THREE ESSAYS ON STOCK MARKET VOLATILITY

by

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Abstract

This dissertation consists of three essays on stock market volatility. In the first essay, we show that investors will have the information in the idiosyncratic volatility spread when using two different models to estimate idiosyncratic volatility. In a theoretical framework, we show that idiosyncratic volatility spread is related to the change in beta and the new betas from the extra factors between two different factor models. Empirically, we find that idiosyncratic volatility spread predicts the cross section of stock returns. The negative spread-return relation is independent from the relation between idiosyncratic volatility and stock returns. The result is driven by the change in beta component and the new beta component of the spread. The spread-relation is also robust when investors estimate the spread using a conditional model or EGARCH method.

In the second essay, the variance of stock returns is decomposed based on a conditional Fama–French three-factor model instead of its unconditional counterpart. Using time-varying alpha and betas in this model, it is evident that four additional risk terms must be considered. They include the variance of alpha, the variance of the interaction between the time-varying component of beta and factors, and two covariance terms. These additional risk terms are components that are included in the idiosyncratic risk estimate using an unconditional model. By investigating the relation between the risk terms and stock returns, we find that only the variance of the time-varying alpha is negatively associated with stock returns. Further tests show that stock returns are not affected by the variance of time-varying beta. These results are consistent with the findings in the literature identifying return predictability from time-varying alpha rather than betas.

In the third essay, we employ a two-step estimation method to separate the upside and downside idiosyncratic volatility and examine its relation with future stock returns. We find that idiosyncratic volatility is negatively related to stock returns when the market is up and when it is

down. The upside idiosyncratic volatility is not related to stock returns. Our results also suggest that the relation between downside idiosyncratic volatility and future stock returns is negative and significant. It is the downside idiosyncratic volatility that drives the inverse relation between total idiosyncratic volatility and stock returns. The results are consistent with the literature that investor overreact to bad news and underreact to good news.

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Essay One: Idiosyncratic Volatility Spread and the Cross-Section of Stock Returns

Abstract

Investors will have the information in the idiosyncratic volatility spread when using two different models to estimate idiosyncratic volatility. In a theoretical framework, we show that idiosyncratic volatility spread is related to the change in beta and the new betas from the extra factors between two different factor models. Empirically, we find that idiosyncratic volatility spread predicts the cross section of stock returns. The negative spread-return relation is independent from the relation between idiosyncratic volatility and stock returns. The result is driven by the change in beta component and the new beta component of the spread. The spread-relation is also robust when investors estimate the spread using a conditional model or EGARCH method.

JEL Classifications: G3, G31, C51.

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1. Introduction

Modern portfolio theory claims that investors should be compensated for bearing only systematic risk. Due to incomplete information, investors may hold imperfectly diversified portfolios. Therefore, they also require compensation for bearing idiosyncratic risk (Merton, 1987). While the literature has not reached a conclusive agreement on the relation between idiosyncratic volatility and stock returns[1], it attempts to analyze the information within the idiosyncratic volatility (IVOL) and study it from different channels. Jiang, Xu, and Yao (2009) document that idiosyncratic volatility is negatively related to future earnings shocks and that the negative IVOL-return relation is induced by information on IVOL about future earnings. Han and Lesmond (2011) model the effect of zero returns and bid-ask spread on stock returns and find that the negative IVOL-return relation becomes marginally significant by controlling the estimation bias on IVOL arising from the failure to account for this effect. Among all these studies, none of them attempts to study the idiosyncratic volatility estimated from different models. Moreover, the intrinsic information in the estimated idiosyncratic volatility is still unknown for investors. In this study, we focus on the idiosyncratic volatility spread that is the difference of idiosyncratic volatility estimated from two models. We contribute to the literature by investigating the cross-sectional relation between idiosyncratic volatility spread and stock returns. The stock returns predictability of idiosyncratic volatility spread is robust using different models and estimation methods. To the

[1] The literature on the relation between idiosyncratic volatility and stock returns has produced mixed results. Malkiel and Xu (2002) find that portfolios with higher idiosyncratic volatility have higher average returns, and this is confirmed by Goyal and Santa-Clara (2003). Spiegel and Wang (2005) and Fu (2009) use EGARCH to estimate idiosyncratic volatility and capture the time-varying properties of idiosyncratic risk. They find a positive relation between idiosyncratic volatility and the expected returns of individual stocks. However, Bali, Cakici, Yan, and Zhang (2005) argue that the positive IVOL-return relation is driven by small stocks in the NASDAQ and by a liquidity premium. Ang, Hodrick, Xing, and Zhang (2006) use OLS regression to estimate idiosyncratic volatility based on the Fama-French three-factor model and find a negative relation between idiosyncratic volatility and stock returns at the firm level. Ang, Hodrick, Xing, and Zhang (2009), Brandt, Brav, Graham, and Kumar (2010), and Hou and Loh (2016) confirm the negative IVOL-return relation.

best of our knowledge, this is the first study to investigate the information within idiosyncratic volatility by analyzing idiosyncratic volatility spread and addressing the spread-return relation.

In fact, investors may choose an asset pricing factor model based on their own knowledge, expertise, and trading experience. For the investors who are aware of one model in idiosyncratic volatility estimation but also select another model with better performance (i.e., a relatively high *R*-square and/or relatively small pricing error) on explaining stock returns, it is interesting to see if they benefit from the additional information captured by the better model and how they gain trading advantages compared with the other investors who only consider one model in their trade. The spread contains additional information that is only directly accessible to the group of investors who select two models to estimate idiosyncratic volatility. Therefore, a model plays an important role in the idiosyncratic volatility estimation and hence in the idiosyncratic volatility spread.

The existing literature has well addressed the relation between idiosyncratic volatility (IVOL) and stock returns, but the results are inconclusive. Merton (1987) and Malkiel and Xu (2002) show that idiosyncratic volatility is positively related to expected stock returns when it is estimated using the capital asset pricing model (CAPM). However, Ang, Hodrick, Xing, and Zhang (2006) find a negative relation between idiosyncratic volatility and stock returns using the Fama-French three-factor model. As Fama and French (1992) claim that the CAPM performs poorly in explaining stock returns, the underlying idiosyncratic risk cannot be correctly captured if the idiosyncratic volatility is estimated by CAPM. Moreover, the idiosyncratic risk captured by models accounting for different factors contain different information and such different information is worthwhile for further investigation. For example, the Fama-French three-factor model, relative to CAPM, incorporates size and value factors, which may account for additional information with systematic risk that is related with stock returns. While we start calculating the spread from the idiosyncratic volatility estimated by the CAPM and Fama-French three-factor model, these models may omit important risk components that are associated with stock returns.[2] To calculate idiosyncratic volatility spread, a model with a better performance is needed. It is possible that idiosyncratic volatility spread calculated from CAPM and Fama-French three factor fail to contain information associated with other risk that is related with stock returns. For example, Spiegel and Wang (2005) find that idiosyncratic volatility is negatively related to liquidity. If a factor model controls for liquidity, investors who use it as an additional model to estimate idiosyncratic volatility will have the idiosyncratic volatility spread that contains liquidity-related information. To this end, we calculate the idiosyncratic volatility spread by pairing the five well-known factor models: (1) the CAPM, (2) the Fama-French three-factor model, (3) the Carhart four-factor model, (4) liquidity adjusted Fama-French three factor model, and (5) the Fama-French five-factor model.

The poor empirical performance of the CAPM may be due to its omission of time-variation in parameters (i.e. alpha and/or betas). Jagnannathan and Wang (1996), Ferson and Harvey (1999), and Lettau and Ludvigson (2001) use lagged macro variables to model the evolution of time-varying alpha and betas to study conditional stock returns. They show that the conditional model (with time-varying alpha and beta) performs better than the unconditional one (with constant alpha and beta). Using the same conditioning variable, Adrian and Franzoni (2009) assume that time-varying alpha and betas reflect investors' learning abilities regarding the unobserved long-run

[2] Studies following Jegadeesh and Titman (1993) show that stock returns also exhibit momentum: stocks that have historically done well tend to continue doing well. Based on this evidence, Carhart (1997) proposes a four-factor model that incorporates a momentum factor in addition to the Fama-French three factors. This momentum factor is priced. Besides these factors, Amihud and Mendelson (1986) find that stock returns are also affected by liquidity. Amihud (2002) develops an illiquidity measure and finds a cross-sectional positive return–illiquidity relation. Similarly, Pastor and Stambaugh (2003) propose a liquidity factor as a state variable based on size, value, and momentum factors. Recently, Fama and French (2015) and (2016) propose a five-factor model that captures the size, value, profitability, and investment patterns in average stock returns. They present evidence that this five factor model performs better than the Fama-French three factor model.

changes in risk factor loadings, and show that the conditional CAPM performs better than its unconditional version. The unconditional version of factor models used in this study, such as the Carhart four-factor model or the Fama-French five-factor model, also fails to capture the time-varying property of alpha and betas. Investors may consider an additional conditional model along with its unconditional version and thus obtain the idiosyncratic volatility spread calculated from these two models (e.g. conditional Carhart model v.s. unconditional Carhart model). Hence, we consider the conditional version of each factor model by introducing the time-varying alpha and betas and explore how such time variation affects the relation between idiosyncratic volatility spread and stock returns.

In a simple theoretical framework, we show that the idiosyncratic volatility estimated by different factor models contains diverse risk. The idiosyncratic volatility spread between two factor models is composed of a change in beta term, a new beta term, and a change in covariance term. Moreover, we derive the idiosyncratic volatility spread as the difference in idiosyncratic volatility estimated by a conditional model and its unconditional version for each factor model. Based on the theoretical and empirical analysis, we obtain the following findings. First, the idiosyncratic volatility spread is negatively related with stock returns on both portfolio and individual stock levels. Stocks with high idiosyncratic volatility spread tend to have low future returns. Second, the negative spread-return relation is independent from the negative relation between idiosyncratic volatility and stock returns. Using a decomposition analysis, it shows that the spread-return relation is mainly driven by the components associated with the change in beta term and the new beta term. Third, the idiosyncratic volatility spread is negatively related to stock returns when calculated from an unconditional factor model and its conditional version. Time-varying alpha, instead of time-varying betas, contributes to this relation. Finally, the negative spread-return relation still holds when we use exponential generalized autoregressive conditional heteroscedastic (EGARCH) to estimate expected idiosyncratic volatility and the spread. The relation is not affected by ordinary least squares (OLS) and EGARCH estimation methods.

This paper is organized as follows. Section 2 provides a theoretical model setup. Section 3 describes the methodology and data. Section 4 provides the empirical results for detailed analysis of the relation between idiosyncratic volatility spread and stock returns. Section 5 shows the results for conditional models and EGARCH estimation method. Section 6 concludes the study.

2. Risk Decomposition and Spread

In this section, we first derive the idiosyncratic volatility based on different factor pricing models, and then we obtain the idiosyncratic volatility spread between each pair of factor models. Next, we derive the idiosyncratic volatility using conditional models with time-varying alpha and betas and present the idiosyncratic volatility spread between a conditional and its unconditional version.

2.1 IVOL obtained from different factor models

We consider a generalized factor model that contains *N* risk factors given by:

$$R_{i,t} = \alpha_i + \sum_{p=1}^{N} \beta_i^p F_t^p + \varepsilon_{i,t}, \qquad (1)$$

where F_t^p is the *p-th* risk factor and β_i^p is its factor loading for stock *i*. For example, F_t^p represents the MKT, SMB, and HML factors in a Fama-French three-factor model (N = 3). The first two terms on the right-hand side of Equation (1) represent the systematic return component of stock *i* and the $\varepsilon_{i,t}$ is its idiosyncratic return component. In line with Campbell, Lettau, Malkiel, and Xu (2001) and Xu and Malkiel (2003), we can calculate the idiosyncratic risk given by:

$$Var(\varepsilon) = Var(R) - Var\left(\alpha + \sum_{p=1}^{N} \beta^p F^p\right) - 2Cov\left(\varepsilon, \alpha + \sum_{p=1}^{N} \beta^p F^p\right) \tag{2}$$

$$IVOL = Var(R) - Var\left(\sum_{p=1}^{N} \beta^p F^p\right), \tag{3}$$

where by definition $Cov\left(\varepsilon, \alpha + \sum_{p=1}^{N} \beta^p F^p\right) = 0$ and the idiosyncratic volatility is $IVOL = Var(\varepsilon)$[3]. Given the expression of idiosyncratic volatility, the idiosyncratic volatility spread[4] estimated by a pair of two-factor pricing models is:

$$Spread_{1-2} = IVOL_1 - IVOL_2 = Var\left(\sum_{q=1}^{M} \beta^q F^q\right) - Var\left(\sum_{p=1}^{N} \beta^p F^p\right), \tag{4}$$

where $IVOL_1$ and $IVOL_2$ are the idiosyncratic volatility estimated by Models 1 and 2 with *N* and *M* factors, respectively (we assume that M>N). With the assumption that Model 2 contains all the *N* factors as in Model 1, Equation (4) is further decomposed into three components[5]:

$$Spread_{1-2} = \text{Delta}_{\text{Beta}} + \text{New}_{\text{Beta}} + \text{Delta}_{\text{Cov}}, \tag{5}$$

where the change in *Beta* term is

$$\text{Delta}_{\text{Beta}} = \sum_{j=1}^{N} ({\beta^{j1}}^2 - {\beta^{j2}}^2) Var\left(F^j\right),$$

the new *Beta* term is

$$\text{New}_{\text{Beta}} = \sum_{j=N+1}^{M} {\beta^j}^2 Var\left(F^j\right),$$

and the change in covariance term is

$$\text{Delta}_{\text{Cov}} = 2\left(\sum_{1 \le j1 < j2 \le N} Cov\left(\beta^{j1} F^{j1}, \beta^{j2} F^{j2}\right)\right) - 2\left(\sum_{1 \le j1 < j2 \le N} Cov\left(\beta^{j1} F^{j1}, \beta^{j2} F^{j2}\right)\right) +$$
$$2\left(\sum_{N+1 \le j1 < j2 \le M} Cov\left(\beta^{j1} F^{j1}, \beta^{j2} F^{j2}\right)\right) + 2cov\left(\sum_{j=1}^{N} \beta^j F^j, \sum_{j=N+1}^{M} \beta^j F^j\right).$$

[3] While idiosyncratic risk is equivalent to the variance of residuals in our model, we use the standard deviation of residuals, $IVOL(s.d.)$, as the proxy for idiosyncratic volatility following AHXZ (2006), Bali and Cakici (2008), Fu (2009), and others.

[4] When we calculate idiosyncratic volatility spread, we use the difference in variance, consistent with theoretical framework. Our empirical results do not change if we use the difference in standard deviation of residual to calculate the spread.

[5] See Appendix A for detail.

β^{j1} is the beta of factor j from Model 1 and β^{j2} is from Model 2. Factor N+1 to M is the new factors in Model 2 relative to Model 1. In Appendix B, we present the idiosyncratic volatility spread between the four pairs of five factor models used in this study.

2.2 Time-varying alpha and betas

In this section, we incorporate the effects of time-varying alpha and/or time-varying betas on idiosyncratic volatility spread for three scenarios.

Scenario 1 (Time-varying alpha and constant betas): When alpha is time-varying and betas are constant, the idiosyncratic volatility is given by:

$$IVOL_{vacb} = Var(R) - \left(Var(\alpha) + 2Cov\left(\alpha, \sum_{p=1}^{N} \beta^p F^p\right) + Var\left(\sum_{p=1}^{N} \beta^p F^p\right)\right). \qquad (6.1)$$

Equation (6.1) reports two additional terms compared to the case of unconditional model above with constant alpha and constant betas: the variance of alpha, $Var(\alpha)$, and the covariance between alpha and return, $Cov\left(\alpha, \sum_{p=1}^{N} \beta^p F^p\right)$. Therefore, the idiosyncratic volatility spread between this model and its unconditional version is given by:

$$Spread_{vacb} = Var(\alpha) + 2Cov\left(\alpha, \sum_{p=1}^{N} \beta^p F^p\right). \qquad (6.2)$$

Scenario 2 (Constant alpha and time-varying betas): When betas are time-varying, while alpha is constant, the idiosyncratic volatility is given by:

$$IVOL_{cavb} = Var(R) - \left(\sum_{p=1}^{N} Var(\beta^p F^p) + 2\sum_{1\le p}\sum_{<q\le N} Cov(\beta^p F^p, \beta^q F^q)\right). \qquad (7.1)$$

In this scenario, the idiosyncratic volatilitly is also affected by the covariance among the return components explained by each risk factor, $Cov(\beta^p F^p, \beta^q F^q)$. The idiosyncratic volatility spread between this model and its unconditional version is given by:

$$Spread_{cavb} = 2\sum_{1\leq p}\sum_{<q\leq N} Cov(\beta^p F^p, \beta^q F^q). \tag{7.2}$$

Scenario 3 (Time-varying alpha and time-varying betas): When both alpha and betas are time-varying, the idiosyncratic volatility is given by:

$$IVOL_{vavb} = Var(R) - \begin{pmatrix} Var(\alpha) + 2Cov\left(\alpha, \sum_{p=1}^{N} \beta^p F^p\right) \\ + \sum_{p=1}^{N} Var(\beta^p F^p) + 2\sum_{1\leq p}\sum_{<q\leq N} Cov(\beta^p F^p, \beta^q F^q) \end{pmatrix}. \tag{8}$$

Compared with the case of unconditional models in Equation (3), the idiosyncratic volatility spread is given by:

$$Spread_{vavb} = IVOL - IVOL_{vavb}$$

$$= \left[Var(\alpha) + 2Cov\left(\alpha, \sum_{p=1}^{N} \beta^p F^p\right) + 2\sum_{1\leq p}\sum_{<q\leq N} Cov(\beta^p F^p, \beta^q F^q)\right]. \tag{9}$$

Equation (9) is equal to the sum of Equation (6.2) and (7.2). It shows this idiosyncratic volatility spread has two components that are the spread from time-varying alpha and the spread from time-varying beta[6]. It is given by:

$$Spread_{vavb} = Spread_{vacb} + Spread_{cavb} \tag{10}$$

3. Methodology and Data

3.1 Methodology

Before calculating the idiosyncratic volatility spread, we need to estimate the idiosyncratic volatility. We use two methods for estimation. AHXZ (2006) uses OLS regression to estimate the idiosyncratic volatility of each stock, defined as the standard deviation of the residuals from a regression of daily stock returns in month $t-1$. By multiplying the daily standard deviation by the square root of the number of trading days in that month, we derive the standard deviation of

[6] Appendix C shows the idiosyncratic volatilitiy of different conditional models.

monthly residuals.[7] Spiegel and Wang (2005) and Fu (2009) use an EGARCH (1, 1) process to estimate the conditional expected idiosyncratic volatility. EGARCH uses an expanding window regression accounting for all information available until month (*t*-1).

We use five factor models to estimate the idiosyncratic volatility, including the CAPM, the Fama-French three-factor model (FF3), and the Carhart four-factor model (Carhart). Spiegel and Wang (2005) find that idiosyncratic volatility annihilates the explanatory power of liquidity in predicting stock returns. Therefore, we introduce the model with Fama-French three factors and liquidity factor (FF3Liq). Recently, Fama and French (2015) have proposed a five-factor model (FF5), which seems to perform better than the Fama-French three-factor model. We also include the Fama-French five-factor model in our analysis. As idiosyncratic volatility spread from each pair of these models may contains overlapping information and exhibit heterogeneous properties, we focus on the following four pairs of models and the corresponding idiosyncratic volatility spread: (1) CAPM v.s. Fama-French three-factor model, spread (*CAPM-FF3*), (2) Fama-French three-factor model v.s. Carhart four-factor model, spread (*FF3-Carhart*), (3) Fama-French three-factor model v.s. liquidity adjusted Fama-French three factor model, spread (*FF3-FF3Liq*), (4) Fama-French three-factor model v.s. Fama-French five-factor model, spread (*FF3-FF5*).

Due to the improved performance of conditional models, we introduce it to calculate idiosyncratic volatility spread. Ferson and Harvey (1999) propose a conditional factor model that explicitly allows alpha and betas to vary with macroeconomic variables (called state variables).[8]

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[7] A similar method is used in French, Schwert, and Stambaugh (1987), Schwert (1989), Fu (2009), and Huang, Liu, Rhee, and Zhang (2010).

[8] Ferson and Harvey (1999) use five state variables. They include the difference between the one-month lagged returns of a three-month and a one-month Treasury bill, the dividend yield of the S&P index, the spread between Moody's Baa and Aaa corporate bond yields, the spread between a ten-year and a one-year Treasury bond yield, and the lagged value of a one-month Treasury bill yield.

It has several advantages. First, this approach can be extended to different factor models since they assume that the return generating process follows a generalized factor model. This allows us to explore whether time-varying alpha and betas affect the IVOL-return relation for each factor model. Second, this method can isolate the effect of time-varying alpha from that of time-varying betas; thus, the risk associated with each component of the spread can be examined separately. Finally, Lewellen and Nagel (2006) assume that the conditional alpha and beta are constant within each short window but Ferson and Harvey (1999) allow alpha and beta to vary within regression window. Following Ferson and Harvey (1999), we estimate the idiosyncratic volatility spread for pairing the five factor models with their unconditional versions. This method is also used by Cao, Simon, and Zhao (2008) to study the relation between growth options and trends in idiosyncratic risk.

3.2 Data

We built the sample from the Center for Research in Security Prices (CRSP) common shares traded on the NYSE, AMEX, and NASDAQ from July 1963 to June 2018. The daily and monthly stock returns are obtained from the CRSP. The stocks with monthly returns greater than 300 percent are excluded from the sample (Fu, 2009). The accounting data are collected from the Compustat database. The Fama-French factors are downloaded from Professor French's website.[9] The liquidity factor is constructed using the methodology introduced by Pastor and Stambaugh (2003). Our results are robust to the choice of different liquidity factors, including Amihud (2002) ILLIQ, Hasbrouck (2009) Gibbs Sampler, and Amivest liquidity ratio.

[9] We thank professor French for making this data available at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

Following Harvey and Ferson (1999), we use four state variables. *TERM* is calculated as the spread between the ten-year and one-year Treasury bond yield (e.g., Campbell and Vuolteenaho, 2004; Adrian and Franzoni, 2009); *Default* is the spread between Moody's Baa and Aaa corporate bond yield (e.g., Avramov and Chordia, 2006); *DIV* is the dividend yield on value-weighted CRSP market portfolio, and *T-bill* is the short term risk-free rate measured by the secondary market rate of the three-month T-bill. Data on bond yields and T-bill rates are downloaded from the website of the Federal Reserve Bank of St. Louis. The dividend yield on value-weighted CRSP market portfolio is directly collected from the CRSP.

4. Spread-Return Relation `

4.1 Summary Statistics

Table 1 reports the descriptive statistics of the idiosyncratic volatility estimated by different models and the corresponding idiosyncratic volatility spread from four pairs of models. Panel A presents the results of estimated idiosyncratic volatility. The mean column shows that the average of idiosyncratic volatilities decreases with the number of pricing factors. The median column demonstrates a similar pattern. With the 3148390 observations, the CAPM model generates the largest idiosyncratic volatility, while Fama-French 5 factor model generates the smallest one. Panel B shows the descriptive summary of idiosyncratic volatility spread. The two groups of spread from the CAPM v.s. Fama-French three-factor model and from the Fama-French three-factor model v.s. five-factor model are larger and more volatile compare with the other two groups. Table 2 shows the time series average of correlations among the idiosyncratic volatility for each factor model and idiosyncratic volatility spread calculated. It shows that the idiosyncratic volatility estimations are highly correlated with each other. The correlations range from 0.9845 (between the CAPM and FF5) to 0.9946 (between the FF3 and FF5). A weaker correlation can be found between idiosyncratic volatility and idiosyncratic volatility spread, with the correlations ranging from 0.5217 to 0.7036. The correlations among idiosyncratic volatility spread are not as high as them. While the highest correlation is 0.5137 between spread (*CAPM-FF3*) and spread (*FF3-FF5*), lowest correlation is only 0.4101 between spread (*CAPM-FF3*) and spread (*FF3-FF3Liq*).

4.2 Idiosyncratic volatility spread

The returns of the stock portfolios that are sorted by size and idiosyncratic volatility spread are reported in Table 3. In Panel A, stocks are first sorted into five size quintile portfolios, and then each size quintile is further sorted into five quintiles based on the idiosyncratic volatility spread (*CAPM-FF3*). As a result, the 5 × 5 equally weighted portfolio returns are obtained. In general, portfolio return is higher with large size when idiosyncratic volatility spread is high. However, it increases as size increases but then decreases when idiosyncratic volatility spread is low. Under each size quintile, we calculate the yield by buying the portfolio with the highest idiosyncratic volatility spread and short selling the one with the lowest idiosyncratic volatility spread. We also report the Jensen's alpha from the CAPM and the Fama–French three-factor model. The Newey–West adjusted t-statistics are in parentheses. For size quintiles 1 to 4, the long-short trading strategy yields negative average returns that are statistically significantly different from zero. No such relation is observed for portfolios under large size quintile 5. The largest two negative yields are observed under size quintile 1 with −0.70 and size quintile 2 with −0.73. Such monthly yield is equivalent to eight percent annually. The yield becomes smaller as the size increases. Jensen's alphas from the CAPM and the Fama–French three-factor model confirm the results. The last column of Panel A reports the 5 quintiles portfolio returns that are sorted solely on idiosyncratic volatility spread. By only considering the idiosyncratic volatility spread, we use a long-short trading strategy and obtain −0.58 as the monthly yield. The Newey–West adjusted t-statistics shows the yield is statistically significantly different from zero. The results from Jensen's alpha are consistent with it. This evidence confirms that idiosyncratic volatility spread (*CAPM-FF3*) is negatively related with the cross section of stock returns at the portfolio level. High spread predicts low future returns. Similar results are observed in Panel B, where stocks are first sorted on size and then sorted on Spread (*FF3-Carhart*). The long-short trading strategy yields similar negative returns for small and median size stocks from size quintiles 1 to 3. The spread is marginally significant for size quintile 4 but is not significant for the large size quintile. The result from last column shows that spread (*FF3-Carhart*) is negatively related with stock returns. Panel C and D reports the portfolio sorting results from spread (*FF3-FF3Liq*) and spread (*FF3-FF5*) respectively. Results are consistent with those in Panel A and B. As size increases, the long-short trading strategy yield shrinks. While this pattern does not exist for large size quintile portfolios, the negative spread-return relation holds in general.

Then we sort stocks on book-to-market ratio and idiosyncratic volatility spread. The portfolio returns and long-short trading strategy yields are reported in Table 4. Panel A to D show the results for the idiosyncratic volatility spread analyzed in this study. Stocks are first sorted into five book-to-market quintile portfolios, and then each quintile is further sorted into five quintiles based on the idiosyncratic volatility spread. As a result, we calculate the 5 × 5 equally weighted portfolio returns. When the spread is controlled, portfolio return is increasing as book-to-market ratio increases. Controlling book-to-market ratio, we obtain the yields from the long-short trading strategy. A statistically significant negative yield is reported with only one exception in book-to-market quintile 4 when stocks are further sorted by Spread (*FF3-Carhart*). All the CAPM alphas and Fama–French three-factor alphas confirm the negative relation between idiosyncratic volatility spread and stock returns. This trading strategy yield is larger and more significant for growth stocks. For example, Panel A reports that the trading strategy yield ranges from -0.87 for book-to-market quintile 1 to -0.27 for quintile 4. Therefore, we find a negative relation between idiosyncratic volatility spread and stock returns on portfolio level.

In order to analyze the cross-sectional spread-return relation on individual stock level, we performed the Fama-MacBeth (1973) regressions to test it. Table 5 shows the results with the Newey-West adjusted t-value reported in parentheses. Models (1) confirms the relation between beta, size, and book-to-market ratio and stock returns. In Model (2) we control for the momentum effect, by including the compound gross return from month (*t*-7) to (*t*-2), the liquidity effect, by including the average turnover (TURN) and the coefficient of variation of turnovers (CVTURN) (e.g., Chordia, Subrahmanyam, and Anshuman, 2001; Fu, 2009), as well as the leverage effect, by including the leverage ratio. Model (3) shows that the idiosyncratic volatility spread is significantly negatively related to stock returns after controlling the variables in the previous model. While the coefficient of Spread (*FF3-FF5*) is the smallest at -0.26, we find that all coefficients on the spread are negative and significant from Model (3) to (6). This result is consistent with the portfolio sorting in Table 3 and 4.

Table 6 compares the spread-return relation with the relation between idiosyncratic volatility and stock returns. Model (1) confirms the negative IVOL-return relation. As Spread (*CAPM-FF3*) is added in Model (2), a significantly negative coefficient -0.15 is still observed for the spread. Similar patterns are observed for other spreads excluding the relation between Spread (*FF3-FF3Liq*) and stock returns when idiosyncratic volatility is controlled. Nevertheless, a negative coefficient is still obtained with its sign consistent with the negative spread-return relation.

We also notice that the negative IVOL-return relation does not change as the idiosyncratic volatility is estimated from different factor models. It is independent from the model used in idiosyncratic volatility estimation. From the analysis above, we conclude that idiosyncratic volatility spread has a significant relation with stock returns on individual stock level. Investors who are aware of this information can use the spread to predict future stock returns better compared to other investors who use only one model in estimating idiosyncratic volatility.

4.3 Decomposition analysis of idiosyncratic volatility spread

Though the spread-return relation is discovered, one question still remains: why is the spread negatively related with stock returns? As the idiosyncratic volatility spread is decomposed into three parts in Equation (5), the change in beta term, the new beta term, and the change in covariance term, the spread-return relation is decomposed and studied in this subsection.

We conduct the analysis by using a 3-stage decomposition method following Hou and Loh (2016). In Table 7, we decompose the negative relation between month t − 1 idiosyncratic volatility spread and month t stock returns into a number of components each is related to a candidate variable and a residual component. Using firm-level Fama-MacBeth cross-sectional regressions, Stage 1 regresses month t returns on month t – 1 ($R_{it} = \alpha_t + \gamma_t Spread_{it-1} + \varepsilon_{it}$). and the average coefficient on *Spread(CAPM-FF3)* is -20.96% with a t-statistic of -2.84 in Panel A. In stage 2, we regresse *Spread* on the candidate variable each month ($Spread_{it-1} = \alpha_{t-1} + \delta_{t-1} Candidate_{it-1} + \mu_{it-1}$) to decompose *Spread* into two orthogonal components: ($\delta_{t-1} Candidate_{it-1}$) and ($\alpha_{t-1} + \mu_{it-1}$).

In stage 3, the γ_t from Stage 1 is decomposed as: $\gamma_t = \frac{Cov[R_{it}, Spread_{it-1}]}{Var[Spread_{it-1}]} = \frac{Cov[R_{it},\ \delta_{t-1} Candidate_{it-1}]}{Var[Spread_{it-1}]} + \frac{Cov[R_{it},\ \alpha_{t-1} + \mu_{it-1}]}{Var[Spread_{it-1}]} = \gamma_t^C + \gamma_t^R$. Since we simultaneously select several candidate variables as the idiosyncratic volatility spread components in Equation (5), γ_t can be further decomposed into different components such that each candidate variable has a γ_t^{Ci} and the γ_t^R is related to the residual component. γ_t^{Ci}'s will be calculated and the time-series average of each γ_t^{Ci} divided by the time-series average of γ_t then measures the fraction of the negative spread-return relation explained by the candidate variable. The average of the γ_t^R divided by the average γ_t measures the fraction of the relation left unexplained by the candidate variables.

In Panel A, the γ_t^{Ci} for change in beta term (Delta β^{MKT}) is 1.83% and the γ_t^{Ci}'s for new beta term (New β^{SMB} and New β^{HML}) and change in covariance term(Delta$_{\mathrm{Cov}}$) are -9.87% , -8.51% and -3.07%, respectively. In total, 87.7% of the negative relation between Spread (*CAPM-FF3*) and stock returns is explained by new beta term and 14.6% is explained by the covariance term. The negative 8.7% fraction of Delta β^{MKT} means it contributes in the opposite direction of the spread-return relation. In Panel B, it shows that both change in beta term (Delta β^{SMB}) and new beta term (New β^{MOM}) explain most of the relation between Spread (*FF3-Carhart*) and returns. We also notice that change in covariance term work in the opposite direction of the relation. Panel C presents that the fraction of Spread (*FF3-FF3Liq*)-return relation attribute to Delta β^{SMB} is 36.6% and New β^{IML} explains another 54.9% of the relation. All other components explain a little of the relation. In Panel D, the relation between Spread (*FF3-FF5*) and stock returns is mainly explained by Delta β^{SMB}, New β^{RMW} and New β^{CMA} with a fraction of 58.4%, 42.7% and 36%, respectively. Delta β^{MKT} and Delta β^{HML} seem to have no explanation on the spread-return relation. The effects of change in covariance term are not consistent throughout the decomposition analysis of different idiosyncratic volatility spread.

Figure 1 plots the percentage of the spread-return relation that is explained by each spread component. We choose the components that have a positive fraction in Stage 3 and re-calculate the percentage as the fraction divided by the sum of all the positive fractions of each components. Based on Equation 5 and the results of Table 7, the components are categorized into the change in beta term, the new beta term, the change in covariance term, and the residual component. While change in beta term has no contribution to Spread (*CAPM-FF3*), 80.7% of the spread-return relation is explained by the new beta term. This is because of the absence of Delta β^{SMB} for the change in beta term. Comparing across the three idiosyncratic volatility spread, Spread (*FF3-Carhart*), Spread (*FF3-FF3Liq*) and Spread (*FF3-FF5*), we see that the change in beta term and new beta term play an equally important role on the spread-return relation. The change in beta term collectively explains 38.6% to 50.3% of the relation while the new beta term captures another 44.3% to 54.9%. The relation attribute to the change in covariance term and residual term is virtually small. Overall, we conclude that the negative spread-return relation can be explained by both change in beta term (Delta β^{SMB}) and new beta term of the idiosyncratic volatility spread.

5. Further Exploration of Spread-Return Relation

In this section, we explore how conditional models with time-varying alpha and betas affect the spread-return relation and whether the relation still holds under EGARCH estimation method. To answer the first question, we estimate idiosyncratic volatility using conditional models. Following Ferson and Harvey (1999), we identify four state variables to estimate time-varying alpha and betas. Recall that Equation (9) shows that with conditional models, the idiosyncratic volatility spread, spread (*vavb*), is composed of two parts. The first part is the spread from the time-varying alpha, spread (*vacb*), as derived in Scenario 1 in Section 2.2. The second part is the spread from time-varying betas, spread (*cavb*), as shown in Scenario 2 in Section 2.2. Figure 2 plots the spreads for conditional models with time-varying alpha and constant betas, constant alpha and time-varying betas, and time-varying alpha and time-varying betas for the five factor models with four different state variables. We find that as the number of factors increases, spread (*cavb*) significantly increases compared to spread (*vacb*). Besides, the spread (*cavb*) is larger than spread (*vacb*) in each model. For example, the spread (*cavb*) jumps from below 1 percent for the CAPM to 1.9 percent for the FF3, increases to 2.4 percent for the Carhart and FF3Liq, and rises to 3 percent for the FF5. However, the spread (*vacb*) is quite stable across different factor models. It does not change significantly by introducing additional factors into the model. Finally, the spread (*cavb*) shows a consistent pattern, as in the case of spread (*vavb*).

Table 8 presents the results for the relation between stock returns and idiosyncratic volatility spread obtained from the conditional Fama-French three factor models. We use the state variable *Term* and conditional Fama-French three factor model as an example to illustrate the results, but our results are not sensitive to the choice of state variables and factor models analyzed in this study. In Panel A, Model (1) reports the cross-sectional regression result for idiosyncratic volatility estimated by a conditional model with time-varying alpha and constant betas. In Model (2), the corresponding idiosyncratic volatility spread is calculated from the unconditional Fama-French three factor model and its conditional version with time-varying alpha and constant betas. When controlling for beta, size, book-to-market ratio momentum, liquidity and leverage, a one percent increase in idiosyncratic volatility spread predicts 0.22 percent decrease in stock returns. This negative relation is statistically significant. It is still at the similar level when idiosyncratic volatility is controlled for in Model (3). This result supports that the negative spread-return relation is different from the relation between idiosyncratic volatility and stock returns. When we switch to idiosyncratic volatility estimated by a conditional model with constant alpha and time-varying betas in Panel B, the IVOL-return relation exhibits similar patterns as in the case of time-varying alpha and constant betas. Moreover, the negative relation between idiosyncratic volatility and stock returns is still statistically significant. Finally, when we apply that both alpha and betas are time-varying (Scenario 3 in Section 2.2) in Panel C, idiosyncratic volatility is still negatively related to stock returns and, similar to Panels A and B, the spread-return relation is independent from the negative relation between idiosyncratic volatility and stock returns. A comparison among all the coefficients of idiosyncratic volatility spread in Models (3), (6) and (9) shows that the magnitude of spread (*vavb*) is close to that of spread (*cavb*). This is consistent with the results in Figure 2 that the magnitude and change pattern of the two are close to each other. However, Model (10) regresses both spread (*vacb*) and spread (*cavb*), the components of spread (*vavb*), on stock returns and shows the negative spread-return relation is mainly driven by the spread of time-varying alpha not that of time-varying beta. Most importantly, we find that the above results are robust when we use the other four different factor models and their corresponding conditional versions to calculate the idiosyncratic volatility spread. Therefore, if an investor use both a factor model and its conditional version to estimate idiosyncratic volatility, the spread calculated is negatively related to stock returns.

In order to answer the second question, we estimated the expected idiosyncratic volatility using EGARCH method and then calculate the spread (E-Spread). Table 9 reports the regression results. Unlike the OLS estimation, the coefficients on the expected IVOL are positive and significant in Model (1). In line with Fu (2009), it shows that, as expected, when the idiosyncratic volatility increases by 1 percent, stock returns increase by 0.16 percent per month. From Model (2) to (5), we find that all coefficients on the spread are negative and significant. While the impact of the expected idiosyncratic volatility spread between FF3 and Carhart is marginally significant, coefficients of other spreads for different factor models are statistically significant. This result is consistent with the spread-return relation when idiosyncratic volatility is estimated using OLS method.

6. Conclusion

Investors who use two factor models to estimate idiosyncratic volatility will have access to additional information in the idiosyncratic volatility spread. Such spread does provide trading advantages to them as the spread predicts the cross section of stock returns. We provide evidence that the idiosyncratic volatility spread is negatively related with stock returns on both portfolio and individual stock levels. Stocks with high idiosyncratic volatility spread tend to have low future returns. A long-short portfolio trading strategy yields approximately 0.6 percent monthly return that is statistically and economically significant. This result is meaningful for a portfolio manager who want to create a new portfolio trading strategy and increase the portfolio return. Besides, the negative spread-return relation is different from the negative relation between idiosyncratic volatility and stock returns. A decomposition analysis shows that the relation is mainly driven by the change in beta term and new beta term derived between two factor models.

A professional investor such as a portfolio manager can also benefit from the spread-return relation by using different techniques when estimating idiosyncratic volatility spread. We show that the idiosyncratic volatility spread is negatively related to stock returns when calculated from an unconditional model and its conditional version. Time-varying alpha, instead of time-varying betas, contributes to this relation. The negative spread-return relation still holds when we estimate expected idiosyncratic volatility and then calculate the spread using EGARCH method. The negative spread-return relation is not affected by the methods when estimating idiosyncratic volatility. Finally, we also find the relation between idiosyncratic volatility and stock return is independent from the model used to estimate idiosyncratic volatility. All the five factor models and their conditional versions provide a similar negative relation between idiosyncratic volatility and stock return, which leaves the idiosyncratic volatility spread-return relation still “puzzling”.

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Appendix A

In this appendix, we show the decomposition of IVOL spread from equation (3) to equation (4). The IVOL spread is further decomposed by expending the variance terms.

$$Spread_{1-2} = Var\left(\sum_{q=1}^{N} \beta^q F^q + \sum_{q=N+1}^{M} \beta^q F^q\right) - Var\left(\sum_{p=1}^{N} \beta^p F^p\right) = \left(\sum_{q=1}^{N} Var(\beta^q F^q)\right) - \left(\sum_{p=1}^{N} Var(\beta^p F^p)\right) + \left(\sum_{q=N+1}^{M} Var(\beta^q F^q)\right) + 2\left(\sum_{1\le q1<q2\le N} Cov(\beta^{q1} F^{q1}, \beta^{q2} F^{q2})\right) - 2\left(\sum_{1\le p1<p2\le N} Cov(\beta^{p1} F^{p1}, \beta^{p2} F^{p2})\right) + 2\left(\sum_{N+1\le q1<q2\le M} Cov(\beta^{q1} F^{q1}, \beta^{q2} F^{q2})\right) + 2cov\left(\sum_{q=1}^{N} \beta^q F^q, \sum_{q=N+1}^{M} \beta^q F^q\right)$$

Appendix B

In this appendix, we provide the idiosyncratic volatility spread between factor models based on Equation (4). The estimated from CAPM and the Fama-French three factor model is given by

$$Spread_{CAPM-FF3} = \left({\beta^{SMB}}^2 Var(SMB) + {\beta^{HML}}^2 Var(HML)\right). \qquad (B.1)$$

Equation (A.1) shows that the spread is associated with variance of size and value factors. Since factor loadings are dependent on factors, this spread is determined by both first and second moment of size and value factors. Similarly, the spread of idiosyncratic risk estimated from the Fama-French three factor model and Carhart four factor model is determined by the extra momentum factor given by:

$$Spread_{FF3-Carhart} = \left({\beta^{MOM}}^2 Var(MOM)\right) \qquad (B.2)$$

The spread of idiosyncratic risk estimated from the Fama-French three factor model and liquidity adjusted Fama-French three factor model is:

$$Spread_{FF3-FF3Liq} = \left({\beta^{Liq}}^2 Var(Liq)\right) \qquad (B.3)$$

Since the Fama-French five factor model contains extra profitability and investment factors relative to the Fama-French three factor model, the spread of idiosyncratic risk between these two models is:

$$Spread_{FF3-FF5} = \left({\beta^{RMW}}^2 Var(RMW) + {\beta^{CMA}}^2 Var(CMA)\right) \qquad (B.4)$$

Appendix C

In this appendix, we show the idiosyncratic volatility spread failing to consider time-varying alpha and time-varying betas. Following Equation (8), CAPM comes with the spread given by:

$$Spread_{vavb}(CAPM) = [Var(\alpha) + 2Cov(\alpha, \beta^{MKT}F^{MKT})] \qquad (C.1)$$

Similarly, the spread from the Fama-French three factor model is:

$$\begin{aligned} Spread_{vavb}(FF3) &\\ = Var(\alpha) &+ 2Cov(\alpha, \beta^{MKT}F^{MKT} + \beta^{SMB}F^{SMB} + \beta^{HML}F^{HML}) \\ &+ 2Cov(\beta^{MKT}F^{MKT}, \beta^{SMB}F^{SMB}) + 2Cov(\beta^{MKT}F^{MKT}, \beta^{HML}F^{HML}) \\ &+ 2Cov(\beta^{SMB}F^{SMB}, \beta^{HML}F^{HML}) \qquad (C.2) \end{aligned}$$

For the Carhart four factor model, the spread is:

$$\begin{aligned} Spread_{vavb}(FF4) &\\ = Var(\alpha) &+ 2Cov(\alpha, \beta^{MKT}F^{MKT} + \beta^{SMB}F^{SMB} + \beta^{HML}F^{HML} + \beta^{MOM}F^{MOM}) \\ &+ 2Cov(\beta^{MKT}F^{MKT}, \beta^{SMB}F^{SMB}) + 2Cov(\beta^{MKT}F^{MKT}, \beta^{HML}F^{HML}) \\ &+ 2Cov(\beta^{MKT}F^{MKT}, \beta^{MOM}F^{MOM}) + 2Cov(\beta^{SMB}F^{SMB}, \beta^{HML}F^{HML}) \\ &+ 2Cov(\beta^{SMB}F^{SMB}, \beta^{MOM}F^{MOM}) + 2Cov(\beta^{HML}F^{HML}, \beta^{MOM}F^{MOM}) \quad (C.3) \end{aligned}$$

For the liquidity-adjusted Fama-French three factor model, the spread is:

$$\begin{aligned} Spread_{vavb}(FF3Liq) &\\ = \big[Var(\alpha) &+ 2Cov\big(\alpha, \beta^{MKT}F^{MKT} + \beta^{SMB}F^{SMB} + \beta^{HML}F^{HML} + \beta^{Liq}F^{Liq}\big) \\ &+ 2Cov(\beta^{MKT}F^{MKT}, \beta^{SMB}F^{SMB}) + 2Cov(\beta^{MKT}F^{MKT}, \beta^{HML}F^{HML}) \\ &+ 2Cov\big(\beta^{MKT}F^{MKT}, \beta^{Liq}F^{Liq}\big) + 2Cov(\beta^{SMB}F^{SMB}, \beta^{HML}F^{HML}) \\ &+ 2Cov\big(\beta^{SMB}F^{SMB}, \beta^{Liq}F^{Liq}\big) + 2Cov\big(\beta^{HML}F^{HML}, \beta^{Liq}F^{Liq}\big)\big] \qquad (C.4) \end{aligned}$$

The spread from the Fama-French five factor model is:

$$
\begin{aligned}
Spread_{vavb}(FF5) \\
&= Var(\alpha) \\
&+ 2Cov(\alpha, \beta^{MKT}F^{MKT} + \beta^{SMB}F^{SMB} + \beta^{HML}F^{HML} + \beta^{RMW}F^{RMW} + \beta^{CMA}F^{CMA}) \\
&+ 2Cov(\beta^{MKT}F^{MKT}, \beta^{SMB}F^{SMB}) + 2Cov(\beta^{MKT}F^{MKT}, \beta^{HML}F^{HML}) \\
&+ 2Cov(\beta^{MKT}F^{MKT}, \beta^{RMW}F^{RMW}) + 2Cov(\beta^{MKT}F^{MKT}, \beta^{CMA}F^{CMA}) \\
&+ 2Cov(\beta^{SMB}F^{SMB}, \beta^{HML}F^{HML}) + 2Cov(\beta^{SMB}F^{SMB}, \beta^{RMW}F^{RMW}) \\
&+ 2Cov(\beta^{SMB}F^{SMB}, \beta^{CMA}F^{CMA}) + 2Cov(\beta^{HML}F^{HML}, \beta^{RMW}F^{RMW}) \\
&+ 2Cov(\beta^{HML}F^{HML}, \beta^{CMA}F^{CMA}) \\
&+ 2Cov(\beta^{RMW}F^{RMW}, \beta^{CMA}F^{CMA}) \qquad (C.5)
\end{aligned}
$$

Table1 Summary Statistics

This table reports descriptive statistics of idiosyncratic volatility and spread estimated from five different factor models, for the period of July 1963 to June 2018. The idiosyncratic volatility, Idio(x), is defined as the standard deviation of regression residuals of daily stock returns in month *t−1* from factor model (x). Monthly Idio(x) is calculated by multiplying the residual standard deviation and the square root of the number of trading days for each month. Spread is the difference of idiosyncratic volatility estimated from two models. A stock should have at least 15 daily returns in a month to be included in estimation. *FF3* refers to Fama-French (1993) three factor model; *Carhart* represents Carhart (1997) four factor model; *FF3liq* denotes liquidity adjusted Fama-French three factor model; and *FF5* is Fama-French five factor model.

	Mean	S.D.	5%	25%	Median	75%	95 %	N
Panel A: Idiosyncratic Volatility								
Idio(*CAPM*)	0.1392	0.1328	0.0321	0.0645	0.1044	0.1707	0.3577	3,148,390
Idio(*FF3*)	0.1298	0.1255	0.0293	0.0595	0.0968	0.1591	0.3359	3,148,390
Idio(*Carhart*)	0.1253	0.1218	0.0279	0.0572	0.0933	0.1537	0.3256	3,148,390
Idio(*FF3Liq*)	0.1281	0.1241	0.0280	0.0586	0.0958	0.1574	0.3319	3,148,390
Idio(*FF5*)	0.1224	0.1190	0.0267	0.0557	0.0912	0.1502	0.3177	3,148,390
Panel B: Idiosyncratic Volatility Spread								
Spread (*CAPM-FF3*) *100	0.2904	0.5641	0.0028	0.0267	0.0924	0.2868	1.2669	3,148,390
Spread (*FF3-Carhart*) *100	0.1239	0.2831	0.0001	0.0044	0.0250	0.1044	0.5950	3,148,390
Spread (*FF3-FF3Liq*) *100	0.1232	0.2871	0.0001	0.0042	0.0243	0.1021	0.5908	3,148,390
Spread (*FF3-FF5*) *100	0.2628	0.5210	0.0014	0.0224	0.0795	0.2554	1.1638	3,148,390

Table 2 Correlations of IVOL and Spread

This table reports time-series means of cross-sectional Pearson correlation for both idiosyncratic volatility and spread for the period of July 1963 to June 2018. The idiosyncratic volatility, Idio(x), is defined as the standard deviation of regression residuals of daily stock returns in month *t−1* from factor model (x). Monthly Idio(x) is calculated by multiplying the residual standard deviation and the square root of the number of trading days for each month. Spread is the difference of idiosyncratic volatility estimated from two models. A stock should have at least 15 daily returns in a month to be included in estimation. *FF3* refers to Fama-French (1993) three factor model; *Carhart* represents Carhart (1997) four factor model; *FF3liq* denotes liquidity adjusted Fama-French three factor model; and *FF5* is Fama-French five factor model. *, **, and *** denote statistical significance level at 10%, 5%, and 1%, respectively based on their time-series standard error.

	Idio (*CAPM*)	Idio (*FF3*)	Idio (*Carhart*)	Idio (*FF3Liq*)	Idio (*FF5*)	Spread (*CAPM-FF3*)	Spread (*FF3-Carhart*)	Spread (*FF3-FF3Liq*)	Spread (*FF3-FF5*)
Idio(*CAPM*)	1.00	0.99***	0.98***	0.98***	0.98***	0.69***	0.58***	0.58***	0.69***
Idio(*FF3*)		1.00	0.99***	0.99***	0.99***	0.62***	0.57***	0.58***	0.70***
Idio(*Carhart*)			1.00	0.99***	0.99***	0.61***	0.52***	0.57***	0.69***
Idio(*FF3Liq*)				1.00	0.99***	0.62***	0.57***	0.53***	0.69***
Idio(*FF5*)					1.00	0.61***	0.56***	0.57***	0.63***
Spread (*CAPM-FF3*)						1.00	0.42***	0.41***	0.51***
Spread (*FF3-Carhart*)							1.00	0.43***	0.51***
Spread (*FF3-FF3Liq*)								1.00	0.50***
Spread (*FF3-FF5*)									1.00

Table 3 Double Sort by Size and Idiosyncratic Volatility Spread

Stocks are first sorted into five size quintile portfolios and then each size quintile is further sorted into five quintiles based on idiosyncratic volatility spread. As a result, it gives the equally weighted returns on 5x5 portfolios. Under each size quintile, the long-short trading strategy yield is calculated. The Jensen's alphas from CAPM and the Fama-French three factor model are reported in the last four rows. The last column reports the portfolio returns when all stocks are sorted into 5 quintiles based on idiosyncratic volatility spread. The Newey-West adjusted t-statistics are in parentheses. Panels A to D present the results for the four different idiosyncratic volatility spreads analyzed in this study. *, **, and *** denote statistical significance level at 10%, 5%, and 1%, respectively.

Panel A

	Size Quintiles					
Spread (*CAPM-FF3*) Quintiles	1-Small	2	3	4	5-Large	All Stocks
1-Low	1.07	1.11	1.20	1.18	1.03	1.10
2	1.09	1.10	1.26	1.18	1.05	1.13
3	1.05	1.15	1.20	1.24	1.06	1.13
4	0.84	1.01	1.04	1.14	1.06	1.01
5-High	0.37	0.38	0.66	0.86	0.89	0.51
High-Low	−0.70***	−0.73***	−0.55***	−0.32**	−0.13	−0.58***
	(−4.93)	(−4.90)	(−3.43)	(−2.10)	(−0.73)	(−3.93)
CAPM Alpha	−0.87***	−0.98***	−0.81***	−0.59***	−0.41***	−0.78***
	(−6.79)	(−7.43)	(−5.93)	(−4.55)	(−2.59)	(−6.05)
FF3 Alpha	−0.92***	−0.99***	−0.75***	−0.47***	−0.31**	−0.80***
	(−8.50)	(−9.03)	(−6.62)	(−4.32)	(−2.15)	(−9.08)

Panel B

	Size Quintiles					
Spread (*FF3-Carhart*) Quintiles	1-Small	2	3	4	5-Large	All Stocks
1-Low	0.98	1.10	1.22	1.17	1.01	1.09
2	0.98	1.13	1.21	1.19	1.05	1.12
3	0.92	1.08	1.16	1.20	1.06	1.09
4	0.89	0.97	1.02	1.13	1.05	1.01
5-High	0.45	0.44	0.75	0.91	0.83	0.58
High-Low	−0.53***	−0.65***	−0.47***	−0.26*	−0.18	−0.51***
	(−4.98)	(−5.43)	(−4.09)	(−1.95)	(−1.24)	(−4.41)
CAPM Alpha	−0.65***	−0.83***	−0.65***	−0.46***	−0.39***	−0.65***
	(−6.53)	(−7.71)	(−6.57)	(−3.98)	(−2.99)	(−6.36)
FF3 Alpha	−0.69***	−0.83***	−0.60***	−0.38***	−0.29**	−0.64***
	(−7.82)	(−8.97)	(−6.88)	(−3.83)	(−2.54)	(−8.89)

Panel C						
	Size Quintiles					
Spread (*FF3-FF3Liq*) Quintiles	1-Small	2	3	4	5-Large	All Stocks
1-Low	0.91	1.12	1.19	1.18	1.04	1.08
2	0.96	1.06	1.23	1.20	1.04	1.10
3	0.90	0.98	1.12	1.19	1.05	1.05
4	0.70	0.93	1.04	1.17	1.09	0.97
5-High	0.25	0.37	0.66	0.84	0.89	0.56
High-Low	−0.66***	−0.75***	−0.53***	−0.34**	−0.15	−0.62***
	(−5. 68)	(−6.42)	(−4.57)	(−2.53)	(−1.03)	(−5.41)
CAPM Alpha	−0.78***	−0.90***	−0.72***	−0.54***	−0.35***	−0.75***
	(−7.51)	(−8.87)	(−7.19)	(−4.67)	(−2.79)	(−7.50)
FF3 Alpha	−0.80***	−0.89***	−0.66***	−0.45***	−0.27**	−0.74***
	(−8.41)	(−9.70)	(−7.02)	(−4.51)	(−2.29)	(−9.59)

Panel D						
	Size Quintiles					
Spread (*FF3-FF5*) Quintiles	1-Small	2	3	4	5-Large	All Stocks
1-Low	1.10	1.18	1.24	1.16	1.03	1.12
2	1.20	1.28	1.37	1.25	1.08	1.22
3	1.06	1.23	1.27	1.28	1.09	1.19
4	0.94	1.03	1.06	1.16	1.07	1.03
5-High	0.38	0.26	0.56	0.72	0.73	0.42
High-Low	−0.72***	−0.92***	−0.68***	−0.44***	−0.24	−0.71***
	(−5.48)	(−6.33)	(−4.78)	(−2.67)	(−1.51)	(−4.79)
CAPM Alpha	−0.88***	−1.14***	−0.92***	−0.71***	−0.58***	−0.88***
	(−7.25)	(−9.20)	(−7.79)	(−5.09)	(−3.45)	(−6.85)
FF3 Alpha	−0.95***	−1.15***	−0.86***	−0.59***	−0.46***	−0.91***
	(−8.87)	(−10.78)	(−8.81)	(−4.99)	(−3.05)	(−9.80)

Table 4 Double Sort by BE/ME and Idiosyncratic Volatility Spread

Stocks are first sorted into five book-to-market quintile portfolios and then each quintile is further sorted into five quintiles based on idiosyncratic volatility spread. As a result, it gives the equally weighted returns on 5x5 portfolios. Under each book-to-market quintile, the long-short trading strategy yield is calculated. The Jensen's alphas from CAPM and the Fama-French three factor model are reported in the last four rows. The Newey-West adjusted t-statistics are in parentheses. Panels A to D present the results for the four different idiosyncratic volatility spreads analyzed in this study. *, **, and *** denote statistical significance level at 10%, 5%, and 1%, respectively.

Panel A

	BE/ME Quintiles				
Spread (*CAPM-FF3*) Quintiles	1-Low	2	3	4	5-High
1-Low	0.88	0.99	1.12	1.18	1.29
2	0.87	1.02	1.12	1.23	1.38
3	0.86	1.05	1.17	1.21	1.45
4	0.63	0.86	1.13	1.22	1.36
5-High	0.01	0.27	0.73	0.91	0.95
High-Low	−0.87***	−0.72***	−0.39**	−0.27*	−0.35**
	(−5.57)	(−4.83)	(−2.49)	(−1.66)	(−2.27)
CAPM Alpha	−1.06***	−0.90***	−0.58***	−0.46***	−0.52***
	(−7.58)	(−6.65)	(−4.14)	(−3.20)	(−3.88)
FF3 Alpha	−1.11***	−0.90***	−0.62***	−0.52***	−0.62***
	(−10.37)	(−8.11)	(−5.80)	(−4.75)	(−5.68)

Panel B

	BE/ME Quintiles				
Spread (*FF3-Carhart*) Quintiles	1-Low	2	3	4	5-High
1-Low	0.82	1.03	1.10	1.16	1.34
2	0.81	0.96	1.15	1.24	1.39
3	0.77	0.97	1.18	1.20	1.36
4	0.66	0.84	1.10	1.17	1.35
5-High	0.04	0.45	0.74	0.99	1.03
High-Low	−0.79***	−0.58***	−0.36***	−0.17	−0.30**
	(−6.72)	(−4.83)	(−2.80)	(−1.13)	(−2.45)
CAPM Alpha	−0.89***	−0.70***	−0.52***	−0.30***	−0.43***
	(−8.01)	(−6.51)	(−4.43)	(−2.64)	(−3.83)
FF3 Alpha	−0.92***	−0.69***	−0.54***	−0.33***	−0.49***
	(−10.56)	(−7.62)	(−5.68)	(−3.58)	(−5.19)

Panel C					
	BE/ME Quintiles				
Spread (*FF3-FF3Liq*) Quintiles	1-Low	2	3	4	5-High
1-Low	0.73	0.95	1.14	1.19	1.32
2	0.72	1.01	1.12	1.24	1.36
3	0.65	0.93	1.07	1.27	1.35
4	0.55	0.78	1.05	1.24	1.31
5-High	-0.14	0.38	0.61	0.86	0.89
High-Low	−0.87***	−0.57***	−0.52***	−0.34***	−0.42***
	(−7.08)	(−4.42)	(−4.33)	(−2.66)	(−3.34)
CAPM Alpha	−0.98***	−0.70***	−0.65***	−0.46***	−0.53***
	(−8.85)	(−5.86)	(−5.99)	(−4.11)	(−4.44)
FF3 Alpha	−0.99***	−0.70***	−0.65***	−0.45***	−0.54***
	(−10.14)	(−6.62)	(−6.97)	(−4.81)	(−5.27)

Panel D					
	BE/ME Quintiles				
Spread (*FF3-FF5*) Quintiles	1-Low	2	3	4	5-High
1-Low	0.87	1.02	1.13	1.19	1.33
2	0.98	1.10	1.23	1.29	1.48
3	0.91	1.03	1.21	1.27	1.51
4	0.63	0.83	1.13	1.25	1.40
5-High	-0.13	0.32	0.49	0.89	0.89
High-Low	−1.00***	−0.71***	−0.64***	−0.30*	−0.45***
	(−6.98)	(−4.51)	(−4.19)	(−1.79)	(−2.99)
CAPM Alpha	−1.16***	−0.88***	−0.81***	−0.48***	−0.59***
	(−8.80)	(−6.71)	(−5.92)	(−3.26)	(−4.41)
FF3 Alpha	−1.20***	−0.89***	−0.84***	−0.52***	−0.66**
	(−11.85)	(−7.51)	(−8.07)	(−4.51)	(−6.06)

Table 5 The Negative Spread-Return Relation

This table reports the results of firm-level Fama-MacBeth (1973) regressions of monthly stock returns on idiosyncratic volatility spread for the period of July 1963 to June 2018. Spread is the difference of idiosyncratic volatility estimated from two models. It is scaled up by multiplying 100. Beta is the regression coefficient of the past three years of monthly returns on market returns. ME and B/M are size and book to market ratio in Fama and French (1992). Ret (-2,-7) is the compound gross return from month (t-7) to (t-2). TURN and CVTURN are the average volume turnover and coefficient of variance of TURN calculated over the past 36 months in Chordia, Subrahmanyam and Anshuman (2001). Leverage is the ratio between total asset and market equity from previous fiscal year end. Newey-West adjusted t-value is reported in parentheses. To avoid the effect of possibly spurious outliers, all explanatory variables below 0.5 (above 99.5) percentile are set equal to 0.5 (99.5) percentile. *, **, and *** denote statistical significance level at 10%, 5%, and 1%, respectively.

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Beta	0.31**	0.40*	0.42**	0.41**	0.41**	0.45*
	(2.10)	(1.97)	(2.04)	(2.02)	(2.00)	(1.96)
Ln(ME)	-0.09**	-0.14***	-0.15***	-0.14***	-0.14***	-0.14***
	(-2.14)	(-3.67)	(-4.32)	(-4.01)	(-4.03)	(-3.77)
Ln(B/M)	0.29***	0.23***	0.22***	0.22***	0.22***	0.24***
	(4.90)	(5.49)	(5.34)	(5.47)	(5.40)	(5.60)
Ret(-2,-7)		0.77***	0.73***	0.74***	0.76***	0.73***
		(5.19)	(5.06)	(5.14)	(5.18)	(4.81)
Ln(TURN)		-0.24***	-0.23***	-0.23***	-0.24***	-0.25***
		(-5.44)	(-5.16)	(-5.24)	(-5.37)	(-5.52)
Ln(CVTURN)		-0.36***	-0.34***	-0.34***	-0.35***	-0.36***
		(-6.59)	(-6.40)	(-6.37)	(-6.44)	(-6.49)
Ln(Leverage)		-0.07**	-0.06*	-0.06*	-0.07*	-0.06*
		(-1.98)	(-1.92)	(-1.91)	(-1.92)	(-1.84)
Spread (*CAPM-FF3*)			-0.41***			
			(-6.46)			
Spread (*FF3-Carhart*)				-0.54***		
				(-5.27)		
Spread (*FF3-FF3Liq*)					-0.43***	
					(-5.67)	
Spread (*FF3-FF5*)						-0.26***
						(-3.72)
Adj. R2	0.05	0.08	0.09	0.09	0.09	0.09

Table 6 Spread-Return Relation with IVOL

This table reports the results of firm-level Fama-MacBeth (1973) regressions of monthly stock returns on idiosyncratic volatility spread when idiosyncratic volatility is controlled for the period of July 1963 to June 2018. Spread is the difference of idiosyncratic volatility estimated from two models. It is scaled up by multiplying 100. Beta is the regression coefficient of the past three years of monthly returns on market returns. ME and B/M are size and book to market ratio in Fama and French (1992). Ret (-2,-7) is the compound gross return from month (t-7) to (t-2). TURN and CVTURN are the average volume turnover and coefficient of variance of TURN calculated over the past 36 months in Chordia, Subrahmanyam and Anshuman (2001). Leverage is the ratio between total asset and market equity from previous fiscal year end. Newey-West adjusted t-value is reported in parentheses. To avoid the effect of possibly spurious outliers, all explanatory variables below 0.5 (above 99.5) percentile are set equal to 0.5 (99.5) percentile. *, **, and *** denote statistical significance level at 10%, 5%, and 1%, respectively.

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8
Beta	0.42**	0.42**	0.42**	0.42**	0.45**	0.45**	0.44**	0.44**
	(2.15)	(2.14)	(2.14)	(2.14)	(2.08)	(2.07)	(2.14)	(2.15)
Ln(ME)	-0.18***	-0.18***	-0.18***	-0.18***	-0.17***	-0.17***	-0.18***	-0.18***
	(-5.91)	(-5.90)	(-5.89)	(-5.87)	(-5.34)	(-5.32)	(-5.74)	(-5.72)
Ln(B/M)	0.20***	0.20***	0.20***	0.21***	0.23***	0.23***	0.21***	0.21***
	(5.07)	(5.08)	(5.07)	(5.11)	(5.11)	(5.10)	(5.12)	(5.11)
Ret(-2,-7)	0.75***	0.74***	0.75***	0.74***	0.66***	0.66***	0.72***	0.73***
	(5.26)	(5.24)	(5.28)	(5.21)	(4.54)	(4.52)	(5.14)	(5.17)
Ln(TURN)	-0.20***	-0.20***	-0.23***	-0.20***	-0.25***	-0.25***	-0.20***	-0.21***
	(-4.73)	(-4.70)	(-4.78)	(-4.74)	(-6.11)	(-6.09)	(-4.83)	(-4.89)
Ln(CVTURN)	-0.31***	-0.31***	-0.32***	-0.32***	-0.33***	-0.32***	-0.32***	-0.32***
	(-6.01)	(-6.04)	(-6.05)	(-6.09)	(-5.98)	(-5.94)	(-6.05)	(-6.12)
Ln(Leverage)	-0.05	-0.05	-0.05	-0.05	-0.07*	-0.06*	-0.07**	-0.07**
	(-1.48)	(-1.46)	(-1.48)	(-1.45)	(-1.86)	(-1.85)	(-1.98)	(-1.97)
IVOL(*FF3*)	-0.04***	-0.04***						
	(-7.02)	(-6.09)						
Spread (*CAPM-FF3*)		-0.15**						
		(-2.23)						
IVOL(*Carhart*)			-0.04***	-0.04***				
			(-6.94)	(-6.40)				
Spread (*FF3-Carhart*)				-0.23**				
				(-2.29)				
IVOL (*FF3Liq*)					-0.04***	-0.04***		
					(-6.62)	(-6.34)		
Spread (*FF3-FF3Liq*)						-0.10		
						(-1.48)		
IVOL (*FF5*)							-0.04***	-0.03***
							(-6.37)	(-5.17)
Spread (*FF3-FF5*)								-0.16***
								(-2.96)
Adj. R2	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09

Table 7 Decomposing the Spread-Return Relation

Based on a 3-stage decomposition method, the negative relation between month t − 1 idiosyncratic volatility spread and month t returns is decomposed into a number of components each is related to a candidate variable and a residual component. Using firm-level Fama-MacBeth cross-sectional regressions, Stage 1 regresses month t returns on month t – 1 *Spread* ($R_{it} = \alpha_t + \gamma_t Spread_{it-1} + \varepsilon_{it}$). Stage 2 regresses *Spread* on the candidate variable ($Spread_{it-1} = \alpha_{t-1} + \delta_{t-1} Candidate_{it-1} + \mu_{it-1}$) to decompose *Spread* into two orthogonal components: ($\delta_{t-1} Candidate_{it-1}$) and ($\alpha_{t-1} + \mu_{it-1}$). In stage 3, the γ_t from Stage 1 is decomposed as: $\gamma_t = \frac{Cov[R_{it}, Spread_{it-1}]}{Var[Spread_{it-1}]} = \frac{Cov[R_{it},\ \delta_{t-1} Candidate_{it-1}]}{Var[Spread_{it-1}]} + \frac{Cov[R_{it},\ \alpha_{t-1}+\mu_{it-1}]}{Var[Spread_{it-1}]} = \gamma_t^C + \gamma_t^R$. Several candidate variables are chosen from the idiosyncratic volatility spread components in Equation (5). Therefore, there is a γ_t^{Ci} for each candidate variable and one γ_t^R for residual component. The γ_t^{Ci}'s will be calculated and the time-series average of each γ_t^{Ci} divided by the time-series average of γ_t then measures the fraction of the negative spread-return relation explained by the candidate variable. The average of the only γ_t^R divided by the average γ_t measures the fraction of the relation left unexplained by the candidate variables. *, **, and *** denote statistical significance level at 10%, 5%, and 1%, respectively.

Panel A: Components of Spread (*CAPM-FF3*)

Stage		Variable	Coeff.	Fraction	t-stat
1	Return on Spread	Intercept	1.26***		(5.47)
		Spread	-20.96***		(-2.84)
2	Spread on Components	Delta β^{MKT}	-0.03***		(-5.45)
		New β^{SMB}	0.61***		(51.52)
		New β^{HML}	0.56***		(41.31)
		$\text{Delta}_{\text{Cov}}$	-0.03***		(-10.13)
		Adj-R2	0.93		
3	Decompose Stage 1 Spread Coefficient				
		Delta β^{MKT}	1.83**	-8.7%	(2.22)
		New β^{SMB}	-9.87**	47.1%	(2.26)
		New β^{HML}	-8.51*	40.6%	(-1.87)
		$\text{Delta}_{\text{Cov}}$	-3.07***	14.6%	(-2.92)
		Residual	-1.34*	6.4%	(-0.97)
		Total	-20.96***	100%	(-2.84)

Panel B: Components of Spread (*FF3-Carhart*)

Stage		Variable	Coeff.	Fraction	t-stat
1	Return on Spread	Intercept	1.24***		(5.27)
		Spread	-36.17***		(-3.37)
2	Spread on Components	Delta β^{MKT}	-0.01		(-0.60)
		Delta β^{SMB}	-0.01**		(-2.39)
		Delta β^{HML}	0.00		(0.12)
		New β^{MOM}	1.27***		(27.41)
		$\text{Delta}_{\text{Cov}}$	-0.01**		(-2.40)
		Adj-R2	0.95		
3	Decompose Stage 1 Spread Coefficient				
		Delta β^{MKT}	-0.27	0.7%	(-0.37)
		Delta β^{SMB}	-33.99**	93.9%	(-2.75)
		Delta β^{HML}	-0.13	0.4%	(-0.13)
		New β^{MOM}	-30.27**	83.7%	(-2.12)
		$\text{Delta}_{\text{Cov}}$	32.13	-88.8%	(1.12)
		Residual	-3.64**	10.1%	(-2.07)
		Total	-36.17***	100%	(-3.37)

Panel C: Components of Spread (*FF3-FF3Liq*)					
Stage		Variable	Coeff.	Fraction	t-stat
1	Return on Spread	Intercept	1.24***		(5.24)
		Spread	-37.33***		(-3.39)
2	Spread on Components	Delta β^{MKT}	0.01		(0.45)
		Delta β^{SMB}	-0.05**		(-2.41)
		Delta β^{HML}	0.01		(1.02)
		New β^{IML}	0.75***		(76.38)
		$\text{Delta}_{\text{Cov}}$	-0.01***		(-10.88)
		Adj-R2	0.96		
3	Decompose Stage 1 Spread Coefficient	Delta β^{MKT}	-0.66	1.8%	(-1.60)
		Delta β^{SMB}	-13.66**	36.6%	(-1.98)
		Delta β^{HML}	-0.07	0.2%	(-0.17)
		New β^{IML}	-20.49***	54.9%	(-2.82)
		$\text{Delta}_{\text{Cov}}$	-0.68	1.8%	(-0.77)
		Residual	-1.75	4.7%	(-1.28)
		Total	-37.33***	100%	(-3.39)

Panel D: Components of Spread (*FF3-FF5*)					
Stage		Variable	Coeff.	Fraction	t-stat
1	Return on Spread	Intercept	1.26***		(5.42)
		Spread	-30.11***		(-3.15)
2	Spread on Components	Delta β^{MKT}	0.03***		(5.71)
		Delta β^{SMB}	-0.01***		(-6.57)
		Delta β^{HML}	0.02***		(5.01)
		New β^{RMW}	0.45***		(39.03)
		New β^{CMA}	0.39***		(35.30)
		$\text{Delta}_{\text{Cov}}$	-0.01***		(-6.59)
		Adj-R2	0.93		
3	Decompose Stage 1 Spread Coefficient	Delta β^{MKT}	0.33	1.1%	(0.71)
		Delta β^{SMB}	-17.59***	58.4%	(-3.02)
		Delta β^{HML}	0.21	-0.7%	(0.29)
		New β^{RMW}	-12.85***	42.7%	(-2.97)
		New β^{CMA}	-10.83**	36.0%	(-2.31)
		$\text{Delta}_{\text{Cov}}$	12.98	-43.1%	(0.96)
		Residual	-2.37	7.9%	(-1.62)
		Total	-30.11***	100%	(-3.15)

Table 8 Idiosyncratic Volatility Spread from Conditional Models

This table reports the results of firm-level Fama-MacBeth (1973) regressions of monthly stock returns on idiosyncratic volatility spread from the Fama-French three-factor model and its conditional version. IVOL(*vacb*) is estimated from the conditional FF3 with time-varying alpha and constant beta. Spread (*vacb*) is the corresponding idiosyncratic volatility spread derived at Scenario 1 in Section 2.2. IVOL(*cavb*) is estimated from the conditional FF3 with constant alpha and time-varying beta. Spread (*cavb*) is the corresponding idiosyncratic volatility spread derived at Scenario 2 in Section 2.2. IVOL(*vacb*) and therefore Spread (*vavb*) are calculated using the conditional FF3 with time-varying alpha and time-varying beta at Scenario 3 in Section 2.2. Spread is scaled up by multiplying 100. Beta is the regression coefficient of the past three years of monthly returns on market returns. ME and B/M are size and book to market ratio in Fama and French (1992). Ret (-2,-7) is the compound gross return from month (t-7) to (t-2). TURN and CVTURN are the average volume turnover and coefficient of variance of TURN calculated over the past 36 months in Chordia, Subrahmanyam and Anshuman (2001). Leverage is the ratio between total asset and market equity from previous fiscal year end. Newey-West adjusted t-value is reported in parentheses. To avoid the effect of possibly spurious outliers, all explanatory variables below 0.5 (above 99.5) percentile are set equal to 0.5 (99.5) percentile. *, **, and *** denote statistical significance level at 10%, 5%, and 1%, respectively.

	Panel A: FF3 with TV Alpha			**Panel B: FF3 with TV Beta**			**Panel C: FF3 with TV Alpha and TV Beta**			
	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9	Model 10
Beta	0.43**	0.40**	0.43**	0.43**	0.41**	0.43**	0.42**	0.41**	0.43**	0.43**
	(2.09)	(1.98)	(2.12)	(2.09)	(1.98)	(2.11)	(2.08)	(2.01)	(2.11)	(2.11)
Ln(ME)	-0.18***	-0.14***	-0.18***	-0.18***	-0.15***	-0.18***	-0.18***	-0.15***	-0.18***	-0.18***
	(-5.65)	(-3.87)	(-5.68)	(-5.61)	(-4.11)	(-5.69)	(-5.56)	(-4.24)	(-5.69)	(-5.62)
Ln(B/M)	0.21***	0.22***	0.21***	0.21***	0.22***	0.21***	0.21***	0.22***	0.21***	0.21***
	(5.16)	(5.49)	(5.16)	(5.18)	(5.41)	(5.15)	(5.20)	(5.39)	(5.16)	(5.18)
Ret(-2,-7)	0.74***	0.79***	0.74***	0.74***	0.77***	0.74***	0.75***	0.76***	0.74***	0.74***
	(5.10)	(5.17)	(5.07)	(5.08)	(5.15)	(5.05)	(5.12)	(5.12)	(5.06)	(5.08)
Ln(TURN)	-0.20***	-0.23***	-0.20***	-0.20***	-0.23***	-0.20***	-0.20***	-0.23***	-0.20***	-0.20***
	(-4.58)	(-5.16)	(-4.58)	(-4.59)	(-5.04)	(-4.53)	(-4.63)	(-4.99)	(-4.45)	(-4.60)
Ln(CVTURN)	-0.32***	-0.35***	-0.31***	-0.32***	-0.35***	-0.31***	-0.32***	-0.34***	-0.31***	-0.32***
	(-6.04)	(-6.39)	(-6.00)	(-6.00)	(-6.47)	(-5.99)	(-6.03)	(-6.41)	(-5.99)	(-6.05)
Ln(Leverage)	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05
	(-1.44)	(-1.48)	(-1.43)	(-1.45)	(-1.44)	(-1.42)	(-1.44)	(-1.44)	(-1.43)	(-1.41)
IVOL(*vacb*)	-0.04***		-0.04***							
	(-6.46)		(-6.05)							
Spread(*vacb*)		-0.22***	-0.16***							-0.15***
		(-5.82)	(-4.99)							(-4.48)
IVOL(*cavb*)				-0.04***		-0.04***				
				(-6.54)		(-6.22)				
Spread(*cavb*)					-0.09***	-0.04***				-0.01
					(-4.87)	(-2.95)				(-0.45)
IVOL(*vavb)*							-0.04***		-0.04***	-0.04***
							(-6.37)		(-5.86)	(-5.88)
Spread(*vavb*)								-0.09***	-0.05***	
								(-5.60)	(-4.21)	
Adj. R2	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09

Table 9 Spread-Return Relation with EGARCH

This table reports the results of firm-level Fama-MacBeth (1973) regressions of monthly stock returns on expected idiosyncratic volatility spread estimated by EGARCH. E(IVOL) is the expected idiosyncratic volatility estimated using EGARCH from past monthly stock returns up to month t-1 on Fama-French three-factor model, where E(IVOL) is defined as the conditional standard deviation for EGARCH (1, 1). A stock should have at least 30 monthly returns to be included in EGARCH estimation. E-Spread is the difference of expected idiosyncratic volatility estimated from two models. It is scaled up by multiplying 100. Beta is the regression coefficient of the past three years of monthly returns on market returns. ME and B/M are size and book to market ratio in Fama and French (1992). Ret (-2,-7) is the compound gross return from month (t-7) to (t-2). TURN and CVTURN are the average volume turnover and coefficient of variance of TURN calculated over the past 36 months in Chordia, Subrahmanyam and Anshuman (2001). Leverage is the ratio between total asset and market equity from previous fiscal year end. Newey-West adjusted t-value is reported in parentheses. To avoid the effect of possibly spurious outliers, all explanatory variables below 0.5 (above 99.5) percentile are set equal to 0.5 (99.5) percentile. *, **, and *** denote statistical significance level at 10%, 5%, and 1%, respectively.

	Model 1	Model 2	Model 3	Model 4	Model 5
Beta	0.27	0.47**	0.46**	0.47**	0.46**
	(1.35)	(2.27)	(2.24)	(2.25)	(2.24)
Ln(ME)	0.11***	-0.16***	-0.16***	-0.16***	-0.14***
	(3.65)	(-4.36)	(-4.27)	(-4.27)	(-4.31)
Ln(B/M)	0.33***	0.20***	0.20***	0.20***	0.20***
	(7.36)	(4.82)	(4.80)	(4.83)	(4.72)
Ret(-2,-7)	0.97***	0.73***	0.74***	0.74***	0.74***
	(6.55)	(4.76)	(4.81)	(4.82)	(4.78)
Ln(TURN)	-0.51***	-0.25***	-0.25***	-0.25***	-0.25***
	(-11.97)	(-5.47)	(-5.52)	(-5.59)	(-5.52)
Ln(CVTURN)	-0.60***	-0.37***	-0.37***	-0.37***	-0.37***
	(-10.63)	(-6.71)	(-6.71)	(-6.72)	(-6.79)
Ln(Leverage)	0.01	-0.04	-0.04	-0.04	-0.04
	(0.19)	(-1.14)	(-1.11)	(-1.21)	(-1.14)
E(IVOL)	0.16***				
	(12.24)				
E-Spread (*CAPM-FF3*)		-0.21***			
		(-6.44)			
E-Spread (*FF3-Carhart*)			-0.13*		
			(-1.72)		
E-Spread (*FF3-FF3Liq*)				-0.24***	
				(-8.17)	
E-Spread (*FF3-FF5*)					-0.22***
					(-6.83)
Adj. R2	0.09	0.09	0.09	0.09	0.09

Figure 1 Components of Idiosyncratic Volatility Spread

This figure plots the contribution of the components of spread to the spread-return relation. Components that explain a positive fraction of the spread-return relation are selected from Table 7 and then categorized into the change in beta term (Delta β'^{s}), the new beta term (New β'^{s}), the change in covariance term ($\text{Delta}_{\text{Cov}}$), and the residual component. The percentage is calculated as the fraction divided by the sum of all the positive fractions of each components. *FF3* refers to the Fama-French (1993) three factor model; *Carhart* represents the Carhart (1997) four factor model; *FF3liq* denotes the liquidity adjusted Fama-French three factor model; and *FF5* is the Fama-French five factor model.

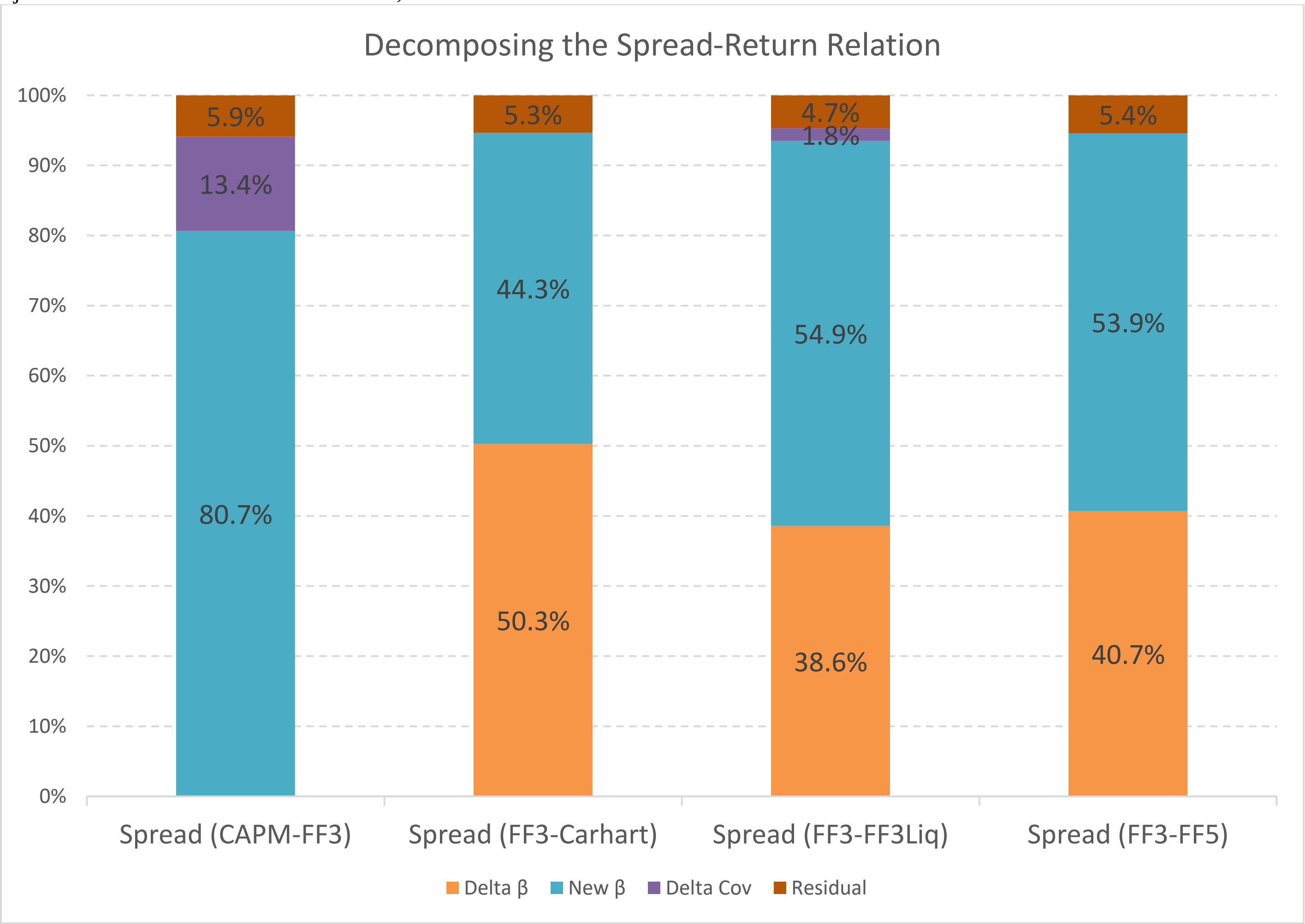
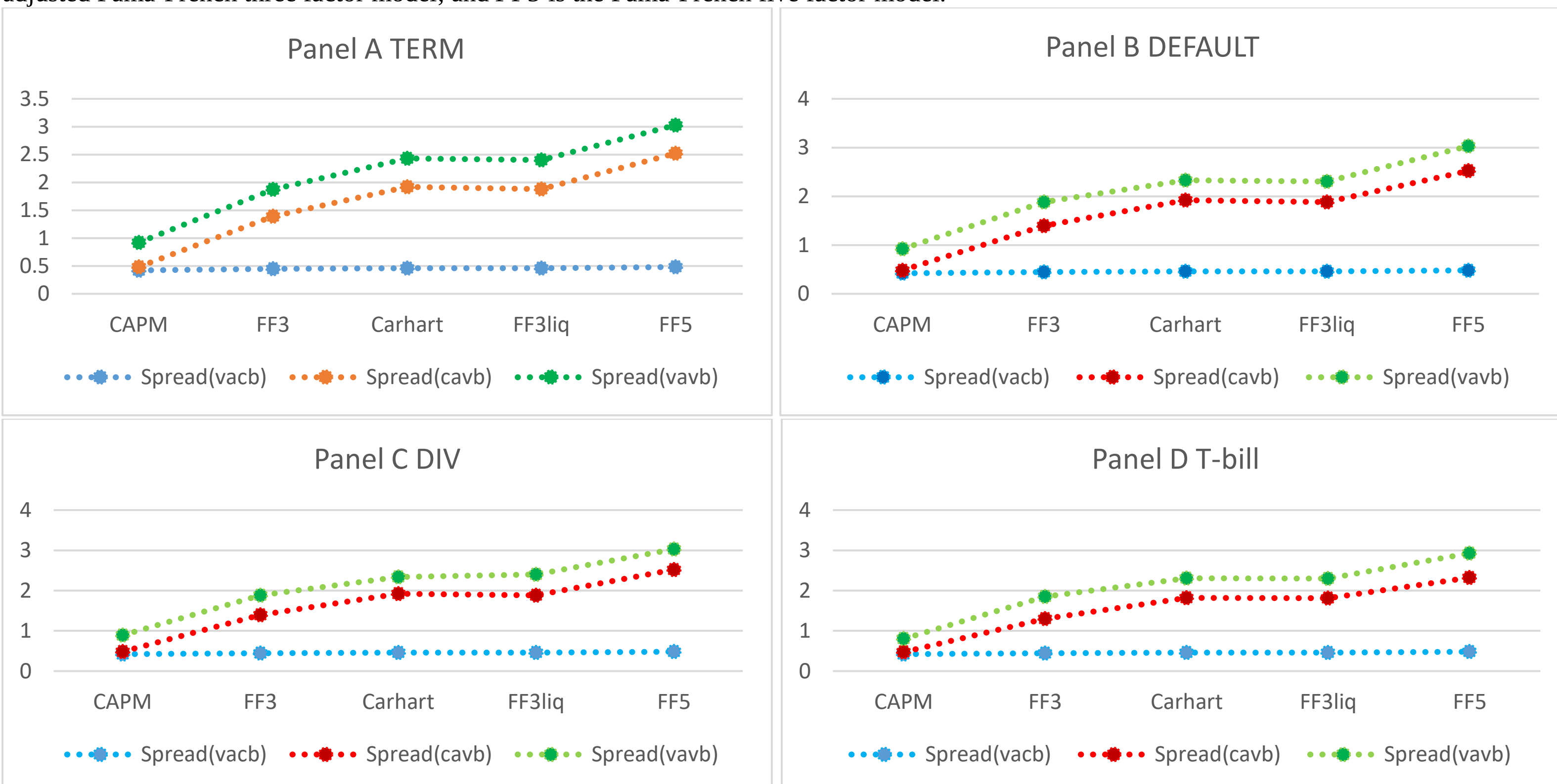


Figure 2 Idiosyncratic Volatility Spread between Unconditional and Conditional Models

This figure plots the idiosyncratic volatility spread for time-varying alpha and betas estimated from different conditional factor models using state variables *TERM*, *DEFAULT*, *DIV* and *T-bill*. The blue dashed line is the spread from an unconditional model and its conditional version where only alpha varies over time. The red dashed line is the spread from an unconditional model and its conditional version where beta varies over time. The green dashed line is the spread from an unconditional model and its conditional version where both alpha and beta are time-varying. The magnitude of spread is reported in percentage. *FF3* refers to the Fama-French (1993) three factor model; *Carhart* represents the Carhart (1997) four factor model; *FF3liq* denotes the liquidity adjusted Fama-French three factor model; and *FF5* is the Fama-French five factor model.



Essay Two: Alpha Beta Risk and Stock Returns

Abstract

The variance of stock returns is decomposed based on a conditional Fama–French three-factor model instead of its unconditional counterpart. Using time-varying alpha and betas in this model, it is evident that four additional risk terms must be considered. They include the variance of alpha, the variance of the interaction between the time-varying component of beta and factors, and two covariance terms. These additional risk terms are components that are included in the idiosyncratic risk estimate using an unconditional model. By investigating the relation between the risk terms and stock returns, we find that only the variance of the time-varying alpha is negatively associated with stock returns. Further tests show that stock returns are not affected by the variance of time-varying beta. These results are consistent with the findings in the literature identifying return predictability from time-varying alpha rather than betas.

JEL Classifications: G12 G32

Keywords: Conditional Model, Time-varying alpha and betas, Alpha beta risk, Idiosyncratic volatility, Stock returns.

1. Introduction

In an influential study, Ang et al. (2006) found that high idiosyncratic risk, defined as the standard deviation of the residuals from the Fama–French (1993) model, predicts low individual stock returns. This finding is puzzling because the modern portfolio theory by Markowitz (1952) suggests that investors should be compensated only for systematic risk in equilibrium. In addition, Merton (1987) and Malkiel and Xu (2002) argue that idiosyncratic risk is positively related to expected stock returns when portfolios are not perfectly diversified. However, the negative relation between idiosyncratic risk and stock returns is confirmed in other studies by Ang, Hodrick, Xing and Zhang (2009); and Hou and Loh (2016). All of these studies use an unconditional Fama–French three-factor model to estimate idiosyncratic risk. However, if the unconditional asset pricing model fails to account for important nature of the parameters such as a time-varying property, the negative relation may be misleading since the estimate of idiosyncratic risk is not purely idiosyncratic (i.e., it contains certain components that may exhibit systematic patterns in predicting stock returns).

In this paper, we decompose total risk based on a conditional Fama–French three-factor model. Using time-varying alpha and betas in this model, we find that four additional terms—the variance of alpha, the variance of the interaction between time-varying component of beta and its respective factors, plus two covariance terms—are the components included in the idiosyncratic risk estimated using an unconditional model. The main task of this paper is to study the relation between those components and stock returns, and how they affect the relation between idiosyncratic risk and stock returns.

The predictability of returns has been widely studied (e.g. Balvers et al. (1990), Patelis (1997), Avramov and Wermers (2006), Avramov et al. (2011), and Banegas et al. (2013), among others). Since Keim and Stambaugh (1986), conditional asset pricing models that incorporate instrumental variables have been studied to predict future stock returns. These models, which contain time-varying alpha or time-varying betas, provide new insight into explaining stock returns. For example, Jagannathan and Wang (1998), Ferson and Harvey (1999), and Lettau and Ludvigson (2001)—all of whom use macroeconomic variables as instrumental variables—show that a conditional model performs better than its unconditional counterpart. Moreover, Avramov and Chordia (2006) use an optimal portfolio strategy derived from mean-variance theory and demonstrate that the conditional model provides predictability for stock returns that is associated with the time-varying property of the model. They find that the optimal portfolio performs better when accounting for time-varying alpha, which implies that the return predictability is determined by the time-varying alpha. The question still remains whether this predictability comes from the additional sources of risk that are linked to the framework of conditional pricing models (i.e., the time-varying alpha) or from the time-varying betas. While the stock return variance has been extensively studied in the literature (see, for example, Barclay et al. (1990), Bollen (1998), Bekaert and Wu (2000), and Vliet (2007), among others) and an increasing number of studies have investigated the relation between variance and return (e.g., French et al. (1987), Campbell and Hentschel (1992), Glosten et al. (1993), Braun et al. (1995), Duffee (1995), Dennis et al. (2006), Bollerslev et al. (2009), and Carr and Wu (2009)), few studies has been conducted from the perspective of idiosyncratic risk components. Due to the fact that the volatility of alpha is a component of idiosyncratic risk in an unconditional model and that the volatility of alpha may be an additional source of risk that comes with a conditional model, we label it "alpha risk" in this study. Since time-varying betas also play an indirect role in the components of idiosyncratic volatility, we define "beta risk" as the volatility of time-varying betas. Although Brooks et al. (1998) also study time-varying beta risk, this study is different in that we use conditional models following Ferson and Harvey (1999) rather than generalized autoregressive conditional heteroscedasticity (GARCH) and Kalmam filter approaches.

The results in literature suggest that investors are supposed to use conditional models instead of unconditional models to capture more information reflected in the stock price. As the conditional one better captures the time-varying property of alpha and beta and relaxes the assumption of conventional asset pricing models, investors will better off when use a conditional model to predict future stock returns. This study contributes to the conditional asset pricing model by analyzing the information captured by it. We find that the volatility of alpha can predict average stock returns, but the volatility of betas fails to do so in general. It is possible that the predictability is driven by the instrumental variable because the alpha and betas evolve directly with the macroeconomic variables. In a conditional model, macroeconomic variables are usually selected from those that predict stock returns. We then explore the negative relation between idiosyncratic risk and stock returns by controlling for these additional terms. Therefore, we also explore how the alpha beta risk interacts with idiosyncratic volatility in determining average stock returns. Last, we investigate whether the return predictability from the alpha risk is affected by the time-varying alpha itself. Supporting Avramov and Chordia (2006), we find that the time-varying alpha predicts average stock returns, but that this relation does not impact the relation between idiosyncratic risk and stock returns.

The findings of this paper are three-fold: (1) alpha risk predicts average stock returns at both the portfolio level and the firm level, which is not true for beta risk in general; specifically, stocks with higher alpha risk will have lower future returns; (2) alpha risk predicts returns for both small and medium stocks with a low book-to-market ratio, but does not predict returns for large stocks; and (3) the return predictability from alpha risk at the firm level is not driven by idiosyncratic risk, macroeconomic variables, or the time-varying alpha itself.

2. Alpha Beta Risk

In this section, we theoretically demonstrate why alpha beta risk is associated with stock returns. We adopt the decomposition approach similar to Campbell et al. (2001) and Xu and Malkiel (2003). First, we have the following relation:

$$Var(R_{i,t}) = Var(R_{M,t}) + Var(r_{i,t}) + 2Cov(R_{M,t}, r_{i,t}) \qquad (1)$$

where $Var(R_{i,t})$ is the total risk of stock i. $R_{M,t}$ is the systematic element of the stock's return and $r_{i,t}$ is its idiosyncratic return component. Because by definition $Cov(R_{M,t}, r_{i,t}) = 0$, the idiosyncratic risk is $Var(r_{i,t}) = Var(R_{i,t}) - Var(R_{M,t})$.

We consider a generalized factor model that contains N risk factors. It is fitted to individual stocks as follows:

$$R_{i,t} = \alpha_i + \beta_i' F_t + \varepsilon_{i,t} \qquad (2)$$

where F_t is an N × 1 vector that contains the N risk factor at time (t) and β_i' is a 1 × N vector that contains the corresponding factor loading for stock (i). Based on Equations (1) and (2), we decompose the variance of stock returns under an unconditional model as follows:

$$Var(R) = Var(\alpha + \beta' F) + Idio = Var(\beta' F) + \ Idio \tag{3}$$

where $Idio \equiv Var(\varepsilon)$. We omit the subscripts (*i*) and (*t*) for simplicity.

Considering that the alpha and betas are time-varying conditional on macro-economic state variables, we assume that both consist of a constant component and a time-varying component as follows:

$$\alpha_t = \alpha_0 + \alpha_1 \times f(z_{t-1}) \tag{4}$$

$$\beta_t = \beta_0 + \beta_1 \times f(z_{t-1}) \tag{5}$$

where $f(z_{t-1})$ is a scalar of a function of the macroeconomic state variable that drives the change in alpha and beta over time. β_0 and β_1 are N × 1 vectors. Note that Equations (4) and (5) nest the unconditional version of the alpha and betas, where α_1 and β_1 are assumed to be equal to zero. In the circumstances of Equations (4) and (5), the variance of stock returns can be decomposed as follows:

$$\begin{aligned} Var(R) &= Var(\alpha) + Var(\beta' F) + 2Cov(\alpha, \beta' F) + \ Idio_c \\ &= \ Var(\alpha) + Var((\beta_0{}' + \beta_1{}' \times f(z_t))F) + 2Cov(\alpha, \beta' F) \\ &\quad + Idio_c \\ &= Var(\alpha) + Var(\beta_0{}' F) + Var(\beta_1{}' \times f(z_t) F) \\ &\quad + 2Cov(\beta_0{}' F, \beta_1{}' \times f(z_t) F) + 2Cov(\alpha, \beta' F) + \ Idio_c \end{aligned} \tag{6}$$

where $Idio_c$ is the idiosyncratic volatility from a conditional model. Comparing Equations (3) and (6), we find that the variance of stock returns consists of four additional terms: the variance of alpha, the variance of the interaction between the time-varying component of beta and the respective factors, and two covariance terms. In an unconditional model, $\beta_1 = 0\ and$ then $\beta = \beta_0$; comparing Equation (3) with Equation (6) gives:

$$Idio = Var(\alpha) + Var(\beta_1{}' \times f(z_t)F) + 2Cov(\beta_0{}'F, \beta_1{}' \times f(z_t)F) + 2Cov(\alpha, \beta'F) + Idio_c \tag{7}$$

Equation (7) shows that the four additional terms are the components of idiosyncratic risk that occur when people fail to account for the conditional pricing model with the time variation of alpha and betas. If the time-varying conditional model is the true model, then these four components represent systematic risk but are mistakenly included in the idiosyncratic risk category.

3. Methodology and Data

The theoretical results above imply that the time variation of both alpha and betas plays a role in the components of idiosyncratic volatility. In this study, we focus on the volatility of the time-varying alpha and betas. While alpha risk is defined as the standard deviation of the time-varying alpha in one month, estimated from a regression of daily stock returns on the three Fama–French factors, we define the beta risk as the standard deviation of the beta for each factor. Since we use a conditional Fama–French three-factor model, we calculate the beta risk for Fama-French three factors- MKT, SMB, and HML[10]- separately. It is common practice to use macroeconomic variables to predict stock returns (e.g. Breen et al. (1989), Chen (2009), among others). Our conditional pricing model is in line with Ferson and Harvey (1999). The time-varying alpha and betas are generated from a linear

[10] MKT is the excess return on the market minus the one-month Treasury bill rate. SMB (Small Minus Big) is the average return on the three small portfolios minus the average return on the three big portfolios; HML (High Minus Low) is the average return on the two value portfolios minus the average return on the two growth portfolios.

function with respect to the instrumental variable. This model is similar to that used by Avramov and Chordia (2006), except that they consider the additional conditional time-variation of the risk premium. The model used by Ferson and Harvey (1999) explicitly models the time-variation of alpha and betas; thus, it is sufficient for studying the risk associated with time-varying alpha and betas.

Using an approach that differs from the optimal mean-variance portfolio strategy used in Avramov and Chordia (2006), we sort stocks directly into portfolios by alpha (beta) risk and use a long-short strategy to study the yield from this strategy. Stocks are sorted based on the characteristics demonstrated at the end of the previous month. Portfolios are held for one month and then portfolio returns are calculated. We also use regression tests at the firm level by regressing stock returns on alpha (beta) risk and firm characteristics.

We choose the data sample from the Center for Research in Security Prices (CRSP) common shares traded on the New York Stock Exchange (NYSE), American Stock Exchange (AMEX), and NASDAQ from July 1963 to December 2015. The daily and monthly stock returns are from the CRSP. A stock should have available monthly accounting data from Compustat. Following French, Schwert and Stambaugh (1987) and Schwert (1989), daily volatilities are converted into a monthly version by multiplying by the square root of the number of trading days in that month. In each month, a stock should have at least 15 daily returns. Following Ferson and Harvey (1999), we use four state variables:

TERM is calculated as the spread between 10-year and 1-year Treasury bond yields (e.g., Campbell and Vuolteenaho (2004); Adrian and Franzoni (2009)).

Default is the spread between Moody's Baa and Aaa corporate bond yield (e.g., Avramov and Chordia (2006)).

DIV is the dividend yield on a value-weighted CRSP market portfolio.

T-bill is the short-term risk-free rate, as measured by the secondary market rate of 3-month Treasury bills.

Data on bond yields and Treasury bill rates are downloaded from the Federal Reserve Bank of St. Louis website. The dividend yield on the value-weighted CRSP market portfolio is collected directly from the CRSP. To avoid the potential spurious effect of outliers, observations with monthly returns higher than the 99.75 percentile or lower than the 0.25 percentile are deleted. Similar to Ang et al. (2006), we measure the idiosyncratic volatility of stock i at month t as the standard deviation of the residuals from a regression of daily stock returns in month $t-1$ on the three Fama–French factors (which are obtained from Kenneth R. French's website).

4. Empirical Results

Table 1 reports the descriptive statistics of the alpha beta risk estimated from a conditional Fama–French three-factor model for the period of July 1963 to December 2015. Alpha risk (volatility of alpha) is defined as the standard deviation of time-varying alpha from the daily stock returns in month $t-1$. Beta risk (volatility of beta) is the standard deviations of time-varying beta MKT, beta SMB, and beta HML. Monthly risk is calculated by multiplying the standard deviation by the square root of the number of trading days for each month. A stock should have at least 15 daily returns in a month to be included in the risk estimation. The alpha beta risk is reported as a percentage.

Both the mean and median of alpha risk are smaller than that of the beta risk. While the magnitude of the beta HML risk comes with the largest mean and standard deviation, the beta MKT has the smallest mean and standard deviation. The average beta SMB risk is three times as large as the alpha risk and one and half times as large as the beta MKT risk.

Table 2 reports the returns of the stock portfolios that are sorted by size and alpha risk. Stocks are first sorted into five quintile portfolios, and then each size quintile is further sorted into five quintiles based on the alpha risk. As a result, equally weighted returns of 5 × 5 portfolios are obtained. Under each size quintile, we calculate the yield by buying the portfolio with the highest alpha risk and short selling the one with the lowest alpha risk. By controlling size, we also report the Jensen's alpha from the capital asset pricing model (CAPM) and the Fama–French three-factor model. The Newey–West adjusted *t*-statistics are in parentheses. For size quintiles 1 to 4, the long-short trading strategy yields negative returns that are significantly different from zero. No such relation is observed for portfolios under large size quintile 5. The largest negative yield is observed under size quintile 2 with −0.87 and this yield becomes much smaller as the size increases. The last column of Table 2 reports the portfolio returns that are sorted by alpha risk. All stocks are sorted into 5 quintiles based on the alpha risk. By only considering the alpha risk, we use a long-short trading strategy and obtain −0.32 as the yields. The results from Jensen's alpha are consistent with the yields. This evidence confirms that alpha risk is negatively related with stock returns at the portfolio level.

The results of the two-way sorting by size and beta MKT risk are shown in Table 3. By controlling for size, we long the portfolio with the highest beta MKT risk and short the one with the lowest beta MKT risk. Similar to the results for the alpha risk, the trading strategy yields are significantly negative across the different size quintiles. The size quintile 2 yields the most negative spread, and this spread decreases as the size increases. When all stocks are sorted solely by the beta MKT risk, this negative long-short trading spread is marginally significant. This result implies that on a portfolio level, the negative relation between the beta MKT risk and stock returns is weak but becomes stronger when size is controlled.

Table 4 reports similar evidence for the negative relation between stock returns and the beta SMB risk. When stocks are sorted only by the beta SMB risk, the spread yielded from the long-short trading strategy is not significantly different from zero. As size is further controlled in two-way sorting, the negative long-short trading spread is only significant for size quintiles 2 to 4 and is much weaker for both small-size quintiles and large-size quintiles. Overall, the negative relation is somewhat significant for certain size quintiles when size is controlled for. The negative relation between the beta HML risk and stock portfolio returns is reported in Table 5. The one-way sorting by the beta HML risk yields a negative spread of −0.30, which is significant at the 5% level. By further controlling for the size, we find a significantly negative relation between the beta HML risk and stock returns. Jensen's alphas from the CAPM and the Fama–French three-factor model provide consistent evidence. This negative relation is weaker for stocks in large size quintiles.

Based on the evidence from the one-way and two-way sorting, the negative relation between the alpha risk and stock portfolio returns is quite significant. While the negative relation between the beta risk and stock portfolios returns is not as significant as that between the alpha risk and stock portfolio returns, it becomes more evident when size is controlled for at the portfolio level. The yield from the long-short trading strategy is larger when stocks are sorted by the alpha risk rather than by the beta risk. One-way sorting by the beta MKT risk and beta SMB risk shows that the relation between the beta risk and stock returns is not robust compared to the alpha risk.

Since the relation between the alpha risk and stock portfolio returns is more significant than that for the beta risk, we further investigate this relation by triple sorting stocks into 3 × 3 × 3 portfolios by size, book-to-market ratio, and alpha risk. We first sort all stocks into three size portfolios, and then each size portfolio is sorted into three portfolios by the book-to-market ratios. Finally, each size-book-to-market portfolio is divided into three alpha risk portfolios. Again, we conduct the long-short trading strategy and calculate the spread of the portfolio returns. Panel A in Table 6 reports the results for low book-to-market portfolios. The negative relation is significant across the three different size portfolios. It shows that the alpha risk has the strongest negative relation with portfolios from small-cap and low book-to-market stocks. Panel B shows the portfolio returns and trading strategy yields from medium book-to-market stocks. Although the negative relation is still significant for small-cap and mid-cap stocks, the magnitude of the relation is small compared to the portfolios of the same size under low book-to-market stocks in Panel A. Furthermore, the relation becomes even smaller as shown in Panel C for high book-to-market stocks. Overall, there seems to be little relation between the alpha risk and stock portfolio returns for large-cap stocks. Based on the Jensen's alphas from the CAPM and the Fama–French three-factor model, the negative relation between the alpha risk and stock portfolio returns is more evident for small-cap stocks, especially those with low book-to-market ratios.

To this point, we have shown a strong negative relation between the alpha risk and stock returns and a weaker relation between the beta risk and stock returns at the portfolio level. It is interesting to investigate the relation between the alpha beta risk and stock returns at the firm level. If a negative relation between the alpha beta risk and stock returns exists at the firm level, investors can use it to predict stock returns.

Table 7 provides the results of the firm-level Fama–MacBeth (1973) regressions of monthly stock returns on the alpha beta risk for the period of July 1963 to December 2015. Alpha risk (volatility of alpha) is defined as the standard deviation of time-varying alpha from daily stock returns in month $t - 1$. Beta risk (volatility of beta) is the standard deviations of time-varying beta MKT, beta SMB, and beta HML. Monthly risk is calculated by multiplying the standard deviation by the square root of the number of trading days for each month. A stock should have at least 15 daily returns in a month to be included in the risk estimation. Beta is the regression coefficient of the past two years of monthly returns on market returns. ME and B/M are the size and book-to-market ratio in Fama and French (1992). Ret (−2, −7) is the compound gross return from month $(t - 7)$ to $(t - 2)$. TURN and CVTURN are the average volume turnover and coefficient of variance of TURN calculated over the past 36 months in Chordia, Subrahmanyam, and Anshuman (2001). The idiosyncratic volatility (Idio_VOL) is the standard deviation of the regression residuals of daily stock returns in month $t - 1$. The Newey–West adjusted t-value is reported in parentheses. To avoid the effect of possibly spurious outliers, all explanatory variables below the 0.5 (above 99.5) percentile are set equal to the 0.5 (99.5) percentile.

By controlling for size, book-to-market, momentum, and liquidity, model 1 demonstrates that the alpha risk is significantly negatively associated with stock returns.

As the alpha risk increases by one percent, the expected stock return will drop by 0.04 percent. When the beta risk is added in model 2, only the beta HML risk is negatively related to stock returns. Although the relation is significant, the magnitude is much smaller (−0.007), being only one tenth of the relation between the alpha risk and stock returns. Since the literature shows that the time-varying alpha predicts stock returns, we control for it in model 3 to see if the relation between the alpha beta risk and stock returns changes. The Newey–West *t*-statistics for the alpha risk and beta HML risk show that the relation is not driven by the time-varying alpha. Idiosyncratic volatility is added in model 4. While the return predictability from the alpha risk is unchanged, the beta HML risk is marginally related to stock returns. This implies that the negative relation between the beta HML risk and stock returns is affected by the negative relation between idiosyncratic volatility and stock returns. Since the alpha beta risk is associated with the macroeconomic risk from the macroeconomic variables, one concern about the negative relation between the alpha risk and stock returns is that this relation may be driven by the macroeconomic variables themselves. The state variable, term spread, and its volatility are controlled for in model 5. The macroeconomic variables and their volatility do not affect the negative relation between the alpha risk and stock returns.

In sum, the negative relation between the alpha risk and stock returns is significant at both the firm level and the portfolio level. This relation is not driven by the beta risk, the time-varying alpha itself, idiosyncratic volatility, macroeconomic variables, or the volatility of the macroeconomic variables. On the other hand, the negative relation between the beta HML risk and stock returns is affected by idiosyncratic volatility. There is no relation between stock returns and the beta SMB risk or beta MKT risk.

5. Conclusion

In this paper, we provide a theoretical framework that shows four additional terms that are components of idiosyncratic risk when people fail to account for the conditional pricing model with the time variation of alpha and betas. If the time-varying conditional model is the true model, then these four components represent systematic risk but are mistakenly included in the category of idiosyncratic risk. Both alpha risk and beta risk play a role as a part of the additional terms. Empirical evidence shows that alpha risk is negatively related to stock returns at the portfolio level. This negative relation is less significant between beta risk and stock portfolio returns. Small-cap stocks and those with low book-to-market ratios are more affected by this negative relation between the alpha risk and stock returns.

We also analyze the relation between the alpha beta risk and stock returns at the firm level. The results of the cross-sectional regressions demonstrate that the negative relation between the beta HML risk and stock returns is driven by idiosyncratic volatility and that the negative relation between the alpha risk and stock returns is independent from beta risk, idiosyncratic volatility, time-varying alpha itself, macroeconomic variables, and the volatility of the macroeconomic variables. This negative relation is robust at both the firm level and the portfolio level.

The empirical results provided in this study suggest that portfolio managers can incorporate volatility of time-varying alpha into trading strategies. A long-short portfolio trading strategy yields a 3.8 percent significant annual return. If other strategies are combined, for example, small size focused, the annual return will even increase to 6.1 percent. While using time-varying beta does not help portfolio managers to make profit in general, a conditional model with time-varying alpha and time-varying beta is still needed for portfolio managers to obtain the volatility of time-varying alpha.

While we mainly focus on the alpha beta risk in this study, future research could be done on how the covariance components of idiosyncratic volatility relate to stock returns. This work is related to the literature about covariance risk and stock portfolio management.

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Table 1 Summary Statistics

This table reports descriptive statistics of alpha beta risk estimated from a conditional Fama-French three factor model using OLS for the period of July 1963 to December 2015. Alpha risk (volatility of alpha) is defined as the standard deviation of time-varying alpha from daily stock returns in month t−1. Beta risk (volatility of beta) is the standard deviation of time-varying beta MKT, beta SMB, and beta HML, respectively. Monthly risk is calculated by multiplying the standard deviation and the square root of the number of trading days for each month. A stock should have at least 15 daily returns in a month to be included in the risk estimation. The alpha beta risk is reported as a percentage.

	Mean	S.D.	5%	25%	Median	75%	95 %	N
Vol_Alpha	3.89	1.65	2.07	2.74	3.48	4.64	6.99	2,334,870
Vol_Bmkt	8.07	4.44	2.93	4.87	7.14	10.13	16.75	2,335,918
Vol_Bsmb	12.11	5.75	4.96	8.13	11.08	14.95	22.28	2,335,918
Vol_Bhml	14.74	7.32	6.31	9.98	13.06	17.83	27.67	2,335,918

Table 2 Double Sort by Size and Vol Alpha

Stocks are first sorted into five size quintile portfolios and then each size quintile is further sorted into five quintiles based on alpha risk. As a result, it gives the equally weighted returns of 5x5 portfolios. Under each size quintile, yield by long the portfolio with the highest alpha risk and short the one with the lowest alpha risk is calculated. The Jensen's alphas from CAPM and the Fama-French three factor model are reported in the last four rows. The last column reports the portfolio returns by single sort by alpha risk. All stocks are sorted into 5 quintiles based on alpha risk. The Newey-West adjusted t-statistics are in parentheses.

	Size Quintiles					
Vol Quintiles	1-Small	2	3	4	5-Large	All Stocks
1-Low	1.02	1.15	1.22	1.21	1.07	1.13
2	1.16	1.19	1.25	1.23	1.04	1.21
3	1.01	1.09	1.19	1.25	1.08	1.20
4	0.99	1.06	1.16	1.12	0.98	1.13
5-High	0.51	0.28	0.57	0.80	0.83	0.80
High-Low	-0.51***	-0.87***	-0.65***	-0.42***	-0.24	-0.32**
	(-4.19)	(-7.41)	(-5.39)	(-3.32)	(-1.68)	(-2.20)
CAPM Alpha	-0.63***	-1.03***	-0.80***	-0.59***	-0.43***	-0.49***
	(-5.46)	(-9.93)	(-7.34)	(-5.48)	(-3.43)	(-3.85)
FF3 Alpha	-0.65***	-1.03***	-0.77***	-0.52***	-0.36***	-0.51***
	(-5.92)	(-11.14)	(-8.08)	(-5.79)	(-3.26)	(-5.46)

Table 3 Double Sort by Size and Vol_Bmkt

Stocks are first sorted into five size quintile portfolios and then each size quintile is further sorted into five quintiles based on beta MKT risk. As a result, it gives the equally weighted returns of 5x5 portfolios. Under each size quintile, yield by long the portfolio with the highest beta MKT risk and short the one with the lowest beta MKT risk is calculated. The Jensen's alphas from CAPM and the Fama-French three factor model are reported in the last four rows. The last column reports the portfolio returns by single sort by beta MKT risk. All stocks are sorted into 5 quintiles based on beta MKT risk. The Newey-West adjusted t-statistics are in parentheses.

	Size Quintiles					
Vol Quintiles	1-Small	2	3	4	5-Large	All Stocks
1-Low	0.93	1.16	1.22	1.21	1.04	1.06
2	1.17	1.22	1.23	1.21	1.05	1.22
3	1.04	1.15	1.20	1.28	1.06	1.23
4	1.00	0.94	1.09	1.17	1.03	1.13
5-High	0.54	0.31	0.61	0.78	0.75	0.75
High-Low	-0.39***	-0.85***	-0.61***	-0.43**	-0.29**	-0.29*
	(-3.35)	(-7.04)	(-5.30)	(-3.52)	(-2.07)	(-1.96)
CAPM Alpha	-0.51***	-1.01***	-0.76***	-0.61***	-0.48***	-0.44***
	(-4.72)	(-9.96)	(-7.25)	(-5.71)	(-3.98)	(-3.56)
FF3 Alpha	-0.58***	-1.03***	-0.73***	-0.52***	-0.41***	-0.48***
	(-5.80)	(-11.35)	(-7.67)	(-5.50)	(-3.80)	(-5.32)

Table 4 Double Sort by Size and Vol_Bsmb

Stocks are first sorted into five size quintile portfolios and then each size quintile is further sorted into five quintiles based on beta SMB risk. As a result, it gives the equally weighted returns of 5x5 portfolios. Under each size quintile, yield by long the portfolio with the highest beta SMB risk and short the one with the lowest beta SMB risk is calculated. The Jensen's alphas from CAPM and the Fama-French three factor model are reported in the last four rows. The last column reports the portfolio returns by single sort by beta SMB risk. All stocks are sorted into 5 quintiles based on beta SMB risk. he Newey-West adjusted t-statistics are in parentheses.

	Size Quintiles					
Vol Quintiles	1-Small	2	3	4	5-Large	All Stocks
1-Low	0.84	1.00	1.22	1.23	1.08	1.13
2	1.08	1.19	1.23	1.26	1.05	1.21
3	1.08	1.17	1.19	1.20	1.02	1.20
4	0.99	0.99	1.15	1.18	1.03	1.17
5-High	0.60	0.38	0.60	0.76	0.77	0.89
High-Low	-0.24*	-0.63***	-0.62***	-0.47***	-0.31**	-0.23
	(-2.07)	(-5.30)	(-5.25)	(-3.92)	(-2.17)	(-1.59)
CAPM Alpha	-0.35***	-0.78***	-0.77***	-0.64***	-0.48***	-0.38***
	(-3.25)	(-7.47)	(-7.26)	(-5.96)	(-3.88)	(-3.07)
FF3 Alpha	-0.40***	-0.79***	-0.74***	-0.56***	-0.41***	-0.41***
	(-3.88)	(-8.31)	(-7.86)	(-6.09)	(-3.65)	(-4.54)

Table 5 Double Sort by Size and Vol_Bhml

Stocks are first sorted into five size quintile portfolios and then each size quintile is further sorted into five quintiles based on beta HML risk. As a result, it gives the equally weighted returns of 5x5 portfolios. Under each size quintile, yield by long the portfolio with the highest beta HML risk and short the one with the lowest beta HML risk is calculated. The Jensen's alphas from CAPM and the Fama-French three factor model are reported in the last four rows. The last column reports the portfolio returns by single sort by beta HML risk. All stocks are sorted into 5 quintiles based on beta HML risk. The Newey-West adjusted t-statistics are in parentheses.

	Size Quintiles					
Vol Quintiles	1-Small	2	3	4	5-Large	All Stocks
1-Low	1.02	1.25	1.26	1.21	1.05	1.20
2	1.11	1.13	1.19	1.27	1.06	1.21
3	1.04	1.04	1.26	1.22	1.02	1.20
4	0.96	0.93	1.02	1.10	1.06	1.14
5-High	0.55	0.41	0.64	0.76	0.74	0.88
High-Low	-0.48***	-0.84***	-0.62***	-0.46***	-0.31**	-0.30**
	(-4.16)	(-7.01)	(-5.29)	(-3.61)	(-2.17)	(-2.07)
CAPM Alpha	-0.60***	-1.00***	-0.77***	-0.64***	-0.50***	-0.46***
	(-5.76)	(-9.83)	(-7.32)	(-5.90)	(-4.10)	(-3.71)
FF3 Alpha	-0.63***	-1.00***	-0.74***	-0.56***	-0.43***	-0.49***
	(-6.24)	(-11.13)	(-8.34)	(-5.96)	(-3.85)	(-5.38)

Table 6 Triple sort by size, book-to-market and vol_alpha

Stocks are first sorted into three size quintile portfolios and then each size quintile is further sorted into three quintiles based on book-to-market ratio. Finally, each size book-to-market portfolio is sorted into three portfolios by alpha risk. As a result, it gives the equally weighted returns of 3x3x3 portfolios. Panel A, B, C reports the portfolio returns under low book-to-market, medium book-to-market, and high book-to-market, respectively. Under each size book-to-market sorts, yield by long the portfolio with the highest alpha risk and short the one with the lowest alpha risk is calculated. The Jensen's alphas from CAPM and the Fama-French three factor model are reported in the last four rows. The Newey-West adjusted t-statistics are in parentheses.

	Panel A			Panel B			Panel C		
	Low Book-to-Market			Medium Book-to-Market			High Book-to-Market		
Vol_Alpha	Small Cap	Mid Cap	Large Cap	Small Cap	Mid Cap	Large Cap	Small Cap	Mid Cap	Large Cap
Low	0.66	0.94	1.05	1.14	1.26	1.07	1.37	1.38	1.20
Medium	0.50	0.96	1.00	1.12	1.25	1.11	1.44	1.49	1.24
High	-0.09	0.37	0.81	0.61	0.98	1.02	0.88	1.08	1.19
High-Low	-0.75***	-0.57***	-0.24***	-0.53***	-0.28***	-0.05	-0.49***	-0.31***	-0.01
	(-7.19)	(-7.14)	(-2.76)	(-4.74)	(-3.06)	(-0.68)	(-4.57)	(-3.07)	(-0.08)
CAPM Alpha	-0.84***	-0.65***	-0.34***	-0.64***	-0.40***	-0.14**	-0.59***	-0.42***	-0.13*
	(-8.33)	(-8.37)	(-4.19)	(-6.19)	(-4.87)	(-2.13)	(-5.82)	(-4.80)	(-1.82)
FF3 Alpha	-0.86***	-0.63***	-0.31***	-0.65***	-0.42***	-0.12*	-0.64***	-0.47***	-0.15**
	(-9.10)	(-8.57)	(-4.21)	(-6.44)	(-5.65)	(-1.90)	(-6.53)	(-6.03)	(-2.35)

Table 7 Firm-level cross-sectional regressions

This table reports the results of the firm-level Fama-MacBeth (1973) regressions of monthly stock returns on alpha beta risk for the period of July 1963 to December 2015. Alpha risk (volatility of alpha) is defined as the standard deviation of time-varying alpha from daily stock returns in month t−1. Beta risk (volatility of beta) is the standard deviation of time-varying beta MKT, beta SMB, and beta HML, respectively. Monthly risk is calculated by multiplying the standard deviation and the square root of the number of trading days for each month. A stock should have at least 15 daily returns in a month to be included in the risk estimation. ME and B/M are size and book-to-market ratio in Fama and French (1992). Ret (-2,-7) is the compound gross return from month (t-7) to (t-2). TURN and CVTURN are the average volume turnover and coefficient of variance of TURN calculated over the past 36 months in Chordia, Subrahmanyam and Anshuman (2001). The idiosyncratic volatility (Idio_VOL) is the standard deviation of regression residuals of daily stock returns in month t−1. The Newey-West adjusted t-value is reported in parentheses. To avoid the effect of possibly spurious outliers, all explanatory variables below 0.5 (above 99.5) percentile are set equal to 0.5 (99.5) percentile. *, **, and *** denote the statistical significance level at 10%, 5%, and 1%, respectively.

	Model 1	Model 2	Model 3	Model 4	Model 5
Ln(ME)	-0.19***	-0.19***	-0.18***	-0.20***	-0.19***
	(-4.74)	(-4.92)	(-4.73)	(-5.52)	(-4.90)
Ln(B/M)	0.16***	0.16***	0.16***	0.15***	0.16***
	(3.07)	(3.06)	(3.12)	(3.02)	(3.05)
Ret(-2,-7)	0.29	0.31	0.31	0.34*	0.31
	(1.46)	(1.61)	(1.58)	(1.83)	(1.60)
Ln(TURN)	-0.08	-0.07	-0.09	-0.06	-0.07
	(-1.01)	(-0.92)	(-1.11)	(-0.76)	(-0.93)
Ln(CVTURN)	-0.38***	-0.37***	-0.38***	-0.36***	-0.38***
	(-6.43)	(-6.44)	(-6.56)	(-6.29)	(-6.46)
VOL_Alpha	-0.04***	-0.04***	-0.03***	-0.03***	-0.04***
	(-4.62)	(-5.90)	(-4.08)	(-4.93)	(-5.78)
VOL_Bmkt		0.005	0.007	0.005	0.004
		(1.06)	(1.55)	(1.38)	(0.98)
VOL_Bsmb		0.002	0.003	0.003	0.002
		(0.66)	(1.06)	(1.04)	(0.68)
VOL_Bhml		-0.007***	-0.007**	-0.005*	-0.007**
		(-2.66)	(-2.56)	(-1.99)	(-2.49)
Alpha			-0.02***		
			(-10.41)		
Idio_VOL				-0.02***	
				(-3.09)	
Z (TERM)					-3.53
					(-1.29)
VOL_Z					1.35
					(0.02)
Adj. R2	0.05	0.05	0.05	0.05	0.05

Essay Three: A Separation Analysis of the Idiosyncratic Volatility-Return Relation

Abstract

We employ a two-step estimation method to separate the upside and downside idiosyncratic volatility and examine its relation with future stock returns. We find that idiosyncratic volatility is negatively related to stock returns when the market is up and when it is down. The upside idiosyncratic volatility is not related to stock returns. Our results also suggest that the relation between downside idiosyncratic volatility and future stock returns is negative and significant. It is the downside idiosyncratic volatility that drives the inverse relation between total idiosyncratic volatility and stock returns. The results are consistent with the literature that investor overreact to bad news and underreact to good news.

JEL Classifications: G12, G32.

Keywords: Market Condition, Idiosyncratic Volatility, Upside Risk, Downside Risk, Investor Reaction

1. Introduction

In this paper we investigate the relation between idiosyncratic volatility and stock returns conditional on the condition of market. Specifically, we measure the idiosyncratic volatility on both "upside" and "downside" and consider the relation separately conditional on positive and negative market returns. This approach is based on the established practice in literature that investors react differently to downside losses than they do to upside gains. The tradeoff between risk and return can be better captured by focusing on the upside or downside of risk. For example, semi-variance is used by Markowitz (1959) to measures downside losses instead of upside gains. Besides, risk measures, such as value-at-risk (VAR) focused on downside of return distribution.

The analysis under positive and negative market conditions is widely applied in literature (see, for example, Cooper, Gutierrez, and Hameed (2004), Chordia, Roll, and Subrahmanyam (2002), and Wang and Xu (2015), among others)[11]. The separation of upside idiosyncratic volatility and downside idiosyncratic volatility originates from the idea that investors react differently between downside losses and upside gains (see, for example, Markowitz (1959), Kahneman and Tversky (1979), and Ang, Chen, and Xing (2006), among others). Thus, it is reasonable to measure upside and downside idiosyncratic volatility separately conditional on positive or negative market returns. Investors may demand different returns due to asymmetric attitudes towards losses and gains. This demand may change under asymmetric market conditions.

[11] Additionally, empirical literature has used the separation of market condition well to study the beta return relation such as up and down market betas.

In our analysis, we use a two-step estimation process to measure the upside and downside idiosyncratic volatility and examine the conditional relation between idiosyncratic volatility and stock returns. To do this, we estimate a Fama-French three factor model separately for up and down markets in the first step. Then in the second step, upside idiosyncratic volatility is measured by the semi-standard deviation of positive idiosyncratic returns, and downside idiosyncratic volatility is measured by the semi-standard deviation of negative idiosyncratic returns. Our results show that conditional on positive market return, high idiosyncratic volatility predicts low future stock returns. This negative relation is also true when conditional on negative market return. More importantly, we find an insignificant relation between upside idiosyncratic volatility and stock returns, which means investors do not misprice upside risk. Meanwhile, there is a negative relation between downside idiosyncratic volatility and future stock returns. Our results show that the downside idiosyncratic volatility drives the relation between total idiosyncratic volatility and stock returns. After controlling the downside idiosyncratic volatility, the negative relation between total idiosyncratic volatility and stock returns turns out insignificant.

Finally, we attempt to explore why upside idiosyncratic volatility is not related to stock returns and why the downside idiosyncratic volatility is negatively related to future stock returns. We relate our finding to the investor reaction to news. As investors react differently to "good" news and "bad" news (see, for example, Hong and Stein (1990), Hong, Lim, and Stein (2000), and Chan (2003)), we test whether the IVOL-return relation is a result of positive news and negative news that offset may each other. We control for the downside risk as a proxy of "bad" news and examine the effect of "good" news, proxied by upside risk, on future stock returns. Based on the results, our results support that investors underreact to “good” news and overreact to “bad news”, which is consistent with Veronesi (1999).

Our study is related to Ang, Chen, and Xing (2006) in that both studies consider the upside and downside risk. While they focus on the systematic risk, we only consider idiosyncratic volatility. This paper is also related to Wang and Xu (2015) in that both studies take market conditions into consideration. The difference is that Wang and Xu (2015) relate systematic risk with momentum strategy, but we focus on the idiosyncratic volatility-return relation itself.

This paper is organized as follows. Section II describes the data and the estimation process of upside and downside idiosyncratic volatility. Section III explores the relation between idiosyncratic volatility and stock return under positive and negative market returns. In Section IV, we further analyze the relation and relate it to investor behavior. Section V concludes.

2. Data and Idiosyncratic Volatility Measurement

In this section, we show the data screening process and method used in the estimation of Idiosyncratic volatility. We first measure the IVOL following the method applied by Ang, Hodrick, Xing, and Zhang (2006). Then we propose the two-step estimation method to measure the upside and downside IVOL.

Data in this paper is mainly obtained from two sources: CRSP and Compustat for the period July 1963 to June 2018. In our analysis, we only include common shares that are traded at NYSE, AMEX, and NASDAQ. Stocks with price less than $1 at previous period are excluded. We also adjust returns of delisted stocks following Shumway (1997).

Similar to Ang, Hodrick, Xing, and Zhang (2006), we measure idiosyncratic volatility as the standard deviation of the residuals from Fama-French 3-factor model:

$$R_t = \alpha + \beta_1 MKT_t + \beta_2 SMB_t + \beta_3 HML_t + \epsilon_t \qquad (1)$$

Where R_t is daily stock returns in excess of the risk-free rate, MKT_t is daily excess market return, and SMB_t and HML_t are daily Fama and French factors obtained from Ken French's website. We use the CRSP value-weighted index return as a proxy for market returns and the 3-month T-bill yield as a proxy for the risk-free rate. IVOL is measured as follows:

$$IVOL = \sqrt{N} Std(\epsilon_t) \qquad (2)$$

Where N is used to normalize the IVOL measure from daily to monthly frequency. We use the IVOL at month t-1 as a proxy for realized IVOL at month t.

The construction of upside and downside IVOL contains a two-step estimation. We first estimate the Fama-French 3-factor model separately for positive and negative market return so all the estimates are asymmetric conditional on the condition of the market:

$$R_t = \alpha + \beta_1 MKT_t + \beta_2 SMB_t + \beta_3 HML_t + \epsilon_t \qquad (3)$$

According to the above specification, two series of residual returns (ϵ_t) are measured conditional on the market return.

$$IVOL^{+MKT} = \sqrt{N} Std(\epsilon_t) | R_{M,t} \geq 0 \qquad (4)$$

$$IVOL^{-MKT} = \sqrt{N} Std(\epsilon_t) | R_{M,t} < 0 \quad (5)$$

Following this step, we define the upside and downside IVOL, respectively associated with negative and positive residual returns for the two market conditions:

$$IVOL_{Up}^{+MKT} = \sqrt{N} SemiStd(\epsilon_t) | \epsilon_t \geq 0, R_{M,t} \geq 0 \ (6)$$

$$IVOL_{Down}^{+MKT} = \sqrt{N} SemiStd(\epsilon_t) | \epsilon_t < 0, R_{M,t} \geq 0 \ (7)$$

Where $Up\ IVOL(+MKT)$ and $Down\ IVOL(+MKT)$ measures the upside (downside) variation in idiosyncratic returns conditional on positive market return. Similarly, these are measured separately when market return is negative as follows:

$$IVOL_{Up}^{-MKT} = \sqrt{N} SemiStd(\epsilon_t) | \epsilon_t \geq 0, R_{M,t} < 0 \ (8)$$

$$IVOL_{Down}^{-MKT} = \sqrt{N} SemiStd(\epsilon_t) | \epsilon_t < 0, R_{M,t} < 0 \ (9)$$

In total, we have 6 different measures of IVOL under different market conditions: total IVOL under positive market return; upside and downside IVOL conditional on positive market return; total IVOL under negative market return; and upside and downside IVOL conditional on negative market return. Our results are robust across different factor models in estimation. For the consistency of reporting, our results based on the Fama-French 3-factor model in the two-step estimation.

In table 1, we report the descriptive statistics of the IVOL measures. The first row shows the classic IVOL directly estimated from a Fama-French 3-factor model. Panel A are the IVOL estimates under positive market condition. The average total IVOL conditional on positive market return is lower than the average of classic IVOL. This is also true for the median and standard deviation of the measures. We expect to see this reduction due to the separation of market conditions in the first step of estimation. It shows a similar reducing pattern for upside and downside IVOL. On average, the upside IVOL is smaller than the total IVOL and it is also less volatile than the latter. The downside IVOL are even smaller in mean and standard deviation compared with the upside IVOL. Panel B presents a similar shrinking pattern from total IVOL to upside IVOL, and then to downside IVOL. A comparison between Panel A and Panel B demonstrates that IVOL is generally lower conditional on negative market return than that under positive market return.

3. Idiosyncratic Volatility and Return Relation

In this section, we examine how IVOL is related to future stock returns. Table 2 reports the one-month value-weighted holding period returns for quintile portfolios sorted on total IVOL. It also presents the portfolio alphas adjusted based on the CAPM, Fama-French 3-factor model, and Carhart 4-factor model. In Panel A, stocks are sorted based on classic IVOL estimated directly from Fama-French 3-factor model. It confirms the inverse cross-sectional relation between IVOL and stock returns documented in Ang, Hodrick, Xing, and Zhang (2006). A long-short trading strategy that hold the portfolio with the highest IVOL and sell the portfolio with the lowest IVOL yields a -0.60 percent return. Newey and West (1987) t-statistics shows the return from the trading strategy is significant. Consistent with Ang, Hodrick, Xing, and Zhang (2006), our portfolio returns exhibit a non-linear pattern in the relation with IVOL. Despite of it, results from CAPM alpha, Fama-French 3-factor alpha, and Carhart 4-factor alpha all confirm the negative IVOL-return relation.

In Panel B, stocks are sorted into quintile portfolios by the total IVOL conditional on positive market return ($IVOL^{+MKT}$). It shows that IVOL is still negatively related to stock returns when the market return is positive. While the return trading yield between the high and low IVOL portfolios is significant at 5 percent level, the magnitude of the yield (-0.5 percent) is smaller than the one (-0.6 percent) based on total IVOL without consideration of market conditions. Panel C demonstrates a similar results. Conditional on negative market return, stocks with high total IVOL,$IVOL^{-MKT}$, tend to have a lower future return. Although one may notice that the trading strategy yield at -0.47 percent is weaker than that conditional on positive market return, the t-statistics shows that such yield is still significant.

Overall, the results in Table 2 suggest that IVOL predicts future stock returns at both positive market condition and negative market condition. The negative IVOL-return relation is confirmed. While the predictive power is weaker in positive and negative market conditions than the unconditional one, it is consistent with the inverse IVOL-return relation in Ang, Hodrick, Xing, and Zhang (2006).

As we are interested in the investors' different reaction toward upside and downside risk, we test the relation between upside and downside IVOL and future stock returns. Table 3 is the upside and downside IVOL-return relation conditional on positive market return. Panel A shows the results of value-weighted quintile portfolios formed based on upside IVOL conditional on positive market ($IVOL_{Up}^{+MKT}$). As upside IVOL increases from portfolio 1 to 5, the portfolio return slightly increases at the beginning and then falls. The yield between high and low upside IVOL is -0.22 percent. However, the Newey and West (1987) t-statistics indicates that such a negative yield is insignificantly different from zero.

So the results show that there is no relation between upside IVOL and stock returns. This insignificant relation is different from the inverse relation between total IVOL and stock returns. It implies that investors do not misprice the upside IVOL. The CAPM alpha, Fama-French 3-factor alpha, and Carhart 4-factor alpha further confirm that upside IVOL fail to predict future returns.

Now we turn to the downside IVOL ($IVOL_{Down}^{+MKT}$) in our analysis. Since the total IVOL is negatively related to future returns and upside IVOL fails to do so, we expect to see such an inverse relation between downside IVOL and future returns. Results in Panel B confirm this expectation. The portfolio with the highest downside IVOL has a one-month holding period return at 0.53 percent and the one with the lowest downside IVOL has a return of 0.92 percent. The high and low portfolio strategy yield of -0.39 percent is significant at 5 percent level. It means conditional on positive market, stocks with high downside IVOL have lower future returns. Additionally, the CAPM alpha, Fama-French 3-factor alpha, and Carhart 4-factor alpha further confirm it. Therefore, under positive market condition, there is a negative relation between downside IVOL, not upside IVOL, and future stock returns.

How about the relation between upside and downside IVOL and stock returns conditional on negative market return? Table 4 presents the results of portfolio returns. Similar to that in Table 3, we analyze the relation separately for upside IVOL,$IVOL_{Up}^{-MKT}$, and downside IVOL, $IVOL_{Down}^{-MKT}$. Panel A shows the yield between high and low upside IVOL portfolio returns is -0.25 percent. The insignificant yield suggests that there is no relation between upside IVOL and future returns. On the other hand, Panel B demonstrates that downside IVOL predicts future returns conditional on negative market. The high and low portfolio yield is -0.33 percent, which is significant but slightly weaker than that conditional on positive market return, -0.39 percent. While the negative downside IVOL-return relation conditional on negative market return is not as strong as that conditional on positive market return, the high and low strategy yield on CAPM alpha, Fama-French 3-factor alpha, and Carhart 4-factor alpha are statistically significant and further confirm the inverse downside IVOL-return relation.

We analyze the effects of market condition on IVOL-return relation by investigating the relation between idiosyncratic volatility and stock returns conditional on the sensitivity of market returns, which is measured by the market beta. Monthly beta is estimated from the previous 5-year monthly data by regressing historical stock returns on the market return. Then, stocks are sorted into quintile portfolios based on idiosyncratic volatility conditional on positive/negative beta. Panel A of Table 5 shows the results. While the long-short strategy yields a significant 0.58 percent month return conditional on positive beta, it doubles the yield to 1.13 percent conditional on negative beta. Consistent with the results in previous tables, the relation between idiosyncratic volatility and stock returns is negative in both conditions. As we further decompose the idiosyncratic volatility into upward and downward components, we sort stocks into quintile portfolios again in Panel B. There is a significant negative relation between returns and downward idiosyncratic volatility. The upside idiosyncratic volatility has only marginal negative relation with stock returns conditional on positive beta or negative beta. It seems that the downside idiosyncratic volatility drives the negative relation between idiosyncratic volatility and stock returns.

Based on the evidence above, we are interested in if the predictive power of total IVOL is driven by the downside IVOL. Because of the weak relation between downside

IVOL and future returns relative to the total IVOL-return relation, we are cautious about drawing this conclusion before we do the analysis. One way to analyze this problem is re-examine the total IVOL-return relation by controlling the downside IVOL. In Table 6, stocks are sorted into a 5x5 portfolios based on downside IVOL and total IVOL. Panel A presents the total IVOL-return relation conditional on positive market return ($IVOL^{+MKT}$) as the downside IVOL ($IVOL_{Down}^{+MKT}$) is controlled. Each raw represents five portfolios that have different levels of total IVOL conditional on positive market return. The column (H-L) reports the strategy yield of high and low total IVOL portfolios after controlling downside IVOL. Four out of the five portfolio groups show an insignificant relation between total IVOL and stock returns. Results from the CAPM alpha, Fama-French 3-factor alpha, and Carhart 4-factor alpha are reported in the following columns, which show a consistent pattern. Panel B shows a similar result conditional on negative market return. As downside IVOL ($IVOL_{Down}^{-MKT}$) is controlled, the relation between total IVOL, $IVOL^{-MKT}$, and future stock returns is none. Therefore, it is safe to conclude that the inverse total IVOL-return relation is driven by the downside IVOL.

To sum up, total IVOL has a cross-sectional predictive power on future stock returns conditional on both positive market return and negative market return. Under each market condition, the upside IVOL fail to predict future returns. However, the downside IVOL is negatively related with future returns. The predictive power of downside IVOL is slightly stronger conditional on positive market return. It is the downside component of IVOL, not the upside one, that contribute to the inverse IVOL-return relation. Our results about the asymmetrically priced idiosyncratic volatility are consistent with literature that investors are more concerned about downside losses than upside gains.

4. Further Analysis of upside and downside IVOL

While the evidence presented so far provides important new insights on the IVOL-return relation, it also raises further questions that is challenging. Why is upside risk not related to stock returns? What drives the negative relation between downside risk and future stock return? Merton (1987) suggests that investors may hold imperfectly diversified portfolio and demand compensation for bearing idiosyncratic risk. However, our results are not consistent with it. In this section, we attempt to explore these questions.

In the analysis, we start from the evidence presented in Chan (2003), among others, that investors react distinctly to good news and bad news. On one hand, we consider that the insignificant relation between upside IVOL may imply that good news associated with upside IVOL is not fully reflected in stock price and thus investors underreact to good news. It is possible that good news is offset by bad news, which would explain the insignificant upside IVOL-return relation. We sort stocks into 5X5 portfolios based on downside IVOL and upside IVOL. The results are reported in Table 7. Panel A presents the relation between upside IVOL and stock returns conditional on positive market return. After controlling downside IVOL, we report the high minus low in the returns for portfolios with distinct level of upside IVOL. Throughout column 6, we find that four out of five portfolio groups demonstrate an insignificant upside IVOL-return relation. The only one exception is the portfolios with the highest downside IVOL. Within this group, stocks with high upside IVOL tend to have a low future return. The portfolio strategy yield is significant at 5 percent level. The respective CAPM alpha, Fama-French 3-factor alpha, and Carhart 4-factor alpha show a consistent result with portfolio returns. Panel B provides a similar insignificant upside IVOL-return relation after controlling for downside IVOL conditional on negative market return. All five portfolio groups in different downside IVOL have an insignificant trading strategy yield by upside IVOL. Overall, we conclude that there is still no relation between upside IVOL and future returns when downside IVOL is controlled. Therefore, we provide evidence that investors underreact to good news.

On the other hand, the finding that downside IVOL predicts future returns may suggest that investors overreact to bad news, resulting in a downward price correction in future. Alternatively, it is possible that bad news is offset by good news. The negative downside IVOL-return relation is studied by controlling the upside IVOL. In Table 8, stocks are sorted into 5X5 portfolios where the sorting is first based on upside IVOL and then by downside IVOL. Panel A shows the downside IVOL-return relation after controlling upside IVOL conditional on positive market return. Under each upside IVOL portfolio group, the value-weighted portfolio return first increase and then decrease as the downside IVOL increases. The negative relation between downside IVOL and stock returns is no-linear. The column H-L shows the trading strategy yield when investors take long position on a portfolio with the highest downside IVOL and short one with the lowest downside IVOL. Although all the yields show a negative relation between downside IVOL and future returns, this relation is insignificant in four out of five portfolio groups. A comparison between Panel A in Table 8 and Panel B in Table 3 demonstrates that the negative downside IVOL-return relation disappears after controlling upside IVOL. It is confirmed by the results based on CAPM alpha, Fama-French 3-factor alpha, and Carhart 4-factor alpha. This result seems in support of that bad news is offset by good news. Moreover, the results of downside IVOL-return relation conditional on negative market return in Panel B further support it. After controlling the upside IVOL by sorting the stocks into five portfolios, we then sort the stocks in each portfolio into another five portfolios by downside IVOL. Similar to the result in positive market condition, downside IVOL has no relation with stock returns after controlling for upside IVOL. Four out of five portfolio groups show that downside IVOL no longer predict future stock returns. The results in alphas show a consistent pattern as the portfolio returns. To sum up, our result is consistent with investor's overreaction to bad news.

5. Conclusion

In this study, we employ a two-step estimation method to measure the upside and downside idiosyncratic volatility. By considering the positive and negative market returns separately, we estimate the idiosyncratic volatility asymmetrically conditional on the condition of the market. We confirm the negative relation between idiosyncratic volatility and stock returns as that in Ang, Hodrick, Xing, and Zhang (2006). We also discover the existence of such negative relation when the market is up and when it is down. Although the magnitude of the negative relation is smaller than that without considering market conditions, the relation is still statistically significant. The yield between the high and low quintile portfolios sorted by ($IVOL^{+MKT}$) and ($IVOL^{-MKT}$) is ranged from -0.5 to -0.47 percent in a month, compared to -0.6 percent of yield for portfolios formed on $IVOL$. After further separating idiosyncratic volatility into upside and downside components, we find that the upside idiosyncratic volatility has no relation with stock returns, which supports that good news is not reflected in stock price. Nonetheless, the relation between downside idiosyncratic volatility and future stock returns is negative and significant.

Portfolio managers who seek to trade based on idiosyncratic volatility should consider the volatility component as it can be more effective to conduct a portfolio strategy on downside idiosyncratic volatility rather than the upside component or the total idiosyncratic volatility. When they select stocks with negative market betas and create a long-short portfolio trading strategy, the potential yield almost doubles compared to choosing stocks with positive betas. Therefore, portfolio managers fail to consider the condition of market may suffer from not be able to reap the profit from IVOL-return relation.

Finally, we provide evidence that the negative relation between total idiosyncratic volatility and stock returns is driven by the downside idiosyncratic volatility. After controlling the downside idiosyncratic volatility, the predictive power of total idiosyncratic volatility, $(IVOL^{+MKT})$ and $(IVOL^{-MKT})$, will be insignificant under its corresponding market condition. Investors who are aware of this conclusion can quickly adjust their trading strategy based on the change in downside idiosyncratic volatility of stock returns. In general, our results are consistent with literature that investors underreact to "good" news and overreact to "bad" news.

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Table 1 Summary Statistics

This table shows the descriptive statistics for different measures of IVOL. IVOL is estimated as the standard deviation of residuals from Fama-French 3-factor model on daily stock returns in a month. $IVOL^{+MKT}$($IVOL^{-MKT}$) is the total IVOL conditional on positive (negative) market return. Under each market condition, upside (downside) IVOL is calculated as the semi standard deviation of positive (negative) idiosyncratic stock returns following the two-step estimation detailed in section II. All IVOL measures are normalized by multiplying the square root of number of trading days in the IVOL estimation period. We use IVOL at month t-1 as a proxy for realized IVOL at month t. We only include common shares that are traded at NYSE, AMEX, and NASDAQ. Stocks with price less than $1 at previous period are excluded. We also adjust returns of delisted stocks following Shumway (1997). N is the total number of observations for IVOL in the study.

	Mean	S.D.	5%	25%	Median	75%	95 %	N
IVOL	0.1160	0.0967	0.0288	0.0566	0.0906	0.1456	0.2870	2,973,014
				Panel A: positive market condition				
$IVOL^{+MKT}$	0.0875	0.0774	0.0194	0.0408	0.0670	0.1101	0.2224	2,809,601
$IVOL_{Up}^{+MKT}$	0.0568	0.0620	0.0093	0.0224	0.0396	0.0702	0.1584	2,594,407
$IVOL_{Down}^{+MKT}$	0.0485	0.0449	0.0093	0.0211	0.0358	0.0609	0.1292	2,652,928
				Panel B: negative market condition				
$IVOL^{-MKT}$	0.0814	0.0717	0.0176	0.0376	0.0621	0.1024	0.2084	2,395,785
$IVOL_{Up}^{-MKT}$	0.0488	0.0519	0.0078	0.0191	0.0339	0.0603	0.1376	2,108,523
$IVOL_{Down}^{-MKT}$	0.0462	0.0432	0.0083	0.0196	0.0340	0.0583	0.1245	2,139,979

Table 2 The Relation Between Total IVOL and Future Stock Returns

This table reports the one-month value-weighted holding period returns for quintile portfolios sorted on total IVOL. Panel A is the results based on IVOL estimated without any condition. $IVOL^{+MKT}$($IVOL^{-MKT}$) in Panel B (C) is the total IVOL conditional on positive (negative) market return. The CAPM_Alpha is the CAPM alphas, and FF3_Alpha and FF4_Alpha represent the Fama-French 3-factor alphas and Carhart 4-factor alphas respectively. The Newey and West (1987) t-statistics are reported in the parenthesis. *, **, and *** denote statistical significance level at 10%, 5%, and 1%, respectively.

Panel A IVOL-return relation

IVOL	Return	CAPM_Alpha	FF3_Alpha	FF4_Alpha
1(Low)	0.88	0.46	0.47	0.48
	(5.6)	(2.87)	(2.88)	(2.79)
2	0.99	0.54	0.57	0.62
	(5.07)	(2.80)	(2.87)	(2.93)
3	0.97	0.5	0.52	0.54
	(3.93)	(2.10)	(2.16)	(2.09)
4	0.87	0.38	0.44	0.50
	(2.77)	(1.25)	(1.47)	(1.48)
5(High)	0.28	-0.25	-0.21	-0.18
	(0.75)	(-0.73)	(-0.61)	(-0.48)
H-L	-0.60**	-0.71***	-0.69***	-0.66**
	(-2.69)	(-2.76)	(-2.59)	(-2.20)

Panel B IVOL on positive market

$IVOL^{+MKT}$	Return	CAPM_Alpha	FF3_Alpha	FF4_Alpha
1(Low)	0.91	0.50	0.51	0.54
	(5.96)	(3.15)	(3.19)	(3.28)
2	0.96	0.52	0.53	0.56
	(5.01)	(2.72)	(2.79)	(2.83)
3	0.94	0.48	0.51	0.53
	(4.02)	(2.08)	(2.19)	(2.13)
4	0.91	0.42	0.47	0.52
	(3.03)	(1.46)	(1.64)	(1.66)
5(High)	0.42	-0.09	-0.03	0.02
	(1.18)	(-0.29)	(-0.08)	(0.05)
H-L	-0.50**	-0.59**	-0.54**	-0.52**
	(-2.24)	(-2.32)	(-2.12)	(-2.15)

Panel C IVOL on negative market				
$IVOL^{-MKT}$	Return	CAPM_Alpha	FF3_Alpha	FF4_Alpha
1(Low)	0.89	0.47	0.49	0.52
	(5.77)	(2.97)	(3.04)	(3.11)
2	0.98	0.53	0.56	0.59
	(5.17)	(2.84)	(2.95)	(2.98)
3	0.97	0.52	0.54	0.56
	(4.20)	(2.27)	(2.36)	(2.27)
4	0.85	0.36	0.40	0.43
	(2.89)	(1.26)	(1.41)	(1.38)
5(High)	0.42	-0.11	-0.08	-0.04
	(1.18)	(-0.34)	(-0.24)	(-0.10)
H-L	-0.47**	-0.58**	-0.57**	-0.56**
	(-2.13)	(-2.44)	(-2.29)	(-2.17)

Table 3 Upside and Downside IVOL Conditional on Positive Market

This table reports the one-month value-weighted holding period returns for quintile portfolios sorted on upside and downside IVOL conditional on positive market return. $IVOL_{Up}^{+MKT}$($IVOL_{Down}^{+MKT}$) is the upside (downside) IVOL measured conditional on positive market return following the two-step estimation detailed in section II. The CAPM_Alpha is the CAPM alphas, and FF3_Alpha and FF4_Alpha represent the Fama-French 3-factor alphas and Carhart 4-factor alphas respectively. The Newey and West (1987) t-statistics are reported in the parenthesis. *, **, and *** denote statistical significance level at 10%, 5%, and 1%, respectively.

Panel A: upside IVOL on positive market

$IVOL_{Up}^{+MKT}$	Return	CAPM_Alpha	FF3_Alpha	FF4_Alpha
1(Low)	0.91	0.49	0.50	0.52
	(5.69)	(2.99)	(3.05)	(3.09)
2	0.95	0.52	0.55	0.60
	(5.14)	(2.78)	(2.9)	(3.02)
3	0.94	0.48	0.51	0.52
	(4.36)	(2.28)	(2.40)	(2.25)
4	0.84	0.36	0.39	0.42
	(3.19)	(1.43)	(1.52)	(1.47)
5(High)	0.69	0.20	0.25	0.30
	(2.21)	(0.65)	(0.84)	(0.93)
H-L	-0.22	-0.29	-0.25	-0.22
	(-1.03)	(-1.43)	(-1.23)	(-0.99)

Panel B: downside IVOL on positive market

$IVOL_{Down}^{+MKT}$	Return	CAPM_Alpha	FF3_Alpha	FF4_Alpha
1(Low)	0.92	0.50	0.52	0.55
	(5.94)	(3.16)	(3.20)	(3.28)
2	0.94	0.51	0.53	0.55
	(5.06)	(2.71)	(2.81)	(2.84)
3	0.95	0.50	0.53	0.55
	(4.39)	(2.32)	(2.45)	(2.41)
4	0.82	0.35	0.40	0.46
	(3.11)	(1.35)	(1.57)	(1.66)
5(High)	0.53	0.02	0.06	0.07
	(1.70)	(0.06)	(0.19)	(0.22)
H-L	-0.39**	-0.49**	-0.47**	-0.48**
	(-2.05)	(-2.39)	(-2.28)	(-2.00)

Table 4 Upside and Downside IVOL Conditional on Negative Market

This table reports the one-month value-weighted holding period returns for quintile portfolios sorted on upside and downside IVOL conditional on negative market return. $IVOL_{Up}^{-MKT}$ ($IVOL_{Down}^{-MKT}$) is the upside (downside) IVOL measured conditional on negative market return following the two-step estimation detailed in section II. The CAPM_Alpha is the CAPM alphas, and FF3_Alpha and FF4_Alpha represent the Fama-French 3-factor alphas and Carhart 4-factor alphas respectively. The Newey and West (1987) t-statistics are reported in the parenthesis. *, **, and *** denote statistical significance level at 10%, 5%, and 1%, respectively.

Panel A: upside IVOL on negative market

$IVOL_{Up}^{-MKT}$	Return	CAPM_Alpha	FF3_Alpha	FF4_Alpha
1(Low)	0.89	0.46	0.49	0.50
	(5.60)	(2.86)	(2.99)	(2.95)
2	0.92	0.49	0.51	0.55
	(5.02)	(2.65)	(2.73)	(2.85)
3	0.90	0.45	0.48	0.50
	(4.08)	(2.04)	(2.17)	(2.12)
4	0.84	0.37	0.41	0.42
	(3.30)	(1.5)	(1.66)	(1.56)
5(High)	0.64	0.13	0.16	0.19
	(2.01)	(0.41)	(0.53)	(0.58)
H-L	-0.25	-0.34*	-0.33	-0.31
	(-1.16)	(-1.65)	(-1.55)	(-1.33)

Panel B: downside IVOL on negative market

$IVOL_{Down}^{-MKT}$	Return	CAPM_Alpha	FF3_Alpha	FF4_Alpha
1(Low)	0.87	0.45	0.48	0.50
	(5.39)	(2.74)	(2.86)	(2.82)
2	0.91	0.47	0.51	0.53
	(4.89)	(2.52)	(2.70)	(2.70)
3	0.91	0.45	0.47	0.52
	(4.25)	(2.14)	(2.21)	(2.32)
4	0.87	0.41	0.42	0.45
	(3.55)	(1.70)	(1.74)	(1.71)
5(High)	0.55	0.04	0.07	0.13
	(1.79)	(0.15)	(0.25)	(0.42)
H-L	-0.33**	-0.41**	-0.41**	-0.37*
	(-1.98)	(-2.23)	(-2.19)	(-1.77)

Table 5 The Relation Between IVOL and Stock Returns Conditional on Beta

This table reports the one-month value-weighted holding period returns for quintile portfolios sorted on IVOL conditional on positive/negative market beta. $IVOL_{Up}^{-MKT}$ ($IVOL_{Down}^{-MKT}$) is the upside (downside) IVOL measured conditional on market beta following the two-step estimation detailed in section II. The Newey and West (1987) t-statistics are reported in the parenthesis. *, **, and *** denote statistical significance level at 10%, 5%, and 1%, respectively.

Panel A IVOL-return Relation

	Portfolio Return		
	$IVOL$	$IVOL^{+Beta}$	$IVOL^{-Beta}$
1(Low)	0.92	0.92	0.56
	(6.26)	(5.96)	(2.22)
2	0.98	0.97	0.26
	(4.57)	(4.96)	(0.71)
3	0.962	0.99	0.39
	(4.16)	(4.12)	(0.99)
4	0.87	0.86	0.25
	(2.53)	(2.73)	(0.49)
5(High)	0.29	0.34	-0.57
	(0.38)	(1.08)	(-1.15)
H-L	-0.63**	-0.58**	-1.13**
	(-2.34)	(-2.14)	(-2.15)

Panel B IVOL-return Relation

	Portfolio Return			
	$IVOL_{Up}^{+Beta}$	$IVOL_{Down}^{+Beta}$	$IVOL_{Up}^{-Beta}$	$IVOL_{Down}^{-Beta}$
1(Low)	0.93	0.94	0.40	1.20
	(6.09)	(5.99)	(1.71)	(6.35)
2	0.95	0.98	0.54	0.95
	(4.94)	(5.10)	(1.66)	(4.44)
3	1.04	0.96	0.45	0.73
	(4.51)	(4.16)	(1.07)	(2.08)
4	0.99	0.85	0.01	0.86
	(3.62)	(3.13)	(0.01)	(0.92)
5(High)	0.57	0.35	-0.48	0.14
	(1.83)	(1.10)	(-1.25)	(-1.31)
H-L	-0.36	-0.59***	-0.88*	-1.06***
	(-1.64)	(-2.70)	(-1.88)	(-2.24)

Table 6 Downside IVOL on the total IVOL-return Relation

This table reports the one-month value-weighted holding period returns for quintile portfolios formed on total IVOL after controlling for downside IVOL in the corresponding market condition. The CAPM_Alpha is the CAPM alphas, and FF3_Alpha and FF4_Alpha represent the Fama-French 3-factor alphas and Carhart 4-factor alphas respectively. The Newey and West (1987) t-statistics are reported in the parenthesis. *, **, and *** denote statistical significance level at 10%, 5%, and 1%, respectively.

Panel A: Driven by downside IVOL on positive market

	$IVOL^{+MKT}$								
$IVOL_{Down}^{+MKT}$	1(Low)	2	3	4	5(High)	H-L	CAPM_Alpha	FF3_Alpha	FF4_Alpha
1(Low)	1.03	1.09	1.07	0.82	1.40	0.37	0.23	0.37	0.54
	(5.75)	(4.82)	(3.61)	(2.1)	(2.14)	(0.62)	(0.40)	(0.63)	(0.91)
2	0.84	1.02	1.03	0.93	-0.12	-0.96***	-1.08***	-1.04***	-0.90***
	(2.25)	(2.99)	(2.47)	(1.40)	(-1.73)	(-3.38)	(-3.38)	(-3.16)	(-2.61)
3	0.84	0.92	0.99	1.06	0.80	-0.04	-0.14	-0.16	-0.10
	(2.09)	(2.37)	(2.25)	(1.90)	(0.75)	(-0.47)	(-0.47)	(-0.53)	(-0.30)
4	1.36	1.21	0.94	0.97	0.95	-0.41	-0.42	-0.32	-0.18
	(2.90)	(2.98)	(1.59)	(1.40)	(1.08)	(-1.03)	(-1.03)	(-0.76)	(-0.40)
5(High)	0.44	0.87	0.94	0.69	0.46	0.02	0.06	0.03	0.03
	(0.40)	(0.20)	(0.59)	(0.55)	(0.61)	(0.18)	(0.18)	(0.03)	(0.03)

Panel B: Driven by downside IVOL on negative market

	$IVOL^{-MKT}$								
$IVOL_{Down}^{-MKT}$	1(Low)	2	3	4	5(High)	H-L	CAPM_Alpha	FF3_Alpha	FF4_Alpha
1(Low)	0.85	0.97	0.94	0.70	0.22	-0.63	-0.79*	-1.09*	-0.90
	(5.23)	(4.44)	(3.26)	(1.94)	(0.03)	(-1.43)	(-1.79)	(-1.76)	(-1.45)
2	0.92	0.97	1.00	0.84	0.69	-0.23	-0.35	-0.31	-0.42
	(5.40)	(4.91)	(3.99)	(2.74)	(1.57)	(-0.60)	(-0.91)	(-0.79)	(-0.95)
3	0.84	0.91	1.05	0.81	0.79	-0.05	-0.12	-0.07	-0.11
	(3.96)	(4.49)	(4.47)	(2.62)	(2.13)	(-0.17)	(-0.39)	(-0.22)	(-0.33)
4	1.05	0.86	0.77	0.76	0.23	-0.82**	-0.82**	-0.83**	-0.93**
	(3.88)	(4.00)	(3.01)	(2.31)	(0.61)	(-2.28)	(-2.29)	(-2.34)	(-2.26)
5(High)	0.73	1.02	0.88	0.79	0.31	-0.42	-0.56	-0.59	-0.65
	(0.36)	(2.23)	(0.87)	(0.77)	(-0.21)	(-0.47)	(-0.44)	(-0.36)	(-0.47)

Table 7 Further Examination of the upside IVOL-return Relation

This table reports the one-month value-weighted holding period returns for quintile portfolios formed on upside IVOL after controlling for downside IVOL in the corresponding market condition. The CAPM_Alpha is the CAPM alphas, and FF3_Alpha and FF4_Alpha represent the Fama-French 3-factor alphas and Carhart 4-factor alphas respectively. The Newey and West (1987) t-statistics are reported in the parenthesis. *, **, and *** denote statistical significance level at 10%, 5%, and 1%, respectively.

Panel A: Positive Market

	$IVOL_{Up}^{+MKT}$								
$IVOL_{Down}^{+MKT}$	1(Low)	2	3	4	5(High)	H-L	CAPM_Alpha	FF3_Alpha	FF4_Alpha
1(Low)	0.95	0.97	1.03	0.70	1.31	0.36	0.30	0.38	0.43*
	(6.34)	(5.59)	(5.35)	(2.88)	(4.29)	(1.46)	(1.22)	(1.51)	(1.67)
2	0.88	0.96	1.08	0.90	1.04	0.16	0.11	0.09	0.09
	(4.84)	(5.05)	(5.14)	(3.61)	(3.57)	(0.81)	(0.53)	(0.42)	(0.44)
3	0.90	0.99	0.93	0.92	1.08	0.18	0.11	0.11	0.15
	(4.64)	(4.62)	(3.92)	(3.49)	(3.53)	(0.91)	(0.58)	(0.58)	(0.70)
4	0.87	0.89	0.74	0.95	0.90	0.03	0.05	0.10	0.15
	(3.59)	(3.38)	(2.65)	(3.07)	(2.70)	(0.12)	(0.18)	(0.41)	(0.56)
5(High)	0.72	0.97	0.96	0.55	0.26	-0.46**	-0.66**	-0.63**	-0.63**
	(2.19)	(3.46)	(3.14)	(1.57)	(0.17)	(-2.14)	(-2.39)	(-2.21)	(-2.05)

Panel B: Negative Market

	$IVOL_{Up}^{-MKT}$								
$IVOL_{Down}^{-MKT}$	1(Low)	2	3	4	5(High)	H-L	CAPM_Alpha	FF3_Alpha	FF4_Alpha
1(Low)	0.9	0.92	0.98	0.93	1.01	0.1	0.02	-0.01	-0.06
	(5.95)	(5.24)	(4.71)	(3.70)	(3.17)	(0.41)	(0.08)	(-0.02)	(-0.22)
2	0.89	1.00	0.89	1.00	0.98	0.09	0.02	0.06	0.05
	(5.17)	(5.35)	(4.01)	(4.14)	(3.29)	(0.48)	(0.12)	(0.28)	(0.21)
3	0.83	0.95	1.02	0.84	0.82	-0.02	-0.06	-0.05	-0.04
	(4.09)	(4.53)	(4.28)	(3.21)	(2.56)	(-0.08)	(-0.31)	(-0.24)	(-0.15)
4	0.91	0.96	0.98	0.85	0.56	-0.34	-0.38*	-0.36*	-0.35
	(3.93)	(3.91)	(3.81)	(2.88)	(1.67)	(-1.61)	(-1.81)	(-1.68)	(-1.45)
5(High)	0.70	0.70	0.79	0.63	0.22	-0.47	-0.56**	-0.48*	-0.47*
	(2.48)	(2.57)	(2.34)	(1.94)	(0.59)	(-1.62)	(-1.97)	(-1.65)	(-1.69)

Table 8 Further Examination of the downside IVOL-return Relation

This table reports the one-month value-weighted holding period returns for quintile portfolios formed on downside IVOL after controlling for upside IVOL in the corresponding market condition. The CAPM_Alpha is the CAPM alphas, and FF3_Alpha and FF4_Alpha represent the Fama-French 3-factor alphas and Carhart 4-factor alphas respectively. The Newey and West (1987) t-statistics are reported in the parenthesis. *, **, and *** denote statistical significance level at 10%, 5%, and 1%, respectively.

Panel A: Positive Market

			$IVOL_{Down}^{+MKT}$						
$IVOL_{Up}^{+MKT}$	1(Low)	2	3	4	5(High)	H-L	CAPM_Alpha	FF3_Alpha	FF4_Alpha
1(Low)	0.95	0.88	0.9	0.88	0.72	-0.23	-0.36	-0.35	-0.32
	(6.33)	(4.85)	(4.64)	(3.63)	(2.19)	(-0.88)	(-1.39)	(-1.40)	(-1.23)
2	0.96	0.96	0.99	0.89	0.96	0.00	-0.07	-0.14	-0.13
	(5.58)	(5.05)	(4.62)	(3.38)	(3.45)	(0.01)	(-0.35)	(-0.67)	(-0.58)
3	1.02	1.08	0.93	0.74	0.96	-0.07	-0.13	-0.11	-0.10
	(5.35)	(5.14)	(3.92)	(2.65)	(3.14)	(-0.31)	(-0.63)	(-0.50)	(-0.43)
4	0.70	0.90	0.92	0.95	0.55	-0.15	-0.17	-0.11	-0.05
	(2.88)	(3.61)	(3.49)	(3.07)	(1.57)	(-0.60)	(-0.68)	(-0.46)	(-0.20)
5(High)	1.31	1.05	1.08	0.90	0.07	-1.24***	-1.32***	-1.36***	-1.37***
	(4.29)	(3.58)	(3.55)	(2.69)	(0.18)	(-4.49)	(-4.95)	(-4.93)	(-4.64)

Panel B: Negative Market

			$IVOL_{Down}^{-MKT}$						
$IVOL_{Up}^{-MKT}$	1(Low)	2	3	4	5(High)	H-L	CAPM_Alpha	FF3_Alpha	FF4_Alpha
1(Low)	0.90	0.89	0.83	0.91	0.70	-0.21	-0.27	-0.34	-0.29
	(5.95)	(5.17)	(4.09)	(3.93)	(2.48)	(-0.92)	(-1.23)	(-1.52)	(-1.19)
2	0.92	1.00	0.95	0.96	0.70	-0.22	-0.26	-0.30*	-0.26
	(5.24)	(5.35)	(4.53)	(3.91)	(2.57)	(-1.22)	(-1.46)	(-1.70)	(-1.44)
3	0.98	0.89	1.02	0.98	0.79	-0.19	-0.26	-0.22	-0.12
	(4.71)	(4.01)	(4.28)	(3.81)	(2.34)	(-0.92)	(-1.23)	(-1.05)	(-0.53)
4	0.93	1.00	0.84	0.85	0.63	-0.30	-0.30	-0.31	-0.35
	(3.70)	(4.14)	(3.21)	(2.88)	(1.94)	(-1.42)	(-1.38)	(-1.38)	(-1.46)
5(High)	1.01	0.98	0.82	0.56	0.22	-0.78***	-0.85***	-0.81***	-0.70**
	(3.17)	(3.29)	(2.56)	(1.67)	(0.59)	(-2.93)	(-3.23)	(-3.04)	(-2.54)