THE THREE NUCLEON SYSTEM INCLUDING ONE DYNAMICAL PION:

A one dimensional test case

by

Thomas Melde

A Thesis
Submitted to the Faculty of Graduate Studies
in Partial Fulfillment of the Requirements
for the Degree of

DOCTOR OF PHILOSOPHY

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A Thesis/Practicum submitted to the Faculty of Graduate Studies of The University of Manitoba in partial fulfillment of the requirements of the degree of Doctor of Philosophy

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ABSTRACT

The Padova model, describing πNNN scattering, is discussed in a new partition notation and a new approximation scheme is proposed. The resulting equations are recast in a standard AGS-form with the inclusion of correction terms to the two-cluster effective potential. These correction terms describe the explicit degree of freedom for one pion. It is shown that one set of correction terms can be interpreted as three-nucleon corrections of the Tucson-Melbourne type. A second set of correction terms is discovered that was previously not included in three-nucleon force calculations. In order to investigate the effect of the correction terms a one-dimensional system is developed that mimics standard two-nucleon physics. The corresponding three-nucleon system is investigated in respect to the $1D$-triton binding energy. It is demonstrated that the TM-type correction terms depend significantly on an adjustable parameter. The range of the effect on the $1D$-triton binding energy makes it possible to correct the underbinding discovered in the standard calculation. However, it is also shown that the new type of correction terms has a non-negligible effect on the $1D$-triton binding energy. Therefore, the adjustment parameter should take this effect into account when gauged to the $1D$-triton binding energy.

Chapter 1

INTRODUCTION

Human thought is so primitive, it's looked upon as an infectious disease in some of the better galaxies. That kind of makes you proud, doesn't it?

Kay (Men In Black)

1.1 Why Few-Body Systems in the year 2000?

Few-nucleon systems have played an important role from the very beginning of nuclear theory studies, because they describe the simplest non-trivial problems in nuclear physics. All models are based on some idealized concepts with rather limited degrees of freedom. In respect to a field theoretical approach, even the simplest nuclear systems require an infinite number of degrees of freedom. This fact probably leads some nuclear theorists with a field theoretical background to believe that Few Body physics is at the end of its useful lifespan. Of course in light of all this, one has to ask why then is a PhD thesis in Few Body Physics of interest in the year 2000. The short answer to this question must be that it is still the best theory around to describe nuclear processes involving three or more nucleons. To my knowledge, calculations based on Few-Body models are still the dominant source for reasonable results in descriptions of scattering processes with more than two nucleons, the triton binding energy or the A_y-puzzle. I would agree that the usefulness of Few Body physics is

about to end, if there would be no problem in the calculations and no improvements needed to be found.

However, this is not the case, because we still have unsolved problems like the A_y puzzle, or even something presumably simple as the triton binding energy. Unless we decide to ignore these problems or wait for the arrival of some new model able to solve them, we should use Few Body models to increase our understanding of the underlying physics. Furthermore, recent applications of Few-Body models in sub-hadronic physics opened another exciting application to this field. Taking all this into account, I believe that Few-Body Physics is still an exciting area that can help understand the physical world better.

Surprisingly, the development of the basic Few-Body models was completed in a span of ten years. It started with Faddeev's [Fad65] first introduction of a mathematical description of the three-body problem in the year 1965. Yakubovsky [Yak67] extended the model to systems that have more than three particles in 1967 and at the same time Alt, Grassberger and Sandhas [AGS67] gave an equivalent description of Faddeev's original model. They based their model on the quasiparticle formalism introduced earlier by Weinberg [Wei63a, Wei63b, Wei64] and used in a practical approximation by Lovelace [Lov64]. Immediately following their three-body paper, Grassberger and Sandhas [GS67] also gave a description of the N-particle problem. The AGS system, which can be found in a comprehensive form in a paper by Sandhas [San72], describes the non-relativistic three-nucleon problem and still remains the model of choice in modern few-body calculations. It is also the starting point for this thesis and I present the model briefly in Chapter 2.

There were only three major improvements of the model. First, the introduction of relativistic equations using the Bethe-Salpeter equation and Blanckenbecler-Sugar reduction. Also, several attempts were given to describe an extension of the Faddeev-AGS system to the πNN system, which I show in more detail later in this introduction. Recently, there also were improvements in including Coulomb forces into the model.

All these improvements were rather successful, but so far they still can not completely explain the aforementioned problems. In this thesis I restrict myself to the non-relativistic models without Coulomb forces and show how one dynamical pion can be included in the three nucleon system. In the next sections I show some relevant developments in the last 25 years.

1.2 Development of high precision NN-potentials

The fundamental input of the model is the nucleon-nucleon interaction and the early models were purely phenomenological in nature. Two typical representatives of that type of potentials are the Reid soft core [Rei68] and Hamada-Johnston [HJ62] potentials. A particulary interesting and simple set of potentials were given by Malfliet and Tjon [MT68]. Their potentials only included S-wave interactions and they used local two-particle interactions to solve the Faddeev equations. The MT-potentials were given as the sum of two Yukawa potentials [Yuk35], one attractive and on repulsive. The nucleon-nucleon scattering is the first obvious place where improvements in the calculations can be achieved. Therefore, it is not surprising that considerable progress has been made in the description of nuclear forces since the early potentials. The first advancement was the introduction of One Boson Exchange Potentials (OBEP), where heavier mesons than the pion where included in the exchange processes. A typical early potential of this type is the Bryan-Scott potential [BS69] and it presented a vast improvement over the earlier OPEP models. Nevertheless, potentials of this type still have conceptual problems like the introduction of the σ-boson in the mass range of 500-700 MeV. The most modern potential of this type is the Nijmegen potential [NRd78], which shows exactly the same problems as the older Bryan-Scott potential. It can therefore be assumed that the inclusion of an increased number of heavier mesons can not solve the problems associated with the potentials.

One major draw-back of the OBEPs was the fact that they did not allow the ex-

change of two or more mesons at the same time. This problem led to the introduction of the Paris potential [LLR$^+$80], which included the 2π-exchange explicitly into the model with OPE and ω exchange terms. They also had to introduce a phenomenological term describing the short range ($r < 1.5fm$) behaviour and were able to describe the $2N$ problems quite well. The Bonn potential [MHE87], which was based on field theory and did not introduce the phenomenological short range term, was the next improvement in the potentials. It has 12 parameters, namely the coupling constants and cut-off masses of the meson-nucleon vertices involved and a very good description of the NN-observables up to $300MeV$ was achieved.

All these models are considered earlier models, because they pre-date the extensive NN-analysis given by the Nijmegen group [SKRdS93] in the early 90's. All the models following this research effort and using the results obtained with it are called high-precision potentials and it is required that they fit the NN-data with $\frac{\chi^2}{datum} \approx 1$. The Nijmegen group gave the first three high-precision potentials, namely Nijmegen 93, Nijmegen I and Nijmegen II [SKTdS94]. The Argonne group gave the V_{18} potential [WSS95] and the Bonn-group gave the CD-Bonn [MSS96] potential, both of which also have a $\frac{\chi^2}{datum} \approx 1$. All the high-precision potentials have approximately 45 parameters that need to be adjusted in order to achieve this high precision. However, it should be apparent that the more parameters have to be fixed, the less fundamental physics can be seen in the process. A very good review on the nuclear forces and nuclear structure was recently given in an invited talk at the *Nuclear Structure '98* conference by R. Machleidt [Mac98].

1.3 Three-nucleon forces

Unfortunately, the apparent success of the high-precision potentials did not translate directly into success for three-body calculations. It turned out that the potentials underbind the triton considerably and have trouble describing the vector analyzing

powers. Already in the early days it was recognized that the few-body problem could allow for the existence of so-called three-body forces, for example Gloeckle [Glo83][pp.118] already indicated this possibility in his textbook. Several groups pursued the concept of three-nucleon forces in the hope of finding the solutions to the apparent discrepancies in the $3N$-calculations. Probably the first three-nucleon force diagram was given by Fujita and Miyazawa [FM57] in 1957 and it is still the archetype of most three-nucleon forces. It is true that modern three-nucleon forces (3NF) are more sophisticated, include more types of mesons and are generally derived from more extensive theories, but the essence still remains the same. We always have the exchange of mesons between two nucleons, where something happens between the exchanged meson and the third nucleon. In the most complicated cases the meson even can change before it is absorbed at the second nucleon. Figure [1.1] shows the generic form of three-nucleon force diagrams due to meson exchange and the Fujita-Miyazawa type diagram with the forward propagating delta isobar. A more recent off-shoot of

Figure 1.1: The generic form of three-nucleon force diagrams based on meson exchange and the Fujita-Miyazawa type representative of this class. The blob in the generic diagram can represent complicated processes.

the Fujita-Miyazawa 3NF is given by the Urbana-Argonne model [CPW83], which is also based on Δ isobars. The second approach used to find 3NF is based on chiral symmetries and the earliest of these is the Yang model [Yan74]. In this approach is one model based on chiral perturbation theory, the Texas model [Ov92, van94], as well as two models based on relativistic field theory, the Brazil [CDR83, Rob87] and the Ruhrpot model [EG96]. Furthermore, the Tucson-Melbourne [CSM+79] model

was also based on chiral constraints using PCAC and current algebra. Recently, it was argued that chiral perturbation theory leads to corrections in the original set of Tucson-Melbourne diagrams [FHv99]. At the present time, efforts are under way to develop a unified model describing the different 3NF and two good reviews were recently given by Friar [Fri99, Fri00]. Even though the modern 3NF are derived from different theories, they all depend on a similar set of diagrams. The only real difference is in the set of strength parameters used within the different models and a clear review on the status of these parameter values is given by Kamada, Hueber and Nogga [KHN99, FHv99]. Since the three-nucleon forces are not derived directly from the model used in the calculations, one has to be careful not to introduce terms that are already taken care of by that model, like iterated OPEPs [CF86]. Also, most models assume some type of cancellation between terms and I discuss the importance of the cancellations in more detail in section 3.3 . It is believed that the 3NF can account for the problems appearing in the triton binding energy calculations, but unfortunately not for the A_y puzzle.

Already in 1994 Saito and Afnan [SA94] challenged the success of the three-nucleon forces in triton binding energy calculations. They also calculated the triton binding energy with a two-pion-exchange 3NF and found the effects to be very small. Their results were questioned because they used separable πN potentials that did not satisfy low energy theorems and did produce soft πNN form factors. However Saito and Haidenbauer [SH00] recently reinvestigated this calculation using an improved meson-theoretical model for the πN interaction. The Juelich model [SDHS94, HHH+98] can not be critized in the same way as the model used in the earlier calculations. Nevertheless, the magnitude of the TM-3NF effects to the triton binding energy remained very small. In light of this calculation, I believe it is important to question the role of the 3NF and their present interpretation. One important point that should be made is the fact that the 3NF are neccessary, because the meson degrees of freedom were frozen out in the few-body equations. Therefore, it seems only natural to investigate

what happens if these degrees of freedom explicitly were included into the underlying model. My thesis is investigating this problem by calculating the triton binding energy in a one-dimensional Toy model.

1.4 Short review of the A_y-puzzle

Historically one of the most important problems in NN-scattering is the A_y-puzzle in low-energy scattering. The analyzing power for $n-d$ scattering shows a peak which is significantly larger than the one found with Faddeev calculations. The discrepancy can be up to thirty percent at low energy and vanishes completely for energies larger than $30MeV$. A detailed review on the status of the A_y-puzzle was recently given by Knutson [Knu98] at the Groningen Few-Body conference. The three-nucleon forces were the obvious candidates to resolve this discrepancy, but it turns out that they were not able to solve this problem. The Brazil and Urbana 3NF introduced an increase of the peak value into the right direction, but not nearly strong enough to account for the discrepancy. The Tucson-Melbourne type 3NF even introduced a shift of the peak value in the wrong direction. At the Autrans Few-Body physics conference in 1999, Hueber [Hue99] argued that it is indeed impossible to find a solution to the A_y-puzzle with existing 3NF. He proposed that the problem is likely to be solved by the introduction of a new type of 3NF.

To my knowledge, none of the existing three-nucleon force groups were able to come up with a new type of 3NF that could solve the A_y puzzle without destroying existing fits to NN-data. However, we (Canton, Melde, Svenne [CMS00]) demonstrated that the inclusion of an explicit pion degree of freedom into the standard AGS-equations results in a new set of diagrams. Recently Canton and Schadow [CS00a] argued that this new set of correction terms could solve the A_y-puzzle. In order to find a definite solution to the A_y-puzzle a full calculation using both types of corrections still has to be performed. Nevertheless, the initial results are very promis-

ing. In this thesis I investigate the roles of these correction terms in a one-dimensional calculation of the triton binding energy.

1.5 *Early πNN models*

The most important meson in the two- and three-body interactions is the pion and it seems desirable to find a model that can include this degree of freedom explicitly. In the first order, which should be the most dominant, one would like to allow for the creation of one dynamical pion at a time. This problem was first investigated for the $NN - \pi NN$ system and Garcilazo and Mizutani [GM90] gave a comprehensive review on this topic in 1990. Traditionally there are two approaches to give a model describing the πNN system. Either a two-body model is extended to include resonances, or a three-body-model is extended to allow for creation and destruction of the pions. The first way Garcilazo and Mizutani call the coupled channel method and is well described in a paper by Green and Niskanen [GN76]. A hybrid model between the CCM and PNNA aproaches is given by Poepping, Sauer and Zhang [PSZ87]. They realized that the nucleon is actually the ground state of a composite system given by the nucleon and the pion. It is therefore possible that the nucleon-nucleon interaction can go through channels that include the excited states of this composite system. The resulting set of equations with this method are quite similar to the ones that were derived with the second approach, which Garcilazo and Mizutani refer to as the coupled $\pi NN - NN$ approach (PNNA). However, it should be observed that while the theory in this paper is well described, the numerical calculations are known to be wrong. On the other hand, the PNNA starts with the three body scattering equations of Faddeev [Fad65] or Alt, Grassberger and Sandhas [AGS67]. The first attempt to solve the problem with this approach was given by Myhrer and Koltun [MK73, MK74], who restricted themselves to the elastic πd channel. However, their findings in the resonance region turned out to be not satisfactory and improvements of

the model were necessary. One problem lies in the existence of the channel $\pi d - NN$, because it allows for the explicit destruction of the pion on the deuteron. Therefore, in the elastic πd scattering system one has to include the possibility of an intermediate state without any pions. Afnan and Thomas [AT74] used the Bound state picture to incorporate the NN intermediate state without losing the requirement of particle number conservation necessary for the Faddeev-AGS systems. Essentially, they treated the NN intermediate state as a bound state of the nucleon and pion in the πN P_{11} partial wave, where the binding energy is equal to the pion mass.

However, it became apparent that this method is troubled by the fact that the two nucleons in the intermediate state are not the same. One of them is a composite body made up by a pion-nucleon cluster, called N'. The other is a elementary nucleon without a pion associated to it, called N. The trouble arises because N can only absorb a pion, but not emit it, and N' can only emit a pion, but not absorb it. Since the two nucleons are clearly distinguishable by this connection to the pion creation and absorption processes, the Pauli principle is broken. The only way out of this problem seems to be that we assume the nucleon to be a superposition of states given in the infinite dimensional Fock space describing all bound states with an arbitrary number of pions present. Clearly, this would lead far away from the original starting point, where the three particles are conserved.

Fortunately, Rinat [Rin77] and Mizutani and Koltun [MK77] derived an equivalent set of equations starting with different techniques that do not use the Bound state picture. However, it turned out that their system is still not fully coupled, because of the form of equations describing the NN sub system. These equations did not have any contributions from diagrams with an intermediate πNN state. The fully coupled two- and three-particle equations were independently given by Thomas and Rinat [TR79] and Avishai and Mizutani [AM79]. Both papers also explicitly addressed the two- and three-body unitarity of the resulting models. Finally, Afnan and Blankleider [AB80] derived a set of coupled integral equations using a time ordered diagrammatic

method. Since this model was the starting point for the extensions to the πNNN system discussed in this thesis I describe this model in more detail.

1.6 Short review of the Afnan-Blankleider model

In 1980 Afnan and Blankleider [AB80] derived the prototype of equations describing the πNN system used later to derive a description of the πNNN system. The main difference between this description and the earlier ones by Avishai and Mizutani [AM79] is the inclusion of a πN interaction. They claim that this inclusion leads to a fully dressed system without changing the basic structure of the equations. They employed a diagrammatic classification scheme utilizing the last (first) cut lemmas in an old fashioned time ordered field theory. The lemmas are used to expose first the two-body and then the three-body unitarity cuts. The first cut leads to a Lippmann-Schwinger type structure for the NN system. However, already at this stage a delicate problem with propagator dressing arises. Depending on the position of the cuts Afnan and Blankleider classified the diagrams into reducible and irreducible cuts. They are classified depending on the connected or disconnected nature of the parts split off by the cut. The careful implementation of these procedures seemingly leads to a fully dressed LS-equation with an effective NN-potential. The next step is to invoke the last (first) cut lemma again and expose the three-body unitarity cuts. Obviously, these cuts can only be found in the parts of the diagrams that do not have already at least one two-body cut. In other words, these cuts have to be found in the potential terms of the LS-type equation. This cut procedure also exposes parts of diagrams that actually have a two-nucleon in-going and a two-nucleon plus one pion out-going state (and vice versa). In a first approximation, these creation (destruction) vertices can be given by the creation of a pion from either one of the nucleons. However, the form factors at this point are not fully dressed and one has to employ the classification into reducible and irreducible diagrams again. Nevertheless, careful classification removes

this problem and leads to an AGS-type description of the πNN system.

Already in 1985 Sauer, Sawicki and Furni [SSF85] showed a nucleon-renormalization problem with this approach, but the criticism went largely unnoticed. In 1994 Kvinikhidze and Blankleider [KB94, BK94] addressed this problem. At that time they realized that the unitary $NN - \pi NN$ approach has problems on grounds of an overcounting due to the particular use of the last cut lemma. In particular, they found diagrams that do not have an unique last cut, which indeed would be troublesome. It is noteworthy that the diagrams in question must have two pions in flight concurrently in order to see this problem. In a theory that truncates the Hilbert space at one dynamical pion this type of diagram would be rather hard to generate. It is true that in general the second pion could be part of pion exchange that occurs in the potential that acts between the two nucleons. However, in a non-relativistic theory that is truncated at the one degree of freedom for the pion the diagrams in question should not be of any consequence. At this point it should be also clear that there is a delicate interplay between the truncation and the relativistic corrections to a non-relativistic theory. For the same reason as in the 3NF the corrections are generated explicitly by the truncation. Kvinikhidze and Blankleider argued that it is crucial to dismiss the unitary model altogether and devised a new theory based on convolution integrals. On the other hand, Phillips and Afnan [PA97] argued that the diagrammatic method can be adjusted to avoid the overcounting problem and found an equivalent set of equations.

1.7 Development of πNNN models

I demonstrated in the last section that the development of the πNN theories is far from being trivial. The πNNN system turned out to be even more complicated due to the addition of another nucleon. The chain of partition formalism has to be employed and the cut structures are even more complicated. A first attempt describing this

problem was given by Avishai and Mizutani [AM80a, AM80b, AM82, AM83], but the description is rather complicated. Cattapan, Canton and Svenne [CCS93] also attempted to generalize the AB unitary model and not suprisingly discovered similar problems with the cut structure. In 1994 Canton and Cattapan [CC94a, CC94b] gave the first in a series of papers where they developed a πNNN theory from a generalization of the Grassberger-Sandhas-Yakubovsky formalism. The problem is to find a connection between the NNN and πNNN sectors in a physically sound way. They also argued that the equations are connected after three iterations, but overlooked a set of graphs that spoiled connectivity. In 1997 they proposed some approximate schemes that were to take care of these unfavourable diagrams. However, in 1998 Canton [Can98] devised a new model that was not plagued by all the problems of the previous attempts. It was based on sound physical assumptions and turned out be connected and of a very simple nature. Recently, we (Canton, Melde, Svenne) [CMS00] gave an approximation to this model that allowed us to recover diagrams in the 3N problem that can be identified with diagrams given by 3NF. In addition we get a diagram that is previously assumed to be cancelled completely in 3NF models. I show the derivation of the Padova model and the approximation scheme in detail in Chapters 2 and 3. In Chapters 4 and 5 I develop a one-dimensional Toy model for the NN system, which is embedded into the $3N$ system in chapter 6. Finally, in chapter 7 I show how the corrections due to the pion dynamics affect the triton binding energy in the one-dimensional model. Chapter 8 gives the conclusion and outlook and the Appendix includes the core codes of the calculations and a section on Jacobi-momenta.

Chapter 2

ALGEBRAIC INCLUSION OF THE PION DYNAMICS INTO THE AGS-FORMALISM

If you build it, he will come

W. P. Kinsella

2.1 The four-body scattering problem

In this section I give a brief review of the traditional four-body problem in a formulation that is easily generalized to the more complicated πNNN scattering problem. The derivation of the multi-particle AGS theory follows closely the derivation given by Canton and Cattapan [CC94a, CC94b]. Generally the starting point is the four body Lippmann-Schwinger equation

$$T|^4 = V|^4 + VG_0T|^4 \tag{2.1}$$

Here $VG_0T|^4$ means that all the operators on the left of the $|^4$ are in the 4-body space and therefore G_0 is the 4-body free resolvent operator in the 4-body space. The idea of Alt, Grassberger and Sandhas (AGS) [AGS67, San72] was to introduce channel equations and channel transition operators in a way that they can be recast into a super Lippmann-Schwinger equation for the $N = 3$ case and Grassberger and Sandhas [GS67, San74, San75] extended the theory to the $N > 3$ case. The first step is the splitting of the potential V into two-body interactions expressing it as the

sum of all two-body potentials present, with A denoting the pair of bodies involved in the interaction (here, I use the term "bodies" to denote both clusters and particles and it can easily be seen that the bodies on this level are particles). The splitting of the potential is defined by the expression

$$V|^4 = \sum_A v_A|^4 \tag{2.2}$$

Before describing the channel LS-equations I should mention that the index A is given by the six possible partitions of 4 particles into three clusters. The channel equations can now be described by the following expression, which is just the two-particle Lippmann-Schwinger equation expressed in the 4-particle space

$$t_A|^4 = v_A|^4 + v_A G_0 t_A|^4 \tag{2.3}$$

The T-matrix is determined by coupling all the channels and introducing the new channel transition operators $U_{AB}|^4$. The amplitude $T|^4$ can be rewritten using the ansatz:

$$T|^4 = \sum_A t_A|^4 + \sum_{AB} t_A G_0 U_{AB} G_0 t_B|^4 \tag{2.4}$$

Following the same arguments as in the traditional AGS-theory this yields the equations for the transition operators

$$U_{AB}|^4 = G_0^{-1}\bar{\delta}_{AB}\Big|^4 + \sum_C \bar{\delta}_{AC} t_C G_0 U_{CB}\Big|^4 \tag{2.5}$$

These equations do not reduce the problem to an effective two-body equation and the same trick has to be used again. It is therefore necessary to recast eq.(2.5) into a Lippmann-Schwinger form on the three-body level

$$T|^3 = V|^3 + V G_0 T|^3 \tag{2.6}$$

This is achieved by the following definitions for the three body operators

$$T|^3 = U_{AB}|^4 \tag{2.7}$$

$$V|^3 = G_0^{-1}\bar{\delta}_{AB}\Big|^4 \tag{2.8}$$

$$G_0|^3 = G_0 t_A G_0 \delta_{AB}|^4 \tag{2.9}$$

Analogously to the $|^4$ in eq.(2.1), here $|^3$ means the operators on the left of it are in the three-body space. The operator $U_{AB}|^4$ describes the transition from a three-body partition to another three-body partition. The partition A' is a two-body partition, which is included in the partition A, if A' can break up into A by breaking just one cluster. Later, in the extended model in which one particle is a pion, A' has a slightly different meaning. However, for now A' defines simply the chain-of-partitions in the standard 4-body problem. A short review of this notation is given in Adhikari and Kowalski [AK91, pp.251-252] and a detailed review is given in the first of a series of papers by Cattapan and Vanzani [CV83, CV84, CV85].

Table 2.1: The Yakubovsky chain space. The table denotes the six dimensional space spanned by the possible three-body clusters, as defined in the column labels. The matrix elements denote all two-body clusters that can break up into either three-nucleon cluster describing the particular matrix element.

	$a=(bc)ad$	$b=(ac)bd$	$c=(ab)cd$	$\alpha=(ad)bc$	$\beta=(bd)ac$	$\gamma=(cd)ab$
a	a',α',d'	d'	d'	α'	a'	a'
b	d'	b',β',d'	d'	b'	β'	b'
c	d'	d'	γ',c',d'	c'	c'	γ'
α	α'	b'	c'	α',b',c'	c'	b'
β	a'	β'	c'	c'	β',a',c'	a'
γ	a'	b'	γ'	b'	a'	γ',a',b'

Table [2.1] shows all possible three body transitions and the allowed two-body transitions common to both three-body transitions involved, where the primed vari-

ables denotes the set of two-body partitions

$$H^2 = \left\{ \begin{array}{lll} (abc)\,d = d' & (dab)\,c = c' & (bc)\,(ad) = \alpha' \\ & (dac)\,b = b' & (ac)\,(bd) = \beta' \\ & (dbc)\,a = a' & (ab)\,(cd) = \gamma' \end{array} \right\}, \tag{2.10}$$

The chain-of-partitions are described by the usual notation aa', but table [2.1] already gives the chains in more detail. The unusual notation (singling out one of the four particles, d) will prove very convenient when I generalize to the πNNN system, where particle d will be identified with the pion. The important observation in this table is that all off-diagonal elements have a unique contribution from the two-body subspace. It is again possible to split the potential in 3-body space, which has only off-diagonal elements, in the following unique way

$$V|^3 = \sum_{A'} (v_{A'})_{AB} \tag{2.11}$$

The terms $(v_{A'})_{AB}$ are only non-zero when the implicit chain relations are satisfied, namely partitions A, B both have to be obtainable by the break-up of A'. The new set of channel equations in respect to the two-body partitions A' is defined by

$$(t_{A'})_{AB} = (v_{A'})_{AB} + \sum_{C,D} (v_{A'})_{AC}\,(G_0)_{CD}\,(t_{A'})_{DB} \tag{2.12}$$

where

$$(G_0)_{AB} = G_0 t_A G_0 \delta_{AB} \tag{2.13}$$

This complicated looking free propagator denotes the free propagation of three bodies, one of which is a composite body. The delta function can be inserted explicitly to get the more familiar form

$$(t_{A'})_{AB} = (v_{A'})_{AB} + \sum_{C} (v_{A'})_{AC}\,G_0 t_C G_0\,(t_{A'})_{CB} \tag{2.14}$$

Because this split is unique and complete the same AGS formalism can be used again on this sub-level. The channel transition equations on the two-body sub-level are

found with the fundamental Ansatz

$$\left(T|^3\right)_{AB} = \sum_{A'} (t_{A'})_{AB} + \sum_{\substack{A',B' \\ C,D,E,F}} (t_{A'})_{AC} (G_0)_{CD} (U_{A'B'})_{DE} (G_0)_{EF} (t_{A'})_{FB} \qquad (2.15)$$

It is again possible to rewrite this equation and give an explicit expression for the new channel transition operators

$$(U_{A'B'})_{AB} = (G_0)^{-1}_{AB} \bar{\delta}_{A'B'} + \sum_{\substack{C' \\ C,D}} \bar{\delta}_{A'C'} (t_{C'})_{AC} (G_0)_{CD} (U_{C'B'})_{DB} \qquad (2.16)$$

These equations act in the three-body subspace and all operators are defined in this sub-space. This solves the problem algebraically, because it can be shown that the equations are connected. It should be noted that, depending on the A', the equations are either 3×3 or 2×2 matrix equations. In the next section I argue that eq.(2.16) can be generalized to the πNNN scattering problem.

2.2 The Padova coupling formalism

In this section I show that it is possible to generalize the traditional AGS formalism to include explicitly one pion degree of freedom. This generalization is by no means trivial and care must be taken to include a physically sound coupling between the pion and no-pion sectors. The first section defines a new notation that allows investigation of the coupling scheme in more detail than the usual few-body chain labels. In the second section the notation is used to describe the explicit coupling between pion and no-pion sectors and the corresponding generalized AGS equations are given. In the following sections a practical approximation scheme is defined and the algebraic implications for the three-body problem with one pion degree of freedom are discussed.

2.2.1 The πNNN partition notation

In this section I define the notation that is used in the derivation of the proposed model for the πNNN scattering problem. There are four particles in the system,

three nucleons denoted by the indices a, b, c and one pion denoted by the index π. These particles are allowed to form clusters and the pion itself does not have to be conserved. The partitions with three clusters are given by the seven unprimed indices $a, b, c, \alpha, \beta, \gamma, \bar{\pi}$. Here, the indices a, b, c denote the following three-cluster partitions $a(bc)\pi, (ac)b\pi, (ab)c\pi$ respectively. Namely, the pion is a spectator and the nucleon-nucleon cluster is denoted by the unique spectator nucleon. The indices α, β, γ denote the three-cluster partitions $(a\pi)bc, a(b\pi)c, ab(c\pi)$ respectively. Once again the pion is always present, but this time it appears in a πN cluster and the partition is denoted by the unique nucleon in the cluster. This notation is very similar to the one used earlier, because the nucleon d is replaced by the always present pion π. This covers all possible three-cluster partitions when the pion is present. The last three-cluster partition is given by a, b, c where the pion is missing and this partition is denoted by the index $\bar{\pi}$ (no pion).

The two-cluster partitions are differentiated from the three-cluster partitions by the use of a prime.The set of possible three cluster partitions is given by

$$H^3_{1\pi} = [A, B, C, \bar{\pi}] = [a, b, c, \alpha, \beta, \gamma, \bar{\pi}]\,. \tag{2.17}$$

Since every element in this set denotes a three-cluster partition it is possible to find exactly three different two-cluster partitions for each one of them. However, it is only possible to have ten distinct two-body clusters, namely

$$H^2_{1\pi} = \left\{ \begin{array}{llll} (abc)\,\pi = \pi' & (\pi ab)\,c = c' & (bc)\,(a\pi) = \alpha' & (ab)\,c = \bar{c}' \\ & (\pi ac)\,b = b' & (ac)\,(b\pi) = \beta' & (ac)\,b = \bar{b}' \\ & (\pi bc)\,a = a' & (ab)\,(c\pi) = \gamma' & (bc)\,a = \bar{a}' \end{array} \right\}, \tag{2.18}$$

where a, b, c, π again denotes the nucleons and the pion respectively. The two body cluster notation is given by the primed symbols and the bar again denotes no-pion states. The notation is very suggestive and allows an easy interpretation of the equations in the proposed model.

2.2.2 The coupling between the pion and no-pion sectors

In the last section I introduced a new notation for the possible three and two-body partitions and it is evident that two-body partitions can break up into a multitude of three-body partitions. This is a well known fact from the four-body scattering problem and it is necessary to define chain-of-partitions. In general, the four-body problem has 18 chains and connectivity of the AGS-equations can be achieved. The reason for this connectivity is intimately coupled to the structure of the two-body partitions. Unlike in the three-body case, where only one type of two-body partitions exists (one two-particle cluster and one free particle), in this case two distinct types of partitioning exist. The partitions are either $3+1$ or $2+2$ and they give distinctive channel equations, because one has three and the other has two chains associated with it. In the four-body problem it is a nice feature that there is no coupling between these two types of chains. This allows the definition of channel equations on the two-body sub-level in order to solve the system. The πNNN system is more complicated, because the pion and no-pion systems have to be coupled in a physically sound way. This problem was first investigated in a paper by Cattapan, Canton and Svenne [CCS93] by generalizing the Thomas-Rinat-Afnan-Blankleider-Avishai-Mizutani (TRABAM) [AT74, Rin77, MK77, TR79, AM79, AB80] equations for the πNN problem. In subsequent papers Cattapan and Canton [CC94a, CC94b, CC97] addressed open problems of the proposed model and recently Canton[Can98] was able to give a model that solved all the aforementioned problems. In this thesis I use the same model in a slightly different notation.

In the system in question everything is assumed to be dressed, i.e. the nucleon and the creation/destruction vertices. All partitions are just groupings and not necessarily bound states and the pion can not be created into a nucleon-pion group from the same nucleon, because this constitutes the dressing of the vertex. This also is the reason that in the three-body partitions no coupling exists between the pion and

no-pion sectors. Furthermore, the clusterings $a', \alpha', \bar{a}'$ are asymptotically not distinguishable since, asymptotically, the pion may be associated with the single nucleon, the two-nucleon cluster, or be absent. This means that arbitrary transitions between the aforementioned clusters are allowed without changing the pole structure of the underlying equations. Therefore the system describing this channel is already a coupled system. However, the AGS formalism demands that the transition between a' and α' does not occur directly, because they represent uncoupled channels. Therefore, a two-body channel equation for the πNNN system can be defined in the following way

$$t_{A'}|^3 = v_{A'}|^3 + v_{A'} G_0 t_{A'}|^3 \tag{2.19}$$

where

$$t_{A'}|^3 = \begin{pmatrix} (t_{A'})_{a'A,a'B} & (t_{A'})_{a'A,\alpha'B} & (t_{A'})_{a'A,\bar{a}'\bar{\pi}} \\ (t_{A'})_{\alpha'A,a'B} & (t_{A'})_{\alpha'A,\alpha'B} & (t_{A'})_{\alpha'A,\bar{a}'\bar{\pi}} \\ (t_{A'})_{\bar{a}'\bar{\pi},a'B} & (t_{A'})_{\bar{a}'\bar{\pi},\alpha'B} & (t_{A'})_{\bar{a}'\bar{\pi},\bar{a}'\bar{\pi}} \end{pmatrix} \tag{2.20}$$

$$v_{A'}|^3 = \begin{pmatrix} (v_{A'})_{a'A,a'B} & 0 & (v_{A'})_{a'A,\bar{a}'\bar{\pi}} \\ 0 & (v_{A'})_{\alpha'A,\alpha'B} & (v_{A'})_{\alpha'A,\bar{a}'\bar{\pi}} \\ (v_{A'})_{\bar{a}'\bar{\pi},a'B} & (v_{A'})_{\bar{a}'\bar{\pi},\alpha'B} & (v_{A'})_{\bar{a}'\bar{\pi},\bar{a}'\bar{\pi}} \end{pmatrix} \tag{2.21}$$

$$v_{A'} G_0|^3 = \begin{pmatrix} (v_{A'})_{a'A,a'B} G_0 t_B G_0 & 0 & (v_{A'})_{a'A,\bar{a}'\bar{\pi}} g_0 \\ 0 & (v_{A'})_{\alpha'A,\alpha'B} G_0 t_B G_0 & (v_{A'})_{\alpha'A,\bar{a}'\bar{\pi}} g_0 \\ (v_{A'})_{\bar{a}'\bar{\pi},a'B} G_0 t_B G_0 & (v_{A'})_{\bar{a}'\bar{\pi},\alpha'B} G_0 t_B G_0 & (v_{A'})_{\bar{a}'\bar{\pi},\bar{a}'\bar{\pi}} g_0 \end{pmatrix} \tag{2.22}$$

Here the A' has a different meaning than in the standard four-body problem, because it simply denotes the channels, which in this case are given by coupled systems (thus, A' plays a similar role as that of s in [Can98]). It is noteworthy that this system collapses to the uncoupled Sandhas system if the coupling between the pion and no-pion sector is set equal to zero and the no-pion sector is ignored. Consequently, this channel equation algebraically makes sense.

The explicit expressions for the operators in the above equation are given by

$$(v_{A'})_{a'A,a'B} = G_0^{-1} \left(\bar{\delta}_{a'}\right)_{AB} \tag{2.23}$$

$$(v_{A'})_{\alpha' A,\alpha' B} = G_0^{-1}\left(\bar{\delta}_{\alpha'}\right)_{AB} \tag{2.24}$$

$$(v_{A'})_{\bar{a}'\bar{\pi},\bar{a}'\bar{\pi}} = \mathcal{V}_{\bar{a}'} \tag{2.25}$$

which defines the usual particle conserving interactions in the three and four particle system and

$$(v_{A'})_{\bar{a}'\bar{\pi},A'B} = \sum_{i=\alpha,\beta,\gamma} f_i\left(\bar{\delta}_{A'}\right)_{iB} \tag{2.26}$$

$$(v_{A'})_{A'A,\bar{a}'\bar{\pi}} = \sum_{i=\alpha,\beta,\gamma} f_i^{\dagger}\left(\bar{\delta}_{A'}\right)_{iA} \tag{2.27}$$

which defines the general form of the creation and destruction vertices. The anti-delta functions have to satisfy the chain-relations in addition to the anti delta-relations. This means that the anti-delta in eq.(2.26) for example is zero, unless the two indices i, α are both contained in A' and not equal to each other. The f_i are the usual elementary vertices, but with the notation of the partitions denoting a pion-nucleon clustering. The elementary vertex is acting on the same nucleon that defines this particular partition. This choice appears to be rather arbitrary, but it guarantues that there is one nucleon that never connects to the other two, therefore, defining the channel. It is helpful to write down the explicit terms for the creation/destruction operators in respect to the usual elementary vertices f_α. The anti-delta function also includes the chain restrictions and the following explicit terms are found

$$(v_{A'})_{\bar{a}'\bar{\pi},a'a} = f_\beta + f_\gamma \tag{2.28}$$

The chain relations require $a = a, \beta, \gamma$ and $i = \beta, \gamma$ and the anti-delta function has no effect. Another set of terms is given by

$$(v_{A'})_{\bar{a}'\bar{\pi},a'\beta} = f_\gamma \tag{2.29}$$

Again, the chain relations require $a = a, \beta, \gamma$ and $i = \beta, \gamma$, but this time β is excluded because of the anti-delta function. The channel A' also includes the following three terms in a similar way

$$(v_{A'})_{\bar{a}'\bar{\pi},a'\gamma} = f_\beta \tag{2.30}$$

$$(v_{A'})_{\bar{a}'\bar{\pi}.\alpha'a} = f_\alpha \tag{2.31}$$

$$(v_{A'})_{\bar{a}'\bar{\pi},\alpha'\alpha} = 0 \tag{2.32}$$

These definitions are also given in a diagrammatic form in figures [2.1,2.2] and it is clear that these vertices are more complicated than the simple creation/destruction vertices $f_\alpha, f_\beta, f_\gamma$ themselves. Correspondingly, similar expressions for the inter-

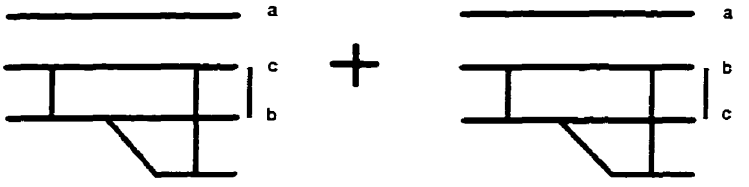

Figure 2.1: The creation vertex $(v_{A'})_{\bar{a}'\bar{\pi},\alpha'a} = f_\beta + f_\gamma$. The vertical bars denote bare partitions and should not be interpreted as interactions. The pion is absorbed on the nucleons c and b respectively.

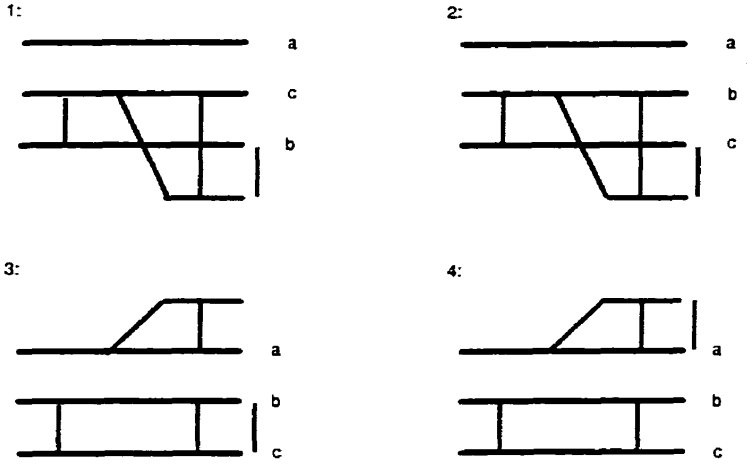

Figure 2.2: The destruction vertices $(v_{A'})_{\bar{a}'\bar{\pi},\alpha'\beta} = f_\gamma$ (1), $(v_{A'})_{\bar{a}'\bar{\pi},\alpha'\gamma} = f_\beta$ (2), $(v_{A'})_{\bar{a}'\bar{\pi}.\alpha'a} = f_\alpha$ (3) and $(v_{A'})_{\bar{a}'\bar{\pi},\alpha'\alpha} = 0$ (4). As argued in the thesis the last vertex is identically zero due to the vertex dressing.

actions in the other two channels B', C' can be given and in general the channel-interactions in chain space are given by table [2.2] (see end of this chapter).

It is also possible to describe the interaction term in the complete chain space, which is made up by the sum of the interactions in the three channels A', B', C' and the interaction in the pion-spectator channel π'. The chain-space interactions for the three channels, where the pion is not a spectator, are shown in table [2.3] (see end of this chapter). The case were the pion is a spectator is usually treated separately with standard three-body techniques. The tables [2.2,2.3] show that the channel splitting is not as simple as in the standard AGS case. However it is also apparent that the suggested splitting does indeed add up to the expected format, namely the standard AGS format in the pion sector, the typical three-body contributions in the no-pion sector and the expected contributions in the connecting sectors. The connecting sectors show that the pion can come from any one nucleon, unless it is partitioned with one nucleon in which case it can only come from any one of the remaining two. In table [2.3] this can be seen, because the sectors connecting the no-pion partitions to the partitions a, b, c have contributions from all three nucleons and the sectors connecting the no-pion partitions to the partitions α, β, γ have only contributions from the remaining two nucleons that do not take part in the partition. It should be noted that the T-matrix also has contributions from the transitions $a'\alpha', \alpha'a', ...,$ which are identically zero in the channel potentials. This is not an unfamiliar result, because in the standard AGS theory it is also possible to get contributions from transitions between the 2+2 and 3+1 partitions. However in the AGS theory it is not possible to get these contributions from a direct transition, or in other words from the driving terms of the LS-equations. In the next section I introduce the channel transition operators of the two-body partitions, which lead to the final set of algebraic equations of the proposed connected kernel model.

2.2.3 The channel transition operators of the two-body sub-space

In the last step of this procedure I define the channel transition operators with the usual Ansatz defined in AGS-systems and solve for them accordingly. The Ansatz is

that the sum of all channels plus the sum of all channel transitions gives the total T-matrix, which gives

$$T|^3 = \sum_{A'} t_{A'} + \sum_{A',B'} t_{A'} G_0^{(3)} U_{A'B'} G_0^{(3)} t_{A'} \tag{2.33}$$

where the summation over the A' involves implicitly a summation over the corresponding intra-channel partitions $a', \alpha', \bar{a}'$. Solving for the transition operator ($U_{A'B'}$) is somewhat tricky, because of the amount of chains involved. It is a rather lengthy, but not too complicted calculation. At this point the newly developed notation does not appear to be simpler than the original notation used by Canton [Can98]. Admittedly, the new notation is rather complicated, but unfortunately so is the underlying system. The problem lies in the fact that the number of explicit elements in the matrices is reduced, which means that the elements now span a bigger subspace than the previous ones. In essence, the four particle space, which was given previously by four elements, is combined into one element. A new partition has to be introduced that denotes the two possible two-cluster partitions a', α' in one partition index. Unfortunately, there is only so much information that can be described by one partition index before it looses its simplicity. Therefore, I define the generic two-cluster partitions $\hat{a}', \hat{\alpha}'$ to describe the set of two-cluster partitions of the four body space belonging to channel A', namely a', α'. The two generic partitions with the hat can be used interchangeably, because they describe the same set. With these definitions and the corresponding ones for the channels B', C' the transition operator is described by the equation

$$\begin{pmatrix} (U_{A'B'})_{\hat{a}'A,\hat{b}'B} & (U_{A'B'})_{\hat{a}'A,\bar{b}'\bar{\pi}} \\ (U_{A'B'})_{\bar{a}'\bar{\pi},\hat{b}'B} & (U_{A'B'})_{\bar{a}'\bar{\pi},\bar{b}'\bar{\pi}} \end{pmatrix} = \begin{pmatrix} (G_0 t_A G_0 \delta_{AB})^{-1} \left(\bar{\delta}_{A'B'} + \delta_{A'B'}\bar{\delta}_{\hat{a}'\hat{b}'}\right) & 0 \\ 0 & (g_0)^{-1}\,\bar{\delta}_{A'B'} \end{pmatrix}$$

$$+ \sum_{\substack{C',C \\ \hat{c}',\hat{\gamma}'}} \begin{pmatrix} \left(\bar{\delta}_{A'C'} + \delta_{A'C'}\bar{\delta}_{\hat{a}'\hat{c}'}\right)(t_{C'})_{\hat{c}'A,\hat{\gamma}'C}\, G_0 t_C G_0 & \left(\bar{\delta}_{A'C'} + \delta_{A'C'}\bar{\delta}_{\hat{a}'\hat{c}'}\right)(t_{C'})_{\hat{c}'A,\bar{c}'\bar{\pi}}\, g_0 \\ \bar{\delta}_{A'C'}\,(t_{C'})_{\bar{c}'\bar{\pi},\hat{\gamma}'C}\, G_0 t_C G_0 & \bar{\delta}_{A'C'}\,(t_{C'})_{\bar{c}'\bar{\pi},\bar{c}'\bar{\pi}}\, g_0 \end{pmatrix}$$

$$\times \begin{pmatrix} (U_{C'B'})_{\hat{c}'C,\hat{b}'B} & (U_{C'B'})_{\hat{c}'A,\bar{b}'\bar{\pi}} \\ (U_{C'B'})_{\bar{c}'\bar{\pi},\hat{b}'B} & (U_{C'B'})_{\bar{c}'\bar{\pi},\bar{b}'\bar{\pi}} \end{pmatrix}$$

(2.34)

and this forms the formal solution to the problem. However, it should be mentioned that solving for the transition operator is not a trivial task. The channel transition operator ($U_{A'B'}$) is defined in the seven dimensional space given by the three-cluster partitions. Therefore, one has to add up the contributions to the matrices from the given channels on the left and the right of the transition operator. When solving for the transition operators the unusual delta function $\left(\bar{\delta}_{A'B'} + \delta_{A'B'}\bar{\delta}_{\hat{a}'\hat{b}'}\right)$ appears. It appears due to the fact that in the derivation equivalent terms on the left hand side of the equation are collected and therefore these terms are missing on the right hand side. Either two distinct channels are given in the definition of the transition operator, which is described by the case $\bar{\delta}_{A'B'}$, or the same channel is used twice, which is described by the case $\delta_{A'B'}$. However, in the second case the left most channel elements in question have to be different, namely $\bar{\delta}_{\hat{a}'\hat{b}'}$ must be satisifed. The $\hat{a}'\hat{b}'$ denote two possible two-cluster partitions in the four-particle space that belong to channel B', namely they both can be b' or β' .

It turns out that this set of equations is indeed connected after a finite number of iterations, which has to be true for any reasonable theory of this type. It should be kept in mind that the operators in this equation are themselves highly non-trivial and that the system is therefore highly complicated. Especially the channel equations should be treated with care, because they already represent matrix equations describing the coupled set of two-body partitions A'. The advantages of the new notation are displayed in the three tables, because of the rather simple form of the possible chains in respect to the new notation. It is also possible to show directly the connectedness of the equation by explicitly investigating all possible chains.

2.3 Quasiparticle formalism

The paper of Canton [Can98] also includes making quasiparticle approximations at the three-cluster and two-cluster levels. Since this is also needed in this work, to arrive at a practical approximation scheme (next chapter), I here review the quasiparticle method. Traditionally the Quasiparticle formalism corresponds to the separable expansion method developed by Weinberg [Wei63a, Wei63b, Wei64, SWW64] in a series of papers. For a comprehensive description of this theory I refer to Weinberg's lectures [Wei65]. The expansion is given in respect to the so called Weinberg states, but I shall denote all possible expansion methods as quasiparticle formalisms, as long as they correctly describe the pole structure of the underlying t-matrices. For a collection of different possible expansion methods I refer to the book by Adhikari and Kowalski [pp.149-169] [AK91]. In this section I assume that I have a valid expansion method, but the type of method is unspecified. In general, if a separable expansion of the potential is given, an explicit solution of the t-matrix can be found. For simplicity, I also restrict myself to rank one expansions and the t-matrix is then given by

$$t_A(z) = \left|A^{(3)}(z)\right\rangle \tau_A^{(3)}(z) \left\langle A^{(3)}(z)\right| \tag{2.35}$$

The explicit form of the τ and the form factors depends on the specifics of the expansion method. Also, the t-matrix is a two-body t-matrix embedded in the four body space and therefore the spectators could be split off. However, I come back to this point in the 1-D Toy model in chapter 5 and for now assume simply that the form factors and τ's in question can be found. For simplification of the notation the following short hand for the form factors is defined

$$\left|A^{(3)}(z)\right\rangle = |A\rangle \tag{2.36}$$

This definition should be no problem, because A denotes a set of three-cluster partitions in the four body space. It is clear that the model can have two distinct types

of form factors, namely the ones corresponding to a NN cluster and the ones corresponding to a πN cluster. In the new notation the NN cluster embedded in the four-body space is denoted by a, the corresponding πN cluster by α and the set of the two is denoted by A. This distinction of the cluster types becomes important in section 3.2 when the practical approximation scheme is introduced. Furthermore, it is obvious that form factors of this kind can only exist in the four body space and not in the three nucleon space. I show that this does not pose any problems, because the t-matrix that is approximated by the quasiparticle terms appears only in the four body part of the three-cluster Green's function.

The first step in the formalism is to insert the quasiparticle t-matrix into the final set of equations derived in the last section. At this point there is no notational advantage to the matrix notation due to the complexity of the matrices involved. Therefore, the result of the calculation for the four components is given explicitly, which gives an equivalent set of equations to the ones previously given by Canton [Can98]. The quasiparticle channel transition equations for the Padova model are therefore

$$\begin{aligned}
(U_{A'B'})_{\tilde{a}'A,\tilde{b}'B} &= (G_0\,|A\rangle\,\tau_A\,\langle A|\,G_0\delta_{AB})^{-1}\left(\bar{\delta}_{A'B'}+\delta_{A'B'}\bar{\delta}_{\tilde{a}'\tilde{b}'}\right)\\
&\quad+\sum_{C',\tilde{c}'}\left(\bar{\delta}_{A'C'}+\delta_{A'C'}\bar{\delta}_{\tilde{a}'\tilde{c}'}\right)(t_{C'})_{\tilde{c}'A,\bar{c}'\bar{\pi}}\,g_0\,(U_{C'B'})_{\bar{c}'\bar{\pi},\tilde{b}'B}\\
&\quad+\sum_{C',\tilde{c}',\tilde{\gamma}',C}\left(\bar{\delta}_{A'C'}+\delta_{A'C'}\bar{\delta}_{\tilde{a}'\tilde{c}'}\right)(t_{C'})_{\tilde{c}'A,\tilde{\gamma}'C}\,G_0\,|C\rangle\,\tau_C\,\langle C|\,G_0\,(U_{C'B'})_{\tilde{\gamma}'C,\tilde{b}'B} \qquad (2.37)\\
(U_{A'B'})_{\bar{a}'\bar{\pi},\tilde{b}'B} &= +\sum_{C'}\bar{\delta}_{A'C'}\,(t_{C'})_{\bar{c}'\bar{\pi},\bar{c}'\bar{\pi}}\,g_0\,(U_{C'B'})_{\bar{c}'\bar{\pi},\tilde{b}'B}\\
&\quad\sum_{C',\tilde{c}',\tilde{\gamma}',C}\bar{\delta}_{A'C'}\,(t_{C'})_{\bar{c}'\bar{\pi},\tilde{\gamma}'C}\,G_0\,|C\rangle\,\tau_C\,\langle C|\,G_0\,(U_{C'B'})_{\tilde{\gamma}'C,\tilde{b}'B} \qquad (2.38)\\
(U_{A'B'})_{\tilde{a}'A,\bar{b}'\bar{\pi}} &= +\sum_{C',\tilde{c}'}\left(\bar{\delta}_{A'C'}+\delta_{A'C'}\bar{\delta}_{\tilde{a}'\tilde{c}'}\right)(t_{C'})_{\tilde{c}'A,\bar{c}'\bar{\pi}}\,g_0\,(U_{C'B'})_{\bar{c}'\bar{\pi},\bar{b}'\bar{\pi}}\\
&\quad\sum_{C',\tilde{c}',\tilde{\gamma}',C}\left(\bar{\delta}_{A'C'}+\delta_{A'C'}\bar{\delta}_{\tilde{a}'\tilde{c}'}\right)(t_{C'})_{\tilde{c}'A,\tilde{\gamma}'C}\,G_0\,|C\rangle\,\tau_C\,\langle C|\,G_0\,(U_{C'B'})_{\tilde{\gamma}'C,\bar{b}'\bar{\pi}} \qquad (2.39)\\
(U_{A'B'})_{\bar{a}'\bar{\pi},\bar{b}'\bar{\pi}} &= +\sum_{C'}\bar{\delta}_{A'C'}\,(t_{C'})_{\bar{c}'\bar{\pi},\bar{c}'\bar{\pi}}\,g_0\,(U_{C'B'})_{\bar{c}'\bar{\pi},\bar{b}'\bar{\pi}}
\end{aligned}$$

$$(g_0)^{-1}\,\bar{\delta}_{A'B'} + \sum_{C',\bar{c}',\bar{\gamma}',C} \bar{\delta}_{A'C'}\,(t_{C'})_{\bar{c}'\bar{\pi},\bar{\gamma}'C}\,G_0\,|C\rangle\,\tau_C\,\langle C|\,G_0\,(U_{C'B'})_{\bar{\gamma}'C,\bar{b}'\bar{\pi}} \quad (2.40)$$

Up to this point the derivation is straightforward and the character of the equations did not change substantially from the equations derived in the last section.

The next step is to fold the equations between appropriate states in order to get the explicit matrix elements. The states in question are given by the expressions $G_0\,|A\rangle$ and in the case of Weinberg states, where $|A\rangle = V_A\,|\phi_A\rangle$, $|\phi_A\rangle$ is given by the two-body bound state. At this point I do not consider the actual interpretation of these terms and simply perform the folding. The resulting equation is given by the expression

$$\mathbf{X}_{A'B'} = \mathcal{G}^{(3)-1}\bar{\Delta}_{A'B'} + \sum_{C'} \bar{\Delta}_{A'C'}\mathbf{x}_{C'}\mathcal{G}^{(3)}\mathbf{X}_{C'B'} \quad (2.41)$$

with the definitions

$$\left(\bar{\Delta}_{A'B'}\right)_{\bar{a}'A,\bar{b}'B} = \delta_{AB}\left(\bar{\delta}_{A'B'} + \delta_{A'B'}\bar{\delta}_{\bar{a}'\bar{b}'}\right) \quad (2.42)$$

$$\left(\bar{\Delta}_{A'B'}\right)_{\bar{a}'A,\bar{b}'\bar{\pi}} = 0 \quad (2.43)$$

$$\left(\bar{\Delta}_{A'B'}\right)_{\bar{a}'\bar{\pi},\bar{b}'B} = 0 \quad (2.44)$$

$$\left(\bar{\Delta}_{A'B'}\right)_{\bar{a}'\bar{\pi},\bar{b}'\bar{\pi}} = \bar{\delta}_{A'B'} \quad (2.45)$$

and

$$\mathcal{G}^{(3)} = \begin{pmatrix} \tau_A\delta_{AB} & 0 \\ 0 & g_0 \end{pmatrix} \quad (2.46)$$

and

$$\mathbf{x}_{A'} = \begin{pmatrix} (x_{A'})_{\bar{a}'A,\bar{\alpha}'B} & (x_{A'})_{\bar{a}'A,\bar{\alpha}'\bar{\pi}} \\ (x_{A'})_{\bar{a}'\bar{\pi},\bar{\alpha}'B} & (x_{A'})_{\bar{a}'\bar{\pi},\bar{a}'\bar{\pi}} \end{pmatrix} \quad (2.47)$$

and correspondingly the expression for $\mathbf{X}_{A'B'}$. These definitions are quite easily seen in the matrix form of the equations as derived in the last section. There seems to be a double contribution of δ_{AB}, because this term appears in the new delta function and the quasiparticle three-cluster Green's function. However, careful calculations show that these two terms indeed appear explicitly, one deriving from the original

definition of the three-cluster Green's function and one from the folding. The explicit expression for the components of these two operators are given by

$$(x_{A'})_{\tilde{a}'A,\tilde{\alpha}'B} = \langle A|\, G_0\, (t_{A'})_{\tilde{a}'A,\tilde{\alpha}'B}\, G_0\, |B\rangle \tag{2.48}$$

$$(x_{A'})_{\tilde{a}'A,\bar{a}'\bar{\pi}} = \langle A|\, G_0\, (t_{A'})_{\tilde{a}'A,\bar{a}'\bar{\pi}} \tag{2.49}$$

$$(x_{A'})_{\bar{a}'\bar{\pi},\tilde{\alpha}'B} = (t_{A'})_{\bar{a}'\bar{\pi},\tilde{\alpha}'B}\, G_0\, |B\rangle \tag{2.50}$$

$$(x_{A'})_{\bar{a}'\bar{\pi},\bar{a}'\bar{\pi}} = (t_{A'})_{\bar{a}'\bar{\pi},\bar{a}'\bar{\pi}} \tag{2.51}$$

and

$$(X_{A'B'})_{\tilde{a}'A,\tilde{b}'B} = \langle A|\, G_0\, (U_{A'B'})_{\tilde{a}'A,\tilde{b}'B}\, G_0\, |B\rangle \tag{2.52}$$

$$(X_{A'B'})_{\tilde{a}'A,\bar{b}'\bar{\pi}} = \langle A|\, G_0\, (U_{A'B'})_{\tilde{a}'A,\bar{b}'\bar{\pi}} \tag{2.53}$$

$$(X_{A'B'})_{\bar{a}'\bar{\pi},\tilde{b}'B} = (U_{A'B'})_{\bar{a}'\bar{\pi},\tilde{b}'B}\, G_0\, |B\rangle \tag{2.54}$$

$$(X_{A'B'})_{\bar{a}'\bar{\pi},\bar{b}'\bar{\pi}} = (U_{A'B'})_{\bar{a}'\bar{\pi},\bar{b}'\bar{\pi}} \tag{2.55}$$

As I mentioned before the folding terms only appear in the four-body components of the system, denoted by A, B. The last three equations show the terms that connect the pion and no-pion sectors and describe the three-nucleon sector. It is straightforward to show that the subsystem amplitudes are also governed by a quasiparticle equation using the same folding procedure. The result is given by

$$\mathbf{x}_{A'} = \mathbf{z}_{A'} + \mathbf{z}_{A'}\mathcal{G}^{(3)}\mathbf{x}_{A'} \tag{2.56}$$

with

$$\mathbf{z}_{A'} = \begin{pmatrix} (z_{A'})_{\tilde{a}'A,\tilde{\alpha}'B} & (z_{A'})_{\tilde{a}'A,\bar{a}'\bar{\pi}} \\ (z_{A'})_{\bar{a}'\bar{\pi},\tilde{\alpha}'B} & (z_{A'})_{\bar{a}'\bar{\pi},\bar{a}'\bar{\pi}} \end{pmatrix} \tag{2.57}$$

$$= \begin{pmatrix} \langle A|\, G_0\, |B\rangle\, (\delta_{A'})_{\tilde{a}'\tilde{\alpha}'} \left(\bar{\delta}_{\tilde{a}'}\right)_{AB} & \langle A|\, G_0\, (v_A)_{\tilde{a}'a,\bar{a}'\bar{\pi}} \\ (v_{A'})_{\bar{a}'\bar{\pi},\tilde{\alpha}'B}\, G_0\, |B\rangle & \mathcal{V}_{A'} \end{pmatrix} \tag{2.58}$$

The components in the second matrix follow directly from the initial definition of the channel potential terms, as described in the last section. There the definition was

given in a more explicit component form distinguishing explicitly between the chains involving the a' and the α' partitions. The off-diagonal terms in these partitions $a'\alpha'$ and $\alpha' a'$ were shown to be zero, which are accounted for by the introduction of the new delta function $(\delta_{A'})_{\hat{a}'\hat{\alpha}'}$. Again, the delta function requires that both $\hat{a}'$ and $\hat{\alpha}'$ have to be contained in A' and have to be equal. Obviously, this also guarantues that they both run only over partitions a', α'.

The next step is to introduce another separable expansion, this time on the two-cluster level. The four sub-amplitudes of the system are defined in the separable form

$$(x_{A'})_{\hat{a}'A,\hat{\alpha}'B} = \left|(A')^{(2)}_{\hat{a}'A}\right\rangle \tau^{(2)}_{A'} \left\langle(A')^{(2)}_{\hat{\alpha}'B}\right| \tag{2.59}$$

$$(x_{A'})_{\hat{a}'A,\bar{\alpha}'\bar{\pi}} = \left|(A')^{(2)}_{\hat{a}'A}\right\rangle \tau^{(2)}_{A'} \left\langle(A')^{(2)}_{\bar{a}'\bar{\pi}}\right| \tag{2.60}$$

$$(x_{A'})_{\bar{a}'\bar{\pi},\hat{\alpha}'B} = \left|(A')^{(2)}_{\bar{a}'\bar{\pi}}\right\rangle \tau^{(2)}_{A'} \left\langle(A')^{(2)}_{\hat{\alpha}'B}\right| \tag{2.61}$$

$$(x_{A'})_{\bar{a}'\bar{\pi},\bar{a}'\bar{\pi}} = \left|(A')^{(2)}_{\bar{a}'\bar{\pi}}\right\rangle \tau^{(2)}_{A'} \left\langle(A')^{(2)}_{\bar{a}'\bar{\pi}}\right| \tag{2.62}$$

Again, the separable approximation can be achieved by introducing an expansion of the potential in the two-cluster channels and subsequently solving for the channel t-matrix. However, since the channel equations are given by a coupled system, the form factors are multi-dimensional. The explicit form of these new form factors is far from being trivial and in the next chapter I give an approximation scheme exactly on these form factors. Without an approximation scheme one would have to solve the complicated channel equations directly, which is rather difficult. At this point I assume that it is possible to find an expansion method and the corresponding two-cluster τ and form factors. It should be evident that for the case π' there are only contributions from the four body sector, defined by the first equation. Now the same procedure as on the three-cluster level is performed. Namely, the quasiparticle expression is inserted and folded with the corresponding form factors. This gives the

final quasiparticle equation

$$X^{(2)}_{A'B'} = Z^{(2)}_{A'B'} + \sum_{C'} Z^{(2)}_{A'C'} \mathcal{G}^{(2)}_{C'} X^{(2)}_{C'B'} \tag{2.63}$$

with the definitions

$$\mathcal{G}^{(2)}_{A'} = \tau^{(2)}_{A'} \tag{2.64}$$

$$\begin{aligned} Z^{(2)}_{A'B'} &= \left\langle A'^{(2)} \right| \mathcal{G}^{(3)} \bar{\Delta}_{A'B'} \left| B'^{(2)} \right\rangle \\ &\equiv \left\langle (A')^{(2)}_{\bar{a}'\bar{\pi}} \right| g_0 \left| (B')^{(2)}_{\bar{b}'\bar{\pi}} \right\rangle \bar{\delta}_{A'B'} + \sum_{\bar{a}',\bar{b}',A} \left\langle (A')^{(2)}_{\bar{a}'A} \right| \tau_A \left| (B')^{(2)}_{\bar{b}'B} \right\rangle \left(\bar{\delta}_{A'B'} + \delta_{A'B'} \bar{\delta}_{\bar{a}'\bar{b}'} \right) \end{aligned} \tag{2.65}$$

and the quasiparticle two-cluster amplitude

$$\begin{aligned} X^{(2)}_{A'B'} &= \left\langle A'^{(2)} \right| \mathcal{G}^{(3)} X_{A'B'} \mathcal{G}^{(3)} \left| B'^{(2)} \right\rangle \\ &\equiv \left\langle (A')^{(2)}_{\bar{a}'\bar{\pi}} \right| g_0 \left(X_{A'} \right)_{\bar{a}'\bar{\pi},\bar{b}'\bar{\pi}} g_0 \left| (B')^{(2)}_{\bar{b}'\bar{\pi}} \right\rangle + \sum_{\substack{\bar{a}',A \\ \bar{b}',B}} \left\langle (A')^{(2)}_{\bar{a}'A} \right| \tau_A \left(X_{A'} \right)_{\bar{a}'A,\bar{b}'B} \tau_B \left| (B')^{(2)}_{\bar{b}'B} \right\rangle \\ &+ \sum_{\bar{a}',A} \left\langle (A')^{(2)}_{\bar{a}'A} \right| \tau_A \left(X_{A'} \right)_{\bar{a}'A,\bar{b}'\bar{\pi}} g_0 \left| (B')^{(2)}_{\bar{b}'\bar{\pi}} \right\rangle + \sum_{\bar{b}',B} \left\langle (A')^{(2)}_{\bar{a}'\bar{\pi}} \right| g_0 \left(X_{A'} \right)_{\bar{a}'\bar{\pi},\bar{b}'B} \tau_B \left| (B')^{(2)}_{\bar{b}'B} \right\rangle \end{aligned} \tag{2.66}$$

Equation (2.63) is similar to the type of equation introduced by Lovelace [Lov64], which is a connected LS-type equation for the standard $3N$ problem with separable potentials. It is clear that the additional pion makes the set of equations more complicated. Nevertheless, the overall structure is of the same type as the original Lovelace equation. Before I attempt to solve this system, I introduce one more approximation on the second level of the separable expansion.

Table 2.2: Chain-space contributions to the effective potential from channel A': The first row in every element denotes the possible two-cluster transitions that are possible in the give three-cluster transition element. The empty rows and columns show that the cannel A' does not contribute to these three-cluster transition elements. The second row in the table denotes the explicit contributions to the effective potential for the given two-cluster transitions

	a	b	c	α	β	γ	$\bar{\pi}$
a				$\alpha'\alpha'$ G_0^{-1}	$a'a'$ G_0^{-1}	$a'a'$ G_0^{-1}	$a'\bar{a}'$ $f_\beta + f_\gamma, f_\alpha$
b							
c							
α	$\alpha'\alpha'$ G_0^{-1}						$\alpha'\bar{a}'$ 0
β	$a'a'$ G_0^{-1}					$a'a'$ G_0^{-1}	$a'\bar{a}'$ f_γ
γ	$a'a'$ G_0^{-1}				$a'a'$ G_0^{-1}		$a'\bar{a}'$ f_β
$\bar{\pi}$	$\bar{a}'a', \bar{a}'\alpha'$ $f_\beta^\dagger + f_\gamma^\dagger, f_\alpha^\dagger$			$\bar{a}'\alpha'$ 0	$\bar{a}'a'$ $f_\gamma^\dagger$	$\bar{a}'a'$ $f_\beta^\dagger$	$\bar{a}'\bar{a}'$ $v_{\bar{a}}$

$a = (bc)a\pi, b = (ac)b\pi, c = (ab)c\pi$ $\alpha = (a\pi)bc, \beta = (b\pi)ac, \gamma = (c\pi)ab, \bar{\pi} = abc$

Table 2.3: Chain-space contributions to the effective potential from channels A', B', C'

	$a=(bc)a\pi$	$b=(ac)b\pi$	$c=(ab)c\pi$	$\alpha=(a\pi)bc$	$\beta=(b\pi)ac$	$\gamma=(c\pi)ab$	$\bar{\pi}=abc$
a				$\alpha'\alpha'$ G_0^{-1}	$a'a'$ G_0^{-1}	$a'a'$ G_0^{-1}	$a'\bar{a}'$ $f_\beta+f_\gamma, f_\alpha$
b				$b'b'$ G_0^{-1}	$\beta'\beta'$ G_0^{-1}	$b'b'$ G_0^{-1}	$b'\bar{b}', \beta'\bar{b}'$ $f_\alpha+f_\gamma, f_\beta$
c				$c'c'$ G_0^{-1}	$c'c'$ G_0^{-1}	$\gamma'\gamma'$ G_0^{-1}	$c'\bar{c}', \gamma'\bar{c}'$ $f_\alpha+f_\beta, f_\gamma$
α	$\alpha'\alpha'$ G_0^{-1}	$b'b'$ G_0^{-1}	$c'c'$ G_0^{-1}		$c'c'$ G_0^{-1}	$b'b'$ G_0^{-1}	$\alpha'\bar{\alpha}', b'\bar{b}', c'\bar{c}'$ $0, f_\gamma, f_\beta$
β	$a'a'$ G_0^{-1}	$\beta'\beta'$ G_0^{-1}	$c'c'$ G_0^{-1}	$c'c'$ G_0^{-1}		$a'a'$ G_0^{-1}	$a'\bar{a}', \beta'\bar{b}', c'\bar{c}'$ $f_\gamma, 0, f_\alpha$
γ	$a'a'$ G_0^{-1}	$b'b'$ G_0^{-1}	$\gamma'\gamma'$ G_0^{-1}	$b'b'$ G_0^{-1}	$a'a'$ G_0^{-1}		$a'\bar{a}', b'\bar{b}', \gamma'\bar{c}'$ $f_\beta, f_\alpha, 0$
$\bar{\pi}$	$\bar{a}'a', \bar{a}'\alpha'$ $f_\beta^\dagger+f_\gamma^\dagger, f_\alpha^\dagger$	$b'\bar{b}', b'\beta'$ $f_\alpha^\dagger+f_\gamma^\dagger, f_\beta^\dagger$	$\bar{c}'c', \bar{c}'\gamma'$ $f_\alpha^\dagger+f_\beta^\dagger, f_\gamma^\dagger$	$\bar{a}'\alpha', b'\bar{b}', \bar{c}'c'$ $0, f_\gamma^\dagger, f_\beta^\dagger$	$\bar{a}'a', b'\beta', \bar{c}'c'$ $f_\gamma^\dagger, 0, f_\alpha^\dagger$	$\bar{a}'a', b'\bar{b}', \bar{c}'\gamma'$ $f_\beta^\dagger, f_\alpha^\dagger, 0$	$\bar{a}'a', b'\bar{b}', \bar{c}'c'$ $v_{\bar{a}}, v_{\bar{b}}, v_{\bar{c}}$

Chapter 3

PRACTICAL APPROXIMATION SCHEME AND THREE-NUCLEON FORCES

I choose a block of marble and chop off whatever I don't need

Francois-Auguste Rodin

3.1 Practical Approximation scheme for Lovelace-type equations

In the last chapter I re-derived a quasiparticle description for the algebraic model that gave a set of connected Lovelace type equations. I already mentioned that the second level of separable expansion is not trivial and I show in this section how one can find a first approximated form for this problem. Like in any other separable expansion method the problem lies in the solution of the channel equations. Approximating the potentials by separable potentials allows one to solve the channel equations analytically. This approach is rather difficult in the present case due to the complexity of the channel equations. However, we recently gave an approximation that makes it possible to find the form factors for this expansion in a fairly direct way [CMS00]. Obviously, this can only give an approximated expression for the form factors. Nevertheless, the simplicity of the procedure outweighs this defect and significant correction terms to the standard three-nucleon AGS problem are found. For later research it is also clear at which points the approximations can be improved to higher order effects, but in this thesis I am only interested in the first order effects.

The main problem is to solve the channel equation on the two-cluster level

$$\mathbf{x}_{A'} = \mathbf{z}_{A'} + \mathbf{z}_{A'}\mathcal{G}^{(3)}\mathbf{x}_{A'} \tag{3.1}$$

In order to describe the approximation scheme in a clear way I first write eq. (3.1) in more detail. The dynamical equations of the sub-amplitudes in component form are given by the set of equations

$$\begin{aligned}
(x_{A'})_{\breve{a}'A,\hat{a}'B} &= \langle A|\, G_0\,|B\rangle\, \bar{\delta}_{AB}\delta_{\breve{a}'\hat{a}'} + \langle A|\, G_0\,(f_{A'})_{\breve{a}'A,\bar{a}'\bar{\pi}}\, g_0\left(x^{\dagger}_{A'}\right)_{\bar{\pi}\bar{a}',\hat{a}'A} \\
&\quad + \sum_{\breve{a}',C} \langle A|\, G_0\,|C\rangle\, \bar{\delta}_{AC}\delta_{\bar{a}'\breve{a}'}\tau_C\,(x_{A'})_{\breve{a}'C,\hat{a}'B} \qquad (3.2)\\
(x_{A'})_{\bar{a}'\bar{\pi},\hat{a}'B} &= \left(f^{\dagger}_{A'}\right)_{\bar{a}'\bar{\pi},\hat{a}'B} + \mathcal{V}_{A'}g_0\left(x^{\dagger}_{\bar{a}'\bar{\pi},\hat{a}'B}\right) \\
&\quad + \sum_{\breve{a}',C}\left(f^{\dagger}_{A'}\right)_{\bar{a}'\bar{\pi},\breve{a}'C} G_0\,|C\rangle\,\tau_C\,(x_{A'})_{\breve{a}'C,\hat{a}'B} \qquad (3.3)\\
(x_{A'})_{\breve{a}'A,\bar{a}'\bar{\pi}} &= \langle A|\, G_0\,(f_{A'})_{\breve{a}'A,\bar{a}'\bar{\pi}} + \langle A|\, G_0\,(f_{A'})_{\breve{a}'A,\bar{a}'\bar{\pi}}\, g_0\,(x_{A'})_{\bar{a}'\bar{\pi},\bar{a}'\bar{\pi}} \\
&\quad + \sum_{\breve{a}',C}\bar{\delta}_{AC}\delta_{\bar{a}'\breve{a}'}\,\langle A|\, G_0\,|C\rangle\,\tau_C\,(x_{A'})_{\breve{a}'C,\bar{a}'\bar{\pi}} \qquad (3.4)\\
(x_{A'})_{\bar{a}'\bar{\pi},\bar{a}'\bar{\pi}} &= \mathcal{V}_{A'} + \mathcal{V}_{A'}g_0\,(x_{A'})_{\bar{a}'\bar{\pi},\bar{a}'\bar{\pi}} \\
&\quad + \sum_{\breve{a}',C}\left(f^{\dagger}_{A'}\right)_{\bar{a}'\bar{\pi},\breve{a}'C} G_0\,|C\rangle\,\tau_C\,(x_{A'})_{\breve{a}'C,\bar{a}'\bar{\pi}} \qquad (3.5)
\end{aligned}$$

Using this set of equations an approximation scheme can be developed that allows for the two-body form factors to be calculated. However, for the channel π' the set collapses to the equation

$$(x_{\pi'})_{\pi'A,\pi'B} = \langle A|\, G_0\,|B\rangle\,\bar{\delta}_{AB} + \sum_C \bar{\delta}_{AC}\,\langle A|\, G_0\,|C\rangle\,\tau_C\,(x_{\pi'})_{\pi'C,\pi'B} \tag{3.6}$$

due to the inclusion principles. This is a standard Faddeev-type equation and can be solved directly for the quasiparticle form factors

$$\left|(\pi')^{(2)}_{\pi'A}\right\rangle = |\pi'A\rangle \tag{3.7}$$

The left hand side indicates that the approximated form factor is actually an exact result given by the Faddeev components of this channel.

The approximated form factors for the remaining channels A', B', C' can be found starting from eq.(3.5). This equation describes the behaviour in the three-nucleon sector and consists of a standard LS-equation for the cluster, given by the first two terms on the right-hand side, and a correction term. The potential in the LS-part is the standard static NN potential, including the OPEP part. The correction term describes one pion dynamics in the $2N$ subsystem and it is clear that this dynamics does not connect in any way to the spectator nucleon. Therefore, this term describes the *disconnected* dispersive effects to the $3N$ dynamics that originate in the $2N$ dynamics. It is straightforward to solve the standard NN problem given by the first two terms. Furthermore, it is known that in first approximation the LS-equation describes the NN problem quite well. Therefore, it is reasonable to approximate the correction term by zero. This is physically sound, because the approximation terms should not contribute in a dominant form to the NN problem, otherwise standard nuclear physics would not be a good approximation to nuclear dynamics.

The approximation scheme results in an elastic channel $(NN)\,N$ for the $3N$ problem, because the pion dynamics are explicitly ignored. Of course, there is still pion-exchange in the potential, but this is a static effect due to the phenomenological potential, not due to the dynamics of the system. Now this channel can be treated with the techniques used in standard $3N$ AGS-type models. The approximated channel equation is now given by

$$(\tilde{x}_{A'})_{\bar{a}'\bar{\pi},\bar{a}'\bar{\pi}} = \mathcal{V}_{A'} + \mathcal{V}_{A'} g_0 \, (\tilde{x}_{A'})_{\bar{a}'\bar{\pi},\bar{a}'\bar{\pi}} \tag{3.8}$$

and the quasiparticle t-matrix component for this channel is

$$(\tilde{x}_{A'})_{\bar{a}'\bar{\pi},\bar{a}'\bar{\pi}} = |\bar{a}'\bar{\pi}\rangle \, \tilde{\tau}^{(2)}_{A'} \, \langle \bar{a}'\bar{\pi}| \tag{3.9}$$

The form factors in this equation can easily be obtained using any of the valid expansion methods. Also, it is noteworthy that there is a strong resemblance between these form factors and the ones used in the first level of the quasiparticle approximation. They both come from solving a two-body problem, but in the A' channel for

example the three-cluster form factor can have the $N\pi$ contribution $\left|\alpha^{(3)}(z)\right\rangle$ and the NN contribution $\left|a^{(3)}(z)\right\rangle$. These two form factors are two-body form factors with two spectators split off. On the other hand, the three-nucleon form factor in the A' channel $|\bar{a}'\bar{\pi}\rangle$ only can have a NN cluster, identical to the cluster in the three-cluster system, but only one spectator is split off. In other words, the three-cluster form factor $\left|a^{(3)}(z)\right\rangle$ and the three-nucleon form factor $|\bar{a}'\bar{\pi}\rangle$ come from the same separable approximation. The proposed approximation requires

$$(x_{A'})_{\bar{a}'\bar{\pi},\bar{a}'\bar{\pi}} \approx (\tilde{x}_{A'})_{\bar{a}'\bar{\pi},\bar{a}'\bar{\pi}} = |\bar{a}'\bar{\pi}\rangle\, \tilde{\tau}^{(2)}_{A'}\, \langle\bar{a}'\bar{\pi}| \tag{3.10}$$

and consequently

$$\left|(A')^{(2)}_{\bar{a}'\bar{\pi}}\right\rangle \approx |\bar{a}'\bar{\pi}\rangle \tag{3.11}$$

and

$$\tau^{(2)}_{A'} \approx \tilde{\tau}^{(2)}_{A'} \approx \tau_A \tag{3.12}$$

This gives the approximated component of the two-cluster quasiparticle form factor and the approximated τ in the three-nucleon space. As I mentioned before, neglecting the pion dynamics on this level is a good approximation due to the success of standard nuclear physics. However, it is not a good approximation to neglect the pion dynamics in the four particle sectors of this system. This means that the remaining form factors in the separable expansion on the two-cluster level cannot be ignored. In order to define an approximation to these remaining form factors I investigate eq.(3.4). The first and third term in this equation do not depend on the $3N$ pole structure, which leads to the choice of the remaining, dominant, term for the approximated quasiparticle equation

$$(\tilde{x}_{A'})_{\tilde{a}'A,\bar{a}'\bar{\pi}} = \langle A|\, G_0\, (f_{A'})_{\tilde{a}'A,\bar{a}'\bar{\pi}}\, g_0\, (x_{A'})_{\tilde{a}'A,\bar{a}'\bar{\pi}} \tag{3.13}$$

Contrary to the approximation in the three-nucleon sector which was based on physical grounds, this approximation is valid on mathematical grounds. Using the approx-

imation for the respective two-cluster quasiparticle channel component yields

$$(x_{A'})_{\bar{a}'A,\bar{a}'\bar{\pi}} = \left|(A')^{(2)}_{\bar{a}'A}\right\rangle \tau^{(2)}_{A'} \left\langle(A')^{(2)}_{\bar{a}'\bar{\pi}}\right| \approx \langle A|\, G_0\, (f_{A'})_{\bar{a}'A,\bar{a}'\bar{\pi}}\, g_0\, |\bar{a}'\bar{\pi}\rangle\, \tilde{\tau}^{(2)}_{A'}\, \langle\bar{a}'\bar{\pi}| \qquad (3.14)$$

and by direct comparison

$$\left|(A')^{(2)}_{\bar{a}'A}\right\rangle \approx \langle A|\, G_0\, (f_{A'})_{\bar{a}'A,\bar{a}'\bar{\pi}}\, g_0\, |\bar{a}'\bar{\pi}\rangle \qquad (3.15)$$

At first glance it appears that the pion sector form factor is given by a scalar product, which is strange. However, a closer inspection shows that this term is not a scalar product, because the bra of this term lives in the three-cluster space while the ket lives in the two-cluster space. For a scalar product the bra and ket would have to live in the same cluster space. Therefore, these terms indeed define a transformation from a three-cluster to a two-cluster space, which is exactly what the form factors are designed to do.

This defines all the ket components of the approximated two-cluster quasiparticle form factors. In order actually to give an expression for the corresponding t-matrix the bra components of the form factors also need to be defined. These components can be constructed by taking the adjoints, which yields

$$\left\langle(A')^{(2)}_{\bar{a}'\bar{\pi}}\right| \approx \langle\bar{a}'\bar{\pi}| \qquad (3.16)$$

and

$$\left\langle(A')^{(2)}_{\bar{a}'A}\right| \approx \langle\bar{a}'\bar{\pi}|\, g_0\, \left(f^{\dagger}_{A'}\right)_{\bar{a}'\bar{\pi},\bar{a}'A}\, G_0\, |A\rangle \qquad (3.17)$$

This is the end of the approximation and I am now in a position to consider explicit calculations. However, it is noteworthy that due to the explicit definition of the creation/destruction vectors following relation is true

$$\left|(A')^{(2)}_{\alpha'\alpha}\right\rangle \approx \langle\alpha|\, G_0\, (f_{A'})_{\alpha'\alpha,\bar{a}'\bar{\pi}}\, g_0\, |\bar{a}'\bar{\pi}\rangle = 0 \qquad (3.18)$$

The fact that this equation holds true is because the relation $(f_{A'})_{\alpha'\alpha,\bar{a}'\bar{\pi}} = 0$ is a property of the original set of algebraic equations. This introduces a whole set of

simplifications in the calculations as I show in a later section. I also argue how the driving term in the Lovelace-type equation can be interpreted in order to recover corrections to the standard $3N$ AGS system. However, first the equations can be simplified even more by projecting out the closed pionic channels.

3.2 Projecting out the pionic channels

In general, the Padova model describes the pion-three-nucleon system, but in this thesis I am only investigating the triton binding energy. This means that the pionic channels of the Padova model are closed and a projection procedure due to Feshbach [Fes62] can be applied. In this section, I outline the projection procedure and show the result for the channels with no free pions. The starting point of the projection procedure is the two-cluster Lippmann-Schwinger equation (2.63)

$$X^{(2)}_{A'B'} = Z^{(2)}_{A'B'} + \sum_{C'} Z^{(2)}_{A'C'} \mathcal{G}^{(2)}_{C'} X^{(2)}_{C'B'} \tag{3.19}$$

Next, I define two projection operators Q and P, which project onto the free pion and no-free pion channels respectively. With these projection operators the LS-equation can be rewritten in the form

$$\begin{aligned} QX^{(2)}_{A'B'}Q &= QZ^{(2)}_{A'B'}Q \\ &\quad + \sum_{C'} \left(QZ^{(2)}_{A'C'}Q\mathcal{G}^{(2)}_{C'}QX^{(2)}_{C'B'}Q + QZ^{(2)}_{A'C'}P\mathcal{G}^{(2)}_{C'}PX^{(2)}_{C'B'}Q \right) \end{aligned} \tag{3.20}$$

$$\begin{aligned} QX^{(2)}_{A'B'}P &= QZ^{(2)}_{A'B'}P \\ &\quad + \sum_{C'} \left(QZ^{(2)}_{A'C'}Q\mathcal{G}^{(2)}_{C'}QX^{(2)}_{C'B'}P + QZ^{(2)}_{A'C'}P\mathcal{G}^{(2)}_{C'}PX^{(2)}_{C'B'}P \right) \end{aligned} \tag{3.21}$$

$$\begin{aligned} PX^{(2)}_{A'B'}Q &= PZ^{(2)}_{A'B'}Q \\ &\quad + \sum_{C'} \left(PZ^{(2)}_{A'C'}Q\mathcal{G}^{(2)}_{C'}QX^{(2)}_{C'B'}Q + PZ^{(2)}_{A'C'}P\mathcal{G}^{(2)}_{C'}PX^{(2)}_{C'B'}Q \right) \end{aligned} \tag{3.22}$$

$$\begin{aligned} PX^{(2)}_{A'B'}P &= PZ^{(2)}_{A'B'}P \\ &\quad + \sum_{C'} \left(PZ^{(2)}_{A'C'}Q\mathcal{G}^{(2)}_{C'}QX^{(2)}_{C'B'}P + PZ^{(2)}_{A'C'}P\mathcal{G}^{(2)}_{C'}PX^{(2)}_{C'B'}P \right) \end{aligned} \tag{3.23}$$

Only the last one of these equations represents an open channel and we can ignore the other three. However, the kernel of the last equation includes a term that depends on $QX^{(2)}_{C'B'}P$. In order to replace this dependence I solve the second equation in respect to this term, which yields

$$QX^{(2)}_{A'B'}P = \left(1 + QZ^{(2)}_{A'B'}Q\mathcal{G}^{(2)}_{B'}\right)^{-1} \cdot \left(QZ^{(2)}_{A'B'}P + \sum_{C'} QZ^{(2)}_{A'C'}P\mathcal{G}^{(2)}_{C'}PX^{(2)}_{C'B'}P\right) \quad (3.24)$$

and the zeroth order contribution gives the approximation

$$QX^{(2)}_{A'B'}P \approx \left(QZ^{(2)}_{A'B'}P + \sum_{C'} QZ^{(2)}_{A'C'}P\mathcal{G}^{(2)}_{C'}PX^{(2)}_{C'B'}P\right) \quad (3.25)$$

This approximation makes sense, because we want to retain only the lowest order terms that connect to explicit pion-channels. The ignored terms all would introduce higher iterations of the pionic channels. Now I substitute this term into the no-free pion sector equation (3.24) and replace the pionic channel by the corresponding π' (recall also eq.(2.64)), which gives

$$\begin{aligned} PX^{(2)}_{A'B'}P &= PZ^{(2)}_{A'B'}P + \sum_{C'} PZ^{(2)}_{A'C'}P\tau^{(2)}_{C'}PX^{(2)}_{C'B'}P \quad (3.26)\\ &\quad + PZ^{(2)}_{A'\pi'}Q\tau^{(2)}_{\pi'}QZ^{(2)}_{\pi'B'}P + \sum_{C'} PZ^{(2)}_{A'\pi'}Q\tau^{(2)}_{\pi'}QZ^{(2)}_{\pi'C'}P\tau^{(2)}_{C'}PX^{(2)}_{C'B'}P \end{aligned}$$

This equation can now be rearranged to give an effective LS-equation with a modified driving term

$$\begin{aligned} PX^{(2)}_{A'B'}P &= PZ^{(2)}_{A'B'}P + PZ^{(2)}_{A'\pi'}Q\tau^{(2)}_{\pi'}QZ^{(2)}_{\pi'B'}P \quad (3.27)\\ &\quad + \sum_{C'} \left(PZ^{(2)}_{A'C'}P + PZ^{(2)}_{A'\pi'}Q\tau^{(2)}_{\pi'}QZ^{(2)}_{\pi'C'}P\right)\tau^{(2)}_{C'}PX^{(2)}_{C'B'}P \end{aligned}$$

Using the usual definition $\hat{A}' = A', B', C'$ for the channels without a free pion this equation can also be given in the simplified form

$$X^{(2)}_{\hat{A}'\hat{B}'} = Z^{(2)}_{\hat{A}'\hat{B}'} + Z^{(2)}_{\hat{A}'\pi'}\tau^{(2)}_{\pi}Z^{(2)}_{\pi'\hat{B}'} + \sum_{\hat{C}'} \left(Z^{(2)}_{\hat{A}'\hat{C}'} + Z^{(2)}_{\hat{A}'\pi'}\tau^{(2)}_{\pi'}Z^{(2)}_{\pi'\hat{C}'}\right)\tau^{(2)}_{\hat{C}'}X^{(2)}_{\hat{C}'\hat{B}'} \quad (3.28)$$

Therefore, I am left with a LS-type equation that has a modified driving term. In the next section I investigate the driving term and show how some of the terms can be interpreted as 3NF terms.

3.3 The approximation scheme and 3N-force diagrams

In this section I argue how the approximation scheme can be used to find correction terms to the standard 3N-AGS type potentials. In order to see this, I interpret the driving term of the quasiparticle equation in more detail using the approximated form factors. I already mentioned that the driving term does not contribute at all to the case $\pi'\pi'$ due to the inclusion principles. Furthermore, in the modified LS-equation I derived in the last section the pion-sector was projected out, which also means that this case is not included. Therefore, the only following three distinct cases are left to interpret:

1. both channels are not π' and equal. The representative case $Z_{A'A'}$ is shown
2. both channels are not π' and they are not equal to each other. the representative case $Z_{A'B'}$ is shown
3. one of the channels is π', the other is not. The representative case $Z_{\pi'A'}$ is shown

The first observation is that the first two cases can only contribute to $Z^{(2)}_{\bar{A}'\hat{B}'}$ and the last case only contributes to $Z^{(2)}_{\bar{A}'\pi'}\tau^{(2)}_{\pi'}Z^{(2)}_{\pi'\hat{B}'}$. In the next two subsections I investigate exactly these two cases.

3.3.1 *Contributions to the driving term from* $Z^{(2)}_{\bar{A}'\hat{B}'}$

The approximated general form of the driving term in the case that $A' \neq \pi'$ and $B' \neq \pi'$ is given by the expression

$$\begin{aligned}\tilde{Z}^{(2)}_{A'B'} \approx{}& \langle \bar{a}'\bar{\pi}|\, g_0 \left|\bar{b}'\bar{\pi}\right\rangle \bar{\delta}_{A'B'} \\ &+ \sum_{\hat{a}',\bar{b}',A} \langle \bar{a}'\bar{\pi}|\, g_0 \left(f^{\dagger}_{A'}\right)_{\bar{a}'\bar{\pi},\hat{a}'A} G_0 \,|A\rangle\, \tau_A \,\langle A|\, G_0 \left(f_{A'}\right)_{\bar{b}'A,\bar{b}'\bar{\pi}} g_0 \left|\bar{b}'\bar{\pi}\right\rangle \left(\bar{\delta}_{A'B'} + \delta_{A'B'}\bar{\delta}_{\hat{a}'\bar{b}'}\right)\end{aligned} \tag{3.29}$$

In case 2, where $A' \neq B'$, only the first of the delta-functions in the second term survives. Also, B' and A' have three common three-cluster break-ups, namely α, β, γ and

there is a unique contribution to each of these from the two-cluster partitions in the respective channels. Taking all the inclusion principles into account and substituting the explicit expressions for the complicated creation/destruction operators therefore leads to the equation

$$\tilde{Z}^{(2)}_{A'B'} \approx \langle \bar{a}'\bar{\pi}|\, g_0 \left|\bar{b}'\bar{\pi}\right\rangle + \langle \bar{a}'\bar{\pi}|\, g_0 f^{\dagger}_{\beta} G_0 \,|\gamma\rangle\, \tau_\gamma\, \langle\gamma|\, G_0 f_\alpha g_0 \left|\bar{b}'\bar{\pi}\right\rangle \tag{3.30}$$

The first term describes the standard AGS-type driving terms and the second term describes corrections to the off-diagonal components of the driving term. The off-diagonal correction terms of the effective potential are also shown in a diagrammatic way in figure [3.1]. The middle structure of this graph describing the pion behaviour

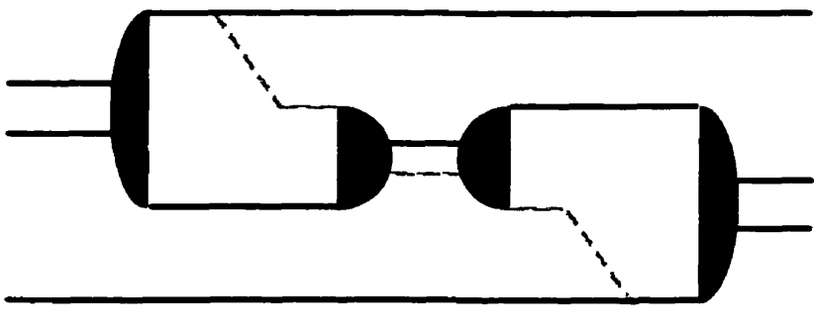

Figure 3.1: The diagrammatic representation of the off-diagonal correction term $\langle \bar{a}'\bar{\pi}|\, g_0 f^{\dagger}_{\beta} G_0 \,|\gamma\rangle\, \tau_\gamma\, \langle\gamma|\, G_0 f_\alpha g_0 \left|\bar{b}'\bar{\pi}\right\rangle$ to the effective potential. The inside structure of the diagram is of the same type as Fujita-Miyazawa

is the same as the 3NF graphs based on the Fujita-Miyazawa model. The pion is created on one nucleon, interacts with a second nucleon and finally is absorbed on the third nucleon. In my calculations this part of the diagram is replaced by the one-dimensional equivalent to the Tucson-Melbourne form of a static 3NF correction. However, the fact that a diagram describing a behaviour similar to modern 3NF is recovered already constitutes an interesting result.

In case 1, where $B' = A'$, only the second term of eq.(3.29) survives and only the part in respect to the second delta function is non-zero. Since in that case the two

partitions have to be unequal, one of them has to be a' and the other α' or vice versa. The possible chains have only one common three-cluster matrix element, namely $a\bar{\pi}$ and therefore only the following terms survive after the explicit expression for the complicated creation/destruction operators are inserted

$$Z^{(2)}_{A'A} \approx \langle \bar{a}'\bar{\pi}|\, g_0\left(f^\dagger_\beta + f^\dagger_\gamma\right) G_0\, |a\rangle\, \tau_a\, \langle a|\, G_0 f_\alpha g_0\, \left|\bar{b}'\bar{\pi}\right\rangle \qquad (3.31)$$
$$+ \langle \bar{a}'\bar{\pi}|\, g_0 f^\dagger_\alpha G_0\, |a\rangle\, \tau_a\, \langle a|\, G_0(f_\beta + f_\gamma) g_0\, \left|\bar{b}'\bar{\pi}\right\rangle$$

These contributions are the diagonal correction terms and they can be represented by four distinct diagrams. In figures [3.2,3.3] the topologically different diagrams are shown. It should be clear that the standard AGS-system has no contributions to the diagonal parts of the driving terms. Therefore, it is valid to interpret these terms as corrections. The corresponding set of diagrams does not appear in modern

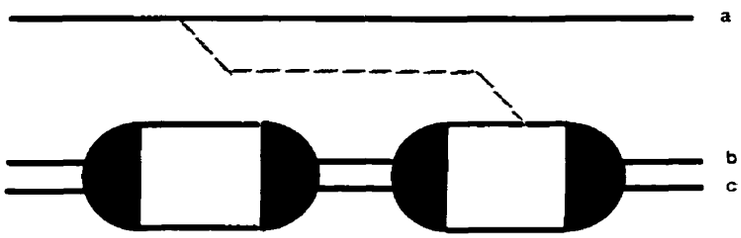

Figure 3.2: The diagrammatic representation of the diagonal correction term to the effective potential $\langle \bar{a}'\bar{\pi}|\, g_0 f^\dagger_\alpha G_0\, |a\rangle\, \tau_a\, \langle a|\, G_0 f_\beta g_0\, \left|\bar{b}'\bar{\pi}\right\rangle$.

3NF, because it is assumed that these terms already are completely described by the underlying theory. However, I show that the cancellation that is usually assumed is not complete and yields a rather strong effect that should not be neglected.

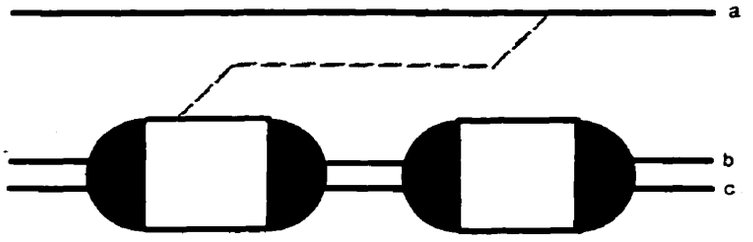

Figure 3.3: The diagram corresponding to the diagonal correction term to the effective potential $\langle \bar{a}'\bar{\pi}|\, g_0 f_\beta^\dagger G_0 \,|a\rangle\, \tau_a \,\langle a|\, G_0 f_\alpha g_0 \,\big|\bar{b}'\bar{\pi}\big\rangle$.

3.3.2 Contributions to the driving term from $Z^{(2)}_{\pi' B'}$

Case 3, where a free pion exists in an intermediate state, is described by a somewhat different equation, namely

$$\tilde{Z}^{(2)}_{\pi' A'} \approx \sum_{\tilde{a}',A} |\pi' A\rangle\, \tau_A\, \langle A|\, G_0\, (f_A)_{\tilde{a}' A, \bar{a}'\bar{\pi}}\, g_0\, |\bar{a}'\bar{\pi}\rangle \tag{3.32}$$

Imposing all the inclusion principles and inserting all the explicit expressions then leads to the equation

$$\tilde{Z}^{(2)}_{\pi' A'} \approx |\pi' a\rangle\, \tau_a\, \langle a|\, G_0 (f_\beta + f_\gamma) g_0\, |\bar{a}'\bar{\pi}\rangle + |\pi' a\rangle\, \tau_a\, \langle a|\, G_0 f_\alpha g_0\, |\bar{a}'\bar{\pi}\rangle \tag{3.33}$$

This correction term is due to the coupling with the pionic sector, which was projected out by the Feshbach procedure. In fig [3.4] one diagram contributing to this correction term is shown. The total contribution to this correction term is the sum of all diagrams of this type, where the pion can connect any incoming with any outgoing nucleon line. This type of correction is another 3NF correction to the effective potential of the $3N$ bound state problem. However, in this thesis I restrict myself to the investigation of the first two types of correction terms in a simple $1D$ model. The effect of this last type of corrections should be investigated in subsequent research efforts, but for now I do not discuss them further in any detail.

These are now all types of correction terms appearing in the approximated system.

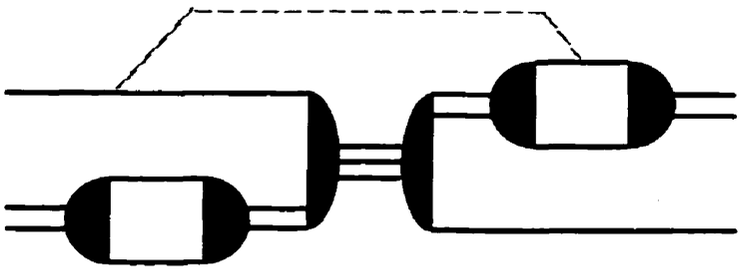

Figure 3.4: The diagram correction term $Z'_{\tilde{A}'\tilde{B}'} = Z^{\pi}_{\tilde{A}'\pi'}\tau^{(2)}_{\pi'}Z^{\pi}_{\pi'\tilde{B}'}$ that appears because the mesonic channels were projected out by the Feshbach procedure.

Consequently, the modified driving term can be given by the expression

$$Z_{\tilde{A}'\tilde{B}'} = Z^{AGS}_{\tilde{A}'\tilde{B}'} + Z^{\pi}_{\tilde{A}'\tilde{B}'} + Z'_{\tilde{A}'\tilde{B}'} \tag{3.34}$$

where the first term is defined by the standard AGS-type driving terms

$$Z^{AGS}_{\tilde{A}'\tilde{B}'} = \langle \bar{a}'\bar{\pi}|\, g_0 \left|\bar{b}'\bar{\pi}\right\rangle \bar{\delta}_{\tilde{A}'\tilde{B}'} \tag{3.35}$$

The second term is defined by the terms of the first two cases that do not belong to $Z^{AGS}_{\tilde{A}'\tilde{B}'}$ and the third term is defined by contributions due to the connection to the pionic channels.

$$Z'_{\tilde{A}'\tilde{B}'} = Z^{\pi}_{\tilde{A}'\pi'}\tau^{(2)}_{\pi'}Z^{\pi}_{\pi'\tilde{B}'} \tag{3.36}$$

The next step in thesis is the derivation of a one-dimensional Toy model in order to investigate the dynamical properties of this system numerically.

Chapter 4

ONE-DIMENSIONAL TOY-MODEL FOR THE NN-SYSTEM

Technology is the knack of so arranging the world that we do not experience it

Max Frisch

4.1 Schroedinger description of the NN Toy-model

The two-body problem is an essential input into the approximated quasiparticle equations as I have shown in the last chapter. In this chapter I describe the one-dimensional Toy-model for the NN problem and derive the corresponding Lippmann-Schwinger representation. I also derive the general form of the NN Toy-potential to be used in the model. In the first part I describe the model in the Schroedinger representation and subsequently derive the general Lippmann-Schwinger form. I choose to do this in coordinate space in order to include the boundary conditions in a transparent way. In the second part I derive the general form of the potential also in coordinate space, which gives a potential that can be compared directly to the coordinate behaviour expected from a "nuclear" potential.

In the first step the quantum mechanical two-body problem for scattering on a line needs to be derived. The Hamiltonian for this problem given a potential V (for

example, see Messiah [Mes76]) has the form

$$h = -\frac{\hbar^2}{2m_1}\nabla_1^2 - \frac{\hbar^2}{2m_2}\nabla_2^2 + V(r_1, r_2) \tag{4.1}$$

where ∇_i are the two derivatives in respect to the coordinates of the particles and m_i are the two masses. In the one-dimensional case one would not explicitly need the ∇, which is an inherently three-dimensional symbol. However, in staying with the ∇ it is easier to compare the derivation with the well-known three-dimensional one. The problem in question is described by the Schroedinger equation of this system, which is given by the expression

$$h\Psi(r_1, r_2) = E\Psi(r_1, r_2) \tag{4.2}$$

I assume that the potential is translationally invariant and that it is possible to separate out the center of mass motion, because it is a free motion. The transformed Schroedinger equation is then given by

$$\left[\frac{1}{m_1 + m_2}\nabla_R^2 + \frac{1}{\mu}\nabla_r^2 - \frac{2}{\hbar^2}V(r)\right]\Phi(r, R) = -\frac{2}{\hbar^2}E\Phi(r, R) \tag{4.3}$$

with R being the center of mass, $r = r_1 - r_2$ being the relative coordinate and $\mu = \frac{m_1 m_2}{m_1 + m_2}$ being the reduced mass. This equation is separable and can be written as

$$\Phi(r, R) = \psi(r)\,\phi(R) \tag{4.4}$$

which separates the equation in the following way

$$\nabla_R^2\phi(R) + \frac{2(m_1 + m_2)}{\hbar^2}(E - E_i)\,\phi(R) = 0 \tag{4.5}$$

$$\nabla_r^2\psi(r) + \frac{2\mu}{\hbar^2}(E_i - V)\,\psi(r) = 0 \tag{4.6}$$

where E_i is the constant of separation given by the total energy of the two particles in the C.M system. However, in eq.(4.5) the second term vanishes and the center of mass is simply given by a motionless point. All the information of the scattering system is given by the wave function of the two-body system $\psi(r)$. The indices r, i

are now being dropped and the one-dimensional Schroedinger equation describing the two body scattering system can be given by the equation

$$\left[-\frac{\hbar^2}{2\mu}\nabla^2 + V\right]\psi = E\psi \tag{4.7}$$

It is apparent from this equation that the units of the potential have to be same as the units of the Hamiltonian, namely the units of an energy E. The next step is the derivation of the Lippmann-Schwinger representation for eq.(4.7) including the boundary conditions that still have to be defined. It is noteworthy that the derivation of eq.(4.7) did not yet take into account that the system is solely living on the line and therefore is a more general result than actually needed. The restriction of the system to the line is important when the boundary conditions are defined.

4.2 *Derivation of the LS-equation in the two-body Toy-model*

In this section I derive the Lippmann-Schwinger equation for the $1D$-scattering system. This task is done using the *Green's function* method, which is described later in this section. However, it is convenient to give another simple transformation in order to simplify eq.(4.7) and find the equation

$$\left[\nabla^2 + k^2\right]\psi = U\psi \tag{4.8}$$

with the definitions

$$k^2 = \frac{2\mu E}{\hbar^2} \tag{4.9}$$

and

$$U = \frac{2\mu V}{\hbar^2} \tag{4.10}$$

The *Green's function* method constructs solutions to eq.(4.8) by the use of the Green's function $G(k, r, r')$ which is a solution of the equation

$$\left[\nabla^2 + k^2\right] G(k, r, r') = \delta(r - r') \tag{4.11}$$

It can be easily shown that the expression

$$\psi_{part}(r) = \int G(k, r, r') U(r') \psi(r') dr' \tag{4.12}$$

solves eq.(4.8). Furthermore, it is clear that U has units of fm^{-2}, which means the Green's function has to have units of fm. In order to see that this is indeed a solution eq.(4.8) is expanded in the following way

$$\left[\nabla^2 + k^2\right] \psi(r) = \int \delta(r - r') U(r') \psi(r') dr'$$

$$\left[\nabla^2 + k^2\right] \psi(r) = \left[\nabla^2 + k^2\right] \int G(k, r, r') U(r') \psi(r') dr' \tag{4.13}$$

where the operator in the square brackets is taken outside the integral, because it does not depend on the integration variable. This shows that eq.(4.12) indeed gives a particular solution of eq.(4.8). The general solution is given by the particular solution plus a solution of the homogeneous equation

$$\left[\nabla^2 + k^2\right] \phi(r) = 0 \tag{4.14}$$

namely

$$\psi(r) = \phi(r) + \int G(k, r, r') U(r') \psi(r') dr' \tag{4.15}$$

The next step is to include the boundary conditions, which is done by an appropriate definition of the Green's function. At this point it is important that the system is restricted to the line, because it defines the boundary conditions. From the theory of differential equations it is clear that the solutions of the homogeneous equation are represented by the plane waves

$$\phi(r) = Ae^{ikr} + Be^{-ikr} \tag{4.16}$$

Since the plane waves form a complete orthonormal set, they can be used as a basis. According to the theory of Fourier transforms the particular solution is expanded in respect to the plane waves in the following way

$$\psi_{part}(r) = \int A(k') e^{ik'r} dk' \tag{4.17}$$

This yields for the Schroedinger equation, using the property of the particular solution given in eq.(4.12), the equation

$$U(r)\,\psi(r) = \left[\nabla^2 + k^2\right] \int A(k')\, e^{ik'r} dk' \tag{4.18}$$

However, the plane waves are eigenstates of the differential operator with eigenvalues $-k'^2$ and interchanging the operator and the integration gives the expression

$$U(r)\,\psi(r) = \int \left[-k'^2 + k^2\right] A(k')\, e^{ik'r} dk' \tag{4.19}$$

Next, the expression is multiplied by $e^{-ik''r}$ and integrated over r in order to make use of the orthonormality of the plane wave states. This calculation yields

$$\int U(r)\,\psi(r)\, e^{-ik''r} dr = \int \left[-k'^2 + k^2\right] A(k') \int e^{i(k'-k'')r} dr dk' \tag{4.20}$$

and expressing the r-integration with the corresponding delta-function and subsequent use of the delta-function yields

$$\int U(r)\,\psi(r)\, e^{-ik''r} dr = (2\pi)\left[k^2 - k''^2\right] A(k'') \tag{4.21}$$

This expression can be used in turn to define the Fourier coefficients of the particular solution as

$$A(k') = \frac{1}{(2\pi)\left[k^2 - k'^2\right]} \int U(r)\,\psi(r)\, e^{-ik'r} dr \tag{4.22}$$

Therefore, the expansion of the particular solution given in eq.(4.17) may now be written explicitly as

$$\psi_{part}(r) = \frac{1}{(2\pi)} \int \frac{1}{\left[k^2 - k'^2\right]} \int U(r')\,\psi(r')\, e^{ik'(r-r')} dr' dk' \tag{4.23}$$

Identifying this expression with eq.(4.12) finally gives an expression for the Green's function, which is given by

$$G(k, r, r') = \frac{1}{2\pi} \int \frac{1}{\left[k^2 - k'^2\right]} e^{ik'(r-r')} dk' \tag{4.24}$$

At this point it would appear that this derivation was in vain, because the integral that defines the Green's function actually does not exist due to the poles on the real line. However, I use the trick of slightly deforming the contour around the poles and give a slightly different set of Green's functions (for example, see Merzbacher[Mer70, pp.223] or Itzykson and Zuber[IZ80, pp.32]). Obviously this new set can not be seen as solutions to eq.(4.11), but the integrals are now explicitly solvable. It is a well known procedure to take the limit of the contour deformation to zero after integration and to recover the solution sought in the beginning. The contour deformation can be done in a number of different ways and it is seen that the different choices correspond to different boundary conditions.

The first deformation that defines the new Green's function is described by the equation

$$G_{+\varepsilon}(k,r,r') = -\frac{1}{2\pi}\int \frac{1}{[k'^2 - k^2 - i\varepsilon]} e^{ik'(r-r')} dk' \tag{4.25}$$

This is a Fourier-integral and it is exactly solvable using the theory of residues. The theory requires to distinguish between two different cases (see Fischer [FK88, pp.569].

1. The case $r > r'$, when the contour is closed with a large semi-circle through the upper half plane. This contour encloses the pole in the positive half plane, namely the pole at $k' = +\sqrt{k^2 + i\varepsilon} \approx +\left(k + \frac{i\varepsilon}{2k}\right)$ which by the use of residues yields the following result for the new Green's function

$$G^{\wedge}_{+\varepsilon}(k,r,r') = 2\pi i Res\left(f^{+}, +\sqrt{k^2 + i\varepsilon}\right) \tag{4.26}$$

with

$$f^{+}(k') = -\frac{1}{2\pi}\frac{1}{[k'^2 - k^2 - i\varepsilon]} e^{ik'(r-r')} \tag{4.27}$$

Calculating the residue explicitly gives (using Maple V)

$$G^{\wedge}_{+\varepsilon}(k,r,r') = -\frac{i}{2}\frac{e^{i(r-r')\sqrt{k^2+i\varepsilon}}}{\sqrt{k^2 + i\varepsilon}} \tag{4.28}$$

and taking the limit yields the Green's function

$$G_{+}^{\wedge}(k,r,r') = -\frac{i}{2}\frac{e^{i(r-r')k}}{k} \tag{4.29}$$

2. The case $r < r'$, when the contour is closed with a large semi-circle through the lower half plane. This contour encloses the pole in the negative half plane, namely the pole at $k' = -\sqrt{k^2+i\varepsilon} \approx -\left(k+\frac{i\varepsilon}{2k}\right)$ which yields the following result for the new Green's function using the properties for Fourier-integrals

$$G_{+\varepsilon}^{\vee}(k,r,r') = -2\pi i Res\left(f^{+}, -\sqrt{k^2+i\varepsilon}\right) \tag{4.30}$$

Again calculating the residue yields this time

$$G_{+\varepsilon}^{\vee}(k,r,r') = -\frac{i}{2}\frac{e^{-i(r-r')\sqrt{k^2+i\varepsilon}}}{\sqrt{k^2+i\varepsilon}} \tag{4.31}$$

and again taking the limits gives

$$G_{+}^{\vee}(k,r,r') = -\frac{i}{2}\frac{e^{-i(r-r')k}}{k} \tag{4.32}$$

The Green's functions have the expected units, which is a good test of the calculation. It should be pointed out that the two exponents of the different cases have the opposite sign, which is important in the inclusion of the boundary conditions. The solution for this wave function is therefore given by the expression

$$\psi(r) = \phi(r) + \int_{-\infty}^{r} G_{+\varepsilon}^{\wedge}(k,r,r')\,U(r')\,\psi(r')\,dr' + \int_{r}^{+\infty} G_{+\varepsilon}^{\vee}(k,r,r')\,U(r')\,\psi(r')\,dr' \tag{4.33}$$

However,the inhomogeneous term is given by a plane wave, which results in the equation

$$\begin{aligned}\psi(r) &= Ae^{ikr} + Be^{-ikr} \\ &+ \int_{-\infty}^{r} G_{+\varepsilon}^{\wedge}(k,r,r')\,U(r')\,\psi(r')\,dr' + \int_{r}^{+\infty} G_{+\varepsilon}^{\vee}(k,r,r')\,U(r')\,\psi(r')\,dr'\end{aligned} \tag{4.34}$$

Inserting the original expressions and the appropriate Green's functions gives

$$\begin{aligned}\psi(r) &= Ae^{ikr} + Be^{-ikr} \\ &\quad -\frac{i\mu}{\hbar^2 k}\left[\left(\int_{-\infty}^{r} e^{-ir'k}V(r')\,\psi(r')\,dr'\right)e^{irk} + \left(\int_{r}^{+\infty} e^{ir'k}V(r')\,\psi(r')\,dr'\right)e^{-irk}\right]\end{aligned} \tag{4.35}$$

The corresponding terms in this equation are now collected, which yields

$$\begin{aligned}\psi(r) &= \left[A - \frac{i\mu}{\hbar^2 k}\left(\int_{-\infty}^{r} e^{-ir'k}V(r')\,\psi(r')\,dr'\right)\right]e^{irk} \\ &\quad + \left[B - \frac{i\mu}{\hbar^2 k}\left(\int_{r}^{+\infty} e^{ir'k}V(r')\,\psi(r')\,dr'\right)\right]e^{-irk}\end{aligned} \tag{4.36}$$

In order to incorporate the boundary conditions in a meaningful way the coefficients A, B have to be choosen. The solutions are compared with the asymptotic states, which is done by taking the limits $r \to \pm\infty$ (for example, see Bolle et. al. [BGW83]). This procedure allows for two distinct choices in a physically meaningful way. The first choice is $A = 1, B = 0$ which yields

$$\psi_1^{-\infty}(r) = e^{ikr} - \frac{i\mu}{\hbar^2 k}\left(\int_{-\infty}^{+\infty} e^{ir'k}V(r')\,\psi(r')\,dr'\right)e^{-irk} \tag{4.37}$$

$$\psi_1^{+\infty}(r) = \left[1 - \frac{i\mu}{\hbar^2 k}\left(\int_{-\infty}^{+\infty} e^{-ir'k}V(r')\,\psi(r')\,dr'\right)\right]e^{irk} \tag{4.38}$$

This represents a plane wave incoming from the right and reflected and transmitted waves travelling left and right respectively. The second choice is given by $A = 0, B = 1$ and yields the solutions

$$\psi_2^{+\infty}(r) = e^{-irk} - \frac{i\mu}{\hbar^2 k}\left(\int_{-\infty}^{+\infty} e^{-ir'k}V(r')\,\psi(r')\,dr'\right)e^{irk} \tag{4.39}$$

$$\psi_2^{-\infty}(r) = \left[1 - \frac{i\mu}{\hbar^2 k}\left(\int_{-\infty}^{+\infty} e^{ir'k}V(r')\,\psi(r')\,dr'\right)\right]e^{-irk} \tag{4.40}$$

This case represents a plane wave incident from the left and transmitted and reflected waves travelling left and right respectively. The fact that this is indeed one solution for the one-dimensional scattering, becomes obvious when the asymptotic states are written the following way

$$\psi_1^{-\infty}(r) = e^{ikr} + s_{12}(k)\,e^{-irk} \tag{4.41}$$

$$\psi_1^{+\infty}(r) = s_{11}(k)\, e^{irk} \tag{4.42}$$

$$\psi_2^{-\infty}(r) = s_{22}(k)\, e^{-irk} \tag{4.43}$$

$$\psi_2^{+\infty}(r) = e^{-irk} + s_{21}(k)\, e^{irk} \tag{4.44}$$

with the definitions

$$s_{12}(k) = -\frac{i\mu}{\hbar^2 k}\left(\int_{-\infty}^{+\infty} e^{ir'k} V(r')\,\psi(r')\,dr'\right) \tag{4.45}$$

$$s_{11}(k) = \left[1 - \frac{i\mu}{\hbar^2 k}\left(\int_{-\infty}^{+\infty} e^{-ir'k} V(r')\,\psi(r')\,dr'\right)\right] \tag{4.46}$$

$$s_{21}(k) = -\frac{i\mu}{\hbar^2 k}\left(\int_{-\infty}^{+\infty} e^{-ir'k} V(r')\,\psi(r')\,dr'\right) \tag{4.47}$$

$$s_{22}(k) = \left[1 - \frac{i\mu}{\hbar^2 k}\left(\int_{-\infty}^{+\infty} e^{ir'k} V(r')\,\psi(r')\,dr'\right)\right] \tag{4.48}$$

This yields the general scattering solution for the $+i\epsilon$ correction.

The solutions for the case of the $-i\epsilon$ correction result from the following Green's function

$$G_{-\varepsilon}(k, r, r') = -\frac{1}{2\pi}\int \frac{1}{[k'^2 - k^2 + i\varepsilon]} e^{ik'(r-r')} dk' \tag{4.49}$$

The Fourier-integrals are solved again with the residue theorem and two cases result.

1. The case $r > r'$, when the contour is closed in the upper half. This time the contour encloses the pole in the negative half plane, namely $k' = -\sqrt{k^2 - i\varepsilon}$, which yields the following result for the Green's function

$$G^{\wedge}_{-\varepsilon}(k, r, r') = 2\pi i\, Res\left(f^-, -\sqrt{k^2 - i\varepsilon}\right) \tag{4.50}$$

with

$$f^-(k') = -\frac{1}{2\pi}\frac{1}{[k'^2 - k^2 + i\varepsilon]} e^{ik'(r-r')} \tag{4.51}$$

Calculating the residue explicitly gives

$$G^{\wedge}_{-\varepsilon}(k, r, r') = \frac{i}{2}\frac{e^{-i(r-r')\sqrt{k^2 - i\varepsilon}}}{\sqrt{k^2 - i\varepsilon}} \tag{4.52}$$

and taking the limit yields the Green's function

$$G^{\wedge}_{-}(k, r, r') = \frac{i}{2}\frac{e^{-i(r-r')k}}{k} \tag{4.53}$$

2. The case $r < r'$, when the contour is closed in the lower half plane. This encloses the pole in the positive half, namely $k' = \sqrt{k^2 - i\varepsilon}$ which yields the Green's function

$$G^{\vee}_{-\varepsilon}(k, r, r') = -2\pi i Res\left(f^{-}, \sqrt{k^2 - i\varepsilon}\right) \tag{4.54}$$

and taking the residue gives

$$G^{\vee}_{-\varepsilon}(k, r, r') = \frac{i}{2}\frac{e^{i(r-r')\sqrt{k^2-i\varepsilon}}}{\sqrt{k^2 - i\varepsilon}} \tag{4.55}$$

which gives after taking the limit

$$G^{\vee}_{-}(k, r, r') = \frac{i}{2}\frac{e^{i(r-r')k}}{k} \tag{4.56}$$

Once again, following a similar argument, this gives a solution for the wave function, but this time it is given by the equation

$$\begin{aligned} \Phi_2(r) &= \left[A + \frac{i\mu}{\hbar^2 k}\left(\int_r^{+\infty} e^{-ir'k} V(r')\,\psi(r')\,dr'\right)\right] e^{ikr} \\ &+ \left[B + \frac{i\mu}{\hbar^2 k}\left(\int_{-\infty}^{r} e^{+ir'k} V(r')\,\psi(r')\,dr'\right)\right] e^{-ikr} \end{aligned} \tag{4.57}$$

Once again, the result allows two different choices for the constants A, B. The choice $A = 0, B = 1$ gives the solutions $\Phi_1^{\pm}(r)$ and the choice $A = 1, B = 0$ gives $\Phi_1^{\pm}(r)$. The corresponding asymptotic states are then given by

$$\Phi_1^{-\infty}(r) = \tilde{s}_{12}(k)\, e^{ikr} + e^{-ikr} \tag{4.58}$$

$$\Phi_1^{+\infty}(r) = \tilde{s}_{11}(k)\, e^{-ikr} \tag{4.59}$$

$$\Phi_2^{-\infty}(r) = \tilde{s}_{22}(k)\, e^{ikr} \tag{4.60}$$

$$\Phi_2^{+\infty}(r) = \tilde{s}_{21}(k)\, e^{-ikr} + e^{ikr} \tag{4.61}$$

with the definitions

$$\tilde{s}_{12}(k) = \frac{i\mu}{\hbar^2 k}\left(\int_{-\infty}^{+\infty} e^{-ir'k} V(r')\,\psi(r')\,dr'\right) \tag{4.62}$$

$$\tilde{s}_{11}(k) = \left[1 + \frac{i\mu}{\hbar^2 k}\left(\int_{-\infty}^{+\infty} e^{+ir'k} V(r')\,\psi(r')\,dr'\right)\right] \tag{4.63}$$

$$\tilde{s}_{22}(k) = \left[1 + \frac{i\mu}{\hbar^2 k}\left(\int_{-\infty}^{+\infty} e^{-ir'k} V(r')\,\psi(r')\,dr'\right)\right] \tag{4.64}$$

$$\tilde{s}_{21}(k) = \frac{i\mu}{\hbar^2 k}\left(\int_{-\infty}^{+\infty} e^{+ir'k} V(r')\,\psi(r')\,dr'\right) \tag{4.65}$$

This second set of functions describes a recombination process, which is highly improbable and therefore generally neglected. This completes the definition of my $1-D$ toy model and in the next section I find a potential that resembles nuclear physics at least in a first approximation.

4.3 General form of the potential for the LS-equation

In the last sections I derived an integral representation for the one-dimensional scattering problem, but so far I have not specified the potential that appears in the Hamiltonian. In this section I give a detailed discussion on the form of this potential from a meson theoretical point of view first suggested by Yukawa [Yuk35].

However, it is necessary to make a preliminary remark on the potential due to the omitted constraints used in the derivation. The potential has to be of a special type in order for the derivation to be valid, namely it has to be a potential belonging to the Faddeev-class. This class is defined by the constraint

$$\int_{-\infty}^{+\infty} (1 + |x|)\,|V(x)|\,dx \quad < \infty \tag{4.66}$$

and the proposed potential has to satisfy this constraint, which, it turns out, it does. In order to derive the potential for the scattering model the free meson field needs to be given. The meson is intrinsically a relativistic particle and therefore a relativistic description should be used. The commonly used description is the Klein-Gordon equation (see Messiah [Mes76, p. 67])

$$\left[\frac{1}{c^2}\partial_t^2 - \nabla^2 + \left(\frac{m_\pi c}{\hbar}\right)^2\right]\psi(x) = 0 \tag{4.67}$$

where again in the one-dimensional case the ∇ operators are actually total derivatives in respect to the variable x. In order to find the potential the static approximation for this equation is used, which gives

$$\left[\nabla^2 - \left(\frac{m_\pi c}{\hbar}\right)^2\right] \psi(x) = 0 \tag{4.68}$$

The important assumption is that the source for the pion is given by the nucleons and therefore include a source term in the static meson field in the following way

$$\left[-\nabla^2 + k_\pi^2\right] \psi(x) = \rho(x) \tag{4.69}$$

where

$$k_\pi = \frac{m_\pi c}{\hbar} \tag{4.70}$$

The source has to be given as a confined small, but extended function with the same dimensional properties as the nucleon, restricted to one dimension. This is not immediately an easy task, but this problem is discussed later. The main interest here is a point source in order to recover the Green's function description. The Green's function for this system is defined by

$$\left[\nabla^2 - k_\pi^2\right] G_\pi(k_\pi, x - x') = \delta(x - x') \tag{4.71}$$

where the Green's function is assumed to be translationally invariant. A solution for the meson field is then given by the formal expression

$$\phi(x) = \int G_\pi(k_\pi, x - x')\, \rho(x')\, dx' \tag{4.72}$$

which means it remains to find the explicit form for the Green's function and the source term. The Green's function can again be found using the theory of Fourier transformations, which gives

$$G_\pi(k_\pi, x - x') = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \tilde{G}_\pi(k_\pi, k'_\pi)\, e^{-ik'_\pi(x - x')} dk'_\pi \tag{4.73}$$

Inserting this expression into the Green's function eq.(4.71) and performing the operation yields

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\left[-(k'_\pi)^2-k_\pi^2\right]\tilde{G}_\pi\left(k_\pi,k'_\pi\right)e^{-ik_\pi(x-x')}dk'_\pi=\frac{1}{2\pi}\int_{-\infty}^{+\infty}e^{-ik_\pi(x-x')}dk_\pi \tag{4.74}$$

where the properties of the delta function and their representation are used (see Messiah [Mes76, p.422]). Comparing the integrands gives the Green's function equation in k_π-space, namely

$$\left[-(k'_\pi)^2-k_\pi^2\right]\tilde{G}_\pi\left(k_\pi,k'_\pi\right)=\frac{1}{\sqrt{2\pi}} \tag{4.75}$$

or in other words

$$\tilde{G}_\pi\left(k_\pi,k'_\pi\right)=\frac{1}{\sqrt{2\pi}}\left[-(k'_\pi)^2-k_\pi^2\right]^{-1} \tag{4.76}$$

In order to give the Green's function in coordinate space it is substituted back into the definition for the Fourier transform, which yields

$$G_\pi\left(k_\pi,x-x'\right)=-\frac{1}{2\pi}\int_{-\infty}^{+\infty}\left[+(k'_\pi)^2+k_\pi^2\right]^{-1}e^{-ik'_\pi(x-x')}dk'_\pi \tag{4.77}$$

This integral can be solved using the theorem of residues, because there are no poles in the denominator. Again, the two Green's functions depend on the sign of $x-x'$, namely

$$G^{\wedge}_\pi\left(k_\pi,x-x'\right) = -\frac{e^{-k_\pi(x-x')}}{k_\pi} \tag{4.78}$$

$$G^{\vee}_\pi\left(k_\pi,x-x'\right) = -\frac{e^{k_\pi(x-x')}}{k_\pi} \tag{4.79}$$

Therefore, the meson field coming from the nucleon source is given by the expression

$$\phi(x)=-\frac{1}{k_\pi}\left[\int_{-\infty}^{x}e^{-k_\pi(x-x')}\rho(x')\,dx'+\int_{x}^{\infty}e^{k_\pi(x-x')}\rho(x')\,dx'\right] \tag{4.80}$$

Recombining the integrals therefore leaves me with the expression

$$\phi(x)=-\frac{1}{k_\pi}\int_{-\infty}^{\infty}e^{-k_\pi|x-x'|}\rho(x')\,dx' \tag{4.81}$$

The explicit expression for the nucleon source at the position x_1 is defined by the equation

$$\rho(x) = \frac{f}{2R_N} e^{-\frac{|x-x_1|}{R_N}} \tag{4.82}$$

This results in the following expression for the meson field

$$\phi(x) = -\frac{f}{2k_\pi R_N} \int_{-\infty}^{\infty} e^{-k_\pi|x-x'|-\frac{1}{R_N}|x'-x_1|} dx' \tag{4.83}$$

and one integration remains. In order to simplify the integration the following definitions are given

$$\chi = \frac{-b}{2a} \tag{4.84}$$

$$a = k_\pi \tag{4.85}$$

$$b = \frac{1}{R_N} \tag{4.86}$$

These definitions are used to give the equation

$$\phi(x) = \chi f \int_{-\infty}^{+\infty} e^{-a|x-x'|-b|x'-x_1|} dx' \tag{4.87}$$

In order to solve this integral two cases have to be considered, namely $x_1 > x$ and $x_1 < x$. The first case $x_1 > x$ gives the following split for the integral

$$\phi^I(x) = \chi f \left[\int_{-\infty}^{x} e^{-a(x-x')-b(x_1-x')} dx' + \int_{x}^{x_1} e^{-a(x'-x)-b(x'-x_1)} dx' + \int_{x_1}^{+\infty} e^{-a(x'-x)-b(x'-x_1)} dx' \right] \tag{4.88}$$

The integrations are rather lengthy, but straightforward, and the result is given by the expression

$$\phi^I(x) = \chi \frac{2f}{a^2-b^2} \left(a e^{-b(x_1-x)} - b e^{-a(x_1-x)} \right) \tag{4.89}$$

The second case $x_1 < x$ works similarly and gives the result

$$\phi^{II}(x) = \chi \frac{2f}{a^2-b^2} \left(a e^{-b(x-x_1)} - b e^{-a(x-x_1)} \right) \tag{4.90}$$

Combining the two cases gives for the meson field the following expression

$$\phi(x) = \chi \frac{2f}{a^2 - b^2} \left(a e^{-b|x-x_1|} - b e^{-a|x-x_1|} \right) \tag{4.91}$$

Consequently, the NN-potential is described by

$$V(x_1, x_2) = \int_{-\infty}^{\infty} \phi(x) \rho_2(x) \, dx \tag{4.92}$$

or more explicitly

$$V(x_1, x_2) = \chi \frac{bf^2}{a^2 - b^2} \left[a \int_{-\infty}^{\infty} e^{-b|x-x_1|} e^{-b|x-x_2|} dx - b \int_{-\infty}^{\infty} e^{-a|x-x_1|} e^{-b|x-x_2|} dx \right] \tag{4.93}$$

where the normalization of the nucleon source has been taken into account. The integration can be performed explicitly and the first term in this equation is given by

$$V^b(x_1, x_2) = \chi \frac{abf^2}{a^2 - b^2} \int_{-\infty}^{\infty} e^{-b|x-x_1|} e^{-a|x-x_2|} dx \tag{4.94}$$

Again, calculation of this integral requires to consider two cases. First, the integration for the case $x_1 > x_2$, which yields

$$V_1^b(x_1, x_2) = \chi \frac{abf^2}{a^2 - b^2} \left[\int_{-\infty}^{x_2} e^{-b(x_1 - x + x_2 - x)} dx + \int_{x_2}^{x_1} e^{-b(x_1 - x + x - x_2)} dx + \int_{x_1}^{\infty} e^{-b(x - x_1 + x - x_2)} dx \right] \tag{4.95}$$

and performing the integrations results in the expression

$$V_1^b(x_1, x_2) = \chi \frac{abf^2}{a^2 - b^2} \left(\frac{1}{b} + (x_1 - x_2) \right) e^{-b(x_1 - x_2)} \tag{4.96}$$

Second, the case $x_1 < x_2$ gives equivalently

$$V_2^b(x_1, x_2) = \chi \frac{abf^2}{a^2 - b^2} \left(\frac{1}{b} + (x_2 - x_1) \right) e^{-b(x_2 - x_1)} \tag{4.97}$$

Combining the two cases results in the equation

$$V^b(x_1, x_2) = \chi \frac{abf^2}{a^2 - b^2} \left(\frac{1}{b} + |x_2 - x_1| \right) e^{-b|x_2 - x_1|} \tag{4.98}$$

The second term in the integrand

$$V^a(x_1,x_2) = -\chi\frac{b^2f^2}{a^2-b^2}\int_{-\infty}^{\infty} e^{-a|x-x_1|}e^{-b|x-x_2|}dx \tag{4.99}$$

is calculated following similar arguments as the ones used for the first term, which gives the expression

$$V^a(x_1,x_2) = -\chi f^2\frac{b^2}{a^2-b^2}\frac{2a}{a^2-b^2}e^{-b|x_2-x_1|} + \chi f^2\frac{b^2}{a^2-b^2}\frac{2b}{(a^2-b^2)}e^{-a|x_2-x_1|} \tag{4.100}$$

Adding up all the terms describes the final expression for the NN-potential in the one dimensional case, namely

$$V(x_1,x_2) = \chi f^2\frac{ab}{a^2-b^2}\left(\frac{1}{b}+|x_2-x_1|-\frac{2b}{a^2-b^2}\right)e^{-b|x_2-x_1|}+\chi f^2\frac{2b^3}{(a^2-b^2)^2}e^{-a|x_2-x_1|} \tag{4.101}$$

which is equal to the equation

$$V(x_1,x_2) = f^2\frac{-b^2}{2a^2-b^2}\left[\left(\frac{1}{b}+|x_2-x_1|-\frac{2b}{a^2-b^2}\right)e^{-b|x_2-x_1|}+\frac{2b^2}{a(a^2-b^2)}e^{-a|x_2-x_1|}\right] \tag{4.102}$$

In the case that the nucleon radius goes to zero this potential collapses to the form

$$V_{R_N=0}(x_1,x_2) = -f^2\frac{e^{-a|x_2-x_1|}}{a} \tag{4.103}$$

or more explicitly

$$V_{R_N=0}(x_1,x_2) = -f^2\frac{e^{-k_\pi|x_2-x_1|}}{k_\pi} \tag{4.104}$$

This is the potential used in the Toy model in the k-space and the form of the potential is the one generally expected for a model of the given type. The units of this potential have to be MeV in order to have a consistent theory. However, k_π has units of fm^{-1}, which requires that the coupling constant has the units

$$f^2 = \left[\frac{MeV}{fm}\right] \tag{4.105}$$

Also, the potential is used in the center of mass system, which requires to express the potential in respect to the relative position. The potential V in standard units in the

center of mass system is therefore given by the equation

$$V = -\frac{\hbar f^2}{m_\pi c} e^{-\frac{mc}{\hbar}|x|} \tag{4.106}$$

with the overall units of MeV, which is also exactly as expected. Therefore, I devised a one-dimensional model for the two-nucleon scattering and the corresponding NN-potential. In the next section I use this potential to find the bound states of the given system numerically.

Chapter 5

THE $2N$ TOY-MODEL BOUND STATE PROBLEM

The creator of the universe works in mysterious ways. But he uses a base ten counting system and likes round numbers.

Scott Adams (Dilbert)

5.1 Integral representation for the $2N$ Toy-model bound state problem

In the last section I derived a description for the Yukawa type NN-potential in one dimension for the Toy model. This potential is the equivalent to the $3D$ Yukawa potential, which usually is $\propto \frac{e^{-kr}}{r}$. However, even though the $1D$ potential only shows an exponential behaviour, I use the expressions Yukawa and Malfliet-Tjon type for the $1D$ potentials. In this section I prepare the Toy-model in order to calculate the form factors needed for the separable expansion. Since I am only interested in the bound state problem and not the scattering problem, I choose the unitary pole approximation (UPA) as expansion method. The UPA and its validity is documented in detail in a review by Levinger [Lev74] and for more information I refer to the literature [Har70, PS77, CHS81].

In order to give the UPA the two-body bound state problem has to be solved and it is given by the homogeneous part of the t-matrix Lippmann-Schwinger equation. In order to find the strength factors of the potentials to give the right 2-body bound state energy Sturmian functions are introduced. Sturmian functions are often used

for separable expansions of the T-matrix and the scattering problem due their explicit dependence on the energy. However, I use the sturmian procedure only to find the bound states and refer for more information on Sturmians and their applications to the literature [Rot62, Raw82, CCP88, RC90, CR91, DCPA94].

The goal in this section is to develop an integral representation of the relevant operator equations and then to prepare the equations for numerical calculations, which are performed afterwards. Once the bound states are found the UPA is implemented, which is done in the subsequent section. In this thesis I restrict myself to the use of two simple potentials, a one term attractive potential V_1 and a two term potential V_2, which is made up of an attractive and a repulsive part of the Yukawa type. Since the potential V_1 is a special case of potential V_2, where the repulsion is zero, I show the convergence behaviour for the potential V_2 only. In general, the functions are satisfying the operator equation

$$V G_0 V \left|\psi_S\right\rangle = \eta V \left|\psi_S\right\rangle \tag{5.1}$$

The momentum state representation of these equations can be found most easily by projecting onto the momentum eigenstates and inserting identities expressed in the appropriate momentum state basis. This procedure yields the integral representation

$$\iint \left\langle k\right| V \left|k'\right\rangle \left\langle k'\right| G_0 \left|k''\right\rangle \left\langle k''\right| V \left|k'''\right\rangle \left\langle k''' \mid \psi_S\right\rangle dk'dk''dk''' = \int \left\langle k\right| V \left|k'\right\rangle \left\langle k' \mid \psi_S\right\rangle dk' \tag{5.2}$$

The operators are described in the k-space with k given in units of fm^{-1}. In this notation the wave function is given by

$$\left\langle k \mid \psi_S\right\rangle = \psi_S(k) \tag{5.3}$$

and the integral equation has to be solved in order to find the explicit expressions for the wave functions. The potential in k space is given by the Fourier transform of the potential in standard units, as defined in the last section. The function in the center

of mass coordinate space is

$$V(x',x) = \langle x'|\, V\, |x\rangle = \delta(x-x')\, f^2 \frac{e^{-a|x|}}{a} \tag{5.4}$$

and the Fourier transform is given by the explicit calculation of the expression

$$\langle k'|\, V\, |k\rangle = \int \langle k'\mid x'\rangle \langle x'|\, V\, |x\rangle \langle x\mid k\rangle\, dxdx' \tag{5.5}$$

The calculation is straightforward and yields

$$\langle k'|\, V\, |k\rangle = \int \frac{1}{\sqrt{2\pi}} e^{-ik'x} f^2 \frac{e^{-a|x|}}{a} \frac{1}{\sqrt{2\pi}} e^{ikx} dx = \frac{f^2}{\pi} \frac{1}{a^2 + (k'-k)^2} \tag{5.6}$$

and the overall dimensions of the potential are $MeV - fm$. The Sturmian function is defined by an integral equation with integration over k. Therefore, the right hand side of the Sturmian equation has the units $MeV - fm$ from the potential and fm^{-1} from the integration. The remaining units, besides the wave function, therefore are MeV. The left hand side must have a similar behaviour and the three k integrations introduce units of fm^{-3} and the two potentials introduce units of $MeV^2 - fm^2$. Consequently, the Green's function has to have units of fm/MeV in order to have a consistent model. The Green's function in k-space is given naturally by the expression

$$G(k',k,E) = \frac{\delta(k'-k)}{E - \frac{\hbar^2 k^2}{2\mu}} \tag{5.7}$$

and indeed the Green's function has the expected units. Inserting the expression into the Sturmian equations gives the following result

$$\int U(k,k',E)\, \psi_S(k') dk' = \int V(k,k')\, \psi_S(k')\, dk' \tag{5.8}$$

where the new function U is given by

$$U(k,k',E) = \int V(k,q) G_0(q,E)\, V(q,k')\, dq \tag{5.9}$$

In this equation the diagonal character of the Green's function has been implemented, which reduced the integrations by one variable. For numerical analysis it is helpful

to write the integral equation in dimensionless variables. This is achieved by the coordinate transformation $u = \frac{\hbar k}{m_\pi c} = \frac{k}{a}$, which introduces a Jacobi factor in the integration measure $dk = \frac{m_\pi c}{\hbar} du = a du$. The following integral equation for the dimensionless wave function is the result of the coordinate transformation

$$\int U(u, u', E)\, \psi(u')\, a du' = \int V(u, u')\, \psi(u')\, a du' \tag{5.10}$$

where

$$U(u, u', E) = \int V(u, u'')\, G_0(u'', E)\, V(u'', u')\, a du'' \tag{5.11}$$

and

$$V(u', u) = \langle u' | V | u \rangle = \frac{f^2}{\pi a^2} \frac{1}{1 + (u' - u)^2} \tag{5.12}$$

The Green's function in the same variables is given by

$$G(u, E) = \frac{2\mu}{(m_\pi c)^2} \frac{1}{\frac{2\mu E}{(m_\pi c)^2} - u^2} \tag{5.13}$$

and consequently the U in dimensionless variables is

$$U(u', u, E) = \frac{2\mu}{(m_\pi c)^2} \left(\frac{f^2}{\pi a^2}\right)^2 \int \frac{1}{1 + (u' - u'')^2} \frac{1}{\frac{2\mu E}{(m_\pi c)^2} - (u'')^2} \frac{1}{1 + (u'' - u)^2} a du'' \tag{5.14}$$

Because the U plays the same role as the potential on the left hand side the units of U and V must be the same, which indeed they are. With the definitions

$$\hat{V}(u, u') = \frac{1}{1 + (u - u')^2} \tag{5.15}$$

and

$$\hat{U}(u', u, E) = \int \frac{1}{1 + (u' - u'')^2} \frac{1}{\frac{2\mu E}{(mc)^2} - (u'')^2} \frac{1}{1 + (u'' - u)^2} du'' \tag{5.16}$$

the integral equation is given by the expression

$$\frac{2\mu a}{(m_\pi c)^2} \left(-\frac{f^2}{\pi a^2}\right)^2 \int \hat{U}(u, u', E)\, \psi(u')\, a du' = -\frac{f^2}{\pi a^2} \eta \int \hat{V}(u, u')\, \psi(u')\, a du' \tag{5.17}$$

This equation is simplified even further and the final equation to solve is

$$-\frac{2\mu c^2 \hbar c}{(m_\pi c^2)^3}\frac{f^2}{\pi}\int \hat{U}(u,u',E)\,\psi(u')\,du' = \eta \int \hat{V}(u,u',E)\,\psi(u')\,du' \tag{5.18}$$

This equation is dimensionless and it remains to develop a code for this system for numerical analysis. The only variables in this system are the eigenvalue, the coupling constant and the energy. The Sturmian procedure fixes the eigenvalue to be one for bound states and the energy of the bound state is $-2.225 MeV$ as for the NN problem in $3D$, where the bound state is the deuteron. This in turn should give the value for the coupling constant.

In the next section I describe the numerical solution of the Sturmian integral equation and fix the variables in order to describe nuclear physics.

5.2 *Numerical solution of the $2N$ bound state problem*

In the last section I described the Sturmian procedure and found an integral equation in order to find the states necessary for a separable expansion of the T-matrix. It is now necessary to develop a numerical procedure that allows the calculation of the integral equation. With the numerical solution the remaining constants are fixed so that the Toy-model resembles nuclear physics. In this section I show how the integral equation can be transformed into a generalized eigenvalue equation using quadratures [LP97, PTVF92]. In the given system this procedure has to be used twice, first in order to find a numerical description of the $\hat{U}$ and second to find a numerical description of the integral equation. The operator $\hat{U}$ was given by the expression

$$\hat{U}(u,u',E) = \int \frac{1}{1+(u-u'')^2}\,\frac{1}{\frac{2\mu c^2}{(m_\pi c^2)^2}E - u''}\,\frac{1}{1+(u''-u')^2}\,du'' \tag{5.19}$$

and the integral is calculated numerically using any quadrature. However, the choice of the particular procedure is important in order to find a satisfactory convergence of the numerical method. For now, I assume that the quadrature in question converges

and the discretization is possible. In this case the integral can be given by

$$\hat{U}(x_i, x_j, E) = \sum_k \frac{1}{1+(x_i-x_k)^2} \frac{1}{\beta^2 E - x_k^2} \frac{1}{1+(x_k-x_j)^2} w_k \tag{5.20}$$

where the x_i are the mesh points of the quadrature and the w_i the corresponding weights. The numerical code for this integration is given by the subroutine 'Colonel' given in the appendix and it returns all matrix elements of $\hat{U}$ for the potential V_2. The potential V_1 can be handled with the same routine, if the relative strength factor is set to zero. The integral equation

$$-\chi \frac{f^2}{\pi} \int \hat{U}(u, u', E)\, \psi(u', k)\, du' = \eta \int V(u, u', E)\, \psi(u', k)\, du' \tag{5.21}$$

is discretized over the same mesh points, which yields the generalized eigenvalue problem

$$-\chi \frac{f^2}{\pi} \sum_j \hat{U}(x_i, x_j, E)\, w_j \psi(x_j) = \eta \sum_j \hat{V}(x_i, x_j)\, w_j \psi(x_j) \tag{5.22}$$

Due to the weight factors the two matrices in this generalized eigenvalue problem are obviously not symmetric if they include the weights. However, the weights are associated with the vectors of the generalized eigenvalue problem and it remains to solve the following system

$$-\chi \frac{f^2}{\pi} \hat{U}(E)\, \tilde{\psi} = \eta \hat{V} \tilde{\psi} \tag{5.23}$$

where

$$\tilde{\psi} = D_w \psi \tag{5.24}$$

and D_w is the diagonal matrix with the appropriate weights. The two matrices in this generalized eigenvalue problem are now both symmetric. Furthermore, for negative E they are both real and without singularities, which simplifies the calculations. However, the matrices are not sign definite, which is clear under the presence of a repulsive contribution as in the Malfliet-Tjon type potential V_2. In the Yukawa type potential V_1 all entries in the matrices have the same sign and for appropriate grids they are even sign-definite, but all grids lose these properties at large N due to tail-contributions. Therefore, it is necessary to give a detailed convergence discussion

and show that the system converges long before it breaks down. Realistic nuclear potentials have a repulsive character at small separation distances and consequently I choose the more general case of the Malfliet-Tjon type potential for the calculations. In conclusion, a routine for the generalized eigenvalue problem has to be developed, where both matrices are real, symmetric and without singularities.

In the calculations I use the following numerical values

- The pion mass $mc^2 = 134.9766 MeV$
- The nucleon mass $\mu c^2 = 938.271998 MeV$
- The Planck factor $\hbar c = 197.3269600 MeV - fm$

Also, the appropriate routines from the IMSL library [IMS] are used in the calculations. It turns out that a strength factor of $f^2/\pi = 2.114450$ is needed for the Yukawa type potential in order to get an eigenvalue of one for the largest eigenvalue at $-2.225 MeV$. Similarly, the Malfliet-Tjon type potential requires a strength factor $f^2/\pi = 10.09496$ and a relative repulsion of 4.57000 with a nuclear radius of $0.700 fm$ as repulsive range parameter. In figure [5.1] the energy dependence of the first two eigenvalues of the Yukawa system is shown. It is apparent that only the largest eigenvalue actually can take on the value one. Therefore the system has exactly one bound state. All other eigenvalues for the Yukawa type potential are even smaller than the second one and can be neglected. However, in nuclear physics it is also known that there should be another bound state just above zero energy and in order to account for this behaviour I postulate that my second eigenvalue should approach the value one at zero energy. This second requirement allows the determination of a second strength factor, which fixes the repulsive character of a Malfliet-Tjon type potential. In figure [5.2] the energy dependence of the three largest eigenvalues of a Malfliet-Tjon type potential is shown. The largest eigenvalue again describes a bound state with the right energy and a second bound state nearly appears at zero energy.

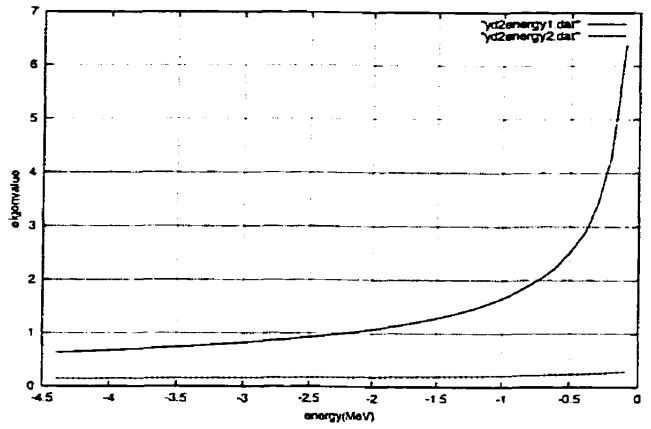

Figure 5.1: Energy dependence of the two largest eigenvalues of the Sturmian system with the Yukawa-type potential

It is important though that the eigenvalue actually does not take on the the value one before it crosses zero. It is also apparent that the third eigenvalue is actually negative and comes from the strong repulsive core of the Malfliet-Tjon type potential. Therefore, the Malfliet-Tjon type potential should give a better description of

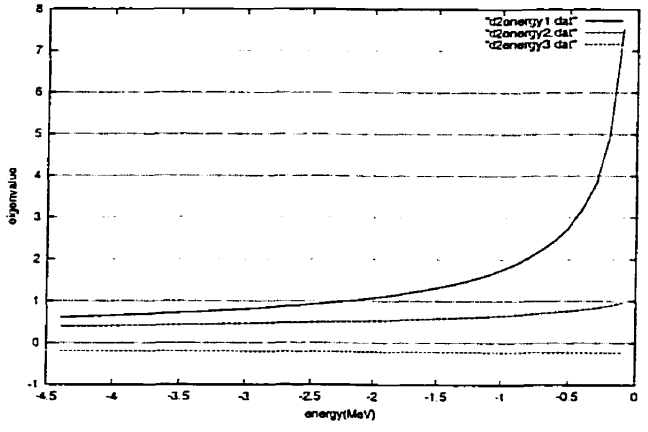

Figure 5.2: Energy dependence of the largest three eigenvalues for the Sturmian system with the Malfliet-Tjon type potential

the nuclear system than the Yukawa type potential. The two potentials are given in

figures [5.3, 5.4] and the potentials indeed show the expected behaviour in coordinate space. As I mentioned before I only have to show a detailed convergence discus-

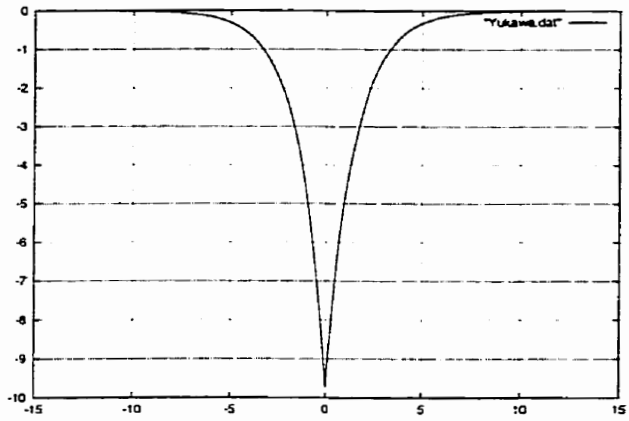

Figure 5.3: Yukawa type potential in coordinate space

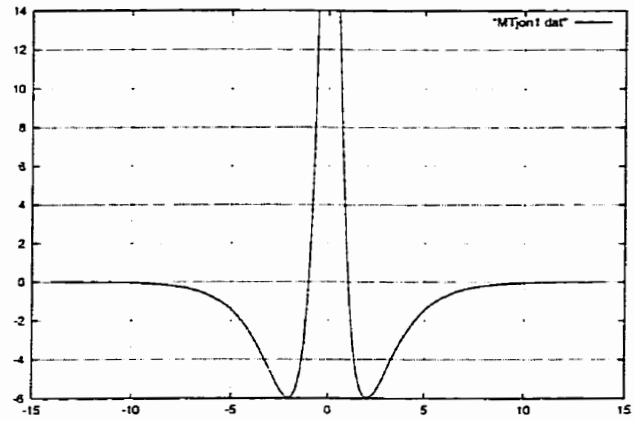

Figure 5.4: Malfliet-Tjon type potential in coordinate space

sion with the Malfliet-Tjon type potential. The Yukawa-type potential has the same type of convergence behaviour and the same calculations can be performed with the code choosing a zero repulsion strength. In figure [5.5] it is shown that the largest eigenvalue at the bound state energy converges rapidly and I choose the system to

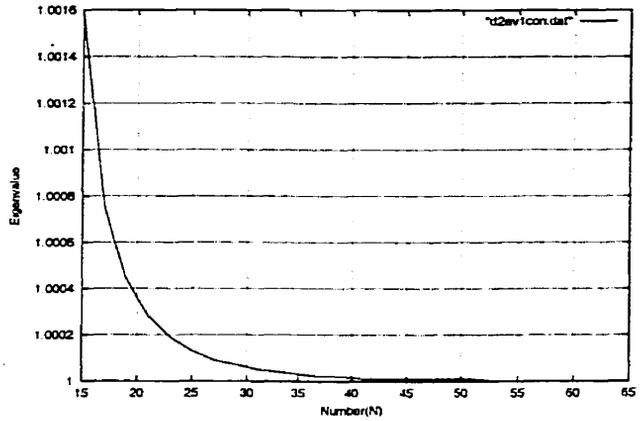

Figure 5.5: Convergence behaviour in respect to number of quadrature points for the largest Eigenvalue at $-2.22MeV$ with the Malfliet-Tjon type potential

be stable at 41 quadrature points. I should mention that these convergence tests have been performed throughout the calculations and the system remained stable at 41 quadrature points. For completeness, I show in fig.[5.2] the bound states for the Yukawa and the Malfliet-Tjon type potential and they do indeed show the expected behaviour. Having found the bound state of the system I now proceed to give the

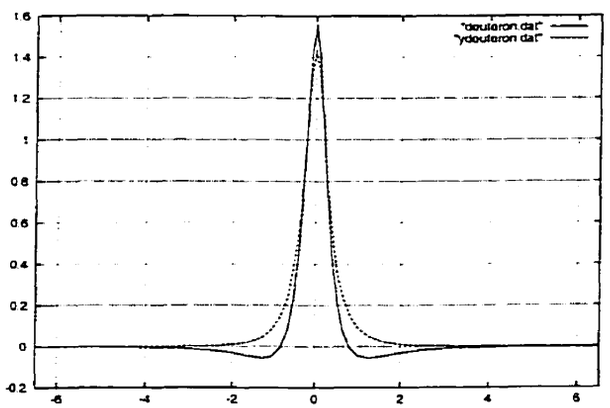

Figure 5.6: $1D$ deuteron type bound state in dimensionless variables

UPA, which is the input for the 1-D AGS system.

5.3 *Unitary Pole Approximation of the 1-D Toy model*

In the last section I defined the input potentials for the $1D$ NN problem and found the only eigenstate of the system. Because only one bound state exists, I choose the unitary pole approximation as the rank-one separable expansion method of choice. This is certainly not the most elegant method available, but its simplicity helps to pinpoint the sources of corrections in the calculations. Up to this point I always assumed that it is possible to find a separable expansion, which gives the following form for the separable t-matrix

$$t_a(z) = |a\rangle \tau_a(z) \langle a| \tag{5.25}$$

with

$$\tau_a(z) = \left(\lambda^{-1} - \langle a| G_0(z) |a\rangle\right)^{-1} \tag{5.26}$$

It now remains to apply the UPA in order to find the explicit expressions for the form factors and the propagators. So far I have given no restrictions on the form factors and consequently they could depend directly on the energy z like in the Sturmian expansion. The UPA is an expansion in respect to the energy independent form factor $|a\rangle$, which satisfies the form factor equation

$$|a(E_B)\rangle = \eta(E_B) V G_0(E_B) |\chi(E_B)\rangle \tag{5.27}$$

The eigenvalue $\eta(E_B)$ is naturally equal to one as shown in the last section. Also it should be clear from the sturmian procedure that the form factor is given by the following definition

$$|a(E_B)\rangle = V |\psi_B\rangle \tag{5.28}$$

where $|\psi_B\rangle$ is the normalized bound state wave function. However, the UPA form factor has to be normalized in the following way

$$\langle a| G_0(E_B) |a\rangle = 1 \tag{5.29}$$

in order to have the right pole behaviour for the t-matrix. Namely, the t-matrix has to have a pole at the binding energy of the deuteron. The normalization of the form factors is explicitly done by the code in the appendix. The rest of this section is a rather technical expose on the behaviour of the form factors, but it helps to understand their properties in more detail. In fig.[5.7] the first two attractive form factors are shown while in fig.[5.8] first two repulsive form factors are shown.

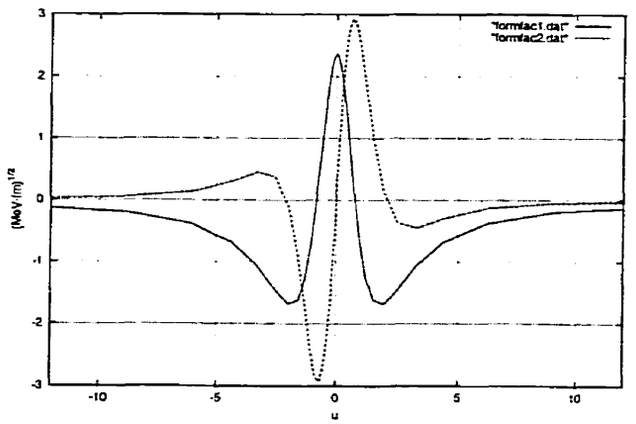

Figure 5.7: The first two normalized attractive form factors for the UPA

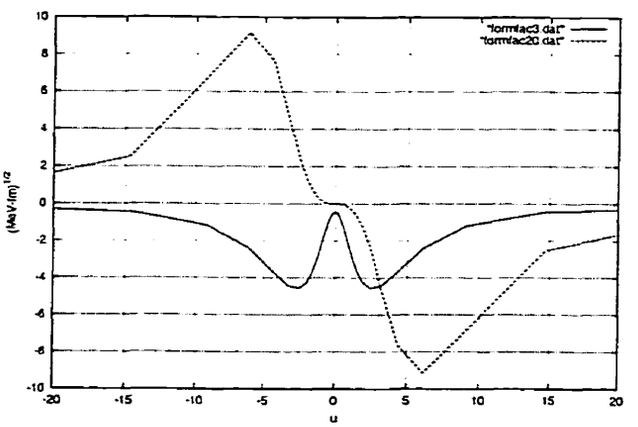

Figure 5.8: The first two repulsive normalized form factors for the UPA

The behaviour of the two form factors belonging to the largest eigenvalues in

respect to their signs is very similar to the one observed by Harms [Har70] in the $3D$ case. It is quite surprising that the $1D$ system behaves so similar to the one described by Harms. This similarity is even more exemplified in the behaviour of the t-matrices, which is described by Harms in the same paper and reviewed by Levinger [Lev74]. In fig.[5.9] three diagonal elements of the t-matrix are shown with the same energy units used by Harms ($s = 41.46 MeV$). The behaviour of the diagonal t-matrices is again

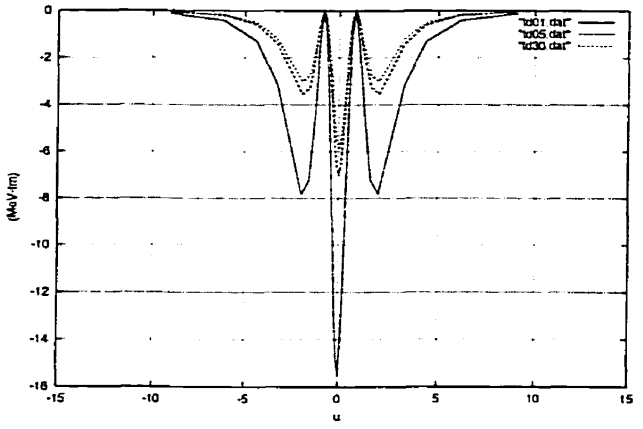

Figure 5.9: The diagonal t-matrices for three energies using Harms' units

the same as the one observed by Harms. The same similarity is also shown in the off-diagonal form factors as can be seen in fig.[5.10], which describes the behaviour of t-matrices that are not far off the diagonal. For completeness, fig.[5.11] shows the off-diagonal t-matrices that are far away from the diagonal with the same energy. In order to use the UPA as input into the three-body equations the $2N$-UPA has to be embedded into the $3N$ space. This procedure is well known for any separable expansion and involves simply a shift in the energy variable. This shift occurs, because the three-body operator is given in respect to the total energy including the energy of the spectator. Therefore, this energy has to be subtracted in order to get the energy of the clustered $2N$ sub-system, which yields in general

$$t_a(z) = |a\rangle \tau_a \left(z - \frac{q_a^2}{2M_a} \right) \langle a| \tag{5.30}$$

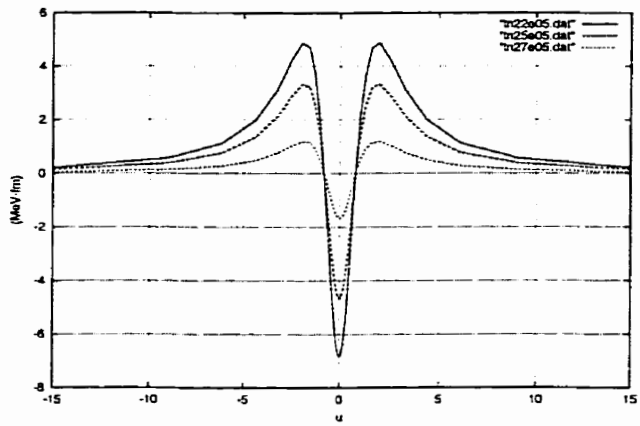

Figure 5.10: The off diagonal t-matrices close to the diagonal with energy $0.5s$

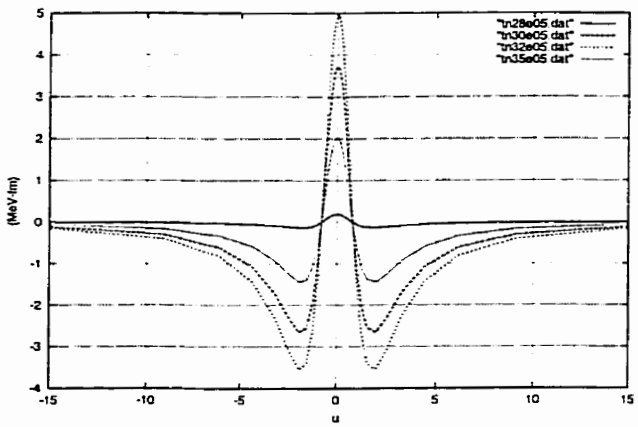

Figure 5.11: The off diagonal t-matrices far away from the diagonal with energy $0.5s$

For now, I simply give this result and the next section the 1-D AGS system is symmetrized and the UPA is incorporated. Once the final form of the resulting effective two-body Lovelace type equation is found I proceed in calculating the $3N$ binding energies.

Chapter 6

ONE-DIMENSIONAL TOY MODEL FOR THE STANDARD $3N$-SYSTEM

A common mistake that people make when trying to design something foolproof is to underestimate the ingenuity of complete fools

Douglas Adams

6.1 Embedding of the $2N$ t-matrices into the $3N$ space

In the last chapter I defined a 1-D Toy model for the two-nucleon scattering problem and applied it to find the strength factors of the Malfliet-Tjon type potential. The $2N$ problem is now the input for the channel equations and behaves exactly the same, except that I have an energy shift due to the spectator nucleon. In this section I describe the embedding of the $2N$ problem into the $3N$ space explicitly and in the next section I show the symmetrization of the 1-D AGS-equations.

The channel equation of the standard $3N$ systems are the main interest in this section and they are given by the expression

$$t_{\bar{a}'}(z) = v_{\bar{a}'} + v_{\bar{a}'} g_{\bar{a}'}(z)\, v_{\bar{a}'} \tag{6.1}$$

or in a different representation by the expression

$$t_{\bar{a}'}(z) = v_{\bar{a}'} + v_{\bar{a}'} g_0(z)\, t_{\bar{a}'}(z) \tag{6.2}$$

All channels are conveniently expressed in their natural set of Jacobi-coordinates $|p_{\bar{a}'}, q_{\bar{a}'}\rangle$, which are explained in more detail in the appendix A. In order to see the effects of the spectator nucleons the relevant terms are given explicitly. The channel potential in its natural Jacobi coordinates is given by

$$\langle p_{\bar{a}'}, q_{\bar{a}'}| \, v_{\bar{a}'} \, |p'_{\bar{a}'}, q'_{\bar{a}'}\rangle = \delta\left(q_{\bar{a}'} - q'_{\bar{a}'}\right) \langle p_{\bar{a}'}| \, \hat{v}_{\bar{a}'} \, |p'_{\bar{a}'}\rangle \tag{6.3}$$

where $\hat{v}_{\bar{a}'}$ denotes the $2N$ potential as defined in the standard two-body problem. The observed split occurs, because the channel potential is only defined in respect to the relative coordinate of the two nucleons in the cluster, namely $p_{\bar{a}'}$. The Green's functions can be given following a similar argument, but the energy of the spectator nucleon has to be subtracted from the three-nucleon energy z in order to get the energy of the cluster system. For the channel Green's function this yields

$$\langle p_{\bar{a}'}, q_{\bar{a}'}| \, g_{\bar{a}'}\left(z\right) |p'_{\bar{a}'}, q'_{\bar{a}'}\rangle = \delta\left(q_{\bar{a}'} - q'_{\bar{a}'}\right) \langle p_{\bar{a}'}| \, \hat{g}_{\bar{a}'}\left(z - \frac{\hbar^2 q^2_{\bar{a}'}}{2M_{\bar{a}'}}\right) |p'_{\bar{a}'}\rangle \tag{6.4}$$

and the free Green's function is given by

$$\langle p_{\bar{a}'}, q_{\bar{a}'}| \, g_0\left(z\right) |p'_{\bar{a}'}, q'_{\bar{a}'}\rangle = \frac{\delta\left(q_{\bar{a}'} - q'_{\bar{a}'}\right) \delta\left(p_{\bar{a}'} - p'_{\bar{a}'}\right)}{z - \frac{\hbar^2 q^2_{\bar{a}'}}{2M_{\bar{a}'}} - \frac{\hbar^2 p^2_{\bar{a}'}}{2\mu_{\bar{a}'}}} \tag{6.5}$$

The channel t-matrix also follows these arguments and the result is the following expression

$$\langle p_{\bar{a}'}, q_{\bar{a}'}| \, t_{\bar{a}'}\left(z\right) |p'_{\bar{a}'}, q'_{\bar{a}'}\rangle = \delta\left(q_{\bar{a}'} - q'_{\bar{a}'}\right) \langle p_{\bar{a}'}| \, \hat{t}_{\bar{a}'}\left(z - \frac{\hbar^2 q^2_{\bar{a}'}}{2M_{\bar{a}'}}\right) |p'_{\bar{a}'}\rangle \tag{6.6}$$

which evidently also displays the energy shift due to the presence of the spectator nucleon. In the two-body case a separable expansion method for the t-matrix is used and results in a similar expression for the separable t-matrix in the three body space. The separable t-matrix has the same type of energy shift and is given by

$$\langle p_{\bar{a}'}, q_{\bar{a}'}| \, t^S_{\bar{a}'}\left(z\right) |p'_{\bar{a}'}, q'_{\bar{a}'}\rangle = \langle p_{\bar{a}'}, q_{\bar{a}'}| \, \bar{a}'\rangle \, \hat{\tau}_{\bar{a}'}\left(z - \frac{\hbar q^2_{\bar{a}'}}{2M_{\bar{a}'}}\right) \langle \bar{a}' \, |p'_{\bar{a}'}, q'_{\bar{a}'}\rangle \tag{6.7}$$

where the form factor does not depend on the inter-cluster momentum, which becomes important later. I am now in the position to describe a symmetrization procedure and an integral representation of the symmetrized system.

6.2 Symmetrization of the AGS-equations

In nuclear physics, the nucleons are treated as identical particles, which introduces a formal simplification in the AGS-equations. In general the nucleons are fermions and the AGS-equations should be anti-symmetrized. However, for simplicity I choose to symmetrize the system instead. This choice is made also, because the "nucleons" in the $1D$ system are spinless and therefore behave like bosons. I could have included a spin degree of freedom, by using helicity states, but choose not to do that in this first model calculation. The choice of helicity for the spin equivalent in a $3D$ system without partial wave analysis is described in a recent paper by Fachruddin, Elster and Gloeckle [FEG00]. Including the helicity into the $1D$-system should be seen as a research topic worth pursuing in the future. Nevertheless, I should get some insight on the importance of the correction terms with the $1D$-system, even without the helicity component. Next I show how the standard AGS-equation can be symmetrized.

The standard three-body AGS-equation is given by the expression

$$U_{\bar{a}'\bar{b}'} = \bar{\delta}_{\bar{a}'\bar{b}'} g_0^{-1} + \sum_{\bar{c}'} \bar{\delta}_{\bar{a}'\bar{c}'} t_{\bar{c}'} g_0 U_{\bar{c}'\bar{b}'} \tag{6.8}$$

where the $\bar{a}', \bar{b}', \bar{c}'$ and the Green's function g_0 indicate that the system is in the three-nucleon only space. In the standard three body problem no connection to the pion space is given, but for reasons of clarity I keep the notation. In order to have a symmetrized system, the input t-matrix $t_{\bar{c}'}$ already has to be symmetrized. The UPA shown in the last chapter takes care of this symmetrization explicitly, because only the symmetric bound state is used in the separable expansion. Now, the AGS-equation describes two distinct cases, the direct scattering where the incoming and outgoing channels are the same, and the rearrangement scattering where they are not. The direct scattering is described by the diagonal elements and the rearrangement scattering by the off-diagonal elements of the AGS-operator U. However, in standard three-body AGS-models all channels are asymptotically the same and should be indistinguishable. Therefore, the form factors in the separable expansion should be the

same for each channel. The standard AGS-equations in separable form is given by the expression

$$\langle \bar{a}' | \, g_0 U_{\bar{a}'\bar{b}'} g_0 \left| \bar{b}' \right\rangle = \langle \bar{a}' | \, g_0 \left| \bar{b}' \right\rangle \bar{\delta}_{\bar{a}'\bar{b}'} + \sum_{\bar{c}'} \bar{\delta}_{\bar{a}'\bar{c}'} \langle \bar{a}' | \, g_0 \, | \bar{c}' \rangle \, \tau_{\bar{c}'} \langle \bar{c}' | \, g_0 U_{\bar{c}'\bar{b}'} g_0 \left| \bar{b}' \right\rangle \tag{6.9}$$

This expression is derived by inserting the separable expansion for the t-matrix and folding the equation between the form factors of the clusters. It is important to note that this equation does not yet depend explicitly on the inter-cluster momenta and this fact allows for the derivation of an integral representation. Furthermore, the equation is of the same form as the well known Lovelace equation and can be given in the abbreviated form

$$X_{\bar{a}'\bar{b}'} = Z_{\bar{a}'\bar{b}'} + \sum_{\bar{c}'} Z_{\bar{a}'\bar{c}'} \tau_{\bar{c}'} X_{\bar{c}'\bar{b}'} \tag{6.10}$$

with

$$X_{\bar{a}'\bar{b}'} = \langle \bar{a}' | \, g_0 U_{\bar{a}'\bar{b}'} g_0 \left| \bar{b}' \right\rangle \tag{6.11}$$

and

$$Z_{\bar{a}'\bar{b}'} = \langle \bar{a}' | \, g_0 \left| \bar{b}' \right\rangle \bar{\delta}_{\bar{a}'\bar{b}'} \tag{6.12}$$

However, in the case of three identical particles, not all of these operators are independent. On the other hand, they are also not all identical due to the anti-delta function. The proposed symmetrization procedure follows naturally the Lovelace procedure and starts with the known constraints.

All channels described by the partitions $\bar{a}', \bar{b}', \bar{c}'$ describe the same physical situation, namely one two-particle cluster and one particle far away from the cluster. Therefore the t-matrices for these situations should be the same and in the separable expansion, therefore, also the corresponding τ's should be the same, which yields

$$\tau \equiv \tau_a = \tau_b = \tau_c \tag{6.13}$$

The effective potential term naturally displays that all diagonal elements are zero, while all off-diagonal elements are the same. in the correction terms contributions

to the diagonal terms can exist. However, the symmetrization procedure can readily be generalized to that case. In order to stay close to Lovelace's original procedure, I continue to treat the diagonal terms as zero for now. In this spirit, the two classes are written in the following way

$$Z^D \equiv Z_{\bar{a}'\bar{a}'} = Z_{\bar{b}'\bar{b}'} = Z_{\bar{c}'\bar{c}'} = 0 \tag{6.14}$$

and

$$Z^N \equiv Z_{\bar{a}'\bar{b}'} = Z_{\bar{a}'\bar{c}'} = Z_{\bar{b}'\bar{a}'} = Z_{\bar{b}'\bar{c}'} = Z_{\bar{c}'\bar{b}'} = Z_{\bar{c}'\bar{b}'} \tag{6.15}$$

The Lovelace equations behave in a similar way and also have two distinct classes, given by the diagonal and the off-diagonal elements. The diagonal elements are explicitly given by the definition

$$X^D \equiv X_{\bar{a}'\bar{a}'} = X_{\bar{b}'\bar{b}'} = X_{\bar{c}'\bar{c}'} \tag{6.16}$$

and the off-diagonal elements by the definition

$$X^N \equiv X_{\bar{a}'\bar{b}'} = X_{\bar{a}'\bar{c}'} = X_{\bar{b}'\bar{a}'} = X_{\bar{b}'\bar{c}'} = X_{\bar{c}'\bar{b}'} = X_{\bar{c}'\bar{b}'} \tag{6.17}$$

With these definitions it is straightforward to show that the symmetrized Lovelace system is given by the following set of equations

$$X^D = 2Z^N \tau X^N \tag{6.18}$$

$$X^N = Z^N + Z^N \tau X^N + Z^N \tau X^D \tag{6.19}$$

This is a coupled set of integral equations and the next step is to decouple the equations. In order to decouple the system two new operators are defined, namely

$$X = X^D + 2X^N \tag{6.20}$$

$$Y = X^D - X^N \tag{6.21}$$

which lead to the decoupled Lovelace set of equations

$$X = 2Z^N + 2Z^N \tau X \tag{6.22}$$

$$Y = -Z^N - Z^N \tau Y \tag{6.23}$$

Lovelace already showed that the second set of equations is not contributing to physical processes, which means only the following equation has to be solved

$$X = 2Z^N + 2Z^N \tau X \tag{6.24}$$

I should mention again that the symmetrized Lovelace equations that include the pion-dynamics are somewhat different, because they have contributions to the diagonal terms of the effective potential. However, the resulting complication is taken care of in the treatment of the diagonal terms. Therefore, I concentrate on the standard case for now and give the corresponding integral representation.

I already stated that the given set of equations does not explicitly depend on the inter-momentum variables $q_{\bar{a}'}$ and I furthermore observe that our final symmetrized equation has the form of an effective Lippmann-Schwinger equation. In order to give this equation in momentum representation I have to sandwich between intra-momentum states and insert intra-momentum identities, which gives the following integral equation

$$\begin{aligned} \langle q_{\bar{a}'}| X(z) |q'_{\bar{a}'}\rangle &= 2\langle q_{\bar{a}'}| Z(z) |q'_{\bar{a}'}\rangle \\ &\quad +2\int \langle q_{\bar{a}'}| Z(z) |q''_{\bar{a}'}\rangle \tau_{\bar{a}'}\left(z - \frac{(\hbar q''_{\bar{a}'})^2}{2M_{\bar{a}'}}\right) \langle q''_{\bar{a}'}| X(z) |q'_{\bar{a}'}\rangle \, dq''_{\bar{a}'} \end{aligned} \tag{6.25}$$

Introducing the explicit expressions for the effective potential and the propagator and inserting appropriate identities into the effective potential, using Jacobi-momentum eigenstates, yields

$$Z_{\bar{a}'\bar{b}'} = \int \langle q_{\bar{a}'}| \langle \chi_{\bar{a}'} \mid p''_{\bar{a}'}, q''_{\bar{a}'}\rangle \langle p''_{\bar{a}'}, q''_{\bar{a}'}| g_0 |p'''_{\bar{b}'}, q'''_{\bar{b}'}\rangle \langle p'''_{\bar{b}'}, q'''_{\bar{b}'} \mid \chi'_{\bar{b}'}\rangle |q'_{\bar{b}'}\rangle \, \bar{\delta}_{\bar{a}'\bar{b}'} dp''_{\bar{a}'} dq''_{\bar{a}'} dp'''_{\bar{b}'} dq'''_{\bar{b}'} \tag{6.26}$$

This term simplifies using some well known relations. First of all, the form factors in momentum representation are given by the expression

$$\langle q_{\bar{a}'}| \langle \chi_{\bar{a}'} \mid p''_{\bar{a}'}, q''_{\bar{a}'}\rangle = \delta(q_{\bar{a}'} - q''_{\bar{a}'}) \, \hat{\chi}^*_{\bar{a}'}(p''_{\bar{a}'}) \tag{6.27}$$

and using the eigenvalues for the g_0 the Green's operator is replaced by its eigenvalue representation, which gives

$$Z_{\bar{a}'\bar{b}'} = \int \chi^*_{\bar{a}'}\left(p''_{\bar{a}'}\right) \frac{\left\langle p''_{\bar{a}'}, q''_{\bar{a}'} \mid p'''_{\bar{b}'}, q'''_{\bar{b}'}\right\rangle}{E + i\varepsilon - \frac{\left(\hbar q'''_{\bar{b}'}\right)^2}{2M} - \frac{\left(\hbar p'''_{\bar{b}'}\right)^2}{2\mu}} \chi_{\bar{b}'}\left(p'''_{\bar{b}'}\right) \bar{\delta}_{\bar{a}'\bar{b}'} dp''_{\bar{a}'} dp'''_{\bar{b}'} \tag{6.28}$$

However, the overlap of the Jacobi-momenta is given by the expression

$$\begin{aligned} \left\langle p''_{\bar{a}'}, q''_{\bar{a}'} \mid p'''_{\bar{b}'}, q'''_{\bar{b}'}\right\rangle &= \delta\left(q''_{\bar{a}'} - q'_{\bar{a}'}\left(p'''_{\bar{b}'}, q'''\right)\right) \delta\left(p''_{\bar{a}'} - p'_{\bar{a}'}\left(p'''_{\bar{b}'}, q'''\right)\right) \\ &= \delta\left(q''_{\bar{a}'} + p'''_{\bar{b}'} + \frac{1}{2} q'''_{\bar{b}'}\right) \delta\left(p''_{\bar{a}'} + \frac{1}{2} p'''_{\bar{b}'} - \frac{3}{4} q'''_{\bar{b}'}\right) \end{aligned} \tag{6.29}$$

which in turn reduces the effective potential term to the expression

$$Z_{\bar{a}'\bar{b}'} = \frac{\chi^*_{\bar{a}'}\left(\frac{1}{2} q''_{\bar{a}'} + q'''_{\bar{b}'}\right) \chi_{\bar{b}'}\left(-q''_{\bar{a}'} - \frac{1}{2} q'''_{\bar{b}'}\right)}{E + i\varepsilon - \frac{\left(\hbar q'''_{\bar{b}'}\right)^2}{2m_N} - \frac{\left(\hbar q''_{\bar{a}'} + \hbar q'''_{\bar{b}'}\right)^2}{2m_N} - \frac{\left(\hbar q''_{\bar{a}'}\right)^2}{2m_N}} \bar{\delta}_{\bar{a}'\bar{b}'} \tag{6.30}$$

At this point the explicit Jacobi-sets are ignored, because all channels are the same. Again it is clear that the diagonal elements of the effective potential are zero. This means the final expression for the effective potential is given by

$$Z^N\left(q, q', E + i\varepsilon\right) = \frac{\chi^*\left(\frac{1}{2} q + q'\right) \chi\left(-q - \frac{1}{2} q'\right)}{E + i\varepsilon - \frac{\hbar^2 q^2}{2m_N} - \frac{\left(\hbar q + \hbar q'\right)^2}{2m_N} - \frac{\left(\hbar q'\right)^2}{2m_N}} \tag{6.31}$$

For bound state problems, which are restricted to negative energies, the potential terms do not have any poles. Consequently, the term $i\epsilon$, which is introduced to avoid any poles on the real line, can be eliminated. Now it is possible to develop a numerical code for these terms, which are used as input in the effective two-body calculation. The propagator follows directly from the explicit form of the separable expansion and depends on the choice of form factors. However, in general the propagator has the form

$$\tau_{\bar{a}'}\left(z\right) = \frac{1}{\left(\lambda^{-1} - \left\langle \bar{a}' \right| g_0\left(z\right) \left| \bar{a}' \right\rangle\right)} \tag{6.32}$$

and in the UPA the parameter λ is fixed to one, while the quadratic form is equal to one for $z = E_B$, the deuteron bound state energy. This means that the propagator has a singularity at the deuteron bound state energy, as is expected.

6.3 *Numerical treatment of the one-dimensional $3N$-AGS system*

In the last section I symmetrized the 1-D AGS-system and gave an integral representation. In order to solve this integral equation numerically I need to perform two calculations first, namely I need to find an explicit representation for the effective potential and the quadratic form appearing in the propagator. Once these two expressions are calcuated they can be inserted into the homogeneous part of the effective Lippmann-Schwinger equation. The Sturmian procedure is then used to find the corresponding bound state energy. The energy shifts in the operators due to the spectators require the use a spline procedure in order to evaluate the form factors on the new grid points. This is a technical complication that can be dealt with. The main pitfall is the use of a wrong set of variables and it is important to specify the same set of variables as the ones defined in the calcuation of the form factors. For these reasons a concise discussion of the $3N$ Sturmian problem is given and the transformations to appropriate variables are shown.

The Sturmian problem is again described by the generalized eigenvalue problem of the LS-kernel, which is given by the expression

$$\int U_T(q,q',E)\,\Psi_T(q',E)\,dq' = \int Z^N(q,q')\,\Psi_T(q',E)\,dq' \tag{6.33}$$

with

$$U_T(q,q',E) = 2\int \frac{Z(q,q'',E)\,Z(q'',q',E)}{1-\langle a|\,G_0\left(z-\frac{(\hbar q'')^2}{2M}\right)|a\rangle}dq'' \tag{6.34}$$

However, the form of the denominator of the kernel does not depend on the specific set of variables and there is no numerical advantage to any one set. The effective potential on the other hand does have a preferred set of variables, because the denominator has units of MeV. In order to get a dimensionless form of the effective form factor I choose to use the same set of variables as the ones used in the $2N$ calculations. Namely, the momentum transformation $u = \frac{q}{a}$ is used, which also introduces the integration measure $dq = adu$. In order to get the dimensionless version of the

generalized eigenvalue problem the relations between the operators in dimensionless and dimensional variables have to be described. These relations are already known for the potential V, the Green's function and the wave function of the $2N$ problem. The only really difficult relation is the one for the form factors and it is shown how to get this relation in detail. The form factor in q-space is defined by the expression

$$\chi(q) = \int V(q,q')\,\psi(q')\,dq' \tag{6.35}$$

which evidently has the units $MeV - fm^{\frac{1}{2}}$. The normalized form factor on the other hand is given by

$$\hat{\chi}(q) = \frac{\int V(q,q')\,\psi(q')\,dq'}{\sqrt{\int \chi(q)\,G_0(q,E_B)\,\chi(q)\,dq}} \tag{6.36}$$

which has the appropriate units of $(MeV - fm)^{\frac{1}{2}}$ in order to expand the t-matrix. Next a coordinate transformation is performed on the un-normalized form factor in order to get the equivalent expression in respect to the dimensionless variables

$$\chi(q) = \frac{f^2}{\pi a^2}\int \hat{V}(u,u')\,\frac{\psi(u')}{\sqrt{a}}\,a\,du' \tag{6.37}$$

or in other words

$$\frac{f^2}{\pi a^{\frac{5}{2}}}\chi(u) = \chi(q) \tag{6.38}$$

In this expression all units are collected in the overall constant and they are given by $MeV - fm^{\frac{1}{2}}$. The normalized form factor in dimensionless variables can now be given by the expression

$$\hat{\chi}(q) = \frac{\frac{f^2}{\pi a^2}\int \hat{V}(u,u')\,\frac{\psi(u')}{\sqrt{a}}\,a\,du'}{\frac{f^2}{\pi a^{\frac{3}{2}}}\sqrt{\frac{2\mu}{(mc)^2}\int \chi(u)\,\hat{G}_0(u)\,\chi(u)\,a\,du}} \tag{6.39}$$

or in other words

$$\hat{\chi}(q) = \frac{mc}{\sqrt{2\mu a}}\hat{\chi}(u) \tag{6.40}$$

where again all the dimensions are collected in the overall constant. Now the $3N$ operators are described in dimensionless variables using the appropriate coordinate

transformations. The effective potential is written in the following way

$$Z^N(q,q',E) = \left(\frac{mc}{\sqrt{2\mu a}}\right)^2 \frac{2\mu}{(mc)^2} \frac{\hat{\chi}\left(u'+\frac{1}{2}u\right)\hat{\chi}\left(-\frac{1}{2}u'-u\right)}{\frac{2\mu}{(mc)^2}E - \frac{\mu}{m_N}\left(u^2-(u+u')^2-(u')^2\right)} \tag{6.41}$$

or in other words

$$Z^N(q,q',E) = \frac{1}{a}\hat{Z}^N(u,u',E) \tag{6.42}$$

where the effective potential in dimensionless variables is defined by the previous equation. The propagator in dimensionless variables is given by the expression

$$\tau^{-1}(u,E) = \left(1 - \int \frac{\chi^*(u')\chi(u')}{\frac{2\mu}{(mc)^2}E - \frac{\mu}{M}u^2 - (u')^2}du'\right) \tag{6.43}$$

and can be used in any given valid set of variables directly. Inserting these expressions in the kernel equation and performing the coordinate transformation on the integration variable yields

$$U_T(q,q',E) = \frac{1}{a^2}\int \hat{Z}^N(u,u'',E)\,\tau^{-1}(u'',E)\,\hat{Z}^N(u'',u',E)\,a du'' \tag{6.44}$$

It should be clear that the kernel again has the same units as the effective potential, which is exactly what is expected. Once this operator is calculated, it is used as input in the generalized eigenvalue equation describing the Triton bound state problem in dimensionless variables

$$\frac{1}{a}\int U_T(u,u',E)\frac{\Psi_T(u')}{\sqrt{a}}a du' = \frac{1}{a}\int Z_T(u,u',E)\frac{\Psi_T(u')}{\sqrt{a}}a du' \tag{6.45}$$

This equation can be simplified even further to the expression

$$\int U_T(u,u',E)\,\Psi_T(u')\,du' = \int Z_T(u,u',E)\,\Psi_T(u')\,du' \tag{6.46}$$

which is now ready for numerical analysis. In the appendix B the code "tritonmain.f" is included, which is the core code for the calculations. I performed the calculations again with a 41 point Gauss-Legendre quadrature and the convergence of the eigenvalue is striking as can be seen in fig.[6.1]. The behaviour for the eigenvalues of the

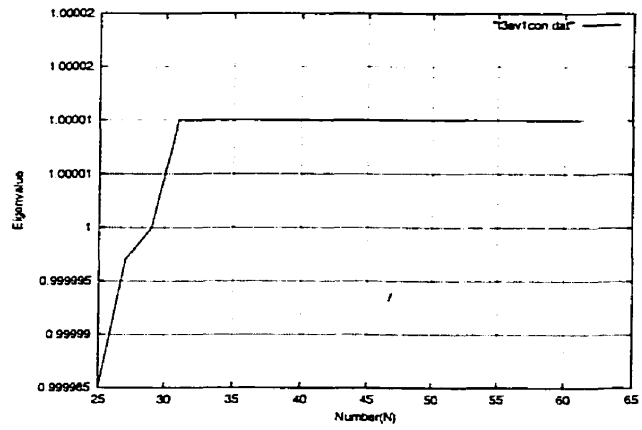

Figure 6.1: Convergence of the largest eigenvalue for the sturmian system at the triton binding energy

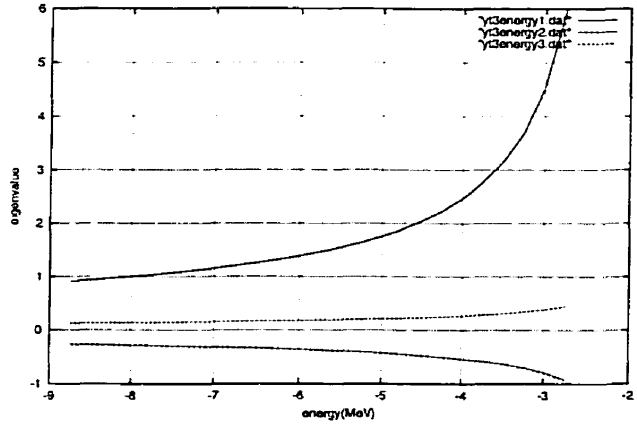

Figure 6.2: The energy dependence of the eigenvalues for the Triton GEV with the Yukawa potential

sturmian system with the Yukawa-type potential in respect to the energy is given in fig.[6.2]. The bound state appears at $-8.01MeV$ and the second largest eigenvalue is negative. It is surprising that the second largest eigenvalue is actually negative considering that the Yukawa has only positive eigenvalues. Nevertheless, it is explained by the fact the the effective potential of the $3N$ case is not as simple as the Yukawa-type potential of the $2N$ case. The crossing of the second largest eigenvalue at -1 is of no importance.

The system using the Malfliet-Tjon type potential is strikingly similar, as can be seen in fig.[6.3]. Here the bound state appears at an energy of $-7.277MeV$. There also

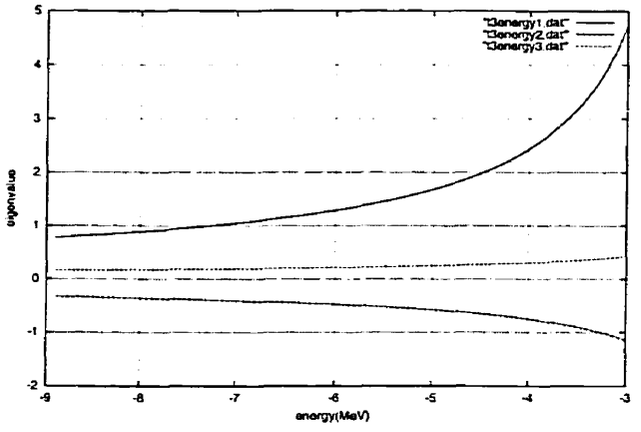

Figure 6.3: The energy dependence of the eigenvalues for the Triton GEV with the Malfliet-Tjon potential

is a difference in the shape of the corresponding solutions of the BS quasiparticle problem, in the inter-cluster momentum q (in shorthand, I call this the q-eigenfunction) as can be seen in fig.[6.4]. While the Yukawa type potential produces a wave function that is sign definite, the Malfliet-Tjon type potential produces two knots in the tail and a negative contribution at intermediate momenta. This behaviour can be attributed to the repulsive contribution from the Malfliet-Tjon type potential and is expected. This result is the benchmark for my investigation of the correction terms in the Padova model, which is done in the next chapter.

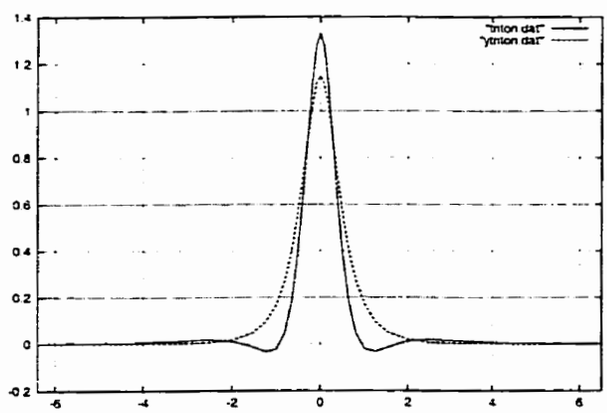

Figure 6.4: The q-eigenfunctions of the bound state problem for the Yukawa type and Malfliet-Tjon type potentials

Chapter 7

THE PION DYNAMICS CORRECTIONS TO THE STANDARD $3N$-SYSTEM IN THE 1-D TOY MODEL

I am a great believer in luck, and I find the harder I work the more I have of it

Stephen Leacock

7.1 *Off-diagonal correction terms to the effective potential* $Z_{A'B'}$

In the quasiparticle approximation scheme I already argued that the pion dynamics introduces corrections to the effective potential. In this section I show how these correction terms can be incorporated in a first approximation. The explicit form of the off-diagonal correction terms is given by

$$Z^{FM}_{A'B'} = \langle \bar{a}'\bar{\pi}|\, g_0 f^{\dagger}_{\beta} G_0 \,|\gamma\rangle\, \tau_{\gamma}\, \langle\gamma|\, G_0 f_{\alpha} g_0 \,\big|\bar{b}'\bar{\pi}\big\rangle \tag{7.1}$$

This type of correction terms is equivalent to the type of diagrams used in 3NF's based on the Fujita-Miyazawa model. It is not a trivial task to actually find an explicit algebraic expression for these terms that are suitable for numerical analysis. The reasons for these difficulties are that both a three and a four body system have to be considered. Furthermore, the πN t-matrix and the incoming and outgoing form factors are given in different partitions.

These diagrams include a part that is explicitly given by modern 3NF. Instead of developing a dynamical description of the pion dynamics, it is possible to replace a

part of the correction term by a static 3NF term. In order to do this, the contact term of the Tucson-Melbourne 3NF adjusted for the one-dimensional model is used. I follow the form of the TM 3NF given by Friar, Hueber and van Kolck [FHv99], but since the $1D$ Toy model is based on a one-dimensional coupling to spinless nucleons, the overall constants are slightly different. In the same set of variables as the ones used in the standard AGS calculations, the TM 3NF type potential is given by

$$\hat{V}_{TM}^{(3)} = \frac{f^2}{(\pi)} \frac{F_{\pi NN}^2(Q^2)\, F_{\pi NN}^2\left((Q')^2\right)}{\left(Q^2 + \frac{m_\pi^2 c^2}{\hbar^2}\right)\left((Q')^2 + \frac{m_\pi^2 c^2}{\hbar^2}\right)} a_1 \tag{7.2}$$

where Q is the pion momentum in the center of mass system. Furthermore,the vertex function $F_{\pi NN}^2(Q^2)$ is assumed to be equal to one, which gives

$$\hat{V}_{TM}^{(3)} = \frac{f^2}{(\pi)} \frac{a_1}{\left(Q^2 + \frac{m_\pi^2 c^2}{\hbar^2}\right)\left((Q')^2 + \frac{m_\pi^2 c^2}{\hbar^2}\right)} \tag{7.3}$$

The strength constant a_1 has units of fm^{-1} in order to get the right units for the off-diagonal correction terms.

Even though only the contact term of the TM 3NF is investigated in this thesis the results should help to understand the pion dynamics better. The correction term for the effective potential itself can now be given in respect to this 3NF term in the form

$$Z_{TM} = \int \frac{\chi(p_a)}{E - \frac{(\hbar q_a)^2}{2M_a} - \frac{(\hbar p_a)^2}{2\mu_a}} \hat{V}_{TM}^{(3)}(Q, Q') \frac{\chi(p_b')}{E - \frac{(\hbar q_b')^2}{2M_b} - \frac{(\hbar p_b')^2}{2\mu_b}} dp_a dp_b' \tag{7.4}$$

In order to solve the integrals the pion momenta are expressed in respect to the Jacobi-momenta of the incoming and outgoing three-nucleon system. The pion momenta are given in respect to the Jacobi-momenta that couple to the nucleon on which the pion is created last. Due to the odd man out notation, this set of Jacobi-momenta is simply defined by the index of the coupling constant describing the creation and destruction of the pion on the particular nucleon. In this case, the pion momentum Q is defined in respect to the Jacobi-momenta q_b, q_b', which define the Jacobi-momenta

on the left and right of the vertex. Since momentum conservation on that vertex has to be satisfied the pion momentum is simply

$$Q = q_b - q_b' \tag{7.5}$$

Similarly, the second pion line is absorbed on nucleon a and the momentum is defined by the expression

$$Q' = q_a - q_a' \tag{7.6}$$

However, the pion momenta have to be given in respect to the Jacobi-momenta of the in-going and out-going states, which are given by the sets p_a, q_a and p_b', q_b'. In other words, it is necessary to find appropriate transformations in order to express the variables q_a' and q_b in respect to a suitable set of Jacobi-momenta. Finding the transformations is straightforward and is shown in appendix A, they are given by

$$q_b = p_a - \frac{1}{2}q_a \tag{7.7}$$

$$q_a' = -p_b' - \frac{1}{2}q_b' \tag{7.8}$$

The pion momenta in respect to the same set of Jacobi variables that is used in the integration are therefore given by

$$Q = p_a - \frac{1}{2}q_a - q_b' \tag{7.9}$$

$$Q' = q_a + p_b' + \frac{1}{2}q_b' \tag{7.10}$$

Consequently, the TM 3NF contribution to the off diagonal correction term of the effective potential is described by

$$V_{TM} = \frac{a_1 f^2}{(\pi)} \int \frac{\chi(p_a)}{E - \frac{(\hbar q_a)^2}{2M_a} - \frac{(\hbar p_a)^2}{2\mu_a}} \frac{1}{\left(\left(p_a - \frac{1}{2}q_a - q_b'\right)^2 + \frac{m_\pi^2 c^2}{\hbar^2}\right)} \frac{1}{\left(\left(q_a + p_b' + \frac{1}{2}q_b'\right)^2 + \frac{m_\pi^2 c^2}{\hbar^2}\right)} \frac{\chi(p_b')}{E - \frac{\left(\hbar q_b'\right)^2}{2M_b} - \frac{\left(\hbar p_b'\right)^2}{2\mu_b}} dp_a dp_b' \tag{7.11}$$

It is clear that the expression depends on the same two variables as the off-diagonal terms in the standard AGS effective potential, namely q_a, q'_b. Therefore, the integration can be perfomed numerically and the constant a_1 can be adjusted. However, the potential is given in the same type of variables as used in the standard $3N$ AGS calculation and consequently a coordinate transformation to the same coordinates u, u' has to be performed. The normalized form factors in dimensionless variables are given by the expression

$$\chi(p_a) = \frac{m_\pi c}{\sqrt{2\mu a}} \hat{\chi}(\tilde{u}_a) \tag{7.12}$$

and the TM 3NF in dimensionless variables is described by

$$Z_{TM} = \frac{a_1 f^2}{(\pi)} \left(\frac{m_\pi}{\sqrt{2\mu a}}\right)^2 \frac{1}{a^4} \left(\frac{2\mu}{m_\pi^2}\right)^2 \int \hat{Z}^K_{TM}(\tilde{u}_a, u_a, \tilde{u}'_b, u'_b)\, a^2 d\tilde{u}_a d\tilde{u}'_b \tag{7.13}$$

where

$$\begin{aligned} \hat{Z}^K_{TM}(\tilde{u}_a, u_a, \tilde{u}'_b, u'_b) &= \frac{\chi(\tilde{u}_a)}{\frac{2\mu}{m_\pi^2}E - \frac{\mu}{M}(u_a)^2 - (\tilde{u}_a)^2} \frac{1}{\left(\left(\tilde{u}_a - \frac{1}{2}u_a - u'_b\right)^2 + 1\right)} \\ &\times \frac{1}{\left(\left(u_a + \tilde{u}'_b + \frac{1}{2}u'_b\right)^2 + 1\right)} \frac{\chi(\tilde{u}'_b)}{\frac{2\mu}{m_\pi^2}E - \frac{\mu}{M}(u'_b)^2 - (\tilde{u}'_b)^2} \end{aligned} \tag{7.14}$$

Collecting the variables in the same way as in the standard AGS yields the following expression for the dimensionless off-diagonal correction terms

$$Z_{TM} = \left(\frac{m_\pi}{\sqrt{2\mu a}}\right)^2 \frac{2\mu}{m_\pi^2} \hat{Z}_{TM} = \frac{1}{a} \hat{Z}_{TM} \tag{7.15}$$

where

$$\hat{Z}_{TM} = \frac{a_1 f^2}{(\pi)} \frac{2\mu}{a^2 m_\pi^2} \int \hat{Z}_{TM}(\tilde{u}_a, u_a, \tilde{u}'_b, u'_b)\, d\tilde{u}_a d\tilde{u}'_b \tag{7.16}$$

This allows for the calculation of the off-diagonal correction terms which are then included in the Triton calculation. However, the explicit expression for the constant a_1 is still not defined. In order to define this parameter in a consistent way a comparison with the previously defined $1D$ scattering equations is made. The transmission coefficients are now playing the same role as the scattering amplitudes, which are used in

the original $3D$ Tucson-Melbourne definition. However, the transmission coefficient in respect to the t-matrix is given in the following way

$$s_{12}(k) = -\frac{2\pi i\mu}{\hbar^2 k} t(p, p', E) \tag{7.17}$$

The definition for the a_1 is now using the scattering threshold expression for the NN t-matrix and assuming that it is hundred times larger than the corresponding πN t-matrix in that range. This is a reasonable assumption, because at the scattering threshold in the $3D$ case only s-wave scattering is allowed and the two scattering amplitudes are related to each other by a factor of roughly 100, see Ericson and Weise [EW88]. The coefficient a_1 is therefore defined by the expression

$$a_1 = -\frac{2\pi\mu}{\hbar^2} t(p = 0, p' = 0, E = 0) \tag{7.18}$$

which is dimensionally exactly the required form. This definition is not the same definition as in the $3D$ TM-definitions, because the $1D$ case is fundamentally different from the $3D$ case. However, the terms can be interpreted as the equivalent of the TM-force terms in the one-dimensional model. I show the explicit effect of these terms in detail in section 7.3 together with the diagonal correction terms.

7.2 Diagonal correction terms to the effective potential $Z_{A'A'}$

In addition to the off-diagonal correction terms I also have a set of diagonal correction terms. These terms do not appear in the Tucson-Melbourne 3NF, because they are assumed to be taken care of completely by the underlying AGS-model. In order to avoid overcounting I have to subtract the separable potential describing boson exchange type diagrams from the corresponding NN t-matrix before calculating the correction terms. This procedure is described in a recent paper by Canton and Schadow [CS00b], which I follow in the definition of the diagonal correction terms. I have argued in a previous section that the diagonal correction terms are given by the

expression

$$Z^D_{A'A'} = \langle \bar{a}'\bar{\pi}|\, g_0 f^\dagger_\beta + f^\dagger_\gamma G_0 \,|a\rangle\, \tau_a \,\langle a|\, G_0 f_\alpha g_0 \,|\bar{a}'\bar{\pi}\rangle + \langle \bar{a}'\bar{\pi}|\, g_0 f^\dagger_\alpha G_0 \,|a\rangle\, \tau_a \,\langle a|\, G_0 f_\beta + f_\gamma g_0 \,|\bar{a}'\bar{\pi}\rangle \tag{7.19}$$

This expression is similar to the off-diagonal one in respect to the fact that it is possible to introduce a static term replacing part of the expression. It is possible to rewrite this expression in the following form

$$Z^D_{A'A'} = \langle \bar{a}'\bar{\pi}|\, g_0 V^{3N}_{\beta\alpha} g_0 \,|\bar{a}'\bar{\pi}\rangle + \langle \bar{a}'\bar{\pi}|\, g_0 V^{3N}_{\gamma\alpha} g_0 \,|\bar{a}'\bar{\pi}\rangle + \langle \bar{a}'\bar{\pi}|\, g_0 V^{3N}_{\alpha\beta} g_0 \,|\bar{a}'\bar{\pi}\rangle + \langle \bar{a}'\bar{\pi}|\, g_0 V^{3N}_{\alpha\gamma} g_0 \,|\bar{a}'\bar{\pi}\rangle \tag{7.20}$$

where

$$V^{3N}_{\beta\alpha} = f^\dagger_\beta G_0 \,|a\rangle\, \tau_a \,\langle a|\, G_0 f_\alpha \tag{7.21}$$

The integral representation in the appropriate Jacobi coordinates is given by

$$Z^D_{A'A'} = \int \frac{\chi(p_a)}{E - \frac{(\hbar q_a)^2}{2M} - \frac{(\hbar p_a)^2}{2\mu}} \left(V^{3N}_{\beta\alpha} + V^{3N}_{\gamma\alpha} + V^{3N}_{\alpha\beta} + V^{3N}_{\alpha\gamma}\right) \times \frac{\chi(p'_a)}{E - \frac{(\hbar q'_a)^2}{2M} - \frac{(\hbar p'_a)^2}{2\mu}} dp_a dp'_a \tag{7.22}$$

The terms $V^{3N}_{\alpha\beta}$ can again be given in a similar way as the corresponding off-diagonal terms, namely the product of two vertices, the meson propagators in the static approximation and what happens in between. However, this time the dynamical term contributing to the diagonal correction is given by

$$V^{3N}_{\beta\alpha}(Q) = \sqrt{\frac{\hbar c}{\pi\omega}} f^\dagger G^{(4)}_0(p,q)\, \tilde{t}_a \left(p_a, p'_a, E - \frac{q_a^2}{2M} - \hat{\omega}_\pi\right) G^{(4)}_0(p',q) \sqrt{\frac{\hbar c}{\pi\omega}} f \tag{7.23}$$

where

$$\omega = \sqrt{Q^2 + \frac{m_\pi^2 c^2}{\hbar^2}} \tag{7.24}$$

and where the relativistic pion energy is given by the expression

$$\hat{\omega}_\pi = \sqrt{(\hbar c Q)^2 + m_\pi^2 c^4} \tag{7.25}$$

The square roots come from the normalization of the pion fields as observed in a second quantization procedure, see for example in Mandl and Shaw [MS84]. Furthermore, since the pion in flight does not interact with the nucleons, only one pion momentum is relevant, namely

$$Q = q_a - q'_a \tag{7.26}$$

The two Green's functions depend on the same variable q, which is due to the fact that while the pion is in flight the relative momentum can change from p to p', but the total momentum stays conserved. Therefore, since only the total momentum of the two nucleons involved in the scattering contributes to the definition of q, the momentum q stays the same until the pion is absorbed again. Also, in this definition all nucleon recoil terms are ignored. Furthermore, the definitions of the renormalized vertex functions and the static approximation of the Green's function in the πNN case recover the original expression of the OPEP potential. The four-body Green's functions are generally given by the expression

$$G_0^{(4)}(p,q) = \frac{1}{E - \frac{p^2}{2\mu} - \frac{q^2}{2M} - \hat{\omega}_\pi} \tag{7.27}$$

However, the static approximation

$$E \approx \frac{p^2}{2\mu} - \frac{q^2}{2M} \approx \frac{(p')^2}{2\mu} - \frac{q^2}{2M} \tag{7.28}$$

gives for both Greens functions the simpler expression

$$G_0^{(4)}(p,q) = -\frac{1}{\hat{\omega}_\pi} \tag{7.29}$$

Since I used the static approximation, the terms now define the static contribution to the diagonal correction terms, which are given by

$$V_{\beta\alpha}^{3N} = \frac{f^2}{\pi}\frac{1}{\hbar c \omega_\pi^3}\tilde{t}_a\left(p_a, p'_a, E - \frac{q_a^2}{2M} - \hbar c\omega_\pi\right) \tag{7.30}$$

where the $\hbar c$ factor is incorporated into the pion energy of the Green's functions

$$\omega_\pi = \sqrt{Q^2 + \frac{m_\pi^2 c^2}{\hbar^2}} \tag{7.31}$$

Also, I set $\omega = \omega_\pi$, because the renormalization comes from the same pion field as the pion field observed in the propagation. The pion momentum in this case is simply given by

$$Q = q_a - q'_a \tag{7.32}$$

because it describes the momentum transferred in the process. The transition amplitude has be adjusted to ensure that no terms are introduced that are already taken care of by the standard AGS theory. Therefore, I exclude all the terms that describe a one-pion exchange and subtract explicitly the separable version of the attractive potential. This yields for the subtracted t-matrix the expression

$$\tilde{t}_a\left(p_a, p'_a, E - \frac{q_a^2}{2M} - \hbar c\omega_\pi\right) = t_a\left(p_a, p'_a, E - \frac{q_a^2}{2M} - \hbar c\omega_\pi\right) - v_a^{attr}\left(p_a, p'_a\right) \tag{7.33}$$

which in dimensionless variables is given by

$$\tilde{t}_a\left(p_a, p'_a, E - \frac{q_a^2}{2M} - \hbar c\omega_\pi\right) = \tag{7.34}$$
$$\frac{(m_\pi c)^2}{2\mu a}\left(\hat{\chi}\left(\tilde{u}_a\right)\tau_a\left(\frac{2\mu}{(m_\pi c)^2}E - \frac{\mu_a}{M}\left(u_a\right)^2 - \frac{2\mu}{m_\pi c}\tilde{\omega}_\pi\right)\hat{\chi}\left(\tilde{u}'_a\right) - \hat{\chi}^{attr}\left(\tilde{u}_a\right)\hat{\chi}^{attr}\left(\tilde{u}'_a\right)\right)$$

The final expression for this part of the diagonal static correction in dimensionless variables is now

$$V_{\beta\alpha}^{3N} = \frac{f^2}{\pi}\frac{(\hbar c)^3}{(m_\pi c)^2}\frac{1}{2\mu}\frac{\hat{t}_a\left(\tilde{u}_a, \tilde{u}'_a, \frac{2\mu}{(m_\pi c)^2}E - \frac{\mu_a}{M}\left(\tilde{u}_a\right)^2 - \frac{2\mu}{m_\pi c}\tilde{\omega}_\pi\right)}{\tilde{\omega}_\pi^3} \tag{7.35}$$

where the 'hat' means the dimensionless t-matrix. The other three terms have to be calculated the same way which yields the following expression for the static contribution to the diagonal correction terms

$$V^{3N} = \frac{f^2}{\pi}\frac{(\hbar c)^3}{(m_\pi c)^2}\frac{1}{\mu}\left(\frac{\hat{t}_a\left(\tilde{u}_a, \tilde{u}'_a, \frac{2\mu_a}{(m_\pi c)^2}E - \frac{\mu_a}{M}\left(u_a\right)^2 - \frac{2\mu_a}{m_\pi c}\tilde{\omega}_\pi\right)}{\tilde{\omega}_\pi^3}\right. \tag{7.36}$$
$$\left. + \frac{\hat{t}_a\left(\tilde{u}_a, \tilde{u}'_a, \frac{2\mu_a}{(m_\pi c)^2}E - \frac{\mu_a}{M}\left(u'_a\right)^2 - \frac{2\mu_a}{m_\pi c}\tilde{\omega}_\pi\right)}{\tilde{\omega}_\pi^3}\right)$$

The correction terms are again given by the integrated form, which is in dimensionless variables given by

$$Z^{D}_{A'A'} = \frac{1}{a}\frac{f^2}{\pi}\frac{2\hbar c}{(m_\pi c)^2}\int \frac{\hat{\chi}(\tilde{u}_a)}{\frac{2\mu}{m^2_\pi c^2}E - \frac{\mu}{M}(u_a)^2 - (\tilde{u}_a)^2}\hat{V}^{3N} \times \frac{\hat{\chi}(\tilde{u}'_a)}{\frac{2\mu}{m^2_\pi c^2}E - \frac{\mu}{M}(u'_a)^2 - (\tilde{u}'_a)^2}d\tilde{u}_a d\tilde{u}'_a \tag{7.37}$$

I am now in a position to calculate the triton binding energy in the one-dimensional system with the correction terms of Tucson-Melbourne type and the "new" type of diagonal correction terms.

7.3 Effect of the correction terms on the triton

In the last two sections I developed the correction terms for the effective potential and described them in a form suitable for calculations. The code for the calculations is given in the appendix B and in this section I describe the detailed results of these calculations. I expect the Tucson-Melbourne type correction terms to give the dominant contribution to the corrections, because the original Tucson-Melbourne 3NF showed some success. The modern 3NF's are usually introduced with some parameters that have to be adjusted and in the given model the adjustable parameter is obviously the strength parameter between the NN and πN threshold scattering t-matrix. According to Ericson and Weise [EW88][p.16] the two scattering lengths relate to each other by a factor of one hundred and I used this to gauge my corrections terms. The resulting binding energy for the triton with this strength factor is $-8.17 MeV$ compared to the previous value of $-7.28 MeV$ without any correction. However, the strength factor is an adjustable parameter and it is important to investigate the effect of varying this parameter. In fig.[7.3] it is evident that the triton binding energy calculated with the TM 3NF depends rather strongly on the particular choice of the strength parameter. I do see the expected asymptotic behaviour for small values of the strength parameters, but the convergence to the binding energy without any TM 3NF corrections is slow.

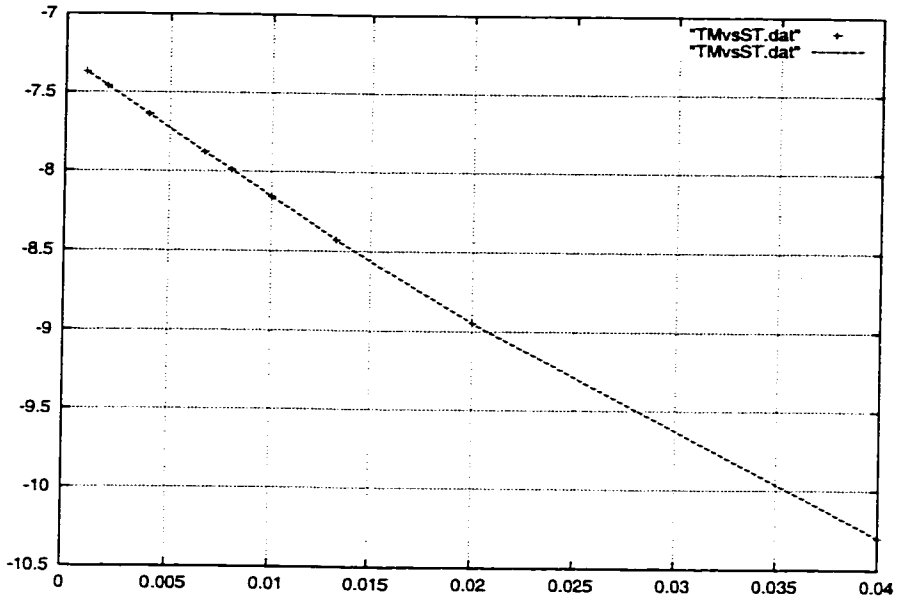

Figure 7.1: Dependence of Triton binding energy on the value of the TM 3NF strength parameter.

This means that this effect is certainly non-negligible. However, it is also clear that the effect of these terms is rather large for larger values of the strength factors. This means that if the πN scattering length is too close to the NN scattering length at threshold, the effect on the binding energy would be comparable to the corresponding effect due to the $2N$ force. In that case nuclear physics as it is known today would not exist. This gives a physically reasonable range for the strength parameter, or at least an upper limit.

The next step is to include the diagonal correction terms into the calculation. Unlike the off-diagonal correction terms, the diagonal ones do not have an adjustable parameter. I calculated the triton binding energy again, this time inlcuding the TM 3NF with the parameter adjusted by 0.01 and including the diagonal correction terms. I observe another shift in the triton binding energy, which now is equal to $-8.49MeV$. However, the fact that the triton binding energy is so close to the actual experimental value of $-8.48MeV$ should not be given to much weight. After all, the model only

describes a spinless, one-dimensional and symmetrized system. However, it turns out that compared to the correction of the one-dimensional TM 3NF adjusted by the parameter value 0.01, the additional shift in the triton binding energy due to the diagonal term is about thirty percent of the one due to the TM 3NF alone. This means that, if the diagonal terms would be ignored and the TM 3NF adjusted to recover the triton binding energy, the TM 3NF would be thirty percent too strong.

For completeness, I show in Figures [7.2,7.3] what effect the correction terms have on the q-eigenfunctions. I observe that the correction terms essentially "flatten"

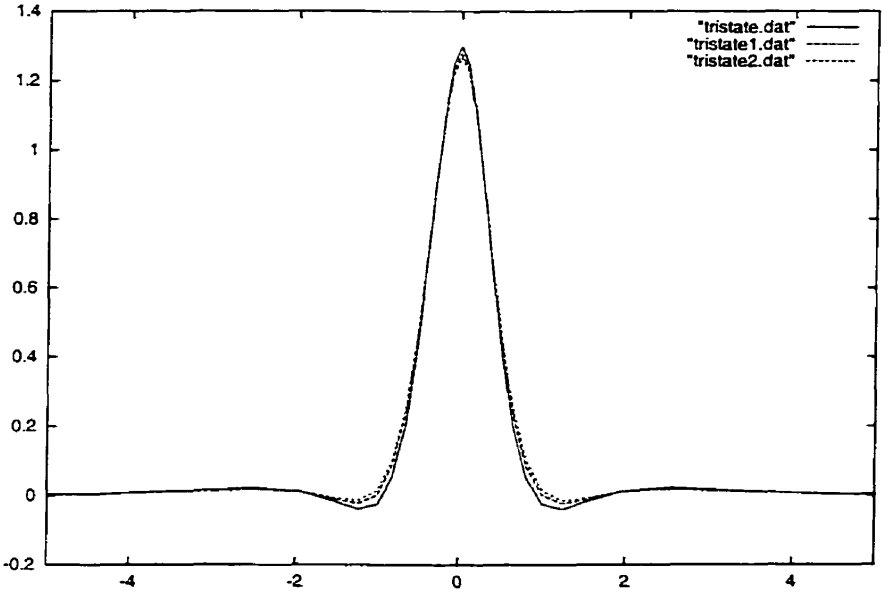

Figure 7.2: The q-eigenfunctions at an energy of $-8.48MeV$. The three graphs describe the eigenfunctions for the uncorrected version, the version with the TM 3NF correction and the version with both corrections.

the normalized q-eigenfunctions. I should note that the three q-eigenfunctions are calculated at the same energy, namely $-8.49MeV$.

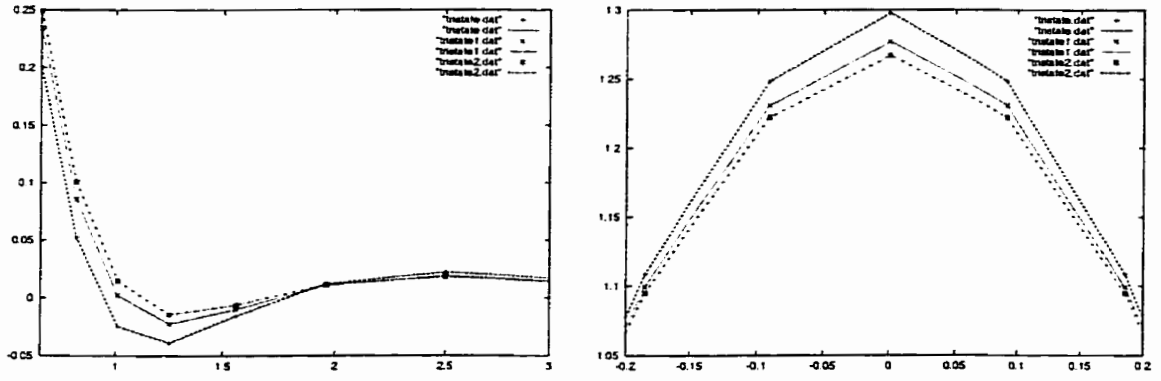

Figure 7.3: Behaviour of the q-eigenfunctions of fig.[7.2] in the regions of the first minimum and the central peak.

Chapter 8

SUMMARY AND CONCLUSION

I don't know why I did it, I don't know why I enjoyed it,
and I don't know why I'll do it again

Bart Simpson

8.1 The Padova model

In this thesis I demonstrated that it is possible to include one pion degree of freedom into a standard $3N$ system. The Padova model, describing the πNNN system, was shown in section 2. I argued that it is possible to find a physically sound coupling between the pion and no-pion sectors and consequently to define a AGS type system describing the πNNN system. Canton [Can98] already showed in the original paper that the resulting theory is connected and I did not repeat these arguments in this thesis. Furthermore, I want to stress that the Padova model is not plagued by the same problems observed in the earlier TRABAM-type πNN models. However, the channel equations of the Padova model on the two-cluster sub-level are rather complicated. They are given by a coupled matrix equation and it is not an easy task to actually find a solution to these equations.

I also showed that it is possible to introduce approximations that are given on physical grounds in order to simplify the channel equations. The quasiparticle approximation leads to a Lovelace type structure for the Padova equations. Further-

more, I argued that it is reasonable to ignore some terms in the channel equations on physical grounds. This allowed me to find directly a separable solution to the two-cluster channel equations of the Padova model. Freezing out the pionic channels in the Lovelace-type Padova equations using the Feshbach procedure resulted in a new system describing the $3N$ problem. The system differed from the standard ones in the fact that the explicit pion degrees of freedom show up in correction terms to the standard effective potential of the two-cluster Lippmann-Schwinger type equation. One type of correction term, which allowed for the propagation of a pion in the presence of a three-cluster interaction was not investigated in this thesis. It should be checked in future research efforts if this type of correction terms produces a non-negligible effect.

8.2 Three-nucleon forces

The other two types of correction terms were investigated in more detail in this thesis. I demonstrated that one type could be interpreted as the equivalent to correction terms described by modern three-nucleon forces. In particular, I argued that part of the diagram describing this type of correction terms could be interpreted directly as the equivalent of a static Fujita-Miyazwa force diagram. In this thesis I replaced this part of the diagram by the corresponding static approximation. Nevertheless, I believe it could be of interest to investigate this type of correction terms in a fully dynamic description instead. This type of description would introduce an energy shift in the πN t-matrix due to the spectator nucleons and it could be interesting to see if this energy shift reduces the effect of the correction terms. This investigation also could help to decide if the three-nucleon force effects are actually small as argued by Afnan, Saito and Haidenbauer [SA94, SH00]. However, besides this already thoroughly investigated type of correction term, I also showed that another type should be of interest. In most cases of three-nucleon force descriptions the correction terms

are not based on a Faddeev-Yakubovsky theory. This means the underlying theory is not the same as the model that is used in subsequent calculations. This introduced a new problem because it is important the correction terms do not produce any terms that are already taken care of by the model used in the calculations. This last type of diagrams is commonly ignored because it is assumed that it is already taken care of by the AGS-model. However, I argued that the terms, which incidentally in this case are a result of the underlying model itself, should not be completely cancelled. In the one-dimensional Toy calculation this cancellation was shown to be indeed rather incomplete. I believe the described model could show some interesting results in actual three-dimensional calculations (and indeed already has [CS00b, CS00a]).

8.3 The Padova model applied to a one-dimensional system

I also developed a one-dimensional system that was able to mimic, at least in a first approximation, nuclear physics. The system was used to test the effects of the correction terms due to the pionic degree of freedom in a triton binding energy calculation. Without any correction terms, the one-dimensional system had a triton bound state energy of $-7.27MeV$. This means the system clearly underbinds the triton whose binding energy should be $-8.48MeV$, in common with results obtained in full $3D$-calculations using realistic NN-interactions. The Tucson-Melbourne type correction term in the one-dimensional model introduced a correction in the right direction. However, the magnitude of the effect depends rather strongly on a free parameter used to adjust the relative strength between NN and πN threshold scattering. At the strength parameter usually used for s-wave threshold scattering in three-dimensions the effect of the correction term pushed the triton binding energy to $-8.17MeV$, clearly closer to the experimental value. Nevertheless, the strong dependence on the strength parameter makes it hard to pin-point the exact size of the effect. In the Tucson-Melbourne approach this strong dependence does not really

matter too much, because obtaining a fit to the triton binding energy actually would be used to adjust the strength parameter.

However, I also showed that the second type of correction terms also introduces a non-negligible effect. At least in the symmetrized, one-dimensional, spinless model the effect is rather large, namely around thirty percent of the TM-type effect. The effect itself does not depend on any adjustable parameter and therefore can not be made small enough to vanish by fiddling with the parameter. The additional correction terms pushed the triton binding energy to $-8.49 MeV$, but this value should not be taken too literally. First of all, the underlying Toy-model only describes nuclear physics in a first approximation and second the choice of the strength parameter value has to be seen as a lucky guess. I already mentioned that a slight change in this parameter has a rather significant effect in the binding energy, which consequently would move the total effect in the same direction. However, it can be argued that the relative effect between the diagonal and the TM-type corrections is rather large. Consequently, if this behaviour translates to the three-dimensional case, then the present choice of parameters for the TM-type three-nucleon forces should yield a force of this type that is too strong.

8.4 Outlook

In conclusion, it is possible to introduce one pion degree of freedom into a standard three-nucleon problem. Furthermore, the present use of three-nucleon forces seems to neglect an important type of corrections. I believe the role of the dynamical pion degrees of freedom is not yet completely understood and more work needs to be done in this field. I also hope to have shown that the field of Few Body physics is still alive and well and could contribute to our understanding of the nuclear world for years to come.

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Appendix A

JACOBI-COORDINATES OF THE SYSTEM IN MOMENTUM SPACE

In this section I investigate the description of the algebraic model in respect to the momentum space. This will be achieved by introducing the concept of Jacobi-coordinates and show their explicit realization in the given chains. Every chain will have a natural realization in respect to the Jacobi-coordinates and it will be necessary also to describe the transformations between the sets. It should be clear already that there is an intimate relationship between the new chain-notation and the Jacobi coordinates of the respective chain. This means the new notation not only allows for the description of the algebraic model, but also is extremely helpful in the representation of the model.

The role of the pion is special in the system and this fact has to be taken into account explicitly when defining the Jacobi-coordinates. The Jacobi-coordinates in the three-nucleon space already play a central role in the development of the Jacobi-coordinates of the πNNN system, because the form the basis of the generalization.

A.1 Jacobi-coordinates for the NNN sub-space

In order to be able to find a generalization of the Jacobi-coordinates to the πNNN system, it is crucial that I give a clear description of the $3N$ Jacobi-coordinates first. The problem of finding a suitable set of coordinate transformations is a long standing one and several different definitions were given in the past. Probably the most commonly used system is the one described by Schmid and Zimmermann [SZ74],

Gloeckle [Glo83] and Adhikari and Kowalski [AK91] (SZGAK). However, in the matrix description the transformation matrix between the natural momenta k_i and the Jacobi-coordinates has a determinant of -1. Even though the resulting transformation matrices between the different sets of Jacobi-momenta have determinants of $+1$, I find this a mathematically strange definition.

Another description given by Lovelace [Lov64] which does not have this initial problem, but Lovelace included the reduced masses into the definition of the Jacobi-coordinates. This procedure is not widely and does not give any advantages in the specific problem, which is why I also do not use this type of definition. However, Garcilazo and Mizutani [GM90] (LGM) also gave a definition for the Jacobi-coordinates that has a determinant of $+1$ for the defining matrices. They used the Lovelace approach, but without the inclusion of the reduced masses into the definition of the Jacobi-momenta. This type of Jacobi-coordinates would be my prefered choice, but for reasons of tradition rather than aesthetics I define the Jacobi coordinates in the commonly used way according to SZGAK.

In the three particle space described by the momenta of the three particles k_a, k_b, k_c I now define three Jacobi momenta. The first momentum is given by the relative momentum between two of the particles and I use the odd man out notation. Namely, the Jacobi-momentum p'_a is given by the relative momentum between the two particles b, c. At this point I have to make a choice in the sign of the relative momentum and decide to use a cyclic permutation in the momenta. There are three possible choices for the first Jacobi-momentum and they all can be derived by cyclic permutation of the indices in the defining equation

$$p'_a = -\frac{1}{m_i + m_c}(m_c k_b - m_c k_c) \tag{A.1}$$

It is evident that the three momenta are ordered in a cyclic permutation of a, b, c as required. The prime in the Jacobi-momentum is only important for the generalization of this system to the πNNN case. If I do not intend to make this generalization, the

prime could be omitted altogether. Up to this point I follow the usual convention used in all definition of Jacobi-momenta described earlier (except Lovelace). However, the SZGAK choice differs from the LGM choice in the next step when defining the second Jacobi momentum. I need to find the relative momentum between the center of mass motion of the first two particles used in the first Jacobi momentum and the third particle. Again, there is a choice of sign and I use the SZGAK choice, which always subtracts the center of mass of the first two particles from the third one, namely

$$q'_a = \frac{1}{m_a + m_b + m_c} \left((m_b + m_c)\, k_a - m_a\, (k_b + k_c) \right) \tag{A.2}$$

The choice of this sign gives a determinant -1 for the transformation matrix and the remaining two Jacobi-momenta of the second type can be given by cyclic permutation of the indices. The last Jacobi-momentum is given by the total momentum of the system and simply yields

$$P = k_a + k_b + k_c \tag{A.3}$$

which is the same for all three sets of Jacobi-momenta. This defines the system of Jacobi-momenta in the $3N$ space. However, it is possible to give a matrix notation for this transformation, which allows for an easy computation of more complicated transformations. The transformations in matrix form are given by the general expression

$$\vec{P}_{\bar{a}} = T_{\bar{a}} \vec{k} \tag{A.4}$$

with

$$T_{\bar{a}} = \begin{bmatrix} 0 & \frac{m_c}{m_b+m_c} & -\frac{m_b}{m_b+m_c} \\ \frac{m_b+m_c}{m_b+m_a+m_c} & -\frac{m_a}{m_b+m_a+m_c} & -\frac{m_a}{m_b+m_a+m_c} \\ 1 & 1 & 1 \end{bmatrix} \tag{A.5}$$

The index $\bar{a}$ is already in preparation for the generalization to the πNNN system. It should be evident that this transformation matrix again has a determinant of -1 and the transformation matrices for the other two sets of Jacobi-momenta follow the

same procedure. What I gained with this matrix notation is an easy way to find the transformation between two different sets of Jacobi-momenta. I simply have to transform one set of Jacobi-momenta back to the original set of particle momenta using the inverse transformation and then transform the set into the required one. In other words I simply have to give the expression

$$\vec{P}_a = T_{\bar{a}} T_{\bar{b}}^{-1} \vec{P}_b \tag{A.6}$$

where I define

$$T_{\bar{a},\bar{b}} = T_{\bar{a}} T_{\bar{b}}^{-1} = \begin{bmatrix} -\frac{m_b}{m_b+m_c} & \frac{m_c(m_b+m_a+m_c)}{(m_b+m_c)(m_a+m_c)} & 0 \\ -1 & -\frac{m_a}{m_a+m_c} & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{A.7}$$

It is clear from this transformation matrix that the total momentum of th system is conserved and all transformation between different Jacobi-momenta can be given in the same way. The determinant of this type of transformations between different sets of Jacobi-momenta is $+1$. Furthermore, this general procedure also applies to the more complicated πNNN system and I only need to find the transformation matrices between the different sets of Jacobi momenta and the original particle momenta.

A.2 Jacobi-coordinates for the πNNN sub-space

The Jacobi-momenta in the πNNN system are more complicated, because of several reasons. The fact that they are defined in a four body system already introduces the complication due to the variety of possible chains. Furthermore, since I have the pion as the fourth particle I also require that the deletion of the pion should recover one of the three-body Jacobi-momenta described in the last section. Another requirement is that I always want to have a determinant of -1 for any of the defining transformation matrices introduced. In this section I define all transformation matrices between the Jacobi-momenta of given chains and the original particle momenta that have the

three-particle system $\bar{a}$ as a subsystem. The transformations that have the systems $\bar{b}, \bar{c}$ as subsystems can be given in an equivalent manner.

The chain of partition systems in the four-body space naturally fall into two categories, chains of the $3+1$ type and chains of the $2+2$ type. First I investigate the chains of the $3+1$ type, which again fall into two categories, the one where the pion is coupled last and the one where it is not. If the pion is coupled last, I only have one chain that has the system $\bar{a}$ as a subsystem, namely the chain $\pi' a$. I define the transformation matrix for the Jacobi-momenta describing this chain in the following way

$$T_{\pi' a} = \begin{bmatrix} 0 & \frac{m_c}{m_b+m_c} & -\frac{m_b}{m_b+m_c} & 0 \\ \frac{m_b+m_c}{m_b+m_a+m_c} & -\frac{m_a}{m_b+m_a+m_c} & -\frac{m_a}{m_b+m_a+m_c} & 0 \\ \frac{m_\pi}{m_a+m_b+m_c+m_\pi} & \frac{m_\pi}{m_a+m_b+m_c+m_\pi} & \frac{m_\pi}{m_a+m_b+m_c+m_\pi} & -\frac{m_b+m_a+m_c}{m_a+m_b+m_c+m_\pi} \\ 1 & 1 & 1 & 1 \end{bmatrix} \tag{A.8}$$

It is straightforward to see that this transformation matrix has determinant -1 and the subsystem $\bar{a}$ as required by the definition. In the case that the pion is not coupled last, I have again two possibilities, it can be coupled first or second. If it is coupled first I have three possible chains with the subsystem $\bar{a}$, namely $a'\alpha, a'beta, a'\gamma$. I only show the transformation of one of these chains explicitly, because the other two can be given accordingly. I define the Jacobi-transformation for the chain $a'\gamma$ by the matrix

$$T_{a'\gamma} = \begin{bmatrix} 0 & 0 & \frac{m_\pi}{m_\pi+m_c} & -\frac{m_c}{m_\pi+m_c} \\ 0 & \frac{m_\pi+m_c}{m_\pi+m_c+m_b} & -\frac{m_b}{m_\pi+m_c+m_b} & -\frac{m_b}{m_\pi+m_c+m_b} \\ \frac{m_\pi+m_c+m_b}{m_a+m_b+m_c+m_\pi} & -\frac{m_a}{m_a+m_b+m_c+m_\pi} & -\frac{m_a}{m_a+m_b+m_c+m_\pi} & -\frac{m_a}{m_a+m_b+m_c+m_\pi} \\ 1 & 1 & 1 & 1 \end{bmatrix} \tag{A.9}$$

Again, this matrix has determinant -1 and the reduces to the tranformation matrix of the subsystem $\bar{a}$, if I delete the pion from the system. The last possibility of chains

in the $3+1$ is when the pion is the coupled in the second step, which is given by the chain $a'a$. I define the transformation matrix for this chain by the the expression

$$T_{a'a} = \begin{bmatrix} 0 & \frac{m_c}{m_b+m_c} & -\frac{m_b}{m_b+m_c} & 0 \\ 0 & -\frac{m_\pi}{m_\pi+m_c+m_b} & -\frac{m_\pi}{m_\pi+m_c+m_b} & \frac{m_b+m_c}{m_\pi+m_c+m_b} \\ \frac{m_\pi+m_c+m_b}{m_a+m_b+m_c+m_\pi} & -\frac{m_a}{m_a+m_b+m_c+m_\pi} & -\frac{m_a}{m_a+m_b+m_c+m_\pi} & -\frac{m_a}{m_a+m_b+m_c+m_\pi} \\ 1 & 1 & 1 & 1 \end{bmatrix} \tag{A.10}$$

The sign for the Jacobi momentum that couples the pion follows directly from the determinant, which is again -1. The only two chains of the $2+2$ type that have $\tilde{a}$ as a subsystem are $\alpha'a, \alpha'\alpha$ and they are defined in following way

$$T_{\alpha'a} = \begin{bmatrix} 0 & \frac{m_c}{m_b+m_c} & -\frac{m_b}{m_b+m_c} & 0 \\ -\frac{m_\pi}{m_\pi+m_a} & 0 & 0 & \frac{m_a}{m_\pi+m_a} \\ \frac{m_b+m_c}{m_a+m_b+m_c+m_\pi} & -\frac{m_\pi+m_a}{m_a+m_b+m_c+m_\pi} & -\frac{m_\pi+m_a}{m_a+m_b+m_c+m_\pi} & \frac{m_b+m_c}{m_a+m_b+m_c+m_\pi} \\ 1 & 1 & 1 & 1 \end{bmatrix} \tag{A.11}$$

and

$$T_{\alpha'\alpha} = \begin{bmatrix} \frac{m_\pi}{m_\pi+m_a} & 0 & 0 & -\frac{m_a}{m_\pi+m_a} \\ 0 & \frac{m_c}{m_b+m_c} & -\frac{m_b}{m_b+m_c} & 0 \\ \frac{m_b+m_c}{m_a+m_b+m_c+m_\pi} & -\frac{m_\pi+m_a}{m_a+m_b+m_c+m_\pi} & -\frac{m_\pi+m_a}{m_a+m_b+m_c+m_\pi} & \frac{m_b+m_c}{m_a+m_b+m_c+m_\pi} \\ 1 & 1 & 1 & 1 \end{bmatrix} \tag{A.12}$$

With this set of definitions it is easy to find transformations between different sets of Jacobi-momenta the same way as in the $3N$ case.

A.3 Examples of transformations between different Jacobi-coordinates

In this section I simply show some of the Jacobi-transformations. I start with the transformations in the three-nucleon system first and continue with the ones in the

πNNN system. In the transformations it is already assumed that the three nucleons have the same mass, namely $m_a = m_b = m_c = m$. The transformations used in the $3N$ system are

$$T_{\bar{a},\bar{b}} = T_{\bar{a}} T_{\bar{b}}^{-1} = \begin{bmatrix} -1/2 & 3/4 & 0 \\ -1 & -1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{A.13}$$

and

$$T_{\bar{b},\bar{a}} = T_{\bar{b}} T_{\bar{a}}^{-1} = \begin{bmatrix} -1/2 & -3/4 & 0 \\ 1 & -1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{A.14}$$

The transformations in the $3N\pi$ are of the following type

$$T_{\pi' b,a'\gamma} = T_{\pi' b} T_{a'\gamma}^{-1} = \begin{bmatrix} 1/2 & -1/2\,\frac{m}{m_\pi+m} & -1/2\,\frac{3m+m_\pi}{m_\pi+2m} & 0 \\ -1/3 & 1/3\,\frac{2m_\pi+3m}{m_\pi+m} & -1/3\,\frac{3m+m_\pi}{m_\pi+2m} & 0 \\ 1 & \frac{m_\pi}{m_\pi+m} & \frac{m_\pi}{m_\pi+2m} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{A.15}$$

$$T_{\pi' a,b'\gamma} = T_{\pi' a} T_{b'\gamma}^{-1} = \begin{bmatrix} -1/2 & -1/2\,\frac{m}{m_\pi+m} & 1/2\,\frac{3m+m_\pi}{m_\pi+2m} & 0 \\ -1/3 & -1/3\,\frac{2m_\pi+3m}{m_\pi+m} & -1/3\,\frac{3m+m_\pi}{m_\pi+2m} & 0 \\ 1 & -\frac{m_\pi}{m_\pi+m} & \frac{m_\pi}{m_\pi+2m} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{A.16}$$

$$T_{a'\gamma,b'\gamma} = T_{a'\gamma} T_{a'\gamma}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -\frac{m}{m_\pi+2m} & \frac{m_\pi{}^2+4\,m_\pi m+3\,m^2}{(m_\pi+2\,m)^2} & 0 \\ 0 & -1 & -\frac{m}{m_\pi+2m} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{A.17}$$

These few examples should show how easy it is to find any transformation between different sets of Jacobi-momenta with this procedure. It is even easier, if Maple V is utilized, because besides calculating all the matrices Maple V also allows the output to be of either Latex or even FORTRAN readable format. In the fully dynamical treatment of the correction terms this should be an invaluable simplification.

Appendix B

FORTRAN CODES

B.1 Fortran code for the kernel of the $2N$ integral equation

```
C     SUBROUTINE TO CALCULATE THE ARGUMENTS OF THE GENRALIZED
C     EIGENVALUE PROBLEM DEFINING THE STURMIAN SYSTEM
C     THE OUTPUT IS THE KERNEL AND THE POTENTIAL WITH THEIR NATURAL SIGNS.
C     IN OTHER WORDS THE ATTRACTIVE PARTS HAVE A NEGATIVE SIGN ATTACHED
C     FINAL VERSION AS OF DEC 22/2000

      SUBROUTINE COLONEL(MC,HBAR,MUC,QX,QW,LDA,N,E,REP,RN,ARG,RARG)

      IMPLICIT REAL*8(A-H,O-Z)

      REAL*8 QX(LDA),QW(LDA),E,REP,RN,R,X,VA,VR,VAP,VRP
      REAL*8 ARG(LDA,LDA),RARG(LDA,LDA),MC,MUC,HBAR
      REAL*8 RANGE,GCONS,SUM,POT,GF
C      PARAMETER(MC=134.9766D0)
C      PARAMETER(HBAR=197.32D0)
C      PARAMETER(MUC=938.271998D0/2.000000D0)
      POT(R,X)=1.000000D0/(R+X**2)
      GF(E,X)=1.000000D0/(E-X**2)
      GCONS=2.00000D0*MUC/(MC**2)
C      write(6,*)GCONS,'Gcons',MC,MUC
      RANGE=(HBAR/(RN*MC))**2
c      write(6,*)'Range',RANGE

      DO 10 I=1,N
         DO 20 J=1,N
            SUM=0.0D0
            DO 30 K=1,N
               VA=POT(1.000000D0,QX(I)-QX(K))
               VAP=POT(1.000000D0,QX(K)-QX(J))
               VR=RN*POT(RANGE,QX(I)-QX(K))
               VRP=RN*POT(RANGE,QX(K)-QX(J))
```

```
              SUM=SUM+(-VA+REP*VR)*GF(E*GCONS,QX(K))
     1 *(-VAP+REP*VRP)*QW(K)
C               WRITE(6,*)'SUM', SUM
 30          CONTINUE
             ARG(I,J)=SUM
             RARG(I,J)=-POT(1.000000D0,QX(I)-QX(J))
     1            +REP*RM*POT(RANGE,QX(I)-QX(J))
C            WRITE(6,*)'COL', ARG,'col2',RARG
 20       CONTINUE
 10    CONTINUE
       RETURN
       END
```

B.2 The Code for the triton calculations

This code is still rather rough and is the core code used in all the calculations shown in the thesis. It allows for different choices depending on the desired calculations. The extended code, which is not shown here because it is even more complicated, also includes several test on symmetries of the potentials, expected pole structure and other similar tests.

```
      PROGRAM TRITONSTATE
      IMPLICIT REAL*8(A-H,O-Z)
      INTEGER N,NFIX

      PARAMETER(LDA=50,LDEVEC=LDA)

      REAL*8 CHI,REP,RM,E,E1,PICONS
      REAL*8 QW(LDA),QX(LDA),ARG(LDA,LDA),RARG(LDA,LDA)
      REAL*8 ARGNORM,BETA(LDA),ALPH,BET
      REAL*8 HBAR,MC,MUC, DEUT(LDA)
      REAL*8 FORMFAC(LDA),NORM,SUM1,SUM
      COMPLEX*16 ALPHA(LDA),EVAL(LDA),EVEC(LDEVEC,LDA)
      REAL*8 POTSEP(LDA,LDA),TSEP(LDA,LDA),FORMATTR(LDA)
      REAL*8 QWI(LDA),QXI(LDA),ACONS,QUAD
      REAL*8 XVEC(LDA),VALUE(LDA),XVEC1(LDA),MVALUE(LDA,LDA)
      REAL*8 MXVEC(LDA,LDA),MXVEC1(LDA,LDA),MVALUE1(LDA,LDA)
      REAL*8 VALUE1(LDA),ZEFF(LDA,LDA),ZNOM(LDA,LDA),QUAD1(LDA)
```

```
      COMPLEX*16 ZALPHA(LDA),ZEVAL(LDA),ZEVEC(LDEVEC,LDA)
      REAL*8 ZETA(LDA),ZARG(LDA,LDA),TAUMOM(LDA,LDA),TRISTATE(LDA)
      REAL*8 DEUTDIM(LDA),FORMFACDIM(LDA),TSEPDIM(LDA,LDA)
      REAL*8 POTSEPDIM(LDA,LDA),ZEFFDIM(LDA,LDA)
      REAL*8 ZK2(LDA,LDA,LDA),ZK3(LDA,LDA,LDA)
      REAL*8 POTSEPY(LDA,LDA),OMEGA(LDA,LDA),OMEGADIM(LDA,LDA)
      REAL*8 QUAD3(LDA,LDA,LDA)
      REAL*8 TAUDIAG(LDA,LDA)
      REAL*8 ZD1(LDA),ZDIAG(LDA,LDA)
      REAL*8 ZOD1(LDA,LDA),ZOD2(LDA,LDA),ZOD(LDA,LDA),ZA
      REAL*8 TRISTATEDIM(LDA),ZDR1(LDA)
      REAL*8 GREEN3(LDA,LDA)

c     OPEN(9,FILE="POTSEPY.DAT",STATUS="NEW")
      OPEN(9,FILE="POTSEPY.DAT",STATUS="OLD")

      PICONS=3.141592654D0
      HBAR=197.3269600D0
      MC=134.9766D0
      MUC=938.271998D0/2.00000D0

      ZA=-8.93909D-02/100.0D0
c      ZA=0.0D0
C     For comparison with Levinger and Harms use these units for the
C     energy of the t-matrix, etc.

c     Harms' units
c      E1=-0.5D0*41.46D0

c     Malfliet-Tjon
c      E1=-7.277D0
c     MT+TM
c      E1=-8.166D0
c     MT+TM+DIAG
      E1=-8.493D0
c     Yukawa
c      E1=-8.001D0
C     Deuteron
c      E1=-2.225D0
c      E1=0.000D0
C     Set the number of Gauss points

      L=41
```

```
      DO 5 N=41,L,2

         QXZ=(N+1)/2

         IWEIGH = 1
         ALPH  = 0.0D0
         BET   = 0.0D0
         NFIX   = 0
C                                  Get points and weights from GQRUL

      CALL DGQRUL (N, IWEIGH, ALPH, BET, NFIX, QXFIX, QXI, QWI)

C     Scale the Gauss points to the real line

      DO 10 I=1,N
         QX(I)=1.2D0*QXI(I)/(1.0D0-QXI(I)**2)
         QW(I)=1.2D0*((1.0D0+QXI(I)**2)/(1.0D0-QXI(I)**2)**2)*QWI(I)
c      WRITE(6,*)QX(I),QW(I)
 10   CONTINUE

C     Define the deuteron binding energy

      E=-2.2250000D0

C     For Yukawa take zero repulsion and chi=2.114450D0
C     For MT take Rep=4.57D0,RN=0.7D0,Chi=10.09496D0

c      REP=0.0D0
      REP=4.57000D0
      RN=0.7000D0

      CHI=10.09496D0
c      CHI=2.1175D0

C     The subroutine to define the matrices of the generalized EV-problem

      CALL COLONEL (MC,HBAR,MUC,QX,QW,LDA,N,E,REP,RN,ARG,RARG)

C     Define the overall constant in the G-EV problem
C     this corrsponds to equation 4.124 in thesis

      ARGNORM=2.0D0*MUC*HBAR/MC**3
```

```
      DO 20 I=1,N
         DO 25 J=1,N
            ARG(I,J)=CHI*ARGNORM*ARG(I,J)
 25      CONTINUE
 20   CONTINUE

C     Find the eigenvalues and eigenstates in dimensionless variables

      CALL DGVCRG (N,ARG,LDA,RARG,LDA,ALPHA,BETA,EVEC,LDEVEC)
C                                  Compute eigenvalues
      DO 30  I=1, N
         EVAL(I) = ALPHA(I)/BETA(I)
c            WRITE(6,*)I,EVAL(I)
c         WRITE(6,*)QX(I),REAL(EVEC(I,1))/QW(I)
 30   CONTINUE
C                                  Compute performance index
      PI = DGPIRG(N,N,ARG,LDA,RARG,LDA,ALPHA,BETA,EVEC,LDEVEC)

c      WRITE(6,*)N,REAL(EVAL(1))

      DO 35 I=1,N
         DEUT(I)=REAL(EVEC(I,1))/QW(I)
 35   CONTINUE

C     Normalize the dimensionless wave functions

      SUM=0.0D0

      DO 37 I=1,N
         SUM=SUM+DEUT(I)**2*QW(I)
 37   CONTINUE

      DNORM=DSQRT(SUM)

      DO 39 I=1,N
         DEUT(I)=DEUT(I)/DNORM
         DEUTDIM(I)=SQRT(MC/HBAR)*DEUT(I)
c         WRITE(6,*) QX(I),DEUT(I)
 39   CONTINUE

c      WRITE(6,*)N,DEUT(QXZ),DEUTDIM(QXZ)
```

```
C     Find the form factors in dimensionless variables.
C     The overall dimensions remain sqrt(MeV-fm), collected in the overall
C     constant in front of the normalized dimensional form factor

      DO 40 I=1,N
         SUM1=0.0D0
         DO 42 J=1,N
            SUM1=SUM1+(RARG(I,J)*DEUT(J)*QW(J))
 42      CONTINUE
         FORMFAC(I)=SUM1
c     WRITE(6,*) QX(I),FORMFAC(I)
 40   CONTINUE

C     Normalize the form factors in dimensionless variables.
c     The dimensions are re-introduced in the overall constant

      SUM=0.0D0
      ACONS=2.0D0*MUC/MC**2

      DO 45 M=1,N
         SUM=SUM+(FORMFAC(M)**2)*QW(M)/(ACONS*E-QX(M)**2)
 45   CONTINUE

      NORM=DSQRT(-SUM)
c       WRITE(6,*)NORM,'Norm'

      DO 47 M=1,N
         READ(9,*)FORMATTR(M)
         FORMFAC(M)=FORMFAC(M)/NORM
         FORMFACDIM(M)=CHI/SQRT(MC/HBAR)*FORMFAC(M)
c          WRITE(6,*)QX(M),FORMATTR(M),FORMFAC(M)
 47   CONTINUE

      CLOSE(9,STATUS="KEEP")

c      WRITE(6,*)N,FORMFAC(QXZ)

C     for completeness we give the explicit numerical description
C     for the separable t-matrix and potential. If interest is only in
C     3N calculations, this can be skipped. On loop is explcitly given
C     for the calculation of the FORMATTR terms
```

```
      SUM=0.0D0
c      DO 50 M=1,QXZ-1
      DO 50 M=1,N
         SUM=SUM+(FORMFAC(M)**2)*QW(M)/(ACONS*E1-QX(M)**2)
 50   CONTINUE

      QUAD=SUM

c       WRITE(6,*) 'QUAD',QUAD,NORM**2

c      SUM=QUAD
c      DO 150 M=QXZ+1,N
c         SUM=SUM+(FORMFAC(M)**2)*QW(M)/(ACONS*E1-QX(M)**2)
c          WRITE(6,*)M,'den',SUM
c 150   CONTINUE
c
c      QUAD=SUM
c
c       WRITE(6,*) 'QUAD1',QUAD,NORM**2

      DO 51 M=1,N
c         WRITE(6,*)M,QX(M)
         DO 55 K=1,N
            POTSEP(M,K)=-FORMFAC(M)*FORMFAC(K)
            POTSEPDIM(M,K)=MC*HBAR/(2.0D0*MUC)*POTSEP(M,K)
            TSEP(M,K)=-FORMFAC(M)*FORMFAC(K)/(1.0D0+QUAD)
            TSEPDIM(M,K)=MC*HBAR/(2.0D0*MUC)*TSEP(M,K)
 55      CONTINUE
c      WRITE(6,*)QX(M),POTSEPY(M,M)
 51   CONTINUE

C     The constant for the TM terms is given by the following
c      AZ1=-2*PICONS*MUC/HBAR**2*TSEPDIM(QXZ,QXZ)
c      WRITE(6,*)'AZ',QX(QXZ),AZ1

C     Return to 3N problem
C     Spline the form factors

      DO 60 L=1,N
         DO 62 M=1,N
            XVEC(M)=QX(M)+QX(L)/2.0D0
            XVEC1(M)=-QX(M)/2.0D0-QX(L)
c         WRITE(6,*)XVEC(M),XVEC1(M)
```

```
 62      CONTINUE

C                                Compute cubic spline interpolant
         CALL DCSIEZ (N, QX, FORMFAC, N, XVEC, VALUE)

         CALL DCSIEZ (N, QX, FORMFAC, N, XVEC1, VALUE1)

         DO 63 M=1,N
            MVALUE(L,M)=VALUE(M)
            MXVEC(L,M)=XVEC(M)
            MVALUE1(L,M)=VALUE1(M)
            MXVEC1(L,M)=XVEC1(M)
 63      CONTINUE
 60   CONTINUE

C     Tests for the splined form factors
C
C      DO 65 L=1,N
C         DO 67 M=1,N
c            WRITE(6,*)MVALUE(L,M),MVALUE1(M,L)
C 67      CONTINUE
c         WRITE(6,*)MXVEC1(L,10),MVALUE1(L,10)
C 65   CONTINUE
C

C     Calculate the kernel ZEFF in dimensionless variables
C     Again, we have a constant with dimensions and including the
C     dimension already present from the form factors, ZEFFDIM has
C     units of fm.

      DO 70 L=1,N
         DO 72 M=1,N
      ZNOM(M,L)=ACONS*E1-0.5D0*((QX(M))**2+(QX(M)+QX(L))**2+(QX(L))**2)
         ZEFF(M,L)=MVALUE(M,L)*MVALUE1(M,L)/ZNOM(M,L)
         ZEFFDIM(M,L)=HBAR/MC*ZEFF(M,L)
c         WRITE(6,*)MXVEC(L,M),ZEFF(L,M)
 72      CONTINUE
 70   CONTINUE

C     the off diagonal correction terms are computed in the 100 routines

      DO 170 L=1,N
         DO 171 M=1,N
```

```
      GREEN3(L,M)=(ACONS*E1-3.0D0/4.0D0*QX(M)**2-QX(L)**2)
c           write(6,*)L,M,GREEN3(L,M)
            DO 172 K=1,N
      ZK2(L,M,K)=1.0D0/((QX(L)-QX(M)/2.0D0-QX(K))**2+1)
      ZK3(L,M,K)=1.0D0/((QX(K)+QX(L)+QX(M)/2.0D0)**2+1)
 172        CONTINUE
 171     CONTINUE
 170  CONTINUE

      DO 175 L=1,N
        DO 176 M=1,N
            SUM=0.0D0
            SUM1=0.0D0
            DO 177 K=1,N
               SUM=SUM+FORMFAC(K)/GREEN3(K,L)*ZK2(K,L,M)
               SUM1=SUM1+FORMFAC(K)/GREEN3(K,M)*ZK3(K,M,L)
 177        CONTINUE
            ZOD1(L,M)=SUM
            ZOD2(M,L)=SUM1
 176     CONTINUE
 175  CONTINUE
      ZODCONS=CHI*2.0D0*MUC*HBAR**2/(MC**4)
c      WRITE(6,*)'ZODCONS',ZODCONS
      DO 180 L=1,N
         DO 181 M=1,N
            ZOD(L,M)=ZOD1(L,M)*ZOD2(M,L)
            ZEFF(L,M)=2.0D0*(ZEFF(L,M)+ZA*ZODCONS*ZOD(L,M))
c            write(6,*)L,M,ZOD(L,M)
 181     CONTINUE
 180  CONTINUE

C     the diagonal correction terms are given in the 200 routines

C     The pion energy/momentum in dimensionless variables

      DO 220 L=1,N
         DO 222 M=1,N
            OMEGA(L,M)=DSQRT((QX(L)-QX(M))**2+1.0D0)
            OMEGADIM(L,M)=MC/HBAR*OMEGA(L,M)
c          WRITE(6,*)L,M,OMEGA(L,M)
 222     CONTINUE
 220  CONTINUE
```

```
C     The Tau term in the separable potential, note the negative value

      DO 225 L=1,N
         DO 227 M=1,N
            SUM=0.0D0
            DO 230 K=1,N
               QUAD3(K,L,M)=1.0D0/(ACONS*E1-QX(K)**2
     C -3.0D0/4.0D0*QX(L)**2-2.0D0*MUC/MC*OMEGA(L,M))
      SUM=SUM+FORMFAC(K)**2*QW(K)*QUAD3(K,L,M)
c      write(6,*)L,M,K,SUM
 230        CONTINUE
            TAUDIAG(L,M)=-1.0D0/(1.0D0+SUM)
c       WRITE(6,*)L,M,TAUDIAG(L,M)
 227     CONTINUE
 225  CONTINUE

C     The integral over the terms depending on the integration variables
C     for the separable t-matrix.
C     Note the square of the form factor, one comes from the t-matrix and
C     and one from the ZOD term.

      DO 240 L=1,N
          SUM=0.0D0
          DO 242 M=1,N
             SUM=SUM+FORMFAC(M)**2*QW(M)/GREEN3(M,L)
 242      CONTINUE
             ZD1(L)=SUM
c             WRITE(6,*)L,ZD1(L)
 240      CONTINUE

      DO 250 L=1,N
         DO 252 M=1,N
            ZDIAG(L,M)=ZD1(L)*ZD1(M)*TAUDIAG(L,M)
c           WRITE(6,*)L,M,ZDIAGINT(L,M)
 252     CONTINUE
 250  CONTINUE

C     The corresponding integral to 240 for the separable potential of
c     the attractive part only. Note the minus sign coming from the
c     negative strength factor of the potential, which yields a plus
c     sign in the "subtracting terms 270/272". Also, this time
c     we have a multiplication of the formfactor coming from ZOD and
```

```
c     a formattr coming form the separable potential

      DO 266 L=1,N
         SUM=0.0D0
         DO 268 M=1,N
            SUM=SUM+FORMATTR(M)*FORMFAC(M)*QW(M)/GREEN3(M,L)
 268     CONTINUE
         ZDR1(L)=SUM
c         write(6,*)QX(L),ZD1(L),ZDR1(L),ZD1(L)-ZDR1(L)
 266  CONTINUE

      DO 270 L=1,N
         DO 272 M=1,N
         ZDIAG(L,M)=ZDIAG(L,M)+ZDR1(L)*ZDR1(M)
c         WRITE(6,*)L,M,ZDIAG(L,M)
 272     CONTINUE
 270  CONTINUE

c     Bring the terms to the same units as used in the previous effective
c     potentials

      ZDIAGCONS=CHI*2.0D0*HBAR/(MC**2*1.0D0)
c      write(6,*)ZDIAGCONS
      DO 280 L=1,N
         DO 282 M=1,N
       ZEFF(L,M)=ZEFF(L,M)+ZDIAGCONS*(ZDIAG(L,M)+ZDIAG(M,L))
     C /(OMEGA(L,M)**3)
c         write(6,*)L,M,ZDIAG(L,M),ZDIAG(M,L)
 282     CONTINUE
 280  CONTINUE
C
C     Tests for ZEFF
C
C      DO 75 L=1,N
C         DO 77 M=1,N
c            WRITE(6,*)'ZNOM',ZNOM(M,L)-ZNOM(L,M),M,L
c            WRITE(6,*)ZEFF(L,12)-ZEFF(12,L),L,M,'test'
c            WRITE(6,*)ZEFF(M,L)-ZEFF(L,M),ZEFF(M,L)*ZEFF(L,M),L,M
c            WRITE(6,*)MXVEC(M,L),ZEFF(M,L),M,L
C 77      CONTINUE
c         WRITE(6,*)MXVEC(21,L),ZEFF(21,L)
C 75   CONTINUE
```

```
C     Calculate the quadratic form in the propagator tau
C     Note that Quad is dimensionless.
C
      DO 80 M=1,N
         SUM=0.0D0
         DO 82 L=1,N
            TAUMOM(M,L)=(ACONS*E1-3.0D0/4.0D0*QX(M)**2-QX(L)**2)
            SUM=SUM+(FORMFAC(L)**2)*QW(L)/TAUMOM(M,L)
c     WRITE(6,*)M,'den',SUM
 82      CONTINUE
         QUAD1(M)=SUM
C
C     Test that we have right normalization with E1=-2.225D0. We
C     expect to see the value -1.0D0 for the midpoint.
C
c      WRITE(6,*)QUAD1(M)
 80   CONTINUE

C     define the RHS kernel U for the generalized EV-problem.
C     Note that U has the proper units of fm.

      DO 90 M=1,N
         DO 92 L=1,N
            SUM=0.0D0
            DO 93 J=1,N
c     WRITE(6,*)SUM,J,L,M
             SUM=SUM+ZEFF(M,J)*ZEFF(J,L)*QW(J)/(1.0D0+QUAD1(J))
c     write(6,*)J,L,M,SUM
 93       CONTINUE

          ZARG(M,L)=-SUM
 92    CONTINUE
 90   CONTINUE

C
C     Tests for ZARG, symmetries, etc.
C

      DO 95 M=1,N
         DO 97 L=1,N
c            WRITE(6,*)ZARG(M,L)-ZARG(L,M),M,L
c            WRITE(6,*)ZARG(M,L)
 97      CONTINUE
```

```
 95   CONTINUE
C
C     Solve the triton generalized EV problem
C
      IWEIGH = 1
      ALPH  = 0.0D0
      BET   = 0.0D0
      NFIX   = 0

      CALL DGVCRG (N,ZARG,LDA,ZEFF,LDA,ZALPHA,ZETA,ZEVEC,LDEVEC)
C                                  Compute eigenvalues
      DO 100  I=1,N
         ZEVAL(I) = ZALPHA(I)/ZETA(I)
C
C     Check eigenvalues
C
            WRITE(6,*)I,ZEVAL(I)
c            WRITE(6,*)QX(I),REAL(ZEVEC(I,1))/QW(I)
         TRISTATE(I)=REAL(ZEVEC(I,1))/QW(I)
 100  CONTINUE
C
C     Normalize the triton wave function
C
      SUM=0.0D0

      DO 110 I=1,N
         SUM=SUM+TRISTATE(I)**2*QW(I)
 110  CONTINUE

      TNORM=DSQRT(SUM)

      DO 115 I=1,N
         TRISTATE(I)=TRISTATE(I)/TNORM
         TRISTATEDIM(I)=SQRT(MC/HBAR)*TRISTATE(I)
c          WRITE(6,*)QX(I),TRISTATE(I)
 115  CONTINUE
C
C     Things to test and play with
C
c      WRITE(6,*)N,REAL(ZEVAL(1)),REAL(ZEVAL(2)),REAL(ZEVAL(3))
c       WRITE(6,*)N,REAL(ZEVAL(1))
c      WRITE(6,*)N,TRISTATE(QXZ),QX(QXZ)
```

```
5     CONTINUE
      END
```