THE UNIVERSITY OF MANITOBA

AN ANALYSIS OF GRADE XI DEPARTMENTAL EXAMINATION RESULTS OF STUDENTS TAUGHT BY FULLY QUALIFIED AND PERMIT TEACHERS FOR JUNE 1956 MATHEMATICS

BEING A THESIS SUBMITTED TO THE COMMITTEE ON POST-GRADUATE STUDIES IN PARTIAL FULLFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF EDUCATION

BY

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THE PROBLEM

This study was concerned with the relative success on the June 1956 Departmental Mathematics Examination, in the lower half of mathematical achievement, of three groups of pupils. The test items on the examination were analyzed with a view to assessing the values assigned questions involving mathematical understanding and those solvable by imitation. Scores made on various items and question types yielded information as to the comparative difficulty and discriminative power of each.

DEFINITIONS OF TERMS USED

"A" Students = those taught by fully-qualified teachers in public schools of School District of Winnipeg Number One.

"B" Students = those instructed by fully-qualified teachers outside District Number One.

"C" Students = those trained by teachers on permit in rural schools.

Source of Data = Grade XI 1956 Mathematics answer-booklets, selected by the Department of Education as representative of the three groups.

Sample Papers of Lower Half of Mathematical Achievement = samples of the papers from the lower 50% of all pupils in Grade XI (for each group).

Questions Solvable by Imitation = questions similar to those asked for June 1955, or worked out in detail in the texts.

Original Question = one for which there was no parallel on the June 1955 examination, nor a detailed explanatory example in the texts.

Easy Question in Terms of Score = a question on which, on the average, all pupils of one group scored 80% of the possible marks.

Difficult Question in Terms of Score = a question on which, on the average, all pupils of one group scored less than 20% of the possible marks.

Easy or Difficult Questions or Question Types in Terms of Rank - For each group, questions and question types were ranked according to the percentages of possible marks scored. Rank 1 = easiest.

Discriminative Power of a Question or Question Type = the ability to discriminate between students at different levels of achievement.

Percentage Figure for Discriminative Power was obtained by dividing the score of the lowest ranking group ("C") by the corresponding score of the top ranking group ("A").

SAMPLE PAPERS FOR LOWER HALF

The Department of Education supplied 103, or 13.8% of total "A" papers written; 103 or 13.7% of total "B"; and 51 or 39.2% of total "C". The fact that the failure rate (15.5%) for the "A" sample was exactly the same as for the entire "A" population was evidence that a well balanced choice of sample papers had been made. Because the better "A" (32.1%) and "B" (18.8%) were exempted, it was decided to choose samples of papers from the lower half of all three groups. Using Christmas and Easter marks from the largest City High School, figures were produced showing that had exempted students written June tests their scores would not have belonged in the lower half.

EXAMINATION PAPER

In the algebra portion of the examination, 31 marks of a possible 50 were assigned for questions similar to those of the previous June; 10, for examples fully explained in the text; and 9, for 'original' questions requiring one or more statements. In the geometry section, 12 marks were given for propositions; 15, for 'original' blank-filling items; and 23, for 'original' questions requiring more than one statement. Through mastering just the previous June test and examples demonstrated in the texts, a students could have made 31% out of a possible 50% in algebra, and 12% out of 50% in geometry. Two-and-a-half times as many marks were given in the geometry as in the algebra section for 'original' questions requiring several statements. It was concluded that geometry would require a more thorough understanding of mathematical principles than algebra.

COMPARISON OF TOTAL SCORES OF THREE GROUPS

(1) The failure rate of "A" students was half that of "B", and one quarter that of "C".

(2) The order of merit was "A", "B", and "C" for all comparisons made as to total scores for:

(a) algebra and/or geometry (means and percentiles 10, 25, 50, 75, and 100)

(b) all question types.

(3) "A"s were even more superior in geometry than in algebra.

COMPARISON OF MARKS ON INDIVIDUAL QUESTIONS AND QUESTION TYPES

<u>Average Marks</u> On the 37 questions the average marks in percentages ranked in an "A","B", "C" order in 30 of the questions. Table I, page 5, summarizes the results. The 37 questions without exception were able to discriminate between the high scoring ("A") group and the low scoring ("C"); in all but two questions, between the middle scoring ("B") group and "C"; and in all but five between "A" and "B".

<u>Discriminative Power of Question Types</u> To find the discriminative power of each question type, the total average marks for "A" were divided by the corresponding total for "C". The quotients in percentages, listed in order of increasing discrimination were:

(1) 'original' geometry questions requiring the filling of a blank with a number	78.0%
(2) questions similar to those for the previous June, and of equal, or less, difficulty	72.9%
(3) questions similar to those fully explained in the texts	70.2%
(4) 'original' questions requiring more than one statement	60.6%

TABLE I

SUMMARY OF COMPARISONS OF RESULTS ON QUESTION TYPES FOR THREE GROUPS

QUESTION TYPE	NUMBER of QUESTIONS	RANK ORDER		
		"A","B","C"	"B","A","C	"A","C","B"
Same as Previous Test				
Equal or less difficulty (algebra only)	10	7	(1b,1e,) 3(1f#)	
Added difficulty (algebra only)	4	4		
Explained in Text				
Algebra	3	1	1(9b#)	1(4b#)
Geometry	2	2		
TOTAL SOLVABLE BY IMITATION	19	14	4	1
Original				
Number in blank (geometry only)	5	5		
Reason in blank (geometry only)	6	6		
More than one statement Algebra	3	2	1(9a#)	
Geometry	4	3		1(7#)
TOTAL ORIGINAL	18	16	1	1
TOTAL ALGEBRA	20	14	5	1
TOTAL GEOMETRY	17	16		1
GRAND TOTAL	37	30	5	2

#Figure in bracket is question number.

(a) algebra only 66.8%

(b) geometry only 51.9%

(5) 'original' geometry questions requiring the filling of a blank with a reason 45.2%

(6) questions similar to those of the previous June but with one added difficulty 44.4%

When the following pairs of question types were compared, the second mentioned in each case proved to be more discriminating.

a)	solvable by imitation	65.7%	'original'	59.8%
b)	algebra	63.4%	geometry	58.1%
c)	number in blank (least discriminating of all types)	78%	reason in blank (second most discriminating type)	45.2%
d)	similar to June 1955	72.9%	same with added difficulty (most discriminating of all types)	44.4%
e)	geometry questions not concerning the circle	68.8%	geometry questions concerning the circle	51.3%

<u>Easy and Difficult Questions</u> Table II on page 7 gives the number of questions classified as easy or difficult for each group. It will be noted that no question was easy for pupils taught by Permit Teachers and that not one 'original' question requiring more than one statement, nor one similar to the previous June but with an added difficulty, was found

TABLE II

QUESTIONS AND QUESTION TYPES FOUND EASY OR DIFFICULT

		"A"	"B"	"C"	TOTAL
ON BASIS OF MARKS					
Easy Questions	J	2	2		4
	J+				0
	T	1	1		2
	O.B.	5	1		6
	O.S.				0
Total		8	4	0	12
Difficult Questions	J	1[#]	1[#]	1[#] + 1	4
	J+			1	1
	T	1[z]	1[z]	1[z]	3
	O.B.			1	1
	O.S.		2	2	4
Total		2	4	7	13
ON BASIS OF RANK ORDER AS TO DIFFICULTY					
Easiest Question					
All test		Geo.5b	Geo.5b	Geo.1a	
Algebra only		2a	1b	1a	
Geometry only		5b	5b	1a	
Most Difficult Question					
All test		Alg.1e	Alg.1e	Geo.5e	
Algebra only		1e	1e	9b	
Geometry only		8	8	5e	
Easiest Question Type					
All test		O.B.R.	O.B.R.	O.B.N.	
Algebra only		J+	J	T	
Geometry only		O.B.R.	O.B.R.	O.B.N.	
Most Difficult Question Type					
All test		O.S.	O.S.	O.S.	
Algebra only		T	T	J+	
Geometry only		O.S.	O.S.	O.S.	

KEY J = Similar to June 1955 (+ = Added Difficulty)
O.B. = 'Original' Blank Filling (R = Reason, N = Number)
O.S. = 'Original' Requiring More than One Statement
T = Explained in Text
# = Algebra Question No. 1e
z = Algebra Question No. 9b

easy for any group of pupils.

Two questions found difficult for all groups were:

(a) "Simplify $X^{2n} \div X^{n-1}$"

(b) the motion problem.

In addition to these two questions "B" and "C" scored less than 20% on two 'original' geometry questions requiring more than one statement. For at least one question of each type "C" scored less than one-fifth of the possible marks.

Easy and Difficult Question Types An analysis of the lower portion of Table II page 7 shows that, except for the easy type for algebra, both groups of students taught by fully qualified teachers found the same question types easiest or hardest. Other than finding 'original' questions with more than one statment difficult, pupils of Permit Teachers had different question types from those of qualified teachers rated as easiest and most difficult.

Conclusion The results on questions similar to those of the previous June but with one added difficulty led one to suspect that "A" students depended least of all three groups on memorized rules without understanding for solution of algebra questions; and "C" pupils, most of all. Results on geometry questions requiring one reason for each answer, seemed to indicate that scholars of Permit Teachers had relatively little understanding of geometry concepts. In algebra, the ranking of question types for "B", and "C" led one to infer that "C" students spent relatively more time studying algebra examples fully explained in the text; and "B", algebra questions for the previous June.

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CHAPTER I

THE PROBLEM

Statement of the problem During the past two years, several comparisons of failure-rates in the Grade XI Departmental Examinations of June 1956 have been published. These surveys have consistently shown that school districts employing larger proportions of qualified teachers have recorded lower failure rates than those engaging teachers lacking prescribed qualifications for high school instruction. "The Failure Study for School District Number One"[1], for example, indicates that the percentages of pupils who failed in mathematics were as follows:

School District Number One	10.5%
Province outside School District No. 1	23%
Province of Manitoba in general	19.8%

Professor C. B. Germain, in his review of the situation[2], quoted the mathematics failure-rate for Winnipeg City Public Schools as 10.6%, compared to a rate for schools outside Greater Winnipeg of 23.8%

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1 Appendix A.

2 Clarence B. Germain, A Statistical Study of School Examinations in Manitoba (November 1957), p.p. 7 and 8.

Three classes of high school teachers are employed in the Province of Manitoba; (1) fully-qualified, (2) permit, and (3) first-class certificate holders. Of the 5120 pupils who sat for the 1956 examinations, 130 received their tuition from permit holders.[3] This study will report results achieved by pupils, in the lower half of mathematical achievement, of three groups of instructors. These groups will be designated as follows:

1. "A" students - or those taught by fully qualified teachers in public schools of School District of Winnipeg Number One.
2. "B" students - or those instructed by fully-qualified teachers outside District Number One.
3. "C" students - or those trained by teachers on permit, in rural schools.

In all high schools of Winnipeg City, as well as those of some other municipalities, students may be excused from writing the Departmental examinations if their term grades are sufficiently high. Since over 500 of the better pupils in accredited schools were promoted without sitting for these tests, it is impossible to make valid comparisons on the basis of <u>all</u> Departmental examinations written. This paper will therefore be confined to the consideration of the mathematics marks earned by the <u>lower-half</u> of the Grade

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[3] See Appendix B.

XI Groups.[4] Mathematics answer-booklets selected by the Department of Education as representative of the three groups will be the source of data used.

Marks, awarded for algebra, and/or geometry, on sample papers written by groups "A", "B", and "C" will be compared. Average scores made by pupils of each group on each question and question type will be set out, in order that a decision may be reached as to which problems can be solved as well by pupils of permit teachers as by those of fully-qualified instructors. For each test item the average marks of the best group will be compared to those of the poorest group, so that questions can be listed in order of discrimination. Using the average percentages of marks made by pupils, questions and question types will be ranked and compared as to the rank difficulty for each and all groups of students.

This study will also be concerned with an analysis of the 1956 test items, with a view to assessing the relative values assigned questions involving mathematical understanding as opposed to those solvable by memory.

Importance of the study

1. The high failure rates of Grade XI students

[4]See Chapter II.

have recently evoked many expressions of concern. During the summer of 1957, for example, a Winnipeg daily newspaper published an article criticizing the variation in failure rates for the City's Public Schools.[5] Evidence showing the results of pupils in these schools (which are staffed by fully-qualified personnel) as compared to those of students taught elsewhere by fully-qualified teachers seems particularly pertinent at this time.

2. Mathematical science is constantly assuming ever-greater importance in the world of today. High-School mathematics, forming as it does, a basis for future work in this field, warrants careful consideration.

3. Over the past few years, there has been considerable criticism of the fact that so many permit holders are engaged in the Province. It should therefore be of interest to compare the degrees of success achieved by these and fully-qualified teachers; as well as to ascertain the types of questions which appear to be answered more-or-less equally successfully by pupils taught by either group.

4. It is believed that the accompanying classification of the 1956 examination questions according to marks scored will serve a useful purpose in the setting of future

[5] Article in The Winnipeg Tribune, July 18, 1957.

examination papers. This breakdown, supplemented by an analysis of the test problems with a view to determining the proportions of marks awarded for mathematical understanding rather than memory, may be of interest to Grade XI mathematics teachers.

<u>Limitations of the study</u> (A.) Since 32% of the better "A" students and 19% of the "B" did not write Departmental examinations, inferences cannot be drawn as to the relative mathematical achievement of the better half of the students. However, it is interesting to note that the medians for the entire sample were as follows:

Median	"A"	"B"	"C"
Total scores	61	56	51
Algebra	33½	31	26
Geometry	29	27	21½

(B.) Differences in achievement cannot be entirely attributed to quality of teaching as similar circumstances do not prevail in the selection of those who take algebra and geometry in Grade XI, e.g.

1 In large high schools not all students are required to take the course requiring algebra and geometry.

2 In rural Manitoba the school leaving age is fourteen years and in Winnipeg City, sixteen.[6]

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[6] School Attendance Act

3 Permit Teachers in the main go to the poorest (poorest in more ways than one) of the high schools.

(C) The number of figures, used to find averages which were compared, were not always equal.

Organization of the thesis Chapter II is concerned with the representativeness of the original sample papers; as well as a discussion of the validity of the scores made on the papers finally selected as representative of the lower-half of the range of mathematical ability in each entire group of Grade XI students. The total scores for the algebra and/or geometry sections, will be discussed in Chapter III. Chapter IV will cover the division of the 1956 test questions under three main headings:

a. those similar to problems on the June 1955 examination.

b. those worked out in detail in the texts,

c. those appearing to require original thought.

For each question in these three classifications, Chapter V will present comparisons of results achieved by "A", "B", and "C" scholars; and, using scores of the best and poorest groups, comparisons will be made as to the discriminative power of each section and question. For each and all three groups questions and question types will be ranked and compared, as to rank difficulty. In the final chapter, conclusions will be drawn as to the relative success achieved by the three groups of pupils and as to the comparative difficulty and discriminative powers of the various questions and question types.

CHAPTER II

SAMPLING

<u>Introduction</u> Samples of Grade XI 1956 Mathematics answer-booklets, selected by the Department of Education as representative of the three groups described in Chapter I, were made available to the Faculty of Education. This Chapter is concerned with:

1. reliability of the sampling as typical of the entire population
2. choice of papers representative of the lower-half of mathematical achievement for each entire group of Grade XI pupils

<u>Reliability of original samples</u> Information furnished with the papers indicated the number of examinations written by each of the three groups was: "A", 750; "B", 750; and "C", 130. It will be noted that there is a slight discrepancy between the foregoing "A" figure and that quoted in "The Failure Study for School District Number One" as indicated in Table XVII, Appendix B. This difference of two, however, will have only a negligible effect on statistics derived for this investigation. The samples consisted of 103, or 13.8% of total "A" papers written; 103, or 13.7% of total "B"; and 51, or 39.2% of total "C".

Inasmuch as papers of almost two-fifths of all "C" Grade XI students, as opposed to about 14% of "A" and "B", were included in the sampling; "C" results are likely to be the most representative of the three groups.

The Winnipeg School Board reported that 116, or 15.5% of the 748 Winnipeg City students writing, failed in Grade XI mathematics.[1] In the "A" sample, 16 (or 15.5%) of the papers bore a grading lower than the pass-mark. The equality in percentages failing for the entire "A" population and "A" sample, led the investigator to conclude that a fairly-well balanced choice of sample-papers had been made.

No comparable "B" group failure figures were available. However, since the proportions of total "A" and "B" papers supplied had been so nearly equal, and the same selection principles had applied, we should expect the scores for sample "B" to be as reliable as those for "A".

Because the "A" and "B" samples each represent 14% of total papers written, it should be possible to combine these results and obtain equally-reliable scores for pupils taught by qualified teachers throughout the Province.

Final samples chosen In view of the fact that different proportions of the better students in groups "A" and "B" were exempt from final examinations because of

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[1] See Appendix A.

superior term marks, it would have been unfair to compare results of even all June "A", "B", and "C" papers. Only 67.9% of "A" pupils wrote Departmental Examinations; 81.2% of "B"; and 100% of "C". Samples of the papers from the <u>lower</u> 50% of <u>all</u> pupils in Grade XI were selected. Table I shows that $\frac{50}{67.9}$, or 76 papers, of "A" sample were chosen; $\frac{50}{81.2}$, or 63, of "B"; and 50%, or 26, of "C". In other words the best 27 "A" papers were rejected; the best 40 "B" papers; and the best 25 "C" papers.

TABLE I

DERIVATION OF FINAL NUMBER OF PAPERS SELECTED

	% of Students Exempted	% of Students Writing	Original Sample Number	Final Sample of Lower Half: Fraction of Original Sample	Number
"A"	32.1%	67.9%	103	$\frac{50}{67.9}$	76
"B"	18.8%	81.2%	103	$\frac{50}{81.2}$	63
"C"	0	100%	51	$\frac{50}{100}$	26

<u>Exemptions</u> All "A", and some "B", schools are accredited if, among other qualifications:-

1. Each teacher has:

   (a) a collegiate certificate

(b) two years successful teaching experience,

(c) two years university training in the subjects taught.

2. The pupil exempted has:

(a) attended regularly throughout the school year,

(b) a complete standing in Grade X,

(c) no Grade XI subject-average lower than 50%,

(d) an average of 67% or better on the full Grade XI General Course.[2]

In 1956, there were 354 Winnipeg City students promoted without having written June examinations.[3] Through an analysis of Christmas, Easter, and June marks in a certain "A" school (to be designated as School I), evidence will be presented attempting to prove that if "A" exempted pupils had written:

1. probably none would have failed
2. the few who might have scored in the lower-half of the range of marks would not have decreased to any extent the marks of the lower range.

Through the co-operation of the Principal and a mathematics teacher in School I, the marks awarded its 1956 Grade XI students were made available for this report.

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[2] Programme of Studies for the Schools of Manitoba Senior High Schools 1958 - 9 (Winnipeg, 1958) pp. 174 -175.

[3] See Appendix B.

Because School I had a failure rate 1.6% above the City average of 10.5% and a registration of 24.7% of the Grade XI students in School District Number One (See Figure 1, page 13) it was thought that its average marks would not be higher than those for the entire District.

Composite scores (computed as 40% of the Christmas and 60% of the Easter marks) were available for 177 pupils eligible for accrediting except on the basis of composite scores, and for whom there were both composite and June marks. Table II indicates that marks earned by the 177 were on the average 9.4% lower on the School tests[4] than on the Departmental Examinations. (Could these lower term marks have been due to more difficult School tests, more severe marking or less opportunity for review?)

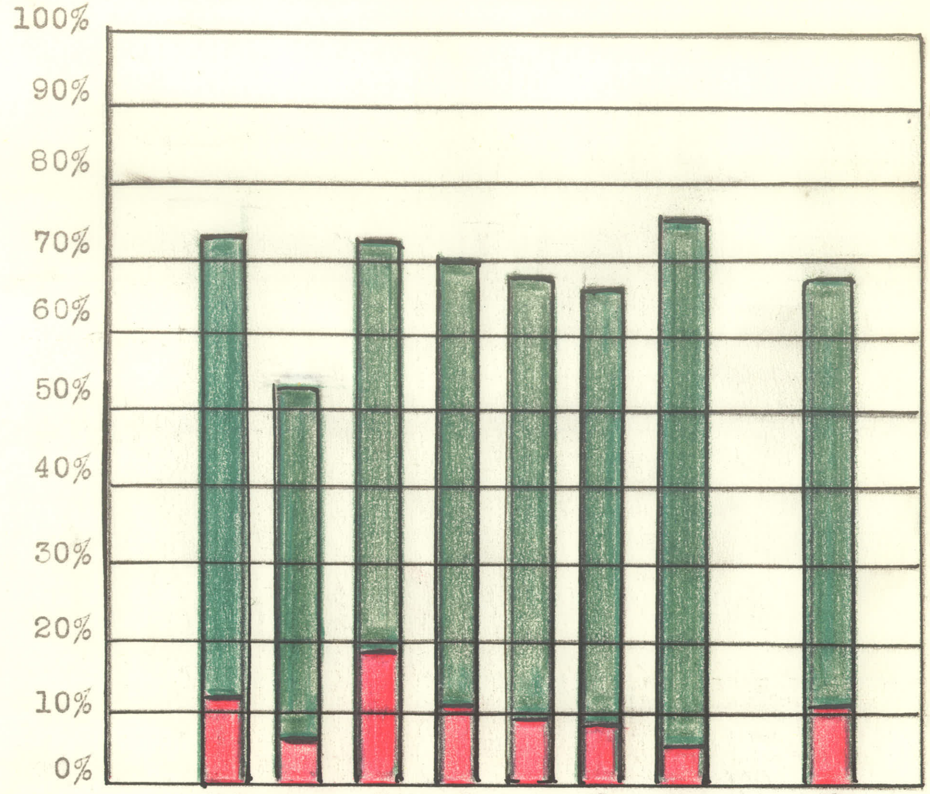
TABLE II

JUNE AND COMPOSITE (XMAS AND EASTER) SCORES FOR SCHOOL I

	No. of Pupils	AVERAGE June %	AVERAGE Composite %	June - Composite
Exempted pupils who did not write in June	69		71.9	
Exempted pupils who wrote in June	11	87.9	84.4	3.5
Pupils qualified for exemptions except on composite scores	177	59.3	49.9	9.4

[4] See Appendix C

Green = % Writing
Red = % Failing



"A" SCHOOLS	I	II	III	IV	V	VI	VII	ALL
% Failing of Pupils in School	12.1	5.8	18.2	10.6	9.	8.5	5.6	10.5
% Writing of Pupils in School	73.5	55.4	73.3	70.2	68.	66.1	75.9	67.9
% of All Grade XI's in Each School	24.7	23.4	16.	14.6	11.1	5.4	4.9	100.
Number Grade XI's in Each School	272	258	176	161	122	59	54	1102

"A" SCHOOLS FAILURE RATES BASED ON NUMBER OF PUPILS IN GRADE

FIGURE 1

In School I pupils' marks on the average went up in June. However, in his study Professor Germain states:

> "The number of such students[5] has been added to obtain the "N" on which the failure rate is based. This obviously tends to make the record of these schools look better. Since some of these students would almost certainly fail one or more examinations."[6]

The evidence at hand would lead one to believe that had School I exempted students written the June 1956 Mathematics examination, practically none would have failed. Of 177 students qualified for exemptions, except on the basis of marks, only one who passed on the composite score (55%) failed in June (41%). She was a special case a European displaced by the Russians, who had suffered great emotional strain. This student had entered Grade X immediately after emigrating from Germany, and while writing the Departmental tests, was preparing to move to New Jersey. This girl had:-

(a) a failing composite mark in social studies

(b) an average composite score on all subjects of 58%

(c) a composite score in mathematics of 55%

(d) a June mathematics score of 41%.

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[5] i.e. exempted students

[6] Germain, op. cit., p.12.

The record for the 69 exempted students would likely be even better, for they had:-

(a) no failing composite score

(b) an average composite mark on all subjects of 67% or more

(c) an average composite mark in mathematics of 71.9%.

As far as School I is concerned, there would seem to be little likelihood that an exempted student would have failed in the June 1956 mathematics test. Because this school, which registered about one quarter of all "A" students, had a failure rate above the "A" average, we should not expect the failure rate in the other three-quarters of the "A" population to be any higher.

Now as to the possibility that had all pupils written in June, the median score for all (or top score for the lower half of the students) would have been lowered to any appreciable extent. The lowest composite mark for exempted students (55%) was made by three individuals. Again assuming an increase of 9.4%, these three would have fitted in the top quartile of the lower half (Q_3 for "A" students equals 63% [a]) Including these three, there was a total of twelve pupils with composite scores of 61% or less who, even with the average increase of 9.4%, might not have reached the entire "A" population assumed-median score of 71%. These twelve had

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[a] See Table III page19.

an average mark of 58.5%. If the expected increase of 9.4% is added to this figure their June mark could have averaged 67.9%

It should be emphasized here that this 9.4% increase was established by non-exempted students. Inas-much as exemptees are the better students, one could reasonably assume that the differential between their June and composite results would have been somewhat better than 9.4%; that is, an expected June mark in excess of 67.9%. The third quartile for the lower half is 63%. We should therefore expect that exempted students would have belonged in <u>at least</u> the upper quartile of the lower group.

The final sample papers of "A" students constituted 6.9% of all "A" Grade XI's[7]. This would mean that .828 (.069 x 12) of a pupil from School I would be included in the sample of the lower half of "A" students. Because School I registered 24.7% of all "A" scholars there could be expected to be 3.4 ($\frac{100}{24.7}$ X .828) exempted students included in the "A" sample.

If one considers that <u>some</u> of the twelve School I exempted students with composite marks below 62% might have had a June mark of 71% or more and that School I had an above average failure rate; there probably would have been

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[7] See Appendix B page 92 .

fewer than 3 such people in the "A" sample of the lower half. Including in the final "A" sample of 76, three or fewer students with an expected average score of 67.9% - plus, would not have affected, to any great extent, the marks of the lower-half (which ranged from 21% to 71% and averaged 55%).[b]

<u>Conclusion</u> The papers selected for "A" and "C" groups as samples of pupils in the lower-half of the scoring range should be expected to yield quite reliable results.

Since term scores for "B" students were not available, it was impossible to produce evidence showing that the June scores of exempted "B" students would not have decreased the marks for the sample of the lower-half of the pupils in this group. However, assuming that the "B" scores would have been lowered, the difference between "A" and "B" ratings would then have been greater; and between "B" and "C", smaller. In any case, valid comparisons for the lower-half of all students could still be made at some of the percentiles below 100.

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[b]See Tables III and XX.

CHAPTER III

COMPARISONS OF SCORES

<u>Introduction</u> The purpose of this chapter is to present a number of representative comparisons of the marks for the three groups of Grade XI mathematics students in algebra, geometry and total June scores[1] The following scores will be found and compared: the means and percentiles 100, 75, 50, and 25. The 100th percentiles of the three lower groups can be considered the medians of all Grade XI pupils - "A", "B" and "C".

<u>Comparison of total scores</u> Table III, page 19, gives comparisons made between the means and five percentiles in total scores. In each case, the order of merit was "A". "B", "C". Figure 2 on page 20, shows graphically that, on the whole, the differences between "B" and "C" were greater than thos e between "A" and "B". However, the differentials at the 100th percentiles of the lower half (median for entire populations) were the same in both instances, while at the tenth percentiles the differential between "A" and "B" was 2% more than between "B" and "C".

The failure-rate of the final sample as shown in Table IV page 21, indicates that the "A" rate was slightly

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[1]Appendixes D, E, and F tabulate algebra, geometry and total June scores for "A", "B", and "C" students.

TABLE III

COMPARISON OF TOTAL MATHEMATICS SCORES FOR PUPILS OF THREE GROUPS

Percentile	A	B	C	A-B	B-C
10th	35	26	19	9	7
25th	50	38	25	12	13
50th	57	50	33	7	17
75th	63	55	40	8	15
100th	71	61	51	10	10
Mean	54.7	44.8	33.3	9.9	11.5

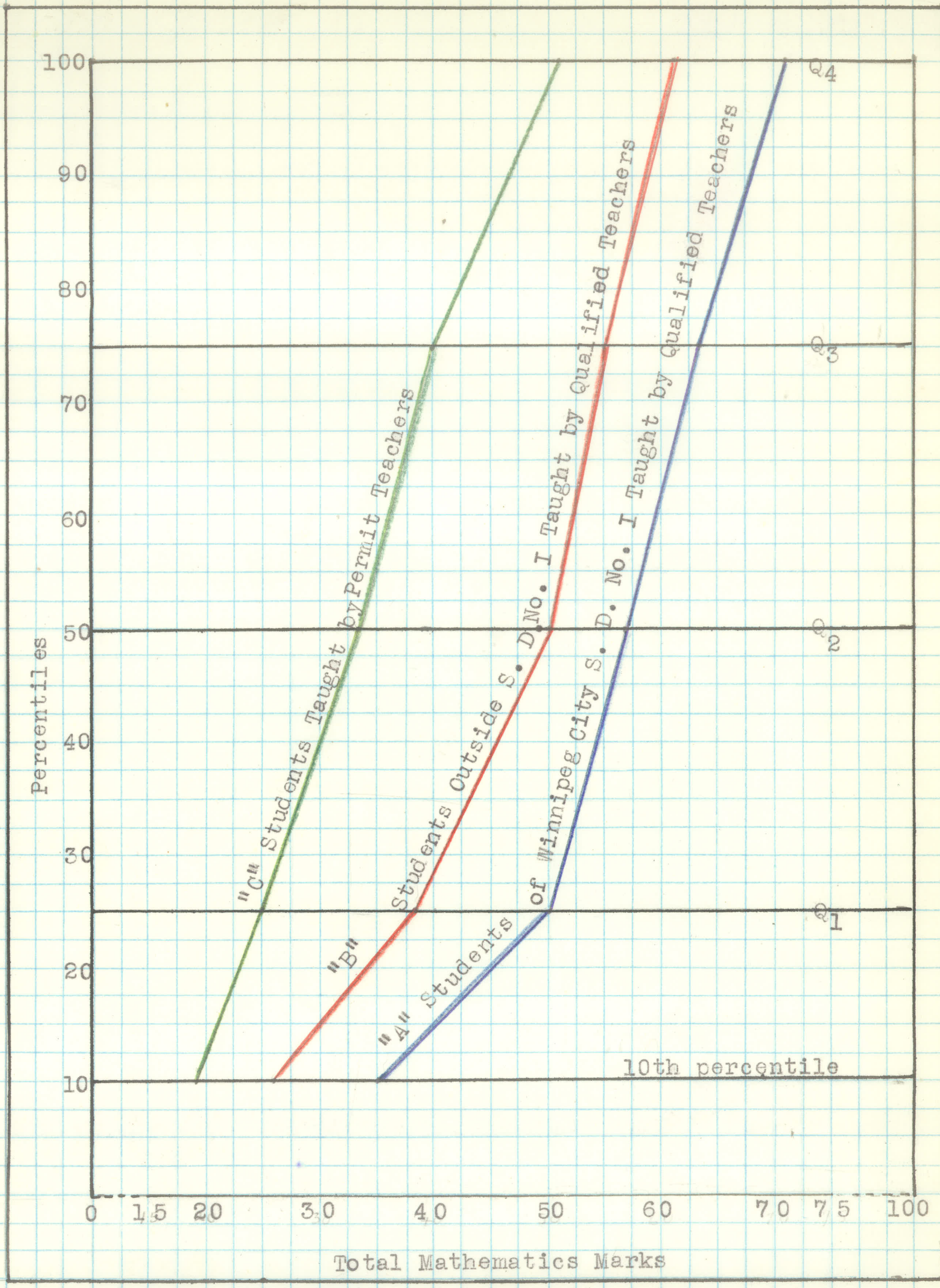


FIGURE 2

MATHEMATICS PERCENTILES 10, 25, 50, 75, and 100 FOR LOWER HALF OF STUDENTS "A", "B", and "C" less than half that for "B"; and the "B" rate slightly more than half that for "C".

TABLE IV

FAILURES IN SAMPLES OF LOWER HALF OF THREE GROUPS OF PUPILS

	A	B	C
Number of Pupils in Sample	76	63	26
Number of Failures	16	28	22
Number of Failures as % of Sample Pupils	21.1%	44.4%	84.6%

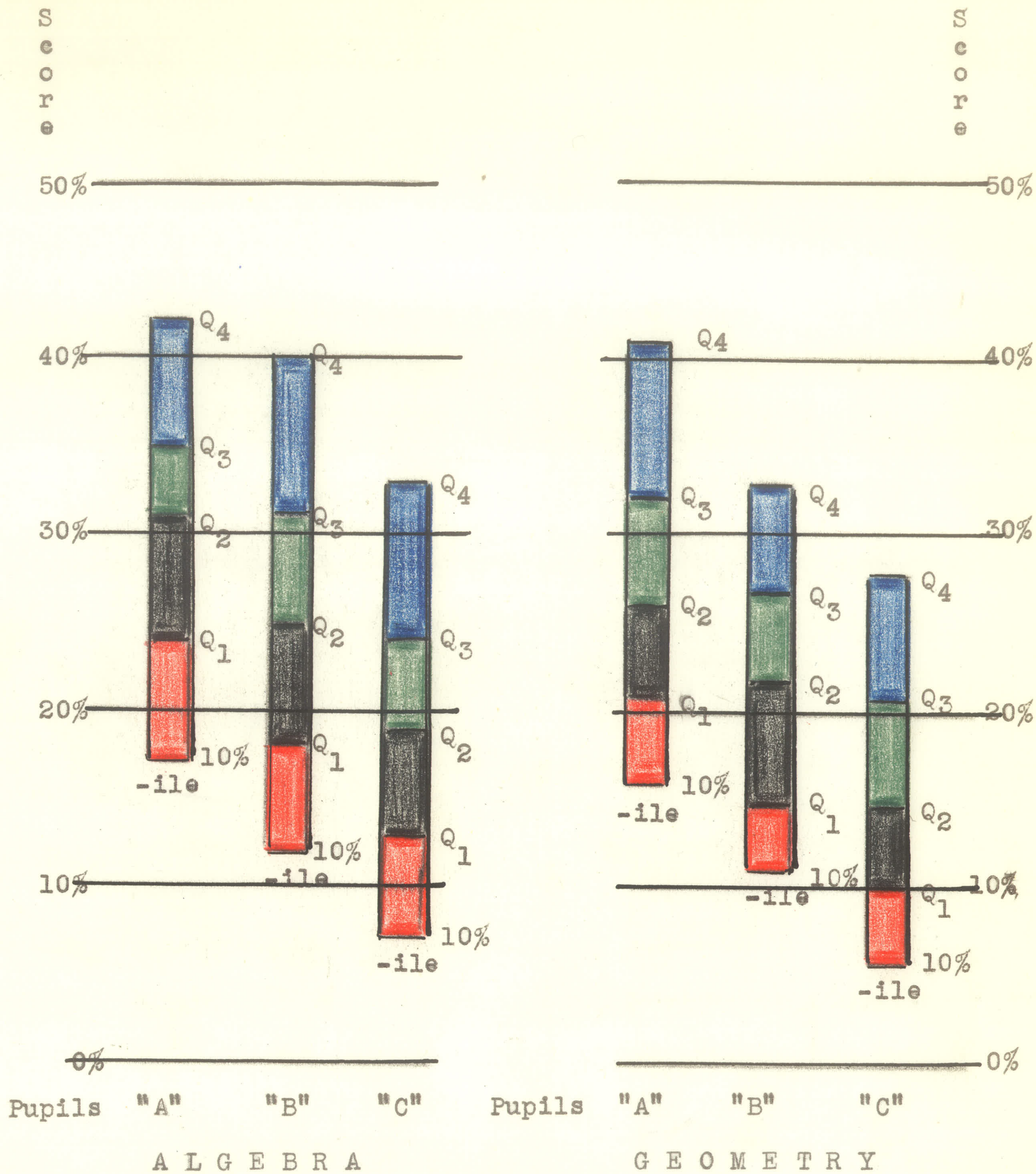
Comparison of algebra scores Table V on page 22 for algebra, as in the case of Table III, page 19, for total scores, shows the same "A", "B", "C" ranking. In particular, at the 75th and 100th percentiles, there were decidedly greater differences between "B" and "C", than between "A" and "B". An examination of Figure 3, page 23, reveals that the 50th percentile for "A" is the same as that for "B"s 75th percentile, and the 50th percentile for "B" exceeds the 75th percentile for "C" by 1.

Comparison of geometry scores Table VI, page 24, shows an "A", "B", "C", rank order for geometry scores, as was the case for algebra. Figure 3, page 23, indicates that, in geometry, the differences between "A" and "B",

TABLE V

COMPARISON OF ALGEBRA SCORES FOR PUPILS OF THREE GROUPS

Percentile	A	B	C	A-B	B-C
10th	17	12	7	5	5
25th	24	18	13	6	5
50th	31	25	19	6	6
75th	35	31	24	4	7
100th	42	40	33	2	7
Mean	28.7	24	18.2	4.7	5.8



GEOMETRY AND ALGEBRA PERCENTILES 10, 25, 50, 75 and 100
FOR LOWER HALF OF STUDENTS "A", "B", and "C"

FIGURE 3

TABLE VI

COMPARISON OF GEOMETRY SCORES FOR PUPILS OF THREE GROUPS

Percentile	A	B	C	A-B	B-C
10th	16	11	6	5	5
25th	21	15	10	6	5
50th	26	22	15	4	7
75th	32	27	21	5	6
100th	41	33	28	8	5
Mean	26.0	20.5	15.1	5.5	5.4

on the whole, equal those between "B" and "C". The assumed median scores for the entire populations showed a differential of 8% between "A" and "B", and 5% between "B" and "C".

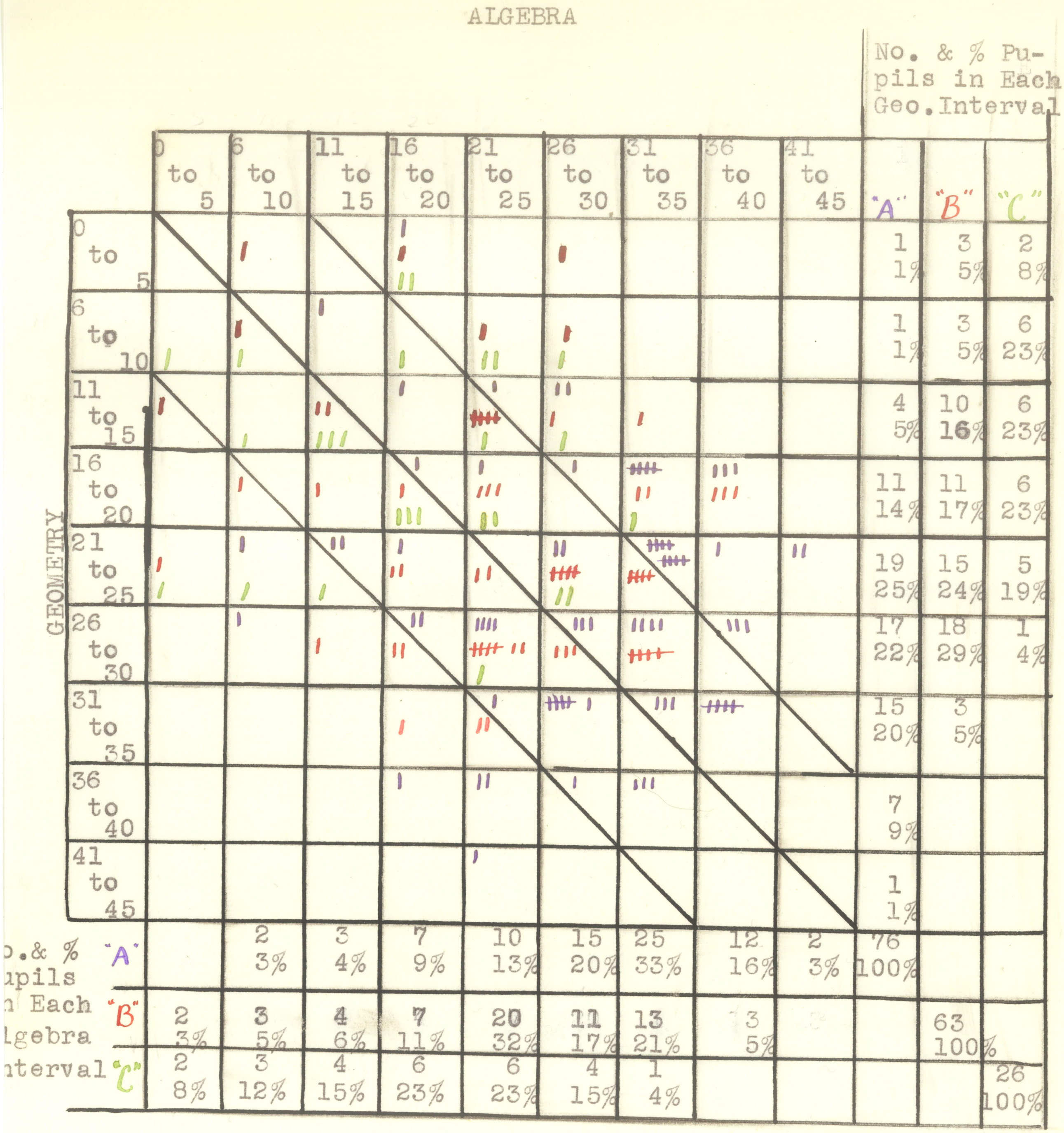
Comparison of algebra and geometry scores When comparisons were made of the means and five percentiles for algebra and geometry, it was found that, without exception, the geometry scores were lower than those for algebra. The data presented in Table VII, page 26, show that in each group the differences between algebra and geometry scores were 1% at the 10th percentiles and 3% at the 25th percentiles. The hundredth percentiles of the lower-half of the students, or median for the entire groups of Grade XI students, showed a differential for group "A" of 1%; for "B" 7%; and for "C", 5%. As judged by examination results, the middle student in the entire "A" group seemed to have mastered geometry almost as well as algebra, whereas "B" and "C" middle pupils found geometry more difficult than algebra.

An examination of Figure 4, page 27, the scatter diagram for algebra and geometry scores, shows that 7 "C" students (27%) were more than two intervals of five per cent lower in geometry; 11 "B" (17%); and 14 "A" (18%). In geometry, 10% of "A" students had higher marks than any in the "B" and "C" groups, as compared to 3% in algebra, (see TableVIII, page 28).

TABLE VII

COMPARISON OF ALGEBRA AND GEOMETRY SCORES

Percentile	"A"			"B"			"C"		
	Alg.	Geo.	Alg.-Geo	Alg.	Geo.	Alg.-Geo.	Alg.	Geo.	Alg.-Geo.
10th	17	16	1	12	11	1	7	6	1
25th	24	21	3	18	15	3	13	10	3
50th	31	26	5	25	22	3	19	15	4
75th	35	32	3	31	27	4	24	21	3
100th	42	41	1	40	33	7	33	28	5
Mean	28.7	26	2.7	24	20.5	3.5	18.2	15.1	3.1



PAIRED ALGEBRA AND GEOMETRY SCORES FOR THREE GROUPS "A" "B" "C"

FIGURE 4

TABLE VIII

PERCENTAGES OF PUPILS IN ALGEBRA AND GEOMETRY SCORING INTERVALS

EXAM MARKS	ALGEBRA A	ALGEBRA B	ALGEBRA C	ALGEBRA TOTAL	GEOMETRY A	GEOMETRY B	GEOMETRY C	GEOMETRY TOTAL
0-5	0	3.2	7.7	2.4	1.3	4.8	7.7	3.6
6-10	2.6	4.8	11.5	4.8	1.3	4.8	23.1	6.1
11-15	3.9	6.3	15.4	6.7	5.3	15.9	23.1[z#]	12.1
16-20	9.2	11.1	23.1[z#]	12.1	14.5	17.5	23.1	17
21-25	13.2	31.7[z#]	23.1	21.8	25	23.8[z#]	19.2	23.6
26-30	19.7[z]	17.5	15.4	18.2	22.3[z#]	28.6	3.8	21.8
31-35	32.9[#]	20.6	3.8	23.6	19.7	4.8		10.9
36-40	15.8	4.8		9.1	9.2			4.2
41-45	2.6			1.2	1.3			.6
46-50								

[z] Interval for mean

[#] Interval for median

	A	B	C	Total
Algebra more than two intervals of five higher than geometry	18.4	17.5	26.9	19.4
Geometry more than two intervals of five higher than algebra	7.9	4.8	7.7	6.7

<u>Conclusion</u> Table IV on page 21 shows that Grade XI students in Winnipeg City Schools had half the failure rate of those taught by qualified teachers outside School District Number One, and one quarter of those trained by Permit Teachers. Whatever led to the consistently superior mathematical achievement of "A" students apparently had a greater effect on geometry than algebra. The facts brought out in this chapter led the writer to wonder whether the superior "A" results were due to one or more of the following reasons:

1. "A" teachers having had more experience
2. more professional stimulation in large Winnipeg High Schools than those smaller institutions found outside Winnipeg City
3. the City's choice of better teachers
4. larger schools having teachers who give instruction only in mathematics.
5. Because in Winnipeg City there are more technical vocational and /or high school leaving courses available, "A" students who elect or are permitted to take the general course may be a more select group than "B".

Furthermore, in every comparison, "C" scores were lower than "B". Could this fact be accounted for as follows:

1. inferior professional and/or academic training of the unqualified teachers

2. permit holders having, on the average, more subjects to prepare
3. "C" pupils who are limited in their choice of Grade XI courses being a less selected group than "B" students who elect to take the General Course
4. poorer schools having more Permit Teachers?

CHAPTER IV

ITEMS ON 1956 TEST[1]

INTRODUCTION

> "Students of mathematics agree that power, ingenuity, resourcefulness in problem-solving is a major objective of instruction. They interpret problem-solving as finding, by reasoning, a satisfactory response to a situation which is novel, for which there is no available recalled response".[2]

According to this definition, questions similar to those asked for June 1955, or worked out in detail in the texts, might not test the problem-solving ability of those pupils who depended chiefly on their memory.

This chapter will be concerned with grouping the June 1956 test items under the following headings:

1. questions similar to those for June 1955[3]
   (a) of equal, or less, difficulty
   (b) with one added difficulty

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[1] June 1956 test is in Appendix G.

[2] John R. Clark and Howard F. Fehr "Learning Theory and the Improvement of Instruction - A Balanced Program", <u>The Learning of Mathematics Its Theory and Practice</u>, TwentyFirst Yearbook of the National Council of Teachers of Mathematics, Chapter II (Washington, D. C., 1953), p.341.

[3] See Appendix H for 1955 examination.

2. questions for which there is a full explanation in the texts

3. 'original' questions:-

   (a) those for which one blank is left for the answer;

      (1) a number required

      (2) a reason required

   (b) those for which more than one line is required.

1. JUNE 1956 TEST ITEMS SIMILAR TO THOSE FOR 1955

According to authorities,

"It has (also) been found upon analysis that on the average, pupils do wellon material that has appeared in a number of previous examinations, although it was not included in the course of study; and that they do poorly with material listed in the course of study, but not included in previous tests".[4]

At the Manitoba Text Book Bureau, the only examination

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[4]"Evaluation of the Progress of Pupils", The Place of Mathematics in Secondary Education, The Final Report of the Joint Commission of the Mathematical Association of America and The National Council of Teachers of Mathematics, The National Council of Teachers of Mathematics Fifteenth Year book, Chapter IX (New York Bureau of Publications Teachers College, Columbia University, 1940), p. 168.

available prior to 1956 was that for June 1955. Since the test for the previous year is usually the most readily available for reference, questions similar to those for June 1955 will be considered solvable by imitation.

<u>June 1956 algebra questions similar to those for June 1955, and of equal, or less, difficulty</u> The following questions would be familar to students who could accurately recall the mechanical manipulations required for the June 1956 test:

Question No. and Value	June 1956 Test Item	Question No.	June 1955 Test Item
1a (1)	How much greater is $7a + 3b - 4c$ than $3a - 2b + 5c$?	1a	Subtract $a - 4b + 5$ from $3a - b - 1$
1b (1)	Evaluate: $a^3 - 2a^2 + 1$ when $a = -2$	1d	If $X = 3$ and $y = 4$, find the numerical value of $8-2\ (5x^2 - y^2)$
1c (1)	Simplify $x^{2n} \div x^{n-1}$	5b	Simplify and express with positive indices: $\frac{2^{n+1}}{(2^n)^{n-1}} \div \frac{4^{n+1}}{(2^{n-1})^{n+1}}$
1d (1)	What value of k will make $x^2 - 5x + k$ a perfect square trinomial?	1e	What number must be added to $x^2 - 3x$ to make it a perfect square trinomial?
1e (1)	$\sqrt{8} + 2\sqrt{32} - 3\sqrt{2}$	6a	Simplify $(\sqrt{8} - \sqrt{6})^2$

Question No. and Value	June 1956 Test Item	Question No.	June 1955 Test Item
			Solve for X:
1f (1)	$k = \frac{bx + c}{x}$	4a	$\frac{x}{m} + n = \frac{x}{n} + m$
4c (4)	$\frac{2}{x-4} - \frac{1}{x-2} = 2$ Questions 4c for 1956 and 4b for 1955 required the use of the quadratic formula. Question 4c for 1956 and 3 for 1955 had "1" in numerator of fraction questions	**	Whole quantities "m" and "n" in question 4a for 1955, require procedures similar to that for "2" in question 4c for 1956
		4b	$2x^2 = 4x + 7$
		3	Simplify $\frac{1}{x^2 + 3xy} + \frac{2}{9y^2 - x^2} - \frac{2}{x^2 - 6xy + 9y^2}$
	Simplify (and express with positive indices)		
5a (3)	$\left\{\frac{9a^{-2}}{25b^4}\right\}^{-\frac{1}{2}}$	5a	$\left\{\frac{36a^{-4}}{25b^{-6}c^6}\right\}^{-\frac{1}{2}}$
5b (2)	$4x^0 + (2^2)^3$	** 1b	$(-2x^3 \; y^2)^4$
			Solve graphically
8 (4)	$x^2 - 6x + 8 = 0$	8a	$x^2 - 4x = 0$

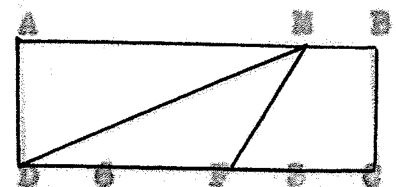
<u>June 1956 algebra questions similar to those for June 1955 but with one added difficulty</u> For students who depend on memorizing mathematical rules without understanding, a small change in a question (such as substitution of a fraction or a surd for a whole quantity, or a binomial expression for a monomial one) can make the solution more difficult to perform

through memorized mechanical manipulations. On the following page, four questions (together with added difficulties) are listed along with similar 1955 questions.

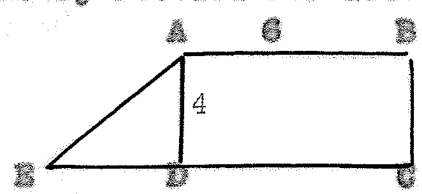
<u>June 1956 geometry questions similar to those for June 1955</u> No June 1956 geometry question was much like any for June 1955. However, four questions had something in common with those for the previous year.

I Questions Involving the Use of Formula $A = \frac{1}{2}\, b \cdot a$

(A) Question 1b for 1956 (value 2)
ABCD is a rectangle with an
area of 36 square units
DF = 6 units
FC = 3 units
(a) BC =
(b) Area of △ FDM =

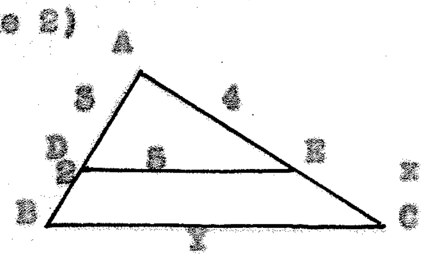


(B) Question 10 (first in geometry section for 1955)
ABCD is a rectangle
∠ E = 45°
AB = 6 units
AD = 4 units
Area of figure ABCE =

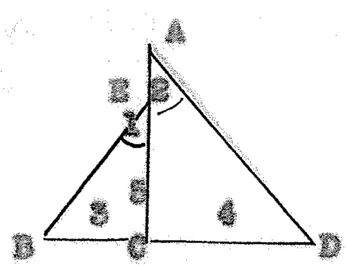


II Questions Involving the Dimensions of Similar Triangles

(A) Question 5b for 1956 (value 2)
DE// BC
AD = 3 units
DB = 2 units
AE = 4 units
DE = 5 units
(b) BC =



(B) Question 12 (3rd in geometry section for 1955)
∠ 1 = ∠ 2
AC is perpendicular to BD
BC = 3 units
CD = 4 units
CE = 5 units
AE =



Question No. and Value	SIMILAR QUESTIONS 1956	Question	1955	Change for 1956
	Factor fully:			
2b (2)	$x^3 - \frac{b^3}{8}$	(2a)	$8x^4 - 27x$	Fraction for whole quantity
	Simplify:			
3 (4)	$\frac{x+3}{x-3} - \frac{x-3}{x+3} + \frac{36}{9-x^2}$	(3)	$\frac{1}{x^2+3xy} + \frac{2}{9y^2-x^2} - \frac{2}{x^2-6xy+9y^2}$	Binomial numerator for monomial numerator
	Rationalize the denominator etc.			
6 (3)	$\frac{2\sqrt{3}-\sqrt{2}}{2\sqrt{3}+\sqrt{2}}$	(6b)	$\frac{4-3\sqrt{2}}{6+\sqrt{2}}$	Binomial with two surds instead of one
	Find the square root of:			
7 (3)	$7a^2 - 6a + a^4 - 2a^3 + 9$	(7)	$16m^6 - 40m^4 - 8m^3 + 25m^2 + 10m + 1$	Not in descending order otherwise 1955 harder than 1956

III Questions Requiring the Reason: "Tangent chord angle equals the angle in the alternate segment"

(A) Question 5e for 1956 (value 1)

(B) Question 14 (5th in geometry section for 1955) required the use of this reason as <u>part</u> of a proof.

IV Proportion Questions

(A) Question 9 for 1956 (value 7) was a proportion one requiring:
  (1) the construction of a diagram from the given description
  (2) the proofs and proportions for two pairs of similar triangles
  (3) the use of the substitution and equality axioms in the sets of proportions for (2)[a]

(B) Question 13 in geometry section for 1955 required :
  (1) the proof and proportions of only one pair of similar triangles

(C) Questions 18 (9th in geometry section for 1955) required:
  (1) the construction of a diagram from a given description
  (2) the proof and proportion for one pair of similar triangles
  (3) the use of the substitution axiom for radii

<u>Conclusion</u> It would seem that, while some of the same principles were involved in the geometry sections of both tests, the diagrams and applications of these principles varied sufficiently to make the questions difficult to solve for pupils depending chiefly on imitation of the solutions required for June 1955.

In the algebra portion of the June 1956 test, there were more questions solvable by imitating those for June 1955.

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[a] Steps used in 100 per cent student answer booklet

(1) Ten questions (worth nineteen marks) were very similar to those on the 1955 test, and of equal, or less, difficulty.

(2) Four questions (worth twelve marks) were similar to 1955 items, but each had one added difficulty, namely:

(a) fraction for whole quantity

(b) binomial numerator for monomial

(c) unorganized arrangement of terms for descending order.

Ten of the similar questions on the examinations for 1955 and 1956 came in approximately the same position on both tests.

2. QUESTIONS FULLY EXPLAINED IN THE TEXTS

In the Algebra Text[5], some examples are worked out in detail, together with checking methods. Propositions are fully explained in the Geometry text[6]. Solutions for such items used as examination questions therefore could not always be considered the product of original thought on the part of the pupil.

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[5] Daniel W. Snader, Algebra Meaning and Mastery (Toronto: John C. Winston.Co., 1954.)

[6] W. J. Oliver and P. P. Winters, A First Course in Plane Geometry (Regina: School Aids and Text Book Publishing Co. Ltd., 1954.)

For the simultaneous fractional equations used in question 4b (value 4), a full page in the text[7] is used to demonstrate the method and check for this identical question. Rote-learning could help provide the responses to two geometry propositions required for questions 6a and 6b (worth 12 marks).

There are also two algebra questions similar to those explained in the text except for a change of sign and/or number. Question 2a: - "Factor fully: ax - by + ay - bx", was worth two marks. In the text, almost a full page[8] is devoted to methods and a check for, "Factor ax + by - bx - ay".

Problem 9b (worth 4 marks) is:

> "A man travelled 15 miles at a constant rate. If he had travelled 2 miles per hour faster, he would have gone the same distance in two hours less time. What was his original rate of travel?"

The distance, rate, and time formula is discussed on pages 13, 30 and 98. On page 143, expressions are written in symbols, together with a given Figure for "d", "t", "d", "r" and "t". In the text, is an example[9] requiring the use of the same formula as that for 9b. Two methods and a check are demon-

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[7]Snader, op. cit., p. 273.

[8]Ibid., p. 212.

[9]Ibid., p.280 - p. 282.

strated for this text question which reads as follows:

> "A certain train has a scheduled run of 100 miles between two cities. If the average rate of travel were decreased ten miles per hour, the run would take one half hour longer. Find the average rate of travel."

There are a few minor differences between the text and test:

	D_1	D_2	R_1	R_2	T_1	T_2
Text	100	100	R	R - 10	T	$T + \frac{1}{2}$
Test	15	15	R	R + 2	T	T - 2

The text question had larger numbers and an added fraction difficulty.

Conclusion For examination questions identical to examples explained in detail in the text, four marks were given for algebra, and twelve for geometry. Two algebra questions (worth six marks) were similar to text examples (for which detailed explanations had been given), other than for changed numbers and /or signs.

3 ORIGINAL QUESTIONS

For a good student who had carefully reviewed the previous June test and examples demonstrated in the texts, the foregoing questions would have been comparatively easy.

The writer therefore, interpreted an 'original' problem as one for which there was no parallel on the June 1955 examination, nor a detailed explanatory example in the text.

"The most valuable achievement in mathematics learning is thorough understanding as demonstrated in adaptation and application at the point of need for use. This is therefore the most appropriate behaviour to evaluate"[10]

One realizes of course that teachers are not limited to explanations of examples from the texts and previous June examination. Good teachers provide wider experience. For their students so called 'original' questions may not be entirely new. Nevertheless the investigator believed that 'original' problems as defined were more likely to evaluate this behaviour than those 'solvable by imitation'. However, for solutions of 'original' questions requiring a straight-forward one-lined answer, one could not always be certain whether the response was a guess, the result of memorizations, or evidence of genuine understanding. It would seem that, the more statements required in a coherent logical sequence, the less guesswork was likely to be resorted to in arriving at the correct solution. Accordingly, 'original' questions will be sorted into categories according to the length of answer required; i.e.:-

(a) involving only the filling in of a blank space with a:

(1) number

(2) reason for a given statement

[10] Donovan Johnson and H. C. Trimble "Evaluation of Mathematical Meanings and Understandings". Emerging Practices in Mathematics Education Twenty-Second Yearbook of the National Council of Teachers of Mathematics, (New York City: Bureau of Publications Teachers College Columbus, 1954), p. 343.

(b) involving more than one written statement.

(a) Answers requiring the filling in of a blank space Geometry questions 1a, 1b, 2, 3a, and 3b each called for an answer in the form of a number. With the exception of question 2, these questions required the recollection of formulae or rules and their application to solve arithmetic questions involving multiplication or division by a one-digit number.

Geometry questions 5a to 5f gave one mark for each reason supplied for a given statement.

(b) Original questions requiring more than one written statement In the algebra portion, there were three 'original' questions requiring more than one line of explanation:

(i) 2c (value 3 marks) - a factoring question, the answer to which comprised three binomials.

(ii) 4a (value 3 marks) - an equation with one unknown, and including three binomial expressions, each of which had an arithmetic-fraction factor.

(iii) 9a (value 3 marks) - a problem quoting the area of a triangle and asking for its altitude.

The geometry section, on the other hand, had one more 'original' question than the algebra (of the type requiring at least two written statements). These were:

(i) 4 (value 5 marks) - concerning areas of overlapping triangles and parallelograms

(ii) 7 (value 5 marks) - a construction question necessitating description, but not proof

(iii) 8 (value 6 marks) - calling for proof of equality of two intersecting chords whose angle of intersection was bisected by a diameter.

(iv) 9 (value 7 marks) - a proportion-question involving two sets of similar triangles and use of the equality axiom.

Conclusion According to the classifications for 'original' questions, the number of items and values assigned for 'original' questions were as follows:-

(a) questions involving only the filling of a blank space with a :

(1) number - 5 geometry questions, worth 9 marks

(2) reason for a given statement - 6 geometry questions, worth 6 marks

(b) questions involving more than one statement

(1) 4 geometry questions, worth 23 marks

(2) 3 algebra questions, worth 9 marks.

Although an original question was more likely to produce evidence of understanding; for the blank-filling answers, it was not always possible to be sure whether the response was a guess, the result of memorization, or evidence of genuine understanding. Nevertheless it would seem that the geometry section was more likely to test genuine mathematical understanding than the algebra.

CONCLUSION

Table IX, on page 45, gives a summary of the number of questions of each type in the algebra and geometry sections and the marks assigned to each type.

In the algebra portion of the 1956 examination, 31 marks out of a possible 50 were awarded for questions similar to those of the 1955 test. Of the 31 marks, 19 were assigned for questions of equal or less difficulty than those of the previous year; and the remaining 12, for questions involving one added difficulty. In the algebra section for both years, a tendency to have the similar questions in the same order was noted. For three algebra questions, worth 10 marks, detailed explanations and checks had been given in the text. In one case, the text and test questions were identical; in another, the test question was easier; and the third, differed only as to signs.

Both the 1955 and 1956 geometry papers carried problems necessitating the employment of the same principles, but their applications varied in both tests. For a pupil depending chiefly on a good memory, propositions in questions 6a and 6b (worth 12 marks) would be easier than those of equal

TABLE IX

VALUES FOR EACH QUESTION TYPE

NATURE OF QUESTIONS	ALGEBRA		GEOMETRY	
	Total No. Questions	Value	Total No. Questions	Value
Similar to June 1935				
(a) of equal or less difficulty	10	19		
(b) with one added difficulty	4	12		
Fully Explained in Text	3	10	2	12
Total Solvable by Imitation	17	41	2	12
Original				
Blank left for				
(a) number			5	9
(b) reason			6	6
More than one line required	3	9	4	23
Total Original	3	9	15	38

difficulty, but requiring original logical reasoning. In geometry, nine marks were given for 'original' questions entailing the insertion in blanks of numbers; six, for filling in blanks with reasons.

On the whole, the algebra questions were much easier to do by imitation than the geometry. Two-and-a-half times as many marks were given in the geometry as in the algebra section for 'original' problems requiring several written statements.

CHAPTER V

COMPARISON OF THE RESULTS ACHIEVED BY GROUPS "A", "B" & "C", ON INDIVIDUAL EXAMINATION QUESTIONS AND QUESTION TYPES

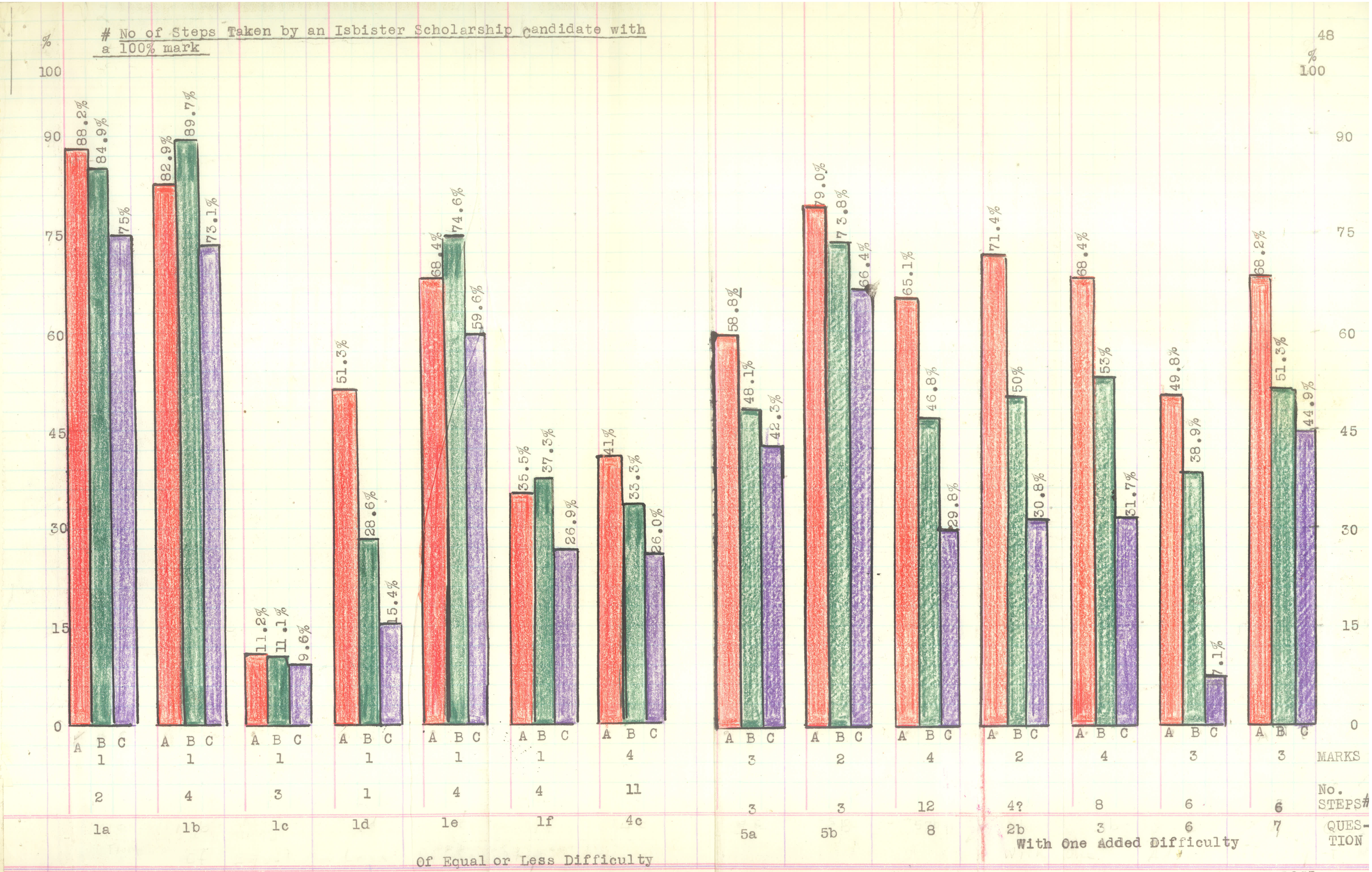
INTRODUCTION

In this chapter, questions as classified in Chapter IV, will be assessed according to the average percentages of marks made by pupils in each group. Using scores of the groups with the highest and lowest marks, comparisons will be made as to the discriminative power of each section, question, and question type.

EXAMINATION QUESTIONS SIMILAR TO THOSE FOR 1955

<u>Questions similar to those for 1955 and of equal or less difficulty</u> Figure 5, on page 48, shows the average marks in percentages of three groups of students on ten June 1956 algebra questions, which were similar to those of June 1955, and of equal or less difficulty. In seven cases the bar graphs diminished from "A" to "B" to "C". However, for three questions (1b, 1e and 1f), each worth one mark, "B", "A", "C" was the order.

<u>Questions similar to those for 1955 with one added difficulty</u> Figure 5 also shows the results of comparisons made for four June 1956 algebra questions which were similar



AVERAGE MARKS IN PERCENTAGES OF THREE GROUPS OF PUPILS ON JUNE 1956 ALGEBRA QUESTIONS SIMILAR TO THOSE FOR JUNE 1955

FIGURE 5

to those for June 1955, but with one added difficulty. In all comparisons an "A", "B", "C" order resulted.

It would seem that the added difficulty helps to differentiate between pupils at different scoring levels.

EXAMINATION QUESTIONS FULLY EXPLAINED IN THE TEXTS

Figure 6, page 50, shows graphically the comparisons made for three questions identical to those fully explained in the texts, and for two algebra questions identical, except for changed sign and/or number, to examples explained in the text. On both propositions (questions No. 6a and b) and the factoring question (No. 2a) the pupils ranked "A", "B" and "C". For the simultaneous fractional equations (question 4b, identical to text example), "A", "C", "B" was the order of merit; and "B", "A", "C" for the motion problem (No. 9b).

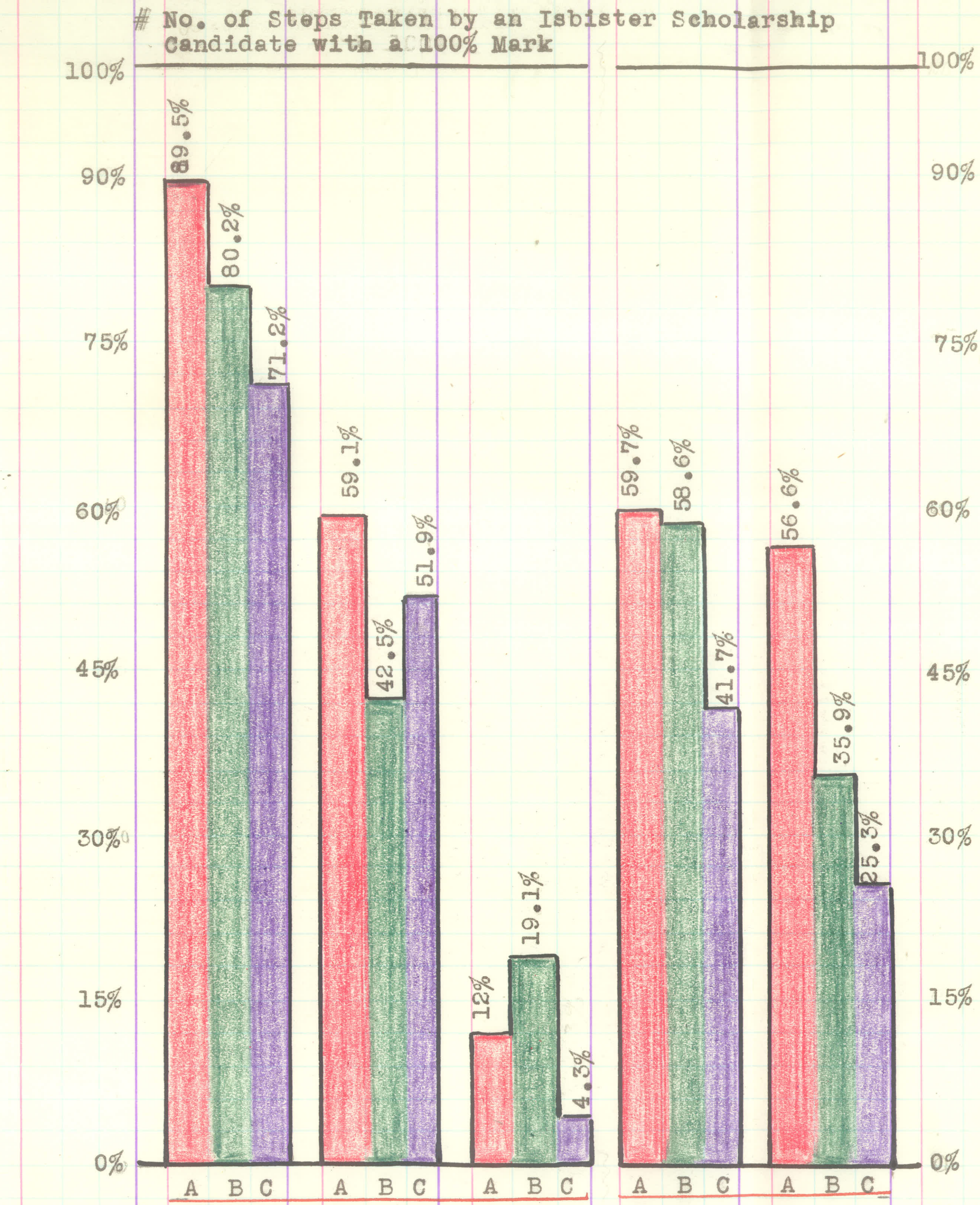
ORIGINAL QUESTIONS

Questions requiring filling in blanks correctly

Figure 7, page 51, "Average Marks in Percentages of Three Groups of Pupils for Filling in Blanks Correctly for 'Original' Geometry Questions" indicates that the bar graphs diminished from "A" to "B" to "C" for each of the eleven questions involved.

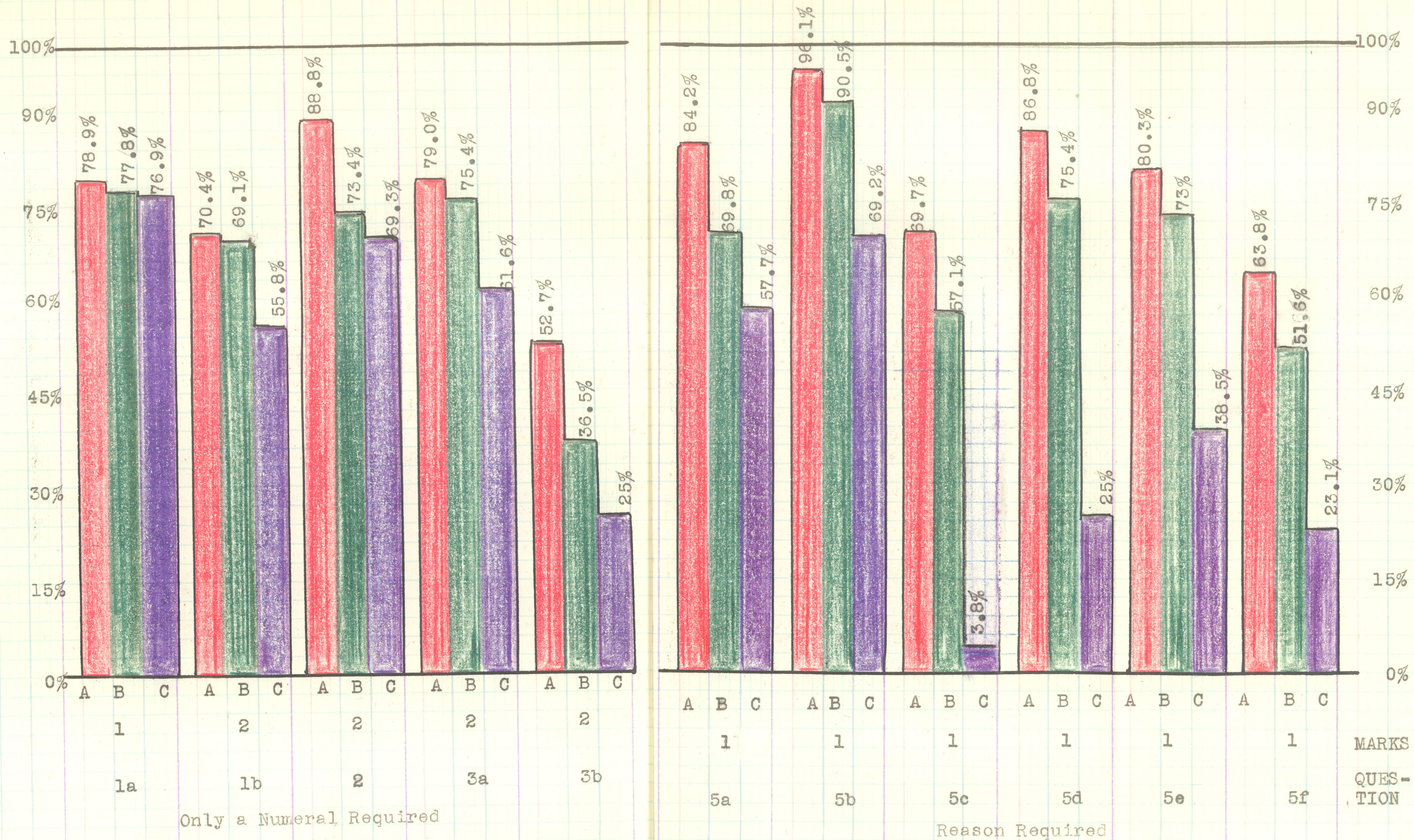
Questions requiring more than one written statement

Figure 8, page 52, concerning original questions involving



MARKS	2	4	4	6	6
No. STEPS#	3	10	14	6	8
QUESTION	Algebra 2a	Algebra 4b	Algebra 9b	Geometry 6 a	Geometry 6b

AVERAGE MARKS IN PERCENTAGES OF THREE GROUPS OF PUPILS ON QUESTIONS FULLY EXPLAINED IN THE TEXTS
FIGURE 6

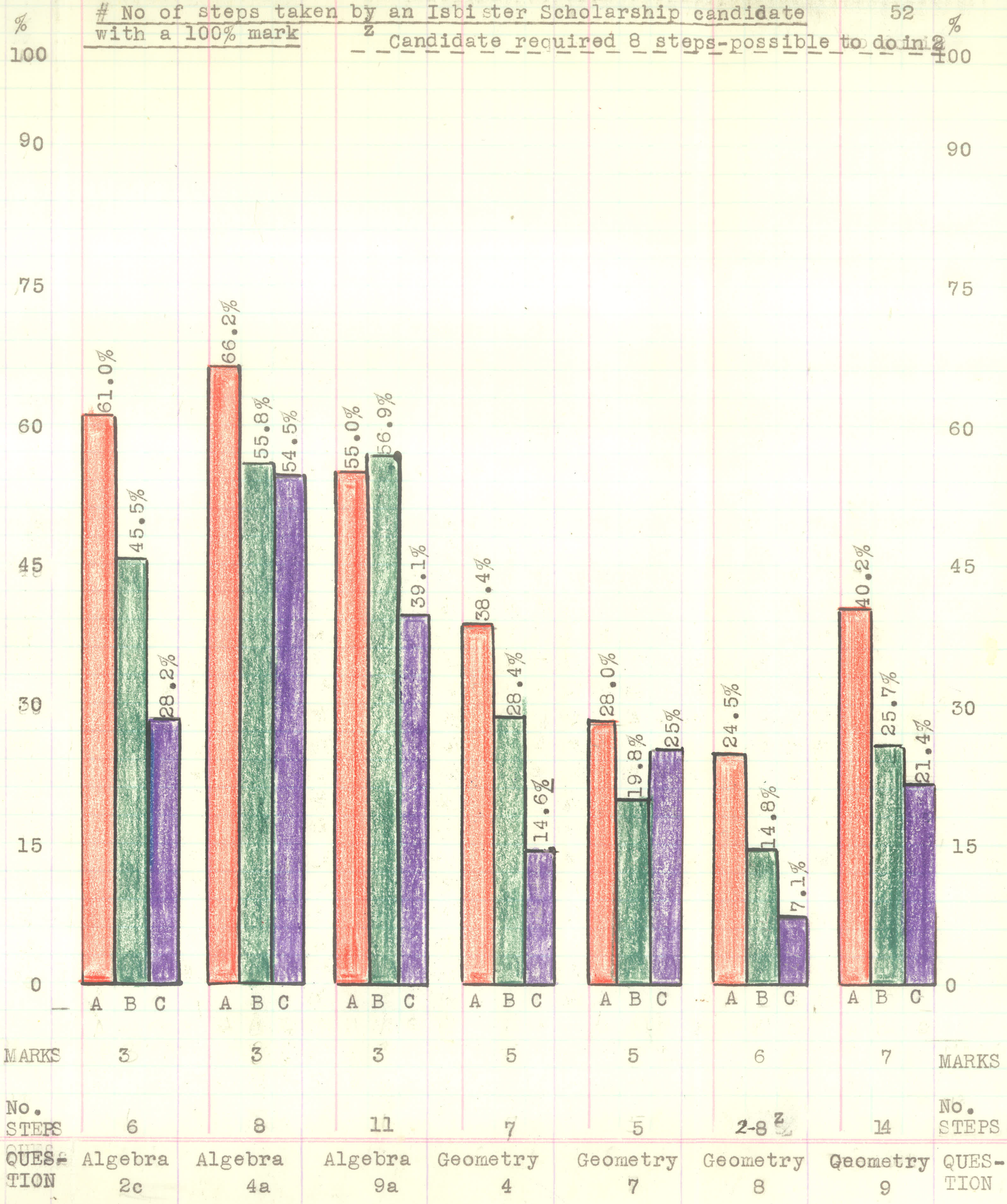


AVERAGE MARKS IN PERCENTAGES OF THREE GROUPS OF PUPILS FILLING IN BLANKS CORRECTLY FOR "ORIGINAL" GEOMETRY QUESTIONS

FIGURE 7

No of steps taken by an Isbister Scholarship candidate with a 100% mark

z Candidate required 8 steps-possible to do in 2



AVERAGE MARKS IN PERCENTAGES OF THREE GROUPS OF PUPILS ON "ORIGINAL" QUESTIONS INVOLVING MORE THAN ONE STATEMENT

FIGURE 8

more than one written statement shows that invariably all groups had less success with the geometry than the algebra items of this type.

For algebra questions 2e and 4a the order was the usual "A", "B", "C". However, the area problem 9a was an exception to the rule, for "B" had an average mark of 1.9% higher than "A". The writer wondered whether this lone exception to the order found for original proofs might have had any connection with the fact that children are taught from Grade VI up to find the area of triangles.[1]

In Figure 8, page 52, one also notes the usual "A", "B", "C" order for thegeometry questions, except for construction question 7 (which required a description and no proof) on which, on the average, "C" pupils got 5.2% more than "B".

<u>Conclusion for 'original' questions</u> For original questions requiring more than one statement there were two exceptions to the "A", "B", "C" order :- one, the algebra problem on area; the other, the geometry construction question for which no proof was required.

For the three groups of pupils, all original geometry questions requiring several statements were harder than any of that type for algebra. For "A" and "B" pupils the blank-filling geometry questions were the easiest of any original

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[1] Guy T. Buswell et al <u>Living Arithmetic Grade Six</u> (Toronto: Ginn and Co., 1951) p. 256.

geometry ones. On only two blank-filling items were "A" and "B" scores inferior to those for some original algebra questions. They were geometry question 3b (on proportion of triangles) and geometry 5f (which asked for only one reason but required more). Group "C" in addition to these two exceptions, had lower scores for Geometry No. 5c, 5d and 5e, than for some original algebra questions.

Conclusion for all types Table X, on page 55, is a summarization of results of comparisons made between the three groups as to percentages of marks received.

The order of merit was usually; first, "A"; second, "B"; and last "C". This held for 30 of the 37 questions. In the 20 algebra questions there were 6 exceptions to the sequence. However, when the 17 geometry items were considered, there was only one exception:- construction problem (no proof required). Only 2 of the 7 exceptions occurred in the 18 original questions. On 5 of the questions "B" did better than "A"; but only on 2 items did "C" excel "B".

On the whole it would seem that geometry rather than algebra, and 'original' questions rather than those 'solvable by imitation', were able to differentiate between groups of varying achievement. The 37 questions without exception, were able to discriminate between the highest scoring ("A") group and lowest scoring ("C"); in all but 2 questions, between the

TABLE X

SUMMARY OF COMPARISONS OF RESULTS ON QUESTION TYPES FOR THREE GROUPS

QUESTION TYPE	NUMBER of QUESTIONS	RANK ORDER "A","B","C"	"B","A",C	"A","C",B
Same as Previous Test				
Equal or less difficulty (algebra only)	10	7	3(1b,1e)(1f#)	
Added Difficulty (algebra only)	4	4		
Explained in Text Algebra	3	1	1(9b#)	1(4b#)
Geometry	2	2		
TOTAL SOLVABLE BY IMITATION	19	14	4	1
Original				
Number in blank (geometry only)	5	5		
Reason in blank (geometry only)	6	6		
More than one statement Algebra	3	2	1(9a#)	
Geometry	4	3		1(7#)
TOTAL ORIGINAL	18	16	1	1
TOTAL ALGEBRA	20	14	5	1
TOTAL GEOMETRY	17	16		1
GRAND TOTAL	37	30	5	2

Figure in bracket is question number.

"B" group and "C"; in all but 5, between "A" and "B".

DISCRIMINATIVE POWER OF QUESTIONS AS JUDGED BY "A" & "C" RESULTS

It has been shown that pupils of School District No. I who were taught by qualified teachers outscored pupils trained by Permit Teachers on every comparison made. To reach a decision as to the discriminative power of each section and each question, a percentage figure will be found by dividing each average raw score for "C" by the corresponding one for "A". A percentage of 63.4% was found for the algebra means; and 58.1%, for the geometry. The percentage figures for each question are listed in order of size in Table XI, on pages 57 and 58. It will be noted that the smaller the percentage figure the greater is the discriminative power.

<u>June 1956 test items similar to those for June 1955</u> With one exception "C"s scores were at least 75% of those for "A" on all questions worth one mark which were similar to those for the previous year and with no added difficulty. The exception was question 1d (on completing the square of a trinomial) on which "C" did less than one-third as well as "A".

Of the items similar to June 1955, the one which discriminated most (14.2%) between the two groups of pupils was that on rationalizing a denominator with two surds instead of one as in 1955

TABLE XI

QUESTIONS RANKED IN ORDER OF DISCRIMINATION BETWEEN "A" AND "C"

Question No.	Algebra	Geometry	Value	IMITATION = +	IMITATION 1955 ed.	IMITATION Text	ORIGINAL Blanks No.	ORIGINAL Blanks Reason	ORIGINAL Statements	"C" / "A" %	RANK
5c		x	1					x		5.5%	1
6	x		3		x					14.2%	2
5d		x	1					x		28.8%	3½
8		x	6						x	28.8%	3½
1d	x		1	x						30.0%	5
9b	x		4			x				36.0%	6
5f		x	1					x		36.2%	7
4		x	5						x	38.1%	8
2b	x		2		x					43.1%	9
6b		x	6			x				44.7%	10
8	x		4	x						45.8%	11
2e	x		3						x	46.3%	12
3	x		4		x					46.4%	13
3b		x	2				x			47.5%	14
5e		x	1					x		47.9%	15
9		x	7						x	53.3%	16
4e	x		4	x						63.4%	17
7	x		3		x					65.8%	18
5a		x	1					x		68.5%	19
6a		x	6			x				69.9%	20

TABLE XI Continued

Question No.	Algebra	Geometry	Value	IMITATION =	IMITATION 1955 + ed.	IMITATION Text	ORIGINAL Blanks No.	ORIGINAL Blanks Reason	ORIGINAL Statement	"C" / "A" %	RANK
9a	x		3						x	71.0%	21
5a	x		3	x						72.0%	22½
5b		x	1					x		72.0%	22½
1f	x		1	x						/75.8%	24
2		x	2				x			78.0%	25½
3a		x	2				x			78.0%	25½
1b		x	2				x			79.2%	27
2a	x		2			x				79.5%	28
4a	x		3						x	82.3%	29
5b	x		2	x						84.0%	30
1a	x		1	x						85.0%	31
1c	x		1	x						85.7%	32
1e	x		1	x						87.1%	33
4b	x		4			x				87.9%	34
1b	x		1	x						88.2%	35
7		x	5						x	89.2%	36
1a		x	1				x			97.5%	37

On questions similar to those of the previous June, "C", on the average, did over two-thirds (72.9%) as well as "A" on items with no difficulty added for 1956. Questions with an added difficulty, brought "C" scores to less than half (44.4%) of those for "A".

<u>Questions fully explained in the texts</u> Table XI, pages 57 and 58, shows that the proportion proposition yielded a decidedly greater differentiation (44.7% as opposed to 69.9%) between "A" and "C" than did the proposition on the cyclic quadrilateral. For questions fully explained in the texts, the algebra distance, rate, and time problem proved the most discriminating (36.0%).

<u>Original Questions</u> Table XI, pages 57 and 58, shows that geometry question 5e (requiring the reason "Tangent chord angle equals the angle in the alternate segment",) was by far the most discriminating item of any on the test. "C" scores were 5.5% of "A"s on this item. Next in order, with 28.8% each, came two more questions concerning circles:-

(a) 5d - requiring the reason angles in the same segment

(b) 8 - requiring more than one reason to prove that two chords were equal

Eight out of eighteen 'original' questions concerned circles: four of these were the most discriminating of the original items with an average 24.8% as opposed to 59.8% for all original questions and 58.5% for all original _geometry_ questions.

Geometry question 1a (width equals area divided by total length) had practically no discriminative power, (97.5%).

Other comparisons Because the geometry section is heavily weighted with original questions (value 38% as opposed to 9% for algebra), it was impossible to make valid comparisons regarding total marks on 'original' questions for algebra and geometry. However, the three most discriminating geometry questions 5c, 5d and 8 (ranks 1, 3½ and 3½) were all of the 'original' type. All three concerned circles and two of them required the filling of a blank with a reason. The three most discriminating algebra questions were concerned with:

(a) rationalizing a denominator with two surds (rank 2)

(b) completing a square (rank 5)

(c) distance, rate, and time problem (rank 6)

Of the five least discriminating geometry items, four (ranks 25½, 25½, 27 and 37) required the filling of a blank with a number; the fifth, discriminating rank 36, (which was a construction question) did not require a proof. Ranks 28 to 35 were all algebra questions of various types.

To find the discriminative power of each question type, the total average marks for "A" were divided by the corresponding total for "C". The quotients in percentages, listed in order of increasing discrimination were:

(1) 'original' geometry questions requiring the filling of a blank with a number	78.0%
(2) questions similar to those for the previous June, and of equal, or less, difficulty	72.9%
(3) questions similar to those fully explained in the texts	70.2%
(4) 'original' questions requiring more than one statement	60.6%
(a) algebra only	66.8%
(b) geometry only	51.9%
(5) 'original' geometry questions requiring the filling of a blank with a reason	45.2%
(6) questions similar to those of the previous June but with one added difficulty	44.4%

<u>Conclusion</u> For the June 1956 Mathematics Test, the following question types have been compared with the result that on the average the second mentioned in each case proved to be more discriminating:

a) Solvable by imitation	65.7%	'Original'	59.8%
b) Algebra	63.4%	Geometry	58.1%

c) Number in blank (least discriminating of all types)	78%	Reason in blank (second most discriminating type)	45.2%
d) Similar to June 1955	72.9%	Same with added difficulty (most discriminating of all types)	44.4%
e) Geometry questions not concerning the circle	68.8%	Geometry questions concerning the circle	51.3%

EASY AND DIFFICULT QUESTIONS

The investigator defined an easy question for any group as one on which that group scored 80% of the possible marks; and a difficult question, less than 20%

<u>Easy questions</u> According to the definition the following questions could be considered easy

(a) Algebra questions similar to June 1955

(14 in all)

1a - "A", 88.2%;[z] "B"; 84.9%[z]; "C", 75%

1b - "A", 82.9%[z]; "B", 89.7%[z]; "C", 73.1%

(b) Examples fully explained in the texts (5 in all)

Algebra 2a - "A" 89.5%[z]; "B", 80.2%[z]; "C", 71.2%.

(c) 'Original' geometry blank-filling questions

(11 in all)

2 - "A", 88.8%[z]; "B", 73.4%; "C", 69.3%

5a- "A", 84.2%[z]; "B", 69.8%; "C", 57.7%

5b- "A", 96.1%[z]; "B", 90.5%[z]; "C", 69.2%

5d- "A", 86.8%[z]; "B", 75.4%; "C", 25%

5e- "A", 80.3%[z]; "B", 73%; "C", 38.5%

(d) 'Original' geometry questions involving more than one statement (7 in all)

The highest mark made on this type was 66.2%

[z]It will be noted that over 80% of the possible marks were scored by "A", on 8 questions; by "B", on 4; and by "C" on none

<u>Difficult questions</u> The following questions produced average marks of less than 20% for at least one of the three groups of students

(a) Algebra questions similar to June 1955 (14 in all)

1c - "A", 11.2%#; "B", 11.1%#; and "C", 9.6%#

1d - "A", 51.3%; "B", 28.6%; and "C", 15.4%#

6 - "A", 49.8%; "B", 38.9%; and "C", 7.1%#

(b) Examples fully explained in the texts (5 in all)

Algebra 9b - "A", 12 %#; "B", 19.1%# and "C", 4.3%#

(c) 'Original' blank filling geometry questions (11 in all)

5c - "A", 69.7 %; "B", 57.1%; "C", 3.8%#

(d) 'Original' questions involving more than one statement (7 in all)

Geometry 4 - "A", 38.4%; "B", 28.4%; "C", 14.6%#

Geometry 7 - "A", 28.0%; "B", 19.8%#; "C", 25%

Geometry 8 - "A", 24.6%; "B", 14.8%#; "C", 7.1%#

It will be noted that less than one-fifth of the possible marks were scored by "A" on only two questions; by "B", on 4 questions; and by "C" on 7.

Conclusion Table XII, pages 65 and 66, lists all items classified as easy or difficult for each separate group. No question was easy for Group "C" and only two questions were hard for "A" (algebra 1c and 9b) When the total number of easy and difficult questions for each group is considered the usual "A", "B", "C" order of merit results.

RANK ORDER OF DIFFICULTY FOR INDIVIDUAL QUESTIONS AND QUESTION TYPES

<u>Introduction</u> Using the average percentages of marks made by pupils, questions and question types will be ranked and compared as to difficulty for each group.

<u>Individual Questions</u> In Table XIII, pages 67 and 68, are listed the 37 questions according to the classifications outlined in Chapter IV together with the rating from the easiest to the most difficult for all three groups.

The following questions were ranked among the eight easiest for all three groups:

Algebra 1a - "How much greater is $7a + 3b - 4c$ than $3a - 2b + 5c$?"

TABLE XII

QUESTIONS FOUND EASY OR DIFFICULT ON BASIS OF PERCENTAGES OF MARKS SCORED BY EACH GROUP

	A	B	C
Algebra 1a[j]	Easy	Easy	
1b[j]	Easy	Easy	
1c[j]	Hard	Hard	Hard
1d[j]			Hard
2a[t]	Easy	Easy	
6[j] +			Hard
9b[t]	Hard	Hard	Hard
Geometry 2[ob]	Easy		
4[os]			Hard
5a[ob]	Easy		
5b[ob]	Easy	Easy	
5c[ob]			Hard
5d[ob]	Easy		
5e[ob]	Easy		
7[os]		Hard	
8[os]		Hard	Hard
Total Easy Questions	8	4	0
Total Difficult Questions	2	4	7

j = question similar to June 1955 (+ = added difficulty)
t = question explained in text
ob = 'original' blank filling item
os = 'original' question requiring several statements

TABLE XII Continued

		A	B	C	Total
Easy questions	j	2	2		4
	j +				0
	t	1	1		2
	ob	5	1		6
	os				0
Difficult-Questions	j	1#	1#	1# + 1	4
	j +			1	1
	t	1z	1z	1z	3
	ob			1	1
	os		2	2	4

#Algebra question No. 1e

z Algebra question No. 9b

TABLE XIII

RANKS AS TO DIFFICULTY OF QUESTIONS, FOR ALL THREE GROUPS

I QUESTIONS SOLVABLE BY IMITATION

QUESTION TYPE	QUESTION NUMBER Alg.	QUESTION NUMBER Geo.	RANKS "A"	RANKS "B"	RANKS "C"
A. Similar to June 1955					
1. Equal or less difficulty	1a		4	3	2
	1b		7	2	3
	1c		37	37	33
	1d		28	31	31
	1e		15	8	9
	1f		33	28	23
	4c		30	25	24
	5a		23	23	15
	5b		9½	9	7
	8		19	24	21
2. One added difficulty	2b		12	22	20
	3		18	19	19
	6		29	27	34½
	7		16	21	14
B Explained in text	2a		2	4	4
	4b		22	26	13
	9b		36	35	36
		6a	25	15	16
		6b	26	30	25

TABLE XIIIContinued

II ORIGINAL QUESTIONS

QUESTION TYPE	QUESTION NUMBER Alg.	Geo.	RANKS "A"	"B"	"C"
A. Blank for answer					
1. Number required		1a	11	5	1
		1b	13	13	11
		2	3	10	5
		3a	9½	6½	8
		3b	27	29	27
2. Reason required		5a	6	12	10
		5b	1	1	6
		5c	14	16	37
		5d	5	6½	27
		5e	8	11	18
		5f	20	20	29
B. Other than blank filling					
	2e		21	14	22
	4a		17	18	12
	9a		24	17	17
		4	32	32	32
		7	34	34	27
		8	35	36	34½
		9	31	33	30

This was equal in difficulty to a similar question of June 1955.

"A" rank, 4; "B", 3; "C", 2

Algebra 1b - "Evaluate $a^3, - 2a^2 + 1$ when a equals - 2".

This question was similar to one for June 1955.

"A" rank, 7; "B", 2; "C", 3

Algebra 2a - "Factor $ax - by + ay - bx$"

In the text, almost a full page is devoted to methods and check for a question identical except for two signs.

"A" rank, 2; "B", 4; "C", 4

Geometry 5b - A given statement required the reason, "Equal arcs subtend equal chords"

"A" rank, 1; "B", 1; "C", 6

The following questions were ranked among the eight most difficult questions for all three groups:

Algebra 1e - "$x^{2n} \div x^{n-1}$"

This question was similar to one for June 1956 with no added difficulty.

"A" rank, 37; "B", 37; "C", 33

Algebra 9b - " A man travelled fifteen miles at a constant rate. If he had travelled two miles per hour faster, he would have gone the same distance in two hours less time. What was his original rate of travel?"

Although a more difficult example had been explained in the text, no item on the entire paper proved to be more difficult than this motion problem, which required 14 steps for a 100% student.

"A" rank, 36; "B", 35; "C", 36

Of all algebra items, this question had the highest percentages of students in each group failing to show any written attempt. The percentages were: "A", 26.3%; "B", 42.9% and "C", 26.9%

Geometry 4 - a problem involving the area of overlapping parallelograms and triangles

"A" rank, 32; "B", 32; "C", 32

Of all geometry items, this problem had the highest percentages of students failing to show a written attempt. The percentages were:

"A", 22.4%; "B", 42;9% and "C", 38.5%

Geometry 8 - A geometry problem required proof of the equality of two intersecting chords whose angle of intersection was bisected by a diameter.

"A" rank, 35; "B", 36: "C", 34½

Geometry 9 - "G is the midpoint of a line MN. PE is parallel to MN. PN is joined. The straight line EFGH cuts PN at F and meets PM produced at H."

Prove: $\frac{EF}{FG} = \frac{EH}{HG}$"

"A" rank, 31; "B", 33; "C", 30

Question Types Using the average percentage mark for each question type, question types were ranked as to difficulty for each group (See Table XIV, on page 72.)

Difficult type. All groups found original questions requiring more than one statement the most difficult of any type.

Variation in Ranking between Groups This section will be concerned with assessing the amount of variation between groups in the ranking as to difficulty of questions and question types.

Algebra questions In only 3 algebra questions was there a variation of 10 or more in rank order of difficulty between the three groups. They were:

1f - "Solve for x: $K = \frac{bx - c}{x}$ "

a question easier than a similar one for June 1955

"A" rank, 33; "B", 28; "C", 23

2b - "Factor $x^3 - \frac{b^3}{8}$"

On the 1955 test the similar item was "$8x^4 - 27x$"

"A" rank; 12; "B", 22; "C", 20

4b - The method and check for this identical simultaneous equation question had been demonstrated in the text.

TABLE XIV

RANKS AS TO DIFFICULTY OF QUESTION TYPES, FOR ALL THREE GROUPS

QUESTION TYPES	TOTAL No. QUES-TION Alg.	Geo.	RANKS "A"	"B"	"C"	Sum of "A","B","C"
Similar to June 1955						
Equal or less difficulty	10		4	3	2	9
One added difficulty	4		3	4	5	12
Explained in Text	3	2	5	5	3	13
'Original'						
Blanks						
Numbers		5	2	2	1	5
Reasons		6	1	1	4	6
Statements	3	4	6	6	6	18

"A" rank, 22; "B", 26; "C", 13

<u>Geometry questions</u> In geometry there were greater variations in rankings as to difficulty of questions than in algebra.

Five of the six questions involving filling in blanks with reasons for given statements were found relatively more difficult for pupils of Permit Teachers than of fully qualified teachers. Three of these with a variation of ten or more (together with the reason required for each) were:

5c - "Chord tangent angle equals angle in alternate segment".

"A" rank, 14; "B", 16; "C", 37

5d - "Angles in same segment"

"A" rank, 5; "B", 6½; "C", 27

5e - "Angle in semi-circle equals 90°."

"A" rank, 8; "B", 11; "C", 18.

Two other questions with decided variations were:

1a - "A rectangle for which the area and two parts of the length were supplied, and for which the width was required."

"A" rank, 11; "B", 5; "C", 1

6a - The proposition on the opposite angles of a cyclic quadrilateral

"A" rank, 25; "B", 15; "C", 16

<u>Question types</u> On the average, there was practically no variation between "A" and "B" pupils as to the relative

difficulty of various question types. The greatest variation between "C" and others was found in 'original' geometry questions requiring a reason for each given statement. This type rated the easiest (Table XIV, page 72) for pupils of fully qualified teachers and in the more difficult half for those of Permit Teachers.

In Chapter IV, page 34, it was stated that for students who depend on memorizing mathematical rules without understanding, a small change in a question (such as substitution of a fraction or a surd for a whole quantity, or a binomial expression for a monomial one) can make the solution more difficult to perform. Both "B" and "C" pupils found questions similar to those of June 1955 of equal or less difficulty ("B" average, 52.8%; "C", 42.4%) easier than similar items with one added difficulty ("B" average, 48.3%; "C", 28.6%). However, "A" students scored on the average 6.4% more on questions with the added difficulty than on those of equal or less difficulty (average 58.1% compared to 64.5%). One might therefore suspect that either "A" students depended least of all three groups on memorized mathematical rules without understanding, or they had studied such questions with added difficulties.

<u>Other Comparisons</u> The three factoring questions, No. 2a, b and c were arranged in increasing order of difficulty

for groups "A" and "C" as were the three equation questions No. 4a, b and c.

Of the two verbal algebra problems the one on motion, No. 9b, was more difficult than that on area 9a. Possible reasons for the differential in rank might have been:-

1. Area of triangles is taught from Grade VI on.
2. Area problems are easier to visualize than motion.
3. In the model paper the area problem required 11 steps; the motion, 14.

Of all four original geometry questions requiring more than one statement, question 8 (the only one of the four involving a circle) was the most difficult for all groups.

CONCLUSION

Table XV, on page 76, lists outstanding examples as to difficulty, for questions and question types. Both groups of students taught by fully qualified teachers found the same questions or question types easiest or hardest, except for the easiest algebra question and algebra type. Other than finding 'original' questions with more than one statement most difficult, pupils of Permit Teachers had different ratings as to the easiest and hardest questions and question types.

In algebra, scholars of qualified teachers found questions explained in the text most difficult. The easiest

TABLE XV

EASIEST AND MOST DIFFICULT QUESTIONS AND QUESTION TYPES, ON BASIS OF RANK ORDER FOR EACH GROUP

	"A"	"B"	"C"
QUESTIONS			
Easiest			
All test	Geo. 5b	Geo. 5b	Geo. 1a
Algebra only	2a	1b	1a
Geometry only	5b	5b	1a
Most Difficult			
All test	Alg. 1c	Alg. 1c	Geo. 5e
Algebra only	1c	1c	9b
Geometry only	8	8	5e
QUESTION TYPES			
Easiest			
All test	O.B.R.	O.B.R.	O.B.N.
Algebra only	J +	J	T
Geometry only	O.B.R.	O.B.R.	O.B.N.
Most Difficult			
All text	O.S.	O.S.	O.S.
Algebra	T	T	J +
Geometry only	O.S.	O.S.	O.S.

KEY J = Similar to June 1955 (+ = Added Difficulty
O.B. = Original Blank Filling (R = Reason N = Number)
O.S. = Original Requiring More than One Statement
T = Explained in Text

for pupils of Permit Teachers was this type; but for "B" students, questions similar to those for June 1955 were easiest; and for "A", the same type but with added difficulty. Perhaps "C" students spent more time studying text examples; and "B", the test paper for the previous June.

Table XVI on page 78, lists the outstanding examples of questions and question types as to discriminative power between groups "A" and "C". No connection could be found between discriminative power and values assigned to, or number of steps required for, questions.

TABLE XVI

QUESTION AND QUESTION TYPES DISCRIMINATING MOST AND LEAST BETWEEN GROUPS "A" AND "C"

	QUESTIONS	QUESTION TYPES
Least Discriminating		
All test	Geo. 1a	O.B.N
Algebra only	1b	J
Geometry only	1a	O.B.N.
Most Discriminating		
All test	Geo. 5e	J +
Algebra only	6	J +
Geometry only	5e	O.B.R.

KEY

J = Similar to June 1955 (+ = Added Difficulty)

O.B. = Original Blank-filling (R = Reason N = Number)

O.S. = Original Requiring More than One Statement

T = Explained in Text

CHAPTER VI

SUMMARY AND CONCLUSIONS

EXEMPTED "A" STUDENTS

From School I results, it might be assumed that had all "A" Grade XI students written the June 1956 mathematics examination:-

(1) probably no exempted "A" student would have failed.

(2) few, if any, would have had scores in the lower-half of the range for the total population.

(3) on the average, the June marks for accredited pupils would have been at the very least 9% higher than their actual scores on the Christmas and Easter tests.

QUESTION TYPES ON JUNE 1956 MATHEMATICS EXAMINATION

From the evidence presented in Table IX, on page 45, it would seem that:

1) A student who had mastered both the examination for the previous year, and the detailed explanatory examples in the text, could have earned a good mark in algebra.

2) The geometry portion of the examination (in which "A" students were decidedly superior) was a much better test of resourcefulness in problem-solving than the algebra section.

COMPARISON OF AVERAGE MARKS SCORED
ON QUESTION TYPES

According to the last column of Table XIV, page 72, question types (as set out in Chapter IV) arranged in order of increasing scale of difficulty were:

(1) 'original questions involving the filling of a blank with a number

(2) 'original' questions involving the filling of a blank with a reason

(3) questions similar to those of the previous June test, and of equal, or less, difficulty

(4) questions similar to those of the previous June with one added difficulty

(5) examples fully explained in the text

(6) 'original' questions requiring more than one statement.

COMPARISON OF AVERAGE MARKS
MADE BY THREE GROUPS

<u>Total</u> <u>Scores</u> The order of merit was "A", "B", "C" for all comparisons made as to total scores for algebra and/or

geometry, i.e. failure rate, percentiles 10, 25, 50, 75, and 100. "A", were even more superior in geometry than in algebra. For all groups, the geometry section was more difficult than the algebra. However, the median pupil of the entire "A" population, unlike "B" and "C", did almost as well in geometry as in algebra.

Individual Questions

Easy questions No question was easy for pupils taught by Permit Teachers. However, students taught by fully qualified teachers got at least 80% of the possible marks on the following questions:

algebra 1a, 1b and 2a

geometry 5b.

Only children taught in School District I found the following questions easy:

geometry 2, 5a, 5d and 5e.

Difficult questions Only algebra questions 1c and 9b were difficult for all groups, while geometry question 8 was difficult for groups "B" and "C". Group "B" alone got less than one-fifth of the marks for geometry question 7. Group "C" had trouble with four more questions: algebra 1d and 6; and geometry 4 and 5c.

Discrimination The 37 questions without exception were able to discriminate between the highest scoring ("A") group and lowest scoring ("C"); in all but 2 questions, (algebra 4b and geometry 7) between the middle scoring ("B") group and "C"; and in all but 5 (algebra 1b, 1e, 1f, 9a and 9b), between "A" and "B". The most and least discriminating items were blank filling ones in the geometry section. They were respectively:-

(a) Question 5e requiring the reason, "Tangent chord angle equals the angle in the alternate segment."

(b) Question 1a required a number resulting from use of the formula; width of rectangle equals area divided by total length.

The most discriminating algebra item, No. 6, required a binomial denominator with two surds to be rationalized. The least discriminating algebra question was No. 1b, "Evaluate $a^3 - 2a^2 + 1$ when $a = 2$".

When the ranks as to difficulty between groups were compared, it was found that there was greater variation between rank orders of difficulty on individual questions in geometry than in algebra.

Question Types

Easy type. For groups "A" and "B" the original geometry questions requiring the filling of a blank with a reason, was the easiest. For "C" the easiest was the type which required a number in a blank.

Difficult type. For all three groups original questions requiring more than one statement was the most difficult type: and the geometry questions of this type proved more difficult than the algebra.

Discrimination of Types Using results of the highest scoring group ("A") and lowest scoring group ("C"), question types listed in order of increasing discriminative power were:

1. 'original' geometry questions requiring the filling of a blank with a number.
2. Questions similar to those for the previous June and of equal, or less, difficulty
3. Questions similar to those explained in the text
4. 'original' questions requiring more than one statement
5. 'original' geometry questions requiring the filling of a blank with a reason
6. questions similar to those of the previous June, but with one added difficulty.

When the following pairs of question types were compared, the second mentioned in each proved more discriminating.

Solvable by imitation	Original
Algebra	Geometry
Number in blank	Reason in blank
Similar to June 1955	Same with added difficulty
Geometry questions not concerning the circle	Geometry questions concerning the circle

CONCLUSION

<u>Exempted Students of School District Number One</u> From the evidence in School 1, the inference seems to be that had exempted students of School District Number One written the June 1956 Mathematics Examination, almost without exception, their scores would have belonged in the upper half of the range for all Grade XI students of that district.

<u>June 1956 Test</u> Through just mastering the previous June test, and examples demonstrated in the texts, a student could have made 31 out of a possible 50% in algebra; and 12 out of 50% in geometry. Two-and-a-half times as many marks were given in the geometry as in the algebra section for 'original' questions requiring several statements.

<u>Relative Ability of the Three Groups of Students</u> When the average marks and five percentile ratings for algebra and/or geometry were compared for the three groups, the order

of mebit was "A", "B", "C" in every instance. Also when questions were grouped as to types, an "A", "B", "C" order resulted for each comparison. The evidence presented from paired algebra and geometry scores would indicate, that "A" students excelled even more in geometry than algebra. The results on questions similar to those of the previous June but with one added difficulty leads one to suspect that "A" students depended least of all three groups on memorized rules without understanding for solution of algebra questions; and "C" pupils most of all. Results on geometry questions requiring one reason for each answer, would seem to indicate that scholars of Permit Teachers had relatively little understanding of geometry concepts. In algebra, the ranking of question types for "B", and "C" would lead one to infer that "C" students spent relatively more time studying algebra examples fully explained in the text; and "B", algebra questions for the previous June.

Discrimination of Question Types The types of questions which discriminated most between students taught by Permit Teachers and those by teachers of School District Number One were:

(a) algebra questions similar to those of the previous June but with an added difficulty,

("C"s average score was 44% that of "A")

(b) 'original' geometry questions requiring a reason in each blank ("C"s average score was 45% that of "A".)

The least discriminating type was the original geometry questions requiring a number in each blank ("C"s average score was 78.% that of "A".)

<u>Difficulty of Question Types</u> All three groups found 'original' questions requiring more than one statement the most difficult type, ("A average, 44.8%; "B", 35.3%; and "C", 27.1%) Of these questions, geometry problems were more difficult ("A" average, 32.8%; "B", 22.2%; and "C", 17.0%) than algebra.

Both "A" and "B" had highest scores for filling blanks in geometry with reasons ("A" average, 80.2%; "B", 69.6%). "C"s highest score, 78.0%, was made in filling in geometry blanks with numbers.

SUGGESTIONS FOR FUTURE INVESTIGATIONS

In order to assess the possible reasons for the consistent "A", "B", "C" ranking as to mathematical achievement for the lower half of all Grade XI students in each group, the following information would have been helpful:

(1) Re. average for each group of teachers for:

(a) experience in years

(b) number of subjects taught during the year

(c) years of professional or academic training

(2) Re. pupils;

(a) average intelligence quotients of pupils in each group

(b) number of students in each group who were not permitted to write the final examinations in their own school but did so at the Departmental Centre

(c) number of weak students in each group who, on the advice of members of the faculty of the school concerned, did not write the final examinations

(d) exemption marks of "B" students.

When comparisons were made of the means and five percentiles for algebra and geometry, it was found that without exception, the geometry scores were lower than those for algebra. The lower scores might have been due to the fact that -

geometry is a more difficult subject
and/or
the geometry section had fewer questions solvable by imitation

It would be interesting to study results of a test with equal numbers of algebra and geometry items for each question type.

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WORKS CITED

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APPENDIX

APPENDIX A

GRADE XI DEPARTMENTAL PAPERS, June 1956

		Lit	Comp.	Soc.Stds.	Maths.	Geom.	Alg.	Chem.
	Papers	808	806	823	748	2	30	704
	Exemptees	353	353	353	354			324
	Totals	1161	1159	1176	1102	2	30	1028
Totals	Failures	262	147	267	116	1	3	176
	%	22.5	12.6	22.7	10.5	50	10	17.1
	Papers	5418	5520	5396	4960	42	118	5214
Provincial	Failures	1700	1082	1770	987	10	26	1273
Total	%	31.4	19.6	32.8	19.8	23.8	22.0	24.4

Failure Comparison **Grade XI**

	Wpg. %	1956 Man. %	Prov. without Wpg. %
Literature	23	31	34
Composition	13	20	21
Social Studies	23	33	37
Mathematics	11	20	23
Chemistry	17	24	26
Physics	14	23	25
Biology	18	25	35
French	23	32	36
Latin	13	20	27
German	39	14	12
Totals	18	26	28

APPENDIX B TABLE XVII

GR. XI JUNE 1956 MATHEMATICS FIGURES FOR: NUMBER OF PUPILS, EXAMINATIONS & FAILURES
I Total Population and Sources of Figures. II Original Samples. III Samples of Lower Half.

	ALL		GROUP "A"		GROUP "B"		Group "C"	
I Total Population	5120	M	1102	# W.T.	924		130	
No. pupils exempted	528	D.A.	354	#	174			
No. pupils exempted as % of total pop.	10.3		32.1		18.8			
No. papers written	4960	#	750 748	x #	750	x	130	x
No. papers written as % of total pop.			68.1 67.9		81.2		100	
No. failures	987	#	120 118	G # W.T.				
No. failures as % of total population	19.3		10.8 10.6	G #				
No. failures as % of papers	19.98	M	15.8					
II Original Sample								
No. Sample Papers	257		103		103		51	
No. sample papers as % of total pop.	5.0		9.3		11.1		39.2	
No. sample papers as % of papers written	5.2		13.7 13.8		13.7		39.2	
No. failures	66		16		28		22	
No. failures as % of original sample			15.5		27.2		43.1	
III Lower Half								
No. papers	165		76		63		26	
No. papers as % of original sample	64.2		73.8		61.2		51	
No. papers as % of total population	3.2		6.9		6.8		20	
No. papers as % of total papers written	3.3		10.2		8.4		20	
No. papers as % of 165 (total lower half)	100		46.1		38.2		15.8	
No failures	66		16		28		22	
No. failures as % of lower half sample			21.1		44.4		84.6	

Key D.A. - Department of Education Annual Report p.18
G. - C.E. Germain's "Statistical Study of School Examinations in Manitoba"
M. - Manitoba School Journal November 1956 p5.
W.T. - Winnipeg Tribune
\# - School District No.1 Annual Failure Study
x - According to paper around original sample papers

APPENDIX C

Grade XI Mathematics Easter 1956

Time: 3 hours

Name ____________________ Room________ Teacher of Subject ____________

Value 50 marks PART A - ALGEBRA ($1\frac{1}{2}$ hr.)

<u>Note: Do all questions on foolscap. Return this printer paper.</u>

1 1.(a) Simplify: $3y^2 - 2y(-3 + 2y) - y$

1 (b) Simplify: $\frac{24a^3x^2 - 48axy + 16a^2xb}{-8ax}$

1 (c) Express in an equivalent form :- $\frac{c - d}{a - b} =$

1 (d) Simplify: $\left[\sqrt[3]{(x-y)^3}\right]^6 =$

1 (e) Express with positive indices: $\frac{a^3b^{-2}c^{-1}}{x^{-2}yz^{-4}} =$

1 (f) Simplify, expressing with positive indices:

$$\left(\frac{27m^6b^3}{8x^{-3}}\right)^{\frac{2}{3}} =$$

1 (g) Simplify: $\left[\left(x^{-1}\right)^{-10} \cdot x^{-5}\right]^{\frac{2}{5}} =$

1 (h) Solve: $x^2 - 2x = 63$

1 (i) Express as one fraction: $\frac{x + y}{2x} - \frac{x - y}{3x}$

1 (j) Solve for M: $h = \frac{M^2}{2g}$

2. Find the square root of:

2 (a) $4x^4 - 12x^3 + 29x^2 - 30x + 25$

APPENDIX C Continued

2 (b) 6 (Correct to hundredths)

3. Factor fully:

2 (a) $8x^3b + a^3b =$

2 (b) $3x^3 - 2x^2 - 12x + 8 =$

2 (c) $x^2 - 6x + 9 - 4y^2 =$

2 4 (a) Reduce to lowest terms: $\dfrac{x^2y - xy^2}{x^3 + 2x^2 - 3xy^2} =$

4 (b) Simplify $\dfrac{x^3 - 27}{(x-3)^3} \cdot \dfrac{2x^2 - 11x + 15}{x^2 + 3x + 9} \div \dfrac{2x^2 + x - 15}{2x^2 - 18} =$

3 (c) Simplify: $\dfrac{1}{2x - 3} - \dfrac{5}{4x + 6} - \dfrac{3x - 11}{9 - 4x^2}$

2 (d) Evaluate $1^5 - 81^{-\frac{3}{4}} + \dfrac{1}{4^{-1}} + 6x^0$

2 (e) Simplify: $\sqrt{18b^2} \cdot \sqrt{3a^2}$

2 (f) Simplify $\dfrac{15}{2\sqrt{3}}$

2 (g) $\left(x^{\frac{1}{2}} - y^{\frac{2}{3}}\right)\left(x^{\frac{3}{2}} + y^{\frac{1}{3}}\right) =$

3 6. (a) Solve for x:

$$\frac{2}{5}(5 - 2x) - \frac{1}{6}\left(\frac{5 + x}{5}\right) = 2 - \frac{1}{3x}$$

3 (b) Solve for x: $(x - 1)^2 + (x + 1)^2 + (x-1)(x + 1) - 28 = 0$

3 (c) Solve for x and y: $ax = by$
$bx + ay = m$

APPENDIX C Continued

Note: Students taking Algebra and Geometry will do either VII or VIII

Students taking Algebra only will do both VII and VIII

4 7. A man travels a distance of 150 miles at a uniform rate of speed. On the return trip he increases his speed by 5 miles per hour and thus requires 1 hour less time. Find his original rate of speed.

-- Or --

8. A man wishes to invest ½ his money at 5%, $\frac{1}{6}$ of it at 6%, and the balance of his money at 4%. How much must he invest at each rate to yield an annual income of $1247?

END OF PART A

Teacher of
Subject_______________ Name _______________ Room _________

Grade XI MATHEMATICS (3) Easter, 1956

PART B - GEOMETRY

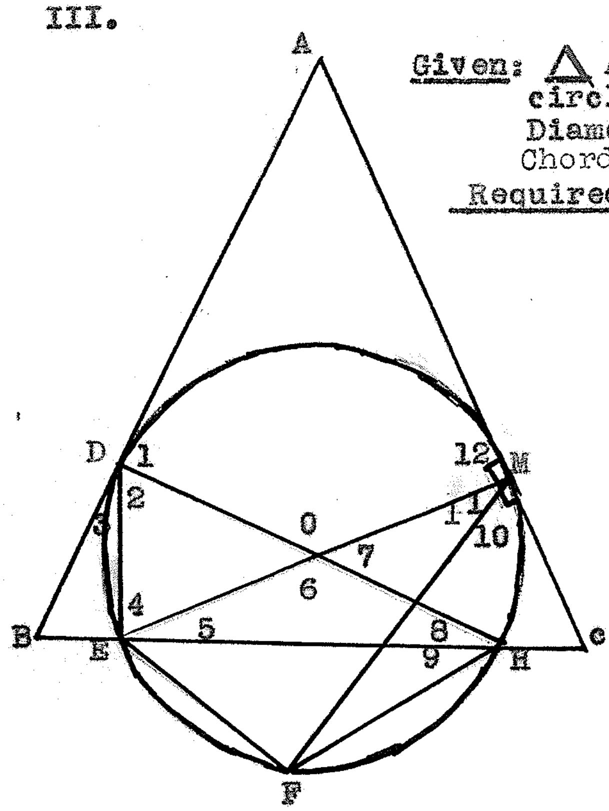
Value: 50 marks. Note DO QUESTIONS I, II, VI, on foolscap.

No. Questions III, IV, V, VII on this paper.

6 I. The opposite angles of a cyclic quadrilateral are supplementary.

8 II. The angles between a tangent to a circle and a chord from the point of contact are equal respectively to the angles in the alternate segments.

III.



Given: △ ABC with AB tangent to a circle with centre O at D. Diameters DH and EM. AC ⊥ EM. Chords ED, EF, FH.

Required: Without further construction to fill in the statements and reasons needed to prove the following:

Proof: Statements	Reasons
1 (a) ∠11 = 9	
1 (b) ∠ 2 = ½∠ 6	

Proof: Statements Reasons

1 (c) DE ⊥ BC

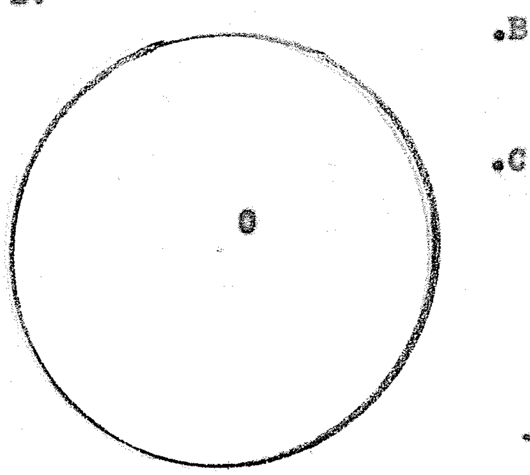
2 (d) AD = AM

1 (e) ∠3 = ∠8

2 (f) Quad. ADOM is cyclic.

1 (g) ∠7 = ∠ A

6 IV



Given: 3 points, A, B, and C outside a circle with centre O.

Required: To find a point P, that is on the tangent from pt. C to the circle with centre O, and that is also equidistant from points A and B.

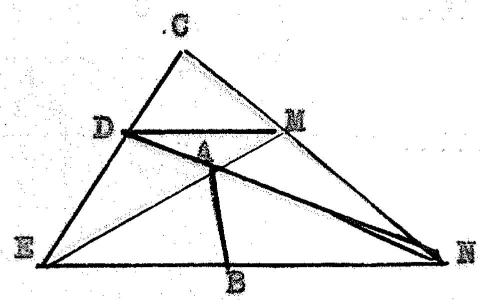
Show clearly all construction marks.

Const.

Proof	Statements	Reasons

Conc

7 V



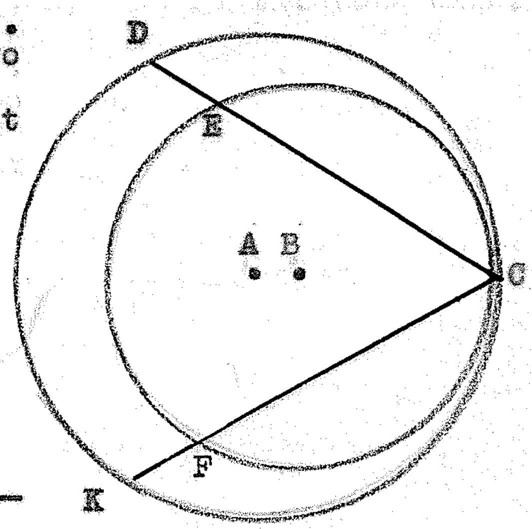
Given: △CEN with DM/ /EN. B if the mid point of EN. Lines DN, EM.

Required: (a) To prove quad. DEBA = quad. MABN in area.
(b) To prove $\frac{CD}{DE} = \frac{CM}{MN}$
(c) If DE = 14.7, CM = 20, MN = 35, find the length of CD.(show work)

Proof	Statements	Reasons

7 VI Given A circle with centreO. 6 points A, B, C, D, E and F on this circle in that order. Arc AB = arc EF.
Chord BD intersects chord EC at M.
Chord AC intersects chord BD at N.
Chord FD intersects chord EC at S.
Required: To prove points C, N, S and D are concyclic.

7 VII. Two circles touch internally. From the point of contact two equal chords of the larger circle are drawn. Prove that the two chords of the smaller circle are equal.



Given

Required:

Const

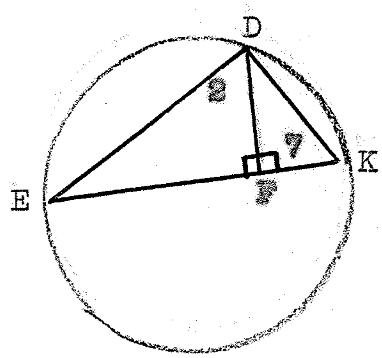
Proof Statements Reasons

Conc.

End of Examination for students taking Algebra and Geometry 1955 - 56.

Students taking Geometry only 1955-56 do the following question.

(Time for these students - hours)



Given: △ DEK inscribed in a circle with DF ⊥ EK, ∠2 = ∠7

Required: To prove EK passes through the centre of this circle.

Proof: (only) to be written on foolscap.

APPENDIX D

TABLE XVIII

ALGEBRA SCORES FOR THREE GROUPS OF STUDENTS

No. of Paper	A		B		C	
	Score	Rank	Score	Rank	Score	Rank
1	31	36	35	4½	33	1
2	38	7½	40	1	22	9½
3	34	23	32	11½	28½	3
4	35	16½	31½	13	26	5
5	38	7½	30	17½	24	6½
6	30	41	39	2	23	8
7	37	11	31	15	29	2
8	37	11	31	15	19	13
9	34½	19½	32	11½	17	16
10	36½	13	33½	7	20	12
11	31	36	25	30	24	6½
12	33½	27	25	30	12	21
13	40	3	29	20½	10	22
14	39	4	32½	9	27	4
15	35	16½	34	6	22	9½
16	22	60	25	30	21	11
17	37	11	36	3	14	18½
18	24	56½	24	34½	5	25
19	34	23	31	15	14	18½
20	42	1	25	30	13	20
21	27	47½	26	26	16	17
22	41	2	32½	9	18	14½
23	30	41	32½	9	18	14½
24	34	23	26	26	8	23½
25	33½	27	29½	19	8	23½
26	26	52	26	26	2	26
27	36	14	22	41		
28	28	45	23	38½		
29	28	45	28	22½		
30	35	16½	21	45		
31	35	16½	25	30		
32	33½	27	28	22½		
33	21	63	35	4½		
34	28	45	24½	33		
35	38	7½	16½	53		

TABLE XVIII Continued

No. of Paper	A Score	A Rank	B Score	B Rank	C Score	C Rank
36	34	23	17	51½		
37	19	67½	22	41		
38	26	52	15	55½		
39	38	7½	16	54		
40	38½	5	27	24		
41	34½	19½	19	48		
42	24	56½	21	45		
43	32	31	24	34½		
44	32½	29	23½	36½		
45	31	36	18	49½		
46	31	36	29	20½		
47	26	52	23½	36½		
48	25	58	21	45		
49	26	52	17	51½		
50	27	47½	23	38½		
51	32	31	21	45		
52	31	36	22	41		
53	32	31	30	17½		
54	21	63	11	58		
55	22	60	21	45		
56	21	63	15	55½		
57	26½	49	5	62		
58	34	23	14	57		
59	31	36	6	61		
60	31	36	10	59		
61	29½	43	18	49½		
62	20	65½	2	63		
63	16	71	6½	60		
64	30	41				
65	19	67½				
66	26	52				
67	25	55				
68	20	65½				
69	8	76				
70	13	72½				

TABLE XVIII Continued

No. of Paper	A		B		C	
	Score	Rank	Score	Rank	Score	Rank
71	10	75				
72	22	60				
73	12	74½				
74	17	69½				
75	13	72½				
76	17	69½				

APPENDIX E

TABLE XIX

GEOMETRY SCORES FOR THREE GROUPS OF STUDENTS

No. of Paper	A		B		C	
	Score	Rank	Score	Rank	Score	Rank
1	40	2	26	19½	18	10
2	32	16	20	37	28	1
3	36	7½	28	12	21½	6
4	34	11	28½	9	24	2
5	31	21½	29	7	20	8
6	39	3½	19	40	17	11½
7	32	16	27	16½	11	18
8	31	21½	27	16½	20	8
9	33½	12½	25	23½	20	8
10	31½	19	23½	26	17	11½
11	37	6	32	2	12	16½
12	33½	12½	31	3	22	5
13	26	39	27	16½	23	3½
14	27	36	22½	28	6	24
15	30	24½	22	31½	10	20
16	41	1	30	4½	10	20
17	26	39	18	44½	15	13
18	39	3½	30	4½	23	3½
19	29	28	22	31½	14	14
20	21	58	28	12	12	16½
21	35	9½	26	19½	8	22½
22	21	58	18	44½	5	26
23	32	16	18½	43	5	25
24	27	36	25	23½	13	15
25	27½	34	21½	35	8	22½
26	35	9½	5	23½	10	20
27	25	42	28	12		
28	32	16	27	16½		
29	31	21½	22	31½		
30	23	51	29	7		
31	23	51	25	23½		
32	24½	44	22	31½		
33	36	7½	15	49		
34	29	28	25½	21		
35	19	63	33½	1		

TABLE XIX--Continued

No. of Paper	A		B		C	
	Score	Rank	Score	Rank	Score	Rank
36	23	51	28	12		
37	33	5	23	27		
38	31	21½	29	7		
39	19	63	28	12		
40	17½	67	15	49		
41	21½	56	22	31½		
42	32	16	19	40		
43	24	46	15	49		
44	23½	48½	15½	47		
45	24	46	21	36		
46	24	46	9	59		
47	28	32	14½	51		
48	30	24½	17	46		
49	27	36	19	40		
50	25	42	12	56		
51	20	60	13	53		
52	21	58	12	56		
53	19	63	1	62½		
54	29	28	19	40		
55	28	32	7	60		
56	29	28	13	53		
57	23½	48½	22	31½		
58	16	69½	12	56		
59	19	63	19	40		
60	19	63	10	58		
61	16½	68	1	62½		
62	26	39	13	53		
63	29	28	3½	61		
64	15	72				
65	22	54				
66	15	72				
67	16	69½				
68	18	66				
69	28	32				
70	22	54				

TABLE XIX--Continued

No. of Paper	A		B		C	
	Score	Rank	Score	Rank	Score	Rank
71	25	42				
72	12	74				
73	22	54				
74	15	72				
75	10	75				
76	4	76				

APPENDIX F

TABLE XX

MATHEMATICS TOTAL SCORES FOR THREE GROUPS OF STUDENTS

No. of Paper	A		B		C	
	Score	Rank	Score	Rank	Score	Rank
1	71	1	61	1	51	1
2	70	2½	60	3	50	3
3	70	2½	60	3	50	3
4	69	5½	60	3	50	3
5	69	5½	59	5	44	5
6	69	5½	58	7	40	6½
7	69	5½	58	7	40	6½
8	68	9½	58	7	39	8
9	68	9½	57	10	37	9½
10	68	9½	57	10	37	9½
11	68	9½	57	10	36	11
12	67	12	56	12½	34	12
13	66	13½	56	12½	33	13½
14	66	13½	55	15	33	13½
15	65	15	55	15	32	15
16	63	18	55	15	31	16
17	63	18	54	17½	29	17
18	63	18	54	17½	28	18½
19	63	18	53	19½	28	18½
20	63	18	53	19½	25	20
21	62	22	52	21	24	21
22	62	22	51	24	23	22½
23	62	22	51	24	23	22½
24	61	25½	51	24	21	24
25	61	25½	51	24	16	25
26	61	25½	51	24	12	26
27	61	25½	50	31		
28	60	28	50	31		
29	59	29	50	31		
30	58	31	50	31		
31	58	31	50	31		
32	58	31	50	31		
33	57	36	50	31		
34	57	36	50	31		
35	57	36	50	31		

TABLE XX--Continued

No. of Paper	A		B		C	
	Score	Rank	Score	Rank	Score	Rank
36	57	36	45	36½		
37	57	36	45	36½		
38	57	36	44	38½		
39	57	36	44	38½		
40	56	42	42	40		
41	56	42	41	41		
42	56	42	40	42		
43	56	42	39	44		
44	56	42	39	44		
45	55	45½	39	44		
46	55	45½	38	47		
47	54	47	38	47		
48	53	48½	38	47		
49	53	48½	36	49		
50	52	51	35	50		
51	52	51	34	51½		
52	52	51	34	51½		
53	51	53	31	53		
54	50	57	30	54		
55	50	57	28	55½		
56	50	57	28	55½		
57	50	57	27	57		
58	50	57	26	58		
59	50	57	25	59		
60	50	57	20	60		
61	46	61½	19	61		
62	46	61½	15	62		
63	45	63½	10	63		
64	45	63½				
65	41	66				
66	41	66				
67	41	66				
68	38	68				
69	36	69				
70	35	70½				

No. of Paper	A		B		C	
	Score	Rank	Score	Rank	Score	Rank
71	35	70½				
72	34	72½				
73	34	72½				
74	32	74				
75	23	75				
76	21	76				

Paper No. 10—June, 1956

GRADE XI MATHEMATICS—Continued

(GEOMETRY AND ALGEBRA)

PART B—GEOMETRY

All questions of Part B are to be done on this leaflet. After putting the answers to the questions in the spaces provided, candidates will detach Part B and fasten it securely inside the answer booklet.

Values

1. ABCD is a rectangle with an area of 36 square units.

   DF = 6 units.

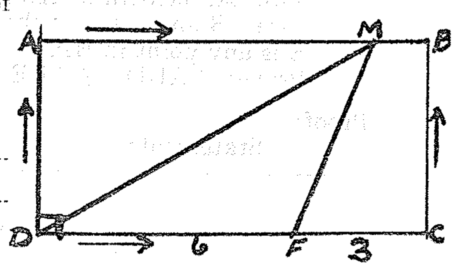
   FC = 3 units.

1 (a) BC = ...

....

2 (b) Area of $\triangle$DFM = ...

....

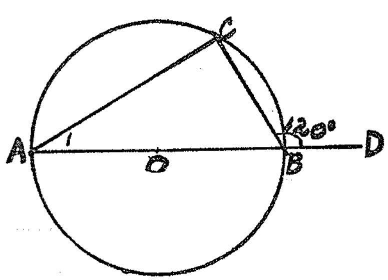


2 2. AB is a diameter produced to D.

$\angle$CBD = 120°.

$\angle$CAB = ...

....



(Over)

APPENDIX G

Paper No. 10

DEPARTMENT OF EDUCATION
MANITOBA

HIGH SCHOOL EXAMINATION BOARD OF MANITOBA
(Representing Department of Education and University of Manitoba)

EXAMINATIONS, JUNE, 1956

GRADE XI MATHEMATICS (GEOMETRY AND ALGEBRA)

Friday, June 22nd, 9.00 to 12.00 a.m.

Examiners: O. T. Anderson, C. L. Kerr, B. Noonan, W. H. W. Walker

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IMPORTANT—All rough work must be done in the answer booklet.

PART A—ALGEBRA

Values

1 1. (a) How much greater is $7a+3b-4c$ than $3a-2b+5c$?

1 (b) Evaluate: a^3-2a^2+1 when $a=-2$.

1 (c) Simplify: $x^{2n} \div x^{n-1}$.

1 (d) What value of k will make x^2-5x+k a perfect square trinomial?

1 (e) Simplify: $\sqrt{8}+2\sqrt{32}-3\sqrt{2}$.

1 (f) Solve for x: $k = \dfrac{bx+c}{x}$.

2. Factor fully:

2 (a) $ax-by+ay-bx$.

2 (b) $x^3 - \dfrac{b^3}{8}$.

3 (c) $x^3-6x^2+11x-6$.

(Over)

4 3. Simplify: $\frac{x+3}{x-3} - \frac{x-3}{x+3} + \frac{36}{9-x^2}$.

4. Solve the following equations:

3 (a) Solve for x:

$$\frac{2}{3}(x-3) - \frac{3}{2}(2x-5) = 1 - \frac{1}{6}(x-1).$$

4 (b) Solve for x and y: $\frac{x}{2} - \frac{y}{10} = \frac{3}{2}$.

$$\frac{x}{3} + \frac{y}{5} = 9.$$

4 (c) Solve for x: $\frac{2}{x-4} - \frac{1}{x-2} = 2.$

5. Simplify and express with positive indices:

3 (a) $\left(\frac{9a^{-2}}{25b^4}\right)^{-\frac{1}{2}}$

2 (b) $4x^0 + (2^2)^3$.

3 6. Rationalize the denominator and simplify:

$$\frac{2\sqrt{3}-\sqrt{2}}{2\sqrt{3}+\sqrt{2}}.$$

3 7. Find the square root of: $7a^2-6a+a^4-2a^3+9$.

4 8. Solve graphically: $x^2-6x+8=0$.

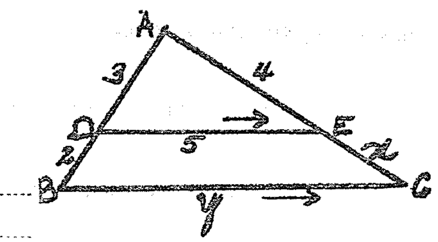
9. Solve algebraically:

3 (a) If the base of a triangle exceeds its altitude by 3 inches, and its area is 27 square inches, what is the altitude of the triangle?

4 (b) A man travelled 15 miles at a constant rate. If he had travelled 2 miles per hour faster, he would have gone the same distance in 2 hours less time. What was his original rate of travel?

(Over)

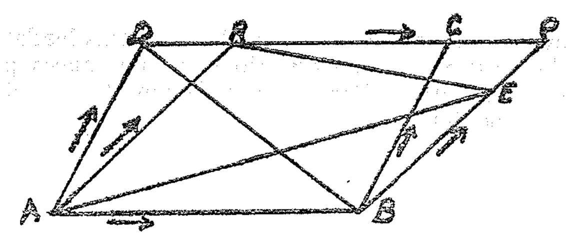
3. DE//BC
AD = 3 units
DB = 2 units
AE = 4 units
DE = 5 units



2 (a) EC = ...

2 (b) BC = ...

5 4. Given:



Two parallelograms ABCD and ABPR on the same base AB and between the same parallels AB and DP. E is any point in BP.
Prove: △ABD = △ABE + △EPR.

Proof:

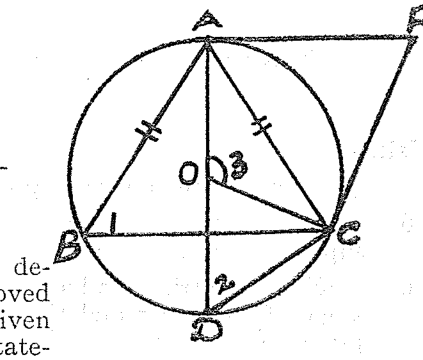
Statements	Reasons

5. ABC is a triangle inscribed in a circle with centre O.

AB = AC.

AD is a diameter.

PA and PC are tangents.



The following true deductions may be proved from the information given above. Beside each statement, give the reason or reasons why you consider it true. (You may consider any previous part true and use it in any part following even though you did not ge the reason for it correct.)

	Statements	Reasons
1	(a) PA = PC	....
1	(b) Arc AB = arc AC	....
1	(c) ∠PAC = ∠1	....
1	(d) ∠1 = ∠2	....
1	(e) AC ⊥ CD	....
1	(f) ∠3 + ∠P = 180°	....

(Over

PART C—GEOMETRY

This part to be done in the answer booklets.

Values

6. Prove the following propositions:

6 (*a*) The opposite angles of a cyclic quadrilateral are supplementary.

6 (*b*) If two triangles have one angle of one equal to one angle of the other and the sides about the equal angles proportionals then the triangles are similar.

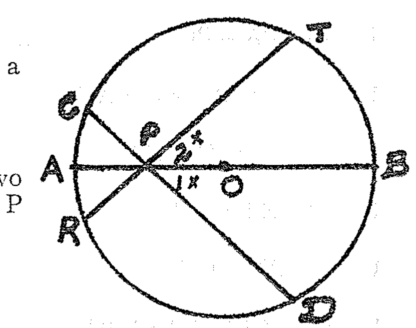
5 7. On the same base as a given triangle construct a triangle, with an angle of 60°, equal in area to the given triangle. (Use compasses and a straight edge or ruler only. Show all construction lines, describe the construction but omit the proof.)

6 8. AB is a diameter of a circle with centre O.

P is a point on AB.

CD and RT are two chords drawn through P so that $\angle 1 = \angle 2$.

Prove that: CD = RT.



7 9. G is the mid point of a line MN. PE is parallel to MN. PN is joined. The straight line EFGH cuts PN at F and meets PM produced at H.

Prove: $\frac{EF}{FG} = \frac{EH}{HG}$.

Total 100

APPENDIX H

GRADE XI MATHEMATICS
(GEOMETRY AND ALGEBRA)

Paper No. 12

DEPARTMENT OF EDUCATION
MANITOBA

HIGH SCHOOL EXAMINATION BOARD OF MANITOBA
(Representing Department of Education and University of Manitoba)

EXAMINATIONS, JUNE, 1955

GRADE XI MATHEMATICS
(GEOMETRY AND ALGEBRA)

Thursday, June 23rd, 9.00 to 12.00 a.m.

Examiners: O. T. Anderson, Miss C. E. Carson, F. A. Hodgkinson, B. Noonan.

IMPORTANT—All rough work must be done in the answer booklet.

PART A—ALGEBRA

Values

1 1. (*a*) Subtract $a-4b+5$ from $3a-b-1$.

1 (*b*) Simplify: $(-2x^3y^2)^4$.

1 (*c*) Divide: $(8a^3b^3-9a^2b^2+ab)$ by $(-ab)$.

1 (*d*) If $x=3$ and $y=4$, find the numerical value of:

$$8-2(5x^2-y^2).$$

1 (*e*) What number must be added to x^2-3x to make it a perfect square trinomial?

2. Factor fully:

3 (*a*) $8x^4-27x$.

3 (*b*) x^3-7x+6.

3 (*c*) x^2-4y^2+9-6x.

4 3. Simplify: $\dfrac{1}{x^2+3xy} + \dfrac{2}{9y^2-x^2} - \dfrac{2}{x^2-6xy+9y^2}$

4. Solve for x:

3 (a) $\dfrac{x}{m} + n = \dfrac{x}{n} + m.$

3 (b) $2x^2 = 4x + 7.$

4 (c) $\dfrac{15}{x} - \dfrac{12}{x+1} = \dfrac{4}{x-1}.$

5. Simplify and express with positive indices:

3 (a) $\left(\dfrac{36a^{-4}}{25b^{-6}c^{0}}\right)^{-\frac{1}{2}}$

3 (b) $\dfrac{2^{n+1}}{(2^{n})^{n-1}} \div \dfrac{4^{n+1}}{(2^{n-1})^{n+1}}.$

6. Simplify:

2 (a) $(\sqrt{8}-\sqrt{6})^2.$

3 (b) Express with rational denominator: $\dfrac{4-3\sqrt{2}}{6+\sqrt{2}}.$

3 7. Find the square root of:
$16m^6 - 40m^4 - 8m^3 + 25m^2 + 10m + 1.$

3 8. (a) Solve graphically: $x^2 - 4x = 0.$

1 (b) Using the graph of (a) solve: $x^2 - 4x = 5.$

4 9. Two men starting walking at the same time fro the same place, one of them going due south and the othe due west. At the end of 5 hours they are 25 miles apar and one of them has gone 5 miles more than the othe Find the rates of walking.

PART C—GEOMETRY

This part is to be done in the answer booklets.

15. Prove the following propositions:

6 (*a*) If two triangles are on the same base and between the same parallels, they are equal in area.

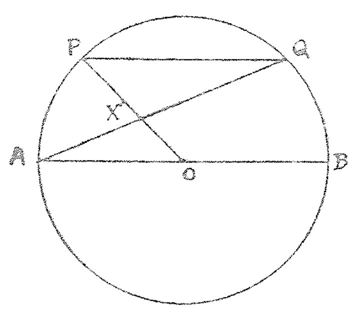
(*b*) If a triangle contains a right angle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.

16. From a point outside a given circle draw two tangents to the circle.

(Show clearly all construction marks, describe construction but OMIT proof.)

17. AB is the diameter of a circle with centre O. PQ is a chord parallel to AB. PO and AQ intersect at X.

Prove $\angle PXA = 3\angle PQA$.



7 18. C is any point in the diameter AB of a circle with centre O. CD is drawn at right angles to AB meeting the circumference in D. DO is joined and CE is perpendicular to OD.

Prove $\frac{AO}{DC} = \frac{DC}{DE}$.

(Over)

Paper No. 12—June, 1955

GRADE XI MATHEMATICS—Continued

(GEOMETRY AND ALGEBRA)

PART B—GEOMETRY

After putting the answers to the questions in the spaces provided, candidates will detach Part B and fasten it inside the answer booklet.

3 10. ABCD is a rectangle.

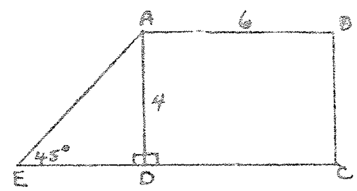
$\angle E = 45°$.

AB = 6 units.

AD = 4 units.

Area of figure ABCE = ...

....

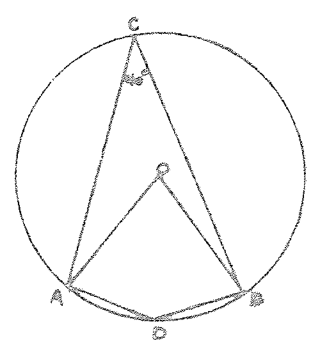


2 + 2 11. O is the centre of the circle. A, B, C and D are points on the circumference.

$\angle ACB = 40°$.

$\angle AOB =$ ....

$\angle ADB =$ ....



3 12. ∠1 = ∠2.

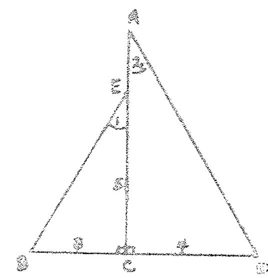
AC is perpendicular to BD.

BC = 3 units.

CD = 4 units.

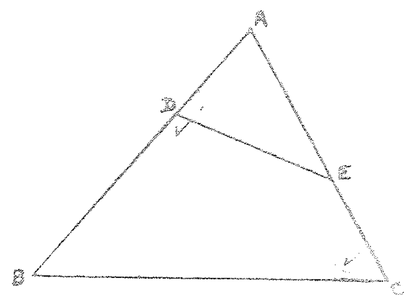
CE = 5 units.

AE = ...



4 13. Given: in triangle ABC, ∠BDE is the supplement of ∠C.

Required: to prove that $\frac{AE}{AB} = \frac{DE}{BC}$.

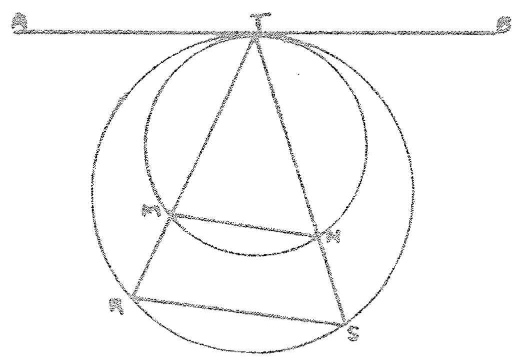


Proof:

Statements	Reasons

4 14. Given: AB is tangent to both circles at T. Chords TR and TS of the large circle cut the smaller circle at M and N respectively.

Required: To prove MN||RS.



Proof:

Statements	Reasons