GROUNDWATER DEVELOPMENT AND ELECTRIC AND DIGITAL MODELLING OF THE UPPER CARBONATE AQUIFER, METROPOLITAN WINNIPEG AREA

A Thesis

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Master of Science

By

FRANCIS WILLIAM RENDER

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ABSTRACT

An extensive confined aquifer, currently pumped at a rate of 3 billion gallons per year, occurs in the fractured and jointed upper 100 feet of the thick Paleozoic carbonate rock sequence underlying metropolitan Winnipeg. The karstic bedrock surface, a controlling parameter of the groundwater flow system, slopes towards the Red River Valley from recharge areas located in uplands along the borders of the Red River Basin. The bedrock surface is mantled by 30 to 200 feet of glacial drift. The drift is composed of 5 to 30 feet of till overlain by lacustrine sediments, mainly clays, that average 40 feet in thickness.

Groundwater has been a source of water supply in the Winnipeg area since the early 1800's. The Upper Carbonate aquifer, which during the first two decades of the twentieth century supplied all the city's water requirements, has a transmissibility ranging between 2,000 and 200,000 gallons per foot per day. The storage coefficient varies from 1×10^{-6} to 1×10^{-3}. Water in the aquifer is generally fresh to brackish, however south of the Assiniboine River the water is brackish to saline and is not potable.

Groundwater withdrawals have created a major drawdown cone in the central industrial area of metropolitan Winnipeg. The drawdown cone is at the center of a lateral radial flow

system. Elevations of the piezometric surface range between 800 and 1,500 feet in the recharge areas and are below 700 feet in the drawdown cones. In most locations the groundwater has a temperature range of 39^{0} to 43^{0}F and is mainly used for cooling purposes. Pumpage varies from 5 million gallons per day in the winter months to 10 million gallons per day during the summer air conditioning period. The increased summer withdrawals cause temporary declines in the piezometric surface of more than twenty feet in central Winnipeg, and approximately one foot along the outer fringes of the urban areas.

A resistor-capacitor analog and a digital computer model of the Upper Carbonate aquifer were tested by simulating the difference between summer and non-summer pumpage. The simulated drawdown correlated reasonably well with field drawdown observations. The digital model indicated that while the Red and Assiniboine River beds have a low hydraulic conductivity they still contribute significant amounts of water to the aquifer. The models can be utilized to indicate the probable effects of increased aquifer development, and as an aid in the prediction of changing water quality patterns.

CHAPTER I

INTRODUCTION

The Winnipeg area (Figs. 1 and 2) is underlain by an extensive carbonate rock aquifer which currently yields approximately 3 billion gallons of water per year. Groundwater from this confined aquifer has been extracted for at least 130 years. During this interval over 200 commercial and industrial wells and thousands of domestic wells have been installed. Total pumpage has varied from less than one million to ten million gallons per day.

The industrial and commercial development of the Metropolitan area has, to a large extent, been influenced by the excellent surface water supply that is brought in by the 85 million gallons per day capacity aqueduct from the Lake of the Woods ninety miles east of Winnipeg. Carbonate aquifer wells provide approximately seventeen percent of the water supply used in the area. Pumpage from this aquifer reached a maximum of 3.6 billion gallons per year in 1918 (Fig. 3). Following the completion of the aqueduct in 1919, pumpage abruptly declined but has gradually increased due to groundwater development for industrial and air conditioning purposes.

The Red River Valley is located along the mid-continent

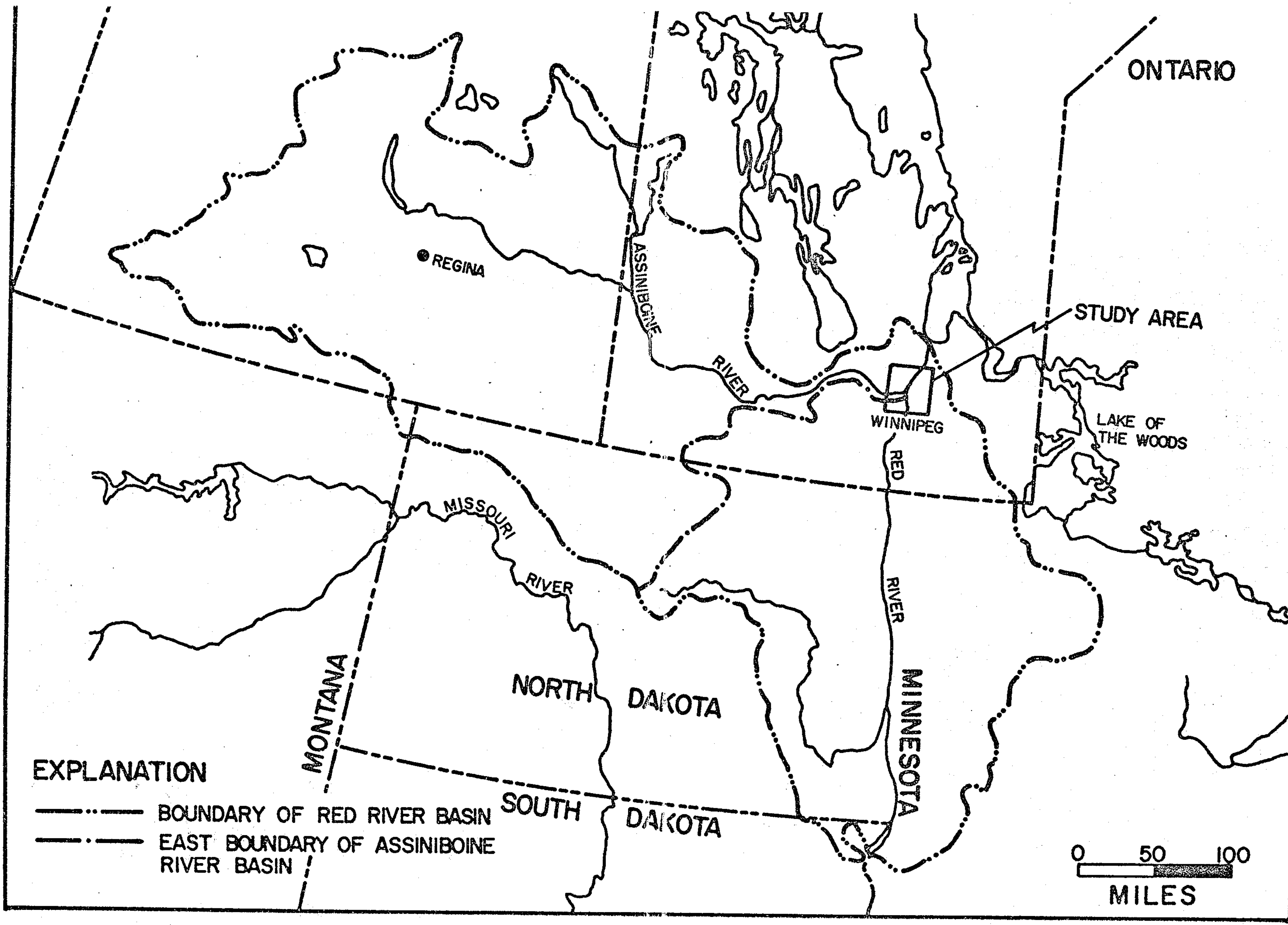

Fig. 1. Location of the study area.

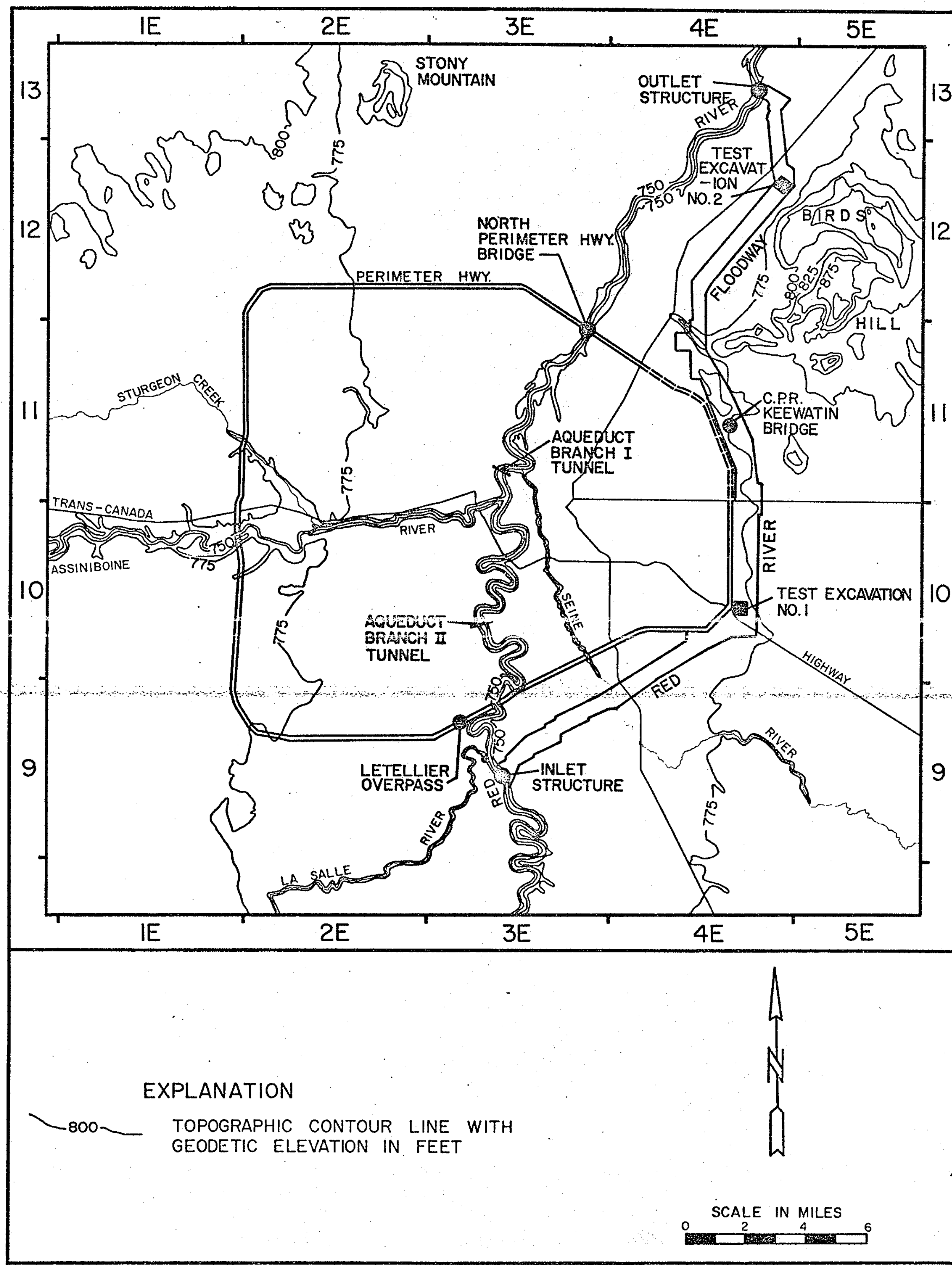

Fig. 2. Topography of the Winnipeg Area.

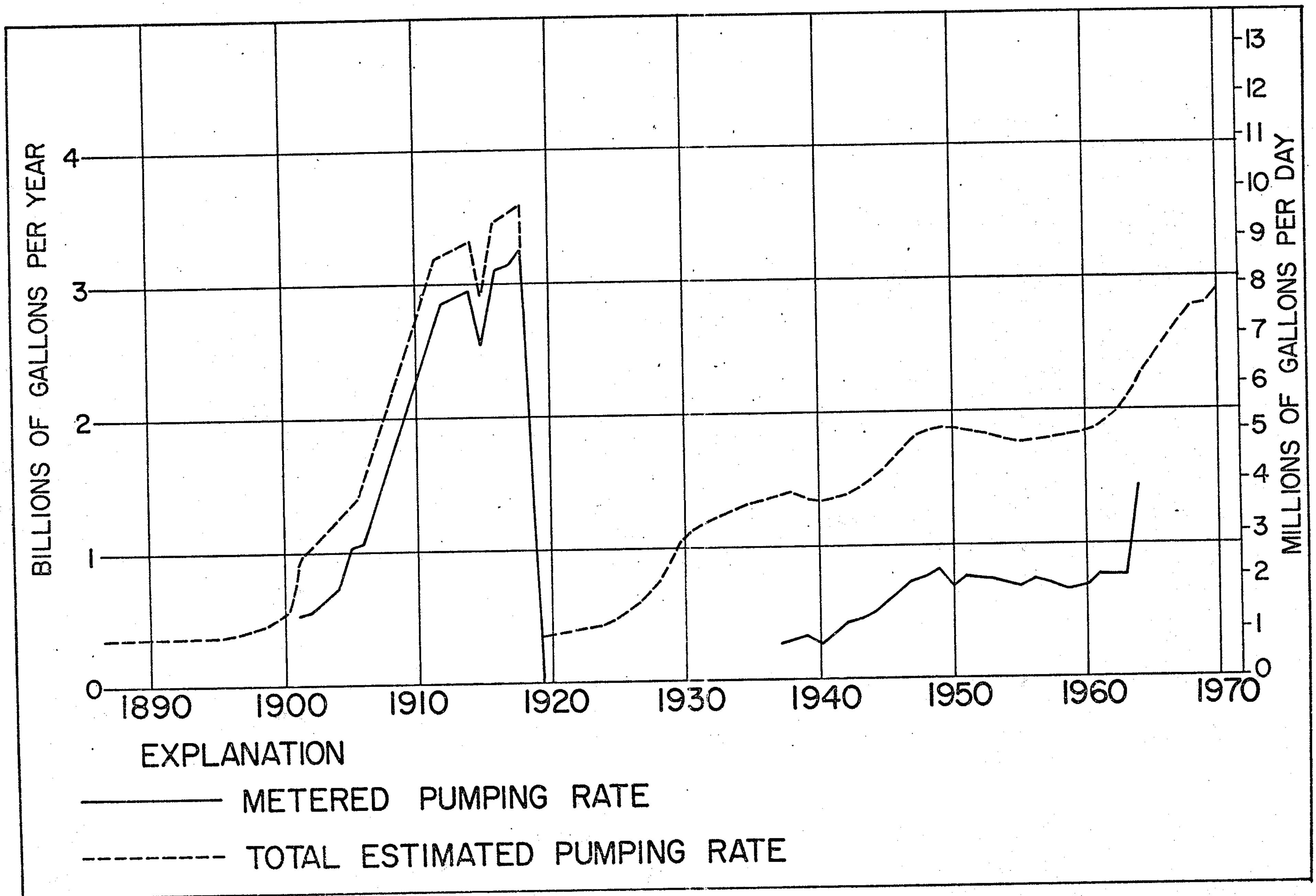

Fig. 3. Groundwater pumping rates.

topographic low. Consequently, surface and groundwater systems flow toward it both from the east and the west.

Although natural drainage of the urban area consists of the two main rivers and several small streams, the extreme flatness of the prairie surface has required the construction of an extensive network of agriculture and flood protection drains. The largest of these is the recently completed Red River Floodway. In the Metropolitan area the sewer systems remove most of the average annual nineteen inches of precipitation before it percolates down to the water table.

Electric analog and digital computer models of the Upper Carbonate aquifer were constructed, based on the available geohydrologic information, in attempts to provide tools for the quantitative evaluation of the aquifer. These models were tested utilizing the drawdown observations, and the estimations of the increase in pumpage for the 1965 air-conditioning season. While the models reasonably duplicated the geohydrologic conditions, the tests on them indicated that more precise field data are required before detailed models of the Upper Carbonate aquifer in the metropolitan Winnipeg area could be constructed.

Acknowledgments

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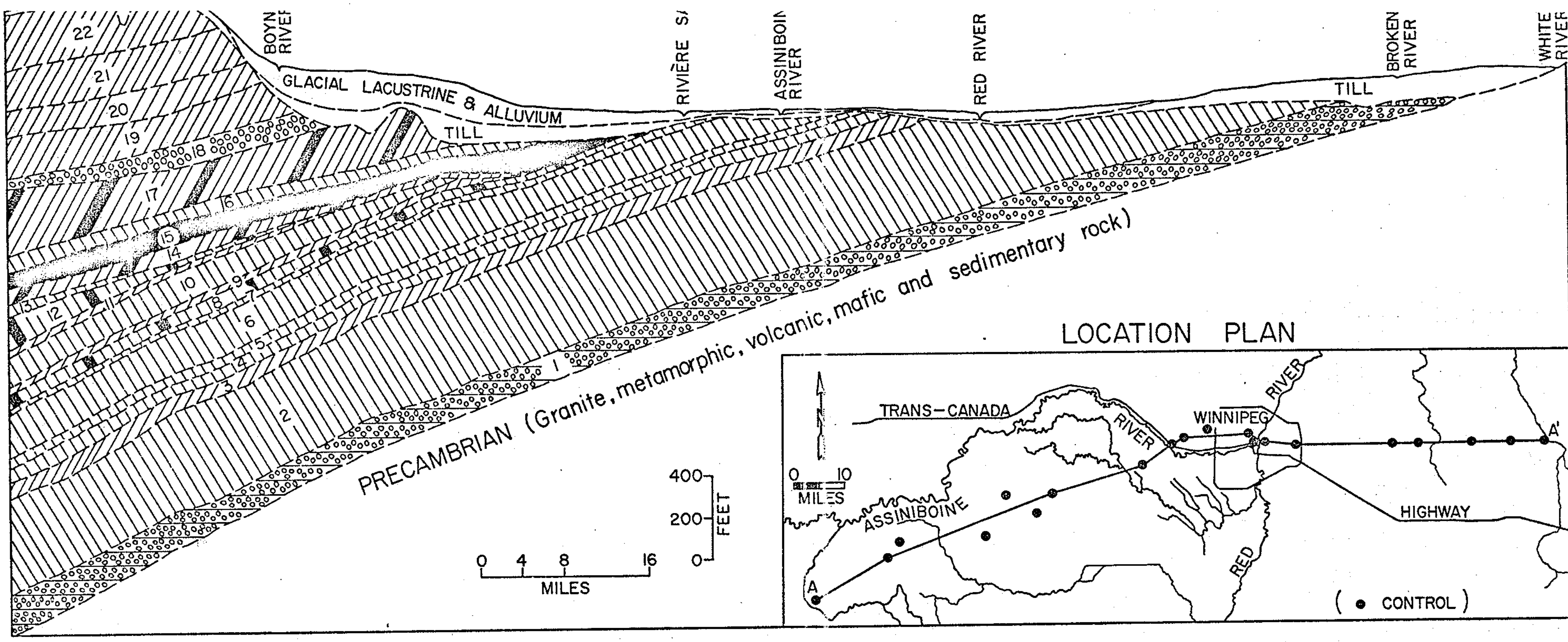
Appreciation is expressed to the residents of the area, and to the Provincial and Municipal agencies, industries, engineers, and water well contractors who generously co-operated in the investigation. The basic data upon which this thesis is based was collected by the author as part of a study of the groundwater resources of the metropolitan Winnipeg area undertaken by the Water Resources Branch, Manitoba Department of Mines and Natural Resources and is available in the open files of the Branch.

CHAPTER II

GEOLOGIC ENVIRONMENT

The stratigraphy of the Red River Basin is a controlling parameter of the groundwater system. The Winnipeg area is located stratigraphically on the northeast fringe of a sequence of gently dipping sedimentary rock units (Fig. 4). The bedrock surface is characterized by Paleozoic carbonate formations (Fig. 5) which under Winnipeg range in thickness from 250 to 750 feet. The bedrock surface (Fig. 6) has a number of closed depressions which suggest a karst topography similar to that observed in rock exposures between Lakes Winnipeg and Manitoba. A mantle of Pleistocene drift overlies the bedrock (Fig. 4). The drift has a maximum thickness over 400 feet, and is generally thicker immediately east of the Pembina Escarpment and in the Sandilands Forest Reserve area (Davies et al, 1962). In the central Red River Basin, the drift comprises Glacial Lake Agassiz clay and recent fluvial deposits overlying till (Fig. 7). Under the metropolitan Winnipeg area the surficial deposits have a maximum thickness of approximately 200 feet. The average thickness is sixty feet (Fig. 8).



MESOZOIC

CRETACEOUS

22 RIDING MOUNTAIN FORMATION (Siliceous shale, minor bentonite)

21 VERMILION RIVER FORMATION (Carbonaceous, calcareous, non-calcareous shale, minor bentonite)

20 FAVEL FORMATION (Calcareous shale, minor limestone and bentonite)

19 ASHVILLE FORMATION (Shale, minor sand, silt and bentonite)

18 SWAN RIVER FORMATION (Sandstone and shale)

JURASSIC

17 MELITA FORMATION (Shale, with minor sandstone, limestone and anhydrite)

16 RESTON FORMATION (Interbedded argillaceous limestone and shale)

15 UPPER AMARANTH FORMATION (Anhydrite)

14 LOWER AMARANTH FORMATION (Red dolomitic shale with minor anhydrite)

PALAEOZOIC

DEVONIAN

13 DUPEROW FORMATION (Limestone and dolomite)

12 SOURIS RIVER FORMATION (Dolomite, limestone, shale and minor anhydrite)

11 FIRST RED SHALE (Shale)

10 DAWSON BAY FORMATION (Argillaceous and fossiliferous limestone)

9 SECOND RED SHALE (Shale)

8 WINNIPEGOSIS FORMATION (Fossiliferous, dolomite and minor anhydrite)

7 ASHERN FORMATION (Argillaceous dolomite and dolomitic shale)

SILURIAN

6 INTERLAKE GROUP (Dolomite, minor sandy argillaceous beds)

ORDOVICIAN

5 STONEWALL FORMATION (Dolomite, dolometic limestone, minor sandy beds)

STONY MOUNTAIN FORMATION

4 GUNTON MEMBER (Dolomite)

3 SHALE MEMBER (Shale and argillaceous limestone)

2 RED RIVER FORMATION (Dolomitic limestone and dolomite)

1 WINNIPEG FORMATION (Shale and sandstone)

CARBONATE ROCK

SHALE

MAINLY ANHYDRITE

SANDSTONE

SHALE – SANDSTONE

CARBONATE ROCK – SHALE WITH MINOR ANHYDRITE

Fig. 4. Geologic cross-section of the Red River Basin through Winnipeg.

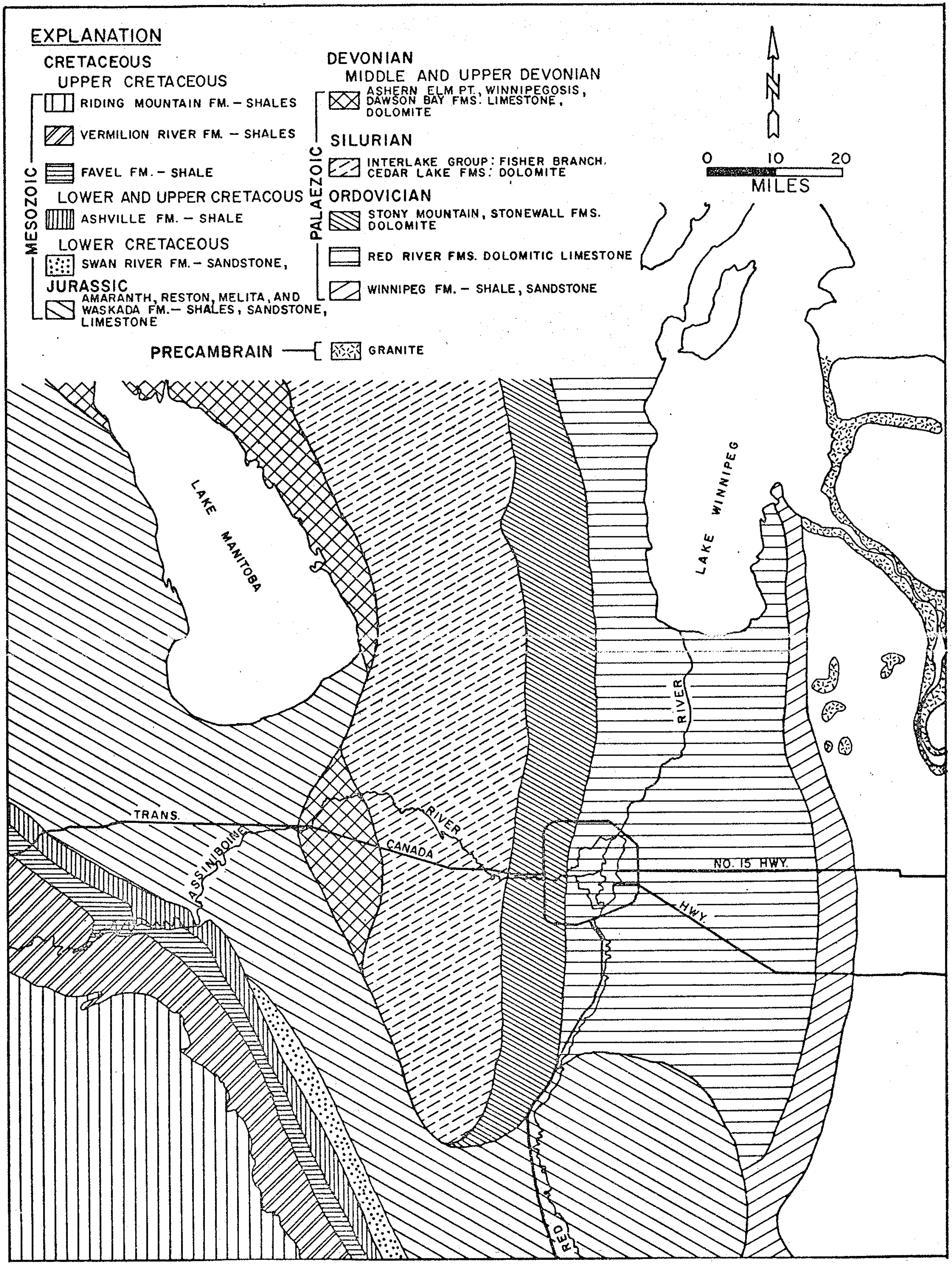


Fig. 5. Bedrock surface geology of south central Manitoba

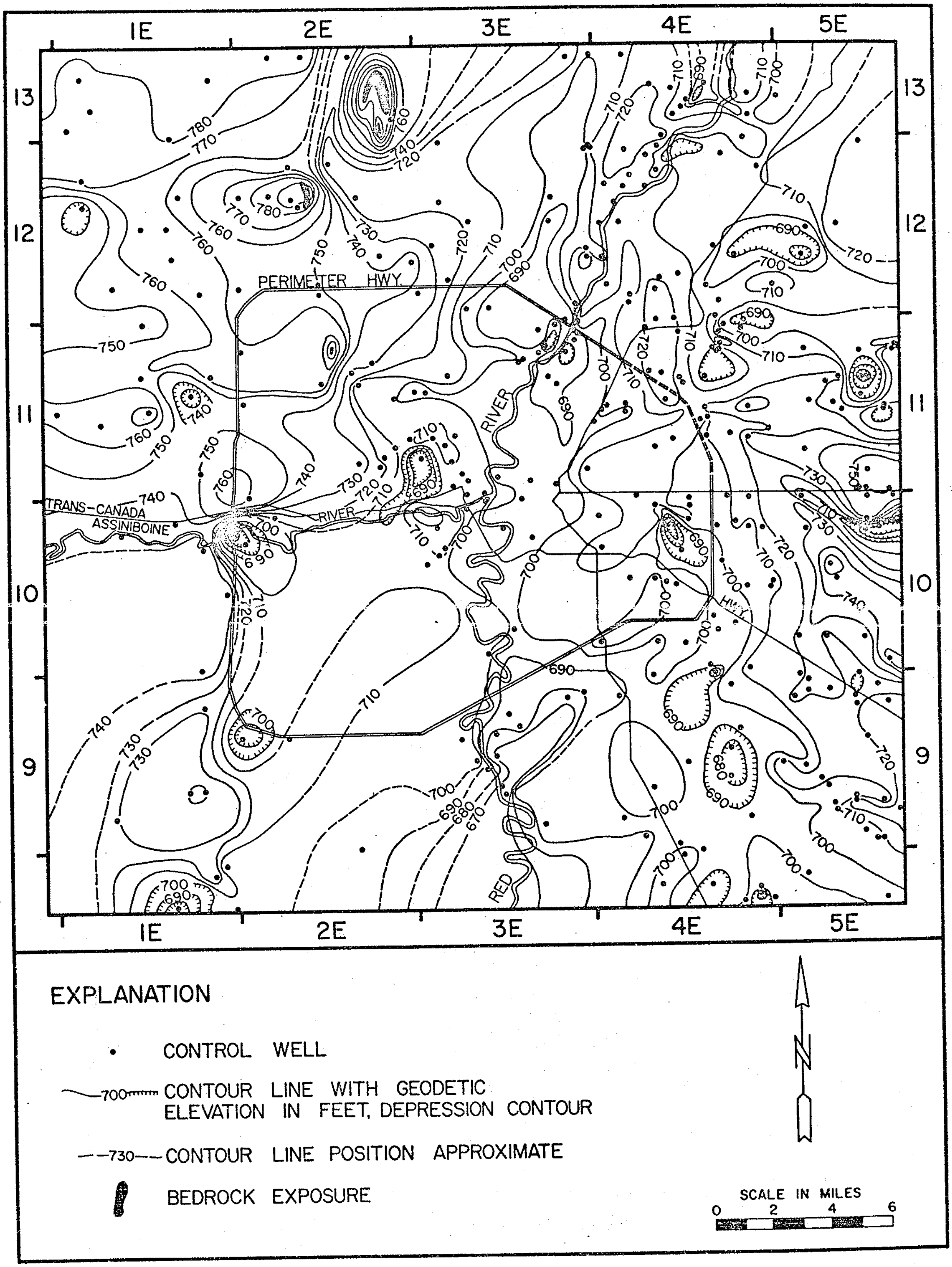


Fig. 6. Topography of the Bedrock Surface

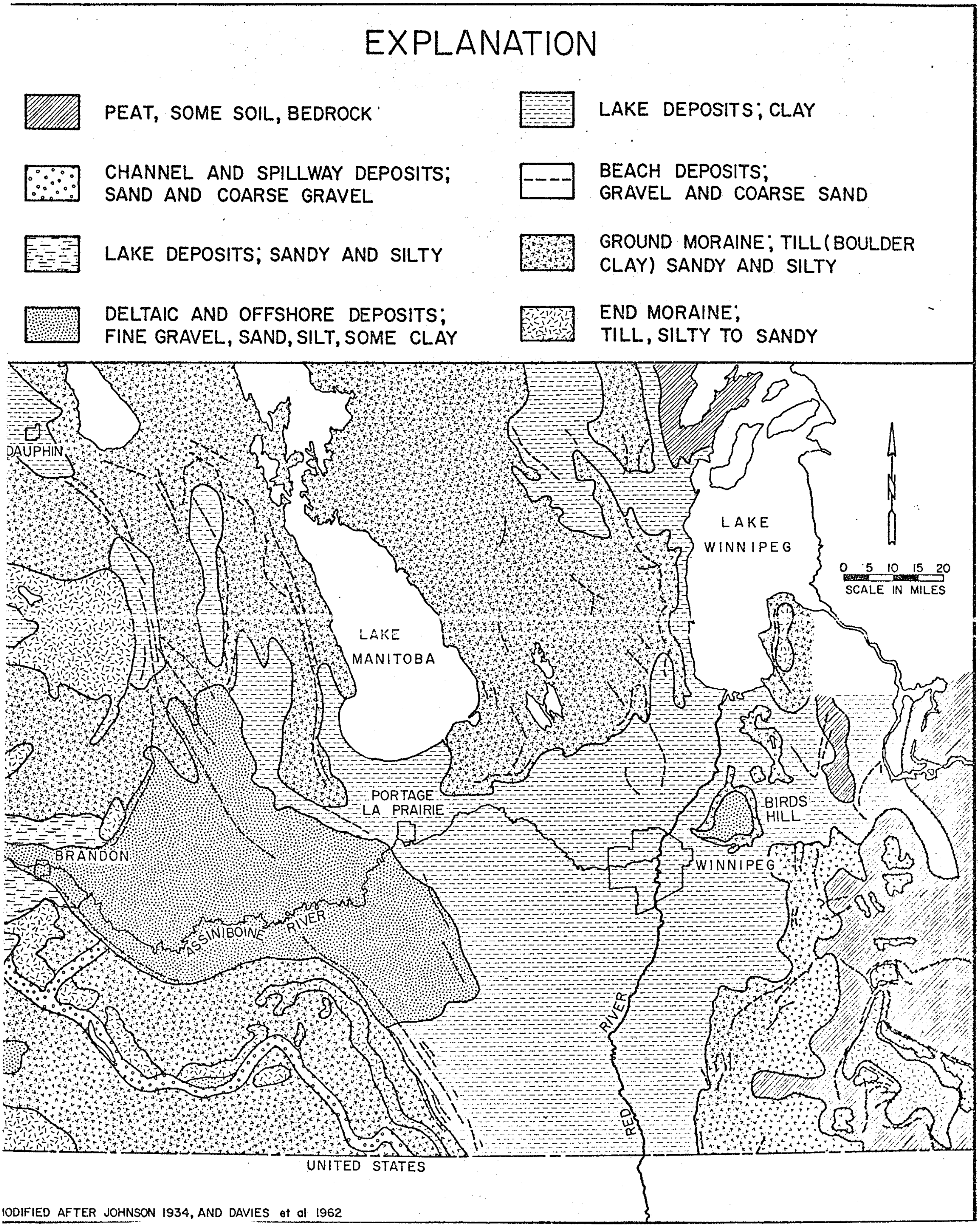


Fig. 7. Surface deposits of south central Manitoba.

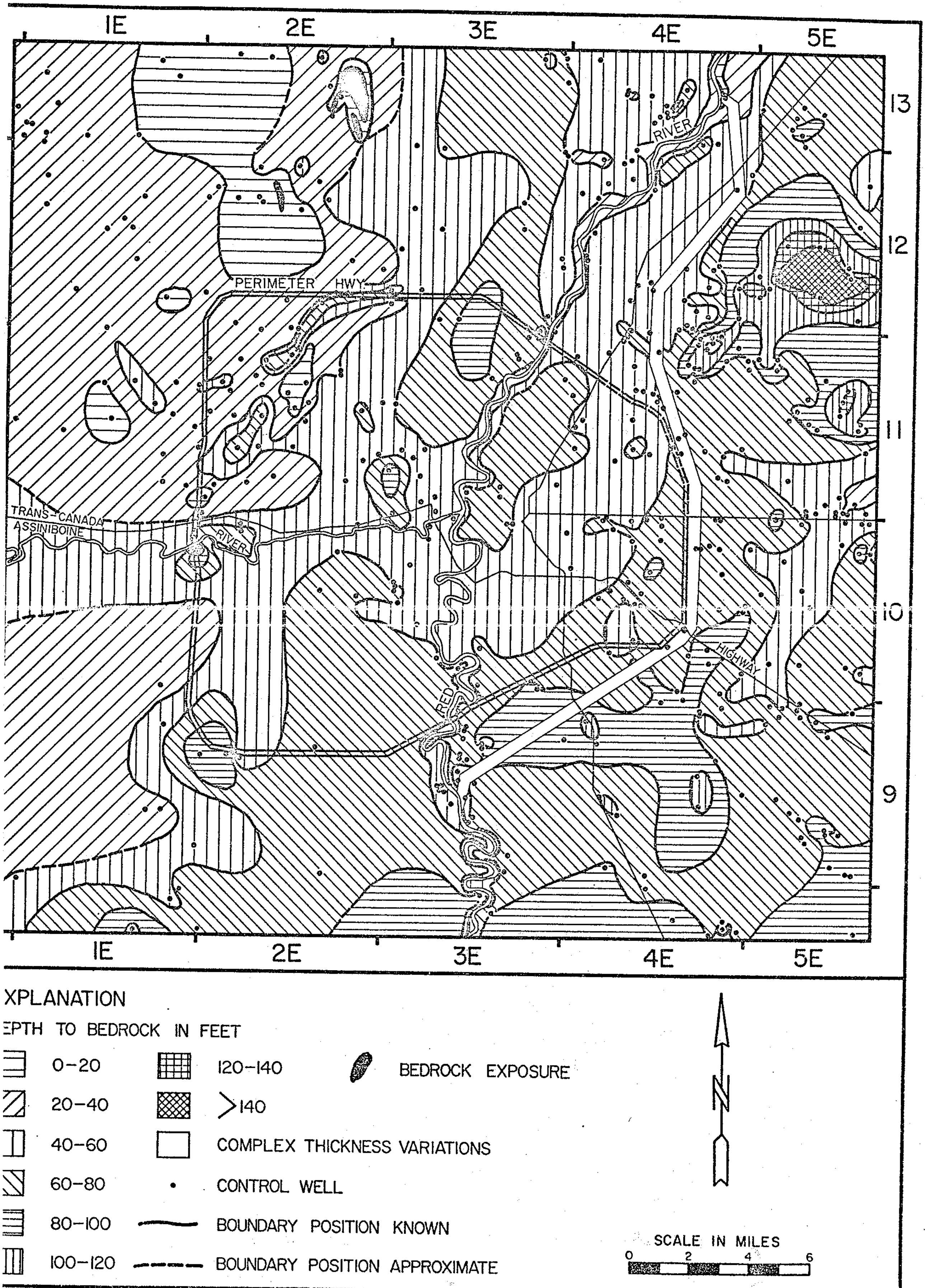


Fig. 8. Thickness of the deposits overlying the bedrock.

CHAPTER III

HYDROGEOLOGY

Introduction

The major aquifer underlying the Winnipeg area, referred to as the Upper Carbonate aquifer, occurs in the top fifty to one hundred feet of the Paleozoic limestones and dolomites (Fig. 9). The aquifer is partially confined above by the glacial drift and below by the slightly pervious underlying carbonate rock. A relatively minor aquifer, called the Lower Carbonate aquifer, occurs in the bottom twenty-five to fifty feet of the Red River formation, along the contact with the upper shale unit of the Winnipeg formation. The Winnipeg formation contains an Upper Sandstone aquifer, 20 to 40 feet thick and a Lower Sandstone aquifer 10 feet thick. Both of these aquifers contain saline water. Recharge occurs through the glacial till and glaciofluvial deposits located in the uplands along the borders of the Red River Basin and in the Birds Hill area. In a few places, large diameter dug wells have been installed to provide domestic water supplies from silt and very fine sand in the upper fifteen feet of the glacial lake clay, or from an uncemented zone in the top of the till.

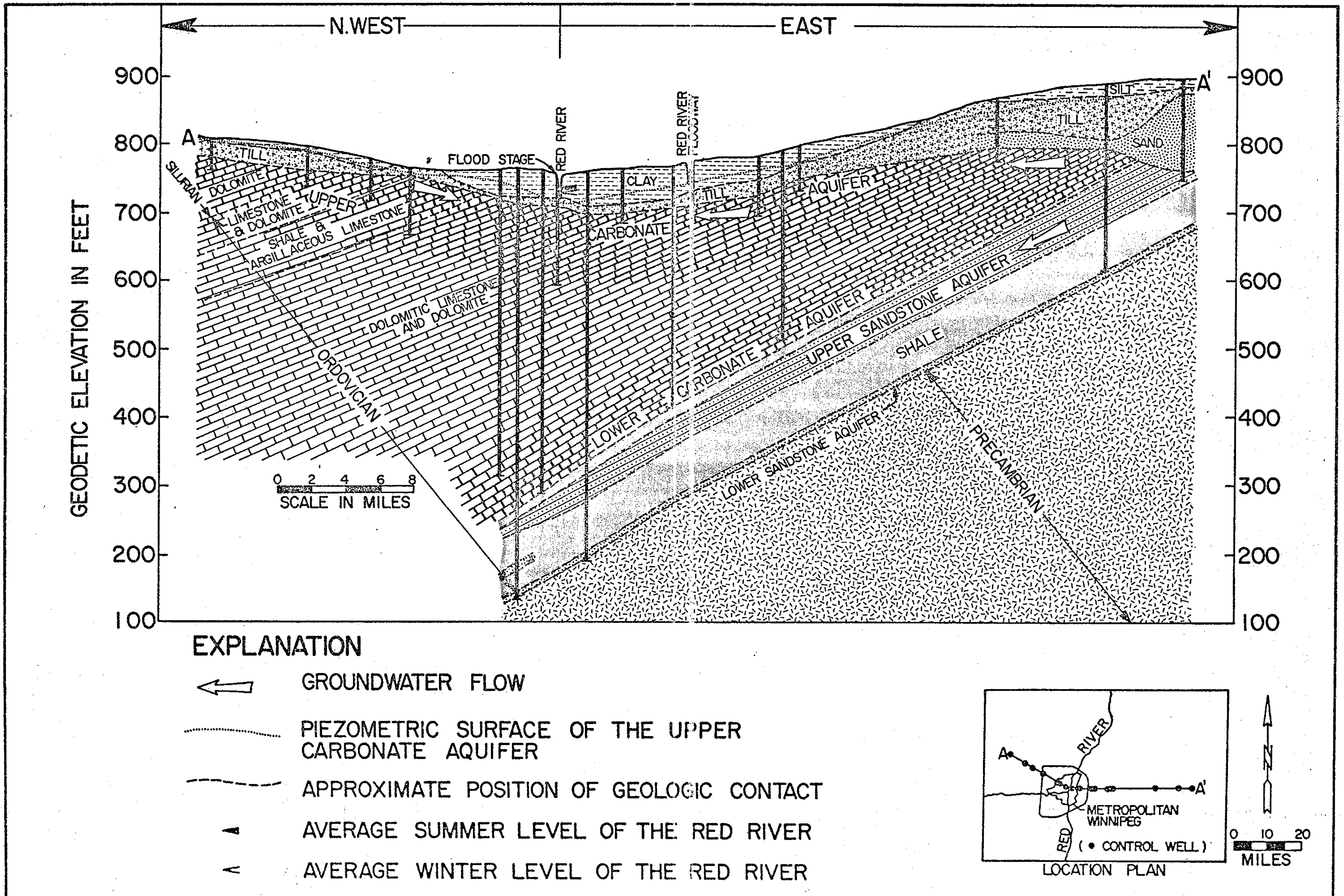


Fig. 9. Hydrogeologic cross-section of the Red River Valley through the Winnipeg area.

Carbonate Aquifers

The Upper Carbonate aquifer is characterized by a network of fractures, joints, and bedding planes which provide aquifer permeability. Geochemical solution processes acting for at least the past several thousand years have enlarged the pore spaces and modified the network. Fracture openings generally have maximum widths at the bedrock surface and decrease in size with depth. The upper 25 feet of the carbonate rock is the major zone of hydraulic conductivity in the aquifer, and is thus the section of most active flow. The character of the fracture permeability in the upper part of the bedrock at the Inlet Structure of the Red River Floodway is shown in Figure 10. While the entire thickness of the carbonate rock is saturated, bore hole pressure testing in the Winnipeg area has indicated that the portion between the Upper and Lower Carbonate aquifers has an exceptionally low hydraulic conductivity and therefore is not a producing zone.

Large solution cavities in the bedrock, typical of karst topography have been encountered. Interception of these solution cavities by wells results in high capacity, whereas, wells that do not intercept these features may have a low specific capacity. As the hydraulic conductivity is highest in the upper part of the bedrock, the slope of the bedrock surface has a controlling influence on the local and regional directions of groundwater movement in the

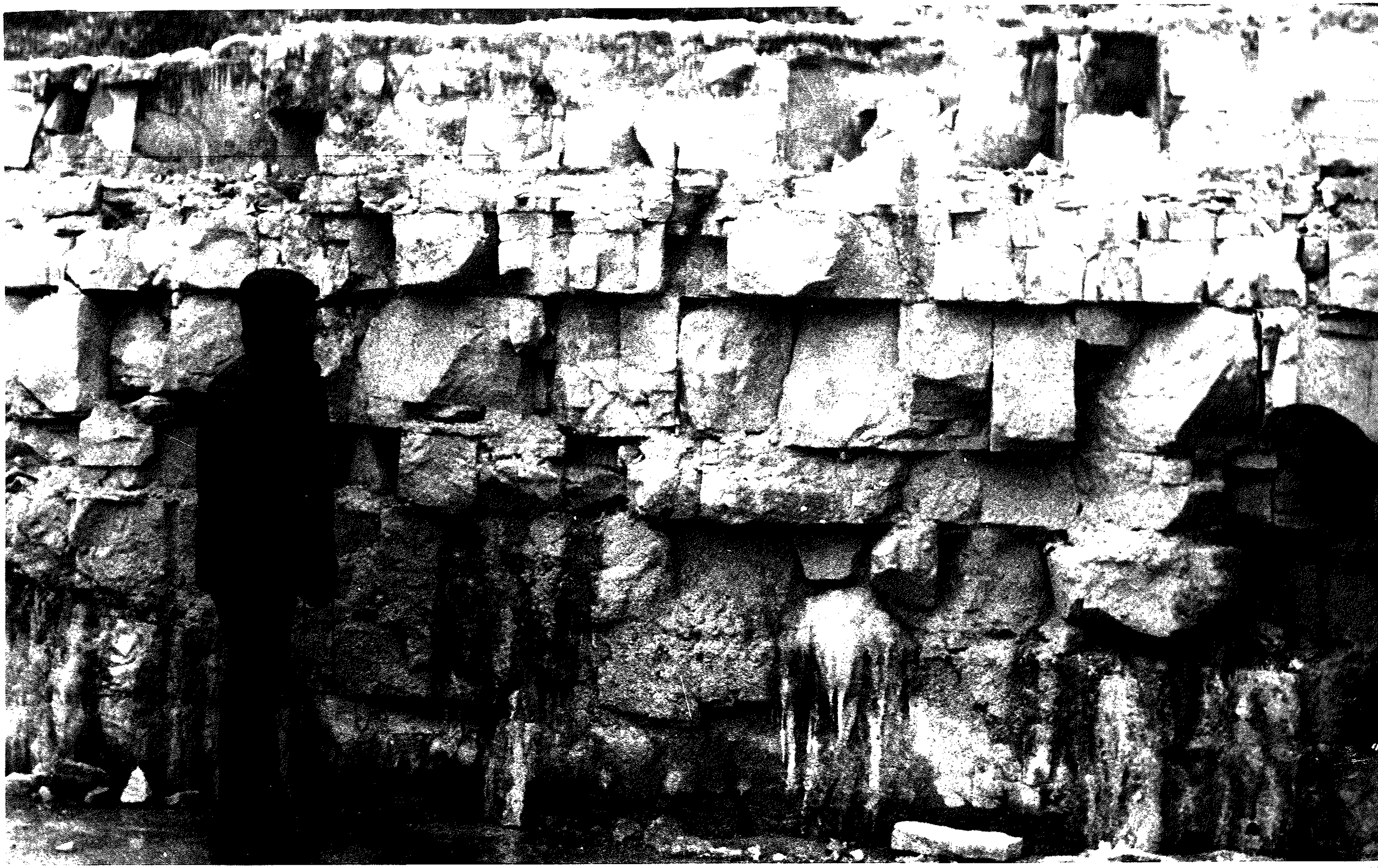


Fig. 10. Upper bedrock zone under the Red River Floodway Inlet control structure.

aquifer. Openings along the bedding planes, observed both in outcrops and in excavations, usually do not exceed one inch in height. Major joint openings strike both northwest and northeast, and vary in size from hairline fractures to openings over one foot wide. The joint blocks are one to ten feet wide.

The transmissibility[1] of the Upper Carbonate aquifer (Fig. 11) ranges from under 2,000 to over 200,000 gallons per foot per day. The storage coefficient varies from 1×10^{-6} to 1×10^{-3}.

Data from bore hole water pressure tests and pumping tests indicate that the hydraulic conductivity of the Lower Carbonate aquifer is much less than that of the Upper Carbonate aquifer. The interception of fractures is less frequent and the groundwater flows are usually small, indicating narrow openings. The maximum transmissibility of this aquifer is probably less than 5,000 gallons per day.

Sandstone Aquifers

The Winnipeg formation contains the Upper and Lower Sandstone aquifers. The ten to twenty feet thick upper shale unit of the Winnipeg formation is a confining layer for the Lower Carbonate aquifer and acts as an aquiclude between this aquifer and the saline water bearing Upper

[1] transmissibility - hydraulic conductivity in gallons per day per square foot multiplied by aquifer thickness in feet.

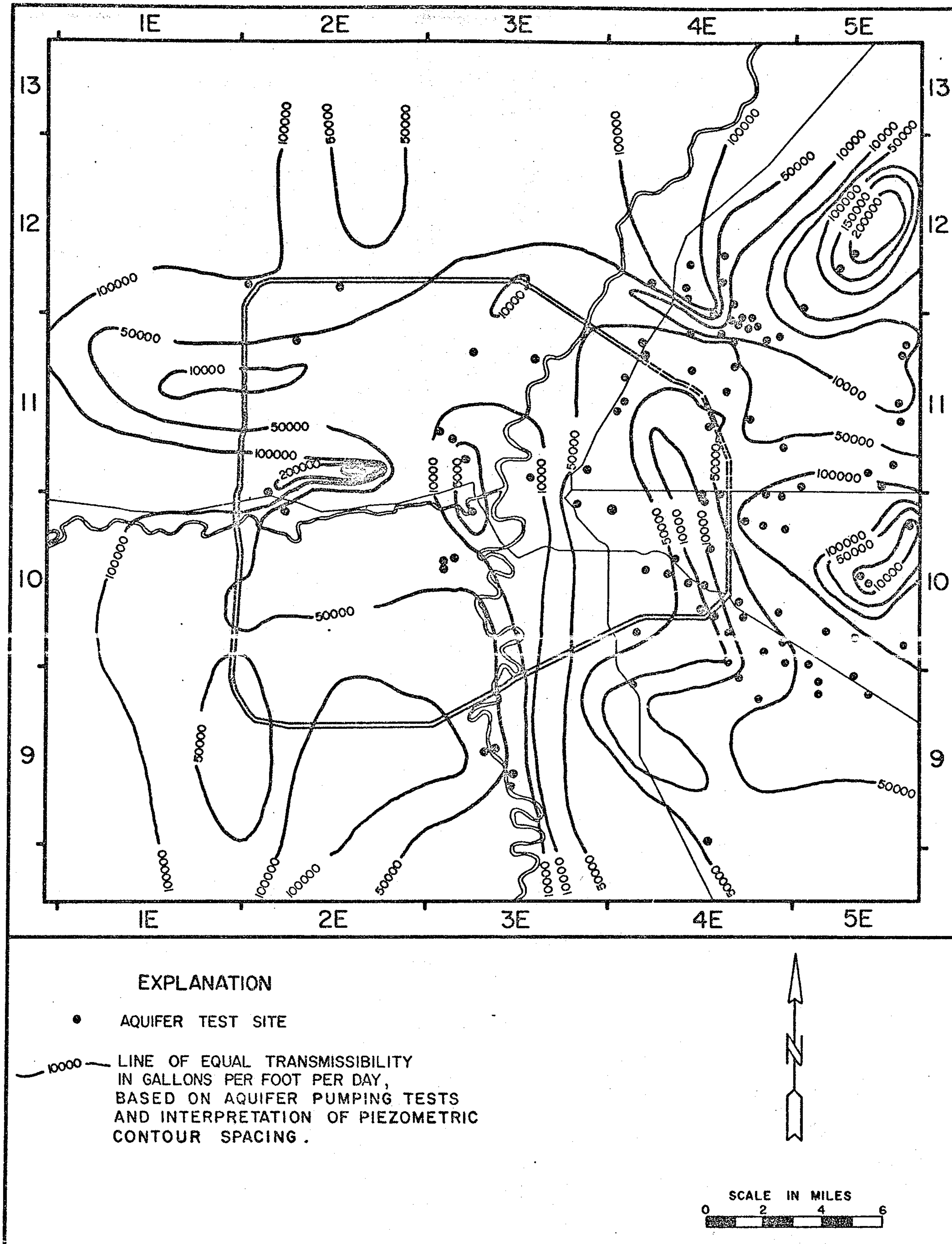


Fig. 11. Transmissibility of the Upper Carbonate Aquifer.

Sandstone aquifer. Beneath the City of Winnipeg the Upper Sandstone aquifer is twenty to forty feet thick (Fig. 9) and consists of beds of very fine silicious sandstone interbedded with thin layers of shale. In the Metropolitan area this aquifer has a transmissibility of less than 1,000 gallons per foot per day. Twenty miles southeast of the city, where the sandstone is thicker and coarser grained, the transmissibility is in the order of 10,000 gallons per foot per day. Thirty miles south of Winnipeg this unit thickens to eighty feet. Between this aquifer and the top of the Lower Sandstone aquifer are sixty to seventy feet of shale. Thin lenses of sandstone occur at some places within this shale unit.

Under the City of Winnipeg the Lower Sandstone aquifer is composed of fine-grained, silica sandstone. This unit extends down dip and in places along the western border of Manitoba is forty feet thick (Andrichuk, 1959). A pumping test on the two sandstone aquifers under central Winnipeg indicated a combined transmissibility of approximately 1,000 gallons per foot per day.

Surficial Deposits

The Upper Carbonate aquifer is overlain by deposits of glacial drift (Fig. 7). Near the rivers and streams alluvium occurs on the drift. The lower portion of the drift consists of boulder till and associated glaciofluvial deposits. In the central basin the upper part of the drift comprises clay and clayey silts deposited in Glacial Lake Agassiz. Deposits consisting primarily of outwash sand and gravel occur in the Birds Hill area ten miles northeast of Winnipeg.

The bouldery till consists of carbonate rock fragments in a clay-silt matrix. The till thickness under metropolitan Winnipeg averages twenty feet and ranges up to forty feet. In places, Lake Agassiz sediments rest directly on the bedrock. Under the Winnipeg area the upper part of the till consists of a soft, wet, uncemented zone that ranges in thickness up to twelve feet, and averages three feet. The underlying till is usually highly cemented with calcium carbonate. The author has observed hairline joints in the cemented till which probably account for most of its permeability. Slightly north of the study area field tests using the Hvorslev method (Hvorslev, 1951) yield hydraulic conductivity values that average 3×10^{-6} centimeters per second (K. Goff, personal communication, 1968). Glaciofluvial deposits, such as the Birds Hill complex, considerably increase the overall hydraulic conductivity of the drift.

The Birds Hill glaciofluvial deposits (Fig. 7) underlie an area of some sixty square miles and form an important partially confined aquifer that contributes recharge to the Upper Carbonate aquifer. The deposits are variable in grain size and structure, ranging from massive sections of fine silty sand to cross-bedded coarse gravels. Till occurs discontinuously under the glaciofluvial materials. An upper till usually overlies the periphery of the deposits (Figs. 7 and 12). The discontinuity of the lower till allows appreciable groundwater recharge from the Birds Hill aquifer into the Upper Carbonate aquifer. The western extension is a larger esker. Pumping tests indicate that the transmissibility of the combined Birds Hill and the Upper Carbonate aquifers ranges between 100,000 and 300,000 gallons per day per foot. The high transmissibility is primarily due to the Birds Hill aquifer. Other glacio-fluvial deposits, deltas, and lake beaches occur along the perimeter and within the Red River Basin (Fig. 7). In general, areas of groundwater recharge are associated with these deposits.

Lake Agassiz deposits which underlie the central part of the Red River Basin in general consist of two distinct geohydrologic units. The upper unit consists of five to fifteen feet of silt and very fine sand interbedded with thin beds of clay. Under Winnipeg the Agassiz sediments have a maximum thickness of eighty feet and an average thickness of forty feet (Fig. 13) but thicken westward to over 200 feet.

Laboratory tests on samples from the lower clay unit indicate an intergranular hydraulic conductivity in the order of 10^{-9} to 10^{-11} cm per second and horizontal permeabilities that are twice the vertical (Baracos, 1960; Mishtak, 1964). Hairline fractures were observed in the lower clay

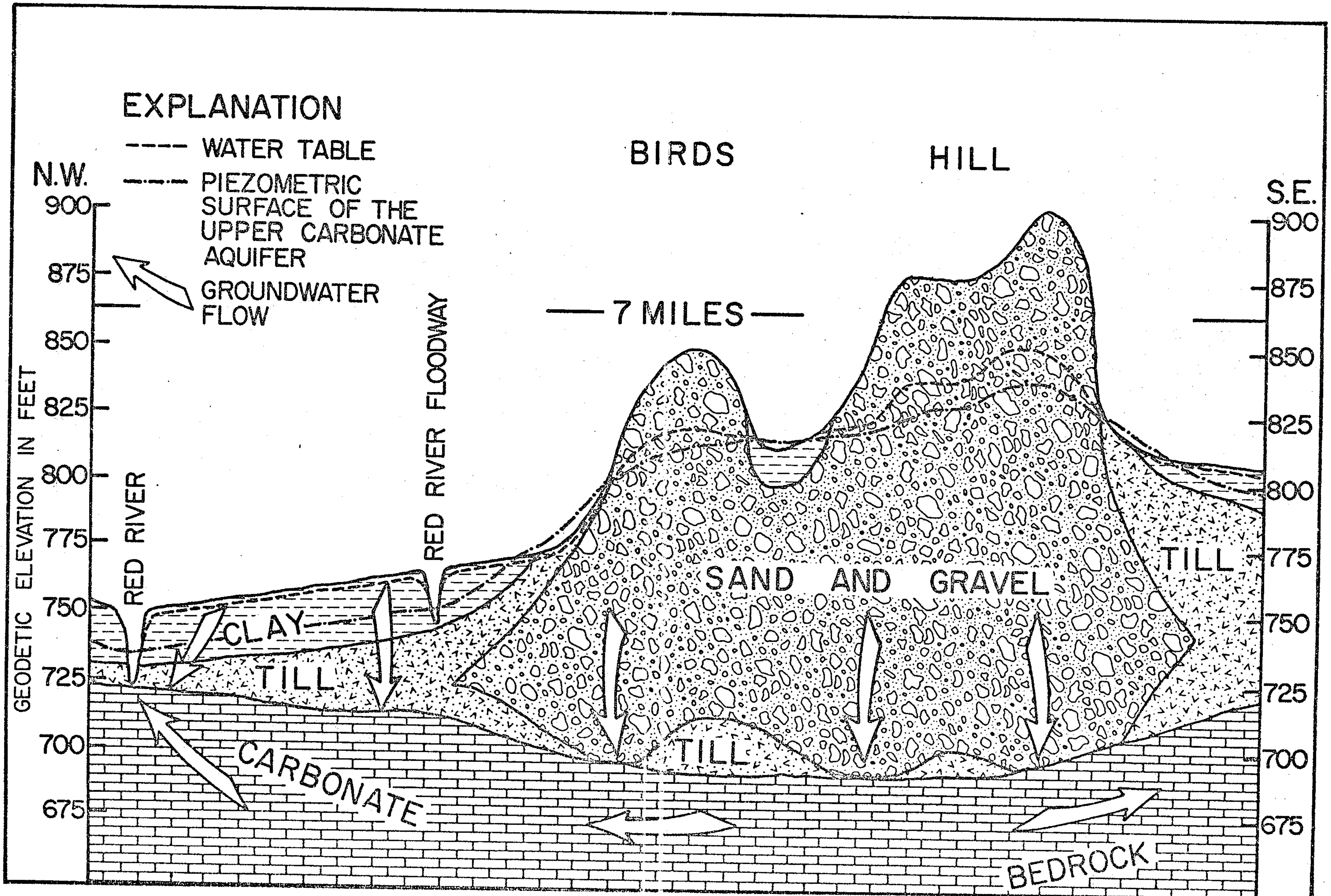


Fig. 12. Schematic geologic cross-section of the Birds Hill Area.

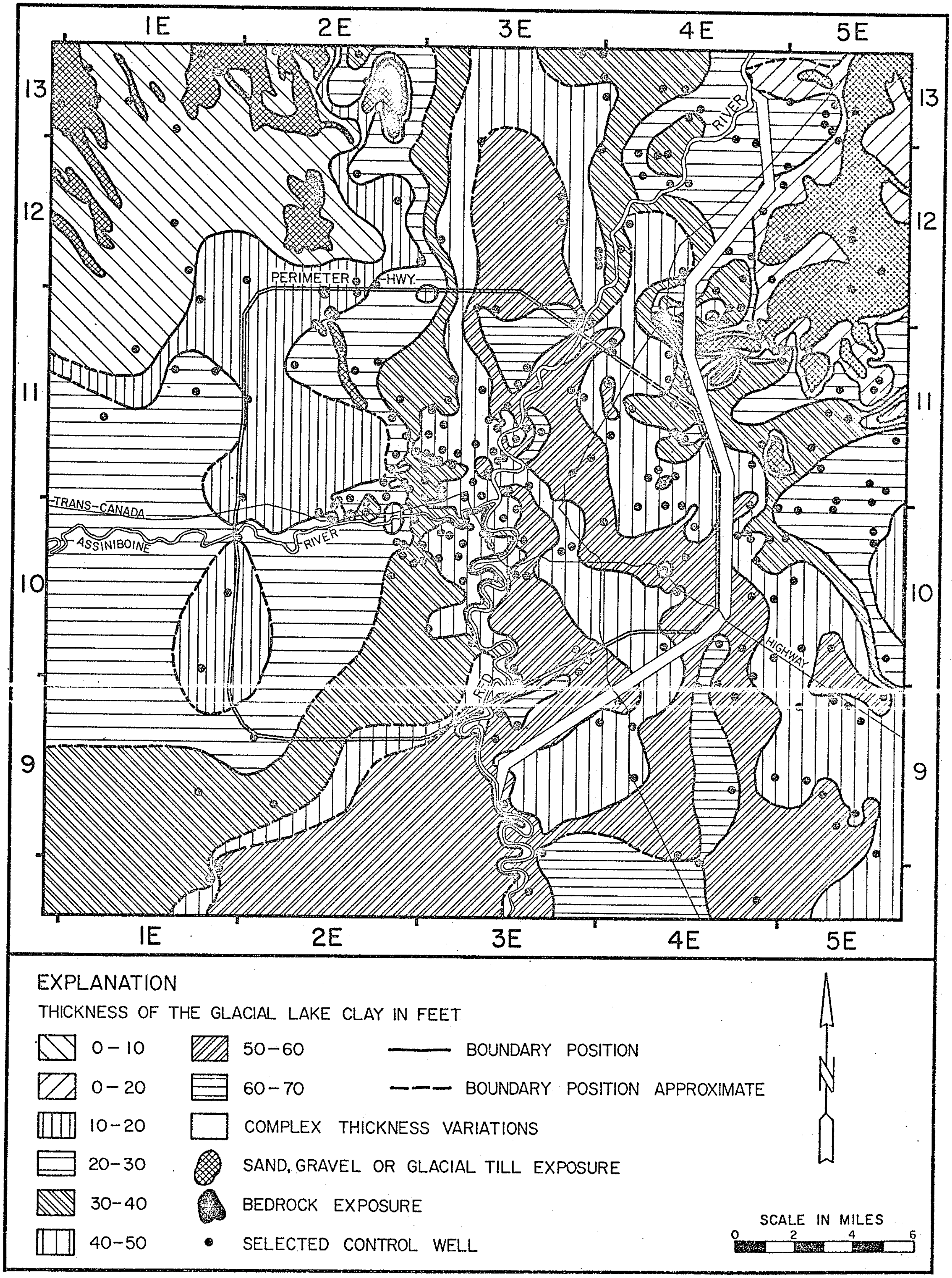


Fig. 13. Thickness of the glacial lake clay.

unit during the floodway excavation. These fractures appear to form a secondary permeability system. Because of the silt and sand beds the upper Agassiz unit has a higher hydraulic conductivity than the underlying clay. The most important hydrogeologic aspects of the lower Agassiz unit are its restriction of recharge to the Upper Carbonate aquifer, and the consequent deterrence of pollution from surface sources.

CHAPTER IV

GROUNDWATER FLOW SYSTEMS

Groundwater Movement in the Upper Carbonate Aquifer

In the Red River Basin the Upper Carbonate aquifer is the most important part of the extensive zone of groundwater movement that occurs within the upper portion of the bedrock. The groundwater regime in this zone can be divided into three segments of predominantly lateral movement: (1) from the east, (2) from the northwest, and (3) from the southwest (Fig. 14). Infiltration in the glacial till upland east of the lacustrine plain and in the Birds Hill aquifer complex recharge the eastern segment (Figs. 4 and 7). Recharge for the northwestern region occurs in the areas of thin glacial till northwest of Winnipeg. The southwestern system appears to be recharged through the thin veneer of glacial till and fluvial deposits that overlie the Mesozoic shale uplands on the western side of the basin.

In 1894 before extensive groundwater pumping occurred, the piezometric surface in the Upper Carbonate aquifer was 1 foot to 4 feet above ground level in northwestern Winnipeg (Johnson, 1934). Adjacent to the Red River in north central Winnipeg the piezometric surface was 10 to 20 feet below

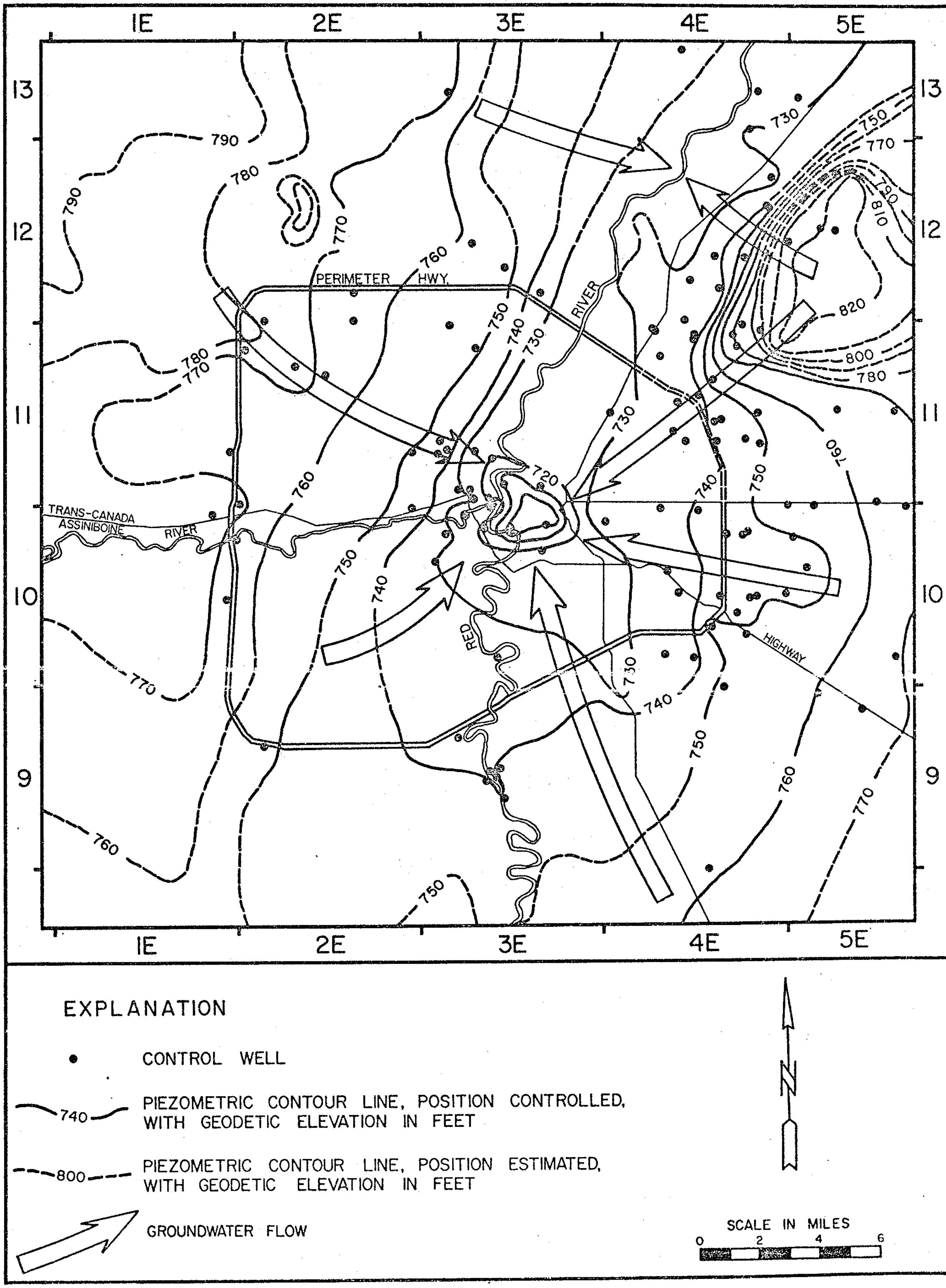


Fig. 14. Piezometric surface of the Upper Carbonate Aquifer.

ground level. Currently in the central part of the city pumping has extensively depressed the piezometric surface. In the central part of the drawdown cone the depths to water range from 70 to 80 feet, and the piezometric elevations are at some places below 700 feet (Fig. 14 and Hydrograph M-105, Fig. 15). Fluctuations of piezometric levels in the Upper Carbonate aquifer during the past six years are shown in the hydrographs in Fig. 15.

On a regional basis the aquifer has been considered to be isotropic and the groundwater flow directions are interpreted as occurring in the direction of the hydraulic gradient (Fig. 14). It is recognized, however, that irregular groundwater flow channels in the joints, fractures, and bedding planes probably created areas of local anistropy.

Evidence of Aquifer Recharge

Major evidence for recharge is provided by the metropolitan pumpage and water level data. Over the eighty year period of major groundwater development (Fig. 3) at least 130 billion gallons of groundwater have been withdrawn from the aquifer. Throughout the basin, springs and flowing wells also discharge large quantities of groundwater. The pumpage in the Winnipeg drawdown cone appears to have affected the piezometric surface of the Upper Carbonate aquifer over an area of 1,300 square miles. If all the water withdrawn had been produced from storage within the aquifer, the average drawdown over this area, assuming a

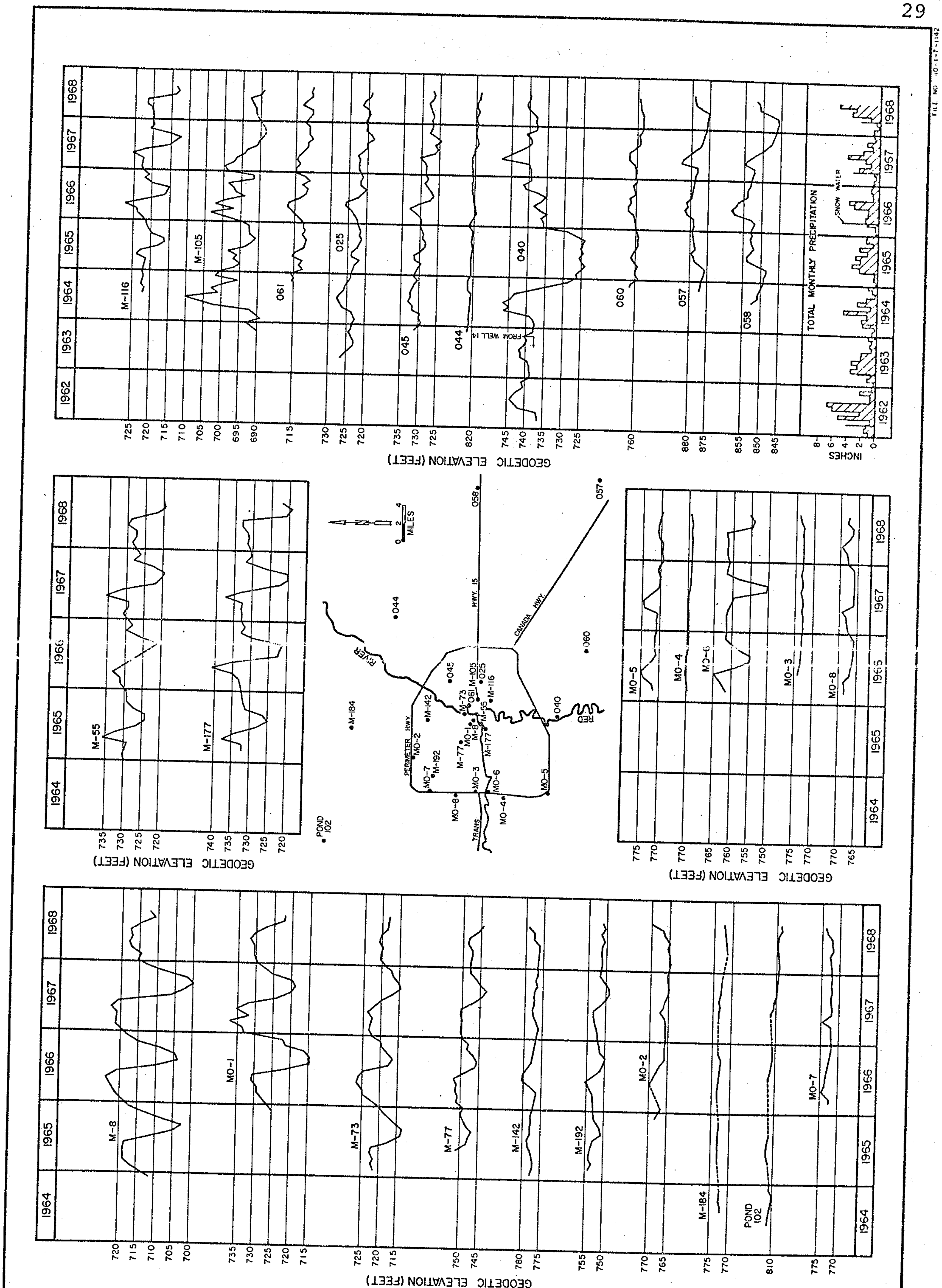


Fig. 15. Selected Upper Carbonate Aquifer groundwater levels.

relatively high storage coefficient of 1×10^{-3}, would have been in the order of 600 feet. The observed maximum drawdown is approximately 60 feet. This storage coefficient and the assumption of lateral recharge produced resonable test results when they were utilized as two of the parameters in a resistor-capacitor analog model (Render et al., 1968).

Since 1947, approximately 50 billion gallons of water have been pumped from the aquifer in the urban area. However, the piezometric surface of the central portion of the drawdown cone has shown only minor fluctuations in elevation. As relatively little water was withdrawn from groundwater storage, the aquifer during this period must have acted primarily as a transmission zone. Another indication that the aquifer is receiving substantial recharge is provided by groundwater data from the floodway area. For over a year, while the floodway has been discharging approximately 3,000 gallons per minute of groundwater, stable water level elevations have been observed in the surrounding observation wells. Furthermore, the line of zero drawdown for the floodway drawdown cone is located near the till upland on the eastern side of the basin. This indicates that there is sufficient recharge available to maintain the draught on the aquifer. Evidence for recharge of the groundwater flow systems in the Upper Carbonate aquifer by precipitation percolating through the glacial till and glaciofluvial deposits located in the uplands along the borders and within the basin is shown by the hydrographs for wells 057 and 058 (Fig. 15). The identification of the Birds Hill as a recharge area is indicated by the dome-shaped piezometric surface of the underlying Upper Carbonate aquifer.

Groundwater Geochemistry

The water in the eastern and northwestern flow regions which cross carbonate rock is fresh in the recharge and slightly brackish and very hard in the discharge areas. The total dissolved solids values range from 300 to 1,500 milligrams per litre and the chloride ion concentrations vary from less than 10 to 500 milligrams per litre (Fig. 16). These two systems are utilized for rural and suburban potable water supplies and for the majority of the urban commercial groundwater pumpage. Only a few miles east of its recharge area the southwestern flow system contains brackish and saline water. The rapid deterioration of the water quality in this system is possibly caused by the water flowing across formations that contain evaporite deposits. Some of the salinity could possibly be due to intermixing with saline waters discharging into the system from a continental flow system (Van Everdingen, 1968). Within the study area total dissolved solids values in the southwestern system range between 2,000 and 10,000 milligrams per litre. As a result of contamination from wells installed into the sandstone aquifers, the chloride ion

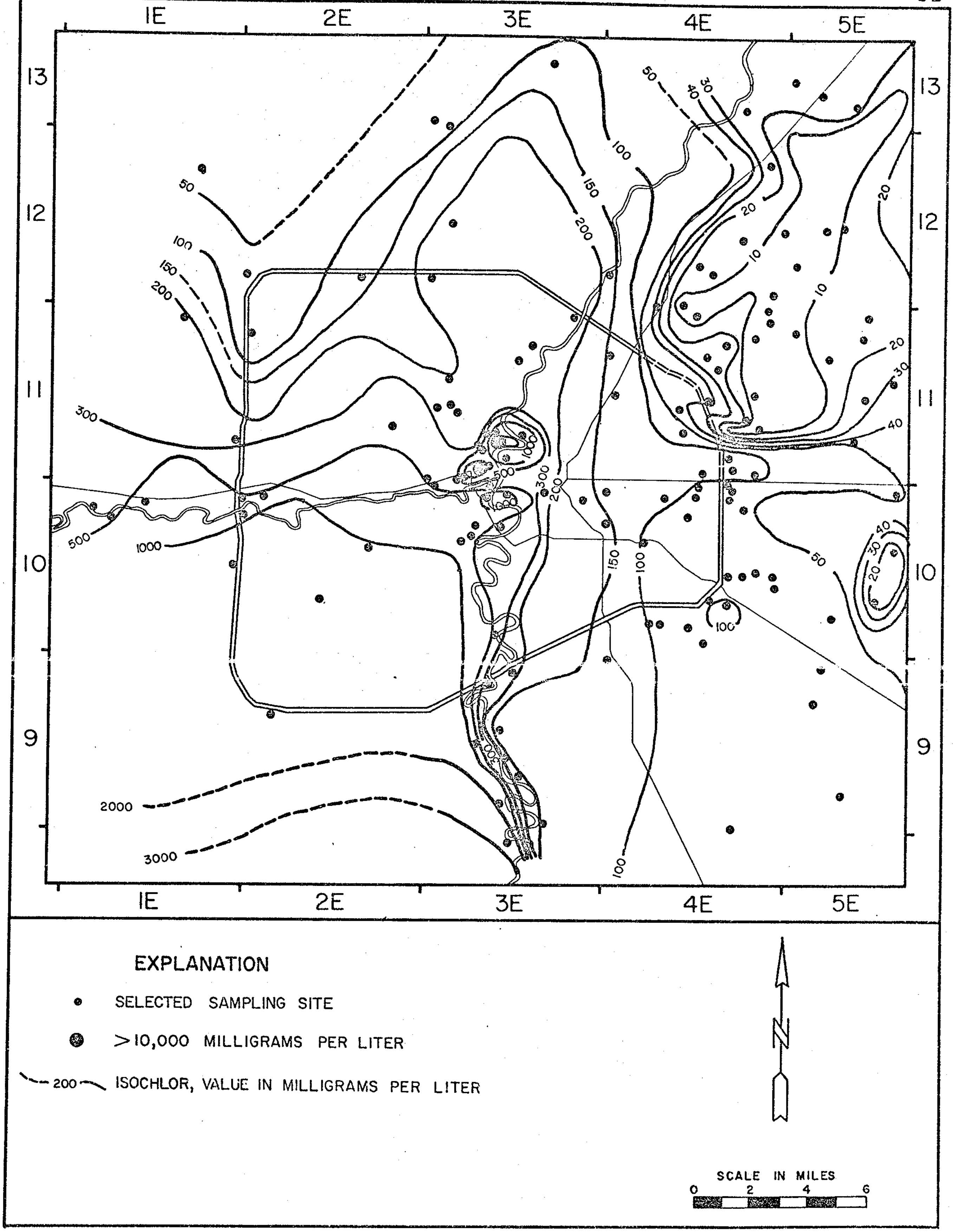


Fig. 16. Upper Carbonate Aquifer chloride ion concentrations.

values under north central Winnipeg are in excess of 500 milligrams per litre (Fig. 16).

The groundwater temperature in the Upper Carbonate aquifer generally varies between 39 and 43 degrees Fahrenheit. At some sites within the urban drawdown cone the temperatures range up to 50 degrees Fahrenheit. Available bacterial data indicate the aquifer water is usually pure.

Vertical and River Recharge

In addition to the regional lateral flow systems there are local vertical flow regimes within the overburden that move water stored in the Upper Agassiz unit down to the Upper Carbonate aquifer (Fig. 12). Because of the permeability of the Lower Agassiz unit the contributions to the water in the Upper Carbonate aquifer are negligible, probably less than one percent of the summer pumpage.

Where the Upper Carbonate aquifer piezometric surface is depressed below the water levels in the Red River in central Winnipeg (Fig. 9) there is a gradient from the river towards the aquifer. An anolog model study of the effect of floodway groundwater discharge on the piezometric surface of the Upper Carbonate aquifer indicated that the river is at least partially connected with the aquifer (Hobson et al., 1964). The anolog incorporated the assumption that the Red River constituted a hydraulic boundary of the aquifer. The assumption was generally verified by the close relationship, in the areas adjacent to the river, of the predicted and observed floodway drawdown cones. The potential difference between the river and the aquifer is greatest during the spring flood period and the summer air conditioning pumpage interval. Within the central Winnipeg area, the quantitative contribution of this flow system appears to be significant.

Groundwater Flow in the Birds Hill Aquifer

The saturated sections of the Birds Hill sand and gravel deposits form an aquifer that is up to 150 feet thick. The water table in the aquifer is at higher elevations than the piezometric surface of the Upper Carbonate aquifer (Fig. 12). Thus water moves down to recharge the underlying Upper Carbonate aquifer. The groundwater flows radially out from under the Hill with the major flow direction being southwest into the urban drawdown cone. The exposed sand and gravel form excellent water intake areas and as a result of the short contact time with the aquifer materials and the continuous downward movement of fresh water, the water in the aquifer has, as indicated by the isochlors on Fig. 16, a low salinity.

Lower Carbonate Aquifer Flow Systems

Though it possibly receives some downward percolation through the overlying slightly pervious carbonate rock, the Lower Carbonate aquifer is primarily recharged where it subcrops, east of Winnipeg under the glacial till (Fig. 9). The direction of groundwater flow appears to be westerly along the top of the upper shale unit of the Winnipeg formation. In the urban area some of the water in the aquifer is discharged through deep wells that penetrate to the top of the Winnipeg formation. The natural discharge area for the aquifer is not known. However, it is possible that vertical discharge occurs through the overlying carbonate rock and glacial drift into the Red River. Chemical tests obtained at three widely separated sites indicate that this aquifer has a significantly higher salinity than the overlying Upper Carbonate aquifer.

Groundwater Movement in the Sandstone Aquifers

The sandstone aquifers are recharged east of Winnipeg where they subcrop under the glacial deposits. The hydraulic gradient is westward towards the Red River Valley. Near the eastern edge of the Agassiz plain the geodetic elevation of the sandstone aquifer piezometric surface is at approximately 890 feet, while in central Winnipeg the piezometric elevation ranges between 740 and 745 feet. The lower values occur under the central section of the drawdown cone in the Upper Carbonate aquifer. The groundwater flows down the dip of the beds at least as far as the centre of the Red River Basin. The areas of natural discharge have not been determined. The down dip phasing out of the Upper Sandstone aquifer into shale severely restricts flow in this aquifer. While the Lower Sandstone aquifer thickens down dip the flow in it has not been evaluated.

Artificial discharge from the sandstone aquifers occurs in southeastern Manitoba where the water in the aquifers is fresh. In metropolitan Winnipeg, where approximately six wells have been drilled through to the Precambrian rock, water in these aquifers has total dissolved solids between 30,000 and 80,000 milligrams per litre. The water is generally unsuitable for most industrial and commercial purposes and at present none of the wells is used for water supply.

As part of the study of the hydrogeologic conditions in metropolitan Winnipeg two observation wells were installed in the sandstone aquifers (Fig. 17). In the central area of the Upper Carbonate drawdown cone the hydrographs indicate that the piezometric elevations in the sandstone aquifers are twenty or more feet above the piezometric surface of the Upper Carbonate aquifer. This hydraulic gradient induces saline water from the sandstone aquifers into the carbonate aquifer through open bore holes and abandoned wells. A further indication of the connection between the carbonate and sandstone aquifers in this area is the correlation in the trend of water level fluctuation in the two aquifers (Hydrographs MO-11 and 061, Fig. 17). Conversely in the northwestern section of the drawdown cone, where no wells have been installed in the sandstone, there is no relationship between the Upper Carbonate and Sandstone aquifer piezometric surface fluctuations. The salinity contamination that has resulted in north central Winnipeg is shown on Fig. 16.

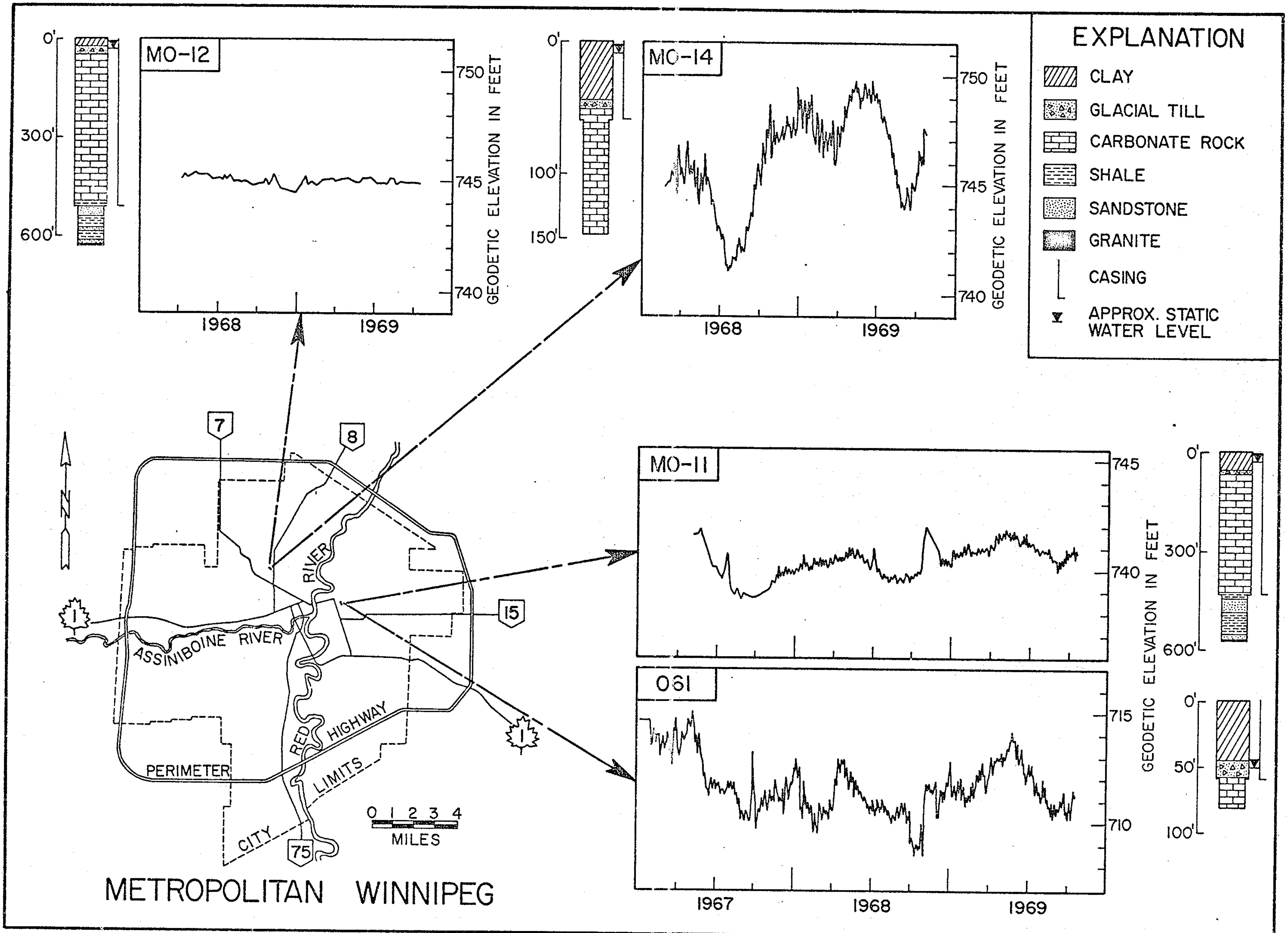


Fig. 17. Comparison of Upper Carbonate and Sandstone Aquifer water levels.

CHAPTER V

GROUNDWATER DEVELOPMENT

During the early development of Winnipeg, the Upper Carbonate aquifer was an important source of municipal and industrial water. From 1900 to 1919 pumpage from the aquifer fulfilled the water requirements of the first city-owned water system. The 1969 estimated annual pumpage of 3 billion gallons per year (Fig. 3) is approximately seventeen percent of the total annual consumption for the metropolitan Winnipeg area. Because of its constant low temperature groundwater is mainly used for commercial and industrial cooling. However, groundwater is also used for irrigation and domestic water supplies. Presently the groundwater withdrawals in Winnipeg vary from five million gallons per day in the winter to ten million gallons during the summer when the air conditioning demand is at a maximum (Fig. 18).

Phase One

The first settlers drew their water from the rivers, however as the settlement moved back from the river, water from individual and community wells was delivered by horse drawn wagons. In 1880 the Winnipeg Water Works Company was

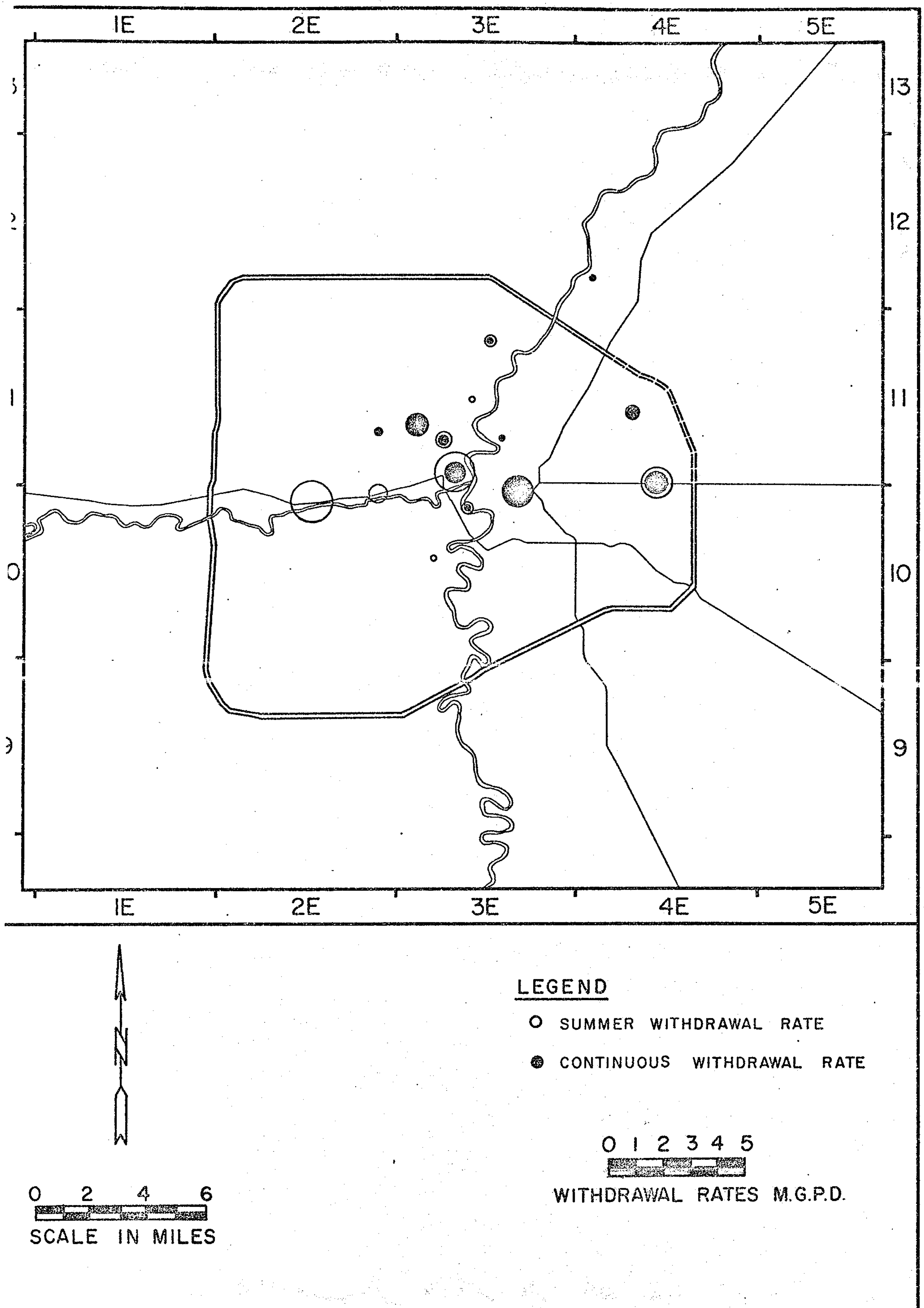


Fig. 18. Upper Carbonate Aquifer withdrawal rates - 1965.

enfranchised for a period of twenty years and began operations in 1882 drawing from the Assiniboine River at Armstrong's Point. During the eighteen hundreds, flowing wells were present in northwest Winnipeg (Johnson, 1934) and it is estimated that from 1882 to 1900 the groundwater withdrawal rate was one million gallons per day (Fig. 3).

In 1889 the City of Winnipeg purchased the Winnipeg Water Works Company and in October,1900, converted the system from the polluted and sediment-laden Assiniboine River water to groundwater from the Upper Carbonate aquifer. From 1890 to 1914 the City of Winnipeg examined as altenate water sources, the Red, Assiniboine and Winnipeg Rivers, the Upper Carbonate aquifer and the Lake of the Woods which was chosen as the supply for the aqueduct system. Immediately prior to completion of the Greater Winnipeg Aqueduct in 1919, a field of 29 wells extending from northwest Winnipeg ten miles north at one-half mile intervals, to Stony Mountain, Manitoba (Fig. 2) produced ten million gallons per day. These wells had capacities up to 500 gallons per minute and ranged in depth from 85 to 300 feet and in diameter from 1.5 to 15 feet. By 1919, as a result of the municipal pumping, water levels in northwest Winnipeg had declined to approximately forty feet below ground level. During this same interval St. Boniface obtained water from a field of six wells located in the north central part of the city. While some of the original city wells are still maintained for emergency purposes, major groundwater withdrawals ceased following the completion of the aqueduct, and the piezometric surface of the Upper Carbonate aquifer recovered to its natural position close to the ground surface.

The groundwater system was abandoned primarily because of excessive natural hardness and sulphate and because groundwater technology had not become sophisticated enough to allow the long term yield of the aquifer to be predicted. In addition, the maintenance cost of the groundwater system was anticipated to be higher than that of the gravity flow aqueduct. Although groundwater was not satisfactory as a long term water supply, it played an important role by allowing the city to attain the size where financing of the aqueduct was feasible.

Phase Two

A second phase of groundwater development began in the 1920's when commercial centres of pumping were created in the central urban area, and also meat packing plants established high capacity wells in St. Boniface. A substantial drawdown cone developed primarily east of the Red River (Fig. 14) and well productivity decreased because of the decline in the piezometric surface. As the water levels in the St. Boniface area are much the same today as they were in 1947, the drawdown cone appears to have stabilized. In central Winnipeg, wells were installed by theatres and restaurants for air conditioning and by cold storage plants which required large quantities of cold water during their peak operating periods. Another feature of this phase of development was the institution of the groundwater sewage tax by the City of Winnipeg. This tax, which still exists, economically encourages the recharging of spent cooling water to the aquifer and between 1940 and 1960 eight recharge systems came into operation.

During the second phase of groundwater development at least four wells penetrated to the Precambrian. Where these wells were of poor capacity there was little contamination of the carbonate aquifers due to saline water discharge. Also, in areas such as northwest Winnipeg where the hydraulic heads in the aquifers are essentially identical (Hydrographs for wells MO-12 and MO-14, Fig. 17) relatively little damage was done. However where the above stated conditions did not prevail (Hydrographs for wells 061 and MO-11, Fig. 17) saline water flowed up into the fresh Upper Carbonate aquifer.

Phase Three

Since 1960 there has been an increase in apartment house and hotel construction. The consequent installation of high capacity wells for air conditioning, has constituted a third phase of groundwater development. Some of these units are supply-recharge well installations which eliminate the effluent sewage tax and conserve groundwater. These systems usually operate on a large volume of water which is raised a few degrees in temperature while passing through the cooling unit.

Systems that are adapted to the corrosiveness of the water allow the brackish groundwater that occurs south of the Assiniboine River to be used for cooling apartment buildings. The development of apartment building air conditioning has also created large summer pumpage centres in the suburban areas located along the Assiniboine River in the western sections of the City (Fig. 18). The air conditioning water demand places a short term high capacity stress on the system, as illustrated by representative hydrographs for the urban area (Fig. 15).

Of approximately two hundred commercial and municipal wells that have been developed in the Winnipeg area, about 115 are still operating. The estimated annual pumping rates for the operating wells, which draw nearly exclusively from the Upper Carbonate aquifer, are presented in Fig. 3. In the central section of the area groundwater withdrawals combined with narrow well spacing have lowered the piezometric surface close to the main producing zones of the aquifer so that there is little remaining available drawdown. At some places during the summer period of heavy pumpage, the top of the aquifer is dewatered and the effective transmissibility is reduced.

Well Construction

The construction of most domestic wells consists of a four inch diameter casing drilled into the top of the bedrock with an uncased shaft extending to the point where sufficient water is encountered. The depth of the domestic wells varies from 25 to 400 feet, with the average being between 100 and 150 feet. The similarly constructed commercial wells are usually of greater diameter and depth, often being extended to the top of the Winnipeg Formation. Commercial well specific capacities range between 0.64 and 205 and average 15 gallons per minute per foot of drawdown. In constructing a high capacity well the casing must be properly seated in solid rock so that the silt and rock flour in the lower sections of the glacial till do not enter the well. At the same time care must be exercised to ensure that the casing does not block off producing zones in the upper portions of the bedrock.

CHAPTER VI

ELECTRIC ANALOG AND DIGITAL COMPUTER MODELS OF THE UPPER CARBONATE AQUIFER

Principle of Resistance - Capacitance Analog Computers

Resistance-Capacitance electric analog computer techniques have been used for reservoir evaluation by the petroleum industry since the early 1940's. However, this method did not come into use for groundwater evaluation until the early 1960's. Bermes (1960) and Skibitzke (1961) converted the electric analog to a tool that could be used in aquifer evaluation. Walton and Prickett (1963) published several papers on the value of analog computers in evaluating the types of aquifers encountered in Illinois. Subsequently, numerous other investigators have used analog computers as a working tool in evaluating aquifer potentials, mainly with respect to urban and industrial aquifer developments. The electrical theory and analog techniques employed by these investigators are essentially those given by Karpulus (1958), and Walton and Prickett (1963).

Theory of Electric Analog Aquifer Models

The electric analog computer is a technique for solving the groundwater flow equations. In order to obtain a unique solution the aquifer parameters and boundary conditions must be specified. The foundation of the technique is a mathematical expression that describes the physics of flow of a compressible fluid through an elastic porous medium. The following discussion is taken from Pinder and Bredehoeft (1968) which gives a review of the basic hydraulic and electrical theory of analog aquifer models.

The partial differential equation governing the non-steady state, two dimensional flows of groundwater in heterogeneous, anisotropic and elastic porous medium is:

$$\frac{\partial}{\partial x_i}\left(T_{ij} \cdot \frac{\partial h}{\partial x_i}\right) = S\frac{\partial h}{\partial t} + W(x,y,t) \qquad (1)$$

where h is the hydraulic potential, S is the storage coefficient, T is the transmissibility tensor, t is time and W is the volume flux per unit area.

Consider the aquifer divided into an equal square area grid (Fig. 19). The intersection of the grid lines are called nodes. The grid width, a, approximates the sides Δx and Δy of a finite-difference grid. That is, the element area, a^2, is very small compared to the area of the aquifer.

The finite difference expression for the flow equation at node ij is:

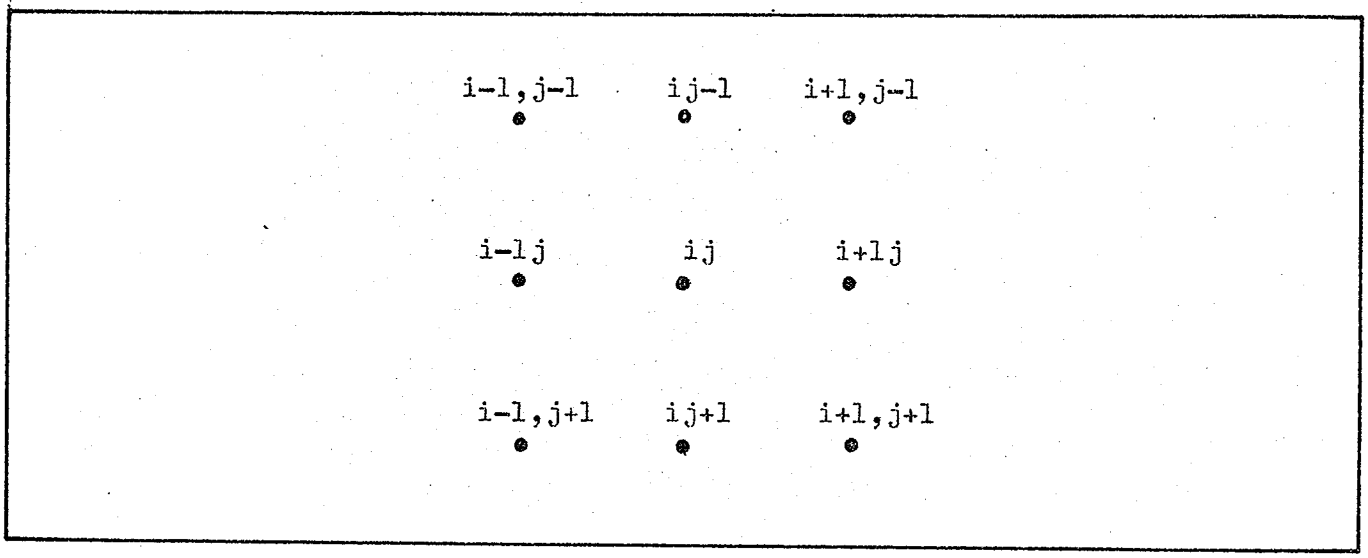


Fig. 19. Nodal array for development of finite difference expressions.

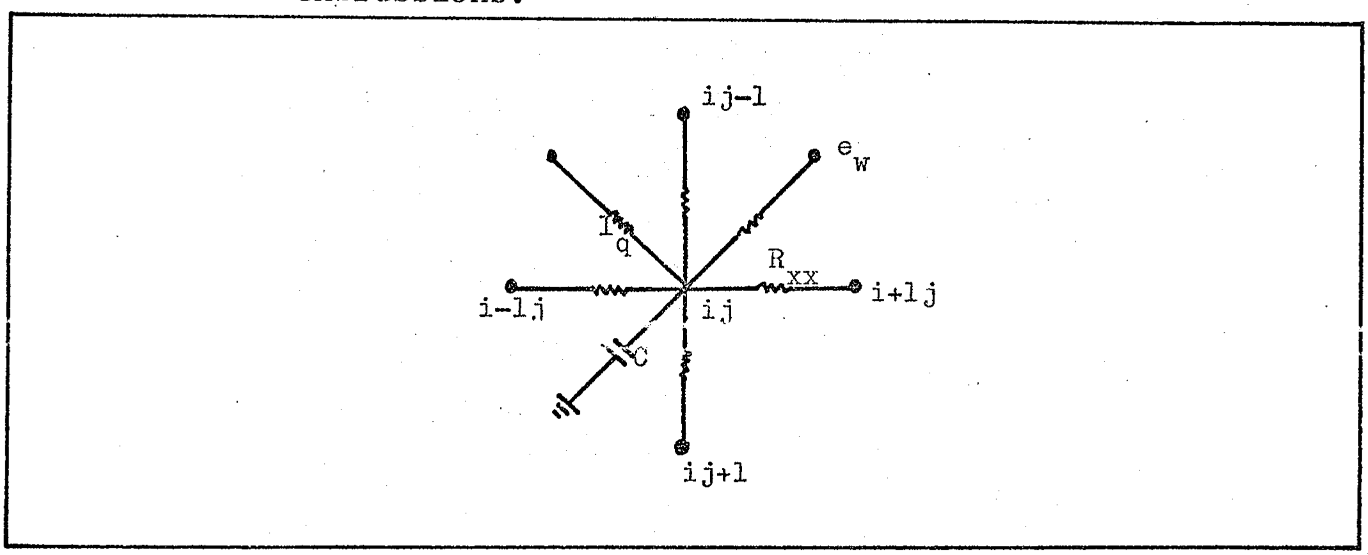


Fig. 20. Resistor-capacitor network for developing electric analog.

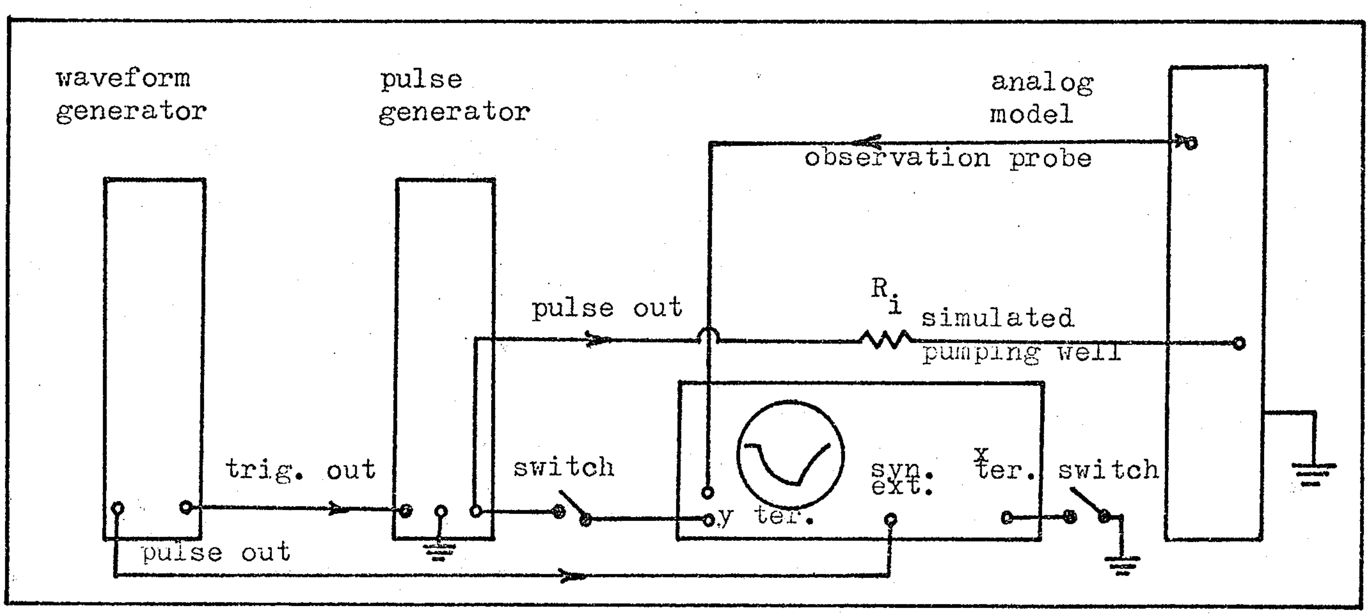


Fig. 21. Schematic diagram of analog computer components.

$$T_{xx(i-\frac{1}{2}j)} \cdot \frac{(h_{i-1jk} - h_{ijk})}{(\Delta x)^2} + T_{xx(i+\frac{1}{2}j)} \cdot \frac{(h_{i+1jk} - h_{ijk})}{(\Delta x)^2}$$

$$+ T_{yy(ij-\frac{1}{2})} \cdot \frac{(h_{ij-1k} - h_{ijk})}{(\Delta y)^2} + T_{yy(ij+\frac{1}{2})} \cdot \frac{(h_{ij+1k} - h_{ijk})}{(\Delta y)^2}$$

$$= S\frac{(h_{ijk} - h_{ijk-1})}{\Delta t} + W(x,y,t) \qquad \ldots (2)$$

where i, j, and k are the indices in the x, y, and t dimensions, and Δx, Δy, and Δt are increments in the x, y, and t dimensions.

In a resistance-capacitor network with a square pattern and network junctions consisting of six resistors and one capacitor (Fig. 20), the relationship of the electrical potential in the vicinity of the junction according to Kirchoff's law is:

$$\frac{1}{R_{xx(i-\frac{1}{2}j)}} \cdot (e_{i-1j} - e_{i,j}) + \frac{1}{R_{yy(i,j-\frac{1}{2})}} \cdot (e_{ij-i} - e_{ij})$$

$$+ \frac{1}{R_{xx(i+\frac{1}{2}j)}} \cdot (e_{i+1j} - e_{ij}) + \frac{1}{R_{yy(ij+\frac{1}{2})}} \cdot (e_{i+1j} - e_{ij})$$

$$= C\frac{\partial e}{\partial t} + I_q + \frac{1}{R_z} \cdot (e_{ij} - e_w) \qquad \ldots (3)$$

where e is the electromotive force, I is the electric current, R is the resistance, and C is the capacitance.

The analogy between the hydrologic and electrical systems is apparent from equations (2) and (3).

Modelling Scale Factors

In addition to the above analogies, the following also exist; the flow of water through the aquifer according to Darcy's law

$$\vec{Q} = -K\ \vec{grad}\ h$$

where Q is the flow of water and K is the hydraulic conductivity, is similar to the flow of electricity as stated by Ohm's law

$$\vec{I} = -\frac{1}{k}\ \vec{grad}\ e$$

The two networks are related in four ways: gallons are related to coulombs, head is related to volts, gallons per day are related to amperes, and time corresponds to time. From the relationship of Darcy's and Ohm's laws it is evident that in order to model the aquifer system, there must be four scale factors. These scale factors, Bermes (1960), are: K_1 which relates gallons to coulombs, K_2 scales feet to volts, K_3 equals gallons per day per ampere, and K_4 converts model time in seconds to aquifer time in days.

The following formulae set forth the scaling relationships:

$$q_{gal} = K_1 C \text{ coulombs} \quad \ldots (4)$$

$$H_{feet} = K_2 V \text{ volts} \quad \ldots (5)$$

$$Q_{gal/day} = K_3 I \text{ amperes} \quad \ldots (6)$$

$$T_{days} = K_4 t \text{ seconds} \quad \ldots (7)$$

In choosing scale factors, the following relationship must always be met:

$$K_3K_4/K_1 = 1 \qquad \ldots (8)$$

To produce a formula for calculating the size of the resistors for the model, Darcy's Law and Ohm's Law are substituted in equation 6 to yield.

$$R = \frac{K_3}{K_2T} \text{ ohms} \qquad \ldots (9)$$

Equation 9 is used in conjunction with vector area methods, Karplus (1958) to calculate the model resistor values. The capacitor size is derived from Coulomb's Law for the electric charge of a capacitor and the relationship of C to a^2S. The resultant formula is:

$$C = 7.48\, a^2S \left(\frac{K_2}{K_1}\right) \text{ farads} \qquad \ldots (10)$$

Simulating Aquifer Boundary Conditions

Vertical Recharge Boundary

Vertical recharge can occur in a semi-confined aquifer through the aquitard from an overlying source. This situation can be either of considerable areal extent, covering many square miles, or limited, as in the case where the source is a river and the leakage was through an underlying aquitard. In either case, there must be a gradient from the source through the confining bed to the aquifer. The general formula for calculating the resistors that represent the vertical permeability of the aquitard is:

$$R_G = \frac{K_3}{K_2\ (P'/M')\ a^2} \qquad \ldots (11)$$

where R_G is the value of the resistor in ohms, K_3 and K_2 are scaling factors as described previously, P' is the vertical permeability of the aquitard, M' is the thickness of the aquitard, and a is the node spacing. In calculating the value of vertical resistors under a river bed, M' becomes the thickness of the aquitard under the river bed, and the term a^2 is replaced by the product of width of the river, and the length of channel, represented by each node simulating a portion of the river channel.

Lateral Boundaries

Special boundary procedures are required where two different grid spacings are in juxtaposition, or where a grid system must be terminated. The boundary techniques utilized in this study are called streamline and equipotential boundaries.

Streamline Boundaries

Resistors along streamline boundaries usually have different values from those representing equivalent conditions in the interior of the model system. The vector area for a streamline resistor is only half the size of that of network interior resistors. Therefore, a streamline boundary resistor represents a component of the original field that has only half the transmitting capability of an interior grid element. Resistor values are proportional to the inverse of the transmissibility. Consequently, along the boundaries, the resistor values must be twice as large as the interior resistors. The streamline boundary techniques were used in this study to effect the matching of coarse and fine grid sections. The procedures used are given by Karplus (1958).

Equipotential Boundaries

In the case of a boundary zone where all nodes are maintained at equal voltage, the resistors along the boundary are eliminated as they are all effectively shorted out.

A portion of the aquifer may directly intercept a surface water body such as a river or lake. If the water level in the water body can be considered to be essentially stable, the model boundary can be represented by an infinite source of electricity at constant potential. This type of boundary is constructed by grounding all the nodes that occur along it and by eliminating all the resistors between these nodes.

In order to represent the horizontal flow of recharge water into the boundaries of the modelled area, terminal resistors are connected from the boundary nodes to the electric ground. The values of these resistors are usually estimated from the permeability of the area they represent. Winslow and Nuzman (1966) used a terminal resistor value that was ten times that of the average value of an internal resistor. This approach would represent either a further ten units of aquifer area, or a ten fold decrease in aquifer permeability within a unit area. When there is a voltage drop within the model interior, these terminal resistors allow a trickle of electricity to enter the boundaries of the model in the same manner that groundwater flows into the edges of an aquifer when there is a potential drop within it. Boundary resistors have a critical effect on the accuracy of the model when the voltage drop along the model boundaries is large.

The Upper Carbonate Aquifer Electric Analog Model

The analog model was constructed and tested by the author and students in the Department of Earth Science at the University of Manitoba. The model was built in an attempt to simulate the aquifer conditions in the metropolitan Winnipeg area. The aquifer was simulated as being anistropic, heterogeneous and semi-confined with lateral and river recharge. Standard construction techniques utilizing peg-board, resistors, and capacitors were used in forming the analog. The difference between the 1965 summer and non-summer pumpage was simulated on the model. The simulated and observed drawdown data for the Upper Carbonate aquifer during the summer of 1965 were compared to test the accuracy of the model.

Design and Construction of the Model

The analog model was constructed with a piece of ¼ inch pegboard perforated with holes on a 1 inch square pattern. A transmissibility contour map of the Upper Carbonate aquifer under metropolitan Winnipeg at a scale of 1 inch = 5,000 feet was fastened over the pegboard. A grid system was established and the transmissibility value for calculating resistor sizes was obtained from the contour map by means of the vector area technique, Karplus (1958). In each case resistor size was determined from equation 9. Brass shoe eyelets was inserted into the pegboard and four resistors and a capacitor were connected to each node. The size of the capacitor at each node was determined from the value of the storage coefficient for the particular area and equation 10.

Transmissibility

The transmissibility values for the detailed model areas were obtained from the data presented on Fig. 11. For the vector areas outside the detailed zones, the transmissibility values were obtained empirically from the spacings of the piezometric contours for the bedrock surface zones.

Storage Coefficient

The storage coefficient value used over most of the model area was 1×10^{-3}. In the detailed zone, this value was ascertained from pumping tests. In the outlying sections of the model the value was based on several pumping tests, piezometric gradients, and on the nature of the geology.

In the central section of metropolitan Winnipeg a storage coefficient of 1×10^{-5} was used. In the Birds Hill area where the sand and gravel are in contact with the Upper Carbonate aquifer a value of 2.5×10^{-2} was used for the storage coefficient.

Model Scale Factors

The values for the model scale factors that were used initially were:

$$K_1 = 1 \times 10^5 \text{ gal/coulomb}$$

$$K_2 = 1 \text{ ft/volt}$$

$$K_3 = 5 \times 10^{10} \text{ gal/day/ampere}$$

$$K_4 = 2 \times 10^4 \text{ days/second}$$

The choice of the constants is essentially an arbitrary procedure. The value of K_2 was chosen as 1 for convenience. The remainder of the scale factors were selected so that readily available and inexpensive resistors and capacitors, and available excitation-response apparatus could be used. Standard tolerance, low wattage fixed carbon resistors with values from approximately 10^2 to 10^7 ohms and capacitors of low voltage rating with values of 10 to $10^{15}/\mu f$ were chosen. During evaluation of the model, it was found that the required voltage for a given pumping rate could not be obtained in the centre of the model and the following changes in constants were made:

$$K_1 = 5 \times 10^{15} \text{ gal/coulomb}$$

$$K_2 = 5 \text{ ft/volt}$$

$$K_3 = 25 \times 10^{10} \text{ gal/day/ampere}$$

Model Boundaries

Resistors around the boundaries of the small scale portion of the model have values twice that of the inside resistors representing the same transmissibility value. This is because the vector area represented by the boundary resistors is ½ as large (Karplus, 1958). Because the pumping was performed in the fine scale grid, it was possible to use a larger grid spacing around the finer one to approximate the large groundwater reservoir. The inaccuracy of using this larger grid is not believed to be significant. A grid spacing of 1 inch = 20,000 feet (4 times that of the fine grid) was used and the aquifer was extended 12 miles on each side. The method used to affect the variation in net spacing was to regard the coarse and fine regions of the analog system as separate fields and to treat the interface between the two as a streamline boundary. The resistors along the coarse grid boundaries were doubled in the same manner as the fine grid boundary resistors.

Capacitors were chosen for the coarse grid in the same manner as they were for the fine grid except that (a) in equation 10 is 20,000 feet instead of 5,000 feet.

Resistors were connected to the nodes around the boundaries of the large scale grid and to ground to simulate the resistance to flow of water from the recharge areas to the model boundaries. On the western boundary, 10 times the estimated average interior resistance was used. The Sandilands Forest Reserve area east of the modelled region is a major recharge zone. Variable resistors were used so that the simulated recharge from the Sandilands area could be varied. For each eastern boundary node, resistance values of 20A, 10A, 5A, and 2A, where A is the average value of the interior resistance, were installed.

At certain localities in the centre of Winnipeg, the Red and Assiniboine Rivers are acting as recharge areas for the Upper Carbonate aquifer. The relationship is dependent upon the river stage. However, if leakage through the confining bed into the aquifer is vertical and proportional to the drawdown, the hydraulic head in the river can be assumed to be more or less constant. This leakage can be simulated in the model by the addition of resistors connected to ground and to each node of the network representing a river location. The size of resistor used in simulating leakage from the Red and Assiniboine Rivers was calculated from equation 11. For each of 15 nodes along the rivers, variable resistors were installed that could simulate vertical permeabilities of 0.009, 0.02, 0.04, and 0.1 gal/day/ft^2 as well as an open circuit. Because values for the coefficient of permeability of the till in the Winnipeg area were not known, the above range of values for the hydraulic conductivity were chosen from Todd (1959).

Evaluation of the Upper Carbonate Aquifer Analog Model

Excitation-Response Apparatus

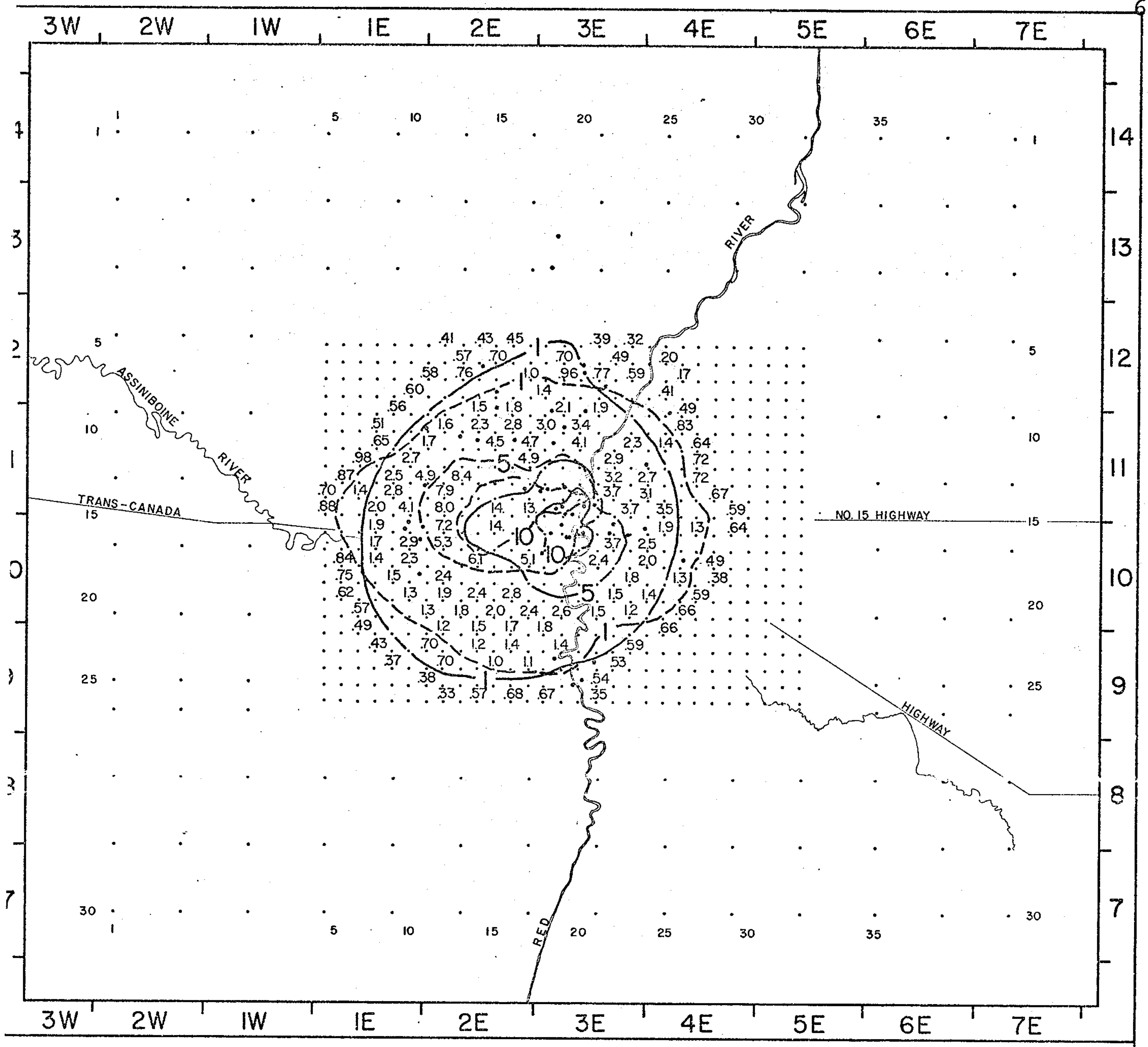
The excitation-response apparatus (Fig. 21) consists of a waveform generator, a pulse generator, and an oscilloscope. The apparatus forces electrical energy, in the proper time phase, into the analog model and measures energy levels within the energy-dissipative resistor-capacitor network. The apparatus is also used to determine "V_r" in equation 12 below. The waveform generator produces a sawtooth wave pulse which synchronizes the sweep of the oscilloscope with a negative rectangular wave produced by the pulse generator. The rectangular wave has an amplitude analogous to the pumping rate (Q) and a wave duration time analogous to the pumping period (t_d). The pulse is passed through a resistor and is then connected to the analog model at a node which represents a pumping center. The pumping rate is related to the voltage and the resistor by equation number 12, (Walton and Prickett, 1963).

$$Q = \frac{V_r K_3}{1.44\ R_i} \qquad \ldots (12)$$

where Q is the pumping rate, R_i is the resistor value through which the pulse is passed, and V_r is the effective voltage across the resistor.

The excitation apparatus is connected to the model and the pulse width set to simulate the pumping period. The voltage drop across the resistor in the excitation connection is adjusted by use of the pulse amplitude until the voltage required to simulate the pumping rate is measured on the oscilloscope. The apparatus is then reconnected, and the oscilloscope scale adjusted so that the voltage drop within the network can be measured against time. The observation probe is then moved to each node in the model and the voltage drop at the end of the pumping interval is recorded as feet of water level drawdown.

Nodes on the analog model (Fig. 22) were numbered 1 to 30 in a north-south direction and 1 to 37 in an east-west direction with the number 1 nodes in the north-west corner of the map sheet. The nodes which were used in the testing ranged from 5 to 33 in an east-west direction and from 5 to 26 in the north-south direction. The reason for this choice being that in the outer areas of the model, voltage drops were insignificant due to the effects of the boundary regions.



EXPLANATION

3.6 ANALOG NODE WITH SIMULATED DRAWDOWN IN FEET

• OBSERVATION WELL

---5--- LINE OF EQUAL DRAWDOWN BASED ON SIMULATED DRAWDOWN DATA

——5—— LINE OF EQUAL DRAWDOWN BASED ON OBSERVATION WELL DATA

DURATION OF PUMPING = 100 DAYS.

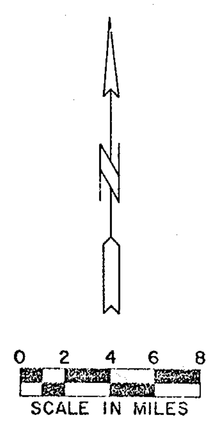


Fig. 22. Analog simulation 1965 summer drawdown.

Model Experiments

Simulated pumping of the model was performed at the nodes which corresponded to major centers of increased groundwater pumping in the city during the summer of 1965 (Fig. 18). Voltage drop observations were taken in order to observe the simulated effect of the increase in pumping on the aquifer piezometric surface. The summer pumpage season is here defined as the average number of frost free days which Winnipeg experiences, approximately 100 days. The pumping rates were obtained graphically from Fig. 18. The centers, in the order they were pumped, are shown in Table I.

The first simulated pumping center tested was located at node 14-19. The following describes the procedure used at this node. The procedure for all other "pumping center" nodes was similar. The values used for each center are listed in Table I.

The pulse generator output was connected at node 14S-19E and the pumping time was set at 100 days to match the length of the summer pumping. The setting for the analog pumping time was determined by substituting the desired pump time (100 days) into the equation,

$$t_d = K_4 t_s$$

where t_d is the pump time in days, t_s is the model pump time in seconds and K_4 is equal to 2×10^4 days/second, t_s was

TABLE I

Analog Model Pumping Rate Data

Test Number	K_3	Pumping Center Location Node	Pump Time (days)	Pump Rate (10^5 gal per day)	Resistance V_r (ohms)	Voltage Drop (volts)
1	$25x10^5$	14S-19E	100	8.	22,000	0.106
2	$5x10^5$	15S-16E	100	6.	33,000	0.571
3	$5x10^5$	16S-14E	100	16.	13,000	0.600
4	$5x10^5$	18S-19E	100	1.25	180,000	0.650
5	$5x10^5$	13S-19E	100	.50	430,000	0.620
6	$5x10^5$	10S-21E	100	.60	330,000	0.570
7	$5x10^5$	16S-20E	100	1.0	220,000	0.634
8	$5x10^5$	15S-26E	100	1.1	110,000	0.633

determined as 5 milliseconds. However, because of extraneous electrical interference, setting the pulse width multiplier at 5 milliseconds did not insure an exact reading of 100 days for the pump time. Therefore, the apparatus was set to give a negative rectangular wave which measures one centimeter horizontally on the oscilloscope scale.

The apparatus was connected so that V_r in equation 12 could be measured. The value of Q for node 14S-19E was equal to 800,000 gal/day; R_i was 22,000 ohms; K_3 was 25×10^5 gal/day/ampere; V_r was calculated as 0.106 volts. The pulse amplitude was adjusted to 19 volts and a voltage drop of 0.106 volts was obtained on the oscilloscope. It was necessary to change to a K_3 scaling factor of 25×10^5 in order to set the voltage drop at 0.106. This did not effect the final readings, however, because K_1, K_2 and K_4 were kept constant and the drawdown observed on the oscilloscope was multiplied by 5 giving a net operative effect of zero.

Now the model was arranged so that current withdrawal and the observation of voltage drop (drawdown observations) could commence. The method of measuring the drawdown was to connect the observation probe to a node and then to measure the indicated drawdown on the oscilloscope. Drawdowns were measured at 638 nodes for each pumping center and recorded on the data sheets. After all the pumping centers had been pumped, the eight drawdowns recorded for each node were totalled so that the total drawdown at each node for all of the pumping centers was obtained. These drawdowns were plotted on a grid of the analog model and the values were contoured (Fig. 22).

Digital Model of the Upper Carbonate Aquifer

Introduction

A digital approach to aquifer evaluation, which was more flexible for the single layer case than the electric analog, was provided by Pinder and Bredehoeft (1968). This technique indicated that an exact and more detailed model of the Upper Carbonate aquifer could be efficiently obtained by the digital method. Accordingly, a model of the groundwater conditions in the bedrock surface zone was developed which encompassed most of the Red River Basin in Manitoba. This model was evaluated by simulating the 1965 summer increase in groundwater withdrawal from the aquifer and then comparing the simulated drawdown of the piezometric surface with that observed in the field. A number of different models were attempted by varying the values for leakage through the confining units and the river bed hydraulic conductivity. The procedure was continued until a satisfactory representation of the field drawdown was obtained. As the pumping rates used were estimations, no attempt was made to accomplish a better fit between the simulated and observed values by altering the transmissibility and storage coefficient parameters.

Theoretical Development of the Aquifer Model

The groundwater flow equations and the discretization of the groundwater field for the digital solution are very similar to those employed for the analog model. Pinder and Bredehoeft (1968) provide a detailed discussion of the theory involved in aquifer modelling. The following theory is taken from their paper.

If the differential equation for non-steady flow of a compressible fluid in an elastic non-homogeneous porous medium is written:

$$\frac{\partial}{\partial x_i}\left(T_{ij}\cdot\frac{\partial h}{\partial x_j}\right) = S\frac{\partial h}{\partial t} + W(x,y,t) \qquad \ldots \ (13)$$

where:

- T_{ij} is the transmissibility tensor $\left(\frac{L^2}{T}\right)$
- h is the hydraulic head (L)
- S is the storage coefficient
- t is time
- W is the volume flux per unit area $\left(\frac{L}{T}\right)$

If the co-ordinate axes of the finite difference grid (Fig. 19) are aligned parallel to the principal directions of the transmissivity tensor, the finite difference approximation of equation (13) may be written:

$$T'_{xx(i-\frac{1}{2},j)} \cdot \left(\frac{h_{i-1,j,k} - h_{ijk}}{\Delta x_i}\right) + T'_{xx(1+\frac{1}{2},j)} \cdot \left(\frac{h_{i+1,j,k} - h_{ijk}}{\Delta x_i}\right)$$

$$+ T'_{yy(i,j-\frac{1}{2})} \cdot \left(\frac{h_{i,j-1,k} - h_{ijk}}{\Delta y_j}\right) + T'_{yy(i,j+\frac{1}{2})} \cdot \left(\frac{h_{i,j+1,k} - h_{ijk}}{\Delta y_j}\right)$$

$$= S\left(\frac{h_{ijk} - h_{i,j,k-1}}{\Delta t}\right) + \frac{q_{w(i,j)}}{\Delta x_i \Delta y_i}$$

$$- K_s \cdot \left(\frac{2H_{r(i,j)} - h_{ijk} - h_{i,j,k-1}}{2m_{r(i,j)}}\right) \qquad \ldots (14)$$

where:

$$T'_{xx(i+\frac{1}{2},j)} = \frac{2T_{xx(i,j)} \cdot T_{xx(i+1,j)}}{T_{xx(i,j)}\Delta x_{i+1} + T_{xx(i+1,j)}\Delta x_i}$$

is the harmonic mean of:

$$\frac{T_{xx(i,j)}}{\Delta x_i}, \quad \frac{T_{xx(i+1,j)}}{\Delta x_{i+1}}$$

and where:

- i is the index in the x dimension
- j is the index in the y dimension
- k is the index in time
- K_s is the hydraulic conductivity of the stream bottom $(\frac{L}{T})$
- m_r is the thickness of the stream bottom (L)
- $q_{w(i,j)}$ is the rate of withdrawal or injection at the node $x_i y_j$ $(\frac{L^3}{T})$
- t is the time increment (T)

$\Delta x, \Delta y$ are space increments (L)

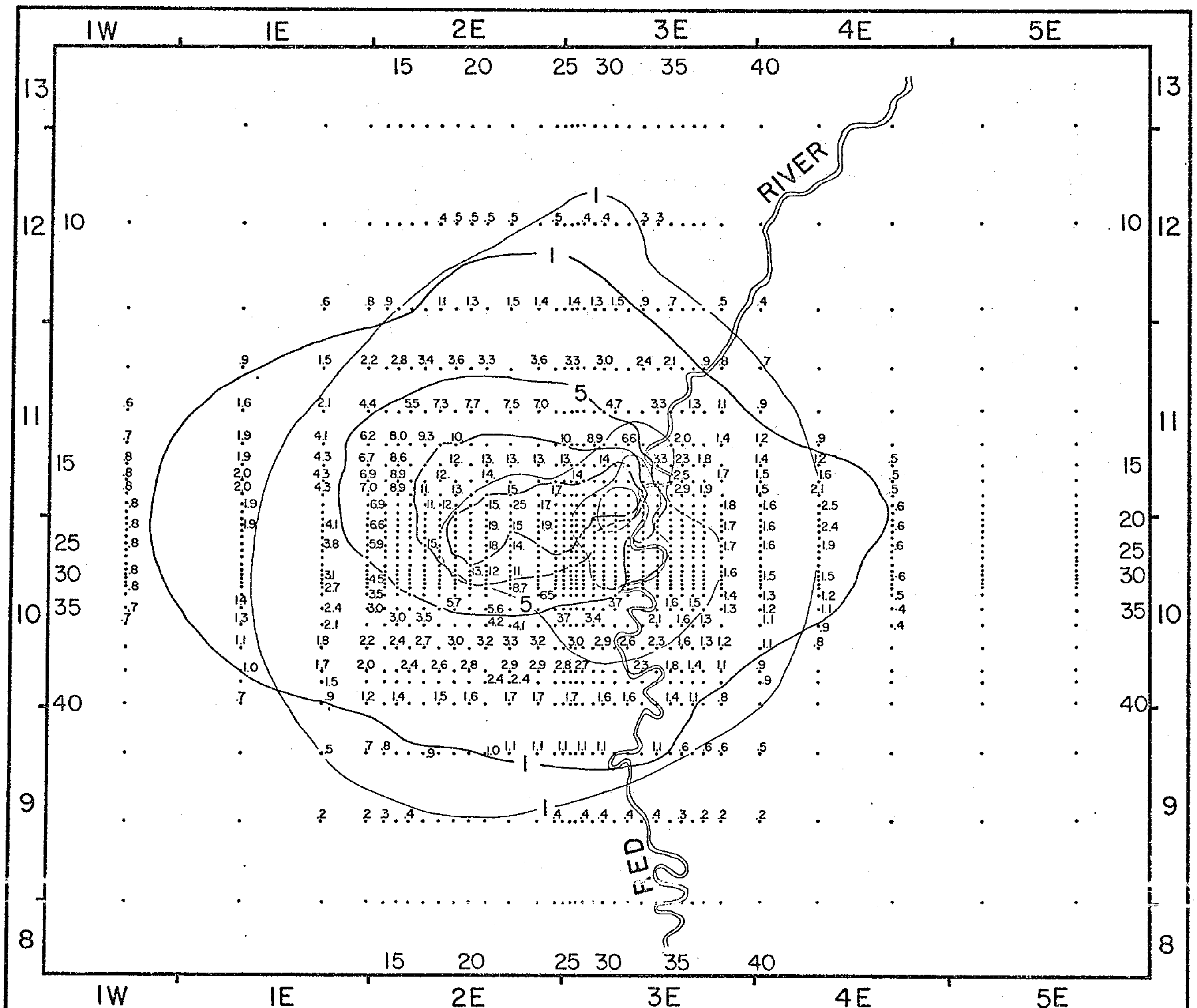
H_r is the hydraulic head in the lakes and streams

The Computer Program

The Computer Program (Appendix A) is written in Fortran IV for the IBM 360 system. The program solves the finite difference approximation (Equation 14) of the ground-water flow equation by an alternating direction implicit procedure (Pinder, 1970). Equation 14 is solved for the desired boundary conditions at each node in the finite difference network after t/2 (where t = the length of the initial time step in seconds) seconds of pumping. These values are then used as initial conditions for the calculation of the head values at time t. The procedure continues until the desired period of pumping has been simulated.

Analytical Procedure

In utilizing the program, a grid is established over the aquifer area (Figures 19 and 23). The centers of each grid element are called nodes. Matrices are formed by recording the transmissibility and storage coefficient for each element and the locations of pumping wells and surface water bodies. The initial head in the aquifer is a required parameter. A deck of cards is prepared, giving the x and y dimensions of the grid. The data set is fronted by a set of



EXPLANATION

· DIGITAL MODEL NODE

.5 NODE WITH SELECTED DRAWDOWN (FEET)

——5—— LINE OF EQUAL DRAWDOWN BASED ON DIGITAL SIMULATED DRAWDOWN DATA (FEET)

——5—— LINE OF EQUAL DRAWDOWN BASED ON OBSERVATION WELL DATA (FEET)

DURATION OF PUMPING = 99 DAYS

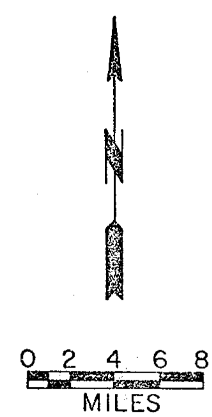


Fig. 23. Digital simulation 1965 Summer drawdown.

parameter cards which list such critical data as the head for surface water bodies, the thickness of aquitards underlying surface water bodies, the flux through the confining units, the length of the initial time step and the maximum duration of pumping. The procedure is detailed in Appendix A. The program requires that all units be in feet and seconds.

Formulation of the Digital Model

The area modelled extended 70 miles west, 50 miles east, 60 miles south and 54 miles north of the confluence of the Red and Assiniboine Rivers. The model grid for the central section of this area was detailed (Fig. 23).

Model Grid

Vector areas were mapped within the region studied. The area centered on the confluence of the Red and Assiniboine Rivers and including the whole of metropolitan Winnipeg was divided into rectangular vector areas of relatively small dimensions (Fig. 23). This was done to obtain a more detailed evaluation of the drawdown and to restrict the size of the vector areas representing pumping wells and river channels. In contrast to the analog approach, the element dimensions were varied as desired. The region, bounding the detailed area and not affected by its grid dimensions was divided into a coarse network of six square mile vector areas. One of the features of the digital grid is that any row or column dimension must be projected through the whole system from boundary to boundary. This is in contrast to the analog model where detailed grid systems are bounded within the coarser grid networks.

Aquifer Coefficients

The transmissibility and storage coefficient values used in the digital model were the same as those described for the analog model. One difference occurred with respect to the transmissibility values. Because the digital model area extends some twenty miles further west than the analog model area, transmissibility values had to be estimated for the regions underlain by Mesozoic shale. The value selected was 10,000 gallons per foot per day. This figure was based on several pumping tests that had been performed in the bedrock surface zone. These pumping tests also indicated that the storage coefficient for the areas underlain by shale was 1×10^{-3}, or the same as had been chosen for the majority of the model area.

The digital model requires that an initial or pre-pumping head value be provided for each node. Providing all the factors affecting flow through the aquifer can be simulated, the prepumping piezometric surface for the aquifer can be incorporated in the model. The difficulties involved in simulating all the factors affecting flow in the aquifer, prevented the pre-pumping piezometric surface from being incorporated into the present model experiments. Therefore, a pre-pumping piezometric surface with zero gradient set at an arbitrary uniform elevation of 750 feet above sea level was used.

Boundary Conditions

The boundary conditions for the model consisted of an impermeable barrier boundary around the outside of the model area, a constant head recharge source in the outer sections of the grid system, recharge by leakage through the river beds along the Red and Assiniboine Rivers, leakage through the confining units and discharge conditions simulated by pumping wells.

The barrier boundary is required to provide a region around the perimeter of the model where no flow occurs. If the model is not bounded in this manner, it is impossible to obtain a solution to the finite difference approximation of the groundwater flow equation. The outlying constant head recharge boundary (rows designated "R", Fig. 24) was established to provide an infinite source of recharge water to the aquifer. The boundary location was chosen so that the lateral boundary of the digital model would be at the same geographic location as that of the analog model. Partial hydraulic connection between the aquifer and the Red and Assiniboine Rivers was simulated within the metropolitan area (Fig. 24). The constant head in the rivers was set at

ITERATION NUMBER= 13

ALPHABETIC CONTOURS FOR DRAWDOWN

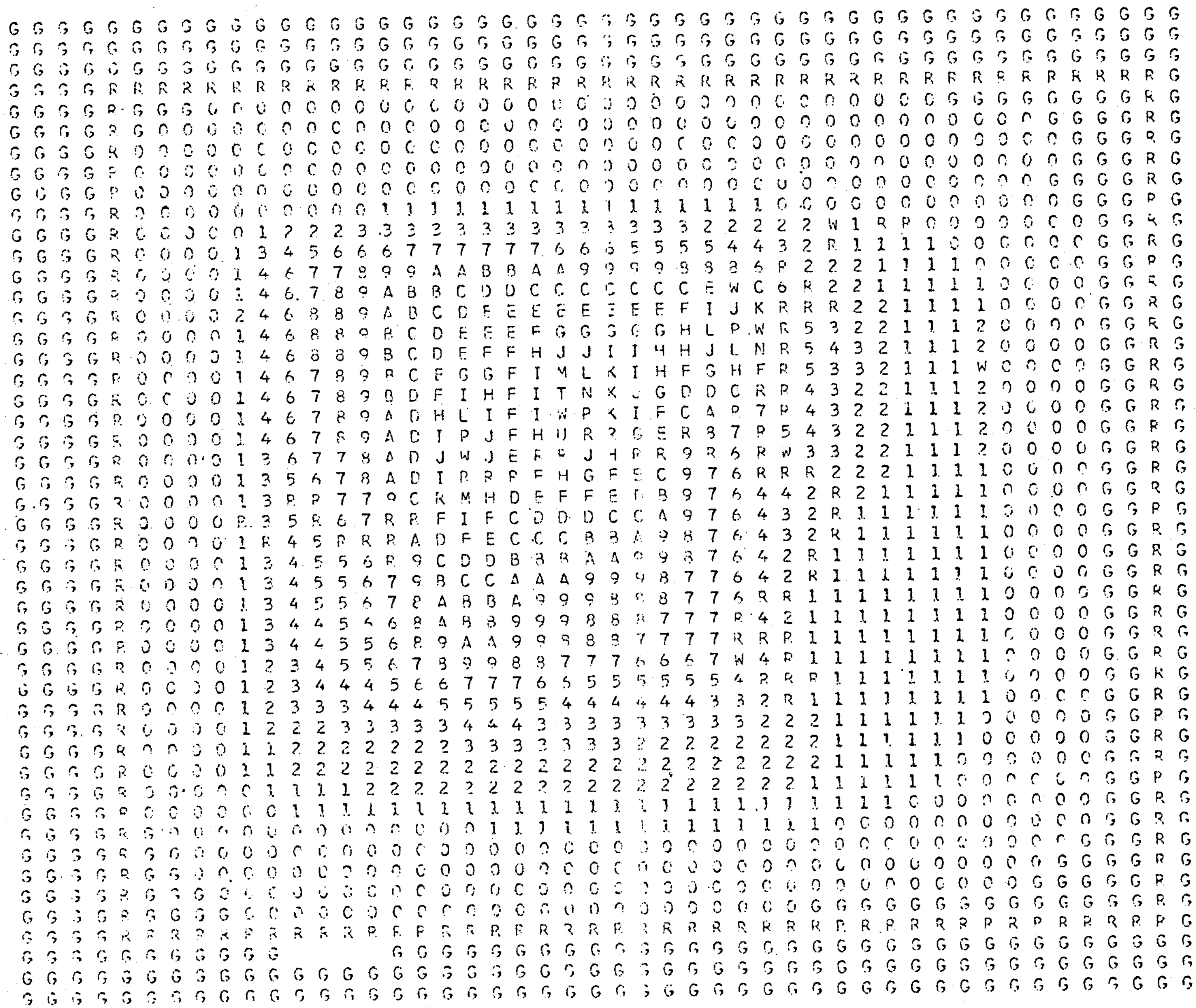
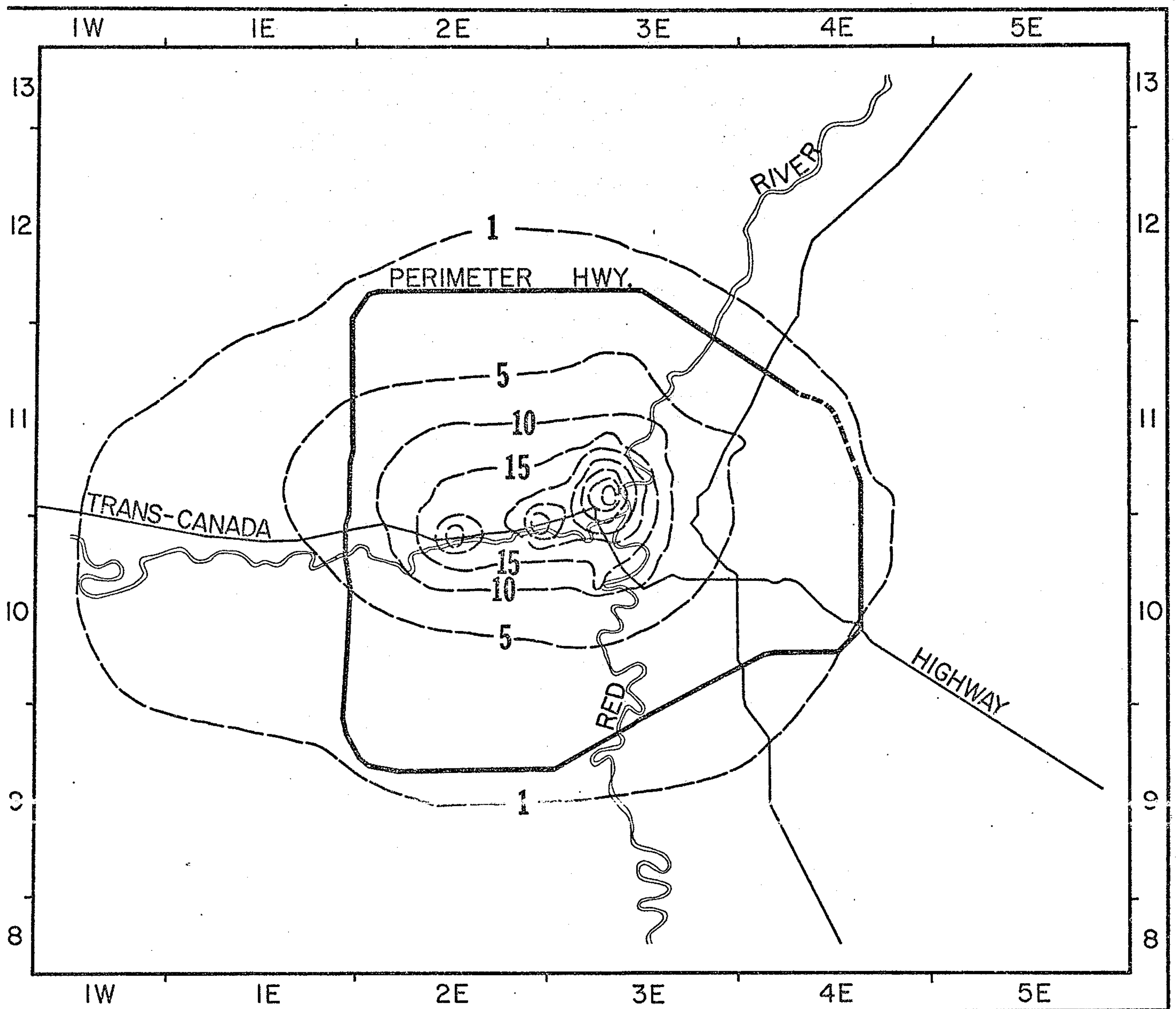


Fig. 24. Alpha-numeric map of digital simulation 1965 summer drawdown.

the same elevation as the initial piezometric surface of the aquifer. Vertical leakage through the confining till and clay and discharge from the aquifer by means of wells that pump at increased rates during the summer, were the other two boundary conditions simulated.

Digital Model Response

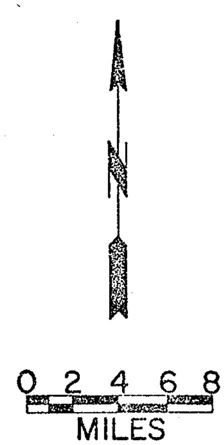
The response of the model was evaluated by simulating the 1965 difference between summer and non-summer pumpage in the metropolitan Winnipeg area for periods of 99 days. Pumping centers rather than individual wells were simulated. These were located at the node nearest to their geographic location (Fig. 18). The pumping center concept was used so that the distortions of the drawdown cone caused by moving wells into pumpage centers, would be similar for the digital and analog models. The first pumping tests were simulated with no recharge through the clay and no leakage through the Red and Assiniboine River beds. The simulated drawdown is shown in Fig. 25. The simulated drawdown was much greater than that observed in the field. Therefore, the aquifer boundary conditions were altered by using different values for the flux through the clay and the hydraulic conductivity of the river beds. The best fit to the field data was obtained with a flux through the clay of 4.88×10^{-13} cu ft/sec/ft^2 and a river bed hydraulic conductivity of 8×10^{-8} ft/sec. The comparison of the simulated and observed drawdown cones are shown on Fig. 23. The Alpha numeric map



EXPLANATION

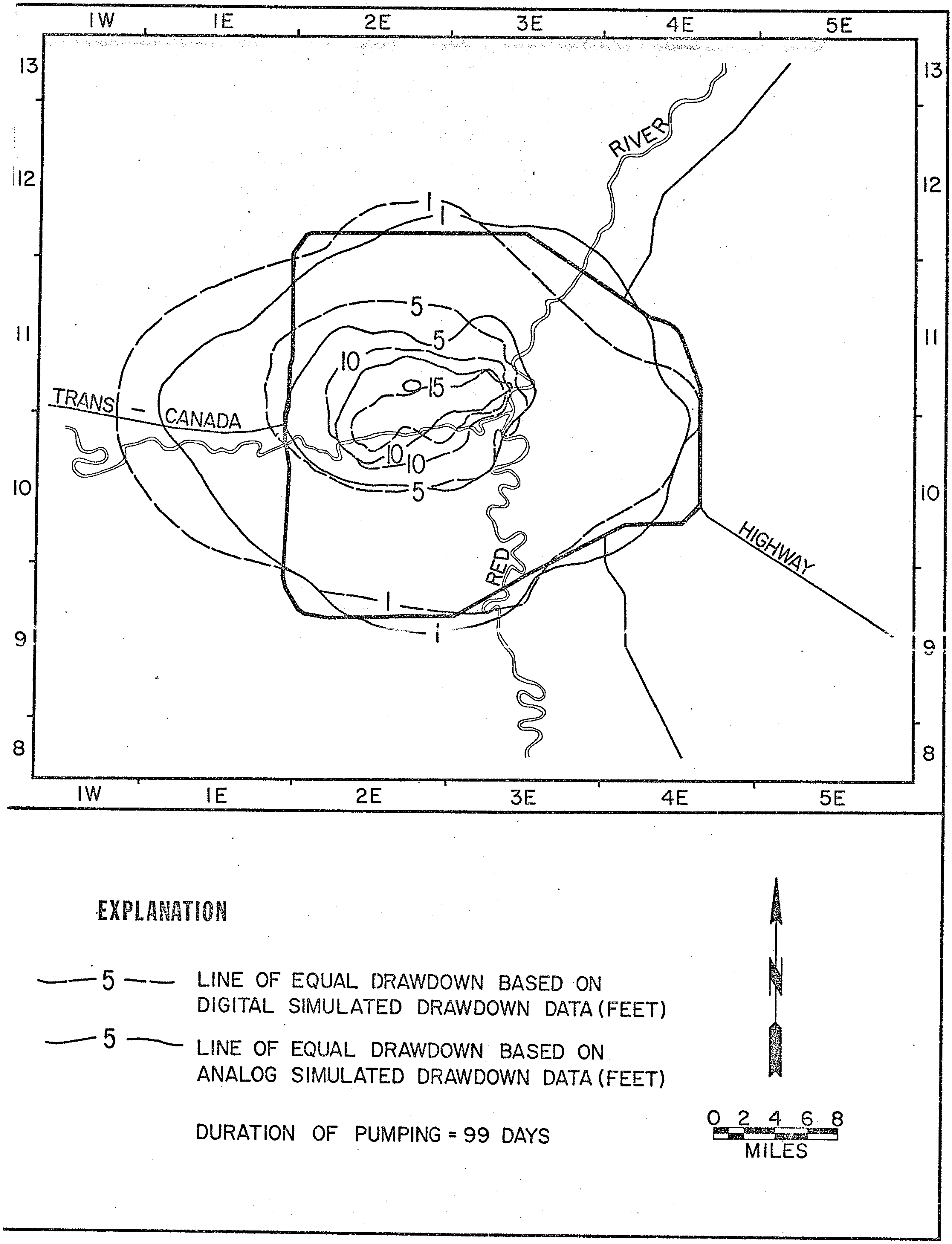
—5— LINE OF EQUAL DRAWDOWN BASED ON SIMULATED DRAWDOWN DATA (FEET) WITH NO RIVER OR VERTICAL RECHARGE IN THE WINNIPEG AREA.

DURATION OF PUMPING = 100 DAYS



ig. 25. Digital simulation 1965 summer drawdown with no vertical leakage.

printed by the computer (Fig. 24) illustrates the bounding river and well locations as well as the drawdown values. Figure 26 illustrates the relationship between the analog and digital model drawdown simulations.



g. 26. Comparison of analog and digital simulations 1965 summer drawdown.

CHAPTER VII

DISCUSSION OF RESULTS

Reasonable simulation of the drawdown in the Upper Carbonate aquifer for the period of increased pumping during the summer of 1965 was obtained from the analog and digital models. Improved accuracy would have been possible had metered pumping rates, and a more dense and more selective observation well network been available.

The drawdown simulated by the analog model closely approximates the field information in the central section and in the outlying portions of metropolitan Winnipeg (Fig. 22). In the west central section of the model grid, the simulated drawdown appears to be much larger than the observed values. However, in this area, there are large air conditioning pumping centers and few observation wells. Therefore, the summer drawdown is likely larger than the present observation system indicates. Also because the pumping rates are estimated, it may be that the pumping values are too large, thus producing a larger drawdown. The one foot drawdown line probably provides the best indication of the accuracy of the model. In the outlying sections of the urban area, the effects of fluctuating pumping rates tend to be damped out and the drawdown is more easily assessed. Similarly in the extremities of the detailed model network, the effects of grouping individual wells into pumping centers has less of a distorting effect on the magnitude of the drawdown. On the eastern side of the drawdown cone, it was difficult, due to the effect of the drawdown caused by the discharge of groundwater into the Red River Floodway, to assess the location of the observed one foot drawdown line. This may, in part, account for the difference between the drawdown line locations in this area.

Preliminary experiments with the analog model indicated that the hydraulic conductivity values chosen for the beds of the Red and Assiniboine Rivers were too large and, therefore, the final tests were carried out with the river nodes all set on open circuit. The relatively close approximation of the test results to the field results is difficult to explain in the face of the fact that when the digital model was operated under similar boundary conditions, the simulated drawdowns were considerably larger than the observed values.

The field conditions for the digital model evaluations were the same as those for the analog model. Therefore, the above comments on the field data apply to the comparison of the digitally simulated and observed drawdowns. The digital model simulation of the 1965 summer drawdown (Fig. 23) that compared most favourably with the observed drawdown was obtained with the following aquifer boundary conditions:

1. Vertical flux through the confining beds of 4.88×10^{-13} cu. ft/sec/ft^2.
2. River bed hydraulic conductivity of 8×10^{-8} ft/sec.
3. Boundary zone simulating a horizontal recharge source.

Generally the simulated drawdown is larger than the observed drawdown (Fig. 23). There is, however, reasonable correlation in the central Winnipeg area and along the one foot drawdown zone. The apparent excessive drawdown in the west central section of the model network is similar to that obtained with the analog model and is probably a function of observation well density and pumping rate estimation. Alterations in the hydraulic conductivity of the Red and Assiniboine Rivers' beds by as little as one-half an order of magnitude has considerable effect on the drawdown values.

The comparisons of simulated and observed time drawdown values (Fig. 27) indicate that the aquifer coefficients and the estimated pumping rates are quite close to the field conditions in the vicinity of observation well M-177. The hydrographs for the areas near observation wells M-55 and 061 indicate that the aquifer coefficients chosen for the model are too small or that the field pumping rates increase gradually over the pumping period. If the latter situation is indeed a significant factor, the final pumping rates

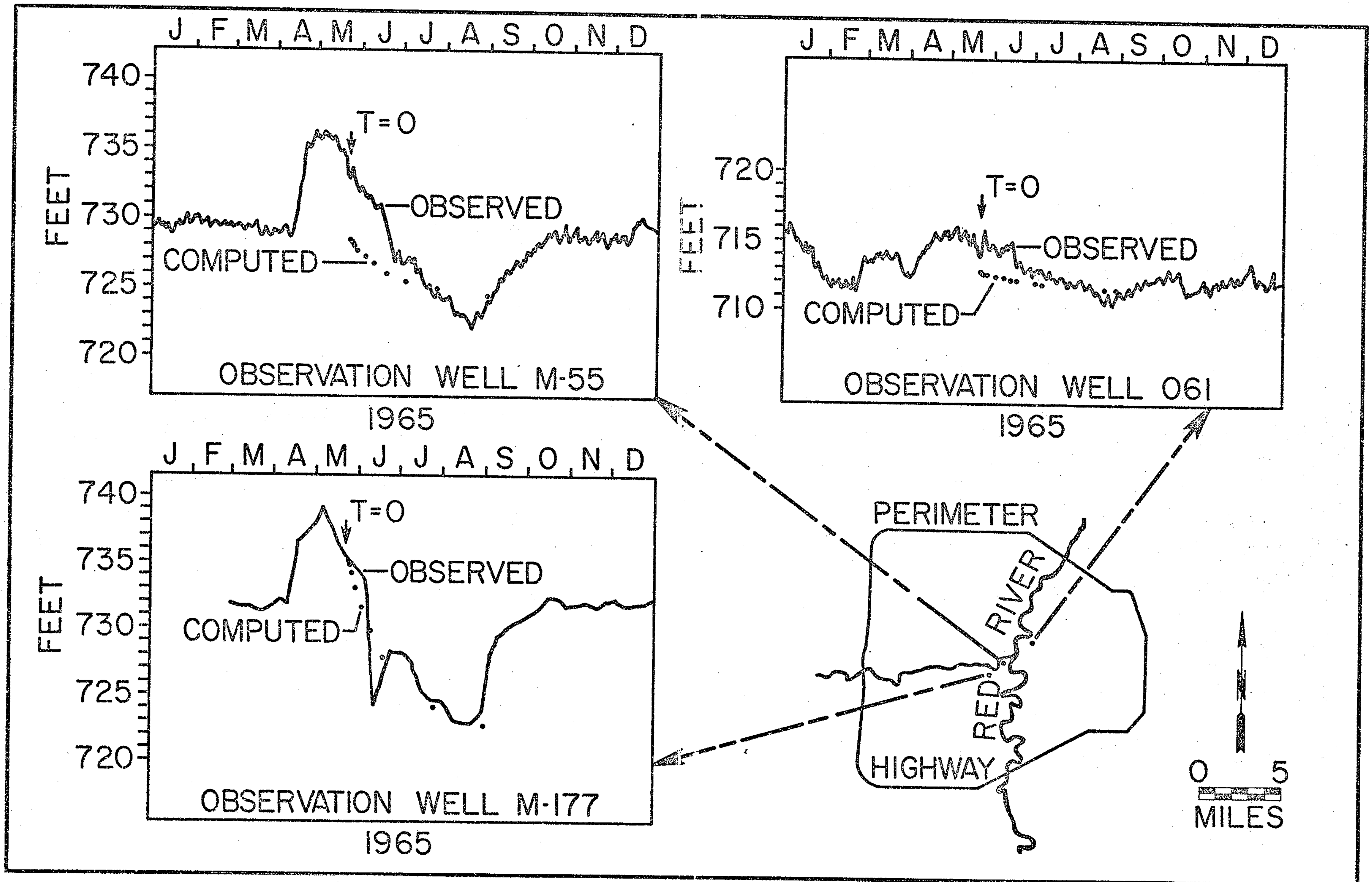


Fig. 27. Comparison of observed and computed drawdown.

must exceed the estimated rates as the observed and simulated drawdown values correlate quite well at the end of the pumping period.

The digital model operated under the stated boundary conditions produces a simulated drawdown cone that very closely resembles that derived from the analog model tests (Fig. 26). However, the drawdown values obtained from the digital model are usually larger, particularly on the western side of the urban area, than those from the analog. When the digital model was operated under the same boundary conditions as those set for the analog model, a much larger, deeper drawdown cone was simulated. The reason for this discrepancy is not known at present. However, it is probably a procedural error, in the analog simulation, involving the wave form generator operation.

CHAPTER VIII

CONCLUSIONS

The Upper Carbonate aquifer, which acts primarily as a groundwater transmission zone, contributes an estimated 17 percent of the annual water supply in the metropolitan Winnipeg area, and will be an increasingly important source of commercial water. The summer rate of groundwater pumpage for cooling is gradually increasing, however, the present rates do not appear to be causing more than a seasonal lowering of the piezometric surface of the Upper Carbonate aquifer. The mantle of glacial drift confining this aquifer is a deterrence to pollution from surface sources in most localities in the urban area. It appears that wells and excavations are the most likely mechanisms by which pollution can occur. Similarly, the upper shale of the Winnipeg formation prevents natural influx of saline water from the Upper Sandstone aquifer. However, salinity contamination caused by open well bores has been detected in north central Winnipeg. The injection of large quantities of warm water into the groundwater system will raise the temperature of the groundwater and destroy its most attractive attribute. This study supports the concept that local detailed assessment of the major aquifers within a groundwater basin requires cognizance of the characteristics of the basin-wide groundwater system.

The model studies corroborate the interpretation that the glacial lake clay and till underlying the river channels form a barrier to aquifer recharge from the surface water sources. The analog and digital models both provided a reasonable simulation of the drawdown observed during the summer of 1965. However, the digital model boundaries are more representative of the geohydrologic conditions and, therefore, it appears to be the superior model. The digital model, for the single layer case, is more efficient at synthesizing the complex geohydraulic conditions that are encountered in the field. The ease with which the digital model can be revised to correlate simulated conditions with observed values make it, in the single layer case, a more powerful experimental and management tool.

To formulate as accurate a model as practical the geologic and hydrologic parameters should be known as precisely as is economically feasible. The piezometric surface of the aquifer, which is the most significant parameter, must be defined in detail. The pumping or recharge rates for each non-domestic well should be metered in accordance with American Water Works Association Standards. Also, it is imperative that the changes in the piezometric surface caused by changes in pumping rates be accurately monitored. In order to meet this requirement at least one observation well should be strategically located within each drawdown cone. Observation wells are required along the boundaries of surface water bodies that lie within the modelled area. While it is feasible to model a developed aquifer without knowing the field values for the transmissibility and storage coefficient, the writer suggests that a more realistic model can be more efficiently acquired if aquifer hydraulic tests are performed within each segment of the flow system. These tests should be located such as to test the aquifer parameters within characteristic segments of the flow net. At least one precise test should be carried out within each segment of the flow system. The remaining data can be acquired from commercial well pumping tests, from simple pumping tests involving only a pumping well, or a pumping well and one observation well, or from water pressure testing of well bores. Water quality considerations require that analyses be obtained throughout the flow system, and particularly in areas where waters from different segments of the flow system may be mixing, or where deleterious water is contaminating sections of the aquifer.

The collection of data on the aquifer should be performed as accurately as field practice allows. Water level measurements should be determined to an accuracy of 0.01 foot. The barometric efficiency of the well is required in assessing water level changes in the order of 1 foot. The meters used in monitoring the discharge or recharge of wells should be calibrated to plus or minus 3 percent of total capacity and should be recalibrated at least every three years. In general, complete chemical analyses should be reported and the cation and anion equivalents should balance to within less than 2 percent of the total equivalents.

In their present state, the two models could be utilized to evaluate the probable effects of new pumping centers and to estimate the quantitative capabilities of the Upper Carbonate aquifer in the metropolitan Winnipeg area. Establishing the new piezometric surface that would result from an increase in groundwater development would be a considerable aid in predicting probable changes in groundwater quality. Further development of the models will require an improvement in the observation well network, particularly in the west central metropolitan area and the metering of the discharge from all non-domestic wells.

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APPENDIX A

AN ITERATIVE DIGITAL MODEL FOR AQUIFER EVALUATION

An Iterative Digital Model for Aquifer Evaluation

By

George F. Pinder

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An Iterative Digital Model for Aquifer Evaluation

By

George F. Pinder

PURPOSE

This program simulates the response of a confined or unconfined aquifer to pumping at a constant rate from one or more wells. The ground-water reservoir may be irregular in shape and non-homogeneous with infiltration from one or more lakes and streams. The program is written in FORTRAN IV for the IBM 360 system. (See Appendix for program listing).

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THEORETICAL DEVELOPMENT

The differential equation for nonsteady flow of a compressible fluid in an elastic non-homogeneous porous medium can be written (Pinder and Bredehoeft, 1968)

$$\frac{\partial}{\partial x_i}\left(T_{ij}\frac{\partial h}{\partial x_j}\right) = S\frac{\partial h}{\partial t} + W(x,y,t) \qquad (1)$$

where T_{ij} is the transmissivity tensor (L^2/T);
h is the hydraulic head (L);
S is the storage coefficient (dimensionless);
t is time (T);
W is the volume flux per unit area (L/T).

If the coordinate axes are aligned with the principal directions of the transmissivity tensor the finite-difference approximation to (1) may be written (see Fig.1)

$$
\begin{aligned}
& T'_{xx(i-\frac{1}{2},j)} \left(\frac{h_{i-1,j,k} - h_{i,j,k}}{\Delta x_i}\right) \\
& + T'_{xx(i+\frac{1}{2},j)} \left(\frac{h_{i+1,j,k} - h_{i,j,k}}{\Delta x_i}\right) \\
& + T'_{yy(i,j-\frac{1}{2})} \left(\frac{h_{i,j-1,k} - h_{i,j,k}}{\Delta y_j}\right) \\
& + T'_{yy(i,j+\frac{1}{2})} \left(\frac{h_{i,j+1,k} - h_{i,j,k}}{\Delta y_j}\right) \\
& = S \left(\frac{h_{i,j,k} - h_{i,j,k-1}}{\Delta t}\right) \\
& + \frac{q_{w(i,j)}}{\Delta x_i \Delta y_j} - K_s \left(\frac{2H_{r(i,j)} - h_{i,j,k} - h_{i,j,k-1}}{2m_{r(i,j)}}\right)
\end{aligned}
$$

where

$$T'_{xx(i+\frac{1}{2},j)} = \frac{2T_{xx(i,j)}T_{xx(i+1,j)}}{T_{xx(i,j)}\Delta x_{i+1} + T_{xx(i+1,j)}\Delta x_i}$$ = the harmonic mean of

$$\frac{T_{xx(i,j)}}{\Delta x_i}, \quad \frac{T_{xx(i+1,j)}}{\Delta x_{i+1}}$$

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i is the index in the x dimension;

j is the index in the y dimension;

k is the index in time;

K_s is the hydraulic conductivity of the stream bottom (L/T);

m_r is the thickness of the stream bottom (L);

$q_{w(i,j)}$ is the rate of withdrawal or injection at the node $x_i y_j$ (L^3/T);

Δt is the time increment (T);

Δx, Δy are space increments (L);

H_r is the hydraulic head in the lakes and streams.

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In an unconfined aquifer the transmissivity is a function of time as well as of space. In order to approximate equation (2) without generating a non-linear set of finite-difference equations, the head from the preceding time step is used, and the transmissivity is given by

$$T_{i,j,k} = K_{i,j}h'_{i,j,k-1}$$

where K is the hydraulic conductivity (L/T);

h' is the saturated thickness of the aquifer (L).

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METHOD OF ANALYSIS

A rectangular net or grid as indicated in figure 1 is superposed on a plan view of the groundwater reservoir.

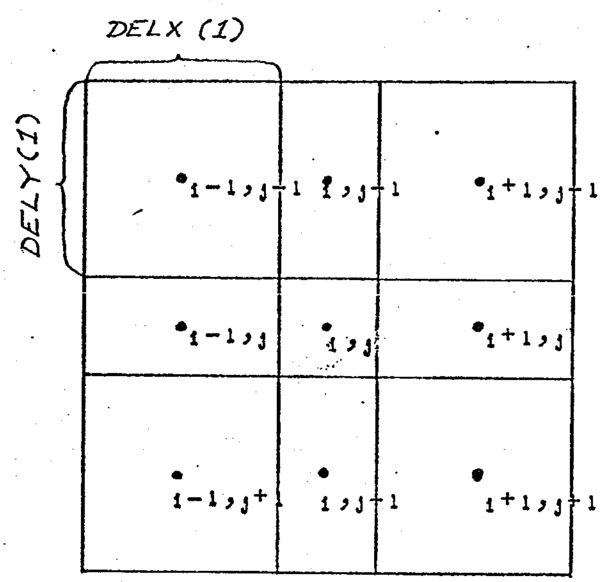

Fig. 1. Nodal array for digital model.

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At each node the transmissivity, storage coefficient, and initial head are recorded. The pumping rate is recorded at each node where a well is located and the hydraulic conductivities of stream or lake beds where they occur. In the case of an unconfined aquifer the transmissivity is replaced by the hydraulic conductivity and the elevation of the base of the aquifer is also recorded. A parameter card is prepared providing the dimensions of the grid, the head in the stream, the thickness of the stream or lake beds, information concerning the maximum duration of pumping and other constants used in the computational scheme. All computer input and output is in a consistent set of units; the program is set-up to use feet and seconds.

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The objective of the program is to solve equation (2) implicitly for the desired boundary conditions at each node on the grid after Δt seconds of pumping. These values are then used as initial conditions for the calculation of the head values at time $2\Delta t$; this procedure is continued until the reservoir has been pumped the desired length of time. The size of the time step is increased exponentially. Numerical values and an alphameric contour map of the drawdown in the aquifer are printed at selected time steps (see Fig. 2).

Figure 2 -- Alphameric contour map of drawdown at Antigonish Nova Scotia

---

In order to determine the precision of the calculations, a mass balance may also be computed at this time if there is no infiltration into the system.

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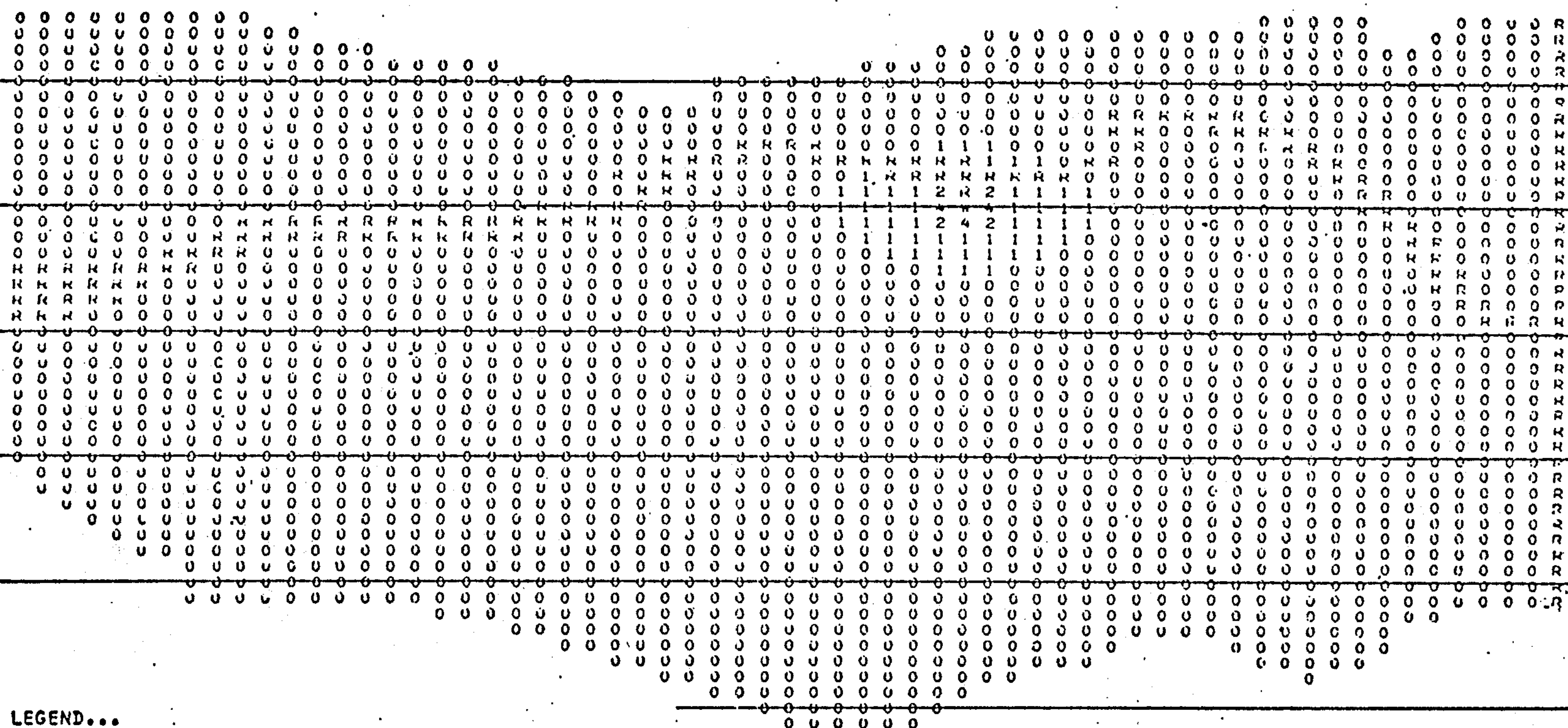
ITERATION NUMBER 15
SIZE OF TIME STEP IN SECONDS= 2.33
DURATION OF PUMPING AT THIS PRINTOUT IN SECONDS= 17.5
MINUTES= 0.291
HOURS= 0.485E-02
DAYS= 0.202E-03

ALPHABETIC CONTOURS FOR DRAWDOWN



LEGEND...
DRAWDOWN FROM 0-35 FEET REPRESENTED BY SYMBOLS 0-Z
DRAWDOWN GREATER THAN 35 FEET REPRESENTED BY THE SYMBOL *
CONE OF IMPRESSION INDICATED BY SYMBOL >
LETTER R INDICATES AREA OF RECHARGE
LETTER W INDICATES LOCATION OF A WELL

Fig. 2

The iterative alternating-direction implicit procedure is the mathematical technique used to solve the N simultaneous equations where N is the number of nodes in the matrix. Hydraulic head values are calculated for each node in the matrix by solving alternately the equations for rows and columns implicitly. Equation (2) for the calculation of rows is

$$T'_{xx(i-\frac{1}{2},j)} \left(\frac{h^{n}_{i-1,j,k} - h^{n}_{i,j,k}}{\Delta x_i}\right)$$

$$+ T'_{xx}(i+\tfrac{1}{2},j) \left(\frac{h^{n}_{i+1,j,k} - h^{n}_{i,j,k}}{\Delta x_i}\right)$$

$$+ T'_{yy(i,j-\frac{1}{2})} \left(\frac{h^{n-1}_{i,j-1,k} - h^{n-1}_{i,j,k}}{\Delta y_j}\right)$$

$$+ T'_{yy(i,j+\frac{1}{2})} \left(\frac{h^{n-1}_{i,j+1,k} - h^{n-1}_{i,j,k}}{\Delta y_j}\right)$$

$$= S \left(\frac{h^{n}_{i,j,k} - h_{i,j,k-1}}{\Delta t}\right) + \frac{q_{w(i,j)}}{\Delta x_i \; \Delta y_j}$$

$$- K_s \left(\frac{2H_{r(i,j)} - h^{n}_{i,j,k} - h_{i,j,k-1}}{2m_{r(i,j)}}\right)$$

$$+ I(h^{n}_{i,j,k} - h^{n-1}_{i,j,k})$$

where I is a normalized iteration parameter;

n is the index indicating iteration number.

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At each time step the row and column equations are solved alternately until the greatest head difference between a row and column computation at any node is less than a prescribed error criteria. When this closure is achieved the program begins solving for the head at the new time $t + \Delta t$. The procedure is continued until the desired period of analysis has been simulated or the aquifer becomes dewatered. When any condition arises which terminates computation the head matrix and the elapsed simulation period are punched on cards. These cards can be used as input if it is necessary to extend the period of analysis at a later time.

For a detailed explanation of the technique used for solving the finite-difference equations see Douglas and Rachford (1956).

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APPLICATION

The following section describes the preparation of parameter and data cards. The arrangement of the assembled program deck is shown in figure 3.

Figure 3. Assembled program deck.

Parameter cards

All values are right justified in the data field as indicated in Fig. 4.

Figure 4. Sample coding form.

Card 1

Column	Variable	Contents
1 - 10	TMAX	simulation period in hours
11 - 20	DIML	number of nodes in a column of the matrix *
21 - 30	DIMW	number of nodes in a row of the matrix *
31 - 40	NUMT	maximum number of time steps *
41 - 50	QRE	vertical leakage into the aquifer in feet per second
51 - 60	DELT	initial time increment in seconds

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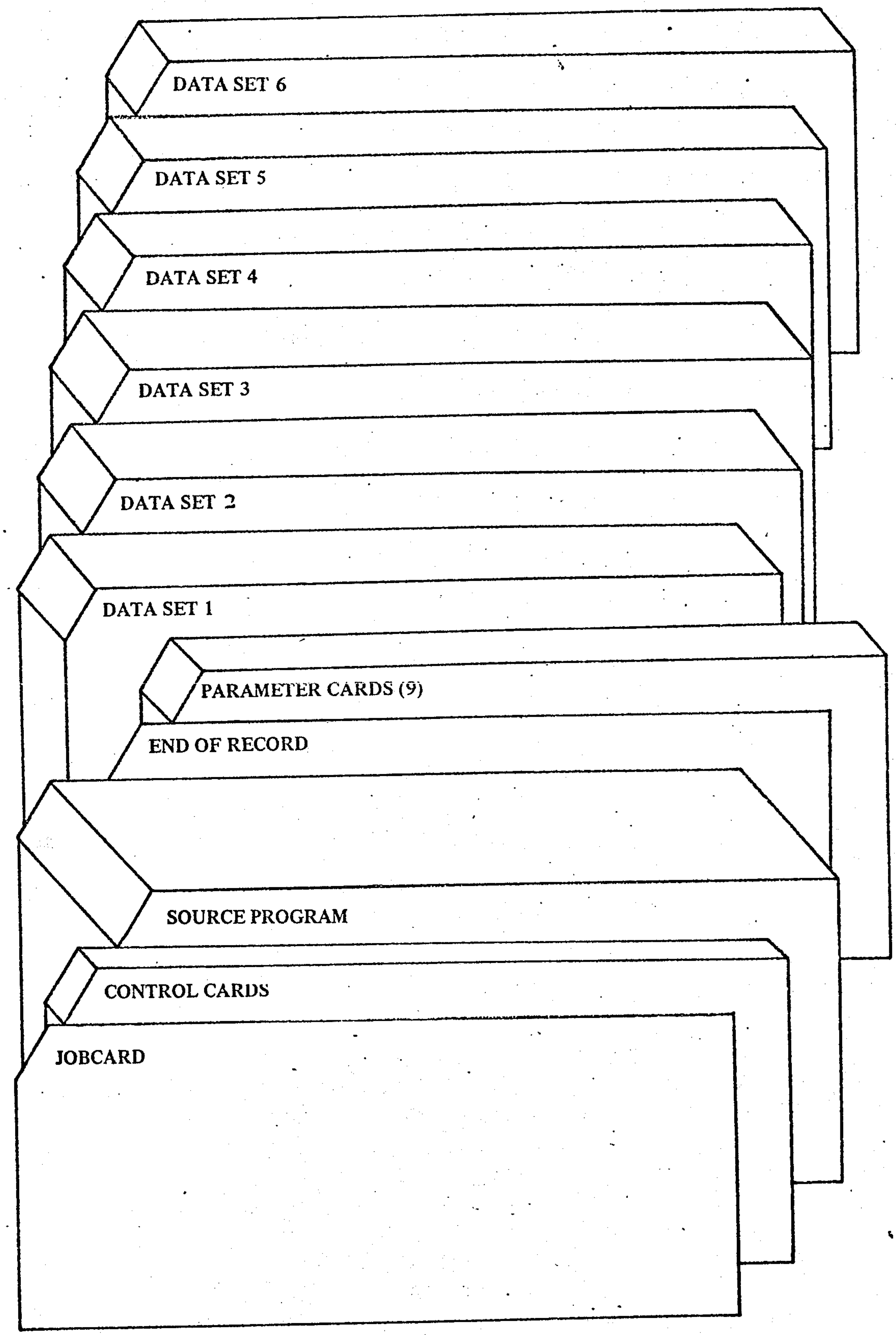
DATA SET 6
DATA SET 5
DATA SET 4
DATA SET 3
DATA SET 2
DATA SET 1
PARAMETER CARDS (9)
END OF RECORD
SOURCE PROGRAM
CONTROL CARDS
JOBCARD

Fig. 3

PROGRAMMER ____ DIVISION ____

LOCATION ____ PHONE ____

U. S. DEPARTMENT OF THE INTERIOR
GEOLOGICAL SURVEY
COMPUTER CODING FORM

NUMBER ____

PROJECT ____

SHEET 1 OF 2

IDENTIFICATION: ITERATIVE DIGITAL MODEL FOR AQUIFER EVALUATION

INFORMATION: SAMPLE CODING FORM

TMAX	DIML	DIMW	NUMT	QRE	DELT		
10000.	5	5	5	0.	10.0		
M	**KTH**	**FACTOR**	**LENGTH**	**ERR**	**FACS**	**FACB**	**FACP**
10.	1	.001	4	.001	.001	1.	.001
FACT	**RIVER**	**SPACNG**					
1.	100.	1.					
SUM							
0.							

PUNCH

WATERTABLE

CØNTØUR

NUMERIC

CHECK

-DATA SET 1; DELX,DELY-

100. 100. 50. 100. 100.

-DATA SET 2; STRT-

100. 100. 100. 100. 100.

FORM 9 - 1633 A
JULY 1967

0 = ZERO Ø = ALPHA O 1 = ONE I = ALPHA I 2 = TWO Z = ALPHA Z / = SLASH | = VERT. BAR - = MINUS _ = HORZ. BAR

DATA CODING FORM

Fig. 4

U. S. DEPARTMENT OF THE INTERIOR
GEOLOGICAL SURVEY
COMPUTER CODING FORM

PROGRAMMER ______ DIVISION ______

LOCATION ______ PHONE ______

PROGRAM NO. ______

PROJECT ______

SHEET 2 OF 2

PROGRAM IDENTIFICATION: ITERATIVE DIGITAL MODEL FOR AQUIFER EVALUATION

PROGRAM INFORMATION: SAMPLE CODING FORM

STATEMENT IDENTIFICATION

```
-DATA SET 3; RATE
   0.   0.  -1.   0.   0.

-DATA SET 4; PERM
   0.   1.   1.   1.   0.

-DATA SET 5; BOTTOM
   0.   0.   0.   0.   0.

-DATA SET 6; S
   1.   1.   1.   1.   1.
```

FORM 9-1634 A
JULY 1967

0 = ZERO Ø = ALPHA O 1 = ONE I = ALPHA I 2 = TWO Z = ALPHA Z / = SLASH | = VERT. BAR - = MINUS _ = HORZ. BAR

PROGRAM CODING FORM

Fig. 4

Card 2

1 - 10	M	thickness of stream or lake beds in feet
11 - 20	KTH	number of time steps between printouts *
21 - 30	FACTOR	multiplier for values of hydraulic conductivity of stream or lake beds
31 - 40	LENGTH	number of iteration parameters *
41 - 50	ERR	error criteria for closure
51 - 60	FACS	multiplier for storage coefficient
61 - 70	FACB	multiplier for aquifer base bottom elevation
71 - 80	FACP	multiplier for hydraulic conductivity of aquifer

Card 3

1 - 10	FACT	multiplier for transmissivity
11 - 20	RIVER	hydraulic head in river in feet
21 - 30	SPACNG	contour interval in feet

Card 4

1 - 10	SUM	elapsed time at beginning of computations (usually 0. unless provided as punched output)

Card 5

1 - 5	PUNCH	indicator for punched output; if punched output is desired at termination of computations write PUNCH, otherwise leave card blank.

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Card 6

1 - 10	WATER	indicator of water-table conditions, if water-table conditions are encountered write WATERTABLE: otherwise leave card blank.

Card 7

1 - 7	CONTR	indicator of contoured printout; if an alphameric contour map of hydraulic head is desired write CONTOUR; otherwise leave this card blank.

Card 8

1 - 7	NUM	indicator of numerical printout; if the numerical head values are desired write NUMERIC, otherwise leave this card blank.

Card 9

1 - 5	CHCK	indicator of computational check; if a mass balance on the system is desired write CHECK; otherwise leave card blank.

* values are fixed point; they must be right justified and not include a decimal point.

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Data Cards

Set 1

Distance between nodes in the prototype in feet: the distance between adjacent nodes is recorded as indicated in figure 1. The values in the x direction, *DELX*, are recorded first and are followed, beginning on a new card, by the values in the y direction, *DELY*. As indicated in figure 1 the values for *DELY* and *DELX* must remain constant for each row and column respectively. Each value may occupy 10 spaces including the decimal point.

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Set 2

Initial hydraulic head, STRT: the initial hydraulic head values are recorded from left to right along the rows beginning with the top row. Each row begins on a new card and each value may occupy 10 spaces including a decimal point (there will be a maximum of eight values recorded on each card). The hydraulic head values punched at the termination of the program can be read as initial hydraulic head values.

Set 3

Locations of wells, lakes, and streams, RATE: the locations of wells, lakes, and streams are recorded left to right along the rows, beginning with the top row. Each row begins on a new card and each value may occupy only 4 spaces including a negative sign and decimal point (no sign is necessary for a positive number). At any node where a lake or stream occurs the hydraulic conductivity of the bed divided by the hydraulic conductivity multiplier is recorded. At any node where a well is located the pumping rate in cfs is recorded preceded by a negative sign.

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Set 4

Hydraulic conductivity: PERM or transmissivity T: this data set consists of the hydraulic conductivity values in the case of a water table problem or transmissivity values in a confined aquifer situation. The values divided by the appropriate multiplier are recorded left to right along the rows beginning with the top row. Each row begins on a new card and each value may occupy up to 4 spaces including the decimal point.

Set 5

Aquifer base elevation, BOTTOM: this data set consists of the elevation of the base of the aquifer measured from the same reference datum as the initial head, divided by an appropriate multiplier. This data set is omitted if a confined-aquifer problem is considered. The values are recorded left to right along the rows beginning with the top row. Each row begins on a new card and each value may occupy up to 4 spaces including the decimal point.

Set 6

Storage coefficient values, S: this data set consists of the storage coefficient at each node, divided by the appropriate multiplier. The values are recorded left to right along the rows, beginning with the top row. Each row begins on a new card and each value may occupy up to 4 spaces including the decimal point.

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PROGRAM FEATURES

Number of nodes:

The dimensions of the nodal array considered in an aquifer problem generally depend upon the storage capacity in the computer and available funds. The larger the nodal array the more computer time is required to simulate a specified period of analysis. This program permits arrays up to 50 X 50 nodes.

Drawdown near the pumping well:

Withdrawal from a well is assumed to occur over the area of influence of the well node. Drawdown values within one node of the well, therefore, will not accurately represent field responses. The use of smaller space increments near the well will improve the accuracy of the solution.

Advantages of iterative technique:

Certain limitations inherent in the alternating direction implicit technique are overcome by introducing a relaxation factor and iteration. Some of the important advantages of an iterative analysis are:

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i. abrupt changes in transmissivity between nodes is permitted,

ii. variable space increments may be used,

iii. any time increment Δt may be used,

iv. the elliptic form of the flow equation can be treated directly (the steady-state problem),

v. the iterative procedure requires less computer time when periods extending tens or hundreds of years are simulated.

Multiplication Constants

Multiplication constants (FACTOR, FACB, FACS, FACP, FACT) are used to decrease the number of data cards when very small or very large numbers must be read. For example, if it was necessary to read a storage coefficient value of .0001 the multiplication factor FACS could be set equal to .0001 and 1. would be recorded on the data card; the program would convert the 1. to .0001 during execution.

Program Diagnostics

Two program diagnostics may be generated if an abnormal program termination occurs. The message EXCEEDED PERMITTED NUMBER OF ITERATIONS is printed when convergence is not achieved within 100 iterations. This situation generally arises when an error in data input results in an impossible physical problem. The message WELL GOES DRY occurs when the cone of depression in an unconfined aquifer drops below the impermeable aquifer base.

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Iteration Parameters

The choice of the number of iteration parameters, LENGTH, and the convergence criteria, ERR, depends upon the physical problem considered. Three to seven iteration parameters and a value for ERR of .001 should provide satisfactory results in most problems.

Mass Balance

The CHECK option permits the calculation of a mass balance when there is no infiltration into the system. The volume of water removed from the aquifer is computed by integration over the cone of depression and by calculating the volume removed from the wells. These values are compared and the deviation expressed as a percent of the volume pumped. If infiltration does occur the deviation is a measure of the volume of water entering the system.

This program is not designed to simulate all of the complicated hydrologic problems encountered in the field. It is intended as a starting point from which more complex models can be developed.

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SELECTED REFERENCES

Douglas, J., and Rachford, H. H., Jr., 1956, On the numerical solution of head conduction problems in two or three space variables, Trans. Amer. Math. Soc., 82, p. 421-439.

Pinder, G. F., and Bredehoeft, J. D., 1968, Application of Digital Computer for Aquifer Evaluation; Water Resources Research, Vol. 4, No. 5, pp. 1069-1093.

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```
C      TO PROVIDE THE POTENTIAL DISTRIBUTION IN A CONFINED AQUIFER AFTER
C      A DESIGNATED PERIOD OF TIME.
C      ***************************************************************
C
C      METHODO
C      THE ALTERNATING DIRECTION IMPLICIT TECHNIQUE IS USED TO SOLVE
C      THE LINEAR EQUATIONS.
C
C      DESCRIPTION OF PARAMETERSO
C      A=COEFFICIENT IN FINITE DIFFERENCE EQUATION
C      B=COEFFICIENT IN FINITE DIFFERENCE EQUQTION
C      BE(I)=DUMMY VARIABLE DEFINED AS C/W
C      BOTTOM(I,J)=ELEVATION OF BOTTOM
C      C=COEFFICIENT IN FINITE DIFFERENCE EQUATION
C      D=DUMMY VARIABLE REPRESENTING PARAMETERS OF EQUATION WHOSE VALUES
C      ARE KNOWN FROM PREVIOUS ITERATION
C      DAYS=NUMBER OF DAYS SINCE PUMPING STARTED
C      DELT= LENGTH OF INITIAL TIME STEP (SECONDS)
C      DELX=DISTANCE BETWEEN NODES IN THE PROTOTYPE IN THE X DIRECTION
C           (FEET)
C      DELY=DISTANCE BETWEEN NODES IN THE PROTOTYPE IN THE Y DIRECTION
C           (FEET)
C      DIML=NUMBER OF NODES IN COLUMN OF MATRIX
C      DIMW=NUMBER OF NODES IN ROW OF MATRIX
C      ERR=CLOSURE CRITERIA FOR ITERATION
C      FACTOR=MULTIPLICATION FACTOR FOR ADJUSTING STORAGE COEFFICIENT
C      FACB=MULTIPLICATION FACTOR FOR BOTTOM ELEVATION
C      FACP=MULTIPLICATION FACTOR FOR  HYDRAULIC CONDUCTIVITY
C      FACS=MULTIPLICATION FACTOR FOR STORAGE COEFFICIENT
C      FACT=MULTIPLICATION FACTOR FOR TRANSMISSIVITY
C      G=DUMMY VARIABLE DEFINED AS (D-A*G(J-1))/W
C      HRS=NUMBER OF HOURS SINCE PUMPING STARTED
C      IMK=ITERATION PARAMETER
C      KEEP(I,J)=STORAGE MATRIX FOR PHI VALUES
C      KTH=NUMBER OF TIME STEPS BETWEEN PRINTOUTS
C      M=THICKNESS OF STREAM BED OR LAKE BOTTOM (FEET)
C      NUMT=MAXIMUM NUMBER OF TIME STEPS
C      PHI(I,J)=HEAD IN AQUIFER AT NODE (I,J) IN FEET
C      PRNT(I,J)=ARRAY FOR PRINTING OUT HEAD MATRIX IN CONTOUR FORM
C      QRE= RECHARGE FLUX FROM THE CONFINING LAYER (CUBIC FEET PER SECOND
C      RATE(I,J)=RATE OF PUMPING IN CFS IF NEGATIVE
C                HYDRAULIC CONDUCTIVITY OF STREAM OR LAKE BOTTOM IF
C      POSITIVE
C      RHO=DUMMY VARIABLE DEFINED AS S(I,J)/DELT
C      RIVER=HEAD IN RIVER
C      RR=VARIABLE ACCOUNTING FOR VERTICAL LEAKAGE
C      RW=VARIABLE ACCOUNTING FOR PUMPING
C      S(I,J)=STORAGE COEFFICIENT (DIMENSIONLESS)
C      STRT(I,J)=INITIAL VALUE OF HYDRAULIC HEAD IN AQUIFER
C      SUM=DURATION OF PUMPING IN SECONDS
```

```
C      SYM(J)=LINEAR ARRAY CONTAINING SYMBOLS FOR PRINTING OUT HEAD
C                 MATRIX
C      T(I,J)=TRANSMISSIVITY (FEET**2/SECOND)
C      TEMP(I)=TEMPORY LOCATION OF CURRENTLY CALCULATED VALUES OF HEAD
C      TMAX=MAXIMUM ALLOTTED PERIOD OF PUMPING
C      W=DUMMY VARIABLE DEFINED AS B-A*BE(J-1)
C
C      ................................................................
C
      COMMON PHI,SUM,DELX,DELY,DIML,DIMW,RATE,S,STRT,SPACNG
      DIMENSION S(50,50), PERM(50,50), BOTTOM(50,50), RATE(50,50), KEEP(
     150,50), G(50), TEMP(50), BE(50), RHOP(25), CHK(10), STRT(50,50), T
     2(50,50), PHI(50,50)
      DIMENSION DELX(50), DELY(50)
      REAL MINS,M,K
      REAL*8 KEEP,IMK,NUM
      INTEGER DIML,DIMW
      DOUBLE PRECISION PHI,KEEP,D,G,TEMP,BE,W,T1,T2,T3,T4,RHO,A,B,C,DELT
     1,RHOP,PARAM,IMK,RIVER,CHK,PNCH,WATER,CONTR,NUM,CHCK
C
C      ................................................................
C
      DATA CHK(1)/5HPUNCH/,CHK(2)/8HWATERTAB/,CHK(3)/7HCONTOUR/,CHK(4)/7
     1HNUMERIC/,CHK(5)/5HCHECK/
C
      READ (5,520) TMAX,DIML,DIMW,NUMT,QRE,DELT,M,KTH,FACTOR,LENGTH,ERR,
     1FACS,FACB,FACP,FACT,RIVER,SPACNG
      READ (5,610) SUM
      READ (5,600) PNCH,WATER,CONTR,NUM,CHCK
      READ (5,680) (DELX(J),J=1,DIMW)
      READ (5,680) (DELY(I),I=1,DIML)
      DO 10 I=1,DIML
      READ (5,620) (STRT(I,J),J=1,DIMW)
      DO 10 J=1,DIMW
   10 PHI(I,J)=STRT(I,J)
      DO 20 I=1,DIML
      READ (5,530) (RATE(I,J),J=1,DIMW)
      DO 20 J=1,DIMW
   20 IF (RATE(I,J).GT.0) RATE(I,J)=RATE(I,J)*FACTOR
      IF (WATER.NE.CHK(2)) GO TO 50
      DO 30 I=1,DIML
      READ (5,530) (PERM(I,J),J=1,DIMW)
      DO 30 J=1,DIMW
   30 PERM(I,J)=PERM(I,J)*FACP
      DO 40 I=1,DIML
      READ (5,530) (BOTTOM(I,J),J=1,DIMW)
      DO 40 J=1,DIMW
   40 BOTTOM(I,J)=BOTTOM(I,J)*FACB
      GO TO 70
   50 DO 60 I=1,DIML
```

```
      READ (5,530) (T(I,J),J=1,DIMW)
      DO 60 J=1,DIMW
   60 T(I,J)=T(I,J)*FACT
   70 DO 80 I=1,DIML
      READ (5,530) (S(I,J),J=1,DIMW)
      DO 80 J=1,DIMW
   80 S(I,J)=S(I,J)*FACS
C
C     ...............................................................
C
C     PRINT PARAMETER VALUES
      WRITE (6,550) DELT,TMAX,NUMT,QRE,DIML,DIMW,LENGTH,ERR,FACTOR,FACS,
     1FACB,FACP,FACT,RIVER,M,KTH
      WRITE (6,690) (DELX(J),J=1,DIMW)
      WRITE (6,700) (DELY(I),I=1,DIML)
      TMAX=TMAX*3600.
      IF (WATER.EQ.CHK(2)) GO TO 100
      WRITE (6,560)
      DO 90 I=1,DIML
   90 WRITE (6,570) I,(T(I,J),J=1,DIMW)
  100 WRITE (6,590)
      DO 110 I=1,DIML
  110 WRITE (6,570) I,(S(I,J),J=1,DIMW)
      WRITE (6,540)
      DO 120 I=1,DIML
  120 WRITE (6,570) I,(RATE(I,J),J=1,DIMW)
      IF (WATER.NE.CHK(2)) GO TO 150
      WRITE (6,630)
      DO 130 I=1,DIML
  130 WRITE (6,570) I,(PERM(I,J),J=1,DIMW)
      WRITE (6,640)
      DO 140 I=1,DIML
  140 WRITE (6,570) I,(BOTTOM(I,J),J=1,DIMW)
  150 JNO1=DIMW-1
C
C     ...............................................................
C
C     COMPUTE ITERATION PARAMETERS
C     COMPUTE HMIN
      HMIN=2.
      XVAL=3.1415**2/(2.*DIMW**2)
      YVAL=3.1415**2/(2.*DIML**2)
      DO 160 I=2,DIML
      DO 160 J=2,DIMW
      IF (T(I,J).EQ.0.) GO TO 160
      XPART=XVAL*(1/(1+DELX(J)**2/DELY(I)**2))
      YPART=YVAL*(1/(1+DELY(I)**2/DELX(J)**2))
      HMIN=AMIN1(HMIN,XPART,YPART)
  160 CONTINUE
      ALPHA=EXP(ALOG(1/HMIN)/(LENGTH-1))
```

```
      RHOP(1)=HMIN
      DO 170 NTIME=2,LENGTH
  170 RHOP(NTIME)=RHOP(NTIME-1)*ALPHA
      PARAM=RHOP(1)
      WRITE (6,650) (RHOP(J),J=1,LENGTH)
      KT=0
      TEST=0
C
C     ..................................................................
C
C     IF TEST EQUALS 1 CONTINUE ITERATION, IF TEST EQUALS 0 GO TO NEXT
C     TIME STEP
  180 IF (TEST.EQ.1) GO TO 250
      IFINAL=0
      IF (KT.GT.NUMT.OR.SUM.GT.TMAX) IFINAL=1
      NTH=C
      IF (MOD(KT,KTH).NE.0.OR.KT.EQ.0.AND.IFINAL.NE.1) GO TO 210
      WRITE (6,580) KT,DELT,SUM,MINS,HRS,DAYS,KOUNT
      IF (CONTR.EQ.CHK(3)) CALL PRNTA
      IF (NUM.EQ.CHK(4)) CALL PRNTI
      IF (CHCK.EQ.CHK(5)) CALL CHECK
      IF (IFINAL.NE.1) GO TO 210
      IF (PNCH.NE.CHK(1)) STOP
  190 DO 200 I=1,DIML
  200 WRITE (7,620) (PHI(I,J),J=1,DIMW)
      WRITE (7,610) SUM
      STOP
  210 CONTINUE
      KT=KT+1
      KOUNT=0
      DO 220 I=1,DIML
      DO 220 J=1,DIMW
  220 KEEP(I,J)=PHI(I,J)
      IF (WATER.NE.CHK(2)) GO TO 240
      DO 230 I=1,DIML
      DO 230 J=1,DIMW
      T(I,J)=PERM(I,J)*(PHI(I,J)-BOTTOM(I,J))
      IF (T(I,J).GE.0.) GO TO 230
      WRITE (6,660)
      GO TO 190
  230 CONTINUE
  240 DELT=DELT+DELT
      SUM=SUM+DELT
      HRS=SUM/3600.
      MINS=HRS*60.
      DAYS=HRS/24.
      GO TO ~~200~~ 270
  250 IF (KOUNT.LT.100) GO TO 260
      WRITE (6,670)
      GO TO 190
```

```
  260 KOUNT=KOUNT+1
      IF (MOD(KOUNT,LENGTH)) 270,270,280
  270 NTH=0
  280 NTH=NTH+1
      PARAM=RHOP(NTH)
      TEST=0.
  290 DO 300 J=1,DIMW
  300 TEMP(J)=PHI(1,J)
      DO 400 I=2,DIML
      DO 360 J=2,JNO1
C
C     ..............................................................
C
C     DETERMINE WHETHER NODE IS OUTSIDE AQUIFER BOUNDARY
      IF (T(I,J)) 310,360,310
C
C     ..............................................................
C
  310 RHO=S(I,J)/DELT
C
C     ..............................................................
C
C     CALCULATE AVERAGE VALUES OF T BETWEEN ADJACENT NODES
C     NODE CONFIGURATION:  T1=LEFT, T2=RIGHT, T3=UPPER, T4=LOWER
      T1=((2.*T(I,J-1)*T(I,J))/(T(I,J)*DELX(J-1)+T(I,J-1)*DELX(J)))/DELX
     1(J)
      T2=((2.*T(I,J+1)*T(I,J))/(T(I,J)*DELX(J+1)+T(I,J+1)*DELX(J)))/DELX
     1(J)
      T3=((2.*T(I-1,J)*T(I,J))/(T(I,J)*DELY(I-1)+T(I-1,J)*DELY(I)))/DELY
     1(I)
      T4=((2.*T(I+1,J)*T(I,J))/(T(I,J)*DELY(I+1)+T(I+1,J)*DELY(I)))/DELY
     1(I)
      IMK=PARAM*(T1+T2+T3+T4)
      K=0.0
C
C     ..............................................................
C
C     CHECK WHETHER NODE IS ALONG A STREAM OR ON A LAKE
      IF (RATE(I,J)) 330,330,320
  320 K=RATE(I,J)
C
C     ..............................................................
C
C     CALCULATE VALUES FOR PARAMETERS A,B,C, AND BE
  330 B=-T1-T2-RHO-K/(2.*M)-IMK
      A=T1
      C=T2
      W=B-A*BE(J-1)
      BE(J)=C/W
C
```

```
C     ..................................................................
C
C     CHECK NODE FOR POSSIBLE WELL LOCATION
      RR=QRE
      RW=0.0
      IF (RATE(I,J)) 340,350,350
  340 RW=-RATE(I,J)/(DELX(J)*DELY(I))
  350 D=-T3*PHI(I-1,J)+(T4+T3-IMK)*PHI(I,J)-T4*PHI(I+1,J)-RHO*KEEP(I,J)-
     1(K/M)*(RIVER-KEEP(I,J)/2.)+RW-RR
      G(J)=(D-A*G(J-1))/W
  360 CONTINUE
C
C     ..................................................................
C
C     CALCULATE HEAD VALUES FOR ROWS OF MATRIX AND PLACE THEM IN
C     TEMPORARY LOCATION TEMP
      NO3=DIMW-2
      DO 390 KNO4=1,NO3
      NO4=DIMW-KNO4
      PHI(I-1,NO4)=TEMP(NO4)
      IF (T(I,NO4)) 380,370,380
  370 TEMP(NO4)=PHI(I,NO4)
      GO TO 390
  380 TEMP(NO4)=G(NO4)-BE(NO4)*TEMP(NO4+1)
  390 CONTINUE
  400 CONTINUE
C
C     ******************************************************************
C
C     FOLLOW SIMILIAR PROCEDURE FOR COLUMNS OF MATRIX AS THAT CONSIDERED
C     FOR ROWS
      DO 410 I=1,DIML
  410 TEMP(I)=PHI(I,1)
      INO1=DIML-1
      DO 510 J=2,DIMW
      DO 470 I=2,INO1
      IF (T(I,J)) 420,470,420
C
C     ..................................................................
C
  420 RHO=S(I,J)/DELT
C
C     ..................................................................
C
C     CALCULATE AVERAGE VALUES OF T BETWEEN ADJACENT NODES
      T1=((2.*T(I,J-1)*T(I,J))/(T(I,J)*DELX(J-1)+T(I,J-1)*DELX(J)))/DELX
     1(J)
      T2=((2.*T(I,J+1)*T(I,J))/(T(I,J)*DELX(J+1)+T(I,J+1)*DELX(J)))/DELX
     1(J)
      T3=((2.*T(I-1,J)*T(I,J))/(T(I,J)*DELY(I-1)+T(I-1,J)*DELY(I)))/DELY
```

```
     1(I)
      T4=((2.*T(I+1,J)*T(I,J))/(T(I,J)*DELY(I+1)+T(I+1,J)*DELY(I)))/DELY
     1(I)
      IMK=PARAM*(T1+T2+T3+T4)
      K=0.0
C
C     ...............................................................
C
C     CHECK WHETHER NODE IS ALONG A STREAM OR ON A LAKE
      IF (RATE(I,J)) 440,440,430
  430 K=RATE(I,J)
C
C     ...............................................................
C
C     CALCULATE VALUES FOR PARAMETERS A,B,C,AND BE
  440 A=T3
      C=T4
      B=-T4-T3-RHO-K/(2.*M)-IMK
      W=B-A*BE(I-1)
      BE(I)=C/W
C
C     ...............................................................
C
C     CHECK NODE FOR POSSIBLE WELL LOCATION
      RR=0.0  QRE
      RW=0.0
      IF (RATE(I,J)) 450,460,460
  450 RW=-RATE(I,J)/(DELX(J)*DELY(I))
  460 D=-T1*PHI(I,J-1)+(T1+T2-IMK)*PHI(I,J)-T2*PHI(I,J+1)-RHO*KEEP(I,J)-
     1(K/M)*(RIVER-KEEP(I,J)/2.)+RW-RR
      G(I)=(D-A*G(I-1))/W
  470 CONTINUE
C
C     ...............................................................
C
C     CALCULATE HEAD VALUES FOR COLUMNS OF MATRIX AND PLACE IN TEMPORARY
C     LOCATION TEMP
      NO3=DIML-2
      DO 500 KNO4=1,NO3
      NO4=DIML-KNO4
      PHI(NO4,J-1)=TEMP(NO4)
      IF (T(NO4,J)) 490,480,490
  480 TEMP(NO4)=PHI(NO4,J)
      GO TO 500
  490 TEMP(NO4)=G(NO4)-BE(NO4)*TEMP(NO4+1)
      IF (DABS(TEMP(NO4)-PHI(NO4,J)).GT.ERR) TEST=1.
  500 CONTINUE
  510 CONTINUE
      GO TO 180
C
```

```
C     ******************************************************************
C
C
C
  520 FORMAT (F10.2,3I10,2F10.2/F10.2,I10,F10.2,I10,4F10.2/3F10.2)
  530 FORMAT (20F4.0)
  540 FORMAT (1H1,61X,11HRATE MATRIX)
  550 FORMAT (1H1,60X,16HINPUT PARAMETERS//40H LENGTH OF INITIAL TIME ST
     1EP IN SECONDS=,E10.3//46H MAXIMUM PERMITTED PERIOD OF PUMPING IN H
     2OURS=,E10.3//40H MAXIMUM PERMITTED NUMBER OF TIME STEPS=,I4//60H F
     3LUX DUE TO VERTICAL LEAKAGE FROM A CONFINING LAYER IN CFS=,E10.3//
     437H NUMBER OF NODES IN COLUMN OF MATRIX=,I4//34H NUMBER OF NODES I
     5N ROW OF MATRIX=,I4//32H NUMBER OF ITERATION PARAMETERS=,I4//28H E
     6RROR CRITERIA FOR CLOSURE=,E10.3//53H MULTIPLIER FOR HYDRAULIC CON
     7DUCTIVITY OF STREAM BED=,E10.3//36H MULTIPLIER FOR STORAGE COEFFIC
     8IENT=,E10.3//33H MULTIPLIER FOR BOTTOM ELEVATION=,E10.3//39H MULTI
     9PLIER FOR HYDRAULIC CONDUCTIVITY=,E10.3//31H MULTIPLIER FOR TRANSM
     $ISSIVITY=,E10.3//15H HEAD IN RIVER=,E10.3//25H THICKNESS OF STREAM
     $ BED=,E10.3//40H NUMBER OF TIME STEPS BETWEEN PRINTOUTS=,I4)
  560 FORMAT (1H1,64X,23HTRANSMISSIBILITY MATRIX)
  570 FORMAT (1H0,I5,(1H ,14E9.1))
  580 FORMAT (1H1,55X,17HTIME STEP NUMBER=,I10/50X,29HSIZE OF TIME STEP
     1IN SECONDS=,E10.3/40X,48HDURATION OF PUMPING AT THIS PRINTOUT IN S
     2ECONDS=,E10.3/86X,8HMINUTES=,E10.3/86X,6HHOURS=,E10.3/86X,5HDAYS=,
     3E10.3/55X,17HITERATION NUMBER=,I10)
  590 FORMAT (1H1,54X,26HSTORAGE COEFFICIENT MATRIX)
  600 FORMAT (A5/A8/A7/A7/A5)
  610 FORMAT (F10.3)
  620 FORMAT (8F10.4)
  630 FORMAT (1H1,52X,29HHYDRAULIC CONDUCTIVITY MATRIX)
  640 FORMAT (1H1,46X,40HELEVATION OF IMPERMEABLE BASE OF AQUIFER)
  650 FORMAT (1H1,56X,20HITERATION PARAMETERS///(1H ,10E12.3))
  660 FORMAT (1H0,13HWELL GOES DRY)
  670 FORMAT (1H0,39HEXCEEDED PERMITTED NUMBER OF ITERATIONS)
  680 FORMAT (8F10.0)
  690 FORMAT (1H1,40X,40HGRID SPACING IN PROTOTYPE IN X DIRECTION///(1H0
     1,12F10.3))
  700 FORMAT (1H0,40X,40HGRID SPACING IN PROTOTYPE IN Y DIRECTION///(1H0
     1,12F10.3))
      END
```

```
C     ******************************************************************
      SUBROUTINE PRNTA
C     ******************************************************************
C     THIS SUBROUTINE PRINTS OUT THE HEAD MATRIX AS ALPHABETIC CONTOURS
      COMMON PHI,SUM,DELX,DELY,DIML,DIMW,RATE,S,STRT,SPACNG
      INTEGER DIML,DIMW
      REAL K
      DIMENSION RATE(50,50), SYM(39), PRNT(60), PHI(50,50), STRT(50,50),
     1 BLANK(60), S(50,50)
      DIMENSION DELY(50), DELX(50)
      DOUBLE PRECISION PHI
      WRITE (6,50)
      DATA SYM/1H1,1H2,1H3,1H4,1H5,1H6,1H7,1H8,1H9,1HA,1HB,1HC,1HD,1HE,1
     1HF,1HG,1HH,1HI,1HJ,1HK,1HL,1HM,1HN,1HO,1HP,1HQ,1HR,1HS,1HT,1HU,1HV
     2,1HW,1HX,1HY,1HZ,1H0,1H ,1H*,1HG/,BLANK/60*1H /
      IND=(65-DIMW)/2
      DO 40 IB=1,DIML
      DO 30 JB=1,DIMW
      K=STRT(IB,JB)-PHI(IB,JB)
      K=K/SPACNG
      IF (K.LT.0) GO TO 10
      K=AMOD(K,36.)
      IF (K.LT.1.) PRNT(JB)=SYM(36)
   10 IF (K.LT.0) PRNT(JB)=SYM(39)
      IF (PHI(IB,JB).EQ.STRT(IB,JB)) PRNT(JB)=SYM(37)
      N=K
      IF (N.LT.1) GO TO 20
      PRNT(JB)=SYM(N)
   20 IF (RATE(IB,JB).GT.0.) PRNT(JB)=SYM(27)
      IF (RATE(IB,JB).LT.0.) PRNT(JB)=SYM(32)
   30 CONTINUE
   40 WRITE (6,60) (BLANK(I),I=1,IND),(PRNT(JB),JB=1,DIMW)
      WRITE (6,70) SPACNG
      RETURN
C
C     ******************************************************************
C
C
C
   50 FORMAT (1H0,50X,32HALPHABETIC CONTOURS FOR DRAWDOWN,////)
   60 FORMAT (1H ,65A2)
   70 FORMAT (10H0LEGEND***/18H0CONTOUR INTERVAL=,F10.3/32H0LOCATION OF
     1RECHARGE BOUNDARY=R/16H0WELL LOCATION=W/21H0CONE OF IMPRESSION=G)
      END
```

```
C     ******************************************************************
      SUBROUTINE PRNT1
C     ******************************************************************
C     THIS SUBROUTINE PRINTS OUT THE HEAD MATRIX IN NUMERICAL FORM
C     ..................................................................
      COMMON PHI,SUM,DELX,DELY,DIML,DIMW,RATE,S,STRT,SPACNG
      DIMENSION RATE(50,50), DDN(60), PHI(50,50), S(50,50), STRT(50,50)
      DIMENSION DELY(50), DELX(50)
      INTEGER DIML,DIMW
      DOUBLE PRECISION PHI
      WRITE (6,30)
      DO 20 I=1,DIML
      DO 10 J=1,DIMW
   10 DDN(J)=STRT(I,J)-PHI(I,J)
   20 WRITE (6,40) I,(DDN(K),K=1,DIMW)
      RETURN
C
C     ******************************************************************
C
C
C
   30 FORMAT (1H1,58X,16HDRAWDOWN IN FEET//)
   40 FORMAT (1H0,I5,(1H ,11E11.3))
      END
```

```
C     ******************************************************************
      SUBROUTINE CHECK
C     ******************************************************************
C     THIS SUBROUTINE COMPUTES THE ERROR IN THE SOLUTION ON A MASS
C     BALANCE BASIS
C     ***WARNING - USE THIS SUBROUTINE ONLY WHEN THERE IS NO INFILTRATIO
C     INTO THE SYSTEM***
      COMMON PHI,SUM,DELX,DELY,DIML,DIMW,RATE,S,STRT,SPACNG
      INTEGER DIML,DIMW
      DIMENSION RATE(50,50), S(50,50), PHI(50,50), STRT(50,50)
      DIMENSION DELY(50), DELX(50)
      DOUBLE PRECISION PHI
      TOTL=0.0
      PUMP=0.0
      DO 10 IC=2,DIML
      DO 10 JC=2,DIMW
      TOTL=TOTL+S(IC,JC)*DELX(JC)*DELY(IC)*(STRT(IC,JC)-PHI(IC,JC))
   10 IF (RATE(IC,JC).LT.0.O) PUMP=PUMP-RATE(IC,JC)*SUM
      DIFF=PUMP-TOTL
      PERCNT=(DIFF/PUMP)*100.
      WRITE (6,20) TOTL,PUMP,DIFF,PERCNT
      RETURN
C
C     ******************************************************************
C
C
C
   20 FORMAT (63H0QUANTITY PUMPED ACCORDING TO CONE OF DEPRESSION IN CUB
     1IC FEET=,E20.10//59H QUANTITY PUMPED ACCORDING TO WELL DISCHARGE I
     2N CUBIC FEET=,E20.10//51H ESTIMATE FROM PUMPING LESS ESTIMATE FROM
     3 DRAWDOWN=,E20.10//42H DIFFERENCE AS A PERCENT OF VOLUME PUMPED=,E
     420.10)
      END
```