Developing Conceptual Understanding of Equality and Equation in Grade 8 Algebra Through Inquiry Based Processes Featuring the Balance

by

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Abstract

Inquiry-based learning is often associated with large-scale thematic projects, and, as a teacher, I have experienced excellent outcomes from enacting inquiry in this manner. Wanting to see how inquiry-based processes can be applied to smaller curricular spaces, I chose Grade 8 algebra as my area of study. Over a three-week period in March and April 2015, I administered an action research project in a colleague's grade eight mathematics class. I addressed a need for comprehension of two term algebraic equations by applying inquiry-based learning practices to a series of lessons, beginning with allowing my students to create actual working balances in the classroom. I utilized these balances to explore depth of knowledge related to solving linear algebraic equations and the basic mathematical concept of equality. Through observations, collection of student work, field notes, and student interviews, I gained knowledge about how students learn the crucial concept of equality and how inquiry impacts their understanding. I synthesized this knowledge by identifying four components or "themes" that are crucial in helping students learn: (1) community and collaboration, (2) time and space to think and discover, (3) connecting hands-on and symbolic learning, and (4) multiple learning pathways.

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Chapter 1: Introduction

I am a teacher at a Manitoba urban high school with Grades 7-12, where I have been teaching Grades 7 and 8 mathematics – among other courses – for nine years. I teach in an alternative program that was created to extend alternative education practices from elementary multiage classrooms. This program features multi-age groupings, student-centered, activity-based, and thematic learning and is available in three other schools in my school division.

I am extremely passionate about inquiry-based learning, which incorporates much of what my school division expects from our program. Inquiry is embedded in much of what I do in the classroom, and I have had excellent experiences with this style of teaching over the years. In my experience, inquiry-based learning has led to high levels of engagement and long-term and short-term student achievement, and it has allowed my students to produce truly amazing exemplars and products of learning.

Along with my teaching partner, I have achieved a great deal of success using inquiry on a macro level. Each year, I co-facilitate two to three large-scale thematic class projects. Starting in the second week of school, my students spend three or four classes planning "our year" in our program. During these days, I first organize students in groups that represent their favorite core area subjects (English, Math, Social Studies, and Science). I provide these groups with many of the specific learning outcomes (SLOs) for these subject areas, which I have rewritten in student-friendly language on cue cards. Using these outcomes, students brainstorm and plan fun and interesting assignments and projects that we can accomplish as a class. I will often spend the most time with the math group, as it is challenging for many students to translate the SLOs into actual assignments.

The next class, students are rearranged into heterogeneous groups that have representatives from all four core subjects. These groups are encouraged to share their assignment ideas and integrate them into plans for cross-curricular projects. Once these groups have generated a few ideas, we congregate as a class and share. Then, as a class, we choose the best ideas to formulate several ideas for projects we can do either as in-class assignments or large-scale thematic units. Ideas can range from dissections to field trip wish lists to complex integrated projects. Two years ago, for example, our largest project was the “Labyrinth of History”. The labyrinth, which encompassed an entire classroom, was a maze with six-foot-high walls that took people who entered it through a gallery walk of ancient societies, from Mesopotamia to the Aztecs. The tour included a model of the Nile with running water, a mummy tomb, eight-foot-tall Roman pillars, a wall of Spartan weapons, a Pagoda one could walk through, and a massive Aztec temple in which students could stand. This project was rooted in the Social Studies curriculum, but it also incorporated outcomes from all of the core subjects as well as from other subject areas such as art, practical arts, and drama. For example, the construction of the labyrinth and its structures required students to learn to work with scale, measurement, and metric conversions – all grade seven and eight mathematics skills. They also applied curricular outcomes related to surface area and volume to their structures and models.

Inquiry-based learning, of course, does not only have to occur through large thematic units or projects. In subjects that I teach and have taught, I have been able to apply inquiry to assignments, projects, and even tests and exams. When I taught English, for example, I allowed students to design their own inquiry projects on any virtually any topic they desired. This task was made easier with an English curriculum that includes open-ended outcomes such as "reading", "writing", and "presenting" (Manitoba Education and Training, 2013). In humanities and science classes, topics of study in the grade seven and eight curriculum such as ancient history, water systems, world geography, and novel studies lend themselves well to inquiry. With scaffolding, students can design projects and assignments around most of the curriculum topics in these areas.

However, my current main subject of instruction is Grade 7 and 8 mathematics, and while inquiry can be applied to many areas of study in in this subject, it is difficult to apply it to some areas in particular. As noted above, students can apply spatial mathematics to projects that involve hands-on models or structures, such as the labyrinth, but there are other meaningful ways in which students can learn content like scale and area, such as designing and building a skate park. Mathematics teachers can also create many hands-on, meaningful experiences for students learning rational numbers, such as tangram activities for fractions and real-world money simulations for decimals and percents. However, designing meaningful experiences for topics such as algebra and order of operations can be quite challenging. Unsure of how to engage students personally with some of the more abstract mathematical concepts, I have too often resorted to textbooks, lectures, and notes to explain them. When this happens, the level of student engagement plummets and teaching can become frustrating.

One purpose of this thesis, then, was to acquire strategies to employ inquiry-based learning into small curricular spaces, particularly areas of mathematics in which I have had a great deal of difficulty implementing inquiry. Specifically, I chose a small-scale unit on Grade 8 algebra that, however important and foundational it may be, has been historically taught by me in a traditional, transmission-based manner. I have felt that students in my past Grade 8 math classes have not clearly established a deep understanding of algebra, and I wanted to find strategies to teach this topic that are more in line with my beliefs and strengths as a teacher, while enabling my students to develop a stronger comprehension of algebra.

My students' experience in elementary school with algebra amounts to studying patterns and relations. In my experience teaching pre-algebra to Grade 7 and 8 students, they often do not understand the link between the work they did in elementary school with graphing linear equations and the one-term or two-term equations they now have to solve (e.g. $3x-3=16$ or $2x-3=16$, solve for x). One reason for this lack of connection may be that they have trouble seeing the concept of equality when working with linear relations. For many students, a linear relation like $y=3x$ is merely a grouping of symbols that signifies points on a graph or numbers on a t-chart (a two-column chart with room in each row for an x value and the y value that matches it). When moving from linear relations to equations, my students tend to view equations as a completely new topic, and have trouble seeing the "equals" sign as essentially a balance between two equal sides. Rather than grasping this concept, my students also seem to have equated the "equals" sign with "the answer", which correlates with other researchers' findings regarding the concept of equality (e.g., Asquith, Stephens, Knuth, & Alibali, 2007).

My students seem to understand the concept of balance once I explain it to them. However, without a deeper level of cognition behind their learning, they seem to be unable to apply it to solving actual equations. They tend to rely on memorizing steps to solve equations, which allows them to be successful in the short term. However, in the long term – understanding more complex equations and polynomials in Grade Nine, for example – my students seem to struggle a great deal. My Grade Eight students performed particularly poorly on their final divisional math exam on questions that asked them to solve algebra through any means other than through memorized steps. Through incorporating inquiry-based learning principles into my teaching of pre-algebra, I hoped to develop my students' understanding of linear equations so that they could solve them in multiple ways, not just through rote memorization and repeated practice of steps. I also hope that a deep understanding of one-variable and two-variable equations will improve their understanding of algebra more generally in the long term.

I am also interested in how students learn mathematics. Despite evidence to suggest that students comprehend mathematical concepts better when they acquire a conceptual understanding before an operational understanding (Silver, 1997), there is little information available on how students develop and comprehend concepts (Davis, Sumara, & Luce-Kapler, 2008). A colleague of mine once described math as a subject in which a teacher throws an enormous amount of information at students over and over and over, and every now and then something will stick. This unfortunate scenario is how I learned math; I discovered, for example, that the "equals" sign means a balanced equation – rather than "the answer" - at some moment on my own while working through a large number of algebraic equations. However, research has shown that the majority of students do not ever experience this moment of realizing the "relational" nature of equality; students tend to understand the equals sign simply as requesting or announcing "the answer" (Asquith et al., 2007). I am interested to know how inquiry-based practices can help facilitate these realizations in a more reliable and structured way.

1.1 Research question

How do students develop conceptual understandings of equality and equation in Grade 8 algebra through inquiry-based learning processes featuring the balance?

Chapter 2: Review of Literature

This research project is based on the passion I have for inquiry-based learning, which is an extremely broad, complex, and imprecise term. The next section is an attempt to explain how I built my understanding of inquiry through many readings and thoughtful deliberations. This research amalgamated with my own experiences, and over time I have constructed a unique understanding of what inquiry-based learning is. I have synthesized this understanding of inquiry into principles that are meaningful to me, my students, and in the context of this research.

2.1 Inquiry and Constructivism

Inquiry's roots intertwine with constructivist learning theories, thus both ideas need to be examined for this project. Constructivism is an umbrella term for theories that "are concerned with how individuals make sense of the world". (Davis et al., 2008, p. 98) There are many common threads to constructivism, but they all support the idea that learning is an active process, unique to the individual, and consists of constructing meaning from information and prior experiences (Davis, 2009). Constructivists argue that we are more than empty vessels that absorb bits of information at a time. Rather, the information we take in interacts with our prior experiences, ideas, and knowledge in a complex manner that creates new ways of knowing (Davis et al., 2008).

In my teaching career so far, I have witnessed how amazing things can happen in a classroom when a group of students truly unites as a learning community. The success of inquiry-based learning, in my experience, hinges on the strength of the community within the classroom. It is no surprise, then, that I am drawn toward the theory of social constructivism. This theory can be traced back to the Russian psychologist Lev Vygotsky, who was concerned with the social aspect of learning, or how people learned together in small groups or in community. Vygotsky created a model for understanding how individuals learn called the Zone of Proximal Development (Vygotsky, 1978). The “zone” in Vygotsky’s model represents the area of a topic or skill that a student cannot reach on his or her own and needs assistance to master a certain skill or gain a certain set of knowledge. This zone is where a person has mastered as much as s/he can on his or her own, and now needs the assistance of others to gain a deeper understanding.

Because of the importance of other people working to help guide someone through the Zone of Proximal Development, Vygotsky is widely associated with social constructivism, which emphasizes the importance of the community in learning. Where constructivists are concerned with how individuals in learning environments make their own meaning out of their own experiences, social constructivists recognize the importance and influence of the learning community in how that meaning is created. A community has its own set of beliefs, systems, language, and culture that influence how knowledge is interpreted and meaning is constructed. Social constructivists “give much more weight to the collective knowledge of culture, language, and interpersonal dynamics involved with learning” (Davis et al., 2008).

2.2 Inquiry-Based Learning

Everyone has experienced inquiry at some point in his or her life. Discovering something new or interesting and inquiring into it is how we naturally experience and understand the world around us from a very young age. When we are uncertain about a topic, idea, or phenomena, “the mental processes involved to resolve this uncertainty” is inquiry (Lemlech, 1998, p. 109). Inquiry learning instruction methods employ these instinctive processes, thus it is impossible to trace the origins of inquiry to a single person.

The term "inquiry-based learning" can be traced back to the 1960s, when inquiry was developed as an instructional method to combat the perceived failures of traditional, rote, fact-based forms of instruction (Bruner, 1961). This method was developed in the wake of research and ideas put forth by Dewey (1938), Piaget (1962), and Vygotsky (1978), among others. In fact, inquiry-based learning as it relates to the sciences can be traced back to Aristotle (Hmelo-Silver, Duncan, & Chinn, 2007).

There have been numerous definitions of what inquiry-based learning entails. Wheeler (2000) complains that the word "inquiry" is too broad of a term, and that it can be used to fit certain people's worldviews. Indeed, people in education circles often use it to describe any type of learning or teaching that incorporates constructivist values. It is often associated with discovery learning and problem-based learning, of which there are aspects that overlap (Hmelo-Silver et al., 2007). Prince & Felder (2006) describe inquiry learning as an "umbrella term" that incorporates several other teaching methods including problem-based learning (p. 127). Hmelo-Silver et al. (2007) argue that inquiry should not be viewed as identical to problem-based learning as it can be applied to any type of learning scenario rather than just to problems to be solved (p. 100).

Inquiry and discovery learning are highly similar but with small differences that set them apart. Like inquiry, discovery learning occurs in problem-solving situations where the learner draws on past experiences and knowledge to discover new information and ideas (Prince & Felder, 2006). However, inquiry emphasizes the learner's role in the learning process, including generating topic ideas, choosing learning outcomes, and generating questions (Hmelo-Silver et al., 2007). Discovery learning also has a reputation of allowing only minimal guidance from teachers (Hmelo-Silver et al., 2007; Kirschner, Sweller, & Clark, 2006; Prince & Felder, 2006).

Prince & Felder (2006) state that in it's "purest form", discovery learning allows teachers to "set the problems and provide feedback on the students' efforts but (they) do not direct or guide those efforts" (p. 132). However, Bruner (1960, 1961), whose writings heavily influenced both inquiry learning and discovery learning, argues that creating a structure for student guidance is paramount.

Another issue that makes it difficult to provide a concrete definition of inquiry is that definitions differ greatly from sphere to sphere and from subject to subject. The arts and humanities allow for models of inquiry that provide a great deal of freedom for the learner. Students in these areas tend to inquire about abstract ideas and art forms instead of physical objects and natural phenomena that lend themselves to a more procedural examination. It is no surprise, then, that in the domain of science – where inquiry first came to prominence - it has acquired specific sets of procedures and steps that have developed over many years (Hmelo-Silver et al., 2007). Quintana, Reiser, Davis, Krajcik, Fretz, and Duncan (2004) define scientific inquiry as "the process of posing questions and investigating them with empirical data, either through direct manipulation of variables via experiments or by constructing comparisons using existing data sets".

In education circles, common themes and procedures have emerged to allow inquiry to exist as a teaching method. Atkins & Karplus (1962) define inquiry as a learning cycle that begins with exploration of a phenomenon. Students should be provided with space and time to explore a new concept or situation. After initial observations, students create an "invention" to explain these observations. An instructor could introduce a new concept or term during this invention phase. The final phase is discovery, where students use their invention to verify or extend their learning. These three phases (exploration, invention, discovery) have been reconfigured and refined over the years by many parties into steps such as “exploration, concept introduction, concept application” (Bell, Smetana, & Binns, 2005), “engage, explore, analyze, extend, assess” (National Research Council, 2001), and “activate, acquire, apply, assess” (MECY, 2006).

Bell et al. (2005) attempt to synthesize the definitions of inquiry from ten separate researchers who provide characteristics of scientific inquiry. Their synthesis defines inquiry as a procedure or a set of nine specific steps. These steps are (1) orienting and asking questions, (2) hypothesis generation, (3) planning, (4) investigation, (5) analysis and interpretation, (6) model creation and exploration, (7) conclusion and evaluation, (8) communication, and (9) prediction (Bell et al., 2005) These steps follow the general inquiry schema that begins with exploration and developing questions, investigating those questions through facilitated investigation, and applying what they have learned. The last step, prediction, is about learners using their new-found knowledge to inquire into other related or unrelated topics to further enrich their understanding. The inquiry process is never complete as new information acquired from research opens new and previously unknown avenues for inquiry. Because of this unfinished process of inquiry, some authors prefer the representation of inquiry in the form of a cycle (Bell et al., 2005).

When inquiry is described or defined outside of scientific inquiry, it is often presented as a more open process with fewer steps. In a support document intended for all teachers of multiage classes, Manitoba Education and Youth (2003) provides a basic process for inquiry: Students:

- pose questions and explore ways to answer them
- locate and manage information from various sources

- process and synthesize their findings
- share their findings on an ongoing basis, supporting each other in their research
- reflect on and celebrate their inquiry findings (Manitoba Education and Youth, 2003)

In the context of the arts, inquiry is often defined in even less restrictive terms. Holzer (2009) refers to inquiry as a process for artists to personally experience and connect with their work. This process includes exploring their medium, creating something new, asking questions, noticing deeply, imagining alternative solutions, and reflecting on their work. Keengwe and Maxfield (1996), define inquiry simply as “an approach to learning that involves a process of exploring the natural or material world, that leads to asking questions and making discoveries in the search for new understandings” (p. 240).

The National Council of Teachers of Mathematics does not provide a specific definition of inquiry but has consistently recommended best practices that adhere to core values of inquiry. Recommendations over the past three decades, such as open-ended questions, meaningful mathematical tasks, hands-on learning, and using problem solving as an over-arching goal of mathematics, are consistent with core values of inquiry (NCTM, 1980, 1989, 2000). Principles and Standards for School Mathematics (NCTM, 2000) stresses the importance of students having the opportunity to “seek, formulate, and critique explanations so that classes become communities of inquiry” (NCTM, 2000, p. 346), and emphasizes the need for students to solve meaningful problems by accessing prior knowledge and constructing new meanings and understandings through the inquiry process:

> Problem solving means engaging in a task for which the solution is not known in advance. In order to find a solution, students must draw on their knowledge, and through this

process, they will often develop new mathematical understanding. Solving problems is not only a goal of learning mathematics but also a major means of doing so. (p. 51)

2.3 The Principles of Inquiry

For the purpose of my research, I have synthesized the countless definitions of inquiry into a holistic set of values, rather than a set of steps, a procedure, or a graphical representation. These values, or principles, represent what I consider to be common themes within the available definitions of inquiry, along with what I have personally experienced as a classroom teacher. They have been organized into five principals for the purpose of this thesis: experiences, scaffolding, classroom community/environment, deep conceptual understanding, and student involvement. Below I explain the philosophical and theoretical underpinnings that support each tenet.

2.3.1 Experiences. Inquiry should draw upon prior experiences, a notion posited by Piaget (1970), who argued that learners need to be "perturbed" by new experiences which call into question past experiences and allow students to then formulate their own conclusions about their worldviews. This principle of inquiry draws heavily from the works of Dewey (1938), who argues that students need to experience curriculum in order for education to reach its potential. He points out that there is an intricate relationship between experience and learning, and that "to learn from experience is to make backward and forward connection between what we do to things and what we enjoy or suffer from things in consequence. Under such conditions, doing becomes trying; an experiment with the world to find out what it is like; the undergoing becomes instruction – discovery of the connection of things" (Dewey, 1938, p. 140).

To Dewey (1938), experience is one of the core concepts of his curriculum theory. Experiencing something is a complex interaction between the mind, body, society, and the environment, as well as the past, present, and future. We do not experience things in isolation; rather, each new experience is affected by past experiences, and it will, in turn, affect future experiences. The surrounding people, structures, and cultural and societal expectations will affect this new experience. This is how human beings are meant to learn and how we learn from very early on in our lives. For instance, if a child experiences a new food for the first time, her decision to eat the food is partially driven by whether or not that food looks and feels like other food she has eaten and enjoyed. Whether she has a positive or negative reaction to the new food will affect how open to new foods she will be in the future. Also, the culture and society in which the child is situated will largely determine the type of food to which she is introduced.

Through Dewey's (1938) lens, traditional, transmission-based teaching is wholly unnatural as it treats learning as a separation of the mind and body, since curriculum is only experienced in the abstract. We cannot separate the intellectual from the physical, nor can we ignore the impact our environment has on our learning. For Dewey, then, education begins with a focus on the experiences of the learner. Rather than separating education into subject areas and have students passively experiencing curriculum as a series of goals, he argues that the aim of curriculum should be to create experiences for students that allow them to develop meaningful relationships with the subject areas. Curriculum should have regard for and focus on accessing students' prior experiences with a subject in order to foster meaningful educational experiences. The "starting point, the center, and the end" is always the child, who should be encouraged into "self-expression" (Dewey, 1972, p. 19). The aim of "the educational process is to supply the material and provide…the conditions so that the expression shall occur in its normal social direction" (Dewey, 1972, p. 229).

Dewey's (1938) assertion that curriculum should be experienced – and that the first consideration of curriculum should be the student – has been extremely influential in the field of education and in inquiry-based learning in particular. Lemlech (1998) describes inquiry as, "an active process that depends on the learner. It is the learner who must connect what is new to him or her to past experience and knowledge" (p. 110). According to Dewey, in order to make student learning meaningful, curriculum should not dictate what these experiences will be. Rather, curriculum should emerge out of these experiences (Dewey, 1972). As students experience learning through hands-on, meaningful activities, content becomes more relevant to their lives as they understand the world around them.

2.3.2 Scaffolding. A second tenet of inquiry is an emphasis on the importance of scaffolding to assist students through that difficult area of learning that one cannot understand on his or her own, as demonstrated by Vygotsky's (1978) Zone of Proximal Development. This principle asks the role of the teacher to facilitate rather than steer learning, always being aware of the appropriate time to intervene to guide students through difficult material (Lemlech, 1998).

Inquiry-based learning is often criticized for its lack of structure and minimal guidance toward learning (Kirschner et al., 2006; Prince & Felder, 2006). This criticism may have some basis in reality, as some educators have interpreted discovery learning as an instruction method with minimum feedback and a lack of guidance (Kirschner et al., 2006). However, Bruner (1960) places a great deal of emphasis on the importance of structure in learning. Deep understanding cannot be attained unless there is a structure in place to strengthen fundamental knowledge amongst learners. Topics cannot be learned without teachers providing "their contexts in the broader fundamental structure of a field of knowledge" (Bruner, 1960, p. 31).

Balancing the need for structure with my desire to give students freedom to discover is something I personally struggled with early on in my teaching career. I misunderstood inquiry much in the same way many people do by assuming that if I allow a great deal of student voice in what we learn and permit a great deal of freedom in how we experience learning, students will create their own meaningful learning experiences. However, although many self-directed students remained engaged and met learning objectives, approximately half of my students became highly disorganized and in a short period disengagement followed. As a result, I would often spend most of my time focusing students and dealing with behavioral issues rather than facilitating learning. I quickly realized that I could not rely simply on student interest to keep them engaged and organized. I had to create frameworks to enable students to remain in the zone of proximal development throughout the inquiry process.

One way in which teachers may employ structure to an inquiry assignment is through mini-lessons or benchmarks. These mini-lessons often take the form of direct instruction, and should occur only when students exhibit a need to know the information presented. These lessons are rendered more meaningful due to their immediate relevance as well as their direct connection to student experience. Edelson (2001) refers to these instances of direct instruction as "just-in-time" interventions (p. 358). A teacher-facilitator must be aware of the optimal time to intervene, which is not only based on the ideal "zone" in which students need guidance to understand a concept or complete a task (Vygotsky, 1978). It is also based on the immediate relevancy and necessity of the information presented. Students understand when a lesson is necessary and relevant to their problem solving and investigations, and a well-timed mini lesson, lecture, or benchmark allows students to construct their knowledge in a meaningful context.

Hmelo-Silver et al. (2007) outline several methods in which teachers practicing inquiry-based learning can provide scaffolding. One method they describe is using scaffolds that structure complex tasks and reduce cognitive load. Inquiry allows students to embark on highly complex assignments and projects and inquire into content far above their grade level. However, without a facilitator providing structure, students often flounder with such a high cognitive load. Teachers can use a number of strategies to structure these complex tasks, such as journals, exit slips, log books, or writing out the process on an overhead or whiteboard. Maintaining these routine tasks is an important part of supporting intellectual discourse, fostering interaction, setting classroom norms, and providing direction for students who need it.

Another scaffolding strategy that Hmelo-Silver et al. (2007) outline is to be explicit about disciplinary thinking and strategies. Teachers in inquiry-based classrooms frequently push students to explain their thinking to strengthen the understanding of causal relationships or to get them to realize the boundaries of their knowledge. Prompting students to repeatedly explain and reflect on their thinking helps them to remain engaged in their learning task and impedes tendencies toward apathy or laziness. This concept of openness can be applied to assessment as well. Manitoba Education, Citizenship and Youth (MECY) (2006) encourages "assessment for learning", which encourages students to be fully involved in the assessment process. In inquiry classrooms, this often takes the form of students taking an active role in designing the rubric for upcoming assignments or projects.

Like the many definitions of inquiry, there are no set rules for scaffolding, but scaffolding remains a key component of nearly every description of inquiry I have encountered. The concept of the teacher as a facilitator of learning is an important aspect of scaffolding during the inquiry process. Dewey stresses the importance of the teacher's role as one who guides students toward goals that are socially desirable. In order to achieve these goals, a structure is required. However, Dewey believes these structures should not be externally imposed, but should emerge out of the activities themselves (Dewey, 1938). In this vision, scaffolding occurs organically out of the process of inquiry, and the learning community creates structures specifically designed for each new assignment, project, or learning task.

2.3.3 Classroom environment/community. Inquiry models place a great deal of importance on the environment in which learning occurs. An inquiry-based classroom must be an environment that encourages interaction among students and teachers in a safe and democratic manner. Dillenbourg (1999) states that “inquiry learning often incorporates an element of collaboration, meaning the engagement of participants in a common endeavor” (p. 84). Inquiry is interconnected with theories of social constructivism, which state that students form knowledge and collaboratively find solutions to problems through social interaction within communities of learners. Vygotsky (1978) emphasized the importance of social interaction in the emergence of cognitive conflicts. Through the lens of social constructivism, peers as well as the teacher-facilitator can access the zone of proximal development. Indeed, Vygotsky defined the zone of proximal development as “the intellectual potential of an individual when provided with assistance from a knowledgeable adult or more advanced peer” (Jones & Brader-Araje, 2002, p. 6). By assisting or scaffolding with peers, students are able to move on to the next level of understanding.

Once again, Dewey (1899) is instrumental in the concept of community being a foundational principle of inquiry. Dewey believes that learning is inherently social and that curriculum should emerge out of a child’s experiences and “social activities” (Dewey, 1899, p. 13). He also argues that the classroom should reflect the community in which it is situated, and it should maintain a strong connection with the democratic society at large. Dewey writes, “democracy has to be born anew every generation, and education is its midwife” (Dewey, 1899, p. 15). The classroom should not be a passive environment of rote learning, but should be a dynamic community of democratic citizens who are inspired into social action. It should “be a miniature community, an embryonic society” (p. 15).

Palmer (2007) also writes extensively about the importance of community. He rejects the traditional classroom structure, where only an “expert” holds knowledge about a subject, who then transmits this knowledge across an invisible divide to students, or “amateurs” as Palmer refers to them in this model (Palmer, 2007, p.103). Palmer offers a different model of an educational setting, one in which a community of learners surround a “Great Thing” or subject of great meaning and interest. These learners create and utilize two-way channels of learning between themselves and the subject all at once, treating every entity involved as something sacred. It is an image of learning that serves to create an ideal model for creating a collaborative learning community in inquiry-based classrooms. Palmer calls this model “the community of truth” (Palmer, 2007, p. 105).

There is also the practical importance of creating a strong classroom community in an inquiry classroom. Inquiry allows for highly complex assignments and projects but relies heavily on students’ ability to be self-motivated and self-directed. Scaffolding alone cannot ensure the successful completion of inquiry projects if collaborative learning falls apart because the students are unable to work with each other due to conflicts. Inquiry teachers must work to ensure students are able to work together collaboratively by creating a strong, safe, caring classroom community. Palmer (2007) writes about classroom structures and settings that create “possibility” (p. 30). Parker’s structures, or “paradoxes”, all honor, first and foremost, the people in the community of truth, and they outline how best to foster the relationships needed to make learning possible. He points out that creating a safe and caring community is paramount to enable possibility because the greatest barrier to possibility is fear. Teachers should not rely so much on technique that they lose sight of matters of the heart.

2.3.4. Deep conceptual understanding. Inquiry classrooms should aspire for deeper conceptual knowledge rather than rote memorization and superficial understanding. This idea, again, has roots in the works of Dewey (1938), who writes, “I use the word understanding rather than knowledge because …knowledge to so many people means ‘information.’ Information is knowledge about things (it is static), and there is no guarantee that understanding - the spring of intelligent action - will follow from it” (Dewey, 1938, p.48). The concept of deep knowledge has been further developed around the framework provided by Bloom (1956). Chapter 2 Taxonomy is a model that ranks certain types of cognitive skills - remembering, understanding, applying, analyzing, evaluating, synthesizing, and creating - from low-level cognition to high-level thinking (Bloom, 1956). Inquiry instruction should address all skills, but should place emphasis on the higher-level abilities, such as evaluating and creating (Kohn, 2008).

Bruner (1960) addresses the need for a “fundamental understanding” of the “underlying principles” of any subject under study (p. 31). In order to achieve this deep understanding, Bruner (1960) argues that students must be exposed to curricula that position topics within the contexts the broader subject. When a topic is taught in isolation, it is difficult for students to apply information to a new set of problems and they are likely to forget what they have learned (p. 32). Deep understanding comes from comprehension of basic, foundational, often simple ideas. Once this idea has been thoroughly understood by a learner, that learner can expand and extrapolate this understanding into newer, more advanced areas. “The more fundamental or basic is the idea (a student) has learned," Bruner (1960) writes, "almost by definition, the greater will be its breadth of applicability to new problems" (p. 18).

Skemp (1976, 1987) and Mellin-Olsen (1987) describe two different types of understanding: relational and instrumental. Relational understanding refers to a deeper level of comprehension, knowing not just how to do something, but why. Instrumental understanding, on the other hand, refers to rote memorization or, as Skemp (1976) defines it, "rules without reason" (Skemp, 1976, p. 89). Instrumental understanding is the type of understanding that many people of my generation have experienced in math. Personally, aside from some mathematics instruction I experienced outside of school, all of my learning in this subject has been the instrumental kind. Addition, subtraction, multiplication, and division were all taught through memorization of steps in my elementary school. Rational numbers, algebra, spatial math, and every other topic were taught the same way throughout high school.

Instrumental understanding is not necessarily a negative aspect of learning. This type of understanding can be useful once students acquire conceptual understanding of a particular topic. For example, the memorization of multiplication tables can be a useful tool for solving higher-level mathematical problems, but it can undermine intuitive understandings of number relationships if this is the primary introduction to multiplication (Sousa, 2008). Another attractive attribute of instrumental understanding is its efficiency. Rote learning and memorization, though less knowledge may be involved, can often lead to the right answer more quickly and reliably than relational thinking, especially in the beginning stages of learning a topic. For instance, 9 x 9 = 81 may be quicker for a student to process by memory rather than calculating nine groups of ten minus nine. Also, within its own context, instrumental mathematics is often easier to understand. Topics such as multiplying two negatives or dividing by a fraction are difficult to understand relationally and are much easier to learn – and to teach - as algorithms (Skemp, 1976).

However, Skemp (1976, 1987) argues that most positive attributes of instrumental understanding are only desirable in the short term or are reliant on a solid foundation of relational understanding. According to Skemp, there are four advantages of relational mathematics. First, it is more adaptable to new tasks. Students with only instrumental understanding will undertake a mathematical problem by assigning a fixed procedure to that particular type of problem. However, if that problem deviates from a fixed pathway, or introduces a slightly different concept, students can become confused. For instance, if students have been taught the algebraic method with two-term equations, they may become stumped when introduced to a third term or like terms. However, if they understand conceptually what equation and variable actually mean, they should be able to solve for x using a variety of methods.

The second advantage of relational understanding is that it is easier to remember (Skemp, 1976). This may seem like a paradox at first, as relational mathematics is often difficult to learn and teach. However, in the long term, relational understanding helps students to remember mathematical concepts by accessing their underlying and prior knowledge. For example, students often have difficulties remembering when to use and not use squared units in their answers when solving problems with spatial mathematics. A student might, for instance, be confused as to whether or not to answer metres or squared metres for a question involving perimeter of a circle. However, if that student understands that area is the number of squares of equal size within a two dimensional object and has learned perimeter through a hands-on activity, such as measuring circular objects with string and then measuring the length of the string, then that student will be better able to access that knowledge when faced with the above question.

The third advantage to relational mathematics is its ability to be effective as a goal in itself, due to the higher level of student engagement it creates. Skemp (1987) argues that with relational learning, "the need for external rewards and punishments is greatly reduced, making what is often called the 'motivational' side of a teacher's job much easier" (p. 159). This argument is related to the fourth advantage, which is that relational learning is "organic" in nature. That is, students who engage in this type of learning are motivated to expand their learning outside of what is being taught. I have witnessed this phenomenon many times. In one instance, my class and I were learning about area of triangles by exploring triangles on grid paper and discovering that they could always be drawn into rectangles and squares (thus developing a relational understanding of the equation A=LxW/2). Near the end of the class, one particularly engaged student cut a sheet of paper into five triangles to disprove our theory that any rectangle could be broken up and reassembled into two triangles of equal size. This student's creation very nearly stumped our class until one by one, students successfully reassembled the pieces into two equivalent triangles. Through engagement and discovery prompted by relational mathematics, this student created a rich learning task from which we all benefitted.

2.3.5 Active Student Involvement. There is a Chinese proverb that says, "Tell me and I forget. Show me and I remember. Involve me and I understand". This proverb succinctly describes the significance of student involvement, which is perhaps the most important tenet of inquiry. Inquiry models encourage student-led education and active, often hands-on participation among learners (Marshall & Horton, 2011). These instructional paradigms are based on the idea that there is a difference between "doing" and "understanding" (Bruner, 1960, p. 29). When students actively participate in their learning, they are not only more engaged in their learning, but they gain a deeper understanding of the content they are covering (Bruner, 1960). Dewey (1972) argues, “education must be experience based, centering on ideals such as open-mindedness and discipline in aim-based activity” (p. 29). He believes these aim-based activities could be accomplished through large-scale, long-term projects that grow out of a child’s interest.

Allowing students to become active in their learning is an essential part of engaging students in the inquiry process. In Boaler’s (2002) studies of mathematics classrooms, she contends that students do not only learn knowledge in mathematics classrooms, but “they also learn a set of practices and these practices come to define their knowledge” (p. 43). If these practices allow students to become active in their learning, they tend to develop “positive, active relationship(s) with mathematics” (p. 46). On the other hand, students in traditional mathematics classrooms tend to develop negative mathematical identities. They begin to see themselves as “received knowers” rather than as individuals who are actively engaged in thinking and learning. For these students, mathematics that is taught in an instrumental manner is at odds with their identities as dynamic, independent individuals (Boaler, 2002).

Through research involving “non-academic” high school students enrolled in Consumer Mathematics classes, Mason & McFeetors (2005) also examined how becoming active learners can positively affect students’ mathematical identities. The experiences of these students suggest that “succeeding in school mathematics is less a matter of learning mathematics content than it is a quest for a more positive sense of identity in mathematics class” (Mason & McFeetors, 2005, p. 17). This positive identity is developed through the allowance of student voice. Whether it is reflecting, describing mathematical thinking, or developing initiative for independent study, these students exhibit a more positive mathematical identity and, consequently, experience success in the subject, when they become active, confident learners.

A key factor in developing active learners is the allowance of choice in their learning. Choice allows students - all people, in fact - to gain greater intrinsic motivation for whatever they are doing (Bruner, 1960; Pink, 2009). Rather than being coerced into completing tasks by systems of rewards and punishments, students who are given an active voice in choosing what and how they will study exhibit greater academic achievement, creativity, reasoning skills, and motivation than students who have less say in what they study (Kohn, 2008). Students who have a sense of control and self-determination in their classroom also develop a greater sense of self-esteem and general well being (Ryan & Grolnick, 1986). Significantly, allowing choice in a classroom has similar effects on teachers, who benefit greatly from working in a dynamic environment, collaboratively working with motivated individuals who are all striving for excellence (Kohn, 2010).

2.4 Inquiry-Based Learning's Value in Mathematics

Inquiry-based learning has become more and more prevalent in mathematics education in North America over the last twenty-five years. Inquiry instruction is one of eight core National Science Standards (NRC, 1996), and according to the National Council of Teachers of Mathematics (NCTM), tenets of inquiry are integral to all five of the National Mathematics process standards (NCTM, 2000). A particular focus in mathematics education has been on deep, rich understandings of foundational concepts. Wiggins and McTighe (1998) refer to this focus as "enduring ideas", while the NCTM calls this an emphasis on "big ideas" (NCTM, 2006). The consensus among many educators is that our modern world is complex and changing, and in order for people to adapt to this dynamic environment, they will need to solve complicated problems, and they will not be able to solve these problems by relying on rote learning or memorization of facts (Freidman, 2005; Pink, 2010; Robinson, 2010). This worldview has existed in the world of education for many years. As early as 1897, Dewey (1972) wrote, “It is an absolute impossibility to educate the child for any given station in life. The school as a vital social institution should rather educate for change” (p. 59).

Researchers have demonstrated the value of inquiry-based learning in mathematics in many studies, especially emphasizing inquiry’s ability to foster deeper levels of understanding among learners. Silver (1997) argues that inquiry instruction fosters greater creativity in students. Vanosdall’s (2002) research suggests that not only does inquiry improve student achievement – in the form of standardized test scores – but it is more beneficial than traditional methods to low-income and minority students. Witt and Ulmer's (2010) study of middle school students' achievement through inquiry-based versus traditional learning methods yielded similar results. Marshall and Horton (2011) focused on two principles of inquiry - student participation and deep conceptual understanding - and then studied the correlation between the two. When middle school students both explored underlying concepts before teachers explained them and were involved in the explanations, “students were more frequently involved at a higher cognitive level” (Marshall & Horton, 2011, p. 93). The researchers also found that the longer students had to explore a certain topic, the higher their cognitive levels became.

2.5 Big Ideas: Balance and Equality

Since inquiry-based learning has been found by many to enhance deeper understanding of concepts, and mathematics education leaders suggest that teachers should focus on the big ideas in math (NCTM, 2006), it stands to reason that inquiry will likely be effective in helping my students gain a deeper understanding of core algebraic principles. The analogy of balance is particularly important to algebraic reasoning, since solving algebraic equations conceptually requires an understanding of the equals sign as a midpoint between two quantities (NCTM, 2006). Throughout elementary school, students are taught that the equals sign means “the answer to”. In algebra, it is more helpful for students to understand the equals sign as equivalence or balance (Mann, 2004). In the Kindergarten to Grade 8 Manitoba Framework of Outcomes (2008), equality is emphasized as an important concept in early years, and teachers are encouraged to use the analogy of a balance to demonstrate equality throughout Grades K-6.

Much work has been done on how students learn and understand the concept of equality in algebra. Many teachers have employed balances in various forms to help students understand algebraic equations. Some have used equal pan balances and have used items such as candy (Mailley & Moyer, 2004) and film containers (Porter, 1995) to demonstrate equality. Polly (2010) used digital versions of equal pan balances with shapes to help students understand equality, while others had students draw tiles and other symbols on pictures of balances (Maida, 2004). Some researchers had students draw mobiles, or balances with shapes hanging from each side, to demonstrate the idea of balance in algebra (Hedin, 2007). My intention is to alter some of these activities featuring balances by incorporating inquiry-based learning tenets into instruction. I will then conduct a qualitative analysis of how these tenets helped to develop a conceptual understanding of equality in my students.

2.6 Summary of Review of Literature

The term “inquiry” can be traced back to the ideas of thinkers and philosophers from thousands of years ago. It is the way human beings learn about new information, and at its core, it is not a highly complex idea. However, when applied to vastly different fields of study, from hard sciences to fine arts, inquiry begins to take many forms. These fields have created countless frameworks for laying out how to inquire about a certain subject in a way that fosters curiosity and knowledge. There is no one, single framework for inquiry, although there are certainly common themes and tenets.

When I began delving into the history, research, and philosophy behind inquiry-based learning, my intention was not to create my own, personal tenets. However, the lack of consensus on how inquiry-based learning has been organized and categorized led me to rethink inquiry in general. Indeed, I came to find the apparent need for individuals and groups to categorize this type of learning rather troubling. At its core, I believe inquiry is powerful because of its foundational principles, and it runs counter to the spirit of these principles to distill inquiry to a series of steps. For my research and my teaching, it made sense to focus on these overarching principles. Once I was able to conceptualize inquiry in this way, I was able to shape my unit and plan my research methodology.

Chapter 3: Methodology and Design

This chapter explains my action strategy in terms of the original intentions for my study. I outline the research methodology used as well as the methods I chose for data collection, interpretation, and analysis. I also explain ethical issues as well as how I ensured the quality of my research. Chapter 4 will detail what actually transpired during the project, which differs in many ways from my original plan.

3.1 Methodology

3.1.1 Action Research. Because I am studying inquiry-based learning, and letting students guide instruction is such a huge component of inquiry, it was a natural fit for me to use action research for this project. Action research is itself a form of inquiry in which the practitioner reflects upon his or her practice to bring about change and contribute to new knowledge (McNiff & Whitehead, 2011). Stringer (2008) refers to action research as simply another term for inquiry learning. The action component is all about the activity, or what we do to enact change. The research component refers to inquiring about what we are doing, analyzing it, and creating new knowledge. In an educational setting, action research can be an intentional way to study solutions to problems.

Through the lens of action research, my study has two major intentions. One intention is to develop and refine teaching practices, materials, and activities, and the second is to allow me to better understand student's mathematical learning. McNiff and Whitehead's (2011) model for planning an action research project provides a useful set of steps for my first intention.

1. Identify an area of concern within my practice;
2. Recognize that my educational values are negated;
3. Assess the situation and craft a solution;

4. Make a plan;

5. Act on my plan; and

6. Reflect and repeat cycle.

Using this model, I identified areas of concern in my practice, namely that I felt as though my students were not developing deep, conceptual understanding of algebra. I recognized that my values - particularly my passion for inquiry - were being negated when teaching this unit. The solution I crafted was to apply inquiry-based learning processes to the teaching of this small curricular space. This plan to implement inquiry into my instruction of equality and balance was my action strategy, while collecting and interpreting data helped me to observe, reflect upon, and systematically understand what happened during my intervention. My research will also inform what I will do in the future, including what teaching strategies I will use, what materials I will develop, and what activities I will employ.

Action research allowed me to reflect and act during the actual intervention. During my project, specifically in my daily notes, I reflected on my observations and acted based on my interpretation of those observations (Stringer, 2008). During my instruction, I continually practiced the small-scale action research sequence of observing, thinking, and reacting. On a more macro scale, this was the first cycle in what I intend to be a series of cycles that last many years. I teach pre-algebra to my seventh grade students, so I will be able to use the insight from the eighth grade class and apply it to my sevens. I will also cycle back to a new and improved version of this unit next year when I re-teach algebra, applying knowledge I have gained from this research project to that year and beyond. Hopefully, these cycles will result in better teaching from me and better long-term understanding and success from my students.

McNiff and Whitehead's (2011) above model also helps to guide the second intention of my study, which is to inquire into student mathematical thinking and learning. Another “area of concern” was that I lacked understanding of how conceptual understanding is achieved in algebra. My “solution” was to apply procedures associated with inquiry-based learning to the teaching and learning of algebra, since I have studied - as well as personally witnessed - the tendency for students to develop deep conceptual understanding when immersed in inquiry. Through engaging in research during this project, I learned about how students develop this understanding through these inquiry-based processes. Analyzing, interpreting, and reflecting on my data not only helped to illuminate student conceptual understanding in this curricular area, but it will hopefully help other interested parties understand it as well.

My four methods of data collection assisted me in developing my intentions. The artifacts that were created, collected, and catalogued helped me to refine and develop learning and teaching tools within my classroom. These artifacts were also useful in showing the progression and proof of learning when I searched for evidence of conceptual understanding. Field notes helped me to document the successes and failures of my unit plan’s activities. They also allowed me to record key moments that showed student understanding. Interactive writing was an incredibly useful tool to receive feedback about my teaching practices, materials, and activities, while simultaneously getting personal descriptions of learning from my students. Finally, narrative inquiry allowed me to receive deep, personal feedback about my teaching and about students’ learning. This study afforded me an incredible opportunity to directly hear my students reflect about my teaching their learning in an extremely detailed way.

3.1.2 Narrative Inquiry. Interviews were conducted using the framework of narrative inquiry, which is a major component of my data collection as well as my data interpretation.

Narrative inquiry is a research methodology that allows a researcher to inquire about certain questions, problems, or life experiences through an emphasis on narrative or story (Clandinin & Connelly, 2000). Chase (2005) describes narrative inquiry as “the interdisciplinary study of the activities involved in generating and analyzing stories of life experiences (e.g. life histories, narrative interviews, journals, diaries, memoirs, autobiographies, biographies) and reporting that kind of research.” (p. 204) Inspired by the historical importance of storytelling throughout all human cultures and by Dewey's ideas about the importance of experience both personal and social, narrative inquirers seeks to understand these experiences through collaboration between researchers and participants through social interaction. Like stories themselves, learning is deeply personal, situated, and takes place over time. Narrative inquiry can be used to define how this learning happens because it asks the researcher to construct a story out of the learning.

The way I utilized narrative inquiry in my research was to take a collaborate and assisted approach to crafting and understanding the stories of my students' learning (Clandinin & Connelly, 2000). Using observations, field notes, artifacts, and interactive writing, I constructed narrative texts of three to five paragraphs for each of the four students I interviewed. These texts were stories of student learning describing the flow of learning as interpreted by me. I provided detailed descriptions of events and key moments in these narrative texts. I then read these texts to my students in brief chunks, using illustrations and detailed examples to allow them to clarify, expand upon, and/or correct my interpretation of their learning.

This process allowed my interviewees to take ownership of their narratives by responding to what I presented to them, essentially becoming co-writers and co-constructors of their own stories. During the interviews - or “conversations” as Clandinin and Connelly (2000) prefer to call them - I also provided participants with the opportunity to reflect on what mattered to them by purposefully leaving gaps in the narrative and inviting them to fill them in with their own experiences. This narrative text approach allows researchers to add value to their data, as participants who are invested in the research validate and solidify what the researcher believes has transpired (Chase, 2005).

3.2 Action Strategy

The “big idea” I focused on is the concept of algebraic balance in mathematics as it relates to the equals sign. In the past, I explained the concept of algebraic balance by drawing models on the board and even demonstrating an equation with a real-life equal pan balance. However, my plan was to introduce a rich learning activity where the students would explore real life balances they would create in class and then use them to comprehend and demonstrate algebraic strategies. The rich learning activity that I chose was an introductory lesson in balance and equality, in which students were instructed to build scales out of materials provided by me. These scales would form the foundation of the unit on algebra that would follow. Throughout the unit, I designed my instruction to follow the five principles of inquiry, which I outlined in the previous chapter. Each principle was addressed in the following ways:

3.2.1 Experiences. Russell & Mokros (1996) found using balances helpful when teaching students concepts of central tendency, such as mean, median, and mode. Maida (2004) and Polly (2010) used visual representations of scales to facilitate the understanding of algebraic equations. I encouraged students to experience curriculum by having them not only create their own balances in the form of mobiles made out of pop bottles, hangers, and string, but I also instructed students to choose their own weights as well. I asked students to create their own balances in order to provide them with a hands-on experience meant to help them formulate their own visual representations of equality.

3.2.2 Scaffolding. In small curricular spaces, scaffolding becomes less intrusive, but it is nonetheless important. As solving algebraic equations is generally not a topic on which Grade 8 students have prior knowledge, I integrated multiple mini-lessons at the beginning of lessons. For example, when I began to introduce one-term algebraic equations, rather than asking students to simply play with their scales, I directed them with specific instructions and examples. Observations of student work also allowed opportunities for just-in-time mini-lessons and worksheets (Edelson, 2001). In situations where I noticed that many students were misunderstanding a specific instruction, for example, I would stop the class and clarify instructions. I also facilitated learning among groups of students who were struggling with the assignment. When circulating the classroom during periods of work, I would often identify students or groups of students who were struggling, and I would sit with them and work through problems with them.

Students also engaged in written reflections on topics that I provided. I replied to these journal entries with entries of my own, commenting on what students had written and/or asking them to further explain some of their thoughts. This interactive writing will allowed me to identify areas of instruction that I needed to address and enabled me to keep as many students as possible in the zone of proximal development.

3.2.3 Classroom environment/community. My original intention of this study was to conduct my research in my own classroom within our alternative program's community. However, due to issues surrounding the ethics of conducting research with my own students (see Chapter 3.7, p. 48), I was fortunate enough to conduct my research in a safe, caring, empathetic environment in my colleague's neighboring Grade 8 math class. Although not all members of my colleague's class bought into what Palmer (2007) describes as the "Community of Truth", the majority of students had a strong commitment to their community of learners. My colleague worked all year to encourage questions and exploration, and he discouraged arrogance and bullying. Students already sat in groups of their choosing. Allowing this grouping helped to establish connections between students as they collaboratively built knowledge.

3.2.4. Deep conceptual understanding. The concepts taught in this unit are foundational for understanding of algebra and I sought to develop relational understanding of these concepts. This unit incorporated learning strategies that scored high on Bloom's (1956) taxonomy of learning. I looked for data that demonstrated a relational - rather than instrumental - understanding of algebra (Skemp, 1976). To demonstrate a relational understanding, participants needed to show an understanding of algebra symbolically, concretely, and pictorially.

The unit on algebra was designed to not only foster comprehension and application of knowledge, but by building their own balances, students also had a hand in creating their own learning experiences, which is the highest level of cognition on Bloom's taxonomy (Bloom, 1956). During the unit, I intervened when necessary to explore themes or ideas that merited further exploration. Again, scaffolding became important as students required interventions from me to help them relate their balance activities to solving equations.

3.2.5. Student involvement. Despite the work done on how best to demonstrate equality, none of the researchers mentioned in the last section of Chapter 2 involved students in the creation of the balances (see Chapter 2.5, p. 25). Abiding by the last tenet of inquiry, students should have as much involvement in their learning as possible. It would have been preferable to have students design their entire investigation into algebra themselves, but since one of my intentions in conducting this research is to illuminate how inquiry can work in small curricular spaces, I did not provide quite that level of student involvement because of the amount of time it would take to start the unit. Also, in order for the students to formulate complex questions about a topic they inquired about, they needed to have moderate interest and prior knowledge about that topic. Most of my colleague's students had little to no interest in, or knowledge about, algebra.

However, students still had a great deal of involvement in the creation of their scales. Although I provided the materials for the scales, I allowed freedom in the students' designs. I also permitted students to choose the weights for their scales. As the unit progressed, I injected moments of freedom whenever I was able to, and I encouraged situations of greater student involvement when I felt it was appropriate. For example, during the scale-building activity, although I did not plan for it, I instructed students to assign their scales a name of their choice.

The following is a daily break down of my initial three-week unit plan (See Table 1).

Table 1

Unit Plan

Week 1	
Monday	After reviewing students' prior knowledge about balances, I will ask them to design and build their own to help them to understand algebra. I will let them explore with the supplies in the room, which I will provide, but I will also allow them to supply their own. There are tutorials on the Internet that explain how to build versions of equal pan balances and students are welcome to look these up and follow them. Some of the supplies I will provide, such as empty coke bottles and string, are items that have been used in these videos, so I will know that they can be used to make working scales.
Tuesday	Students will continue to create their balances. They will complete them in class or finish them for homework.
Wednesday	Students will present their working balances. Students who have not completed

	their balances or left them at home by mistake will be provided equal pan balances. Students will get to choose their "weights". These can be any small, uniform manipulative, as long as there are enough of them. Pennies, gram weights, and pattern blocks are examples that can work. Students will also receive small paper coin envelopes and small clear Ziploc bags. Once they have their balances and weights in working order, we will start to work with them. Students will place their weights into paper bags to simulate an unknown quantity (or "variable"). I will guide them through a few examples. (For example, I might present two variables (bags of an unknown number of weights) are on one side, and four variables are on another. I might say: "What if I didn't know the weight of either variable? How would I figure out the weight of either one? I would have to know information about one side. So say I did know that the weights on the left were ten g each? What would the other weights be? Now how did you know? That logical reasoning is basic algebra – you're given some information and you have to use logic along with that information to find an unknown quantity.") Give a few more examples and then have them solve them with other groups. Pairs can guess the weight of each figure based on the weight of one figure provided by its owner/creator (in the form of weights in Ziploc bags). The units of measurement are arbitrary, so students can have fun with naming their units of weight measurements.
Thursday	Figures and balances will be redistributed. Review activity from yesterday. Keep instructing students to try more and more difficult configurations. Have them record their thinking on worksheets by drawing out what they are building.
Friday	Review last class's activity. Have students draw examples on the board. As a class, we will decide on which symbols work the best. For example, they may choose shapes like squares and rectangles to represent different bags of different numbers of weights. Unknown bags may be represented by a question mark. Have students build equations and draw out how they are solving them, step-by-step.
Week 2	

Monday	Discuss the term “variable”. Explain that in algebra, we use letters to identify unknown variables, particularly "*x*". Ask them if x is the best letter. For instance, why not use a question mark or other letters? Give examples from previous classes, but now enumerate with variables. Ex: $10+10 = x+x+x+x$, which can be written as $20 = 4x$. Provide small, clear Ziploc bags. Demonstrate how they can use these clear bags to show known quantities and the paper bags to show unknown quantities. Give them numerical equations and have them build them with their weights and balances. Ex: Build 2x=30. Model one or two of these problems, then ask them to record these equations on a worksheet by drawing what they see using symbols as well as writing the equations. <u>Students will not solve these equations yet.</u> Move on to more complex equations to build. Ex: $23 = 4x + 3$.
Tuesday	Review. Have students solve simple equations (for *x*) that I provide. Students will build them and draw them on an accompanying worksheet. On this worksheet, they will have to show the steps by drawing each one and explaining their thinking. Instruct them to come up with some common strategies (or "rules") for solving algebraic equations. These rules will be written on their worksheets in the "explain your thinking" column. During this lesson, I will look for a-ha moments. As a whole class, compile different rules and see what general rules we can create together.
Wednesday	Review and synthesize rules. Continue with more of the activity from last class. Build simple equations and show numerically what we are actually doing when we are solving. Ex: In $20 = 2x + 4$, we are taking away *4* from both sides, etc. Model how to build equations, as well as solve them pictorially as well as numerically. Ask them to build a series of equations, and write down numerically what is going on when they are solving them. Accompanying worksheet will get them to draw the weights on a line balance, and show the steps one-by-one they used to solve the equation, using the rules we have compiled as a class.

Thursday	Continue and complete Wednesday's lesson.
Friday	Have students jigsaw around the classroom. Get them to work with others they haven't sat beside and get them to create equations for each other, then build what they see. Accompanying worksheet will get them to draw the weights on a line balance, and show the steps one-by-one they used to solve the equation, using the rules we have compiled as a class.
Week 3	
Monday	Present them with problems too complex, or with too large of numbers to solve concretely. Have them solve pictorially and numerically. Ex: $3(t+4) = 30$. If time permits, I will introduce negative numbers into the drawings.
Tuesday	Start to move away from drawings to numerical only. Give examples on the board. Give students equations for them to solve numerically only, showing each step.
Wednesday and Thursday	Give students time to explore this concept, providing more questions and guidance. Students will solve algebraic equations numerically only on worksheets.

3.3 Data Collection

I collected data throughout the research project in four ways: artifacts, field notes, written student reflections, and interviews. The first part of my data was derived from artifacts such as the scales, weights, and worksheets that my students completed. The scales and other tangible artifacts helped to demonstrate student understanding as well as engagement in activities. Students demonstrated relational and conceptual understanding of the content by drawing and writing their answers to various questions and word problems on worksheets. These worksheets

– along with the tangible items – helped record the progression of learning throughout the study and were used in conjunction with teacher observations, student writings, and interviews. I took photographs of their balances and collected the worksheets that they completed in class.

3.3.1 Artifacts. Artifacts have been shown to be useful in educational research but are often difficult to interpret (Cohen, Manion, & Morrison, 2007). Tangible items may suggest what a participant has done but cannot explain why. The worksheets showed a progression of understanding, but without explanation from students they could not describe how conceptual understanding was achieved. That is why I utilized artifacts in conjunction with other data gathering methods. For instance, in many instances I observed exemplars from worksheets that demonstrated that students had gained some understanding. I then specifically asked those students about what helped them gain this comprehension during class or during an interview if any of those students happened to be one of my interviewees.

3.3.2 Field Notes. The second type of data I drew upon was written reflections in the form of teacher observation field notes. These day-to-day notes were reflections on how well students were understanding algebraic concepts, what transpired during class, and any other pertinent observations I may have noticed. These notes were sometimes recorded during class time, but it was rarely possible for me to do this. I recorded most notes in after class ended, in blocks of about thirty minutes. I utilized LeCompte and Preissle's (1993) guidelines to direct my observations, focusing on the following questions:

-How are activities being described, justified, explained, organized, labeled?

-What is being said, and by whom?

-What is being discussed frequently/infrequently?

-What appears to be the significant issues that are being discussed?

-What non-verbal communication is taking place? (Cohen et al., 2007)

The third form of data I utilized was written student reflections. At five points during the unit, I employed interactive writing techniques to gather reflections and observations from students in the form of written student journals. Mason and McFeetors (2002) describe interactive writing as a way to garner rich understandings of student learning. Interactive writing involves prompts from the teacher for students to reflect upon in the form of short paragraphs in journals. The teacher then writes responses in these journals in order to prompt further reflection, validate feelings, clarify information, and verify interpretation (Mason & McFeetors, 2002). For the purposes of my research, I asked students to reflect on questions related to how they were developing conceptual understanding of equality and equation through the tenets of inquiry that they encountered throughout the unit.

3.3.3 Interviews. Before the unit began, students indicated whether or not they wished to be interviewed on an information form collected by my teaching partner. Once the unit was over, I selected students to be interviewed based on purposeful sampling techniques to encourage diverse perspectives (Stringer, 2008). I sought interviewees who represented diverse perspectives and were "typical" of people in the setting (Stringer, 2008). Based on my knowledge of the participants gained from student products and field notes, I selected two high achieving students, one struggling student, and one student with a moderate understanding of the content. Interviews were conducted immediately after the end of the unit. Four students were asked to comment and reflect on the unit they just finished and what they thought the balance exercise did for their understanding of algebra.

Spradley's (1979) framework of questioning techniques helped to guide me in eliciting natural, neutral, and authentic responses from my participants as I engaged in the interview process. This framework guides researchers to begin interviews with a broad, open-ended "grand tour" question, such as, “Tell me about your experience” or “What happened during this unit?” (Spradley, 1979). These grand tour questions allowed interviewees to frame their experiences in their own terms. However, because I used narrative inquiry, and the students’ experiences were already synthesized in part from their own writings, my grand tour question was simply to ask participants if the narrative text I presented to them was accurate and if there was anything they wanted to add or change. The second phase of questions, or "mini-tour" questions, were derived from the content of the grand tour answers. During the answering of the first phase question, I will be listening for references to inquiry, learning concepts, and/or developing skills. For the second phase, I mentioned these references and asked students to, “Tell me more about this (experience with inquiry, learning task, etc.)”. I attempted to extend participant responses by instigating prompt questions, such as, “Is there anything else you can tell me about (your experience with inquiry)?” When I believed there was more insight to be given, I would ask interviewees to provide examples.

3.4 Data Analysis and Interpretation

My quest in analyzing my data was to restate the data as a handful of powerful themes that represent an account of the impact that our inquiry unit had on the learning community and the understanding of mathematics. During the data analysis phase of my research, I looked for key experiences or illuminative moments that allowed me to understand the learning processes of my students. After I identified these key experiences, I used Stringer's (2008) framework to organize and unpack these experiences, ascertaining what was significant about each experience and what its features were. This framework for analysis required me to:

- Review information acquired from teacher and students in the data gathering phase

- Identify significant or key experiences within each participant's/group's data
- Deconstruct or "unpack" those events to reveal the detailed features and elements of key experiences.
- Use those features and elements to construct individual accounts describing how selected individuals experience and interpret the issue investigated.
- Use the features and elements within individual accounts to construct joint accounts revealing the perspectives and experiences of students and teacher.
- Use the joint accounts to provide the material for a collective account chronicling events by comparing and contrasting the perspectives of students and teacher. The collective account identifies points of commonality among perspectives and experiences, and points of discrepancy, diversity, or conflict (Stringer, 2008).

The last point underscores the importance of joint accounts when collecting, interpreting, and analyzing data. The process of narrative inquiry is a method of collecting data, but it is also useful in analyzing and interpreting data. By co-authoring narrative texts that represent student learning, the data that I collected through field notes, artifacts, and interactive writing was studied and analyzed by the research subjects themselves. This collaboration was driven by data and steered by my interest in how my students built their understandings of Grade 8 algebra through inquiry-based learning. By asking participants to verify and expand upon their learning stories, I sought to add value and authenticity to my data and my interpretation of this data.

This framework was not the only method of identifying key moments. It was important for me to not let the framework completely dictate my data analysis process. I strived to be alert to the meaningful experiences that were seen and felt by myself and by my students. These moments arose in many situations, including during class discussions, periods of work, and in one-on-one conversations. Other key findings came to light when studying student work, observations (field notes), interactive writing, and interviews. Interactive writing helped to highlight key experiences as students wrote about moments that were meaningful to them. By writing back to them each time, restating their experiences and checking to see if I interpreted their accounts correctively, interactive writing also allowed me to interpret as the data progressed. Field notes were also helpful in conjunction with my students' writing, as I cross-referenced them with interactive writing data. Once these experiences were identified, interviews allowed me to delve deeper into why these experiences were meaningful and what they meant in regard to my research. Artifacts, such as worksheets and tangible project items, helped identify as well as explain key experiences all throughout the data analysis process.

3.5 Criteria for Assessing Validity in Qualitative-Oriented Action Research

Anderson and Herr (2015) suggest that most action research traditions agree on the following objectives: (a) the achievement of action-oriented outcomes, (b) the generation of new knowledge, (c) the education of both the researcher and participants, (d) results are relevant to the social setting, and (e) research methodology is sound and appropriate. (Anderson and Herr, 2005). Based on these traditions, Anderson, Herr, and Nihlen (2007) developed five indicators of quality for action researchers:

3.5.1 Outcome Validity. One test of the validity of action research is to observe if the action taken results in a resolution of the problem that prompted the study. In my study, I was able to address the problems of a lack of understanding and interest in algebra from my students as well as answer my research question, which inquired into the conceptual understanding of equality and equation in Grade 8 algebra. During my research project, I also engaged in "rigorous" action research by continuously evaluating my instruction and reframing the problems of my study as new information arose (Anderson & Herr, 2005).

3.5.2 Democratic Validity. All parties who have a stake in the resolution of the problems under investigation should be allowed to participate in the study (Anderson et al., 2007). In my study, participants were highly invested in developing conceptual understanding of equality and equation. Although I did not provide marks for this unit, the knowledge they developed over the course of the project was foundational to their understanding of future units in future levels of mathematics. I included the voices of my students in this study through interviews, interactive writing, and recording quotes from participants in my field notes. Including these stakeholders' voices ensures that their perspectives are included.

3.5.3 Catalytic Validity. Anderson and Herr (2005) explain catalytic validity as "degree to which the research process reorients, focuses, and energizes participants toward knowing reality in order to transform it" (p. 24). This test of validity relates to the education of both the researcher and participants. Both researcher and participants should reorient their "view of reality" from action research (Anderson & Herr, 2005, p. 24). In my study, as I continually reassessed how I enacted inquiry-based learning processes into my instruction, I would enable students to also reorient their learning. Participants also had opportunities to reorient their learning as a result of interactive writing. After reflecting and writing about their learning, many students came to realizations about their learning reality. I found that the interviewees, in particular, gained particularly deep understanding about how they came to learn algebra. Through the interview process, the interviewees provided insight into their learning that was helpful for my research, but, I believe, also helpful for their own education in the future.

This study has also prompted me to alter my teaching practices in profound ways. I will be teaching this unit again, likely many times, and I will continue to use the scale activity and refine the learning experiences that follow it. On a macro level, this study will also impact how I will alter my teaching of all subjects in the future. For example, regarding inquiry-based learning through the lens of the five principles has already changed the way I approach inquiry, as I now see it as a more holistic approach to instruction on every aspect of daily school life. See Chapter 7 for further discussion about how this study has and will impact my growth as a teacher and, hopefully, other teachers.

3.5.4 Dialogic Validity. Diagnostic validity is similar to democratic validity but differs in that "the focus is less on broad inclusion than on the validation – both during and after the study – that methods, evidence, and findings resonate with a community of practice" (Anderson & Herr, 2005, p. 25). I will achieve dialogic validity by publishing my study on the University of Manitoba's MSpace database and making it the property of my community of practice. Colleagues in the school in which I work were privy to the successes and setbacks of the entire process of my project. I have shared and will continue to share my learning with colleagues in my school as well as in the broader education community through conferences and workshops.

3.5.5. Process Validity. Process validity asks that the researcher has designed a study that has been carried out in a competent manner (Anderson & Herr, 2005). I demonstrated my competence throughout my study, and my committee is in charge of checking that my research is competent and complete. Anderson and Herr (2005) also state that process validity asks that problems are framed and addressed "in a manner that permits ongoing learning of the individual" researcher. As stated above, during my research project, I continuously re-evaluated and adapted my instruction in reaction to my data. As a teacher-researcher, I will continue to rework and refine my instruction as I work in the topic of Grade 8 algebra and in other subjects.

Guba (1981) suggests that there are four aspects of a study's trustworthiness. It is important that research is "credible", "transferable", "dependable", and "confirmable" (p. 80).

3.5.6 Credibility. My research is "credible" because I participated in the study myself over a long period, I persistently observed my students, and I had many data sources to triangulate and crosscheck with one another. I debriefed with my students in the form of interactive writing, allowing them to reflect on my own observations and confirm whether or not they were accurate or there was more information to be added. I also utilized narrative inquiry to create narrative texts from my other data and read these texts to my interviewees. The interviewees were then able to comment on these narrative texts, which helped to further strengthen my data.

3.5.7 Transferability. Although my study was primarily meant for my own personal growth as a learner, there are "transferable" aspects. I taught an area of the Manitoba curriculum that is mandatory for all regular and advanced eighth grade students and my students were not members of my alternative education group, which can be viewed as a specialized class. The participants in my study were diverse with a wide variety of backgrounds, learning styles, and abilities.

Other teachers, researchers, and teacher-researchers working with this age group and within this topic will be able to find meaningful, transferable knowledge within my study. Through describing every step of my research design, I have provided a detailed description of my action strategy that other members of my community of practice can understand. Other teacher-researchers may wish to follow my action strategy in their own instruction or research, while others may wish to focus on individual aspects of it.

3.5.8 Dependability. Stringer (2008) states that trustworthiness of qualitative research also "depends on the extent to which observers are able to ascertain whether research procedures are adequate for the purposes of the study" (p. 50). If insufficient information is available through a lack of data, conclusions may not be dependable. I used many methods to collect data, including field notes, interactive writing, and interviews. I also collected a great deal of artifacts to establish an "audit trail" of data (Mills, 2007, p. 84).

3.5.9 Confirmability. My research is "confirmable" because my raw data was continuously re-examined throughout the course of writing my thesis. It was, and is, available for my advisor to review. As stated above, I triangulated and crosschecked my data, and I practiced reflexivity (Mills, p. 85). That is, I included in my notes regular reflections on my own biases and assumptions that affected my research question and interpretations.

I also referenced Wolcott's (1994) relatively simple strategies for ensuring the validity and quality of my research. Since my research was founded on inquiry-based learning, for which letting the students participate more in discussions is one of its tenets, I "talk(ed) little and listen(ed) a lot". I also "record(ed) observations accurately" and "(wrote them) early" by ensuring I wrote down reflections immediately after class and after interviews. I will "report fully" all my findings and I worked hard to see what other stories my data told me. I also "(sought) feedback" from the participants in my study, including students, parents, administrators, and my teaching partner in detailed letters sent to these parties. Finally, I tried to write this report in a way that allows readers to "see (conclusions) for themselves" (Mills, 2007, p. 94).

3.6 Ethical Considerations

The chief ethical issue with my study was that I wanted to conduct research on my own students. This was problematic because of the power-over relationship between my students and me. In order to mitigate this issue, I worked with my colleague's Grade 8 Math class rather than my own students. Although I did not teach these students mathematics, I had a relationship with many of them because of many factors, including proximity, extra-curricular activities, and relationships they had (and have) with my own students. I also taught many of them the previous year in classes such as Grade 7 Technology and Grade 7 Drama. I had a strong rapport with most students in the class, and I knew the practices of the teacher who teaches them math. They all knew me well and understood my reputation as a teacher who tends to value hands-on learning. All participants had experience with establishing t-charts from numerical patterns and graphing them. They did not have any experience with variables in algebraic equations.

It was important that I protected anonymity and confidentiality. To achieve this, I acquired informed consent from parents and informed consent from participants (see Appendices B and C). I also ensured safe, secure storage of data on a password-protected computer. Students who chose to participate in this study by allowing me to use their work as data were unknown to me until after my study was complete, as a colleague of mine introduced the project to them and collected all consent and assent forms. I also did not know which students volunteered to be interviewed until after the unit was over.

I ensured that the benefits to the participants far outweighed the risks in this study. There were no discernable risks to the participants, but there were many benefits to the students. They were able to participate in a unit that was extremely well thought out down to every considerable detail. I designed this unit to maximize comprehension and provide meaningful learning experiences that would motivate and inspire students to enjoy mathematics. Those students who chose to participate in interviews also had the rare opportunity to discuss their learning of mathematics in a one-on-one (approximately) 30 minute session with the teacher.

In this chapter, I have outlined my study's design and methodology. In the next chapter, I describe what actually transpired during my 18-class project, outlining key moments from my data in chronological order.

Chapter 4: Project Implementation and Data Description

4.1 The Teaching Experiment

From the beginning stages of planning for my project, I understood that there would inevitably be changes to the way my intervention, or teaching experiment, would be implemented. Planning is always problematic when one gathers data in a complex, often chaotic environment such as a middle school. Choir practices, student absences, field trips, and impromptu assemblies all impacted the plans I had for this project. Unexpected challenges arose in the midst of data collection and I often had to alter plans partway through the period. For instance, on the third day of data collection, I arrived early to print off the worksheets for the day. However, the school's internal computer systems were down, so I had to improvise. I had the students write their own worksheets by hand, following my directions from the front of the classroom. Unfortunately, this procedure took nearly half of the period, so I had to completely change my plans for instruction and data collection that day. It also resulted in my project being further pushed back and delayed. This unexpected but all-too-common occurrence is an example of numerous similar occurrences throughout the intervention.

I altered my project for practical reasons as well. The scales were, and have always been, the main component of my teaching project. It was important to me that these were designed and created by the students themselves. My reasoning for this was that creation scores high on (1956) Taxonomy, and thus sufficiently supports the fourth tenet of inquiry, which is deep conceptual understanding (see Chapter 2.3.4, p. 19). Having students design their own scales would also allow the highest level of student involvement that I could conceive, which would satisfy the fifth tenet. Nevertheless, one must often balance ideals with reality, and when I attempted to build the scales on my own in the months before the unit began, I was faced with the reality that students designing and building the scales according to their own images of balance was simply untenable. It would have no doubt been a valuable experience to have students design working scales from their own ideas, drawings, and supplies. I may even revisit this idea in the future, or in other classes, such as Science. However, the process of designing and building the scales would have taken many classes to complete, and due to the time restrictions, I decided to provide students with supplies for the scales as well as a basic design.

Despite the compressed timelines, I found the data collection to be a delight. It was immensely enjoyable and fulfilling to be able to finally put my plans into action. The last minute alterations often resulted in a better-structured unit than I had planned. For example, when nearly half of my class was away for a field trip that I had not foreseen, I was unable to move forward as planned. The field trip forced me to slow down and re-evaluate my project up until that point, and I was able to provide my students with an extra work period, which many of them needed.

Over the 18-class intervention, I observed and experienced an array of key moments of understanding, illumination, and reevaluation. These moments went beyond just answering my research question. They gave me insights into how my students learn and how my teaching impacts that learning.

In this chapter, I first outline these moments of understanding in chronological order using my field notes and student work as data. In this lengthy section, I also provide context for these key moments by describing activities and occurrences in each class of the unit. The next part summarizes further key moments of understanding found in interactive writing activities. The third part briefly describes the interview portion of my data collection, setting the stage for a detailed set of student profiles gleaned from the interviews in Chapter 5.

4.2 Part 1: Field Notes and Student Work

This section outlines key moments gleaned from student work and field notes and organizes these moments into four chronological blocks of classes, or days. The first block, days 1-3, represents the introductory activity of building scales. The second block, days 4-9, represents the development of understanding of algebraic terms. The third block, days 10-12 represents moving from concrete to symbolic understanding, and the last block, days 13-17, corresponds to activities meant for students to establish understanding of solving equations symbolically as well as move on to more advanced algebraic equations. Each block is structured by first displaying a table briefly summarizing the daily learning experiences and the data collected in each class. These tables are followed by sections that provide further explanation of key moments by providing examples from each day of the unit.

4.2.1 Days 1-3: Building the scales and "What's in the Bag?"

Table 2

Timeline of Teaching Experiment Days 1-3

Day	Learning Experiences	Data
Monday, March 16, Day 1	• Students were provided with supplies required to create scales after a brief demonstration of the scale I built for myself. • Students spent the entire class crafting their scales. Most students chose to build their scales in pairs, although a few created their own individual scales. • In order to help foster ownership over their work, I asked students to name their scales whatever they wanted to.	• Teacher field notes • Photographs of work
Tuesday, March 17 Day 2	• I demonstrated how to play "What's in the Bag?" • Once students understood how the game worked, they acquired their scales, went back to their tables, and played the game in pairs.	• Teacher field notes • Photographs of work

	• Students had the opportunity to choose the types of weights they wanted to work with. Options included screws, aluminum cubes, glass stones, and pennies.	
Wednesday, March 18, Day 3	• I had the students gather at the front of the class in their chairs. • Using the scale I built, I played “What’s in the Bag?” and had students brainstorm ways in which they could draw what I was building. Three different students came up to demonstrate how to draw the scales, weights, and bags. • Students once again played “What’s in the Bag?” in pairs with their own scales. This time, however, they had to record their steps by drawing their scales out on Worksheet 1. • With 15 minutes left in the period, students wrote in their math journals, responding to the prompt, “What are your thoughts about this unit so far?” I collected journal entries. Later in the day I read the entries and responded to them in writing.	• Teacher field notes • Photographs of work • Worksheet 1 • Interactive Writing

As mentioned above, I decided to provide participants with supplies for their scales as well as a basic design. The final design of the scales was inspired by a YouTube video (Whatisthescience, 2009). Out of the surprisingly large number of online tutorials of how to build homemade scales, the scales in this video seemed the most practical. These scales are made by cutting two 2L pop bottles in half and hanging them with string from the opposite sides of a hanger (see Figure 1). When experimenting with the bottles, I found it challenging and slightly dangerous to poke holes in the bottles for the string to go through, so I pre-made these holes. I still wanted students to experience building the scales as much as possible, so I left them in charge of feeding the string through the holes and around the bottles. I also had them adhere the string to their hangers and fix any other issues with the scales they might notice. It was their responsibility to keep their scales well balanced, since the big idea of balance is so foundational to their understanding of algebra (see Chapter 2.5, p. 25). I kept this instruction purposely vague and open-ended to allow for some creative freedom for students in their designs. When it came time to weigh objects, students could hang their scales from the bar under their desks or, if they chose, they could find another spot for hanging them elsewhere in the room.

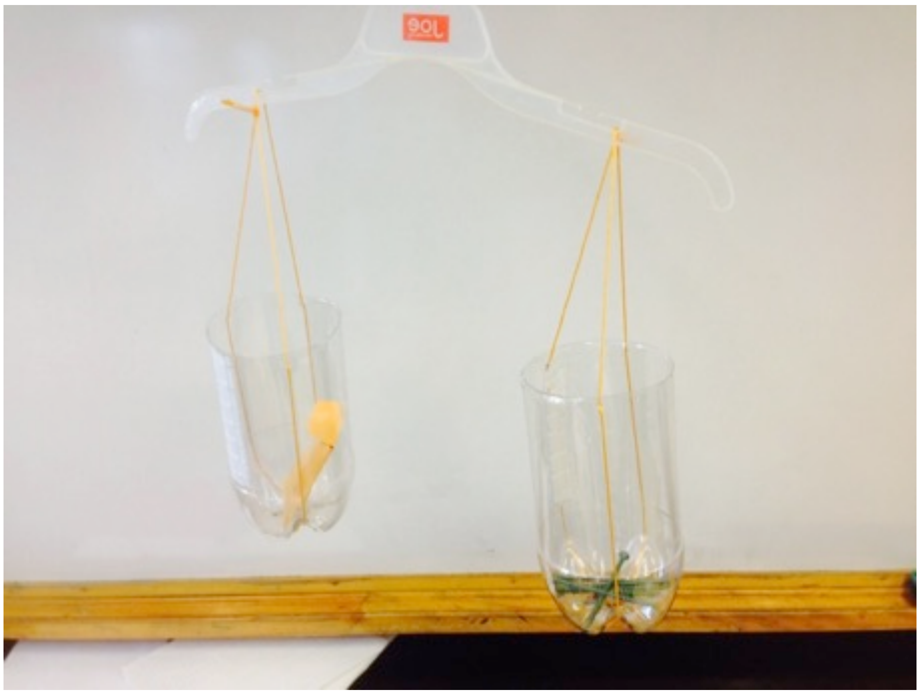

Figure 1. Pop Bottle Scale. One student group's pop bottle scale with screws representing whole numbers and a bag representing a variable.

When it came to actually building the scales, things went smoother and quicker than I anticipated. Many students completed their scales in less than ten minutes. The ones who completed their scales sooner were able to help others. I observed that students were engaged in the scale building and were enjoying the task. Although I demonstrated how to construct the scales, some students veered away from my design. Rather than loop the string under the bottle and secure it with tape, like I had demonstrated, two students hung the string directly from the holes to their hangers. I did not correct this deviation from my instruction, as their scales worked just as well as the others. In fact, their design actually allowed for quicker construction, and if not for the fact that their long stings prohibited them from hanging the scales on their desks, I would have preferred this design.

When in a classroom, I am often aware of the tenets of inquiry-based learning and I am often looking for opportunities to infuse them into my teaching. However, as I began my research project I was struck by how extra mindful I was about inquiry, particularly how I could get my pupils to be more involved in their learning. In this first period of my research, I had the idea of offering students to name their scales whatever they wanted to. Some students simply wrote their names on their scales but most opted for creative monikers like “Beyonce and Jay-Z” and “Blue Steel”. I was actually taken aback by how engaged the group was with this aspect of the design process.

The scales were all completed by the end of the first day. In the second class, we started to play “What’s in the Bag?” I developed this game from my personal view of algebraic equations. In my view, solving algebraic equations is like solving a little mystery. One is given an unknown quantity and provided with a set of clues, which, through the power of deductive reasoning, can reveal the unknown. In order to tap into this basic game-like structure of equation, I devised a very simple game in which a student would drop a number of weights into a brown paper bag on one side of a scale, and then have a partner guess the number of weights in the bag based on how many weights the partner needed to drop into the opposite side of the scale to balance it. The game was developed with the five principles of inquiry in mind. It was meant to develop community by encouraging play between individuals, be experienced as a hands-on activity, actively involve student in their learning, and help create a foundation on which to build scaffolding for students’ conceptual understanding.

Despite the inquiry-based underpinnings of the game, I was still – like the naming the scales activity – largely taken aback by how engaged students were with it. Several of the most engaged students happened to be the weaker ones. Grant, an outwardly confident individual passing mathematics by a slim margin, particularly enjoyed the activity. Stronger students, like Mesego and James, were enthusiastic about the game as they tried hard to stump each other.

Another rather impromptu decision I made in the days leading up to the intervention was to provide students with multiple options for what they could use for their weights. In experimenting with my scale, although I had planned on using aluminum cubes, I found that many items could be used as weights. Items such as glass stones, poker chips, and pennies all worked, so I placed as many as these as I could on the classroom's front table in order to give students as many options as possible. Unfortunately, the coke bottle scales are not very precise and, more importantly, the students were generally not very precise in the way they built and worked with the scales. When experimenting with weights before the unit began, I was using a scale built on my own with meticulous care and I was testing weights with the fine motor skills of an adult. The students' scale strings were often uneven and off-centre, and many students lacked the dexterity to gingerly place weights in the bottles, rather than dropping them in. The result was that many of the weights I had deemed satisfactory were problematic for the students.

However one student, Samuel, had brought in a large box of three-inch screws for an unrelated project. After struggling with the other weights, he discovered that these screws weighed enough that they negated the imprecise effects of the scales. Word quickly spread that screws worked well, and I stopped the class to make the announcement about Samuel's discovery. I immediately noticed a change in the way the class interacted with me after I shared this discovery. Students were suddenly approaching me to tell me about other discoveries they were making about their weights. One student, for example, was taping stones together to create heavier ones. Another wanted to show me how he was adding bags to the other side of the scale to help balance it out. By the end of the second class, screws became the weight of choice for most of the group.

In this classroom, students sit at table groups. At the beginning of the third class, I had students move their chairs up to the area immediately around the whiteboard and sit closely together in one large group. Educators often use this strategy to refocus the class as a whole. I find that this strategy also encourages more interaction between students and between the students and the teacher, mostly due to the close proximity between individuals. When a teacher is open to their students' ideas and asks them open-ended questions, rich conversations about mathematics often occur, thus contributing knowledge to the community of learners (Palmer, 2007). I continued to use this strategy more often than I usually do throughout the unit, as I remained aware of ways in which to instill inquiry into my instruction.

In this particular instance, students gathered at the front to brainstorm and demonstrate ways to model pictorially – i.e. to draw – what they were accomplishing during their "What's in the Bag?" games. I modeled playing a game with a student and asked another to come up to the board and draw what was transpiring. The student, Lily, is a talented artist, and she drew a very detailed picture of the type of scale one would see at a grocery store. Another student drew an upside-down "T" to symbolize the scale, circles to symbolize the weights, and squares to symbolize the bags. A third student kept the circles as weights but drew the bags as larger circles. This student drew the scale as a straight line with a triangle at the midpoint at the bottom.

All three students represented moving weights out of the scales by circling them and drawing an arrow, which attached to the circles and pointed away from the scale. A brief, respectful discussion about the pros and cons of each drawing strategy followed. Students generally accepted that the third scale was the quickest way to draw a scale and that it was important to have different shapes symbolizing the bags and the weights. Nevertheless, I left it open to the students to decide which ways of representing weights and balances they deemed best. Again, I allowed for this small moment of choice in order to foster a sense of ownership over their work. Students then played "What's in the Bag?" and drew out the steps to discovering the bag's quantity on Worksheet 1.

This activity was carried over to the third class. Student kept building their own equations on their scales, drawing them out, and solving what was in the bag. Most groups seemed content with keeping the equations very simple, with only one bag and a number of weights outside of it on one side and a handful of weights on the other side. However, seeing that some groups were becoming bored with the activity, I suggested they try placing their weights and bags in different arrangements and configurations, perhaps using more than one bag. Assuming that some students would build equations like $3x = 12$ or $4x + 3 = 15$, I was surprised when one pair, Otis and James, attempted to play "What's in the Bag?" with envelopes on both sides of their scale. On their worksheets, both students showed signs of struggling through the question they had developed but finally wrote "IMPOSSIBLE" over the equation. Through this open-ended activity, they discovered that one could not have unknown variables on both sides of an equation and still hope to solve the value of the variable.

4.2.2 Days 4-9: Applying algebraic terms.

Table 3

Timeline of Teaching Experiment Days 4-9

Day	Learning Experiences	Data

Thursday, March 19, Day 4	• Students came up to the front. The term "variable" was discussed and defined. • I modeled two questions from Wednesday's class, asking for input from students along the way with questions such as, "What's my next step?" and "How do I draw "take away"? • We brainstormed how to draw and write different combinations of bags and weights. For instance, three bags on one side of the scale and six weights on the other can be written as $3x = 6$. • Students acquired their scales and went back to their tables in their groups with the instruction to keep playing "What's in the Bag?", except this time showing what they were doing by drawing their steps in one column and writing it mathematically (symbolically) in another. • Realizing that this step was integral to their understanding of equations and I wanted to ensure everyone was able to complete it, I spontaneously asked all students to complete the same questions for their first and second questions on their worksheets. I built and drew the equations $x + 3 = 4$ and $x + 4 = 9$, and asked them to "solve" it – i.e. discover what is in the bag – on their worksheets. • Groups who completed these first two questions were encouraged to try building more challenging equations, such as $2x + 1 = 7$.	• Teacher field notes • Photographs of work • Worksheet 2
Friday, March 20	• No classes due to inservice.	
Week 2:		
Monday, March 23, Day 5	• Many students were late or absent due to poor weather conditions. This was also the day the students' Science Fair projects were due, so many of them were distracted and asking to finish their projects. The school's printer was down, so students were directed to write their own worksheets. • I brought the students up front to work through	• Teacher field notes • Photographs of work • Worksheet 3

	the equation $x + 4 = 9$. With student-input, I modeled how to explain in writing how I solved this equation. I gave them another two questions to solve and had them go back in their groups to build the equation with their scales, and solve them pictorially and symbolically on Worksheet 3. For this worksheet, students also had to explain in words how they solved each equation. • As there was little time left in class, most students only started the second equation before class change. • NOTE: Some students opted to not use their scales and I allowed this to happen.	
Tuesday, March 24, Day 6	• This class started with handing back math journals and having students respond to the questions and prompts I wrote in response to their journal entries from March 18. I allotted approximately 15 minutes for this activity. • Next, I called students up to the front once again and reviewed the activity from yesterday, modeling how to solve $x + 4 = 7$ by drawing it out, solving symbolically, and explaining how to solve it. I then asked them to solve $3x = 6$ and explain what was going on. Students shared many interesting ideas. • Students went back to their table groups and continued with Worksheet 3, solving equations by building, drawing, symbolically writing out, and explaining.	• Teacher field notes • Worksheet 3 • Interactive Writing
Wednesday, March 25, Day 7	• Due to a science symposium, many students were absent this class. • I distributed Worksheet 4, which directed students to do the same set of steps as Worksheet 3: Build and draw the equation on the left side of the table, write the equation symbolically on the right side of the table, and explain the steps used to solve the equation in the space below the table. • I asked them to solve a set of six equations: $3x = 15$, $3x + 4 = 10$, $5x + 4 = 9$, $2x + 6 = 16$,	• Teacher field notes • Photographs of work

	$3 + 4x = 15$, and $13 = 2x + 9$.	
Thursday, March 26, Day 8	• Students continued to work on Worksheet 4. Only two students, James and Anouk, finished.	• Teacher field notes
Friday, March 27, Day 9	• I asked students to write in their math journals once again with the prompt, "In a few sentences, tell me what you understand about equality." • I allowed students to work on Worksheet 4 for the rest of the class. I wrote six additional questions on the board to be completed if students completed their worksheets.	• Teacher field notes • Interactive Writing • Worksheet 4

The first three classes of the unit were meant to simply let students explore the concept of balance by building and playing with their scales. The next step was to begin to apply the terms and concepts of algebra to their "What's in the Bag?" game.

We began the fourth class at the front once again, and I introduced the term "variable" by relating it to the unknown quantities in the bags. I recognized another opportunity to provide a choice to the students and asked them, "What letter should we use to symbolize variables?" I was fully prepared to allow different students to use different letters and to have a discussion about why it might make sense to use certain letters over others ("*b*" for bag, for example). However, students unanimously chose the letter "*x*". When I inquired as to why *x* should be the letter of choice, they informed me that prior teachers had instructed them to use this letter.

Prior knowledge no doubt helped the lesson move along quickly when I modeled some examples of what I saw students building and drawing the previous class and asked the class to translate them to algebraic language. For example, students were able to quickly translate three weights and one bag on one side of the scale and six weights on the other to $x + 3 = 6$. After two more examples, I instructed them to go back into their groups and had them construct an equation that I built at the front as well as express it pictorially and symbolically (algebraically) on Worksheet 2. Knowing that they wouldn't be able to see what was in the bottles, I also drew this equation.

I decided on the spot to have all students do this particular one together as their second question, which is an example of the kind of "just-in-time" intervention discussed in Chapter 2. (see Chapter 2.3.2, p. 15). Inquiry is, after all, not complete freedom and experimentation. When following an inquiry model of instruction, a teacher must always be alert for opportunities to intervene when necessary (Hmelo et al., 2007). In the above example, I observed that many students were still unsure of how exactly they should write out their algebraic equations. Although I wanted to leave as many opportunities for creativity as I could, there are some conventions of mathematics that I must enforce. For example, the convention of writing 3 multiplied by x should be written algebraically as $3x$, not $x3$ or $3 \times x$. I wanted to make sure that these conventions were being met and thus walked my students through one more problem. Approximately two thirds of the students completed the first two questions quickly and immediately moved on to four other simple equations I had written on the board.

The next stage in the unit was to have students add the step of writing how they are going about solving equations in sentences below each problem. We began this process with students coming to the front once again and modeling how to describe in words the steps needed to solve equations. Students were given another set of simple equations to solve pictorially, symbolically, and now in writing and were to record these problems on Worksheet 3. However, a high rate of absenteeism and other extraneous factors (such as an assembly) led to a fractured lesson over two classes.

I set aside the sixth period to refocus the group and to lay the groundwork for a block of exploration classes. In this beginning stage of the class, we discussed how to draw and solve the question $3x = 6$. The students who spoke during the discussion understood that one should take away three bags from one side and then take away four weights from the other side in order to balance the equation. We discussed the mathematical language of what is transpiring through solving an equation like the one above. Darcy explained, "You're taking away two thirds of one side and then two thirds of the other". I then asked students to build upon Darcy's thoughtful statement and think about what mathematical operation is being used. Several students correctly stated that it was division. In a non-inquiry classroom, the next step might be to demonstrate one or more ways to correctly draw division, but based on the inquiry principle of experiencing constructivist learning (see Chapter 2.3.1, p. 15), I gave students time to work on some equations involving multiplication and division without any further direct instruction to see how they would pictorially represent these types of equations. I also introduced the whole class – rather than just a select few who were working ahead - to two step algebraic equations, such as $3x + 4 = 10$.

Recognizing the importance of giving my students time and space to allow them to experience their understanding of what they had learned up to this point, I did not introduce any new concept over the next four classes. I handed out Worksheet 4, which, like the previous worksheet, required students to solve equations pictorially and symbolically as well as explain their steps in writing below each question. At this point, I still asked students to take their balances to their desks, but I informed them that I did not require them to use them. I had already observed in the previous two classes that students were using their scales less and less. However, I still wanted them to have their scales and weights in front of them in case they needed them.

My reasoning for this was that students were more likely to use the scales when they encountered a challenging question if they did not have to physically acquire their scales from across the classroom.

During these exploration periods, I investigated problems with many students. Some students, like partners Nolan and Melissa, were struggling with understanding the content of the unit so far. I worked through a few questions with each of them, building the equations and solving them one step at a time. For the first question, $3x = 15$, they really didn't know what to do. Clearly, these two students were not able to learn well from the discussions the class and I had been having at the front of the room. Nolan figured out that there were five weights in each bag but had a difficult time explaining why. When I asked Melissa what to do she said "Take away two bags" and Nolan then said, "Take away 10 (from the other side)". It was clear that Nolan and Melissa knew what to do but could not explicitly verbally identify that division was taking place. Neither did they know how to draw or symbolically express division but knew what the equation would look like at the beginning and at the end (see Figure 2.) Nolan finally explained that he knew there were five in each bag, so he knew 5 goes into 15 three times, so he knew that 5 plus 5 is ten, so he took away 10 weights. He never used the term "division".

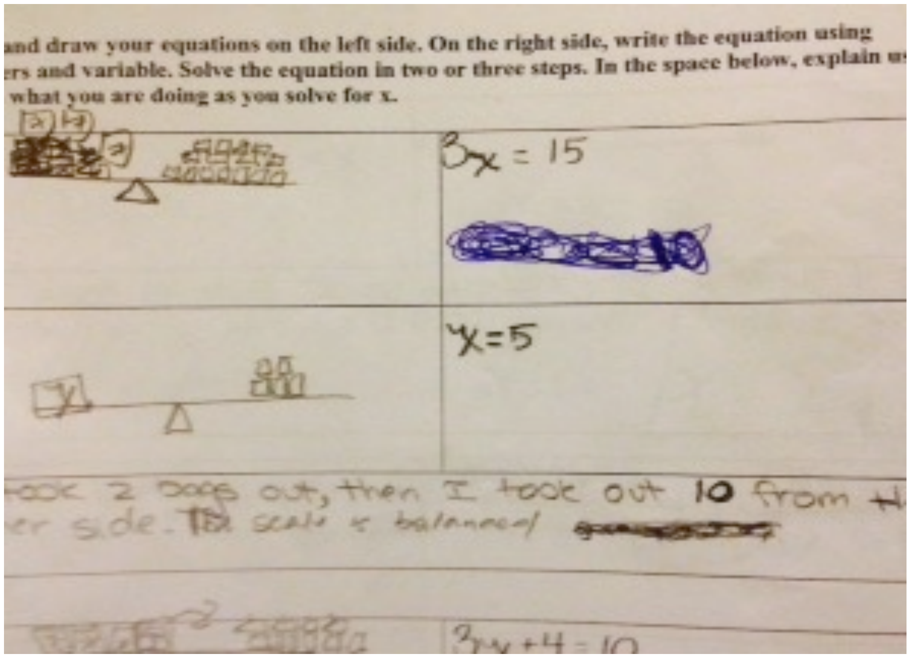

Figure 2. Nolan and Melissa's First Question, Worksheet 4. Students knew how to represent the equation pictorially and symbolically at the beginning and at the end but did not how to show operations like division.

The discussion between Nolan, Melissa, and me was typical of many discussions I had with students over Days 7 to 9. During these periods, orally explaining understanding became extremely important, as many students struggled to explain in writing what was going on mathematically when they balanced an equation with multiple variables. Nevertheless some students clearly understood that division was transpiring in Question 1, as they explicitly wrote this on their worksheets (see Figure 3.)

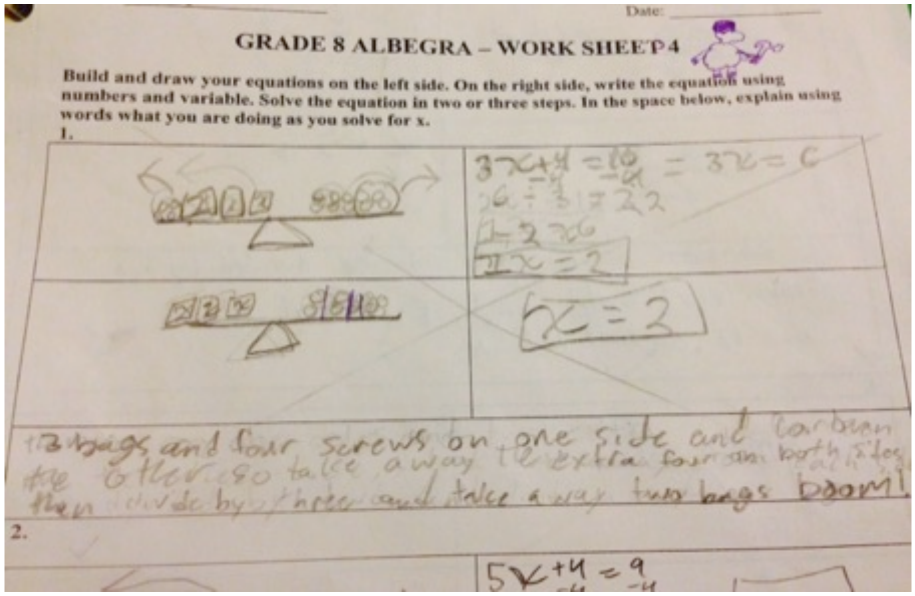

Figure 3. Otis's First Question, Worksheet 4. Otis demonstrates understanding of division.

While a few students had a strong understanding of the content I had presented in the unit -correctly expressing algebraic equations concretely, pictorially, symbolically, and verbally - most students struggled in at least one of these areas. Many students knew the value of variables in their equations, but did not correctly express their steps algebraically. In the written section of their questions, many explained that in an equation involving multiplication of variables, each side of the equation needed to be divided by the number of variables.

Some students, like Samuel, came to the realization partway through the worksheet that division, rather than subtraction, was occurring. In his first question, $3x = 15$, Samuel writes that he "subtracted 2/3 off each side then fund (sic) that five weights are equal to the bag". By his third question, $5x + 4 = 9$, he wrote that he "subtracted four from both sides then divided both by 5 to achive (sic) my answer". Darcy was one of the few students to successfully represent division pictorially. Most students drew the division of variables and the removal of groups of weights on the other side of the scale as subtraction. This understanding was similar to the way I described Melissa and Nolan's orally explanation above.

Through working with students during these periods, I also observed that when I related what they were trying to accomplish to their physical scales, they often quickly understood the content. For example, Cathy was stuck on how to start the first two questions, $4x + 3 = 15$ and $5a + 6 = 10$. Even though she had completed questions like this previously, she expressed frustration with the amount of steps she had to remember and execute. To prompt her to think, I simply asked her to imagine how she would go about solving this equation with her scales. As she explained how her scales would look and what she would do to balance them, I asked her to calmly draw out and express symbolically what she saw in her mind and she finished both questions with ease. I realized that this strategy of relating the more abstract work back to their concrete scales was extremely effective and I used this strategy quite often over the next several classes.

4.2.3 Days 10-12: Moving from concrete to symbolic.

Table 4

Timeline of Teaching Experiment Days 10-13

Day	Learning Experiences	Data
	Week 3	
Monday, April 6, Day 10	• This was the first day back from Spring Break. I gathered the class at the front and we spent time reviewing and discussing what we had done so far in the unit. • We brainstormed ways in which we could draw division on our scales. Two students demonstrated two unique ways of modeling division. An interesting discussion about drawing division ensued.	• Teacher field notes • Photographs of work

	• I directed students to start Worksheet 6. (This worksheet should have been Worksheet 5, but I made an error.) This worksheet had the same directions and format as their previous worksheet, except I explicitly asked them to write down as many "rules" that they could think of that would be helpful when solving equations. • Students were still encouraged to use their scales, but I made it clear that this was optional. Only two students continued to work with their scales.	
Tuesday, April 7, Day 11	• I gathered students to the front and asked each of them to keep Worksheet 6 handy for reference. I asked them to brainstorm "rules of algebra" we could create based on what we knew up to this point. Working through examples from previous worksheets, students were able to collaboratively create five rules in their own words: 1. Isolate the variable 2. Do the opposite operation to isolate the variable 3. Whatever you do to one side you do the exact same thing to the other 4. All *x*s have to be equal 5. Add and subtract before divide and multiply • We then briefly tackled questions with negatives by building one with the scale. Students had innovative ideas about how to represent negatives concretely and pictorially and an interesting discussion ensued. • Students continued to work on Worksheet 6. I asked them explicitly to adhere to the rules of algebra they created.	• Teacher field notes • Photographs of work
Wednesday, April 8, Day 12	• I instructed students to respond to the prompt, "Describe a moment of understanding you have had in this unit."	• Teacher field notes • Photographs of work • Interactive Writing • Worksheet 6

	• Students continued with Worksheet 6. I wrote a few more equations on the board, adding more negatives and one with division. I collected all worksheets so that I could look over their work and comment on it.	

Mathematics teachers must always be mindful of their subject's linguistic and symbolic conventions, and algebra is no exception. It was clear to me that an intervention from me was needed to help students express algebraic division in a more formalized method of expression. Samuel's conception of algebraic division ("I subtracted 2/3 off each side") was not necessarily wrong, but it was not convention. Samuel's and others' conception of division as subtraction was also clearly developed because when working with weights and scales, one really does "take away", or subtract, a fraction of their variables and the equivalent fraction of their weights from their physical scales. I felt that it was important for everyone in the class to know the algebraic convention of expressing division.

Nevertheless, I ensured that I involved my students as much as possible in coming to this "correct" convention, rather than simply "telling" them about it in a direct teaching style of intervention. At this juncture, it was paramount to keep students embedded in the metaphor of a balanced scale so, as much as possible, I was trying to facilitate moments in which students could expand their comprehension of equation by extrapolating from the understanding they had developed from their scales. I had students come up to the whiteboard and demonstrate how they had been representing division pictorially and we discussed, as a class, the degree to which these illustrations established true division. The first student, Lily, drew lines between the variables and divided groups and humorously wrote “Poof” and “Flop” to show which values were “taken out” of the equations (see Figure 4.)

Figure 4. Lily’s Illustration of Division.

Another student, Adam, also used lines to show the act of dividing, but went a step further by labeling the corresponding groups on both sides of the equation with letters (see Figure 5).

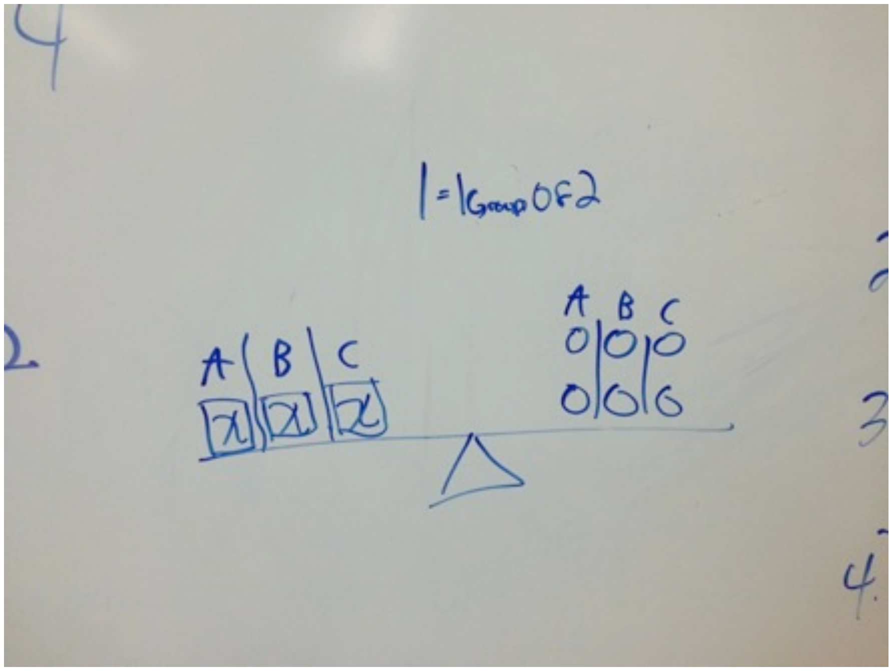

Figure 5. Adam's Illustration of the Division of Variables.

A third student, Mesego, demonstrated what he deemed his "Hanukah" method. This method demonstrated division by showing how many "weights" group together in each variable through a mass of curved arrows (see Figure 6, p. 72). To Mesego's eyes, this mass of arrows resembled a menorah, hence the name he attributed to this strategy.

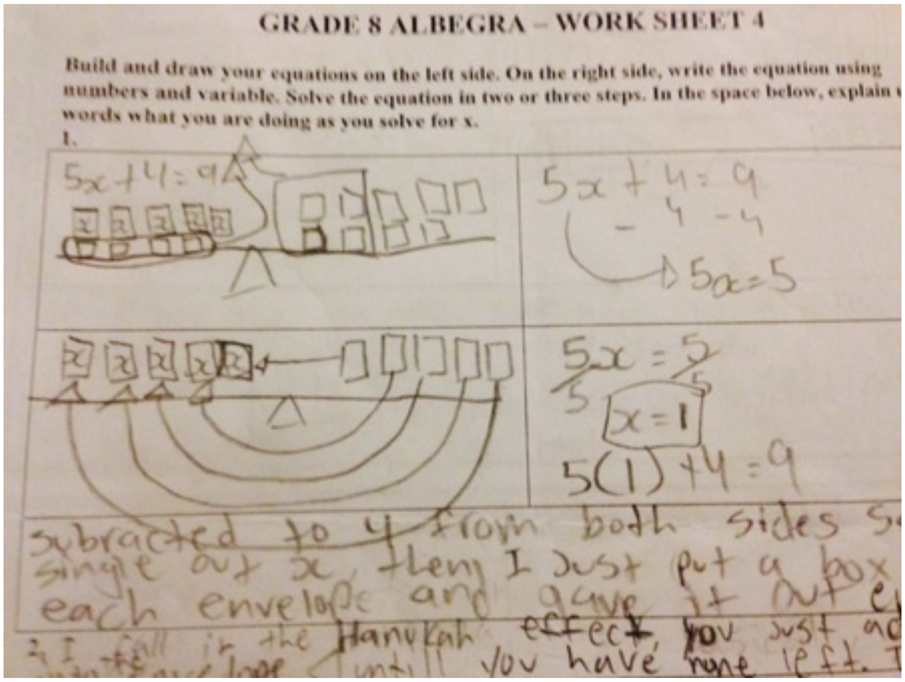

Figure 6. Mesego's "Hanukah" Method

After brainstorming different ways of illustrating division, the class went about working on their next worksheet with the instruction to jot down any rules they could think of while solving their equations. This task was extremely difficult for most of the students, as they were already solving two-step equations in many ways and had to think about them in yet another way. Nevertheless, I observed that when I prompted students with questions, they were able to devise interesting rules. For example, Rianna did not know what to write for a rule, so I related her drawing of her scale back to her physical scale by asking, "If this were a real scale and you took away a group of weights from one side, what would you do the other?" She replied that she would take away, or subtract, the same group from the other side. After a few more exchanges like this, she caught on that one rule could be, "Whatever someone does to one side of an equation, someone has to do the same thing to the other".

The next day (Day 11), when I had the class come up to front and brainstorm the rules of algebra, many students had very thoughtful ideas and suggestions, especially after I had Grant and Rianna share their examples of rules. The students already knew from prior knowledge that one needs to, in their words, “get the variable by itself”. I suggested the word “isolate” replace “by itself”. Two students, Mesego and Lily, offered creative stories to explain how they understand this rule. Lily explained that she sees isolating the variable as if she was walking to school and wanted to see where she came from. In order to do that, she would take the same number of steps backward to see where she originated. She then related this story to solving for x, as, in a way, one must work backwards in order to discover what x is. Another student offered an analogy that made sense to him, in which x is a child, the coefficient is their parent, and the constants are people that the parents do not want around their child.

Within 10 minutes, the class had generated 5 rules in their own words: 1) isolate the variable; 2) whatever you do to one side, you do the exact same thing to the other side; 3) do the opposite operation to isolate the variable; 4) all x’s (sic) have to be equal; and, 5) subtraction and addition before multiplication and division. The last two rules, were quite unexpected. The rule “all x’s have to be equal” happened to be generated by Tara, a student who is generally weak in Math.

At this point in the unit, I was aware that there were only a handful of classes remaining in my planned block of time. The expectation of the Manitoba curriculum is that students need to know how to solve two-step algebraic equations involving division and integers (Manitoba Education, 2013). Knowing that I would be giving questions with negatives to students who are ahead in their work, I briefly discussed how to build an equation with negative numbers and how they might represent it. I wrote the equation $3x - 4 = 5$ and asked students how to build it with a scale. Since at this point most students had stopped using their physical scales, I assumed that they would not have any ideas about how to build this equation and I would have left it alone.

However, several students had extremely interesting ideas. Lily suggested different colored blocks to symbolize negatives, for example. Darcy suggested a separate container. Another student suggested using different types of weights. However, we agreed as a group that physically building equations with the scales was not helpful in helping to understand and solve them.

I designated the rest of Day 11 and the entirety of Day 12 to allow students to work through the problems on Worksheet 6 (I meant this to be Worksheet 5, but skipped a number by accident). Again, to correspond to principles of inquiry such as building community, allowing students to experience understandings, and involving students in their learning, I left students with little direct instruction regarding how to draw negative numbers in equations. I wanted them to struggle through this task and to see what they would construct without too much teacher intervention. From my observations, I saw many interesting representations of negative numbers in equations. Some drew negatives under or beside the scales, some used legends to designate positive and negatives, and some shaded in the negative numbers. Some students had no idea how to draw negative numbers at all. When students were struggling with how to draw negatives, I observed that when I related what they were trying to accomplish to their physical scales, they quickly understood. For instance, Emma struggled with how to represent the equation $4c - 7 = 1$. I asked her what she would have physically done to balance her scale if the equation had been something like $4c + 7 = 10$. She replied that in order to balance it, she would have taken 7 weights from each side. I then asked her what rules of algebra this first step would be addressing, and she replied that the second (“Do the opposite operation to isolate the variable”) and the third rule (“Whatever you do to one side, you do the exact same thing to the other.”) was being followed. From this point, Emma was able to understand that she needed to add 7 to both sides of her equation. Through drawing her steps, she devised an interesting way to represent negatives as well. She drew -7 as seven circles with negative symbols and showed them being cancelled out by positive circles drawn above.

4.2.4 Days 13-17: Advanced equations.

Table 5

Timeline of Teaching Experiment Days 13-17

Day	Learning Experiences	Notes
Thursday, April 9, Day 13	• Students came up to the front. We went over the previous day's work. I gave students time to read the feedback that I wrote on their work. • I wrote examples of some mistakes I saw as well as some interesting things I observed, such as some interesting ways students had devised to symbolize negative numbers on their scales. I had them correct their work. • We had a class discussion about how to represent our understanding in the best ways possible. These ways include boxing answers, drawing neatly, and verifying answers. • We spent some time defining what "verifying" is, and how verifying ensures the correct answer is achieved. • I wrote the last question that I had written on the board for Worksheet 6, (x/5 + 17 = 19), and challenged the class to build it. After some struggles, we realized that it was not helpful to build an equation like this one using the scale and so I challenged them to draw it. James and Shawn demonstrated interesting strategies to draw division of variables. • I handed out Worksheet 7, the final one of the unit. This worksheet directs students to solve equations symbolically and	• Teacher field notes • Photographs of work

	pictorially as well as explain their thinking. I added an extra row to their tables for more complex problems. These two-step algebraic equations were written on the board and included negatives and division as well as questions involving the distributive property (3(2x+1) = 15) and combining variables (5n+2n = -14). • Class was cut short by Bus Ridership presentations.	
Friday, April 10, Day 14	• Students continued to work on Worksheet 7. I circulated the class, assisting and observing students.	• Teacher field notes • Photographs of work
	Week 4:	
Monday, April 13, Day 15	• Students had time to complete Worksheet 7. Unfortunately, many did not finish the advanced questions involving combining variables and the distributive property. • I returned students' math journals and asked them to respond to my feedback regarding their April 8 entries.	• Teacher field notes • Photographs of work • Interactive Writing
Tuesday, April 14, Day 16	• I collected students at the front and I handed back Worksheet 7 with feedback and commentary from me. We corrected questions as a class, working through each question at a time.	• Teacher field notes • Photographs of work
Friday, April 17, Day 17	• Algebra test given. I included two advanced questions on this test, which I informed students were not for marks and which would be included in my data collection.	• Amended question sheet from test.

The final expectation of the curriculum that this unit was designed to address was to introduce division and multiplication of negative numbers (Manitoba Education, 2013). On Day 13, I had the group come up to the front once again to discuss how to represent negatives. After handing back their worksheets, correcting common mistakes, and reviewing two examples of how to draw negatives, I decided to discuss how to answer Question 10 on Worksheet 6, $x/5 + 17 = 19$. This was the first equation that I had introduced with division of a variable. Once again, I tried physically building the equation with my scale, incorrectly assuming students would not have any ideas about how to assist me. However, several students had ideas about how one could build this equation. These ideas included a color-coded bag and cutting up bags into equal parts with a pair of scissors. I had Lily come up to the front and physically cut up the bag to demonstrate the latter idea.

Despite the innovative ideas, the group realized that building this equation was not helpful and we went about the challenge of how to draw what was happening when a variable is divided. James volunteered to demonstrate his method of illustrating division of variables, so I had him draw it out for us on the whiteboard (see Figure 8.) James began with a legend that labeled the part of x that was “present” and the part “not present”. I asked him to explain this, and he explained that even though x is being divided or “cut” into 5 equal pieces, the “answer” to x divided by 5 is still only one of these five pieces, hence piece labeled as “present”. This explanation led to him and me discussing what “x divided by 5” actually means, and what division actually means in a larger context. The whole class was witness to our discussion and several students offered their own input into the nature of division as it related to this particular question and to mathematics as a whole. For me, this rich conversation about mathematics was one of the most memorable and impactful moments of the unit.

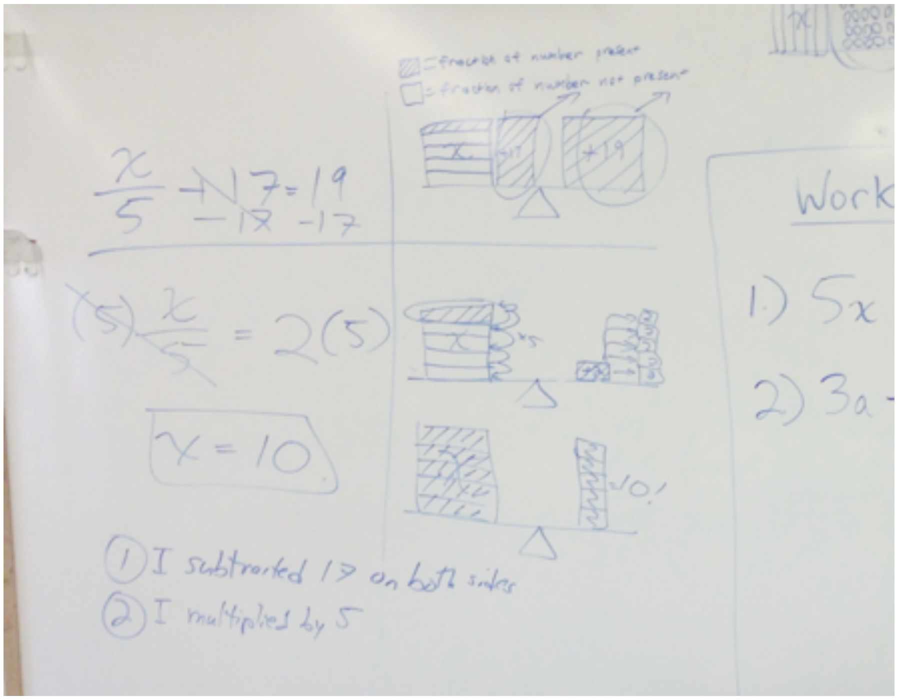

Figure 7: A Collaboration to Draw Division of Variables.

The final stage of the unit was to have students solidify their abilities to successfully demonstrate one and two-step algebraic equations pictorially and symbolically involving negative numbers, division, and multiplication. I also was curious as to how strong their conceptual understanding of algebra was, so I included advanced, Grade 9-level equations on this last worksheet (Worksheet 7) to see if they could solve them correctly. These questions included combining like terms (e.g. $5x + 2x = -14$) and the distributive property (e.g. $3(2x + 1) = 15$). I observed that the students who were able to complete the questions that required them to combine like terms were able to clearly identify the like terms. For instance, when Madison drew out the equation $5x + 2x = -14$, she verbalized that it became quite clear that she should combine $5x + 2x$ into $7x$ as soon as she drew out the equation (see Figure 8.)

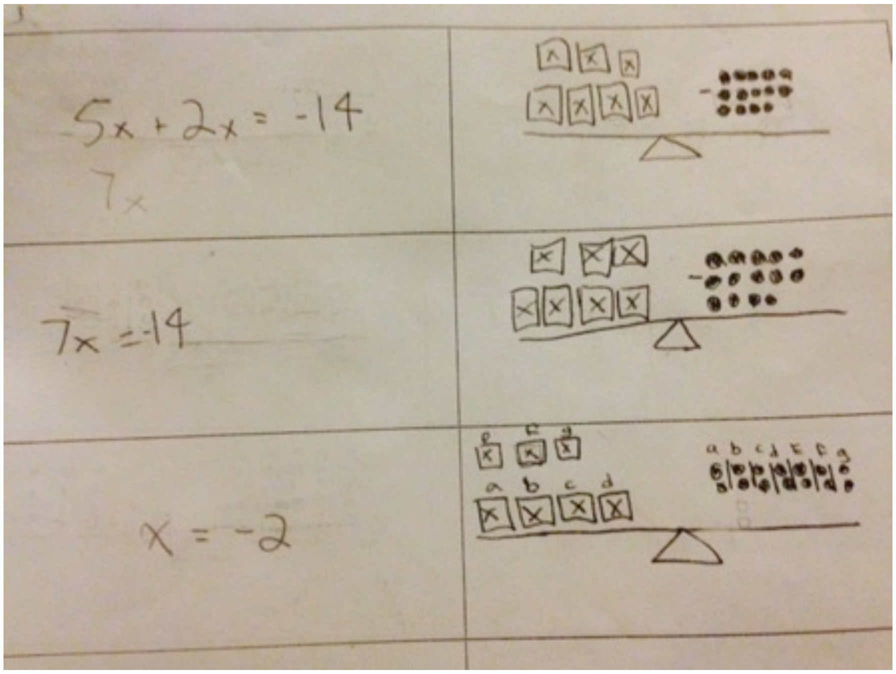

Figure 8. Question 3 From Madison's Worksheet (7).

Students had a little more difficulty solving questions with the distributive property. Samuel, for example, was unsure about how to draw and solve $3(2x + 1) = 15$. However, after I reminded him that the brackets separating 3 and $2x + 1$ mean "multiplication", he was able to understand this expression as three groups of $2x + 1$. He was then able to illustrate this expression as $6x + 3$, pictorially and symbolically.

Because I left the advanced questions to the end of the worksheet, many students did not have a chance to work on them. To give more students an opportunity to complete advanced questions, I wrote two of these types of questions on their unit test, informing them that they were optional and not worth marks. Although I expected most students to skip these questions, about half of the class ended up attempting them and many of those who tried these questions completed them successfully. The first question, $3(t + 1) = t + 7$, included the distributive property as well as combining like terms. Like Samuel earlier, James was able to understand a type of equation he had never seen before by drawing three groups of *t + 1*. His method of combining the like terms and solving for t was unconventional, but effective (see Figure 9).

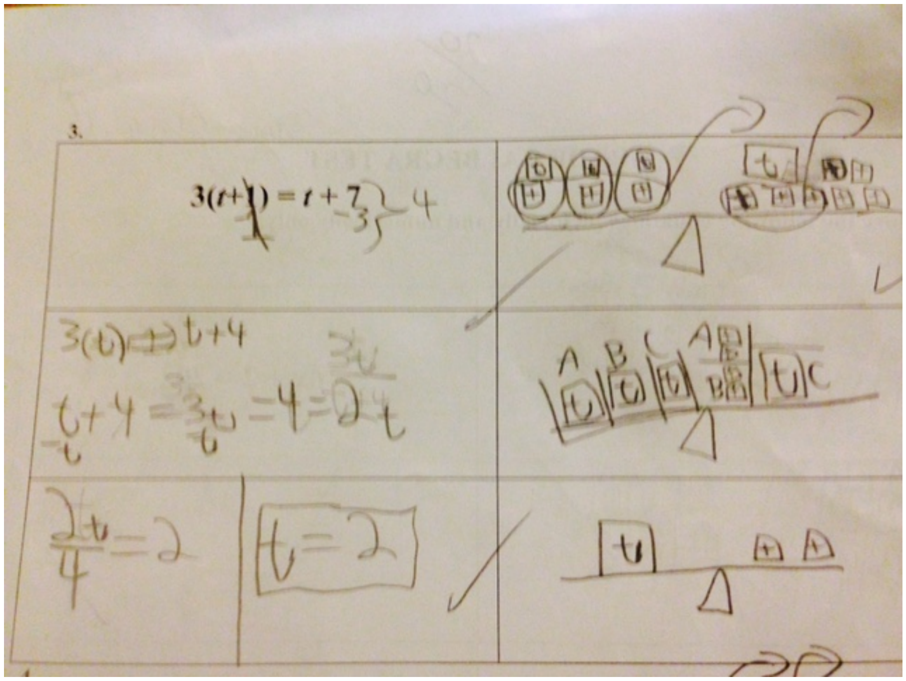

Figure 9: James's Solving of Question Involving Distributive Property.

Some students correctly solved for t and tried different configurations of what the equation would look like on a scale to make sense of the problem. Otis drew his scale out eight times before being able to make sense of the problem. In the space provided for his answer, one can see that he used trial and error to draw out the equation, using multiple strategies that he learned from his peers and me during the unit on algebra, including "Adam's method" of showing division by assigning letters to like groups (see Figure 10). It is extremely messy, but Otis correctly solves for *t* and his illustrations, when analyzed closely, correctly show the steps to solving the equation.

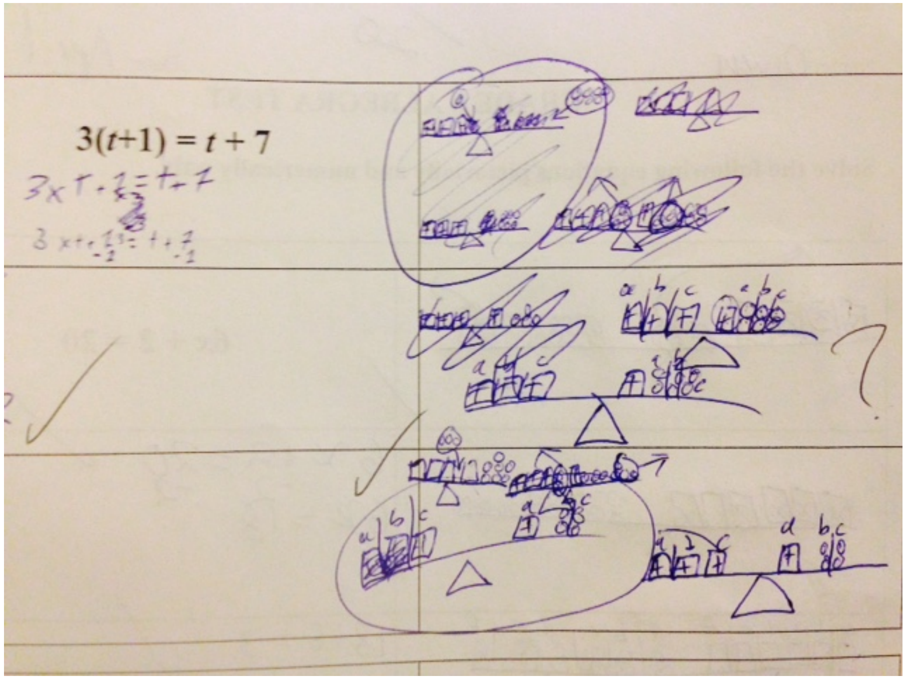

Figure 10: Owen Using "Adam's Method" to Solve Equation.

I also included questions on the test that directed students to solve equations symbolically only. Students were extremely successful on this part of the test, even students who had been struggling in the unit. Nolan, for example, solved every question successfully. Grant, who often does not complete tests or assignments, achieved full marks on this section. Melissa, who missed several classes during the unit, did not complete these questions, but achieved partial marks by drawing out two of the equations before attempting to solve them. Clearly, she was not ready to completely move away from the pictorial stage, but it was encouraging to see that she was using conceptual tools that were given to her during the unit.

By the end of the unit, I had several pages of field notes and hundreds of pages of student work. The above lengthy section provides merely a snapshot of the data that my students generated, and only a few dozen descriptions of key moments of understanding. There were many more moments that I recorded in my field notes and collected through student work and there were many more still that I did not observe at all. In addition to my field notes and student work, I also possessed interactive writing from which to gain understandings, and I had yet to further explore my understanding through four closure interviews.

4.3 Part 2: Interactive Writing

I employed interactive writing as another method of data collection. As described in Chapter 3, interactive writing is a strategy to implement writing for learning in a mathematics classroom. Rather than simply ask students to reflect on teacher-initiated prompts in their journals, interactive writing allows the teacher to respond to the journal entries with a brief reply. These replies are often starting points for rich, written conversations between teacher and student (Mason & McFeetors, 2002). Although his process was instructional and relational, it was also an effective form of data gathering as a researcher. I employed it at five points during the unit.

Table 6

Schedule of Interactive Writing

Day	Prompt
Day 3, March 18	What are your thoughts about this unit so far?
Day 6, March 24	Individualized prompts based on responses from March 18
Day 9, March 27	In a few sentences, tell me what you understand about equality
Day 12, April 8	Describe a moment of understanding you had in this unit
Day 15, April 13	Individualized prompts based on responses from April 8

I had originally intended to use interactive writing more frequently, but I found that extraneous circumstances often left me short on time, and interactive writing was often the activity that would often be left out of the data collection plan because of time constraints. The first time I employed interactive writing was at end of the third class, when I gave students time to write reflections in their math journals. The prompt I wrote on the board was simply, “What are your thoughts about this unit so far?” The responses I received were varied, and my written replies were different for each one. In these written replies, I also asked another question as a prompt and asked students to reply when I handed back their journal entries two classes later. The data that I received from this round of interactive writing were very useful and interesting for me. I grouped the journal entries into three categories: entries that provided generally positive, but vague, feedback; entries that provided generally positive, but detailed, feedback; and entries that provided generally negative feedback.

Most entries provided positive, but vague, feedback. Maria’s responses were typical of many students. In response to my first prompt, she wrote, “I like it and the scales are fun. I find it fun and easy and usually math is pretty hard for me.” I responded with, “Can you please elaborate? What is it about this unit/the scale activities that make it more easy than usual?” In her reply, she stated that she “like(s) it” and is “more motivated to do it.” Maria also stated, “doing things instead of writing and listening helps me learn better.” Many of the student comments were similar to Maria. They wrote that they enjoyed the hands-on aspect of the beginning scale activities, and identified hands-on learning as something that helps them learn and understand content. Emma, for example, wrote, “(hands-on learning) helps me learn better and faster.” Five students also commented on the ability to work in groups as being something they appreciated. For instance, Ellie wrote, “I think this unit so far is pretty cool. Being able to do group work is fun. Being not alone helps me focus better." I was unprepared for so many students to write about working in groups, and I found it interesting that this was at the forefront of their minds when asked to simply write their thoughts about the unit so far.

Some students offered slightly more descriptive positive feedback about how hands-on learning helps them learn better. Darcy wrote that building the scales helped her to "visualize the problem", which "helps (her) to solve the question easily." Nancy wrote, "I like drawing the scales because then you can see what you are actually doing and it's a lot easier to do the math when you have a diagram or something real to help you understand what you are sposed (sic) to do." Samuel commented on how building the scales make learning easier than "from a textbook or in a handout", but also appreciated being able to create his own questions. "The idea that you are making the question rather than it be preset" is valuable to Samuel.

After prompting from me, both Otis and James wrote about the instance described earlier when they tried experimenting with bags on both sides of their equation, and then finally wrote the word "IMPOSSIBLE" over that question. Otis wrote that the question was impossible because when there were bags in both bottles, "we didn't know what was in the first bottle apart from the known four (weights)". James, wrote,

> What my partner and I meant by IMPOSSIBLE is that the equation is impossible to solve. This is because the equation $x = y + 4$ only tells us that one side is equal to the other side. x could be 5 or x could be 5,138,976. All we can ever know is y is 4 more than x. Thus it is impossible to know with certainty what either side of the equation is.

Three students included negative opinions in their responses about the unit so far. Amy found the scales "very irritating…because they are shoddy and inaccurate (sic)". Katherine described the unit so far as "boring" and stated that she hadn't learned anything from working with the scales. She also stated that she couldn't "think of anything from real life to apply it to". When I asked her if she saw value in the actual process of building the scales and working with partners, she wrote back that she sees value in working with other people, because "in real life you can't do everything alone and every group project I do helps me learn how to work in groups and how to divide the work amongst people". James' journal response was mixed. On one hand, he found the scale activity to be an inefficient use of time. On the other hand, he admitted that he "(has) no idea what is necessary for the teaching of…algebra and therefore command no authority within" this topic. "In short," James wrote, "let's see where this goes".

I initiated the next interactive writing activity on March 27, with the prompt, "In a few sentences, tell me what you understand about equality". I quickly noticed that many students were defining equality in terms of human rights, so I verbally asked them to relate their answers to mathematics and what we had been doing so far in our unit on algebra. Many students wrote journal entries that vaguely related the concept of equality to the scales. Hebem wrote, "What I understand about equality is that the balances make uneven and even at the same time." Responses like Hebem's were typical of many students who understood that there was a connection between the activity of balancing their scales and the mathematical definition of equality. However, these students were unable to articulate this connection.

Other students were more successful in elucidating this connection. James wrote, "(Equality) means the two sides must have the same sum for it to apply". Amy wrote that her understanding of equality is that "Both sides of an equation are equally balanced". Despite the wide range of writing abilities in the class, it was significant to me that in their responses, not one student mentioned the "equals" sign or defined equality as "the answer", except for Bonnie, who wrote, “When I was younger I thought the equal sign always meant the answer. Now I know it as each side of the sign is equal or the same”.

For the final round of interactive writing, I asked them to respond to the prompt, “Describe a moment of understanding you had in this unit”. This prompt resulted in a multitude of unique responses. Some students wrote about drawing negative numbers, like Anna, who wrote, “I understood how to draw questions with negatives in them yesterday” and Emma, who wrote, “I understood that the minus numbers pretty much work the same way as the plus numbers”. Others, like Katherine, identified the class discussions we had about division as something that “helped her alot (sic)”. Some students like Madison appreciated explanations for problems given by their peers. In most of my questions and responses, I asked students to elaborate on what they had written in response to my first prompt. Their replies were richer and more varied this time. This round of interactive writing was extremely illuminating for me. I will comment further on their responses in Chapter 4.

4.4 Part 3: Interviews

When the unit was over and I discovered which students had agreed to participate in interviews, I was quite surprised at the low number of individuals who wanted to be interviewed. I had anticipated that several of the more quiet and shy students would not be interested, but there were many students who I expected to be enthusiastic about sharing their learning experiences who declined the opportunity to be interviewed. Nevertheless, I was able to choose four interviewees who represented the diversity of learners in the group (Stringer, 2008). Some of the interviewees are more mathematically confident than others, some are more articulate, and all four have very unique insights into how they learn and how inquiry impacted their learning during the project.

The interviews were conducted in ways that were most agreeable to the participants. All four chose to be interviewed in either my classroom or the classroom immediately across the hall from mine. These conversations were carried out at various times, depending on the availability of the students. Two interviews were conducted during lunch hours, one was conducted after school hours, and one was conducted during a second period slot. Interview duration varied between 20 and 30 minutes. For each student, I wrote down interview questions on a printed out hard copy, and I had a duotang that included student worksheets and interactive writings for convenient reference. Conversations were recorded on a digital recording device in MP3 file format. I then transcribed interviews into Microsoft Word documents. These interviews will be the featured data in the next chapter, where I will present the themes that I have gleaned from my data as four detailed participant profiles.

Chapter 5: Data Interpretation and Discussion of Themes

5.1 Summary of Themes

During my data collection, I began noticing events, moments, and circumstances that caused me to reflect deeply about teaching and learning. These events were not all necessarily directly connected to my research question, but they occurred in the complex web of connections that existed between students, teacher, and inquiry-based learning processes. These impactful observations eventually developed into four major interconnected themes: (1) community and collaboration: a strong learning community is essential to learning, and this community is fostered in many small, varied ways; (2) time and space to think and discover: having the time and space to think, discover, and reflect allows opportunities for incredible insights; (3) Connecting hands-on and symbolic learning: hands-on learning and symbolic mathematics need to be connected to each other in highly personal ways for both the concrete and symbolic understanding to be meaningful; and (4) multiple learning pathways: students learn differently in astonishingly vast ways.

5.2 Theme 1: Community and Collaboration

As I became more aware of what a learning community is and that it is intertwined with the other principles of inquiry, I attempted to foster a community of learners with my participants as best I could. What I did not anticipate was how absolutely crucial building a learning community was to the participants in this study. I realized I had taken the community I had carefully built in my own class for granted as I struggled early on to establish connections to students I hardly knew. I also did not anticipate how many varied, seemingly insignificant instances contributed to this sense of community. Through analysis of my data and, particularly, the interviews, it became clear that participants not only valued community, but they could also specifically pinpoint occurrences and instances that contributed to it.

These instances and occurrences varied between participants, but each one could identify several key factors of the unit that contributed to his or her sense of belonging to a learning community. Chief among these factors was the ability to work with other students in group settings. However, the importance they placed on working with their *friends*, and not just their peers, cannot be understated. As the interviews underscored, the opportunity to work with friends created a safe environment for students, where they felt they could ask for help or offer assistance without the embarrassment or trepidation one would expect between 13-year-olds who do not know each other well (see Chapter 6.2, p. 98).

Another comparatively small intervention that helped build community was the strategy of asking students to carry their chairs to the front of the class and gather closely around the whiteboard. This strategy was not new for me, but because of the intentionality of my instruction during my data gathering, I utilized it a great deal more than usual. The intimacy of the setting allowed students to be more focused, and as the unit progressed, I noticed that the discussions at the front of the class became richer. In my field notes, I recorded that I felt having students at the front was useful as a tool of community building, as the more focused, intense discussions allowed students to see how invested everyone was in this unit.

Through this strategy, many rich conversations about the nature of mathematics arose, and through these conversations, students were not only able to construct meaning through social interaction, but they began to see me as more than just an instructor. Through conversing with them, working through problems, and validating their ideas, I positioned myself as a fellow learner of mathematics. Students saw me as someone who was on the same journey as they were, and this view allowed them to be more open to my assistance. More importantly, however, this view also contributed to a general atmosphere of curiosity and exploration within the classroom.

5.3 Theme 2: Time and Space to Think and Discover

Through intentional application of the principles of inquiry through my instruction, I observed that students were developing a high number of interesting ideas and concepts independently during periods of individual or group work, and sometimes during class discussions. I realized that having the time and space to think, talk, and discover allowed students to come up with amazing insights that they would not likely have initiated without that time and space. Of course, by “space” I do not mean literal physical three-dimensional space. I am instead referring to the mental real estate I purposely left alone so that students could take the information I presented and fill this space with their own constructed meanings. More often than not, when given the opportunity, students filled this space with insightful discoveries. Sometimes, these insights helped the entire class view something in new and exciting ways, such as in James’ representation of division. Other times, ideas might not have been mathematical convention, but they revealed rich contextual thinking about mathematics. An example of this thinking was on display when Samuel initially saw division as subtraction of fractions (see Chapter 4.2.2, p. 66).

This “time and space” theme is intimately connected to the previous theme of community. One must belong to a strong learning community before one can take advantage of having time and space to think and learn with his or her peers. Many participants commented on the importance of having the time and freedom to talk with friends in table groups, even if much of the conversation can be off-task (see Chapter 6.2, p. 105). This structure encouraged social interaction among peers, which not only developed relationships among students who were already friends, but it also allowed deep, rich conversations about mathematics to occur. Having a deep, trusting relationship with their table partners yielded authentic learning experiences. In some ways, working in friend groups helped push students into the zone of proximal development, as friendly competition between companions kept students pushing the boundaries of their capabilities. Students were then able to work through these difficult problems because they were able to ask their companions for help.

5.4 Theme 3: Connecting Hands-on and Symbolic Learning

Throughout undergraduate education programs, teachers are taught to value hands-on learning, and it is well known that student engagement generally increases with kinesthetic learning (Begel, Garcia, & Wolfman, 2004). However, my study allowed participants to provide insight into why hands-on learning is so valuable. The students I interviewed had incredibly nuanced and varied reasons for appreciating the kinesthetic aspects of the unit and of hands-on work in general. These reasons were highly personal, varying from student to student. Some students reported that they enjoyed the opportunity to work with their hands, some students appreciated being able to get up and move around, and many students identified themselves as "hands-on learners" who knew that they learned best when "doing" (Lemlech, 1998). A common theme among many students, in particular, was that they appreciated the ways in which the early activities in the unit allowed them to take more ownership of their learning, which they linked partially to their understanding of equality and equation. This sense of ownership helped create active, rather than passive, learners, which contributed to the environment needed to conceptually learn the mathematical tasks in this unit.

Although not specifically articulated by students in their interactive writing or in their interviews, I observed that having success in the content of the unit contributed to the high level of student engagement and involvement. Cathy, for instance, gave positive feedback early on in her journal, referring to the scale-building activity as “easy”. However, in her March 27 entry, she was unable to answer the prompt, “Describe what you understand about equality”, and expressed frustration toward the content of the unit at that point. However, Cathy achieved success when I was able to help her see the connection between concrete and symbolic. After this point in the unit, Cathy had a much better attitude toward algebra and she achieved success in understanding the content (see Chapter 4.2.2, p. 66).

Students such as Cathy and others who will be described in the next chapter gained confidence and ownership of their learning through being able to connect the concrete to the symbolic. However, many of these students needed help from me to be able to apply their understanding of the hands-on activities of the unit to the more abstract aspects of it. For these students, the inclusion of drawing out equations before the introduction of the symbolic was not enough for them to move into the zone of proximal development (Vygotsky, 1978). They required prompts from me and discussion with their peers or me in order to make the connections between the concrete and the symbolic that they needed to fully conceptually comprehend the content of the unit. Once they made these connections, they were able to gain much more success and confidence throughout the rest of the project.

This theme is not just about hands-on learning leading to student understanding and involvement in their learning; it is also about instructional patience. The scale building activity took a great deal of patience from a teaching perspective. As mentioned in Chapter 1, in past units on algebra I condensed the three-class scale activity into a 15-minute demonstration at the front of the class involving an equal pan balance and some weights. Like myself in my early years of teaching, some students, including one of the interviewees, did not see the value in spending three classes on concepts that could take a short period of time to explain or demonstrate through direct teaching methods. However, as I learned, the scale activity was not just about transmitting information, it was about experiencing learning and creating strong foundations for conceptual understanding.

5.5 Theme 4: Multiple Learning Pathways

It should have been no surprise to me that students learn differently. As educators, the idea that there is a wide range of learners in every classroom is one that presented to us during our undergraduate degrees – often in the form of influential educational research, such as Gardner's (1983) work on multiple intelligences – and it is continuously reinforced by the reality of our student bodies. However, as I read and responded to the first round of interactive writing (see Chapter 4.3, p. 82), I was struck by the diversity of responses from students, and I realized that I had hugely underestimated the degree to which students learn differently.

This theme of diversity in learning continued to develop throughout my data gathering and analysis. By the end of the unit, it was clear that most students in the class arrived at a sound conceptual understanding of equality and equation, but each of them had taken a unique path to achieve this objective. Some valued the hands-on work more than others and appreciated the scale-building activity as a vehicle for building understanding. Some students enjoyed the smaller moments of free choice, such as personalizing their scales, while others were indifferent toward these moments. Some students relied on their scales to solve complex problems while others forgot about them completely. Nevertheless, the methods and strategies employed by the students, no matter how varied, were all extremely important to them and their learning. This discovery has led to many insights and implications about my own instructional strategies, which I will expand upon in Chapter 7 (see Chapter 7.2, p. 133).

The four themes described above were further developed during my conversations with the four interviewees. Their insightful responses helped me to view my relatively small-scale inquiry-based intervention as a major learning experience for the students. Conversely, for me, the interviewees’ reflections revealed sophisticated insight into how inquiry-based processes impacted their learning. In the next chapter, I will present my themes through four profiles of the interviewees.

Chapter 6: Interview Narratives and Themes Revisited

Through profiles of each of the four interviewees, I will provide a brief narrative about the learning journey of each student as they attempted to conceptually understand equality and algebraic equations. Four longer sections representing each of the four major themes will follow these profiles. Each of these sections will include specific excerpts and examples from some or all of the interviewees that help to support each theme.

6.1 Student Narratives: Grant, Bonnie, James, and Mesego

Grant. Of the four interviewees, Grant is by far the student who struggles the most in mathematics, as well as in other academics. Although he is able to express his ideas orally very well, Grant's reading and writing level is three or four grades below grade level and it is difficult for him to express his ideas in writing. He struggles with fine motor skills, which impedes his ability to express math problems pictorially and/or symbolically. This lack of fine motor skills further impedes his ability to express himself when writing in his journals, although he was able to communicate some key thoughts through these journals. In his interactive writing, he described himself as a student who is "bad at math" and a generally poor student. In his interview, he described himself as "not the brightest". His self-esteem regarding academics is extremely low and his mathematical identity is highly entrenched in his lack of confidence in this subject as well as in other school subjects.

However, despite his past struggles, I observed that Grant was quite engaged in this unit and he generally performed well on most tasks. In the first three classes, Grant was perhaps more engaged in the scale-building activity than any other student in the group. He was the first student I observed who actually changed the design of his scale after he struggled with tying the strings in the way that I demonstrated. His engagement remained high after we started drawing the scales on Worksheets 1 and 2, although he found the act of drawing challenging. Despite the struggles, he regularly asked for help from his table partners and from me, and he rarely showed signs of frustration or despair. He found the leap from pictorial to symbolic easier than from concrete to pictorial, although he had difficulties when faced with questions involving negatives and division. However, once again, he remained focused and open to help, and he found assistance from his peers and me. In fact, I observed that as the unit progressed, Grant relied on his friends to assist him much more often than he relied on me. By the last worksheet and the unit test, Grant had demonstrated a strong conceptual understanding of Grade 8 level algebraic equations and was even able solve advanced equations that he had not yet experienced.

Bonnie. Bonnie is a student who struggles in some academic subjects but is a high achiever in Math. Her mathematical identity is connected to her confidence in mathematics. In her interview, she stated, “Throughout my years of learning it’s been really easy for me to learn math. It’s just my best subject.” She enjoys working with and thinking about numbers, and she consistently engages in class discussions about mathematics. She is also a social learner, preferring group work to individual tasks. She enjoys helping others, and others are drawn to her patient and caring nature.

Bonnie consistently demonstrated a keen understanding of the content covered in the unit on algebra. She was engaged in hands-on tasks, involved in class discussions at the front of the room, and provided positive feedback verbally and in her journals. She was occasionally off-task with her partner, Katherine, but she successfully completed nearly all of her assignments. She also took full advantage of the social learning that was available to her. I observed that she and the people at her table discussed their work with each other more than any other group in the class. Bonnie spent a great deal of class time working through problems with her partner and other people at her table, often reviewing and essentially teaching concepts covered in class. Grant, the previous interviewee, sat at her table with two others students, and he was the recipient of much of Bonnie's help.

James. James is an extremely strong student in mathematics and in all other academic subjects. He is extremely intelligent, thoughtful, and articulate, and he takes school very seriously. He can learn independently, but he also learns well in group settings, where he is often the most on-task student at his table group. When working with his friends – all of whom are strong students in mathematics - he asks a great deal of questions and also provides a great deal of answers. He enjoys thinking about and working with numbers, and he enthusiastically engages in class discussions about mathematics.

James began the unit with a high level of engagement during the scale-building activity. For this activity, he worked in partnership with Otis, another strong mathematics student. This pair was one of the first groups to complete their scale and begin to experiment with it. They were also one of the first groups to immediately understand how to play "What's in the Bag?" and were the first to finish the first and second worksheet. Later in the unit, James worked in partnership with Mesego. These two had many rich conversations about mathematics, and their often-competitive relationship tended to help push Josh into deeper understandings about algebra and other mathematical concepts.

Mesego. Mesego is a moderately successful mathematic student. He is a bright student with a great sense of humor who loves to talk. He loves to talk so much, in fact, that it often gets him into a great deal of trouble. He can be highly distracted and highly distracting to other students if he is not engaged in what the class is learning, which affects his ability to be as successful in mathematics as he could be. He has admitted to purposely derailing lessons of other teachers because he was bored. However, if Mesego is engaged and interested in what is happening in class, he can be an extremely active learner who participates in all aspects of group and individual learning. He can also enjoy conversing about mathematics and can display an intellectual tenacity when trying to figure out mathematical concepts. Mesego is also a social learner, who prefers group work to individual tasks. He will ask for help from his friends and me when needed, and he enjoys explaining how to accomplish mathematical tasks to others.

During this unit, Mesego was generally engaged in the early stages of this unit but was occasionally off-task. He worked with two different partners: Adam for the scale-building activity, and James during more advanced questions that did not involve the scales. He was generally involved in class discussions at the front of the room, and he provided positive feedback about the unit in his journals. He demonstrated a strong understanding of the content covered in the unit on algebra, successfully completing nearly all of his assignments. Mesego sat at a table with three other students who are all strong students in math, one of which was James. He was an ideal subject for an interview because Mesego is very talkative and he had a great deal to say about the unit and his learning.

6.2 Theme 1 – Community and Collaboration - Revisited Through Interview Data

Grant. Early on in my data gathering, I observed that Grant bought into the community I was trying to create quite quickly, and that he thrived in his particular group setting. He referenced the importance of social learning in his April 8 interactive writing. During our conversation, I brought forth his journal entry, in which he wrote about the importance of working with his friends at his table.

Me: Do you like working with people in, like, a setting like this, generally?

Grant: Well, I like people, so that generally helps when you work with people because people are unique and have their own personalities and stuff so … I know this might seem surprising but before I came to Churchill I wasn't the most social person. I was a more … angry person. Yeah. So it's easier to just work with people who sorta just understand and help you cuz they're friends.

Me: Bonnie said this in one of her journals, that she likes working with people and likes to be able to help and teach and stuff like that, and then even by her showing people how to do stuff she learns too. Do you find that helpful as well?

Grant: Yeah. She's like my mini study buddy.

Me: Do you teach her things too? Do you think you do?

Grant: I don't know about that cuz I'm not the brightest but I still sorta try to get things done.

Here, Grant revealed many aspects of his mathematical identity. He identified himself as a social learner who "likes people" and credited a great deal of his success to an interpersonal approach to instruction. By describing himself as a former "angry" person, he also continued his personal narrative about his belief that he used to be a poor learner and a poor student in general. Because he sees himself as a weak student, he sees an unbalanced relationship with his friend Bonnie, who often helps him with his work. Nevertheless, Grant has found a way to be valuable in this relationship by making sure his group stays on task and "get(s) things done".

Later in the interview when I asked him to explain how he was able to comprehend the concept of combining like terms when I had not previously taught him how, he once again attributed his success to interpersonal opportunities:

Grant: Once again, this is all because of how my friends help me out. I also sort of solved this on my own a bit. I just did what I normally do, I divided them both equally, and I got 2 in the end. I did what I normally do for normal questions so just instead of getting a positive I got a negative.

In this instance and others, Grant received unspecified assistance from his friends. The relationships between Grant's table partners and him allowed Grant to recognize if his arithmetic made sense and worked in whatever mathematical context that was presented. Even though he was working out answers on his own, he saw his friends as a legitimizing factor in understanding his work.

During the unit, I asked students to name their scales and to choose their own weights because they seemed like small ways in which to honor the inquiry principle of student involvement. However, I soon observed that these small offerings of choice were engines of community building. I asked Grant to comment on the act of naming the scales and choosing weights. He talked at length about specific names that other students named their scales, even correcting me when I wrongly stated a group's name. He commented on the weight choice in a similar fashion. Instead of reflecting on how this choice impacted his learning, he described the process:

Grant: You took out the colored cubes cuz they were too light, same with the other light blocks, then you just ended up just using the black cubes and the screws.

When asked about having the option to choose weights, Grant replied that it was "pretty cool". Although Grant's responses did not specifically address how these two small choices impacted his learning, I believe that they were integral ingredients for creating the kind of learning community in which Grant could thrive. Grant, more than any of the other interviewees, seemed to appreciate these morsels of free choice.

Humor is another important aspect of creating a learning community, according to Grant. During the interview, he referenced what he described as the "poofs". What Grant is referring to is an instance during the tenth class of the unit, when I asked students to approach the whiteboard and show how they expressed division and multiplication pictorially. One student, consistent with others, drew circles and arrows on values that were to be removed from the equation/scale. However, this student, Lily, also wrote the word "poof" to emphasize the point that these values now ceased to exist. Grant clearly appreciated this occurrence, as he began to write the word "poof" on his equations after this class. He also mentioned this occurrence twice during the interview. It is clear that this small injection of humor resonated with Grant, further fostering a sense of community, which in turn allowed Grant to feel safe and comfortable while he learned.

Bonnie. Like Gabe, Bonnie communicated that she enjoyed the opportunities to work with her peers and I observed that she had a tendency to fully engage in interpersonal learning. I shared my observations with Bonnie during our interview, showing her two journal entries from March 18 and March 24, in which Bonnie writes about the importance of being able to talk and learn with her friends. She expressed an appreciation for table groups rather than rows of desks. In Bonnie's opinion, this structure encourages social interaction among peers, which she credits to developing relationships among students who are already friends or who are strangers to each other. According to Bonnie, without these relationships, "you'll likely get bored and depressed and you won't focus too much on work".

I asked Bonnie to talk about how working in these social groups might help her learn.

Me: Do you think helping Kathy and Grant or whoever's at your table helps you learn, well, algebra better, but, in general, do you think it helps you learn better?

Bonnie: I do, yes, because you go through it again in your head and try and explain it in an easier way in, like, not-big words and slowly go through it again and again.

Me: Because sometimes I phrase a mathematics question a certain way. Does it help to talk it out with your friends and maybe say it in a different way to them and have them say it in a different way to you and make sense of it in your head?

Bonnie: Yeah it makes it easier to understand.

Me: Do you learn better when the person you are talking to gets it right away or when you have to work hard to teach and they struggle through?

Bonnie: I guess it would be easier to learn if you teach through it, because then you go over it again and again in your head and it kind of stays there.

Through this series of questions and answers, Bonnie confirmed for me what I had observed through observations which I had recorded in my field notes, which is that Bonnie not only enjoyed working with her peers because of social reasons, but it also benefitted her by helping her gain a deeper understanding of the content. By teaching what she has learned to her friends, Bonnie becomes an active learner who builds upon her understanding as she explains concepts to others. In accordance with Davis's (2009) definition of constructivism, Bonnie constructs meaning through a continuously expanding loop of knowledge.

Bonnie also commented on the oft-used strategy of having students come to the front and gather around the whiteboard. "Coming up to the front," Bonnie stated, is "easier for us because we get more involved". For Bonnie, the experience of learning in a more intimate way allowed the class to be more active in their learning. This, in turn, contributed to an atmosphere of engagement, where all members of the class were striving to learn as one cohesive learning community.

James. Over the course of the unit, James positioned himself as an important and embedded member of our learning community. Some of the small instances that Bonnie and Grant appreciated, like naming the scales, were not meaningful for James. Instead, being able to talk freely with his friends was an important ingredient in fostering this community. I asked James about what he thought about working and talking with people while he was learning.

> James: I think (working and talking with people) definitely helps. Like, if you don't understand a question then you can ask around, assuming it's not a test. If you don't understand something from the lesson you can just ask, and it really helps to have access to the possibility of reinforcement throughout the class. Because the teacher does the lesson at the beginning and then if you don't understand something or if you forget something, you can just ask the people around you.

Like Bonnie, James also appreciated the opportunity to help other struggling learners, as helping people not only contributed to our community of learners, but it also often uncovered misconceptions about what we were covering.

> James: (Assisting other students) just helps me understand it a lot better, explaining things. And it's actually helped me uncover some mistakes I was having, because when I explained it to people. It's just, like, "Oh, that doesn't make sense; I've been doing it wrong."

Although James valued the opportunity to talk to his friends, he admitted that they were often off-task. When I asked what percentage of time was actually devoted to math, James answered "about ten percent". Despite this low percentage of time spent on discussing mathematics, James cherished this time as an opportunity to expand his conceptual understanding through discourse.

Another example of James contributing to our learning community was when he demonstrated division of variables on the whiteboard (see Figure 7, p 78). In the eleventh class, James spent many minutes at the front struggling through a question in which he had to draw an equation involving division of a variable. James and I started working collaboratively to solve the equation, and then other students started to join in the discourse. In this instance, James attributed the safe, intimate atmosphere that had been created over multiple whole-group trips to the front of the classroom. When asked how this occurrence was helpful, James attributed his ability to struggle through the problem with the support he received from our community of learners.

> James: Well it's coming back to the group thing, with being in a group helps to understand new things. So this was a new thing, this was something I didn't know how to draw, so being in a group – even though we weren't talking that much – it just reassured me and pushed me to go on and keep thinking about it, instead of just drifting off like I would have if I was alone, I would have just skipped to a different question, like if it was a test. I would have just started thinking about something else.

In another instance of group discussion at the front of the class, the group generated five rules of algebra. In his interactive writing, James wrote that these rules were helpful, particularly in the way they were generated as a learning community. If I had given the rules to the class on a sheet, James states that this would not have been as personally meaningful or impactful.

> James: I think that I would have got it if it was on a sheet but I don't think I would've paid too much attention to it. And I think I would've strayed from the rules if something

didn't make sense to me. But having the class come up with them just… It took more time, so I just remembered it a lot more clearly.

Mesego. Like James, Mesego had a very sophisticated grasp of what contributed to our group's sense of community. Some contributions were seemingly minor, like being given the freedom to name their scales.

Me: Was it meaningful to you to be able to name your scales? Did you name your scale?

Mesego: Yeah, I called it Beyonce. It gave us a personal feel to it.

Me: When you came up (to the front to acquire a scale) did you always use that scale?

Mesego: I think somebody else had a crush on my scale so they took it away from me.

Me: (Laughs.) So, you seem to have liked naming your scales. Why do you think it was important?

Mesego: Well it makes a personal connection with the thing so it just made us feel more engaged.

Mesego also appreciated how working in table groups with friends contributed to classroom community. He participated in many rich conversations with his friends regarding mathematics, particularly with James, who named Mesego as someone who really helped him to understand algebraic division. Mesego addressed the point James made about there being a high percentage of off-task behavior in these types of group settings. Mesego guessed the percentage of productive conversation as a much higher value than James – 51% - but he confirmed that if he did not work with his friends, productivity would drop even further.

Mesego: (When you are) alone, you think about other things and get really bored. You don't really care, because when you're not having fun, you don't really care. Like, doing a job that you don't really care about, you're not going to do as well as you should. And

> the good thing about sitting with a group is that you can talk about what you're doing. I came up with a bunch more other theories than the Hanukah effect and I talked about them with James and Otis, and they told me it was stupid, and so I had to come up with something else. But if it wasn't for them I would have been like "Oh, okay, my idea sounds good in my head, I'll just go with it", and I could have been wrong. So it's good to have a group of people you can talk to and share your ideas with.

For Mesego, sitting with his friends creates a greater range of conversation and thinking. Though he talked about topics completely unrelated to math more often in his friend group, he also engaged in rich mathematical discussions more often than if he was sitting alone in a desk. Despite the struggle to remain on task, Mesego and his friends had "some good products at the end".

I asked Mesego to compare working in groups settings with friends versus someone he does not know well.

> Mesego: Well, there's two factors to that. The good part would be, you know, talking to somebody you don't know as well, you'll be more serious, because you don't have any inside jokes or whatever so you get right on topic and you'd start talking. And the bad part could be you don't… You're not really comfortable with that person, you don't like them, or you don't want to share your ideas with them because you don't trust them. That's another part about talking to people who you know; you can share your ideas with them.

Mesego valued being able to talk to people he knew, but he also understood the importance of fostering a community of learners. For Mesego, this community is essential if there are to be rich conversations about learning.

> Mesego: I think it's good to build a community with people that you learn with because when you build a community with someone, you're not afraid to be wrong. That's one of the reasons why people don't want to talk out loud in a big class. They don't want to be wrong and embarrassed. But people learn more from their mistakes. So if you're not afraid to make mistakes in a group of people who you're learning with, then you'll learn better and be a better learner and everybody else will learn from your mistakes. And you'll learn from your mistakes and it'll benefit everybody because of that community.

Mesego also commented on my role in the community of learners. The fact that I was clearly learning as the unit progressed and was open to new ideas helped foster an environment of respect and scholarship.

> Mesego: Like, other teachers from Grade 1 to Grade 6, they've always wanted to teach a class, but they've always wanted to be the kings or the queens. Like, what they teach you is right and nothing else. Like, you can't correct them. They're always right. But with you, you're open to hearing new ideas, and you're not going to shut them down and saying they're stupid and they should stick to the algorithm. When you allow people to think more and give them more freedom with these theories – how you accept them and think about them – it strengthens the community and people just learn more from it. Because, going back to the personal connection thing, people will remember the conversations we had here. When everybody's closer together and we're all like talking back at each other, we're just – we're going to remember those moments. We're thinking, we're learning, we're engaged, everybody's talking, nobody's scared to talk, nobody's afraid to be wrong, because nobody's going to be shut down.

6.3 Theme 2 – Time and Space to Think and Discover - Revisited Through Interview Data

Bonnie. As I mentioned in Chapter 1, I used to begin the Grade 8 unit on algebra by demonstrating how to build and balance equations with an equal pan balance. I essentially condensed the scale-building activity into a 15-20 minute demonstration and would have students solve equations pictorially and symbolically from that point. Early on in the interview, and in Bonnie's interactive writing, she informed me that her fourth grade teacher had done the same scale demonstration as me when she was teaching her students pre-algebraic concepts. As she was the only student I knew of who had experienced both forms of scale interventions, I was curious to hear Bonnie's insights into the two lessons.

> Bonnie: I thought (the scale-building activity) was interesting because I did learn something like it in Grade 4 I believe, except it wasn't as advanced and I was kind of wondering more about what you could do with the scales how some of the stuff works.
>
> Me: Did you use scales?
>
> Bonnie: No we didn't use scales but the teacher had a scale of her own – an actual scale – with weights and stuff. She just told us to balance stuff out…
>
> Me: Did she demonstrate it?
>
> Bonnie: Yeah. She did demonstrate a few times. She even taught us that the "equals" sign isn't "The answer is…" but that it's about both sides being balanced, although I forgot about what the equals sign actually is. So when you re-taught it I remembered about what I learned back then and that I was wondering if there was anything more.
>
> Me: When I used to do this unit before, I would demonstrate using a balance. One of the different things I did for this unit is that I wanted a hands-on, inquiry lesson with this, and that's why you guys made the coke bottles and you had a couple of classes where you spent time making them and playing with them, right?

Bonnie: Yeah.

Me: So, what do you think the difference is? What do you think building and working with the scales did for you versus being shown how they work?

Bonnie: Well, working with it lets you kind of experiment more on your own. And making it more your own, it kind of makes me feel better to earn what I do and get it on my own rather than other people, like, handing it to me: "And there you go, there's what you do, you do that and you work with this." As I make it myself, I'm more happy with what I do with it.

Even though Bonnie understood the concept of balance and equality when it was demonstrated to her in Grade 4, she admitted that these concepts did not stay in her memory. Despite the fact that Bonnie quickly remembered and relearned these concepts during my demonstration, she still appreciated being able to build the scales and experiment with them. Bonnie valued the time and space afforded to her as an opportunity to understand something on her own. As she constructed her own meaning of balance and equality, she found that it made her "happ(ier)" than if she had been simply "told" the concepts.

James. James also utilized the time and space afforded to him to construct meaning, but stretched his abilities even more than Bonnie, especially once James felt comfortable and supported in the community of learners. In his interactive writing, he wrote that a moment of understanding he had occurred on the third day of the unit when he wrote "IMPOSSIBLE" on his worksheet when his partner and he attempted to solve an equation with variables of different values. I asked him to talk about what he was thinking about when he and his partner attempted this problem.

> James: Well that equation was the first one I'd written so I didn't really know how to formulate equations. I really didn't know the rules. So I just wrote down a random equation – or I built a random equation on the scale – and I was seeing if I could solve it and I couldn't, and I realized that you can't solve something when you only know that one side is four more than the other side but there's an unknown number on both sides.

James came to the realization that one could not solve equations with unlike terms through playing and experimenting with his scale. He showed his comprehension of like and unlike terms later in the unit when asked to combine *5x* and *7x* on Worksheet 7. I observed that being given the time and space to build and draw the equations and to think about these equations in different ways allowed James to construct an understanding of combining like terms. His insights allowed him to better comprehend advanced questions later in the unit, particularly ones which included combining like terms.

Another moment of understanding James wrote about was when Mesego showed James his "Hanukkah" method for showing division (see Figure 6, p. 72).

> James: Well, up until then I had understood that division was going on. I understood how to solve it numerically but I didn't understand what was actually going on when you divided it. So when Mesego taught me his method, something clicked, and I realized how division was actually happening and how I could draw it.

In this instance, being given the time and space to openly talk with his friends led to a rich discussion about the nature of algebraic division between Mesego and James. This discussion greatly expanded James' understanding of division in general.

Mesego. When given the time and space to think and discuss within a strong community of learners, Mesego was also able to generate incredible ideas about algebra. One clear example of these kinds of ideas was what Mesego referred to as the “Hanukah effect” (see Chapter 4.2.3, p. 72). In this example, Mesego drew a series of curved arrows to pictorially demonstrate how weights fit into divided slots. He created this image after I instructed the group to try their best to draw division of variables while working on their worksheet, and I gave them a whole class to experiment with their ideas. Mesego’s model gave him a better understanding of algebraic division and of division in general.

> Mesego: (The Hanukah model) made (division) more visual. Like, I never actually drew division, I always did long division with the numbers, and now that I drew it, I can see it.

Mesego saw this moment as crucial in his understanding of algebra. Moments like the one above came about because Mesego was given time and mental “real estate” to construct his own personal meanings to the content we were covering. He appreciated being left alone to think about math. He understood that it is more powerful when he discovered concepts on his own rather than if I had explained or demonstrated these concepts to him.

> Mesego: (Being given the time and space to experiment with math)’s like giving a kid a box of Lego, and just leaving him alone for a couple of minutes. Like, he’ll come up with something, he’ll build, like, a space ship, and it’s just amazing to see that.

During my conversation, I specifically described my theme of the importance of time and space to Mesego, informing him of my observations that students can come up with amazing ideas when given the freedom to do so. Mesego validated my observations:

> Mesego: Yeah, it’s like, the more freedom you give to kids, the more chance they have of doing something with the freedom. The more specific and the more direct you give your instructions to what you want them to do, the less they have to think because they think

they have to do what you are telling them to and they're afraid to branch out because they don't want to lose marks or whatever.

6.4 Theme 3 – Connecting Hands-on Learning and Symbolic Learning - Revisited Through Interview Data

Grant. Early on, Grant clearly identified himself as a hands-on learner, both verbally and in his math journal. I also made note of how this historically weak student was immediately engaged with the hands-on scale-building activity. I began Grant's interview with pointing out how engaged he was in activity and the unit in general. After he confirmed my observations, I asked him to provide his thoughts about why this was the case.

Me: What was it about the beginning two – or a couple of – classes that you really appreciated?

Grant: Um, well the first class we started building the scales, so that was a bit fun because it was funny to watch people screw up with the strings and it was quite funny. I got it right away though cuz I build things. It's fun. But, um, it was still a lot easier than normal math because you can see it, you can adjust it, you could, like, manipulate the way you do your own stuff, so… that was helpful.

In Grant's interview responses, he immediately identified hands-on learning as the reason behind his engagement early on in the unit. He showed a strong mathematical identity, as he explained that he learns best in a hands-on environment. By boldly saying, "I build things," he revealed an extremely strong, personal belief about who he is as a learner. He also revealed circumstances in which he does not learn as well, as he decried "paperwork" earlier in the interview. By "paperwork", I believe that Grant was not necessarily referring to all worksheets, only those which require rote answers, no discourse with peers, and numerical answers only.

He continued to speak to the scale-building activity as an opportunity for him to take ownership over his work:

> Grant: Well you sort of have the pride in building something. Well that's just with me cuz when I build things I'm sort of happy that I can build them and say that I build this and such, but it helped me a bit more cuz with the scales it was like oh I built mine a bit different from everyone else so I could tell that it was mine personally.

Grant continued to speak about the importance of having ownership over his scale. He spoke about his appreciation for the opportunity to choose his own weights as well as being able to name his scale. He recalled the name of his scale as well as several others in the class. He spoke further about how having a sense of ownership is important in his work and in his life and how naming things helps develop that ownership:

> Grant: Yeah even when I do, like, regular every-day stuff. Like, if I build something – same with you giving us the choice for us to name something, even when I play games? Like, video games? When I run into something I really like, I'll name it just so you have more of a sorta sentimental feel in the game. Just so you get more attached to the thing that you're using, and the people that are playing, like … with some games you can get people that follow you around and if you don't like their name you can just pretend their name is something else or give them a nickname or something and that just allows you to get, uh, attached to your character. With most games, when you name your character that's sort of a big thing to me cuz I get attached to a character when I name it. That's a big thing for me.

Once Grant achieved ownership over the hands-on aspects of his learning, he was able to make connections to the symbolic. During the unit, when Grant was stuck on a problem, I would often relate the equation he was working on back to how one would physically build that equation with a scale. I shared my observation that when I related his work concretely in this manner, he would often immediately see how to solve the equation and finish the problem. Grant confirmed that when he has something concrete to rely on like the scales, even in his imagination, he is often successful in solving equations. When I asked him how he solved his first equation on Worksheet 6, when the class had stopped using the scales, he replied,

> Grant: Well, since we couldn't get any scales at the time just tried figuring things out in my mind cuz I'm good at picturing detailed objects in my head – I can do that with most things – and I just imagined that it was a really good scale and it had the same question on it, one "x" or whatever the number is and it's chopped into groups and it's just the way I tried figuring it out and I guess it worked.

Bonnie. Like Grant, Bonnie clearly understands the power of hands-on work and its importance in her personal growth, but she framed it differently than he did. For Bonnie, being given the opportunity to physically play and experiment with concrete learning devices allowed her to struggle through her learning and eventually "earn" her understanding. I asked her to elaborate on the concept of earning understanding. She stated, "Yeah you pretty much (laughs) "earned" the right to use it, I guess. And when you make it your own it's more yours than something the teacher just handed to you." Learning in this way helps Bonnie stay interested and helps her remember concepts better in the long term.

The theme of ownership continued as I asked Bonnie to comment on the opportunity I gave the class to name their scales:

> Bonnie: (Naming the scales) makes it a little more "yours" and you can learn better, remember it, even if you get a funny name. You'll be able to remember it easier.

Bonnie and her partner showed further ownership of their scale by consistently using their scale, rather than using scales made by others. Despite the fact that the screws became the weight most commonly used by their classmates, Bonnie and her partner also always used the black cube weights throughout the unit, rather than the screws. I was interested to know if their weight choices were connected to their sense of ownership over their work. I asked Bonnie to comment on this, but she could not explain why they used these weights nor did she assign any significance to this occurrence.

Bonnie did not need my assistance in seeing the connection between the hands-on and symbolic, but it is clear that having an intimate understanding of the concrete helped her understand more advanced equations. On Bonnie's test for example, she explained that she was able to reference the scale activity to solve a question involving combining like terms. In her interview, Bonnie described how she was able to visualize, in her mind, how she would go about building the particular equation. She then solved it pictorially and, curiously, drew it out last.

6.5 Theme 4 – Multiple Learning Pathways - Revisited Through Interview Data

Grant. The story of Grant's learning is completely unique to him, although at the beginning of this project, his learning journey would have been the type of narrative I would have hoped to ignite through inquiry. He began the unit with little to no prior knowledge about algebra and had a negative view of himself as a mathematics learner. However, he was engaged by several inquiry-based instructional strategies, such as building the scales and being able to name them. This engagement, along with the opportunity to work with his friends, allowed him to incrementally gain conceptual understanding of equality and equation. Whenever he stepped out of his comfort zone – or his zone of proximal development - he was able to access the scale-building activity, which had provided for him a solid foundation of knowledge.

When asked about what he had learned about equality in the unit on algebra, Grant replied,

> Grant: Like the equals sign? Like, it doesn't mean, like, this plus this equals this. It's not just the answer, this is what happens when you take this out of an item and you do a certain manipulation, like you manipulate this. I understand it, like, this is the product of you manipulating two things together so you're getting a product from mixing or adding certain things together to get a new item or whatever you call it.

Although his response lacks cohesion, Grant shows, in his own words, a conceptual understanding of equality. It is a statement about a much more mature comprehension of equality. He understands that when one manipulates variables and constants, one must keep the balance of the equation. Equality does not just mean "the answer"; it is also a symbol used to signify balance, and one can manipulate this balance to find the value (or "product", as Grant defines it) of *x*.

Grant reveals more of his mathematical identity when explaining how he was successful on these questions from later in the unit, despite not being able to use his scale:

> Grant: Well I just try to think things through logically. I try to think things out the best I can and if I can't I get help but most of the time I don't try to get help cuz I'm a bit stubborn.

He related a similar experience when answering the questions at the end of the unit, which I asked him to solve symbolically only, without building or drawing the equation.

> Grant: I relied on my thoughts just to figure this out. Sometimes I think too fast and I sorta talk too fast and I just figured it out with equations quickly. It's not because I'm

> rushing I can just figure it out quicker than some people. Sure I may write messy, but I get the answer right. Isn't that what really counts in the end, really?

In these two instances, Grant reveals a method of learning that has been helpful for him. He begins undertaking a problem through cognition, or "(relying) on (his) thoughts". However, he is flexible enough and aware enough of his weaknesses that he will ask for help. He describes himself as "stubborn", but one could describe this attribute as "persistent".

I showed Grant a set of questions that he had solved using arithmetic only, and asked him if he was thinking of anything. I was trying to find out if he was still relying on his scales or if his conceptual understanding was strong enough that he had moved beyond the concrete into a comprehension of the abstract.

> Me: You just did these questions really well. Did you think about anything when you solved these?
>
> Grant: Not really, no. I didn't think of anything. I just thought of equation and how do I get the answer. That's all I thought about.

While Grant demonstrated a strong conceptual understanding of algebra concretely, pictorially, and symbolically, he also demonstrated a sophisticated understanding of how and when to use this knowledge. Like training wheels on a bike, he was able to reference the scales when he needed to, such as when he was introduced to a new and/or difficult equation. However, he also knew enough about equation and equality to not have to rely on the scales to correctly solve most one and two-term equations.

Bonnie. Bonnie found an extremely successful formula for learning during this unit. She would first obtain knowledge from direct instruction, and then she would work experimentally with her partner to further construct meaning. This construction of meaning was intricately connected to social interactions with her partner as well as her friends at her table. She found that as she explained her thinking to others, she gained new insights into her understanding of algebra. When she needed help, she would take advantage of "just-in-time" interventions from me or would ask her friends or me direct questions about what she did not understand. Humor was important for her – she also mentioned the "poofs" as being significant – but other learning opportunities, such as having the opportunity to choose her own weights – carried little meaning for her.

In later assignments, Bonnie demonstrated an array of tools that she used to solve problems. One tool she employed was observing and choosing conceptual models that made the most sense to her. For instance, she enjoyed and appreciated the class discussions and demonstrations about division. According to Bonnie, she had a weak understanding of division, but Adam's strategy of breaking values into sections and then labeling these sections with letters made the most sense to her. Once we started working with equations without building them with the scales, another strategy that Bonnie discovered was working backward. Bonnie's mathematical thinking was sound enough that she was able to guess the value of variables, and solve many of the equations we studied by working backward. She employed this strategy on the last worksheet as well as on her test.

Bonnie's most valuable learning attribute is her ability to be open to any strategy she needs to successfully solve problems, even if that strategy is brand new. I asked her to talk about how she successfully solved the equation $9a – 4 = 23$ on her last worksheet, as she drew her scale in a way I had not yet seen. She replied that she drew it in a way she had never done before. However, she was confident enough in her understanding of division and negative numbers to try something different. This strategy helped her successfully solve this problem.

In other instances, Bonnie shows that she can utilize her understanding of algebra in different ways depending on what the question requires of her. When she came across a question on the test that she had not seen before, she needed a quick explanation about what the brackets meant before she went on to solve it.

> Me: Once you understood (that brackets meant multiplication), did you solve it first or did you draw it first?
>
> Bonnie: I think I solved it first.
>
> Me: And what about (Question 4)?
>
> Bonnie: That one? I think I solved it as I went. I (wrote it out numerically here) and went here and drew it, back and forth.

When solving a group of questions that only required solving numerically, she employed different methods for different questions, even though she found solving numerically "easier" than drawing. For two of these questions, she solved numerically only, but for two others, she referenced the scales in her mind when she needed to double-check her work. One of the questions was an advanced equation involving combining like terms. (see Chapter 6.4, p. 115).

James. Like Bonnie, James acquired several strategies for being successful at solving equations during the unit, but he employed these strategies differently and he attributed different activities to their development. At the beginning of the unit, James felt he did not need the scale activity in order to understand equations.

> James: (In the first few scale demonstrations) it was just really easy to visualize the scale you had at the front of the class and after you did two or three of them I pretty much got it and I was able to extrapolate most of those rules onto the questions. And I think that I just grasped it sooner from those initial demonstrations.

Me: Okay so it was the demonstrations. Once you started to work with that, do you think (working and playing with the scales) solidified your understanding or do you think you already got it?

James: I think I already got it.

After the first few classes, James related in his journal that he saw the scale-building activity as an inefficient way to learn concepts that he quickly grasped. On March 18, James wrote,

> This unit has given me mixed thoughts. On one hand, the scale based activities take a long time and result in entire math classes being spent on scale operations which seem to simply be reinforcing the little algebra I learned in sixth grade. On the other hand, I have never been exposed to difficult algebra, and have no idea what is necessary for the teaching of advanced algebra and therefore command no authority within the topic of algebra education. In short let's see where this goes.

I read James the above quote then asked him if he saw the activity's value now.

James: I think the scale activity definitely helped, yeah. I don't think I would've gotten … I don't think I'd have been able to grasp the more complicated concepts if I had not started with the scales.

James conceded that the scales helped to develop a foundational understanding and they did help him when he faced more difficult problems later in the unit. James demonstrates this later when solving an advanced equation on the test.

James: Well I couldn't get this numerically right off the bat so I decided to try pictorially because numerically is a lot faster but when you don't get something it can get really – it's a lot easier to grasp something. So I started by putting the ts and the 1s in blocks –

> three of them cuz it's three – and then I took the 1s as you do with any question, and I took the equivalent three from the other side and I was left with 1t and 4 1s.

In this example, James demonstrates how relational understanding helped him come to the correct answer on a problem he had never seen before. He was able to combine his knowledge of the concrete scales and his ability to express equations pictorially to successfully comprehend the equation he had to solve. Once he accessed his foundational knowledge of equation, he was able to answer this question symbolically.

Mesego. Mesego was able to be very frank and articulate about his learning journey throughout the unit. He believed that his success was due to several factors, including the freedom to think and experiment independently and in groups, the ability to experiment with hands-on learning, and the opportunity to be creative in his learning. He valued the hands-on aspect of the scale activity, but when he was faced with questions he had not yet seen, he relied on his ability to solve equations numerically first, and then pictorially when this strategy did not work. On his test, he solved both bonus question equations correctly. I asked him what strategy he employed to solve them.

> Mesego: I tried to do numerically, then realized that that wasn't working out for me, so I took your advice and I started drawing it out and when I started drawing it out it started to click in my head, like it started to actually make sense.

Mesego enjoyed working with the scales early on with his partner because he liked the challenge of stumping his partner during "What's in the Bag?" He would do things like "crunch(ing)" up the bags to make it seem like there were more weights in them. He also identified the opportunity to help other students with their work as something that facilitated his learning, especially when he was helping them with a topic he did not totally comprehend.

> Mesego: I've gone through many cases when I've been explaining something to somebody, and they wouldn't understand and I'd be like, "Oh my gosh, you're so dumb" and then I'd keep explaining and I'd be in the middle of my train of thought and I'd be, like, "Oh, what I'm saying actually doesn't make sense; I'm the one who's stupid!" Then I'd stop talking and I'd be like "Oh, you don't get it cuz it doesn't make sense" and then we'd just talk it out and we'd come up with something better that does make sense.

Mesego further appreciated working with someone who understood concepts better than he did. When he was in this position, he was appreciative of the way in which his partner and he added on to each other's understanding of any given concept.

I observed during the unit that Mesego had a tendency to craft creative narratives or stories to explain mathematical concepts or his own mathematical thinking. For example, in the eleventh class, I asked students to experiment with different ways to pictorially represent the manipulation of negative numbers in solving equations. Mesego struggled at first to make sense of negatives in this context, but he soon created an explanation, which he deemed the "factory" rule. He explained:

> Mesego: It's kind of like the factory turns negatives into positives and it also goes back in time. So the negatives go back to where positives, cuz that was before… Cuz that's what algebra basically is, it's going backwards in time and retracing your steps to find out what "a" was before you did all this stuff to it.

The above example is not the only instance of Mesego connecting mathematical concepts with stories. During the tenth class, Mesego explained a rule of algebra as a story about how x was a child who needed to lose bad influences around him. On his last worksheet, he described two negative numbers as "Two survivors turned evil with bad attitudes". Stories are very important for Mesego. They help him remember concepts and they help him construct his own understanding. Mesego also stressed that these stories are much more powerful and meaningful to him when he generates them rather than someone else, like a teacher.

6.6 The Impact of the Interviews

I set out to keep the interviews casual and conversational, but I still kept a list of questions at my disposal that were taken from the narrative text I had created for each interviewee (Chapter 3.1.3, p. 31). However, I was taken aback at how quickly the interviews often veered off of my “script” and into pure conversation. It was a pleasure to sit with the four interviewees and discuss Grade 8 mathematics in a deep, personal, academic manner. As a teacher, it was the type of rewarding experience that I wish to replicate in order to gain insight into student learning in other areas. As a researcher, the conversations with the four interviewees provided detailed and profound insight into their learning. Through their frank, honest responses, I was able to further develop the observations I had recorded in my field notes into what became the four major themes.

Chapter 7: Conclusions

7.1 Synthesis of Themes

The four themes (A strong learning community is essential to learning, having the time and space to think and discover allows opportunities for incredible insights, hands-on learning and symbolic mathematics need to be connected in order for both to be meaningful, and students learn very differently in vast ways) are merely my attempt at compartmentalizing the significant insights I developed over the course of my project as I attempted to address my research question, "How do students develop conceptual understanding of equality and equation in Grade 8 algebra through inquiry-based learning processes featuring the balance?" Separately, these themes speak to my research question directly in some cases and indirectly in others. But they are, in fact, intimately interconnected, and none of them can exist without the other. The theme of differentiation among learners, in particular, both incorporates and encompasses the other three themes. These themes also existed within a broader, complex, interconnected framework of inquiry. Inquiry-based learning processes no doubt caused learning, but there was not a single process that caused learning for every student. In the end, inquiry created an environment in which a high number of students were able to find their own unique access points to understanding.

7.1.1 Community and collaboration. One of the reasons the participants in this study developed conceptual understanding of algebra is because they belonged to a strong learning community. As Palmer (2007) attests, when students identify themselves as part of a strong learning community, they are able to experience richer and deeper levels of comprehension. From the beginning of my data collection, it was striking to me how much they valued being part of a community. Early on, when asked to simply reflect on the unit so far, a significant amount of students wrote in their journals about the importance of social learning (see Chapter 6.2, p. 101). They emphasized that their high engagement and high levels of understanding up to that point in the unit was largely due to them being able to work in small groups with their friends. Many students identified the ability to discuss questions and topics with their peers as helpful for developing their understanding. Dillenbourg (1999) refers to this reinterpretation under the light of what a peer does as “appropriation”, which allows individuals to develop their comprehension by appropriating beliefs, opinions, and thoughts of their partner.

As the unit progressed, students began to value other ways of social learning. Bringing students to the front for instruction and class discussion, for example, became a defining feature of the unit (see Chapter 5.1, p. 89). Students began to feel safe and validated, and over the course of the unit, they opened up and took chances. Rich conversations about mathematics arose from this courage, and a culture of learning was formed. The “community of truth” that Palmer (2007) describes may not have been achieved during the three-week unit, but the foundation of developing this community was definitely formed.

This culture was not only formed because of the allowance of group work. Social learning did not just occur once students were given freedom to work in groups and talk, although this was definitely a part its development. This learning was carefully fostered through the community-minded tone of the unit. This tone began with the scale-building activity and continued as I allowed choice, encouraged students to speak their minds, and took steps to learn collaboratively. It is also important to note that these students already belonged to a strong community of learners and that I had relationships with many of them and a reputation for being an open-minded teacher before I stepped into their classroom.

Unfortunately, there is not one formula that can develop an effective learning community (Palmer, 2007). It was not one single offering or event that allowed students to feel comfortable enough to experiment with problems and engage in risky conversations about math. In reality, it was many varied processes that helped create this community-minded tone, and the degree to which these processes were meaningful varied greatly among participants. Some students, like Mesego and Grant, valued small freedoms like being able to name their scales or choose their own weights. Others, like James, appreciated the opportunity to brainstorm and discuss the rules of algebra as a class, rather than being provided a rule sheet. These small changes had enormous collective impact on the atmosphere of the classroom.

Many of these small changes amounted to minor alterations to my instruction. In the past, I had already employed the strategy of bringing students to the front, but I utilized this strategy more often during my project. I had also allowed freedom and creativity in past math lessons, but during the unit, I was more intentional about allowing students to use their imagination and make decisions, like the naming of scales and drawing seemingly silly illustrations on their work, like Lily's "poof", or Mesego's story of *x* (see Chapter 6.5, p. 122). Most importantly, I believe, I slowed down my instruction during class discussions, and I really took the time to value my students' voices. When I allowed myself the time to validate their ideas and have them share their different mathematical strategies, an important by-product occurred; participants developed inferences and insights and became immersed in discovering new knowledge. Through allowing the students to be the intelligent, insightful individuals that they are, I was able to be enamored enough to organically become a part of the community of learners. Once entrenched in this community, students no longer saw me as "sage on the stage", but as a facilitator and teacher-learner, and they responded to this reality by being more honest, creative, and open to developing new understanding (Jones, 2006).

7.1.2 Time and space to think and discover. Once students felt connected to our learning community, all they needed were opportunities to gain insights on their journey toward constructing their own understandings. Insights like Mesego's "Hanukah" method (see Chapter 4.2.3, p. 72) or James' division model (see Chapter 4.2.4, p. 78) are only two examples of many. Samuel, for example, discovered on his own that one must divide both sides of an equation when presented with a variable being multiplied by a coefficient (see Chapter 4.2, p. 85). Anna, while struggling through advanced problems early on, discovered the distributive property through a guess-and-check strategy (Field Notes, Day 15).

Most discoveries were not as dramatic. Melissa and Noah, for example, slowly figured out how to represent their scales pictorially and symbolically over several classes of steady work (Student artifacts, Days 7-13). Habem experimented with different ways of illustrating scales before he finally settled on one that worked for him. Nevertheless, it was the freedom to experiment with and struggle through their equations that allowed them to successfully construct understanding.

Just because students are given time to work does not mean that they will learn and this theme of constructing understanding when given the time and space to do so is, once again, complex (Davis et al., 2008). It is also highly connected to the first theme of community. My students were able to discuss and struggle through algebraic equations because they felt the collective will of the classroom community to gain knowledge (Palmer, 2007). Through class discussions and engagement in tasks, students were able to foster a strong community of learners, and it was difficult for students who were not engaged in mathematics to not get swept up in this atmosphere. I believe that I was an integral part of this atmosphere, as many of my instructional strategies collectively helped to create an environment of openness, creativity, and respect for differing ideas. Once students felt immersed in this environment, they were able to take risks with their learning when given the opportunity to do so, and the stage was set for them to construct their own meanings.

The time and space for thinking and discovering allowed what Dewey (1938) refers to as rich experiential learning. Dewey (1938) valued continuity in quality educational experiences. Continuity is the idea that ideas need to build upon past experiences, and that these ideas can propel individuals to search out new experiences. In my study, I observed students achieving continuity on many occasions. For example, Mesego achieved continuity of experience when he applied previous experiences with division and attempted to draw these experiences through his "Hanukkah method" (see Chapter 4.2.3, p. 72). He then applied this idea to a new situation when he answered advanced questions involving Grade 9-level content. Allowing for time and space to think also gave students the ability to reflect and gain understanding, such as when James discovered a new way to draw division (see Chapter 4.2.4, p. 78). Without time to reflect and gain mental growth, Dewey (1938) considers learning experiences to be "non-educative" (p. 19).

7.1.3 Connecting hands-on and symbolic learning. Another part of making meaning that was relevant for the participants in this study was the hands-on aspect of the scale activity. Algebra is a concept that is resistant to relevance in the minds of middle school students. In her earliest journal entry, Kyla wrote, "I can't think of anything from real life to apply (algebra) to". The scales were my attempt at making algebra as meaningful as possible while not compromising the foundational concepts of equality and equation. Even then, however, some students failed to see the relevance of the scales. Habem "hat(ed)" the scales while Anna found them “irritating… shoddy, and inaccurate”. Nevertheless, overall most students found value in the hands-on aspects of the early classes. Students, like Samuel and Nahlia, described the scale building as making algebra “fun” and “easy to understand”. Others, like Darcy, were more specific about how the scales helped her understand algebra. “(The scales) have helped me visualize the problems”, she wrote, “which helps me solve the question easier”.

Other students appreciated my attempts at making algebra more “real world” through the scale activity. Mesego valued hands-on learning because it was concrete and real world. For Mesego, having concrete activities made math meaningful and gave him motivation to learn.

> Mesego: Well, I like to relate math to real life situations because that’s what math should be used for…I think comparing (math) to your life will give you a more personal connection, and when you have a more personal connection to things, you’ll care. And that’s one of the big problems people have in school. (Students) don’t care so they won’t try.

In follow-up writings and in the interviews, it became clear to me that there was more to the scale activity than it just being beneficial for the hands-on learners in the class. Students needed something concrete to make meaning out of what would soon be abstract mathematical problems, whether or not they enjoyed or connected with the scale activity. As Sousa (2008) states, connecting non-symbolic exercises with symbolic arithmetic can help students master symbol systems. Meanings of symbols must be “firmly rooted in experiences with real objects”, otherwise students’ comprehension of symbolic operations will be limited to “rote repetitions of meaningless memorized procedures” (p. 83).

Neither Madison nor Melissa reported in their journals that they enjoyed building and working with the scales. However, the foundational knowledge that they learned through the scale activities helped both Melissa (Chapter 4.2.2, p. 64) and Madison (Chapter 4.2.4, p. 79) move from understanding of the concrete to the symbolic. These two students, as well as many others, needed scaffolding from me to be able to see the connections between what we were doing with the scales and algebraic language. The scales provided a foundational, real-world example of a way to contextualize the workings of an algebraic equation, but in order for students to see how these hands-on experiences connected to symbolic equations, I needed to translate what we had done with our scales into algebraic language. This would involve demonstrations and/or detailed explanations with examples.

Other students, like Bonnie (see Chapter 6.1, p. 96) and Mesego (see Chapter 6.1, p. 98), did not require assistance from me to make the connections from hands-on to symbolic. However, they nonetheless utilized their experiences with the concrete to be successful when they felt they needed it, such as when they were presented with equations they had never seen before. It was also apparent by the end of the unit that even students who seemingly did not "need" this concrete experience to understand algebra benefitted from it in the long term. James, for instance, felt that the scale activity was an inefficient way to learn what could have been taught in a fraction of the time using direct teaching strategies. However, in his interview, which occurred seven school days after the end of the unit, James understood that the scales created a solid foundation of conceptual understanding for him to fall back on (Chapter 6.5, p. 120). The scales also served as a gateway to understanding for students who historically struggled in math. Grant, for example, was able to successfully solve advanced equations by relying on his knowledge of how equations would look and feel on physical scales (see Chapter 6.4, p. 114).

The importance of hands-on learning is connected to the other themes. The scale activity was an effective community-building exercise. It immediately injected a jolt of energy and creativity into a group of learners, many of whom were historically disengaged. Within minutes, the atmosphere of the class changed from being dull and monotone to boisterous and exciting. Despite the small-scale nature of this activity, students immediately responded to it by showing high levels of engagement and enjoyment. Just being able to move around and play with objects that related to math was enough for most of the class's tone to change to a tone that was community-minded. After they were able to move around, interact, and engage playfully with inquiry, students began to experience the rewards of coming to an understanding about the subject by engaging with the hands-on tasks in teams. This positive experience helped them to buy into this form of learning early on in the unit. Once this community-minded tone was established, students utilized the time and space they were given to build upon their foundational knowledge, which was established through the hands-on, concrete scale activity (Skemp, 1976; Sousa, 2008).

7.1.4 Multiple learning pathways. For the final journal entry, I asked students to reflect on a moment of understanding they had in the unit. In my personalized responses, I asked even more general questions about which parts of the unit they thought had impacted their understanding of algebra. Tara wrote about the different ways of drawing negatives in scales that seemed helpful to her: "When I was looking at #6 (from Worksheet 7), I was confused… but then when you were showing the scale to us, you shaded in the cubes to tell the difference from the others which made sense". Darcy valued conversation. "When I talk," she wrote, "I process things better." Emma identified the discussions at the front of the class as being important, stating that they helped her understand negative numbers. Wynona appreciated the scale activity and mentioned that it helped her personally connect to the rules of algebra.

Others did not find the scale activity helpful at all. Darcy wrote that although enjoyed it, the scale activity “did not help me”. Emma wrote that the scales seem “childish” and “if anything (they) confuse me more”. Some students wrote silly responses about imaginary creatures and the fact that they learned that they disliked Johnny Cash. I do not discount these responses, because they show that these students felt that they could be themselves in their writings to me. Palmer (2007) refers to the importance of feelings of safety and acceptance within the community of truth. The students’ responses demonstrate to me that they felt safe and comfortable within the classroom community, and that this was something they clearly valued.

The above journal entries, along with several others, confirmed my observation throughout the project that it is absolutely astonishing how differently my students learned. The interviewees further supported this observation (see Chapter 6). Grant had a negative view of himself as a learner of mathematics, but he gained confidence and understanding incrementally as he became engaged with the hands-on, inquiry-based aspects of the unit and received support from his classmates and me. Bonnie took full advantage of the multitude of learning opportunities during the project, finding value in the inquiry, social, and conceptual aspects of the unit. James showed little interest in the inquiry-based processes of the unit until he found deeper understanding through some of these processes. He valued the time and space I provided to think about underlying concepts but considered the small instances of choice, such as naming the scales, to be trivial. Mesego, on the other hand, found significance of the small moments of choice, using them as opportunities to add humor and joy to his learning. He utilized stories and narratives to understand concepts and he attributed much of success to the communal learning at play during the project.

The students in the class who came to understand equality and algebraic equations each took unique paths to achieve this objective. Some valued the hands-on work more than others, some enjoyed personalizing their scales while some were indifferent, some identified social learning as important, and some students relied on their scales to solve complex problems while others did not. Despite the responses from participants, it was clear that no matter what strategies they valued and employed to learn algebra, and no matter how different they were from student to student, every strategy was vital to each individual's understanding.

Once again, this should not have been a surprise. Research has shown that personality differences exist and can be measured that allow for taxonomy and theory (McCrae, Costa, de Lima, Simões, Ostendorf, Angleitner, & Piedmont, 1999). These theories, such as Gardner's (1983) theory of multiple intelligences, have been staples of education for decades now. However, what I observed was not just about me diversifying instruction to allow access points for different students with different personalities and learning styles. Rather, it was about small but profound instances of learning, unique to each participant.

The four themes all answer my research question in various ways, but it is the fourth theme of diversity of learning that binds all the themes together and answers how students develop conceptual understanding of equality and equation. Simply, students develop conceptual understanding very, very differently. It is clear that hands-on learning, interpersonal connections, and being given time and space to learn were all important for the participants in this study. However, the importance of each theme varied greatly between students and, in the end, it was not the overarching themes that they identified as being key aspects of their learning. Rather, it was the collective influence of many small instances and strategies that led to their successful understanding.

Personally, the theme of multiple learning pathways impacted my research in a way that cannot be minimized. The amount of unique, distinct routes that students traversed to success have given me many insights beyond answering just my research question, and these insights are all connected to inquiry. I originally viewed my research question as rather specific, as the scale activity seemed like a simple activity and I was anticipating rather straightforward data in terms of how this activity impacted learning. However, because I viewed my question – as well as my data – through the lens of inquiry, I amassed broader, richer answers, which addressed my research question as well as provided me with deeper insights into my own instruction. The four themes, particularly the theme about learning diversity, could not have been reached without the framework of inquiry.

7.2 The Impact of Inquiry

This project began as a way for me to see how inquiry could be applied in small curricular spaces. I had witnessed my students achieve so much knowledge and confidence throughout large-scale inquiries and I wanted to see these positive effects in areas of academics that are resistant to inquiry. When teaching inquiry-based learning on a large scale, a major part of the inquiry process was to tailor the curriculum to concrete endeavors that students were interested in or even passionate about. The scale activity, then, was an intervention designed to engage students in mathematical concepts with which they had little or no meaningful connection. They were largely not passionate about nor were they even interested in building scales, but having a concrete, hands-on experience enabled them to grasp onto something meaningful and real world. This was a good starting point for an inquiry-based unit.

The key to understanding the overall success of this unit – if "success" is measured in student learning and engagement – is that the scale activity was only one small part of inquiry-based instructional strategies that I employed over the unit. On its own, it was an engaging intervention based on the principles of inquiry. However, had I abandoned the principles after this intervention, or had I studied the activity in isolation, I do not believe that I would have seen the true, overall value of applying inquiry-based strategies. The success of the inquiry-driven scale activity informed how successful the rest of the unit would be. The scale activity, in turn, would not have been as effective if not viewed in terms of how an overarching focus on inquiry impacted the manner in which the activity was carried out. By the end of this project, it has become clear to me that there is no such thing as a small-scale inquiry. Once the principles of inquiry are applied to instruction in an intentional way, they permeate every aspect of teaching and learning.

My beliefs about inquiry and the way I see how it works in a classroom have changed. Before this project, I saw two types of teaching and learning environments in my classroom. One environment consisted of large and smaller-scale inquiries as described in Chapter 1. These projects often involved two, sometimes three classes of students, and incorporated many academic and non-academic subjects. Occasionally, I would engage in smaller inquiry projects and assignments. In math, these assignments were rare, as the constant push for the inclusion of content overrode my deeply held beliefs about how best to teach and learn. When these projects and assignments did arise, however, students always responded positively. When engaging in inquiry projects, it often felt like heaviness had been lifted from the class. Engagement immediately rose, and, in turn, so did the amount of authentic learning experiences, as disengaged students cannot effectively learn.

The second type of environment in which I situated myself was one in which I occasionally injected small-scale hands-on activities into direct instruction. These activities included dice games when introducing probability, measuring items around the classroom while learning about area, and demonstrating volume by displacing 3-dimensional objects in water. Often I would include games and hands-on assignments that would exist as one-day lessons within broader, overarching units. These activities were mostly engaging for students, and were largely effective as instruments of information transmission. However, I did not see them as "inquiry-based" activities, and they did not exist in the context of a larger unit based on the principles of learning.

This project has shown me that once I am committed to inquiry, its benefits pervade all aspects of my teaching and learning environment. Inquiry existed on both the macro and micro level of the learning that transpired during the unit on algebra. It was the engine that drove the four themes as well as the small experiences within these themes. This realization of the significance of the themes and the importance of all the collective small moments of learning was a by-product of my intentional introduction of inquiry-based learning processes. While attempting to employ as many of the five principles of inquiry as possible, at all times, learning and teaching became organically enriched. Moments such as allowing different choices of weights, bringing students to the front more often, and giving voice to new ideas from students came about because I was keenly aware of implementing inquiry-based strategies, instead of focusing on how fast and efficiently I can teach learning outcomes.

The value of inquiry has changed in my mind. I have always valued inquiry as a method of achieving high engagement among students. It has also yielded consistently deep understanding from students about a wide range of subjects and topics. However, I now also see inquiry as a means to provide a rich, varied tapestry of learning opportunities for students, through which they can best achieve deep, conceptual understanding of content. The differences between the way individuals learn is extraordinary, and inquiry provides enough diverse access points for understanding that every student in a group should have their own unique opportunities for learning.

7.3 Limitations and Benefits of Methodology

There are several limitations to my study. For one, I have a small sample size of one class of students. Second, although my participants come from diverse backgrounds and are a diverse group of learners, students of high socio-economic status are over-represented as a sample of the general Canadian population. Third, at first glance, people may not pay much attention to my findings since I teach in an alternative program, and there are assumptions about students in the program that they are more studious, more motivated, and come from better socio-economic backgrounds than other students. The reality is that, unlike some other alternative program, my school does not have any process of admittance into the program. Also, my participants in the study were students from the regular non-alternative program in my school. Nevertheless, people may dismiss any findings I may discover based on their own assumptions.

There are other aspects of my study that may be interpreted as limitations, but these limitations also reveal strengths. For example, like many action research studies in education tends, my project was executed as a means to improve my own personal teaching and learning (Stringer, 2008). It was specifically designed for my own individual goals, personality, and teaching philosophy. This can also be considered a strength, however, as my study will have a significant impact on my personal growth as an educator. Other limitations that can be viewed partly as strengths are two methods that I developed to collect my data, interactive writing and interviews. On one hand, these methodologies enable students to communicate their thoughts in meaningful, intentional ways. On the other hand, because participants volunteered their input to be used as data, students who were less engaged in their work tended to not volunteer to allow their data to be used. As a result, my data flow included a greater percentage of high achieving and engaged students than was represented by the actual class.

Nevertheless, my study has many benefits. As mentioned in Chapter 3, my study is credible, dependable, and confirmable (Anderson & Herr, 2005) (see Chapter 3.5, p. 46). Significantly, I believe my study is also highly transferable. Individuals who are in the field of education may incorporate aspects of my study into other research projects. Both my original unit plan and the unit plan I eventually followed are outlined in detail here. Teacher-researchers can follow either plan exactly as it is or they can choose aspects that they would like to utilize. I believe my study could be of particular use for teachers interested in incorporating inquiry into small curricular spaces, whether in the subject of mathematics or in other subjects.

7.4 Implications for Teachers

Teachers are often intimidated by inquiry-based learning practices for many reasons, but one of the reasons is that inquiry often manifests itself in larger-scale interdisciplinary projects that often seem overwhelming. However, if teachers can adhere to the larger values of inquiry – building community, carefully scaffolding, valuing experience, fostering conceptual understanding, and promoting student involvement – they can apply these values to any area of teaching, even in the smallest of scales and in the smallest of curricular spaces. Keeping these values at the forefront of teaching can seem like a difficult endeavor, but this was not the case with my study. Although I began my project being very intentional in introducing these principles, after a short time, they organically permeated every aspect of teaching and learning. Once I began to see myself as an inquiry teacher, rather than an occasional purveyor of inquiry projects and assignments, the principles of inquiry became naturally interwoven throughout my teaching.

This ease of instigating inquiry when one is committed to it comes about because of the interconnectivity of inquiry. Involving students creates active members of a learning community. Once a learning community is established, the teacher cannot help but scaffold learning, as he or she responds and reacts to students who are vocal about their misunderstandings and triumphs. Scaffolding naturally occurs through valuable, often hands-on experiences, and conceptual understanding is reached though all the above tenets. The themes that revealed themselves to me were also directly and indirectly related to establishing the principles of inquiry throughout my study.

In order to address the need to foster success for all learners, creating an inquiry-based classroom is one way to meet the vastly different needs and interests of a student body. Overall, in my study, inquiry worked by connecting to a great many students in ways that most uniquely addressed their learning needs. Grant and James represent an example of two very different students who found unique paths to success. Grant, one of the weakest students in the class, was able to find access points of success through the hands-on nature of the scales. He also was able to create a meaningful foundation of algebraic knowledge out of the concrete nature of the early activities. Grant was later able to recall this concrete knowledge, which helped him to better understand algebraic equations when faced with more advanced questions (see Chapter 6.4, p. 114). James, on the other hand, was already an extremely successful mathematics student, and he was at first skeptical about some of the early inquiry processes that I established. However, James was able to access deeper conceptual understanding through intelligent discourse with his peers and his teacher than he would have likely had accessed otherwise. He appreciated this mental exercise, and through it he developed insightful ideas about the nature of mathematics, which went beyond the simple outcomes of Grade 8 math (see Chapter 6.5, p. 120). Both Grant and James valued being members of a learning community, but in different ways. Grant felt safe to ask for help and developed confidence through working with caring friends. James, on the other hand, was aided by the rich conversations that developed because of our strong learning community.

There are no simple, easy ways to develop the intangible concept of "community", but many small strategies can collectively help. Involving students in their learning is of course a good way to begin. However, the participants in my study surprised me by how much they valued working with their friends. Much has been written about how best to group students, but despite years of research, there is still little consensus about it (Yee, 2013). I have been advised in the past that the optimal groupings in math are groups of two. In these groups, students are paired with others who are at or near the same level of mathematical aptitude. The assumption is that pairing students this way creates an optimal environment for helping each other and for rich mathematical conversation. However, my study has allowed me to rethink this assumption as well as how I group my students. Having learners at different ability levels at the same tables seemed to have helped both the strong students and the weak ones. The weak students gained confidence and understanding by getting help from their peers and the strong students appeared to strengthen their understanding as they taught concepts. Also, in my study, students tended to be more open and willing to engage in mathematical conversations because of proximity to people they knew well. Unsurprisingly, more off-task behavior occurred when students were sitting with their friends. However, according to the data generated from this project, a greater volume of in depth, meaningful conversations about math occurred as well. A teacher must be willing to embrace both extremes to truly foster a learning community.

Hands-on tasks are also effective at establishing principles of inquiry. The small, simple "What's in the Bag?" game immediately contributed to a feeling of community, engaged students in experiencing learning, created a foundation for conceptual understanding, and involved individuals in their learning. As mentioned above, students were not largely interested in building scales, but having a concrete, hands-on experience allowed them to connect to something tangible and real world. Sometimes, as an educator, this a good enough starting point for inquiry. In the case of my unit on algebra, the metaphor of the scale worked because it began with tactile and visual processes. If teachers can find real world experiences for their students to connect with, no matter how small, the impact on learning can be enormous.

The final major shift in the way I will teach inquiry came from the moments of learning and understanding I observed from students when they were given the time and space to think, experiment, and discuss. Teachers are notorious micro-managers, and I am no exception. As much as I hate to admit it, I am quite impatient and I value efficiency. However, students need to be given time and space to be left alone to construct their own understanding. Mesego stated this idea perfectly and even revealed how it connects to classroom community: "When you allow people to think more and give them more freedom with these theories – how you accept them and think about them – it strengthens the community and people just learn more from it."

This insightful quote along with the quality data I received from interviews and oral responses during class discussions show that students also benefit from being given different ways to express their learning and understanding. It is very difficult for most students of this age to explain their mathematical thinking through writing or arithmetic. They also need ways to discuss and converse about their learning process. For this to happen they need time and space, and this time and space must exist within a larger community of fellow learners. It is essential for there to be a trusting relationship between students and between the students and the teacher in order for learning to be processed in this manner.

Inquiry is not perfect. There were many students who expressed negative opinions, either verbally or through interactive writing, during my unit on algebra. I was unfortunately unable to interview any of the four students who admitted to disliking the unit, although all four were successful in developing conceptual understanding of algebraic equations. There were two students, however, who did not develop conceptual understanding. One of the students, Harley, generally did not perform well on her work at all. She was disengaged for the entire unit and did not show any desire to understand the content. Another student, Nolan, revealed a lack of understanding of equality in his interactive writing when I asked the class what equality now meant to them. While most students wrote sufficient responses that showed conceptual understanding of this concept, Nolan's April 9th journal response reads: "I don't know a whole lot about equality. Just that it gives me the answer". He also wrote, "I wish there was (a moment of understanding he could write about). I'm really struggling with this unit. Almost everything about this unit is hard for me." Responses like Nolan's are heartbreaking to me. Despite all of my intentionality of instilling principles of inquiry to reach every student's unique learning needs, some students still struggled, and, in Harley's case, failed to learn concepts outright.

Nevertheless, in my study, inquiry was valuable for the vast majority of my students on a number of levels. It helped them develop into a learning community, it created hands-on experiences that they valued, and it allowed a diverse set of access points of understanding for a diverse group of learners. These gains collectively contributed to most students developing a strong conceptual understanding of equality and equation.

Learning is extremely complex, and finding an instructional practice that meets every learner's needs is impossible. The notion that there is one teaching method that addresses the need of every student is even more questionable when one considers that learning occurs within intense, infinitely complex social systems such as classes, schools, and the complicated lives of hormonal teenagers (Davis et al., 2008). Perhaps, then, we should reexamine the common practice of pushing certain instructional strategies onto teachers. Inquiry-based instructional practices were effective in my classroom, but these strategies might not work for every teacher. I highly recommend inquiry, but I am not prepared to identify it as a one-size-fits-all teaching methodology.

7.5 Concluding Remarks

Throughout my journey toward completing my research, I have realized that the more I learn about teaching and learning, the more I realize how complex the field of education really is. Inquiry has allowed me to be successful in the past in terms of surface-level outcomes like high student engagement and exemplary artifacts of learning. However, through my research, I now understand inquiry to also be a pathway for me to pay respect toward the complexities of my students as well as my profession. Teachers, like students, are complex human beings, and they exist within the complexities of the systems mentioned above. Personally, inquiry slows my instruction in a positive way and holds me accountable to a wider range of learners. Inquiry methodologies have given me an access point to unlocking student potential, provided the rewarding experience of seeing students develop conceptual understanding, and have helped me develop into something near to the teacher I hope to become one day.

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Appendix A

Example of a Mobile Representation