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# PROPERTY TAXATION
# AND HOUSING MARKET ANALYSIS

by

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ABSTRACT

Property Taxation and
Housing Market Analysis

The study attempts an investigation of the impacts of taxation of residential property on three major aspects of housing, namely, (1) the supply and demand for housing-services, (2) the demand for ownership of residential dwellings, and (3) the supply of new residential accommodations. The study undertakes an estimation of these effects with an econometric analysis of cross-section housing data by Canadian municipalities. To the extent allowed by available data, the analysis attempts to indicate the relative tax effects on the owner and rental sectors as competing segments of the residential market.

A simple theoretical framework precedes the empirical investigation to facilitate achievement of the empirical objectives of the study.

TABLE OF CONTENTS

# CHAPTER I

## OBJECTIVES AND SCOPE OF THE STUDY

### A. Introduction

There are at least two categories of policy that the government can pursue to influence the demand for existing stock of residential dwellings and for new addition to existing dwellings. The first category may be considered monetary in nature and the other category, fiscal.

The monetary policy category includes such tools as interest rates(mortgage rates, in particular); volume and availability of mortgage credit; and subsidies in the form of government guaranteed loans, lower interest charges, and relatively longer payment period of principal and interest than ordinary. The fiscal policy category includes such tools as the property-value taxation of residences; the sales taxation of new residential construction; the tax treatment of capital gains and losses from sale of residences; the tax treatment of imputed rent of owner-occupied dwellings; and the tax treatment of accelerated depreciation and recaptured depreciation charges for income tax purposes. Since these tools affect the cost of borrowing for housing and the carrying cost of ownership of residences, they are important decision-variables in the housing market.

Further, since these tools can be changed at the government's discretion, they are generally considered exogenous determinants of the demand for housing.[1/]

In Canada, the great bulk of government programs and policies toward housing are monetary in nature and have operated mainly via the mortgage market. This trend in policies and programs toward housing, perhaps, largely explains why the major studies made about housing in this country are oriented toward the financial aspect of the housing industry.[2/] There are, of course a few exception---such as the minor studies on housing which relate to the taxation of income and the

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[1/] The term "housing" is technically defined and analyzed in the next chapter.

[2/] L.B. Smith's studies are predominant in this area. See, for instance, Housing in Canada: Market Structure and Policy Performance, Central Mortgage and Housing Corporation, Ottawa, January 1971; Housing and Mortgage Markets in Canada, Bank of Canada Staff Research Studies, 1970; with G. Sparks, "On the Economic Implications of the Yield Ceiling on Government Insured Mortgages", Canadian Journal of Economics, August 1967, pp. 420-431; "A Bi-Sectoral Housing Market Model", same journal, November 1969, pp.557-569; "The Interest Sensitivity of Canadian Mortgage Flows", same journal, August 1970, pp.407-421; "A Model of the Canadian Housing and Mortgage Markets", Journal of Political Economy, September/October 1969, pp.795-816; "Postwar Canadian Housing Policy in Theory and Practice", Land Economics, August 1968, pp.339-349.

tax treatment of capital gains and losses.[3] However, so far as I can discover from learned publications, no Canadian study has yet been made about residential property taxation.

The present volume is a study of housing and property taxation of residential dwellings with special reference to the Canadian situation. The study is both theoretical and empirical in approach.

In any study, it is instructive to introduce first what the paper is all about and what it aims to accomplish. I, therefore, present Chapter I (a) to clarify the problems and issues to be examined, (b) to identify what the study hopes to achieve in the process of investigation, (c) to delineate the scope and limitations of the study, and (d) to state the methods to be employed in the investigation.

B. The Problems

Let us pose the following questions. Does the taxation of residential property have any impact on the

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[3] See, for example, S. McFadyen, "British Columbia's Homeowner Grant Program: Its Impact on the Allocation of Resources and the Distribution of Income", Canadian Tax Journal, November/December 1970, pp.533-539; H. Kitchen, "Imputed Rent on Owner-Occupied Dwellings", same journal, September/October 1967, pp.482-491; H.G. Wolff, "The White Paper: Tax Treatment of Principal Residences", same journal, July/August 1970, pp. 263-276; F.A. Clayton, "Bill C-259: Is There a Conflict with Canada's Housing Goals?", Canadian Tax Foundation, 1971 Conference Report, pp.458-470.

demand for the existing stock of residential dwellings and for new residences? If so, what is the nature of its impact and is the impact quantitatively important?

To elaborate, are there any noticeable changes in the communities' demand for residential dwellings as a result of the changes in the residential property tax rates? Do such changes affect the rate of construction of residential dwellings? Do investors in residential dwellings normally consider the real property tax when making investment decisions or are other factors (e.g., risk, credit availability) more important to them?[4/]

These certainly are significant questions. Real property tax collections constitute a large percentage of the actual or imputed rental incomes of residential dwellings as shown in Table 1 for a few representative metropolitan areas in Canada. The ratios shown in the table appear high relative to the

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[4/] Apparently, these are the same types of question that preoccupy monetary economists regarding the relative importance of the rate of interest in investment decisions about which many research studies have been devoted to verify their empirical content. See, for example, the survey article by R. Eisner & R.H. Strotz, "Determinants of Business Investment", in Commission on Money and Credit, Impacts of Monetary Policy, Englewood Cliffs, N.J., 1964, Ch. 5; J. Meyer & E. Kuh, Investment Decision, Harvard University Press, 1957; W.H. White, "Interest Inelasticity of Investment Demand: The case from Business Attitude Surveys Re-examined", The American Economic Review, September 1956, pp565-587.

Table 1

PROPERTY TAX-(IMPUTED) RENTAL INCOME
RATIO FOR AN AVERAGE-SIZE SINGLE FAMILY
DWELLING UNIT IN SELECTED METROPOLITAN AREAS,
1969

| | Imputed Monthly Rent (in $)*/ | Average Monthly Property tax Payment (in $) | Property Tax-Imputed Rent Ratio (in %) |
|---|---|---|---|
| Toronto | 320 | 48 | 15 |
| Quebec | 213 | 36 | 17 |
| Montreal | 219 | 47 | 21 |
| Winnipeg | 248 | 34 | 14 |
| Regina | 233 | 41 | 18 |
| Calgary | 261 | 42 | 16 |
| Vancouver | 273 | 30 | 11 |

*/ The imputed rent figures are based on the formula $R = gC$, where R=imputed rent, C=construction cost plus land cost, and g=discount rate representing the sum of the mortgage rate of interest, the depreciation rate, and the property tax rate. The details of the procedure for calculating imputed rent are discussed in the main text of the study.

SOURCE OF DATA: Central Mortgage and Housing Corporation, Canadian Housing Statistics, 1970, Table 104, p. 83; for the 2.56% depreciation rate used in the calculation of the imputed rent, see E.H. Oksanen, "Housing Demand in Canada: Some Preliminary Experimentation", Canadian Journal of Economics and Political Science, August 1966, Table 3, n.3, p. 318.

general sales tax rates in all Canadian provinces which range from 5% to 8% of retail price, as well as to the 12% Federal sales tax rate levied on the manufacturer's selling price or duty-paid value of imports.[5]

While these figures help indicate the relative importance of the property tax in housing costs, they do not tell us anything about the significance of the property tax as a determining variable. These figures are only snap-shot information about a situation at a point in time. A 15%, 25%, or even 40% property tax-rental income ratio becomes meaningful only if it can be shown that the corresponding levels and changes in the property tax rate matter to the investors and consequently, to housing construction activities. Ideally, therefore, insights on the importance of the real property tax on the demand for housing largely hinge upon the information regarding the behavioural response of the volume of housing demand and residential construction to different property tax rate situations.

## C. Purpose and Objectives

This study will attempt to accomplish three things, namely: (1) to construct a simple model of the housing market which will focus on the real property

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[5] Dominion Bureau of Statistics (DBS), Principal Taxes and Rates: Federal, Provincial, and Local Governments, 1971, Table 4, pp. 20-21, cat. no. 68-201.

tax rate as a policy tool for analysis; (2) to analyze in a qualitative way the nature of the impact of the property tax rate on the demand for the existing stock of residential dwellings and on the construction of residences; (3) to arrive at some quantitative measures of this impact from actual Canadian housing data. In the process, the study hopes to shed light on the qualitative and quantitative aspects of the property-value taxation of residences and invite further research into the same field of investigation.

The simple model presented herein has parts that have been dealt with by other writers and ways of approach that are characteristic of already developed and published models. These published references are cited at the appropriate sections in the chapters of this study. However, a claim to originality can be made in three aspects, namely: (1) the selection of the relevant pieces of information from published references; (2) the development of an explanatory framework out of these pieces of information; and (3) the econometric analysis of the housing market based on this explanatory framework.

D. Scope and Limitations

This study is not designed to investigate a wide range of problems in the housing sector. It is specifically designed for the analysis of residential property taxation and, therefore, cannot be expected at the same time to illuminate the many other issues that are of interest to my readers. For example, the simple model cannot offer an analysis of such issues as: (1) the proportion of national investment expenditures that should be devoted to housing construction; (2) the role played by residential construction as an anti-cyclical economic activity; (3) the welfare aspects of public housing, urban-renewal, and housing subsidies to the urban poor; and (4) the comparative efficiency of the private and public sectors in meeting national or regional housing needs.

The other limitations of this study are less obvious. For instance, the demand-impact of mortgage credit availability can be analyzed in the model only upon the explicit introduction of variables that are financial in nature. These financial variables include the interest rate differentials between mortgage rate and, say, the Federal long-term bond rate; the Federal loan guarantee and insurance program which in L.B. Smith's[6/] judgement, has virtually eliminated the default risk involved on mortgage funds mainly advanced by the

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[6/] "Postwar Canadian Housing Policy in Theory and Practice", Land Economics, vol.44, no. 3, August 1968, p.340.

private lending institutions. Though one may occasionally refer to the demand-for-quality aspect of housing, the model is handicapped in this respect, because the "quality" concept is subsumed under the more general concept "housing-services" discussed in the next chapter.

The model has also a level of aggregation which renders it hardly useful for exploring the areas of residential mobility, zoning, spatial pattern of residential land use, and choice of residential locations--aspects that are capable of sharp focus in a highly disaggregated model of the housing market. Further, the model is short of analytical power to deal with the role of speculation and expectations in the housing market. To explore these matters requires a treatment of how market participants form their expectations, an issue which is beyond the scope of this thesis.

What the simple model cannot do may be considered as its predictive limitations. Accordingly, the empirical results of this study do not pretend to be conclusive in all cases, even if the results appear to conform with the predictions of the simple model attempted here. This study can only hope to avoid the inclusion of its model in the category of those models which, in Fritz Machlup's view, "have been condemned or rejected because they could not be used for purposes for which they had not been designed."[7]

E. A Note on the Methods Used in the Study

In the theoretical portion of this study, the reader will come across such concepts as housing-services, Q, and its conceptual unit price, E; physical dwelling-structures, H; physical land-site, L; property value tax rate, r; market rate of interest, m; and so on. As part of the whole theoretical framework, it should be emphasized that these concepts are theoretic constructs for analytical purposes.[8] It should be emphasized further that, although they are theoretic constructs, they correspond with real phenomena in a meaningful way. It is this meaningful correspondence between the theoretic constructs and observed reality or facts of experience that gives usefulness to the whole theoretical framework in illuminating the complex problem at hand.

The correspondence need not be direct and one-to-one. For example, take the case of the theoretic concept

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[7] F. Machlup, "Theories of the Firm: Marginalist, Behavioral, Managerial", The Americal Economic Review, vol. 57, March 1967, p.8.

[8] Ibid, pp. 9-10, 27. See also, R.M. Cyert & C. Hendrick, "The Theory of the Firm: Past, Present, and Future: An Interpretation", Journal of Economic Literature, vol. 10 no. 2, June 1972, p. 398.

QE/H , which represents the average gross rent per theoretic dwelling-structure, H.[9] This theoretic average rent meaningfully corresponds to the actual average gross rental income of an average-size residential unit in a local market area, and for that matter, to the average gross rental income of an average size residential unit on a regional or national basis. The theoretic average gross rent can also refer to either the average gross rental income of tenant-occupied residences or the average imputed rent of owner-occupied residential units, or an average of both averages. Take also the case of the theoretic physical dwelling-structure , H. It could correspond to an average-size physical dwelling unit as defined by the Dominion Bureau of Statistics(DBS) Census of Canada,[10] or to an average-size room, or to any other possible division thereof.

It may be noted further that whenever we speak of an "average", say, of any series of actual data in any given period, that "average" is in itself "an abstraction which is never directly observed nor experienced by"[11]

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[9] Discussed in Chapter III.

[10] "A dwelling unit is a structurally separate set of living quarters, with private entrance either from outside or from a common hall or stairway inside. The entrance must not be through anyone else's living quarters". See, DBS(1961 Census of Canada), Housing: Values and Rents, cat. no. 93-528.

[11] Karl A. Fox, Intermediate Economic Statistics, John Wiley & Sons, 1968, p.10.

an economic unit. In fact, statistical theory even goes beyond this level of abstraction, for the computed average is considered only as a mere estimate(biased or unbiased) of the yet highly abstract or theoretical "true" population average.

The various theoretic constructs are not specifically associated either directly or uniquely to anyone of their meaningful counterparts in the real world. The crucial point is that these theoretic constructs are means to the analysis, not mere duplicates of any specific instance, of empirical data or facts of experience.[12] Accordingly, the analytical explanations and relationships that emerge among the theoretic constructs are methodologically being brought to bear as testable implications upon the relationships governing the ultimate market data.

F. The Organization of the Thesis

The organization and development of the approach of this study may be summarized as follows. The physical land-site, the physical dwelling-structures, and the various types of home appliances, furnitures, draperies, etc., contained in the structure are seen as necessary

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[12] M. Friedman, "The Methodology of Positive Economics", Essays in Positive Economics, University of Chicago Press, 1966, p.35.

stocks of "input" for the production of a complex bundle of consumer goods hereby called "housing-services". The bundle of housing-services is complex because it has many dimensions. The dimensions include such characteristics as shelter, comfort, privacy, prestige, quality, etc., that are not easily susceptible to measurement even in an arbitrary way.

The conceptualization of the complex bundle of housing-services and the identification of the relevant costs associated with its production are discussed in Chapter II. The discussions of Chapter II are formally developed in Chapter III into a simple analytical framework of the housing market, under a specific set of assumptions. In the formal development of the framework, it proves necessary to explore the technical economic principles governing the relationships among the market aggregates in the production and consumption of housing-services. Chapter III, therefore, also discusses the principle of derived-demand for the various inputs to the production of housing-services; the market pricing of these inputs, with special emphasis on the pricing of physical dwelling-structures; and the economic relationship of the volume of residential construction to the volume of physical dwelling-structures already in stock.

Chapter IV introduces the residential property tax as a policy variable into the model. The introduction of the tax and the examination of its effects on the residential housing market appear intractable without the benefit of a background analysis of the individual homeowner's reaction to the property tax. Therefore, the initial sections of Chapter IV are focused on the individual homeowner and attempt to bring out the essential features of his response to residential property taxation. With this background analysis, the chapter proceeds with the investigation of the tax-effects from the viewpoint of a local market area. It concludes with a general statement of the body of hypotheses concerning the impact of property taxation on housing markets. Chapters V and VI provide empirical tests of these hypotheses using Canadian housing data. Chapter VII contains a summary of the major findings and conclusions of the study.

# CHAPTER II

# HOUSING AND THE COST OF HOUSING

This chapter deals with the definition and explanation of the most fundamental terms I shall use throughout the study. Other terms and concepts are defined in the appropriate sections of the chapter in which they are introduced.

## A. Definition of Housing

### A.1. Housing as a "Package"

In general usage, the term housing refers, as a noun, to a place of abode, residence, or habitation. Technically, its physical counterpart is a set of physical objects, namely, (a) the building or dwelling-structure with certain legal standards of sanitation facilities, wiring, and heating installations; (b) the home appliances, furniture, draperies, etc., contained in the structure; and (c) the land-site of the building-structure. We may call this set of physical objects the housing-package or simply, a dwelling or residential unit. Part or all of the components falling under (b) may or may not be present at the time of the purchase or acquisition of the housing-package.

### A.2. Housing as Analytical Definition

Real estate people maintain that "in a sense, real property may be thought of as 'factories' producing services — shelter, protection, privacy, comfort, convenience, etc.[1/] Indeed, this way of thinking is demonstrated even by the man on the street who is fully aware of the services of his wife or of a law-enforcement officer or of a plumber.

We wish to formalize this way of thinking, as it applies to housing, through the use of the concept of the production function. From such formalization, further technical and analytical subdivision of the housing-package can be done to facilitate the sharp focusing of the particular investigation at hand.

Let the various components of the housing-package be represented by the following:

H = dwelling-structure in physical units;

L = land-site in physical units;

V = other factor inputs such as house appliances, furniture, draperies, etc., in physical units.

In the application of the production function concept, these various physical components are considered as stocks of durable inputs that are technically combined to produce an output (flow) of housing-services per period. Thus, if the flow is represented by Q, the production

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[1/] A.M. Weimer, et.al., Real Estate, sixth edition Ronald Press, 1972, p. 122. E. M. Fisher, Urban Real Estate Markets: Characteristics and Financing, National Bureau of Economic Research, 1951, p. 5.

function may be written as:[2]

$$Q = F\ (H, L, V)$$

The right hand side of the equation is the functional expression for a housing-package which, in the likeness of a factory, produces a quantity of housing-services for an indefinite period. Chapter III, which introduces the simple model of the present study, develops from the above equation.

Speaking of consumer's demand in particular, we can now unambiguously talk about the consumer's "demand for housing services" for the ultimate satisfaction of, say, the "shelter" needs of the consumer. Similarly, it is unambiguous to refer to the producer's "supply of housing-services" for final consumption. Applying subsequently the principle of derived demand, there is a clear-cut meaning to such phrases as: the producer's derived "demand for the physical dwelling structures, H"; "demand for home appliances, V" and "demand for physical land-site, L." Further, if H, L, and V can be treated as one composite input-- i.e., they are combined in fixed proportion-- then we can also unambiguously refer to the producer's derived "demand

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[2] In the equation, Q is a flow whereas H, L, and V are stock inputs. As to the equivalence of a production reckoned as a flow to a production expressed in terms of stock inputs under special implicit conditions, see N. Georgescu-Roegen, "The Economics of Production", (Papers and Proceedings) The American Economic Review Vol. 60, May 1970, pp. 1-9.

for the housing-package".

At the present level of discussion, it should be apparent that such terminology as "the demand for (or supply of housing" is a highly ambiguous term and, therefore, will not be used throughout this study.

The output of housing-services Q per period is not as simple as it looks in the expression of the above equation. Instead, the housing-services Q is a complex bundle of various attributes. In W.F. Smith's view and that of most of real estate people, these attributes include such characteristics as shelter, privacy, prestige, location, and accessibility to the great complex of urban facilities and activities such as transportation, school, private businesses, etc., that add usefulness and value to the housing-package.[3]

This view, elegant as it is, is very difficult to translate into an empirical framework. Many of the characteristics forming part of the output of housing-services are not directly observable nor easily susceptible to measurement. For this reason, the complex bundle of characteristics are lumped together in this study under the one general term "housing-services" and are treated as one composite homogenous

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[3] W.F. Smith, Housing: The Social and Economic Element, University of California Press, 1970, Ch. 1; Weimar, et. al., op. cit.

commodity -- i.e., as though all housing-packages have these attributes in the same proportions.[4] Thus, the distinction between different conceivable characteristics or attributes of housing-services are ruled out at this level of abstraction. Since many of the characteristics forming the complex bundle of housing-services are not directly observable anyway, "it is not possible to argue conclusively and with precision for or against this assumption .... other than by reference to the testable implications"[5] of the simple model in which this assumption finds particular analytical use.

In the most simplified (traditional) theory of consumer behaviour, the quantity bought of an item or the dollar value of a group of items[6] may be taken as the direct determinant of the consumer's satisfaction. This particular relationship or assumption, however, applies only to commodities that are "consumable"-- in the sense of destruction of the commodities[7]-- during

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[4] This is the implicit procedure in R.F. Muth, "The Demand For Non-Farm Housing", in A.C. Harberger, ed., The Demand for Durable Goods, University of Chicago Press, 1960, pp. 32-34f.

[5] E.O. Olsen, "A Competitive Theory of the Housing Market". The American Economic Review, vol. 59, Sept. 1969, p.613 (emphasis mine).

[6] In constant prices of a base period.

[7] K. Boulding, A Reconstruction of Economics, John Wiley and Sons, Inc., 1967, p. 135.

the period within which the consumer's flow demand curve is defined. It does not apply to commodities or assets the life and use of which exceed the unit of time within which the demand curve is defined. Indeed, in the sense of "consumption" or "destruction", it is not meaningful to say "the demand for physical quantities of television set or refrigerator per month". The application of the traditional approach to the durable housing-package (or to any of its durable components) is particularly objectionable methodologically, since the life of the package usually exceeds the 50-year period.

Of course, "the distinction between durables and non-durables can be purely notional"[8/] and is much determined by the unit of time within which the consumer's demand (flow) curve is defined. For a time unit as short as a day, we shall find similar objections to the application of the traditional approach on items which we normally consider as non-durables such as clothing, shoes, and even food.

The analytical distinction between the "flow of services" from the durable goods and the "stock of

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[8/] H. Aaron, "Income Taxes and Housing", The American Economic Review, vol. 60, December 1970, pp. 789-800 (emphasis mine).

durable goods" is thus a very useful one. Instead of the quantity or dollar value of a housing-package or of a television set, it is now the flow of services deriving from them that can be specifically related to the consumer's satisfaction per period. In terms of the analytical procedure of the present study, the flow of housing-services, Q, is basic to the consumer demand; and whatever demand there is for the physical dwelling-structures, H, the land-site, L, and the other factor inputs, V, such demand is a derived stock demand, in the sense of ownership, for these durable inputs.

B. The Cost of Housing

Various types of cost can easily be enumerated, the most apparent of which are the price paid for the housing-package, real property tax charges, insurance cost,[9] maintenance and repair expenditures, interest charges on borrowed funds for the acquisition of the housing-package, etc. However, since we are distinguishing between the "flow of housing-services" and the various "stock" components of a housing-package, a more refined

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[9] It appears to be a standard practice among home owners to have some form of fire insurance for their buildings. In some cases, insurance against theft, tornadoes, and other hazards is also carried by many property owners. There are also instances when homeowners choose to have their gas or oil burner insured which includes free repairs and replacement of parts by the insuring agency. (Weimer, et.al. op. cit. p. 523).

view of the cost elements than the usual one is necessary for our purposes. In doing so, a further distinction between the flow cost and the stock cost concepts emerges.

B.1. Flow costs: The Cost of Supplying Housing-Services

All housing packages must be owned by some individuals, families, households, corporations, collective cooperatives, and governmental units at any period of time. A homeowner acquires housing-package(s) either by full payment or by installment payment over time of the market price of the housing package.

The basic difference between renting and owning is that the renter buys the flow of housing-services from month to month, whereas the owner buys the housing-package(s) in order to secure the right over the whole stream of services for an indefinite period. The owner of the housing-package(s) is thus the supplier of housing-services. He supplies housing-services either to himself or to other ultimate consumers. Obviously, it is the owner of the housing-package(s) who incurs the flow of costs associated with the provision of housing-services.

An installment buyer of a housing-package for his own occupancy (supplying himself with housing-services) is responsible for the interest and principal payments of the mortgage, for real property tax, for insurance, repairs and maintenance, and for such operating expenses as the cost of light, heat, water, etc. If the homeowner is at the same time a landlord or owner of rental housing-packages, he is also responsible for these same expenses.[10] As a landlord, he is likely to spend extra money for the management and supervision of the rental apartments, such as for caretaking, collecting rents, purchasing and using supplies for maintenance and repairs, and many other related activities.

If the homeowner buys the housing-package with his own funds, the annual or monthly interest earnings foregone by the funds if it had been invested elsewhere constitute an implicit flow cost or expense of ownership. If the purchase is accomplished through a combination of equity financing and debt financing, the interest cost element is the sum of the implicit interest earnings foregone on owner's equity and the actual interest payment on the borrowed funds. In general, it makes no difference to the analytical classification whether or not the owner borrows the money from a lending agency

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[10] Different practices exist, however, among landlords in various cities and areas as to the exact coverage of the contract rent per month. There are cases where tenants pay separately for the cost of repairs, water, heat, light, and cooking facilities.

or from himself. Either case of financing will involve an interest cost element as long as the opportunity cost of owned funds is fully accounted for.

The annual or monthly flow cost elements associated with the quantities of housing-services which are produced from a given housing-package (or equivalently produced from a given stock of inputs H, L, and V) may be summarized as follows:

(1) interest cost on borrowed funds and/or interest forgone on owner's equity;

(2) real property tax charges;

(3) depreciation costs of the dwelling-structures, H, and the other durable inputs, V, net of the possible appreciation in their values;

(4) less appreciation in the value of the land-site, L;

(5) operating expenses such as[11/]

(a) repairs and maintenance costs,

(b) caretaking and other supervisory costs,

(c) costs of insurance, light, heat, and water.

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11/ Initially, the stock inputs V is defined to represent the other durable items such as home appliances, draperies, etc. For all practical purposes, we shall include in V such factor inputs as supervisory and caretaking services, light, heat, and water some of which are not necessarily stock in category. This is admitted as a questionable procedure which is ignored for purposes of this study.

Some of these flow costs need not be incurred in the operation of a housing-package. For example, an owner-occupied housing-package does not require the services of a caretaker or supervisor and, therefore, will not incur the cost of these types of labour service normally present in a relatively large-scale rental apartment operation.

In Section C of Chapter III, the short-run and long-run periods of production will be explicitly considered. The consideration of the period of production necesssitates the further division of the flow cost elements into <u>fixed</u> cost category and <u>variable</u> cost category. Thus, some of the flow cost elements will be fixed or variable, depending on whether the counterpart input items are variable or not in the short-run production period.

B.2. <u>Stock cost: The Cost of Ownership of the Housing-Package</u>

The cost of a housing-package in the stock sense is simply the present value of the monthly or annual streams of costs enumerated above. This present value is not equal to the full price paid for or acquisition cost of a housing-package.

There are many reasons why the acquisition cost differs from the present value or stock cost of a housing-package. Some of them may be mentioned. Firstly, in real situations, individuals have different ways of evaluating the durability and quality of the package, causing differences in the valuation of the needed maintenance and repairs to be incurred. Individuals also have different expectations of future events and phenomena and, therefore, will have different market discount rates or "conversion rates"(see below). Secondly, even if we abstract from uncertainties, expectations, and market imperfections(e.g., limited knowledge of the market and the product, product heterogeneity and indivisibility), an individual buyer of a housing-package usually demands deductions and allowances for the capitalized value of the property taxes, maintenance and repairs which certainly form part of the future carrying-cost of ownership of the housing-package.

In arriving at the present value of the flow cost elements, the rate of interest plays the role of a "conversion factor" for bringing flows into stock, or stocks into flow, within a reasonable approximation.[12/] There are, of course, a number of interest rates in the money market that can play this role. The rate of

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[12/] M.A. Danmole, "Theories of Money: An Attempt at Synthesis", Journal of Business and Social Studies, vol. 1, September 1968, p.97; J. Hirshleifer, Investment, Interest, and Capital, Prentice-Hall, Inc., 1970, pp. 35-37.

interest for conversion purposes may well be represented by the mortgage rate or by the Federal long-term bond rate, since ownership of the housing package is usually financed by a mortgage and commits funds for longer periods.

The market rate of interest is not the only conversion factor. If the flow is measured on a gross basis the conversion factor will include allowances for depreciation, maintenance and repairs, net of the appreciation in the market value of the property. The conversion factor will also include allowances for property taxes, risks, and shortcomings in the marketability of the residential property. These allowances are contingent upon the stability, predictability, and level of the future streams of income deriving from the housing-package. For instance, if the expected streams of income are stable and the property is durable and highly marketable for reasons of its physical composition, location and style, the risk and depreciation allowances will be lower. Indeed, in appraising a particular dwelling one may begin with a conversion factor close to the riskless rate, say 6% on long term government bonds. To this may be added a 2% depreciation rate minus a 1.5% appreciation rate, 2% risk rate, 2.5% property tax rate, and a 1% rate for the shortcomings in the marketability of the property -- or a total conversion rate of 12%.

The conversion rate differs among individuals, families, and institutions. It differs because economic units do not pay similar costs of borrowing; nor do they have the same investment opportunities. Even if the costs or the investment opportunities are the same, economic units will apply one conversion rate to residences on first or second mortgage and another to residences without these encumbrances.

The physical and economic status of the residential property also affects the conversion factor. For instance, a particular dwelling-structure may wear out physically and economically in 25 years so that in order to recover the investment in the building a recapture or depreciation rate of at least 4% (100% ÷ 25) will be used. In contrast, another dwelling-structure is expected to last for 50 years so that the investor may settle for a depreciation rate of at least 2%. In the case of land, since the land does not deplete in use over time there is no need to "load" the conversion factor with a percentage for depreciation; indeed, it may even need "unloading" by a percentage for the expected appreciation in the market value of the property over the course of time.[13/]

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[13/] Weimar, et. al., op.cit.

It needs pointing out here that the level of aggregation in the empirical portion of this study is by Canadian cities or municipalities. The aggregation requires the averaging of the individual conversion factors and their components to the municipality level. Due to data limitations, however, we must resort to a single rate across municipalities for some of the components of the conversion factor in these communities. For instance, national rates are used for depreciation and for the market rate of interest, respectively. The depreciation rate employed here is the 2.56% used by the Dominion Bureau of Statistics in the calculation of the national housing stock series. The market rate of interest is represented by the average of provincial mortgage rates, which have very little variation across Canadian provincial boundaries. The available data, however, allow the construction of conversion rates by municipalities that differ in residential property tax rates.

The streams of cost converted into stock represent the total cost of homeownership expressed at the current period. If this total cost is paid for by the owner and the property is not conveyed on a mortgage, the owner is entitled, within the boundary of statutory restrictions on rights of private property,[14] to the exclusive right to dispose or occupy, use and enjoy the services of, the housing-package; or, if not, to sell the services to tenants for the rest of the life of the package or for a shorter period if subsequent resale of the property is contemplated.

Apart from the limitations imposed by statutory laws on rights of private property, further restrictions on these rights are imposed upon the owner whenever the property is pledged on a mortgage to secure debt. A legal right is created in favour of the mortgagee to hold the housing-package or to have it sold for payment of his claim. Indeed, in case of default in mortgage payment due to, say, prolonged unemployment of the owner-mortgagor, the mortgagee can foreclose the property leading to its sale by order of the court. During the severe economic crisis of the 1930's millions of families in the United States lost their homes through foreclosures.[15]

B.3. Statutory vs. Effective Property Tax Rate

It can generally be said that the current United

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[14] E.g., the state's right of eminent domain, statutory rights of dowers, zoning by-laws, statutory building codes, mineral or waterway rights.

[15] L.J. Gordon & S.M. Lee, Economics for Consumers, 5th ed., Van Nostrand Reinholt Co., 1967, p.411.

States and Canadian real property tax system applies the statutory tax rates on the combined assessed value of both the land-site and the building structure.

In theory and in practice, it is desirable that the assessment of the property value for taxation purposes be uniform and close to the property's normal or competitive market value. Unfortunately, in practice, the actual "methods of assessing real property value vary widely", although assessed values are ultimately related to market values.[16] It is not uncommon that properties of similar or identical characteristics are assessed in an extremely different way from one county or municipality to another.[17]

The actual methods of assessment of property value vary due to a number of reasons. Appraising principles and techniques, for instance, differ among assessors, real estate agents, and appraisers. Although similar techniques and principles may have been used, the theoretical principles generally provide only a

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16/ Dominion Bureau of Statistics(DBS), Principal Taxes and Rates: Federal, Provincial, and Local Governments, 1971, p.39, cat. #68-201.

17/ D. Netzer, "Taxation: Property Taxes", International Encyclopedia of Social Sciences, vol. 15, 1968, pp.545-550; see also Netzer's The Economics of Property Tax, Brookings Institution, 1966.

rough guide to the actual valuation process.[18] Ultimately, assessors and appraisers are guided by their own training and informed judgement or opinion of property values in particular situations and circumstances. In addition, assessment statutes usually allow assessors a wide latitude to decide how to determine the "market value" of property. This latitude "varies quite considerably (in Canada) from province to province and even between municipalities in the same province".[19]

The statutory property tax rates, therefore, cannot be taken as one and the same as the "effective" property tax rates. The effective rate refers to the actual tax charges on the property as a percentage of its true market value. In this study, the theoretical and the empirical property tax rates refer to the effective rates. Thus, if the statutory rate is, say, 5%, and the property is assessed at only 50% of its true market value, the effective property tax rate is only 2.5%. In Canada, the statutory rates are quoted

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[18] See, for instance, J.I. Stewart, Real Estate Appraisal in a Nutshell, University of Toronto Press, 1962; R. Steacy, Canadian Real Estate: How to Make It Pay, Peter Martin Associates, Ltd., 1968.

[19] F. H. Finnis, Property Assessment in Canada, Canadian Tax Foundation, March 1970, p.69.

in mill rates, i.e., in mills per dollar(or dollars per $1,000) of the assessed value of the property. Further details on the Canadian residential property taxation are discussed separately in Appendix A of this study.

C. Summary

Housing-services are seen here as consumer goods whose production requires the technical combination of three physical inputs namely, (1) the land-site (L), (2) the dwelling-structure (H), and (3) a variable (V) representing household appliances, furniture, utilities, etc. The output of housing-services is a periodic quantity (a flow), whereas the physical inputs are properly classified as stocks.

A residential dwelling is a package or combination of the physical inputs and should be distinguished from the housing-services it yields. The market value of these services is seen as the periodic rental income of the dwelling. The periodic rental income is a gross or net figure depending on whether the costs of depreciation, maintenance, and repairs of the package are subtracted out.

The cost of production of housing-services is different from the acquisition cost of the physical inputs or of the dwelling. The acquisition cost is also a stock concept, whereas the cost of production of housing-services is a flow. The acquisition cost may be converted into a periodic cost of production through the use of an appropriate conversion or discount factor.

In this discussion, we can distinguish two highly interrelated markets -- the market for housing-services and the market for stocks of physical inputs or for the composite of these inputs, the dwellings. The next chapter will present the necessary theoretical framework for analysis of the underlying relationships between these markets and the volume of residential construction. Specific attention, however, will be focused on the market for housing-services and the market for the physical dwelling-structures, H.

# CHAPTER III

## MODEL: AN APPROACH TO THE ANALYSIS OF THE HOUSING MARKET

This chapter will present a model of the housing market. The model will be developed from the concept of the production function for housing-services already introduced in Chapter II. After its presentation, the model will be used in the next chapter to analyze the impact of residential property taxation on the housing sector.

In the model, we shall be concerned with the interrelationships between two markets: (1) the market for housing-services, Q, and (2) the market for the physical inputs with special reference to the dwelling-structures, H. The volume of residential construction will of course play a part in these interrelationships.

Speaking of housing-services, Q, in particular, the volume of market demand for these services per period is dependent upon their unit market price, the average incomes of individuals or households, the existing tastes and preferences of the general public, the number of households or families. The volume of market supply of housing-services is a function of their unit market price, the existing stock of housing-packages, and the willingness of owners of these packages to vary one or more of the package's components (land, L, dwelling-structures, H, and other inputs, V) to effect a change in the quantity supplied of Q. Depending on the relative size of the quantity demanded and quantity supplied of housing-services, Q, the market for Q may or may not be cleared at the ruling unit market price. In consequence, the existing stock of residential land, L, of physical dwelling-structures, H, and of other inputs, V (e.g., washing machines, oil burners), may or may not be optimal during the period, necessitating specific changes in the derived demand for L, H, or V.

Since the discussion of the demand aspect of the market for housing-services is more or less a repetition of the standard economic theory of market demand for a commodity, it may be advisable to focus closely on the supply side of the market for housing-services. The present chapter thus develops the model with a detailed presentation of the market supply of housing-services. In the portion dealing with the derived demand for factor inputs, the discussion focuses only on the derived demand for the physical dwelling-structures, it being the most important aspect of derived demand for purposes of this study.

A. Production of Housing-Services

In Chapter II, it was noted that housing-services can be viewed in a perspective comparable to any other producible commodities in our contemporary economic system. Thus, the union of the physical dwelling-structure, the physical land-site, and such other factor inputs as home appliances, etc., is said to be a necessary input-combination in the technical sense to the production of housing-services.

Due to the degree of abstraction in which housing-services are specified, the quantity output of housing-services is not directly observable. For this reason, it seems impossible to empirically observe that housing-services are being pricedin the residential housing market. In fact, there can only be a theoretical or conceptual reference to the unit price and the quantity output of housing-services. However, the product of these two theoretically distinguishable concepts finds meaningful and ultimate correspondence to the actual market rental of residential units. Because of this correspondence, the conceptualization of the unit market price and the quantity output of housing-services is both analytically and empirically useful.

A.1. <u>Definition of Terms (Theoretic Constructs)</u>:

$Q$ = quantity output of housing-services per period;

$L$ = land-site in physical units;

$H$ = dwelling-structures in physical units;

$V$ = a variable representing all other factor inputs such as house appliances, carpets, maintenance and repairs, light and heat facilities, caretaking, etc.;[1]

m = discount rate (continuously compounded market rate of interest);

E = unit market price of housing-services.

A.2. Assumptions:

(a) each physical unit of H depreciates exponentially its n-year life and in a technologically determined manner;

(b) the price per unit of land is determined by the total land supply and over-all demand for land in various alternative and competing uses;

(c) purely competitive markets for Q, L, H, and V;

(d) homogeneity of degree one in the production function;

(e) homogeneous output of housing-services, Q.

These assumptions are obviously unrealistic, particularly the assumption of purely competitive markets. However, they formally facilitate (a) an economical presentation of the theory, (b) a simpler method of bringing out the testable hypotheses of the theory on the impact of residential property taxation on the residential

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[1] For all practical purposes, the factor inputs represented by V are assumed to be one composite factor input, i.e., they enter the production function in fixed proportions.

housing market, and (c) the testing of these hypotheses against actual market data. If the results appear odd and need further examination, appropriate qualifications and adjustments may still be made in the model. In contrast, if the assumptions were blue prints of the actual housing market and of the actual physical characteristics of housing-packages, it would be too difficult and unweildly to accomplish the theoretical and empirical objectives of this study.[2/]

These restrictive assumptions are not strictly followed throughout the study. Eventually some of them are either discontinued or adopted only with modifications in the latter part of the dissertation. For instance, in Section B of Chapter VI, we allow for the possibility that appreciation in the market value of residences may well offset the value-reducing effect of the physical depreciation of the dwelling-structures. The imperfection

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[2/] For a detailed discussion of: (a) the role of assumptions in a theoretical framework; (b) such related issues as whether the relevant test of the validity of a theory or hypothesis is the realism of its assumptions or the predictive power of its assertions about the class of phenomena which it intends to "explain"; and (c) the consistency of a particular fact or observed experience with the same hypothesis brought about under different sets of assumptions, see M. Friedman, "The Methodology of Positive Economics", in M. Friedman (ed.) Essays in Positive Economics, The University of Chicago Press, 1966, pp. 3-46.

of competition in the production and sale of new dwelling-structures is alluded to in Section D of this chapter and is subsequently tested empirically in Chapter VI. We also modify the assumption of homogeneous outputs of housing services. In Section D of Chapter IV, for example, we allow for the imperfection in the substitution between rental and owner housing-services and investigate the owner and the rental sectors as two competing housing markets.

A.3. Statements of Relationships

Let us re-state the production function already introduced in Chapter II, i.e.,

$$Q = F( L,H,V ) \tag{1}$$

Its partial derivatives are:

$$\frac{\partial Q}{\partial L} = F'_L \quad , \quad \frac{\partial Q}{\partial H} = F'_H \quad , \quad \frac{\partial Q}{\partial V} = F'_V$$

representing the marginal outputs of housing-services of L, H, and V, respectively. Accordingly, given first degree homogeneity,

$$Q = F'_L L + F'_H H + F'_V V \tag{2}$$

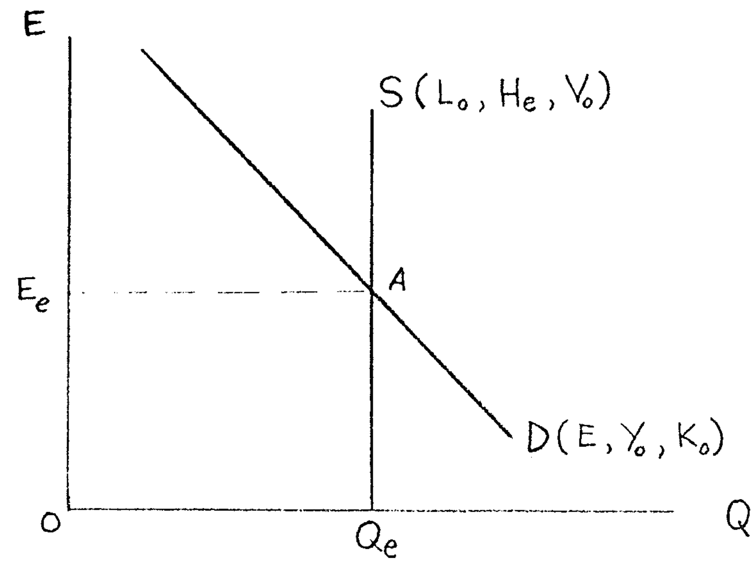
Equation (2) is a statement of the functional distribution of the output of housing-services, Q. The distribution is based on the factors' conceptual marginal productivities.

In the actual housing market, we know that physical dwelling-structures are built and housing-packages are traded and sold at a price; others are rented and occupiers do pay market rentals for them per period. It can be argued that these housing-packages are neither desired nor demanded for their own sake, but for the housing-services they provide the ultimate consumers. When housing-packages are bought or rented, this transaction is also indicative of the implicit market valuation of housing-services taking place in the market. The rental-housing sector is a clear example of the active market for housing-services. We may, therefore, re-express equation (2) in value terms per period as:

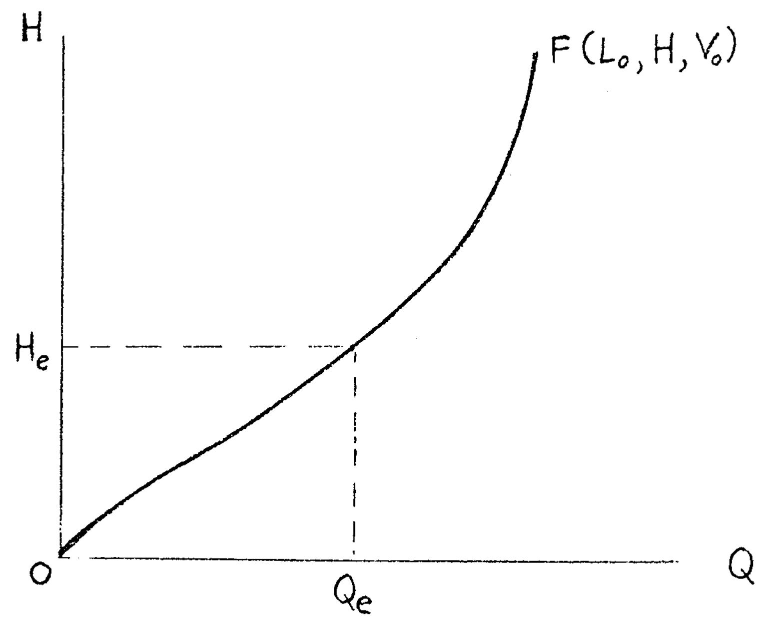
$$G(t) = R_L(t)L + R_h(t)H + R_v(t)V \qquad (3)$$

where E = unit market price of Q, EQ = G, $EF'_L = R_L$, $EF'_H = R_h$, $EF'_V = R_v$, t = time period. The value G represent the total gross rental income generated by the stock of housing-packages or their components H, L, and V per period.

The market pricing as well as the production of housing-services, Q, at time t are illustrated graphically by Figures 1 and 2. The function $D(E, Y_o, K_o)$ represents the market demand curve for housing-services, Q, appearing as a function of unit market price, E, evaluated at a given level of money income, $Y_o$, and




(Figure 1)




(Figure 2)

given levels of all other demand determining factors represented by $K_o$. The function $S(L_o, H_e, V_o)$ is the market supply curve of Q which, for given levels of $L = L_o$ and $V = V_o$, appears perfectly inelastic when evaluated at $H = H_e$.[3/] The aggregate equilibrium level of $Q = Q_e$ is cleared at the unit market price $E = E_e$. The corresponding equilibrium stock of H needed to supply $Q_e$ is $H_e$.

Ceteris paribus, if the consumers of Q have attained their optimal flow of housing-services, the number of physical structures $H = H_e$ will not be augmented by new construction, though, of course, construction will occur for replacement purposes. The area $OQ_eAE_e$ represents the aggregate gross rental income generated by the housing market in an economy or local market area at time t. This aggregate value will increase, decrease, or remain the same over time depending upon whether the market is expanding, contracting, or stationary.

Let

$$G(t) - R_v(t)V = R_L(t)L + R_h(t)H. \tag{4}$$

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[3/] In Section C of this chapter, it is argued that V is variable with respect to the unit market price E, i.e., $V=V(E)$. When the variability of V in response to E is introduced into the model the aggregate supply curve of Q will be upward sloping with finite supply-price elasticity (see Section C below).

The values $R_L(t)$ and $R_h(t)$ are the rental incomes of L and H, respectively. Because the physical dwelling-structure, H, has a life of n years, the values $R_L(t)$ and $R_h(t)$ are, say, annual streams of return for an n-year period. Further, let $R_h(t)=R_{ho}e^{-bt}$, where b is the exponential rate of decline of the rental income of H due to its physical deterioration.[4/] These future streams of value return can be discounted to their present value,[5/] i.e.,

$$\int_0^n [\bar{G}(t)-R_v(t)V] e^{-mt}dt = L\int_0^n R_L(t)e^{-mt}dt + H\int_0^n R_{ho}e^{-(m+b)t}dt \qquad (5)$$

Abstracting from the imperfections and rigidities (e.g. limited information, zoning by-laws) in the market for land, the competitive allocation of land, L,

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[4/] Even though regularly repaired, building structures physically wear out through time. "Most buildings are also eventually being overtaken by what may be called design obsolescence, resulting from improvements in building technology, changes in architectural style, developments in new home appliances which older buildings do not readily accommodate". J. Heilbrun, Real Estate Taxes and Urban Housing, Columbia University Press, 1966, pp. 29-30.

A housing-package may have demonstrated appreciation in market value over time; but the appreciation may be mainly due to the appreciation in the value of the land component of the package, more than offsetting the value depreciation of the structure.

[5/] Since the average life of physical dwelling-structures usually exceeds the 50-year period, for purposes of clear-cut results, it may be instructive to assume $n=\infty$. This assumption is adopted throughout the rest of the study, unless specified otherwise.

for various uses assures that, in equilibrium, the integral of the first term of the right side of equation (5) not only represents the capitalized value of a unit of land, but that the integral also reflects the land's opportunity cost in alternative uses. If a plot of land is receiving below the average rent of similarly situated and productive land, the activity using the plot will earn above normal profit. Other activities will seek to use the plot and bid up its rental income towards the average level prevailing. Conversely, the plot purporting to receive above the average rent will not be used, unless its rent is reduced to the level of the average rent prevailing.[6] Thus, if $P_L$ is the price per unit of land at t=0, we say that in equilibrium

$$P_L(0) = \int_0^n R_L(t)e^{-mt}dt. \qquad (6)$$

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[6] Further and extensive information on the theory of urban land rent and location theory may be found in: W. Alonso Location and Land Use: Toward a General Theory of Land Rent, Harvard University Press, 1964; "A Theory of the Urban Land Market", in W.H. Leahy, et. al., (eds.), Urban Economics: Theory and Development Planning, Collier Macmillan, 1970, pp. 55-03; H.W. Richardson, Regional Economics, Weidenfeld & Nicolson, 1969; R.L. Richman "The Incidence of Urban Real Estate Taxes Under Conditions of Statistic and Dynamic Equilibrium", Land Economics, vol. 43, May 1967, pp. 172-180; R. Turvey, The Economics of Real Property Tax, Allen & Unwin, 1957.

Under the same abstraction from imperfections in and aberrations to the underlying rent-price relationships, the market will establish the unit price of dwelling-structures at the equilibrium level

$$P_h(0) = R_{ho}\int_0^n e^{-(m+b)t}dt = \frac{R_{ho}}{m+b}\left[1-e^{-(m+b)n}\right]$$
$$= \frac{R_{ho}}{m+b} \qquad (7)$$

for $n=\infty$. At this price level, the excess demand for the existing stock of H is zero.

It can be argued that the ratio $R_{ho}/P_h$ is a measure of the gross rate of profit (or return) by the homeowners on their investment outlays in physical structures. In equilibrium, this rate of profit is equal to the sum of m(representing the rate of interest foregone on alternative uses of the same outlay) and b (representing the depreciation rate which guarantees the recapture of the invested funds).[7]

The sum (m+b) is also a measure of the "conversion" or discount rate already discussed in Chapter II. A number of other factors are not accounted for in this rate such as risks, marketability or liquidity considerations, capital gains or losses, taxes, and uncertainty

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[7] M.J. Gordon, "The Pay-Off Period and the Rate of Profit", The Management of Corporate Capital, (ed.) Ezra Solomon, Free Press, 1959, pp. 48-55.

with regard to the amount of the more distant future incomes of the property. Generally speaking, these other factors are sources of variations in the rates of return to capital in various uses, even though monetary funds and physical resources can freely move from one type of use to another. Imperfect information and artificial restrictions or concessions to entry of capital in given areas tend also to amplify the differentials in rates of return among sectors or industries.[8/]

A related distinction that need not concern us here is the differentiation between the rate of return from the viewpoint of the private investors and the rate of return from the viewpoint of the society as a whole-- whether or not they are net of risk or of capital gains and losses. The private rate of return will differ from the social rate of return since private investors do not normally take account of the external benefits (e.g., reduction in crime and fire risks due to urban renewal) and the external costs (e.g., smog , pollution, consumption foregone due to

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[8/] A.C. Harberger, "Taxation: Corporate Income Taxes", International Encyclopedia of Social Sciences, (ed.) D.L. Sills, Macmillan, 1968, pp. 538-545.

transfer of funds) of a given project or activity.[9/]

B. The Aggregate Demand and Supply of H (Stock)

B.1. The market pricing of Stock

The demand for the physical dwelling-structure, H, is derived from the demand for the flow of housing-services.[10/] In J.G. Witte's terminology, this derived

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[9/] For an empirical study of private and social rates of return from capital in Canada by industry, see Glenn P. Jenkins, "Rates of Return and Taxation from Private Capital in Canada", Ontario Economic Review, November/December 1972, vol. 10, no.5, pp.4-19.

[10/] If $P_h$, $P_L$, and $P_v$ are the current market prices of H, L, and V, then the total stock cost of a given amount of these factor inputs at t=0 is:

$$C = P_h H + P_L L + P_v V \quad (8)$$

The present value of the rental-revenue streams derivable from these inputs is

$$\int_o^n G(t)\, e^{-mt}\, dt = \int_o^n [EF(H,L,V)]\, e^{-mt}\, dt \quad (9)$$

(referring to equations 1-5). Let the discounted profit function at t=0 be:

$$\Pi(0) = \int_o^n [EF(H,L,V)] e^{-mt} dt - P_h H - P_L L - P_v V \quad (10)$$

Maximizing $\Pi$ with respect to H, L, and V:

$$\frac{\partial \Pi}{\partial H} = \int_o^n [EF_H'] \, e^{-mt}\, dt - P_h = 0 , \quad (11\text{-a})$$

$$\frac{\partial \Pi}{\partial L} = \int_o^n [EF_L'] \, e^{-mt}\, dt - P_L = 0 , \quad (11\text{-b})$$

$$\frac{\partial \Pi}{\partial V} = \int_o^n [EF_V'] \, e^{-mt}\, dt - P_v = 0 \quad (11\text{-c})$$

demand is a stock-demand and not a continuous flow demand for physical dwelling-structures per period.[11] Further, in this study, the stock-demand for the physical H and for the other stock components of the housing-package is seen as a demand for their ownership. The aggregate stock-demand for H is the sum of all the individual derived demands, actual or potential.

The stock-demand for physical dwelling-structures depends upon the current decisions of participating economic units. Individuals or families desire physical dwellings for the housing-services they yield, as well as for the expected future returns(e.g., income from capital, including capital gains or losses) they

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for n = some fixed real number representing the $n^{th}$ year life of the structure reckoned today (t=0), and where

$$\frac{\partial^2 \Pi}{\partial H^2} < 0 , \quad \frac{\partial^2 \Pi}{\partial L^2} < 0 , \quad \frac{\partial^2 \Pi}{\partial V^2} < 0 . \qquad (12)$$

The derived-demand functions for inputs are obtained by solving from (11-a) to (11-c) for H,L, and V as functions of $P_h$, $P_L$, and $P_v$ and the unit market price E of housing-services. In particular, the derived demand curve for H is obtained by graphing the demand function for H as a function of $P_h$ alone, on the assumption that $P_L$, $P_v$, and E are given parameters.

[11] J.G. Witte, Jr., "The Micro-Foundation of the Social Investment Function", Journal of Political Economy, October 1963, pp. 441-456.

bring to the owner. Under a specified state of engineering know-how, of architectural design, and of expectations regarding future prices of assets and commodities, the individual's demand for physical dwelling-structures will be a function of their current market price. Other things remaining the same, an economic unit is willing to hold a larger stock of dwelling-structures the lower is their unit market price relative to other assets. The demand to own a stock of dwelling-structures normally slopes downward, whether described in terms of an individual economic unit or described in terms of aggregated economic units.[12]

In contrast, the aggregate stock-supply of dwelling-structures at any moment is perfectly inelastic. The quantity in supply is simply the total stock of dwelling-structures that has accumulated through time. The total stock is being altered gradually by future births and deaths of dwelling-structures, "but if a sufficiently short time period is considered, addition to, or depletion of, existing stock will be negligible".[13]

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[12] R.W. Clower, "An Investigation into the Dynamics of Investment", The American Economic Review, vol.44, March 1954, pp.64-81.

[13] Ibid.

In fact, even if the time unit is a one-year period, the annual increment to physical H in stock (current production net of replacement demand) would still be a minor portion of the total stock. The construction- or completion-period of a single family dwelling-structure has a minimum average ranging from four to nine months; whereas, the average life of the same structure would usually exceed the fifty-year period. For this reason, over the life of a structure, a tremendous amount of dwellings can accumulate through time, despite depreciation. Indeed, even a time unit of a year period is relatively too short for the new output of structures, net of replacement, to dominate over the total stock available for supply in the market.

The minor numerical importance of newly completed dwelling-structures relative to the total available stock on a national and regional basis is shown in Table 2. The table summarizes the annual gross addition to existing (occupied) stocks for Canada and selected Canadian metropolitan and urban areas. With the exception of Calgary, Saskatoon, Toronto, and Guelph, the percentage gross addition is barely twice as much as the 2.56% depreciation or replacement rate[14] being

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[14] E.H. Oksanen, "Housing Demand in Canada: Some Preliminary Experimentation", Canadian Journal of Economics and Political Science, August 1960, Table 3, n.3, p.318.

Table 2

ANNUAL AVERAGE NUMBER OF NEWLY COMPLETED DWELLINGS AS A PERCENTAGE OF TOTAL OCCUPIED DWELLINGS, 1961 AND 1966

| | 1961[a] | 1966[b] |
|---|---|---|
| Calgary | 5.12 | 5.75 |
| Halifax | 3.57 | 2.82 |
| Hamilton | 3.16 | 4.23 |
| London | 3.64 | 5.02 |
| Montreal | 4.45 | 3.94 |
| Regina | 4.39 | 3.62 |
| Saskatoon | 4.08 | 6.27 |
| Toronto | 4.24 | 5.07 |
| Vancouver | 3.40 | 4.96 |
| Winnipeg | 2.76 | 2.89 |
| Guelph | 3.10 | 6.15 |
| Sault Ste. Marie | 4.35 | 2.35 |
| Shawinigan | 1.31 | 0.42 |
| Peterborough | 1.54 | 2.68 |
| Sarnia | 1.96 | 3.76 |
| CANADA | 2.98 | 3.33 |

[a] Annual average number of newly completed dwellings in 1962-64 as a percentage of total occupied dwellings in 1961.

[b] Annual average number of newly completed dwellings in 1967-69 as a percentage of total occupied dwellings in 1966.

SOURCE OF DATA: Central Mortgage and Housing Corporation, Canadian Housing Statistics, 1971, Table 8, and other relevant years.

used by the Dominion Bureau of Statistics in the calculation of dwelling stock series. A very small percentage is, therefore, being added on a net basis to the existing stock per annum. Further, these figures are overstated because the calculation did not include vacant dwellings.

To W.G. Grigsby, the dominance of the existing stock over net additions to it need not be limited to sheer physical quantity. The dominance could take the form of quality, say, in terms of durability, style, etc. He observes that "..new dwelling-structures are frequently no better than the old"; the new ones are "often much worse". Considering the inconvenience and cost involved in moving, Grigsby further argues that "millions of existing homeowners find nothing in the newly constructed dwellings that is sufficiently superior to their existing quarters. This fact is well expressed in the frequent admonition to builders that their stiffest competition...are the homes which potential buyers now occupy".15/

As an initial observation, it would appear that the volume of residential construction plays a very subordinate role in the actual determination of the

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15/ W.G. Grigsby, Housing Markets and Public Policy, University of Pennsylvania Press, 1963, pp. 182-183.

market price of dwelling-structures on either a regional or national basis. In the model, the short-run equilibrium unit market price of dwelling-structures is thus seen to be determined by the interplay of competitive forces built into the market stock demand for and supply of dwelling-structures.

The market pricing process is illustrated graphically by Figure 3. The market stock-supply is represented by the vertical line $H^s$, the current stock being $H_e$. The market stock-demand is represented by $H^d$. The equilibrium market price $P_h = P_{he}$ is determined at the intersection of the $H^s$ and $H^d$ curves. Figure 3 is also the market-pricing representation of the equilibrium condition expressed by equation (7) above.

One inherent weakness of the empirical application of this market-pricing approach is that it will methodologically force all physical dwelling-structures of different ages and durability to carry one equilibrium market price.[16/] There is a feasible way to circumvent this one-market-pricing problem associated with the heterogeneity in the actual stock of physical dwelling-structures. Instead of expressing the measure of the stock of dwelling-structures in terms of physical numbers, a measure can be made in terms of

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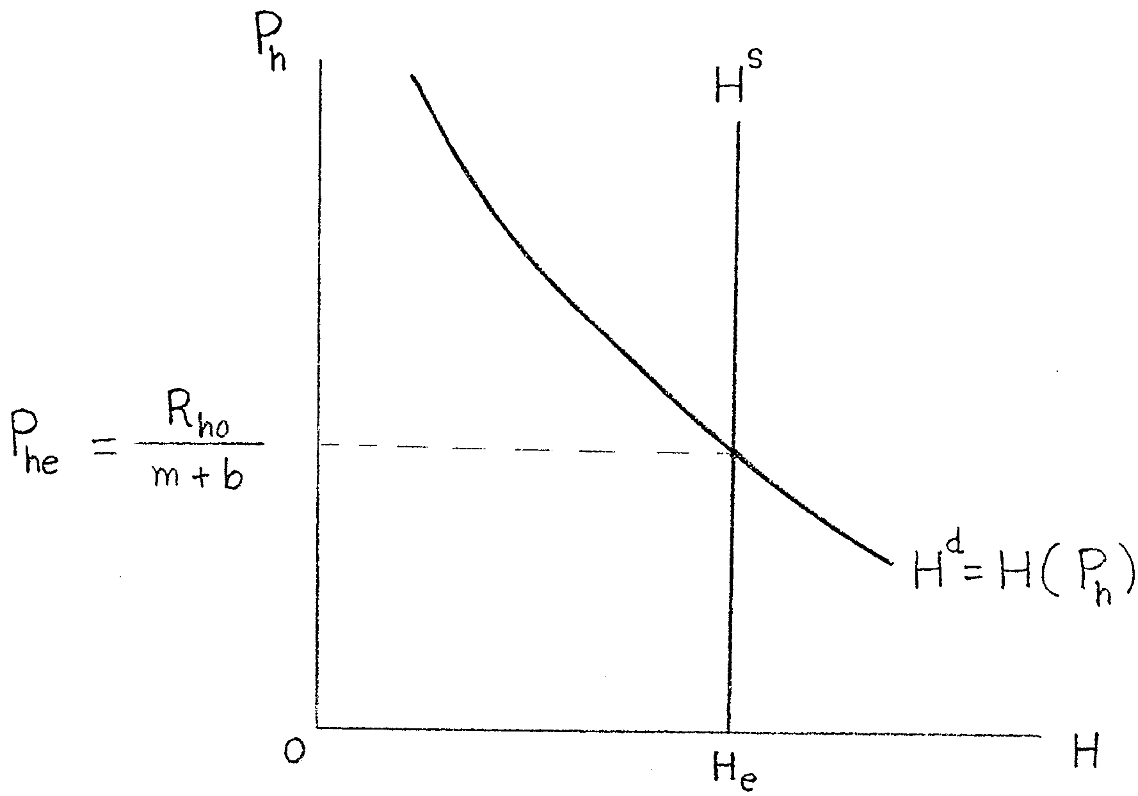
[16/] J.G. Witte, Jr., op.cit.




(Figure 3)

"efficiency units".[17/] Since the efficiency unit is a standardized unit of measure, the stock of dwelling-structures in efficiency units is an aggregate of homogeneous units. Accordingly, the single unit market price may now be represented by the price, $\bar{P}_t$, per efficiency unit. However this method cannot be employed in this study because there are no price data on structures of different ages or vintages.

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[17/] "Efficiency units" may be defined in the following manner. Let us adopt R.G. D. Allen's vintage approach and let the dwelling-structures of successive vintages become more efficient with technical progress. The technical progress in this case would be in the form of new architectural designs and engineering know-how that are embodied in the dwelling-structure of a particular vintage. Also, let the heterogeneity apply only on dwelling-structures of different vintages, so that dwelling-structures of the same vintage may be treated as homogeneous. Further, let $T(\tau) = e^{n\tau}$ represent the output-augmenting-effect of technical progress that is embodied in dwelling-structures of vintage $\tau$. The rate n is the speed at which technical progress takes place. Under these assumptions, if $H(\tau)$ is the physical number of dwelling-structures of vintage $\tau$, then

$$\bar{H}(\tau) = e^{n\tau} H(\tau) \qquad (13)$$

is a measure of all dwelling-structures of vintage $\tau$ in efficiency units. Accordingly, the aggregate stock of dwelling-structures of different vintages in efficiency units at t=0 is:

$$\bar{H}(t=0) = \int_{-\infty}^{t} \bar{H}(\tau)d\tau = \int_{-\infty}^{t} e^{n\tau} H(\tau)d\tau \qquad (14)$$

The aggregate stock of dwelling-structures in efficiency units, $\bar{H}(t=0)$, is thus a weighted sum of the physical number of dwelling-structures of different vintages with the factor $e^{n\tau}$ serving as the appropriate weight. See R.G.D. Allen, Macro-Economic Theory, Macmillan, 1968, pp. 285-287.

B.2. Market Pricing of Stock and Fractional Release of Stock to the Market

It can be argued that not all of the existing stock of dwelling-structures is offered for sale in the market. In fact, there is a greater likelihood that, from the time of their construction to their total disintegration, some dwelling-structures are only offered for sale once. Thus, it seems inappropriate to approach the market-pricing problem in terms of the total stock in existence. Instead, the pricing problem may well be approached in terms of only those structures that are fractionally released and traded on the residential housing market.

In the subsequent discussion, an attempt is made to demonstrate that both approaches yield the same price and quantity determination.

To start with, let us note that, at any moment of time, every physical dwelling-structure in stock must be owned by someone in the ex post sense. However, the actual quantity being held by economic units need not be the desired (ex ante) quantity. If there is a divergence between actual and desired quantities, it means that some economic units are willing to own less (more) than the actually held stock. Since any economic unit is at "liberty to decide for himself --on the basis of relevant market information"-- what quantity of physical structures to own, exchanges between economic units are certain to occur.[18] To the extent that the predisposition to own less (more) is not equally matched by the predisposition to own more (less), there will be competitive excess demand or supply which will lead to variations in the current market price of the physical dwelling-structures.

Gradually, the market price will settle at that level which allows all buyers and all sellers to exactly fulfill their requirements, thus clearing the market. If market clearing is achieved, it means that the actual quantity holdings of existing stock by all economic units is the same as the quantity they desire to own in the aggregate.

Perhaps an example will be helpful. For illustrative purposes, let there be only five economic units A,B,C,D, and M. Let each economic unit own (*ex post*) an initial stock of dwelling-structures in the following quantities:

| | | | |
|---|---|---|---|
| A | owns | 55 | units |
| B | " | 35 | " |
| C | " | 15 | " |
| D | " | 5 | " |
| M | " | 0 | " |
| Total Stock | | 110 | units |

[18] R. W. Clower, "An Investigation into the Dynamics of Investment", pp. 65-66.

Further, assuming a perfect market, let the trading costs be known beforehand and let there be no search costs or information costs involved that are characteristics of an imperfect market. (Some discussion of the imperfect market case are undertaken in the next sub-section, qualifying the market-pricing process discussed in the present sub-section). Finally, let the trading period be short enough to avoid consideration of net additions to the existing stock.

Each economic unit at any particular time has its own eagerness to buy or sell in the open market. This eagerness will determine its decisions as to when to be or not to be in the market. The individual eagerness is represented in Table 3 for each participant by columns (2) to (6). Mr. A, for instance, is predisposed to sell all his stocks of 55 units if the ruling unit market price is $13,000; but is inclined to sell only 20 units if the market price is $10,000. If the market price is very low, say $6,000, Mr. A not only is willing to keep his original stock of 55 units, but also is willing to add 10 more units by offering to buy 10 units on the market. The rest of the hypothetical data in columns (3) to (6) have the same interpretation.

It is possible to extract a realistic interpretation of columns (2) to (6). Unlike participant M, participants A, B, C and D are dealers in large numbers

Table 3

| UNIT PRICE | QTY. OF H TO BUY(+) OR SELL(-) | | | | | MARKET | |
|---|---|---|---|---|---|---|---|
| | A | B | C | D | M | DEMAND | SUPPLY |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| $13,000 | -55 | -35 | -15 | - 5 | 0 | 0 | -110 |
| 12,000 | -50 | -30 | -10 | - 2 | 0 | 0 | - 92 |
| 11,000 | -30 | -20 | - 5 | + 1 | 0 | + 1 | - 55 |
| 10,000 | -20 | -15 | - 3 | + 6 | + 1 | + 7 | - 36 |
| 9,000 | -10 | - 8 | - 2 | + 8 | + 1 | + 9 | - 20 |
| 8,000 | - 6 | - 6 | 0 | +10 | + 2 | +12 | - 12 |
| 7,000 | - 1 | 0 | + 6 | +14 | + 2 | +22 | + 1 |
| 6,000 | +10 | +20 | +10 | +20 | + 2 | +62 | 0 |
| 5,000 | +20 | +25 | +15 | +25 | + 2 | +87 | 0 |

Table 4

| UNIT PRICE | INDIVIDUAL STOCK DEMAND | | | | | TOTAL STOCK | |
|---|---|---|---|---|---|---|---|
| | A | B | C | D | M | DEMAND | SUPPLY |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| $13,000 | 0 | 0 | 0 | 0 | 0 | 0 | 110 |
| 12,000 | 5 | 5 | 5 | 3 | 0 | 18 | 110 |
| 11,000 | 25 | 15 | 10 | 6 | 0 | 56 | 110 |
| 10,000 | 35 | 20 | 12 | 11 | 1 | 79 | 110 |
| 9,000 | 45 | 27 | 13 | 13 | 1 | 99 | 110 |
| 8,000 | 49 | 29 | 15 | 15 | 2 | 110 | 110 |
| 7,000 | 54 | 35 | 21 | 19 | 2 | 131 | 110 |
| 6,000 | 65 | 55 | 25 | 25 | 2 | 172 | 110 |
| 5,000 | 75 | 60 | 30 | 30 | 2 | 197 | 110 |

of units. The case of these first four participants is probably typical of corporations and individuals whose main source of income is the real estate business. A typical household does not belong to this category. Instead, the typical household's behaviour is most likely represented by the case of M. Participant M will choose to remain a tenant at market prices ranging from $11,000 up. Either he feels he cannot afford to own a house in this price range, or he feels the cost of moving to be incurred is too high a price to pay for the extra utility derivable from homeownership.[19/] However, if the market price is $10,000, household M will be predisposed to buy a house for ownership, indicating that the marginal utility of a $'s worth of the fact of homeownership is higher than the marginal

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[19/] Farrell and Cramer advocated the idea that consumers derived utility from the fact of ownership of durable goods, in addition to the utility derived from the consumption of the services of durable goods. See J.S. Cramer, <u>The Ownership of Major Consumer Durables</u>, Cambridge University Press, 1962; M.J. Farrell, "The Demand for Motor Cars in the United States", <u>Journal of Royal Statistical Society</u> (Series A) vo. 117, 1954 cited by Cramer.

utility of a $'s worth of other consumer goods.[20]

The sum of all the individual offers-to-buy and-to-sell at various alternative prices is presented in columns (7) and (8). If these last two columns of Table 3 are plotted (neglecting signs), the usual supply and demand curves will emerge. The market is cleared at price $8,000 and quantity 12 units.

The hypothetical data in Table 3 include only those structures actively traded on the market. These same data can be interpreted in a different manner to permit consideration of both those in active trade and those that are potentially tradable. For instance, at the unit price $10,000, it is shown that Mr. A is eager to sell only 20 units of his total stock of 55 units. This eagerness to sell 20 units equivalently means that

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[20] The analytical argument behind this statement may be elaborated as follows. Assume that the house the tenant now occupies is exactly identical in all respect to the alternative house the tenant can buy (for ownership) in the market. Assume further that all other circumstances remain the same except for the fact of homeownership. The only way the tenant can be induced to buy a house and change residence is the extra utility to be derived from homeownership, which of course he has to "pay" for by an amount X equal to the cost of of moving. If the amount X would yield him higher extra utility in an alternative use, he will not be induced to become homeowner. However, as the price of houses declines, the marginal utility of a dollar's worth of alternative consumer goods becomes smaller relative to the marginal utility of a dollar's worth of the fact of homeownership.

Mr. A is eager to hold in stock 35 units (Table 4) at the $10,000 market price. To take another example, participant B is willing neither to augment nor to deplete his initial stock of 35 units if the market price is $7,000, being satisfied with his present stock. B's stock demand in this case is 35 units (Table 4). The rest of the data in columns (2) to (6) of Table 4 have the same interpretation.

Column (7) of Table 4 represents the sum of the stock ownership demands of the individual economic units at various alternative prices. The total stock available for ownership amounts to 110 units (column 8). The graph of columns (7) and (8) of Table 4 is similar to Figure 3 above. The equilibrium market price is again $8,000. At this price, the actual holdings of the 110 dwelling units exactly equal the stock that all economic units desire to own in the aggregate. It is only at this price that all buyers find enough sellers and all sellers find enough buyers to clear the 12 units actively traded on the market.

B.3. Some Qualifying Observations[21]

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[21] The discussion of this sub-section is based mainly on the articles by E.S. Phelps, "Introduction: The New Microeconomics in Employment and Inflation Theory"; A.A. Alchian, "Information Costs, Pricing, and Resource Unemployment"; D.F. Gordon & A. Hymes, "On the Theory of Price Dynamics"; D.A. Nichols, "Market Clearing for Heterogeneous Capital Goods"-- in E.S. Phelps, et. al., (eds.) Microeconomic Foundations of Employment and Inflation Theory, W.W. Norton & Co., Inc., 1970.

The introduction of market imperfections (e.g., limited knowledge of the market and of the product, product heterogeneity and indivisibility) necessitates some observations qualifying the preceding analysis. At least three can be mentioned. First, there are aggregation problems. Dwelling-structures are a continuous web of competing but heterogeneous units. It is not correct to add together a bungalow and an apartment unit. One way to get around some of these aggregation problems is the vintage approach already suggested in sub-section B.1. above, although of course this approach is still faced with the problem of adding together heterogeneous dwellings constructed on the same date. Further, most of the pure theory of economics deals with individual decision making units such as households and business enterprises. It is these individuals that respond to market disturbances; and it is the understanding of these responses that is essential to the succesful anticipation of the further effects of the initial market disturbances. All too often, however, the individual responses get concealed or buried in the process of aggregation, making it difficult to identify the coefficients of aggregate relationships with those of the basic decision-making units.[22]

<u>Second</u>, the market clearing process becomes relatively slower and, indeed, more unpredictable because imperfect market knowledge may cause individuals to make random or non-optimal responses. Further, since the actual and potential market participants have only limited market information, they do not know all possible bids and offers. They may not conclude transactions at the first offer-price or bid-price that comes along. Instead, the participants will want to abstain from making transactions in favor of a more intensive search of the market environment. Within the limitations of the bearable carrying-cost of the property (e.g., interest foregone for not selling or renting an unoccupied dwelling, property taxes, etc.) this tendency to wait is further intensified by the very durable nature of the property, along with the expectation that future market bids will be more favourable.

<u>Third</u>, new types of cost get introduced. Costs are incurred in transactions and in the gathering and collating of information about current and possible future market bids. There are also costs involved in

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[22] Karl A. Fox, <u>Intermediate Economic Statistics</u>, John Wiley & Sons, Inc., pp. 10,13 and Chapter 14.

the inspection and appraisal of the various characteristics and dimensions of dwelling-structures. Inspection is important because the general indivisibility of the structure will not allow separate transactions in specific characteristics that are of primary interest to the buyer. Needless to say, the authenticity of and encumbrances to the documents concerning rights and title over the property require close scrunity. All of these cost time, effort, and money.

It has been argued that product heterogeneity and indivisibility, limited information, and transaction and search costs are the most tenable explanatory factors for the presence of an optimum rate of unemployment in economic resources. Since these factors are characteristic features of an imperfect market, one may well argue that voluntary and frictional unemployment of economic resources is due to market imperfections.

The optimum rate of unemployment of resources is very much analogous to the "desired" inventory of unsold goods and raw materials in many types of manufacturing operations.[23/] Further, it would seem

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[23/] The application of the intended or desired inventory principle to housing may be found in S. Maisel, "A Theory of Fluctuations in Residential Construction Starts", The American Economic Review, June 1963, pp.366-367. See also W.F. Smith, Housing: The Social and Economic Elements, University of California Press, 1970, p.100.

that the normal or optimal unemployment rate will move up and down depending upon the relative offsetting effect of transaction and search costs and of revenues resulting from a marginal reduction in the volume of unemployed units. This relationship is the same marginal condition for equilibrium, with the transaction and search costs introduced into the analysis and with the understanding that these costs are likely to vary with the degree of market imperfection.

In particular, there is no reason why 100% occupancy of apartment units cannot be achieved, if landlords are willing to spend on advertisement and promotional schemes and to depress rental charges for as long as there are vacant units. But if the reduction in vacancies can only take effect through substantial rent reduction, with the further cost of similar across-the-board reductions for the already occupied quarters, past a certain point the landlords are better off operating with some normal vacancy rate.

This normal vacancy rate does not imply a corresponding fractional amount of excess supply of physical dwelling-structures in the housing market. The vacant or unsold portion of the total stock must still be owned by somebody. If at the current market price there is a desire to keep a portion of the total stock in an unoccupied or unsold status, the owners of these units believe (call the belief illusion or whatever) that they are at an optimal stock position-- given their expectations, past experience, limited information, and impression of the marginal costs and revenues associated with a further reduction of the unsold or vacant units. Moreover, if market prices that are higher (lower) than expected lead owners to reduce (expand) the number of unsold or vacant units to an abnormally low (high) level, then there will be a positive elasticity of supply of housing-services in the short-run. This positive elasticity is associated with variations in the volume of short-run variable inputs (V) used in the production of housing-services.

C. Short-Run Variable Inputs in the Production of Housing-Services

In Section A, it was shown that if all the factor inputs -- dwelling-structures(H),land-site (L), and the variable inputs (V), representing home appliances, furnitures, maintenance and repairs, etc.---are fixed in amount, the supply curve of housing-services, Q, is perfectly inelastic. In this section, if H and L are fixed and V is variable, variations in V will introduce a finite degree of supply-price elasticity to the output of housing-services, Q.

The crucial role played by this section in the theoretical development and empirical application of the basic model relates to three aspects. First, it facilitates the clear-cut introduction of the property tax variable into the model(discussed in Chapter IV). Second, it provides a strong basis for determining whether the property tax forms part of the short-run fixed cost or the variable cost of production of housing-services. Third, it adds justification for inclusion of the unit price of housing-services in the short-run supply function for housing-services, as well as in the long-run.

Let us consider the production of housing-services in the short-run. In the short-run production period, factor inputs are classified as either fixed or variable. The short-run production period is not tied to a specific time interval. The period could take anywhere from a day to a couple of years or more. The only criterion for determining whether the production period is short or long is the fixity of at least one input in the production process. The period is said to be short-run if at least one production input is fixed. In the present case, there are two fixed inputs--the physical dwelling-structures, H, and the physical land,L. The basis for choosing H and L as the short-run fixed inputs is the observational appeal to the fact that the owners of housing-packages are less prone to vary H and L than V. The factor inputs represented by V are seen as short-run variable inputs.

If the fixed inputs land(L) and dwelling-structures (H) become variable, particularly H, the period is considered long-run. The long-run period is generally associated also with changes in the number of families, households, population, etc., as well as with changes in tastes and preferences, building technology, locational pattern of residences , etc.[24/]

The short-run production process for housing-services is analogous to the accepted short-run theory of the firm.[25/] The physical dwelling-structures H, and the physical land, L, are fixed inputs to which the owner applies additional amounts of variable inputs V to generate additional output, Q, of housing-services

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[24/] It is useful to distinguish two types of long-run market period. The first refers to a period long enough to permit full adjustment in the quantity of factor inputs to underlying market conditions. The second refers to a period longer than the first one in that it not only permits full adjustment in quantity, but also in the form (architectural and engineering styles, location patterns) of factor inputs to underlying market conditions. See F. de Leeuw and N. Ekanem, "The Supply of Rental Housing:Reply", American Economic Review, June 1973, pp. 437-438.

[25/] This is Heilbrun's fundamental approach to the analysis of the effects of alternative types of real estate taxation on the maintenance and rehabilitation of urban rental housing. See his, Real Estate Taxes and Urban Housing, Columbia University Press, 1966; Reforming the Real Estate Tax to Encourage Maintenance and Rehabilitation", in A.P. Becker (ed.) Land and Building Taxes: Their Effects on Economic Development, University of Wisconsin Press, 1969, pp. 63-79.

albeit at a diminishing rate.

The variation in the quantity of factor inputs V is seen to be price motivated. In particular, any profit-seeking landlord aspires to find the most profitable way to operate his rental dwellings. Generally, the higher the unit market price E, the larger is the volume of variable inputs V he will put into the production process.

The jth landlord, of course, cannot observe his output of housing services, $Q_j$, nor its conceptual unit market price E. However, he can readily observe and measure his gross rental income, $EQ_j$, and his total flow cost per time period. Thus, the landlord's objective is to maximize the spread between these two observable and measurable figures.[26] But this objective is equivalent to saying that the preferred level of output is where an extra unit of housing services, $Q_j$, brings exactly the same amount of extra revenue to the landlord as it costs. Since the landlord is presumed to be a price-taker in a local market area, the most profitable level of production of $Q_j$ is where the marginal cost is equal to the unit market price E.

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26/ J. Heilbrun, Real Estate Taxes and Urban Housing, 1966, pp. 16-17.

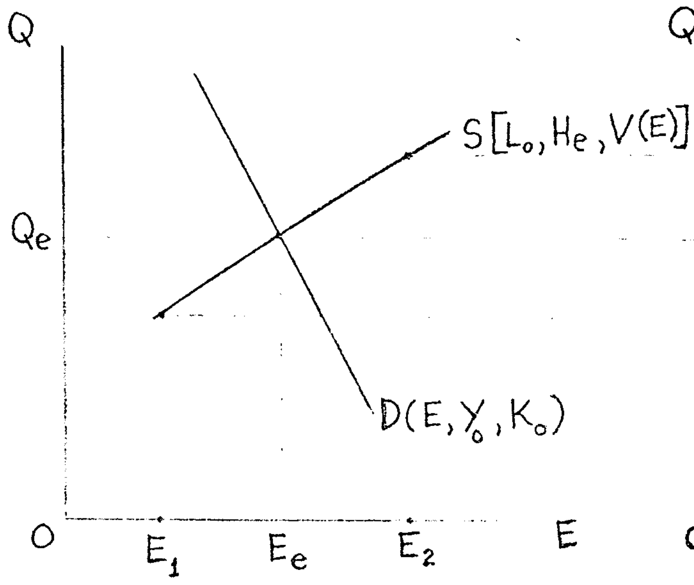
It is not difficult to see that the technical production of housing services is subject to the principle of diminishing marginal returns for a given series of variations in V. In a twenty-suite apartment block, the hiring of the first caretaker to look after the complaints and requests of tenants adds more housing-services per suite than the second or third caretaker. Every suite, whatever its relative size can accommodate only so much carpeting, draperies, and decorations beyond which these items are nuisances.

Perhaps the case of adding more and more washing machines illustrates the principle vividly. Conceivably, one can fill the dwelling-structures with washing machines in a month or year. Obviously, unless the structure is intended for conversion to a warehouse, even before the structure's filling-capacity is reached, the marginal contribution of the last unit of machine would become zero or negative.

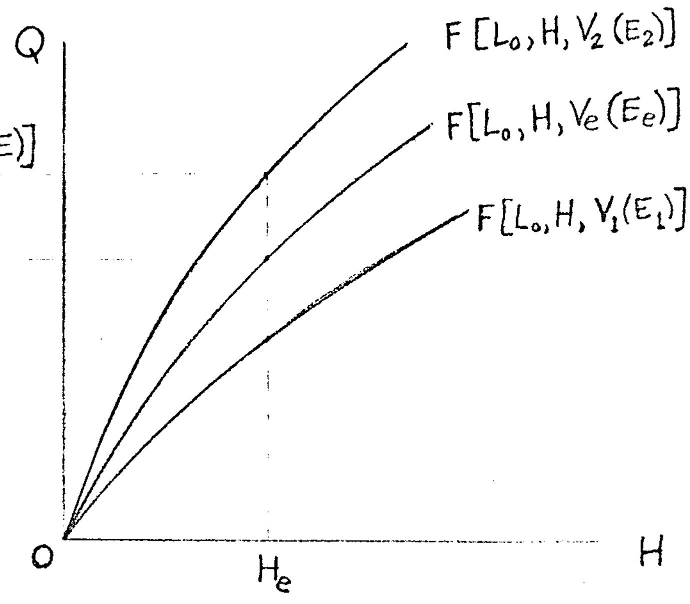
If a year is arbitrarily chosen to be the unit of analysis, the life of an individual washing machine undoubtedly exceeds this period. In this case, the total flow cost variable associated with the machines is not the acquisition cost of these machines, but only a fraction thereof. This fraction is the sum of the true depreciation cost of the machines and the interest foregone on the owner's equity and/or the interest charges on borrowed funds used for the acquisition of the machines. Assuming that the washing machines are homogeneous, their true individual depreciation costs are more or less equal per period. However, since each extra machine put into installation and use adds a smaller contribution to total output, the marginal cost of production of housing-services must increase with output. The short-run aggregate supply of housing-services, therefore, should exhibit a finite degree of supply-price elasticity.

The necessary changes in the diagrammatic exposition of the basic model due to the introduction of variations in V are presented by Figures 4 to 6. In contrast to Figure 1, the market supply curve of housing-services, Q, in Figure 4 is now a function of the unit market price, E. The supply function $S[\bar{H}_e, L_o, V(E)]$ is evaluated at given stocks of $H=H_e$, $L=L_o$, and at various amounts of V.

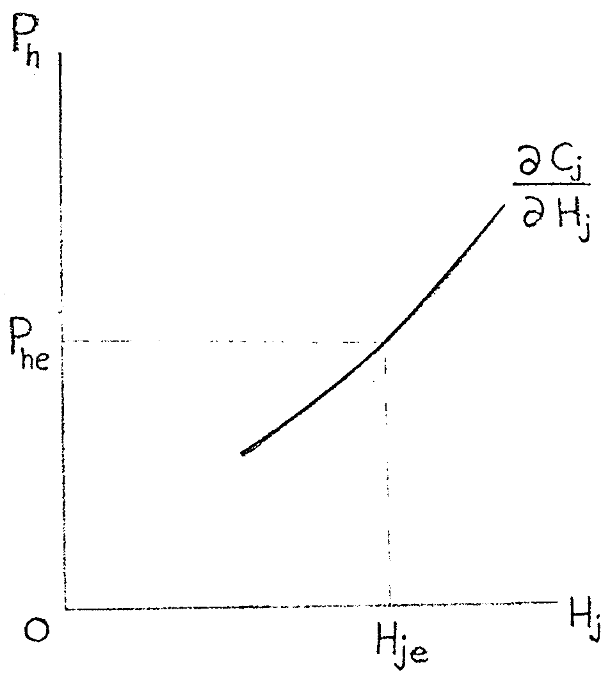
Figure 5 shows the corresponding shifts in the production function for housing-services, Q, at various levels of variable inputs V. These shifts in the production function trace corresponding shifts in the derived stock-demand for dwelling-structures, H. The increases in the amount of V have the corresponding effect of increasing the marginal productivity of the



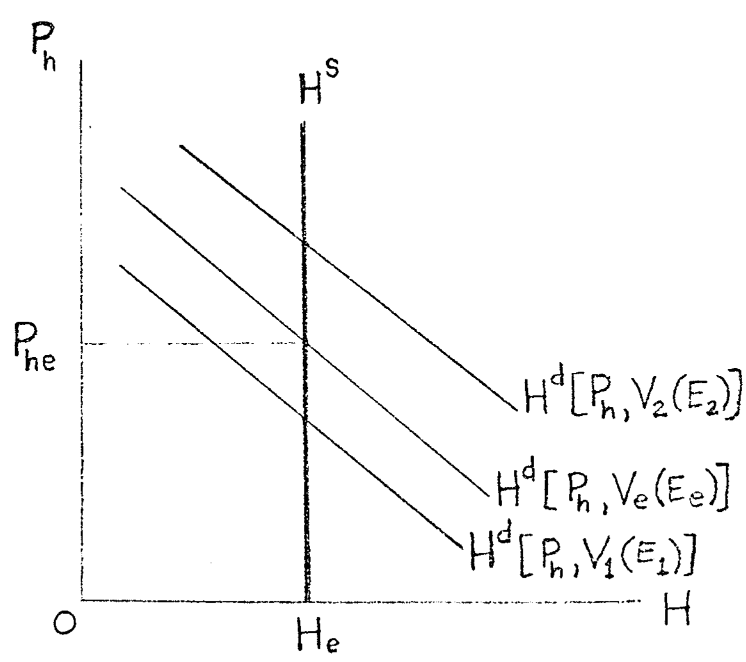

(Figure 4)




(Figure 5)




(Figure 7)




(Figure 6)

given stock of $H=H_e$. Therefore, the schedule of capitalized or discounted values of the marginal productivity of H (derived demand curve) will shift upward. Further, since a larger amount of V is associated with a higher unit market price, E, the higher derived demand curve for dwelling-structures, H, must also be associated with a higher E. The shifts in the derived demand curve for H are shown in Figure 6.

Figures 4 to 6 show that the equilibrium market levels of Q and E are $Q_e$ and $E_e$, respectively. At $E=E_e$, the corresponding optimal amount of V is $V_e$. The equilibrium market price, $P_{he}$, of physical H is determined by the intersection of the stock-supply curve $H^s$ and the stock-demand curve $H^d[\overline{P}_h, V_e(E_e)]$.

Figure 7 shows the level of output of <u>new</u> H for the jth builder, appearing as a price-taker. The discussion of the next section expands on Figure 7.

D. <u>The Production of Physical Dwelling-Structures, H</u>

Let the ruling market price, $P_{he}$, be high enough to warrant the construction of new H. The profit function for the jth builder may be expressed as

$$\Pi_j = P_{he}H_j - C_j(H_j, W, Z) \qquad (18)$$

where $\partial C_j/\partial H_j > 0$; $\partial^2 C_j/\partial H_j^2 > 0$; $H_j$ = output of (newly completed) dwelling-structures; $C_j$ = total construction costs appearing as a function of $H_j$, of W (= wage rate in the construction industry and of Z (=cost of construction materials). In an amplified version of equation (18), $H_j$ will appear explicitly as a function not only of the price of H, but also of the prices of land, L, and of the variable inputs, V. The short-run equilibrium level of output for the jth builder $H_{je}$ is determined by the equation,

$$P_{he} = \frac{\partial C_j}{\partial H_j} , \qquad (19)$$

as is shown in Figure 7. The individual builder is presumed to be a price-taker. The individual output of new dwelling-structures, $H_{je}$, is seen as a negligible fraction of the existing stock. If there are <u>k</u> builders, the total short-run output, $I_g$, of physical dwelling-structures is

$$I_g = \sum_{j=1}^{k} H_{je} \qquad (20)$$

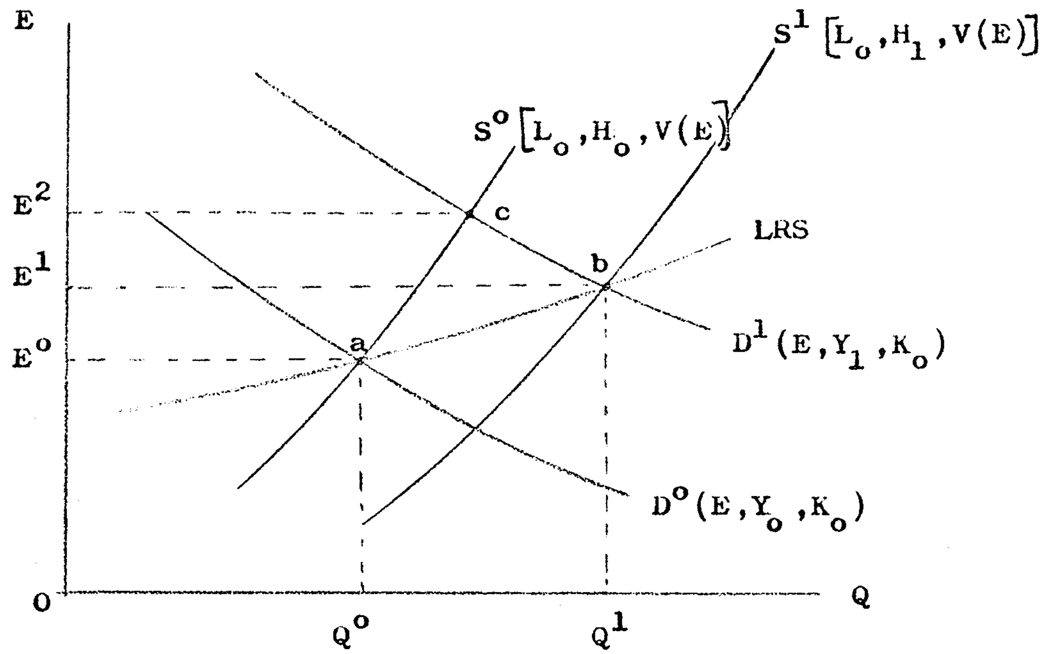
Earlier, it was assumed that $I_g$ is a negligible fraction of the total stock of dwelling-structures, H. In support of this assumption, we have presented the data in Table 2 above. This table shows that the volume of newly completed dwellings per annum in selected Canadian urban areas ranged from 0.42% to 6.27% of total occupied dwellings in 1966. We have also expressed the view that these percentages are overstated because the data on occupied dwellings do not include vacant units. In Section B of this chapter we alluded to the measurement of physical dwellings in efficiency units. It may now be argued that the overstatement arising from the exclusion of vacant units is not analytically conclusive. In terms of efficiency units, a new dwelling may well count more than an old residence. In efficiency units, the resulting percentages may be higher than the percentages shown in Table 2. However, we cannot verify this possibility from available data.

Generally speaking, the total volume of newly constructed dwelling-structures, $I_g$, is larger the higher is the unit price, $P_h$, at which new structures can be cleared in the market. Similarly, $I_g$ is larger the longer is the period of time used in measuring $I_g$. The larger is $I_g$ the greater is its potential influence on the unit price, $P_h$, of the existing stock. This potentiality is also greater in a longer-run period of production, which involves changes in the scale of operation and permits entry and exit of builders to and from the construction industry.

The short-run and long-run effects of the volume of residential construction on the unit price, $P_h$, can be seen in two ways. First, the periodic additions to total stock will accumulate and increase the available stock over time. Second, the net additions will also cause shifts in the aggregate market supply of housing-services. These shifts will affect the unit price, E, of housing-services and thereby, the derived stock demand for physical dwelling-structures. If these effects happen, there will be a feedback on the unit price, $P_h$ , and on the volume of residential construction per period.

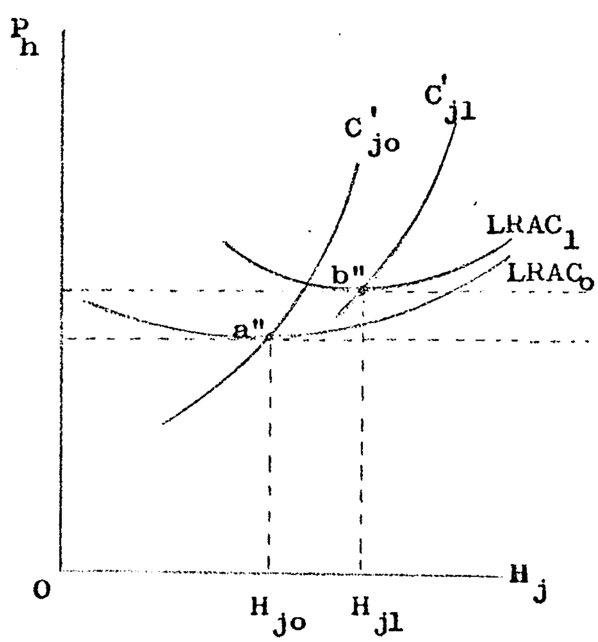
The system of interrelationships between the volume of residential construction and the stock of dwelling-structures is analyzed here under two different circumstances. The <u>first case</u> assumes that the prices of housing services and dwelling structures adjust instantaneously to changes in demand and supply. In particular, this assumption implies that any initial difference between the desired stock and the actual stock of H easily gets corrected through instantaneous price adjustments. The <u>second case</u> assumes that the prices adjust with some delays. The more sluggish the unit price adjustments, the greater tends to be the difference between the desired and the actual stock of H at any time and the slower is the process with which the gap is closed overtime.

Let us consider Figures 8, 9, and 10. In Figure 8, it is assumed that the long-run market supply, LRS, of housing services, Q, is upward sloping(increasing cost case) and that at point <u>a</u> the market for Q is in initial short- and long-run equilibrium. In Figure 9,

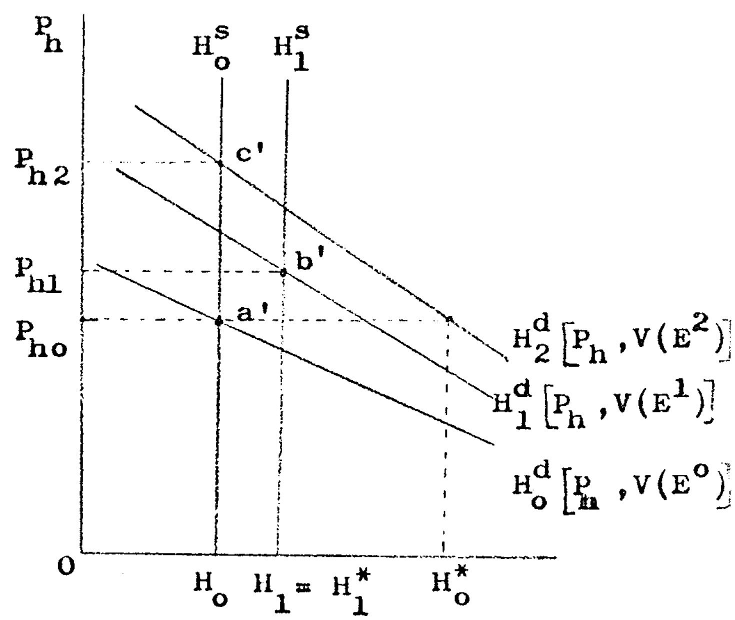



(Figure 8)

$C'_{jo}$ and $C'_{j1}$ are long-run marginal costs, and $LRAC_o$ and $LRAC_1$ are long-run average costs.




(Figure 10)




(Figure 9)

the corresponding initial long-run equilibrium for physical H is represented by point <u>a</u>'. In Figure 10, the jth builder is shown to be in initial long-run equilibrium at point <u>a</u>". There is the implicit assumption here that the construction industry is an increasing cost industry.27/ In Figure 9, the initial long-run equilibrium price of H is $P_{ho}$. At the unit price $P_{ho}$ and level of stock $H_o$, $\sum_{j=1}^{k} H_{jo}$ exactly meets the replacement requirements to keep the actual stock at the level $H_o$.

Let there be an autonomous and permanent increase in aggregate income from $Y_o$ to $Y_1$. Let this increase in income cause an upward shift in the market demand for housing services from $D^o$ to $D^1$ in Figure 8. In response, the market for housing-services will establish its short-run equilibrium position at point <u>c</u>. At this point $E=E^2$. Since at <u>c</u> the unit price(E) of housing-services is higher, the derived demand for dwelling-structures, H, in Figure 9 will be shifted upwards, say, from $H_o^d$ to $H_2^d$, thereby creating an excess quantity of stocks demanded equal to $H_o^*-H_o$. Here, $H_o^*$ represents the desired stock of H when $P=P_{ho}$ and $Y=Y_1$.

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27/ An expanded discussion of the increasing cost case in the production of Q and H is pursued in Chapter IV.

If the unit price, $P_h$, adjusts instantaneouly to the excess quantity demanded, the stock price level will immediately increase from $P_{ho}$ to $P_{h2}$. If this happens, the excess stock demanded will be zero. At $P_h = P_{h2}$, builders will produce new H over and above the total replacement requirements. The higher price will also encourage entry of new builders in the construction industry.[28] A greater volume of new dwelling-structures will periodically be added to the existing stock. The periodic net additions to the existing stock will accumulate and cause rightward shifts in the short-run market supply, S, of housing-services in Figure 8. As S shifts rightward, say, from $S^o$ to $S^1$, the unit price of housing-services will decline from $E^2$ towards the new long-run equilibrium level $E^1$. This decline in E will pull downward the derived demand for H from $H_2^d$ towards $H_1^d$ in Figure 9. $P_h$ will not persist at $P_{h2}$ and will decline towards $P_{h1}$. As long as the stock price, $P_h$, exceeds the minimum long-run average cost (LRAC) of individual builders, there will be entry into the construction industry. As long as the total output of new dwelling-

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28/ U.S. data indicate that both entry and exit of firms were easier and far more rapid in the construction industry than in any other industrial group during the period 1945-1950. See R.F. Muth, "The Demand for Non-Farm Housing", pp. 42-45.

structures exceeds the periodic replacement demand, net additions to existing stock will accumulate and trigger further changes in price and quantity in the markets for housing-services and dwelling-structures. These adjustments will continue until a new long-run market equilibrium is achieved.

In Figure 8, the new long-run market equilibrium is represented by point b . The equilibrium quantity of housing-services $Q^1$ requires an optimum stock of dwelling-structures, $H_1^*$ , equal to $H_1$. This required or desired stock $H_1^*$ gets fulfilled in the market for physical stock. In Figure 9, $H_1^*=H_1$ is determined at point b' and at a unit price $P_h = P_{h1}$. At the equilibrium price, $P_{h1}$, the individual builders are operating at their lowest long-run average cost (LRAC). In Figure 10, the lowest LRAC for the jth builder is shown by point b". Also $\sum_{j=1}^{k} H_{j1}$ is equal to the volume of replacements required to maintain H at $H_1$.

Even if the stock price, $P_h$, adjusts sluggishly the process by which the new long-run equilibrium is achieved will be similar to that of the instantaneous price adjustment case.[29/] However, due to the delayed response of $P_h$ to the excess quantity demanded, the desired and actual stock of H will differ during periods of long-run disequilibrium. A stock-price-adjustment model may then appear more applicable. Further, since

29/ Except that it has different implications for output, particularly the volume of construction.

there is a delayed response in the unit price, a longer time interval will generally be required to complete the market adjustments towards long-run equilibrium.

If there is a rapid growth in the market area, the stock price, $P_h$, and the volume of new residential structures completed are potentially interrelated in a simultaneous way. The volume of new residential construction will respond to changes in $P_h$. As the volume of new residential construction adjusts to price changes, it will also affect the flow supply of housing-services and the stock supply and demand for physical dwelling-structures. There will be a consequential feedback on the unit price, $P_h$. However, as long as there are delays in response, the system of inter-relationships may well be recursive.

The analysis may be extended under the assumption that the individual house builders are not price takers. In this case, the system of interrelationships is necessarily simultaneous. In Chapter VI, our cross-section construction regressions appear to support this assumption.

E. Summary

This chapter has presented a theoretical framework which illuminates the interrelationships between (1) the market for housing-services, (2) the market for the stock of physical inputs with special reference to dwelling-structures, and (3) the volume of residential construction. The derived demand for physical dwelling-structures or residential housing packages is seen as a demand for ownership. The outputs of housing-services have been assumed homogeneous or perfect substitutes so that the analysis may proceed initially with only one market for housing-services. In particular, we have ignored the distinction between rental and owner housing-services as well as the categorical nature of the demand for ownership of a residential dwelling, which may be either for owner-occupancy or for tenant-occupancy, or both.

The next chapter will introduce into the theoretical framework the residential property tax which is of specific interest to our investigation. Here, we shall distinguish between owner and rental dwellings and, later in the chapter, we shall allow for the imperfection in the substitutability of rental and owner housing-services.

# CHAPTER IV

## RESIDENTIAL PROPERTY TAXATION AND THE HOUSING MARKET

This chapter will introduce residential property taxation as a policy tool into the model. The short- and long-run effects of the property tax will be investigated. In the process, we shall expand the theoretical framework by constructing a two-market model for housing-services, thereby treating the outputs of rental and owner housing-services as imperfect substitutes. The two-market approach will elucidate the relative effects of the tax on the owner and the rental sectors, as well as the factors that determine the extent of these effects.

The analysis of the effects of residential property taxation on the housing market appears intractable without a background analysis of the individual homeowner's reaction to this tax. Therefore, the first three sections of this chapter will focus on the individual homeowner and attempt to bring out the essential features of his response to changes in the property tax rate, in the price of housing-services, and in his income. With this background analysis, the chapter will investigate the tax-effects from the viewpoint of a local market area and underline some testable hypotheses concerning the impact of the property tax on the residential market. Chapters V and VI of this study will test these hypotheses against actual Canadian housing data.

A. General Statements About a Homeowner's Choice

All existing housing-packages must be owned at any moment of time by corporations, government, collective cooperatives, individuals and families or households. Let these owners be called homeowners. A homeowner operates housing-package(s) to produce housing-services either for his own consumption or for sale to the residential market (or both). In the first case, the homeowner may be called a pure-homeowner. In the second case, he is a homeowner-rental-housing-operator or simply a landlord. The following are generalized statements about the maximizing behaviour of a representative homeowner of whichever category.

Let

$M_o$ = homeowner's fixed money income from sources other than that connected with the production of housing-services;

$E$ = unit market price of housing-services;

$Q_{jo}$ = the output of housing-services for own consumption of the homeowner;

$Q_{ji}$ = the output of housing-services for sale by the homeowner on the residential market;

$C(Q_{jo},Q_{j1})$ = total short-run cost of production of housing-services;

$EQ_{j1}$ = total revenue or gross rental income from market sales of housing-services.

The homeowner's money income devoted to other goods and services and savings is

$$Y = M_o - C(Q_{jo},Q_{j1}) + EQ_{j1} \quad (1)$$

Let the homeowner's utility function be of the form:

$$U = U(Q_{jo},Y), \quad (2)$$

with $\partial U/\partial Q_{jo} > 0$, $\partial U/\partial Y > 0$, and with the second-order condition that U be convex to the origin. For an extremum problem like the case at hand, let there be Z with the Lagrangian multiplier $\lambda$ such that

$$Z = U(Q_{jo},Y) + \lambda \left[ M_o - Y - C(Q_{jo},Q_{j1}) + EQ_{j1} \right]. \quad (3)$$

The first-order conditions for a utility maximum are thus:

$$\frac{\partial Z}{\partial Q_{jo}} = \frac{\partial U}{\partial Q_{jo}} - \lambda \frac{\partial C}{\partial Q_{jo}} = 0 \quad (4\text{-a})$$

$$\frac{\partial Z}{\partial Y} = \frac{\partial U}{\partial Y} - \lambda = 0 \quad (4\text{-b})$$

$$\frac{\partial Z}{\partial Q_{j1}} = - \lambda \frac{\partial C}{\partial Q_{j1}} + \lambda E = 0 \quad (4\text{-c})$$

$$\frac{\partial Z}{\partial \lambda} = M_o - Y - C(Q_{jo},Q_{j1}) + EQ_{j1} = 0 \quad (4\text{-d})$$

which can be summarized into:

$$\frac{\partial U/\partial Q_{jo}}{\partial U/\partial Y} = \frac{\partial C}{\partial Q_{jo}} \qquad (5)$$

and

$$\frac{\partial C}{\partial Q_{j1}} = E, \qquad (6)$$

subject to

$$M_o = Y + C(Q_{jo}, Q_{j1}) - EQ_{j1} . \qquad (7)$$

Expressions (5) to (7) means that the homeowner (landlord) maximizes both utility and profit.[1/] The maximum (profit) net rental income from open market sales of $Q_{j1}$ forms part of the landlord's money income. This money income, in turn, is allocated between $Q_{jo}$ and Y (for other goods and services) in the manner expressed by equation (5) in order to maximize utility. This expression says that the marginal rate of substitution between $Q_{jo}$ and Y should be equated to the marginal cost (or opportunity cost) of $Q_{jo}$. If the

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[1/] With profit appearing directly as only one of the arguments in the utility function, T. Scitovsky has shown that profit and utility cannot be maximized simultaneously by the economic unit. See T. Scitovsky, "A Note on Profit Maximization and Its Implications", <u>Readings in Price Theory</u> (eds.) G. Stigler & K. Boulding, R.D. Irwin, 1952, pp 353-358. This argument, however, is not relevant for the utility function given in expression (2) above.

economic unit is a pure homeowner, then $\partial C/\partial Q_{j1} > E$ and $Q_{j1} = 0$ in all of the above relations.

The foregoing relationships facilitate the analysis of some of the essential features of the individual homeowner's behavioural response to changes in prices, income, and property taxation. The analysis is presented in subsequent sections.

For simplicity in diagrammatic exposition, let us assume for the moment that $Q_{jo}$ and $Q_{j1}$ are homogeneous outputs. Further, let the short-run cost of production of housing-services, $C(Q_{jo}, Q_{j1})$, be classified as either separable or inseparable. Separation in space need not mean separation in cost functions. However, it may well be a reasonable approximation in the case of housing-services. It is, therefore, assumed here that the homeowner's costs of production of $Q_{jo}$ and $Q_{j1}$ are <u>separable</u> if his place of residence is a separate entity in space from the rental housing-package(s) he operates. Accordingly, it is assumed here that the costs of production of $Q_{jo}$ and $Q_{j1}$ are <u>inseparable</u> if the housing-services are generated from the same housing-package, i.e., the dwelling is partly owner-occupied and partly tenant-occupied. The assumptions may be re-stated as follows:

$$Q_j = Q_{jo} + Q_{j1} \tag{8}$$

i.e., $Q_{jo}$ and $Q_{j1}$ are homogeneous outputs,

$$C(Q_{jo}+Q_{j1}) = F + W(Q_j) \text{ inseparable costs,} \quad (9)$$

where $W'(Q_j) > 0$, $W''(Q_j) > 0$

$$C(Q_j) = C_o(Q_{jo}) + C_1(Q_{j1}) \text{ separable costs,} \quad (10)$$

$$C_o(Q_{jo}) = F_o + W_o(Q_{jo}) \quad (11\text{-a})$$

$$C_1(Q_{j1}) = F_1 + W_1(Q_{j1}) \quad (11\text{-b})$$

and where $W_o'(Q_{jo}) > 0$, $W_o''(Q_{jo}) > 0$

$W_1'(Q_{j1}) > 0$, $W_1''(Q_{j1}) > 0$ .

The terms $F$, $F_o$, $F_1$ represent the fixed flow cost elements, and the functions $W$, $W_o$, $W_1$ represent the variable flow costs, in the production of housing-services.

In particular, should the residential property tax payments form part of the fixed cost or the variable cost of production of housing-services? In actual practice, the residential property tax generally applies only on the combined assessed value of land and buildings. Thus, in our model, the tax is seen to apply only on the combined assessed value of the land-site, L, and the physical dwelling-structures, H. The variable inputs, V, representing home appliances, furnitures, draperies, etc., are not subject to the tax. In the short-run period of production of housing-services, the physical L and H are fixed amounts of inputs. In effect, without any change in the residential property tax rate, the tax cost can vary during the period only through changes in the assessed values of L and H. Again, in the model, it is advisable to treat the assessed values of L and H as constant because, in actual practice, land and buildings are usually re-assessed every four or five years and, for taxation purposes, their assessed values are considered fixed during any taxation period(one year). Thus, in our analysis, the property tax payment per time period forms part of the fixed costs F, $F_o$, and $F_1$.

B. Tenant's and Pure-Homeowner's Choice of Tenure

Here and in subsequent discussions, the analysis abstracts from all other factors affecting the choice of tenure, such as weather considerations, family size, impermanence of job, social and cultural environment, expectations about possible changes in job-site, etc. In short, we are isolating the basic economic principles that govern the economic unit's choice due to changes only in E, in his income, and in the property tax rate, all other factors remaining the same.

B.1. Changes in the Unit Market Price of Housing-Services

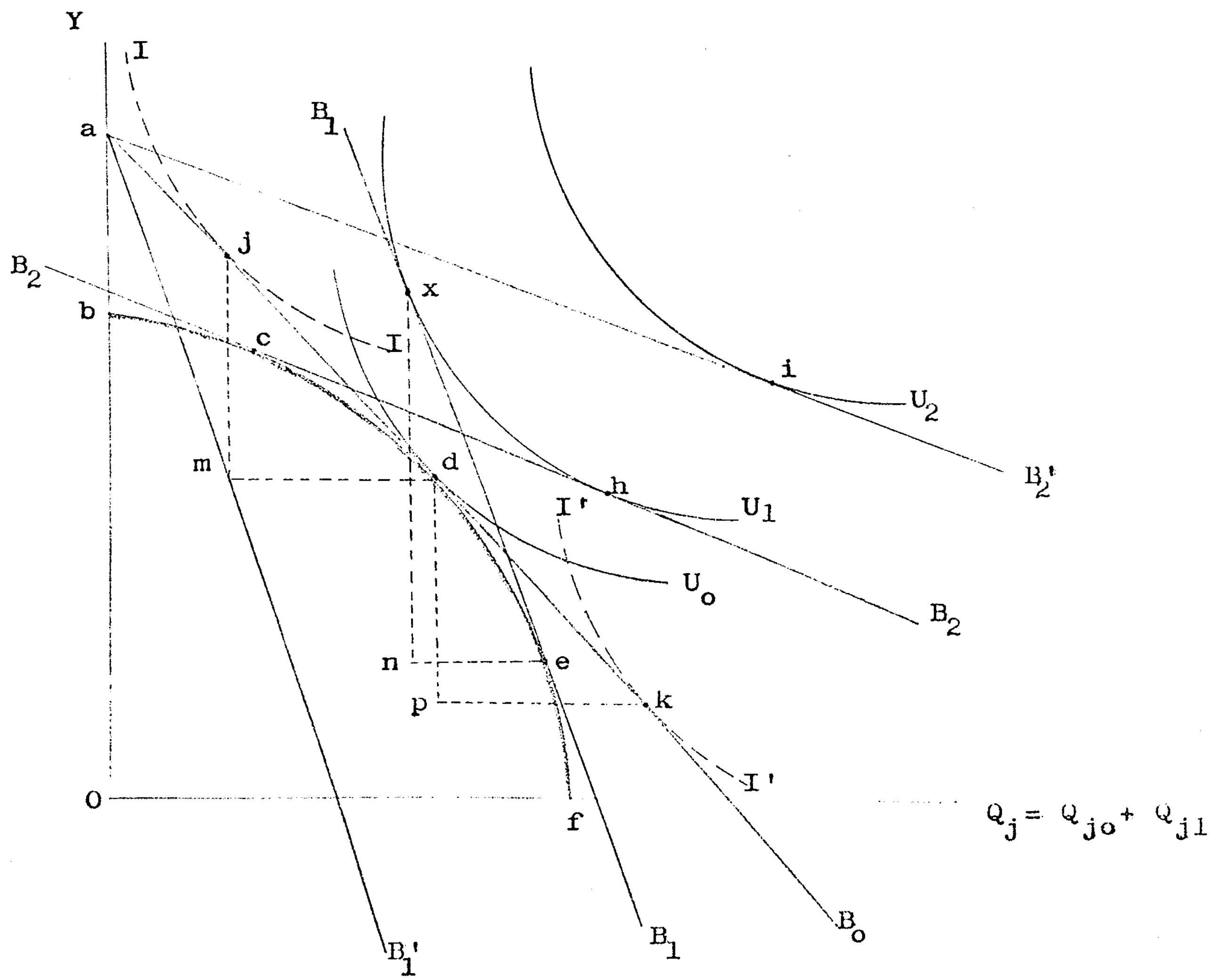
Let there be an economic unit who can choose to be either a pure-homeowner or a tenant. The question is: under what price and cost conditions will he choose each of these two possibilities? Further, if he chooses to be a pure-homeowner, are there any price and cost situations which will induce him to sub-divide his housing-package into owner-occupied and tenant-occupied quarters and sell housing-services on the market?

The graphical analysis is summarized in Figure 11. It is to be noted that although explicit reference is made only to a pure-homeowner (or tenant), the analysis applies implicitly to a landlord whose costs of production are inseparable. The budget or production-possibility-line bcdef is the graphical representation of

$$Y = M_o - F - W(Q_j) \qquad (12)$$

and is obtained by subtracting the total cost function of equation (9) from $M_o$ = aO at various levels of production of $Q_j$. Let us call this line bcdef the <u>pure-homeowner's budget line</u>. The pure-homeowner's budget line is shown to be tangent to the indifference curve $U_o$. As a pure-homeowner, the economic unit will choose the combination Y and $Q_j = Q_{jo}$ at point <u>d</u>.

Let the ruling market price be $E = E_o$, such that if the economic unit had been a tenant his budget line would be $aB_o$ in Figure 11. Let us call this line $aB_o$ the <u>tenant's budget line</u>. The tenant's budget line is purposely shown to be tangent to $U_o$ at the same point <u>d</u> where the pure-homeowner's budget line is tangent to $U_o$.




(Figure 11)

It means that the economic unit is <u>indifferent</u> between renting and owning the same housing-package for own consumption of the relevant quantity of housing-services, $Q_j = Q_{jo}$.

If $E = E_1 > E_o$, the tenant's budget line is $aB_1'$. If the economic unit is a tenant, it is more expensive for him to buy the required quantity of housing-services in the market than to own the housing-package and produce himself the same quantity of housing-services. The economic unit will <u>elect homeownership</u>. As a pure-homeowner, he can choose the same combination Y and $Q_j = Q_{jo}$ at <u>d</u>. However, at the price ratio implicit in $aB_1'$ or equivalently in $B_1B_1$, the pure-homeowner could improve his position further by choosing to produce $Q_j = Q_{jo} + Q_{j1}$ at <u>e</u>, consume $Q_{jo} = (Q_j - Q_{j1})$ and sell ne= $Q_{j1}$ of $Q_j$ in the market, thus attaining a higher indifference curve $U_1$ at point <u>x</u>. In short, the unit market price E is so high that, in addition to choosing to be a pure-homeowner, the economic unit is better off subdividing his housing-package into owner-occupied and tenant-occupied quarters and selling housing-services in the market.

If $E = E_2 < E_o$, the tenant's budget line is $aB_2'$. In this case, the opposite choice of tenure will occur. The economic unit will <u>prefer tenancy</u> to homeownership. If the economic unit is initially a pure-homeowner, he could improve his position by choosing to produce at point c and buy additional housing-services in the market to attain $U_1$ at h. However, the pure-homeowner is much better off being a tenant, since he can attain $U_2$ at i, instead.

Based on the foregoing analysis, let us define a pure-homeowner to be in a marginal, below-marginal, or above-marginal position, by referring to his choice of tenure and his relative cost of acquiring housing-services for his own consumption. The pure-homeowner is said to be at a marginal position, if he is indifferent between owning and renting a housing-package at the current price and cost situation. The pure-homeowner is said to be at an above marginal position, if, at the ruling price and cost situation, he is better off remaining as a pure-homeowner and becoming a landlord selling housing-services in the rental market. The pure-homeowner is said to be at a below-marginal position, if, at the ruling price and cost situation, he is worse off being a homeowner than being a tenant. In subsequent discussions, frequent reference will be made to marginal, below-marginal, or above-marginal pure-homeowners.

It may be pointed out that the change in the market value of the housing-package is directly proportional to the change in the unit market price E, all other factors affecting the market value of the property remaining the same. The decrease in E to $E_2$ and its continuous persistence at that level has decreased also the market value of the pure-homeowner's property. This loss in the capital value of the property occurs whether or not the property is sold. Provided that the loss in capital value is expected to be permanent, there will be an incentive for the below-marginal pure-homeowner to sell his residence, since he is attracted by the compensating gain from tenancy.

This tendency to change tenure will not result if the loss in capital value is only expected to be temporary. Indeed, the below-marginal pure-homeowner may fear a larger capital loss from the immediate sale of his residence than from the continuous retention of the property until the market recovers; or he may retain the property just the same if he takes the myopic viewpoint that there is capital loss only upon sale of his residence. Under these circumstances, the below-marginal pure-homeowner is said to be locked-in to his property. If the lock-in effect is prevalent, the unit price E can move up and down over a certain range without causing a noticeable change in the ownership of existing residences.

Further, the possibility of an economic unit shifting from owner-occupancy to a rental dwelling (or vice versa) will entail moving and transaction costs. Unless the price differential between rental and owner accommodations is viewed from a longer term perspective and the present value of the periodic price differentials exceeds the transaction and moving costs,[2] there will be no inducement to change tenure.

In the discussion we have ignored the indivisibility of factor inputs and the consequential discontinuity in the individual cost of production of housing-services.[3] However, there will be continuity if the individual cost functions are aggregated and divided by the number of economic units operating residential dwellings. The resulting individual cost function may be taken as the representative cost function used here. The aggregation and averaging procedure, however, cannot readily be applied to individual utility functions. Utilities are neither comparable nor additive across economic units. However, for present purposes, this aggregation problem has been ignored.

There is no a priori reason why the indifference map of the economic unit should be that represented

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[2] Abstracting from such non-monetary factors as social and cultural environment, family ties, job-site, etc.

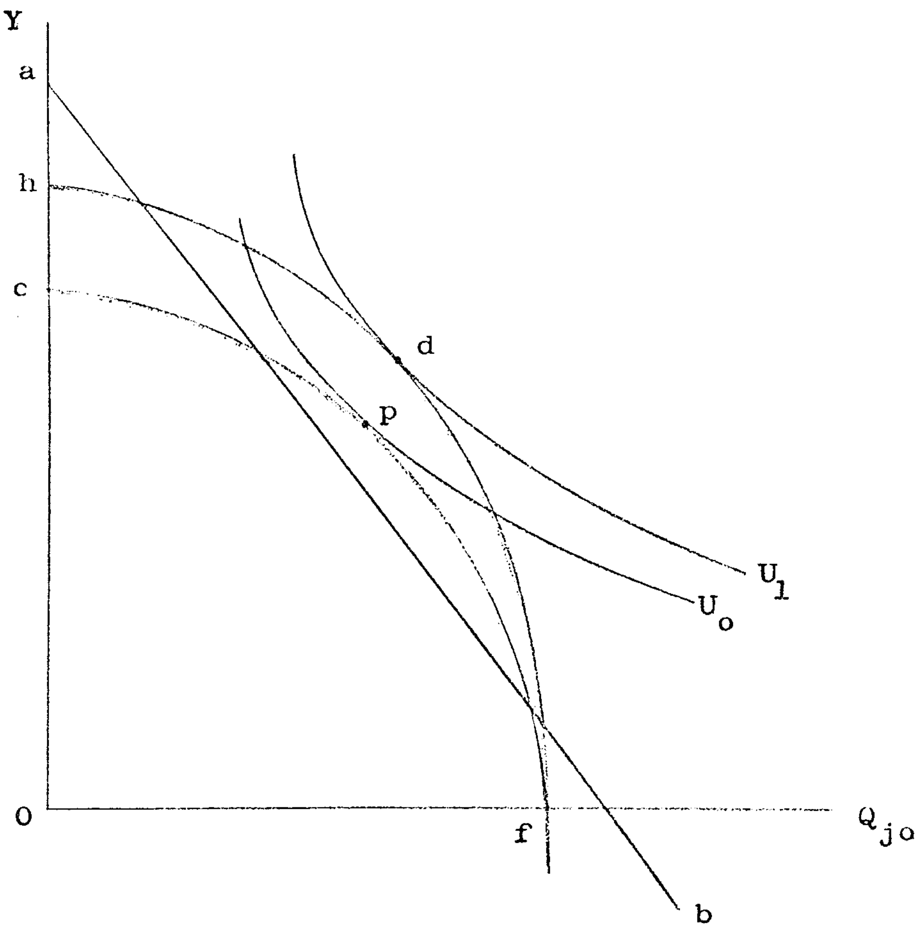
[3] The indivisibility tends to restrict the possibility of dwelling-substitution in response to marginal changes.

by $U_o$, $U_1$, $U_2$ in Figure 11. For example, the economic units indifference map could be the one implied by either the indifference curve II or the indifference curve I'I'. In the former case (II), when $E = E_o$ the economic unit will choose to produce housing-services at d, and at the same time sell quantity md in the open market. It still requires a level of E lower than $E_o$ to push this economic unit into a below-marginal position. In the latter case (I'I'), when $E = E_o$ the economic unit will also choose to produce at d, but at the same time will buy pk units of housing-services in the market. This situation is illustrated by a pure-homeowner who rents a dwelling-unit for the occupancy of a member of his household or family.

B.2. Changes in the Real Property Value Tax Rate

If the property tax rate is increased or is imposed for the first time, the fixed cost element F per period will increase by the amount of the incremental property tax payment. The whole cost function for the production of housing services will be shifted vertically upwards by the same amount. Consequently, the pure-homeowner's budget line will shift vertically downwards by the full amount of the incremental tax payment.

In Figure 12, if the pure-homeowner is at the marginal position d before the imposition of the tax,




(Figure 12)

he will be pushed by the tax toward a below-marginal position. If he is pushed far enough, he will be better off being a tenant, unless there occurs at the same time an offsetting increase in his money income. The more penalizing the tax rate is, the larger is the number of pure-homeowners who will be pushed towards below-marginal positions inducing them to change tenure.[4/]

The pure-homeowners who remain at a above-marginal position will not be induced to change tenure. Nevertheless, the downward shift in their budget lines will motivate them to prefer a smaller quantity of housing-services for own consumption than before the tax imposition. If in Figure 12, cpf is interpreted as the after-tax budget line of the above-marginal pure-homeowner, the movement of his equilibrium position from d to p represents the effect of the incremental tax payment.

Like a decline in the unit price E of housing-services, the tax may also cause a decrease in property values if capitalization of the tax takes place in the market. If this is so, the tax also has a lock-in effect due to the loss in capital value. However,

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[4/] From the market viewpoint, this large-scale inducement to change tenure may affect the current unit market price E. Please see Section D below.

unlike a decline in E that may revive at any future period, the property tax is more or less seen as a permanent feature of the housing market. For this reason, the tax may have a smaller lock-in effect than an equivalent decline in E, since there is slight possibility for speculation on future changes in the property tax rate.

C. Landlord's Production and Consumption Choice

Since the preceding section has covered both the pure-homeowner's choice and the landlord's choice under the inseparable cost case, the following discussion will analyze the separable cost-structure situation.

C.1. For Changes in Price (E)

Under the assumptions expressed by equations (10), (11-a), and (11-b), the landlord's budget line is:

$$Y = M_o - F_o - W_o(Q_{jo}) + EQ_{j1} - F_1 - W_1(Q_{j1}) \qquad (13)$$

where $\prod(Q_{j1}, E) = EQ_{j1} - F_1 - W_1(Q_{j1})$ represents the landlord's rental income per period from the sale of housing-services $Q_{j1}$ in the market. Accordingly, the counterpart condition for maximum utility is

$$\frac{\partial U / Q_{jo}}{\partial U / Y} = W'(Q_{jo}) \qquad (14)$$

subject to $W_1'(Q_{j1}) = E$ and equation (13).

To show these relations in a diagram (Figure 12), let $E = E^*$ and let $Q^*_{j1}$ be the level of the output where $W'(Q^*_{j1}) = E^*$ such that the maximum (profit) net rental income of the landlord is $\Pi^*(Q^*_{j1}, E^*) > 0$. Equation (13) can then be rewritten as:

$$Y = M_o + \Pi^*(Q^*_{j1}, E^*) - F_o - W_o(Q_{jo}), \qquad (15)$$

so that the relevant extremum condition is now equation (14) subject to equation (15). From (15) the landlord's budget line can easily be drawn to obtain line hdf of Figure 12. The landlord chooses the combination Y and $Q_{jo}$ at the point indicated by <u>d</u>.

In addition to hdf, we can imagine another budget line cpf for the landlord when the unit market price E is such that $\Pi(Q_{j1}, E) = 0$. The budget line cpf is thus the landlord's consumption-possibility-line when equivalent to that of a pure-homeowner. We can imagine further that the money income $M_o = 0a$ from all sources other than the sale of housing-services $Q_{j1}$ will buy possible levels of $Q_{jo}$ and Y (representing all other goods and services) in the manner shown by <u>ab</u>.

The direction of the movement of the budget lines <u>hdf</u> and <u>ab</u> for any given directional movement of the unit market price E is obvious. As E increases, <u>hdf</u> moves upward farther from the origin, whereas <u>ab</u> rotates clockwise at <u>a</u> toward the origin. Conversely, as E decreases, <u>hdf</u> moves downward towards the origin, whereas ab rotates counter-clockwise at a away from the origin. At any time that ab is above hdf and cpf and persists in that relative position in every period of time, the landlord would be better off to cease operation of housing-packages and become a tenant.

C.2. For Changes in Property Value Tax Rate

The short-run profit maximizing level of output $Q_{j1}$ will not be affected by the tax since the tax charges form part of the fixed cost elements. In short, the tax has no effect on the short-run marginal cost of production of $Q_{j1}$. However, the absolute level of profit, $\Pi$, from the operation of rental housing will decrease by the full amount of the tax, causing the landlord's budget line hdf to shift vertically downwards by the same amount. A further vertical downward shift in hdf will occur that is attributable to the tax charges on the landlord's own residence. If cpf in Figure 12 is interpreted as the new hdf after the imposition of the new tax rate, the movement of the landlord's equilibrium position from d to p represents the effect of the incremental tax charges.

In particular, if the landlord is pushed by the tax to operate at a loss equal to his short-run total fixed cost and persists in that situation every period, the landlord will be indifferent between continuation or suspension of the operation of his rental dwellings.

In this case, the landlord is said to be in a marginal position. If the total short-run loss exceeds his total fixed cost and the excess persists every period, the landlord would be better off not only to cease operation of rental dwellings but also to have them sold in the market. The landlord in this case is said to be in a below marginal position. In the long-run, all costs of operation are variable. From a long-run point of view, the landlord is in a marginal or below-marginal position if his long-run profit is zero or negative.

D. Market Supply and Demand for Housing-Services

The preceding discussions have assumed that owner and rental housing-services are homogeneous outputs and, therefore, are perfect substitutes. In effect, the analysis has proceeded as if there was only one market for housing-services. In this section, our analysis will involve two competing markets. It is assumed here that although rental and owner housing-services are imperfect substitutes the degree of substitution between them is very high. On this assumption, the two markets will be closely interrelated and the quantity demanded of either type of housing-services will be very responsive to their relative price.

In an actual situation the only apparently active market for housing-services is the rental sector.[5] Trading in housing-services does not generally take place in the owner sector. If there is any trading in this sector, the transaction is in the housing-packages (or physical dwellings) that generate housing-services. In practice, the value of owner housing-services is imputed on the basis of the relevant market price data generated by the rental sector and the market transactions in physical dwellings. In principle, however, no buyer will pay for a physical dwelling a price exceeding the present value of its future stream of expected income, net of expected costs of maintenance and repairs. Since the demand for a physical dwelling derives from the ultimate demand for the housing-services it yields, trading in physical dwellings may be taken as equivalent to an implicit market pricing of stream of housing-services.

D.1. Two-Market Model

In the analysis that follows, we introduce a system of structural equations which will be used, with modifications, in the empirical investigations of Chapter V. This procedure provides a close link

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[5] "Active" market is used here in the sense that there are regular transactions between economic units and the suppliers are different from the consumers of housing-services. This interaction and distinction between sellers and consumers of housing-services is generally absent in the owner sector.

between the present and the subsequent chapters. The structural equations are:

Rental Sector:

$$Q_1 = Q_1(E_1, E_o, Y, N, F) \quad \text{Demand} \quad (16)$$

$$E_1 = E_1(Q_1, Q_o, C_1, r) \quad \text{Supply} \quad (17)$$

Owner Sector:

$$Q_o = Q_o(E_1, E_o, Y, N, F) \quad \text{Demand} \quad (18)$$

$$E_o = E_o(Q_1, Q_o, C_o, r) \quad \text{Supply} \quad (19)$$

where $Q_o$, $Q_1$ = quantities of owner and rental housing-services;

$E_o$, $E_1$ = unit prices of $Q_o$, $Q_1$;

$Y$ = average family or household income;

$N$ = number of persons per family or household;

$F$ = " " families or households;

$C_o$,$C_1$ = implicit indices of prices of factor inputs to the production of $Q_o$, $Q_1$;

$r$ = effective residential property tax rate.

For all practical purposes, the other demand determining factors such as tastes and preferences, and relative prices of alternative goods and services are omitted due to the data limitations to be discussed in Chapter V.

The effects of Y and F on the quantity of housing-services demanded are straightforward. An increase in either Y or F will shift the market demand curve upwards. The effects of such other demographic factors as differences in the size and age composition of families or households will not be represented by F. Indeed, the larger the number of persons in a household, the higher tends to be the quantity of housing-services demanded at the same levels of income and relative prices. The variable N takes account of this type of demographic demand effect.

Both the owner and the rental supply equations include $Q_1$ and $Q_o$, as explanatory endogenous variables. This inclusion takes account of the inseparability of the costs of production of $Q_o$ and $Q_1$ mainly in cases where the dwelling units are partly owner-occupied and partly tenant-occupied. If the costs of production are inseparable, the level of output of one type of housing-services will affect the cost of production of the other and hence its supply-price. Presumably, the cross-quantity-effects on the supply-prices in equations (17) and (19) are greater than zero. It may be surmised that the smaller are the outputs of $Q_o$ and $Q_1$ under inseparable costs relative to the total volume of transactions in housing-services, the smaller are the numerical values of these cross-effects.

The aggregate production of housing-services is assumed to be subject to increasing costs, i.e., the entire set of short-run average cost curves of every housing-operator will be shifted upward as expansion in the scale of production takes place in the whole market. This means that the prices of inputs (represented by $C_i$ in index form, i=o,1) such as costs of land, dwelling-structures, utilities, and labor will increase with expansion in the scale of operation. The supply of these inputs may be so inelastic that a shift in the aggregate demand for them would result in a higher price increase.[6/] Thus, an increase in $C_i$ will shift the short-run market supply curve of the ith sector upwards. Equivalently, this means that the long-run market supply curves implicit in equations (17) and (19) are upward sloping.

In the long-run, the price elasticities of the market demand and supply of housing-services are higher in absolute value than in the short-run. The full adjustment to a given market disturbance usually gets completed only after the lapse of a longer period of time. A given permanent change in relative prices will undoubtedly have an immediate effect on the rate of production and consumption of housing-services. However, the effect will be greater in a longer period. It takes time for consumers to make budgetary adjustments. Similarly, all inputs to the production of housing-

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[6/] For an extended discussion, see F. de Leeuw and N.F. Ekanem, "The Supply of Rental Housing", American Economic Review, December 1971, pp. 806-817.

services could conceivably be varied in a shorter period. However, the cost of production associated with a quick variation in all productive inputs will be larger than the cost of production associated with a delayed variation in some of these inputs. It certainly costs more to finish a residential unit in a month than to finish the same dwelling in four or five months. The costs of overtime construction labour and construction materials that must be supplied at an unusually high rate are obviously prohibitive. The shorter the period, the more uneconomical it is to vary all inputs at the same time in response to a given market disturbance.

Since the property tax rate, r, is of main interest in the investigation, let us concentrate on the effects of this variable on the residential market.

There are many ways in which the residential market will respond to residential property value taxation. The property tax may eventually shift the market supply and demand for housing-services. It may also shift the market supply and derived demand for residential dwellings. These shifts will bring changes in prices and outputs of housing-services and residential accommodations. These changes in turn may allow property owners to pass on the tax cost partially or wholly to another party. The degree to which there will be changes in prices and outputs as well as tax-shifting is partly governed by various elasticities of market supply and demand for housing-services, including the degree of substitution between rental and owner housing-services.

Generally speaking, the partial or full forward shifting of the property tax in the rental sector will occur if, due to the tax, the unit price, $E_1$, rises as a result of a leftward shift in the market supply curve. This shift is implicit in equation (17), as well as in equation (19). The tax rate enters as a shift parameter in both equations. Forward tax shifting will also occur in the rental sector if the tax triggers an upward shift in the market demand curve for rental housing-services via the substitution process. It is argued below that the tax will initially increase the price of owner housing-services relative to the price of rental housing-services, thereby inducing a substitution of rental for owner housing-services.

It has been argued earlier that the property tax enters as a fixed cost element in the short-run production period. Therefore, the tax has no effect on the short-run marginal cost of production of housinng-services. In particular, the aggregate supply curve of rental housing-services will remain at the same position in the short-run despite the tax. In the absence of an upward demand shift triggered by the tax, there will be no immediate short-run forward shifting of the tax in this sector.

In the owner sector, the tax constitutes an increase in the periodic carrying cost of ownership of residential dwellings. Since there is no active market for owner housing-services, there is no market mechanism for the tax shifting in this sector. The owner-occupiers will pay and bear the full amount of the tax cost. The initial short-run impact of the tax therefore, will be recorded in a change in the relative prices of owner and rental housing-services. This relative price effect may induce a substitution of rental for owner housing-services. The higher the property tax rate, the larger is the initial relative price effect. Moreover, holding moving and transaction costs constant, the greater is the tendency to make the substitution. If the substitution takes place, there will be an upward demand shift in the rental sector, thereby permitting short-run forward tax shifting in this sector.

On a priori grounds, the property tax is generally shiftable to tenant occupiers in the long-run. The increased carrying cost of ownership of residential dwellings applies to both the rental and the owner dwellings. As a result, the tax tends to curtail the current willingness to own the existing stock of residential property which may well intensify the desire to sell, thereby creating a competitive excess supply of dwellings in the market. If an excess supply obtains, a reduction in the general level of prices of dwellings will take place. Since the competitive market clearing of the excess stock leads to a declining level of price, a lower volume of residential construction per family will be forthcoming. In the long-run, the markets will have completed the necessary response to the tax with a lower stock of dwellings per family and with higher unit prices of rental and owner housing-services. In particular, as the stock of rental dwellings per family gets reduced in the long-run, a larger fraction of the periodic property tax cost gets shifted forward to tenants. The long-run forward shifting of the tax will be complicated by the time lag required to shift leftward the supply of rental housing-services relative to the periodic independent behavioural shifts in the aggregate market demand for housing-services, say, due to demographic factors.

To gain a better perspective on the factors determining the extent of the long-run forward tax shifting in the rental sector, let us proceed with a more formal application of the structural equations (16) to (19). We shall limit our comparative static analysis to a partial equilibrium framework.

Let the structural equations (16) to (19) be linear in logarithmic form and let all the variables in these equations be measured in logarithms. Taking differentials we find,

| Elasticities | Rental Sector | Owner Sector |
|---|---|---|
| Price-elasticity of demand | $\frac{\partial Q_1}{\partial E_1} = -Q_{11} < 0$ | $\frac{\partial Q_o}{\partial E_o} = -Q_{oo} < 0$ |
| Cross-price-elasticity of demand | $\frac{\partial Q_1}{\partial E_o} = Q_{1o} > 0$ | $\frac{\partial Q_o}{\partial E_1} = Q_{o1} > 0$ |
| Income-elasticity of demand | $\frac{\partial Q_1}{\partial Y} = Q_{1y} > 0$ | $\frac{\partial Q_o}{\partial Y} = Q_{oy} > 0$ |
| Demographic-elasticities of demand, re: | | |
| (a) persons per household | $\frac{\partial Q_1}{\partial N} = Q_{1N} > 0$ | $\frac{\partial Q_o}{\partial N} = Q_{oN} > 0$ |
| (b) number of households or families | $\frac{\partial Q_1}{\partial F} = Q_{1F} > 0$ | $\frac{\partial Q_o}{\partial F} = Q_{oF} > 0$ |
| Price-elasticity of supply | $1/\frac{\partial E_1}{\partial Q_1} = 1/E_{11} > 0$ | $1/\frac{\partial E_o}{\partial Q_o} = 1/E_{oo} > 0$ |
| Cross-quantity-effect on supply-price | $\frac{\partial E_1}{\partial Q_o} = E_{1o} \gtrless 0$ | $\frac{\partial E_o}{\partial Q_1} = E_{o1} \gtrless 0$ |
| Tax-elasticity of supply-price | $\frac{\partial E_1}{\partial r} = E_{1r} > 0$ | $\frac{\partial E_o}{\partial r} = E_{or} > 0$ |
| Input-cost-elasticity of supply-price | $\frac{\partial E_1}{\partial C_1} = E_{1c} > 0$ | $\frac{\partial E_o}{\partial C_o} = E_{oc} > 0$ |

The signs of these elasticities are based on the *a priori* theoretical expectations already discussed in the earlier part of this section.

Solving simultaneously for the reduced forms of the structural equations, we find that

$$Q_i = F_i(Y,N,C_1,C_o,r) \qquad i=0,1 \qquad (20)$$
$$E_i = f_i(Y,N,C_o,C_1,r)$$

Evaluating the derivatives of these equations with respect to the tax rate, we find the reduced-form tax coefficients to be[7]

$$\frac{dQ_o}{dr} = \frac{1}{/G/}\left\{E_{1r}[\bar{Q}_{o1} + E_{o1}(Q_{11}Q_{oo} - Q_{o1}Q_{1o})] - E_{or}[\bar{Q}_{oo} + E_{11}(Q_{11}Q_{oo} - Q_{o1}Q_{1o})]\right\} < 0 \qquad (21)$$

$$\frac{dQ_1}{dr} = \frac{1}{/G/}\left\{E_{or}[\bar{Q}_{1o} + E_{1o}(Q_{11}Q_{oo} - Q_{o1}Q_{1o})] - E_{1r}[\bar{Q}_{11} + E_{oo}(Q_{11}Q_{oo} - Q_{o1}Q_{1o})]\right\} < 0 \qquad (22)$$

$$\frac{dE_o}{dr} = \frac{1}{/G/}\left\{E_{or}(1 + Q_{11}E_{11} - E_{1o}Q_{o1}) + E_{1r}(E_{oo}Q_{o1} - E_{o1}Q_{11})\right\} > 0, \text{ and} \qquad (23)$$

$$\frac{dE_1}{dr} = \frac{1}{/G/}\left\{E_{1r}(1 + E_{oo}Q_{oo} - E_{o1}Q_{1o}) + E_{or}(Q_{1o}E_{11} - E_{1o}Q_{oo})\right\} > 0, \qquad (24)$$

where

$$/G/ = [1 + E_{11}Q_{11} + E_{oo}Q_{oo} + (E_{11}E_{oo} - E_{1o}E_{o1})(Q_{11}Q_{oo} - Q_{1o}Q_{o1}) - E_{1o}Q_{o1} - Q_{1o}E_{o1}] > 0. \qquad (25)$$

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[7] The other reduced-form coefficients are not of immediate interest here and are presented only in the Appendix to this chapter.

The signs of the individual structural elasticities are already accounted for in the reduced form tax coefficients. The signs of the reduced form tax coefficients are specified on the basis of the assumptions that the cross-elasticities of demand are less than the price-elasticities of demand, i.e., $Q_{11} > Q_{1o}$ and $Q_{oo} > Q_{o1}$, and that the cross quantity supply-price elasticities $E_{1o}$ and $E_{o1}$ are very close to zero. For our purposes, we have assumed here that $E_{o1} = E_{1o} = 0$. It has also been assumed that $E_{1r}$ and $E_{or}$ are similar in size, thus accounting for the negativity of $dQ_o/dr$ and $dQ_1/dr$ in (21) and (22), respectively.

It can be seen from (21) to (24) that the extent of the long-run price and quantity effects of the property tax in both sectors is determined by the absolute numerical values of the various supply and demand elasticities. To cite some examples with reference to the rental sector, in (22), the lower the price-elasticity of supply $(1/E_{11})$, the larger is $/G/$ and hence the lower is the decline in $Q_1$. Further, the lower is $/Q_{11}/$, the more the numerator of (22) tends to zero and hence, the smaller also is the decline in $Q_1$. In addition, in (24), the lower is $/Q_{11}/$, the larger is the increase in $E_1$. The higher is the cross-elasticity of demand, $Q_{1o}$, in the rental sector, the smaller is /G/ and the more the numerator of (22) tends to zero (or to a positive number if $E_{or} > E_{1r}$). Thus, in (22) the smaller will be the decline in $Q_1$. In (24), the higher is $Q_{1o}$, the higher is the numerator relative to /G/ so that $E_1$ will increase by a larger amount.

We have argued earlier that the tax is not shiftable in the owner sector since there is no active market for owner housing-services. Thus, the slower the process of forward tax shifting in the rental sector <u>via</u> the supply side, the greater is the tendency to substitute rental for owner housing-services. The substitution, of course, tends to expedite the process of forward tax shifting in the rental sector <u>via</u> the demand side. On the other hand, the quicker the response in the supply of rental housing-services to the tax, the smaller is the initial relative price effect of the tax, and the smaller is the tendency to make the substitution. Thus, the smaller will be the quantity effect of the tax in the owner sector.

Assuming that $E_{1r} = E_{or}$, the long-run effects of the tax on rental incomes in both sectors are best seen in terms of the combined-reduced-form tax coefficients[8/]

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[8/] For details in the derivation of (26) and (27) please refer to equations A-36 to A-41 in the Appendix to this chapter.

$$\frac{dQ_o+dE_o}{dr} = \frac{E_{or}}{/G/}\,(1+E_{11}Q_{11})(1-Q_{oo})+Q_{o1}(1+E_{oo}+E_{11}Q_{1o}) \qquad (26)$$

and

$$\frac{dQ_1+dE_1}{dr} = \frac{E_{1r}}{/G/}\,(1+E_{oo}Q_{oo})(1-Q_{11})+Q_{1o}(1+E_{11}+E_{oo}Q_{o1}), \qquad (27)$$

where /G/ has been simplified to

$$/G/ = 1+E_{11}Q_{11}+E_{oo}Q_{oo}+E_{11}E_{oo}(Q_{11}Q_{oo}-Q_{o1}Q_{1o}). \qquad (28)$$

It follows that the revenue effects of an increase in the tax rate may be positive or negative depending upon the signs of (26) and (27), given the assumptions that $E_{o1} = E_{1o} = 0$, and $E_{1r} = E_{or}$. If the cross-elasticities of demand are small relative to the price-elaticities of demand (that is, if $Q_{o1}$ is small relative to $Q_{oo}$ and $Q_{1o}$ is small relative to $Q_{11}$), then it will be approximately true that

(a) if $/Q_{oo}/=/Q_{11}/=1$, then $\frac{dQ_o+dE_o}{dr} = \frac{dQ_1+dE_1}{dr} = 0$; (29-a)

(b) if $/Q_{oo}/ > 1$ and $/Q_{11}/ > 1$, then $\frac{dQ_o+dE_o}{dr} < 0$ and $\frac{dQ_1+dE_1}{dr} < 0$; (29-b)

(c) if $/Q_{oo}/ < 1$ and $/Q_{11}/ < 1$, then $\frac{dQ_o+dE_o}{dr} > 0$ and $\frac{dQ_1+dE_1}{dr} > 0$. (29-c)

The revenue effects of tax changes depend simply on the price-elasticities of demand. These effects may, of course, be partly offset by the cross-elasticities.

D.2. One-Market Model

Because of data limitations, a one-market approach to housing-services will also be employed in Chapter V. This approach, however, requires the assumption that the owner and rental housing-services are perfect substitutes. Assuming perfect substitution, $E_o = E_1 = E$ and $Q_o + Q_1 = Q$. In effect, we then have a one-sector model with the structural relations

$$Q = Q(E,Y,N,F) \qquad \text{Demand} \qquad (30)$$

$$E = E(Q,C,r) \qquad \text{Supply} \qquad (31)$$

Let us also assume here that (30) and (31) are linear in logarithmic form and that the variables are measured in logarithms. Taking the differentials, we find that

price-elasticity of demand $= \frac{\partial Q}{\partial E} = -Q_E < 0,$

income-elasticity of demand $= \frac{\partial Q}{\partial Y} = Q_y > 0,$

demographic-elasticities of demand $= \left\{\frac{\partial Q}{\partial N} = Q_N > 0,\ \frac{\partial Q}{\partial F} = Q_F > 0\right\},$

price-elasticity of supply $= \frac{1}{\partial E/\partial Q} = \frac{1}{E_Q} > 0,$

input-cost-elasticity of supply-price $= \frac{\partial E}{\partial C} = E_C > 0,$

tax-elasticity of supply-price $= \frac{\partial E}{\partial r} = E_r > 0.$

Solving simultaneously for price and quantity, we find the reduced form expression to be

$$\begin{bmatrix} Q \\ E \end{bmatrix} = \begin{bmatrix} 1 & Q_E \\ -E_Q & 1 \end{bmatrix}^{-1} \begin{bmatrix} \bar{Q} & Q_Y & Q_N & Q_F & 0 & 0 \\ \bar{E} & 0 & 0 & 0 & E_C & E_r \end{bmatrix} \begin{bmatrix} 1 \\ Y \\ N \\ F \\ C \\ r \end{bmatrix} \qquad (32)$$

where $\bar{Q}$ and $\bar{E}$ are constants.

Evaluating (32) and solving for the reduced form coefficients, we find that

$$\Delta = (1 + Q_E E_Q) > 0,$$

$$\frac{dQ}{dY} = \frac{Q_Y}{\Delta} > 0 \quad , \quad \frac{dE}{dY} = \frac{E_Q Q_Y}{\Delta} > 0 \ ,$$

$$\frac{dQ}{dN} = \frac{Q_N}{\Delta} > 0 \quad , \quad \frac{dE}{dN} = \frac{E_Q Q_N}{\Delta} > 0 \ ,$$

$$\frac{dQ}{dF} = \frac{Q_F}{\Delta} > 0 \quad , \quad \frac{dE}{dF} \quad \frac{E_Q Q_F}{\Delta} > 0 \ ,$$

$$\frac{dQ}{dC} = \frac{-Q_E E_C}{\Delta} < 0 \ , \quad \frac{dE}{dC} = \frac{E_C}{\Delta} > 0 \ ,$$

$$\frac{dQ}{dr} = \frac{-Q_E E_r}{\Delta} < 0 \ , \quad \frac{dE}{dr} = \frac{E_r}{\Delta} > 0 \ .$$

Thus, in a regression of the logarithms of total gross rent EQ on the logarithms of the property tax rate and the input cost index, we expect that

$$\frac{dQ + dE}{dC} = \frac{E_C(1-Q_E)}{\Delta} \gtreqless 0 \quad \text{and}$$

$$\frac{dQ + dE}{dr} = \frac{E_r(1-Q_E)}{\Delta} \gtreqless 0 \ ,$$

depending whether $/Q_E/ \lesseqgtr 1$.

One last note before we proceed to the next section. Although there generally is no active market for owner housing-services which permits forward tax shifting in a fashion similar to the rental sector, there are active market transactions in principal residences. In this market, the possibility of forward tax shifting is fairly remote. Since property taxes are recognizable future obligations, buyers usually capitalize them to form a subtraction from the capital value of the property. However, there may be a partial offset to this in the long-run if, due to the tax, the unit prices of residences eventually increase via a gradual reduction in the stock of dwellings per family or households. If this occurs, upon sale of his principal residence, the owner may recover from his buyer a portion of the property taxes he had already paid during the period of his ownership of the property. The next section discusses the principle of residential property tax capitalization.

## E. Property Tax Capitalization and Its Impact on the Volume of Residential Construction

Earlier a residential dwelling (package) has been defined as a set of three physical components, namely: (1) the land-site; (2) the dwelling-structure with a minimum standard of heating and sanitary installations; and (3) other household facilities like furnitures, electric appliances, draperies, etc. Since the components falling under category (3) are provided invariably either by the seller or by the buyer, their value contributions may or may not be excluded from the market valuation of a residential unit, as the case may be.

In principle, the current market value of a residential dwelling is equal to the sum of the present value of all the property's expected or imputed future rental incomes, net of the cost of expected stream of maintenance and repairs, throughout its n-year life. The land component of a dwelling is permanent and does not physically depreciate in use over time. Therefore, the market value of a residential unit will include the present value of the expected rental incomes of the land-site beyond the life of the dwelling-structure.

If $R(t)$[9] is the residential property's net rental income per period, its current market value, $P(0)$, is:

$$P(0) = \int_0^n R(t)e^{-mt}dt \qquad (20)$$

where $t$ = time period, $n$ = life of the property in years, and $m$ = nominal rate of interest which can be earned by the funds for the acquisition of the dwelling in a comparable alternative use.

Let there be an effective property tax rate, $r$, applicable per period on the market value, $P(t)$. The tax capitalization theory asserts that the future property taxes payable on the property will be capitalized to form a subtraction from the capital value of the property.[10] In other words, the current market value, $P(0)$, of the property will be equal to the

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[9] Equation (4) in Chapter III above shows that the combined average rental income (net of the costs associated with the variable inputs V) of a physical dwelling-structure and the average units of land utilized per structure is:

$$\frac{G(t) - R_v(t)V}{H} = \frac{L}{H} R_L(t) + R_h(t) \qquad (20\text{-a})$$

Since the tax rate, $r$, is seen to apply only on the combined values of the land-site, L, and the physical dwelling-structures, H, the $R(t)$ of equation (20) is assumed here equal to equation (20-a).

[10] Luigi Einaudi, "Capitalization and Amortization of Taxes," Readings in the Economics of Taxation, (eds.) R.A. Musgrave & C.S. Shoup, Allen and Unwin, 1959, pp. 389-392.

present value of the property's stream of net rental incomes after tax deduction, i.e.,

$$P(0) = \int_0^n N(t)e^{-mt}dt \tag{21}$$

where $N(t) = R(t) - rP(t)$ is the after-tax net rental income, and $rP(t)$ is the property tax payment per period. Equation (21) indicates that, if capitalization of future property taxes takes place, it would reduce the current market value of the land and dwelling-structure components of residential properties.

The housing market determines the price of residential dwellings. It is, therefore, necessary that the tax capitalization theory explains how the housing market reacts to residential property taxation. More specifically, the theory should explain how the price reduction indicated by equation (21) obtains in the housing market.

Generally speaking, the tax constitutes an increase in the periodic carrying costs of ownership of residential dwellings. As a result, the tax tends to curtail the current willingness to own the existing stock of residential property. This tendency may well intensify the desire to sell, thereby creating a competitive excess stock supply of dwellings in the market. If an excess supply obtains, the necessary price reduction should take place if the excess stock of dwellings is to be cleared in the market. In short, the vehicle through which tax capitalization takes place is the dimunition in the willingness to own, or, alternatively, the intensification in the desire to sell, the existing stock of dwellings.

How far will the market price decline? Ignoring all other tax-effects to be discussed later, except the market disturbance brought about by the reduced willingness to own, the competitive market clearing of the excess stock will take place at a price reduction equal to the capitalized value of the future stream of property taxes payable.[11] This is the required price reduction if the market is to hold the same amount of stock of dwellings as before. In this case, the price reduction can be clearly identified with the effect of full capitalization of future property taxes.

The tendency of the market price to decrease by a magnitude equal to the capitalized value of future property taxes may be offset by other adjustments also taking place in the market. Since the competitive market clearing of the excess stock leads to a declining level of prices, dwelling units about to be

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11/ Alternatively, if there is property tax capitalization, the property tax rate should enter as part of the discount rate for bringing the future stream of rental income from the property to its present value.

completed cannot be cleared at the prevailing prices before the tax imposition. In response, builders and developers will cut down the volume of residential construction per capita (or per family). In the long-run there will be a consequential reduction in the available stock of dwellings per capita that may well increase residential rent and thus, partly offset the price-reducing effect of full tax capitalization.

The decline in the volume of residential construction may reduce the loan requirements for building purposes. The capital fund market will tend to respond in the form of a lower level of interest rate. This tendency is stronger the lesser the sensitivity of savings to a decline in the interest rate. This interest rate effect may increase the market value of residences, because a lower-capitalization or discount rate now applies to the expected future rental incomes of residential properties.[12/]

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[12/] C.S. Shoup, "Capitalization and Shifting of the Property Tax," Property Taxes (ed.) Tax Policy League (now Tax Institute) of America, Little & Ives, Co., 1940, pp. 190-193. For a technical exposition of the same point in relation to the shifting and incidence of property tax, see H.A. Simon, "The Incidence of a Tax on Urban Real Property," Readings in the Economics of Taxation, pp. 416-435; See also, H.G. Brown, The Economics of Taxation, Henry Holt & Co., 1924, pp. 178-212; T.S. Adams, "Tax Exemption Through Tax Capitalization: A Fiscal Fallacy," American Economic Review, Vol. 6, June 1916, pp. 271-287.

A decline in the interest rate, however, may simply be offset by the increase in the property tax rate.

F. Benefit Effects of Local Public Spending

The preceding sections have ignored the expenditure effects of local property taxes. Property taxes form a major source of finance for a number of urban public services such as road improvements and construction, school and recreation facilities, courts and legal services, fire and police protection, garbage collection and sewerage, etc. These urban public services tend to improve the location and accessibility value of residential properties. They enhance the quality of urban living, measured say in terms of the availability of parking services, good roads, public recreation and cultural centers. Thus, the expenditure of property tax revenues will entail benefits[13/] that may well partially offset the

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[13/] Edwin H. Spengler, "Property Tax as a Benefit Tax," Property Taxes, 1940, pp. 165-169; C.F. Bickerdike, "Taxation of Site Values," Readings in the Economics of Taxation, 1959, pp. 377-388. For a cross-section study of the benefits to property by local government spending in the United States, see W.E. Oates, "The Effects of Property Taxes and Local Public Spending on Property Values: An Empirical Study of Tax Capitalization and the Tiebout Hypothesis," Journal of Political Economy, Vol. 77, November/December 1969, pp. 957-970.

negative effects of property taxes discussed herein, as well as those identifiable under a more general equilibrium framework of analysis.[14] In particular, some of these benefits may ultimately be capitalized to compensate partly for the value reducing effect of property tax capitalization and thereby reduce the deterrent effects the tax may have on the volume of residential construction.

In this study, it is not possible to test empirically the expenditure effects of local property taxes because of lack of Canadian data at the municipal level. The problem of unavailability in municipal data is, perhaps, most emphatically expressed by the technical staff of the Economic Council of Canada.[15]

> "Surprisingly,..despite the fact that the annual budgets of some cities exceed those of several provincial governments, no uniform financial data have yet been published in Canada to permit nation-wide comparison and analysis of the operations of even the largest of our cities. The Dominion Bureau of Statistics

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14/ For instance, the interest rate effect of property taxes discussed in the preceding section may be viewed as a partial backward shifting of the property tax. See H.G. Brown, The Economics of Taxation, Henry Holt & Co.,1924, pp. 178-262.

15/ Economic Council of Canada, The Canadian Economy from the 1960's to the 1970's, Fourth Annual Review, Ottawa: Queen's Printer, 1967, pp. 215-225.

> has done a great deal of useful work on municipal financial statistics, but the lowest level of disaggregation for which uniform data are available is the consolidated total of municipalities in each province".

Generally speaking, empirical work pertaining to the benefits derived from public spending is faced with measurement problems. The monetary costs of public goods and services are relatively easy to measure since they are only a matter of public record.[16/] In contrast, the benefits from local public expenditures, represented by the value of public goods and services rendered, are neither directly observable nor easily quantifiable.[17/] More often, the monetary cost of public goods and services is used as a proxy for the value of these outputs. However, as Oates[18/] has

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[16/] Following Hirsch, it is useful to distinguish between social and agency costs. Social costs account for all the resources required to produce and supply the necessary public services in terms of the value in their alternative use. Agency costs include only the actual payments made by the government and the value of government owned resources utilized to secure the production of urban services by respective government agencies. Social costs need not be equal to agency costs. See W.Z. Hirsch, "The Supply of Urban Public Services", Issues in Urban Economics, eds. H.S. Perloff & L. Wingo, Jr., John Hopkins Press, 1968, pp. 477-526.

[17/] For an extensive discussion of this issue, see A.R. Prest and R. Turvey, "Cost and Benefit Analysis: A Survey", Surveys of Economic Theory, volume III, Macmillan, 1966, pp. 155-207. Some of the measurement problems relating to the estimation of production functions for urban public services are discussed in W. Hirsch, op.cit.

[18/] W.E. Oates., op.cit., p. 961.

pointed out, this procedure is unsatisfactory. The magnitude of and variation in the monetary cost may not measure the level and variation in the 'quantity' and 'quality' of public goods and services rendered. Indeed, in collective types of goods where extra benefits need not require additional costs and vice versa, costs and benefits tend to be related irregularly. Although there may well be competition among adjacent communities as to the mix and type of tax-expenditure programs available to actual and prospective residents, there is no guarantee that the costs of local public goods and services will come closer to the benefits derived from them.

In the investigation of the property tax-expenditure effects on property values, there is also the immediate problem of distinguishing between local public expenditures that benefit residential properties (i.e., those that cause increases in residential property values) and expenditures that do not, although both types of expenditures ultimately benefit property owners and the community as a whole. A further complication is the differentiation in assignable values to a given degree of urban public services. Communities and property owners will assign different values to them depending upon the nature of the urban environment, as well as the physical composition of residential properties. To cite an example, fire and police protection services will be more valuable to crime infected areas than to areas that are not. These same public services will likely have less value to owners of residences already secured by appropriate building insurance, or to owners of residences built with bricks and equipped with foolproof fire and theft protection devices.

Urban public services are also financed in varying degrees by sources other than property taxes. These other sources of finance include such revenues as user-charges (e.g., parking fees, charges or special assessments on publicly improved driveways and alleys, and gasoline taxes earmarked for road construction and improvements);[19] aids and grants from senior governments, and direct Federal participation in local projects. To analyze properly the offset and expenditure effects of property taxes, it seems necessary to isolate the fraction of the total benefits that is attributable only to the property tax costs. The adjustment will require the subtraction from total

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[19] During the early 1960's some municipalities in the province of Quebec were reported to have been collecting a fixed amount of garbage tax per dwelling. See Dominion Bureau of Statistics (DBS), Principal Taxes and Rates: Federal, Provincial, and Selected Municipal Governments, 1960-1968, Cat. No. 68-201.

benefits the benefits traceable and attributable to sources of finance other than property taxes.

G. Summary

In this chapter we have introduced property taxation as a policy tool for analysis. The short-run and long-run effects of the tax on the residential market have been investigated. In the process, we have expanded our analytical framework by resorting to a two-market model for housing-services, thereby allowing for imperfect substitution between rental and owner housing-services.

We have argued here that the property tax constitutes an increase in the carrying cost of ownership of residential dwellings. In the owner sector, the owner-occupiers are seen to be carrying the full amount of the tax, since there is no market mechanism here with which to pass the tax on to another party. In the rental sector, the landlords are seen to be bearing a larger or smaller fraction of the tax depending upon the extent of the forward tax shifting taking place in the rental market. The shifting process is a short-and long-run phenomenon. The extent of the forward tax-shifting will be governed by the relative elasticities of the market supply and demand for rental housing-services and the cross-elasticities of demand between rental and owner housing-services.

In a short-run period, the tax is generally not shiftable in the rental sector. During this period, therefore, the rental housing-services will be less expensive than the owner housing-services. Because of this relative price effect, a substitution of rental for owner housing-services may well be induced. If the substitution takes place in the short-run, the market demand for rental housing-services will be shifted upwards, thereby permitting a partial forward tax-shifting in the rental sector.

The property tax is generally shiftable forward in a long-run period. The period is long enough to permit the markets to adjust fully to the tax situation. We have argued that the long-run adjustments will be associated with a reduction in the demand for ownership of residential dwellings which tends to pull down their market values. In an alternative interpretation, this decline in market values means that property tax capitalization is taking place in the market. Since the market values of residences are lower, there will be subsequent reductions in the volume of residential construction per capita and eventually a decrease in the per capita stock of residential dwellings. As the per capita stock of dwellings declines in response to the property tax, an increasing fraction of the tax will be shifted forward to tenant-occupiers to the extent permitted by the relative elasticities of the market supply and demand for rental housing-services.

It may well be that the various tax effects discussed here are partly offset by the benefits from the local expenditures of the property taxes.

## APPENDIX TO CHAPTER IV

Let the structural equations for housing-services in the owner and rental sectors be

Rental Sector

$$Q_1 = Q_1(E_1, E_o, Y, N, F) \qquad \text{A-1}$$

$$E_1 = E_1(Q_1, Q_o, C_1, r) \qquad \text{A-2}$$

Owner Sector

$$Q_o = Q_o(E_1, E_o, Y, N, F) \qquad \text{A-3}$$

$$E_o = E_o(Q_1, Q_o, C_o, r) \qquad \text{A-4}$$

Taking the total differentials, we find

$$dQ_1 = \frac{\partial Q_1}{\partial E_1}dE_1 + \frac{\partial Q_1}{\partial E_o}dE_o + \frac{\partial Q_1}{\partial Y}dY + \frac{\partial Q_1}{\partial N}dN + \frac{\partial Q_1}{\partial F}dF \qquad \text{A-5}$$

$$dE_1 = \frac{\partial E_1}{\partial Q_1}dQ_1 + \frac{\partial E_1}{\partial Q_o}dQ_o + \frac{\partial E_1}{\partial C_1}dC_1 + \frac{\partial E_1}{\partial r}dr \qquad \text{A-6}$$

$$dQ_o = \frac{\partial Q_o}{\partial E_1}dE_1 + \frac{\partial Q_o}{\partial E_o}dE_o + \frac{\partial Q_o}{\partial Y}dY + \frac{\partial Q_o}{\partial N}dN + \frac{\partial Q_o}{\partial F}dF \qquad \text{A-7}$$

$$dE_o = \frac{\partial E_o}{\partial Q_1}dQ_1 + \frac{\partial E_o}{\partial Q_o}dQ_o + \frac{\partial E_o}{\partial C_o}dC_o + \frac{\partial E_o}{\partial r}dr \qquad \text{A-8}$$

where $\frac{\partial Q_1}{\partial E_1} = -Q_{11} < 0, \quad \frac{\partial Q_1}{\partial E_o} = Q_{1o} > 0, \quad \frac{\partial Q_1}{\partial Y} = Q_{1y} > 0,$

$\frac{\partial Q_1}{\partial N} = Q_{1N} > 0, \quad \frac{\partial Q_1}{\partial F} = Q_{1F} > 0, \quad \frac{\partial E_1}{\partial Q_1} = E_{11} > 0, \quad \frac{\partial E_1}{\partial Q_o} = E_{1o} \gtrless 0,$

$$\frac{\partial E_1}{\partial C_1} = E_{1C_1} > 0, \quad \frac{\partial E_1}{\partial r} = E_{1r} > 0, \quad \frac{\partial Q_o}{\partial E_1} = Q_{o1} > 0, \quad \frac{\partial Q_o}{\partial E_o} = -Q_{oo} < 0,$$

$$\frac{\partial Q_o}{\partial Y} = Q_{oY} > 0, \quad \frac{\partial Q_o}{\partial N} = Q_{oN} > 0, \quad \frac{\partial Q_o}{\partial F} = Q_{oF} > 0, \quad \frac{\partial E_o}{\partial Q_1} = E_{o1} \geqslant 0,$$

$$\frac{\partial E_o}{\partial Q_o} = E_{oo} > 0, \quad \frac{\partial E_o}{\partial C_o} = E_{oC_o} > 0, \quad \frac{\partial E_o}{\partial r} = E_{or} > 0.$$

The signs of these partial derivatives are based on *a priori* theoretical expectations.

Let the structural equations be linear in logarithmic form and let the variables be in logarithms. Under these assumptions, all the partial derivatives are measures of elasticities and the structural equations may be expressed as

$$\underset{G}{\begin{bmatrix} 1 & Q_{11} & 0 & -Q_{1o} \\ -E_{11} & 1 & -E_{1o} & 0 \\ 0 & -Q_{o1} & 1 & Q_{oo} \\ -E_{o1} & 0 & -E_{oo} & 1 \end{bmatrix}} \underset{H}{\begin{bmatrix} Q_1 \\ E_1 \\ Q_o \\ E_o \end{bmatrix}} = \underset{P}{\begin{bmatrix} \bar{Q}_1 & Q_{1Y} & Q_{1N} & Q_{1F} & 0 & 0 & 0 \\ \bar{E}_1 & 0 & 0 & 0 & E_{1C_1} & 0 & E_{1r} \\ \bar{Q}_o & Q_{oY} & Q_{oN} & Q_{oF} & 0 & 0 & 0 \\ \bar{E}_o & 0 & 0 & 0 & 0 & E_{oC_o} & E_{or} \end{bmatrix}} \underset{S}{\begin{bmatrix} 1 \\ Y \\ N \\ F \\ C_1 \\ C_o \\ r \end{bmatrix}} \qquad \text{A-9}$$

where $\bar{Q}_1$, $\bar{E}_1$, $\bar{Q}_o$, and $\bar{E}_o$ are constants. The signs of the elasticities are already accounted for in A-9. Solving for H,

$$H = G^{-1} P S \qquad \text{A-10}$$

Equation A-10 is the reduced form of A-9. Evaluating A-10 and taking its derivatives, we find

$$/G/ = \Big\{1+E_{11}Q_{11}+E_{oo}Q_{oo}+(Q_{11}Q_{oo}-Q_{1o}Q_{o1})(E_{11}E_{oo}-E_{1o}E_{o1}) -E_{1o}Q_{o1}-E_{o1}Q_{1o}\Big\} > 0 \qquad \text{A-11}$$

$$\frac{dQ_o}{dY} = \frac{1}{/G/}\Big\{Q_{1Y}(Q_{o1}E_{11}-Q_{oo}E_{o1})+Q_{oY}(1+Q_{11}E_{11}-Q_{1o}E_{o1})\Big\}>0 \qquad \text{A-12}$$

$$\frac{dQ_o}{dN} = \frac{1}{/G/}\Big\{Q_{1N}(Q_{o1}E_{11}-Q_{oo}E_{o1})+Q_{oN}(1+Q_{11}E_{11}-Q_{1o}E_{o1})\Big\}>0 \qquad \text{A-13}$$

$$\frac{dQ_o}{dF} = \frac{1}{/G/}\Big\{Q_{1F}(Q_{o1}E_{11}-Q_{oo}E_{o1})+Q_{oF}(1+Q_{11}E_{11}-Q_{1o}E_{o1})\Big\}>0 \qquad \text{A-14}$$

$$\frac{dQ_o}{dC_1} = \frac{1}{/G/}\Big\{E_{1C_1}\big[Q_{o1}+E_{o1}(Q_{11}Q_{oo}-Q_{o1}Q_{1o})\big]\Big\}> 0 \qquad \text{A-15}$$

$$\frac{dQ_o}{dC_o} = \frac{-1}{/G/}\Big\{E_{oC_o}\big[Q_{oo}+E_{11}(Q_{11}Q_{oo}-Q_{o1}Q_{1o})\big]\Big\}< 0 \qquad \text{A-16}$$

$$\frac{dQ_o}{dr} = \frac{E_{1r}}{/G/}\Big\{Q_{o1}+E_{o1}(Q_{11}Q_{oo}-Q_{o1}Q_{1o})\Big\} - \frac{E_{or}}{/G/}\Big\{Q_{oo}+E_{11}(Q_{11}Q_{oo}-Q_{o1}Q_{1o})\Big\} \gtreqless 0 \qquad \text{A-17}$$

$$\frac{dQ_1}{dY} = \frac{1}{/G/}\Big\{Q_{1Y}(1+Q_{oo}E_{oo}-E_{1o}Q_{o1})+Q_{oY}(Q_{1o}E_{oo}-Q_{11}E_{1o})\Big\}>0 \qquad \text{A-18}$$

$$\frac{dQ_1}{dN} = \frac{1}{/G/}\Big\{Q_{1N}(1+Q_{oo}E_{oo}-E_{1o}Q_{o1})+Q_{oN}(Q_{1o}E_{oo}-Q_{11}E_{1o})\Big\}>0 \qquad \text{A-19}$$

$$\frac{dQ_1}{dF} = \frac{1}{/G/}\left\{Q_{1F}(1+Q_{oo}E_{oo}-E_{1o}Q_{o1})+Q_{oF}(Q_{1o}E_{oo}-Q_{11}E_{1o})\right\} > 0 \qquad \text{A-20}$$

$$\frac{dQ_1}{dC_1} = \frac{-E_{1C1}}{/G/}\left\{Q_{11}+ E_{oo}(Q_{11}Q_{oo}-Q_{o1}Q_{1o})\right\} < 0 \qquad \text{A-21}$$

$$\frac{dQ_1}{dC_o} = \frac{E_{oCo}}{/G/}\left\{Q_{1o}+ E_{1o}(Q_{11}Q_{oo}-Q_{o1}Q_{1o})\right\} > 0 \qquad \text{A-22}$$

$$\frac{dQ_1}{dr} = \frac{E_{or}}{/G/}\left\{Q_{1o}+ E_{1o}(Q_{11}Q_{oo}-Q_{o1}Q_{1o})\right\}$$

$$- \frac{E_{1r}}{/G/}\left\{Q_{11}+ E_{oo}(Q_{11}Q_{oo}-Q_{o1}Q_{1o})\right\} \lesseqgtr 0 \qquad \text{A-23}$$

$$\frac{dE_1}{dY} = \frac{Q_{1Y}}{/G/}\left\{E_{11}+ Q_{oo}(E_{oo}E_{11}-E_{1o}E_{o1})\right\}$$

$$+ \frac{Q_{oY}}{/G/}\left\{E_{1o}+ Q_{1o}(E_{oo}E_{11}-E_{o1}E_{1o})\right\} > 0 \qquad \text{A-24}$$

$$\frac{dE_1}{dN} = \frac{Q_{1N}}{/G/}\left\{E_{11}+ Q_{oo}(E_{oo}E_{11}-E_{1o}E_{o1})\right\}$$

$$+ \frac{Q_{oN}}{/G/}\left\{E_{1o}+ Q_{1o}(E_{oo}E_{11}-E_{o1}E_{1o})\right\} > 0 \qquad \text{A-25}$$

$$\frac{dE_1}{dF} = \frac{Q_{1F}}{/G/}\left\{E_{11}+ Q_{oo}(E_{oo}E_{11}-E_{1o}E_{o1})\right\}$$

$$+ \frac{Q_{oF}}{/G/}\left\{E_{1o}+ Q_{1o}(E_{oo}E_{11}-E_{o1}E_{1o})\right\} > 0 \qquad \text{A-26}$$

$$\frac{dE_1}{dC_1} = \frac{E_{1C_1}}{/G/}\left(1 + E_{oo}Q_{oo} - Q_{1o}E_{o1}\right) > 0 \qquad \text{A-27}$$

$$\frac{dE_1}{dC_o} = \frac{E_{oC_o}}{/G/}\left(Q_{1o}E_{11} - E_{1o}Q_{oo}\right) \gtrless 0 \qquad \text{A-28}$$

$$\frac{dE_1}{dr} = \frac{1}{/G/}\left\{E_{1r}(1+E_{oo}Q_{oo}-Q_{1o}E_{o1})+E_{or}(Q_{1o}E_{11}-E_{1o}Q_{oo})\right\} > 0 \qquad \text{A-29}$$

$$\frac{dE_o}{dY} = \frac{Q_{1Y}}{/G/}\left\{E_{o1}+ Q_{o1}(E_{oo}E_{11}-E_{1o}E_{o1})\right\} + \frac{Q_{oY}}{/G/}\left\{E_{oo}+ Q_{11}(E_{oo}E_{11}-E_{1o}E_{o1})\right\} > 0 \qquad \text{A-30}$$

$$\frac{dE_o}{dN} = \frac{Q_{1N}}{/G/}\left\{E_{o1}+ Q_{o1}(E_{oo}E_{11}-E_{1o}E_{o1})\right\} + \frac{Q_{oN}}{/G/}\left\{E_{oo}+ Q_{11}(E_{oo}E_{11}-E_{1o}E_{o1})\right\} > 0 \qquad \text{A-31}$$

$$\frac{dE_o}{dF} = \frac{Q_{1F}}{/G/}\left\{E_{o1}+ Q_{o1}(E_{oo}E_{11}-E_{1o}E_{o1})\right\} + \frac{Q_{oF}}{/G/}\left\{E_{oo}+ Q_{11}(E_{oo}E_{11}-E_{1o}E_{o1})\right\} > 0 \qquad \text{A-32}$$

$$\frac{dE_o}{dC_1} = \frac{E_{1C_1}}{/G/}\left(E_{oo}Q_{o1} - E_{o1}Q_{11}\right) \gtrless 0 \qquad \text{A-33}$$

$$\frac{dE_o}{dC_o} = \frac{E_{oC_o}}{/G/}\left(1 + Q_{11}E_{11} - E_{1o}Q_{o1}\right) > 0 \qquad \text{A-34}$$

$$\frac{dE_o}{dr} = \frac{1}{/G/}\left\{E_{1r}(E_{oo}Q_{o1}-E_{o1}Q_{11})+E_{or}(1+Q_{11}E_{11}-E_{1o}Q_{o1})\right\} > 0 \qquad \text{A-35}$$

The signs of equations A-11 to A-35 are based on the theoretical expectations that

(a) $E_{o1} \cong E_{1o} \cong 0$,

(b) $Q_{11} > Q_{1o}$ and $Q_{oo} > Q_{o1}$ ,

and on the assumption that

(c) $E_{or} = E_{1r}$ ,

thus accounting for the negativity of $dQ_o/dr$ and $dQ_1/dr$ . Moreover,

$$\frac{dQ_o+dE_o}{dr} = \frac{E_{or}}{/G/}\left\{(1+E_{11}Q_{11})(1-Q_{oo})+Q_{o1}(1+E_{oo}+E_{11}Q_{1o})\right\} \gtrless 0 \qquad \text{A-36}$$

and

$$\frac{dQ_1+dE_1}{dr} = \frac{E_{1r}}{/G/}\left\{(1+E_{oo}Q_{oo})(1-Q_{11})+Q_{1o}(1+E_{11}+E_{oo}Q_{o1})\right\} \gtrless 0 \qquad \text{A-37}$$

where /G/ has been simplified to

$$/G/ = 1 + E_{11}Q_{11} + E_{oo}Q_{oo} + E_{11}E_{oo}(Q_{11}Q_{oo}-Q_{o1}Q_{1o}). \qquad \text{A-38}$$

It follows that the revenue effects of an increase in the tax rate may be positive or negative depending on the signs of A-36 and A-37, under the assumptions that $E_{o1} \cong E_{1o} \cong 0$, $Q_{11} > Q_{1o}$ , $Q_{oo} > Q_{o1}$ and $E_{or} = E_{1r}$ .

Finally, in the special case where the cross-elasticities of demand, $Q_{o1}$ and $Q_{1o}$, are equal to zero, we find that

$$/G/ = 1 + E_{oo}Q_{oo} + E_{11}Q_{11} + E_{oo}E_{11}Q_{oo}Q_{11} \quad , \qquad \text{A-39}$$

$$\frac{dQ_o + dE_o}{dr} = \frac{E_{or}}{/G/}(1 + Q_{11}E_{11})(1 - Q_{oo}) \qquad \text{A-40}$$

and

$$\frac{dQ_1 + dE_1}{dr} = \frac{E_{1r}}{/G/}(1 + Q_{oo}E_{oo})(1 - Q_{11}) \ . \qquad \text{A-41}$$

It can be seen from A-40 and A-41 that

(a) if $/Q_{oo}/ = /Q_{11}/ = 1$, $\frac{dQ_o + dE_o}{dr} = \frac{dQ_1 + dE_1}{dr} = 0$ ;

(b) if $/Q_{oo}/ > 1$, $/Q_{11}/ > 1$, $\frac{dQ_o + dE_o}{dr} < 0$ and $\frac{dQ_1 + dE_1}{dr} < 0$;

and

(c) if $/Q_{oo}/ < 1$, $/Q_{11}/ < 1$, $\frac{dQ_o + dE_o}{dr} > 0$ and $\frac{dQ_1 + dE_1}{dr} > 0$.

The revenue effects of tax changes depend simply on the price-elasticity of demand.

# CHAPTER V

# EMPIRICAL EVIDENCE: HOUSING-SERVICES AND PROPERTY TAXATION

In our analyses, we have distinguished between the market for housing-services and the market for residential dwellings (or its physical components with special reference to the dwelling-structures). This chapter will investigate the empirical content of the theoretical market supply and demand for housing-services with the use of cross-section data by Canadian municipalities. In this investigation, we shall apply to the data the two-market and the one-market models for housing-services we have introduced in the preceding chapter. The empirical investigation will be directed towards the estimation of the various elasticities of the market supply and demand for housing-services, particularly the tax elasticity of supply-price. To this end, our discussion will proceed as follows: (a) observations on the limitations and complications involved in empirical work in this area, (b) regression model specification for estimation purposes, and (c) presentation and interpretation of empirical results.

## A. Limitation and Data Problems

We have argued in Chapter III that the dollar value commonly referred to as the rental income of a housing-package is a product of two theoretically distinguishable entities, namely, (a) the average quantity flow of housing-services, and (b) its unit price. Taken individually, both entities cannot be observed directly. For this reason, it is impossible to determine to what extent the changes in average rent have stemmed from changes in the output of housing-services as opposed to changes in its unit price. Since this chapter will use cross-section data by municipalities, it is helpful, therefore, to make some general comments on the behaviour of these two entities in both the inter-city (municipal) and intra-city (municipal) viewpoints.

A city or municipal area may be viewed in a broad sense as a local market for both housing-packages and housing-services. Every housing-package in this local market differs in many respects such as in terms of age, architectural style, size, landscaping, location, etc., down to the last nail. Regardless of their differences, all packages have one common characteristics far more important than their differences. They all produce housing-services, the demand for which is ultimate to all consumers.

In a city or municipal area, "every dwelling (package) may be considered a substitute for every other dwelling".[1] The degree of substitution varies, so that all the dwellings in the area may be said to form a market characterized by a continuum of substitution phenomena. In particular, the market pricing of housing-services is effected through a continual interaction between occupancy rates and the over-crowdedness of available accommodations, changes in relative prices, and population movements within the local market area. The effects of this interaction are diffused within the market <u>tending</u> to equalize the unit prices of housing-services for the whole city or municipal area.[2] In this case, equilibrium assumptions may be adopted in the empirical investigation. Moreover, differences in the average rent level within a city or municipal area may be taken as acceptable measures of the differences in the average quantity flow of housing-services. These differences originate from variations in the size and quality of

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[1] W. G. Grigsby, <u>Housing Markets and Public Policy</u>, University of Pennsylvania Press, 1963, p. 34.

[2] <u>Ibid</u>., citing L. Winnick, D. Blank & Chester Rapkin, <u>Housing Market Analysis</u>, Washington, 1953, pp.9-10. Grigsby had extended in detail the analysis of substitution phenomena. He started from the metropolitan level as the local market unit and proceeded to divide it into submarket units that were linked together by various substitution channels.

the housing-packages (or of its major components L, H, and V).[3]

In general, the assumption of price equalization may not be appropriate in the inter-municipality (or city) case, since actual equalization is less likely to take place between municipalities. At least two major explanatory factors can be cited, namely: (a) the extreme distance among Canadian cities and municipalities, particularly among those municipalities belonging to different metropolitan areas; and (b) the non-transportable nature of housing-services. These two factors complement each other.

The supply of housing-services is localized and cannot be transported from one place to another. The permanence of the land-site and the physical dwelling-structure explains the locational fixity of the flow of housing-services.[4] Since housing-services cannot be transported, the consumers must come to them. If these services are not utilized as fully as possible,

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[3] The 1961 Dominion Bureau of Statistics (DBS) Census of Canada revealed wide variations in size of dwelling-units measured in terms of average number of rooms per dwelling. As to the (DBS) Census definition of a dwelling-unit, see Appendix A of this study.

[4] E.M. Fisher, Urban Real Estate Markets: Characteristics and Financing National Bureau of Economic Research, 1951, p.9.

"the stream of <u>housing services may be assumed</u> to have been permanently lost".[5] In particular, consumers can take advantage of inter-city price differentials if they are willing to change residence. To do so is to incur moving costs which presumably vary directly with distance. Very few consumers will be willing to move just to take advantage of inter-city price differentials.

Distance and the locational fixity of housing-services may well have the important effect of segmenting cities and municipalities as market units. Consequently, inter-market disequilibrium is most probably present under these circumstances.

There are developments that tend to correct inter-market disequilibrium. For instance, inter-market equalization of prices may take place in the long-run *via* construction activities, inter-city population movements, and family and non-family household formation. In the absence of these corrective forces, a local market excessively supplied with housing-packages and housing-services must provide for its own adjustment mechanism. The mechanism may proceed through (1) a reduction or complete cessation of construction of new dwelling-structures, and (2) a reduction in the rate of maintenance and repairs, in the caretaking and supervision of apartment units,

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[5] *Ibid*.(emphasis mine).

etc.,which accelerates the physical deterioration of dwelling-structures and certainly reduces the aggregate supply of housing-services in the market area. Lowry, Olsen, and Grigsby have analyzed this corrective mechanism by specific reference to the technical term "filtering process", rather than the usual terms, "depreciation", "deterioration", or "dilapidation" of dwelling-units. 6/

It may be concluded that in the short-run, there is less likelihood of inter-market equilibrium than intra-market equilibrium. While there may be only one unit price, E, at the intra-market level in any period, there will most likely be wide differences in inter-city(municipal) unit prices of housing-services. It is also likely that inter-market disequilibrium will persist even in the longer-run period due to market segmentation.

The inter-city price differentials and variations in the quantity of housing-services should facilitate, instead of hinder, empirical analysis if both prices and quantities are separately observable. Where they are not, the estimation of the various elasticities of

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6/ Ira S. Lowry, "Filtering and Housing Standards: A Conceptual Analysis", Land Economics, vol.36, November 1960, pp. 362-370; E.O. Olsen, "A Competitive Theory of the Housing Market", The American Economic Review, vol. 59, September 1969, pp. 612-621; W.G. Grigsby, op. cit.

the demand for and supply of housing-services becomes relatively difficult. In particular, the investigation will generally require more data and more information about the structural relations of the model. The difficulty should become clear as we proceed.

For the moment, let us concentrate on the additional data requirements. Since E (or Q) is not directly observable, a special kind of rent data will be needed to serve as a proxy for the E variable, in addition to the readily available market rent data from the decennial census of Canada. Ideally, these special rent data should be generated from samples of similar dwellings of particular specifications in various municipalities -- i.e., dwellings whose outputs of housing-services, Q, are supposedly equalized and whose rent differentials are thus measures of inter-municipality price (E) differentials. These special rent data are not available from Canadian sources. However, one such body of rent data can be extracted from United States sources, except that the data are by metropolitan areas.

In 1966 and 1967, the United States Bureau of Labour Statistics (BLS) undertook surveys on the gross rental cost of dwellings of particular specifications in 39 metropolitan areas in the United

States.[7] The moderate standard dwellings have the following specifications:[8]

> "---an unfurnished five-room unit (house or apartment) in sound condition and with a complete private bath, a fully equipped kitchen, hot and cold running water, electricity, central or other installed heating, access to public transportation, schools, grocery stores, play space for children, and located in residential neighborhoods free from hazards or nuisances."

The dwellings (package) falling under these specifications can be considered to generate the same average quantity flow of housing-services, since the specifications have in effect equalized and standardized their outputs. Of importance, the inter-metropolitan average gross rent differences in the survey data may be taken as measures of the differences in the unit price level, E, per standard bundle of

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[7] U.S. (BLS), City Worker's Family Budget for a Moderate Living Standard, Autumn 1966, Bulletin # 1570-1; Three Standards of Living for an Urban Family of Four Persons, 1967, Bulletin # 1570-5.

The cost of such items as water, heat, light, cooking fuel, etc., was added to the contract rent for dwellings whose tenants payed separately for them. This was done by the BLS to even-out differences in practices as to household facilities included in the contract rent.

In 1960, the BLS issued an interim report on city workers family budgets in 19 metropolitan areas. This report also included gross rent data for what were considered moderate standard dwellings. See H.H. Lamale & M.S. Stotz, "Interim City Worker's Family Budget", Monthly Labour Review, August 1960, pp. 785-808.

[8] Ibid (1966), Bulletin # 1570-1, p. 18.
Ibid (1967), Bulletin # 1570-5, pp. 5, 42.

housing-services.

The above interpretation of the survey rent data, of course, ignores possible errors of measurement as well as criticisms that can be laid down to the standard specifications. Further, the data were collected on a metropolitan area basis which certainly constitutes a market unit very much larger than the city (municipality) market unit. Nevertheless, these survey data are the best information available that closely approximate the kind of rent data needed to facilitate the empirical work of this study. In this chapter, I shall use the U.S. BLS survey data in the application of my regression model and take the U.S. regression results "as if" they were Canadian results. Without this procedure, no Canadian regressions will allow separate estimates of the parameters of the supply and demand equations for housing-services.

## B. Regression Model Specifications for Housing-Services Market

This section will deal with the specification of the regression models used here in empirical investigation of the market for housing-services. The basis of the specifications are the bi-market and the uni-market approaches already introduced in Section D of Chapter IV.

## B.1. Two-Market Model

### B.1.(a) Structural Equations

Let us re-state here the structural equations of this model.

Rental Sector

$Q_1 = Q_1(E_1, E_o, Y, N, F)$ Demand (1)

$E_1 = E_1(Q_1, Q_o, C_1, r)$ Supply (2)

Owner Sector

$Q_o = Q_o(E_1, E_o, Y, N, F)$ Demand (3)

$E_o = E_o(Q_1, Q_o, C_o, r)$ Supply (4)

where $E_o$, $E_1$ = unit prices of housing-services $Q_o$, $Q_1$;

$Q_o$, $Q_1$ = quantities of owner and rental housing-services per period;

$Y$ = average family or household income;

$N$ = number of persons per household;

$F$ = number of families or households;

$C_o$, $C_1$ = implicit indices of prices of factor inputs to the production of $Q_o$, $Q_1$;

$r$ = effective residential property tax rate.

Economic theory calls for the inclusion in the demand equations of such demand determining factors as tastes and preferences and prices of alternative goods and services[9] that may have significant variations across municipalities. Due to data limitations they are not included in the equations.[10] The exclusion of these variables in the structural equations has possible consequences for the empirical results of the investigation. In particular, if the population coefficients of these excluded variables are different from zero, and if the excluded variables are correlated with the included variables, then there will be bias in the estimated coefficients of the

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[9] The alternative goods and services are those that are close substitutes or are complementary to housing-services. In particular, the prices of closely substitutable commodities will be positively inter-related with the unit prices of housing-services. However, even the prices of unrelated commodities tend to do so because of the across-the-board income effect on the quantities demanded of these items. There seems to be no clear-cut explanation as to which other goods and services are close substitutes to housing-services.

The tastes and preferences of economic units are not directly observable and are, therefore, difficult to measure. If data were available, an indirect measure would be employed. For instance, an index of the relative quantities of housing-services and of other goods consumed per period may be used; but this would be confounded with price-effects in any case.

[10] The available Canadian consumer price index for inter-city comparison purposes is limited to eleven cities at the most. The index includes only food price levels, although it has been extended recently to cover clothing, transportation, health, and personal care. For details see Dominion Bureau of Statistics (DBS), Prices and Price Indices, November 1968, Cat. # 62-002.

included variables in the regression.[11]

B.1.(b) Modified Structural Equations

To allow an application of the structural framework to the available data, we propose to use here a modified version of the system of equations (1) to (4). The modified system is:

$$\frac{Q_1}{F} = D_1(E_1, E_o, Y, S) \qquad (5)$$

$$E_1 = S_1\left(\frac{Q_1}{F}, \frac{Q_o}{F}, C, r\right) \qquad (6)$$

$$\frac{Q_o}{F} = D_o(E_1, E_o, Y, S) \qquad (7)$$

$$E_o = S_o\left(\frac{Q_1}{F}, \frac{Q_o}{F}, C, r\right) \qquad (8)$$

where $Q_i/F$ $(i=0,1)$ = a measure of the quantity flow of housing-services per family or household in the ith sector,

S = number of persons per room (replacing N),

C = implicit index of input prices (replacing $C_o$ and $C_1$).

The other variables have already been defined above.

The reasons behind the proposed modifications are as follows. First, the available data on the average number of persons per family(to represent N)

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11/ A. Goldberger, Econometric Theory, John Wiley & Sons, 1964, pp. 196,197.

will not be representative of household size in the rental sector. Most families live in owner-occupied dwellings, whereas rental units are heavily occupied by non-family households which include young single people, bachelors, widows, widowers, divorcees, and spinsters. We find that the number of persons per room (S) is more representative of household size in the markets for both types of housing-services. S will also serve as a density index reflecting the shortage (or abundance) of accommodation relative to population in both markets.

Second, the prices of inputs (labour, utilities, construction materials, etc.) will hardly differ within a municipal area or across the rental and owner sectors thereof. A single index of input prices, C, may replace $C_o$ and $C_1$, respectively.

Third, the use of the transformed quantity variable $Q_i/F$ *in lieu* of the aggregate quantity $Q_i$ is commendable from purely statistical considerations. Canadian cities and municipalities differ extremely in demographic size. In a cross-section study by municipalities, the variance of the disturbances may well be positively correlated with the demographic size of the sampled communities. The use of the transformed quantity variable is one way to minimize possible heteroscedasticity in the disturbance terms of the regressions.[12]

Fourth, the actual rent data, $R_i (i=0,1)$, are available on a per dwelling unit basis. Thus, $R_i$ may be interpreted as

$$R_i = \frac{E_i Q_i}{H} \qquad i=0,1 \qquad (9)$$

where $Q_i/H$ = quantity flow of housing-services per dwelling in the ith sector and H = total number of dwellings in stock. Consequently, the theoretical rent variable of our structural relations is expressed on a per household basis, i.e.,

$$\frac{E_i Q_i}{F} = \text{rent per household } (i=0,1) \qquad (10)$$

so that it can be represented by $R_i$ in the actual regressions.

Will $R_i$ be an adequate proxy for $\frac{E_i Q_i}{F}$ ? In the 1961 DBS Census definition, the total number of occupied dwellings is equal to the total number of households, F. On the other hand, the total number of dwellings in stock, H, is equal to F plus vacancies. Therefore, $R_i$ differs from $\frac{E_i Q_i}{F}$ by the vacancy variable. This difference is not important to the extent that the variations in vacancy rates across

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[12] J. Johnston, Econometric Methods, 2nd ed., McGraw-Hill, 1972, pp. 214-221.

metropolitan areas and municipalities thereof are differences in "normal" vacancies rather than differences in the degree of tightness in residential accommodations. In this case, rent levels will not be related to vacancies. The cross-section study by F. de Leeuw and N. Ekanem supports this observation. Their reduced form estimators indicate that vacancy rates across U.S. metropolitan areas do not have a statistically significant effect on rent levels.[13] Similar circumstances may well be present in the Canadian housing market and therefore in the data used herein that are generated from this market. If this is so, $R_i$ will adequately represent $\frac{E_i Q_i}{F}$ in the ith sector.

B.1.(c) Specification of the Regression Model

Since the actual rent available $R_i (i=0,1)$ is a product of two dependent variables $E_i$ and $Q_i/H$ and since the available Canadian data do not measure them separately, we shall resort to the use of $R_i$ as an alternative dependent variable. Here we find it necessary to use a log linear specification for the modified structural system. The choice of this type

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[13] F. de Leeuw & N.F. Ekanem, "The Supply of Rental Housing", The American Economic Review, December 1971, p. 812.

of specification is mainly due to data limitations and the multiplicative nature of the dependent variables and should not suggest that the "true" relationships in the housing market are, in fact, linear in logarithmic form. The form of the "true" relationships is unknown and can only be approximated by a number of specifications. The logarithmic form is only one of them and the behaviour of the consumers and suppliers of housing-services is "not expected to conform exactly to this theoretical pattern".[14/]

Using a logarithmic specification and introducing disturbance terms, our regression models for the modified system of equations (5) to (8) are:

Rental Sector

$$q_1 = \bar{q}_1 - q_{11}E_1 + q_{1o}E_o + q_{1y}Y + q_{1s}S + U_1 \qquad (11)$$

$$E_1 = \bar{E}_1 + e_{11}q_1 + e_{1o}q_o + e_{1c}C + e_{1r}r + V_1 \qquad (12)$$

Owner Sector

$$q_o = \bar{q}_o + q_{o1}E_1 - q_{oo}E_o + q_{oy}Y + q_{os}S + U_o \qquad (13)$$

$$E_o = \bar{E}_o + e_{o1}q_1 + e_{oo}q_o + e_{oc}C + e_{or}r + V_o. \qquad (14)$$

The term "log" is omitted before all relevant variables and such constants as $\bar{q}_1$, $\bar{E}_1$, $\bar{q}_o$ and $\bar{E}_o$; the coefficients before the variables are elasticities.

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14/ W.W. Walters, An Introduction to Econometrics, Macmillan, 1968, pp. 212, 213.

The $U_i$ and $V_i$ $(i=0,1)$ are the disturbance terms of the regression model. It is assumed here that the disturbance terms are log-normally distributed with logarithmic mean equal to zero [i.e., $E(U_i) = E(V_i) = 0$] and $E(U_i U_i') = \sigma_{u_i}^2 I_n$, $E(V_i V_i') = \sigma_{v_i}^2 I_n$. 15/ (The operator E for expected value in this particular instance should not be confused with the price variable for housing-services.) It is also assumed that the disturbances are uncorrelated across equations.

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15/ These assumptions must be qualified in that, due to data limitations, such demand determining variables as tastes and preferences and relative prices of alternative goods and services are excluded from the model. If W represents the excluded variables, say, in equation (11) and B is its corresponding vector of non-zero population coefficients, then for example

$$U_1 = BW + U_1^*$$

where $U_1^*$ is the "true" disturbance term, and

$$E(U_1) = BE(W) + E(U_1^*)$$

$$= BE(W) \neq 0.$$

If this is the case, there will be specification error in the regression model and consequently bias in the actual regression results.

See A. Goldberger, op. cit., pp. 196, 197, 214, 215.

The reduced form regression models for the modified version (with the term "log" omitted) are:

$$q_1 = \bar{T}_{q_1} + T_{q_1 y} Y + T_{q_1 s} S + T_{q_1 c} C + T_{q_1 r} r + \bar{U}_1 \qquad (15)$$

$$E_1 = \bar{T}_{e_1} + T_{e_1 y} Y + T_{e_1 s} S + T_{e_1 c} C + T_{e_1 r} r + \bar{V}_1 \qquad (16)$$

$$q_o = \bar{T}_{q_o} + T_{q_o y} Y + T_{q_o s} S + T_{q_o c} C + T_{q_o r} r + \bar{U}_o \qquad (17)$$

$$E_o = \bar{T}_{e_o} + T_{e_o y} Y + T_{e_o s} S + T_{e_o c} C + T_{e_o r} r + \bar{V}_o \qquad (18)$$

Here, $\bar{T}_{q_i}$ and $\bar{T}_{e_i}$ and $\bar{U}_i$ and $\bar{V}_i$ $(i=0,1)$ are the corresponding constants and disturbance terms of the reduced form regression models, respectively. Their compositions are not shown here nor in the Appendix to this chapter, but they can be determined from (11) to (14). The compositions of the reduced form parameters are shown in detail in the Appendix. Under the assumptions that

(a) the cross-elasticities of supply ($e_{1o}$ and $e_{o1}$) are close to zero,

(b) the price-elasticities of demand are greater than the cross-elasticities of demand (i.e., $q_{oo} > q_{o1}$ and $q_{11} > q_{1o}$), and

(c) the tax and input-cost elasticities of supply-prices in both sectors are similar (i.e., $e_{1r} \cong e_{or}$ and $e_{1c} \cong e_{oc}$), we find that

$$\left.\begin{array}{ll} T_{q_i y} > 0, & T_{q_i c} < 0 \\ T_{q_i s} > 0, & T_{q_i r} < 0 \\ T_{e_i y} > 0, & T_{e_i c} > 0 \\ T_{e_i s} > 0, & T_{e_i r} > 0 \end{array}\right\} \quad i=0,1 \qquad (18\text{-a})$$

It is shown in the Appendix to this chapter that the system of structural equations (11) to (14) is just identified. Thus, either the Indirect Least Squares method or the Two-Stage Least Squares method could be used in the estimation of the structural parameters. However, if in fact $e_{1o} = e_{o1} = 0$, the price equations (12) and (14) are over-identified and Two-Stage Least Squares would be the appropriate method of estimation.

Since the Canadian data do not include separate measures of $E_i$ and $q_i$, only the combined-reduced-form model (with the term "log" omitted)

$$(E_i + q_i) = (\bar{T}_{e_i} + \bar{T}_{q_i}) + (T_{e_i y} + T_{q_i y})Y + (T_{e_i s} + T_{q_i s})S$$

$$+(T_{e_i c} + T_{q_i c})C + (T_{e_i r} + T_{q_i r})r + (\bar{V}_i + \bar{U}_i) \qquad (19)$$

$$i=0,1$$

can be used. In this regression model, however, all the structural equations (11) to (14) will be under-identified. For this reason, the empirical investigation using a bi-market model cannot be pursued here rigorously until such time as the necessary data become available. Under the assumption that the cross-elasticities of supply ($e_{10}$ and $e_{01}$) are equal to zero and with the help of outside information available from F. de Leeuw and Ekanem's cross-section study, we are able to arrive at an estimate of the price-elasticity of supply, $1/e_{11}$, of Canadian rental housing-services. This will be discussed in Section C below.

The expected signs of the combined reduced coefficients in (19) may be verified from (18-a) and from equations (A-8) to (A-24) in the Appendix to this chapter. The expected signs are

$$(T_{e_i y} + T_{q_i y}) > 0, \qquad (T_{e_i s} + T_{q_i s}) > 0,$$

$$(T_{e_i c} + T_{q_i c}) \gtreqless 0, \text{ and } (T_{e_i r} + T_{q_i r}) \gtreqless 0.$$

The specific signs of the combined-reduced-form coefficients of the tax and input cost variables will depend upon the relative numerical values of the various supply and demand elasticities. For a detailed discussion of this point, please refer to Section D in Chapter IV as well as to its appendix. It needs pointing out here, however, that since the bi-market model has many structural parameters (or elasticities), a large number of these parameters gets built into the combined-reduced-form coeficients. In consequence, the signs of the combined-reduced-form tax and input cost coefficients cannot be specified unless the numerical values of some of the structural elasticites are known a priori.

B.2. One-Market Model for Housing-Services

Since the investigation is constrained to the use of combined-reduced-form estimators, we find it advisable to use also a one-market model. This model has fewer parameters, thereby permitting an easier and more specific interpretation of its combined-reduced-form coefficients.

B.2(a). Structural Equations

It will be recalled that our one-market model has the following system of equations, namely,

$$Q = Q(E,Y,N,F) \qquad \text{Demand} \qquad (20)$$

$$E = E(Q,C,r) \qquad \text{Supply} \qquad (21)$$

Adopting the necessary modifications already discussed in the preceding sub-sections, the modified version of the one-market case is

$$q = \frac{Q}{H} = D(E,Y,S) \qquad (22)$$

$$E = S\left(\frac{Q}{H}, C, r\right) \qquad (23)$$

Here, $q = {}^{Q}/_{H}$ replaces ${}^{Q}/_{F}$ and S is used in lieu of N.

B.2(b). Regression Model Specification

Using also a logarithmic specification and omitting the term "log", the regression model is

$$q = \bar{q} - q_E\, E + q_y\, Y + q_s\, S + U \qquad (24)$$

$$E = \bar{E} + E_q\, q + E_c\, C + E_r\, r + V \qquad (25)$$

where $\bar{q}$ and $\bar{E}$ are constants,

$-\, q_E$ = price-elasticity of demand,

$q_y$ = income-elasticity of demand,

$q_s$ = demographic-elasticity of demand,

$1/E_q$ = price-elasticity of supply,

$E_c$ = input cost-elasticity of supply-price,

$E_r$ = tax-elasticity of supply-price,

U and V are disturbance terms, and $E(U)=E(V)=0$, $E(UU')= \sigma_u^2 I_n$, $E(VV')= \sigma_v^2 I_n$ . In this instance, the operator E for expected value should not be confused with the price variable for housing-services.

The reduced form of (24) and (25) are

$$q = \bar{T}_q + T_{qy}\, Y + T_{qs}\, S + T_{qc}\, C + T_{qr}\, r + U^* \qquad (26)$$

$$E = \bar{T}_E + T_{EY}\, Y + T_{ES}\, S + T_{EC}\, C + T_{Er}\, r + V^* \qquad (27)$$

where $\bar{T}_q$ , $\bar{T}_E$ are constants and $U^*$, $V^*$ are the reduced form disturbance terms, and where

$$\Delta = ( 1 + q_E E_q ) > 0, \quad \bar{T}_q = \frac{(\bar{q} - q_E \bar{E})}{\Delta} \gtreqless 0, \quad \bar{T}_E = \frac{(q\bar{E}_q + \bar{E})}{\Delta} \gtreqless 0,$$

$$T_{qy} = \frac{q_y}{\Delta} > 0, \qquad T_{EY} = \frac{E_q q_y}{\Delta} > 0,$$

$$T_{qs} = \frac{q_s}{\Delta} > 0, \qquad T_{ES} = \frac{E_q q_s}{\Delta} > 0,$$

$$T_{qc} = \frac{-q_E E_c}{\Delta} < 0, \qquad T_{EC} = \frac{E_c}{\Delta} > 0,$$

$$T_{qr} = \frac{-q_E E_r}{\Delta} < 0, \qquad T_{Er} = \frac{E_r}{\Delta} > 0.$$

The combined-reduced-form of (26) and (27) is

$$(E+q) = (\bar{T}_q + \bar{T}_E) + \frac{q_y(1+E_q)}{\Delta} Y + \frac{q_s(1+E_q)}{\Delta} S + \frac{E_c(1-q_E)}{\Delta} C + \frac{E_r(1-q_E)}{\Delta} r + (U^*+V^*) \tag{28}$$

Here, it can be seen that

$$\left.\begin{array}{l} \dfrac{E_c(1-q_E)}{\Delta} \gtreqless 0 \\ \dfrac{E_r(1-q_E)}{\Delta} \gtreqless 0 \end{array}\right\} \text{ depending whether } /q_E/ \lesseqgtr 1.$$

Both the structural equations (24) and (25) are overidentified. For instance, the application of Indirect Least Squares to (26) and (27) will result in two estimates of $q_E$ and $^1/E_q$, respectively. The two estimators for

$$q_E \quad \text{are} \quad -\hat{T}_{qc}/\hat{T}_{EC} \quad \text{and} \quad -\hat{T}_{qr}/\hat{T}_{ER},$$

while the two estimators for

$$^1/E_q \quad \text{are} \quad \hat{T}_{qy}/\hat{T}_{EY} \quad \text{and} \quad \hat{T}_{qs}/\hat{T}_{ES}.$$

It follows that Indirect Least Squares is not a viable estimation procedure, but that Two-Stage Least-Squares is. Indeed, if the Canadian data had included separate measures of E and q, the Two-Stage Least Squares method should be employed. Since these data are unavailable, only the combined-reduced-form regression model (28) can be used. The structural equations will be under-identified. However, the interpretation of the combined-reduced-form of the one-market model is simpler than that of the two-market model. In the one-market model, the empirical signs of the combined-reduced-form tax and input-price coefficients will immediately suggest whether the demand for housing-services is elastic or inelastic. Further, by resorting to U.S. data and taking them "as if" they were Canadian data, tentative estimates of the structural parameters of the one-market model can be obtained. This will be discussed in the next section.

## C. Empirical Results: Cross-Section by Cities and Municipalities

All the data used herein are taken from the Dominion Bureau of Statistics (DBS) 1961 Census and the 1969-1970 DBS publications of the effective property tax rates for various municipalities across Canada. For empirical purposes, the study uses a sample size of 59 observations on major and minor Canadian cities, including two non-incorporated municipalities. To avoid excessive footnotes and qualifications, the detailed explanations of the sources as well as the nature of the data used herein are separately discussed in Appendix A of the dissertation.

The major shortcoming of the data used herein lies in the nine-year gap between the 1961 Census of Canada data and the 1970 property tax rate data. This gap has serious implications for the credibility of the statistical results that emerge from the study. The property tax rate is an explanatory variable. The appropriate tax rate data, therefore, should at least be concurrent to the 1961 Census data, or earlier if the tax has lagged explanatory effects. The use of the 1970 tax data, in effect, tends to defeat the logic of causation implied in the theoretical framework. Further, there is no guarantee that the relative levels of the available 1970 tax rate data are fairly representative of the unknown 1960 or 1961 tax rates. Since the gap is an eight-year period, there is a great possibility that the two sets of tax rates differ substantially. Consequently, the statistical results reported here may well be wide of the mark.

The 1971 Census of Canada data on housing would be ideal to use here since they are more concurrent with the 1970 tax rate data. However, the publication of these data has been delayed and was not available at the time of writing. We, therefore, proceed on the assumption that the 1970 tax data are "adequate" proxies for the unknown 1960 or 1961 tax rates. In the process, the 1970 tax data appear to be good proxies. The 1970 tax rates demonstrate statistically significant effects on the market values of residential dwellings and on the volume of residential construction. This significant explanatory performance of the 1970 tax rates may well suggest that better results would have emerged from the study if more concurrent data had been available.

It was not possible to construct an index of input prices represented by C. Thus, the actual combined-reduced-form regression analysis is constrained to only the three remaining independent variables Y(average family income), S(number of persons per room), and r(effective residential property tax rate). It can be presumed that changes in C have significant effects on the market prices and on the quantities of housing-services demanded and supplied. Since it is likely that C differs significantly across municipalities in various Canadian provinces, the absence of C in the actual regressions probably injects bias into the statistical results.

Further, there are no available market-generated rental income data for owner-occupied dwellings. I have taken 10% of the median (market) value of owner-occupied dwellings as a proxy variable for the average rental income of the owner-occupied category. This 10% rate applies on all median values across sample observations. From a pure statistical and mathematical point of view, it is immaterial whether one uses median (market) value or 10% of it. The accompanying interpretation, however, allows substantive differences between the two. The median (market) value data is a stock concept. What we need is a value flow of housing-services per annum. The 10% rate is thus used as an approximate discount rate to convert the median (market) value data into value flows of housing-services.

Since there is only one discount or conversion rate across sample observations, to choose any rate other than 10% does not affect the empirical results. In principle, however, the conversion rate should reflect the average gross rate of return on investments in the residential market. (See Chapter II, sub-section B.2.) During the 1960's the weighted average mortgage rate of interest by Canadian provinces ranged from

7.00% (Saskatchewan) to 7.68% (Prince Edward Island), indicating a relatively small inter-provincial variation in rates. The simple average of the provincial mortgage rates amounts to 7.14%.[16] The sum of this simple average and the 2.56% DBS depreciation rate on the housing stock series[17] comes to 9.7%, which is quite close to the flat 10% conversion rate used herein.

Before presenting the empirical results, one very important question needs clarification. Is a cross-section study such as the present one a study of the behaviour of the housing market in the short-run or in the long-run? Alternatively, are cross-section data for a given year short-run or long-run market data? Taken separately and without any reference to inter-municipality differences, housing data in these communities in any given year are far from being long-run market data. However, it is not the length of time *per se* that governs the short-run or long-run characteristic of a given set of market data. A closer examination of the inter-community housing data reveals

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[16] Central Mortgage and Housing Corporation, Canadian Housing Statistics, 1962, Table 50, pp. 47-48.

[17] E.H. Oksanen, "Housing Demand in Canada: Some Preliminary Experimentation", Canadian Journal of Economics and Political Science, August 1961, Table 3, n.3, p. 318.

tremendous differences among municipalities as to size, incomes, prices, tax costs, and stocks of dwellings. In the 59 observations of this study, population ranges from 10,946 to 1,191,062, average family income from $4,330 to $8,371, gross rental income from $660 to $1,296 per annum. These differences tend to persist over time, even for decades. From the econometric point of view, since these differences are the cumulative effects of long-term differences among municipalities, the 59 sample observations used in this study may well be interpreted as long-run market data. In F. de Leeuw and N. Ekanem's judgment, "studying differences among cities and municipalities amounts to studying how housing markets behave in the long-run in the sense of having ample time to adjust to such basic market forces" 18/ as changes in population, income, tastes and preferences, stocks of dwelling-structures, and property tax rates.

C.1. Regression Results

The results of the regressions using the combined-reduced-form models are summarized in Table 5.

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18/ F. de Leeuw and N.F. Ekanem, "The Supply of Rental Housing", The American Economic Review, December 1971, vol. 61, no. 5, pp. 806-817.

Table 5

ESTIMATED COMBINED-REDUCED-FORM
EQUATIONS FOR HOUSING-SERVICES

| | DEPENDENT VARIABLES | INTER-CEPTS | INDEPENDENT VARIABLES | | | $\bar{R}^2$ | F-Ratio |
|---|---|---|---|---|---|---|---|
| | | | $\log_{10} r$ | $\log_{10} Y$ | $\log_{10} S$ | | |
| I | $\log_{10} R$ | -0.747 | -0.165<br>(1.987) | 1.048<br>(8.265) | 0.256<br>(1.948) | 0.75 | 23.53 |
| II | $\log_{10} R_1$ | -0.051 | -0.093<br>(1.016) | 0.819<br>(5.891) | -0.071<br>(0.494) | 0.65 | 15.05 |
| III | $\log_{10} R_o$ | -0.130 | -0.304<br>(2.740) | 1.123<br>(6.629) | 0.483<br>(2.757) | 0.69 | 18.04 |
| IV | $\log_{10} R^*$ | -0.707 | -0.035<br>(0.346) | 0.856<br>(5.606) | 0.515<br>(3.259) | 0.60 | 11.28 |
| V | $\log_{10} R_1^*$ | -0.044 | -0.080<br>(0.704) | 0.634<br>(3.677) | 0.197<br>(1.100) | 0.43 | 4.84 |
| VI | $\log_{10} R_o^*$ | -0.211 | -0.235<br>(2.142) | 1.024<br>(6.130) | 0.736<br>(4.256) | 0.66 | 15.92 |

Notes: Numbers in parenthesis are absolute values of the test statistics for coefficients different from zero;

$N$ = (59) number of observations;

$R_1$ = weighted average gross rental income of tenant-occupied dwellings;

$R_o$ = 10% of median market value of owner-occupied dwellings;

$R = \frac{1}{2}(R_1 + R_o)$ = simple average rental income for all dwellings;

$R^*$, $R_1^*$, $R_o^*$ are average rents per room;

$Y$ = average family income;

$r$ = effective residential property tax rate; and

$S$ = number of persons per room.

SOURCES OF DATA: See Appendix A of the study.

The definitions of the data used appear as footnotes to the table. In these results, there are two types of regressand, namely, (a) the average rental income on a per dwelling-unit (Census definition) basis; and (b) the average rental income on a per room basis.

Regressions I to III report the result using rental data on a dwelling unit basis. Regression I indicates that the tax rate, r, has a very minimal negative impact upon the value of output of housing-services, R, representing all dwellings. The tax coefficient of this regression is significant from the statistical point of view at the 5% level (one-tail). A different result emerges when the rental income is separated into tenant-occupied and owner-occupied categories. Only the rental income of the owner-occupied category is significantly responsive to the tax (regression III). The gross rental income of the tenant-occupied category is hardly responsive to the tax (regression II). Its tax coefficient is short of being statistically significant even at the 10% level (one-tail).

Regressions IV to VI report the results using rental data on a per room basis. The rental income of the owner-occupied category continues to demonstrate the same statistically significant response to the tax rate (regression VI). The gross rental income of the tenant-occupied category demonstrates no response at all from the statistical point of view. The income and the demographic factor coefficients retain their high statistical significance in all regressions, with the exception of the latter in the tenant-occupied regressions.

The average imputed rent, $R_o$, of owner occupied dwellings is obtained by taking 10% of its median market value. Thus, the interrelationship between $R_o$ and the tax variable r is an interrelationship between the market value and the residential property tax rate. The negative and statistically significant association between $R_o$ and r in regressions III and VI may well be an empirical demonstration of the property tax capitalization hypothesis. This point will be pursued in detail in the next chapter.

It can also be argued here that the negative interrelationship between $R_o$ and r stems from a dependence of the effective tax rate on property value. For instance, assuming a target property tax take, the higher the property value the lower will be the effective property tax rate. Using cross-section data from Northeastern New Jersey municipalities in the United States, W.E. Oates had examined the simultaneity in the interrelationships between residential property tax rates and residential property values. He found no statistically significant indication of simultaneous interrelationships.[19/] The absence of a significant simultaneous interrelationship between tax rates and market values may well be applicable to the Canadian data used here. Indeed, it may be argued that the interdependence between the 1961 market value data and the 1970 tax rate data is quite remote because of the nine-year gap between them.[20/] For these reasons, it is unnecessary to apply Two-Stage Least Squares regressions to the Canadian data.

In Section B of this chapter we have suggested two combined-reduced-form specifications with which to interpret the regression results reported in Table 5. The first uses a two-market model and the second uses a one-market model which assumes perfect substitution between rental and owner housing-services. The corresponding combined-reduced-form regressions for

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[19/] W.E. Oates, "The Effects of Property Taxes and Local Public Spending on Property Values: An Empirical Study of Property Tax Capitalization and the Tiebout Hypothesis", Journal of Political Economy, vol. 7, no. 6, November/December 1969, pp. 957-971.

[20/] This, of course, strengthens the case against the use of sets of Canadian data with a nine-year gap.

the two-market model are nos. II and III (per dwelling unit basis) and nos. V and VI (per room basis). The corresponding combined-reduced-form regressions for the one-market model are nos. I (per dwelling unit basis) and IV (per room basis). The combined-reduced-form tax coefficients generated by these regressions generally conform to our theoretical expectations. It should be noted, however, that there is no definite theoretical expectation on the sign of the combined-reduced-form tax coefficient in either models, the sign being determined by the relative and absolute values of the structural elasticities built into these coefficients.

All the structural elasticities are irretrievable from the estimated combined-reduced-form coefficients. The results, however, offer some intelligible implications for the relative numerical values of some of the structural elasticities; although the more specific implications are limited to the one market-model since it has fewer structural elasticities than the two-market model.

In the one-market model, the combined-reduced-form tax coefficient is equal to $E_r(1-q_E)/(1+q_E E_q)$, where $E_r$ = tax-elasticity of supply-price, $q_E$ = price-elasticity of demand, and $1/E_q$ = price-elasticity of supply. In regression no. I in Table 5, the estimated combined-reduced-form tax coefficient is negative and significantly different from zero at the 10% level (two-tail). Therefore, by implication,

(1) the absolute value of $q_E$ is greater than one, and

(2) the absolute value of $E_r$ is greater than zero.

A similar deduction is not possible from the estimated combined-reduced-form tax coefficients for the two-market model because a larger number of structural elasticities is built into these coefficients.

For lack of the necessary Canadian data, we shall attempt in the next sub-section to retrieve the structural parameters of the Canadian regressions with the help of regression results from U.S. data. This is admittedly a very crude procedure. Without this procedure, however, no structural parameters from the Canadian regressions can be identified.

## C.2. Retrieving of Structural Parameters and Interpretation of Results

### C.2(a). Two-Market Model

In a recent cross-section study of U.S. rental housing-services by metropolitan areas, F. de Leeuw and N.Ekanem employed a log linear reduced-form price equation.[21/] The rent data used by the authors were the special U.S. rent data already described in

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[21/] F. de Leeuw and N. Ekanem, op.cit.

Section A of this chapter. The estimators obtained by F. de Leeuw and N. Ekanem for the moderate standard rental housing were the following ( in logarithms but with the term "log" omitted):

$$E_1 = \underset{(1.8)}{2.50} + \underset{(2.6)}{1.26}\,P + \underset{(3.3)}{.60}\,Y_r + \underset{(2.4)}{.24}\,C + \underset{(0.7)}{.07}\,O - \underset{(2.4)}{.04}\,H \qquad R^2=0.60 \qquad (29)$$

$$E_1 = \underset{(1.5)}{2.06} + \underset{(3.6)}{0.653}\,(Y_r\,P) + \underset{(2.9)}{.29}\,C + \underset{(1.0)}{.102}\,O - \underset{(2.4)}{.04}\,H \qquad R^2=0.59 \qquad (30)$$

where the numbers in parenthesis are absolute values of the test-statistics, and where

N = 39 (number of observations),
$E_1$ = unit price of a standard bundle of rental housing-services,
C = an index of prices of capital inputs,
O = an index of prices of operating inputs,
P = an index of the cost of living ( excluding housing) representing prices of alternative goods and services,
$Y_r$ = real income per household,
$Y = Y_r P$ = money income per household,
H = number of households.

The C and O indices were constructed from data on land prices, wage rates of construction workers, financing costs, utility prices, and property taxes in different U.S. metropolitan areas.

The first regression (equation 29) uses log P and log $Y_r$ as separate regressors. The second regression (equation 30) uses log $(Y_r P)$. The coefficients of log $Y_r$ and log $(Y_r P)$ hardly differ in numerical values. The negative relationships between log $E_1$ and log H is to be expected since $E_1$ is a measure of the standard bundle of rental housing-services per dwelling or household, and measurement errors may be present. The explanations of the signs of the rest of the price reduced-form parameters are straightforward and they conform to a priori theoretical expectations.

In their interpretation of these regressions, F. de Leeuw and N. Ekanem implicitly assumed that the rental and owner sectors in a metropolitan area are non-interacting. We have argued here that the two sectors are inter-dependent markets. It is, therefore, proposed that we re-interpret F. de Leeuw and Ekanem's estimators in terms of a two-market approach. In this case, the coefficient of the money income variable in equation (30) would be

$$\frac{\partial \log E_1}{\partial \log(Y_r P)} = 0.653 = \text{an estimate of } T_{e_1 y} \tag{31}$$

where $$T_{e_1 y} = \frac{1}{/A/} \left\{ q_{1y} \angle\overline{\phantom{x}} e_{11} + q_{oo}(e_{oo} e_{11} - e_{o1} e_{1o}) \underline{\phantom{x}}/ + q_{oy} \angle\overline{\phantom{x}} e_{1o} + q_{1o}(e_{oo} e_{11} - e_{o1} e_{1o}) \underline{\phantom{x}}/ \right\} \tag{31-a}$$

and $/A/ = 1 + e_{11} q_{11} + e_{oo} q_{oo} + (q_{11} q_{oo} - q_{o1} q_{1o})$

$$(e_{11} e_{oo} - e_{o1} e_{1o}) - e_{o1} q_{1o} - e_{1o} q_{o1} \qquad (31\text{-b})$$

In Table 5, the Canadian combined-reduced-form regression for rental housing (regression II) yields

$$\frac{\partial \log E_1 q_1}{\partial \log Y} = 0.819 = \text{an estimate of } (T_{q_1 y} + T_{e_1 y}) \qquad (32)$$

where $T_{q_1 y} = \frac{1}{/A/} \left[ q_{1y} (1 + q_{oo} e_{oo} - e_{1o} q_{o1}) + q_{oy} (q_{1o} e_{oo} - q_{11} e_{1o}) \right]$ (32-a)

Taking the U.S. regression by F. de Leeuw and N.F. Ekanem "as if" it were a Canadian regression and assuming $e_{1o} = e_{o1} = 0$ (cross-elasticities of supply) we find that 22/

$$\frac{\partial \log E_1 q_1 / \partial \log Y}{\partial \log E_1 / \partial \log (Y_r P)} = \frac{0.819}{0.653} = \text{an estimate of } \left(1 + \frac{T_{q_1 y}}{T_{e_1 y}}\right)$$

$$= \text{an estimate of } \left(1 + \frac{1}{e_{11}}\right) \qquad (33)$$

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22/ In equations A-35 and A-36 in the Appendix to this chapter, we have shown that if $e_{1o} = e_{o1} = 0$, then

$$\left.\begin{aligned} e_{11} T_{q_1 y} &= T_{e_1 y} \\ e_{11} T_{q_1 s} &= T_{e_1 s} \end{aligned}\right\} \quad \text{so that} \quad \frac{T_{q_1 y}}{T_{e_1 y}} = \frac{T_{q_1 s}}{T_{e_1 s}} = \frac{1}{e_{11}} .$$

Thus, if S had been included in de Leeuw and Ekanem's reduced-form price estimator, the rental price-elasticity of supply, $1/e_{11}$, would be overidentified since

$$\frac{\partial \log E_1 q_1 / \partial \log Y}{\partial \log E_1 / \partial \log S} = (1 + T_{q_1 s} / T_{e_1 s}) = (1 + \frac{1}{e_{11}}). \qquad (34)$$

Thus, $\frac{0.819}{0.653} - 1 = 0.254$ is an estimate of $^1/e_{11}$, the price-elasticity of supply in Canadian rental housing-services by municipalities. The rest of the structural parameters of the Canadian rental market remain underidentified.

C.2(b). <u>One-Market Model</u>

The focus of interest here is the Canadian regression no.I in Table 5. The combined-reduced-form equation estimated by this regression is (in logarithms, but with the term "log" omitted):

$$(E+q) = (\bar{T}_q + \bar{T}_E) + \frac{q_y(1+E_q)}{\Delta} Y + \frac{q_s(1+E_q)}{\Delta} S + \frac{E_r(1-q_E)}{\Delta} r \qquad (35)$$

where $(\bar{T}_q + \bar{T}_E)$ = constant and $\Delta = (1+ q_E E_q)$. Here, we shall also resort to the aid of U.S. data in order to unscramble the estimated structural elasticities built into the coefficients of regression I.

The available information by metropolitan areas from U.S. sources include: (1) the 1966 or the 1967 special rent data which can be taken as a measure of E, (2) the 1967 data on per capita personal income($Y_p$), (3) the 1966 data on effective residential property tax rates(r), and (4) the 1966 data on average annual payments of interest and principal (M) for mortgaged residences. To these U.S. data we apply a regression analysis which, in effect, will estimate a reduced-form price equation of the form (in logarithms):

$$E = \bar{T}^*_E + \frac{(q_y E_q)^*}{\triangle} Y_p + \frac{(E_r)^*}{\triangle} r + \frac{u}{\triangle} M \qquad (36)$$

where $Y_p$ , r, and M are as defined above, E= unit price per standard standard bundle of housing-services, and $\bar{T}^*_E$= constant. The result of the U.S. regression is (in logarithms):

$$E = -0.79 + \underset{(4.392)}{0.589}\, Y_p + \underset{(1.226)}{0.02}\, r + \underset{(1.364)}{2.179}\, M \qquad (37)$$

$$\bar{R}^2 = 0.626, \quad F=8.81$$

(numbers in parenthesis are absolute values of the test statistics for coefficients different from zero)

SOURCE OF DATA: See Appendix A of the study

For lack of data, S is not in the U.S. regression. If it was, the price-elasticity of supply, $^1/E_q$ , would be overidentified. Since it is not, the resulting U.S. regression based on (36) may bear coefficients different in meaning to those built into regression I (Table 5) based on equation (35). It would seem, however, that the omission of S in the U.S. regression has only a minimal impact on our results. The partial correlation coefficients between S and Y and between S and r in the Canadian regressions are relatively small, i.e., 0.293

and 0.216, respectively. This could very well be the case for similar U.S. data, including $Y_p$ which is certainly highly correlated with average family income, Y.

In the U.S. regression, the empirical signs of the coefficients are also as expected theoretically. The tax rate coefficient, however, is a little short of being statistically significant at the 10% level (one-tail).

Both the Canadian regression no. I in Table 5 and the U.S. regression (equation 37) allow the identification or retrieval of all the structural elasticities built into their respective coefficients. The calculated elasticities are summarized in Table 6. Table 6 shows that the absolute numerical values of the estimated income and price elasticities of demand are high. In contrast, the estimated price-elasticity of supply and tax-elasticity of supply-price are very low. If these findings are specifically applied to the rental sector, they would suggest the conclusion that only minimal forward tax shifting may take place in the rental sector. In particular, the rather low absolute numerical value of the estimated tax-elasticity of supply-price suggests that increases in the tax rate hardly result in sufficient upward movements in the market supply curve of housing-

Table 6

ESTIMATED MARKET SUPPLY AND DEMAND
ELASTICITIES FOR HOUSING-SERVICES
ONE-MARKET MODEL

| | | |
|---|---|---|
| Price-elasticity of demand | $(-q_E)$ | -9.25 |
| Price-elasticity of supply | $(^1/E_q)$ | 0.78 |
| Income-elasticity of demand | $(q_y)$ | 5.91 |
| Tax-elasticity of supply-price | $(E_r)$ | 0.26 |
| Demographic demand elasticity | $(q_s)$ | 1.45 |
| Input-cost elasticity of supply-price | $(u)$ | 28.04 |

services to allow forward tax shifting.

The same conclusion is also apparent in the numerical value of the estimated price-reduced-form tax coefficient in equation 37. This coefficient indicates that a 10% increase in residential property tax rate will increase the equilibrium unit price of housing-services by only 0.2%, the other independent variables in the market remaining the same. This minimal price-effect of the tax rate stems from low price- and tax-elasticities of the market supply and a high price-elasticity of the market demand for housing-services. Moreover, since the estimated tax-elasticity of supply-price ($E_r$) is very low, a 10% increase in the property tax rate will result in a smaller decline in the equilibrium quantity of housing-services. The quantity-effect of a 10% increase in the property tax rate is estimated to be - 1.85%.[23]

The general implications of the calculated elasticities presented in Table 6 are subject to a number of qualifications. <u>First</u>, the nine-year gap between the set of 1970 Canadian tax data and the set

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[23] In the reduced-form quantity equation (26), $\frac{\partial q}{\partial r} = \left(\frac{-q_E E_r}{1+q_E E_q}\right)$ . Substituting here the calculated numerical values of the appropriate elasticities, we find an estimate of $\partial q/\partial r$ equal to -0.185.

of 1961 Canada census data used here is a strong case against the credibility of our results. Second, the absence of such other relevant variables as the relative prices of alternative goods and services and an index of input costs in the Canadian regressions may have injected serious bias into our statistical results. Third, if not for purposes of overcoming the identification problems in all Canadian regressions, resort to U.S. data is inappropriate since the structure of the U.S. housing market need not resemble that of the Canadian market. Fourth, the tax coefficient in the U.S. reduced-form price equation has very minimal statistical significance so that the retrievable price- and tax-elasticities of supply are not conclusive. Fifth, the perfect substitution assumption of the one-market model is probably illegitimate; if not for data limitations and the consequential identification problems, the bi-market model is a more useful approach for the investigation of the housing market. Sixth, the very low absolute numerical values of the estimated price- and tax-elasticities of supply may have resulted from the cross-section data being only partly reflective of a long-run type of market data.

D. Summary

We have attempted to estimate here the various elasticities of the market supply and demand for owner and rental housing-services with the use of cross-section data on Canadian municipalities. We initially have applied to the data a bi-market regression model. However, the available Canadian data permit only a combined-reduced-form estimation. With only the combined-reduced-form parameters to work with, all the structural parameters (elasticities) of the bi-market model remain underidentified.

Despite the identification problem, the combined-reduced-form regressions for the bi-market model should facilitate intelligible interpretations of results. But since the model has many structural elasticities, a large number of the elasticities gets built into the individual combined-reduced-form coefficients. A concrete interpretation of the combined-reduced-form results is virtually impossible without *a priori* information on the absolute numerical values of some of the structural elasticities. Since the necessary *a priori* information is not available, we have been unable to interpret rigorously our results or to derive from them any firm conclusions. However, by resorting to the U.S. reduced-form price regression of F. de Leeuw and N. Ekanem to aid the identification, we have estimated a 0.254 price-elasticity of supply for the Canadian rental sector. The rest of the structural elasticities of the bi-market model remain irretrievable from the combined-reduced-form regression equations.

As an alternative approach, we have resorted to a one-market regression model which has fewer parameters and treats owner and rental housing-services as perfect substitutes. Although the application of this model is also limited to a combined-reduced-form estimation and its parameters are underidentified, the regression results are capable of rendering simple and more specific interpretation. Further, we have avoided here the identification problems by resorting to the aid of a reduced-form price equation from U.S. data.

The estimated elasticities for the one-market model are: (1) price-elasticity of demand (-9.25), (2) income-elasticity of demand (5.91), (3) price-elasticity of supply (0.78), (4) tax-elasticity of supply-price (0.26). If these results are applied to the Canadian rental sector, they would suggest the conclusion that only a minimal tax-shifting may take place in this sector. These findings, however, are inconclusive because the data used herein have many serious shortcomings. The critical ones include the nine-year gap in the Canadian data, the use of U.S. data to calculate the parameters of the Canadian housing market, and the possibility that the data used here are not of the long-run type.

# APPENDIX TO CHAPTER V

## STRUCTURAL EQUATIONS FOR HOUSING-SERVICES AND THEIR IDENTIFICATION

### Structural and Reduced-Form Equations

The structural equations presented below are modified versions of the structural equations presented in Section B and the Appendix of Chapter IV. The elasticities appear in small letters to indicate that the quantity variables, $q_1$ and $q_o$, are on a per household basis. It is assumed that the structural equations are linear in logarithmic form. The system of structural equations presented here is the basis of the two-market regression model discussed in the text.

### Rental Sector

$$q_1 = \bar{q}_1 - q_{11}E_1 + q_{1o}E_o + q_{1y}Y + q_{1s}S \qquad \text{A-1}$$

$$E_1 = \bar{E}_1 + e_{11}q_1 + e_{1o}q_o + e_{1c}C + e_{1r}r \qquad \text{A-2}$$

### Owner Sector

$$q_o = \bar{q}_o + q_{o1}E_1 - q_{oo}E_o + q_{oy}Y + q_{os}S \qquad \text{A-3}$$

$$E_o = \bar{E}_o + e_{o1}q_1 + e_{oo}q_o + e_{oc}C + e_{or}r \qquad \text{A-4}$$

where $\bar{q}_1$, $\bar{E}_1$, $\bar{q}_o$, and $\bar{E}_o$ are constants. The variables (in logarithms) are:

$q_o$, $q_1$ = quantities of housing-services per household per period,

$E_o$, $E_1$ = unit prices of housing-services, Y = average family income,

S = number of persons per room,

C = implicit index of prices of factor inputs, and

r = effective residential property tax rate.

The endogenous variables are $E_o$, $E_1$, $q_o$, $q_1$ and the remaining variables are considered to be pre-determined. The coefficients appearing before the variables in A-1 to A-4 are elasticities.

The simultaneous solution for the endogenous variables may be summarized as

$$\underset{X}{\begin{bmatrix} q_1 \\ E_1 \\ q_o \\ E_o \end{bmatrix}} = \underset{A^{-1}}{\begin{bmatrix} 1 & q_{11} & 0 & -q_{1o} \\ -e_{11} & 1 & -e_{1o} & 0 \\ 0 & -q_{o1} & 1 & q_{oo} \\ -e_{o1} & 0 & -e_{oo} & 1 \end{bmatrix}^{-1}} \underset{B}{\begin{bmatrix} \bar{q}_1 & q_{1y} & q_{1s} & 0 & 0 \\ \bar{E}_1 & 0 & 0 & e_{1c} & e_{1r} \\ \bar{q}_o & q_{oy} & q_{os} & 0 & 0 \\ \bar{E}_o & 0 & 0 & e_{oc} & e_{or} \end{bmatrix}} \underset{Z}{\begin{bmatrix} 1 \\ Y \\ S \\ C \\ r \end{bmatrix}} \qquad \text{A-5}$$

Evaluating A-5 we find

$$\underset{X}{\begin{bmatrix} q_1 \\ E_1 \\ q_o \\ E_o \end{bmatrix}} = \underset{T}{\begin{bmatrix} \bar{T}_{q_1} & T_{q_1y} & T_{q_1s} & T_{q_1c} & T_{q_1r} \\ \bar{T}_{e_1} & T_{e_1y} & T_{e_1s} & T_{e_1c} & T_{e_1r} \\ \bar{T}_{q_o} & T_{q_oy} & T_{q_os} & T_{q_oc} & T_{q_or} \\ \bar{T}_{e_o} & T_{e_oy} & T_{e_os} & T_{e_oc} & T_{e_or} \end{bmatrix}} \underset{Z}{\begin{bmatrix} 1 \\ Y \\ S \\ C \\ r \end{bmatrix}} \qquad \text{A-6}$$

where $A^{-1} B = T$ and $\bar{T}_{q_1}$, $\bar{T}_{e_1}$, $\bar{T}_{q_o}$, $\bar{T}_{e_o}$ are the constants of the reduced form equations the composition of which will not be shown here. Moreover,

$$/A/ = (1+e_{11}q_{11}+e_{oo}q_{oo}+(q_{11}q_{oo}-q_{1o}q_{o1})(e_{11}e_{oo}-e_{1o}e_{o1}) - (e_{1o}q_{o1}+q_{1o}e_{o1}) > 0 \qquad \text{A-7}$$

$$T_{q_o y} = \frac{1}{/A/}\left\{q_{1y}(q_{o1}e_{11}-q_{oo}e_{o1})+q_{oy}(1+q_{11}e_{11}-q_{1o}e_{o1})\right\} > 0 \qquad \text{A-8}$$

$$T_{q_o s} = \frac{1}{/A/}\left\{q_{1s}(q_{o1}e_{11}-q_{oo}e_{o1})+q_{os}(1+q_{11}e_{11}-q_{1o}e_{o1})\right\} > 0 \qquad \text{A-9}$$

$$T_{q_o c} = \frac{1}{/A/}\left\{e_{1c}\left[q_{o1}+e_{o1}(q_{11}q_{oo}-q_{o1}q_{1o})\right] - e_{oc}\left[q_{oo}+e_{11}(q_{11}q_{oo}-q_{o1}q_{1o})\right]\right\} < 0 \qquad \text{A-10}$$

$$T_{q_o r} = \frac{1}{/A/}\left\{e_{1r}\left[q_{o1}+e_{o1}(q_{11}q_{oo}-q_{o1}q_{1o})\right] - e_{or}\left[q_{oo}+e_{11}(q_{11}q_{oo}-q_{o1}q_{1o})\right]\right\} < 0 \qquad \text{A-11}$$

$$T_{q_1 y} = \frac{1}{/A/}\left\{q_{1y}(1+q_{oo}e_{oo}-e_{1o}q_{o1})+q_{oy}(q_{1o}e_{oo}-q_{11}e_{1o})\right\} > 0 \qquad \text{A-12}$$

$$T_{q_1 s} = \frac{1}{/A/}\left\{q_{1s}(1+q_{oo}e_{oo}-e_{1o}q_{o1})+q_{os}(q_{1o}e_{oo}-q_{11}e_{1o})\right\} > 0 \qquad \text{A-13}$$

$$T_{q_1 c} = \frac{1}{/A/}\left\{e_{oc}\left[q_{1o}+e_{1o}(q_{11}q_{oo}-q_{o1}q_{1o})\right] - e_{1c}\left[q_{11}+e_{oo}(q_{11}q_{oo}-q_{o1}q_{1o})\right]\right\} < 0 \qquad \text{A-14}$$

$$T_{q_1 r} = \frac{1}{/A/}\left\{e_{or}\left[q_{1o}+e_{1o}(q_{11}q_{oo}-q_{o1}q_{1o})\right] - e_{1r}\left[q_{11}+e_{oo}(q_{11}q_{oo}-q_{o1}q_{1o})\right]\right\} < 0 \qquad \text{A-15}$$

$$T_{e_1y} = \frac{1}{|A|}\left\{q_{1y}[\bar{e}_{11}+q_{oo}(e_{oo}e_{11}-e_{o1}e_{1o})] + q_{oy}[\bar{e}_{1o}+q_{1o}(e_{oo}e_{11}-e_{o1}e_{1o})]\right\} > 0 \qquad \text{A-16}$$

$$T_{e_1s} = \frac{1}{|A|}\left\{q_{1s}[\bar{e}_{11}+q_{oo}(e_{oo}e_{11}-e_{o1}e_{1o})] + q_{os}[\bar{e}_{1o}+q_{1o}(e_{oo}e_{11}-e_{o1}e_{1o})]\right\} > 0 \qquad \text{A-17}$$

$$T_{e_1c} = \frac{1}{|A|}\left\{e_{1c}[1+e_{oo}q_{oo}-q_{1o}e_{o1}] + e_{oc}(q_{1o}e_{11}-e_{1o}q_{oo})\right\} > 0 \qquad \text{A-18}$$

$$T_{e_1r} = \frac{1}{|A|}\left\{e_{1r}(1 + e_{oo}q_{oo} - q_{1o}e_{o1}) + e_{or}(q_{1o}e_{11} - e_{1o}q_{oo})\right\} > 0 \qquad \text{A-19}$$

$$T_{e_oy} = \frac{1}{|A|}\left\{q_{1y}[\bar{e}_{o1}+q_{o1}(e_{oo}e_{11}-e_{o1}e_{1o})] + q_{oy}[\bar{e}_{oo}+q_{11}(e_{oo}e_{11}-e_{o1}e_{1o})]\right\} > 0 \qquad \text{A-20}$$

$$T_{e_os} = \frac{1}{|A|}\left\{q_{1s}[\bar{e}_{o1}+q_{o1}(e_{oo}e_{11}-e_{o1}e_{1o})] + q_{os}[\bar{e}_{oo}+q_{11}(e_{oo}e_{11}-e_{o1}e_{1o})]\right\} > 0 \qquad \text{A-21}$$

$$T_{e_oc} = \frac{1}{|A|}\left\{e_{1c}(e_{oo}q_{o1}-e_{o1}q_{11}) + e_{oc}(1 + q_{11}e_{11}-e_{1o}q_{o1})\right\} > 0 \qquad \text{A-22}$$

$$T_{e_or} = \frac{1}{|A|}\left\{e_{1r}(e_{oo}q_{o1}-e_{o1}q_{11}) + e_{or}(1 + q_{11}e_{11}-e_{1o}q_{o1})\right\} > 0 \qquad \text{A-23}$$

The signs of the reduced form parameters are specified under the assumptions that

(a) $e_{1o} = e_{o1} = 0$ (cross-quantity effects on supply-price),

(b) $q_{oo} > q_{o1}$, $q_{11} > q_{1o}$ (i.e., the price-elasticities of demand are greater than the cross-elasticities of demand), and

(c) $e_{1r} \cong e_{or}$ and $e_{1c} \cong e_{oc}$ (i.e., the supply-price elasticities with respect to the tax rates and input costs are similar in the two sectors).

Identification

Let us present the necessary notation for the concise statement of the rank and order conditions for the identification of our structural equations.

Since $T = A^{-1} B$, then

$$AT = B \tag{A-24}$$

Supposing that we are interested in the identification of the jth structural equation, then A-24 may be re-arranged such that the jth rows of the matrices A and B are partitioned as follows:

$$A = (A_* : A_{**}) \tag{A-25}$$

$$B = (B_* : B_{**}) \tag{A-26}$$

where $A_*$ = one by M* vector of non-zero coefficients, and M*=number of endogenous variables included in the jth structural equation;

$A_{**}$ = one by M** vector of zero coefficients, and

$M^{**}$ =number of endogenous variables <u>excluded</u> from the jth equation;

$B_{*}$= one by $K^{*}$ vector of non-zero coefficients, and $K^{*}$=number of pre-determined variables <u>included</u> in the jth equation;

$B_{**}$= one by $K^{**}$ vector of zero coefficients, and $K^{**}$=number of pre-determined variables <u>excluded</u> from the jth equation.

The reduced-form coefficient matrix, T, may also be accordingly re-arranged and partitioned so that

$$T = \begin{bmatrix} T_{*,*} & \vdots & T_{*,**} \\ \cdots & \vdots & \cdots \\ T_{**,*} & \vdots & T_{**,**} \end{bmatrix} \quad \text{A-27}$$

where the first subscript refers to the endogenous variables and the second subscript refers to the pre-determined variables and where

$T_{*,*}$ = $M^{*}$ by $K^{*}$ matrix,

$T_{*,**}$ = $M^{*}$ by $K^{**}$ matrix,

$T_{**,*}$ = $M^{**}$ by $K^{*}$ matrix, and

$T_{**,**}$ = $M^{**}$ by $K^{**}$ matrix.

Pre-multiplying A-27 by A-25, we find that

$$A_{*} T_{*,*} = B_{*} \quad \text{A-28}$$

$$A_{*} T_{*,**} = 0, \quad \text{A-29}$$

since $A_{**} = 0$ and $B_{**} = 0$.

We may now state the rank and order conditions for the identification of the jth structural equation, namely,

$$\text{(a)} \quad \text{rank}(T_{*,**}) = M^{*}-1 \qquad \text{A-30}$$

$$\text{(b)} \quad K^{**} \geqslant M^{*}-1 \qquad \text{A-31}$$

If A-30 holds, the jth structural equation is said to be identified; if, in addition, inequality holds in A-31, the jth structural equation is over-identified, while the additional stipulation of equality in A-31 implies that the jth structural equation is just-identified. If the rank condition holds and equality holds in the order condition (the just-identified case), there is a unique solution for $A_*$ in A-29 up to a factor of proportionality which upon normalization of the jth equation yields a unique solution for the individual coefficients in $A_*$. Given that there is a unique solution for $A_*$ in A-29, this unique solution may be substituted in A-28 to give a unique solution for $B_*$.

To demonstrate, let us re-state here A-24 showing its individual components, i.e.,

$$\overset{A}{\begin{bmatrix} 1 & q_{11} & 0 & -q_{1o} \\ -e_{11} & 1 & -e_{1o} & 0 \\ 0 & -q_{o1} & 1 & q_{oo} \\ -e_{o1} & 0 & -e_{oo} & 1 \end{bmatrix}} \overset{T}{\begin{bmatrix} \bar{T}_{q_1} & T_{q_1 y} & T_{q_1 s} & T_{q_1 c} & T_{q_1 r} \\ \bar{T}_{e_1} & T_{e_1 y} & T_{e_1 s} & T_{e_1 c} & T_{e_1 r} \\ \bar{T}_{q_o} & T_{q_o y} & T_{q_o s} & T_{q_o c} & T_{q_o r} \\ \bar{T}_{e_o} & T_{e_o y} & T_{e_o s} & T_{e_o c} & T_{e_o r} \end{bmatrix}} = B \qquad \text{A-32}$$

where

$$B = \begin{bmatrix} \bar{q}_1 & q_{1y} & q_{1s} & 0 & 0 \\ \bar{e}_1 & 0 & 0 & e_{1c} & e_{1r} \\ \bar{q}_o & q_{oy} & q_{os} & 0 & 0 \\ \bar{e}_o & 0 & 0 & e_{oc} & e_{or} \end{bmatrix}$$

Please note that the components of T in A-32 are not re-arranged according to the re-arrangement of A-27. Pulling out the relevant sub-matrices of T in A-32, we find that the solutions for the coefficients of the first structural equation, A-1, are

$$\underset{A_*}{\underbrace{(1 \quad q_{11} \quad -q_{1o})}} \underset{T_{*,*}}{\underbrace{\begin{bmatrix} \bar{T}_{q_1} & T_{q_1y} & T_{q_1s} \\ \bar{T}_{e_1} & T_{e_1y} & T_{e_1s} \\ \bar{T}_{e_o} & T_{e_oy} & T_{e_os} \end{bmatrix}}} = \underset{B_*}{\underbrace{(\bar{q}_1 \quad q_{1y} \quad q_{1s})}} \qquad \text{A-33}$$

$$\underset{A_*}{\underbrace{(1 \quad q_{11} \quad -q_{1o})}} \underset{T_{*,**}}{\underbrace{\begin{bmatrix} T_{q_1c} & T_{q_1r} \\ T_{e_1c} & T_{e_1r} \\ T_{e_oc} & T_{e_or} \end{bmatrix}}} = \underset{B_{**}}{\underbrace{(0 \quad 0)}} \qquad \text{A-34}$$

It can be seen that the rank of $T_{*,**}$ in A-34 is equal to two. Therefore, in A-34, there is a unique solution for $q_{11}$ and $q_{1o}$ and, subsequently, for $B_*$

in A-33. Further, in A-1, $M^{*}-1 = 3-1$ and $K^{**}= 2$. Therefore, A-1 is just-identified. It can also be verified that the other three structural equations A-2 to A-4 are all just-identified provided that $e_{1o} > 0$ and $e_{o1} > 0$. If this is so, the Indirect Least Squares method could be appropriately applied in the estimation of the structural parameters.

However, it has been argued earlier in the text that $e_{1o}$ and $e_{o1}$ may be very close to zero so that for all practical purposes we have assumed that $e_{1o} = e_{o1} = 0$. If this is the case, A-2 and A-4 would be over-identified since $(K^{**}= 2) > (M^{*}-1 = 2-1)$ for both the structural price equations. The Two-Stage Least Squares is an appropriate method of estimating the structural parameters in this case.

Note that the rank condition holds in A-2 and A-4 even if $e_{1o} = e_{o1} = 0$. To demonstrate the rank of $T_{*,**}$ for A-2, let us consider the second row of A-32. Here, if $e_{1o}= 0$,

$$\underset{\uparrow A_*}{\left( -e_{11} \quad 1 \right)} \quad \underset{\uparrow T_{*,*}}{\begin{bmatrix} \bar{T}_{q_1} & T_{q_1c} & T_{q_1r} \\ \bar{T}_{e_1} & T_{e_1c} & T_{e_1r} \end{bmatrix}} = \underset{\uparrow B_*}{\left( \bar{e}_1 \quad e_{1c} \quad e_{1r} \right)} \qquad \text{A-35}$$

$$\underset{\uparrow A_*}{\left( -e_{11} \quad 1 \right)} \quad \underset{\uparrow T_{*,**}}{\begin{bmatrix} T_{q_1y} & T_{q_1s} \\ T_{e_1y} & T_{e_1s} \end{bmatrix}} = \underset{\uparrow B_{**}}{\left( 0 \quad 0 \right)} \qquad \text{A-36}$$

It can be verified from A-12, A-13, A-16, and A-17 that if $e_{1o} = 0$, then

$$( T_{q_1 y} \quad T_{q_1 s} )\, e_{11} = ( T_{e_1 y} \quad T_{e_1 s} )$$

i.e., the second row of $T_{*,**}$ in A-36 is a multiple of the first row. Therefore, the rank of $T_{*,**}$ in A-36 is (M*-1)=(2-1). Moreover, the over-identification of $e_{11}$ is obvious in A-36. There are two equations for one unknown ($e_{11}$), and restrictions on the reduced form parameters are implied. It can also be verified that in A-4, if $e_{o1} = 0$, the rank of the corresponding

$$T_{*,**} = \begin{bmatrix} T_{q_o y} & T_{q_o s} \\ T_{e_o y} & T_{e_o s} \end{bmatrix}$$

is (M*-1)=(2-1) because $( T_{q_o y} \quad T_{q_o s} ) e_{oo} = ( T_{e_o y} \quad T_{e_o s} )$.

<u>Combined Reduced-Form</u>

If $q_i$ and $E_i$ (i=0,1) are not separately observable, the empirical application of the model is limited to its combined-reduced-form equations. In A-6, we find the combined-reduced-form equations to be (in logarithms):

$$\begin{bmatrix} (\bar{T}_{q_o}+\bar{T}_{e_o}) & (T_{q_o y}+T_{e_o y}) & (T_{q_o s}+T_{e_o s}) & (T_{q_o c}+T_{e_o c}) & (T_{q_o r}+T_{e_o r}) \\ (\bar{T}_{q_1}+\bar{T}_{e_1}) & (T_{q_1 y}+T_{e_1 y}) & (T_{q_1 s}+T_{e_1 s}) & (T_{q_1 c}+T_{e_1 c}) & (T_{q_1 r}+T_{q_1 r}) \end{bmatrix} \begin{bmatrix} 1 \\ Y \\ S \\ C \\ r \end{bmatrix}$$

$$= \begin{bmatrix} (E_o + q_o) \\ (E_1 + q_1) \end{bmatrix} . \qquad \text{A-37}$$

If only estimators of A-37 are available, it is obvious that all of the structural equations A-1 to A-4 would be under-identified. However, if the price reduced-form equations can be estimated separately from available data, they could be subtracted from the estimators of A-37 so that

$$\begin{bmatrix} \widehat{(E_o + q_o)} \\ \widehat{(E_1 + q_1)} \end{bmatrix} - \begin{bmatrix} \hat{E}_o \\ \hat{E}_1 \end{bmatrix} = \begin{bmatrix} \hat{q}_o \\ \hat{q}_1 \end{bmatrix} . \qquad \text{A-38}$$

A-38 together with $\hat{E}_o$ and $\hat{E}_1$ will yield indirect estimates of all the structural parameters. Again, if $e_{1o} = e_{o1} = 0$, the structural price equations A-2 and A-4 would be over-identified.

# CHAPTER VI

## EMPIRICAL EVIDENCE : IMPACTS OF PROPERTY TAXATION ON THE DEMAND FOR DWELLING-STRUCTURES AND ON RESIDENTIAL CONSTRUCTION

In this chapter, we shall extend our empirical investigations to the area of derived market demand for the physical inputs required for the production of housing-services. The derived demands for the physical land-site (L) and the physical dwelling-structures (H) are of particular interest to us since the residential property tax generally applies only to them. Although such other inputs as utilities, household appliances and furnitures are not subject to residential property taxation, they will of course be affected indirectly by the tax. Due to data limitations, however, we shall confine our empirical investigation to the derived demand for physical dwelling-structures, H.

Across the set of sample observations, the measured stock of residential dwellings in the package sense will probably mis-state the actual volume of dwelling-structures. In recognition of this possible mis-statement, we shall measure the stock of dwelling-structures, H, by the data on the number of rooms. This procedure will empirically distinguish the dwelling-structures, H, from the housing-packages or dwellings. The measurement of H' in terms of rooms is also one way to achieve a certain degree of standardization in the available housing stock data. The standardization will also help minimize the aggregation problems arising primarily from the heterogeneity of the residential stock data. Beyond this level of standardization, no further attempt to remedy the aggregation problems will be carried out.

In the process of investigation, it is necessary that we make an appropriate price specification for the physical dwelling-structures. In particular, should we use a price measure <u>with</u> or <u>without</u> capitalization of future property tax payments? Clearly, the residential property tax capitalization hypothesis is not only an interesting subject in itself, but its empirical content is also instrumental to the construction of a better way of specifying the price variable to be used in the estimation of the derived demand for physical dwelling-structures. This chapter, therefore, will include a section which deals with the empirical testing of the residential property tax capitalization hypothesis.

We have argued earlier that property tax capitalization plays an important role in the explanation of the tax-impact on the volume of residential construction. If there is property tax capitalization, a higher tax rate will be associated with a lower demand-price at which newly constructed dwellings can be cleared in the market. The lower the market clearing price, the smaller is the volume of residential construction. This chapter will include a section investigating the quantitative effects of the tax on the volume of new residential construction.

Due to data limitations, we shall mainly use a one-market approach. In particular, no specific importance will be assigned to any unavoidable distinctions between rental and owner dwelling-structures that emerge from the discussion. Subsection C.3. of this chapter, however, is an exception to this approach. Here, we shall attempt to measure the impact of the property tax on the desire for pure-homeownership (owner-occupancy) relative to renting, as well as the degree of substitution between rental and owner dwellings (package).

## A. Limitations and Availability of Price Data

In the estimation of the derived market demand for physical dwelling-structures, H, the ideal price data to use are those which pertain only to the market value of the H component of the dwellings. However, data on the market value of H are not available. The publications on housing statistics by the Central Mortgage and Housing Corporation (CMHC)[1/] contain data on land prices for at most 26 metropolitan and major urban areas in the early 1960's. These data on land prices could be subtracted from the median market values of owner-occupied dwellings and from an estimated set of average prices for rental dwellings to obtain a rough measure of the unit price of the H component of the residential package.

The CMHC land price data are not used for the above purpose because they are not representative of the average prices of already occupied land sites in the city or municipality limits. The reasons are as follow: (a) the land price data pertain only to the average prices of land sites of newly constructed bungalows; (b) new bungalows are usually built at the fringe areas of cities, particularly cities which have already reached a certain degree of population density or overcrowdedness; (c) the land price data are on a metropolitan area basis, whereas the unit of observation in this study is the city or municipality. The land price data, however, are particularly useful as supplementary means to test empirically the tax capitalization hypothesis.

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[1/] CMHC, Canadian Housing Statistics, (Annual) 1960-1965.

As an alternative (proxy) variable for the price of the dwelling-structure, H, I have used the price pertaining to the housing-package. This price is represented by the data on the median market value of owner-occupied dwellings. Since the median value data cover only the owner-occupied category, it seems desirable to obtain another set of prices representing the market values of tenant-occupied dwellings. The market values of tenant-occupied dwellings can only be estimated from available information. The arithmetic average of the median (market) value data (owner-occupied category) and the set of estimated market values (tenant-occupied category) will be used as the unit market price representing that of all occupied dwellings.

## B. Empirical Test of the Property Tax Capitalization Hypothesis

We have presented a theoretical discussion of property tax capitalization in Section E of Chapter IV. It was argued that the current market value of a residential dwelling is based on the present value of the future stream of net rental incomes of the property after deducting the periodic property taxes payable, i.e.,

$$P(0) = \int_0^n N(t)e^{-mt}dt = \int_0^n [R(t) - rP(t)]e^{-mt}dt \qquad (1)$$

where $P(0)$ = current market value, $R(t)$ = periodic net rental income, $r\ P(t)$ = periodic property tax payable, and m= nominal rate of interest which could be earned in comparable alternatives by the funds used for acquisition of the dwelling. In this section, it will be assumed that the mortgage rate of interest is an adequate measure of the opportunity cost of mortgage funds.

For empirical purposes, a working hypothesis about the precise specification of $R(t)$ is necessary. As an approximation, let $R(t) = R_o e^{xt}$, where $R_o = R(t=0)$, x = continuous rate of change in the property's net rental income per period, and where $-b \leq x < m$ and b is a finite number representing the rate of depreciation. The rate $x \lesseqgtr 0$ depending upon the relative strength of two offsetting effects, namely: (a) the rental-appreciating effect of inflationary demand pressures in the housing and land markets;[2] and (b) the rental-depreciating effect of the obsolescence and physical deterioration in the dwelling-

[2] In Toronto, for instance, H. Kitchen estimated a \$256 increase (from \$11,903 to \$12,159) in the average market value of occupied dwellings in 1961-1963 period. Kitchen, "Imputed Rent on Owner-Occupied Dwellings", Canadian Tax Journal, September-October 1967, pp.482-491. For the same area, the Toronto Real Estate Board reported a more than \$2,000 average annual increase in house prices during the period 1966-1969. H.G. Wolff, "The White Paper: Tax Treatment of Principal Residences", Canadian Tax Journal July-August 1970, pp. 263-276

structures over time. Under this assumption, the market value, $P(0)$, in equation (1) can be specified as

$$P(0) = \frac{R(0)}{r + \frac{(m-x)}{[1-e^{-(m-x)n}]}} \qquad (2)$$

provided that one also assumes that $P(t)=P(0)e^{xt}$. [3/]

Under the *a priori* restriction that $m > x$, equation (2) can be further simplified into [4/]

$$P_o = P(0) \cong \frac{R(0)}{m-x+r} \qquad \text{for } n \rightarrow \infty. \qquad (3)$$

The restriction $m > x$ means that even if $R(t)$ appreciates at a rate $\underline{x}$ due to inflationary demand pressures, this rate will in normal circumstances be less than the market rate of interest, $\underline{m}$. This restriction is justifiable because the market rate of interest usually includes an inflationary premium. The arbitrary use of $n \rightarrow \infty$ is also reasonable since

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[3/] The steps in the derivation of equation (2) are as follows:

$$P(0) = \int_0^n [\bar{R}(t) - rP(t)] \, e^{-mt} dt$$

$$= \int_0^n [\bar{R}(0) - rP(0)] \, e^{-(m-x)t} dt$$

$$= [\bar{R}(0) - rP(0)] \frac{1}{m-x} [1 - e^{-(m-x)n}]$$

$$= \frac{R(0)}{r + \frac{m-x}{[1 - e^{-(m-x)n}]}} \cong \frac{R(0)}{m-x+r} \quad \text{as } n \rightarrow \infty \text{ with } m > x.$$

[4/] In our interpretation, full property tax capitalization simply means the inclusion of $\underline{r}$ in the discount or conversion factor.

the average life of dwelling-structures usually exceeds a 50-year period. Even if the remaining life of an old dwelling-structure is only, say, 20 years, the assumption of $n \rightarrow \infty$ may still be applied because the land component of the dwelling unit may be expected to earn rental income after the end of the life of the dwelling-structure.

The preceding discussion underlines a simple procedure for testing the residential property tax capitalization hypothesis empirically. A regression analysis may be employed relating data on the market value of residential dwellings and data on the effective property tax rate. If future property tax payments are in fact being capitalized, these two sets of data should demonstrate a significant inverse relationship from the statistical point of view. This inverse association, however, will not in itself conclusively establish the extent of the property tax capitalization. The empirical findings reported below support the tax capitalization hypothesis. A further interpretation of the results suggests that these findings are evidence of the full capitalization of property taxes.

B.1. Data and Methods Used

The regression analysis uses the same 59 observations employed in the empirical investigation of the market for housing-services in Chapter V. The dependent variable in the regression is the median value of owner-occupied dwellings. It appears in two forms: (1) $P_o$ = median value per dwelling and (2) $P_o^*$ = median value per room. The use of $P_o^*$ as an alternative dependent variable is intended to eliminate the variations in the median value data emanating from apparent differences in the average number of rooms per dwelling across the sampled observations. This procedure, in effect, removes the negative impact of the tax rate, r, if there is any, on median value data which is not due to tax capitalization, but mainly to the tendency of the tax to deter ownership of bigger dwelling accommodations. In the 59 observations, the range of variation in the average number of rooms per dwelling was quite high -- from 4.4 rooms in Rouyn to 6.9 rooms in Sillery.

The main focus of investigation is the effective property tax rate, r, as an explanatory variable. However, two other explanatory variables are included in the regression. They are: (1) Y = average family income, and (2) S = number of persons per room. Y and S are important determinants of the market value of residential properties. Y influences not only the volume of demand for dwelling accommodations (say, number of rooms), but also the quality (say, modernity and durability) of accommodations families or households can afford. The quantity and quality aspects of dwelling accommodations are presumably built into the median value data. S serves as an index of occupancy density or of the relative shortage (or abundance) of stock of space accommodations in the community. It is also expected that the value effects of S are recorded in the median value data. Since Y and S have demonstrated large variations across the 59 observations used here, they are included as additional explanatory variables, even though they are not the main focus of investigation. In the 59 observations, Y ranges from $4,138 (St. John's) to $8,371 (Pointe Claire) per annum and S from 0.58 (Stratford) to 1.04 (Rouyn).

There are other important sources of variation in median value across municipalities. These sources include such factors as differences in the locational value of owner-occupied dwellings, differences in amenities and public services provided by local governments, dissimilarities in the state of expectations, and differences in vacancy rates. No attempt is made to incorporate them here due to data limitations. Their absence in the actual regressions may inject bias into the empirical findings.

It is also possible that the median-value and the property tax rate are simultaneously determined. For instance, the same amount of local revenue requirements may call for a lower effective tax rate, particularly during periods of increasing tax base (market value), and vice versa. W. Oates investigated this possibility with the use of U.S. data by municipalities.[5/] He found no statistically significant indication of simultaneous interrelationships between the effective residential property tax rate and the median-value of owner-occupied dwellings. In Chapter V, we have expressed the opinion that a similar result may well be the case for the Canadian data, noting further that there is a nine-year gap between the 1970 tax rate data and the 1961 market value data used here. We find it unnecessary to use a Two-Stage Least Squares regression analysis for the Canadian data.

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[5/] W.E. Oates, "Effects of Property Taxes and Local Public Spending on Property Values: An Empirical Study of Tax Capitalization and the Tiebout Hypothesis", Journal of Political Economy, November-December 1969, pp.957-971; See also, Oates & J.D. Heinberg, "The Incidence of Differential Property Taxes on Urban Housing: A Comment and Some Further Evidence", National Tax Journal, March 1970, pp. 92-97.

## B.2. Presentation and Interpretation of Results

The empirical findings are summarized as regressions I to VI in Table 7. The relevant regressions for the direct empirical test of the residential property tax capitalization hypothesis are regressions I and II.[6] The rest of the regressions in the table are presented to support the interpretation of the results of the first two regressions. Regressions III to VI will be explained later.

Regressions I and II shows that the tax coefficients are negative, significantly different from zero from the statistical point of view, and are quite close to a common numerical value despite differences in the dependent variables used. These results support the inverse relationship already discussed and certainly suggest the presence of capitalization

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[6] It should be pointed out that regression nos. (I) and (II) in Table 7 are the same regressions appearing as nos. III and VI in Table 5 of Chapter V, except for the addition of one to the numerical value of the intercept. It will be recalled that $R_o$=[ 10% of median market value ] is interpreted in Chapter V as a representation of the value of the quantity flow of housing-services of owner-occupied dwellings per annum. Thus, if $P_o$ is the median market value of owner-occupied dwellings,

$$\log_{10} R_o = \log_{10}(P_o/10) = \log_{10} P_o - 1.$$

Hence, 1 is added to the intercept of regressions III and VI in Table 5 to appear as regressions I and II in Table 7.

Table 7

TAX CAPITALIZATION HYPOTHESIS

| | DEPENDENT VARIABLES | INTERCEPTS | INDEPENDENT VARIABLES $\log_{10} r$ | $\log_{10} Y$ | $\log_{10} S$ | $\bar{R}^2$ | F-Ratio |
|---|---|---|---|---|---|---|---|
| I | $\log_{10} P_o$ | 0.870 | -0.304 (2.740) | 1.123 (6.629) | 0.483 (2.757) | 0.69 | 18.04 |
| II | $\log_{10} P_o^*$ | 0.789 | -0.235 (2.142) | 1.024 (6.130) | 0.736 (4.256) | 0.66 | 15.92 |
| III | $\log_{10} \hat{R}_o$ | -1.091 | -0.059 (0.534) | 1.178 (7.456) | 0.457 (2.829) | 0.70 | 19.53 |
| IV | $\log_{10} R'_o$ | -1.052 | -0.004 (0.033) | 1.260 (7.606) | 0.456 (2.691) | 0.71 | 19.80 |
| V | $\log_{10} R_1$ | -0.051 | -0.093 (1.016) | 0.819 (5.891) | -0.071 (0.494) | 0.65 | 15.05 |
| VI | $\log_{10} L$ | -5.250 | -0.994 (2.458) | 2.419 (3.533) | | 0.61 | 7.99 |

Notes: Numbers in parenthesis are absolute values of the test statistics for coefficients different from zero; number of observations for regressions I to V is N=59, and for regression VI, N=26;

$P_o$ = median market value of owner-occupied dwellings;

$P_o^*$ = median market value per room;

$\hat{R}_o$ = $(m-x+r)P_o$, $R'_o = (m+r)P_o$, m= 7.14% mortgage rate of interest, x= -2.56% depreciation rate, and r= property tax rate;

$R_1$ = weighted average gross rental income of tenant-occupied dwellings;

L = average cost of land of newly completed single-detached dwellings;

Y = average family income;

S = number of persons per room.

SOURCES OF DATA: See Appendix A of the study.

of future property tax payments.

The average of the tax coefficients of regressions I and II is -0.27, indicating that a 10% increase in the effective property tax rate, r, will result in a 2.7% reduction in the median value of owner-occupied dwellings. This reduction may or may not represent the effect of full tax capitalization. The following discussion focuses on this issue.

If equation (3) is taken as an adequate alternative representation of the median value data, the interpretation of the tax coefficients of regressions I and II can be extended. Conceptually, these coefficients may be broken down into two basic components:

$$\frac{\partial \log P_o}{\partial \log r} = \frac{\partial \log R_o}{\partial \log r} - \frac{\partial \log (m-x+r)}{\partial \log r} \qquad (4)$$

The first term on the right side of (4) represents the effect of the property tax differentials on the imputed rent, $R_o$, across communities. The second term may be called the <u>discount rate effect</u> of the property tax differentials. Thus, the tax coefficients in regressions I and II in Table 7 represent the sum of these two effects. In a cross-section study, the numerical value of the discount rate effect would only be a measure of the effect of the full capitalization of property tax differentials if, in fact, (m-x) is independent of the property tax differentials.

The rate m, represented by the mortgage rate of interest, is most likely to be invariant to property tax differentials. Loanable funds are very mobile across provincial lines and market rates of interest are generally determined at the national level. Besides, the relevant tax magnitudes here are inter-community property tax differentials. Although these differentials may have caused differences in the volume of residential construction, presumably the variations are not so large as to initiate variations in mortgage rates of interest on funds for building purposes in these communities.

The rate of change, x, of imputed rents will differ across communities. The extreme distances across municipalities and the permanence of residential dwellings have the effect of segmenting local housing markets. As a result, individual communities are faced with differing degrees of inflationary housing demand pressures, not to mention different potentialities to depreciate property values over time. Similar dwellings across municipalities will exhibit varying rates of change in rental incomes or market values. Part of these variations may be due to property tax differentials. But we cannot possibly verify this effect because there are no available time series data on imputed rental incomes or on median values of owner-occupied dwellings for the 59 sample observations used herein. Hence, one must simply assume that m-x is constant across the set of sample observations.

Since $-\partial \log(m-x+r)/\partial \log r = -r/(m-x+r)$, it is clear that the discount rate effect of property tax differentials varies with the size of m-x relative to r. For example, if m = 7.14%, and r = 2%, then the values of $-r/(m-x+r)$ associated with (a) x = 0.00% (b) x = -2.56% and (c) x = 2.38% are, respectively, (a) -0.22, (b) -0.17, and (c) -0.30.

In these examples m = 7.14% represents the simple average of the weighted mortgage rates of interest in Canadian provinces during the period 1960-1962.[7] The provincial weighted average mortgage rates during this period ranged from 7.00% (Saskatchewan) to 7.68% (Prince Edward Island). The small inter-provincial variation in mortgage rates indicates the high mobility of mortgage fund across Canadian provinces. In example (a), it is assumed that x = 0.00%, i.e., the imputed rental income will be constant over time. Alternatively, this means that the market participants expect a full offset between the deflationary effect of obsolescence and physical

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[7] Central Mortgage and Housing Corporation (CMHC), Canadian Housing Statistics, 1962, Table 50, pp. 47-48.

deterioration of owner-occupied dwellings and the inflationary effect of demand pressures on land and dwelling accommodations over future period. In example (b), it is assumed that the imputed rental income is expected to decline over time by a constant rate equal to the -2.56% depreciation rate used by the DBS in the calculation of the national housing stock series.[8/] In example (c), the imputed rental income is expected to appreciate at a rate equal to the 2.38% average annual increase in the land cost of newly completed bungalows over the period 1960-1962.[9/]

A 10% one-tail test reveals that the -0.304 tax coefficient of regression I is not significantly different from any of the elasticities calculated in these examples. However, it cannot be concluded from this that the -0.304 tax coefficient is a measure of only the discount rate effect of the property tax differentials. The -0.304 tax coefficient of regression I is a measure of the sum of the imputed rental income effect and the discount rate effect of the tax changes across municipalities. To determine whether

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[8/] E.H. Oksanen, "Housing Demand in Canada: Some Preliminary Experimentation," Canadian Journal of Economics and Political Science, August 1966, Table 3, n. 3, p. 318.

[9/] CMHC, Canadian Housing Statistics, 1970, Table 85, p. 70.

the -0.304 tax coefficient represents only the discount rate effect, it remains to be shown whether in fact the tax differentials do not have a significant impact on imputed rent. To this end, I present regressions of $\hat{R}_o=(m-x+r)P_o$ with $x=-2.56\%$ and of $R'_o=(m+r)P_o$ with $x=0$ on r, Y, and S. They appear as regressions III and IV in Table 7. In both cases, the imputed rent-tax coefficients are not significantly different from zero. Hypothesizing that the set of market generated weighted rental income data, $R_1$, for tenant-occupied dwellings is a better proxy for the imputed rent of owner-occupied residences, I also run a regression of $R_1$ on r, Y, and S.[10/] The result appears as regression V in Table 7. The rent-tax coefficient in this regression is also not significantly different from zero. It would appear that the -0.304 tax coefficient in regression I is an evidence of only the discount rate effect of the tax differentials. We have suggested earlier that m is most likely invariant to tax differentials across communities. To the extent that x is also unaffected by the tax differentials (on which no information can be provided), the -0.304 elasticity value in regression I is a measure of the full capitalization effect of property tax differentials.

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[10/] This is regression II in Table 5 above.

The last regression shown in Table 7 uses the land cost, L, of newly completed bungalows as the dependent variable. In this regression, the land cost data are by metropolitan areas and the rest of the data are by municipalities. Despite this shortcoming, the result of this regression proves useful for present purposes. Regression VI also shows that the land cost L is negatively associated with the property tax rates. The tax coefficient is significantly different from zero and very close to -1. This result suggests that a 10% increase in the tax rate will result in a 10% decline in land value, other value determining factors remaining the same.

At least two reasons explaining the high tax elasticity of land value may be cited. The first is the two-tier assessment practice in effect in most Canadian municipalities. Land is being assessed and taxed at full market value, whereas buildings are not.[11/] The second relates to the absolute permanence of land sites and the relatively inelastic nature of the institutional constraints to the uses made of urban land. Usually, urban land can be transferred from one type of activity to another only if it is re-zoned

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[11/] See the DBS commentaries in Principal Taxes and Rates: Federal, Provincial and Local Governments, 1960-1970, Cat. No. 68-201.

by urban authorities. In contrast, man-made capital can move more or less freely from one type of use to another in response to property tax differentials. Thus, it can be argued that land values bear the brunt of the penalizing effect of property taxation not only in terms of tax capitalization, but also in terms of the tax effect on land's rental income.

In this section we tentatively conclude that the tax capitalization revealed in the data is an indication of full capitalization of property tax differentials. For lack of data, however, the other important variables that affect market values have not been included in the regression analysis. These variables include the tax-expenditure effects of property taxes, vacancy rates, and differences in the state of expectations across municipalities that may well lead to dissimilarities in the components of discount rates other than property tax rate differentials. Their absence in the actual regressions may inject serious bias into the empirical results and this should, therefore, be considered to be an important qualification.

B.3. Implications for Empirical Results in Housing-Services

In Chapter V, we used a one-market approach to housing-services and estimated a combined-reduced-form equation of the following specification(in logarithms) from Canadian data:

$$R = (\bar{T}_q + \bar{T}_E) + \frac{q_y(1+E_q)}{\Delta} Y + \frac{q_s(1+E_q)}{\Delta} S + \frac{E_r(1-q_E)}{\Delta} r \qquad (5)$$

where $R=\frac{1}{2}(R_1 + R_o)$, $R_1$= weighted average gross rental income of tenant-occupied dwellings, $R_o$ = 10% of median-value ($P_o$), and $\Delta = (1+E_q q_E)$. With the aid of U.S. data we arrived at the following numeral values:

$-\hat{q}_E = -9.25$, estimated price-elasticity of demand;

$1/\hat{E}_q = 0.78$, estimated price-elasticity of supply;

$\hat{q}_y = 5.91$, estimated income-elasticity of demand;

$\hat{E}_r = 0.26$, estimated tax-elasticity of supply-price;

$\hat{q}_s = 1.45$, estimated demographic-elasticity of demand.

We applied these findings to the Canadian rental sector and arrived at the conclusion that a minimal forward tax shifting may take place in this sector.

The use of the flat 10% conversion rate for imputing the rental income of owner-occupied dwelling is questionable if there is capitalization of property taxes. Since our investigation provides evidence of full tax capitalization a more appropriate conversion rate should vary with the property tax rate across sample observations. I have re-estimated equation (5) under a full property tax capitalization assumption and the result is as follows (in logarithms):

$$\hat{R} = 0.76 + \underset{(8.355)}{1.053\,Y} - \underset{(0.762)}{0.068\,r} + \underset{(2.106)}{0.272\,S} \qquad (6)$$

$$\bar{R}^2 = 0.75,\ F = 25.08,\ N = 59$$

(Numbers in parenthesis are absolute values of the test statistics for coefficients different from zero)

where $\hat{R} = \frac{1}{2}(\hat{R}_o + R_1)$, $\hat{R}_o = (m-x+r)P_o$, $m = 7.14\%$, $x = -2.56\%$, and r = variable tax rate across sample observations.

In contrast to the results using the flat 10% conversion rate, the tax coefficient in equation (6) is not significantly different from zero from the statistical point of view. Generally speaking, the signs and numerical values of the coefficients of equation (6) are consistent with any of the following three cases, namely,

Case I : $/q_E/ = 1$ and $/E_r/ > 0$,

Case II : $/q_E/ = 1$ and $/E_r/ = 0$, and

Case III : $/q_E/ > 1$ and $/E_r/ = 0$.

The signs are not consistent with the <u>fourth</u> case $/q_E/ > 1$ and $/E_r/ > 0$. Since the structural elasticities built into the coefficients of equation (6) are all underidentified, there is no way of knowing which of the first three cases is most likely to be present in the regression results. We can say, however, that under Cases I and II, $\hat{q}_y$=1.053 and $\hat{q}_s$=0.272. If we assume that $/q_E/=1$ and resort to the aid of the U.S. regression (equation 37 in Chapter V), we would arrive at 0.80 and 0.05 as estimates of $^1/E_q$ and $E_r$, respectively.

Again, if these findings are specifically applied to the Canadian rental sector, they would suggest minimal forward tax shifting. Indeed, the tax-elasticity of supply-price estimated here is even lower than the previous estimate. It should be noted, however, that our findings here are also subject to a number of qualifications already enumerated at the conclusion of Section C in Chapter V.

## C. Tax Impact on the Derived Demand for Physical Dwelling-Structures, H

In Canada and the United States, homeowners usually own both the dwelling-structure and the land site of the residential package. The derived stock demand for the package or for its components is thus seen as a demand for homeownership.

A distinction was made in Chapter IV between homeownership for owner-occupancy and homeownership for rental operation. The first type of ownership was called pure-homeownership.

All homeowners derive income and utility from own consumption and/or sale of housing-services on the market. Homeowners are seen to maximize utility subject to the constraints attendant to the production, consumption, and sale of housing-services. These constraints include such factors as income, relative prices, and costs of production. Included in the limiting constraints in the aggregate are, of course, the actual stocks of dwelling-structures. These actual stocks currently owned may or may not be equal to the desired or optimal stocks. It is the desired demand aspect that is of empirical interest in this section.

There are a number of highly aggregated empirical studies of housing already done for Canada and the United States on a national basis. Using stock-adjustment models, these studies indicate that, on the average, around $^{1}/3$ of the desired additions to the existing stock of dwelling-structures are being put in place every year via construction activities. These studies have suggested, therefore, that at any point of time the actual stock need not be equal to the desired stock. For instance, the empirical studies for Canada by L.B. Smith and E.H. Oksanen, and for the United States by R.F. Muth and T. Lee have indicated a speed of adjustment, $\sigma$ , which ranges from 0.23 to 0.32 on an annual basis.[12] The range is not very wide despite considerable differences in the data used by these writers,[13] and differences

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[12] L.B. Smith, Housing in Canada, CMHC, January 1971, p. 53; E.H. Oksanen, "Housing Demand in Canada: Some Preliminary Experimentation", p. 315; R.F. Muth, "The Demand for Non-Farm Housing", p. 52; Tong Hun Lee, "The Stock Demand Elasticities of Non-Farm Housing", The Review of Economics and Statistics, February 1964, p. 88.

[13] Smith used for Ig the number of dwelling-units started and for H the number of dwelling-units in stock, both deflated by the number of families. Oksanen used for Ig the current dollar value of gross investment in all housing, deflated by an index of construction cost and by the number of families. It is not clear whether Oksanen's Ig also includes expenditures on alterations, maintenance, and repairs. For H Oksanen used an estimated dollar value of all housing stock deflated by an index of construction cost and by the number of families. It is not clear whether Oksanen's housing stock series included the value of the housing-package or only the value of the dwelling-structure component of the package.

Muth used for Ig the current dollar value of all new non-farm dwelling-structures, including additions and alterations made to existing structures, maintenance and repairs, and capital improvements on raw land sites. His Ig is deflated by the population of the United States and by the Boeckh construction cost index. For H he used the current dollar value of only the dwelling-structures, deflated by the Boeckh index and by population. Lee used the same type of data as used by Muth, except that Lee deflated his data by the number of families instead. The H series used by Muth and Lee was taken from L. Grebler, D. Blank, & L. Winnick, Capital Formation in Residential Real Estate: Trends and Prospects, National Bureau of Economic Research, Princeton University Press, 1956, Table D-1, pp. 360-361.

in the numerical value of the rate of replacement demand assumed in order to arrive at an estimate of $\sigma$.[14]

Stock adjustment models properly belong to time series analysis. The data at our disposal permit only the application of a cross-section analysis by municipalities, since time series data on effective residential property tax rates are not available. In the cross-section estimation of the stock demand equation for residential structures, I have adopted an equilibrium assumption thereby ignoring disequilibrium conditions.

The equilibrium assumption requires the measurement of the unobservable "desired" stock figures in the sampled communities used here. Since the data on the actual stock of dwelling-structures in these communities may not be equal to the "desired" stock figures, it might be necessary to make some adjustments to the actual stock data of the given year in order to approximate the optimum or "desired" quantity figures

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[14] For instance, Oksanen's estimate of $\sigma$ is 0.236 under the assumption of a 0.0256 rate of replacement demand, whereas Lee's estimate of $\sigma$ is 0.285 under the assumption of a 0.035 rate of replacement demand. If we apply the 0.0256 assumption in Lee's result and the 0.035 assumption in Oksanen's result, Lee's and Oksanen's estimates for $\sigma$ will be 0.276 and 0.245, respectively.

under existing market conditions during the year in question. For instance, it may be necessary to add to the existing stock data those dwelling-structures that have been completed, net of replacement demand, in subsequent periods, thereby filling in the gap between the actual and the desired stock figures.

The adjustments cannot be pursued here for lack of data and of empirical studies at the city or municipal level. In making the addition, for example, we would be left wondering how far to proceed with it. The empirical speed of adjustment per annum for Canada of one-third may roughly indicate that the addition required is over the three consecutive years. But this procedure implies the very restrictive and indeed questionable assumption that the desired stock will not change over the three consecutive years. Even if this procedure serves as a first approximation, the speed of adjustment of $^{1}/_{3}$ is a national average and, therefore, need not apply to each of the 59 sampled communities used in this study. There is no available empirical evidence on the speed of adjustment at each community level nor any data to utilize to this end. We, therefore, resort to the additional assumption that the actual stock data are "adequate" proxies for the desired stock figures.

Generally speaking, there are reasons why actual cross-section stock data can generate information that is suitable for the estimation of structural relations under equilibrium assumptions First, cross-section data by municipalities or cities can be considered to be long-run market data. The tremendous differences among municipalities with regard to population size, income, rent, tax cost, land cost, etc., are themselves measures of the long-term differences among these communities. Indeed, these differences persist for years, even for decades. From an econometric point of view, these differences may be treated as equivalent to observations on how markets adjust to long-run changes in such basic determinants as prices, income, tastes and preferences, population, property tax cost, land cost, etc.[15]

Second, the averaging of variables in each city or municipality, which happens in cross-section data, offers an "excellent chance for the random elements to cancel out", leaving a closer measure of the "true" values of variables. Indeed, Ekanem and

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[15] F. de Leeuw & N.F. Ekanem, "The Supply of Rental Housing", The American Economic Review, December 1971, vol. 61 no. 5, pp. 806-817.

de Leeuw, Muth, and Reid[16] have considered the average family income data in a city or metropolitan area as a measure of Friedman's permanent or normal income.[17] Along the same line of argument, it can be presumed too that data on the average number of rooms per person or per household in each city or municipality will serve as a good proxy for the equilibrium or desired average amount of room space demanded in the community.

C.1. Stock Demand Regression Model Specification:

The specification follows a very simple procedure. The consumer unit is either a person, family, or a household (family and non-family). The physical dwelling-structure H is measured on a per room basis. To bring out as much as possible the effects of the tax, price, and income variables, the specification proceeds with normalization of the supply and demand equations in respect to demographic factors. The number of rooms in the community is thus expressed

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[16] F. de Leeuw, "The Demand for Housing: A Review of Cross-Section Evidence", Review of Economics and Statistics, vol. 53, no. 1, February 1971, pp. 1-10; Ekanem & de Leeuw, op.cit.; R.F. Muth, "The Demand for Non-Farm Housing", in A.C. Harberger (ed.), The Demand for Durable Goods, University of Chicago Press, 1960, pp. 29-95; Margaret Reid, Housing and Income, University of Chicago Press, 1962.

[17] M. Friedman, A Theory of Consumption Function. Princeton University Press, 1957.

on a per capita, per family, or per household basis.

In a cross-section study, the existing volume of room space as well as the number and size of households and families are given features and characteristics. The quantity in supply of rooms per capita, per family, or per household is thus also given. If this quantity in supply can be properly classified as a pre-determined variable, the system of equations is not simultaneous and estimates of the tax, price, and income elasticities of the stock demand for physical room space can be obtained via a single equation approach.

The stock demand regression model in this case may be expressed as follows:

$$P^* = P^*(Y, r, H, U) \qquad (7)$$

where $P^*$= market price per room of dwelling-structure;
$Y$ = average family income;
$r$ = effective residential property tax rate;
$H$ = number of rooms per capita, per family, or per household;
$U$ = the disturbance term representing all other possible explanatory and/or random variables not explicitly accounted for in the regression model.

Due to the data limitations such variables as the expenditure effects of property taxes, prices of alternative assets or goods and services, and vacancy rates are not included in the specification. The absence of these variables in the actual regressions reported here may inject bias into the empirical results. In particular, vacant dwellings form part of the stock of residential units available for occupancy in the communities and should therefore be added to the number of occupied dwellings. The data on rooms per capita (per family or household) used here do not include the rooms of vacant units. It cannot be verified whether the presence of a vacancy rate variable in the actual regression would make a significant difference to the empirical results reported here. The cross-section study on U.S. rental housing by F. de Leeuw and N. Ekanem[18/] indicated that vacancy rates in U.S. metropolitan areas have no statistically significant effect on rent levels in these communities. This finding may well be applicable to the Canadian situation.[19/]

C.2. Empirical Evidence: Stock Demand for Room Space

The unit of measure is room space. The Census dwelling unit proves to be a very poor explanatory

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[18/] F. de Leeuw & N. Ekanem, "The Supply of Rental Housing", pp. 806-817.

[19/] Although data on vacancy rates in Canadian metropolitan areas are available from the 1961 Census, no attempt is made here to run a separate Canadian regression by metropolitan areas incorporating the vacancy variable.

variable. The results of regressions using the Census dwelling unit as one of the explanatory variables are not reported in this study.

The observations are the same 59 cities or municipalities already used in the preceding chapter. The results of the regressions are summarized in Table 8. The notation used is defined in the footnotes to the table. The demand-price specification is based on the empirical evidence supporting the full tax capitalization hypothesis.

There are three types of room space variable used, namely: (1) $H_p$ = number of rooms per capita; (2) $H_h$ = number of rooms per household; and (3) $H_f$ = number of rooms per family. Two types of demand-price variable are used as dependent variables, namely, (1) price, P, per Census dwelling unit; and (2) price, P*, per room.

It is advantageous to use the price P as an alternative dependent variable. It allows one to check for possible bias in the estimates of the demand parameters that could arise from errors in measurement in the average number of rooms data. The room data are built into $H_p$, $H_h$, and $H_f$. At the same time, the room data enter as deflators of the price per Census dwelling unit in order to obtain price, P*, per room. It is expected that the regressions using $\log_{10}$ P and

Table 8

STOCK DEMAND REGRESSIONS

| DEPENDENT VARIABLES | | INTER-CEPTS | INDEPENDENT VARIABLES $\log_{10}Y$ | $\log_{10}r$ | $\log_{10}H_p$ | $\log_{10}H_h$ | $\log_{10}H_f$ | $\bar{R}^2$ | F-Ratio |
|---|---|---|---|---|---|---|---|---|---|
| I | $\log_{10}P$ | 0.28 | 1.047 (8.378) | -0.286 (3.244) | -0.291 (2.215) | | | 0.78 | 31.20 |
| II | $\log_{10}P^*$ | 0.36 | 0.842 (5.826) | -0.299 (2.939) | -0.543 (3.669) | | | 0.68 | 16.83 |
| III | $\log_{10}P$ | 0.50 | 1.005 (7.380) | -0.262 (2.891) | | -0.160 (0.759) | | 0.76 | 27.62 |
| IV | $\log_{10}P^*$ | 0.56 | 0.983 (7.152) | -0.269 (2.935) | | -1.131 (5.298) | | 0.74 | 24.32 |
| V | $\log_{10}P$ | 0.57 | 0.979 (7.730) | -0.275 (2.921) | | | -0.112 (0.638) | 0.76 | 27.48 |
| VI | $\log_{10}P^*$ | 1.06 | 0.787 (5.572) | -0.346 (3.296) | | | -0.703 (3.597) | 0.67 | 16.56 |

Notes: Numbers in parenthesis are absolute values of the test statistics for coefficients different from zero; N = (59) number of observations;
$P = \frac{1}{2}\left( P_o + \frac{R_1}{(m-x+r)} \right)$ = estimated average market value for all dwellings;
$P_o$ = median market value of owner-occupied dwellings; $R_1$ = weighted average gross rental income of tenant-occupied dwellings; $(m-x+r)$ = sum of mortgage rate(7.14%), depreciation rate(x= -2.56%), and property tax rate(r);
$P^*$ = market value per room; Y= avergage family income; $H_p$, $H_h$, and $H_f$ are number of rooms per capita, per household, and per family, respectively.

SOURCE OF DATA: See Appendix A of the study.

$\log_{10} P^*$ as alternate dependent variables will show tremendous changes in the coefficients of $\log_{10} Y$ and $\log_{10} r$ from one regression to another if in fact there are considerable errors in measurement. Based on the results, it would seem that the possibility of error-bias is very minimal. The coefficients of $\log_{10}Y$ and $\log_{10}r$ are quite stable in all regressions, except for the regression using $H_f$ as one of the explanatory variables. The coefficients are significantly different from zero from the statistical point of view.

Other possible serious consequences of measurement errors in variables are those arising from the property tax rate data. Since the appropriate demand-price specification must be based on full tax capitalization, of necessity, the property tax rate, r, has to be built into the dependent variable, while the rate also appears as a separate independent variable. Again this arrangement is unsatisfactory from a purely statistical viewpoint. Errors of measurement in r may inject bias into the estimated coefficients. One way to check for this possibility is to compare the results of regressions of variables,

$$P_o, \; P_1 = R_1/(m-x+r), \quad \text{and} \quad P = \tfrac{1}{2}(P_o + P_1)$$

on Y and r. To this end, I present the following regression results (in logarithms, but with the term

"$\log_{10}$" omitted):

$$P_o = 0.87 \underset{(2.740)}{-0.304}\, r + \underset{(6.629)}{1.123}\, Y + \underset{(2.757)}{0.483}\, S \qquad (8)$$

$$N = 59,\ \bar{R}^2 = 0.69,\quad F\text{-Ratio} = 18.04$$

$$P_1 = 0.94 \underset{(3.334)}{-0.298}\, r + \underset{(6.006)}{0.823}\, Y + \underset{(0.459)}{0.065}\, S \qquad (9)$$

$$N = 59,\ \bar{R}^2 = 0.73,\quad F\text{-Ratio} = 20.45$$

(Numbers in parenthesis are test statistics for coefficients different from zero).

In terms of their standard errors, the coefficients of $\log_{10}Y$ and $\log_{10}r$ in these regressions and in regression I (Table 8) are not significantly different from each other. This would indicate that errors in measurement do not inject serious bias into the estimated coefficients.

It can be seen from the regression results summarized in Table 8 that both the $H_h$(rooms per household) and the $H_f$(rooms per family) variables do not show a statistically significant effect on the demand-price except when the price variable is expressed on a per room basis. This can easily be seen by comparing the test statistic of $\log_{10}H_h$ and $\log_{10}H_f$ in regressions III and IV and regressions V and VI. In other words, $H_h$ and $H_f$ are picking up the variation in $P^*$ due to the normalization of the equation.

In contrast, the coefficient of $\log_{10}H_p$ remains statistically significant with or without the deflation of the demand-price, P, by the average number of rooms. On this basis, it may be surmised that the regressions using $H_p$ offer more meaningful results. These regressions (I and II in Table 8) should also provide a more reliable basis for deriving a stock demand equation for physical accommodations.

The demand equation derivable from regression I which uses price per dwelling, P, as the dependent variable is

$$H_p = 4.47 \quad Y^{3.60} \quad r^{-0.98} \quad P^{-3.44} \qquad (10)$$

The per capita demand for room space is elastic with respect to income, Y, and unit price, P. For all practical purposes, the estimated tax-elasticity of demand is unitary. This means that a 10% increase in the property tax will result in a 10% decrease in the quantity demanded of room-space per capita in the municipalities.

The demand equation derivable from regression II (Table 8) which uses price per room, P*, as the dependent variable is

$$H_p = 5.69 \quad Y^{1.55} \quad r^{-0.55} \quad (P^*)^{-1.84} \qquad (11)$$

The estimated elasticities in this case are lower. The reduction is mainly due to the normalization which has increased the coefficient of $\log_{10} H_p$ in regression II. In contrast, the coefficients of $\log_{10} Y$ and $\log_{10} r$ are hardly affected by the normalization. However, the more credible estimate of the community demand function for room-space per capita may well be equation (11).

C.3. Empirical Evidence: Demand for Pure-Homeownership:

In the context of national or regional housing policy, widespread ownership of residential dwellings has always been considered a socially desirable goal. It should be interesting to specifically analyze the space demand of the pure-homeowners.

Farrel and Cramer had advocated the hypothesis that in addition to the utility derived from the consumption of housing-services, an owner occupant derives further satisfaction from the fact of home-ownership (emphasis mine).[20] The urge to possess and to own is emotionally strong and quite universal. Individuals demonstrate this feeling in many aspects of their day to day life. All other considerations remaining

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[20] J.S. Cramer, The Ownership of Major Consumer Durables, Cambridge University Press, 1962; M.J. Farrell "The Demand for Motor Cars in the United States", Journal of Royal Statistical Society, Series A, vol. 117, 1954, pp. 171f cited by Cramer.

the same, a person who can afford to either buy or rent a house of his choice will most likely choose to own the property. Perhaps, we may generally state that were if not for monetary (e.g., income) and institutional constraints (e.g., legal prohibitions to own certain kinds of property) beyond the consumers' control, consumers would choose to own most of the assets that attract them now and in the future.[21/]

At the level of aggregation of this study, it would seem that there are at least three major factors or constraints that affect the community's choice or preference for pure-homeownership of residential dwellings. They are (1) income, (2) relative prices or costs of rental and owner dwellings, and (3) the property tax cost which cannot be shifted to a second party[22/] by a pure-homeowner, but may be partly shifted to a tenant by a landlord.

The higher the average family or per capita personal income in a community, the higher is the

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[21/] L.J. Gordon & M.S. Lee, Economics for Consumers, 5th ed., Van Nostrand Reinhold Co., 1967, pp. 400-401.

[22/] In this connection, Le Roy & Brockschmidt have criticized the taxation of residential property because the tax discourages pure-homeownership, see "Who Pays the School Property Tax?", Monthly Review, Federal Reserve Bank of Kansas, November 1972 pp. 3-13.

expected degree of pure-homeownership. Conversely, the higher the price or annual carrying costs of owner dwellings relative to that of rental dwellings, the lower is the degree of pure-homeownership. Since the property tax cost enters as part of the carrying cost of ownership, it can be subsumed in the relative price or cost variable.

The ratio of owner-occupied to tenant-occupied dwellings may serve to indicate the degree of pure-homeownership in the community. The higher the ratio, the more prevalent is the degree of pure-homeownership. This ratio was relatively high in Canadian municipalities in 1961. The mean ratio for the 59 observations in this study was 1.571.

The specification of the regression model for the demand for pure-homeownership in this sub-section follows the same pattern of reasoning as that of the regression model for the stock demand equation. Thus the relative price (or cost) is treated as dependent upon the property tax rate, average family income, and the ratio of owner-occupied and tenant-occupied dwellings. The regression in turn allows the derivation of a demand equation in the sense of pure-homeownership relative to renting.

The result of our regression appears as equation 12 (in logarithms, but with the term "$\log_{10}$" omitted):

$$\left(\frac{P_o}{P_1}\right) = -1.019 + \underset{(3.653)}{0.599\, Y} - \underset{(0.513)}{0.052\, r} - \underset{(6.455)}{0.188} \left(\frac{H_o}{H_1}\right) \qquad (12)$$

$N = 59$, $\bar{R}^2 = 0.654$, F-Ratio $= 15.08$

(numbers in parenthesis are absolute values of the test statistics for coefficients different from zero).

from which one may derive:

$$\left(\frac{H_o}{H_1}\right) = 0.0000263\, Y^{3.186}\, r^{-0.277} \left(\frac{P_o}{P_1}\right)^{-5.319} \qquad (13)$$

where $(H_o/H_1)$ = a measure of the degree of pure-homeownership,

$H_o$ = total number of owner-occupied dwellings,

$H_1$ = total number of tenant-occupied dwellings,

$P_o$ = median market value of $H_o$,

$P_1$ = $R_1/(m-x+r)$ = estimated average market price of $H_1$,

$R_1$ = weighted gross rental income of $H_1$,

$(m-x+r)$ = sum of mortgage rate(7.14%), depreciation rate($x=-2.56\%$), and the variable property tax rate($r$), and

$Y$ = average family income.

As a separate regressor, the property tax rate has a statistically insignificant coefficient. However, the tax effects are incorporated in the relative cost or price variable,

$$\frac{P_o}{P_1} = \frac{(m-x+r)P_o}{R_1} = \frac{\hat{R}_o}{R_1}\,. \qquad (14)$$

$\hat{R}_o$ is a measure of the imputed rental cost of owner-occupied dwellings.

It can be seen from equation (12) that the association between $P_o/P_1$ and $H_o/H_1$ has a high degree of statistical significance. In addition, the association is negative, as theoretically expected, meaning that the higher the annual carrying cost of owner-occupied dwellings relative to that of rental dwellings, the lower is the degree of pure-homeownership in the community.

Equation (13) presents the derived demand function in the sense of pure-homeownership relative to renting. It appears very elastic with respect to income and relative prices.[23/] This result suggests that income and the annual carrying cost of ownership (the main bulk of which is in the form of interest payments[24/] and property taxes) are very important decision variables to an economic unit who already owns or is contemplating to buy a residence for his own occupancy.

Since all dwellings are subject to the same

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23/ In an alternative interpretation, the estimated numerical elasticity of -5.319 in equation (13) measures the elasticity of substitution between owner and rental dwellings.

24/ Interest income foregone if the acquisition of dwellings is self-financed.

property taxation, it may well be that both $\hat{R}_o$ and $R_1$ include the tax cost. However, the relative extent of tax inclusion differs between the two rent variables. $\hat{R}_o$ includes the full amount of the tax cost per dwelling since owner-occupiers pay and bear the full amount of the tax themselves. In contrast, $R_1$ includes only a fraction of the tax cost the size of which depends upon the extent of forward tax shifting taking place in the rental sector. The extent of forward tax shifting in this sector is primarily governed by the relative elasticities of the demand for and supply of housing-services.

In Section C of Chapter IV and Section B of the present chapter, we have attempted to estimate the numerical values of the demand and supply elasticities of housing-services. Due to data limitations and the consequent identification problems, we have failed to arrive at conclusive findings on the numerical values of these elasticities. The estimated elasticities for the one-market model of housing-services have suggested that only a minimal forward tax shifting may take place in the rental sector. This is a tentative conclusion since it is subject to a number of qualifications already discussed in Section C of Chapter V. In particular, the cross-section data used for the housing-services regression model may not be of the long-run type. Instead, the data may be of the short-run type and thus, may have captured only the initial outcomes of the whole process of forward tax shifting which is a long-run phenomenon.

Although no conclusive evidence on the extent of long-run forward tax shifting has emerged from the study, it can still be argued that a fraction of the property tax cost is borne by the landlords in the long-run. This means that even if full quantity adjustments takes place in the market, the ratio $\hat{R}_o/R_1$ will still reflect relative differences in the tax cost components. Indeed, it is doubtful that the 1961 cross-section data used herein "sufficiently" reflect long-run market outcomes. But our findings seem to indicate that the long-run market supply of (demand for) rental housing-services is far from being perfectly elastic (inelastic). If this is so, the full amount of the tax is not shiftable forward in the rental sector and households can avoid payment of a fraction of the total property tax cost associated with their consumption of housing-services by choosing to be tenants. Whether or not this fraction is significant enough to warrant a change in tenure for existing home-owners, as well as the payment of moving costs, remains an unresolved issue until more information and better data become available. Moreover, because tenants also have moving costs to incur, landlords are put in a better position to formally charge a higher rent in response to property taxation, over and above what might actually be shiftable without these costs. If this is the case, there may be a further reduction in the tax cost differential between $\hat{R}_o$ and $R_1$.

## D. Tax Impact on the Volume of Residential Construction

This section will investigate the empirical content of the hypothesis that the property tax reduces the volume of residential construction. Our general findings here support this hypothesis, thereby suggesting also the presence of forward tax shifting in the rental sector.

### D.1. Regression Model Specification for the Residential Construction Sector

(a) Gross Flow Demand:

Let us take the familiar stock-flow model in its simplest form and start with the idea of the desired addition to the existing stock of dwelling-structures, H. This desired addition is represented by the difference between the desired stock, $H^*$, and the actual stock, H, i.e.,

$$I_n^* = H^* - H. \qquad (15)$$

H* is a variable because it is determined by such factors as prices, income, property taxes, and tastes and preferences, that do change over time.

Strictly speaking, $I_n^*$ should not imply any particular time dimension, since it is a difference between two stocks at the same point in time. Further, the closing of the gap between H* and H need not be fulfilled within a specified time period. In fact, in a growing market area it may be doubted whether the closing of the gap is ever accomplished at all. Investors may want to close the gap only at a certain speed per annum, depending upon their reaction to various economic indicators that affect their decisions. For instance, if they consider the increased demand for H to be temporary, they would produce H in a smaller volume than if they view the increase to be permanent. Indeed, in simple stock-flow models, only a certain fraction of $I_n^*$ is accomplished every year, over and above the demand for replacement purposes, i.e.,

$$I_g - D = I_n = \sigma I_n^* \tag{16}$$

where $0 \leqslant \sigma \leqslant 1$ is the speed of adjustment per annum,
$D = dH$ = replacement demand,
$d$ = fractional rate per annum at which replacement takes place,
$I_n$ = actual net addition to existing stock,
$I_g$ = actual gross addition to " " .

In a growing economy, $I_g$ will represent the annual gross flow demand for new dwelling-structures.

Let the desired stock be:

$$H^* = \phi\ (Y, P, r, K) \tag{17}$$

where P = price of H, Y = average family income, r = effective property tax rate, and K = all other demand determining factors such as the number of families, family size, tastes and preferences, and relative prices of alternative assets. Equation (16) can thus be re-stated as:

$$I_g = \sigma\ [\phi(Y, P, r, K) - H] + dH \tag{18}$$

representing the gross flow demand for new H.

(b) The Supply of New H:

The volume of newly completed dwelling-structures is seen to depend upon the unit price, P, construction cost, C, and property tax, r, which can be built into P due to tax capitalization. The supply of new H is:

$$I_g = f(P, C) \tag{19}$$

In flow equilibrium,

$$f(P, C) = \sigma [\phi(Y, P, r, K) - H] + dH \tag{20}$$

(c) Regression Model for the Supply of New Dwelling-Structures

Equation (19) can be re-stated as:

$$P = J(I_g, C) \qquad (21)$$

if, in fact, the individual supplier of new dwelling-structures or the residential construction industry is not a price-taker. It means that part of the variation in P could be explained by the volume of $I_g$.

Substituting (21) for P in (20) and solving for $I_g$,

$$I_g = X_1(C, Y, r, H, K) \qquad (22)$$

Solving for P in (20),

$$P = X_2(C, Y, r, H, K) \qquad (23)$$

Equation (22) and (23) are the reduced-forms of the structural equation (18) and (19).

If there is no simultaneous relationship between (18) and (19), i.e., the residential construction industry is a price-taker, the only relevant equation for present purposes is (19). The Ordinary Least Squares model (OLS) for the supply of new dwelling-structures is thus:

$$I_g = X_3(P, C, U_3) \qquad (24)$$

where $U_3$ is the random disturbance. However, if there is simultaneity in relationship, the Two-Stage Least

Squares(TSLS) method is more applicable. The Two-Stage Least Squares model for the supply of new dwelling-structures is

$$I_g = X_4(\hat{P}, C, U_4) \qquad (25)$$

where $U_4$ is the random disturbance, and $\hat{P}$ is the estimated price resulting from a regression of P on C, Y, r, K, and H (see equation 23).

All the regressions reported in Table 9 below are logarithmic linear approximations of (24) and (25). Due to data limitations, all the other demand determining factors represented by K are not included in the actual regressions. The impact of population, however, is accounted for in the regressions because the stock of H is expressed on a per capita basis.

D.2. Empirical Evidence

For empirical purposes, the volume of new dwelling-structures refers to newly completed dwelling units on the Census definition. It is not possible to achieve further standardization of units, say, in terms of room space as in the earlier regressions. There are no published data on the average number of rooms of newly completed structures by cities and municipalities.

To be able to make use of available data on construction costs[25/] per 100 square feet, the original

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25/ Do not include property tax (inquiry from personnel of the Central Mortgage and Housing Corporation, Winnipeg regional office).

59 sample observations are reduced to the number of metropolitan and major urban areas for which construction cost data are available. Consequently, there remain only 26 observations to use in this sub-section. Although the construction cost data are by metropolitan and major urban areas, it is reasonable to assume that construction costs do not differ across cities or municipalities thereof. In our regressions, therefore, municipalities or cities belonging to the same metropolitan area have the same construction cost.

The results of the regressions are summarized in Table 9. The notation used is defined in the footnotes to the table.

Regressions I and II are obtained by Ordinary Least Squares (OLS) methods. In regression I, the tax is built into the price (P) variable; whereas in regression II, the tax is unscrambled and appears as a separate regressor. Thus, $\bar{P}$ is a measure of the unit price without tax capitalization.[26/] It must

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[26/] The average unit market values of owner and rental dwellings <u>without</u> tax capitalization are, respectively,

$$\frac{R_o}{(m-x)} = \frac{(m-x+r)P_o}{(m-x)} \quad \text{and} \quad \frac{R_1}{(m-x)} = \frac{(m-x+r)P_1}{(m-x)} .$$

Therefore, the simple average $\frac{1}{2(m-x)}[(m-x+r)P_o + R_1]$ is a measure of $\bar{P}$. In Table 9, since the (m-x) factor is constant across sample observations, the use of $\frac{1}{2}[(m-x+r)P_o + R_1]$ to represent $\bar{P}$ will not affect the regression results except for the intercepts.

Table 9

SUPPLY OF NEW DWELLING-STRUCTURES

| DEPENDENT VARIABLE $\log_{10} I_g$ | INTERCEPTS | INDEPENDENT VARIABLES | | | | | | $\bar{R}^2$ | F-Ratio |
|---|---|---|---|---|---|---|---|---|---|
| | | $\log_{10} C$ | $\log_{10} P$ | $\log_{10} \hat{P}$ | $\log_{10} \bar{P}$ | $\log_{10} \hat{\bar{P}}$ | $\log_{10} r$ | | |
| | | A. ORDINARY LEAST SQUARES VERSIONS | | | | | | | |
| I | -13.07 | -1.069 (0.357) | 4.799 (3.704) | | | | | 0.56 | 6.88 |
| II | -12.07 | -0.951 (0.317) | | | 4.546 (3.449) | | -1.827 (1.580) | 0.58 | 4.95 |
| | | B. TWO-STAGE LEAST SQUARES VERSIONS | | | | | | | |
| III | -24.34 | -1.378 (0.574) | | 7.834 (5.861) | | | | 0.75 | 17.2 |
| IV | -23.07 | -1.333 (0.541) | | | | 7.526 (5.327) | -2.050 (2.162) | 0.74 | 10.9 |

Notes: N=26; numbers in parenthesis are absolute values of test statistics for coefficients different from zero; $\log_{10}\hat{P}$ and $\log_{10}\hat{\bar{P}}$ are obtained from a regression of $\log_{10}P$ and of $\log_{10}\bar{P}$ on the common logarithms of C, Y, r, and $H_p$; $I_g$ = newly completed dwelling-structures; $P_o$ = median market value of owner-occupied dwellings; $P_1 = R_1/(m-x+r)$ = estimated average market value of tenant-occupied dwellings; $\bar{P} = \frac{1}{2}[(m-x+r)P_o + R_1]$ = average price for all dwellings without tax capitalization; $R_1$ = weighted average gross rental income of tenant-occupied dwellings; $(m-x+r)$ = sum of mortgage rate (7.14%), depreciation rate ( x=-2.56%), and property tax rate (r); C= construction cost per 100 square feet; Y = average family income; $H_p$ = rooms per capita; and $P = \frac{1}{2}(P_o + P_1)$.

SOURCE OF DATA: See Appendix A of the study.

be noted that the relevant OLS estimate of the supply equation is regression I since the earlier findings support a price specification with full tax capitalization. Regression II merely serves to cross-check the explicit statistical significance of the tax rate as a separate regressor.

Regressions III and IV are Two-Stage Least Squares (TSLS) versions of the first two OLS regressions. Again, regression IV serves to cross-check the explicit performance of the tax rate as a separate explanatory variable. The relevant TSLS estimate of the supply equation is regression III.

The usefulness of the TSLS equations can be appraised in two ways. First it is one way to correct for possible simultaneous equation bias built into the OLS equations if, in fact, the residential construction industry is not a price-taker. In other words, the price variable P may not be independent of the volume of newly completed dwelling-structures. In this connection, it may be noted that the price variables are 1961 data, whereas the volume of newly completed dwelling-structures, $I_g$, is a simple average of the construction data for the 1960-1965 period. Although $I_g$ is an average centered on the 1960-1963 period, this average includes the year 1961. Therefore, the price variable cannot legitimately be taken as a pre-determined variable in the regression.

<u>Second</u>, the estimated price, $\hat{P}$, in the TSLS version can be interpreted as a measure of the "normal" price at which builders expect to clear their output of new dwelling-structures in the market. The market clearing output would include an optimal or normal vacancy rate in new units completed. (Please refer to the discussion of this point in sub-section B.3., Chapter III). In effect, $\hat{P}$ depends on the builders' impressions and observations on the relative magnitudes of construction costs, property taxes, incomes, prices, etc., prevailing in the communities.

In all regressions, the construction cost variable has very limited explanatory power, although its coefficient bears the theoretically expected sign. The tax rate coefficient in the OLS version is a little short of statistical significance at the 10% level (one-tail). In the TSLS version, the tax rate coefficient emerges with high statistical significance.

In general, the TSLS equations show a marked improvement in the explanatory power of both the price and the tax variables. The results offer support to the simultaneous nature of the interrelationship between the price and the volume of newly completed dwelling-structures. Part of the reason, however, could have been due to the positive role played by $\hat{P}$ as a representation of the "normal" market clearing price, plus some other covert factors that I may have failed to take into consideration.

The results of the TSLS regressions indicate that the supply of newly completed dwelling-structures is very price-elastic. In particular, regression III indicates that a 1% decrease in the market price of new dwellings (package) will result in a 7.83% decrease in the volume of newly completed residential structures.

In terms of the 1961 data, a 1% decrease in the mean market price of dwellings calls for a 4.65% increase in the property tax rate.[27] The 7.83% decrease in the volume of $I_g$ in regression III due to a 1% decrease in $\hat{P}$ (brought about by a 4.65% increase in r) is quite close to the effect of the tax on $I_g$ that is implied in regression IV, where the tax, r, appears as a separate regressor. In IV, the 4.65%

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[27] The mean market price $\hat{P}$ in the 26 samples is \$10,505 for the year 1961. The mean tax rate is 2.65%. The sum (m-x) = 9.7% is constant across sample observations. Therefore, the average rental income for all dwellings is $\hat{P}(m-x+r)$ = \$10,505 (.1235) = \$1, 297. If this average rental income remains unchanged when r changes, then $-d\log \hat{P} \div d\log r = -d\log(\$1,297/\ m-x+r\ ) \div d\log r = d\log(m-x+r) \div d\log r = r/(m-x+r)$ = 2.65% ÷ 12.35% = 0.215. Thus, a 1% decrease in $\hat{P}$ is associated with a 4.65% (i.e., 1% ÷ 0.215) increase in r.

increase in the tax rate, r, will result in a 9.53% decrease in the volume of $I_g$. Using a 90% confidence interval, the supply price-elasticity in regression III ranges from 5.10 to 9.94. Therefore, in terms of the standard error of the estimated supply price-elasticity, 7.834, the 9.53% decrease predicted in regression IV is not significantly different from the 7.83% decrease predicted in regression III.

Since we are able to average construction data over a six-year period (1960-1965), this averaging may have transformed the data to make them reflect long-run market outcomes more closely. Indeed, the estimated price- and tax-elasticities of newly completed dwelling-structures are high in absolute values. In particular, our findings here suggest the presence of forward tax shifting. Other factors remaining the same, if property tax rates are increased, the construction regressions predict a reduction in the per capita stock of residential dwellings and thereby in the per capita supply of housing-services.

# CHAPTER VII

## SUMMARY AND CONCLUSIONS

This study has been concerned with an examination of the nature of the impact of residential property taxation on three major aspects of housing, namely, (1) on the supply and demand for housing-services, (2) on the stock demand for ownership of residential dwellings, and (3) on the supply of newly completed dwelling-structures. By the "nature of the impact" of the property tax we mean: Does the property tax influence decisions to produce and consume housing-services and decisions to own existing dwellings and build additional dwellings? If it does, how and why does the influence occur?

An attempt is made to approach these questions formally. A simple model of the housing market is constructed for this purpose. The model starts with the conceptualization of the bundle of housing-services. The bundle is a complex composite of consumer goods. It has many dimensions such as shelter, comfort, prestige, privacy, quality, etc., that are not easily susceptible to measurement. Since these dimensions are not easily quantifiable, they are subsumed under one general term "housing-services", both in the model and in the empirical application of the model.

Housing-services are seen as a continuous flow of output per period. The major productive inputs are: (1) the physical land site, L, (2) the physical dwelling-structure, H, and (3) the variable inputs, V, representing household facilities and appliances built and contained in the structure, including repairs and maintenance, caretaking and similar supervisory functions.

Technically speaking, these factor inputs are complementary to the production of housing-services. However, an advantage can be gained by substituting more of one type of input for less of another type depending upon their relative prices. For instance, if available residential lands are highly scarce or expensive to develop (partly magnified by restrictive zoning by-laws), a landlord is better off putting up land-saving high-rise apartments.

Further, it is also argued that of the three major inputs, the V components are relatively more variable than the others. In effect, V is seen as a short-run variable input. The short-run supply curve of housing-services is thus seen to demonstrate a certain degree of positive supply-price elasticity. An upward sloping long-run supply curve of housing-services is also adopted in the theoretical investigation. This assumption is supported by the results presented herein, as well as by earlier empirical studies done on U.S. housing.[1/]

Two distinguishable concepts of derived demand emerge from the study, namely, (1) the derived demand for each of the three major input categories, and (2) the derived demand for the residential package, i.e., for all the input categories as though they can be treated as one composite producer good. As a composite producer good, the housing-package is interchangeably referred to as a residential dwelling.

The main focus of investigation in the model is the homeowner. Homeowners are classified as either pure-homeowners or landlords. They are the suppliers of housing-services for their own consumption and/or for sale to the market.

The impact of the property tax is investigated from both a short-run and a long-run point of view. The periods implied in the investigation are also the same theoretically distinguishable short- and long-run production periods of housing-services. The production period is short-run if, during the period, only the V components are variable and the land sites and

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[1/] F. de Leeuw and N. Ekanem, "The Supply of Rental Housing", American Economic Review, December 1971, pp.806-816; same authors, "The Supply of Rental Housing: Reply", same Journal, June 1973, pp.437-438; R. Grierson, "The Supply of Rental Housing: Comment", same Journal, June 1973, pp. 433-436.

dwelling-structures are fixed in amount. If the fixed inputs, particularly the dwelling-structures, become variable due to, say, demand pressures or changes in cost, the period is considered long-run.

The long-run period is generally associated also with changes in number of families, households and population, as well as with changes in tastes and preferences, building technology, and locational patterns of residences. In a more rigorous study, it may be useful to distinguish two types of long-run market period. The first refers to a period long enough to allow full adjustments in the quantity of factor inputs to underlying market conditions. The second refers to a period longer than the first in that it not only allows full adjustments in the quantity, but also in the form (architectural and engineering styles, location patterns) of factor inputs to underlying market conditions.[2/] Only the first type of long-run is used in the process of investigation.

In the investigation, two types of housing-services are distinguished, namely, owner and rental housing-services. They are seen as competing and closely substitutable consumer goods. A bi-market approach is then employed. The market for owner

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[2/] *Ibid.*

housing-services and the market for rental housing-services are treated as different, but highly competing and interrelated markets.

In actuality, there are no active market transactions in owner housing-services. Thus, there is no market mechanism to enable owner-occupiers to pass the property tax cost on to another party. Owner-occupiers pay and bear the full amount of the tax cost. In contrast, market transactions in housing-services regularly take place in the rental sector thereby allowing the possibility of forward tax shifting to tenant-occupiers in this market. To the extent that the tax cost is not fully shiftable in the rental sector, property taxation will affect the relative price of owner and rental housing-services. The relative price effect may trigger a substitution of rental for owner housing-services.

In the short-run, the tax enters as a fixed cost element in the total flow cost of production of housing-services. As such, the tax merely shifts the short-run total cost curve vertically upwards. With a series of increases in property tax during the short-run, the several short-run total cost curves will appear as vertical displacements of each other. In other words, the tax does not affect the short-run marginal cost of production of housing-services. In particular, the short-run level of operation of rental dwellings will not be revised due to the tax. This means that, in the short-run, the property tax costs associated with rental housing operation are generally not shiftable to tenants. The short-run penalizing effect of the tax costs will be borne by the landlords. However, due to the substitution effect, the market demand for rental housing-services may shift upwards, thereby permitting a partial short-run forward shifting of the tax.

Notwithstanding the neutrality of the property tax on the short-run marginal cost, the tax has an output-reducing effect on owner housing-services. The tax constitutes a reduction in the net income of every taxpaying homeowner. Assuming a diminishing marginal utility of income, the disutility of having to give up more and more of other goods and services increases at the margin as the tax cost is augmented. Because of this short-run income effect, homeowners will decrease their own consumption of housing-services through a reduction in the V components, until the marginal utility of an extra $'s worth of other goods and services becomes equal to the marginal utility of an extra $'s worth of housing-services. This short-run reduction in the consumption of housing-services is equivalent to a short-run reduction in the production of owner housing-services.

The long-run impact of the tax is best seen through changes in the decision-making processes of all homeowners who are in a marginal position before the imposition of the tax. A marginal homeowner is either a pure-homeowner who is in a position of indifference between renting or owning a dwelling, or a landlord who is breaking-even in rental housing operation, given the ruling price and cost situation.

In general, the higher the tax rate the stronger is the push pressure on marginal homeowners towards a below-marginal position (i.e., they are worse off being homeowners) and the larger also is the expected number of below-marginal homeowners. Consequently, there will be a lesser predisposition to own the existing stock of dwellings, creating a competitive excess supply on the market. This excess supply will trigger a downward pressure on the general level of prices of dwellings. Since the market price of dwellings is declining, the volume of residential construction will be lower than before the tax imposition. _Ceteris paribus_, the long-run market equilibrium will be restored at a lower level of prices of dwellings. At the same time, the market will leave the community with a smaller quantity of housing-services and stock of dwellings per household or per family. In particular, since the general reduction in the stock of dwellings per family applies to a degree to the rental sector, a portion of the annual tax cost per rental dwelling will eventually be shifted forward to tenants through a leftward shift in the market supply curve and a higher unit price of rental housing-services. However, the extent of the long-run forward shifting of the tax will be primarily governed by the relative price-elasticities of supply and demand, as well as the tax-elasticity of the supply-price of rental housing-services.

In Chapter IV we attempted to estimate the demand and supply elasticities of owner and rental housing-services from cross-section data by Canadian municipalities. These data are interpreted as long-run market outcomes so that the estimated elasticities are treated as long-run elasticities.

We initially adopted a bi-market regression model. But due to data limitations, only its combined-reduced-form equation could be estimated. Consequently, all the structural elasticities of this model remained underidentified. Even with the help of a U.S. reduced-form price regression (treated as Canadian regression), the only identifiable structural parameter in the bi-market model is the price-elasticity of supply of rental housing-services which is estimated to be 0.254.

As an alternative, we resorted to a one-market model. In this model, owner and rental housing-services are treated to be perfect substitutes, thereby ignoring imperfection in substitution as well as the consequential aggregation problems. Although the application of this model was also limited to a combined-reduced-form regression, resort to U.S. data permitted the indirect estimation of the structural elasticities. The need to resort to U.S. data, however, greatly reduces the credibility of our results.

In the initial application of the one-market model, the imputed rental income of owner-occupied dwellings in each sampled community is taken as 10% of median market value. Under this assumption the calculated elasticity values are: (1) price-elasticity of demand (-9.25) (2) income-elasticity of demand (5.91), (3) price-elasticity of supply (0.78), and (4) tax-elasticity of supply-price (0.26).

The use of the 10% flat rate across municipalities for imputing the rental income of owner-occupied dwellings is questionable if capitalization of property taxes takes place in the market. Since we found evidence supporting the property tax capitalization hypothesis, a more appropriate rate for imputation purposes must vary with the property tax rate across sample observations. Assuming full capitalization of property taxes in each community, we re-estimated the various supply and demand elasticities for housing-services. The results are: (1) price-elasticity of demand (-1), (2) income-elasticity of demand (1.053), (3) price-elasticity of supply (0.80), and (3) tax-elasticity of supply-price (0.05).

If these findings were applied to the rental sector, they would suggest very minimal forward tax shifting or no forward shifting at all. A number of reasons for qualifying this conclusion were discussed. In particular, it remains doubtful whether the Canadian and U.S. data used herein should be taken as long-run market outcomes. If the data partially reflect long-run market outcomes, the regression results would capture only the initial impact of the whole long-run process of forward tax shifting. This is probably a major reason why the estimated price-elasticity of supply and tax-elasticity of supply-price are low.

In the construction regression, we were able to average construction data over a six-year period (1960-1965). This averaging may have transformed the data into the long-run type. The estimated supply elasticities of newly completed dwelling-structures are larger and have high statistical significance.

The estimated price-and tax-elasticities of the volume of newly completed structures are 7.83 and -2.05, respectively. With other supply determining factors remaining the same, if property tax rates are increased, the construction regression predicts a reduction in the per capita stock of residential dwellings and therefore a fall in the per capita supply of housing-services in the communities. In short, the construction regression results indicate forward tax shifting.

Our analysis has shown how the property tax affects the demand-price of dwellings in stock. The tax has a price-reducing effect, primarily because of property tax capitalization. In Chapter VI, we calculated the tax-elasticity of the per-capita demand for room-space in the municipalities. The estimated tax-elasticity is unitary, meaning that the partial effect of a 10% increase in the property tax rate is a 10% reduction in the per capita demand for rooms, all other demand determining factors remaining the same. In the same chapter, we also calculated the income and price elasticities of the per capita demand for room-space. The estimated elasticities are 1.55 and -1.84, respectively. The price variable used in this calculation is price per room.

We examined also the roles played by property taxes, income, and relative flow costs in the communities' preference for pure-homeownership relative to renting. The evidence indicates that the higher the average family income in the community, the higher is the relative preference for pure-homeownership. This relative preference is also very responsive to the relative flow costs of owner and rental dwellings, indicating a considerable degree of substitutability between them. The estimated elasticity of substitution between owner and rental dwellings is -5.319, meaning that a 1% change in relative flow costs will result in a 5.319% change in the pure-ownership of dwellings relative to renting.

Part of the relative flow cost effect may well be due to property tax. The periodic tax is borne fully by owner-occupiers. Tenants, on the other hand, pay a larger or smaller fraction of the tax depending upon the extent of forward tax shifting taking place in the rental sector.

In the viewpoint of prospective occupiers, the lesser the shiftable portion of the tax in the rental sector, the lower is the periodic cost of renting relative to pure-homeownership. Indeed, the relative cost effect of the tax is pronounced in a short-run period which permits minimal forward tax shifting in the rental sector.

Generally speaking, the tax is shiftable forward in the long-run. However, the relative flow costs would still vary with the property tax rate as long as the tax is not fully shiftable in the rental sector. Although our findings on the supply and demand elasticities of rental housing-services are not conclusive, they seem to indicate that the long-run market supply of (demand for) rental housing-services is far from being perfectly elastic (inelastic), thereby precluding full forward tax shifting. A fraction of the tax is most likely borne by landlords. Whether or not this fraction is important enough to warrant a change in tenure from owner-occupancy to tenancy (which involves moving and transaction costs) remains an unresolved issue for future study.

The capitalization of property taxes plays a very important role in the explanation of the tax-impact on the volume of residential construction. The higher the property tax rate, the lower is the demand-price at which newly constructed dwellings can be cleared in the market and therefore, the smaller is the volume of residential construction. The empirical findings demonstrate that the volume of newly completed dwelling-structures is responsive to property tax differentials among municipalities. A 1% increase in the tax rate will result in a 2.05% decrease in the volume of newly completed residential structures per annum. The empirical supply curve of new dwelling-structures is also very elastic with respect to the price of dwellings. A 1% increase in the price of dwellings will result in a 7.83% increase in newly completed dwelling-structures per annum.

The contribution of this study can be appraised in terms of the validity of its analytical framework. The framework is "valid" in the sense that we have tested some of its hypothetical assertions and found these assertions consistent with actual market data. In other words, the model of this study is capable of explaining the impact of the property tax on the supply and demand for housing-services, on the demand for ownership of residential dwellings, and on the volume of residential construction. The approach promises worthwhile applications to future housing data in Canada and in other countries.

The study also demonstrates one very significant phenomenon of the research process. It illustrates the important interaction between model and data, i.e., not only does the model help determine what data are relevant for the particular investigation at hand, but the data and evidences from the data do suggest some modifications and/or extensions of the model to improve its analytical and predictive power. A number of cases demonstrating this interaction are apparent throughout the study, particularly in Chapters V and VI. To cite some examples, the evidence supporting the tax capitalization hypothesis suggests a more appropriate price specification for the stock demand equation and for imputing the rental income of owner-occupied dwellings. The theoretical distinction between the quantity of housing-services and its unit price adds perspective and dimension to the analytical interpretation of the empirical relationships among rental income, average family income, and property taxes. The Two-Stage Least Squares (TSLS) version of the construction regression indicates that the residential construction industry is not a price-taker, contrary to the initial assumption of the earlier part of the study. Indeed, the TSLS version suggests the need for a more rigorous integration of the stock of dwelling-structures with the flow of new construction output in the explanation of the market pricing of residential properties.

# APPENDIX A

## SOURCES AND EXPLANATION OF DATA

The following are description and explanation of the sources, nature, and limitations of the data used in the empirical portion of the study. The major source of data is the Dominion Bureau of Statistics (DBS) 1961 Census of Canada. The DBS 1971 Census should provide useful and additional information for testing the assertions of the model used herein. Unfortunately, the publication of the 1971 Census data on housing has been delayed and was not yet available at the time of writing.

I. 1961 Canada Census Data on Housing

The 1961 Census data on housing are contained in Volume II of the DBS census publication. This volume is divided into two parts, namely:

(a) Household and Families, Vol.II, Part 1,
Cat. Nos. 93-510 to 93-522; and

(b) Housing Characteristics, Vol. II, Part 2,
Cat. Nos. 93-523 to 93-535.

All the data contained in Part 1 of Volume II were based on a 100% count of households and families; whereas those contained in Part 2 were based on a 20% sample, with the exception of the data in Tables 1-5 of Cat. No. 93-535 which were based also on a 100% count.

The specific data taken and/or computed from Volume II of the 1961 Census publication are:

| DATA | TAKEN FROM |
| --- | --- |
| total number of households, families, dwellings, and average family income. | Volume II, Part 1: Wage-Earner Families, Cat. No. 93-519. |
| average number of rooms per dwelling, persons per room, rooms per capita, average gross rent, median-value. | Volume II, Part 2: Basic Dwelling Characteristics, Cat. No. 93-523; Rooms per Dwelling, Cat. No. 93-524; Values and Rents, Cat. No. 93-528. |

The following are the notation (used in the text), definitions and commentaries applicable to the data taken from Volume II. Most of these definitions are direct quotations from the explanatory notes of the Volume. Unless otherwise specified, the data are by municipalities or cities.

Y = average family income. This was obtained "by dividing the aggregate earnings of all wage-earner family members by the number of wage-earner heads reporting stated earnings".

S = number of persons per room.

S*= average number of rooms per occupied dwelling. "Only rooms used or suitable for living purposes are counted. Sun rooms, summer kitchens, recreation rooms, attic rooms are counted only if they are finished off and suitable for year-round living quarters. Not counted as rooms are bathrooms, clothes closets, pantries, halls, rooms used for business purposes".

$H_p$ = number of rooms per capita = inverse of S.

$H_h$ = number of rooms per household. In the Census definition, it is equal to $S^*$.

Household --"consists of a person or group of persons occupying one dwelling. It usually consists of a family, with or without lodgers, employees, etc. It may consist of a group of unrelated persons, two or more families sharing a dwelling, or of one person living alone. Every person is a member of some household and the number of households equals the number of occupied dwellings."

Dwelling --" a structurally separate set of living quarter, with a private entrance either from outside the building or from a common hall or stairway inside. The entrance must not be through anyone else's living quarters".

The total number of occupied dwellings includes all private dwellings and excludes all collective dwellings.

Collective dwellings --"include hotels, large lodging-houses, institutions of all types, military camps, and other establishments of similar nature".

Private dwellings-- "include single detached; duplex; multiple rows; apartment and flats; mobile homes such as trailers, railway cars, or boats if occupied by persons with no other usual residence; private and separate living quarters of staff members and servicemen in large penal institutions, hospitals, and military camps".

$H_f$= average number of rooms per family. It is equal to S*(H)/F, where

S*= average number of rooms per dwelling,

H = total number of occupied dwellings or total number of households, and F= number of families.

Family --"consists of a husband and wife (with or without children who have never married) or of a parent (with one or more children never married), living together in the same dwelling. Adopted children and step children are counted as own children, as are guardianship children under 21 years of age".

$R_1$= weighted average gross rent of tenant-occupied dwellings. It "refers to the amount of cash rent payable for the month of May 1961, plus any additional amount paid for services such as water, electricity, gas or fuel in the same period". The figures for gross rent "refer to non-farm dwellings only, regardless of type".

$P_o$= median-value of owner-occupied dwellings. This was computed from data on the expected market values of dwellings if they were to be sold by their owners to willing buyers. The data on the expected market values were supplied by the owners themselves in answer to question #22 of Form 2B of the 1961 Census questionaire which reads, "If you were selling this dwelling now, what (amount) would you expect to get for it?"

$P_1$= estimated average market value of tenant-occupied dwellings.(Please see the text, particularly Sections B and C of Chapter VI).

$P^*_o = P_o/S^*$ = median-value per room (owner-occupied dwellings).

$P^*_1 = P_1/S^*$ = estimated price per room (tenant-occupied dwellings).

II. Other Canadian Data and their Sources

The following are taken from Central Mortgage and Housing Corporation(CMHC), Canadian Housing Statistics, 1962 :

L = average cost of land of newly completed single-detached dwellings (bungalows), by metropolitan areas, 1960-1962. The explanatory notes of the source did not specify the land area. The data could vary with the changing proportions of fully-serviced, partially-serviced, and unserviced lots, as well as with the methods of financing--i.e., either municipal financing or full financing by builders or developers. (Table 97, p.72.)

m = (7.14%) average mortgage rate of interest in conventional lending institutions, for all Canada, 1960-1962. Falling under the conventional lending institution category are chartered banks, loan and trust companies, and savings banks. The average mortgage rate was weighted by the amount of loans, number of loans, and the average terms of payments of the loans. (Table 50, p.47.)

C = average construction costs per 100 square feet, by metropolitan areas. (Table 97, p.72.)

<u>except</u>

x =-2.56% depreciation rate for dwellings, for all Canadian residences. This rate has been used by the DBS for calculating the aggregate dwelling stock series. See E.H. Oksanen, "Housing Demand in Canada: Some Preliminary Experimentation", <u>Canadian Journal of Economics and Political Science</u>, August 1966, Table 3, n.3, p.318.

$I_g$ = average number of new dwelling units completed, by municipalities, 1960-1965. See DBS, <u>New Residential Construction</u>, relevant months and years, Cat. No. 64-002.

III. U.S. Data and their Sources, by Metropolitan Areas:

$E_1$ = average gross rent of moderate standard tenant-occupied dwellings (1966).

r = effective residential property tax rate (1966), i.e., average property tax payments per dwelling as a percentage of median-value.

M = sum of mortgage interest and principal payments per dwelling (1966).

$E_1$, r, and M are all taken from U.S. Bureau of Labour Statistics, City Workers Family Budget, Autumn 1966, Bulletin No. 1570-3, Table 4, p.27.

$Y_p$ = per capita personal income (1967); taken from U.S. Department of Commerce, "Metropolitan Area Income in 1967", Survey of Current Business, vol.49, May 1969, Table 1, pp. 19-20.

IV. Commentaries on the Canadian Effective Residential Property Tax Rate Data

Until 1968, the DBS had been publishing annual data on general property tax rates, all expressed in mill rates per $1000 of the taxable assessed value of property. The data were compiled from submission reports by selected municipalities to the DBS. The following discussions are primarily drawn from commentaries and explanatory notes in DBS, Principal Taxes and Rates, 1960-1970, Cat. No. 68-201.

The assessed value refers to the taxable value which obtains from a multiplication of the "real" or "true" market value of a property by a specified statutory percentage rate, whenever the specification is so regulated by municipal law. During the period 1960-1968, some of the reporting municipalities had indicated a statutory percentage rate ranging from 40% to 100% of the "true" market value of property. For the reporting municipalities whose percentage rates are usually specified by municipal law, the estimation of the assessed value of a property generally involves only an application of the statutory percentage rate on the property's "true" market value. If the market for the property is limited and market transactions are irregular, the market will not readily provide information on the "true" market value of the property. In any case, the "true" market value must be estimated.

The "true" market value of a property may be taken synonymously with its sales price, since sales prices are generally "considered to be fairly representative of market prices".[1/] The statutory percentage rate, therefore, can be interpreted as a percentage applicable either to the sales price of the property or to its estimated market value.

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[1/] Frederic H. Finnis, Property Assessment in Canada, Canadian Tax Foundation, March 1970, p.71.

The estimated market value of a property may differ extremely from its "true" market value or sales price. It can be argued that the less active is the market for a particular property and the more the market transactions are limited to the services deriving from the property, the more difficult becomes the estimation of the property's sales price. Moreover, the estimation problems are likely to be confounded by factors associated with the durability of the property. For instance, more durable properties are usually subject to a higher degree of speculative attack and uncertainty in their future market values. The longer is the life of the property, the more unpredictable are the future events that affect the economic status and usefulness of the property. This is not to mention the fact that a slight difference in the (interest) capitalization rate means a substantial variation in the capitalized value of the property.[2/]
A tremendous deviation from sales or "true" market values can thus be expected in the estimated market values of such durable properties as residential dwellings. Notwithstanding the statutory percentage rates for calculating the taxable assessed value, a residential

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[2/] John A. Zangerle, "Taxing Real Estate on Its Income", Property Taxes (Symposium of the Tax Policy League, now Tax Institute of America), J.J. Little & Ives, Co., 1940, p. 211. To cite a very simplistic example, a perpetual annual rental income of $2,500 capitalized at 5% and 6% yields estimated market values of $50,000 and $41,667 , respectively.

property may, in effect, be over-assessed or under-assessed for property taxation purposes.

In a municipality, there could be one or more statutory percentage rates applicable to residential dwellings. For instance, the municipalities in the province of Quebec had one percentage rate for both land and buildings; whereas the reporting municipalities in the western provinces had one rate for land and another rate for buildings. The specified statutory rates would differ also depending on whether the property is commercial or residential in classification.

Here are examples of statutory percentage rates that were applicable to residential properties in some Canadian municipalities during the period 1960-1968. Rouyn and Sherbrooke in the province of Quebec had a 70% rate for both land and buildings. In Winnipeg, Manitoba, the statutory percentage rates were 66 $^2/_3$% for buildings and 100% for land. The City of Brandon in Manitoba reported a 40% rate for buildings and a 60% rate for land. Vancouver and Victoria in British Columbia had 75% and 100% rates for buildings and land, respectively.

Theoretically, information on both the mill rates and the statutory percentage rates for calculating assessed values provide estimates of the effective mill rate, i.e., mill rate per $1,000 of "true" market value. Thus, if the taxable assessed value is 60% of "true" market value and the mill rate is, say, 50 mills per $1,000 of assessed value, then 60% of 50 mills (=30 mills) would be an estimate of the effective mill rate. The effective mill rates should provide additional and useful set of data for testing the model used herein. However, in a cross-section study by municipalities, data on the effective mill rates are useful only if the municipalities have a standard procedure for arriving at estimates of the "true" market values of similar properties. Where the procedure differs and the estimated market values inaccurately reflect the "true" market values of properties, the effective mill rates are suspect for comparative purposes.

In actual practice, the methods for determining "true" market values vary so widely [3/] that the estimated market values of similar properties differ extremely in municipalities. "The Report of the Ontario Committee on Taxation published in 1967 concluded, for example,..... that the majority of dwellings in the municipalities concerned for the year 1961-1963 were being assessed with a degree of inaccuracy that deserve public censure" [4/] In the United States, Dick Netzer [5/] observed that in the best administered jurisdictions, the valuation of single family houses with similar market prices is likely to

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[3/] F.H. Finnis, op. cit.

[4/] Volume II, Chapter 13, p.225 quoted in Ibid., p.71.

[5/] D. Netzer, "Property Taxes", International Encyclopedia of Social Sciences, ed., D.L.Sills, Macmillan, 1968, vol.15, p.548.

have 10% to 15% variation; in the worst jurisdiction, "the average variation may be far in excess of 50%". Thus, the effective mill rates are probably as inaccurate as the statutory mill rates and, therefore, are not useful for comparative purposes.

In 1969 and 1970, the DBS compiled new sets of data on property tax rates. These sets were better than the earlier ones in that the general real property tax was expressed for the first time as a percentage of actual market value, i.e., actual tax payment as a percentage of actual market value. These percentages were undoubtedly better measures of the effective real property tax rates. In particular,"the tax-market value relationship obviates many of the difficulties inherent in the use of the mill rates..for comparative purposes".6/

The necessary information were obtained from a survey of municipalities "requesting details on the amounts of real property tax payable on representative residential and commercial properties comprising land and buildings of given market values".7/ Of the two sets of data on effective property tax rates for the sampled municipalities, the 1970 data were better than the 1969 data since the former were derived from a wider range of properties than were the latter. For this reason, I have used the 1970 tax

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6/ Principal Taxes and Rates, 1970, p.39.

7/ Ibid., 1969, p.37.

data in this study.

Two more comments deserve attention. First, it is not clear in the DBS commentaries whether the data on the 1969 and the 1970 property tax rates had taken account of the property tax rebates currently then in effect in the province of British Columbia, Alberta, Saskatchewan, and Manitoba. British Columbia introduced a homeowner's grant system in 1957. This system was followed by Manitoba in 1964 and then by Alberta and Saskatchewan in 1966. The maximum amount of grant for homeowners in British Columbia started from $28 in 1957 and came up to $110 in 1966. In Manitoba, the maximum grant was $50 under the Revenue Act of 1964. A $50 grant was also available to homeowners in Alberta and Saskatchewan in 1966. [8/]

Second, during the period 1960-1968, many municipalities that are included in the 59 observations of this study had recorded substantial changes in their mill rates. I made calculation of these changes for each of at least 32 municipalities and found a variation ranging from 2.82% to as high as 20.53%. [9/] In my calculation,

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[8/] Harry Kitchen, "Homeowners Grant", Canadian Tax Journal, January-February 1967, vol. XV, no.1, pp.63-67; Stuart McFadyen, "British Columbia's Homeowner Grant Program: Has It Reduced Property Taxes?", same journal, September-October 1970, pp. 420-434; Douglas Clark, "Property Tax Problems in Canada and How They Are Being Met", Property Taxes: Problems and Potentials (Symposium by the Tax Institute of America), Princeton, 1967, pp.402-416.

[9/] Based on two averages, namely, $X_1$= average mill rate for the period 1960-1962 and $X_2$= average mill rate for the period 1967-1968. The percentage for each municipality was computed as follows: $\Delta X(100)/X$ where $\Delta X = X_1 - X_2$ and $X = \frac{1}{2}(X_1 + X_2)$.

however, I did not take account of the fact that in some municipalities, the increases (decreases) in the mill rates were usually accompanied by decreases (increases) in the assessment ratios. Although the mill rates are a poor indicator of the effective property tax rates, the large variation in the mill rates over the period 1960-1968 is suggestive of the strong possibility that the unknown effective property tax rates in the early 1960's were different from the 1970 tax rates used in this study.

Apart from not knowing the extent of the difference, I am not aware of a yardstick for indicating when the difference would be critical. The most I can say is that the results of this study must be viewed in the light of the limitations and shortcomings in the data used herein.

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