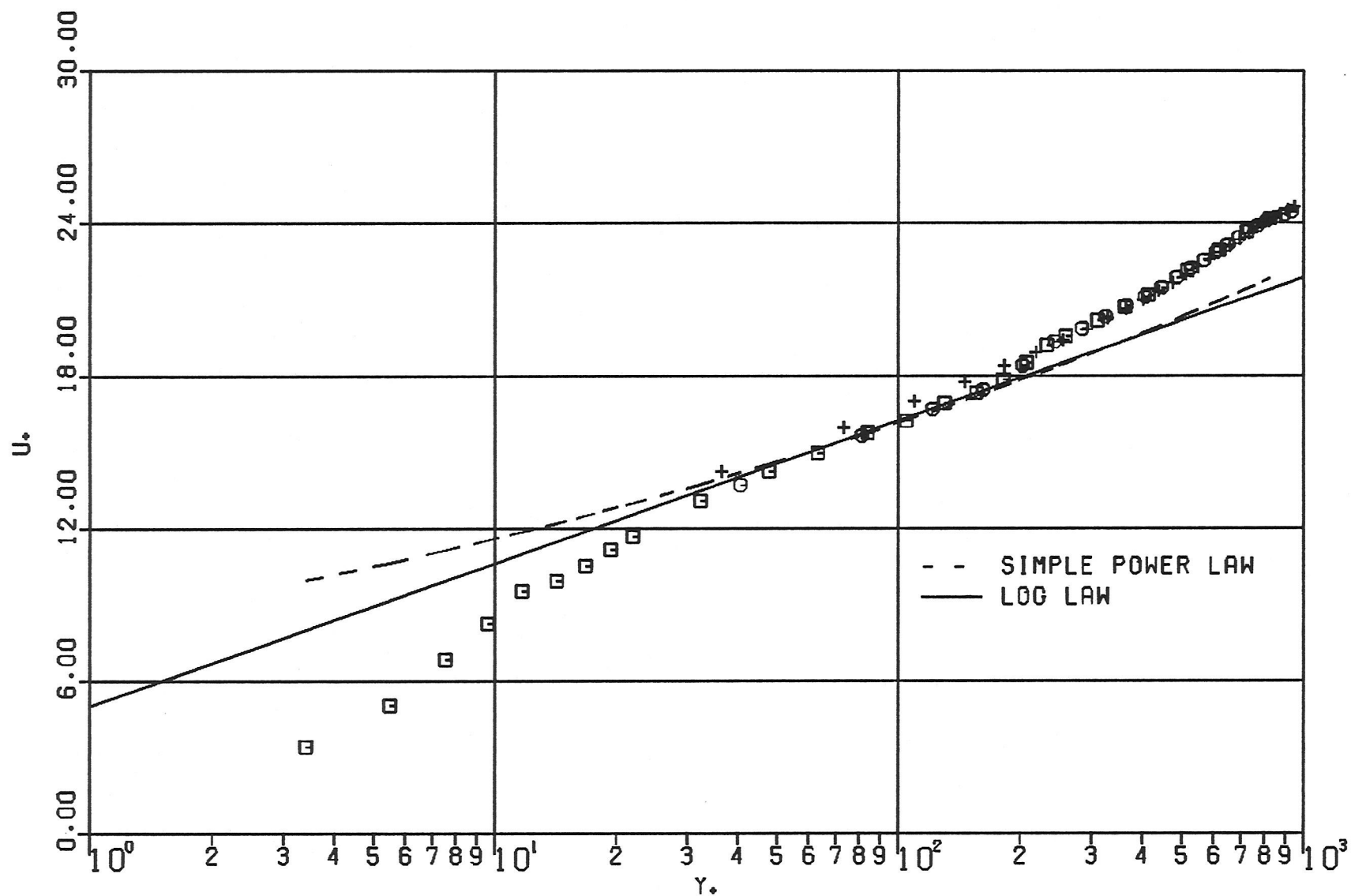


Figure 10: Distribution of U_+ versus Y_+ in the Turbulent Boundary Layer flow at Station 2.27 m

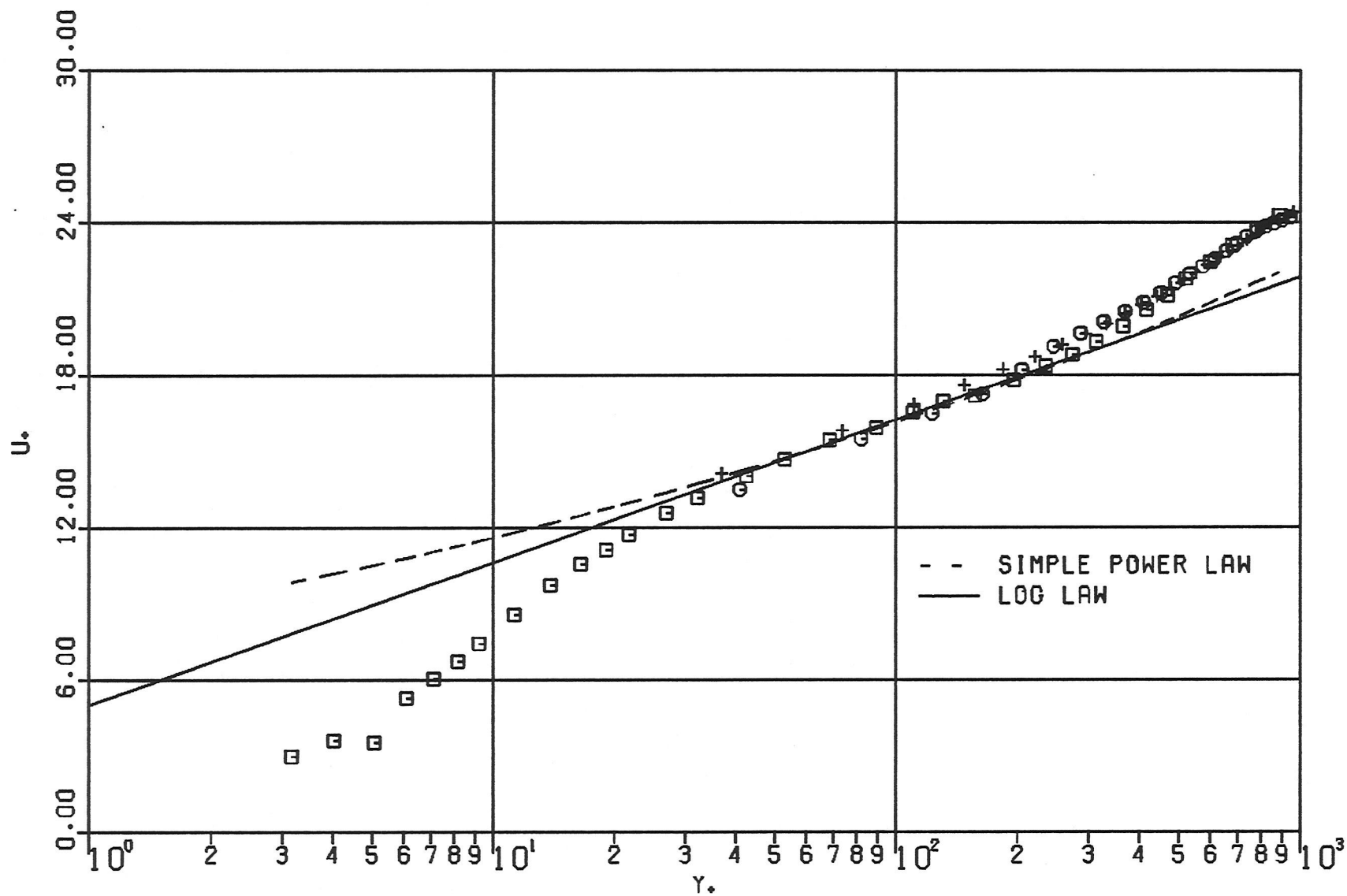


Figure 11: Distribution of U^+ versus Y^+ in the Turbulent Boundary Layer flow at Station 2.29 m

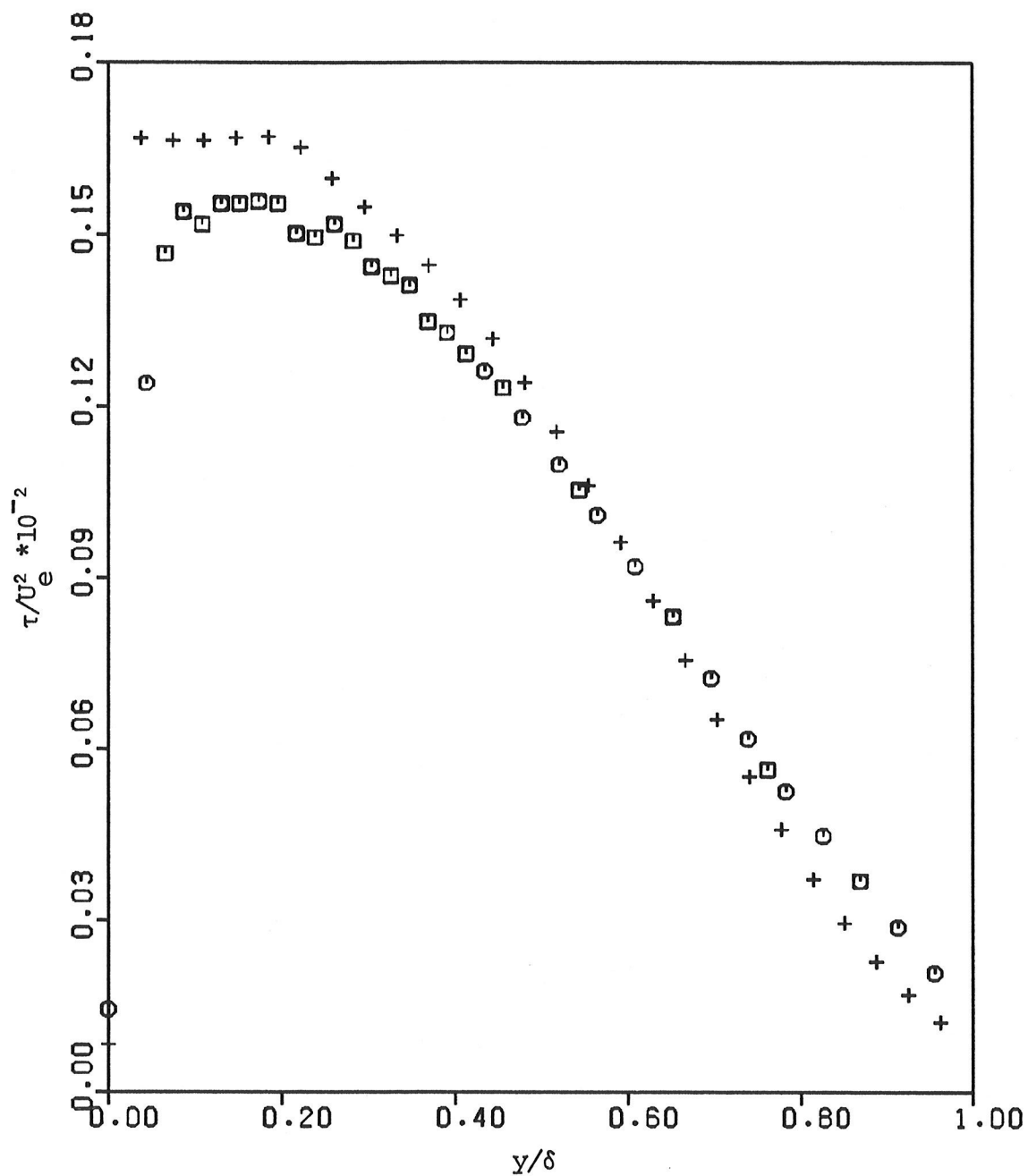


Figure 12: Shear Stress distribution at Station 2.27 m in Turbulent Boundary Layer flow (non-dimensionalized with free stream velocity 8.03 m/sec)

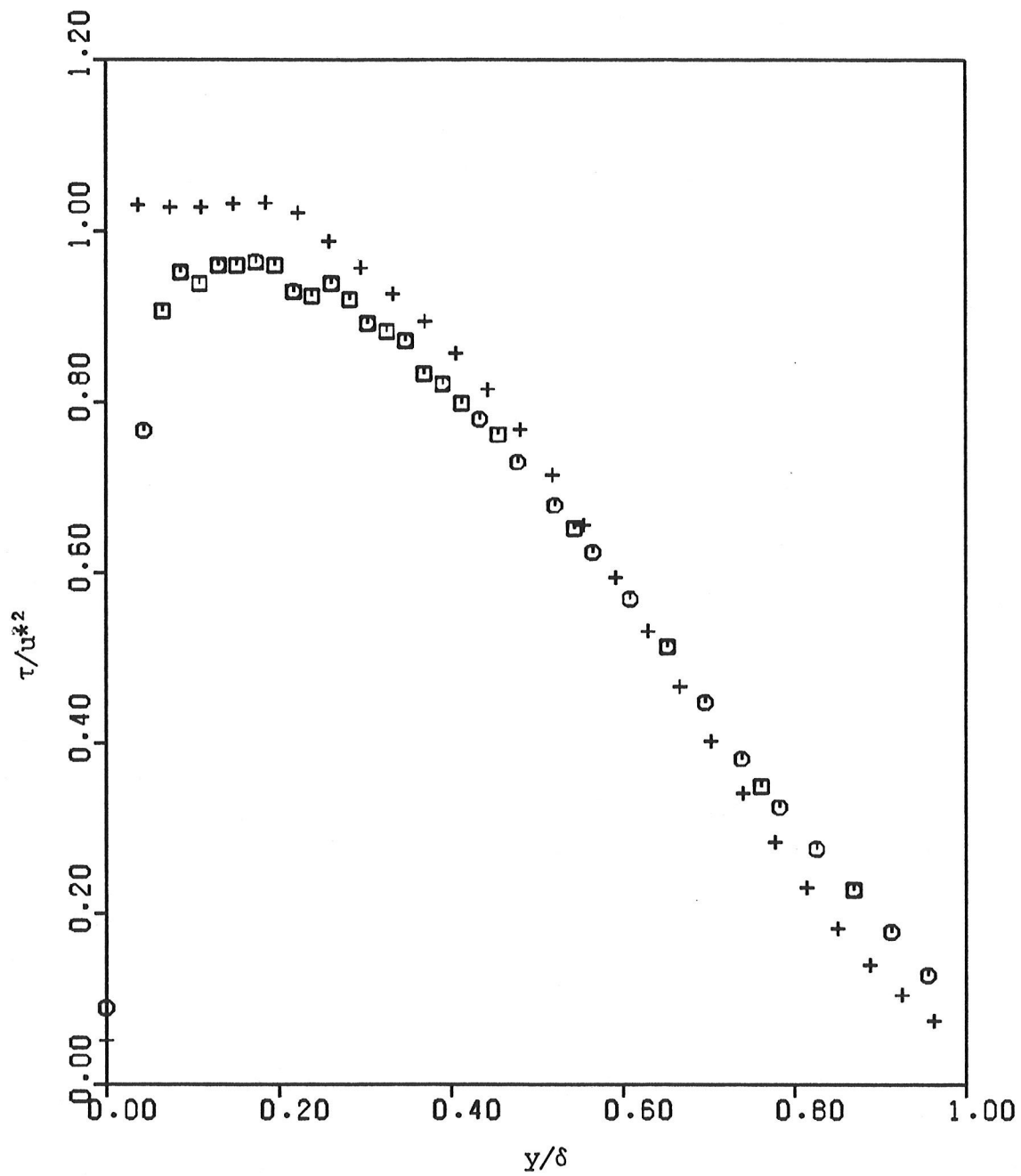


Figure 13: Shear Stress distribution at Station 2.27 m in
Turbulent Boundary Layer flow
(non-dimensionalized with frictional velocity)

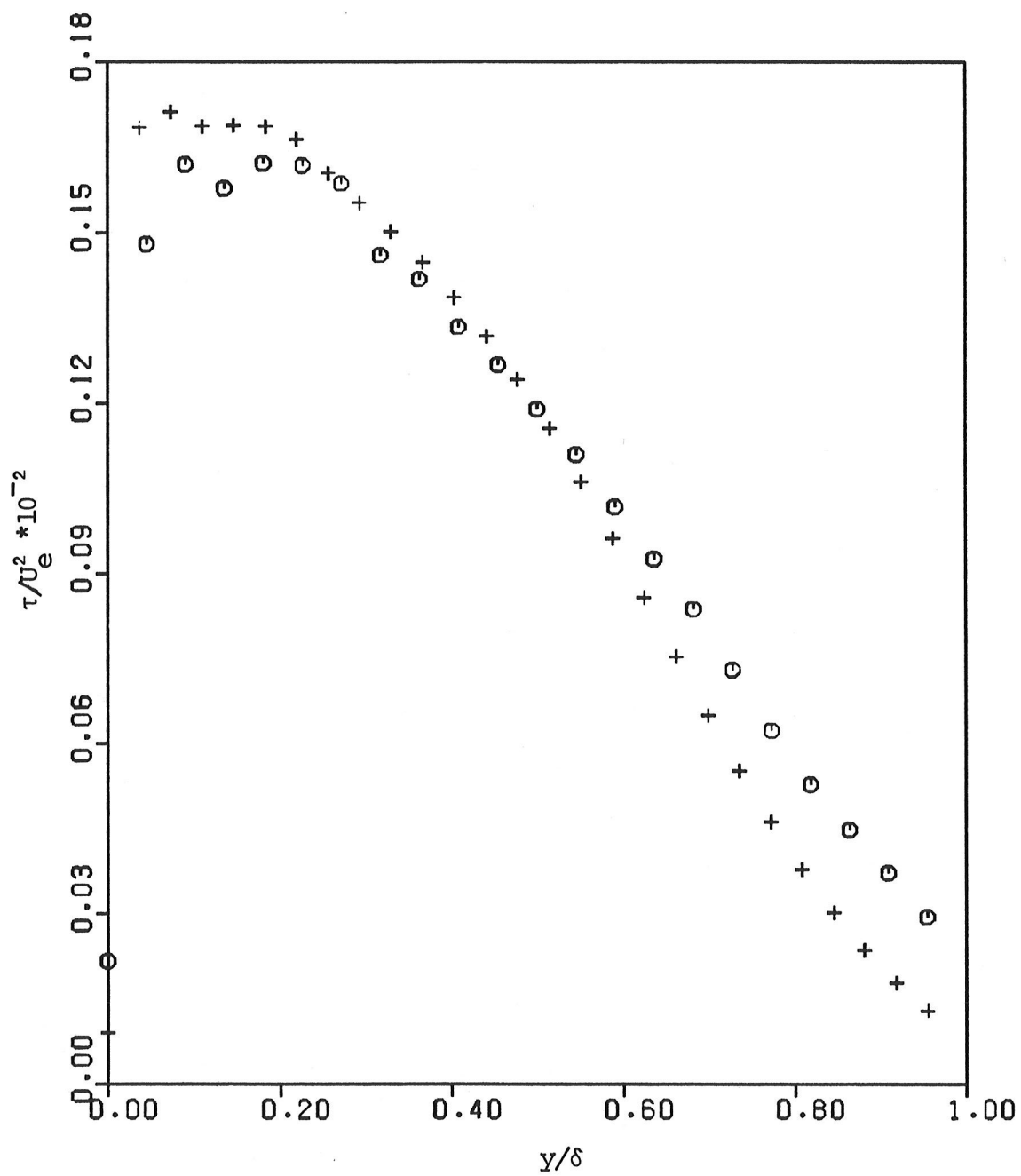


Figure 14: Shear Stress distribution at Station 2.29 m in Turbulent Boundary Layer flow (non-dimensionalized with free stream velocity 8.03 m/sec)

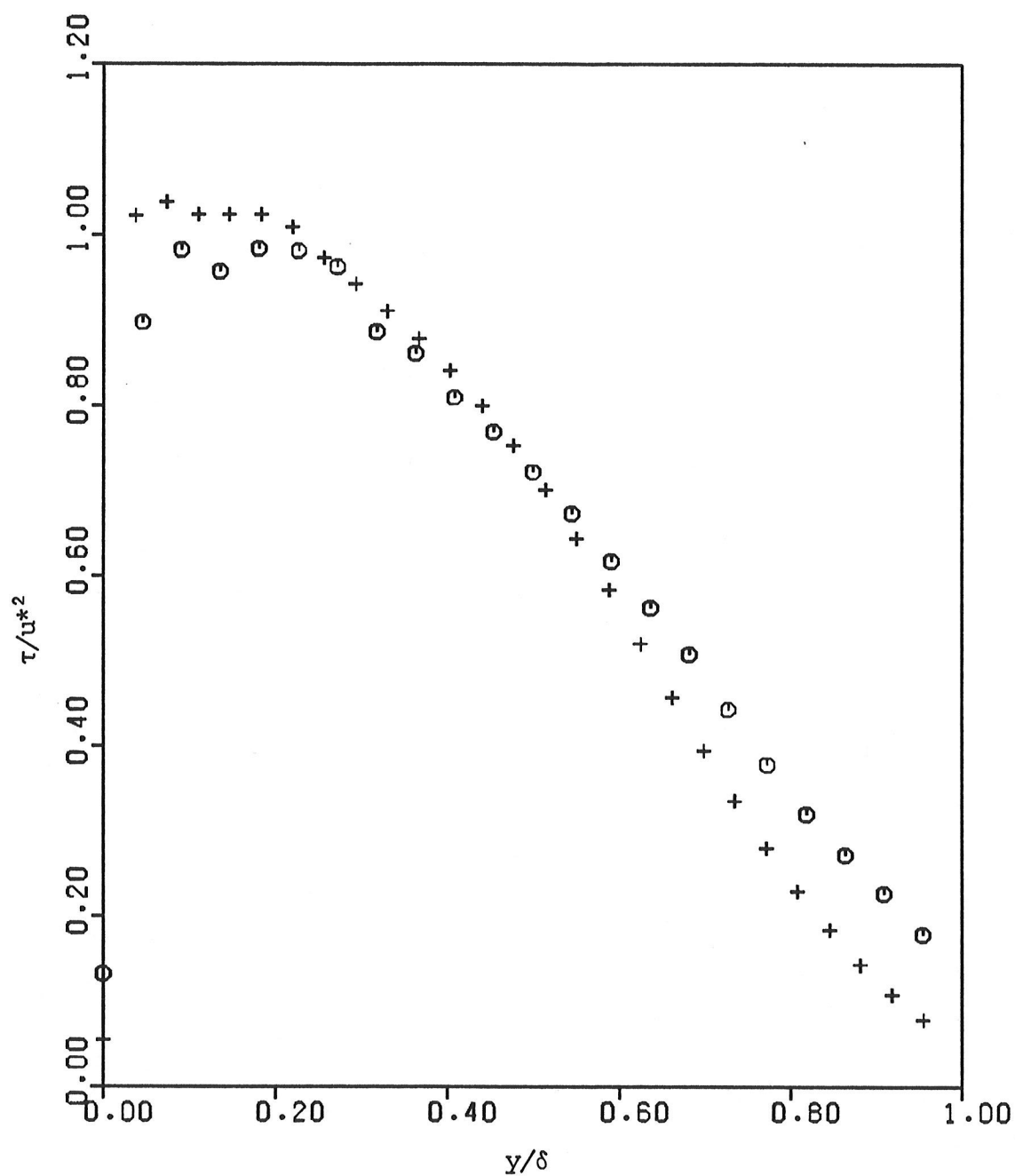


Figure 15: Shear stress distribution at station 2.29 m in
Turbulent Boundary Layer flow
(non-dimensionalized with frictional velocity)

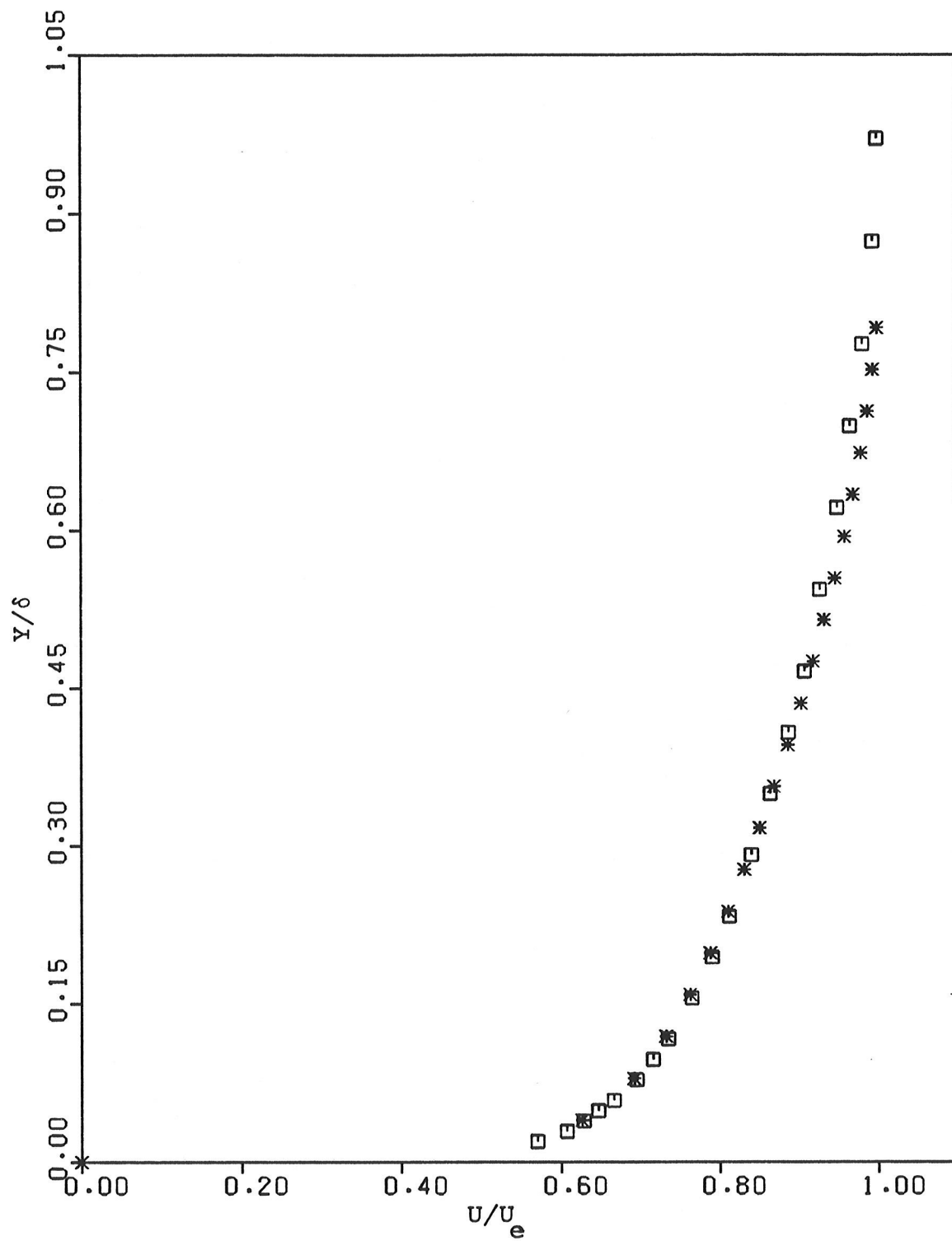


Figure 16 : Velocity profile in 8° conical diffuser at Station 0

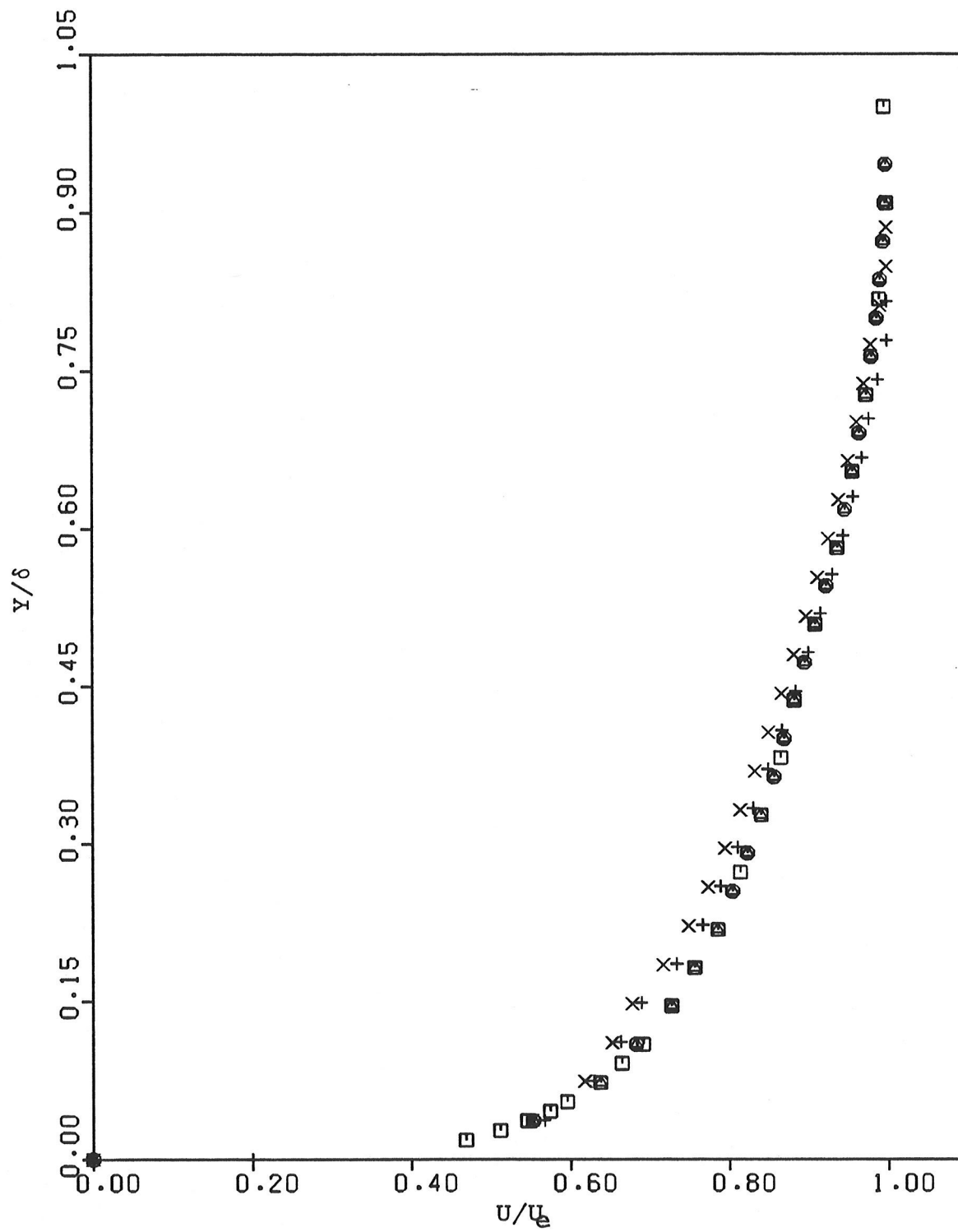


Figure 17: Velocity profile in 8° conical diffuser at Station 6

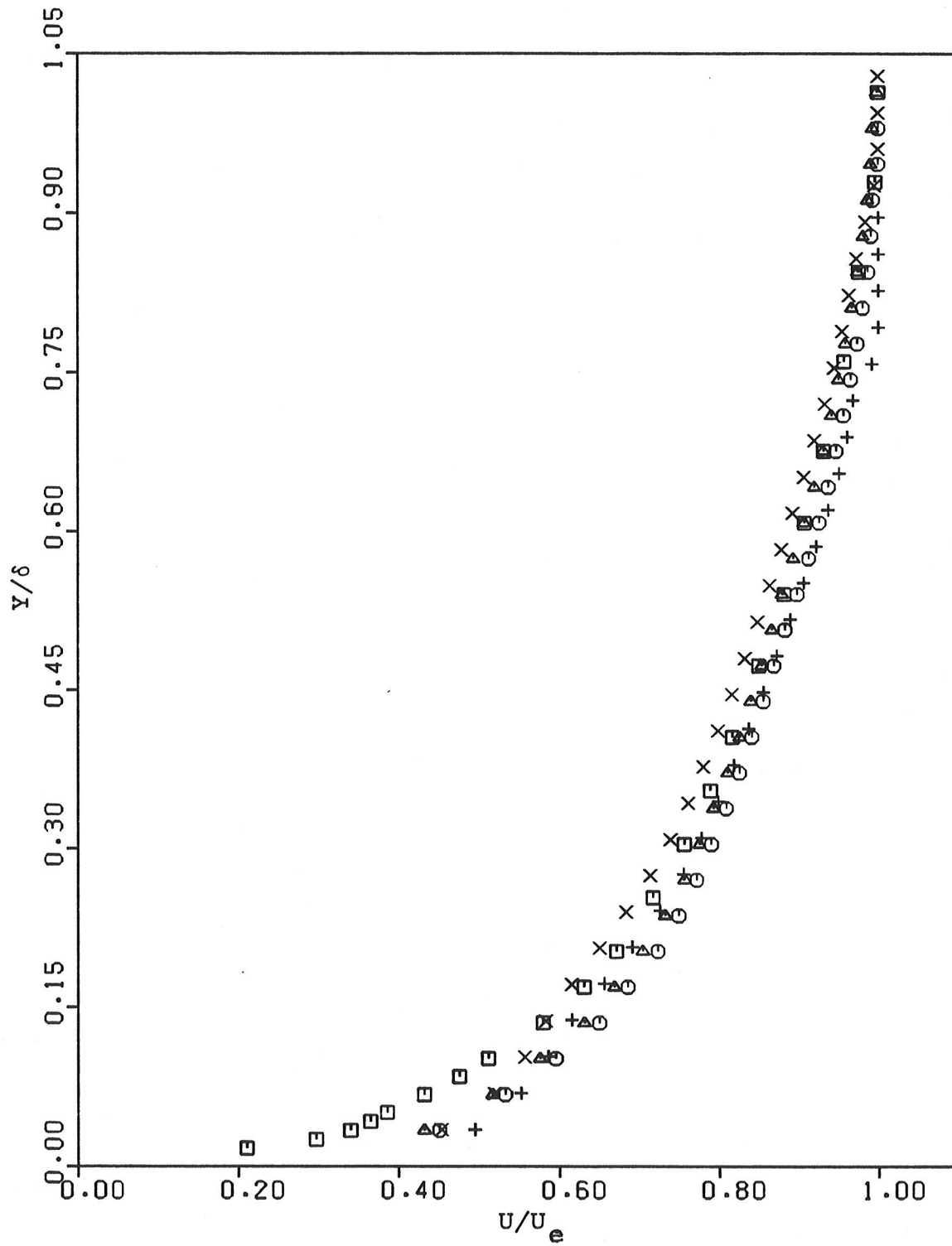


Figure 18: Velocity profile in 8° conical diffuser at Station 12

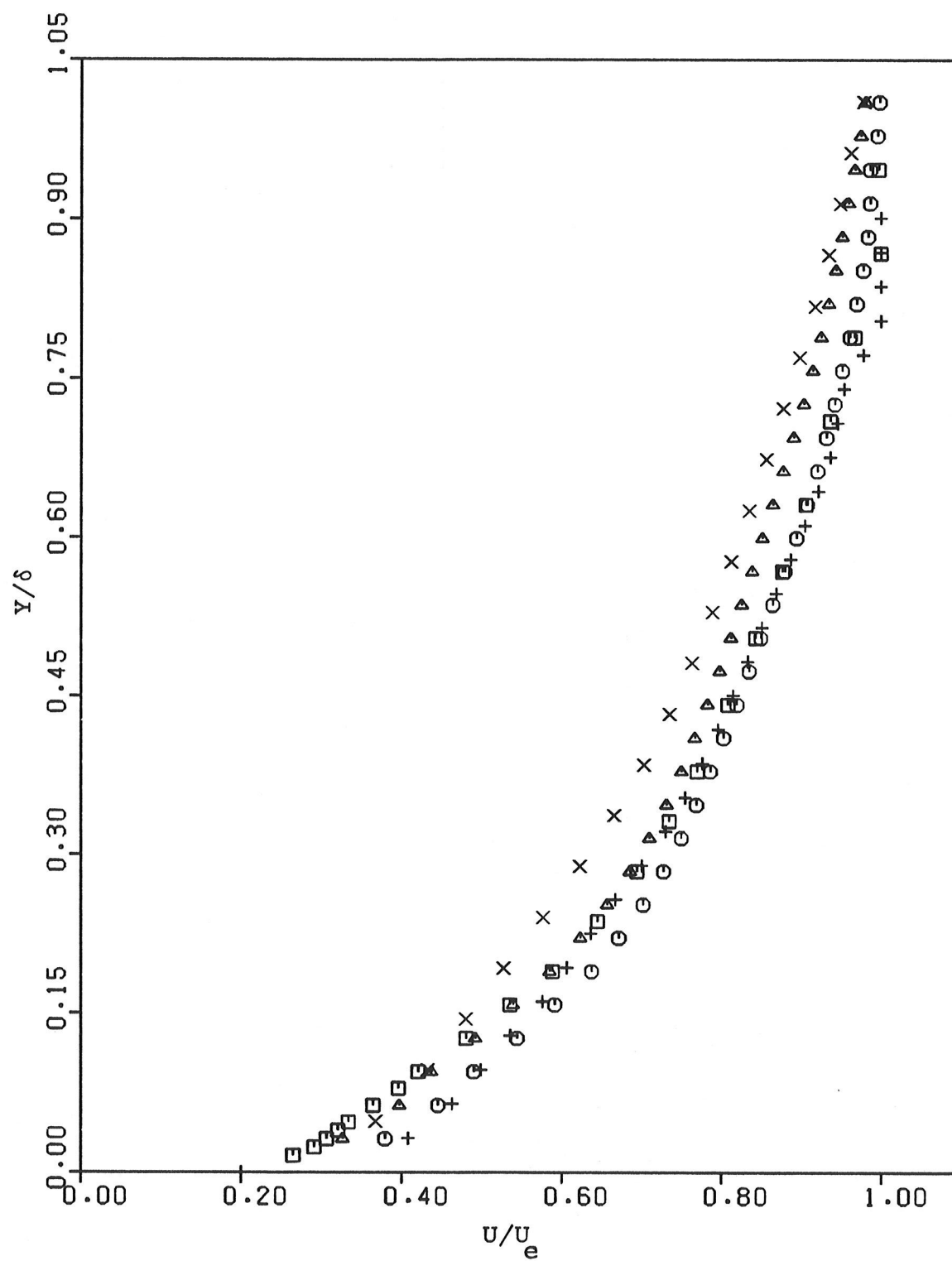


Figure 19: Velocity profile in 8° conical diffuser at Station 18

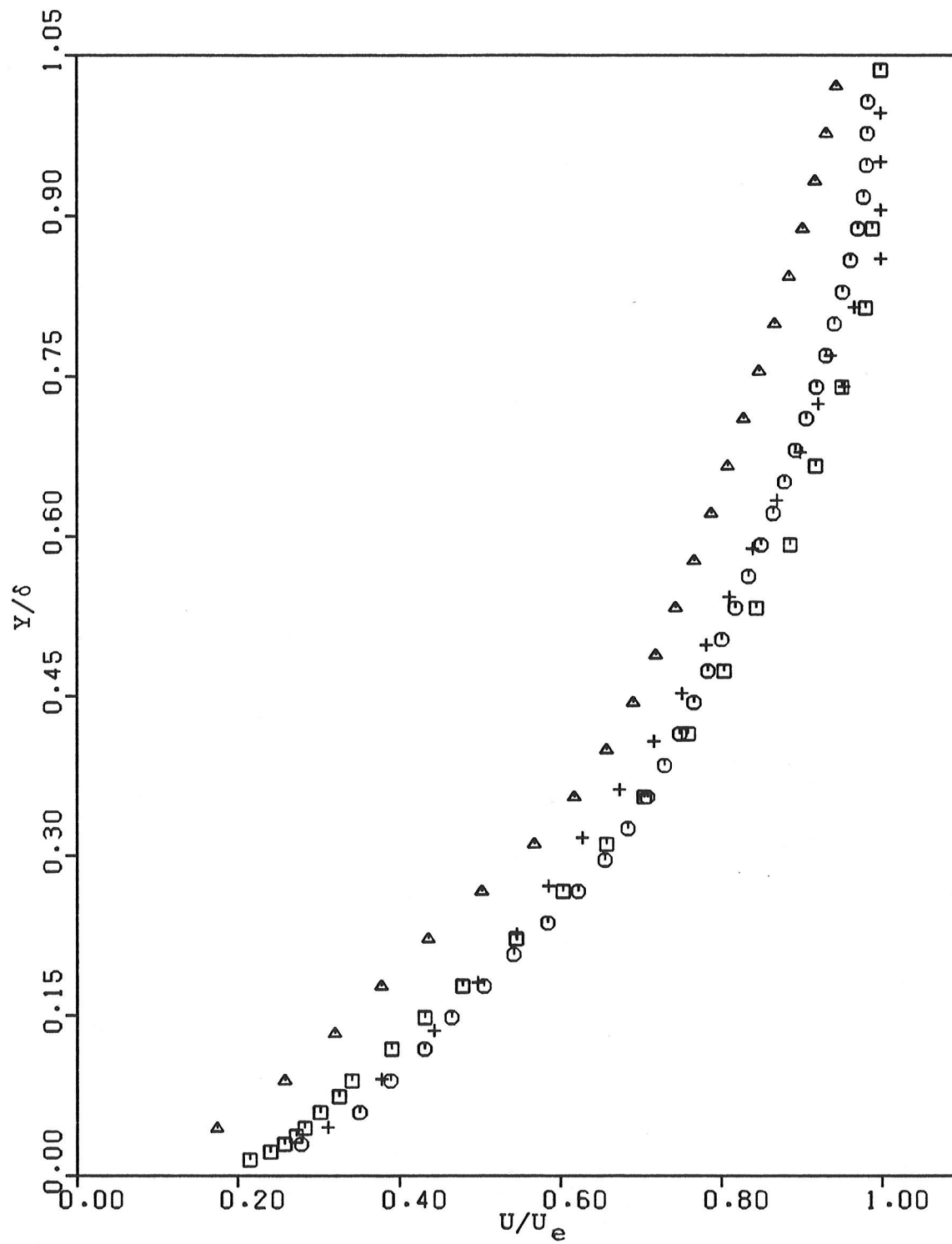


Figure 20: Velocity profile in 8° conical diffuser at Station 24

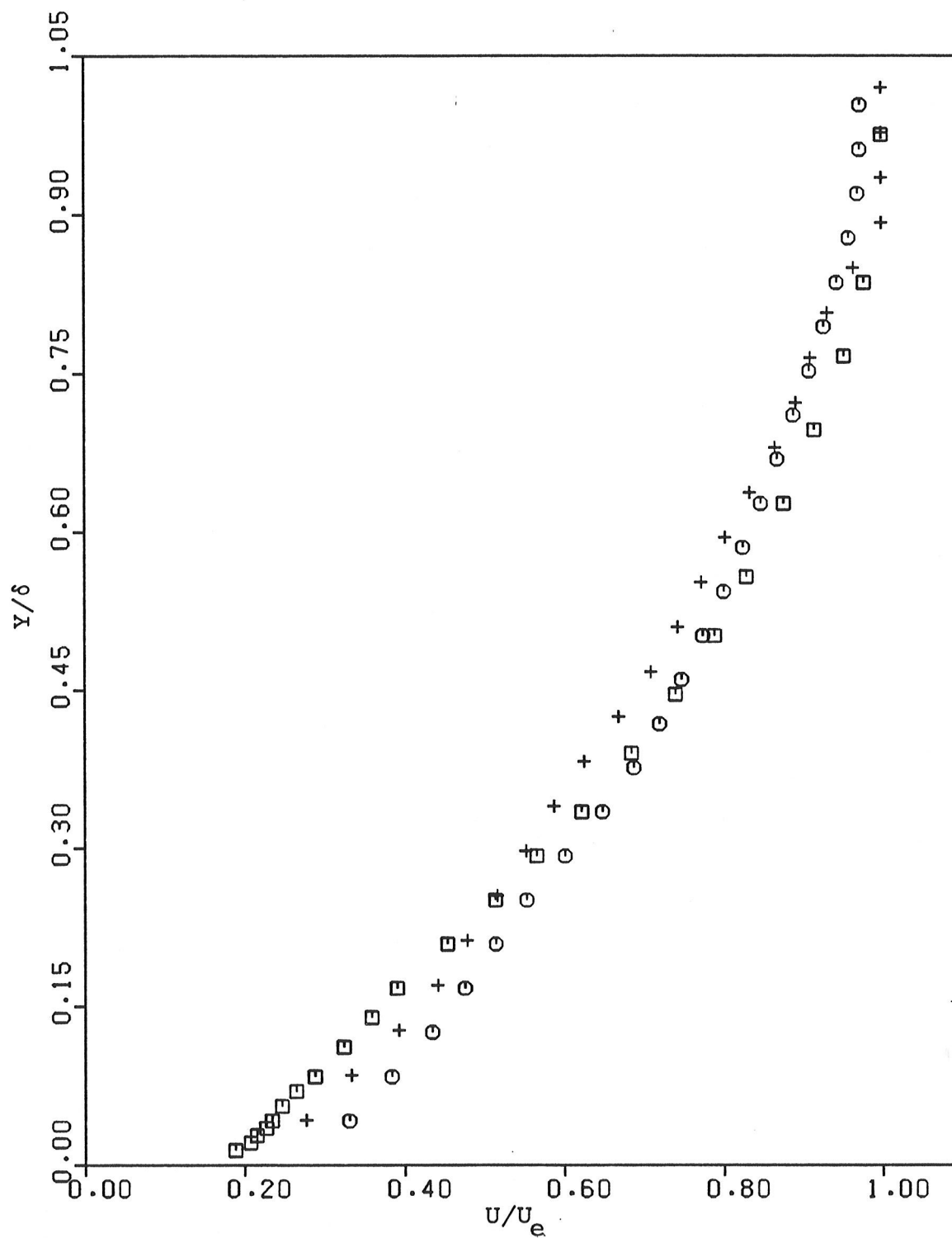


Figure 21: Velocity profile in 8° conical diffuser at Station 30

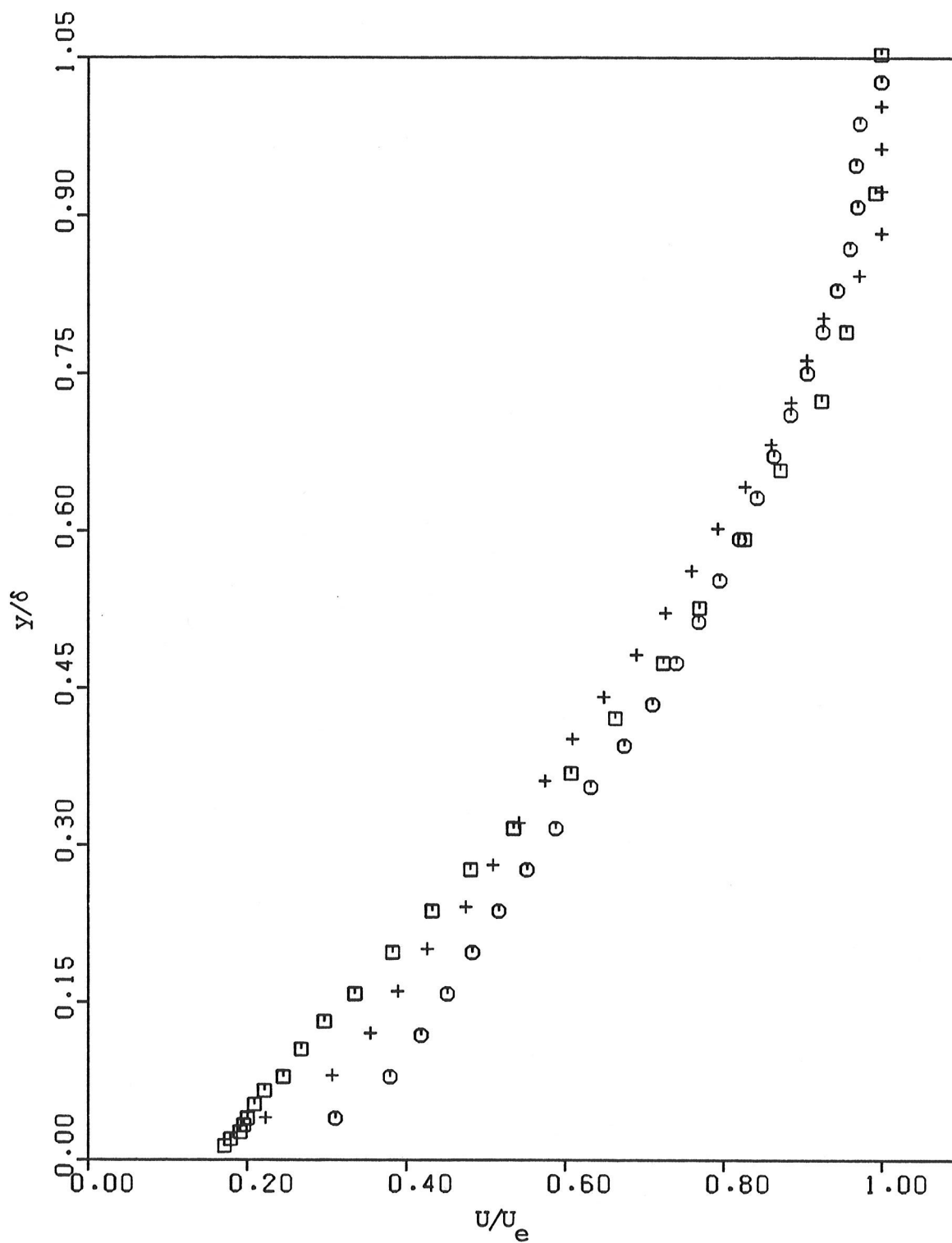


Figure 22: Velocity profile in 8° conical diffuser at Station 36

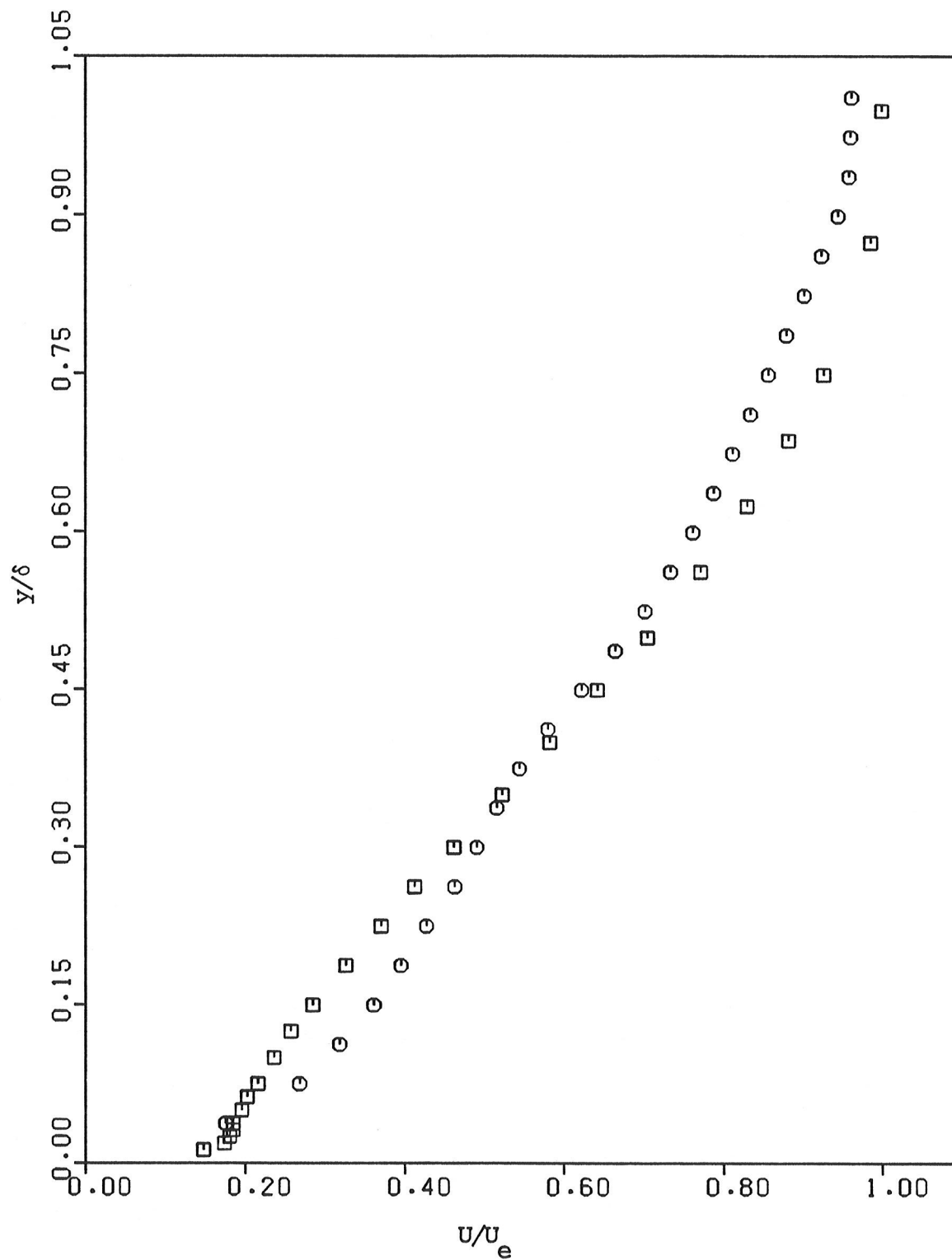


Figure 23: Velocity profile in 8° conical diffuser at Station 42

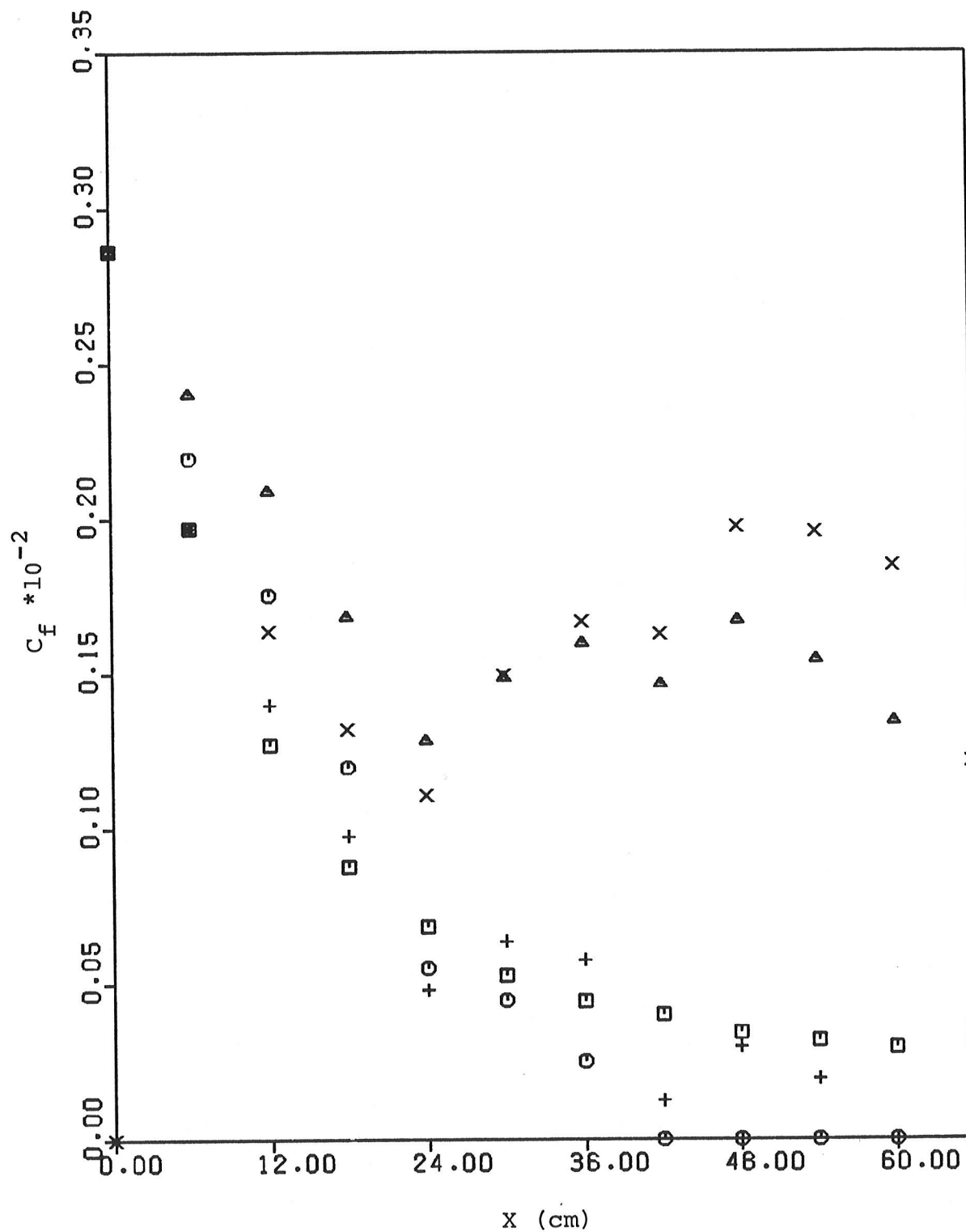


Figure 24 : Variation of C_f in 8° Conical Diffuser in the stream wise direction at Station 30

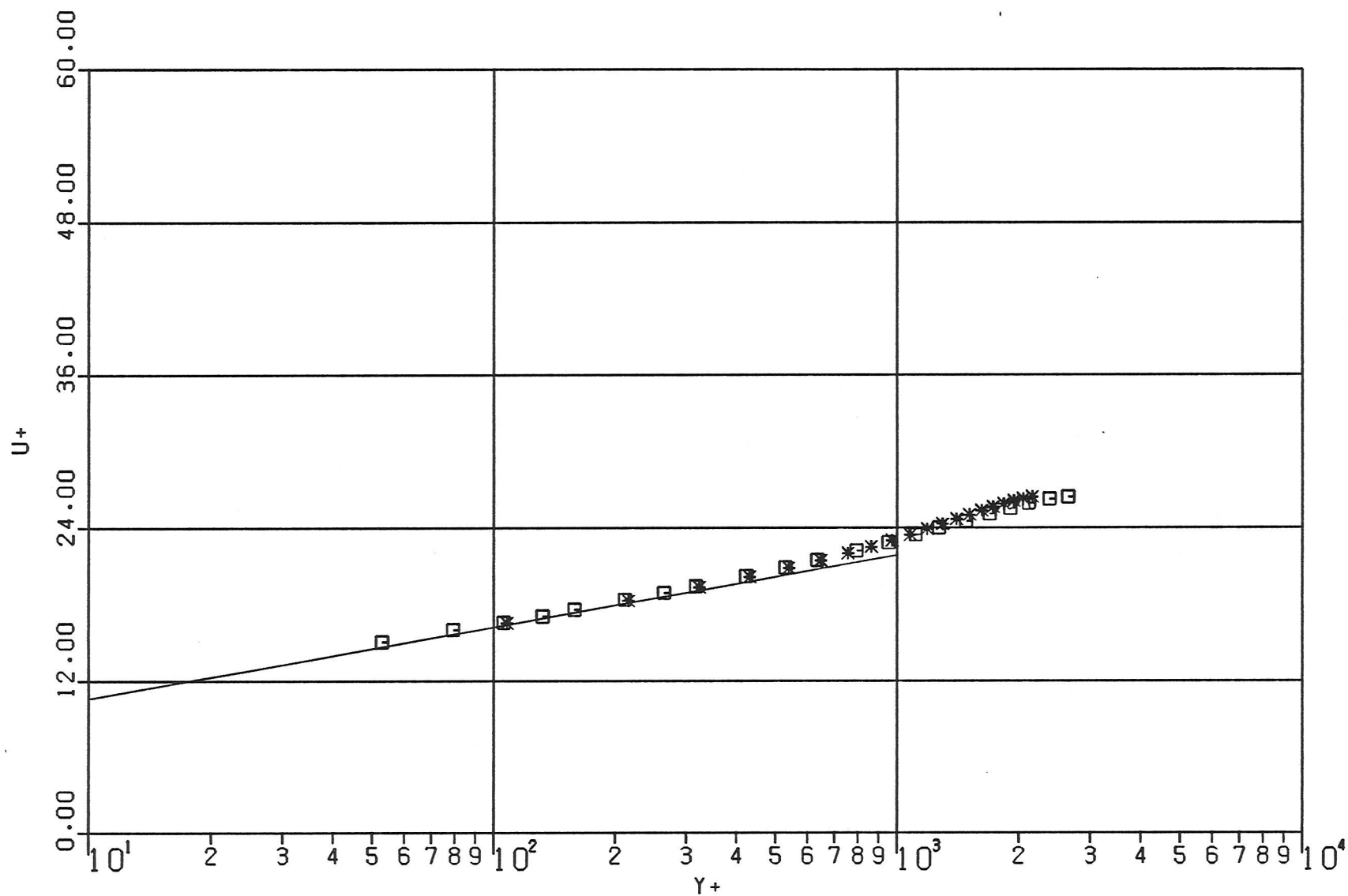


Figure 25: Distribution of U_+ versus Y_+ in 8° conical diffuser at Station 0

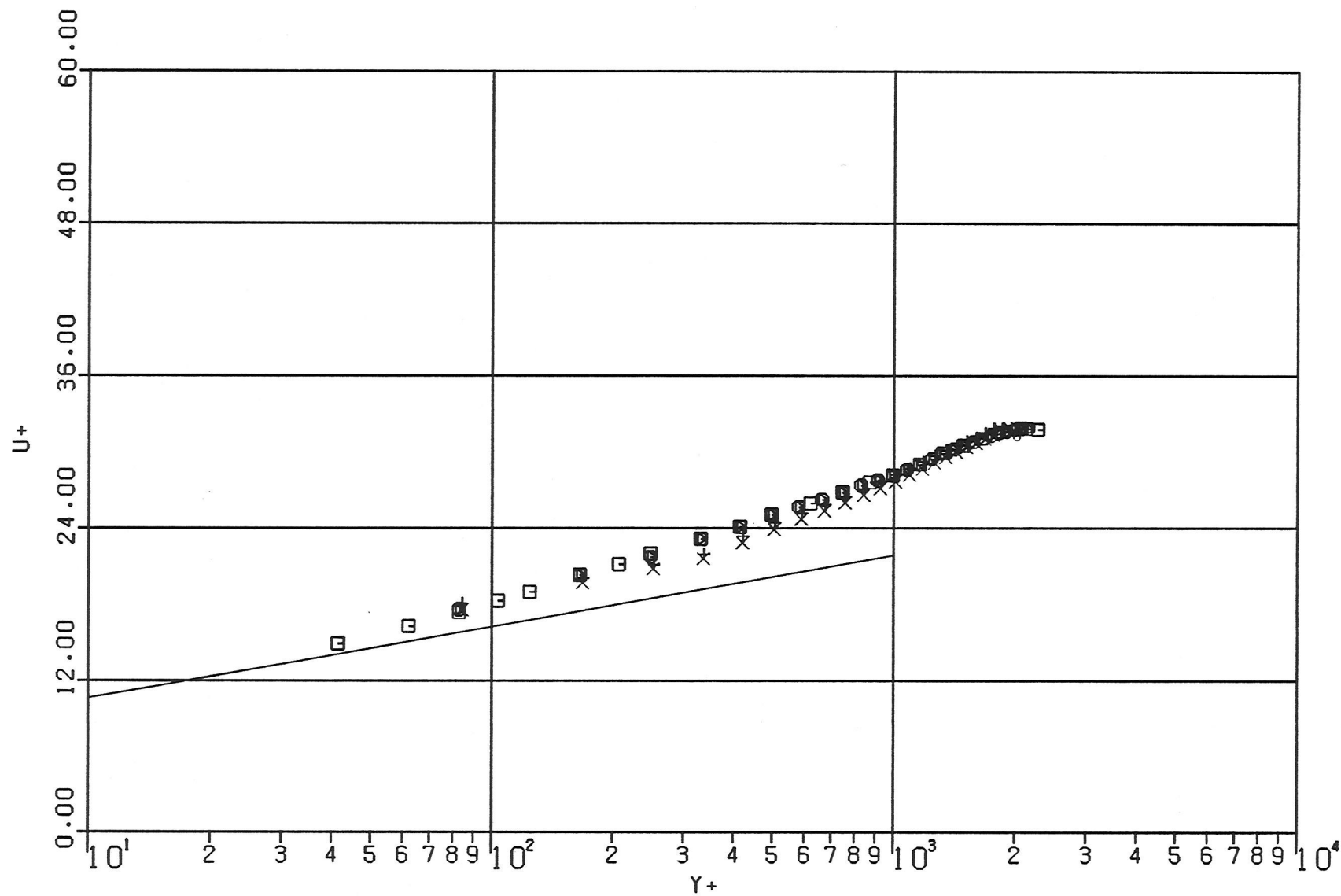


Figure 26: Distribution of U^+ versus Y^+ in 8° conical diffuser at Station 6

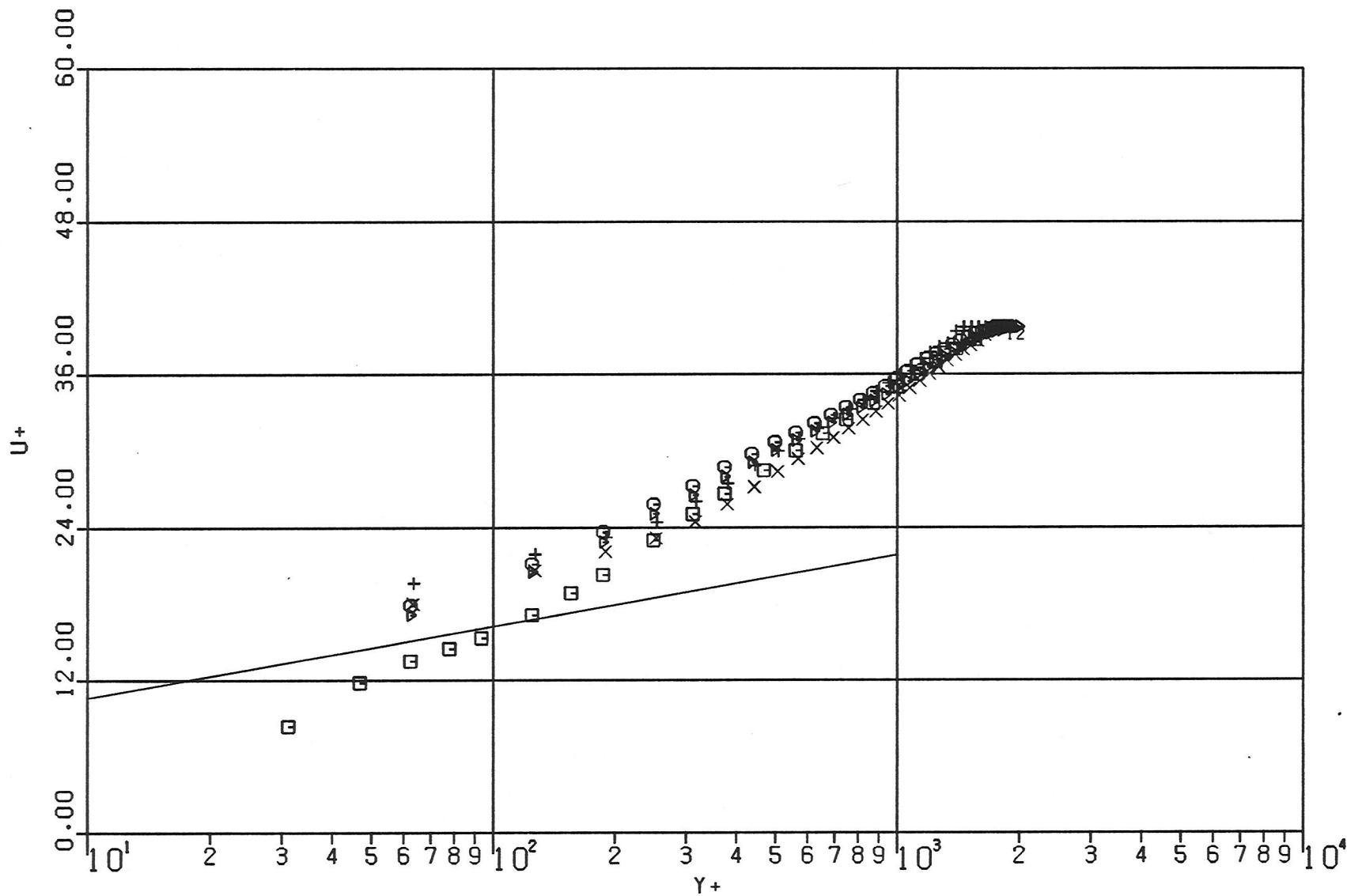


Figure 27: Distribution of U_+ versus Y_+ in 8° conical diffuser at Station 12

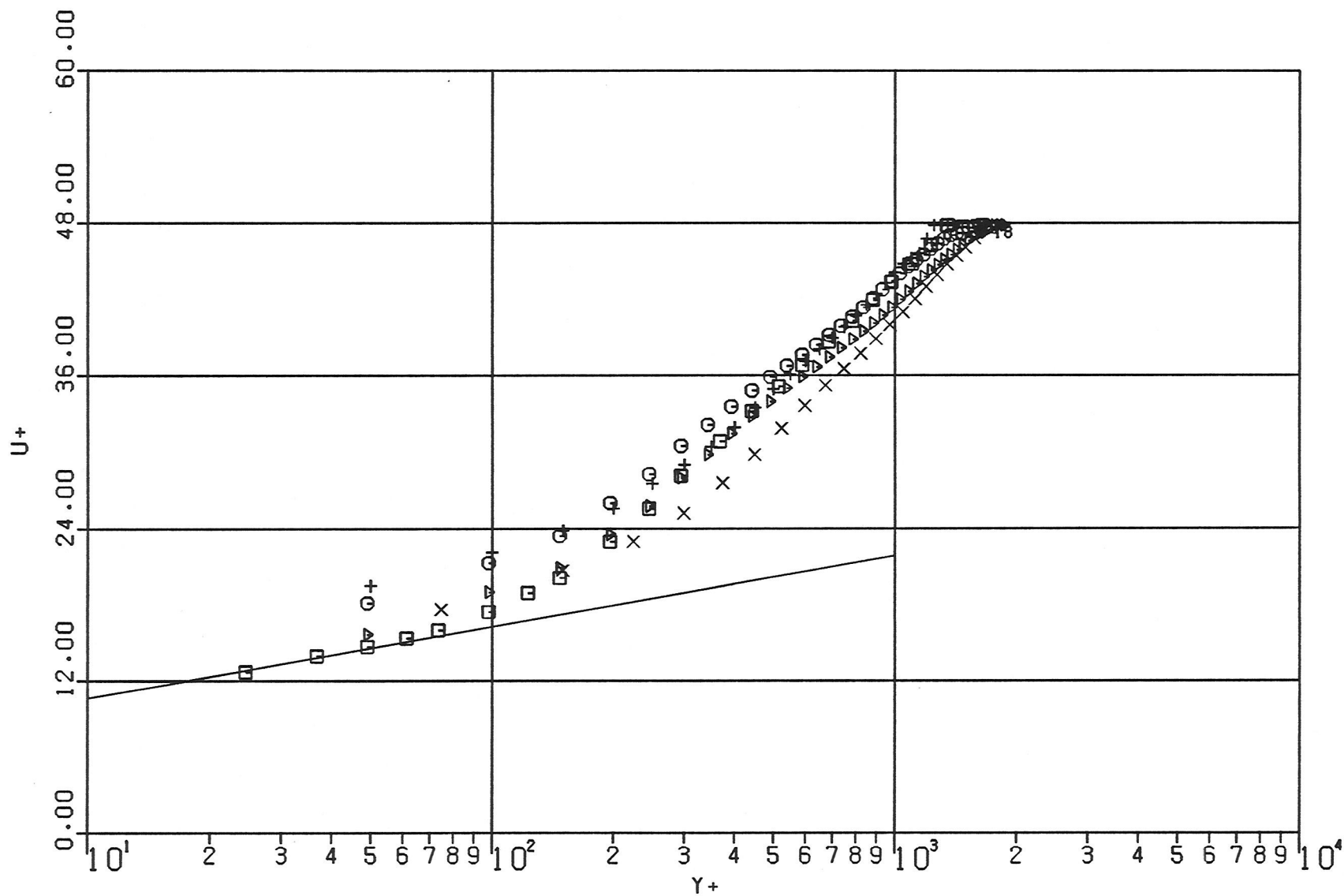


Figure 28: Distribution of U_+ versus Y_+ in 8° conical diffuser at Station 18

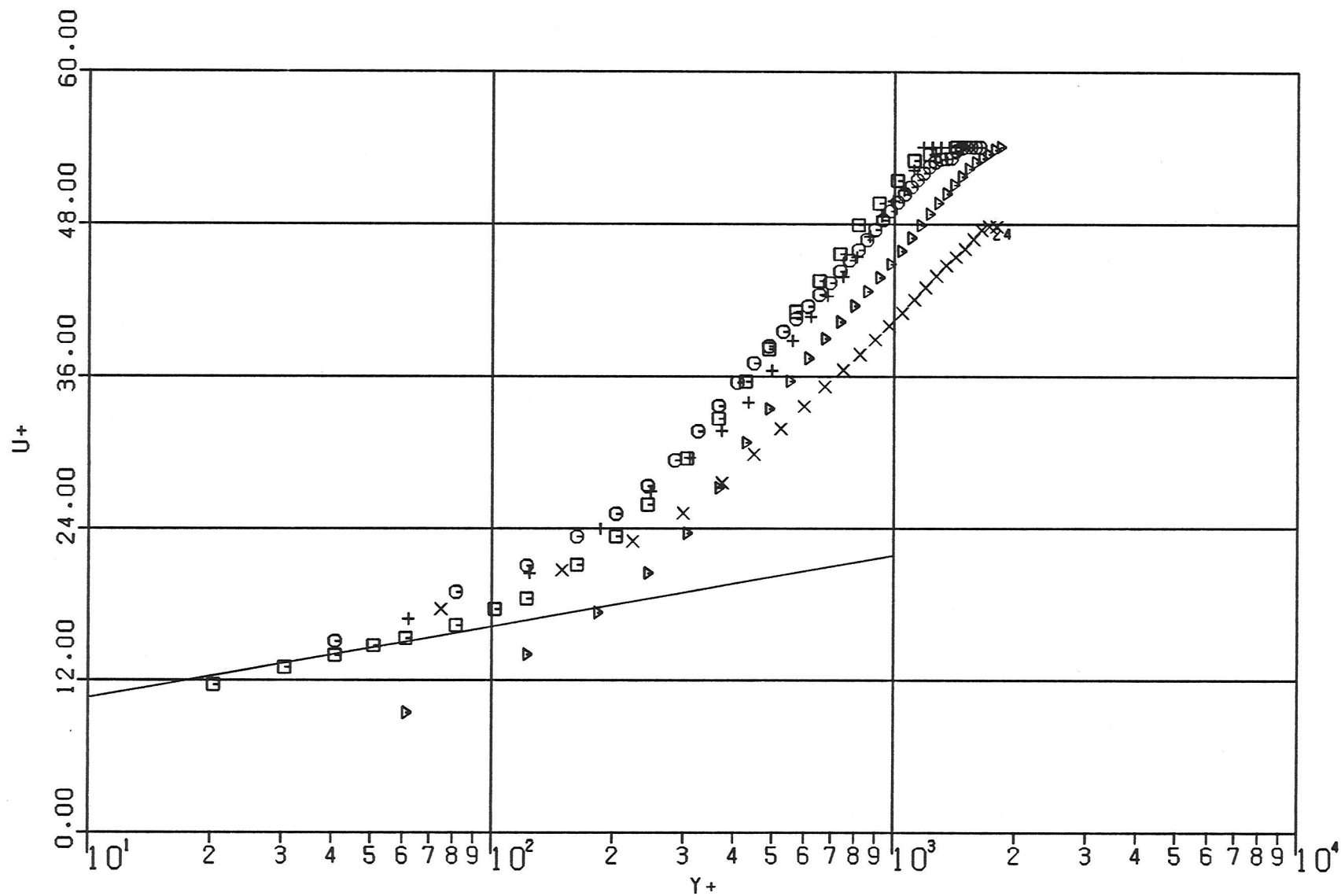


Figure 29: Distribution of U_+ versus Y_+ in 8° conical diffuser at Station 24

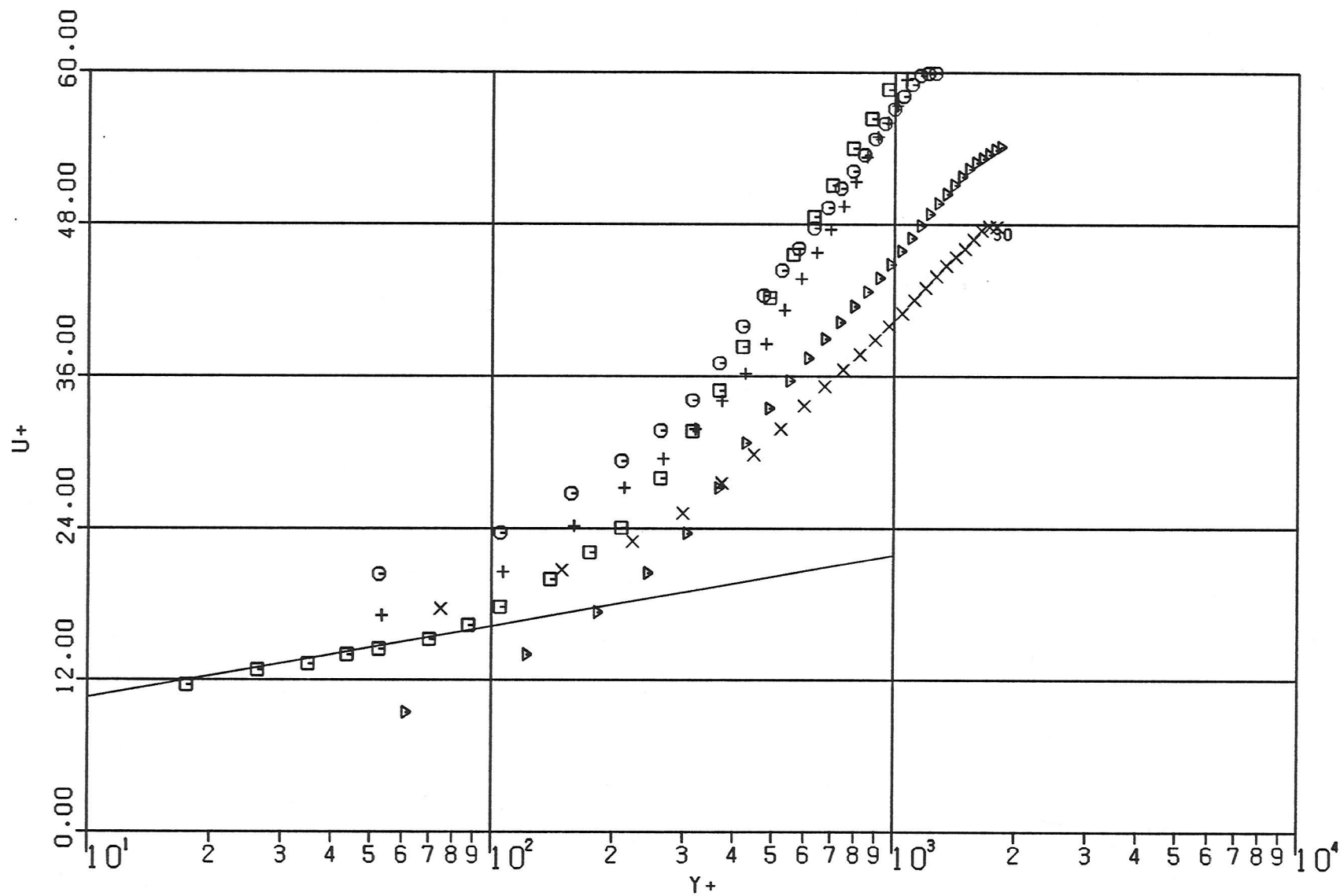


Figure 30: Distribution of U_+ versus Y_+ in 8° conical
-diffuser at Station 30

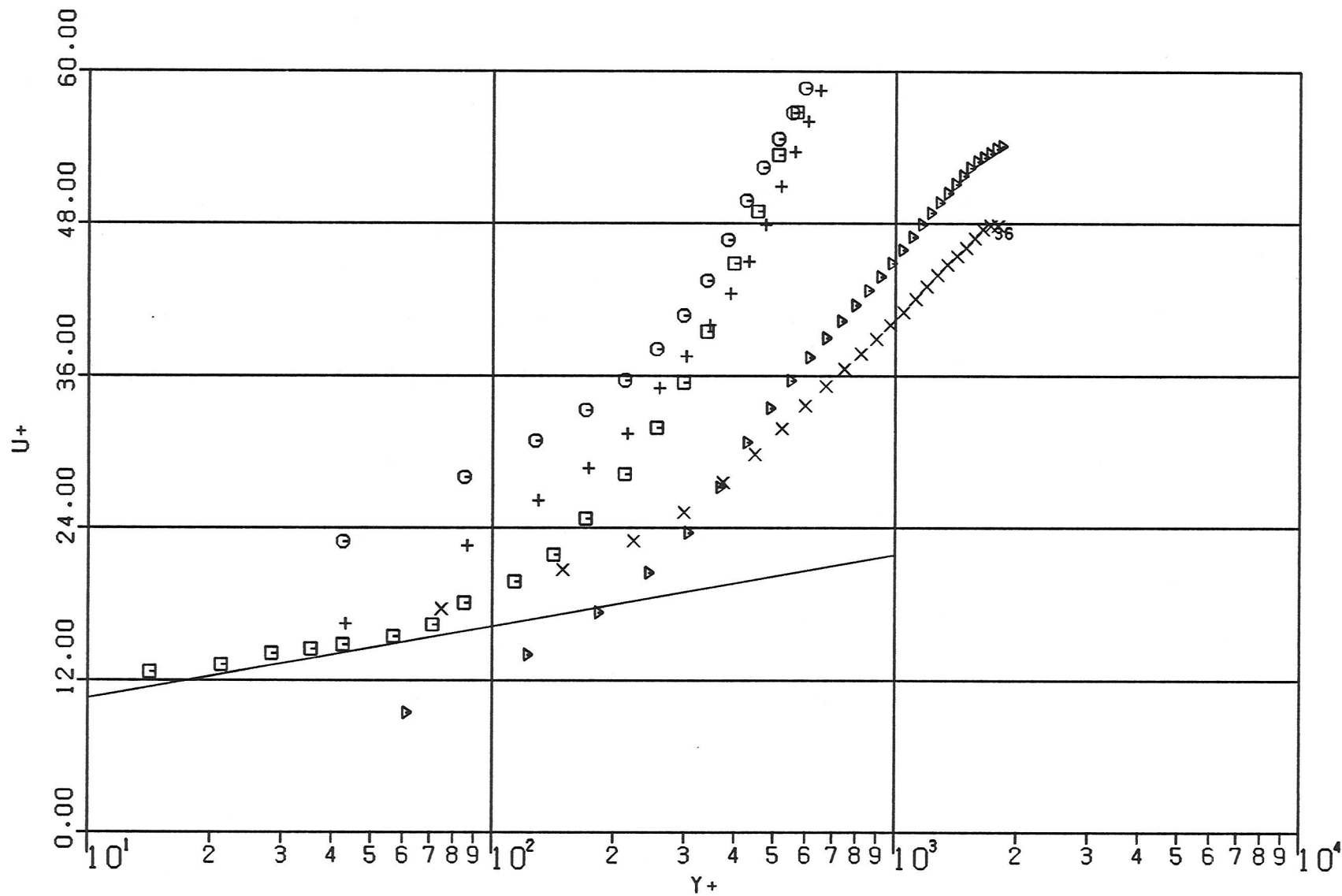


Figure 31: Distribution of U_+ versus Y_+ in 8° conical diffuser at Station 36

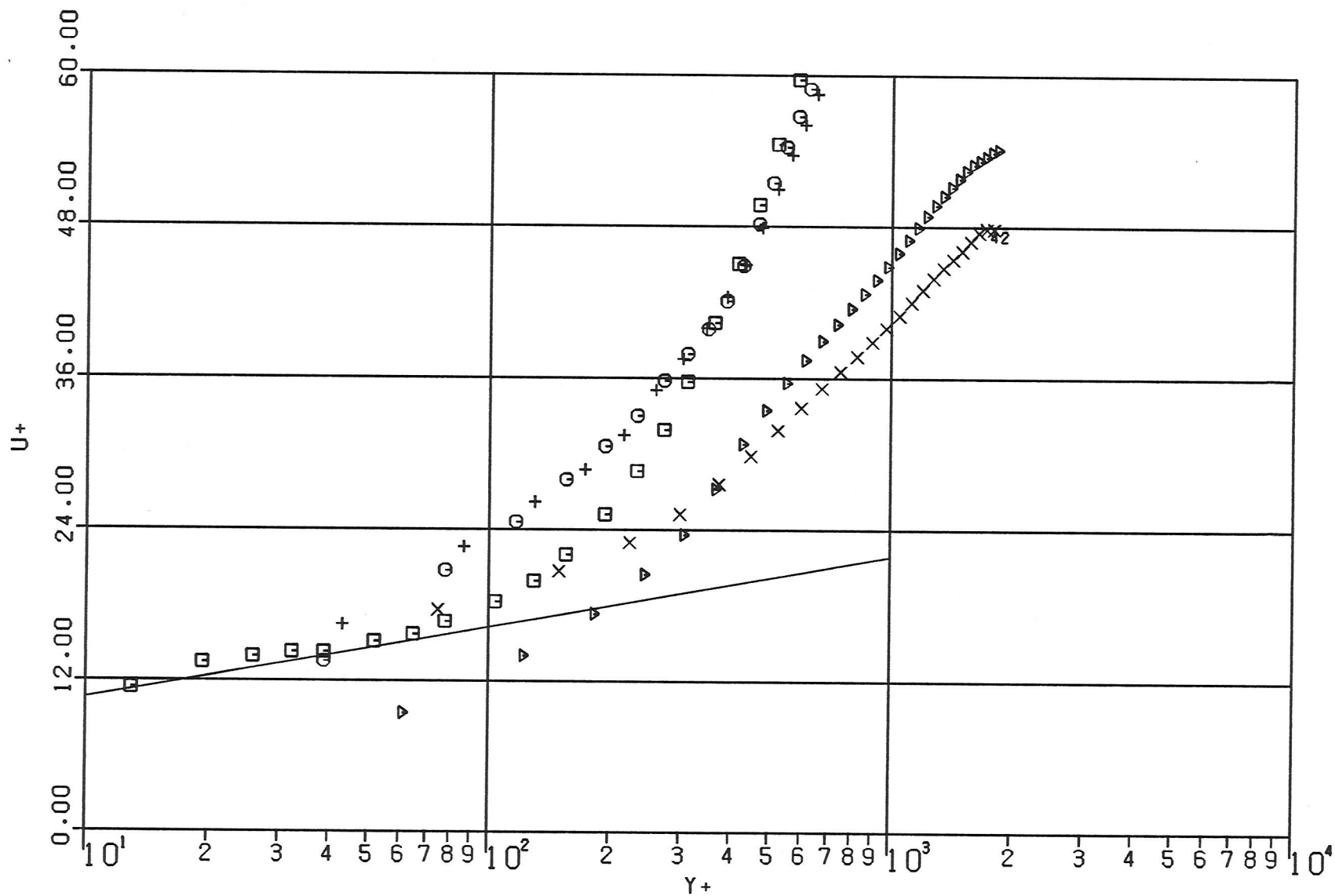


Figure 32: Distribution of U_+ versus Y_+ in 8° conical diffuser at Station 42

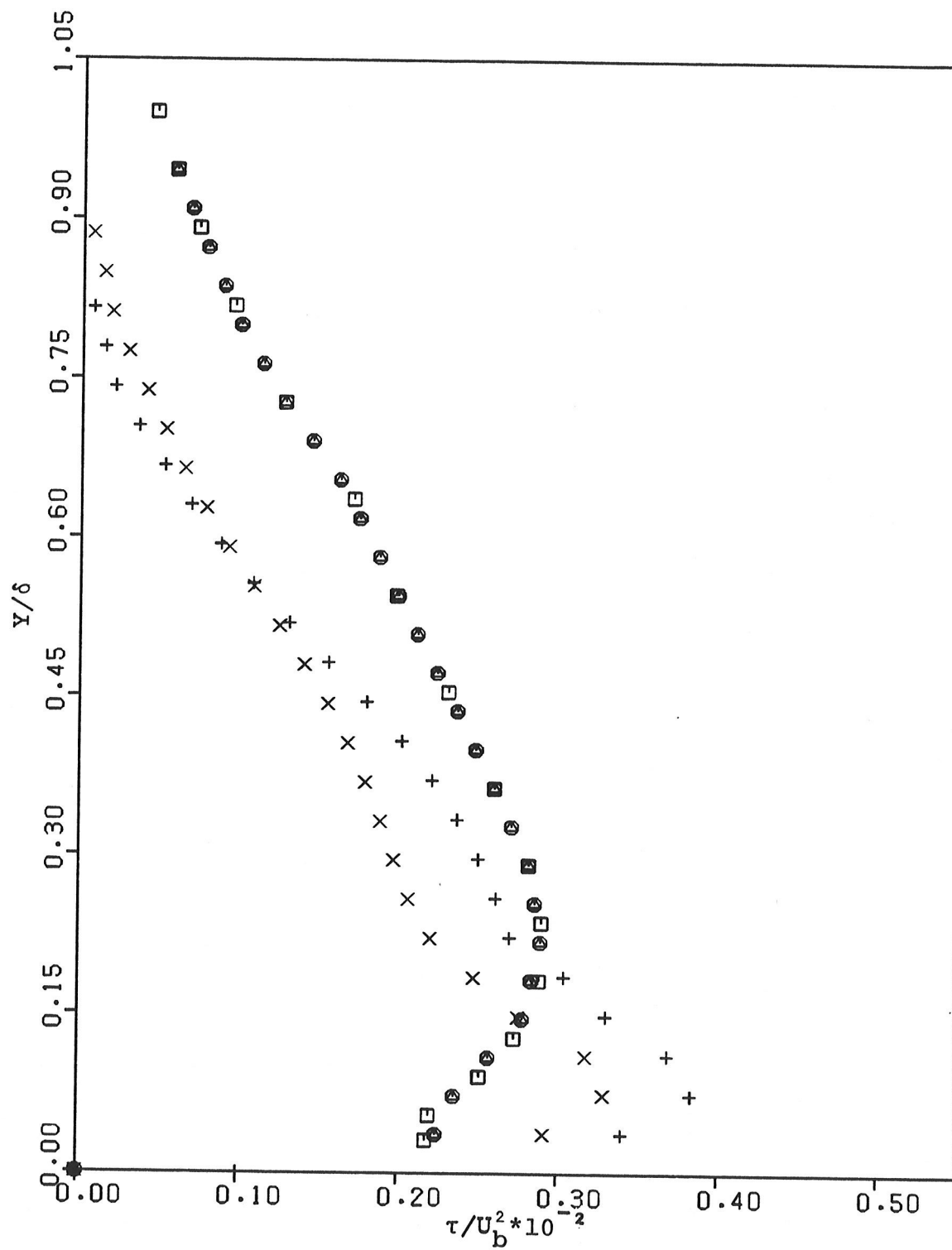


Figure 33: Shear stress distribution in 8° conical diffuser at Station 6

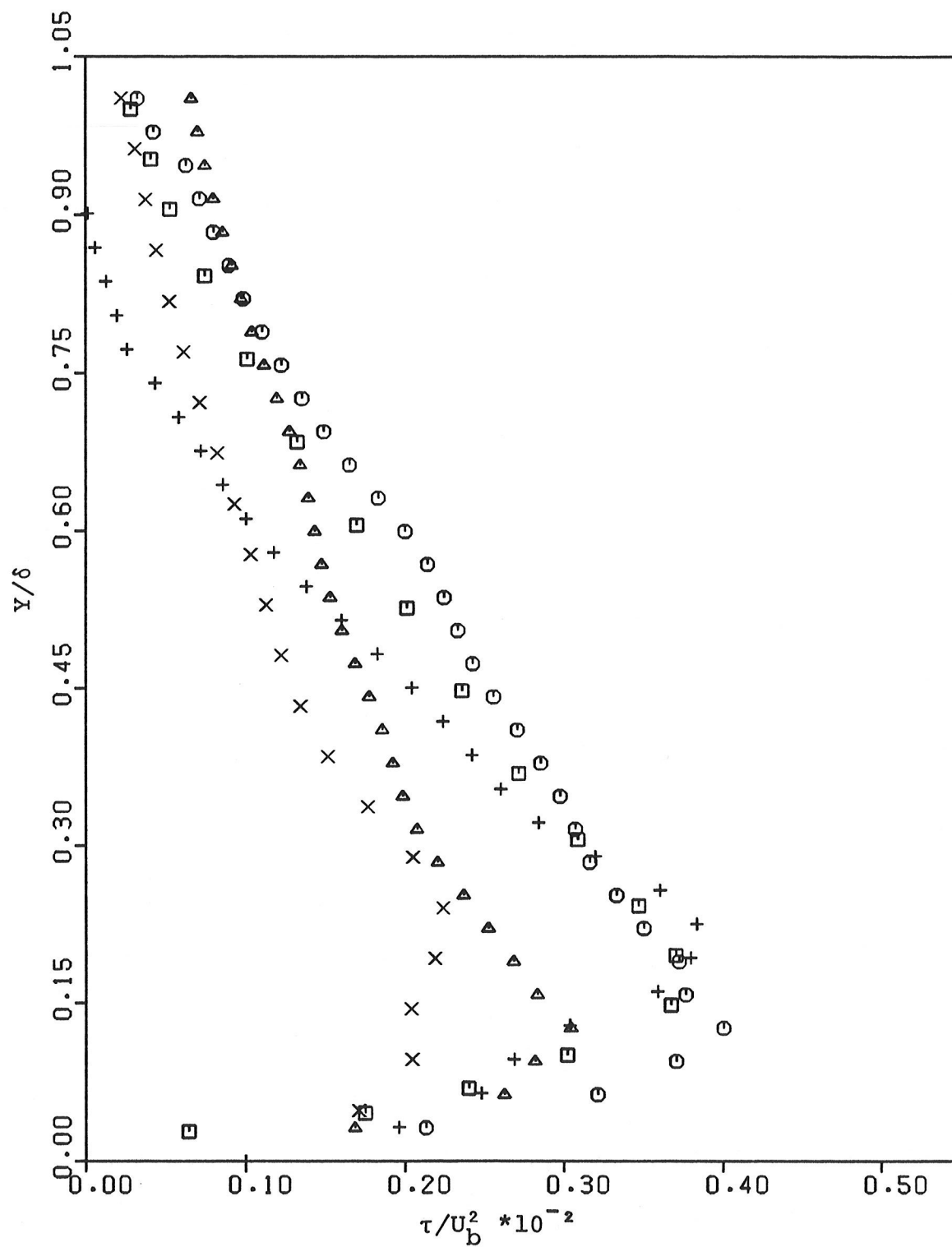


Figure 34: Shear stress distribution in 8° conical diffuser at Station 18

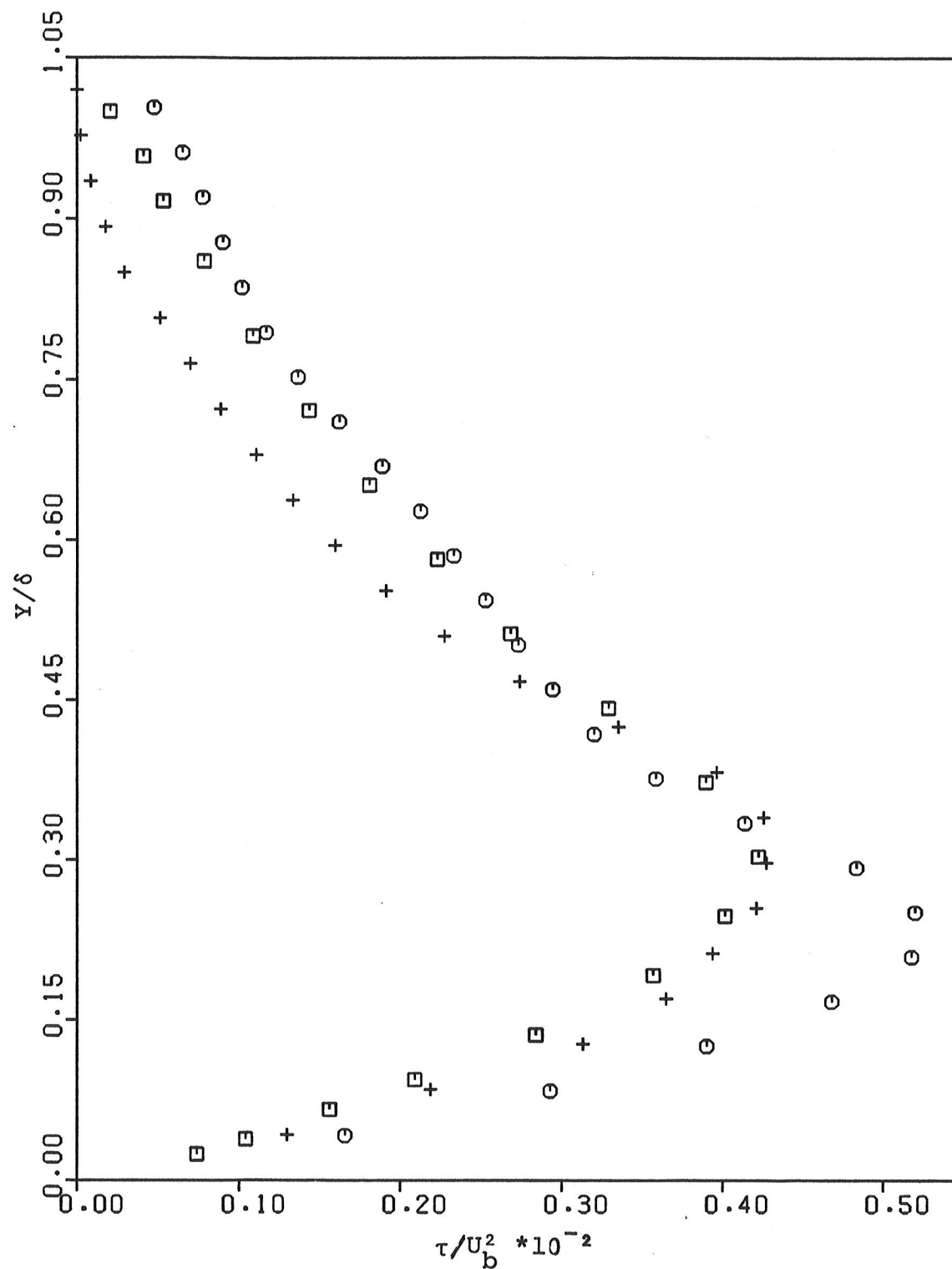


Figure 35: Shear stress distribution in 8° conical diffuser at Station 30