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THE NATURE OF INFERENCE

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A Thesis

Presented to

The Faculty of the Department of Philosophy

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In Partial Fulfillment

of the Requirements for the Degree

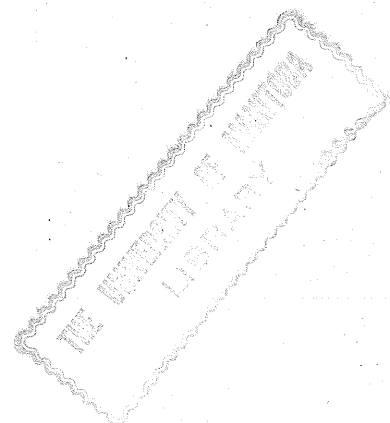
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by

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## ANALYSIS OF CONTENTS

### Chapter 1

#### Introduction

In the introduction to our investigation of the nature of inference we set forth the problem by first asking the question: What is inference? The question is asked from a naive point of view and the ready answers that are forthcoming are stated so that we may bring out the complexity of the subject with which we are dealing. We discover that if we are to find a satisfactory answer to our question it will be necessary to investigate the various logical theories upon which the several inferential theories are based. As we embark upon our investigation we soon discover that, in the main, traditional logic is discredited, and that its importance to-day lies more in historical interest than in any hope that such a logic can yield fruitful results for the investigator who desires to discover the true nature of inference.

We find that, generally speaking, modern logical theories can be grouped under three main heads, i.e., Idealist, Realist and Pragmatist. Also, that while there are shades of differences amongst the various writers and schools within the main groupings, yet, for our purpose, it will be

permissible to give a satisfactory account of logical theory and the nature of inference by discussing the main tendencies and features of each school as these tendencies appear to be a common property of the particular school investigated. In the body of the thesis we shall allow each main school of thought to speak for itself, and in conclusion we shall state our reasons for accepting one of them as most satisfactory.

## Chapter 11

### Idealism

We begin our investigation of the logical theories of the three main schools with a discussion of idealism. We point out that, for idealism, logic cannot be understood apart from metaphysics, ethics, aesthetics, epistemology, etc. Logic is shown to be a system within the whole of Reality, and as such is a difference within an identity of the whole. The attempt to abstract is shown to be erroneous. Next, we show how, for the idealist, the study of logic is the study of the activity of thought in constructing its world. Knowledge is shown to be a judgment, an affirmation. We illustrate the development of knowledge as a mental construct with reference to B. Bosanquet.

We turn to a consideration of 'naming' and point out



how form and content go hand in hand. We show, according to F. H. Bradley, how the proposition differs fundamentally from the judgment, and how he is interested in inference rather than in implication. It is pointed out that our understanding of inference will depend upon our understanding of the constituents of inference, i.e., idea and judgment.

In our discussion of idea we follow the main lines of Bradley's thought. An idea, for judgment, is shown to be a universal meaning, not occasional imagery, and still less the whole psychical event. For logic, ideas are symbols and a symbol is that which has existence, content and meaning. Ideas are not psychological phenomena amenable to scientific investigation. They are not in time, as is a mental image, i.e., they are logical, not psychological. It is shown how Bosanquet agrees with this general theory.

In our discussion we define judgment according to Bradley, as the act which refers an ideal content (recognized as such) to a reality beyond the act. Judgment, however complex, is shown to be a single idea and on account of this the distinction between judgment and idea disappears. Idealism cannot conceive of an idea that is not at the same time a judgment. Subject and predicate in a judgment are not two ideas. It is pointed out that the distinctions of traditional logicians belong more to the mechanisms of language than to the vital processes of thought. In judgment S and P are

differences within an identity, in the proposition they are isolated parts of an extended whole. The mechanisms of language are in time. Judgment is not so conceived to be in time. We pass on to consider that since judgment within the system of judgment is a difference within an identity it follows that such distinctions as those between universal and particular judgments, categorical and hypothetical judgments, qualitative and quantitative judgments are not absolute in any sense. Indeed, we show that judgment is really inference.

When we turn to a discussion of inference, we show how idea is implicit in judgment and how judgment is implicit in inference. We point out what is meant by explicit inference. We accept Bosanquet's definition of inference, i.e., that inference is the mediate reference of an ideal content to reality and then turn to Bradley's account of inference as the ideal self-development of an object. The conclusion of inference represents a development of the object with which we started. There is no suggestion of an inferential leap. There is the process of self-development. Inference makes explicit in its conclusion the differences that were implicit at the beginning. We discuss novelty in inference and show how novelty is an accident of inference and not an essential. Next we discuss, according to Bosanquet, the necessity of a concrete universal that is operative for

inference.

## Chapter lll

### Realism

This discussion begins with an examination of the foundations of symbolic logic in the work of George Boole. We seek to show the reasonableness of expressing the processes of the mind in the symbolic language of a calculus. The logical background of the realist symbolic logician is then put forward. We use Russell's argument for the ontological status of logical essences to show in what sense the realist understands the nature of the entities with which he deals.

Passing on to a definition of inference, we find that it is both a process and a conclusion, and that it must be conceived as psychological, rather than logical. We discover that 'implication' is more important to the realist than 'inference.' The question of validity and invalidity is raised, and we seek to show the nature of validity and invalidity for realism. Validity is seen to rest upon the relation of implication. If this relation holds, our inferential processes are valid. If it fails to hold, they are invalid.

After this discussion, we proceed to show how the

symbolic logician translates the traditional inferences into the symbolic language of a calculus. We first deal with immediate inferences, and then pass on to a discussion of the syllogism interpreted in a calculus of classes with a description of the use of the Venn Diagram. We next show the extension of the calculus of classes to non-syllogistic reasoning. From the calculus of classes we pass on to a discussion of the development of a calculus of propositions and show how the class calculus is reinterpreted as a calculus of propositions.

We continue our investigation with a discussion of the deductive nature of inference for the symbolic logician, and, in passing, refer to the question of mathematical induction. We conclude this section of our discussion by restating the main points as they bear upon the nature of inference for realism.

## Chapter IV

### Pragmatism

We begin with a discussion of the general background of pragmatic logic. Logic is next discussed, according to J. Dewey, as the method of inquiry. It is found that pragmatic logic is essentially of an empirical nature. In pass-

ing, we refer to C. S. Peirce's use of symbols.

The relation of psychology to logic is next discussed with reference to F. C. S. Schiller and John Dewey. We find that Schiller and Dewey are at variance on this point. The question of validity and invalidity is then raised and it is shown that this question rests for its solution upon the theory of 'working.'

The nature of thinking is the next subject of inquiry and we discover, in contrast to idealism, that pragmatism sets very definite limits to human thinking. This matter leads to a discussion of the nature of meaning and it is found that meaning is a function within inquiry. The importance of relevance is next discussed. This concept, as brought forward by F. C. S. Schiller, seems to distinguish pragmatic logic from both realist and idealist logic respectively. Truth, for pragmatism, is next investigated, and we find that Dewey follows Peirce and puts forward a different criterion from that of Schiller.

Leaving the general background, we begin an investigation of the specific constituents of pragmatic inference. First we deal with the nature of 'idea' and point out Dewey's distinction between 'idea' and 'suggestion.' Ideas are data for logic, suggestions are not. This discussion leads us to define more precisely Dewey's understanding of the nature of reasoning.

Judgment is next discussed from the point of view of both Schiller and Dewey. The truth of our judgments is found to be psychological, according to Schiller, while Dewey follows the criterion of Peirce.

Having discussed the constituents of inference, we turn to an examination of inference itself. We find that Schiller's view of inference is essentially psychological and personal, and its value is found in its working. Dewey's theory of inference is discussed and we close the chapter by drawing out the distinction between inference and reasoning.

## Chapter V

## Conclusion

The purpose of the concluding chapter is to show, as clearly as we are able, that both realist and pragmatist theories of inference break down at vital points and that the only tenable theory of the nature of inference is that of idealism.

We begin with a discussion and criticism of realist theory and we soon turn our attention to a criticism of the logical essence theory as put forward by Bertrand Russell. We seek to show that logical essences do not exist apart from mind. We show that inference needs the operation of a

concrete universal.

We draw attention to the informality of formal logic and examine and criticize its truth-claim. Next, we examine and criticize the realist correspondence theory. We contrast the correspondence theory with the coherence theory, and seek to show that the coherence theory is more tenable than the correspondence theory. We next examine and criticize what the realist claims he knows, and on the basis of this discussion turn to a criticism of his grounds for valid conclusions. We seek to show that there is no infallibility in realist truth-claim.

In contrast to realism, we put forward the idealist criterion of truth. Our arguments are based mainly on F. H. Bradley and B. Bosanquet and we seek to show that, for idealism, the criterion is one of reality. We discuss the hypothetical nature of all judgments and find that the claim of realism to infallibility is an exploded myth.

After our criticism of the realist theory of inference we turn to a criticism of pragmatic theory. We seek to show how Dewey violates the canon of pragmatism in his discussion of the nature of inference and point out the inconsistency of his definition of truth with his definition of inference. We criticize Dewey's subordination of truth to inquiry and Schiller's subordination of truth to relevance. The coherence theory is set over against the theories of Schiller and

Dewey and the thesis ends by pointing out that the weaknesses of both realist and pragmatist theories of inference are due to the faulty logic upon which they are based.



## CHAPTER I

### INTRODUCTION

A simple question demands a simple answer, to a mind unaccustomed to logical and metaphysical subtlety. If we naively ask: What is inference? We expect an answer that is clear, free from ambiguity, and one that satisfies the common-sense canon of explanation; i.e., this is a case of that. According to the Oxford Dictionary, "to infer is to deduce or conclude." This answer seems to satisfy the common-sense canon, in that inference is described as both a process and a result. To infer is to deduce, i.e., inference is a process of deduction. To infer is to conclude, i.e., the end of the deductive process is completed: a conclusion is reached, and inference is a result.

The foregoing explanation seems to be clear enough, and it would, apparently, satisfy the casual inquirer. If, however, the inquirer determined to govern his future inferences on the basis of this simple explanation, and submit his inferences to examination, he would find, all too soon, that the explanation was quite inadequate to account for seeming inferences that were neither deductive in process, nor conclusions arrived at by deduction. For instance, if he were a medical doctor investigating an epidemic of scarlet fever,

he would arrive at certain conclusions on an inductive basis, rather than on a deductive one. Would such processes and conclusions be inferences? Suppose, however, he submitted to this simple explanation; then, such an inference as, "All tables have two legs, our janitor is a table, therefore our janitor has two legs" would be an inference which was both deductive in process, and a conclusion arrived at by deduction. This inference satisfies the dictionary explanation, but it is doubtful whether our inquirer would be convinced that it satisfied common-sense.

Let us suppose that our inquirer has been stimulated by the apparently unsatisfactory answer to his first inquiry into the nature of inference, to inquire further, to see if he can find a more satisfactory explanation of this process and result which seems to be one of the most normal phenomena of human intelligence. It would appear, on the most casual investigation, that human beings have been arriving at conclusions as far back as the history of human thought will carry us. The primitive savage, on an animistic level of thought, concluded that trees and rocks, rivers and lakes etc., possessed spirits. The early thought processes of more developed peoples, on the level of anthropomorphic thinking, arrived at the conclusion that the Gods were like men, both in character and form. Are such conclusions inferences? One might expect to hear the reply that, they are not, but that

they are beliefs. But what are beliefs? Are they not conclusions arrived at by certain mental processes? The primitive savage concludes that the tree possesses a spirit because such a conclusion is the ground of explanation for many things that would otherwise be inexplicable to him. How does he arrive at such a conclusion? Is it by induction or deduction? In the modern world he would find many champions for both methods. But, say you, the savage certainly does not use any valid inductive or deductive method. He merely jumps to a conclusion. Our inquirer, on examination, will find that, even this 'jump' is inference, according to some modern schools of thought. He will find such expressions as, "the inferential leap," in current logical treatises.

What is inference? Leaving the Oxford Dictionary, let us join the inquirer and turn to the Dictionary of Philosophy (1) to see if we can find a more satisfactory answer to our question. According to the Dictionary of Philosophy, inference is stated to be: "The process of reasoning whereby starting from one or more propositions accepted as true, the mind passes to another proposition or propositions whose truth is believed to be involved in the truth of the former. Inference is a psychological process connecting propositions asserted to be true and is to be distinguished from implication, the log-

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1 Dagobert D. Runes. Ed. The Dictionary of Philosophy.

ical relation which holds between the same propositions when the inference is valid. An inference is valid when a genuine implicative relation holds between the propositions, invalid when there is no such implicative relation. Inference is deductive or inductive according as the underlying logic is deductive."

Let us examine this statement to see if it yields a satisfactory answer as to the nature of inference. The very first thing we notice about it is that inference seems to be merely a process. This account seems to be inadequate, since when I see spots of rain on the sidewalk, I infer, 'It has rained.' My inference plainly is: 'It has rained.' 'It has rained' is a conclusion and an inference. Common usage, as we learn from the Oxford Dictionary, would agree with this assertion. How is it that the Dictionary of Philosophy disregards common usage and confines inference to a process only? We begin to suspect that Ledger Wood, who defines inference for us in this instance does so in the interests of some particular school of thought. He maintains that inference is a psychological process connecting propositions. It is difficult to determine what is meant by this.

If we take the term 'psychological' in the sense in which a psychologist, i.e., an empirical scientist, would take it, then a psychological process is a process that is amenable to scientific observation, i.e., empirical observa-

tion. Let us examine an inference to see if we really observe the process. We begin with the proposition: 'There are spots of water on the sidewalk.' Next, I infer: 'It has rained.' I seem to observe two things. First, I observe the propositional form and matter of my judgment: 'There are spots of water on the sidewalk.' Secondly, I observe the propositional form and matter of my conclusion: 'It has rained.' I do not seem to observe any process whatsoever. Have I taken an inferential leap? If so, is a leap a process? If it is, why call it a leap? A leap, in distinction from a process, would seem to suggest an activity that was not observable. The plain empirical fact of the matter suggests that it is not so observable.

Shall we then, be driven to maintain that the act of inference is a leap? It may be, but there is apparently no ground for such an assertion except ignorance of any other possible empirical data to suggest otherwise. An argument from ignorance will not induce us to accept the alleged truth of the assertion that, inference is a leap.

Thus far, it seems that we have found little satisfaction in our endeavour to discover the nature of inference. At this point we begin to suspect that the nature of inference is involved in logical theory, and that no satisfactory answer can be found as to its nature until we have investigated logical theory as it bears upon the nature of infer-

ence.

We shall suppose that we have made rapid advance at this point from the time of our first naive question: What is inference? We shall assume that we have read fairly widely amongst the contemporary logical schools, and have come to the conclusion that each has a reasonable case for the position he maintains. We have discovered that, in the main, traditional logic is discredited, and that its importance to-day lies more in historical interest than in any hope that such logic can yield any fruitful results for the investigator who desires to discover the true nature of inference. We have found that, generally speaking, modern logical theories can be grouped under three main heads, i.e., Idealist, Realist and Pragmatist. Also, that while there are shades of differences amongst various writers and schools within the main groupings, yet, we feel that, for our purpose, it will be possible to give a satisfactory account of logical theory and the nature of inference by discussing the main tendencies and features of each school as these tendencies appear to be common property of the particular school investigated. We shall let each main school of thought speak for itself and then state our reasons for accepting one of them as most satisfactory. (2)

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<sup>2</sup> Cf E. A. Burtt: "The Problems of Philosophic Method," Philosophical Review, 1946, pp. 505-533.

Perhaps we should put forward a measure of justification for our assertion that it is possible to give a satisfactory account of logical theory by confining our attention to the three general schools of thought. It may seem to some that a satisfactory account of logical theory could only be given after a detailed study of the many different emphases that one discovers within the streams of the main tendencies. We feel that such an undertaking, while very informative, would not yield the result at which we aim; i.e., a general theory of inference, that seems, in the main, to be satisfactory, yet allows for the possibility of slightly differing viewpoints within the general scheme. Our aim is to seek to understand the nature of inference, and we think that it is impossible to understand anything if one is immersed in a welter of detail. We do not believe that detail is unimportant, but it is maintained that, only when we view a whole is it possible to place the particular in its proper context. We shall try to understand the nature of inference from the 'top' rather than from the 'bottom.' This 'bird's eye view,' we believe, will show us, generally speaking, three chief systems of thought which result in three different interpretations of the nature of inference.

## CHAPTER II

### IDEALISM

For the idealist, logic cannot be understood or appreciated apart from metaphysics, ethics, esthetics, epistemology, etc. The idealist delights to see things whole, and to find within that whole a series of interrelating systems. Logic is a system within the whole of Reality. The idealist does not believe that logic can be abstracted from the whole and at the same time retain any significance. Logic is a difference within the identity of the whole. For purposes of thought one may attempt to abstract a system, but the idealist believes that a pure abstraction is impossible. The attempt to abstract distorts the subject-matter into something which, before, it was not. It seems, to the idealist, that, at the very moment we attempt to abstract a system, that system takes upon itself a concreteness as a different system within the whole. Logic, so it would appear to the idealist, can only be truly considered as a system which implies other systems. It can only be identified as logic in its difference as a system.

The idealist abhors abstractions. Abstractions would appear to be meaningless. The idealist's objection to abstraction is to be found in the meaning given to abstraction



by the formal logician. To the idealist, as, indeed, to the pragmatist, the attempt of the formal logician to develop an abstract system is futile. Logic is not an abstract system. It is a system within the whole, i.e., Reality. Outside the whole, i.e., in abstraction, it cannot be said to exist at all.

The study of logic, for the idealist, is the study of the activity of thought in constructing its world. Indeed, the world, for knowledge, is a logical construct. Knowledge is a judgment, an affirmation. The idealist does not overlook the fact that some judgments are negative, but even the negative judgments have a positive ground, and in that sense assert positively, if not affirmatively. It would seem to be necessary to point this out, since a bare negation could hardly be supposed to be constructive in any sense. Knowledge, for the idealist, involves the ideas of truth and meaning. While truth and meaning are not actually separable, yet for our immediate purpose we may, quite legitimately, consider the latter in separation from the former.

With the dawn of consciousness, whether in the race or in the individual, the world of persons and of things is gradually constructed into a world of meanings. It would seem to be true to say that, to give a name was the first step in the construction of the world of meaning. In the act of naming we see the first signs of the developing construct of judgment. F. H. Bradley in his Principles of Logic,

points out that, a single word is not really isolated, it always implies a sentence. Indeed, we may say that, a single word implies the whole world of meaning, and the world of meaning, in its turn, implies the whole, i.e., Reality.

B. Bosanquet in his Logic, illustrates the development of knowledge on the analogy of the morphology of plants, and in what immediately follows we shall, in our own words, tell Bosanquet's story. Thallophytes form the lowest great division of the plant kingdom. A thallus is a plant body in which there is little or no differentiation of vegetative organs. Among the group of thallophytes the beginnings of structures are found that are observed to become modified in various ways in higher groups. (1) While this analogy holds to some extent, we must bear in mind that, in the case of the morphology of knowledge we must not exclude the physiology of thought, i.e., both the external form and the internal activity must be considered. When we name anything, we give form to a meaning.

Naming may be considered as the content of the vital process of mind. The two cannot be separated. Let us illustrate. A mere word either written or spoken is a non-entity, unless it has meaning. It would seem to be impossible, by psychological tests, to produce from any conscious being a

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1 Cf. J. M. C. Coulter. A Textbook of Botany.

truly nonsensient syllable. (2) Words without meaning cannot possibly enter into the realm of knowledge as a construct. Form and content must go hand in hand. This position implies that grammatical construction bears no necessary relation to logical construction. The form alone is not knowledge. Here we see a fundamental cleavage between idealist and realist logicians. Knowledge is a judgment, and a judgment is an activity that expresses itself in certain forms. These forms are determined for judgment by its own proper interests and purpose. This fact, however, does not preclude the possibility of the application of these forms to all sorts of partial interests. To the idealist, this consideration would seem to suggest that the various forms of the syllogism, and the algebra of symbolic logic, are, in essence, really external to the proper development of the thought construct.

Knowledge, as a construct, does not necessarily suggest that the propositional form is in itself the construct. The enunciative sentence - the unit of language which represents a judgment - is called a proposition. It would be absurd to imagine that the ideal of intelligence lies in the direction of a severance of thought from words. Yet, the spoken or written proposition differs fundamentally from

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<sup>2</sup> Cf. R. C. Lodge. The Questioning Mind. p. 38.

the judgment. Bradley (3) suggests that, since judgment maintains itself as an inference, this is the test that has been held to distinguish a judgment from a proposition. The proposition is the actual spoken or written enunciative sentence; while the judgment is the intellectual act which depends in various degrees upon words or other symbols whether, heard, read or remembered. It is the judgment itself that is the true construct of knowledge.

We may, with interest, follow the development of our knowledge as it expresses itself in the various forms of judgment, but we must never make the mistake of believing that these forms, apart from the content, express the true form of knowledge. There are no jumps in this development, and the various steps of judgment, from the judgment of quality to the disjunctive judgment, are more for the purpose of our convenience in understanding the process of thought than truly representative of the process itself.

Knowledge is a construct, but that construct, as a judgment, is an ever expanding system within the larger system of Reality. The world comes to be known by us as we construct that world as a meaningful world in judgment. It

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<sup>3</sup> Principles of Logic. p. 404. Also, it is to be noted that W. E. Johnson, Logic, Pt. 1, p. 2, defines 'proposition' in such a way as to include Bradley's judgment. He states: "It would thus be more material to speak of passing judgment upon a proposition proposed in thought, than to identify judgment as such with the proposition."

would seem that we cannot come to know our world by a mere series of presentations in perception. The world can only be known to us through the activity of a universal. The universals that we use in judgment form the nexus that binds into a systematic whole what would otherwise be chaos.

Knowledge, as a construct, is judgment that expresses the unity of different parts in a whole, or of differences in an identity. This is the real meaning of 'construction' in knowledge.

With this general background of the nature and process of thought, as it bears upon logical considerations, the idealist seeks to show what is the true nature of inference. Perhaps the best spokesman on behalf of the idealist position is F. H. Bradley, who expresses himself in The Principles of Logic. We shall, in the main, follow his arguments as we seek to put forward what the idealist believes to be the nature of inference. We shall, however, draw from other sources where it seems necessary to press some particular point.

Bradley divides his logical works into two main sections. The first volume of The Principles of Logic deals with judgment, the second with inference. (4) He does not

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<sup>4</sup> It is to be noted that Bradley is interested in inference, rather than in implication. This interest suggests a great point of difference between the idealist logician and, particularly, the mathematical logician. The term 'implication' does not make its appearance in The Principles of Logic until we arrive at the Terminal Essays.

attempt to treat logic as a formal discipline devoid of ontological significance; rather, he thinks of logic as having some intimate relation to the nature of the real.

The basic issue in The Principles of Logic is the nature of inference. Bradley deals with his subject from two main avenues of approach, i.e., he treats of the elements of inference; ideas and judgments: and of its operations.

To understand what an idealist means by inference, it is necessary, first of all, to understand what he means by an idea. When we have arrived at an understanding of what an idea is, we shall then be in a position to appreciate his conception of judgment. After arriving at a position where we know what a judgment is, we can then discuss the nature of inference.

(a) Idea.

Bradley maintains that there is "no judgment until we use ideas as ideas," (5) and he suggests that, "The idea in judgment is the universal meaning; it is not ever the occasional imagery, and still less can it be the whole psychological event." (6) This latter position distinguishes Bradley, and idealism generally, from such psychological logicians as F. C. S. Schiller. Bradley suggests that, for logic, ideas are symbols. But what are symbols? Apparently, a symbol is

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<sup>5</sup> The Principles of Logic. p. 2.

<sup>6</sup> Ibid. p. 7.

that which has three sides: i.e., existence, content and meaning. He explains the matter in this way. (7) Since there are two sides in all that is; i.e., we perceive 'that' it is and 'what' it is: - familiar to readers of Appearance and Reality - so, a symbol is endowed with a third side over and above the 'that' (existence), and the 'what' (content). This third side is 'meaning'. He suggests that a flower exists and has qualities, but "the flower itself cannot be what it means." (8) A sign or symbol "is any fact that has meaning, and meaning consists of a part of the content (original or acquired), cut off by the mind, and considered apart from the existence of the sign." (9) For logic, then, all ideas are signs. There must be no confusion, however, of idea with mental image. A mental image may be said to have existence and content, i.e., be a fact; but it is given. An idea, on the other hand, "is neither given nor presented, but is taken." (10) Bradley refuses to allow that ideas, as existences, function in judgment. They must possess significance. He does not deny that ideas are mental existences, states, images, etc., but he does deny that such ideas

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7 Cf. Ibid. p. 7

8 Ibid. p. 4

9 Ibid. p. 4.

10 Ibid. p. 7.

are ideas for judgment. A mental image is not, for Bradley, a sign. In our own words, it seems that Bradley desires to drive home the point that, mental images are not logical entities. They are psychological entities, and, as such, are not to be considered as signs, or ideas, for judgment. Ideas, for judgment, are not psychological phenomena amenable to scientific observation. They are not in time, as is a mental image, i.e., they are logical not psychological.

Bosanquet agrees with Bradley on this point when he states:

"Mr. Bradley has shown that these images are not ideas in the logical sense, not significant, not meanings. We use them merely as starting points." (11) Bosanquet also upholds the general theory of ideas, for idealists, when he states: "If human intelligence is a continuous judgment there are no non-symbolic ideas." (12) In summing up, "we might say that in the end, there are no signs save ideas, for logic at least, all ideas are signs." (13)

(b) Judgment.

The foregoing discussion of an idea, for judgment, will serve as a simple introduction to our discussion of judgment. In judgment we use ideas and, "Judgment proper is

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11 Logic. Vol. 1. p. 7.

12 Ibid. p. 71.

13 F. H. Bradley. The Principles of Logic. p. 5.



the act which refers an ideal content (recognized as such) to a reality beyond the act." (14) Judgment is essentially an act and, for idealism, it is an act which concerns but one idea, however complex the judgment. As Bosanquet says: "The judgment however complex is a single idea." (15) This understanding of the nature of judgment is very important if we are to appreciate what idealism means by a judgment. In the first place, it is important to notice that if judgment is a single idea, then the dividing line between an idea, as such, and a judgment, as such disappears. Idealism cannot conceive of an idea that is not at the same time a judgment; except, of course, for purposes of logical investigation. The distinction is not ultimate. We may, of course, consider judgment as a system and ideas as a system, but there is nothing 'atomic' about a judgment or an idea.

Subject and predicate in a judgment are not two ideas that are somehow connected by a copula. A judgment is not a proposition. A proposition may represent a judgment, but it can never be a judgment itself. As Bosanquet says: "It is absurd to imagine that the ideal of intelligence lies at all in the direction of a severance of thought from words. Yet the spoken or written word differs fundamentally from

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14 Ibid. p. 10.

15 Logic. Vol. 1, p. 76.

the judgment." (16) Let us illustrate this point. If I judge: 'The rose is red,' I refer an ideal content to a reality beyond the act. The immediate subject of my judgment is reality as it presents itself to me in my series. The ultimate subject is Reality, but I accept as real that which is present to perception. This reality is the actual subject of my judgment. The predicate of the judgment is the ideal content which is referred to the reality beyond the act. Bradley, speaking of the ideal content says: 'It is not a fact but a wandering adjective.' (17)

The rose is red, considered as a proposition, contains subject, copula and predicate. But, this subject is the grammatical, not the logical subject. The distinctions of the traditional logician, for the idealist, belong to the mechanisms of language than to the vital processes of thought. "In judgment S and P are differences within an identity, in the proposition they are isolated parts of an extended whole." (18)

The mechanisms of language are in time, and a proposition, either written or spoken, is in time. Judgment,

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<sup>16</sup> Ibid, p. 74.

<sup>17</sup> The Principles of Logic, p. 10. We note, however, that Bradley corrects this view later, and admits the justice of Bosanquet's criticism that ideas are never 'floating adjectives.' Indeed, since idea is identical with judgment it never 'floats' or 'wanders,' but is always 'referred to reality.' Cf. pp. 16-17.

<sup>18</sup> B. Bosanquet, Logic, Vol. 1, p. 78.

for the idealist logician, is not so conceived to be in time.

"It is absolutely impossible that priority in time should subsist between the parts of a completed judgment." (19)

When I judge: 'The rose is red,' my judgment, considered as a psychological process is in time, and, "we appear to begin with a ready made subject, to which P is added by subsequent transition." (20) But, when we consider this matter from a logical, rather than from a psychological, point of view, we find that the subject has no significance at all unless P is already referred to it. Let us consider this matter for a moment. Suppose I merely say: 'The rose.' A hearer might ask: 'What about it?' My reply might be: 'Ah!' The point of the matter is this; when I say: 'The rose,' my judgment is pregnant with predication. The tacit suggestion is that, my judgment has infinite predication, and, like Spinoza, I do not wish to negate by attribution. Indeed, we recall that Hegel maintains that, the judgment: 'this,' is the most universal of all judgment. The predication becomes explicit in the psychological process, but the psychological process is not to be identified with the logical activity. "It is clear that we are led to regard the completion of the

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19 Ibid, p. 79

20 Ibid, p. 80

judgment as an arbitrary distinction. The completed judgment like the processes by which it is obtained obviously possesses duration, i.e., it can be dwelt upon." (21) But, "The duration of a judgment as a transition in time is its external aspect. This duration is capable of any degree of extension." (22)

As the distinctions within judgments are quite arbitrary for the idealist, so are the distinctions between judgments. "The whole of consciousness may be regarded as a continuous judgment." (23) Judgments within the system of judgment, as a whole, are differences within an identity. From this point of view, such distinction as those between Universal and particular judgments, categorical and hypothetical judgments, qualitative and quantitative judgments, dear to the heart of traditional logicians, are not 'atomic' and absolute in any sense. Indeed, we shall see that judgment is really inference. But, we must wait on 'seeing things clearly, and seeing them whole.' As we could not see the true significance of an idea until judgment was considered, so, we shall not be able to appreciate what is meant by judgment until we have considered inference.

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21 Ibid, p. 81

22 Ibid, p. 82

23 Ibid, p. 84.

(c) Inference.

System is a fundamental principle for idealism. For Bradley, the universe of discourse and the universe of existence are really aspects of the same thing. In the Principles he seeks to put forward the system that is discourse, in Appearance and Reality the system that is existence. Bradley's general position, which is the position common to idealism, will be readily appreciated when we recall the discussion on knowledge as a construct.

We have already sought to show how idea and judgment are to be thought of as one. The Idealist delights to show to what extent two things can be said to be different and to what extent they must be considered the same. The principle of identity in difference is the basic principle of system. Thus, we may say that, we have the system of idea, the system of judgment, and the system of inference. Yet, these systems imply each other, and each implies the whole. While idea is implicit in judgment, and judgment is implicit in inference, yet, each becomes explicit in the processes of thought, and, for purposes of logical investigation, we may say that, on psychological grounds, idea comes before judgment, and judgment before inference. Indeed, when Bradley treats of inference he states: "explicit judgment and inference, acts both of which end in an asserted truth, and one of which starts with a truth laid down as the foundation of its process.

And in this sense it is true that we judge before we reason, since we become possessed of an affirmation, when we cannot produce any other affirmation on which this stands. Thus the distinction which we made remains unshaken. Explicit judgment comes before explicit inference. And supposing that both are really and in the end two sides of one act, then the above conclusion is what we might have expected. Here as everywhere the product comes to consciousness first, and the process afterwards." (24)

In simple terms, let us ask: What is inference? Bosanquet maintains that, "Inference shares the essence of judgment, but qua explicit inference, has in addition a differentia of its own." (25) This statement is in keeping with that at which we have already hinted, i.e., there is an identity between judgment and inference, but that this identity does not exclude the possibility of difference. We recall that the essence of judgment is the reference of an ideal content to reality. Is inference too, the reference of an ideal content to reality? It would seem to be, yet it has a differentia. Bosanquet finds the differentia of inference in the mode of the reference to reality. Whereas judgment appears to be a direct reference, inference seems to

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24 Principles of Logic, p. 481.

25 Logic, Vol. 11, p. 1.

contain an element of mediation. Bosanquet is led to define inference as "the mediate reference of an ideal content to Reality." (26)

It may be somewhat confusing to note how idealist writers use the terms 'reality' and 'Reality;' yet, if one remembers that 'Reality' is the ultimate subject of judgment and 'reality' the immediate subject, the confusion disappears. It is on the ground of this distinction that Bradley is able to show that, in the end, all our judgments are inferences. Although a judgment is the reference of an ideal content to reality, yet, the ultimate subject is Reality. Judgment, in this sense, may be said to be mediated. But this is not what we mean by explicit inference, e.g., I say: 'The rose is red.' I predicate of the rose, in presentation, that it is red, and I assert of reality, yet this reality is taken for Reality. In this sense my judgment is an implicit inference since reality mediates the implied reference to Reality. In explicit inference, certain psychological processes are involved that are absent in the case we have cited. The enunciated syllogism will illustrate this point, i.e., the inferential process becomes externalized, it is in time and may be dwelt upon.

We turn to Bradley again for help. In the first of

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26 Ibid.

his Terminal Essays, Bradley discusses the essential nature of inference. "Inference being a process I will state at once what I take to be its essential nature. This may be set down as the ideal self-development of an object." (27) Bradley's language is a little difficult, yet his meaning seems to be clear. He explains further on: "Now, this object, like all objects, is taken, we may say, as referred to Reality, the real universe; or to speak more correctly, the object is taken as in one with this Reality." (28)

The 'object' that develops itself in inference is the ideal content. It is the universal at work in the inferential process. The conclusion of inference represents a development of the object with which we started. There is no suggestion of an inferential leap. There is the process of self-development. But, we must remember that, for idealism, self-development cannot be conceived as the mere development of an identity without reference to the differences implicit within that identity. Inference makes explicit, in its conclusion, the differences that were implicit at the beginning. Does this mean that there is nothing new brought to light in inference? No. "The general solution of the problem raised by the essence of inference is found, I think, so far as logic is concerned, in the double nature

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27 Principles of Logic, p. 597.

28 Ibid. p. 600.



of the object. Every inference, we saw, both starts with and is confined to a special object. Now, this object, like all objects, is taken, we may say, as referred to Reality, the real Universe, or, so to speak, more correctly, the object is taken as in one with Reality. Hence the object not only is itself, but is also contained as an element in a whole; and it is itself, we must add, only as being so contained. And the difference of the object from, and its essential identity with a whole beyond itself - a whole which logic takes as a system both ideal and real - is the key (so far as logic is concerned) to this puzzle of self-development. On the one side the special object advances to a result beyond the beginning, and yet its progress throughout is nothing beyond the intrinsic development of its proper being. For that which mediates and necessitates its advance is implied within its own self." (29) We illustrate this by pointing out that the object that develops in inference is a difference within an identity. If I say: 'The rose is red,' the rose develops, as it were, in analytic expansion, and one of its differences is made explicit. If the rose is red, then red is implied in the object, and the judgment is the self-development of the object. Since judgment maintains itself as inference, our remarks refer also to inference proper.

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29 Ibid. p. 599 f.

Idealism, generally, maintains this position with regard to the nature of inference. In inference we discover something new, but this discovery "is an accident of inference and not an essential of inference." (30) There is no suggestion of a 'leap' to something new, neither is inference a passage in time, whether instantaneous or extended. The conclusion does not cease to be an inference the moment that it becomes familiar: it merely ceases to be a discovery. Since inference is essentially the operation of a universal, the novelty must be implied from the beginning, yet, psychologically, the novelty emerges in time at the end of the process. Logically, the novelty is a difference within the identity of the universal. This difference becomes explicit in the inferential process. The novelty, psychologically considered, is a discovery, but logically considered, it is still novelty, i.e., an implicit difference within a universal that has become explicit.

Inference without the operation of a universal idealism believes to be impossible. Idealism maintains this position against all schools of thought that would seek to base the nature of inference on other grounds. Perhaps it will be convenient to discuss the manner in which idealism conceives inference to be the operation of a universal. Bosanquet is very clear that valid inference necessitates the operation of

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30 E. Bosanquet. Logic, Vol. 11, p. 8.

a concrete universal. Inference, we recall, is defined by Bosanquet as the mediate reference of an ideal content to Reality. But the question arises: How can one content claim to be true of Reality on the strength of another content distinct from the first? "All turns on the distinction between the abstract or powerless and the concrete or dominant universal. To reduce the former to the latter is a fatal tendency of popular logic." (31) It may be that Bosanquet exaggerates when he speaks of an abstract universal. No such entity could be said to exist for idealism. All universals are concrete. Nevertheless, for the purposes of exposition, it may be permissible to speak of relatively universal. We shall follow Bosanquet's argument to make clear what idealism means by maintaining that valid inference can only be forthcoming when a concrete universal pervades the whole process and result.

How, then, is it possible to proceed from content to content? It is only when these contents are the differences of a universal. But are these differences the differences of a universal? Suppose that I find in a room a hundred different objects such as books, guns, china etc., all marked with the owner's name. The common identity, the universal that all of them have in common is the label of the owner.

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31 Ibid, pp. 2-3.

But what can I infer from the universal except that object 'a' belongs to Mr. X., and object 'b' belongs to Mr. X., and so on? In other words, what can I infer from the universal beyond itself? I cannot infer, for instance, that the twentieth article I shall examine is a tea-pot or a book. True it is that, the tea-pot, or book, will contain the universal 'mark;' but the tea-pot, or book, will not be leavened, as it were, by the universal. In this sense, the universal may be called abstract.

But the case must be examined further to see what we may expect from a universal. A tea-pot, as a tea-pot, is decidedly particular; a book, as a book, is also decidedly particular when considered as the nineteenth or twentieth object of an aggregate. Now, a universal to be concrete must pervade its differences, and, as it were, leaven its differences so that its differences are known as the differences of an identity. When I ask: What is the twentieth object? Is it a tea-pot or a book? My question cannot be answered by an inference because any object considered as a unit and discrete can only have a predication when the unit ceases to be a unit and becomes a universal with its own differences within its own system. For example, we may begin to predicate something of a particular, but in the very moment we begin to do this the particular ceases to be a particular: it becomes an individual, and, as an individual,

possesses differences within its identity.

In order to make this discussion clearer, let us suppose that there are a number of assorted objects in a room. All the objects belong to Mr. X. I begin to examine each object. I know that the tenth object I shall come to will be an object belonging to Mr. X, because, if all these objects are Mr. X's, the tenth must be. I am able to make a synthetic judgment a priori, because all the percepts are implicitly synthesized under the concept. In my Judgment: The tenth object is Mr. X's, we find analytic expansion from the universal, in that one of its differences becomes explicit.

It would seem to be true that, in inference, we must work from the side of the universal, and, as it were, draw out from the universal in analytic expansion. But, there is also the process of synthesis in inference. I say: These books (meaning all I indicate) are Mr. X's. Then I say: This is one of 'these books.' Therefore, this book is Mr. X's. In the minor premise I have indicated a particular book, but in the very moment I indicate its particularity I synthesize it with the common universal, i.e., it is one of the differences of 'these books.'

It will be obvious from the foregoing discussion that, only when we work from the side of the universal, and only when our universal is concrete, may we expect valid inference.

All universals that are actually operative in inference are concrete, i.e., they are systems such that a certain principle pervades each, holding the details together and interrelating them in such a way as to control and account for their differences within the system.

In the foregoing, we have attempted to describe, as briefly as possible, what the idealist logician understands to be the nature of inference. We must now turn to consider the nature of inference according to the most modern school of realist logicians, i.e., the symbolic school.

## CHAPTER III

### REALISM

Actually, formal logicians of the symbolic school seem to have little to say about inference. "Inference is undoubtedly a mental process. If, therefore, our conception of logic were such as to restrict logic to the theory of propositional forms, we should no more need to consider the nature of inference than the mathematician needs to consider the psychological process whereby a student comes to apprehend a mathematical theorem. (1)

In contrast to the idealist, the mathematical logician stresses implication rather than inference. Let us, however, follow the symbolic logician as he sets forth his system. We have intimated previously that the nature of inference for the various logical schools is determined by the general logical theory held by that particular school. It will be convenient, therefore, to discuss briefly the general background of this school. Our examination will be restricted

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1 L. S. Stebbing. A Modern Introduction to Logic, p. 210. Also W. E. Johnson. Logic, Pt. II, p. 3, says: "Every inference is a mental process, and, therefore, a proper topic for psychological analysis."

to the theory of the symbolic logicians. (2) We begin with George Boole's Laws of Thought and culminate with certain modern writers.

In order to gain a clear insight into the basis upon which the structure of symbolic logic is reared, it will be necessary to go back to the 'Father' of the algebra of logic, George Boole, and learn from him the nature of the assumptions upon which the system is based.

In that part of Boole's collected logical works known as An Investigation of the Laws of Thought, Boole begins by describing the nature and design of the work. He states: "The design of the following treatise is to investigate the fundamental laws of those operations of the mind by which reasoning is performed: to give expression to them in the symbolic language of a Calculus, and upon this foundation to establish the science of logic and construct its method: to make that method itself the basis of a general method for the application of the mathematical doctrine of Probabilities; and finally, to collect from the various elements of truth brought to view in the course of these inquiries some of the probable intimations concerning the nature and constitution

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<sup>2</sup> It is to be noted that not all symbolic logicians are 'realist' in background and outlook. H. B. Smith, for instance, accepts Hegelian idealism as fundamental, yet sees in symbolic logic a technical way of approaching certain problems.



of the human mind." (3)

The first point of importance to notice is the fact that Boole proposes to investigate the fundamental laws of those operations of the mind by which reasoning is performed. He sets about his task by assuming that a science of the intellectual powers is possible. He asks; "How shall we obtain knowledge of this? He answers; "Like all other sciences, that of the intellectual operations must primarily rest upon observations, - the subject of such observations being the very operations and processes of which we desire to determine the laws." (4)

It is obvious, from the foregoing, that the foundation of symbolic logic rests upon observation in the first instance. This observation, according to Boole, yields a knowledge that the processes of the mind in reasoning are of a mathematical order. If this is so, then, the general laws to be deduced from these observations are of a different order from the general laws of nature. Indeed, Boole makes clear this contrast, when he states: "The general laws of nature are not for the most part, immediate objects of perception. They are either inductive inferences from a large

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3 p. 1, f.

4 p. 3.

body of facts, the common truth in which they express, or, in their origin at least, physical hypotheses of a causal nature serving to explain phenomena with undeviating precision, and to enable us to predict new combinations of them." (5) Over against the general laws of nature, Boole sets the laws of the mind. "On the other hand, the knowledge of the laws of the mind does not require as its basis any extensive collection of observations. The general truth is seen in the particular instance, and it is not confirmed by the repetition of instances." (6)

Boole's next aim is to give expression to these laws in the symbolic language of a Calculus. He maintains that the laws of the operations of the human mind are mathematical in form, and that, because of this, the laws of the symbols of logic are deducible from a consideration of the operations of the mind in reasoning. On the basis of Boole's method, symbolic logic was launched upon its way. Since the time of these first beginnings, about the middle of the last century, the Calculus has been developed and improved to such an extent that, it has resulted in such monumental works as Principia Mathematica.

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5 p. 4.

6 Ibid.

It seems reasonable to suppose that, if the laws of the human mind are mathematical, then they can be expressed in the symbolic language of a calculus. This is exactly the assumption upon which symbolic logic has its beginning. Consequently, the validity or invalidity of any 'piece of reasoning' can be demonstrated symbolically, according to the laws of the calculus. The symbolic logician feels sure that, if the relation of implication holds, then his reasoning is valid. The search of realism for the ultimate reign of law is seen, from the point of view of logic, in the belief that in symbolic logic we have the certain laws of the operations of the human mind in discourse. Boole is sure that, because a general law of the operations of the human mind is, in fact, established from one instance, that this fact guarantees the strict necessity of that law from which deductions can be made which are absolutely true. (7)

It will be seen quite readily that it is quite reasonable to maintain that the processes of the mind in reasoning can be expressed in the symbolic language of a calculus. Processes of thought, then, are amenable to mathematical manipulation, i.e., there is a one to one correspondence between the mathematical manipulation of symbols and the function of mind in valid reasoning. Later developments,

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<sup>7</sup> Cf. p. 425.

since Boole, have been concerned to establish the truth of this contention, and a general theory of the operations of the human mind in reasoning has resulted in the logical essence theory.

Bertrand Russell puts forward his argument for the ontological status of logical essences in the following manner. He first discounts the arguments of the 'pure' empiricists that our mathematical knowledge is derived by pure induction from particular instances, by pointing out, "that the validity of the inductive principle itself cannot be proved by induction; secondly, that the general propositions of mathematics, such as 'two and two always make four' can obviously be known with certainty by consideration of a single instance, and gain nothing by enumerations of other cases in which they have been found to be true. Thus our knowledge of the general principles of mathematics (and the same applies to logic) must be accounted for otherwise than our (merely probable) knowledge of empirical generalizations such as 'all men are mortal.'" (8)

Russell attacks the Kantian position on the problem of a priori knowledge by suggesting that, "since our a priori knowledge depends upon the constitution of our nature as such, it might happen, if Kant is right, that tomorrow our

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<sup>8</sup> The Problems of Philosophy, p. 131.

nature would so change as to make two and two become five."

(9) Russell, of course, realizes that this statement does not do justice to the Kantian view that time itself is a form imposed by the subject upon phenomena, so that our real self is not in time and has no tomorrow. Yet, he maintains that the substance of the Kantian argument rests on the assumption that the time order of phenomena is determined by characteristics of what is behind phenomena.

Russell's argument for the ontological status of logical essences seems to depend upon logical relations being laws of things and not laws of thought. He argues that the so-called law of contradiction is a law about things rather than a law about thought. (10) It would seem that the modern development of Boole's general position resulted in the establishment of another sphere which is not mind, nor the operations of the mind, but a 'world,' as it were, of logical relations which the mind intuits in its operations. As Russell states it: "The fact seems to be that all our a priori knowledge is concerned with entities which do not, properly speaking, exist, either in the mental or the physical world. These entities are such as can be named

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<sup>9</sup> Ibid. p. 135.

<sup>10</sup> Cf. Ibid. p. 188.

by parts of speech which are not substantives." (11)

If we ask what sort of ontological status these logical essences possess, we find that Russell gives us a ready answer. "Consider such a proposition as 'Edinburgh is north of London.' Here we have a relation between two places, and it seems plain that the relation subsists independently of our knowledge of it. When we come to know that Edinburgh is north of London, we come to know something that has to do only with Edinburgh and London. We do not cause the truth of the proposition by coming to know it, on the contrary we merely apprehend a fact which was there before we knew it. The part of the earth's surface where Edinburgh stands would be north of the part where London stands, even if there were no human minds to know about north and south.....This is of course denied by many philosophers, either for Berkeley's reasons or for Kant's. But we have already considered these reasons, and decided that they are inadequate. We may therefore now assume it to be true that nothing mental is presupposed in the fact that Edinburgh is north of London. But this fact involves the relation 'north of' which is a universal.....Hence we must admit that the relation, like the terms it relates, is not dependent upon thought, but belongs to the independent world which thought apprehends

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11 Ibid. p. 189.

but does not create." (12) Further Russell states: "The world of universals, therefore, may also be described as the world of being. The world of being is unchangeable, rigid, exact, delightful to the mathematician, the logician and the builder of metaphysical systems and all who love perfection more than life." (13)

We have thought it advisable to discuss the theory of logical essences at some length, since the relation of implication is all important for the symbolic logician. As realism approaches the field of logic, it does so with the following general background. All reals are physical reals. Within these physical reals, we find sense data, minds, logical essences and a physical world that corresponds to our sense data.

Equipped with a realist universe, we can proceed to seek an answer to our question: What is inference? "The act of positing a supposed implicate on the ground of its supposed implicans is called inference." (14) In short, this is the definition of inference given in one of the more recent text books on formal logic. But let us ask: Do modern formal logicians conceive inference to be merely an

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12 Ibid. pp. 152-153.

13 Ibid. p. 156.

14 Bennett and Baylis. Formal Logic. p. 12.

act? No. Bennett and Baylis explain that: "Just as the term perception sometimes means 'the act of perceiving' and sometimes 'that which is perceived,' so 'that which is inferred' is sometimes called an inference." (15)

Inference, however, from the point of view of the realist, i.e., the symbolic logician, appears to be psychological rather than logical. Cohen and Nagel suggest this, when they state: "it is essential to distinguish inference, which is a temporal process, from implication, which is an objective relation between propositions." (16) Also, Bertrand Russell states explicitly: "There is always unavoidably something psychological about inference: inference is a method by which we arrive at new knowledge, and what is not psychological about it is the relation which allows us to infer correctly; but the actual passage from the assertion of p to the assertion of q is a psychological process, and we must not seek to represent it in purely logical terms." (17)

As we have previously intimated; it is implication rather than inference, that is of central importance to the formal logician. Inference is the psychological act, and the psychological conclusion. The interest of the symbolic

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15 Ibid. pp. 12-13.

16 An Introduction to Logic and Scientific Method.  
pp. 7-8.

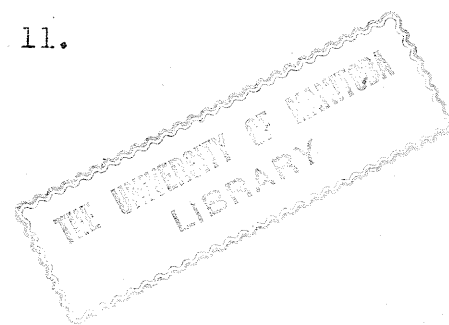
17 Mathematical Philosophy. P. 149.



logician, as far as inference is concerned, is to know if his inferences are valid. The validity, or invalidity, of his inferences may be tested by the application of the laws of the calculus, whether that calculus be a calculus of classes or of propositions. It is the relation of implication that binds the inference together, if it is valid. "The relation between premises and conclusion which justifies an assertion that the latter follows from the former is a relation of implication. When it holds, the premises imply the conclusion follows from the premises." (18) We shall not, at this point, investigate the grounds upon which the formal logician establishes the truth-claim for his inferences; rather it will be our purpose to set forth (as faithfully as we are able) those logical considerations that bear upon the nature of inference apart from the truth-claim. It is not to be supposed, however, that all formal logicians treat of inference apart from the truth-claim. For example: W. E. Johnson maintains the position that a knowledge of the truth of the premises is necessary before an inference can consistently be said to be valid. He states the matter as follows: "Conditions for the Validity of the inference 'p therefore q' Constitutive conditions: (i) the proposition 'p' and (ii) the proposition 'p would imply q' must both be true.

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18 Bennett and Baylis, Formal Logic, p. 11.



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Epistemic conditions: (i) the asserting of 'p' and (ii) the  
 asserting of 'p would imply q' must be permissible  
without reference to the asserting of 'q.'" (19)

On the other hand writers such as Bennett and Baylis, permit inferences from false or inconsistent premises. The two usages appear to be quite common and the choice of usage depends upon the preference of the writer. For the present we shall follow the school of Bennett and Baylis.

The formal logician is concerned with the statement of rules which justify formal inference, although there is a general recognition that many of our inferences are material, i.e., they depend upon the content rather than the form. Yet, even in these cases their validity depends upon certain tacit assumptions. "Thus we should usually infer from 'A is twenty-five years old' that 'A is not a grandparent,' but our inference would not be valid did we not tacitly limit the conditions of our problem so as to exclude domestic animals, step-parents, and extremely youthful mating." (20)

It is the validity of our inferences with which we are concerned, and we can only be certain of their validity when we know that the relation of implication holds. To be certain of this, we must cast all our inferences into sym-

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19 Logic, Pt. 11, Chap. 1, p. 8.

20 Bennett and Baylis, Formal Logic, p. 23.

bolic form, since we are convinced that the truth about reasoning is mathematical, and mathematical forms are the medium whereby we know if our reasoning is valid. From this point of view, the traditional forms of propositions are merely verbal and quite inadequate to express those valid forms of reasoning which are non-syllogistic.

The symbolic logician delights to show how the immediate inferences of the traditionalist, together with the A, E, I and O forms of the syllogism, in all its figures and moods, may be recast into symbolic form and reduced to comparatively simple terms. Let us give one or two examples to show how the symbolic method operates with such power as to make the traditional logic and theory of inference of mere historical interest. We begin with immediate inference in the 'A' form. If we are given 'All S is P,' we may reduce it to the form ' $SP' \neq 0$ .' The obverse, 'No S is non-P' has, obviously, the same form, i.e., ' $SP' \neq 0$ .' Translated, ' $SP' \neq 0$ ' means that the class 'S' in logical multiplication with the class P-complement is materially empty. In this manner we may proceed to reduce all the traditional immediate inferences to symbolic form. The square of opposition, likewise, becomes quite useless under this more powerful method. Indeed, "Immediate inference, when it pretends to be formal, is either trivial or only imperfectly formal, according as it concerns a mere verbal rephrasing of a given logical

relation or as it involves tacit assumptions." (21)

A type of inference which is thought to be non-trivial when cast into symbolic form is the syllogism. L. S. Stebbing commenting on Aristotle's definition of the syllogism states: "In accordance with this definition we might say that a syllogism is a form of implication in which two propositions jointly imply a third." (22) It is to be noted that symbolic logicians consider the two premises a compound implicans, and when one thinks of the two premises in this way it would seem to make the relation of implication between the premises and conclusion the essential operative factor in the inferential process.

All the forms, figures and moods of the syllogism can be recast into algebraic form. By this method the sixty-four possibilities can be reduced to thirty-six algebraic possibilities. The validity of these syllogisms can be tested by the manipulation of algebraic formulae directly, or by use of a Venn diagram. The rules for interpreting the syllogistic diagrams are intuitively obvious.

Let us take an example. A syllogism in the first figure and in the mood Barbara would give us: - MaP: SaM: SaP. Recast into algebraic form, this would give us: -

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21 Ibid. p. 159.

22 A Modern Introduction to Logic, p. 81.

$$MP' \doteq 0$$

$$\underline{SM' \doteq 0}$$

$$SP' \doteq 0$$

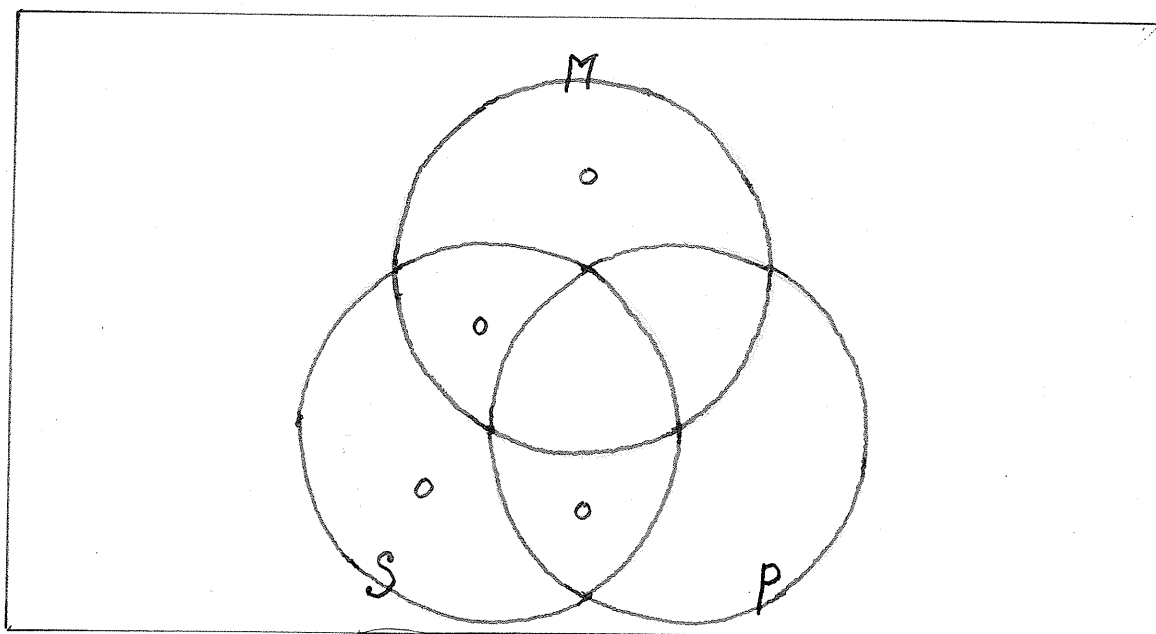
The algebraic explanation of this matter is as follows: -

Since  $MP' \doteq 0$ , therefore  $SMP'$ , and  $\acute{S}MP' \doteq 0$ .

Since  $SM' \doteq 0$ , therefore  $SMP$  and  $SMP' \doteq 0$ .

Since  $SMP' \doteq 0$  and  $SMP \doteq 0$ , therefore  $SP' \doteq 0$ .

If we desire to verify this syllogism by use of a Venn diagram, we have the following: -



In the diagram we see immediately that  $SP' \doteq 0$ , since  $MP'$  and  $SM'$  are both empty.

It will be readily seen that, by the application of the symbolic method, all the varieties of syllogistic reasoning can be brought out of their verbal confinement, and reduced to algebraic form. This method, the symbolic logician believes, gives far greater certainty and clarity in reasoning. So long as we are tied to verbal forms, we are inevitably hindered by verbal interpretations. But, once we move into the abstract realm, these shackles are loosened and we know with certainty whether our reasoning is valid or invalid.

The syllogism, however, because of the strict limitations on its possible forms, is but one of the many types of valid argument. There are many types of argument whose validity can only be discovered by a study of their form alone. It is obvious that the application of other criteria than the syllogism is desirable in cases where the use of the syllogism would so distort the original propositions that we could never be sure whether the conclusion did actually follow from the original premise. The symbolic logician feels that we should not put too much reliance on ordinary verbal forms; hence, there has been developed a calculus of classes which sets forth more powerful and general validating forms.

There are many problems which cannot be solved by the syllogistic method, and the development and extension of

a calculus of classes to formal non-syllogistic class inference ensures valid reasoning in the solving of such problems. We give one example, suggested by Bennett and Baylis. The members of a certain collection are classified in three ways, so that each is, or is not, in X, is, or is not, in Y, and is, or is not, in Z. It is found that class Z is constituted precisely of those members of Y which are not in X, and of those members of X which are not in Y. How is the class constituted?

The data reduce to  $Z = XY' \vee X'Y$ .

Hence  $Z(XY' = X'Y) + Z'(XY' + X'Y) = 0$ .

Hence  $Z(XY + X'Y') + Z'(XY' + X'Y) = 0$ .

Hence  $XYZ + XY'Z' + X'YZ' + X'Y'Z = 0$ .

This last equation is obviously symmetric in X, Y and Z.

This symmetry suggests the conclusion  $X = YZ' + Y'Z$ .

It is, however, in the development of a calculus of propositions that the symbolic logician devises his most powerful validating forms. We must explain this development in greater detail. The traditional logic, based on Aristotle, expressed propositions in forms analyzable into subject, predicate and copula. In the algebraic recasting of the Aristotelian proposition, the subject and predicate terms are treated as classes and the copula and quantifier together as indicating a relation between classes. In reinterpreting a calculus of classes as a calculus of propositions, the

propositions are treated as wholes in a similar manner in which a class is treated as a whole. In this way, two propositions,  $p$  and  $q$ , may be treated as related by implication, i.e.,  $p$  implies  $q$ . "In order to be able validly to infer the truth of a proposition we must know that some other proposition is true, and that there is between the two a relation of the sort called 'implication,' i.e., that (as we say) the premise 'implies' the conclusion." (23) It will be seen that Russell does exactly as we have intimated, i.e., propositions are treated as wholes, and are related. He goes on to say: "When our minds are fixed upon inference, it seems natural to take 'implication' as the primitive fundamental relation, since this is the relation which must hold between  $p$  and  $q$ ." (24)

The possibility of reinterpreting a calculus of classes as a calculus of propositions depends upon the fact that propositions may be treated as the terms of traditional logic. From this point of view, the Boole-Schroder algebra may be regarded, in its class interpretation, as a systematic statement of a logic of terms. We have previously shown that this algebra is applicable to cases not envisaged by traditional logicians. In like manner, the application of

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<sup>23</sup> B. Russell. Introduction to Mathematical Philosophy, p. 146.

<sup>24</sup> Ibid. p. 146.



a calculus to propositions is a further development beyond the confines of the Aristotelian system. As the Aristotelian system was deductive in its inferences, so in symbolic logic, from its recasting of the syllogism, through its non-syllogistic reasoning in a calculus of classes, and finally to its propositional calculus, the whole system is deductive.

Inference, for the symbolic logician, is essentially deductive, and in the calculus of propositions, we may find powerful devices for arriving at valid conclusions. "In deduction we have one or more propositions called premises, from which we infer a proposition called a conclusion." (25) Deduction may be regarded as a process by which we pass from knowledge of a certain proposition, the premise, to knowledge of a certain other proposition, the conclusion. Such deduction, however, must have a relation between the premise and conclusion before it can be called logical deduction. When the relation between premise and conclusion holds, we may be sure that our inference is valid.

Some logicians have considered the words 'infer' and 'deduce' to be synonymous, "But the correct synonym of 'deduce' is formally infer." (26) Under this definition of 'deduce,' we see that there is the possibility of infer-

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25 Ibid. p. 145.

26 L. S. Stebbing. A Modern Introduction to Logic, p. 212.

ence being an inductive process. From a psychological point of view, this may be so; but inference, in the sense of formal inference, must be deductive; i.e., the logical process of arriving at a valid conclusion is mathematical. As L. S. Stebbing points out: "It may be the case that only deductive inference is valid inference.....we cannot, however, even raise the question as to the validity of induction without assuming that there is such a thing as inductive inference. It is noteworthy that Mr. Russell, who, in 1903, did not distinguish between inference and deduction, in 1927, speaks as though all inference were inductive. Cf. The Principles of Mathematics, p. 11, n., with An Outline of Philosophy, Chaps. VII and XXV." (27)

As far as formal inference is concerned, the symbolic logician must speak of it as deductive. Inductive inference can yield only probable conclusions. "In illustration, we shall roughly formulate the inductive principle: 'What can be predicated of all examined members of a class can be predicated, with a higher or lower degree of probability, of all the members of the class.'" (28) A typical inductive inference would be; 'All examined crows are black, therefore, with a higher or lower degree of probability, all crows are black.'

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27 Ibid. n.

28 W. E. Johnson. Logic, Pt. 11, p. 22.

It would seem, then, that inference, as the process and result of the manipulation of a calculus of propositions, is formal inference. Such formal inference will yield necessary and valid conclusions. "Inference is a method by which we arrive at new knowledge, and what is not psychological about it is the relation which allows us to infer correctly; but the actual passage from the assertion of  $p$  to the assertion of  $q$  is a psychological process, and we must not seek to represent it in purely logical terms." (29)

In Principia Mathematica, Russell enumerates the five formal principles of deduction. We shall illustrate by quoting the second of them, i.e.,  $t: q \supset p \vee q$ . This means. It is asserted that, the disjunction  $p$  or  $q$  is true when one of its alternatives is true. Russell refers to such a formal principle as a schema of inference. In such a schema of inference, as we have mentioned, we have a proposition  $q$  which implies the disjunction of  $p$  or  $q$ . These propositions, related by implication, yield necessary truth.

It would seem that valid inference, for the symbolic logician, was deductive. Empirical induction, as we have pointed out, can yield probable conclusions only. There is, however, the matter of mathematical induction to consider. Something like empirical generalization seems to occur in

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29 Bertrand Russell. Introduction to Mathematical Philosophy, pp. 194-5.

mathematics. "It is noted in the series of natural numbers that 0 (or 1) has a certain property, and that, for each number  $n$  which has this property,  $n$ 's immediate successor,  $n + 1$ , has the property, and from this it is inferred, without examining each member in turn, that every natural number has the property." (30) After giving an example of mathematical induction, Bennett and Baylis go on to say: "The basis for this inference lies in the definition of the natural numbers and particularly in the properties of the relation immediate successor of. From the definitions of the numbers in the series of natural numbers, the principle of mathematical induction follows by strictly deductive reasoning.

Such 'induction' is not at all an empirical generalization."

(31) This passage would suggest that valid inference is essentially deductive. Our previous statement that inference, for the symbolic logician, is deductive still stands, in spite of 'so-called' mathematical induction.

In summing up, we see that inference, for the most modern formal logicians, may be considered from two points of view. As a psychological process, the conclusions of inference may, or may not, be valid. As a logical process, the conclusion of an inference must be valid, if the relation

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30 Bennett and Baylis. Formal Logic, p. 371.

31 Ibid. p. 371.

of implication holds. We have noted the fact that some logicians of this school deem it necessary, for valid inference, that the premises be true, while others have little concern for the truth-claim of the premises, and consider a conclusion valid if the relation of implication holds. It is to be noted that both schools stress the importance of the relation of implication, so we feel justified in saying that, generally speaking, a conclusion is valid, if the relation of implication holds.

## CHAPTER IV

### PRAGMATISM

We come now to a consideration of the nature of inference for the pragmatist. Again, we must point out that, the theory of inference for the pragmatist, like the theory of inference for the idealist and the realist, can be understood against the general background of the logical school under consideration.

Pragmatic logic, if we take John Dewey's criterion, begins and ends "as the method of inquiry." (1) Speaking quite generally, it would seem that pragmatist logical theory can best be understood if we make a broad contrast with that of the symbolic logician. It will, however, be necessary to point out that both symbolic and pragmatist logicians follow the tendencies of modern science. Modern science, it will be recalled, has two main emphases, i.e., the mathematical and the empirical. The symbolic logicians follow the mathematical tendency of modern science, and the pragmatists follow the empirical tendency. We realize that this statement is so general that it is quite likely to be misunderstood. We must, therefore, explain more exactly what we

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<sup>1</sup> P. A. Schilpp, Ed. The Philosophy of John Dewey. p. 109.

mean. Pragmatist logicians maintain that their system is scientific, but "it is the experimental character of scientific logic that distinguishes it from scholasticism, mediaeval or modern." (2) The pragmatist logician would conceive the symbolic logician to be a sort of modern scholastic. His mathematico-deductive method is too far removed from the experimental character of life to yield anything of real value for creative living. On the failures of such formalism in logic, Dewey writes: "The issue of strict logical formalism, of any theory which postulates forms apart from matter of logical forms versus forms-of-matter, comes to a head in the question of the relation to method in the natural sciences.....It would at first seem if pure formalism should lead those who accept that doctrine to abstain entirely from any reference whatever to method in the natural sciences, since that method is truistically concerned with factual materials. Such, however, is not the case. Formalistic logic is not content to leave the topic of method in the existential sciences severely alone. Belief in some sort of connection is usually expressed by the phrase 'logic and scientific method.'" (3)

The symbolic logician feels, however, that the

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2 A. W. More. Creative Intelligence, pp. 77-8.

3 Logic. The Theory of Inquiry, pp. 374-5.

mathematical stress of modern science, i.e., in the physical sciences, is the really important tendency in scientific method. Modern science, on its empirical side, however, includes the activity of the experimentalist in his laboratory and, on account of this, we feel justified in making our broad distinction that the symbolic logician follows the mathematical emphasis of modern science, while the pragmatist tends towards the empirical.

At this point it will be necessary to say that the latest developments in pragmatic logic are connected with symbolism; yet, the symbolism of the pragmatist differs from that of the strictly formal logician. The symbolic logicians of the type represented by Bertrand Russell have "retained the faculty of intelligence knit into certain indefinables such as implication, relation, class, term and the like, and have transported the faculty from the human soul to a mysterious realm of subsistence whence it radiates its ghostly light upon the realm of existence below." (4) The most modern pragmatists of our day follow the logical theories of C. S. Peirce. "Logic, in its general sense, is, as I believe I have shown, only another name for semiotic...the quasi-necessary, or formal doctrine of signs." (5) This

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<sup>4</sup> H. C. Brown. Creative Intelligence, p. 118.

<sup>5</sup> Justus Buchler. The Philosophy of Peirce, p. 98.



formal doctrine of signs, however, is not to be divorced from the practical concerns of men; and inquiry is the ultimate source of logical subject matter. Dewey is a follower of Peirce in this matter, and recognizes his indebtedness to him. (6) The Pragmatist abhors the merely formal. His logic must be close to life. It must be a living and moving thing, like experience itself. Some pragmatic logicians, like F. C. S. Schiller, maintain that logic is essentially psychological, and that it is a voluntarist and normative science. Schiller feels that to appreciate this role of valuation, we must have a voluntarist or activist psychology. Valuation seems to be personal and 'subjective,' and relative to the circumstances of the case. "The comparison and evaluation of values, is plainly a psychological fact." (7)

On the other hand, some pragmatic logicians do not so equate logic and psychology. Dewey, for instance, states: "Personally, I doubt whether there exists anything that may be called thought as a strictly psychical existence." (8) He maintains that logic is an autonomous science, and as such precludes the idea that it has psychological foundations. He does not mean, however, that psychology bears no relation

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6 Cf. John Dewey. Logic. The Theory of Inquiry, p. 9, n.

7 Logic for Use, p. 41.

8 Logic. The Theory of Inquiry, p. 21.

to logic. Indeed, he maintains: "Its general relation to logic is found in the light that it, as a branch of inquiry, may throw on what is involved in inquiry. Its generic relation to logic is similar to that of physics or biology." (9) However, Dewey feels that psychology stands closer to logical theory than other sciences. Logical theory, then, for Dewey, is not so much bound up with psychology as "(i)...the systematic formulation of controlled inquiry, and (ii) that logical forms accrue in and because of control that yields conclusions which are warrantably assertable." (10)

The pragmatist logician is not concerned with questions of formal validity. For Schiller, validity cannot be separated from value. If our reasoning has value, its validity is dependent upon its value for a person, or for persons. There is, for Schiller, also a sociological, as well as a personal, side to value. We must not suppose, however, that any value is intrinsic. All values are relative to the social situation within which they function. "Where and when do we come across processes of which we can confidently predicate 'validity?'....To Formal Logicians an argument is valid or not. If it is not valid it matters not to Formal Logic how probable it may be in point of fact." (11) Dewey,

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9 Ibid. p. 22.

10 Ibid. p. 22.

11 Logic for Use, p. 46.

on the other hand, discusses validity in relation to inquiry rather than in relation to value as such. "Validity-invalid-ity....cannot be logically adjudged, therefore, merely on the basis of its formal relations to other propositions. The syllogism 'All satellites are made of green cheese; the moon is a satellite; therefore, it is made of green cheese' is formally correct. The propositions involved are, however, invalid, not just because they are materially false, but because instead of promoting inquiry they would, if taken and used, retard and mislead it." (12)

Thinking, for Schiller, is essentially psychological. For Dewey, thinking is an inquiry, or the result of an inquiry. Inquiry seems to occupy an intermediate and mediating place in the development of our experience. However, Dewey recognizes that thinking needs a temporal context. Without this temporal context there cannot be any real thinking. Now, if thinking has an intermediary place in knowledge, this fact implies a prior stage, i.e., there is a type of experience that is not knowledge. "Men experience illness. What they experience is certainly something very different from an object of apprehension, yet it is quite possible that what makes an illness into a conscious experience is precisely the intellectual element which intervenes." (13)

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12 Logic. The Theory of Inquiry, p. 288.

13 Essays in Experimental Logic, pp. 3-4.

It is this conscious experience that constitutes thinking, and thinking occurs when a situation becomes tensional. We do not reflect until some problem is set before us. Reflection seems to be the dominant trait in a tensional situation, i.e., when there is something the matter, some trouble, due to active discordance, some conflict among the factors of a prior non-intellectual situation.

Let us give an example of this. Suppose that I have a cold. It is possible that I experience a cold without having knowledge of the cold, i.e., without reflecting upon my experience. When I reflect upon this prior non-intellectual experience, I may be said to have knowledge of a cold, and I may judge accordingly: I have a cold. It is these judgments and inferences arising out of a prior non-intellectual experience that are the proper subject matter for logical inquiry. "The theory, in summary form, is that all logical forms (with their characteristic properties) arise within the operation of inquiry so that it may yield warranted assertions. This conception implies much more than that logical forms are disclosed or come to light when we reflect upon processes of inquiry that are in use. Of course it means that; but it also means that the forms originate in operations of inquiry. To employ a convenient expression, it means that while inquiry into inquiry is the causa cognoscendi of logical forms, primary inquiry is

itself the causa essendi of the forms which inquiry into inquiry discloses." (14)

Pragmatic logic feels itself to be close to life's situations, and its subject matter cannot be separated from the meaning its judgments and inferences give to those life situations. The pragmatist logician discovers many lacunae in the more formal logics. Such matters as meaning and its communications, understanding, context, truth, error, relevance, selection, risk, interest and purpose are not given the important place in formal logic that they receive from the pragmatist.

Abstraction from meaning makes logic a meaningless science. The pragmatist believes his logic to be pregnant with meaning. F. C. S. Schiller believes meaning to be a biological function: it is "the great animistic postulate that stirs us up to understand the world." (15) It seems that Schiller, in this matter, takes up different ground from that of Peirce. C. S. Peirce commenting on Schiller's position says: "The brilliant and marvellously human thinker, Mr. F. C. S. Schiller, who extends to the philosophic world a cup of nectar stimulant in his beautiful Humanism, seems to occupy ground of his own, intermediate, as to this quest-

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14 J. Dewey. Logic. The Theory of Inquiry, pp. 3-4.

15 Logic for Use, p. 50.

ion between those of James and mine. I understand pragmatism to be a method of ascertaining the meanings, not of all ideas, but only of what I call 'intellectual concepts,' that is to say, of those upon the structure of which arguments concerning objective fact may hinge." (16) Peirce previously explains the limits he attaches to the scope of meaning when he writes: "Consider a state of mind which is a conception. It is a conception by virtue of having a meaning, a logical comprehension; and if it is applicable to any object, it is because that object has the characters contained in the comprehension of this conception." (17) He further explains that the pragmatic method of ascertaining the meaning of concepts is that of experimentation, a method which, he understands, is that of all successful sciences "in which number nobody in his senses would include metaphysics." (18) Dewey defines meaning as "primarily a property of behaviour, and secondarily a property of objects," it "is not indeed a psychological existence." (19) In this definition of meaning, we see that Dewey is at one with Schiller in maintaining a biological basis for meaning. In chapter three of his

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16 J. Buchler. The Philosophy of Peirce, p. 272.

17 Ibid. p. 235.

18 Ibid. p. 271.

19 Experience and Nature, p. 179.

Logic, Dewey discusses the existential nature of inquiry. This matrix is essentially biological, and meaning is a function within inquiry.

Originally, it seems to the pragmatist, man acts without thinking. This is the prior stage of experience without reflection, or, what Dewey would call inquiry. Thinking is acquired with difficulty, and only arises when man is aware of a tensional situation. Speaking biologically, the scientific level is reached when man reflects and "ascription of meaning is the first and most indispensable step in scientific procedure which is hallowed by biological need and vindicated by success." (20)

Meaning, for the pragmatist, is a living, moving thing. Schiller takes the point of view that every actual meaning must be new and words are personal and social meanings. Verbal meaning is really the raw material for personal meaning. Whatever fixity words may seem to have is due to socially established meanings. But there is no real fixity even in this sense. When we say that meaning is personal, we imply that it has a psychological aspect, and can only be understood from a psychological point of view. There is, for Schiller, nothing objective about meaning, although meaning is ascribed to 'objects.' Meaning seems to be the re-

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20 F. C. S. Schiller, Logic for Use, p. 53.

action of a mind to the course of events, and is 'subjective' in the sense that, without minds, meaning would not exist.

The pragmatist logician scorns the classical battle between realists and nominalists. (21) Universals and particulars are mere abstractions. Both alike are concrete in their use. Universals and particulars are verbal meanings; they function as instruments.

It will be recalled that in our treatment of idealism we discovered that, for valid inference, we needed a concrete universal which was operative throughout the process. This concrete universal was a sine qua non of inference. In our treatment of realist logic, we found that we needed a relation to bind our inference together in order that we might know our inference was valid. The relation of implication is the important relation in realist inference, as the concrete universal is the important operative factor in idealist inference. When we search for the important factor in pragmatist inference, we discover that it is relevance. We shall discuss Schiller's position in this matter first.

Relevance seems to be a practical need, and Schiller charges that, "most prevalent logic still consists in a flat denial of relevance." (22) The idealist would, of course,

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21 'Realism' is to be understood in the sense of Anselm's realism.

22 Logic for Use, p. 75.



answer that, what is really relevant must be relevant to the whole. But this is certainly not what Schiller means by relevance. Relevance, in his view, is selection of the relevant part. He claims nothing absolute for it, and confesses that it is risky and disputable. From the psychological whole, a part is extracted, which seems to solve the problem most satisfactorily to the person, or persons, concerned. He maintains that any given situation for a thinker never embraces the whole of reality. Therefore, if any process of thinking is to be relevant, it must be relevant to a part.

Relevance is more important than truth, and it would seem that, in the order of the actual situations of life, we do not assert things merely because they are true, but rather because they are relevant to the particular situation. "Relevance is prior to the discovery of truth both in time and urgency. The theory of induction has suffered much from the slurring over of the conception of Relevance, even as that of deduction has suffered from the omission of a study of meaning." (23) Relevance seems to be the key to the understanding of pragmatic inference. The question of truth or validity is secondary to that of relevance. Indeed, the truth or validity of a process of thinking depends upon its

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23 Ibid. p. 90.

relevance to the given situation. Says Schiller: "Relevance is the best way of expressing the crucial contrast between the voluntarist and the intellectualist treatment of thinking." (24) It would seem from this point of view, that an inference is true and valid, if it is relevant to the given situation.

But, let us ask more specifically: What does the pragmatist conceive to be truth? It must, of course, be a practical question. Our judgments and inferences are true if they work. It is in connection with the question of truth that we find pragmatists in line with one of the fundamental aspects of modern science. It will be recalled that Bacon maintained in, perhaps, his most famous aphorism that, 'knowledge is power.' This emphasis on power, which is the power of prediction and control, marks a distinction between the Aristotelian basis of science and the dominating interest of modern scientific method whether of the empirical or mathematical type. Schiller maintains that his logic is radically empirical, and in his treatment of truth, we see this radical empiricism brought to the forefront. The significance of our statement may be seen immediately from his remark: "It is safer and sufficient to take the assumption of 'law' merely as a methodological device which has

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24 Ibid. p. 94.

worked." (25)

Truth is a value which has reference to prediction, and a scientific truth which is a value of this kind suggests that science has a practical origin. At this juncture we must point out that, up to the present time, pragmatism has emphasized the empirical aspect of modern science at the expense of the mathematical. Since the explosion of the atom bomb in 1945, formulae, such as  $E=MC^2$ , arrived at several decades ago, may be treated more seriously by pragmatists in the future. However, for the present, we shall follow Schiller's account of the nature of truth.

If science has a practical origin then, "the truths of science are tested by this power to predict the course of events, and their claims to be true by their success in doing so." (26) As far as truth is concerned, science distinguishes between a prediction which comes true and one which fails to do so. Schiller maintains that science cannot identify truth with a truth-claim. It must be pointed out, however, that such a position is radically empirical and in opposition to the mathematical stress of modern physical science. Truth, for the modern physicist, is not a truth that awaits empirical verification. Truth-claim is made

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25 Ibid. p. 95.

26 Ibid. p. 96.

for a conclusion arrived at by the manipulation of mathematical formulae. This is evident on very short acquaintance with such writers as Einstein, Jeans and Eddington. It does seem clear, however, that science cannot content itself with a merely formal truth-claim, since there is always the equally important empirical aspect of modern science. The pragmatist seems to be justified in his attack upon the symbolic logician, in so far as a purely mathematical system is almost meaningless apart from an empirical content. Schiller observes that: "a logic which is observant of scientific usage must test 'true' and 'false' as terms of valuation." (27) It seems quite reasonable to urge that truth, if it is to be truth for human beings, must be of value to them.

With this background of the general nature of pragmatic logic, we may now proceed to investigate the nature of inference. But, first, we must say something of its constituents, i.e., idea and judgment.

(a) Idea.

The theory of ideas developed since the time of Locke is not what pragmatism understands by idea in logical theory. Ideas must be defined functionally as they refer to the solution of a problem. An idea is not merely mental. Of the nature of ideas Dewey says: "An idea is first of all

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27 Ibid. p. 98.

an anticipation of something that may happen, it marks a possibility." (28) Apparently, at first, an idea is something that is rather vague, and occurs in a situation as a suggestion. "Suggestions just spring up, flash upon us, occur to us." (29) They are not mental at this stage but may become stimuli for further activity. At this primary stage, an idea has no logical status as such. "Every idea originates as a suggestion, but not every suggestion is an idea. The suggestion becomes an idea when it is examined with reference to its functional fitness; its capacity as a means of resolving the given situation." (30) So far, in the matter of idea, we seem to have: first, a suggestion. But, this suggestion is not to be confused with a present given existence. Let us explain this. As I look at my desk there are many objects which come into my scope of vision, e.g., a lamp, a bottle of ink, some books, etc. I perceive all these things, yet the situation at present has yielded no suggestion. But, as I gaze, the bottle of ink 'just springs up, flashes upon me, occurs to me.' There is no tensional situation at present, but there is a suggestion. The suggestion may, or may not, ripen into an idea; nevertheless, there

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28 Logic. The Theory of Inquiry, pp. 109-10.

29 Ibid. p. 110.

30 Ibid.

is a functional situation here that is not present in given existence as such. If the suggestion develops into an idea, it implies that a symbol is involved, i.e., the idea becomes explicit as a bottle of ink. This does not mean that I add the symbol 'bottle-of-ink' to a physical thing that now takes on a meaning. The thing is a function, and the symbol is a function. The two become one functional situation at the level of idea. If, then, I say that I have an idea of a bottle of ink, I do not mean that my idea is a mental content, but, that, mind is functioning, i.e., the function at this level is the function of mind.

The foregoing seems to be what Dewey means by an idea, i.e., an idea is a cue to action. Ideas are data for logic. They are logical subject matter in the sense in which we have defined idea. Before passing to a discussion of judgment, it will be necessary to make a few remarks as to what Dewey considers to be the nature of reasoning. He states: "Reasoning is sometimes used to designate inference as well as ratiocination. When so used in logic the tendency is to identify inference and implication and thereby seriously to confuse logical theory." (31) It appears that inference, for Dewey is something totally different from what an idealist, or realist, would conceive it to be. Since reasoning

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31 Ibid. p. 110, n.

is often equated with inference, Dewey makes a distinction between the general sense of reasoning and reasoning in the sense of ratiocination. He states: "The process of operating with symbols (constituting propositions) is reasoning in the sense of ratiocination or rational discourse." (32)

This matter has a bearing on the pragmatist view of inference which we shall develop later on. For the present, we must discuss the matter of judgment.

(b) Judgment.

We shall turn first to Schiller. He conceives judgment to be the central theme of logic, and maintains that it is essentially experimental, the end of deliberation and a personal act. As against idealism, he would urge that: "There is much thinking before there is any judging." (33) As to the genesis of thinking, he conceives it to be an abnormality arising out of a disturbance. "When it becomes biologically important to notice differences, impulse breaks down and thought, reason, reflection emerge." (34) Even in developed minds, Schiller maintains, judgment is relatively rare in thinking, but when a tensional situation arises and we judge, we do so in the hope that our judgment will

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32 Ibid. p. 111.

33 Logic for Use, p. 194.

34 Ibid. p. 195.

solve the immediate problem for us. But, how shall we know if our judgment is true? We may be assured that if our judgments are true they will not refer to anything absolute or transcendent, since "we must not abstract from the personal side of knowing." (35) Truth and falsity are correlative for Schiller, and truth is merely the success of an enterprise. Let us give an example. I tell my child that Santa Claus will come on Christmas Eve. The enterprise is highly successful. My judgment that Santa will come on Christmas Eve, functions as truth in that situation. Since truth and falsity are correlative, my false judgment is true. "Truths are relative to the state of knowledge they formulate." (36) Truth is psychological, and satisfies a cognitive purpose. If a judgment works, it is true. On empirical grounds it is justified by its consequences, and the consequences of telling a child that Santa will come are empirically justified. "The true is always good to believe, it is what is best to believe." (37) We must point out, however, that the pragmatist does not make truth merely convertible with what works, nor identify it with mere usefulness. Truth is personal, and, in that sense, my judgment is true as it refers

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35 Ibid. p. 145.

36 Ibid. p. 150.

37 Ibid. p. 154.



to the child and the functional situation in which I judge. As to error Schiller urges that "Truth becomes an error when it is found out and abandoned." (38) Our illustration of the child and Santa Claus would show how truth becomes error when Santa is 'found out' and abandoned. When we turn to Dewey, we find that he follows Peirce in the matter of the definition of truth. "The best definition of truth from the logical standpoint which is known to me is that of Peirce: 'The opinion which is fated to be ultimately agreed to by all who investigate is what we mean by truth, and the object represented by this opinion is the real.'" (39) This definition would involve, in its scope, our previous discussion of truth.

We return, more specifically, to our discussion of judgment. Dewey defines judgment as "the settled outcome of inquiry." (40) He distinguishes between judgment and proposition, by accounting the latter representative, inasmuch as it is carried, as it were, by symbols. Judgment, for Dewey, has existential import. A literal instance of judgment, in this sense, is provided by the judgment of a court of law which settles an issue which has been in con-

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38 Ibid. p. 185.

39 Logic. The Theory of Inquiry, p. 345, n.

40 Ibid. p. 120.

troversty, i.e., a tensional situation is settled. A judgment of this sort is settled by inquiry. The whole proceeding is of a practical nature with a practical end in view. The case is disposed of, as it were, and the disposition takes effect in existential consequences.

The subject of judgment is that which is selected in the course of inquiry. This selection depends upon its evidential significance. The matter, perhaps, is better understood from a negative point of view. Judgment, for Dewey, and for pragmatism generally, is not the reference of an ideal content to a reality beyond the act. Neither is judgment a determination in thought of the character of an object present to thought: i.e., it is not mere predication. The subject of judgment is functional. It is not substantial, except in the sense where substance would indicate a logical determination. "Sugar, for example, is a substance because through a number of partial judgments completed in operations which have existential consequences, a variety of qualifications so cohere as to form an object that may be used and enjoyed as a unified whole." (41) From this statement, it would appear that logical subject cannot be separated from logical predicate. Indeed, Dewey states: "The logical meaning of predication has been

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41 Ibid. p. 128.

anticipated in the discussion of the logical subject, because of the strict correlativity of respective existential and ideational contents." (42) The predication contents of judgments are the meanings which are suggested as possible solutions of a problem. These meanings become predicates as they are used to direct further operations of experimental observation. Subject and predicate are related as the actual is to the possible. But, when judgment is completed, subject and predicate are unified in an existential whole.

The nature of the copula, too, is determined by that of the subject and the predicate. It is not an independent element, neither does it merely affect the predicate. It is, "the complex of operations by means of which (a) certain existences are restrictively-selected to delimit a problem and provide evidential testing material, and by which (b) certain conceptual meanings, ideas, hypotheses, are used as characterizing predicates." (43) Copula seems to be a name for the functional correspondence between subject and predicate. E.g., If I judge: The rose is red; 'the rose,' an indeterminate situation and 'red' also an indeterminate situation are transformed into one determinate situation. In this sense the copula, in distinction from the term of

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42 Ibid. p. 131.

43 Ibid. p. 133.

formal relation, is a name for the functional correspondence.

(c) Inference.

We must now pass on to consider the nature of inference. Turning first to Schiller, we find that inference is essentially psychological. "Logical inference is, in fact, always rooted in psychological soil, and derives its impetus from psychological forces." (44) Questions of formal validity and necessity, for Schiller, are merely matters that have developed from the logician's will to power. Inference is a natural, psychological act, and logical inference is merely that sort of inference that is brought to the logician's notice and approbation. From a psychological point of view, Schiller conceives inference to be "any passage of thought from one judgment to another." (45) For example: When I look out of my window and see the side-walk wet, I infer that it has rained, i.e., I judge: It has rained. The judgment is representative of the passage of thought from a previous judgment: The side-walk is wet. As to the value of such an inference; it is confirmed by its working. Its truth-claim has to be tested and verified like any other truth-claim. This is, of course, a radically empirical point of view, but, it is only on the grounds of such

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44 Logic for Use, p. 247.

45 Ibid. p. 248.

empiricism that pragmatic inference can be understood. Like the realist, the pragmatist is afraid of confusing inference with implication. Inference is a practical, personal matter. It is a good inference, if its consequences are good to those concerned. It is bad if the consequences are bad. Schiller feels that this is all that needs to be said about the genesis and true nature of inference. "Having traced both the occurrence and the value of every inference to a personal train of thought, and to the reaction of a human mind to a vital situation as 'predicament,' have we not completed the material history of inference? What need is there to superimpose on it any further account in the name of logic, to fabricate a fictitious scheme of 'logical relations,' and to concatenate artificially, sequences of judgments which already cohere naturally? Has not the whole formal conception of inference become unreal and superfluous?" (46) Schiller proceeds to show how, from his point of view, the fictions of formal inference are false, needless, useless and harmful. It is interesting enough to follow him in this diatribe, yet, one must confess that he is not always quite fair in his estimation of formal inference.

Schiller conceives that the whole value of inference, as a thought process, is decided by its working. Inference

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<sup>46</sup> Ibid. p. 250.

does not differ greatly from judgment. Both judgment and inference have their origin in vital situations. Both are essentially experimental and purposive, and their ultimate aim is the control of experience. The difference between judgment and inference is to be found in the fact that the truth-claim of inference is mediated by other judgments, whereas the truth-claim for a judgment is direct.

But, the truth-claim for our inferences is not dependent upon any question of formal validity. "Our reasoning need not be valid to be valuable, a good proof need not be coercive to be persuasive and even convincing, and a truth may grow more probable and certain throughout the ages, without ever becoming absolute." (47) Does this mean that, for the pragmatist, there is a great gulf between the empirical and mathematical methods of knowing? Schiller denies this and is in line with Dewey and pragmatism generally on this point. "It can no longer be conceded that mathematical truth is sui generis, purely a priori, ultimate and unanalysable." (48) As an example of this, we may point to the statement that, 'the angles of a triangle are equal to two right angles.' This depends for its truth upon a basis of Euclidean geometry. It would not be true if the geometrical basis were

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47 Ibid. p. 319.

48 Ibid. p. 357.

that of Riemann or of Lobachevsky. Modern mathematicians seem quite clear to-day that, mathematical definitions and postulates admit of alternatives. Like other hypotheses, mathematics is stimulated by some empirical cue. (49)

Pragmatism seems to be agreed that the claim to be a priori, on the part of mathematics, depended upon its usefulness. But, to-day it is found in some cases, such as in Einstein's physics, that a non-Euclidean basis is more useful. On this basis, it is no longer true that the interior angles of a triangle are equal to two right angles. It is the success of the mathematical sciences which authenticates their principles, not the infallibility of the principles themselves. To make logic mathematical, and to make valid inference a matter of implication is to distort the true nature of inference.

Let us now sum up very briefly. For the moment, we shall turn to P. A. Schilpp, who edits The Philosophy of John Dewey. According to Donald A. Piatt, inference is from the mind to the world. For example, a perceived cloud is the means whereby we infer rain as the end. The perceived cloud functions as a meaning. The meaning is in the thing. Dewey does not deny that the perception is cognitive. He does deny, however, that it needs to be judgmental. Some inferences,

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<sup>49</sup> Cf. E. Kasner and J. Newman. Mathematics and Imagination.

apparently, do not rest upon judgments. Piatt states the position as follows: "Inference from clouds as a natural sign to rain as what is indicated is...cognitive; but it is not judgmental, or does not have to be. It is a matter of habit and expectation. We do not have to think to make such inferences, for the expectation is grounded in material occurrences. Meanings are in things before they are in minds." (50) This instance would not be a case of valid inference. Valid inference has to be established by inquiry. "Dewey admits that reasoning is important but claims that it is distinct from inference and powerless apart from inference." (51)

Reasoning is powerless apart from inference, i.e., reasoning involves inference, but is not to be equated with it. Dewey states: "Just as the sign-significance relation defines inference, so the relation of meanings that constitutes propositions defines implication in discourse, if it satisfies the intellectual conditions for which it is instituted." (52) Let us explain this matter. In the case of inference, the sign 'cloud' has the significance 'rain.'

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50 The Philosophy of John Dewey, pp. 120-1. Also, Cf. W. McDougall, Social Psychology, pp. 30-31, where McDougall describes the reaction of birds to material occurrences. McDougall does not think that they think.

51 Ibid. p. 124.

52 Logic. The Theory of Inquiry, p. 54.



The primary relation of inference is the extrinsic office by which one thing points to another. When we reason, symbol-meanings function in relation to each other, i.e., either ideas (as we have described the nature of idea previously), or judgments as propositions (also previously described). Now, let us give an example of reasoning. The concept 'blue' implies the concept 'colour.' Here we have a case of an intrinsic office, whereby one symbol-meaning points to another. We must bear in mind that natural sign and symbols have in common the fact that they are existential events; i.e., the sign blue (the blue I see), the symbol-meaning blue, whether concept or word, are existential events which function in inference and reasoning, in Dewey's sense of ratiocination.

In the realm of ratiocination we have language, and the advantage of language is that it provides tools for thought that can be manipulated quite independently of external events. Language gives us formal logic, which is only valid, if it is useful. With language, inference no longer has to be a matter of luck; it becomes grounded in science and previous inquiry, and it is subject always to fresh inquiries.

In the foregoing, we have given an account of pragmatic inference, and by way of concluding this chapter, we merely add that, all the traditional and formal distinct-

ions, such as immediate inferences, extension and intension, induction and deduction, moods and figures, etc., are relatively meaningless to the pragmatist.

## CHAPTER V

### CONCLUSION

When we compare the theories of the nature of inference we have discussed above, it seems to us that the idealist theory is the most acceptable. We find grave faults with both the realist and pragmatist theories, on the ground that their respective theories rest upon a faulty logic.

Let us make a brief criticism of the realist position first. We have previously stated that symbolic logic finds its most important beginnings in the work of George Boole. Boole heartily believed in the certainty of mathematics, and since he also believed that the processes of the mind are essentially mathematical, it followed that a logic based upon mathematical principles would yield absolute certainty. Mathematical logicians generally believe that only on the basis of a mathematical logic can the true nature of valid inference be discovered. Now, the important aspect of valid inference, for the realist, is the relation of implication. This position immediately raises the question as to whether the realist logician has any grounds at all for the truth-claim of his system. "Formal logic is little concerned with problems of truth. Its rules govern the valid-

ity of arguments rather than the truth of premises. To be sure, most of the time we wish to reason from premises which we know are true, because we can be sure in that case that any valid conclusion we may draw from them will also be true." (1)

The formal logician is sure that the relation of implication from true premises will yield a true conclusion. But on what grounds is he sure of this? Is it because he has discovered a world of sheer necessity? Boole, indeed, maintained that, because a general law of the operation of the mind is, in fact, established from one instance, this fact guarantees "the strict necessity of that law from which deductions can be made which are absolutely true." (2) But it will be recalled; the data upon which the law is established are empirical data. If the data are empirical it makes little difference whether the law is established by one instance, or by many. The apparent compulsion of the mind to establish causal necessity in the case of many instances, and its apparent compulsion to establish logical necessity in the case of one instance rests ultimately on the same basis of empirical observation. This criticism suggests that, in the last analysis, mathematics is an

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1 Bennett and Baylis. Formal Logic, p. 9.

2 G. Boole. Laws of Thought, p. 425.

a posteriori science, rather than an a priori science. If, however, what one observes in one instance can be established as certain and absolute, then the mathematical logician has gained his point. But we must look further into this matter.

We have previously discussed, in our treatment of realism, Russell's argument for the ontological status of logical essences. If the logical essence theory is true, then realism has discovered the world of strict necessity, where it can be stated categorically that, "what follows from a true premise must be true." We find Russell stating: "Hence we must admit that the relation, like the terms it relates, is not dependent upon thought, but belongs to the independent world which thought apprehends but does not create." (3) This newly discovered land of logical essences is evidently a place where all is bright and beautiful, a land, the contemplation of which generates within the human breast feelings akin to religious fervour. Russell speaks with feeling when he says: "The world of universals, therefore, may also be described as the world of being. The world of being is unchangeable, rigid, exact, delightful to the mathematician, the logician, the builder of metaphysical systems and all who love perfection more than life." (4) This language is strange, coming as it

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3 The Problems of Philosophy, p. 153.

4 Ibid. p. 156.

does, from a formal logician. It would seem to be a sound criticism to suggest that the reality feeling evidenced in the above quotation somewhat prejudices the claim to a strict formality. Stranger still is the statement that this world of universals is appreciated most by those who love perfection more than life. What is meant here is extremely obscure. One thing, however, seems clear. It is living mind, and living mind alone that is able to bring to life these bloodless universals, if they subsist. It is doubtful, in a biological sense, whether a bloodless and fleshless skeleton is closer to perfection than the whole man.

Esthetically it is doubtful whether dry bones are to be preferred to life. Logically it is doubtful whether rigid, unchangeable essences can yield necessity in a world of living thought. Can these dry bones live? It does not seem unjust to Russell to suggest that, it is only when the dry bones are brought to life, granting that they do subsist, by the activity of living thought that their proximity to perfection is realized. A strictly - bloodlessly formal logic is impossible. Russell betrays his feelings, and it is the sheerest nonsense to suppose that he escapes anthropomorphism when contemplating his world of logical essences. This world of logical essences, if it subsists, cannot be known by finite mind except when brought down and

couched in the fleshly veil of materiality. (5)

We have, in our treatment of realism, put forward Russell's argument for the independent status of such a relation as 'north of' in the judgment; Edinburgh is north of London. Russell may, in his own estimation, have successfully dealt with Berkeley and Kant. Let us assume that he has. He has not, however, dealt at all with Hegel, Bradley and Bosanquet. The words 'north of' are significant for mind. Surely it is mind and mind only that gives significance. 'Edinburgh is north of London' has no meaning whatsoever except as it is the externalized propositional form - dead and lifeless in itself - of living thought in judgment. Without mind there could not be Edinburgh, nor north of, nor London, since each is significant. 'North of' could not be, unless it were significant. To say it is significant (which it plainly is) and then to say it subsists apart from mind is tantamount to saying that significance subsists apart from significance, (6) i.e., in symbolic form we would have: 'A' (The class of significance); and it is axiomatic with symbolic

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<sup>5</sup> Cf. pp. 92 ff for further criticism of Russell's views. We might also raise the question whether a theory that demands correspondence with fact (physical fact) can properly apply to 'essences.'

<sup>6</sup> We note that realists always claim that we recognize, when we judge that 'A is B', that the truth (factuality) of AS being B is independent of the fact that we are in process of apprehending it (it is objective not subjective). Cf. H. A. Prichard, Kant's Theory of Knowledge, p. 118 and Cook-Wilson, Statement and Inference, Vol. 1, p. 36.

logicians that every class is a sub-class of itself, i.e.,  $A \subseteq A$  ( $A$  is included in  $A$ ). But, if significance subsists apart from significance, then, in the language of symbolic logic  $A \not\subseteq A$ . To say that,  $(A \subseteq A) = (A \not\subseteq A)$  is absurd. It is the same as maintaining that  $A = A'$ .

The truth would seem to be that nothing can be said to exist, or subsist, apart from mind. All things that can be thought are surely significant. Logical essences are significant, i.e., we may think about them; they may delight us, and we may love them more than life. Yet, they are significant and in consequence mental. The world as we know it, in whatever fulness and variety it has for each one of us, is a construct of thought. The world as we know it is a part of reality itself. "If the object-matter of reality lay genuinely outside the system of thought....thought itself would be unable to lay hold of Reality." (7) Surely, of all self-evident truths, this is the most self-evident. The world, for knowledge, is a logical construct, since it is plain that no other world could be known. "The real world for every individual is emphatically his world; an extension and determination of his present perception, which perception is to him not reality as such, but his point of contact with reality as such." (8)

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7 B. Bosanquet, Logic, Vol. 1, p. 2.

8 Ibid. p. 3.



It would seem, from what has been said, that the ontological status of logical essences subsisting apart from mind is a delusion. The relation of implication is not a delightful, rigid and exact essence which the mind apprehends and uses for purposes of yielding strict necessity. More correctly, it might be described as a finite tool of finite mind which the mind observes in studying its own operations. It might be said that, by induction, from one such instance, the observing mind flies to generalization. Deductions from such generalizations cannot yield strict necessity, since the universal used is abstract, not concrete. If probability is what realism says it is, then we must admit that there is a greater degree of probability in conclusions from such generalizations than there is in conclusions from ordinary scientific generalizations. But there is no necessity in any ultimate sense exhibited here. It will be obvious from what we have said in our discussion of inference, from the idealist standpoint, that only when we work from the side of the universal, and only when the universal is concrete may we expect valid inference.

Symbolic logicians seem to delight in abstractions, and the more abstract a system is, the more powerful, so they believe, it becomes in yielding valid, i.e., strictly necessary conclusions. Traditional logicians, also, with their whole apparatus of A.E.I.O. forms, and the systematic

patterns of formal interrelations of such propositions in Syllogisms, Dilemmas, and the rest, seek to give their students a control over the various entities which may furnish the concrete filling-in of these abstract forms. They regard these abstract forms as giving power over the content, whatever that content may be. A realist logician like Cook Wilson, however, would maintain a more conservative position. He would insist that 'form' and 'matter' are correlative; that neither means anything whatever, except in its counter reference to the other. The true reality for Cook Wilson, as for Aristotle, is always both universal and particular. He shows, taking the actual forms of the traditional syllogism, precisely what its 'matter' is, and that this is in no case completely covered by the alleged abstract form, but requires a genuine judgment, if it is to be operative in thought.

Formal logic is really very informal. It takes up a good deal of space even to prove the 'Principle of Tautology.' We take an example from Principia Mathematica. Second Edition, Vol. 1, p. 96. Primitive Propositions, No. 1, 2.

$\vdash: p \vee p. \supset. p$

This proposition states: "If either  $p$  is true or  $p$  is true, then  $p$  is true. To test the truth of this proposition we use the matrix method as follows: -

p	p	$p \vee p$	$p \vee p. \supset. p$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

We feel constrained to add, also, that a tautology is a case where thought has not moved. The foregoing example is not a tautology at all. The principal relation of implication relates differences, and where there is difference there can be no tautology.

But let us proceed to examine the truth-claim of the symbolic logician. The result of our inquiries may cast considerable doubt on the claim of the symbolic logician to have discovered a system which, by the manipulation of mathematical symbols, yields certain and necessary truth. In Principia Mathematica (9) certain 'Primitive Propositions' are set forth. The first of these is stated thus: "Anything implied by a true elementary proposition is true." An explanation is appended to this proposition which states: "We cannot express the principle symbolically partly because any symbolism in which p is a variable only gives the hypothesis that p is true, not the fact that it is true." We are apparently dealing with facts that are true.

Before proceeding with this examination any further, it will be necessary to clarify our minds as to what the symbolic logician conceives to be a fact that is true. We shall appeal to Russell's The Problems of Philosophy to instruct us. Russell states: "(i) Our theory of truth must be such as to admit of its opposite falsehood.....(ii) It seems fully evident that if there were no beliefs there could be no falsehood, and no truth either.....In fact, truth and falsehood are properties of beliefs and statements.....(iii) But, as against what we have just said, it is to be observed that the truth or falsehood of a belief always depends upon something which lies outside the belief itself." (10)

Russell continues the argument by saying: "If I believe that Charles I died on the scaffold, I believe truly, and not because of any intrinsic quality of my belief, which could be discovered by merely examining the belief, but because of an historical event which happened two and a half centuries ago. If I believe that Charles I died in his bed, I believe falsely: no degree of vividness in my belief, or of care in arriving at it, prevents it from being false, again because of what happened long ago, and not because of any intrinsic property of my belief. Although truth and falsehood are properties of beliefs, they are properties dependent upon the relation of the beliefs to other things,

not upon any internal quality of the beliefs." The upshot of the matter is that Russell believes that truth consists in some form of correspondence between belief and fact, i.e., a belief is true when it corresponds to a certain associated complex, and false when it does not.

The theory of correspondence seems to be obscure. There is one thing, however, that is plain. It is this. When Russell asserts that a belief is true when it corresponds to a certain associated complex he is, in fact, believing a belief. His argument may be discredited by an argument he uses against the 'pure' empiricists, (11) i.e., the validity of the correspondence theory cannot be proved by the correspondence theory. (12) Russell believes the correspondence theory. It cannot be maintained that the correspondence theory itself is a fact of acquaintance which just is, and concerning which the question of truth and falsity does not apply. The question of truth or falsity may be asked concerning the correspondence theory itself. If I believe that the correspondence theory is true; by what criterion shall my belief be judged except by the correspondence theory itself? But the correspondence theory maintains that,

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11 Russell states that the inductive theory cannot be proved by induction.

12 Cf. R. C. Lodge, "Tests of Truth," in the Journal of Philosophy, Vol. XVII, 1920, pp. 71-77.

for a belief to be true it must correspond to a certain associated complex. On Russell's own argument there could be no associated complex to correspond with the correspondence theory. The correspondence theory is a belief that cannot be tested, hence we do not know if the theory itself is true or false. But Russell believes it to be true in spite of the criterion of truth he sets up for himself.

Surely the criterion of truth should be wide enough to include itself as a difference within the identity of the whole. It is evident that the correspondence theory, as a belief, does not, for the realist, cohere with what he himself believes to be a belief. It would seem to be more satisfactory to maintain that the truth or falsity of our beliefs are to be tested by their coherence with the whole of our knowledge. The coherence theory seems to escape the doom of the correspondence theory in that the truth of the theory is its own test.

But let us examine what the realist maintains he knows to be true or false. Can we ever know anything at all, or do we merely sometimes by good luck believe what is true? First, let us be clear as to what the realist means by 'knowing.' Russell explains it in this way: "If a man believes that the late Prime Minister's last name began with a B, he believes what is true, since the late Prime Minister's name was Sir Henry Campbell Bannerman. But, if he believes that

the late Prime Minister was Mr. Balfour, he will still believe that the late Prime Minister's last name began with a B, yet this belief, though true, would not be thought to constitute knowledge." (13) This statement of Russell's is nothing short of fantastic. If Russell means what he says, then, his statement condemns the three mighty volumes of Principia to the limbo of things that may contain truth, but cannot be used to give knowledge.

Let us proceed to an examination of the statement to see what Russell implies. He states: "If a man believes that the late Prime Minister's last name began with a B, he believes what is true, since the late Prime Minister's name was Sir Henry Campbell Bannerman." (14) Now, according to the correspondence theory, a belief is true when it corresponds to an associated complex. This complex is the fact that guarantees the truth of the belief. Have we, in this statement, the necessary factors for a true belief? It would seem that we had. The complex in this case is the class B, which is the class of all those late Prime Ministers whose last name began with B. It is axiomatic with symbolic logicians that universals subsist. "As in the case of concepts the existence ascribed to classes is of a very minimal kind,

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13 The Problems of Philosophy, p. 205.

14 Ibid.

quite unlike the spatio-temporal existence of material individuals." (15) "A class should be distinguished from its members, for classes have properties which are possessed by no one of their members nor by all of them jointly, and, conversely the members of a class possess characteristics the class lacks." (16) Now, the class B is a single membered class. The member, Sir Henry Campbell Bannerman, is not to be confused with the class. Russell courts paradox when he denies knowledge of classes and extends it only to class membership. But, is not this the implication of his statement? The whole structure of Principia is based upon the possibility of a class calculus being reinterpreted as a calculus of propositions.

It would seem, in the instance given by Russell that there are two judgments, not one. The first judgment states belief that the late Prime Minister's last name began with B. It is difficult to see why Russell does not count this as knowledge, since it possesses all the necessary factors for knowledge on the correspondence theory. The only ground for the denial of this knowledge, by the realist, would be on the basis of the non-existence, or non-subsistence of classes. But, a class is a universal, and, for the realist a logical

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15 Bennett and Baylis, Formal Logic, p. 107.

16 Ibid. p. 83.



essence which subsists.

In technical work it may be permissible to disregard the subsistence of such entities, as indeed Whitehead and Russell do, when the class calculus is reinterpreted as a propositional calculus. "The....theory of classes, although it provides a notation to represent them, avoids the assumption that there are such things as classes. This it does by merely defining propositions in whose expression the symbols representing classes occur...." (17) But, in a question concerning knowledge, it is surely not permissible to disregard basic assumptions arbitrarily. To deny that it is knowledge to know that the late Prime Minister's last name began with B, is tantamount to the denial of any knowledge whatsoever under the realist's correspondence theory of knowledge.

But, let us pass on to another point that awaits our criticism in the realist's account of knowledge as put forward by Russell. He states: "In like manner a true belief cannot be called knowledge when it is deduced by a fallacious process of reasoning, even if the premises from which it is deduced are true. If I know that all Greeks are men and that Socrates was a man, and I infer that Socrates was a Greek, I cannot be said to know that Socrates was a Greek,

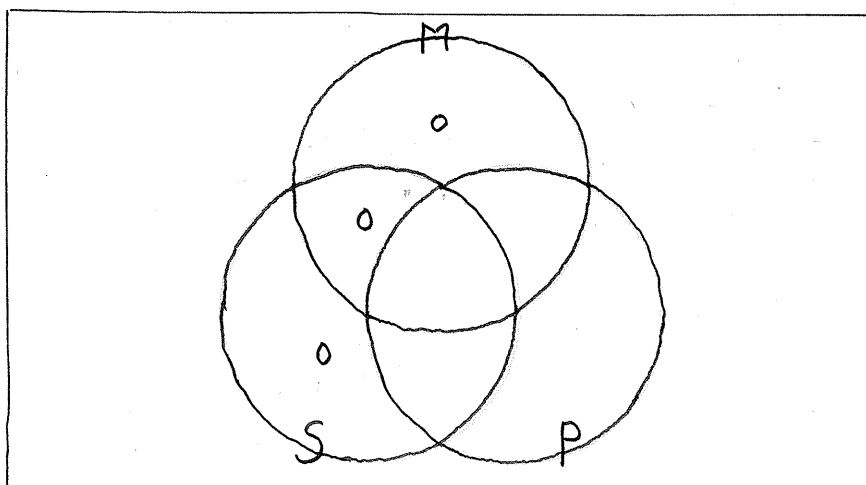
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17 Principia Mathematica, p. 187.

because, although my premises and my conclusion are true, the conclusion does not follow from the premises." (18)

In the foregoing syllogism, both realist and idealist would agree that we do not possess the conditions for a valid conclusion, although the conclusion may be true. But it is for quite different reasons that the realist and the idealist would deny the validity of the conclusion to yield true inferential knowledge. The realist would deny the validity of the conclusion on the ground of the fallacy of affirming the consequent: he would maintain that the relation of implication does not hold. The proof of this fallacy may be exposed, in realist manner, by the algebraic method, with the aid of a Venn diagram.

All Greeks are men	$MP \subseteq O$
<u>Socrates was a man</u>	<u><math>SP \subseteq O</math></u>
Socrates was a Greek	$SM \subseteq O$



With the aid of the Venn diagram we have "one of the easiest methods of determining implied relations and one of the most convincing devices for corroborating formal demonstration."

(19) At this juncture we must ask the question: On what grounds do we know, assuming that we know the truth of our premises, that our conclusion is false? The realist reply is that, the relation of implication does not hold, and this reply is made convincing by formal demonstration. Knowledge, for the realist, is what we firmly believe to be true, if it is intuited or inferred from intuited knowledge from which it follows logically. Intuitive knowledge, for the realist, is that knowledge which has the highest degree of self-evidence, i.e., when our belief or judgment corresponds to fact. "Our theory of truth.....supplies the possibility of distinguishing certain truths as self-evident in a sense which insures infallibility." (20) Given, then, known premises in the highest degree of self-evidence, in the realist sense, we have a guarantee that our derivative knowledge is infallible. In other words: "what follows from a true proposition must be true." (21)

Let us examine this statement to see if it really

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19 Bennett and Baylis, Formal Logic, p. 121.

20 Ibid. p. 210.

21 Ibid. p. 184.

stands up to what the realist would call facts. It is a self-evident truth, in the realist sense, that there are more fractions than integers. If I assert this in propositional form, it implies that the contradictory of the proposition could not be true. We also recall Russell's amazing statement that: the law of contradiction is a law of things rather than a law of thought. The laws of mathematics are likewise laws of things, and not laws of thought. They are rigid, exact and delightful. Beginning with the most elementary assertion that two and two are four; a self-evident truth about which there can be no mistake, i.e., we know that two and two are four; mathematical implication leads us to the astounding conclusion that there are no more fractions than integers. "Cantor discovered a simple but elegant proof that the rational fractions form a denumerably infinite sequence equivalent to the class of integers. (22) We have then, according to the realist theory of truth, two propositions that contradict each other, and both propositions we know to be infallibly true, since, one is self-evident and the other is implied as a valid result from basic mathematical principles which we intuit and know to be true. No one as yet, has been able to detect whether Cantor has erred in his reasoning. He has used the exact and rigid relations

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22 E. Kasner and J. Newman, Mathematics and Imagination, p. 48.

of mathematics and the result finds common sense in alien corn.

Also one could point to the work of the distinguished Polish mathematicians, Banach and Tarski, who extended the implications of Hausdorff's paradoxical theorem to three-dimensional space with the incredible result that, "There is a way of dividing a sphere as large as the sun into separate parts, so that no two parts will have any points in common, and yet without compressing or distorting any part, the whole sun may be fitted snugly into one's vest pocket." (23) This conclusion is "rigorous and unimpeachable." (24) "Surely no fairy tale, no fantasy of the Arabian nights, no fevered dream can match this theorem of hard mathematical logic." (25)

Also we may point to the perennial interest in the paradoxes of Zeno, to support our contention that the claim of realism to infallible knowledge is quite unwarranted. (26)

It will be apparent, from the foregoing discussion, that the correspondence theory of truth fails to show that ultimate truth is attainable. The claim of the formalist to

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23 Ibid. p. 207.

24 Ibid. p. 205.

25 Ibid. p. 207. Italics mine.

26 Cf. Andrew Ushenko in Mind, Vol. LV, No. 218, April 1946, pp. 150 ff.

infallibility, and the extravagant language of absolute certainty, is an exploded myth. We have previously intimated that a more satisfactory theory of truth is to be found in the coherence theory. It remains to say something about this theory in order that our statements may be more readily acceptable. In what follows, we shall appeal largely to the writings of F. H. Bradley.

While it is true that one may detect in Bradley's writings three different theories of truth, i.e., correspondence, coherence and identity, yet, it is nevertheless true that the theory of coherence is the one that is treated at greatest length and which, for finite mind, Bradley conceives to be the most satisfactory.

As far as Bradley's treatment of the correspondence theory is concerned, it would seem that such truth is but the appearance of truth as viewed from the particular point of appearance. Sometimes, however, when he is treating of the doctrine of such as James, Dewey and Russell it might seem that he is making concessions to them, but this is really not the case. What James and Dewey and Russell have to say may be true enough in certain cases on the level of appearance. As we have previously intimated, it would seem that infallible truth could be obtained if it were possible, as in the case of Russell, to isolate a system entirely, i.e., 'pure' mathematics and mathematical logic would yield infal-

lible conclusions if it were really possible to isolate these systems, i.e., really abstract them. In such a case they would be complete wholes containing within themselves their own necessity. But, such an abstraction would seem to be impossible to be known by finite minds. The philosophies of James, Dewey and Russell can be regarded by Bradley as 'elements included within our view of the Absolute;' but, these systems cannot, by their very nature, yield truth that is in any sense ultimate. "Within limits and in their proper place our relative view insists everywhere on the value and on the necessity of absolute judgments both as to right and wrong and as to truth and error. Life in general and knowledge in particular rest on the distinction and on the division of separate regions....so far as we can assume this, we of course can have simple error and mere truth. Thus the doctrine which I advocate contains and subordinates what we have called the absolute view, and in short justifies it relatively. (27) The 'relative view' is the view that parts are relative to the Absolute. Of course, the correspondence theory, from the relative point of view, is only one possible theory of truth. An instrumental theory is equally valid.

In a different section of the Essays we find a much more comprehensive statement of the matter. "If we now

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27 F. H. Bradley, Essays on Truth and Reality, p. 265.

return to that view for which truth is a mere copy of things, we have seen that in the end no such doctrine is admissible. But from a lower point of view it may be convenient to speak of truth as corresponding with reality and as ever reproducing facts. In the first place the individual in truth seeking must subject himself. He must suppress ideas, wishes, and fancies, and anything else in his nature which is irrelevant to and interferes with the process of truth seeking. ....Our understanding has to cooperate in the ideal development of reality, and it has not, like will, to turn ideas into existences. And thus following the object the ideas of the individual in a sense must conform to it." (28)

"In the above sense truth may be spoken of as corresponding to facts, and it is right and proper as against one-sided views to insist upon this correspondence." (29)

It must be remembered in connection with the foregoing that, what Bradley means by a fact is something quite different from what Russell would conceive to be a fact. A fact for Bradley is in the nature of an ideal object. Also, we must point out that, in the second sentence of the first quotation above, where Bradley says: 'in the end,' he apparently means from the point of view of Reality.

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28 Ibid. p. 118 ff. Italics mine.

29 Ibid. p. 119 ff. Italics mine.



Also, when he speaks of a 'lower point of view' he apparently means from the point of view of appearance. From what Bradley has to say, we would gather that the possibility of error lies in the confusion of appearance with reality.

The realist criterion of truth is correspondence with an associated complex. For the idealist, however, if he is able to adjudge anything true or false, his criterion is not a criterion of truth, but rather a criterion of Reality. We can never be absolutely certain that our judgments are infallible since, in our small corner of Reality, we can never predicate immediately of Reality itself, but only of that portion of reality which appears to us in presentation. For the idealist, truth and falsehood are not only concerned with judgment as such, but with phenomena as well. This statement is very clear when we consider that, for the idealist, knowledge is a logical construct, immanent within experience. He is not embarrassed, as is the realist, by a something, he knows not what, to which his judgment is said to correspond. It will be recalled that Bosanquet says that, 'knowledge is the mental construction of reality.'

If the idealist logician is to adjudge anything as true or false, his criterion will be one of reality rather than of truth. Both judgment and phenomena are differences within the identity of the whole construct of knowledge, which is the activity of mind in its self-development. As

judgment coheres with the rest of our growing knowledge, so those judgments are coming to be true or false. But, as far as we can know, the judgments are hypothetically, not categorically, true or false. Since we condemn the inconsistent, it would seem that we should uphold the consistent.

"If we can be sure that the inconsistent is unreal, we must be logically sure that reality is consistent." (30) It should be noted that the point of Bradley's statement depends upon the condition with which it begins.

It remains to view, very briefly, the pragmatist position in the light of all that we have said regarding what we believe to be the true nature of inference. Our examination will be brief, because we find pragmatic theory, like realist theory, breaks down on many of the issues we have raised in defending our own point of view.

As an example of the inadequacy of pragmatism to evolve a satisfactory theory of inference, let us examine Dewey's theory. We have noticed that he refuses to allow that inference and reasoning are one. (31) Also, he maintains that inference is from one thing as means to another as end. For example, from drops of water on the sidewalk I infer: It has rained. Dewey suggests that we do not have

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30 F. H. Bradley, Appearance and Reality, p. 123.

31 Cf. Logic. The Theory of Inquiry, p. 110, n.

to think to make such inferences. Such a position violates the very canon of pragmatism itself, i.e., it simply does not function. From the point of view of common sense a man thinks he thinks when he makes such an inference as the foregoing. (32) If one wants the pragmatic proof of this assertion: ask the next man you meet. We did, and it worked. (33) It should be true, on pragmatic grounds, that we think when we infer. Dewey does not deny that the process is cognitive. This makes the matter worse, from an idealist point of view.

Dewey does not seem to realize that his definition of inference and his definition of truth cannot be held consistently together. Truth, for Dewey, is warranted assertion. We have previously stated how Dewey follows Peirce in this matter. It will be recalled that Peirce states that truth is the opinion which is fated to be ultimately agreed upon by all who investigate. It seems to be satisfactory to most thinking organisms that, when they infer, they believe that they think. They apparently investigate the matter when asked, i.e., they attend to the process of their inference and conclude that they think. When it is suggested

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32 Cf. E. D. Starbuck, Look to This Day, pp. 1-10. "The Human Animal That Thinks it Thinks." Starbuck says: "When thinking becomes a lovely art then a man may think he thinks without being ashamed." p. 10.

33 Our respondents were university professors.

to them that they do not think, such a judgment seems unsatisfactory. Hence it would appear that Dewey's theory of inference cannot be established in the light of his theory of truth.

Dewey, of course, subordinates truth to inquiry. Logic, as we have previously pointed out, is inquiry into inquiry, i.e., is an attempt to find out the truth about logic. If this is not the case, then what is Dewey doing or striving to do in Logic. The Theory of Inquiry? Inquiry itself should yield truth on a pragmatic basis. But this does not seem to be so. When one asks the question: Is Dewey's theory of inference true? One is perplexed if one attempts an answer as a disciple of Dewey's. If we inquire into inquiry, i.e., ask our question on a logical basis, there is no answer forthcoming. When will any opinion be fated to be ultimately agreed upon by all who investigate? If on the other hand we merely inquire, i.e., ask the next person we meet, we find that his judgment is in violent opposition to Dewey's theory, which, by his own definition of truth, can never be substantiated, since it would demand a static social judgment.

We find ourselves in agreement with Russell, in his criticism of Dewey's theory of truth, although, we hasten to add, not for Russell's reasons. Russell criticizes Dewey's theory on the grounds of correspondence with an associated

complex. (34) We criticize it on the basis of coherence. Dewey should not discuss the question of truth at all. Indeed, he prefers the term 'warranted assertion,' but it seems impossible to say when an assertion is warranted. It is no answer to this difficulty to assert that it is the opinion of all who investigate, since his own assertion is unwarranted on the ground of his own criterion. His theory of inference does not cohere with his theory of truth.

When we turn to Schiller, we seem to be in no better position. Schiller maintains that relevance is more important than truth. But, on pragmatic grounds, when does one know if an inference is really relevant? Schiller admits that the search for safety is vain. (35) An inference may seem to be relevant, but, apparently, we are forbidden to think it at all safe to believe that it is relevant. Surely, such a position is the quintessence of skepticism, and Schiller lays himself open to Plato's classic reply to Protagoras, i.e., that he is certain about being uncertain. To assert that it is vain to search for safety, suggests that Schiller has gone the whole way in the quest, i.e., he has arrived at an ultimate, and found that ultimate very shaky and unsafe. His argument turns back upon himself.

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34 Cf. A History of Western Philosophy, p. 820 ff.

35 Cf. Logic for Use, p. 83.

Anyone in so unsafe a position would be well advised to maintain a discrete silence. But Schiller proceeds to discuss the question of truth at some length.

Truth, he maintains, is a practical question. It is a "value referring to prediction." (36) "The truths of science are tested by this power to predict the course of events." (37) Successful prediction, according to Schiller, is what science means by truth. One is led to wonder if modern science would entirely agree with Schiller on this point. On its radically empirical side, science might stop short at such a definition of truth, but on its mathematical side, it is doubtful whether science would say that this was all there is to truth. The physicist would certainly disagree. He was certain that energy was equivalent to mass times the speed of light squared long before it was empirically demonstrated in an atomic explosion.

Also, it is questionable whether Schiller can successfully maintain that truth must be treated as a term of valuation. If truth and value are equated and are to be understood as merely relative to individual psychic experience, such a criterion seems to violate that which the majority of people conceive to be truth. For example, there

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36 Ibid. p. 96.

37 Ibid.

are many things that I conceive to be true, yet, they seem to be quite neutral, i.e., they have no particular value, for me, as such. I take it as true that the 'Radio Quiz Kids' possess a good deal of unrelated pieces of knowledge, but this truth does not seem to enter into a realm of what I would describe as valuable truth for me. Also, one would suppose that a great many people hold the same opinion. It is true for them that this truth has no real value.

Schiller asks: Is a criterion of truth possible? He states: "no general criterion of truth could guarantee itself." (38) In this contention, Schiller is wrong. The coherence theory of truth does guarantee itself, inasmuch as this theory does cohere with the rest of our knowledge.

Schiller's criterion of truth is 'working.' If the criterion of truth is working "then the test of working must itself work, (which it does)." (39) This criterion sounds very plausible, but, who is to know when a truth is working? Also, does a truth stop working? And again, what happens if a truth does stop working? Apparently, if a truth stops working it is no longer a truth. Is it, then, false? But, pragmatic theory would maintain that anything so undynamic as an entity that has ceased to function is no entity at all.

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38 Ibid. p. 112.

39 Ibid. p. 115.

But, falsity, to be falsity, must function, i.e., must be working. It would appear that the criterion of falsity is working too. If, then, truth and falsity are both known by working, it suggests that there is a more ultimate criterion than mere working itself, whereby we judge the true to be true, and the false to be false. (40)

Space does not permit to continue this investigation at any greater length. We think that we have satisfactorily shown that the only reasonable theory of inference is that of the Idealist. The other theories that we have examined seem to fall down because they cannot be maintained against the background of the faulty logic which is their basis.

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40 Schiller falls into the same trap as J. S. Mill when the latter developed his theory of Hedonism, i.e., Mill unwittingly appealed to a higher criterion than pleasure.



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