Balancing Understanding and Skills Development in Mathematics Teaching: Giving Voice to Student Perceptions

by

Anneliese Reimer

A Thesis submitted to the Faculty of Graduate Studies of
The University of Manitoba
in partial fulfilment of the requirements of the degree of

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MASTER OF EDUCATION

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This has been a remarkable journey for me. Not only have I experienced tremendous personal and professional growth, but I have had the wonderful privilege of meeting and working with a great number of people who have enriched the whole experience.

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Anne Reimer
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Abstract

This research study, designed with the principles of phenomenography in mind, examines student perceptions of their learning experiences in two different grade 12 mathematics contexts in Manitoba: Applied and Pre-Calculus. Through synergetic focus groups, follow-up interviews, and interactive writing, twenty-one students reflected on their learning experiences in mathematics. Analysis of the data revealed three qualitatively different categories of description: 1) students' perceptions of the nature of mathematics, 2) their perceptions of the nature of learning, and 3) their attitudes towards the use of technology. Within each category of description, students depicted a wide range of experiences. For example, some saw math as being simply a static collection of equations and procedures while others recognized it as a dynamic interplay between the numerical and physical world. Perceptions of learning ranged from rote memorization and drill to hands-on exploration and making use of all tools available. Their attitudes to technology encompassed everything from seeing the use of a graphing calculator as cheating to whole-heartedly embracing it as a learning tool.

Interpretation of the data centered on constructing meaning as a set of overarching themes that spanned the categories of description and provided a starting point for considering how the data could inform planning and instruction in the classroom. Initially, three themes seemed important to the students. They saw considerable value in having the choice between two different math courses. They stressed that continuity and connections among mathematical concepts were crucial to their learning. They underscored the complementary nature of the two courses and the potential benefit of taking the two courses in combination. A fourth theme centered on the issue of communication between student and educator, and became apparent only when the focus of interpretation shifted from what students were saying to what they were not saying. Students are important stakeholders in their own education, yet they remain unaware that their voices could play a vital part in the educational process. Their perspective is largely missing from the current movement in/towards curriculum reform. But as demonstrated in this study, their voices have the potential to offer an important perspective in finding balance between conceptual understanding and skill development. It is up to educators to find ways of opening pathways of communication so that their voices may be heard.

Table of Contents

Figures and Tables

Introduction

What is mathematics? Is it a mysterious code of symbols accessible only to a few or is it a symphony of ideas whose nuances continuously give delight to those who know how to look for them? What is the study of mathematics? Is it the accumulation of knowledge of theorems and algorithms and proofs, or is it the recognition of patterns and the explorations of interconnectedness among concepts? What is the purpose of studying mathematics? Is it to get a credit for graduation or is it to gain facility with the tools of examination, analysis, interpretation and communication? What is the best way to study mathematics? Is it through drills and practice and memorization, or is it through investigating and hypothesizing and working together with others?

As teachers, we are in the daily position of being able to share our love of mathematics or to turn students off the subject completely. We are in a position to fill empty heads with our expertise, or cause students to construct their own knowledge. We can be talking at the students or we can be learning with them. Our students spend time with us collectively, yet each comes away with a unique learning experience in our classrooms. And that learning experience may or may not have resulted in the student becoming a mathematical thinker, in becoming a person who can enter the "real world" with confidence in his or her problem solving ability, or in becoming someone who appreciates the importance and the beauty of mathematics. Descriptions of those learning experiences can go a long way to providing a student perspective on the questions posed above.

I believe student perspective to be important in dealing with some key questions in the teaching and learning of mathematics. As I completed the research study upon which this thesis is based, I collected, analyzed and interpreted the descriptions of students' experiences as they reflected on their journey through school mathematics, particularly Senior 4 mathematics. This thesis, then, will provide a glimpse into those journeys, and allow at least one group of students to give voice to what they think mathematics and learning are all about.

Chapter 1

Looking for Balance

Setting the Stage

I came into teaching mathematics by accident – when I started my teaching career, jobs were scarce; beginning teachers accepted any position, regardless of their content-area qualifications. They often struggled to review (or learn) unfamiliar content the evening before they implemented the lesson in their classroom – and this is exactly what I did when I accepted a job teaching high school mathematics. That had not been my area of specialty in my university studies, but over the course of many years I have come to see this move into teaching mathematics as a happy accident. My journey as a mathematics teacher has been one of emerging knowledge both mathematical and pedagogical, of discovery and joint learning with students, of trial and error, of collaboration with colleagues, sometimes of frustration in attempting to reconcile long-standing beliefs or practices with reform curricula, and great pleasure and sense of accomplishment at seeing students engage wholeheartedly with mathematics.

My learning and early teaching history is one of traditional instruction where mathematics was seen as a static set of formulas and procedures to be memorized and practiced. Since then, however, I have gradually come to view the learning and teaching of mathematics in a very different light. No longer are students seen simply as receptacles into which I can pour my expert knowledge in the hope that they can recall the information later on demand. Rather, my philosophy of education has evolved, and continues to evolve, in a way that incorporates constructivist principles. I believe that all students need to be active participants in their own learning. Individual students must interact with a concept and construct their own meaning of that concept in relation to the world in which they live. Learning takes place within an active social environment, and understanding must underpin procedural knowledge. As a teacher, I am no longer the content expert in the classroom; instead, I am a planner of activities designed to provide the learning experiences within which each individual can construct mathematical knowledge. I am a facilitator of learning who guides and challenges and questions. I am a colearner with the students as we investigate mathematical problems and engage in mathematical thinking together.

While constructivism holds much promise and has been implemented successfully in many English and Social Studies classes, the reconciliation of experience-based learning and individual meaning-making with the need for developing a good grasp of content knowledge and symbolic language is a great dilemma in mathematics classes. Formulas, theorems, symbol manipulation, procedural skills – all remain important to continue one's study in mathematics. Yet for me, the decontextualized practice and memorization that is sometimes required to develop good facility with these content skills seems at odds with constructivist learning; that has been my greatest challenge in adopting and implementing reform curricula. Somewhere there must exist a balance between focusing on inquiry-based learning and on learning that targets acquisition of content and skill development. What is it that is important for students to know with respect to mathematical content? What skills form the basic skill set students will need for further study, and how much emphasis do I place on formula knowledge? How then do I balance that with my desire to have students truly understand the mathematics they are doing, to recognize the patterns they are working with, to develop a good set of problem solving and analytical skills, to become mathematical thinkers?

Researchers, educators and curriculum designers have long been concerned with finding a balance between procedural and conceptual learning. Not surprisingly, Huntley et al. (2000), in comparing student performance in a traditional mathematics curriculum and a reform-based curriculum, found that students will typically learn what they are taught. Depending on the focus of instruction, they will likely develop proficiencies in specific areas, yet be left with shortcomings in others. For example, if the focus is on symbol manipulation, students will become skilled in the procedures needed to solve equations, but may have difficulty in applying these skills to a contextual problem. If the focus is on reasoning with multiple representations, students will be able to move comfortably among equations, graphs or tables to solve problems, yet they may have difficulty in following through to determine an exact algebraic solution. According to Huntley,

> the heart of the controversy is almost always the balance between conceptual and procedural knowledge in algebra. Proponents of change argue that students need not acquire as much symbolic-calculation skill as formerly; opponents of change

> argue that automaticity of such skills is essential to problem solving and further mathematical learning. (2000, pp. 356-357)

It is apparent that consensus among educators has not yet been reached; indeed, many educational professionals and researchers continue to examine the question of balance from a variety of perspectives. But it seemed to me as I reviewed the literature that one crucial perspective was missing – no one seemed to be asking what the *students* thought.

My own journey towards finding a balance between conceptual understanding and procedural skill has included things such as studying the curriculum documents, carefully perusing support material and research findings, participating in inservices, discussing ideas with colleagues, experimenting with different ways of teaching various topics, and listening to students as they work on mathematical problems or activities. And while I have begun to develop a sense of what 'works' in my classes and what I am comfortable with in regards to my own beliefs about learning mathematics, I remain keenly aware that this is an ever-shifting balance as I work with each new group of students. For if I believe, according to the theory of constructivism, that each student must construct knowledge for himself or herself, how best to facilitate that meaning-making will depend on the individual student. And unless I know how that student makes sense of mathematics, providing the best possible inquiry experiences to ensure understanding of the material remains but a reflection of what I think learning is, and that may or may not coincide with what the student needs. If I am going to be a co-participant in the endeavour of learning mathematics, in keeping with constructivist principles, I need to be listening to the voices of the learners to help determine where the balance should lie.

Thus students have an important role to play in this search for balance. Although many may not be able to articulate it with complete clarity, they already have a sense of how they learn and what habits of mind they have developed. They can specify, for example, whether they look for connections, memorize formulas or catch on more easily through hands-on activities. They can say whether it usually takes five practice questions or twenty-five before they feel confident about understanding a procedure. They can speculate on what type of problem solvers they are. They know whether or not a particular mathematical concept has direct relevance to their lives, and whether they can figure out a way of constructing personal meaning for that concept. They have a sense of their own attitudes toward learning and know their own rationale for studying mathematics. Their perspective is one that seems to be missing in the search for balance, yet theirs are voices that need to be listened to.

As will be detailed later, most of the mathematics students I teach are taking both Applied Mathematics and Pre-Calculus Mathematics (hereafter simply referred to as Applied and Pre-Calculus) in the same year. They are learners in two quite different environments: Pre-Calculus in our school is fairly traditional, focusing on skill development, whereas Applied is more inquiry-based, focusing on discovery and application. It has been my sense as a teacher for some time that the two courses complement each other, with Pre-Calculus providing the theoretical basis and Applied amplifying the conceptual understanding. Students who take both seem to have a better grasp of mathematics than do students who take only one or the other; they seem to be achieving a good balance between procedural knowledge and conceptual understanding, making their experience of learning mathematics a richer one. But how does my perception align with the perceptions of the students taking those math courses? Do the students themselves think that taking both courses provides them with a better learning experience? Do they perceive a connection between the courses that is different from my understanding of it? Or for that matter, do they see the two as being quite separate entities?

As I considered different research topics for this thesis, I saw a remarkable opportunity to learn from these students, to hear from them the missing perspective on what is important to them in learning mathematics. I believe that students in my classes each have their own way of thinking about mathematics and the way they learn it. They would be able to shed light on the importance of algorithms and procedural skill development, and on inquiry to develop understanding, in a way that was possibly different from the way that teachers and researchers understand it. Their recounting of their experiences had the potential to illuminate a whole breadth of ways of understanding and doing mathematics that could serve to inform educational practice. For the more we understand about the variety of ways in which students experience mathematics, the better we will be able to structure learning experiences that will advance their development as mathematical thinkers. The better we understand the scope of the variations, the more effectively we can tailor activities and assignments to be of value to all students in our classes.

Context of the Study

I teach in a small community near Winnipeg. The school comprises about 250 students from Grade 7 through Senior 4. Despite its small size, it has a strong tradition of offering a full slate of academic courses in the higher grades, including Physics, Chemistry, Biology, Mathematics, English and History. In the past, students had always had the option of choosing between two math courses: Mathematics 40S and Mathematics 40G. In the mid-1990s, Manitoba Education and Youth introduced a new triple-stream mathematics curriculum comprising Consumer, Applied, and Pre-Calculus math for the Senior 2 through Senior 4 levels. Students could now choose among three math courses at each level instead of only two – providing schools chose to offer all courses. Given our small size, it was questionable whether or not our school could sustain all three courses at each of the Senior 2 through Senior 4 levels. Each grade level only had an average of 40 students, and assuming an equal division of students among the three courses, that meant a mean class size of only 13 students. An even more likely scenario, based on the academic tradition long established in the community, was that a considerable majority of students starting Senior 2 would want to 'keep their options open' and register for Pre-Calculus, a smaller number of 'weaker' students would opt for Consumer, leaving but a tiny handful who might consider exploring what Applied had to offer. With natural attrition, that tiny handful could dwindle to almost nothing by Senior 4. And patterns of student enrollment were not the only issues facing the administration. Adding to the list of complications were the logistical difficulties presented by introducing three new courses into an already full timetable, with no extra teacher time having been granted. Indeed, the viability of the entire Applied math option was in serious question. Some schools in the division, faced with similar issues, made the decision not to introduce the Applied course. But that was not an option for the principal in this school. As he reflected on his thinking during that period, he wrote, "I wished for our students to have every opportunity they should have and not be disadvantaged by attending our school. I also could not ignore the fact that the community colleges were beginning to add Applied math as a prerequisite for a list of courses they offered.[1]"

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[1] In order to provide the context within which the student reflections could be examined, I requested a written reflection from the former principal in which he outlined his rationale for implementing the Applied math course in the way he did. Details of this data collection process are provided in Chapter 3.

To that end, the principal devised a strategy that he hoped would foster a successful implementation of the Applied math course. He set about structuring a timetable that would steer the higher-achieving Senior 1 students into the new course by ensuring a "lack of any real compelling options to Applied Mathematics". He poured considerable financial resources into providing equipment and technology to the course, and then went to great lengths in promoting the Applied course by strongly encouraging students to consider Applied and Pre-Calculus as a package when they registered for their courses. He did this for several years in a row and the practice of dual registration seemed to catch on. After a while, the encouragement to register for both courses in Senior 2 came not only from teachers and administration, but from older students as well. Currently, many Senior 2 students who take Pre-Calculus at the school also take Applied, and vice versa; in other words, there is a relatively high degree of overlap in registration between the two courses. By the Senior 4 level, the degree of overlap has usually decreased, but still remains well over 50%. For example, in the 2002/2003 school year, 62.5% of students in Applied 40S were also registered in Pre-Calculus 40S, whereas 100% of students in Pre-Calculus were also registered in Applied (see Figure 1 below).

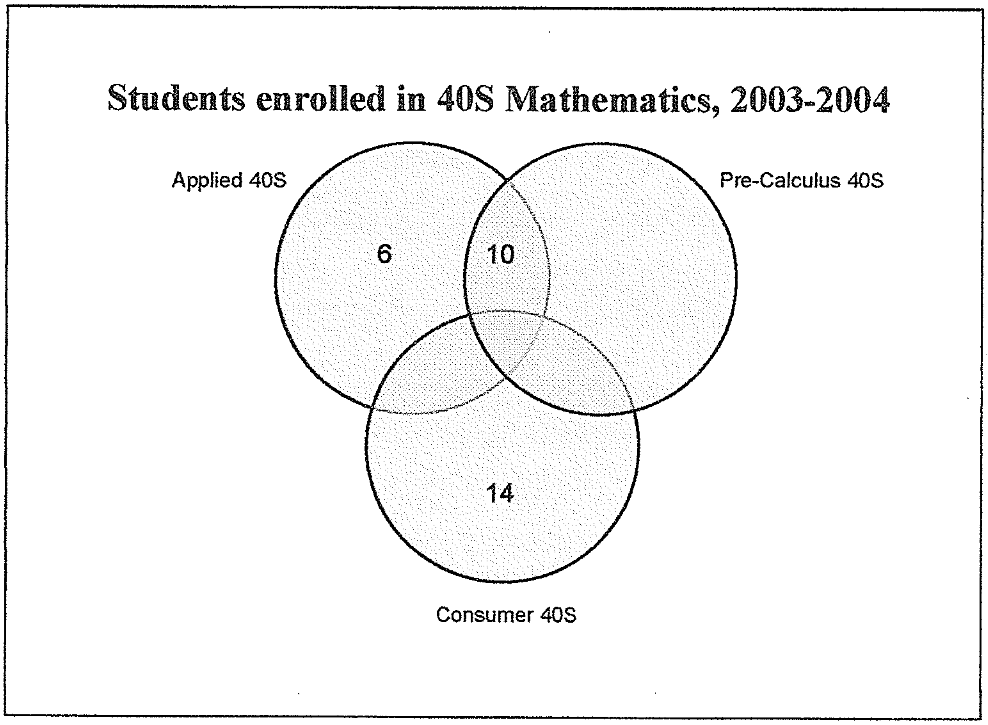

Figure 1: Overall 40S Math Enrollment, 2003/04

In our school, I taught all the Applied courses, and another teacher taught all the Pre-Calculus classes. The two courses have considerable content overlap. Many units cover the same material, but do so in very different ways, as will be detailed in the next chapter. Students respond to these distinctive approaches in different ways as well. As I have taught Applied, I have heard many different comments from students, including statements such as "I learn better hands-on" and "why can't you just give us the formula" and "those who didn't take Applied are having a really hard time [in Pre-Calculus]" and "so that's where the 4/3 comes from in the volume formula". I have noticed that each individual adopts a personal stance towards learning that is unique, and that each comes away from a lesson with an experience that is different from that of everyone else in the class. Collectively, students have a wide range of experiences as they go through one or both courses. These experiences may or may not coincide with the intended purposes and instructional practices envisioned by their teachers. It seems to me, then, that if we wish to increase our effectiveness as teachers, the students' interpretations of these varied experiences are worth listening to as we contemplate what is important in the teaching and learning of mathematics.

The Goal of the Study

It is my belief that good mathematics teaching and learning rests on achieving a balance between strong conceptual and contextual understanding and a good grounding in theory and procedural skill. In Manitoba, the former seems to be emphasized more in Applied; the latter is emphasized more in Pre-Calculus. As stated previously, it has been my perception as a teacher that students who take both courses develop better mathematical understanding than those who take only one, and so I continue to encourage my students to take both courses. However, my perception is only one part of the picture; I think leaving the students out of the discussion about learning is overlooking a very important contribution. Their descriptions of their learning experiences in the individual courses as well as the combination of the two can serve to illuminate how and what students are learning in mathematics classrooms. Their individual stories can be combined to start painting a picture of what mathematical learning looks like for them and what conceptions of the nature of mathematics they are forming. This picture, once it has been painted, can help to position the balance between conceptual understanding and theoretical and procedural competence. It can be shared with others, so that the broader community of mathematics educators can consider how the range of experiences among their own students might serve to inform their practice and their beliefs.

In order to hear and foster voice in my students, then, I undertook a phenomenographic study that explored and detailed the range of experiences as described by Senior 4 students as they considered their learning experiences in Pre-Calculus and Applied. The two-part question that framed my research intentions and guided the development of phenomenographic categories of descriptions was: **1) What perceptions do students form of their mathematical learning in Pre-Calculus Mathematics 40S and Applied Mathematics 40S, and 2) What do students perceive as the nature of the interrelationship between the two courses?** A third question developed out of the inquiry: **3) What are the implications of these student perceptions for developing and teaching high school mathematics courses?**

My intention in this inquiry was to listen to students as they considered and described their learning experiences. I examined those descriptions through a variety of lenses and developed three categories of description that detail the range of experiences this group of students had in learning mathematics. From there, I synthesized a set of overarching themes that highlight some of the implications student voice might have on teaching and course development in high school mathematics.

As I pondered how I would go about the study, I likened the process to an artist painting a picture. The canvas is first prepared by painting a background. The background provides the context for the object of the picture and brings that object into sharper focus. Next, the focal image is painted, with careful attention being paid to the individual details, to how one part of the painting relates to another, and to how the picture fits together as a whole. When the painting is complete, others may examine and study the painting, each person interacting with it in a unique way, each recognizing subtle nuances or coming away with general impressions that may be different from the way in which the artist intended or in which another person saw it. Regardless of how the picture is viewed, each person who sees it has an opportunity to pause and reflect on the image and its personal message to him or her.

The picture in this study, an image of student perceptions of their mathematical learning, has indeed been painted by me and as such, will depict that which I considered to be prominent and will leave in the shadows that which I thought to be of minor consequence. Yet even though the picture is painted through my eyes, I trust it will provide other educators with an opportunity to glimpse mathematics education reform from a different perspective: that of the students.

Chapter 2

Mathematics Reform

Imagine a classroom, a school, or a school district where all students have access to high-quality, engaging mathematics instruction. There are ambitious expectations for all, with accommodation for those who need it. Knowledgeable teachers have adequate resources to support their work and are continually growing as professionals. The curriculum is mathematically rich, offering students opportunities to learn important mathematical concepts and procedures with understanding. Technology is an essential component of the environment. Students confidently engage in complex mathematical tasks chosen carefully by teachers. They draw on knowledge from a wide variety of mathematical topics, sometimes approaching the same problem from different mathematical perspectives or representing the mathematics in different ways until they find methods that enable them to make progress. Teachers help students make, refine, and explore conjectures on the basis of evidence and use a variety of reasoning and proof techniques to confirm or disprove those conjectures. Students are flexible and resourceful problem solvers. Alone or in groups and with access to technology, they work productively and reflectively, with the skilled guidance of their teachers. Orally and in writing, students communicate their ideas and results effectively. They value mathematics and engage actively in learning it (National Council of Teachers of Mathematics, 2000, p. 3).

The Promise of Reform

Such is the vision that has been articulated by the National Council of Teachers of Mathematics (NCTM), the largest organization of mathematics teachers in North America. NCTM has been a key player in advocating reform, starting with its pivotal *Curriculum and Evaluation Standards for School Mathematics,* first published in 1989, and following with numerous other publications aimed at realizing the dream of transforming mathematics education. The quote above, taken from the opening pages of *Principles and Standards from School Mathematics,* depicts a contemporary representation of what mathematics reform looks like and the image of education that might result from it. A solid grounding in procedural skill must be accompanied by an emphasis on understanding.

Curriculum reform has been a long time in the making, and has oscillated between applied or contextual mathematics and pure mathematics a number of times over several generations. Usiskin (1997) tracked the movement between curricular aims and content in the

United States over the last half century, beginning with the space race in the 1960s and its resulting emphasis on pure mathematics ("new math") to produce highly trained people in mathematics, science and engineering. The 1970s saw a backlash of the "back-to-basics" movement that sought to address declining standardized test scores. By the mid 1980s, the development of applications-based mathematics curricula had started and still continues. Many educators today echo the ideals summarized in the opening quote to this chapter: that mathematics must be useful and grounded in applications (Usiskin, 1980), it must emphasize understanding over rote memorization and procedural skill (Hirschhorn et al., 1995), it must incorporate technology fully (Burrill, 1999; Waits & Demana, 1996), and above all, it must produce students who are numerate and possess a high degree of quantitative, interpretative and technical communication skills (National Research Council, 1990; Steen, 1999).

Another element of the current reforms in mathematics education is that it must be accessible to *all* students (NCTM, 2000: Price, 1995). At first, many may have interpreted this as a call for one unique curriculum for all students, but accessibility need not necessarily mean the *same* mathematics for all (Usiskin, 1997). For example, Noddings (2000) maintains that although many occupations are built on mathematical foundations, workers themselves need very little mathematics for their routine tasks. Instead, she envisions a truly useful high school mathematics course as one which includes, in addition to practical mathematical skills, some discussions along political dimensions: "what it means to live in a mathematicized world,...the difference between knowing a subject and having a credential,...how students contribute to their own lower economic status by unreflective resistance to mathematics courses, and what intelligent resistance might look like" (p. 2). Steen (1999), while acknowledging the need for some individuals to become highly trained in mathematics, points out that this is an ill-advised focus for the mathematics most of us need in order to understand the world in which we live:

> [D]espite widespread evidence that numeracy is more than mathematics and that practical wisdom is not the same as classroom learning, anxious parents and politicians push students into the narrow gorge of early algebra and high school calculus in the misguided belief that these courses provide the quantitative skills appropriate for educated citizens...Numeracy, not calculus, is the key to understanding our data-drenched society (p. 9).

I agree with the writers quoted above that it is important to recognize that not everyone needs the same level of mathematics, and that to require that all students demonstrate competence and understanding at the highest level is doing them a disservice. Instead, students need to learn mathematics that is relevant and meaningful for them, and to develop problem solving skills that will serve them well in many diverse situations. In other words, students must have the opportunities to learn *different* mathematics.

As stated earlier, Manitoba introduced new mathematics curricula in the 1990s. In creating three separate and distinct courses, the curriculum authors were able to encompass the vision of mathematics for all, but *enact* it as different mathematics for all. All three courses, Consumer, Applied, and Pre-Calculus Mathematics, are designed with sufficient rigour that a student may pursue post-secondary education with a firm mathematical foundation. An online search of Manitoba universities, for example, showed that successful completion of any of the three courses would qualify a student for general admission at any of the institutions (albeit with some qualifications). Pre-Calculus continues to be a prerequisite for mathematics-intense faculties such as Engineering, Medicine, and Commerce (U of M website, retrieved August 15, 2003). In contrast, however, the view of different mathematics as enabling students to develop strong competencies and conceptions seems to be far less accepted beyond Manitoba. Other Canadian universities, such as the University of Toronto (U of T website, retrieved August 15, 2003) or the University of Alberta (U of A website, retrieved August 15, 2003), still recognize only the equivalent of the Pre-Calculus course as qualifying for admission. But attending university is not, nor should it be, a goal for all students. The fact that a student doesn't aspire to a profession requiring a university degree should never result in that student remaining innumerate, it should never exclude them from receiving a strong foundation in mathematics. The courses now offered in Manitoba are a solid attempt at ensuring that strong mathematical learning for all, regardless of their educational objectives.

Thus students can choose among the three courses on the basis of their own goals, perceived learning abilities and interests, and be assured that they will come away from any one of them with some mathematical learning. But by choosing one over another, are they shortchanging themselves in any way? Do all the courses enact the vision of the *Standards* equally, providing rich and engaging opportunities to explore and construct mathematical knowledge in a meaningful way? Or do some of the courses focus on specific areas to the detriment of other important mathematical ideas and skills?

Examining how the Consumer 40S course fits into the picture is beyond the scope of this study, but in the next section, I will explore the Applied 40S (Manitoba Education and Training (MET), 2000) and Pre-Calculus 40S (MET, 2000) curriculum documents in light of how they seek to incorporate ideas of mathematics reform, comparing and contrasting them with respect to their stated rationale and general or specific learning outcomes.

The Implementation of Reform

Reviewing the Rationales

It becomes clear when examining the rationale given in each of the curriculum documents that both curricula rest on the same reform underpinnings, for both cite the *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) as a major impetus for the direction of the course. And indeed, as shown in Table 1 below, the mathematical foundations that are outlined in each rationale mirror those that frame some of the current NCTM standards as outlined in *Principles and Standards for School Mathematics* (2000):

NCTM Principles and Standards	Pre-Calculus Mathematics 40S	Applied Mathematics 40S
Problem Solving	Problem solving	Problem solving
Reasoning and Proof	Mathematical reasoning	
Communication	Communication	Technical communication
Connections	Connections	Applications and connections
Representation	Visualization	
Technology	Technology	Using information technology
	Mental mathematics and estimation	

Table 1: Comparison of Mathematical Standards and Foundations

It is clear that problem solving is a cornerstone of each curriculum, with close attention being paid to the importance of content applications, connections to the real world and the incorporation of technology. Communication is considered a key skill, particularly as a facilitator of conceptual understanding.

Students' active participation in each course is a vital component in the respective rationales. Compare, for example, statements like "...doing mathematics rather than just knowing mathematics" (p. 4) and "active student engagement in meaningful mathematical tasks" (p. 11) from the Pre-Calculus rationale with statements like "completing hands-on investigations and discussing interesting questions" (p. 3) and "students are encouraged to be responsible for their own learning" (p. 8) from the Applied rationale. In both cases, attention is drawn away from the traditional paper-and-pencil tasks to a dynamic involvement on the part of the student. An examination of the verbs used can also highlight the type of activity that is expected of students. Pre-Calculus uses words such as *examine*, *represent*, *transform*, *solve*, *apply* and *explore*. Applied supplements these with words such as *hypothesize*, *experiment*, *measure*, *analyze*, *assess*, *discuss*, *write*, *explain* and *justify*. All are active verbs that require thinking, planning, noticing and enacting; activities which all contribute to the construction of knowledge and understanding. If the courses are implemented according to the promise in their rationales, students cannot simply sit back and absorb – they are required to be engaged participants.

Reviewing the Introductions to the Units

Just as the verbs in the rationales describe active student engagement, so do the verbs of the unit introductions. For example, consider some of the verbs and phrases used in the respective introductions to the units dealing with sinusoidal functions and sequences in Table 2 below.

Pre-Calculus Mathematics 40S	Applied Mathematics 40S
Distinguish	Relate
Draw and sketch	Do experiments
Analyze	Collect data
Investigate	Use technology
Connect	
Solve	

Table 2: Verbs and Phrases in the Curriculum Document Introductions

Once again, according to the curriculum documents, students are expected to be completely involved in their learning, to use opportunities to explore mathematical ideas, to make connections to real world situations, to incorporate technology, and to use multiple representations. It is conceivable, from the statements made in the Pre-Calculus and Applied curriculum documents, that Manitoba has been successful in implementing reform in the two different mathematics curricula. Students are meant to move beyond simply developing procedural competence to actively probing mathematical problems from a variety of perspectives. Yet in practice (at least in my school), this does not seem to be the case.

A Promise Not Yet Met

The promise of reform-based mathematics education is indeed apparent in the respective rationales and introductions to the curriculum documents. The Pre-Calculus curriculum writers maintain "that mathematical competency involves more than mastering a collection of skills and concepts" and "instructional settings and strategies [should] create a climate reflecting the constructive, active view of the learning process" (MET, 2000, p. 4), yet the curriculum remains remarkably decontextualized and delivery often traditional. Skill development continues to take precedence.

To help portray what classes were like, I asked four teachers by way of questionnaire[2] to describe general strategies, instructional techniques, and types of assessments they would use in their Pre-Calculus classes. One teacher described a typical class as one where she would write "notes on the overhead, first stating the skill or concept, then providing concise steps for completing that skill, and then a variety of examples that are typical to the types of questions that use the skill or concept". Technology was used by some teachers, but others simply did not allow graphing calculators to be used, insisting that everything be done using pencil and paper. Context and application were a low priority; one teacher stated that "very little real life application is given to the problems. The use of algebra is stressed over the answer obtained." Assessment was mostly "pencil and paper" assignments, quizzes and tests.

While some of the teachers indicated in their responses that they are trying hard to incorporate at least some of the reform focus on conceptual understanding into their Pre-Calculus courses, they find it very difficult to do so. Course content is structured from a theoretical perspective and stresses the development of formulas, theorems and algorithms. Symbol manipulation and procedural skill are considered essential components. Technology, if used at all, is used primarily in a supportive role. In fact, the Senior 4 Pre-Calculus Mathematics Standards Test prohibits students from using the graphing calculator or computer for two-thirds

[2] Quotes in this section are from the teachers' responses to the questionnaires. Details of how the teacher data was collected are provided in Chapter 3.

of the exam. Textbooks and other curricular support documents are arranged in a typical way: a short introduction to a problem, the development of a formula to deal with the problem or a proof to substantiate a mathematical statement, and a few examples that show how to implement the formula or how the proof applies. The lesson is followed by several pages of practice questions, arranged to start with dozens of context-free exercises that provide students with an opportunity to practice the procedure under review, and closing with a handful of contextual application-type problems or challenging procedural questions. So although the Pre-Calculus curriculum calls for "new forms of classroom organization, communication patterns, and instructional strategies" (MET, 2000, p. 5), a typical lesson, as illustrated through the questionnaires, often remains teacher centered and lecture driven.

Applied Mathematics takes quite a different approach, and as such, realizes more fully the conceptual component of the reform ideal. The focus is on inquiry, on situating mathematics in a real-world context, and on integrating technology as fully as possible to facilitate understanding of mathematical concepts.

> The emphasis is on collaborative explorations, tolerance toward alternative solutions, probable inferences, and the testing of speculations…Every effort should be made to ensure relevance through the use of practical, applied problem solving and to decrease the use of drill exercises and the traditional memorization of formulas, algorithms and theorems." (MET, 2000, p. 3)

When I asked two Applied teachers about their general strategies, instructional techniques and types of assessment, they stressed the extensive role that technology played, both in the presentation of new material, and in how the students worked with problems and were assessed. Problems were situated within a context. For example, one teacher wrote of sine functions that students "may need to determine all or parts of the equation, but this is done mostly through real life common sense determination (i.e. the crest of a wave is 10 feet above the trough, therefore a=5)".

Textbook and curricular materials provide many investigations and activities in which students are expected to explore the mathematics of a situation, to look for patterns and relationships, to develop multiple representations, alternate strategies and ways of looking at problems. Procedural skill and symbol manipulation, while considered important, are not a primary focus, and are dealt with on an 'as-needed' basis. Graphing calculators and computer software are always available and are primary investigative and problem solving tools. Both are fully available during the Senior 4 Applied Mathematics Standards Test. A typical lesson in Applied begins with an inquiry activity. This activity invites groups of students to explore a mathematical concept through data collection, manipulatives or technology in order to support conceptual understanding. These groups work through the activity and consider several questions designed to get them thinking about the mathematics involved in the situation. Class discussions then serve to elicit and expand their understanding of the material. Follow-up problems are almost always contextual in nature. In addition to regular classroom activities, students in Applied Math are often required to complete unit projects, where they apply the major mathematical concepts learned in that unit to a real-life context.

It is clear, based on the descriptions of even just a few teachers, that the two courses are approached very differently. But in practice, neither course seems to truly realize the ideal of reform – that students will come away with skills *and* understanding, not just one *or* the other. So why, with the curriculum documents themselves both showing such great promise of a departure from rote paper-and-pencil tasks and a redirection towards constructivist learning, do the two courses continue to diverge so significantly, so much so that Pre-Calculus remains very much a skills-oriented course while Applied focuses almost too much on conceptual ideas?

The Experience of Reform

One possible reason for the divergence of the two courses may be in the way teachers view the purposes. Whatever the curriculum rationales may state, teachers have their own interpretation of why a student is or should be taking a particular course. The teachers I asked saw the Pre-Calculus course as a gatekeeper to university studies (although one teacher expressed grave misgivings about this function and sought to align her own sense of purpose more with the student goals as stated within the curriculum document). In order to prepare them for university, teachers would cover a vast amount of material at a fast pace. Students would need to be "gifted" or be "high achievers" in order to pass the course, willing to spend a lot of time on their own, completing homework assignments in order to "fully comprehend the material". There was little time for the 'luxury' of inquiry and experimentation, so the more traditional teacher-centered, lecture-based approach prevails.

Applied, on the other hand, was not considered to be preparatory to university studies and as such was seen to have a "more comfortable pace". It was therefore possible to "provide the student with some real life applications" and to "give them confidence in using technology to solve unique problems they will encounter in their everyday life". While the pressure remains to cover material for the provincial standards test, teachers are able to spend more time with inquiry and experimentation, and hence implement the reform strategies aimed at conceptual understanding more easily or more fully. Interestingly, one teacher suggested that one of the biggest hindrances to the inquiry approach was the students themselves. Many were "too lazy or unwilling" to "'play' with the math", and remained too focused on the more traditional 'finding the answer' as the sole purpose of a math problem.

I believe another reason for the divergence in the courses lies in the prescribed learning outcomes as defined for the individual units of each course. While the rationales prefacing the curriculum documents may have been similar, the specific learning outcomes suggest different actions. Consider, for example, Table 3 below which shows the learning outcomes as specified for the unit incorporating a study of sequences in the respective courses, and notice the highlighted verbs (bold emphasis mine).

Pre-Calculus 40S Prescribed Learning Outcomes	Applied 40S Prescribed Learning Outcomes
Derive and apply expressions to represent general terms for geometric growth ❖ **Distinguish** between arithmetic and geometric sequences ❖ **Generate** the first three terms of a geometric sequence ❖ **Write an exponential function** if the geometric sequence is given ❖ **Examine** a geometric sequence **recursively** ❖ **Derive the general term** of a geometric sequence ❖ **Calculate a stated term** in the sequence without expanding *Solve problems involving finite geometric series* ❖ **Distinguish** between a geometric sequence and a geometric series ❖ **Find the sum** of a geometric series ❖ **Develop a formula** for the sum of a finite geometric series *Apply infinite geometric processes to solve problems* ❖ **Find the sum** of an infinite geometric series	*Use technology to generate and graph sequences that model real-life phenomena* ❖ Introduce the concept of sequences ○ **Explore** the decreasing bounce height of a bouncing ball ○ **Explore** the sum of angles in a sequence of triangles that share a common side with the previous triangle ○ **Explore** the effect of doubling one's allowance repeatedly ❖ **Use spreadsheets to solve problems** involving sequences *Use technology to construct a fractal pattern by repeatedly applying a procedure to a geometric figure.* ❖ **Follow directions** to create a fractal pattern. *Use the concept of self-similarity to compare and/or predict the perimeters, areas, and volumes of fractal patterns.*

Table 3: Prescribed Learning Outcomes

Teachers typically focus on the prescribed learning outcomes and so the actions implicit in these outcomes are often reflected in the learning activities that are planned and implemented. As discussed in the previous section, the verbs and phrases in the rationales and introductions of both Pre-Calculus and Applied were action words, but there is a marked divergence between the two curricula when one examines verbs and phrases used in the prescribed learning outcomes and suggestions for instruction. The Pre-Calculus outcomes often return to more traditional theoretical and procedural skills, centering on answer generation, symbol manipulation, formula derivation and development of algorithms while the Applied outcomes for the same unit emphasize hands-on exploration, data collection and the use of technology. Indeed, the examples that illustrate each learning outcome in the curriculum documents further lead teachers and lesson planners along divergent paths between the two courses. Examples in Pre-Calculus are almost exclusively decontextualized, focus on derivations and manipulations of formulas and show step-wise procedures for solving problems. Examples in Applied provide many application-type problems, detailed investigations and connections to a wide variety of disciplines. Even apart from the contribution that an individual teacher might make in the implementation of the different curricula, Pre-Calculus is already skewed far more towards the traditional model of mathematics teaching and learning than is Applied.

Where are the Students?

Since the publication of the NCTM Standards in 1989, many studies have examined the impact of new reform curricula on students (e.g. Berger, 1998; Graham & Thomas, 2000; Huntley et al., 2000; Tate, 1997; Wilson, 1994). These studies have found that when the reforms were successfully implemented, students typically achieved a greater degree of conceptual understanding in mathematics, often scoring as well as or better than students from traditional classrooms on standardized tests. Yet the key phrase in the preceding sentence is 'when successfully implemented'. The literature abounds with impediments to reform: for example, the order and time frame in which concepts are learned (Pesak, 2000), the lack of appropriate resources and curricular materials (Hirschhorn et al., 1995), the overcrowded nature of the curriculum (Usiskin, 1997), and the effects of high-stakes assessment (Passman, 2001). In particular, many articles examine the teacher's stance as a source of interference, including teachers' beliefs about teaching, learning, and mathematics (Edwards, 2000), inconsistencies between beliefs and practice (Gregg, 1995; Raymond, 1997), inadequate teacher preparation (Wilson, 1994), and differing perceptions between teachers and students (Nathan & Koedinger, 2000). I was able to find only study that examined the traditional versus reform issue from a high school student perspective. Based on the student conceptions that were revealed in her study, Szydlik (2000) concluded that neither a traditional nor a reform based approach to learning was sufficient in and of itself.

> The results of this work offer support for both views. Some students are not ready to hear proofs or arguments; they are not convinced that these are important, and they admit that they do not attend to them…Another group of students is frustrated when they are not provided with formal structure…Without the rigor, these students may be left powerless to make arguments (p. 274).

It is evident that just as there is great diversity among teachers and their stances towards reform, students react to differences in mathematical teaching and learning in a wide variety of ways. Their learning styles, attitudes and beliefs about the nature of mathematics can also profoundly affect the successful implementation of reform.

Manitoba is attempting to implement reform-based instruction and learning, but it is understandably floundering in light of the numerous hurdles to overcome. Yet we certainly cannot call the effort a failure, far from it. Recognizing that the balance between procedural and conceptual mathematical knowledge may be different for different people, it may even be reasonable to conjecture that Manitoba might, in fact, have found a feasible way in which students can pilot their own mathematical learning, through whichever course or combination of courses affords them the best understanding and fits best with their perceptions of the nature of mathematics or with their capabilities and future goals. One single mathematics reform curriculum may be an unattainable ideal, given the multitude of constraints facing reform efforts, but I believe Manitoba is making some big strides to realizing the spirit of reform. Mathematics for all is indeed a realistic target, but it is as a different mathematics for all, enacted through the three different math courses.

But where are the students in this plan? It seems to me that curriculum design and implementation has a tendency to be very top-down and driven by professionals. Is that not at odds with a student-centered philosophy? In a constructivist paradigm, the focus is on student learning rather than on teaching. So if the reform curricula are centered on students and their construction of knowledge, it should follow that their voices on how they construct that knowledge should also be heard. If we wish to illuminate the success or failure of our attempts at changing the way we teach and the opportunities we provide for learning, it becomes crucial to listen to the students, particularly with respect to their perceptions of learning in the different courses and the interrelationships among them. Students' descriptions of their experiences can lend an important missing perspective to where Manitoba stands with respect to mathematics reform. The next chapter details the methodology and the method I used to enable me to listen carefully as students described their mathematical learning experiences in Pre-Calculus and Applied.

Chapter 3

Gathering and Interpreting the Data

Teacher as Researcher

This chapter will detail the methodology and structure of the study upon which this thesis is based, but an important issue needs to be considered first: that of a practitioner being a researcher simultaneously. Can a teacher conduct an inquiry in her own classroom and still fulfill both her roles as a teacher and as a researcher? The Wilson-Wong debate (Wilson, 1995; Wong, 1995) argued this point in depth, with Wong maintaining that researcher and teacher are operating with two distinct purposes, one being to examine a classroom interaction with a critical eye and contribute to theoretical knowledge, and the other to transmit information to students through a variety of approaches and contribute to practical knowledge. According to Wong, the activity and intention of being a teacher were at odds with the activity and intention of being a researcher; in other words, these different purposes cannot be reconciled within a single person if the interests of the students are to be served. Wilson (1995), in rebuttal to Wong, contended that critical inquiry and teaching go hand in hand. One is not a teacher *or* a researcher, rather one is a teacher *and* researcher *in relation*. Both facets are, in fact, embodied within any teacher who reflects on her own teaching and continually seeks to improve it for the sake of her students' learning. Both as inquirer and as teacher, she is constantly asking questions, analyzing and looking for ways to enhance learning.

Lampert (2000) argues that the rise of qualitative research as an inquiry method "helped to open educational research to questions of meaning, perspective, ownership, and purpose" (p. 88). Research moved from being done *on* teachers to including inquiries conducted *by* teachers, and in so doing, provided valuable "insider knowledge", an important perspective of daily practice to the questions under consideration. She echoes Wilson's (1995) contention that teaching and inquiry are relational – they are not two separate entities. In describing a tradition of action research that developed in the United Kingdom, for example, Lampert refuses to countenance the label of teacher-researcher for those who produce and communicate knowledge of teaching, for "they do what they do as part of their everyday practice, accepting the study of teaching and the solving of its problems as a professional responsibility" (2000, p. 91).

As a teacher, I am not only implementing lesson plans and providing learning opportunities for the students in my classes, but I am continually engaged in reflection on how they are coming to know, on reviewing how the structured learning activities were or were not effective in helping students construct knowledge, and in assessing and interpreting this knowledge construction. Hence I am already acting both as a teacher and as an inquirer in my daily practice. This study simply formalizes something that I have been wondering about more and more; it asks questions that I am interested in both from a research perspective as well as a teaching perspective.

Just as student perspective is important to our thinking about curriculum reform, we cannot disregard student perspective of the practitioner/researcher relation, as outlined by Ainley (1999). She sees teaching and research as being complementary, but does find it useful to describe the two as being separate roles. She recognizes that there may be situations in which the roles may be in conflict, and where, in order to be effective, a person must position herself consciously in one role or another. Of particular interest to this study is her discussion of how students perceive these roles. Although a teacher may have a research purpose in posing a particular question, "[i]t is very likely that pupils will see a teacher's questions as…testing questions" (p. 46), and thus feel uncomfortable or confused when being asked to clarify responses. Ainley therefore underlines the importance of ensuring that students clearly understand when a person is being a teacher and when a person is being "not-a-teacher" (p. 47). This issue did not manifest itself within the context of this study – students seemed very aware of my dual roles, and were able to distinguish clearly between them.

Methodology: Phenomenography

Students each learn mathematics in a unique way, and the perceptions of their own learning vary from person to person. I believe that knowing the range of these perceptions can help to illuminate our consideration of the efforts at curriculum reform and the direction those efforts need to take in the future. The focus of this study, then, was on listening to students in order to establish a description of the breadth of their collective experiences and thus an understanding of the conceptualizations they had reached. It sought to examine the wide range of ways in which students perceived their mathematical learning. Phenomenography, a research methodology that concerns itself with an exploration of variation in described experience, was therefore a suitable methodology for an investigation of the variations of student perceptions of learning in two different mathematics courses. In this section I will discuss phenomenography as methodology, some of the issues surrounding it, and the adaptations I introduced to better suit this study.

Background

Phenomenography emerged as a research methodology in the 1970s, championed by Ference Marton and his colleagues at the University of Göteborg in Sweden. It has been used primarily in educational settings, although other disciplines occasionally refer to it as well. The term itself is derived from two Greek words: "phainemenon", meaning *appearance*, and "graphein", meaning *description*. Hence phenomenography is about the description of things as they appear to us (Marton & Pang, 1999). Marton stated that phenomenography is "an empirically based approach that aims to identify the qualitatively different ways in which different people experience, conceptualize, perceive and understand various kinds of phenomena" (as quoted in Richardson, 1999, p. 53). Learning becomes a central focus "because it represents a qualitative change from one conception concerning some particular aspect of reality to another." (Richardson, p. 53). Säljö describes phenomenography as generating "a picture of the variations in human conceptions of phenomena" (as quoted in Willmett, 2002, p. 5). More recently, phenomenographic research has begun to extend its focus beyond simple mapping variation in experience to identifying the structure of awareness underlying the varying experience of phenomena. (Åkerlind, 2002)

The phenomenon under investigation in this study was students' perceptions of their experiences in their study of high school mathematics. Naturally, it is impossible for researchers or teachers to know expressly the experiences or conceptualizations of their students, for there is no direct conduit to their thoughts and feelings. Instead, researchers must rely on what is generally known as a *second-order approach*, where "the emphasis [is] on experience that has been reflected on to the extent that it [can] be discussed and described by the experiencer" (Ashworth, 1998, p. 415). In a variety of ways, then, study participants would have to be willing to reflect on their experiences, to be prepared to share them with the researcher, explaining and re-explaining until the researcher was confident that she understood the nature of the experience. This second-order approach underlies data interpretation as well, because

> [i]t is not focused on the nature of the phenomenon, or the processes by which people develop these perceptions and conceptions, but rather it is focused on the discovery and description of the links constructed by people to describe their relationship to phenomena in the world around them (Willmett, 2002, p. 7).

The crucial idea is that data are not the actual experiences of the research participants, but rather the students' perception of their experiences, and thus the interpretation of the researcher is one of the *experiences as described.*

Phenomenography, then, is exploratory in nature. Its ultimate goal is to characterize variations in experience and to describe the architecture of this experience (Richardson, 1999). Phenomenographic research typically results in an *outcome space* which differentiates a limited number of qualitatively different ways of experiencing the phenomenon called *categories of description*, and may include the structural relationships between these different ways of experiencing. Svensson clarifies the categories of description:

> This form of the results [categories of description] means a favouring of abstraction, reduction and condensation in relation to the richness of the object (and data). The favouring of this form is based on the assumption about the objects that they have whole-characteristics which are representing the central meaning of the objects and the most important similarities and differences between objects and between conceptions" (1997, p. 167).

Thus categories of description do not comprise a seemingly endless list, rather they are a relatively small number of descriptors distilled from many, to feature the critical aspects of the variations in experience.

Phenomenography brings specific strengths to educational research, particularly with respect to its focus on student conceptualization:

> The interviews which provide the data are designed to encourage respondents to reflect on their own experience. As the analyses concentrate on interpreting the respondent's meaning, rather than on linguistic forms or pre-defined technical concepts, the analyses are readable and accessible to non-specialists. The interview extracts used to delimit the categories of description communicate [how teaching is affecting students] in a very direct way through the differing perspectives found among students. (Entwistle, 1997, p. 129).

As teachers, we strive to encourage the development of conceptual understanding in our students. A methodology such as phenomenography, with its vivid portrayal of the differing conceptualizations for the range of students in our mathematics classes, is therefore well suited to give us an image of whether (or how) this conceptual understanding is developing. Teachers discover what the students know, and thereby have a foundation for further development. And if we believe that knowledge is relational, we can begin to see the dynamics of knowledge construction as an interaction between the student, the mathematical learning material, and the learning environment.

Booth, in reviewing the 1988 Svensson and Högfors study, points to the specific benefits that accrue to students in a phenomenographic inquiry:

> Although incomplete, the students' awareness of his or her own conceptions together with some alternative conceptions seemed to liberate him or her in relation to the theoretical structure presented in the teaching. Students appeared to acquire the ability to investigate the structure taught from different viewpoints. (1997, pp. 140-141).

By engaging in reflective thinking and by interacting with others who have thought reflectively about their learning, students have the opportunity to expand their own ways of thinking about various phenomena, and thus enrich their learning experiences.

Bracketing

Rigour in phenomenography is achieved when the research participants' interpretations of their experiences are not permitted to become overshadowed with existing theories, personal researcher conceptions or commonly accepted views. In order to ensure that the perspective being described is that of the student, then, and not of the researcher, it becomes crucial that measures be taken to prevent intrusion of external conceptions on the descriptions given by students and that the individuality of each student's experience be recognized and respected. Here phenomenography utilizes a practice known as epoché or bracketing, a concept first developed by Edmond Husserl in the field of phenomenology, where the researcher deliberately identifies and then sets aside any theories, presuppositions, personal beliefs or foregone conclusions in order to focus on the lived experience of each participant. The practice of epoché is not a one-time only event – it must be continually implemented throughout the research process to remain true to the lived experiences of the individual students.

Ashworth and Lucas (1998, 2000) have written several articles that detail some of the things that need to be put out of consideration by the researcher in order to focus on the second-order reality of what they call the student life world. Several of them were directly relevant to my study.

1. *Bracketing presuppositions based on theories or earlier research findings*: Ashworth and Lucas (1998) stress that the analysis in phenomenography is a process of discovery, and as such, predetermined categories are counterintuitive. They maintain that the danger of familiarity with relevant research literature "is that the attention of the researcher is drawn to aspects of what students say which concord with previous research" (p. 421) and suggest that one may therefore need to limit one's review of prior research before embarking on data analysis. While I understood the validity of this form of bracketing, I believe that a researcher has a responsibility to be conversant with relevant research findings in order to be aware of the issues surrounding the area of investigation, and to properly situate the study with respect to the related literature. Therefore, rather than postponing a literature review, I found it important to be aware of what previous research findings were (see Chapter 2), but I made a conscious effort to set these findings aside in an effort to hear what the participants in *this* study were saying about their experiences in the mathematics classes in my school.
2. *Setting aside the tendency to construct hypotheses and prior constructs*: as one proceeds with the research, one needs to take care that elements from earlier stages do not preclude further descriptions in later stages. In other words, I needed to listen to what *each* individual student was saying and the conceptions that he or she was forming *before* I began to look at how the conceptions of many students fit together. But because I was also collecting independent data from multiple sources, there was a second dimension to this issue. As soon as I had collected the first set of data, I naturally began to form a sense of some possible categories. In order to ensure that these potential categories would not influence my understanding of the descriptions of the life world of the next set of participants, I had to deliberately set those aside while collecting the subsequent data sets. Yet they were never set aside completely, for they provided an opportunity to enlist the help of the later set of participants in clarifying or validating the emerging categories of description from the previous data sources.
3. *Bracketing assumptions from the investigator's personal knowledge and beliefs*: this particular set of bracketing was in some ways one of my greatest challenges, for I have a strong conviction that there is a meaningful connection between the two mathematics courses for students. However, if I was listening only for confirmation of my own beliefs

on the matter, I may have silenced the students or overlooked some crucial aspect of their conception of the courses. Ashworth and Lucas (1998) note "how easily one can be distracted by immediate reactions to student comments and the need to immediately make sense of them in terms of one's own understanding" and how important it is "to be able to recognize research participants' conceptualizations as interesting in their own right and worthy of careful explication" (p. 422), even if it does not correspond with researcher beliefs or with textbook conceptualizations. Therefore I meticulously examined my own beliefs ahead of time. I tried to be vigilant and to listen unconditionally to students. I took careful note of the times when student statements caught me by surprise, and spent considerable time later comparing what they had said with what I believed. As various themes emerged, I always tested them against the data, looking to see if my interpretations were really reflections of what the students had said, rather than what I wanted them to have said.

4. *Setting aside questions of cause*: in addition to bracketing a personal belief that a positive relationship exists between the two mathematics courses, I had to be careful to avoid suggestions of causality. For example, *if* a student takes both Applied and Pre-Calculus, *then* a better understanding of mathematics emerges. A causal relationship might indeed exist, but that could not be a predetermined stance on my part that could be allowed to influence the outcome of the study. Once again, by being aware of my own beliefs, I could consciously set them aside as I gathered and analyzed the data.

Data Collection

Traditionally, the primary data collection method associated with phenomenography is the open-ended interview. This individual meeting provides a context in which the researcher may have a rich encounter with participants as they share their reflections on the phenomenon under study. Ashworth and Lucas (2000) describe the interview as a conversational partnership in which the researcher does not lead the discussion, rather he or she assists the participant in a process of reflection. Questions may be prepared in advance, but they are used minimally so as to give both researcher and participant freedom to follow trains of thought as necessary. Prompts are used when the researcher requires clarification or wishes the participant to elaborate on a description.

Phenomenographers, however, do not limit themselves to a single method of data collection. The phenomenographic tradition allows for other means of obtaining descriptive accounts of a phenomenon as well: "Marton recognized that there were other sources of information by means of which researchers could understand how people conceived of different aspects of their world ... [including methods such as] 'group interviews, observations, drawings, written responses, and historical documents.'" (Richardson, 1999, p. 64). The forms of expression are varied, but all "have the same evidential status as oral accounts" (Richardson, 1999, p. 64). Choosing the appropriate means of obtaining a descriptive account allows the researcher to make adaptations to suit the particular goals and the context of a study. By varying the methods of data collection and therein allowing more freedom for the research participant to describe his or her experience, the researcher may indeed have greater access to the full range of perceptions of experience. Two variations that were particularly useful in this study are described below.

Synergetic Focus Groups. Willmett (2002) offers a significant variation of classical phenomenographic technique: the synergetic focus group. This approach has its roots in traditional focus group interviews, but unlike the traditional model which uses predetermined questions where the moderator plays an important guiding role, synergetic focus group discussions rely heavily on participant-initiated discussion: Russell (quoted in Franz, Ferreira & Thambiratam, 1997) describes the synergetic focus group method as offering "the researcher a variety of unsolicited conceptions through non-directed discussion. As lived experiences related to the phenomenon are shared by participants over an hour or so they explore qualitatively different conceptions of the phenomenon" (p. 1). As Willmett points out, "synergetic focus groups are useful when the researcher is not sure what the important issues might be" (2002, p. 8). Given that phenomenography is engaged in the process of discovery, giving the participants freedom in the range of topics they wish to discuss as pertaining to their experience of learning mathematics seemed to be a particularly good way of eliciting a wide variety of descriptions.

Willmett (2002) goes on to outline the researcher's role in the synergetic focus group. At the beginning of the session, the participants are welcomed, set at ease and assured of confidentiality with respect to their discussion; the moderator provides an outline of the purposes of the discussion and a definition of relevant terms; and finally sets the topic by presenting a wide range of ideas related to the courses and mathematical learning (not necessarily confined to those that the researcher sees as appropriate) that could be starting points for discussion. At this point, the researcher turns the discussion over to the participants, withdrawing to take notes and minimizing any further participation in the dialogue.

Specific advantages to this research approach, according to Willmett, include the fact that "the focus group interview allows participants to listen to other views, to be influenced by the conversation and to make informed decisions" (2002, p. 18). Hearing other views encourages participants to become more reflective of their own experiences as they compare their own descriptions or interpretations of events with those they hear from others in the group. It can help to remind participants of other incidents that they might not have considered previously, but which prove to be germane to the discussion. Furthermore, it allows the researcher to investigate ideas that might not have been anticipated beforehand: participants can explore, probe and clarify issues that emerge within the group discussion.

Naturally, this approach requires skill on the part of the moderator: to create a comfortable climate, to use open-ended questioning, to know how to deal with silences, to know when to encourage further probing or when to move on to a new topic, and to minimize the discussion of irrelevant issues. This could be daunting to someone who has never conducted a formal focus group discussion. Yet the skills involved in moderating a discussion closely parallel what a teacher does as she uses class or small groups discussions to follow up an inquiry activity. She creates an environment in which students feel free to participate, she uses open-ended questions and encourages full student involvement. She probes where necessary and she promotes inter-student discussion. She develops a sense of how far ranging she can allow a discussion to become before she needs to redirect it. In other words, the skills that a teacher has developed as part of her everyday practice will be the skills that will allow her to moderate a focus group effectively.

Interactive Writing. A second variation from classical phenomenographic technique involves the use of interactive writing. In the interactive writing process described by Mason and McFeetors (2002), the teacher provides her students with a specific writing prompt and the students respond to the prompt with one good paragraph. The prompts can be varied in their intention: for example, the goal may be to encourage a student to express a particular mathematical concept in words or it may be to draw a student's attention to his or her use of a particular mathematical process or learning strategy. Students are asked to be reflective and thoughtful in their responses, and can choose for themselves what they want to share with the teacher within the bounds of the prompt. The teacher then responds in writing to the specific statements, issues or concerns raised by the students.

Using interactive writing as a form of gathering perceptions of learning experiences has several advantages. Written responses are private, to be shared only between the student and teacher. As a result, students can feel safe about sharing what they choose to write about. Furthermore, written responses also invite students to be reflective, to consider their experiences and to think carefully about their responses. Therefore, as the students choose their words with care, the descriptions of their experiences may be more deliberate and detailed.

Data Analysis and Interpretation

As stated previously, the goal of data analysis in any phenomenographic study is a set of "categories of description" of the various conceptions of a phenomenon. It is important at this stage to underscore once again that phenomenography focuses on experience *as described*. As a result, the categories characterize the descriptions of the experience, not the experience itself.

Data analysis in phenomenography mirrors some of the techniques of grounded theory (Strauss & Corbin, 1990), in that it uses an iterative and comparative approach – notes and transcripts and written responses are continually read and reread, constantly sorted and resorted, and viewed from a variety of perspectives in an effort to discover the various conceptualizations that students have of learning mathematics. As discussed in the section on bracketing above, the researcher must remain open-minded throughout the process, continually looking for variations in the data and categorization. Äkerlind describes the researcher's open-minded approach to data analysis when she states:

> In the early stages, reading through transcripts is characterized by a high degree of openness to possible meanings, subsequent readings becoming more focused on particular aspects or criteria, but still within a framework of openness to new interpretations, and the ultimate aim of illuminating the whole by focusing on different perspectives at different times" (2002, p. 3).

Initial categories of description are established and are subsequently subjected to further review and modification, before being examined in relation to each other to look for related meanings and underlying structure. Marton describes the procedure as "categories [being] tested against the data, adjusted, retested, and adjusted again. There is, however, a decreasing rate of change and eventually the whole system of meanings is stabilized" (as quoted in Äkerlind, 2002, p. 4).

The outcome space that is generated as a result of the analysis will ideally represent, for a particular population as represented by the sample and at a particular point in time, the full range of possible ways of experiencing the phenomenon under consideration. Marton & Booth define a quality outcome space as one where each category reveals something distinctive about a way of understanding the phenomenon, one in which the categories are logically related (usually in some sort of hierarchy), and one in which the outcomes are parsimonious, or in other words, where the number of categories is limited to as small a number as possible and yet still reflect the critical variation in observed experience (Äkerlind, 2002).

More than just being descriptive, however, the phenomenographic researcher will seek to develop "as deep an understanding as possible of what has been said, or rather, what has been meant" (Marton, 1994, p. 4428). This is often done by interpreting the underlying structure of the categories of description as hierarchical levels of awareness (for example, deep versus surface awareness). Those levels of awareness certainly manifested themselves within this data set, but it seemed to me that this type of interpretation would simply add another layer of description. That layer of description may have proven interesting, but I did not see it as being particularly meaningful to me or to my students. Instead, I have chosen to interpret the results of the study in a way that I trust will amplify the richness of meaning across categories. I believe the results point to an emerging consciousness among students of their own learning experiences, and an emerging awareness within myself as a teacher of the meaning and implications of how students make personal sense of these experiences.

Method: Data Gathering and Analysis in This Study

Painting the Picture

In order to establish and interpret the categories of description that would provide a glimpse of student perceptions of their learning in mathematics, it was necessary for me to create opportunities for the participants of the study to reflect on their experiences and then to describe them as fully as possible to me as researcher. I also needed to examine the context from within which they were describing their experiences – the courses, the prevailing culture, and so forth.

In order to accomplish this, the study was broken down into three parts. The three components helped provide a more comprehensive image of the study of Applied and Pre-Calculus mathematics in Manitoba. It allowed me to adapt data collection methods to particular (groups of) participants and to triangulate the data collected.

Part 1: Preparing the Canvas

Prior to the collection of any data from students, I wanted to establish the backdrop against which the student data would be depicted by examining the curriculum documents, the reflections of some other Pre-Calculus and Applied teachers, and the administrative rationale behind the way student course selections developed in my school. I also wanted to carefully examine my own perceptions with respect to the two math courses, so that these could be bracketed before I engaged in student research. This preliminary research was completed in four steps:

Part 1A: The curriculum documents for Applied 40S and Pre-Calculus 40S were examined. In particular, I compared and contrasted the rationales, outcomes and processes, and then examined specifically the units on sinusoidal functions and geometric sequences.

Part 1B: A short questionnaire was sent to a small number of Manitoba high school teachers who were familiar with both Applied and Pre-Calculus. The sample used was a convenience sample, in that it involved people with whom I had professional contact. This included three other Applied and/or Pre-Calculus teachers in my division, as well as one teacher from another school division. The sample was not intended to be representative – it was included simply to reflect the thoughts of at least some teachers, against which the views of the students could be depicted. Figure 2 below illustrates the types of questions asked.

Teacher Questionnaire

1. What do you consider to be the primary purpose of the Pre-Calculus 40S (Applied 40S) course with respect to students? Consider who is taking the course and why, as well as the stated goals of the curriculum.
2. Consider the unit on sine function or the unit on geometric sequences. Briefly describe (in general terms) some of the general strategies, the instructional techniques, and the types of assessment you would use.
3. Briefly describe what learning strategies you think students will need to use in order to be 'successful' in the Pre-Calculus (Applied) 40S course. You may also wish to comment on what you consider success to be in this particular course.

Figure 2: Teacher Questionnaire

Part 1C: I invited the former principal of my school to submit a written outline of his rationale for establishing the tradition of taking both mathematics courses.

Part 1D: I began a reflective journal in which I examined my own beliefs about mathematics learning. The reflective journal was an ongoing endeavour throughout the research process.

The data gathered in this section was used to put together a picture of the expectations and assumptions surrounding the two courses, thereby setting the context for the student data. The results have already been outlined in the previous chapters.

Part 2: Graduates Thinking Back on Their Experiences

My purpose in this stage of data collection was to explore through small group discussions and follow-up interviews, the perceptions of students who had already graduated and had been away from the public school system for a short while. The format for these small group discussions followed the format of the synergetic focus group as outlined previously. I saw my role as providing possible discussion topics, eliciting individual students' comments about whatever was currently being discussed, encouraging further exploration or clarification of an idea that had come up, or redirecting the conversation if it got off topic. Because class discussions had been a normal part of the Applied classroom culture, I was confident that students would feel comfortable about listening to each other and responding to each other as they consider their mathematical learning experiences. As it turned out, my role became a little more overt than I had expected, as will be described in the following chapter.

Most of these participants had taken both Applied and Pre-Calculus for two or three years and were able to provide a retrospective look at their learning experiences and at the interrelationship between the courses. The audio-taped discussion was transcribed, then analyzed for different categories of description that emerged.

Part 2A: All graduates from my school who took either Applied Math 40S or Pre-Calculus Math 40S or both in the 2002/2003 school year were invited by letter to participate in the study. Based on the response, I conducted a single small group discussion, lasting approximately one hour, with a group of former students, some who had taken both Applied and Pre-Calculus, and some who had taken only Applied in their final year. There were no students in the grade who had taken only Pre-Calculus, so that viewpoint was not represented. The discussion took place at a quiet location and time that was mutually agreed upon by all involved. Focusing questions (see Figure 3 below for examples) invited participants to consider and describe their experiences with learning in Pre-Calculus and Applied, both in general terms and with respect to specific examples, and to consider the interrelationship between the two courses. Participants were also asked to discuss the reasons for taking only one or the other or both. The discussion was audio-taped and I took field notes at the same time.

Examples of Prompts used in the Small Group Discussions

1. Tell me why you took Pre-Calculus (Applied) (both).
2. Think back to Pre-Calculus (Applied) in general, and tell me about how you think you learned mathematics. What were some of the things you did to learn the material? What helped you to understand? What did you do when you got stuck? How did you prepare for a test?
3. Tell me about how the material was presented in class. How do you think this related to the way you learn?
4. Tell me about your interaction with other students during Pre-Calculus (Applied). How did you feel about collaborative work? How did this help you (or hinder you) in your mathematical learning?
5. Tell me about your preparation for the Standards Test you wrote at the end of the course.
6. Think back to a specific learning experience in Pre-Calculus (Applied) that you considered to be positive (negative) or successful (unsuccessful). Describe it. Why was it positive (negative)?
7. Consider a sine function (a geometric sequence). If you were to think about it strictly from what you consider to be a Pre-Calculus (Applied) perspective, what would you be able to tell me about it?
8. What benefits (disadvantages) are there to taking both courses?
9. Consider the year(s) in which you took both Applied and Pre-Calculus. Do you think the order in which you took the courses made a difference to your understanding of the mathematics you were learning? Explain.
10. Pretend a younger student came to you for advice about course selections, starting in Senior 2 (Grade 10). What advice would you give that student about the choices he or she should make with respect to mathematics courses? Explain.

Figure 3: Focusing Questions for Small Group Discussions

Part 2B: Follow-up interviews lasting approximately forty minutes were conducted with all the participants from the small group discussion. Although the original research design called for individual interviews with a few selected students only, I elected instead to regroup the students in pairs or triples according to the types of remarks they had made during the small group discussion. In so doing, I hoped that further synergy would develop when a few people were discussing ideas. These interviews served to get feedback and provided and opportunity to explore emergent ideas or expand on my interpretation and analysis from Part 2A. Figure 4 below shows some of the types of questions that were asked.

Examples of Questions used in the Follow-Up Interviews

1. After examining and thinking about our small group discussion, I really got the sense that people experienced mathematics learning in the following way: [fill in based on analysis of small group discussion transcripts]. Do you agree or disagree with this interpretation? Please explain.
2. During our group discussion, you mentioned [a specific learning experience]. Please elaborate.
3. Think about the mathematics you have encountered in the last [six] months since you graduated. Tell me about one experience. How would you relate that experience to either the Applied or the Pre-Calculus course?

Figure 4: Focusing Questions for Follow-Up Interviews

Part 3: Students Thinking During Their Experiences

The final data set was collected near the end of Semester 1 and during Semester 2 of the current school year. My purpose here was to explore, through interactive writing and a follow-up small group discussion, the perceptions of students who were currently enrolled in Pre-Calculus 40S, Applied 40S or both. The small group discussions followed the same procedure outlined for the small group discussions with the graduates. The students were able to lend some immediacy to their perceptions as they focused on very specific mathematical ideas and their sense of experience with them. In a manner similar to that used for the small group discussion data, I analyzed the interactive writing for different categories of description that emerged, and invited students to participate in the analysis through the follow-up small group discussion. As with the graduates, I was not necessarily looking for commonalities among participants' experiences, but rather for richness and diversity in the experiences that students had in their study of mathematics. Appendix A on page 133 provides a complete list of the prompts given.

Part 3A: From all students enrolled in the Pre-Calculus 40S course in Semester 1, I collected four interactive writing assignments during the two different units in which geometric sequences and sinusoidal functions were taught. I did not teach the Pre-Calculus course, but had an agreement with the teacher to become involved with the class in the following way: the interactive writing process became part of *all* students' learning experiences for that unit. During the Sequences unit, they responded to the prompts I gave them, and I wrote back to each of them. However, I used as data sources only the writings from the students who were participants in the study. Because the unit on trigonometric functions was taught early in the year, the interactive writing for the sinusoidal functions took place only as a single session during the exam review at the end of the course.

Part 3B: From all students enrolled in Applied 40S in Semester 2 (the class had some of the same students and some different students from Part 3A), I collected a total of five interactive writing assignments during the geometric sequences and sinusoidal functions units. The manner of collecting written data was the same as for Part 3A for the Sequences unit. The data collection for the Sinusoidal Data unit was slightly different in that interactive writing was done on a number of specified days throughout the unit, rather than only once during the exam review. Again, the interactive writing was done as part of the normal learning process for all students, but I used as data sources writings from only those students who were participants of the study.

Part 3C: In a manner similar to the method used in Part 2A, I conducted a follow-up small group discussion, lasting approximately one hour, with eleven students. This provided students with an opportunity to discuss ideas about learning mathematics as it came up within their own discussion, and for me to get feedback and to explore emergent ideas and expand on my interpretation and analysis from all previous data collection.

Putting It All Together

Analysis and interpretation of data at each stage took place before the next phase of data collection so that possible categories of description could be further explored in subsequent interviews, discussions or interactive writings. All data were examined individually, then as part of the set they came from, and finally as a part of the overall data set that made up the study. I read and reread the transcripts and writings according to the procedural guidelines set out by Äkerlind (2002), looking at the data through a variety of lenses, establishing categories of description and subjecting them to repeated review and modification, and finally examining them once again in an effort to uncover related meaning and emerging awareness.

Recall that in Chapter 1and again in this chapter, I have likened the process of this study to painting a picture. It is worth reminding the reader that the picture that is being painted here of student perception of their experiences in studying mathematics is one that has emerged through my eyes and ears and mind as the researcher. Steps were taken through bracketing to minimize any skewing of the results, yet the fact remains that we only have access to others' experiences through external means, that is, we can only begin to understand how someone experiences something by asking that person to describe it, and to probe thoroughly enough to be reasonably certain that our understanding of that description approximates what that person hoped to convey through the description. The researcher *experiences* the data, and if we follow through with the basic principles of awareness in phenomenography, there will be variation in the experiences that any researcher will have with the same set of data. Hence it is acknowledged at the outset that my interpretation of the outcome space generated by this study will be but one of many possible interpretations.

Chapter 4

The Underpainting: Sketches of the Participants and the Data

In this chapter, the reader will be introduced to the students who participated in this study and get a sense of the types of responses they generated throughout the various phases of data collection. I will draw attention to some of the ways in which the methods of data collection as outlined in the previous chapter served to further the purpose of reflection and carefully considered descriptions of experiences, and I will point out some of the barriers I encountered. Throughout, I will make liberal use of quotes from all sources of student data so that before the analysis and interpretation in the subsequent chapters narrows the focus, the reader will have a strong sense of the nature, the depth, and the breadth of the comments that formed the data set.

Introducing the Participants

The student participants in this phenomenographic study all lived in or near a small rural community not far from Winnipeg. They were attending the local high school, or had attended it the previous year. The school comprises approximately 250 students, from grade seven through Senior 4 (grade twelve). As is the case in most small high schools, course selections are somewhat limited, but students did have the opportunity to choose among the three different math courses in each of their Senior 2 to Senior 4 years. Data for this study were collected during the 2003-2004 school year, and focused on students who had taken or were taking Pre-Calculus 40S and/or Applied 40S. Two distinct groups of students were invited to participate: those who had graduated in 2003 (referred to as former students), and those who were expecting to graduate in 2004 (referred to as current students).

Because phenomenography seeks to examine as wide a range of variation in lived experiences as possible, I specifically chose these two particular year-groups of students as data sources. In a small rural community such as this one, many students spend almost their entire school career with the same classmates. Over the years, they develop some shared values, attitudes, and priorities with respect to various subjects, teachers, or work habits, and therefore also tend to engage in some joint decision-making. Choosing courses, for example, becomes a matter of animated discussion amongst the students, where they evaluate (according to their own set of criteria) the importance of or the value in taking any particular course. The attitudes and decisions among peers may factor as heavily into a student's own course selection as would personal goals for continued education or career.

The attitudes towards the study of mathematics differed considerably between the two groups and were, I believe, affected considerably by the group dynamic that had developed within each grade. In the older grade (the former students), nineteen out of thirty students had registered for Pre-Calculus in Senior 2, and fourteen had registered for Applied. The school administration was counseling students to take both courses, and so thirteen had registered for both (see Figure 5 below).

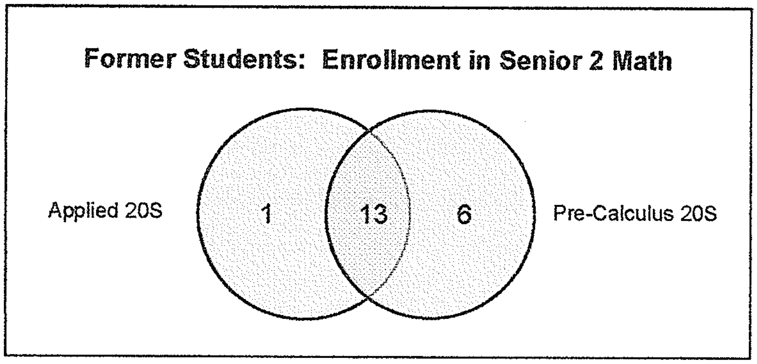

Figure 5: Senior 2 enrollment patterns for former students

The students were academically slightly above average, with a few exceptions; high marks were generally thought to be a good thing, but not a top priority for most. While some were looking for an easy way out, most worked willingly and diligently when required. They were creative and enjoyed tactile experiences. Collaboration, both formally and informally, was an important foundation to the way they learned. They embraced the hands-on exploratory style of Applied math, and became less enamoured with the abstract thinking required in Pre-Calculus. By the time they reached Senior 4, there were only ten students left in Pre-Calculus (all co-registered in Applied), whereas the Applied class had grown to sixteen students (see Figure 6 below).

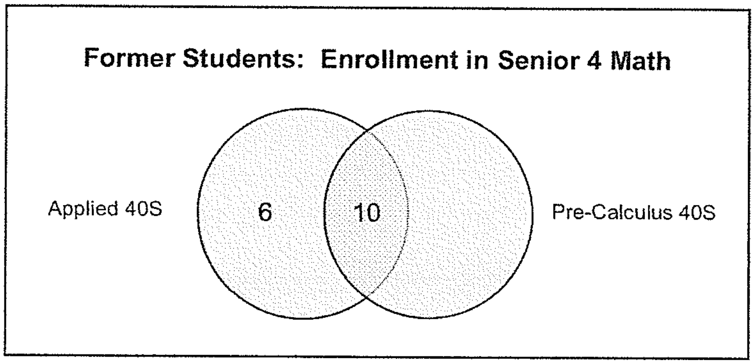

Figure 6: Senior 4 enrollment patterns for former students

Like those in the year before, the students in the younger grade had been counseled by the administration to register for both courses. In their Senior 2 year, twenty-seven out of thirty-seven students registered for Pre-Calculus, twenty-six for Applied, and twenty-six were taking both (see Figure 7 below).

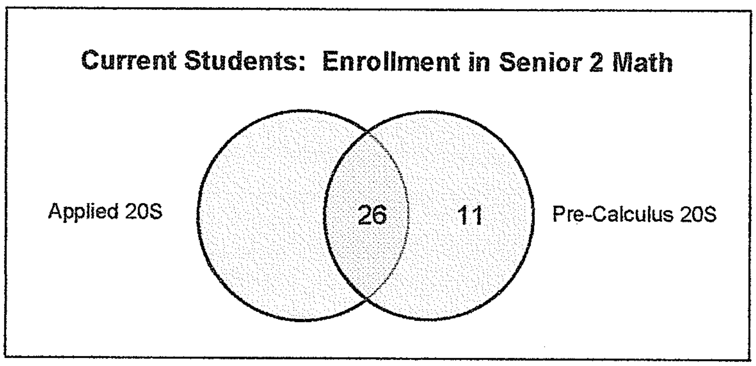

Figure 7: Senior 2 enrollment patterns for current students

While this group, too, had some students who wanted to do as little work as possible, most of the current students took pride in their work and put forth considerable effort to get the results they wanted. Success for many, however, seemed to be defined more in terms of high marks than in terms of wanting to understand the material. The skill sets they had developed over the years tended to focus on memorization and implementation of methodology. Some joked that they only 'learned' things long enough to pass the exam with a good mark, then promptly forgot everything because it was useless information anyway. Pre-Calculus appealed to many – they saw the concepts taught as logical progressions of reasoning that could be accomplished successfully as long as one became proficient with the method. Applied, although somewhat interesting, came to be seen by many as being too ambiguous, requiring too much interpretation, and being too repetitive (covering the same concepts they had already learned in Pre-Calculus). Collaborative work, an important component in Applied, was difficult for some due to the presence of several distinct cliques – crossovers often led to resentments. For students in this grade, then, registration in Pre-Calculus remained strong throughout their high school years, but registration in Applied dropped to only eight students by grade 12 (see Figure 8 below).

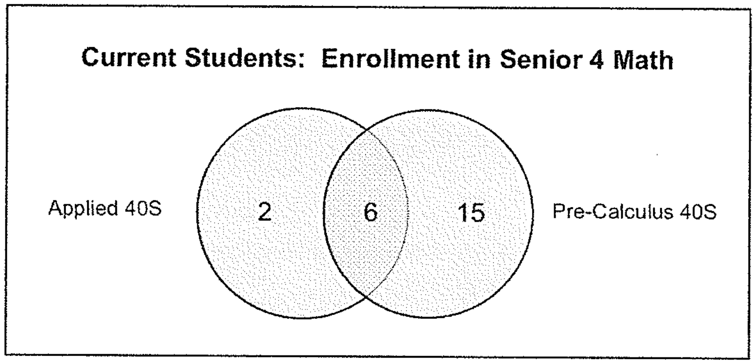

Figure 8: Senior 4 enrollment patterns for current students

It was from these two distinct groups that I invited students to participate in this study. From the group of former students, nine students agreed to participate in a focus group and follow-up interviews (two others were out of town and sent their regrets). All of these students had completed the Applied course, and of these, ten had also completed the Pre-Calculus course. None took only Pre-Calculus in their grade twelve year. From the group of current students, twelve students agreed to participate in the study, engaging in four interactive writing sessions in each of the math courses they took. Ten also took part in the focus group discussion later.

Having portrayed some of the group dynamics that existed in the classes, I now turn to describe some of the individual student participants. Group dynamics and personal motivations seem to guide course selections, so I will use the Senior 4 course selections as a way to categorize types of participants. Within each category, I will describe the participants and their motivations. For clarity, I will designate each student from the former group with an (F) and each student from the current group with a (C).

Pre-Calculus only

Four students fit this category. All had taken both courses in their Senior 2 year, but decided either in a subsequent year to concentrate their efforts in Pre-Calculus only. **Ophelia** (C), **Randi** (C), **Gabrielle** (C) were above average to honours students who took pride in their work. Achieving high marks was a priority for them, and they concentrated on doing what was necessary to achieve that goal. They were all very comfortable with a traditional focus on formulas and procedure as part of their math learning, but seemed to have difficulty making the transition to a focus on interpretation and the extensive use of technology in Applied math. As their marks began to suffer, or as they realized Applied math was not a requirement for them, they chose to drop it at the end of grade eleven, preferring to take only Pre-Calculus in grade twelve.

Taylor (C) began with both courses, but discontinued Applied after his Senior 2 year. He seemed to base his concept of learning on doing whatever was necessary to achieve the credit, rather than finding connections between what he was learning and his own life or surroundings. As a result, he found Applied repetitive, and the activities superfluous to what he had already learned in Pre-Calculus the semester before.

Applied only

Six students fit this category. With the possible exception of one student who transferred in to the school in grade twelve, all the students in this category had started off with both Applied and Pre-Calculus in grade ten. Most of these students can be characterized as having struggled with math in the past, and were looking for an easier alternative. Only some intended to pursue post-secondary education in the near future.

Thomas (F) immediately took to the Applied emphasis on contextual problem solving and the use of technology. While he had done reasonably well in grade ten Pre-Calculus, he had little use for the more theoretical and abstract approach, and did not pursue this course in later grades.

Angela (F) and **Ashley** (F) were both bound for post-secondary education, but had struggled with Pre-Calculus in grades ten and eleven. Once they determined that their respective courses of study did not require a Pre-Calculus credit for admission, they dropped it in favour of Applied only. Both students appreciated the fact that they could rely heavily on technology in that course, and felt that the hands-on approach was more in line with the way they learned.

Kiera (C), having struggled a lot with Pre-Calculus in grade ten, was looking for an easier alternative, but one that still provided a bit more of a challenge than Consumer math. However, she continued to struggle with the mathematical ideas in Applied. Of all the students in this group, she most keenly felt that the students had who had taken Pre-Calculus the semester before had some advantage over her. For them, the concepts were review, but for her they were new, and she did not always feel she had enough time to absorb them before a new concept was introduced.

Simone (F) and **Nic** (C) were students who seemed to have a very weak work ethic and appeared to be looking for the highest level course they could take which required the least amount of effort. Simone placed a strong emphasis on the role of the teacher in determining whether or not she passed a course, whereas Nic probably would have been happy to take Consumer, were it not for the pressure his parents were exerting on him. Both enjoyed the hands-on aspect of Applied.

Pre-Calculus and Applied

Eleven students fit this category. Course selection patterns were more varied here, but the group can be characterized in general as students who liked math, who recognized some benefit in the different perspectives of the two courses, or who wanted to 'keep their options open' for post-secondary education. Most were headed towards post-secondary studies within the next few years.

Michael (F), **Lillian** (F), and **Kathleen** (F) generally liked math and considered Pre-Calculus to be the "pure" math. Since all were headed to post-secondary education after graduation that involved some mathematical study, Pre-Calculus was a requirement, and they appreciated the challenge it offered them. However, they all enjoyed the variety of perspectives and alternate approaches available to them through Applied and so maintained the dual path through to the end of grade twelve.

David (F) and **Cody** (F) liked math as well, taking both courses throughout high school. They liked the rigour of Pre-Calculus, but became particularly fond of the technology component in Applied. They were among the few students who became quite skillful with their graphing calculator, and were able to use the technology in a variety of applications beyond specific tasks in class. **Morgan** (C), having taken only Pre-Calculus before grade twelve, was interested in exploring it from an Applied perspective. She experienced a bit of a culture shock with respect to the use of technology, though – feeling like she was abrogating her responsibility to think when she used the graphing calculator. Once she became more familiar with it, however, she was able to start using it quite effectively as a learning tool.

Heather (C), **Erin** (C), **and Wendy** (C) were high achievers who took pride in maintaining top marks. They generally liked math, and took both courses throughout high school. They, like others, enjoyed the hands-on approach of Applied and the different perspective it afforded, but tended to equate understanding of mathematics with procedural competence rather than conceptual understanding. All three seemed to suffer some kind of 'learning fatigue' in the last few months of school – they often expressed a lack of desire to learn anything new, and were only interested in finishing.

Brady (C) took both courses in grade ten. He was a very bright student and caught on to things quite easily. However, he did not like to exert any extra effort just for the sake of achieving higher marks, and was quite open about having a poor work ethic. He chose to focus only on Pre-Calculus in grade eleven, finding the hands-on activities and projects in Applied to be too much effort. Yet he switched back to taking both in grade twelve. His reasons were somewhat unclear, but I believe he needed to fill his timetable with something that at least had the potential of being interesting to him, even if it did require a bit of work on his part.

Leanne (C) took both courses throughout high school. She too liked math in general, but tended to be a little careless in her work. She often seemed to think she had learned everything she could about a topic after working with it for only a short time, and did not have the patience to explore something in more detail. She considered Applied "fun" math, a way of rounding out what she had learned in Pre-Calculus.

Introducing the Data

Data were collected in a variety of ways, as has been outlined in the previous chapter, with each method providing participants with a slightly different way in which to express themselves. With the students, data collection included synergetic focus groups, follow-up interviews, and interactive writing. My own reflective journal and field notes served as a way of recording my own thoughts with respect to what I was hearing or reading from the students. Each method of data collection had its own advantages and disadvantages, and each provided a different level of richness to the data that was generated. A brief discussion of each method follows, along with examples of the type of data resulted.

Synergetic Focus Groups

At the core of my research was my desire to have students reflect on their learning first, to contemplate their experiences before describing them to me. This was accomplished through the use of synergetic focus groups with both classes of students. Two sessions were conducted according to the guidelines set forth by Willmett (2002) and as described in the preceding chapter. After some opening remarks and the occasional prompt from the moderator, the discussions were allowed to move in any direction as students listened and responded to each other. The expectation was that instead of following a preset agenda, students would eventually end up talking about that which they considered important in their mathematical learning experiences. The intention was that as some students shared their thoughts, this would jog the memories of others and lead them to share their own experiences with the group. I also hoped that there would be some disagreements that would arise as a result of the discussion, thereby underlining some of the differences that existed among the ways students thought about learning math. Nine former students participated in the first session, and ten current students took part in the second session. Both focus group discussions lasted for approximately one hour.

It came as a bit of surprise to me to find a noticeable difference in tone between the two focus groups. The former students seemed to feel a bit more awkward or shy about initiating and following through with any discussion. They had a hard time getting started talking about their experiences, so I suggested that they might want to go around the circle to start, with each describing why they had chosen to take a particular math course. While this got them talking, it also seemed to set a 'go around the circle' tone for the rest of the hour, and participants fell into the pattern of 'waiting their turn' before responding to the ideas under discussion. Students in this group relied heavily on leadership from the moderator - they would respond openly to a question, yet spontaneous discussion never seemed to last for very long. Rather than taking their cues from each other and exploring ideas in greater depth, students would often make short work of the topic, then look to me for the next prompt. Although the students had important things to say, the synergy I had hoped for in the discussion did not materialize to the extent to which I had hoped. Fortunately, I had scheduled follow-up interviews with all the members of this group, where they seemed to be a little more willing to engage freely in discussion.

The focus group with the current students, held several months later, proved to be quite different. There was a definite sense of synergy that developed over the course of the hour. Students relied very little on the moderator – they responded immediately to the opening prompt and proceeded to engage in a lively discussion from there. Most students seemed comfortable in getting involved in the discussion; their comments built on the ideas of previous speakers and they were quite open in sharing their thoughts and describing their experiences. A few students who were more shy by nature were content to listen for the most part, yet it was obvious that they were taking an interest in the discussion, nodding or shaking their heads in response to what was being said, or murmuring agreement or disagreement with what they heard.

The reasons for the marked difference in synergy between the two groups could be many. Group dynamics could have played a role, although the group dynamics turned out to be quite different from those I described earlier in this chapter. The former students, who had once shared a strong sense of class camaraderie, did not seem completely comfortable with each other; the current students on the other hand, who had tended to stick to their friendship groups, participated freely with each other. Perhaps the passage of time (about six months) for the former students had already begun to take its toll not only on the connection they felt with each other but also with the subject of discussion itself, particularly for those who had already entered the work force. In part it might have just been my own increasing experience and skill in moderating these discussions that permitted me to set a more suitable tone in my opening remarks. Yet despite the differences in synergy, students in both groups contributed rich thoughts and ideas about their learning experiences in mathematics.

I believe that the strength of the synergetic focus group as a data-gathering method is its scope. There are few predefined limits that restrict the range of discussion that participants choose to follow. The moderator's preconceived ideas do not set the agenda; instead, participants determine the course of the conversation based on their own notions of priority and importance. Reflection, disagreement, the ability to go back over something that was previously talked about – all provided the participants with the opportunity to think about and to talk about a wide variety of ideas and experiences, thereby contributing richly to the 'range of experiences' that phenomenography seeks to describe. What follows are numerous examples which will illustrate not only various types of interactions that occurred among students, but will also give a flavour of the student conversations themselves.

When reading the data excerpts, it may occasionally be useful to be reminded of a student's course background. To that end, and for the remainder of this document, students who are quoted will be identified with this expanded shorthand notation: former student (F), current student (C), followed by their grade twelve course selections of Pre-Calculus only (PCM), Applied only (APM), or both (BOTH). Furthermore, wherever students are directly quoted, all grammatical or spelling errors made by students have been left intact.

Sharing Differing Perspectives

The fact that different ideas have different meanings for different people was certainly evident in the two focus groups. The transcripts of both focus group sessions are replete with examples of how various people saw things differently. Students would listen to what others were saying about a specific topic, and then add to the description by putting their own interpretation on it. For example, some of the former students were discussing how they felt about working collaboratively in math. Whether they liked group work or not, each had his or her own experience with it:

> **Angela** (F-APM): When we had little experiments, I don't know if this was, I think this was Applied math, and we had to do some ex.., like projects or whatever, and I wouldn't understand what we were doing. It really helped when we were in a group because I could ask someone who understood what they were doing and they could explain it to me and then, you know, I could do it then. So it helps to have groups sometimes.
>
> **Ashley** (F-APM): I don't really like groups, with like with math, because I think for me math, if I don't understand, I like to be one on one with the teacher. Friends can help, but they can't help me as well as a teacher can, so I prefer to kind of figure it out with the teacher by myself.
>
> **Michael** (F-BOTH): With groups, like, say you're working on a problem. With a group, somebody, I don't know, I find this, somebody will get the answer and everybody will like, oh ok, that's how you do it, whatever, good (general laughter and agreement). But with a teacher, you're saying it takes 15 minutes or so to work it out, he, but he like, I don't know, shows you how to do it. He's trying to teach you how to do it, instead of, I don't know, just getting the answer off somebody, just the solution.
>
> ...
>
> **David** (F-BOTH): That's another advantage of having answers beforehand because then if you are just going to take the answers right away, you can just go to the back, but if you're working as a group, trying to figure it out, then you're

> not trying to figure out the answer, you're trying to figure out how to do it, you already had the answer.

The second focus group, too, was able to collaboratively paint a picture of their learning experiences, this time in Pre-Calculus math, where each student highlighted what was important to him or her:

> **Wendy** (C-BOTH): And I liked that we had the answer keys, cuz then if you didn't get the right answer you could just sort of try different things and figure out why or how you get the answers.
>
> **Heather** (C-BOTH): I like it that it's very, um, like formula-based, like you have a certain methodical way of doing everything, and you have your little formula for this and your formula for that, and you just, like, maybe not in every case, but most of the time you can use that way of doing it. So once you master that, then you know.
>
> **Leanne** (C-BOTH): I didn't mind that it was formula-based and stuff, but sometimes I'd want to know why, and then that would just take to long to go through the explaining and then I'd just, I didn't learn the next part because I was too busy concentrating on the why on the other one. I don't know, cuz you had to go very fast from one thing to another, so it's like, I mean it made sense, but if I started thinking about the whys of it, then I'd get too caught up i that and then, yeah.
>
> ...
>
> **Erin** (C-BOTH): I thought that sometimes when he did examples on the board, he did easier examples. And I would have rather had him do harder ones so we would have known how to do some of them in the exercises.

Opportunity for Elaboration

Students sometimes used the discussion to elaborate on themes that arose in the discussion. This was a little different from offering varying perspectives in that students provided a fuller description of the topic under consideration, but did not necessarily take a personal stance towards it. For example, the former students were discussing what they did when they were having difficulty with a particular problem:

> **David** (F-BOTH): It really helped to, ah, have people nearby that you could just ask without necessarily going to the teacher.
>
> **Angela** (F-APM): Yeah, cause it's kind of nerve-wracking going up to the teacher sometimes. You get nervous, cause, then you feel stupid, cause you can't do a question.

Lillian (F-BOTH): Or sometimes a teacher explains it and you still don't get it. They're so smart that you can't really keep up with them, so it's easier to ask a friend. I ask a lot of friends.

David (F-BOTH): In Pre-Calculus especially, the teacher took a long time to get around to you because it took so long to work through each problem, that, you know, you could be sitting there for fifteen minutes waiting to have your question answered before you could go any further, getting nothing done. In Applied math, each question could be answered quite quickly because you could go quickly through technology.

At times the opportunity for elaboration within the synergetic focus group was almost amusing, as a number of students completed a string of thoughts into a coherent whole! In this example, the current students are discussing provincial standards tests, which were being written in both math courses:

Heather (C-BOTH): And you can't be, like, completely prepared for it, because you don't really know what to expect. And whereas if you have an exam that your teacher writes, you know ---

Morgan (C-BOTH): Yeah, you know the teacher's train of thought when he ---

Heather (C-BOTH): You've written his tests all year, you know, like you ---

Kiera (C-APM): And you know that it's only going to be stuff that you learned on the exam. Provincial exams can be so scary, there can be all sorts of surprises.

Wendy (C-BOTH): And they can be worded differently, so ---

Randi (C-PCM): And yeah, questions are written so differently.

Heather (C-BOTH):And if they say, like, if they have a different couple of words to describe a method of doing something, and they say use this and this and this to do this, like who knows what that could mean.

Opportunity for Disagreement

A strong statement made by one student would sometimes be a trigger for others to oppose it and offer an alternate point of view. The former students, for example, had embraced the use of technology in Applied math while they were taking it, yet there was at least one dissenting voice showing considerably less enthusiasm for it than the others:

David (F-BOTH): I've always found it very much easier. I've always found it a lot easier to, um, use technology to do long math problems because I find if I'm writing things out a long, long way, I always, ah, tend to make a small mistake somewhere down the line, so I knew that it would be a lot better for me using technology in math.

...

Michael (F-BOTH): I don't, technology, I don't know. With Applied math it helps you unders.. kind of understand what's going on, but you don't, with mathematics, you don't know why you're doing it? Like Pre-Cal, like when you're writing it on paper, like you understand what's going on, but with Applied math, you just punch it into a calculator and you like, oh there you go, that's the answer. I don't know, Pre-Calc helps you, I don't know, you know what you're doing, like you have to think, you have to think, like you have to know what you're doing to get the right answer.

Opportunity for Reflection

A particular strength of the synergy that developed within the discussion groups was that students had opportunities to listen to each other and to think about what others had said before formulating their own responses. Again, many examples abound where students have taken a moment to reflect on their own stance. For instance, **Randi** (C-PCM) struggled with her own question of how to define math, knowing that she was taking it simply because it was a prerequisite to graduation. It was only after several other students had addressed the question that she attempted to express her own position. And even as she was framing her response, it was evident that she was continuing to examine the question from various aspects within her own mind.

> It's not like I go to Pre-Cal like thinking that I hate this all the time, but I would definitely enjoy other subjects more. But sometimes I like the way how in math you don't always have to discuss things and there's absolutely definitely a solution that you come to {others: yeah} and that's really nice because it's sure. And usually there's a method of doing things too. That's what I like about it.

Sometimes the reflection was a group effort, as in the next example. The current students had been discussing some of the frustrations they had experienced with the previous semester's Pre-Calculus exam. The conversation then migrated into musings on the overall merits of provincial exams:

Kiera (C-APM): I just don't think they should have provincial exams.

Wendy (C-BOTH): I don't think they should have any exams. Exams are pointless because I write my exam, because I cram the night before, and then I go onto a new subject and then I forget what I learned. Like I mean, I remember the basic most of it, but the majority....

...

Kiera: You don't really learn anything more in exams. Like, it's, yeah, like the most important stuff, like the basics, you'll know regardless.

Randi (C-PCM): Sometimes I learned things, though, for exams that I never got. Like before the exam, the Pre-Calc exam this year, there was a few things that I didn't understand the entire way through, and then finally I sat down, and like, ok, now I have to know this. And I actually learned some stuff. Like that doesn't make doing the exam worth it or anything, but...

Brady (C-BOTH): I think I agree with the idea behind provincial exams.

Randi: Me too.

Brady: And the kind of the leveling of the playing field in that everyone has to do it.

Randi: But that's not to say that I enjoy writing them or anything.

Occasionally, students' reflections were something of an "aha moment". The participants had been asked what advice they would give to younger students about which math courses to take in Senior 2. **Morgan** (C-BOTH) had been listening quietly to the conversation for a few minutes, when she suddenly came to an epiphany of sorts. She realized, seemingly for the first time, that her success in Pre-Calculus had come about due to her determination to ask as many questions as she felt necessary to understand why something was done the way it was. This had remarkable implications for her, as will be illustrated in Chapter 5 (see page 88).

Taking other perspectives into consideration

By listening to each other, students had many opportunities to consider how another person's opinion or experience differed from their own. It was evident that they not only recognized the difference, but respected it as well, allowing that the other person's viewpoint was as valid for him or her as their own was for themselves. For example, as the discussion proceeded, **Randi** (C-PCM) and **Leanne** (C-BOTH) realized that their concept of understanding math was very different from each other's. Here are the threads of a conversation that took place between them and a few others over the course of about twenty minutes:

Leanne: I think it would be easier if we had Applied, or for me at least, it would be easier to have Applied before Pre-Cal because then you understand the basics. And often when I understand the basics, then developing on that is so much easier.

...

Leanne: Like I feel like I understand way more in Applied than in Pre-Cal. (...) But, like the Applied was a bit more understandable and the Pre-Cal was more, I don't know, it was more, you sort of had to understand basics to understand what they were doing. And I was trying to understand the basics, so I couldn't really understand anything what they were saying, it was like another language.

...

Wendy (C-BOTH): ...for me Applied is harder, so having the Pre-Cal first really was more of a benefit than if I would have had Applied first.

Randi: And Pre-Cal seems so conceptual. Like it makes sense to learn the concepts or something before you are applying it to different situations.

...

Randi: But maybe for some people like Leanne, then maybe doing the application stuff first helps.

Leanne: Well when you learn the application, then you kind of like, I think back on how I did it and why I did it before and then that kind of gets me like a mindset of that, and then I just do it from that. And it just reminds me of all the stuff that I have to do. I don't know.

Randi: Yeah. So maybe it just depends on the way you learn and the way you think.

Clarifying Contradictory Comments

On rare occasions, students in the focus groups made what seemed to be contradictory statements over the course of the hour-long discussion. For example, **Kathleen** (F-BOTH) maintained that the hands-on and contextual approach to problem-solving in Applied was a much better way of learning for her (see quote on page 113), yet at another point in the discussion, she challenged the idea that contextualizing a problem led to a more meaningful understanding of a concept. In her mind, she would still only be applying a formula or equation, and would view the answer as just another number that had no meaning beyond being a solution to the problem (see quote on page 99). It was, in other words, no different that Pre-Calculus.

The comments certainly seemed contradictory, calling into question the purpose or the usefulness of contextualizing problems. Fortunately, the research design allowed opportunity for follow-up, at least with the former students. Kathleen was able to clarify her position, as will be discussed in greater detail in Chapter 6.

Follow-up Interviews

I had several reasons for wanting to conduct follow-up interviews. Most importantly, I was looking for an opportunity to investigate in greater depth some of the individual comments or general themes that had arisen in the focus group. I hoped that students would provide more clarification of the things that had been said earlier. Secondly, I wanted to 'read back' my perceptions of the focus group transcripts to the students, inviting them to comment on my interpretation of what I had heard. They could provide legitimacy to my interpretation of the emerging themes, or they could correct any misunderstandings on my part. Finally, lest any student had been somewhat intimidated or shy in the larger group setting, I wanted to provide a more intimate setting for them, thereby hopefully allowing them to speak more freely. Like the focus group before them, the follow-up interviews proved to be a strong source of data, generating more of the rich insights I had hoped to collect.

I made four deliberate pairings among the students, hoping to take advantage of existing friendships (thereby instantly achieving a higher level of comfort), course choices, or shared mathematical experiences. **Angela** (F-APM) and **Ashley** (F-APM), formed the first group, and were paired because they were both taking only Applied in Grade twelve, and so would be able to focus on the learning experiences in a single course. **Kathleen** (F-BOTH) and **Lillian** (F-BOTH) were paired to provide a Pre-Calculus only perspective, because their remarks during the group discussion seemed to convey the idea that Pre-Calculus was the "real" math. **Michael** (F-BOTH) and **Thomas** (F-APM) had shown strong preferences for Pre-Calculus and Applied math, respectively, and knowing that they were good friends but with polarized views, I was hoping they would engage in a bit of a debate. **Cody** (F-BOTH) and **David** (F-BOTH) were not only close friends, but had also embraced the underlying principles of the Applied math course more than any other students. I was hoping to come to understand what made that particular course so important to them as part of their mathematical learning experience. Some excerpts will demonstrate how these follow-up interviews served to illuminate some of the themes emerging from the original focus group.

Exploring a concept or theme in greater depth

The role of technology in mathematical learning had appeared as one of many themes in the focus groups, but it became quite prominent in the follow-up interviews as it was explored from a variety of different viewpoints. **Angela** (F-APM) and **Ashley** (F-APM) depicted technology as one of the defining characteristics of the Applied course:

> **Angela**: Well, I would say, Applied math was about, I mean we learned the basic math concepts, … and how to do it on the calculators, how to, how to find out the

answers on the calculators and how to graph it. And we learned how to process that, and, um…

Ashley: I think it was basically more technology-sided than PreCal is, because like she said, we did lots of work on the graphing calculator and on spreadsheets and then we, like we took the math and we kind of used it, like in projects and stuff like that, kind of applied it, so, you know, hence applied math. So I think it just, it's more to the technology side of things than just knowing formulas and boring stuff like that. That's how I would basically describe it.

Students went on to describe the many ways in which they used the calculator. They generally acclaimed technology as being an important means of learning mathematics, believing that knowing how to use it effectively would be an essential skill in their lives outside of school. Yet they exhibited mixed feelings about how much to rely on technology. Some distrusted it, feeling that using a graphing calculator was like cheating, but others firmly rejected that notion, believing it to be an important tool in a technological society. Chapter 5 will explore the theme of technology in more detail.

Clarifying a theme (individual)

During the focus group, **Angela** (F-APM) had stated that she was "just not a math person", a statement echoed by **Ashley** (F-APM) in the follow-up interview. I was curious to know what they thought a "math person" was.

Angela: Yeah, like a math person, I don't know, I think they appreciate math. And they can see the deeper side of it. Like in our last conversation, Michael started talking about how, I don't remember exactly what he said, but he said something about math, there's something deeper in math, like it's not just numbers. And like, I was like, how can you think that, cuz he's a math person, he loves math. Like he can see a deeper side in math than most people can.

Ashley: I think people who are good at math enjoy math because they can do it, and they enjoy it and I guess they have fun doing math, right? But then there's people that struggle with it, and then we end up hating it, because we just cannot grasp it and do good, so we just end up hating all math.

Angela: He said, math is life, that's what he said---

It was also interesting to pursue what they thought would be different for them if they were "math persons":

> **Ashley**: Things would just click all the time. I would hear something once, or be taught it once, and then I would just know. And then I could do the questions and I could do the test and I would be happy about it, because I understand something and I'm proud of that.

In another conversation, the quest for clarification happened spontaneously. Here **Thomas** (F-APM) and **Michael** (F-BOTH) engaged in a bit of debate, as I had been hoping they would. We had been talking about the role of technology in learning mathematics, and Michael was quite reluctant to cede to it the prominence that Thomas wanted to give it, believing strongly that understanding the process was as important as using the tool.

> **Thomas**: It is good to know it on your own and with the technology. But most of the time technology is faster and more efficient.
>
> **Michael**: Is faster better? Like why is that better?
>
> **Thomas**: Why is it better? Cuz...
>
> **Michael**: You want to, you have stuff to do, or...?
>
> **Thomas**: Yeah, well normally if you are using technology forever, if you want to get it done, you don't want to just sit there looking at numbers...
>
> **Michael**: No you don't, you want to think about it. You want to...
>
> **Thomas**: Oh, in the workplace you don't want to spend all day calculating numbers. You want to get it done quickly. Adding machine or computer. (pause) If I had to write down all my cheques at work, then add them up...
>
> **Michael**: Well you're thinking of, you're not thinking of a, you're thinking of just number-crunching. I mean, yeah, you want to get number-cr...like say you have stuff to add up, yeah.
>
> **Thomas**: Yeah, technology helps.
>
> **Michael**: But if you're doing different stuff.
>
> **Thomas**: Such as...?
>
> **Michael**: Indefinite integrals. (laughs) You want to understand what you're doing.

What **Michael** succeeded in doing in this exchange was to differentiate between using mathematics and learning mathematics, and how the role of technology might be quite different for the two. **Thomas** was by this time already in the work force, employed by a financial institution, and so working with numbers constituted a large part of his day. Efficiency and accuracy were important in his job. Technology was the medium by which to achieve these, but the procedures used by the technology were no longer of issue to him. **Michael**, on the other hand, was a first-year engineering student, so he was still very much engaged in the learning of mathematics. To him this meant becoming intimately familiar with the procedures and processes by hand before he would feel comfortable using technology to solve problems.

Clarifying a theme (general)

During the focus group I began to develop a sense that most students in this group viewed Pre-Calculus as the harder course, and Applied as the easier course. And while this seems to be a prevailing attitude among high school students in general, I had had recent reason to challenge it in my own mind already:

> I overheard three Grade ten students talking at the end of [Applied] class today about the differences they perceived between PreCalc and Applied. Two of them maintained that PreCalc was really hard compared to Applied, but the one student, who incidentally had the highest mark among the three, begged to differ. She maintained that PreCalc was way easier, because she didn't have to think. All she needed to do was learn how a formula worked and when to use it – after that, it was simply a matter of plugging in numbers, repeatedly. In Applied, she said she actually had to think about what the problem was all about and how all the various parts connected before she could begin to solve it. She found that a far more difficult endeavour, and hence she found Applied harder. [my reflective journal, January 12, 2004]

When I put the question to the students in the follow-up interviews, most seemed to disagree, maintaining their opinion that Pre-Calculus was the harder course and explaining why:

> **Lillian** (F-BOTH): Cause it just, it asks more of you. You know, PreCal just, it makes you, you have to think, you can't just go through the course. To be honest, Applied's a bit easier because with the projects and it's more people oriented so you can work off of your friends a lot more. Like I did in PreCal, but it was different. It wasn't the same. Like I didn't work with my friends the same way in PreCal that I did in Applied, and in Applied it was more fun, so you didn't have to work as hard, and the math wasn't as hard, like the concepts weren't as hard. Yeah.

> **Kathleen** (F-BOTH): Yeah, I think also in Applied math it was easier because it was more broken down. Like in our textbook and everything else like that, each of the different steps was a different answer, so if you got 'a' wrong, you could still get 'b' right, kind of thing. Like it was very divided up like that so the marks were easier, but in Applied, I mean in PreCal, if you got a question wrong, you got a question wrong. But Applied math, there were so many different aspects to the question that you could get some right and some wrong, so in that way it was kind of easier, because things were divvied up a bit easier.

Yet students did put some effort into thinking about why the student described in my journal might have thought Applied was harder, considering things like what people might want to get out of math, or what type of learners they might be. This will be discussed in further detail in the next chapter.

Reading back

The final way in which I used the follow-up interviews was to 'read back' my perceptions to the students and ask for their comments. This would serve to either legitimize my interpretation of a particular topic of discussion, or students would have a chance to set me straight. Two examples will suffice. In the first instance, I had gotten the sense that for students who took both courses, the order in which they were taken was important. This was verified by most of the students whom I asked, yet not always for the reasons I had anticipated. I had thought that students would prefer one order over the other in keeping how they preferred to learn. To some extent, this was true, as will be shown in the next chapter, but they also placed high priority on the marks that they would be able to achieve in either course:

> **Angela** (F-APM): I think that's very important, because we definitely learned far better with Applied math first and then Pre-Calculus second. Because Applied math was just learned a bit, I don't know, it was learned differently. We were taught differently how to do the different problems. And then in Pre-Calculus we could use that in the class. Whereas when we had Pre-Calculus first, it was harder. It was, we did a lot worse markswise. And of course, and then in Applied math we could do it okay, because we had already learned it in Pre-Calculus, but for the sake of getting better marks in Pre-Calculus, it was easier to take Applied first.

That they could usually achieve better marks in the second course was a common consensus among the students, but it was interesting to note that they also placed a greater priority on achieving those higher marks in Pre-Calculus, rather than Applied.

In the second instance, I was looking for feedback on my sense of disappointment at the responses to the final question posed in the focus group. I had asked students to reflect on how they used mathematics now that they were out of high school. I recall feeling quite deflated when almost none of them could find any sense of relevance between the mathematics they had learned and their daily lives now. They seemed to connect mathematics only with a classroom context, and had difficulty relating how they could possibly use it outside of a classroom. In the follow-up interviews, students assured me that it was only a matter of perspective, that some distance was required before they began to really use mathematics:

> **David** (F-BOTH): Right now, you know, when you're a kid, where we are, you know you don't have to do a lot outside of school. You know, you go to school, you do all your work there, you go home and you watch TV or play computer. You know, it's later on in life when you're trying to calculate your yield off a field or, you know, how much profit you'll make, it will come in handy. It's just up to the point where you really start doing things and being involved in complicated things, it really doesn't seem to apply to you at all.

They declared that mathematics was important and that they used it all the time, they just weren't always aware of it:

> **Cody** (F-BOTH): I think most people do tend to actually use math, they just don't even really realize that they are. You know, it's like a quick subconscious little calculation; it doesn't give you an exact number. Just a quick little subconscious calculation tells you generally something. You know, and you don't even realize that you're doing it, what you do. So I mean if you didn't go through all this math, you wouldn't necessarily be able to do some of this stuff. You know, I think a lot of people don't really realize is how much they actually use math without really, not necessarily even in a math context. You know, just in anything else.

As is evident from the excerpts quoted in this section, the follow-up interviews provided rich opportunities to explore the attitudes and experiences of students in much greater depth than would have been possible in the focus group. It would have been good to do follow-up interviews with the second focus group, the current students, as well, and I might have been tempted to do so, even though it wasn't part of the original research design, but I chose not to, for several reasons. First, their interactive writings (which are described in the next section) were providing me with some additional insight into the way they were thinking about mathematics. Second, the synergy in the second focus group already invited more in-depth exploration and opportunity for clarification within the discussion itself. Third, the ideas that were being stated by the current students were already echoing those of the former students, so there was reason to believe that I was reaching saturation in my data collection. And fourth, by the time I had finished transcribing the recording of the second focus group, I had many hundreds of pages of data that would need to be submitted to phenomenographic analysis, and the simple logistics of that seemed to dictate putting a limit on how much more I would want to collect.

Interactive Writing

My reason for including interactive writing as a data source was to move students away from metacognition and consideration of personal experiences, and to focus instead on how they actually thought about a newly introduced concept or solved a mathematical problem. It was meant to lend an element of immediacy to the data – rather than asking students to reflect on past experiences, they were focused in the present on solving a mathematical problem. Being given only a limited amount of time (about 10-15 minutes at the end of a math period), they needed to respond with the first thing that came to mind. The interactive writings provided me with some insight into their mathematical thinking, and allowed me to draw some comparisons among the students and the approaches they were taking. A few examples will suffice.

The differences in how students thought about a new concept were illuminated as students responded to prompts after their introductory lesson(s) in the unit on sequences in both Pre-Calculus and Applied. Of course, the responses were a reflection of how the lesson had been presented, but nonetheless, it does show a little of how the students were approaching their thinking about the subject. I had anticipated that most of the Pre-Calculus students would explain the day's lesson on sequences in terms of definitions, formulas, and perhaps some examples, whereas the Applied students would write more generally about sequences as being number patterns whose shape and form could be explored in a variety of ways. The responses from the Pre-Calculus students after their first class in sequences were indeed quite homogenous. They showed little evidence of considering sequences conceptually; rather they were content to simply summarize their notes from the lesson, in a manner similar to this example:

> **Gabrielle** (C-PCM): Today in Math class I learned that a sequence is a function whose domain is the set of consecutive natural numbers. I learned about the different types of sequences there are. These include arithmetic and geometeric sequences. An arithmetic sequence is where the difference between consecutive

terms is a constant. A geometric sequence is a sequence where the ratio between two consecutive terms is a constant.
I also learned different methods of representing sequences. These methods include ordered pairs (1, 5), (2, 10), (3, 15); t_1, t_2, t_3 ie) t_3=15 t_n, a_n or f:k F:1=F(1)

A far more varied response to the first question was evident among the Applied students after their introductory activities in the sequences unit. They had spent a couple of classes working on creating fractal patterns before beginning a more detailed study of sequences. Although students had recognized numerical patterns in the iterations, there had not yet been a discussion of mathematical formulas that might be applicable. **Kiera** (C-APM) based her sense of sequences solidly in her work with fractals (spelling mistakes appeared in the original).

> In math we've been learning about fractals. Fractals are repepeted patterns on a line over and ovr again. Each fractal has some sot of math equation that can be used to discover the length, # of sides, and whatever else might involed, so one can correctly predict certain things without actully making the pattern. Each fractal starts with an original and every repreitiontin after is called an itieration. Each iteration is realted to the pervious ones and to the followig ones. Fractals can be used to make crazy desings. Fractals can be made with paper, but also on computer.

Morgan (C-BOTH) sought to draw some connection between her work with fractals and a mathematical representation:

	Folds	Corners	Sides
1st	2	13	4
2nd	3	7	8
3rd	4	15	16
4th	5	31	32
5th	6	63	64
6th	7	127	128
7th	8	255	256

I'm going to show you the dragon fractal. As you can notice, the # of sides – 1 = # of corners iteration + 1 = # of folds so for this fractal, if you know the iteration # and you want to find the # of sides or corners, you use this formula. $2^{(\text{iteration\#} + 1)}$ = # of sides. There is a pattern with all fractals. You can get a formula or predict the next step by seeing a pattern.

And while most students liked the investigative approach to starting a new topic, some students seemed rather lost. **Leanne** (C-BOTH) seems not to have developed a sense of pattern and mathematical relationships in her work with fractals. She knew they had something to do with mathematics, but couldn't find the connection:

> I think that fractals are basically really complex patterns and/or sequences. I think that understanding fractals probably is a key to understanding harder and more difficult mathematics, but we haven't worked with them (fractals) enough for me to understand well enough to explain it, yet.

The second interactive writing assignment illustrates how students dealt with contextual problems in both classes, and the reasoning processes they used. Pre-Calculus students had just finished a lesson on the sums of sequences and were familiar with the formulas, but because they had been working only with equations, I expected at least some of them would have trouble relating the formulas to a real-world scenario. Half the students were able to answer the question and explain their reasoning quite clearly, as shown in Figure 9 below.

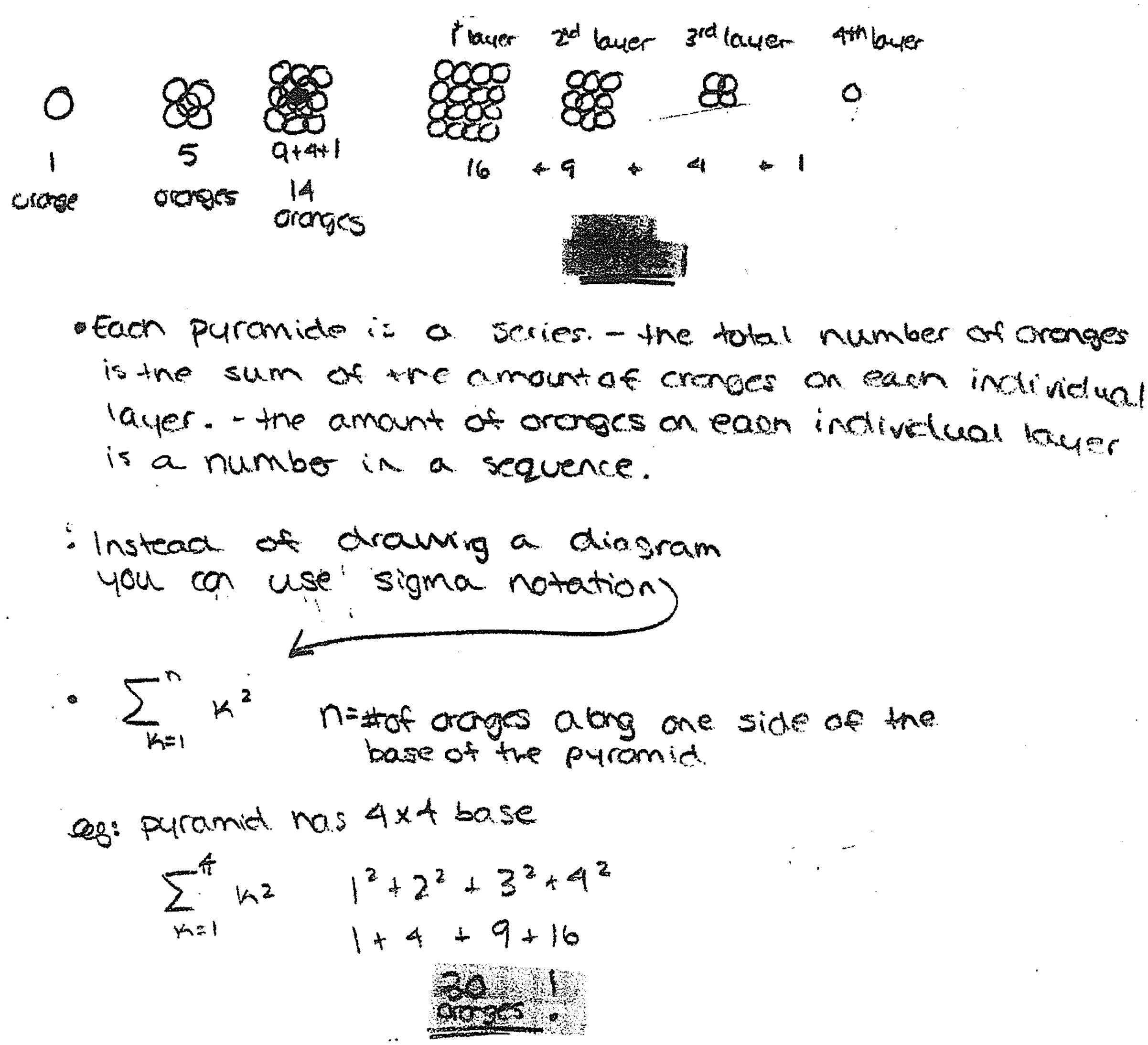

Figure 9: Heather's response, January 12, 2004

But others like **Ophelia** (C-PCM) assumed that the formulas they had learned that day had to fit the situation and struggled to find a way to use them, as shown in Figure 10 below:

You first have to decide on a formula for the pyramid.

1, 4, 9...

$t_n = n^2$

This formula tells you how many oranges are in each layer of the pyramid.

The formula for a finite geometric sequence is:

$$S_n = \frac{t_1(1-r^n)}{1-r}$$

$$S_n = \frac{1(1-2^n)}{-1}$$

So if there was a 5-layered pyramid of oranges you'd say:

$$S_n = \frac{1(1-2^5)}{1-2}$$
$$= \frac{1(1-32)}{-1}$$
$$= 32$$

CHECK:
1, 4, 9, 16, 25
$1+4+9+16+25 = 55$ X

Never mind Sam. it didn't work. I don't know how to do this one.

Figure 10: Kate's response, January 12, 2004

Applied students started the semester with a unit on periodic functions. They spent several classes working on contextual problems before they received their first prompt. I expected that since they were more familiar with this type of problem solving, and especially because they had had more opportunity to practice it, most of them would have no difficulty in linking the information they were given to a sinusoidal expression that would allow them to predict future values. All students did in fact recognize that links existed, but to my surprise, only a quarter of the students were able to make these links correctly. Yet in spite of getting incorrect results in the end, the reasoning that the others used in making the connections between data and equation was for the most part sound: their mistakes were typically not errors in connecting the information with an equation, but errors in manipulating that information first. For example, **Kiera** (C-APM) knew that the '*a*' value in the equation (amplitude) was determined from the maxima and minima, but she recorded the amplitude as the full difference between them, rather than the half it should have been. **Leanne** (C-BOTH) recorded the sunrise time for December 21 as 21:12 in her work (rather than 9:12), and was able to determine an equation that fit her data, but was unreasonable with respect to the context. Interestingly, students who made these types of mistakes were quite vague about their predictions for other sunrise times, choosing to explain how to get the value instead of making an actual prediction:

> **Kiera**: To predict what the time of sunrise is for today enter the equation into your graphing calc. Using the trace feature find day 57 (that's what Feb. 26 is) and record the y-coordinate.

I wondered whether this was because they knew that something was wrong with their equation model and chose not to record an unreasonable answer, or whether they had simply not tested it (either through oversight or lack of time). My response to **Kiera** raised the question of whether she had checked her equation for reasonableness:

> You have come up with an equation using values for amplitude, period and median. How would you go about testing to see whether this is a reasonable estimate of the sunrise pattern? Are there any points you could use as test points? For example, if I graph your equation, I get a maximum at (92, 12.2). This would seem to indicate that somewhere around the beginning of March, the sun would rise shortly after noon, yet we are told in the problem that the latest sunrise time is 9:12 a.m. Similarly, the minimum is at (277, 0.2), meaning that sometime in September, the sun would rise shortly after midnight. Is this reasonable, given your knowledge of sunrise times in Saskatchewan?

The interactive writings proved to be a surprisingly rich source of data, supporting the statements students eventually made in the focus group discussion, particularly with respect to how or what they thought about mathematics. The final piece of information that formed the body of my research data was minor in comparison to the preceding three, yet it served its own purpose.

Personal Journal and Field Notes

During each of the audio taped focus group and follow-up interview sessions, I kept field notes. They were never intended as a data source, but would provide backup in case of technological glitches, and as way a recording thoughts and ideas that I might want to return to later in the conversation. More important was my personal journal which I kept prior to and during the data collection of this research project. Here I recorded my own thoughts and perceptions with reference to the topics I was asking the students to discuss. I recorded anecdotal information of conversations I was part of or that I overheard (as shown in the example given in the previous section). I recorded my reaction to how a session had gone and my impressions of what the important themes of discussion had been. I listed topics that I wanted to explore in greater depth. By recording these thoughts and impressions, I was able to bracket my own perceptions prior to beginning data analysis. The journal provided me with a powerful planning tool for sessions that had not yet taken place.

In summary, then, multiple forms of data collection allowed me to probe the described experiences of the research participants from a variety of perspectives. It allowed students to share ideas, both with me and with each other, to illuminate their sense of what learning math had been like for them in the two math courses. The discussions were far-ranging, but I am confident that the structure of the data collection allowed students to focus on that which they considered important. Many themes emerged, but as will be portrayed in the next chapter, common threads related many of these themes.

Chapter 5

The Emerging Image: Categories of Description

Phenomenography probes the described experiences of research participants, examining as wide a range of lived experiences as possible, and then searching for the common elements across those experiences, distilling them to a few *categories of description* that feature the critical aspects of the variations among them. As I read and reread the data, examining it from various perspectives, trying to look at it through different lenses each time I read it, distinctive themes began to emerge. These themes eventually coalesced into three categories of description: two were more general in nature, and one was more targeted.

Two categories focus directly on the students and their interactions with mathematics. The first provides an overview of what students perceived the **nature of mathematics** to be – what math was all about, and what it was good for. The second focuses on what they perceived the **nature of learning** to be – what it meant to understand something, and how one went about achieving that understanding. The third category also focuses on student perception, but this time it is deliberately viewed through the lens of one of the key differentiating factors between the two mathematics courses: the **role of technology**. It looks at what students did with technology and how they perceived technology to help them (or hinder them) in learning mathematics.

In this chapter, then, each category of description will be detailed. The many quotes from the focus group discussions and the follow-up interviews will hopefully give the reader a glimpse of the wide range of experiences and perceptions described by the students as they reflected on their study of mathematics.

Student Perceptions of the Nature of Mathematics

Throughout the focus groups, follow-up interviews, and interactive writings, it became increasingly evident that student perceptions of learning in either or both of the math classes were filtered through their opinions of what mathematics was all about – its nature, its purpose, and its relevance. There was common agreement when we spoke of the nature of mathematics in general terms, but when we got a little more personal, some differentiation began to show. Some saw math as being relegated only to the classroom and their learning of it simply as a prerequisite to graduation, whereas others saw it as a dynamic force that played a significant part in their daily lives. Whatever they thought of the nature of math, it seemed to influence the experiences they had with learning it and the attitudes they brought to the courses. It defined their stance towards mathematics, and delineated for them distinct purposes of the Pre-Calculus and Applied courses.

Because people's underlying notions of the nature of mathematics seemed so important, I took the opportunity to pose some direct questions about what they thought mathematics actually was, and how they used it. As expected, perceptions among the students were wide-ranging, as will be illustrated by numerous excerpts from the data set. Outlined below are some of the key ideas students voiced about their understanding of the nature of math, both generally and more personally, and how this understanding translates into their perception of what the two math courses are all about.

The general nature of mathematics

In their initial reactions to the questions about the nature of mathematics, students started by stating the obvious: their comments ranged from **Brady's** (C-BOTH) flippant "fun with numbers" to **Thomas's** (F-APM) more staid phrasing "the study of numbers and relations between numbers". **Kathleen** (F-BOTH) saw math as "doing algebra and everything else like that, solving problems and stuff." **Morgan** (C-BOTH) described learning mathematics as being a progression from simple to more abstract, culminating in a collection of formulas and procedures:

> I think, well when you're like a kid, like this is your math, like one, two , three, four, five, six, seven, eight, nine, ten. But when, I guess when you get older, then when you hear about logarithms and stuff, then you see it in your head. And then, like for me, that's how I know, that's how I see math. Like I see, whenever I do tests and stuff, I see my notes in my head, and I see that and then I kind of try to remember what was on the page and then do the stuff. So, I don't know how you guys see it but I usually see math as formulas and, yeah.

And once they had become familiar with formulas and equations, understanding them and developing procedural competence with them was considered by many to be an integral part of the nature of mathematics:

> **Cody** (F-BOTH): ...to understand math, would be to...look at equations and understand how these various equations work. And to be able to manipulate different equations into various things so you can understand these various

> variables and various numbers and equations and graphs and setups and everything. To understand how they work, how you can manipulate them, and I guess even in some cases, how you can change one to the other and you know, back and around and so forth.

As students became more reflective in their thinking, they uncovered other aspects of what they perceived the nature of mathematics to be. Many students commented that mathematics seemed to induce a different way of thinking. **Angela** (F-APM) stated that for her, thinking mathematically was "abstract in some way, cause you can't see it, you can only just think it up in your head and write it down." **Lillian** (F-BOTH) didn't like the term "abstract", preferring to characterize her thinking as "very structured". **Kathleen** (F-BOTH) agreed, describing mathematical thinking as being "a very systematic way of thinking, because you always had to do the first step, and then the next step was always the second step because you couldn't get to the third step without doing the second step." **Randi** (C-PCM) also echoed that sentiment, stating that "... I think mainly math for me has been... like using your brain in a different way than you do in other courses because it's, you have to think differently, because it's a logical progression from step to step and that's really important because you can't just do this abstract thing." **Kiera** (C-APM) liked the way that mathematical thinking allowed her to organize information: "Something that I've learned a lot from math, maybe, is like getting all the information that you need to work through a question. Like separating what you need and what's useless." And **Morgan** (C-BOTH) saw mathematical thinking as being a great form of mental exercise:

> ... to make your brain think like that, that's, like, I might sound weird saying this, but that's healthy. Like it's healthy to make your brain think [background laughter]. Because, I'm serious, like otherwise if I didn't take math, I'd be like the stupidest kid in the whole world. But because I make my brain think, I can, I know in just everyday situations, I can think better. Like, it's weird.

Problem solving was considered a major component of mathematics by most of the students, whether it was thought of in terms of types of questions posed in math class, or as a general skill that was developed as a result of their math studies. **Kathleen** (F-BOTH) thought of problem solving in a contextual sense, where mathematical processes already learned had to be applied to a specific scenario:

> Well, I think PreCal was very systematic, like I said before. How you have a formula and you work through it and you get the answer. And to me that's not, I guess, true problem solving for me. Like problem solving for me was like, you're given a monthly budget, and find out how, with your salary, you can, you know, you can accommodate your needs and stuff like that. And in that we needed to use formulas, and we needed to use math, but it was, I guess, I don't know, a real-life situation, that you had to work through. So to me that was problem solving.

But **Lillian** (F-BOTH) preferred to generalize it, thinking that any instance where mathematical thinking needed to be applied, whether in a contextual situation or in a more abstract sense, could be considered problem solving:

> I guess I'd kind of have to disagree with [Kathleen], because I was just thinking back to all of the problems that we did. Like to me it's the same thing, it's just that one has, you know, a different name or a different situation to it, but I guess it's just more, you know, in a mathematical sense, you're thinking in mathematics, not in life situations, but I'd still look at it as problem solving. It's just in math language.

Cody (F-BOTH) saw problem solving as a more global skill that had been honed in the study of mathematics:

> I think I learned most of my problem solving skills through math, when it comes to anything. You know, you look at the problem, you know, adapt it, in a sense. The way I used to do math, or some of the PreCal stuff is, if I didn't understand how to do this, this equation or something, look to the back, get the answer and work backwards. You know, figure out how they got that. You can almost do that in real life too. Look at what you're supposed to be able to do out of this, and how it's working now, and now it's not working. So you know what you need to do so you can kind of figure it out to how you need to simply have it done. You know, you work backwards in a sense.

In addition to problem solving, students viewed mathematics as being all about making connections, where as **David** (F-BOTH) so delightfully stated, one could "take information and use it with other information to get new information." Students who took Pre-Calculus, for example, commented favourably on its cumulative structure. Rather than being a series of seemingly unrelated topics, it allowed them to build connections between the new concepts they were learning with those they had studied previously:

> **Lillian** (F-BOTH): ...that's part of PreCal, is not looking at it as individual little pieces, as being able to look at it as one big picture, putting it all together.

> **Kathleen** (F-BOTH): Which I think is why our course was cumulative, where it all, cuz slowly we figured out how it all works together, how we can use things we learned at the beginning of the year to work through problems at the end of the year.

Not only could connections be made among various mathematical concepts, but they could be made between math and real world phenomena. For example, **Cody** (F-BOTH) commented on how his understanding of the sine function was strengthened by an exercise they had done on the swing set in the park:

> So, you know, you're still working with the same equation, but because you're looking at it with this real life interpretation it seems easier. And so it's not that you couldn't do it before, it's just now you can do it now because it clicks in somewhere. That you know, you can see an actual example, like the swing distance to the ground kind of thing. You know, it's like you can kind of understand why you've got that sine shape. You know, because you can think about a swing and you can think about the point that you are looking at.

As they considered it further, students seemed to identify a duality in the nature of math: they drew a clear distinction between the mathematics of the classroom and the math that they used every day. With the exception of the units on personal finance, few students could think of ways in which they used math in their daily lives. Other than using simple operations such as addition, subtraction, multiplication, and division, students thought they rarely used any of the mathematics they had learned in their senior years. **Angela** (F-APM) referred to classroom math as "in your face type of math" and remarked that she found little use for it in her own life. **Kathleen** (F-BOTH) pointed out that even basic number operations needn't be used often, "because usually technology does it for me, so I don't really have to think mathematically because something is doing it for me." In only one instance was a student able to point specifically to using a concept learned in high school math that went beyond simple number operations:

> **Brady** (C-BOTH): Oh, I use math all the time. Um, trigonometry is probably my big thing, because I build ramps a lot. And I'm still in the process of trying to figure out some three-dimensional trigonometry stuff, and that we didn't learn, but I don't know if anyone should really have to learn that unless they need to.

Responses to a question in the interactive writings also showed that students sometimes have considerable difficulty relating the mathematics of the classroom and the mathematics of real life, even while they are in the process of learning it. For example, after completing the unit on sequences, **Taylor** (C-PCM) could think of only one reason to learn about sequences:

> One day if you want to become a grade 12 math teacher you will be required to know how to do this so you can teach it properly. Other than that I can't think of much of a use for it.

Yet even though they couldn't always see the immediate practicality of classroom mathematics, students tended to remain relatively open-minded about why they should learn about the various concepts. **Brady** (C-BOTH), the ramp builder quoted above, elaborated on this in his interactive writing:

> I think that learning things you don't necessarily find useful is still healthy. It expands your capacity for learning and keeps your brain sharp. Sequences in and of themselves are something you do run across often and learning about them might come in handy. I doubted the relevancy of trigonometry and circle measure until I started building ramps and now I use those skills on a daily basis. May be someday we will find sequences and series useful as well.

Indeed, many students seemed to recognize that even though they could not always draw direct correlations between the classroom and what they did in their daily lives, that in no way diminished the value of what they learned. The nature of math might in fact be of a dual nature for them, but the bridges between theory and practice did exist, even if they were not explicit. **Kathleen** (F-BOTH) summarized this quite eloquently:

> I think, like I think if we wouldn't have the math education, I don't think we'd be able to do stuff that we can do now, but we just don't realize that we're using that kind of math, that we're using the skills that we learned in those math classes outside of math class, because we're not using it directly. So to us it's not processed as, oh, you know, I took this, or I remember this math class when we learned this. It's just something that we naturally do, but I think without the math courses that we've taken, I don't think we'd be able to do them. We just don't consciously recognize that we are doing math.

The personal interaction with mathematics

The nature of mathematics also had an affective component – students either liked math or they did not. Through the many interactions they had had with mathematics as they studied it, some became very comfortable with it, enjoying its many different concepts and techniques. Others became increasingly frustrated, becoming overwhelmed with the procedures or questioning the relevance of math.

Those who enjoyed math wouldn't necessarily categorize themselves in any particular way. They simply considered their enjoyment of math a personal quirk, a good fit between the way math worked and how they liked to think. **Kathleen** (F-BOTH) liked algebra because she enjoyed "arriving at an answer". She states, "it made me feel like a scientist… I kind of enjoyed, you know, figuring it out on my own, using my own…mind, my own abilities, to figure out problems…" The challenge of mathematics was what appealed particularly to **Lillian** (F-BOTH): "I just liked that I kind of stepped up to the challenge…it just kind of made me feel good to know that I could do that." **Michael** (F-BOTH) liked the fact that he could approach a problem from a variety of perspectives: "I appreciate math when you can find the solution any kind of way." And **Heather** (C-BOTH) enjoyed seeing how far-ranging mathematics actually was

> It was interesting to see like how many different things are related by math things? Like, when you go farther into math, then you can see so many different, like there's a formula for pretty much everything. And like, you don't know, we don't even know, like, we like touched the surface or whatever, but it's just interesting to see, like you can use logarithms for this and you can use this for this and like you can figure out pretty much anything.

In contrast, those who didn't enjoy math or found it difficult quickly labeled themselves as being "not a math person". For some, this self-categorization came about because of a sense that math didn't have any personal relevance. **Randi** (C-PCM), for example, bluntly states "I'm not a math person…I do math because I want to graduate, not because I particularly enjoy it." But more often than not, students who referred to themselves as not being math persons did so because of previous difficulties they had encountered. **Angela** (F-APM) recounts: "in the previous two years I had found it really really hard and um, I'm just not a math person, so in grade 11 especially, I struggled a lot." And compounding the frustration they faced was the fact that they had painted a picture for themselves of what it meant to be a math person and how unattainable that would be for them:

> **Ashley** (F-APM): I think it means that when it's taught, you just, you just get it. And when you are given questions, you can do and you just understand it all the time. And then on tests, you barely have to study and then you get like 98%. That is a math whiz, I think. And for me, math is just, it takes me a while to grasp the concept, and once I do, I can do it, but then if I get, if suddenly you throw in a different worded kind of question, I'm lost and I don't know what to do and then I would start failing tests. I'm just not a math person.

As students continued to examine their own personal stance towards mathematics, they also took into consideration what they thought it meant to understand mathematics. Many students situated their sense of understanding firmly in the procedural realm, believing that to understand math was to know how to do it. Others preferred to know first why things were done as they were, they wanted to understand the broader concepts before spending time on the details. This differentiation between how and why will be discussed further in the section on the nature of learning.

The nature of the Pre-Calculus and Applied courses

Just like they formed distinct opinions about the general nature of mathematics, students in this study identified definite differences between the two math courses – in their rationales, in their approaches, and in their perceived level of difficulty.

Differences in rationale

Opinions about the purpose of each course were almost unanimous. As pointed out by **Heather** (C-BOTH), Pre-Calculus was the precursor to advanced study: "…a lot of it's preparation for if you want to go further into that, like in university or whatever". This proved to be true for people like **Michael** (F-BOTH), who was now a student in the faculty of engineering:

> It gives you the basis of what's to come later. Like now, like with university math, like I know why I took PreCalc…it helped a lot, understanding what I'm doing now, I can actually apply it.

Of course, not all students anticipated going to university, especially when they were first making course selections in grade ten. Yet many considered it an important part of their program of studies, should further education be even a remote possibility for them in the future. As **Lillian** (F-BOTH) states, "I took it because I wanted to keep my options open again for university." But as their plans began to take shape and they realized it wasn't a requirement, it became a dispensable course for some. For example, **Angela** (F-APM) determined that advanced mathematical studies were not part of her university plans, and so the rationale for taking Pre-Calculus was lost: "…so I decided, I had looked into what I had wanted to go into university and saw that I did not need Pre-Calc and so right there, I said ok, well, I don't have to take it, so I won't."

But even more important than being a precursor to university studies, several students talked about Pre-Calculus providing a basic foundation. **Kathleen** (F-BOTH) sometimes referred to it as "pure" or "raw" math, the underpinnings that allowed her to understand the mathematics that she did in other courses. **Heather** (C-BOTH) mentioned how she did not think she would be able to do very well in Applied without the "basics of certain things" that she had learned in Pre-Calculus. And **Lillian** (F-BOTH) reflected on similar experiences in being able to use the skills she had learned in Pre-Calculus in other classes like Chemistry.

Applied math, on the other hand, was more immediate and more personally relevant. As **Lillian** (F-BOTH) suggests, "Applied, it's more getting you ready for life". When asked why it might be useful to take a course such as Applied, students often mentioned the practical skills they learned, referring frequently to those dealing with personal finance issues. **Morgan** (C-BOTH), for example, remarked that Applied

> "… helps me be more comfortable for in the future, cuz I know that when I'm going to be investing in RRSPs or whatever those other things are, or even if I want to find something where I can invest my money properly, then I know how to do it and I won't feel like I'm totally out there.

It was interesting to note that not once in any of the conversations did students ever associate Applied math with continued studies in university, other than it allowed them to satisfy general entrance requirements. It was obvious that Pre-Calculus solidly occupied that niche in their perceptions.

Differences in approach

When students considered the approaches taken in both courses to teaching and learning, the list for Pre-Calculus was very short. It was all about routine and repetition. Aside from the material covered, classes did not vary in their structure. **Lillian** (F-BOTH) describes a typical class as

> ..we would get into class and then um, our teacher would go over some problems that we had with our last questions and then he would go over some stuff that we were, that might help us in our next worksheets, and then we were just left to work. That was pretty much it.

Students characterized the approaches taken in the Applied course as being much more varied. In addition to typical worksheet-style or textbook assignments, they mentioned things like "using technology", "tutorials", "group learning", "experiments", "math projects", "creativity and imagination", "hands-on". **Brady** (C-BOTH) seemed to appreciate the variety:

> I like that in coming to Applied I'm not exactly sure what to expect...I like a bit of the mixing it up and, it's a bit more variety...you show up at Applied and like, I don't know what we're going to do today.

Differences in the level of difficulty

Students perceived a marked difference in the level of difficulty with the two courses, although their rankings and the reasons behind them varied considerably from person to person. Curiously, students' ideas of whether a course was hard or easy seemed to split fairly neatly between the two groups. Recall that in the opening pages of Chapter 4, I described the group of former students as seeming to connect well with the Applied way of doing things, whereas the group of current students found the Pre-Calculus approach preferable. Consequently, most of the former students considered Pre-Calculus to be the harder of the two courses, while many of the current students found Applied the more difficult. Reasons for these differences can be many, but after reviewing the data repeatedly, I believe a key reason lies in each person's (or, it seems, each group's) concept of what it means to think mathematically.

For many of the former students, the hard part about thinking mathematically seemed to be in learning formulas and equations and how to manipulate them algebraically to achieve an end result. Being able to relegate those calculations to technology freed them to concentrate on the outcomes instead of the calculations themselves – students seemed to find this, and by extension the Applied course, easier. **Michael** (F-BOTH), responding to my impression that students thought Applied was easier, was in full agreement:

> Yup. Definitely...cuz you use so much technology in Applied. It does everything for you...Pre-Cal is just more thinking on your own.

Angela (F-APM) could see the purpose in learning procedure, but obviously struggled with it:

> Well, also in class, um, we would learn the hard way first. We would learn the long way and it would drive us insane but we would have to get the answer the hard way. And then once we had learned it, then we were taught the quick way [referring to technology], because I guess we had learned it so then we could understand the fundamentals of it. I think that was a good idea.

This particular group also had many students who described themselves as having difficulty understanding mathematical concepts without a context in which to think about them, as would happen often in Pre-Calculus classes. **Cody** (F-BOTH) drew this comparison between the two classes:

> Sometimes it seems harder, although sometimes it's better to learn abstractly first. But it's harder to learn it first when you can't think of how to apply something … what a lot of people probably did in Pre-Cal is, you know, just to run with the sine graph thing for a little longer here, you know, they looked at a sine graph and think, you know, what's the point of a sine graph, you know, what's this thing? You know, what the heck am I ever going to use this kind of thing, what's the point? But you go to Applied math, you can see the point, so in a sense you're more open to learning about it.

In contrast, the group of current students had long ago developed facility with formulas and equations and memorization of terms, and many found this type of mathematical thinking relatively straightforward. Instead, they had a lot more trouble looking beyond simply doing a calculation to focus on isolating the relevant information, determining the problem solving strategies they might need, or considering the implications of a result. Consequently, many had a harder time in Applied. **Wendy** (C-BOTH) shared her opinion about Applied in this way:

> It's a lot harder, I think…I mean, Pre-Cal was: you know your formula, this is this and this is this. In Applied you have to do a lot more thinking and connecting to the different scenarios.

Randi (C-PCM) struggled with the interpretive element:

> I think I think Applied is harder, and I think I think that because I really don't like the Applied textbook. Cuz it seems like just to get a question done, you have to go through all this like ridiculous wording and like situation, scenario kind of stuff. And I find that very hard because I like when it's just laid out to me, like here are your values and here's what you should come up with and when I have to read through, like, a paragraph and find those values myself and sometimes do things to get those values, then I think that's quite a bit harder.

These students liked to have the information they needed laid out neatly in front of them, ready to be processed according to the methods they were already comfortable with. Since Applied didn't do that, many students in this group tended to find Pre-Calculus the easier course, since it appealed to their sense of step-by-step thinking.

So as we have seen in this section, students have many different ideas about the nature of mathematics, what it is, what it means, of what use it is, and how the two math courses reflect the different natures. These ideas have developed through their experiences in mathematics education, and through their own use of mathematics in their daily lives. But in the same way as each student has personalized his or her idea of what mathematics is all about, so have they formed opinions about what it means to learn math and how to go about doing it. These perceptions will be explored as we examine the second category of description.

Student Perceptions of the Nature of Learning

By the time they reach high school, many students have developed a strong sense of what type of learners they are, based in large part on the experiences they have had up until then. As mentioned previously, mathematics education for most of the students in this study had been fairly traditional, and they had developed strategies for learning and a certain level of comfort (or discomfort) with the way things were taught. Starting in Senior 2, these students were exposed to a different way of learning math in the Applied class, and as a result, they were better able to clarify for themselves how they learned best, or what methods worked for them in order to achieve the goals they had set for themselves. They quickly recognized that learning styles are unique to each person and pointed out that having a choice between the two mathematics courses allowed each individual to capitalize on the differences in approach to help him or her learn math.

Whether they were taking both courses or not, students felt they had a good idea of what they needed to do, and what needed to happen in the classroom in order for them to learn and understand the concepts they were being taught. They had individually developed sets of learning strategies that helped them achieve their goals, and they had begun to recognize what might lead to frustration and failure for them. They made choices about math courses based not only on prerequisites for further study, but on what they thought might be interesting and fit with the way they liked to learn. In this section, we will look more specifically at what these students had to say about what it meant for them to learn mathematics and how they went about doing it.

But just before we examine these ideas, it might be helpful to know what the students' goals were in understanding mathematics. So aside from it being a prerequisite to graduation, I asked them to consider why they were studying mathematics, what it meant for them personally to come to understand it.

Students expressed a variety of viewpoints on what it meant for them to understand math, as illustrated by these representative comments. **Lillian** (F-BOTH) and **Randi** (C-PCM) thought understanding math was all about knowing the process and being able to perform the procedures necessary in solving problems. **Michael** (F-BOTH) felt he understood math when he could do the questions and get the answers right. **Heather** (C-BOTH) believed that she understood when she could do a question without reliance on a calculator. **Kathleen** (F-BOTH) needed to know how she was doing something, why she was doing something, and how it applied. **Cody** (F-BOTH) agreed, stating that understanding math was about making equations and manipulating them, but also about understanding why. **Thomas** (F-APM) wanted to see the meaning in numbers, not simply manipulate them. And **David** (F-BOTH) considered exploration or inquiry a key component of coming to understand mathematics. As will be seen, what students thought it meant to understand mathematics underpinned the learning strategies they implemented.

How and why learners

In general, students seemed to characterize themselves as being either "how" learners or "why" learners, or some combination of the two. Most considered themselves to be primarily "how" learners: learning math meant learning formulas and developing procedural competence with equations. **Lillian** (F-BOTH) described it as having to "go through the process and go step by step to understand how you got to the answer". **Michael** (F-BOTH) spoke of "going through a method" and feeling that he was learning well when he "started getting the answers right". **Kathleen** (F-BOTH) talked of having to think very systematically, "because in a lot of the formulas, it had to do with, um, procedure and the way things go and making sure all your steps are right".

But whether or not developing proficiency with procedures was really learning and understanding math was a subject of some debate. For some students like **Michael** (F-BOTH), the answer was obvious (quoted earlier on page 57). For him, knowing a formula, knowing a method, and being able to manipulate numbers and equations in a manner to achieve a desired result involved a lot of thinking and formed the core of his understanding of mathematics. Others were not so sure that procedural proficiency really demonstrated real mathematical thinking:

> **David**: (F-BOTH): You just take the equation you're given, and you put the numbers into it and you get the answer. And what you're really doing there is, you're not understanding why it works, you're just making it work.
>
> **Cody** (F-BOTH): Just doing what you have to do. Not understanding, yeah exactly, not understanding why you're doing it. You're just doing it because you know this is how it is supposed to go. So eventually once you know, that it's supposed to, you know, that's how that's supposed to go, you don't have to think about it anymore, you just insert whatever you have to insert. Babing-baboom, you're done.

Wendy (C-BOTH) seemed to agree with the idea that becoming procedurally competent was not the end of mathematical thinking, but she was often quite content to stop it there. Speaking of the Pre-Calculus class, she stated:

> I liked it that you didn't need to know the why, cuz then you just gotta put the numbers in and you get your answer and no more thinking about that.

But regardless of what they thought about where mathematical understanding lay, many of the students who described themselves as being "how" learners agreed that they needed to thoroughly learn formulas and procedures first, and only once they had gained familiarity with them would they be prepared to extend the concepts to contextual situations:

> **Randi** (C-PCM): Like it makes sense to learn the concepts or something before you are applying it to different situations.
>
> **Heather** (C-BOTH): Yeah, it just seems like a logical progression. Like you learn this basic thing and you like pretty much kill it cuz you do it so much. And then you learn like why that can maybe be useful, or---

In contrast to the "how" learners, a smaller number of students considered themselves to be "why" learners instead. **Simone** (F-APM) attributed her difficulties in her math studies to the disconnect between how she learned and how math was typically taught:

> I think also why I never really caught onto things much is I always wanted to know why things were the way they were, not just, ok, this is the equation, you figure it out.

Seeing the big picture first was important before even beginning to think about the details that went into solving a particular problem. **David** (F-BOTH) drew this analogy:

> ... it's like trying to teach someone how to fly, or how a plane flies, before they see a plane flying. You know, they can't comprehend that it's even going to work. You know, if you understand that it does work, and how it does work in real life, then it's easier to understand the why of why it works.

Thomas (F-APM) stated that for him, understanding the relevance of an idea or concept provided him with a means of interpreting the results:

> I have to understand why it's needed to get how it's done... So I just, it helps me find the answer I'm looking for, what it's used for. If I'm punching in numbers in a calculator, I don't know what they mean, the answer doesn't mean anything to me. But if I know what it's used for, then to punch them in, it just means more.

Of course, many students considered themselves to be a combination of "how" and "why" learners. To that end they had definite opinions about in what order Pre-Calculus and Applied should be offered, should someone want to take both. They tended to consider the order of the courses primarily in terms of the marks they were able to achieve, agreeing that in this respect, it didn't really matter much which course preceded the other. The first course would provide a foundation for the second, thereby making it easier to get higher marks in the second. But when I asked students to consider the order of the courses without regard to marks, the "how" and "why" distinction once again seemed to make a difference to them.

For those who considered themselves "how" learners first, it made sense to start with Pre-Calculus. **Kathleen** (F-BOTH) and **Heather** (C-BOTH) maintained that this provided "the basics" for them to understand the applications later. **Brady** (C-BOTH) appreciated how things would fall into place for him:

> And I think Applied, really, has just clarified a lot of things that I learned in Pre-Calc. And I understood how to do them, but I didn't really understand why they worked or for what reason they worked, and then Applied just kind of gets to the root of things and clarifies it a bit, makes it a bit more relevant.

In contrast, the "why" learners preferred Applied first, believing it provided the context for the abstractions that would follow in Pre-Calculus. **Cody** (F-BOTH) explained:

> So to take Applied math first...you have a better understanding of where it's coming from in a real sense, so when you go over to Pre-Calculus and you learn about abstract things, you've kind of had, you've worked with it in real life, so when you get these abstract numbers coming at you, you can actually think about it in terms of a real thing. You can adapt a situation you had in Pre-Cal, or I mean

Applied, and you can adapt it to the numbers you have here and you can think about it in that way.

Finally, as students came to understand what type of learners they were through the exposure to different learning opportunities in the two math courses, they sometimes reflected on what things might have been like for them in the past had they been more aware of how they learned. **Morgan** (C-BOTH), for example, had come to realize that she was no longer satisfied to just accept a procedure as taught, instead she wanted to know why. This realization had important implications for her:

> I think one thing, if I knew that's how I learned better, I wish I could go back to grade 7 and redo my whole math career or whatever. Because I, I never, I'm one of those kind of people, it takes me a little bit longer than everyone else to learn, and I just realized this last year in Pre-Cal, I tried asking questions... I don't get this, like explain it in detail and tell me why... I think the number one thing I would tell kids to do is ask if you don't know, because I learned so much better this year.

Strategies in learning

Regardless of whether students were consciously aware of what type of learners they were, they had each developed some specific strategies to help them learn math that were consistent with the type of learner they were. I asked the students to think specifically about what they did to learn mathematics in either or both of the courses, and they identified several of these strategies they had developed. With the exception of only one student, they all agreed that some independent effort was a key element in learning:

> **Heather** (C-BOTH): ...you're kind of responsible for a lot of how well you do. Because you, it's very much like, you're very responsible for how much work you get done and how well you do it and you're responsible for understanding stuff. So then come test time, like sometimes, you wouldn't be as prepared as you could have been.

What form that independent work took varied from person to person. For some like **Michael** (F-BOTH), it was a matter of listening to the teacher and completing all assignments:

> I just made sure I did every question. Like, try and complete every question the best that I could. I don't know, I just did my homework. I asked questions if I didn't know if I was doing it right and I just listened to what he had to say and then I'd try by myself a few examples, and then eventually I got it.

Lillian (F-BOTH) felt that she "didn't learn well in class", so she found that "kind of just sitting down with it by myself for many hours at night, that helped too." **Ashley** (F-APM) followed a similar pattern, spending a lot of time outside of class reviewing and practicing in an effort to understand:

> Well, I would go over all the questions we did in detail so I would know, so I would know exactly how I got it ... I would read through the, ah, the textbook, and I would actually do questions that weren't given. So then I could, of course the questions that had the answers at the back, right, so I could just do questions that weren't assigned and we didn't go through in class, and I would try to do them, and then I'd check my answer.

Others needed to go beyond just doing what the teacher had assigned. They needed to mull over the ideas in their own mind and perhaps explore the topic from different angles before they were satisfied that they understood a concept. For someone like **David** (F-BOTH), the teacher's explanation was just a starting point:

> I always found in Pre-Calc, and this could just be my personal opinion, but I had to end up working with things before I understood them. You know, if you'd asked me right after the teacher had done all his talking and explaining, you know how to do something, I wouldn't have the foggiest idea. I had to do things, and then try and relate that to what he had said... I often did that. If I couldn't get or couldn't understand a question, I'd go back and I'd change a number. You know, okay it does that, if I go the other way, it does this. You know, I'd play with it just to understand how the thing works.

Brady (C-BOTH), too, felt the importance of trying to make personal sense of a concept:

> I think some of the stuff I learned better because I had to figure it out a lot myself. And in doing that it kind of got pounded into my head pretty good. And because I went through all the processes of figuring it out and it was kind of very much clear in my head, when it came time to do it again, it wasn't like I had to go and remember all of the teachings that we had gotten about it.

Sometimes the learning strategies students employed were very specific to the course they were taking. **Kathleen** (F-BOTH), for example, described Pre-Calculus as being "all repetition" and **Heather** (C-BOTH) spoke of the endless exercises as "pounding through it", yet many students saw a lot of value in that. **Cody** (F-BOTH) states that "...through the repetition of doing those equations that are, you know, kind of meaningless numbers, whatever, ... you can get the right answers. So now you know how to do it, cuz you've done it so much." **Morgan**

(C-BOTH) liked the booklets of exercises they worked through: "I found I learned very well from them…cuz it's lots of practice and you just learn." And repetition had the benefit of familiarity – students were used to learning math this way. **Heather** (C-BOTH), recognizing the different approach taken by Applied, spoke of finding comfort in doing things in the way to which they were accustomed:

> And I'm just not as comfortable or just not as comfortable with the new way of doing it because in Pre-Calc, you like, dissect things and you do it over and over and over again. And then you're kind of, like, stuck in that rut, kind of? I don't know, you're just, you're used to that. You're used to that pattern of doing things and then, like in Applied, you're like, actually there's another way of looking at this…I don't know, it's hard to get into a new habit of doing it.

In contrast to the repetition of the Pre-Calculus routine, Applied offered a more hands-on approach that appealed to some students. **Kathleen** (F-BOTH) comments: "…but in Applied math you actually had to do stuff to learn it and to me that was a better way of learning for me." **Thomas** (F-APM) connected the approach with his sense of himself as a learner:

> Um, I felt that I learned better in Applied math because I'm an applied learner. I can't just sit there and read about something and I have to do it and look, see what the actual situation would be. I'm that way in everything. It's just easier to do it than to read about it.

Being able to contextualize something was considered an important learning strategy as well, not only in the sense that it could help them make connections and relate abstract mathematical expressions to real-life situations, but in terms of being able to assess the reasonableness of an outcome. This was illustrated in an exchange between **David** (F-BOTH) and **Cody** (F-BOTH):

> **David**: In Applied math, you know where you should be going with things. You know, you can relate and say well, if I got an answer of a million, that's probably not right, because I'm looking for how much they're going to charge for candy. You know, so it gives you a bit of perspective…In Pre-Calculus math, you can get it, but it doesn't make a lot of sense all the time.
>
> **Cody**: Yeah, like with Applied math, you can get the real life situations. You can kind of understand generally if your number is way out of whack or not. Even if it's some, you know, hard equation that doesn't seem to make any sense, but you're applying it to a real life situation, you can kind of understand where it's supposed to end up in general. So that definitely helps.

Visualization was one more important skill identified by several students. This skill was particularly enhanced through the use of the graphing calculator and will appear in more detail in the section on the role of technology.

Resources in learning

Students seemed to believe that learning was very much an individual effort, but they did mention several additional resources that played an important role. These included teachers and/or tutors, other students, and written resources such as textbooks and the Internet.

Teachers, of course, introduced new materials, and provided explanations and examples. Students saw teachers' roles in different ways. For example, an interesting comment came from **Simone** (F-APM). She seemed to believe that her learning was completely dependent on the teacher, and required no input on her part:

> I went into the exam, and I didn't study, because I wanted to see how much I actually knew from the teacher teaching me. I came out with a 53 or something like that, and I was like, well yeah, the teacher never taught us anything.

But others, recognizing their part in their own learning, used the teachers' explanations as a springboard to individual learning. **Thomas** (F-APM) stated "it helps to get you started and explain what you're going to do. And from then on you can work on your own or with someone else." **Ashley** (F-APM) liked to work one-on-one with the teacher, going through examples:

> Cause if teachers go through the whole thing with you and do examples …and then do them with you and if you have problems, they help you till you understand. They don't just give you booklets and here, do this, you know. They, they actually do it on the board and help you with it.

Tutors fulfilled a similar role. Some students turned to them outside of regular classroom time for extra help, and the intense one-on-one interaction with someone outside of the classroom context seemed to work for them. For example, **Angela** (F-APM) struggled with the unit on matrices, and turned to her older brother for help:

> He showed me, he explained it to me, over and over, about three times and he got me to do it. And I did it by hand, like not on the calculator. And, you know, once I got it down, I could do it, like. And I ended up getting, like, in the 90s on that test, because I finally got it, because I had gone to someone else to get it explained to me.

But students also seemed to feel that the role of the teacher was not simply to help them learn mathematics, but to ensure that they passed the course and graduate. If this meant teaching to the exam, then it should be done. As **Lillian** (F-BOTH) stated, "I think it's important that the teacher teaches you in regards to how the final is going to be". **Nic** (C-APM) was even more direct:

> If [the exams are] all the same, somehow teachers should be able to use, should know the exam, and like, I would like to just for teachers to teach the whole course straight off the exam [background voices: Yeah. Yeah. Yeah]. So that we know exactly what to expect on the exam. Use all the same words, and all the same questions.

Others reiterated this idea of consistency. Recall from Chapter 4, for example, how students had expressed their frustration that the provincial standards tests could deviate so substantially from the form and structure they were used to from their teachers (see quote on page 56). Students seemed to tune their way of making sense of mathematics to the explanations of the teachers. As **Randi** (C-PCM) states, "...I usually kind of know how they think and how they think compared to the way I think." Whether this equated with understanding, or whether it was just memorizing enough to pass the exam and/or the course, was as individual as the students themselves.

But the teacher was not the only resource students turned to. Indeed, they often turned to their friends as they worked together on assignments. Collaboration with other students happened spontaneously in both classes and ranked highly among the participants as being an important component in coming to understand the material they were learning. **David** (F-BOTH) noted that when a teacher was busy helping other students, there were still others nearby who might be of assistance. He liked the fact that working with someone allowed him to find some direction in working through a problem that he might have been struggling with:

> You could just work so far down the line and then find out that you'd gone wrong somewhere and you know, you couldn't figure out where you'd gone wrong, so if you just ask a friend, how did you do this, you could see how they did it without you know, testing a million different possibilities.

Heather (C-BOTH) thought collaborative work allowed her to learn better by offering different perspectives:

> Oh, I really like doing that. Because then if you don't understand a question, you can talk it through with someone and maybe they are looking at this concept from like a different way or something. And then you can totally understand something if you work with someone and it just, it's just better if you have two people thinking about it rather than one.

The alternate explanations offered by students were sometimes even better than those offered by the teachers:

> **Wendy** (C-BOTH): And a lot, but a lot of the time the teacher didn't really have time to go, like they'd go one on one but we'd run out of time. So it helped me a lot to have people around me that understood it in a different way, because they could explain it. And sometimes that was better than the way the teacher actually looked at it.
>
> **Heather** (C-BOTH): And sometimes just by hearing someone else's way of thinking it, you can make connections to the way you would maybe think of it and then, yeah.

Thomas (F-APM) remarked that the collaborative work they were doing in class enabled them to build skills they would need later in life, stating that it "… helps for when you are out of school. At work you always work in groups, and if somebody has a problem, it can help with them through it [sic. Awkward phrasing in original]."

But although the students recognized that learning this way was something they liked to do, they also noted its sinister side. **Brady** (C-BOTH) suggested that working collaboratively might have a tendency to remove the focus from learning to simply getting the answer:

> It's also very easy to abuse it, though. Like if you don't know how to do a question, then just rely on another person to do it, and you don't bother learning it, because you're like, well I could ask them how they did it and figure out how it all works, but nah, I'm tired and I didn't really want to come today and I just want to go home and ---

Cody (F-BOTH) agreed, stating that working in groups sometimes allowed one to simply coast along, focusing on getting the assignments done rather than learning from them:

> Well that's the danger of groups is you tend to just float when people, somebody else figures out, ok, write it down move on the to next one and you may not bother to ask how to figure it out and stuff like that because you're just trying to get the assignment done, cause it's a big assignment or something. It's just the danger of floating along, and that's not good.

It was interesting to find, though, that while students were generally in favour of collaborative work, believing that shared explanations promoted understanding, they preferred to have that happen naturally, rather than to have assigned groups. **Kathleen** (F-BOTH) pointed out that on some of the larger projects in Applied math, for example, responsibilities are delegated, but learning is not necessarily shared among the group members:

> I think when we did our Applied math projects, I think oftentimes then, someone would come up with the answer, at least in the most groups I was in, someone would come up with the answer and it would be like, such a big project that you're like, ok, yeah, sure that's a good answer, you know, whatever, we'll go with that. And so I think with those bigger projects, I think it was easier to go along with what other people said, or with the answers that they came up with. But I think with smaller problems, that we did like in class or stuff like that, then it was, then I think it was easier to figure it out on our own and to actually want to learn how to do it because it was essential, you know, cause we needed those individual ideas and whatever for our tests. But for group projects, it was a group project, so if someone came up with the answer, it doesn't matter, as long as it is on the project. You know, the group is still going to get the good mark, cause someone came up with the answer.

Randi (C-PCM) expressed frustration that assigned groups often consisted of people with unequal abilities and motivations:

> Like, I don't like group work because, I don't know, I just don't do well in group work. But I think lots of the time there's usually one person that's smarter than the other, because there's very rarely two people that are, you know, exactly the same at a given question, or whatever. And I think lots of times the smarter person is determining the rate at which you are working. And that not as smart or not as, whatever, person can kind of get left behind.

And **Lillian** (F-BOTH) indicated that with assigned groups, mistakes would negatively affect the entire group:

> … sometimes when we were doing those group projects, and what we screw up on our data or something, which I know has happened a couple of times, and then we'd just kind of get discouraged. And the problem is when you get discouraged as a group, you're pretty much sunk because then you would just try to do whatever we could to get out of class at that point.

As an additional way of coming to understand the material they were learning, students pointed to their texts and other published resources. Not all students liked referring to the textbooks, but for some, this was a valuable additional source of information and guidance:

Lillian (F-BOTH): Well, it's because they show problems and they work through them in that textbook, and it just really helped me and it has more color in it: that also helps. You can, it's just a boring textbook, and of course I'm reading it all the time 'cause that's the best way I learn so it's really important that the textbook is good for me.

Angela (F-APM): Yeah, like the tutorials and stuff? How they have the question and then they have the solution and they go through every single thing. So even if you had trouble, you can go to the solution and figure it out if the teacher's not there.

The solutions at the back of the book were part of the problem solving strategies students developed:

Kathleen (F-BOTH): I think for me also it helped to have answers in the back of the textbooks because then when, it's not just you go through the problems and you arrive at an answer and you move on to the next one. You can check if you've gotten it right or not, and if you don't, then obviously you know that you have to rework it and you have to find out where you come to whatever wrong thing, so for me answers at the back were really helpful.

A last resource mentioned by at least one student was math websites:

Lillian (F-BOTH): I actually went onto the Internet and did research, because that's the only way I could do it. I actually found a lot of stuff that helped me there.

Thus as we have seen in this category of description, student perceptions of the nature of learning are filtered through the different senses that they have of themselves as learners, the varied strategies they have developed to learn, and the resources they turn to in order to learn new concepts. The third category of description shifts the focus from a description of mathematical learning experiences in general, to a more targeted focus on student experiences with technology.

Student Perceptions of the Role of Technology

A key distinguishing feature between the Pre-Calculus and Applied math courses is the extent to which technology is incorporated as a teaching and learning tool. Students rarely used a graphing calculator or spreadsheets in Pre-Calculus (although they were permitted to use them if they so wished). In the Applied course, on the other hand, the use of technology was an almost daily event. This use of technology centered mainly on the graphing calculator and spreadsheets, but also included the use of calculator-based laboratories (CBLs) with various probes for data collection, and a variety of software packages such as Geometer's Sketchpad and income tax preparation software.

As students reflected on their mathematical learning experiences in the discussion groups, it became evident to me that their perceptions of learning mathematics were very much influenced by their own personal attitudes towards technology, whether it be in a general sense, or whether it was more specifically aimed at the tools that were used in class. Once again, the variety of experiences that the students described was broad, ranging from a strongly stated dislike of all things technological, to a wholehearted embracing of the same. The section that follows will look at this issue from two perspectives: the students' actual use of technology (primarily the graphing calculator), and their expressed attitudes toward technology. It should be pointed out, though, that since Pre-Calculus made little formal use of the graphing calculator, the comments that were generated were often made within the context of a conversation about Applied math.

Using the graphing calculator (and other technology) as a tool

Students talked about a variety of ways in which they used technology. Their descriptions closely paralleled those outlined in a study by Doerr and Zanger (2000) where the researchers examined how students (and teachers) were using graphing calculators as a tool in their mathematics classes. Five distinct patterns and modes of use were identified. Applying this list to the comments made by the students in this study provided an effective framework for the descriptions of how students utilized graphing calculators. I suggest a sixth mode of use as well that applies more specifically to the use of spreadsheets.

1. The most common use of the graphing calculator, just like any other type of calculator, was as a **computational tool**. Students used it to evaluate numerical expressions quickly, efficiently, and accurately, or they would use it as a quick and easy way to generate graphs:

> **Lillian** (F-BOTH): Um, in some ways it is easier. I mean, when we were doing stuff that we had done in PreCal, like you know, any graphs or anything we had to do by hand in PreCal, then I definitely saw the difference. You know, it was a lot easier, it was faster, and I didn't have to think as much about it.

Using the calculator as a computational device became so routine for students that they would automatically reach for it even when the calculations might be relatively simple:

> **Randi** (C-PCM): … there's all kinds of simple things like multiplying decimals and reducing fractions and then changing them into decimals. Like all kinds of stuff like that that I never ever do myself.
>
> …
>
> **Wendy** (C-BOTH): … I tried to help my brother, like when he was in grade 8 or something, and I didn't even remember how to do long division.

Other technological tools such as a spreadsheet also served to increase the efficiency of tedious calculations:

> **Thomas** (F-APM): … for Excel, once you do the first formula, you just fill down and so you don't have to do each separate equation. It does it all at once.

But using the calculator frequently, or coming to rely on it too heavily also had its insidious side, as **Kathleen** (F-BOTH) pointed out:

> I think most people look at it as a way to get the answer without actually having to think or work through the problem. To just use it and get the answer and write it down on paper.

Many students seemed to be aware that they could easily be lulled into a sense of complacency of accepting any result that a calculator might generate and not even think about it. They underscored the need to recognize the limits of the graphing calculator, realizing that unless they knew something about the process the calculator was using to generate an answer, they would have no way of judging the reliability of that answer. Practicing a procedure by hand was an important adjunct to using it on a calculator:

> **Thomas** (F-APM): I could put any number in there and it could come up with a wrong answer. If you didn't know what you were doing, you'd have no idea if it was right or wrong. So you do have to know what you are doing to use a spreadsheet or a calculator even.
>
> …
>
> **Ashley** (F-APM): And I figure if something goes wrong on the calculator, then you could go back doing it by hand and then maybe you can find your, what you did wrong, and then you can correct it on the calculator. Because if you only know, like, what numbers to type in, and then it's wrong, and you don't know, why is it wrong? And then you have to ask for help because you don't know how to convert it kind of in your mind to how it works? So I think that helps with getting to know the math better.

The calculator had one additional drawback as a computational tool, in **Morgan's** (C-BOTH) opinion, especially if a calculation were being evaluated for marks on a test:

> Yes, calculator. And if you get one number wrong, you can't see that we got that number wrong, right? But you, but maybe we got the whole concept right and we knew what we were doing, except just that one number was wrong and then we're screwed for that question. That's what I didn't like.

As she points out, the calculator as a computational tool is about generating a final answer, not about the process of achieving that answer. So unless a student has provided input parameters and/or recorded keystrokes, a teacher will not be able to easily assess where computational errors might have been made.

2. A second key use of the graphing calculator, especially in Applied math, was as a **transformational tool**, where the results of computational tasks were transformed into interpretations. In other words, students' attention was focused on the meaning of results, rather than on the process of the computation. The calculator was used as a bridge linking a mathematical expression with its counterparts in a physical context; where students would, for example, link the coefficients and constants in a sine equation to specific parameters of a swinging pendulum, such as period of swing, or its displacement from rest position. Students seemed to appreciate the ability to relegate the tedious computations to technology, so that they could get on with understanding the applications:

> **Angela** (F-APM): ...we wanted to get the answer so that we could use that answer to find out something else... you can kinda focus on applying the answer to life and to different things that, I don't know, that we did in class, and that's why I think that they use technology so that we don't have to spend all our time figuring out what the answer is, we can just quickly just get the answer and then apply it to whatever we're applying it to.

Moving from dull computational tasks to interpreting results from a graphing calculator was not only restricted to connecting math with the physical world, but led students to explore mathematical concepts further in an attempt make sense of them:

> **David** (F-BOTH): You can figure out that you need a 5 there to get the answer you need. And on pen and paper that might be where you'd stop. Because, you know, you didn't know if you changed a number exactly what you'd get or

whatever. But with a calculator you could, you know, quickly understand how things actually worked, as opposed to the fact that they just did work.

This opportunity to explore "what-if" scenarios allowed **David** (F-BOTH) to make some important connections between previous work and future application. In our conversation we recalled how he had used the graphing calculator to solve a quadratic equation in a chemistry class quickly while the rest of the class struggled with the algebra involved in using the quadratic formula.

> I think what it was is it struck me that it was similar to a lot of questions we had done in math where you were trying to find a value using an equation. So all you had to do was relate that value to zero, if I'm thinking of it right, and then you could find the answer. And it just, it was a slight modification and because, you know, I had played around a lot, changing numbers and stuff like that, I understood how it was that you could go about doing that.

Yet making the transition from computation to interpretation was not always as fruitful as a teacher might have wished, as evidenced by an interchange during the former students' focus group discussion. I had given the students a sample exponential functions problem that involving the exponential growth of bacteria. I asked them to consider the problem and describe how they would approach it from both Pre-Calculus and Applied perspectives. Students did indeed distinguish between the approaches within the two courses, referring to using formulas and tables in the Pre-Calculus setting, and to creating spreadsheets and graphs in the Applied setting, but when I asked them what the result meant to them, the distinction disappeared. Regardless of in which class they did this problem, the result to them was just another answer to just another math question.

> **Cody** (F-BOTH): … basically you don't really think about it in terms of the bacteria. In Pre-Cal you'd think, ok, you've got your graph, they want it at such and such a time, so you find that on your x-axis and then you find it on there and you find the point on your graph that you need and it's just another number.
>
> …
>
> **Kathleen** (F-BOTH): But I think in Applied math it's the same way too, you know? It's just an answer which we arrive at. Like, in Applied math the same question I think still wouldn't have any meaning to me either. It would just be, I just need to find this number and I need, whatever, that's my answer.

> **Lillian** (F-BOTH): Yeah, the only big difference is that with Pre-Calc you do it manually and in Applied you do it through the calculator, but either way, it means the same thing to me.

Clearly, simply placing a problem into a particular context and using technology to investigate it is not always sufficient to encourage students to think about the connections between mathematics and potential applications in their own lives.

3. The graphing calculator was often used together with a CBL unit and probes such as motion sensors, force sensors, and temperature probes. As such, the students used the calculator as a **data collection and analysis tool**, whereby the data would be collected and stored in the calculator, and could be retrieved for further analysis and manipulation at a later time. If, as was sometimes the case, a simple rewording of a problem to place it within a context was insufficient to get students to think about the mathematics related to a particular situation, then physically placing them into that situation with an efficient means of collecting data certainly expanded their opportunities (and motivation) to explore the connections. For example, spending a period in the park with their graphing calculators and some motion sensors allowed students to investigate the relationship between their movement on the swing set and a sinusoidal function.

> **Lillian** (F-BOTH): Well, I think it does let us, kind of expand what kind of situations we can learn about, because you know, if we were doing, if we were using the motion sensors or something like that, we can't do that with our head, you know. So, you know, then it lets us learn about all these different situations that we otherwise wouldn't have been able to learn about.
>
> . . .
>
> **Angela** (F-APM): . . .there was technology to help you along, like you know, make things easier, to figure things out. You know, like on the swings, those distance things, you know, they help you to figure stuff out.

4. Students strongly endorsed the graphing calculator as an effective **visualizing tool** which helped them understand how a mathematical expression and its graphical representation were related, as the comments below illustrate.

> **Thomas** (F-APM): I found that in Applied it's just a lot easier because it's visual. You can see what you are doing and know what it's about. You have an idea of what the answer is supposed to be close to and what it's related to.
>
> . . .

David (F-BOTH): With a calculator, if I want to figure out how doing different things affects a graph, you know, you can very quickly go through and adjust things. I often did that. If I couldn't get or couldn't understand a question, I'd go back and I'd change a number. You know, okay it does that, if I go the other way, it does this. You know, I'd play with it just to understand how the thing works.

...

Cody (F-BOTH): ... you can manipulate to see exactly how it works, whereas, like with the sine function, we always had to make, we had a chart of a couple of different points and then a chart and then get the general pattern, whereas here you could just change one number in the equation, and so. That would make a big difference in just learning the aspects of how it goes from equation to visual very quickly.

5. The graphing calculator would also be used as a **checking tool** – to verify or discount mathematical conjectures that had been made. Again, the comments speak for themselves.

Randi (C-PCM): Lots of times I'd want to, like, check the way I thought was right and do it on the graphing calculator and then if it was, then I wanted to work it out. But sometimes it was good, because then I could kind of see if what I was doing was even kind of anywhere near being right. Because you can work way faster with a graphing calculator than you can working it out manually.

...

Wendy (C-BOTH): ...so I would do it algebraically but I would just use my graphing calculator as a check up, like you said.

6. The study by Doerr and Zanger (2000) was restricted to an examination of how graphing calculators were used in the mathematics classroom. However, particularly in Applied math, the spreadsheet was also an important tool, and so I would venture to add one more item to the list of how students used technology: as an **organizational tool**. Students remarked on how much information they had to process in some of the contextual problems they worked on. For some students like **Thomas** (F-APM), tools which could help him sort out this information were useful in helping him to make sense of it:

And with a spreadsheet. You have it all laid out in front of you, with labels on it. You see what each column or number represents.

Student attitudes towards technology

Students had been using calculators as a tool in math classes for many years before ever taking Pre-Calculus or Applied. As such, they had come to see it as something that would help them to get an answer quickly and efficiently, where they would not have to concern themselves with remembering procedural details in order to solve a problem. The graphing calculator, then, was simply an extension to the basic calculator – for most of the students, they could now do more complex problems, but once again, they could do it quickly and efficiently. Most students were very comfortable with technology in this context and could not really see themselves wanting to work without it. In fact, as has been previously mentioned, some students realized that they had become so dependent on the calculator as a quick way of getting answers to problems that they had forgotten how to do things like long division manually. But beyond accepting the calculator as an answer-generator, student attitudes towards the use of technology in mathematics began to diverge. Some liked what it enabled them to do, as evidenced in the quotes in the previous section, seeing it as an important tool to understanding mathematical concepts. Others, however, were very reluctant to incorporate it carte blanche.

For those students who embraced the use of technology in mathematics, it represented a new and different way of learning. **Cody** (F-BOTH) compared it to the more traditional pen and paper way of learning:

> … you have to learn to use the technology to get you that answer, so it's not any less legitimate than writing it on paper, it's just a different means to get to the same end. By no means is it, I don't think it's less, you know, less effective of a way, it's just a different way. Cuz we're in a technological century or millennium now, so I mean, that is a perfectly legitimate way to do it now, just as pen and paper was before.

What they could now use a graphing calculator for went far beyond the answer generation that they had used basic calculators for in the past. In fact, **David** (F-BOTH) suggested that referring to it as a 'calculator' might in fact be misleading – it was far more than just a quick way of arriving at an answer:

> If they called it like a graphing tool or something like that, you know, it might make people realize it a little bit more that, you know, it's not just for finding the answers, it's a way of really exploring, you know. It has many functions.

Many students were excited by all the different ways in which they could use technology and quickly came to rely on it. **Cody** (F-BOTH) spoke of the graphing calculator as a "lifeline"; others agreed and couldn't imagine doing without it anymore. In particular, the ability to explore math concepts in greater detail, or from different perspectives was appealing and helpful. As quoted earlier, **David** (F-BOTH) would use the graphing calculator to thoroughly explore a new concept, he would "play with it just to understand how the thing works". Or, as will be illustrated in the next chapter, **Morgan's** (C-BOTH) chance to explore through technology finally helped her to understand the more abstract concepts she had learned earlier in Pre-Calculus.

Students seemed to appreciate that technology allowed them to work with numbers in different ways. Having to collect data and analyze it, for example, gave them the opportunity to make connections to the physical world; it allowed them to think about interpretation rather than procedure. Many thought back to some of the data collection activities, recalling them as being "fun" and a "more interesting" way of learning math, and recognized that some of the activities they had done would not even have been possible without the use of technology. They saw spreadsheets as being a great way of doing recursive tasks, organizing information, or being able to see the big picture. Others liked the fact that using technology in the classroom was good preparation for the technological workplace they would someday face.

All students admitted that technology was an important tool for them in learning mathematics, but the extent to which they were willing to use it varied considerably. In contrast to students like **Cody** and **David** (as described above) who thoroughly embraced the possibilities that technology afforded them, there were those who expressed a wariness of it and were reluctant to allow it a dominant role in their learning. This included a few participants from the group of former students, and a substantially higher proportion among the group of current students. Their concerns seemed to center around what they thought it meant to understand math, and around developing a dependency on technology.

Among these students, understanding math included being adept at making equations and being able to work with them, knowing when and how to apply formulas, and understanding the how and why of manipulating mathematical expressions. Being able to give meaning to results, or to make real-world connections was an important adjunct, to be sure, but without procedural competence as a foundation, they could express no confidence in their interpretations. In other words, to allow a calculator to perform a mathematical procedure was, in **Kathleen's** (F-BOTH) words, a "cheater way" of solving a problem. Unless you were capable of doing the calculations yourself, and were relying on the calculator simply to achieve the answer more quickly, then you did not really understand the math that you were doing. Learning to use the calculator merely equated to learning more procedures, but did not promote any conceptual development. **Randi** (C-PCM) put it this way:

> When I was in PreCalc, I felt like I understood what we were doing and I felt like I was actually learning something. But when I was in Applied, then I always felt like I was just memorizing methods to get to an answer instead of actually understanding why I was doing what I was doing.

Lillian (F-BOTH) had a similar experience when she entered an equation into the graphing calculator, but had no idea of how the graph was generated:

> Well, just as one example. Um, when we would do like the y=, and we would come up with a graph. I never knew before why the equation we put into y= made the graph. I just knew you put the equation in there, you put the numbers in the table and it makes the graph, you know. But in Pre-Cal we actually had to make out the table from the equation, and then I knew, oh, that's why, you know, that's where these numbers are coming from, I understood them. Because we actually used the equation, we didn't just get the calculator to use the equations.

So these students needed to know "what the calculator was doing", they needed to be able to do it themselves by hand if necessary, before they would feel reasonably confident that they actually understood what they had been learning. But many of them were also very reluctant to allow themselves to become too dependent on technology. I believe that this was due in large part to the fact that part of the Pre-Calculus exam was to be completed without the aid of a calculator. This had taken some of the former students by surprise, because they had not realized until shortly before that they would not have access to calculators for the entire exam, and had been left scrambling at the last minute trying to learn the procedures they had always relegated to the graphing calculator before. The current students had known about the no-calculator section a lot sooner, and had therefore been making conscious efforts throughout their Pre-Calculus course to become less reliant on technology:

> **Brady** (C-BOTH): And like we had a whole chunk of our exam where you couldn't use your calculator. And I kind of like that because, I know in grade eleven I got really dependent on my calculator. And then this year it kind of forced me to dig into things a little bit and figure out exactly what was going on, not just which buttons to punch into my calculator.

The no-calculator section opened the eyes of a lot of students, when they realized they were second-guessing themselves on even "simple multiplying and dividing". As **Randi** (C-

PCM) remarked, "I had to think if I was doing that right, because I relied so much on my calculator." **Kathleen** (F-BOTH) summarized it this way:

> I kind of enjoyed, you know, figuring it out on my own, using my own, you know, my own mind, my own abilities, to figure out problems and so I think that it helped me be less reliant on a calculator in Applied math because I didn't use … a calculator, like a graphing calculator in Pre-Cal, so I have the formulas and everything else in my head, so it taught me more to work independently from technology more in Applied math, so I think I enjoyed that more.

So within this category of description, we have seen that the role of technology as a learning tool in mathematics is multi-faceted. But the degree to which students are willing to use that tool was governed both by their own attitudes towards technology and by their ideas of what it meant to understand math. Whatever their feelings about technology, students remained very aware that they had to be careful of how they used it – it could never completely replace their own thinking.

The three distinct categories of description outlined in this chapter demonstrate that students have formed wide-ranging perceptions on the nature of mathematics, on the nature of learning, and on the role of technology. But how can what they have to say on these matters inform us as teachers or researchers? What implications might their thoughts have on the development of curricula in Manitoba? How can we give credence to their voice? These are the things that will be considered in the next chapter.

Chapter 6

In the Eye of the Beholder: An Interpretation

Phenomenography, with its categories of description, seeks to break down a body of data into a limited number of qualitatively different ways of experiencing a phenomenon. As I have shown in the previous chapter, the experiences of learning mathematics that the students described to me could be viewed from three distinct perspectives, each depicting a broad range of experiences within it, yet each markedly different from the other perspectives. Students' experiences with Applied and Pre-Calculus math, as they recounted them to me, seemed to be shaped by their perceptions of what they considered math to be all about, what they thought they needed to do to learn it, and how they were willing to incorporate technology in their learning. Yet phenomenography looks not only to categorize and describe, but also to search for an underlying structure, or a way of tying together the categories of description.

Äkerlind (2002) suggests that in contrast to the development of the categories of description, which can be accomplished through a systematic process, the interpretation of those categories and the resulting structure inevitably reflects a relationship between the researcher and the data. In other words, I have analyzed the interactive writings and transcripts of the focus groups and follow-up interviews, I have generated categories of description that I feel encompass the experiences as told to me. But what I have gleaned from this comes not only from the words in the transcripts, but also from the daily interactions I have had with these students in the classroom. The type of structure I perceive is necessarily intensely coloured by the history I have had with them, and may therefore be different from what others may perceive. But according to Äkerlind, this is in keeping with the principles underlying phenomenographic research. The outcome space is "the data *as experienced by* the researcher" (p. 10, italics in original). My interpretation then may not be the only possible outcome; nevertheless it remains grounded in the data and seeks to remain focused on the students.

Given the relationships, then, that existed among me and the participants and the data that resulted, it seemed inappropriate in this case to apply the traditional interpretive patterns of phenomenography which would develop a hierarchy among the levels of awareness shown by study participants. Instead, an awareness emerged within myself, both as a teacher and as a researcher, that in order for this research to be meaningful, action would need to stem from it. Students do not often have or take the opportunity to be deliberate in thinking about their learning, but the nature of the comments within each focus group or follow-up interview suggested to me that students were beginning to think about their learning in ways that were new to them, or that they discovered things about their own learning that they had never considered before. Many of their casual comments after our formal conversations implied a recognition of some benefit to themselves from the exercise. Their continued interest in my progress with the analysis and interpretation of the data long after it had been collected emphasized to me that they had provided their input in the full expectation that something useful would result from it. Consequently, I came to see it as my responsibility to ensure that the results of this study went beyond mere description, that instead they could go on to become the basis for action.

Thus the interpretation that follows is my understanding of what the students have described to me. As I read and reread the transcripts, I strove to not only listen to the words, but to re-listen to the meaning that that lay behind the words. I have constructed that meaning as a set of overarching ideas that arise from the categories of description. They imply no hierarchy, they provide no definitive links among categories; instead, they convey a series of ideas or considerations that appeared important among students, regardless of where they might position themselves within each category. They convey starting points for me as a teacher to develop an action plan to improve the mathematical experiences of the students in my classes. Some would argue that this focus is too narrow, that the interpretation can be made only in light of one particular teacher and two particular classes and therefore gives only a partial understanding, but I believe that interpreting the structure in this way underscores the idea that teaching and learning involves a two-way communication between teacher and student. It extends to other teachers an invitation to listen to their own students, to hear what messages their own students have about what teaching and learning need to look like for them.

I present these overarching ideas as a series of four messages from my students to me about what is important to them in learning mathematics: 1) making choices, 2) making connections, 3) taking courses in combination, and 4) communication.

Making Choices

The first message centers on the opportunity to choose among mathematics courses. Students verified that Manitoba's current practice of offering three distinct math courses in senior high school was important to them, and they made specific recommendations about how students should go about thinking about their choices regarding Applied and Pre-Calculus.

Whether or not they had ever consciously considered how they thought about math prior to participating in this study, students were given the opportunity here to listen to others' experiences and to reflect on their own. They formed clear distinctions in their own minds about Applied and Pre-Calculus, about the rationales behind each of them, of the different approaches each course took. They also recognized that as individuals, students each had their own ideas about what they wanted to get out of a math course, or how they liked to learn. Being presented with the opportunity in their senior high school years to choose among a variety of math courses opened new doors for them.

In the past, students who were bound for university didn't have an option – they had to take Mathematics 40S (the former 'higher-level' math course) as opposed to Mathematics 40G (the former 'lower-level' math course). But along with the introduction of the triple-stream math courses in the mid-1990s came new admission standards at the universities and community colleges. For students, this meant that they no longer needed to focus exclusively on their post-secondary plans when making choices about which math to take. Instead, they now had the luxury of at least thinking about how the courses aligned with how they wanted to learn math before they made their decisions. Students would be studying 'high-level' math regardless of whether they took Applied or Pre-Calculus, but now they could base their choices on whether they preferred hands-on learning or a very structured, drill-based approach. They could make choices based on whether they wanted to work a lot with technology or focus instead on developing mental math strategies. They could make choices about whether they wanted to explore mathematics through real-life applications, or whether they were more interested in a more abstract or theoretical approach. Either/or courses were a possibility, as was a combination of the two courses.

All students in this study had first-hand knowledge of both the Applied and Pre-Calculus courses, each having taken at least one level of each course during his or her high school years. Students had different experiences with each course, and made different decisions about which course(s) to continue with, but all seemed very appreciative of having had exposure to the different learning approaches and strategies that enabled them to become more aware of their own needs, likes, and preferences. Based on their experience in both courses (this would have been the Senior 2 year for most students), they could make more informed choices about what direction their math learning would take for the remainder of their high school tenure. This translated into some telling advice for younger students who were making choices about which math course(s) to register for:

> **Michael** (F-BOTH): I think choice is good. Like, I think they should take both of them, in grade like ten, and then after that they can choose which one, or they can take both or whatever. But, yeah, I'd say, yeah, force it for people to take both first, just to try it out. They might like, they might have this idea of what this is like, but then they realize, oh, maybe it's like this. You know, or maybe like this. I like this, I guess I'll stay.
>
> ...
>
> **Kathleen** (F-BOTH): I think I would tell them to do both, too, because in the first year you have to, I think you have to find out where, in which way you learn best, and PreCal and Applied math, there are two different learning methods, and so taking them both would help them to figure out which one they do learn best in, and how they can better, you know, and then from that they can decide which one they further want to go into, or if they want to continue taking them both.

Randi (C-PCM) intimated that taking both in grade ten was akin to 'test-driving' the courses:

> ... but now that I've taken Pre-Calc and Applied and whatever else, then it's kind of been like, ok, I've taken this and I can do it. You know, ok, and it's just not something that I want to continue with. And I think maybe that's one of the purposes of taking it is just kind of to rule it out.

Indeed, among the many students who expressed an opinion on the matter, the recommendation for students was unanimous: if a younger student was considering a 'higher-level' math course, he or she should take both Applied and Pre-Calculus in grade ten. Only after having experienced both would they be in a position to evaluate which course better suited their purposes and met their needs. Only then would they be able to realistically to commit to one course or the other or both.

To me, the message of the students is clear. They see the different math courses as an opportunity: a chance to explore different methods and approaches. They consider it important that students have both the opportunity and the encouragement to take both courses, at least for one year. By experiencing both, they can become thoughtful about their own learning, they can begin to consider important aspects about how and what they want to learn, rather than simply follow a narrow path prescribed for them by institutional authorities. They can become informed decision makers as they consider the direction their future mathematics studies will take.

Making Connections

The second message underscores the importance of being able to make connections. Students generally liked the idea of being able to connect the math they were learning with real-world scenarios, but more notably, they underscored the need to be able to make connections within mathematics as a way of being able to make sense of what they were learning. Students maintained that when math was presented as a series of unrelated units, the relevance of studying any one of those units was questionable. But when they were able to relate one unit to another, they could begin to see how one concept would tie into another, how each part contributed to a growing understanding of the whole.

In the eyes of the students, Applied did a reasonably good job of relating mathematics to everyday life, but certainly didn't do as well as Pre-Calculus in connecting one mathematical concept to another. Applied was seen as being more fragmented, as indeed it is, particularly at the 40S level. For example, it is difficult to find logical connections among units on matrices, personal finance, and vectors! This compartmentalized nature of Applied was a source of considerable frustration for some:

> **Morgan** (C-BOTH): … in PreCal, like you work your way up and you always reuse the stuff. I didn't really like that in one way, but I was kind of glad, because at the exam, it's like you still remember it, because you've been doing it all the time. But for Applied, it's like you move from one thing to another to another. And that's where I'm going to, like, major downfall, because I do not have a good memory.
>
> **Brady** (C-BOTH): It's not so much progressive, it just sort of kind of jumps all over the place. Like going from a sequences unit straight into an investment unit is a bit of a shift. And we did a bit of that in Pre-Calc. We would jump pretty, um, pretty big jumps between things, but it always tied back to something and there was always, I don't know, recursiveness in it.

Pre-Calculus was more likely to base new knowledge on previous concepts. For example, students recalled how the unit circle, covered early in the course, formed a foundation for many other concepts they subsequently learned. **Lillian** (F-BOTH) and **Kathleen** (F-BOTH) discussed how it helped them to relate ideas and make connections among the various topics they were studying:

Lillian: I found that even just with, um, oh what's that circle thing called?

Kathleen: Unit circle?

Lillian: Yeah, it's been so long (laughs). I was amazed at how many different things we would do, like graphs and stuff like that, even things that the teacher didn't mention that was related to that. And then I'm like, oh yeah, you know, the unit circle, I'd think of the unit circle and I'd remember how it was related. I can't think of an example right now, but I just remember a lot of the times where I was just, like, wow, it all ties in.

Kathleen: Yeah, we used, we made a unit circle at the beginning of the year and we used it all the way to the end.

Other students made similar observations, commenting how it was important to their understanding of mathematics that they not only become familiar with the individual math concepts, but also be able to figure out how they connected to each other. In other words, they needed to learn the bits and pieces, but until they could put all those bits and pieces together to build a whole picture, they wouldn't fully understand it.

Students in both groups referred to the cumulative review exercises at the end of each Pre-Calculus unit as a prime example of how connections between units could be made. These exercises constantly recalled to them what they had learned before, and in so doing it allowed them to notice for themselves what links there might be between what they had learned previously and what they were currently studying. Furthermore, the cumulative exercises also provided a neat little exam review for them.

So once again, a message is clear to me. Students are not content to study a set of isolated topics; rather, they recognize their need to see the interconnectedness of mathematical concepts as part of achieving a fuller understanding of them. Mathematics is something of a jigsaw puzzle to them. They can examine each individual piece and become thoroughly familiar with it. But a truer image doesn't begin to emerge until the pieces are linked to each other, or as each piece is placed in reference to the bigger picture. A mechanism to build the connections already exists to some extent in Pre-Calculus, particularly in the cumulative exercises, but there is considerable room for improvement in the Applied course.

Two Courses in Combination

The third message deals with how the two math courses can complement each other. As students considered the nature of math, the nature of learning, and the role of technology, they identified for themselves the strengths and weaknesses of each course. For those students in particular who were interested in developing a clearer understanding of mathematics, rather than just earning a credit for graduation, it became evident that neither course on its own was sufficient. To a chorus of general agreement, **Randi** (C-PCM) stated it this way: "Probably somewhere between Applied and Pre-Cal is the perfect math course." But there is no in-between course, so students would need to take both to achieve the balance they needed for a fuller understanding of the math concepts they were learning.

In Pre-Calculus students learn the theories, the formulas, and the procedures of mathematics; in Applied they learn the applications and make extensive use of technology. The two courses are distinct, but because of the considerable content overlap, less than half the content of the second course could be considered 'new' material. That little bit of new material hardly seems worth the time and effort of spending another whole semester in a math course, yet students obviously see a benefit to taking both courses. In fact, their reflections suggest that it is not the new material that entices them into a second course; rather it is the opportunity to revisit the old material that makes the second course worthwhile to them. They are less focused on the breadth of mathematics that the combination can give them; instead, they are interested in the depth of understanding they can achieve by examining previously learned concepts from different perspectives. Instead of adding to their knowledge, they are multiplying it. Four examples will serve to illustrate how students perceived the combination.

Brady's (C-BOTH) comments were perhaps the most direct. Applied, followed by Pre-Calculus, allowed him to return to previously learned concepts and to think about them in new ways, thereby deepening his understanding (part of this quote appeared in the previous chapter, but bears repeating here):

> And I think Applied, really, has just clarified a lot of things that I learned in Pre-Calc. And I understood how to do them, but I didn't really understand why they worked or for what reason they worked, and then Applied just kind of gets to the root of things and clarifies it a bit, makes it a bit more relevant… I like that Applied is after Pre-Calc this year. Because I find that when we're learning stuff in Applied, I completely reflect back on all the Pre-Calc of when I learned it before. And then I kind of like, well I know how to do this, but then there's just different subtle ways of doing things. And I kind of make up my mind as to

which one, like some maybe better ways of doing things, and then I'm thinking, oh, that works better for me. And it kind of relates and I put the pieces together with the other way of doing things and then I get a whole nice big picture of how to do it and I can take the pieces of each one that work better for me. And then in some cases I just, well like, that's a good way to do it but I like the way that I already know how to do it. And I find that it's not so much learning new stuff as just clarifying the stuff I already know and refining it a bit. And so I really quite like taking both.

For Brady, taking both courses meant having the opportunity to examine things from different angles, and then to construct his own knowledge based on what he had learned. This allowed him to more fully realize at least one part of the ideal that was presented at the outset of Chapter 2: he could "draw on knowledge from a wide variety of mathematical topics, sometimes approaching the same problem from different mathematical perspectives or representing the mathematics in different ways until [he finds] methods that enable [him] to make progress" (National Council of Teachers of Mathematics, 2000, p. 3).

For **Kathleen** (F-BOTH), taking both courses allowed her to reconcile the apparent rift between what she perceived the nature of math to be and how she preferred to learn it. As already mentioned in previous chapters, Kathleen tended to view Pre-Calculus as the "pure" math, the collections of formulas, equations, and procedures as being the underpinning to understanding how mathematics fit in the broader context.

... because Pre-Calculus was, it was like the basis, the math, whatever, kind of stuff that I needed to learn and I enjoy that, you know, doing algebra and everything else like that, solving problem and stuff... it was learning, you know, formulas and the basis of math, which I could use, you know, if I want to further my math or whatever.

But Pre-Calculus involved a lot of drill and memorization, a learning style that she did not particularly care for. Applied offered her the opportunity to learn differently:

See, for me, I think Applied math, it incorporated more or different learning techniques than what Pre-Calculus did, because Pre-Calculus was all repetition, you know, you do equations, you do this and that, but in Applied math you actually had to do stuff to learn it and to me that was a better way of learning for me. I understood it more when I had to learn it and apply it instead of just, you know, learning and memorizing equations and formulas and working out problems. For me it helped to apply it to something.

Like Brady, Kathleen was able to take advantage of the combination of courses to deepen her understanding of mathematics. The fact that she was learning the same material over again in Applied that she had already learned in Pre-Calculus was not an issue since she was getting something else out of it:

> ...so it was two different perspectives even though it was the same math and sometimes there was overlap, but it was almost like there was two different results to each of them...

By re-examining a previously learned concept from a learning stance that she was more comfortable with, she was able to better construct her knowledge of it the second time around. The formulas and procedures she had learned in Pre-Calculus now had meaning and purpose in Applied. She had the background knowledge; she could now further develop it by figuring out how it connected with other aspects of mathematics or everyday life.

Another telling example is **Morgan's** (C-BOTH) story, as it emerged through her interactive writing and her participation in the small group discussion. Parts of this story have already appeared previously in this document, but I think it is worthwhile to pull it all together here. Recall that Morgan had no history with Applied prior to her Senior 4 year, but had taken Pre-Calculus all the way through high school and had scored well in the course. In her final semester, and for reasons unknown to me, she decided to enroll in the Applied course. She experienced something of a culture shock, particularly with respect to the heavy reliance on technology in some of the units. The unit on sinusoidal functions, for example, an early unit which had a great deal of overlap with Pre-Calculus, proved to be a great source of consternation for her. Part of her frustration came from not being familiar with the graphing calculator (which all the other students were, since they had taken Applied before), but she struggled more with the idea that she was letting a machine do her thinking for her. She made comments in class to the effect that by using the calculator, she wasn't using her brain, and reiterated this in one of her interactive writings about sinusoidal functions:

> This is such a weird class. It's like you don't really have to understand the graphs, you just draw them on your calculator and interpret them. Pretty much I don't understand it. It's way different than Precalc. It feels like I'm superstupid. All I know is that you can find the max, min, period & amplitude. Otherwise I'm in the dark.

In learning to use the graphing calculator, Morgan was focused on the tool itself, rather than on using the tool to further her mathematical understanding. The machine was a black box to her. Understanding graphs, it seems, meant knowing how to generate them from initial values. But the graphing calculator moved from input to final representation in an instant, so she couldn't follow the procedures it implemented. And since her idea of mathematical thinking and understanding seemed to be tied strongly to process and procedure, the absence thereof caused a void, which she then associated with lack of thinking and lack of understanding. It seems reasonable to assume that she must have questioned her own wisdom in taking this second math course. After all, she didn't seem to be furthering her understanding of previously learned concepts; in fact, it seemed she was just muddying the waters.

Over the course of the semester, Morgan scaled the learning curve for the graphing calculator, and gradually was able to start thinking about technology differently. The black box slowly became transparent; she no longer necessarily needed to know what was inside in order to make use of the information she was presented with on the other side. Rather than thinking about how to construct a graph, she could let technology make graphs quickly, and she could focus instead on manipulating them and experimenting with them, playing with parameters to see how they affected the graph. She could put her energies into thinking about the information she could get from a graph, what it represented, or how it could be connected to tangible things in the physical world. Moving past the process and procedure of creating the graph allowed her to think beyond the graph, thereby deepening her understanding of the entire concept. Here is how she summarized it:

> And I've never gotten graphs throughout my whole PreCal thing. And I've always had a really hard time to understand them...I didn't get it, but now I get it because of Applied. I see it on my graphing calculator and I know how it goes now. And I know what is what and I find it easier.

Having had the opportunity to go back to a topic previously covered, to explore it more thoroughly using technology, to examine it from different perspectives and to think about its results not only as a final answer, but as something that could be interpreted and linked to other phenomena allowed Morgan to continue constructing her knowledge of that topic, to develop a richer understanding of it in a way that made sense to her.

In the previous three examples, students all spoke of how Applied helped them understand the concepts they had learned in Pre-Calculus. But for some, it was evident that the courses were mutually complementary, as will be shown in this next example. To **David** (F-BOTH) and **Cody** (F-BOTH), there was a clear distinction between the courses:

> **David:** That's kind of the difference between the two, is that in Pre-Calculus you learn really how to do it, you know, how to physically manipulate the numbers, and Applied math you find out why, and how to use it.

The fact that the two courses were different, however, meant that each could complement the other. They could use the knowledge they learned in both to construct a deeper understanding of a concept. **David** used Pascal's triangle as a specific example:

> Well in Pre-Calculus math we learned about that and he showed us how to make the triangle and everything. So we all understood how to make the triangle, but we were never told the use of it. Right? So later on when we got these grid questions, and he wanted to know how many ways there were to get from point A to point B, we just kind of sat there scratching our heads. As soon as we were informed that you could, you know, apply it to the grid, it made complete sense. And that's a very extreme situation, but that's how a lot of it is... In Pre-Calculus math, you can get it, but it doesn't make a lot of sense all the time... We got to Applied math, and it's like, oh, that explains a lot! Cuz in Pre-Calculus math, you get people, or yeah in Pre-Cal you get people there with their pen trying to trace every single route from A to B. You know, it just, you know, as soon as we understood what the application of it was, it was incredibly useful.

As in the previous examples, Applied certainly helped them to make sense of something they had learned in Pre-Calculus. But during the conversation, they illustrated that the complementary nature was not unidirectional. Pre-Calculus also helped them to make sense of Applied.

> **Moderator:** ...But is there a point to learning the manipulation [in Pre-Calculus]?
>
> **Cody:** Yes. Because even through the repetition, like I think I said this before, through the repetition of doing those equations that are, you know, kind of meaningless numbers, whatever, but you can get the right answers. So now you know how to do it, cuz you've done it so much, you kind of, you know how to do it, so when you go to Applied, you, while you do, you know you understand where it's supposed to go, so you know if you're off, but you also have all this repetition practice, this rote memorization of how it should generally work. So it helps you to work with that Applied equation in that sense.

The process learned in Pre-Calculus helped them get to what they knew to be a reasonable answer in Applied. But they also pointed out that one of the dangers of Applied math was that people might begin to associate certain mathematics with specific situations, and not be able to make the leap to other contexts. In that sense, studying mathematics more in abstraction or isolation had its benefits:

> **Cody:** I guess, to have a simply conceptual idea of something instead of a physical, more concrete idea, just generally helps you to continue to build on that conceptual idea until eventually it reaches a point where it is appliable. So it's easier to make it more complicated, I guess, if you keep it in the conceptual stage. To continue to build on it in that way, I guess, maybe.
>
> **David:** You don't have to strip away some of your set ideas from applying it directly to something.
>
> ...
>
> **Cody:** Or where you assume it should be going when it's trying to take you somewhere else. So if you keep it in the conceptual stage, then I guess you're not really applying it to anything so it's easier to build on it and change it than it is to if you've already applied it to something.
>
> ...
>
> **David**: If I could only relate it to, you know, how much someone should charge for boat rides, then I would have had a very difficult time doing it, probably. But because I knew how the equation worked in its basicness.

Knowing a formula or procedure in general allowed them to apply it in many different contexts, rather than falling into the trap of thinking it applied to only one. They would have the skills to manipulate it in whatever way was necessary to fit any scenario.

These examples illustrate how the complementary natures of Pre-Calculus and Applied work both ways. Taking both courses in combination offers students the opportunity to amalgamate the knowledge they glean from each, thereby allowing them to broaden the scope of their mathematical thinking. They can use the approaches and skills from either course to supplement what they are learning in the other, constructing for themselves a more complete basis for understanding and application.

As stated in the opening chapter of this thesis, I have long had the sense as a teacher that taking the Applied and Pre-Calculus courses in combination was beneficial to students. I believe on the basis of the data in this study that many students seem to share that sense. Once again, the message is clear. If one wants to develop a deeper understanding of a concept, one needs to explore it deeply. One needs spend time with it, and examine it from different perspectives. One needs to learn how to think about it in different ways. This cannot be done well in either of the two courses alone. Pre-Calculus covers too much material and moves too quickly to afford students the luxury of time for exploration. And Applied skims over the processes in favour of interpretation and contextual problem solving. But by taking both courses, students can give themselves the time they need to explore the different approaches and take advantage of the opportunities to work with concepts in diverse ways. The skills learned in one course can complement those learned in the other. The same topic can be viewed from a variety of perspectives, and thereby a fuller understanding of it can be achieved.

As a final note to this section, it needs to be pointed out that student sense of caring about what they were learning was a key factor in their personal decisions about taking courses in combination. For those for whom studying mathematics was just a hoop they needed to jump through on their road to graduation, the combination of courses was nothing more than a waste of time, and most of them eventually elected to concentrate on only one course. Yet even those students who chose not to pursue a combination of courses for themselves, still projected the benefits of the combination onto those who *did* care about understanding the mathematics they were learning.

Pathways to Communication

The fourth message I heard from the students is more subtle in the sense that there are few instances where students addressed this issue directly. Instead, it is the nature of the comments and their richness that conveys the message to me: students have valuable things to say about how and what they learn, and we as educators need to find ways of listening to them and taking their ideas into consideration as we plan our courses. What students have to say can challenge, for example, both what is happening in the classroom, and what is happening in curriculum design.

Sometimes the comments made by students caused me to challenge my own presuppositions. Allow me to illustrate this with two of several possible examples. As part of the bracketing procedure prior to collecting any data, I had spent time reflecting on my own perspectives on mathematics, learning, and teaching. After analyzing the student data, I returned to my reflections and compared them with what the students had been saying. In some areas, such as the idea that there are compelling reasons to take both courses, there was close correlation between what I thought and what a good proportion of the students were saying. In other areas, however, my perceptions were challenged, and as a result, I need to spend some time reconsidering them in light of these data. For instance, I was surprised by the attitudes that some students displayed with respect to technology (described in the previous chapter). As a teacher, I recognized the tremendous potential of the graphing calculator as a learning tool, particularly as an instrument of investigation and as a way of getting past the procedures so that one could focus on the interpretation of the result instead. In fact, all students did use the calculator as a learning tool to a greater or lesser degree (at least in Applied). But it was those students who were not realizing its potential that gave me pause. Some students did not want to let go of procedures, thinking that using the calculator was 'cheating' and therefore they would not understand the math fully by relying on it. Some never got to the stage where the calculator became transparent – they focused so much on the methods of using the technology that they could not move on to using it effectively as a learning tool. Some continued to use it solely as an answer generator – it was a way of quickly finishing an assignment, but they were not interested in exploring the additional learning opportunities it afforded them. As a teacher, I want to consider what I can do to overcome this reluctance that some students have with respect to using technology.

A second example that illustrates how my ideas were challenged concerns my perceptions of the two courses as compared to the perceptions of the students. In my zeal to embrace the constructivist philosophy, I had begun to elevate the Applied course over the Pre-Calculus course in my own mind. I saw Applied as 'living' math, as opposed to a sterile climate in Pre-Calculus which consisted of mindless memorization and drill. My phrase for Pre-Calculus had become 'monkey see, monkey do, monkey don't think' – in my mind, learning how to manipulate formulas wasn't necessarily equated with mathematical thinking. While some procedural competence was necessary for good mathematical understanding, it was not necessary to be so singularly focused on it as Pre-Calculus seemed to be. But the students in this study have suggested that they believe otherwise. Pre-Calculus may be more theoretical and abstract, but for many that had a very important place in their idea of understanding math; in fact, Pre-Calculus was the "pure" math. Pre-Calculus kept a narrow focus and allowed them to develop reasoning skills; some described it as a "logical progression", and others spoke of the "cleanliness" of the step-by-step methodology by which they learned the concepts. Familiarity with procedures and processes was for many a crucial foundation that needed to be in place before they could begin to expand into various applications or different perspectives. Indeed, for many, understanding a mathematical concept centered on being familiar with the formula and how to use it and manipulate it, not on how it might connect with the broader world.

This raises some interesting questions for me. In creating the Pre-Calculus curriculum, the developers targeted those students whose intentions were to continue with post-secondary mathematics studies – a relatively small number, to be sure. Most students will never directly use the mathematics they learn in Pre-Calculus, particularly at the higher levels, yet they continue to flock to the course. Why? Do they want a challenge? Do they want to keep doors open? Is the structure of Pre-Calculus part of their comfort zone? Do they see it as rounding out their understanding? Why do a lot of students not consider Applied to be as 'real' a math as Pre-Calculus? Have they been too conditioned by prior practices to learn to think differently about mathematics? I cannot begin to answer any of these questions here; indeed, they would be the topic for a whole new thesis. But the fact that I have taken the opportunity to listen to students, and the fact that their comments have surprised me and made me think in new directions, argues strongly that there is a need to keep the lines of communication open between teacher and learner.

Sometimes the comments made by students had a direct and immediate impact on the way I was teaching. For example, **Lillian** (F-BOTH) had commented on how she had difficulty in relating equations and graphs when using the graphing calculator (see the quote on page 104). She talked about using an equation in Pre-Calculus to generate a table of values, and then plotting the values from that table to produce a graph. In that way, she could clearly follow how the graph was another representation of the original equation. But somehow that idea did not translate when she used technology: she had difficulty making the connection between the equation she entered into a graphing calculator, and the graph that resulted from it. In other words, by not doing the intermediate step of creating a table of values, Lillian seemed to have lost sight of the idea that a graph is simply a set of (x, y) coordinates that satisfy a given equation. It occurred to me that other students might also be missing this bridge between the different representations, that they would not be able to see that a graphed line was a collection of individual points generated by the equation. This lead to changes in the way I approached functions on the graphing calculator with my classes. When we had worked with these in the past, I had focused primarily on the equation screen and the graph screen. I would have reminded students that they could think of an equation as a table of values that would generate a graph, but I would not have taken a lot of time going through that step. But after considering Lillian's comments, I now very deliberately include the table of values screen along with the equation and graph screens when we talk about functions, so that the idea that there is a direct connection between equation and graph can be reinforced. This happens most often in the Applied 20S course, when the students are just learning how to use the graphing calculators and coming to understand the idea of functions and multiple representations, but I have found it beneficial in some of the other classes as well.

Sometimes the comments made by students were directed at the system, and it is here that a rather sad picture emerges of a group of people with no sense of opportunity to provide feedback to those who are designing their programs for them. Consider the following excerpt from the current students' discussion, noting particularly *who* is being referred to:

> **Morgan** (C-BOTH): I think, like I think for, like sometimes I would come into Pre-Cal, like this last semester, and I would seriously like roll my eyes and think, like, why on earth do we have to learn this, it is so ridiculous. But then sometimes I would be like, oh yeah, this, I could see myself using this in the future. So I think, I don't know, the big people out there should like learn how to decipher what, like, what is useful or not. Because sometimes it's just dumb. Like you don't use it ever.
>
> **Nic** (C-APM): You think it's just fill.
>
> **Morgan**: Yeah. It's just dumb. And then I'd rather learn better, like learn longer time on the stuff that maybe is a little bit more helpful.
>
> **Brady** (C-BOTH): I think a lot of the stuff that you might not find relevant, I think it is important just that it's teaching you to think in ways that you're not normally used to thinking in.
>
> ...
>
> **Nic**: It's just expanding your mind, I guess...But they should expand our minds but then throw in something relevant at least maybe once in a while so we don't feel like quitting. Because I always go through a phase of we're not learning anything that I can apply anywhere, that I get so blaaah. I just want to quit. Like this has no relevance. This, I'll never ever use this. What am I doing? And then I just think it's fill, but then they just throw a little thing that you need to use once in your life. That helps.

At first glance, this excerpt would appear to be concerned with the question of relevance, but I believe there is a deeper issue, one that centers on voice (or lack thereof). Students are the most important stakeholders in their own education, but I was astonished at their perception of how predetermined their educational path was, and of how little input they could have. As I analyzed the body of data for this study it became evident to me that students have very clear ideas about what they want to know, about what they need to know, and about how they should go about coming to know. They can match these ideas to specific courses or combinations, but beyond making course selections, they do not seem to see any further options. Students *should* have some say in what courses are available to them, but they often see the limitations of their particular school context as absolute. Students *should* have some say in what is in those courses, but they see the curricula as set in stone. Students *should* have the right to demand excellent and diverse instruction, but they do not think to actively challenge the status quo. They do not perceive that a way exists in which their needs can be effectively communicated to the education providers.

So once again, I perceive a clear message. There need to be feedback mechanisms that exist between student and teacher, and between student and administration, where the carefully considered ideas of the students can be taken into account as timetables are planned and courses are developed. Pathways to communication between planner and recipient need to be opened. Students need to have a voice, and they need to know that they have a voice. Surely the cause of mathematics education in Manitoba can only be furthered if students were to have the chance to communicate some of their good ideas with those "big people out there". Perhaps studies such as this one will give some support to recognizing the importance of encouraging student voice and input.

Chapter 7

Impressions: A View from the Gallery

Research Intentions – A Summary

This phenomenographic study provided former and current students from my school an opportunity to think about their mathematical learning experiences in Pre-Calculus and Applied, and to describe them within the context of a small group discussion or paired interview, or to write about them in their interactive writings. Analysis of the data resulted in three categories of description that, according to the principles of phenomenography, encompassed the body of data, yet within each category revealed the broad range of experiences as described by the participants:

Student perceptions of the nature of mathematics. Within this category, students revealed what they thought mathematics was all about. They depicted a wide range of perceptions, going from math being nothing more than a collection of equations and formulas that needed to be memorized, to math being a way of thinking that was intimately connected with every aspect of daily life. Students recognized a personal relationship with mathematics, categorizing themselves and others as being anything from 'not a math person' to being a 'math whiz'. They differentiated between the Pre-Calculus and Applied courses, describing their perceptions of the two in terms of the purposes of each course, the different approaches, and the level of difficulty in each. Again, student perceptions varied widely.

Student perceptions of the nature of learning. Students depicted a clear awareness that there were many ways of learning and that no single learning style could apply to everyone. Collectively they described a wide range of learning styles, going from emphasizing drill and practice to become familiar with a procedure, to working collaboratively with others, to engaging in independent exploration of a topic to come to understand it.

Student perceptions of the role of technology. As they discussed the role technology played in their math educations, students described how they used technology and they revealed a wide range of attitudes towards it. Some were reluctant to incorporate it, fearing that developing a dependency on it would come at the cost of understanding a concept. At the other end of the spectrum were those who embraced technology as an important learning tool that could help them explore a concept in greater depth. Technology was considered a major factor that distinguished Applied from Pre-Calculus.

The categories of description depicted the range of experiences among the students, but the meaning behind the students' comments was constructed as a set of overarching ideas, interpreted as a series of messages to me as an educator:

Making choices. How students feel about the nature of mathematics, the nature of learning, and to what extent technology should be involved in learning plays an important role in the choices that will be made. Students in this study recognized the different approaches taken by Pre-Calculus and Applied and the different learning styles that they tended to use in each. They stressed that before one could make an informed choice about which math course to choose, one needed to experiment with both in order to know what approach or combination of approaches worked best. To that end, participants were unanimous in their recommendation that all Senior 2 students who were interested in higher-level math courses not only have the opportunity to take both Pre-Calculus and Applied, but in fact be strongly encouraged to do so.

Making connections. In the eyes of most of the students in this study, mathematics was seen as a progression of ideas, a structure in which each concept learned would eventually provide a foundation for further learning. In a comparable manner, learning was seen as a progression, where new knowledge was best built on prior understanding. Educators have long been aware of this, but students themselves underscored this by emphasizing the importance of being able to make connections to previously learned concepts. A series of disconnected units often left them floundering, but if they could connect a new topic to one they had learned earlier, they had a basis upon which to build. Students thought Pre-Calculus did fairly well in this respect, but faulted the Applied course. Although it excelled at making connections with real-world phenomena, it offered few links among units that would allows students to deepen their understanding of previously learned concepts within the course.

Two courses in combination. Students didn't think consciously about the categories of description outlined in Chapter 5, but their internalized stance within each category affected their how they feel towards Pre-Calculus and Applied. Some students opted for the one course that is most closely correlated with the way they felt about math, learning and technology. But others, particularly those who were interested in developing a more complete understanding of mathematics rather than in simply fulfilling graduation requirements, recognized the limitations of a single course and chose instead to take both courses in combination. Far more than just repeating material, taking both courses allowed them to approach the same topics from new perspectives, to re-view and clarify concepts, or to pool the knowledge and then construct individual meaning from it. The complementary nature of the two courses worked in both directions, each supporting the knowledge being built in the other. Taking both courses not only broadened the knowledge base for the students, but added considerable depth.

Pathways to communication. Students' individual beliefs about the nature of mathematics, the nature of learning, and the role of technology are internalized. As teachers, we like to think that we can 'read' these individual perspectives through their day-to-day comments, attitudes, work habits, and so forth. However, when students are presented with a formal occasion where they are invited to reflect on their stance and given the opportunity to discuss it with others, surprising results may emerge to challenge teacher suppositions, to impact how certain lessons are constructed, or to open the door to broader implications.

Personal Impact

As outlined in Chapter 1, my rationale for undertaking a study of this nature was to continue my search for balance between content-driven instruction and inquiry-based learning in mathematics; to look at it not from a pedagogical stance, but rather to examine it from the point of view of students as they described the experiences they had had in the two math courses. The interactive writing provided some glimpses into their thought processes, but it was the discussion groups and the follow-up interviews that were most illuminating to me about what students thought about their mathematics learning experiences. Their reflections supported some of my perceptions and challenged others, they validated some of my approaches to teaching and caused me to rethink a few, and they certainly opened a whole new world of questions for me. I had the sense that these data alone could support a dozen further research studies.

As I wrote about pathways to communication in the previous chapter, I outlined how this study had impacted my thinking and teaching, even in the midst of the research. But the greatest personal impact of this study for me lies in the fact that after listening to these students, after spending considerable time analyzing what they had said and considering the implications of what they were saying, I cannot, in good conscience, go back to *not* listening to students. Their input is far too important to ignore. As I stated earlier, it is easy for me as a teacher to become complacent and think that I can 'read' the students well enough to know where they stand, but this exercise has taught me that that is not good enough. Inviting students to become very deliberate about reflecting on their learning experiences and about discussing them in a focused setting will provide a far better 'read' than anecdotal tidbits ever can, and will provide a far better basis upon which to plan my lessons and to think about how I want to implement the curriculum. The students who responded to my invitation did so wholeheartedly and with gusto – in fact, many seemed quite excited about having the opportunity to talk about their experiences. In some cases I even had the luxury of just being able to sit back and listen as they talked with feeling about what mattered (or did not matter) to them in their mathematics education. Just because I was their teacher, I was not spared any criticism if someone felt it was warranted. In fact, that indicated to me that the students were comfortable with being open with their thoughts.

I will, of course, have to find other ways of listening to students besides the methods I used in this study. The sheer volume of data generated and the time involved in transcribing and analyzing the conversations make repeating this on a regular basis impractical, but the synergetic focus groups were a wonderful way for students to think about their experiences, compare them with others, and to consider them from various perspectives. Perhaps these can be transformed into shorter, targeted classroom discussions. The interactive writings, already integrated as part of the instructional plan of the classes, provided vehicles through which individuals could express their thoughts. These need to continue. Other avenues can be explored to give students more voice in what and how they are learning.

Going Beyond

Of course, the data collected for this study provides but a snapshot representing two particular groups of students at a particular time and place. The interpretation has been done through my eyes. The messages I gleaned from the students are situated within the teacher-student relationship I had with them, and cannot be extended to any other set of students or teachers. The balance between whether to focus on skill development or on understanding in mathematics has not been defined for anyone as a result of this study – it will continue to shift even for me as each new group of students walks through my door.

But I believe there is a message in this study for the broader educational community. Consider again the last quote in Chapter 6 (page 121), where the students made reference to those who were designing the math curricula. To me, the very absence of any notion that students' opinions about the content of a course or how it is delivered could matter is very concerning. Theirs are simply bleak statements of reality without any hope for change. Students seem so mired in the pattern of letting others control their education that it does not even occur to them that they could or should have a say. **Morgan** (C-BOTH) talked about how the "big people out there" should learn how to decipher what is useful or not, but the idea never seems to come to mind that her thoughts might help in that deciphering process. **Nic** (C-APM) talked about what "they" should do, but in using that ubiquitous term, he underlines the great divide that exists between himself and those that make decisions about his learning, those that have the power and the influence to direct the course of his education. There is no hint that a bridge might or even should be established so that both parties might become involved in the decision-making. Indeed, students seem quite convinced that the decisions others have made about their mathematics education are inflexible and that the people who have made those decisions are faceless and unreachable.

But the students *do* have some good ideas to contribute. As is evident from the data presented in this thesis, they can be very thoughtful and reflective about mathematics education *when given the opportunity*. Their diverse ideas on the nature of mathematics, the nature of learning, and the role of technology, for example, illuminated for me what was happening in my own classroom and in my own school. It underscored for me what we were doing right, and highlighted areas that needed to be rethought. But even more important is that in providing an opportunity for students to speak about their educational experiences, particularly in a forum that promises to carry their voices beyond just the local context, students can be reminded of their stake in the educational process and alert them to the fact that their voices can and should be heard.

As educators, we need to find more ways of listening to students, more ways of including them as we plan our courses and develop our curricula. They may not have the benefit of hindsight like those who have reached their educational destination (e.g. university degree in mathematics), but they are current travelers on that road, and have good ideas of what it will take for them to keep going. Their input is valuable, both to themselves and to others; indeed it is vital if they are to take ownership of their stake in education.

So I believe this study extends an invitation to everyone in the educational community. I invite teachers to find their own ways of giving voice to their students. I invite administrators to make every effort to ensure that all possible course options requested by students are available and accessible through the timetable, and that students are encouraged to consider combinations as a way of expanding their horizons. I invite those who are working with teacher candidates to help them consider students voice and its role in how teachers teach and students learn. I invite those who are responsible for determining the course of education in Manitoba to include students directly in their decision-making, not only as 'subjects' upon which to pilot new courses, but as participating voices in the planning and implementing of new curricula.

Indeed, everyone mentioned in the preceding paragraph already has some say in education. Based on the rich information I was able to gather in this study, I think it is time students had some say as well.

Closing Remarks

This study has been a remarkable leg of the educational journey for me. I have been reminded of how unique each individual student is, and how wide-ranging their collective experiences are. My conviction has been reinforced that learning is meaning-making that varies from person to person, and that opportunities must be provided to foster that meaning-making. My belief that our path to mathematics education reform should continue has been confirmed, although it is worth reconsidering its exact direction as well as who should be on it.

I have been challenged to cultivate in my students a voice, and to encourage them to use it. I am grateful to the students in this study for being such willing participants, sharing openly and seemingly without reservation about their learning experiences. Their rich descriptions and their willingness to explore themes in greater detail permitted me insights into their thinking about mathematics and about learning that would have otherwise been lost to me. It has convinced me that their voices should not be silent.

I am glad to have taken this journey. My course has been altered, and there are new travelers on the road with me. I am encouraged that this research might entice others to embark on a similar path, and I am excited by the prospect of where the journey might lead us all.

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Appendix A

A-1: *Interactive prompts used after the introduction to a concept*

Pre-Calculus (Friday, January 9, 2004)
In today's class, you were introduced to sequences. Your friend Sam wasn't there and has emailed you, asking you to help him get caught up before Monday's class. In your response (one paragraph or so), explain sequences to him, based on your understanding of them after today's class.

Applied (Tuesday, February 10, 2004)
You have started your study of periodic functions. In particular, you are focusing on sinusoidal functions. Write a short letter to your pen pal to explain what you have learned about them.

Applied (Wednesday, March 3, 2004)
You have spent several classes working on fractal patterns, as a lead-up to your study of sequences. Write a paragraph to your pen pal, describing your sense of sequences so far. Consider commenting on how you think they relate to mathematics.

A-2: *Interactive prompts requiring process writing*

Pre-Calculus(Monday, January 12, 2004)
Sam considered your response to him last Friday and thought he was able to figure out sequences pretty well over the weekend. However, he's having problems with the question below and has emailed you again to ask for help. Respond to him, explaining how you would go about working out the solution. (Actually work out the problem as far as you can, explaining your reasoning in each step.)
Problem: One way of arranging oranges for display is to stack them in square-based pyramids. Find a formula that describes the sequence of the total number of oranges in square-based pyramids of increasing size (i.e. find a formula that will allow you to determine the total number of oranges in a square-based pyramid of any size).

Applied (Thursday, February 26, 2004)
Your pen pal wrote back to you to thank you for your explanation of periodic functions a week or two ago. Now he (she) is having some difficulty, though, figuring out how to apply the function. He (she) writes to you, asking how you would go about doing the problem below. Take him (her) through the problem step by step. If you choose to use the graphing calculator in your explanation, assume that your friend is familiar with the basic operations that might be required (e.g. lists, SinReg, etc.).
Problem: At Estevan, Saskatchewan, the latest sunrise time is at 09:12 on December 21. The earliest sunrise time is at 03:12 on June 21. Sunrise times on other dates can be predicted from a sinusoidal equation. There is no daylight savings time in Saskatchewan, and the period is 365 days.

Write a sinusoidal equation that relates the sunrise time to the day of the year; then use the equation to predict the time of sunrise for today (February 26).

Applied (Monday, March 15, 2004)
Your little sister (assume you have one in junior high) sees you working on sequences and is interested in what you are doing. You indulge her and take her through this problem step by step, explaining your reasoning. Write down what you would say to her to help her understand.
Problem: Logging is a multi-million dollar industry in Canada. Calculations of quantities of lumber are often based on sequences. For example, logs are often stacked in such a way that their ends form a triangular shape. These logs sometimes need to be counted, but this would be easier to do if one could determine how many are in the pile based on how many logs are on the bottom layer, rather than counting each log individually. Find a way of determining the total number of logs in a pile with 7 logs in the base layer, and then with any number of logs in the base layer.

A-3: Interactive prompts where students consider relevance

Pre-Calculus(Tuesday, January 13, 2004)
Sam is holding his own in the sequences unit, thanks in part to your help. He complains to you, though, that he thinks that he is maybe missing something. He knows the formulas now, and can figure out how to use them, but he isn't sure he truly understands what or why he is learning about arithmetic or geometric sequences, or of what use they might be. You ponder his dilemma and decide to email him later in the day with your thoughts on the matter. What will you write to help him figure things out?

Applied (Wednesday, March 3, 2004)
There was no corresponding question given here.

A-4: Interactive prompts about test preparation

Pre-Calculus(Thursday, January 15, 2004)
You will be writing a provincial Pre-Calculus exam next Tuesday. Tell me how you are going to prepare for this exam. In a paragraph, outline one or two strategies you plan to use to prepare for the exam in general, and then consider one or two specific strategies you might use when reviewing the trigonometric function in particular (sine functions, etc.).

Applied (Tuesday, March 16, 2004)
How did you prepare for the test today? What were the things you considered important to think about?

Appendix B

Chronology of Data Collection

September, 2003 – Data collection began

Examination of curriculum documents
Ongoing reflective journal

Semester 1
Collected reflections from former principal
Conducted focus group with graduates
Conducted follow-up interviews with graduates
Conducted interactive writing with Pre-Calculus class
Ongoing reflective journal

Semester 2
Conducted interactive writing with Applied class
Conducted focus group with current students
Collected surveys from teachers
Ongoing reflective journal

June, 2004 – Data collection ended